Public debt in England (1737-1750): policy and market expectations

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Abstract

A year after the end of the war of the Austrian succession (1743-1748), the prime minister Samuel Pelham reduced the interest rate from 4 to 3 percent on most of the english public debt. It is shown here, using the prices of the 3% and 4% annuities, that the market was not surprised by the terms of the policy although it was surprised by the rapid decline of interest rates after the war. War loans were for the most past long-run but were redeemable. Their contingent feature contributed to lower the borrowing cost in England and generated an additional incentive for good government policy.

1 Introduction

In 1749, the English prime minister, Pelham, lowered the interest rate on the outstanding public debt from 4 to 3 percent without any disruption of the financial market. In 1770, the french finance minister, the abbé Terray, reduced the interest rate on some government liabilities, caused a furor and a crash of the rente (that was not affected by the interest reduction). A policy that went smoothly in England was a bankrupcy in France. Outcomes were dissimilar because the loan contracts, *ex ante*, were different. In England, the loan strictures allowed for such a reduction, albeit in terms that were not fully specified, thus enabling the market to anticipate the policy and incorporate it in the asset price, long before its implementation. No such provision existed for the french loans.

The market expectations before the Pelham interest reduction are analyzed here with the prices of the main debt instruments used in England during the war of the Austrian succession (1743-1748): the 3% annuity that became the consol in 1752 and the 4% annuity that was subject to the reduction of 1749. These assets were traded most days except sundays and holidays.

The war of the Austrian succession was the first war of the 18th century where England relied on debt financing in a textbook fashion with long-term annuities that were actively traded in a market. This regime was the outcome of a 50 years process that been has been called the "financial revolution" by Dickson (1967) in his superb study that remains a reference today[1]. In this war, as in the seven years war (1757-63) and the american war (1776-1783), most of the debt finance was in long-term instruments that were issued in the middle of the war when the long-term interest rate was at its highest. The part of the short-term debt with post-war refinancing was minor. As a critical property, these long-term financial instruments were redeemable at par. The redemption stricture was equivalent to a provision for the contingent reduction of the coupon rate.

When the interest rate fell after the end of a war, the value of the bonds would increase and reach par thus providing the government with the possibility to pay back the loan through refinancing at a lower interest rate. In order to save on transaction costs, it was more efficient to reduce the interest rate the outstanding

[1]That period has been the subject of important studies recently, among others by Stasavage (2003), (2006), Sussman and Yafeh (2003).

loans that had a higher rate. *De facto*, the debt at 4% or higher was converted into debt at 3%. Because the redeemability was written in the loan contract, the interest reduction could be anticipated by the market and was not perceived as a default.

As is well known, England was able to issue much more debt than France for an interest service that was about the same. The traditional explanation is the better credibility of the english government with a parliament that exercised a control over expenditures and provided the credibility of future tax revenues. This paper shows that the flexibility of reducing the interest rate in the loan contract lowered significantly the debt service.

In Section 2, the market instruments that were issued during the war of the Austrian succession are described. Both were callable bonds, redeemable at par, and the callable property is taken into account for the computation of the actual cost of the loans. The dominant instruments were long-term bonds with coupons of 3% and 4%. Using the characteristics of the loan issues, a correct value of the cost of borrowing *ex ante* is computed, taking into account the callable feature of the new loans[2].

In 18th century England, the optimal rule to call the bond immediately when it reaches par (Ingersoll, 1977) could not be applied because of the fixed costs in floating large amount of new debt, and also because of issues of moral hazard and fairness that are discussed in Section 3. The recall of the high coupon debt could not and was not expected to take place at par.

Market participants viewed the 4% debt as a contingent asset that paid a coupon of 4% as long as the price of the 3% was below par. When the 3% reached par and was stable, the 4% debt could be redeemed. In order to save on the transaction cost of refinancing with new 3% bonds purchased by the same individuals holding the 4% debt, the government announced an "interest reduction": the coupon of the 4% was reduced gradually to 3% during a relatively short transition period. That reduction was equivalent to a conversion of the 4% into a 3%, one for one, with an additional payment equal to the present value of the coupons over 3% during the transition (an amount equal to about 4 in 1749). A 4% bond was therefore a derivative financial asset of the 3% bond: all its payments depend solely on the price of the 3%: it would pay 4 pounds per face value of 100 per year until the price of the 3% reached par.

[2] The impact of the callable feature on the effective cost of borrowing has been emphasized by Harley (1977) and Klovland (1994) for the computation of the long-term rate at the end of the 19th century when the government exercised the call and lowered the interest rate to 2.75% first, and then to 2.5%, the rate in effect today. Harley does not use any model but makes assumptions about the call. Klovland evaluates critically these assumptions and rejects some of them by comparing the *ex post* mean returns of the redeemable and the non redeemable debt for some time intervals.

Around that date, it was converted into a 3% bond with a additional payment that was not specified at the time of issue but could be anticipated by the market.

In April 1748, peace negotiations started at Aix-la-Chapelle (April 24)[3]. Although some military actions (especially at sea) continued until the signing of the treaty in the following October, "the end was in sight". The price of bonds continued to vary randomly but in assessing its future, the trend of interest rates was definitely downwards and the main issue was about the convergence to the peace time level. From the political context, we can assume that the asset prices, subject to shocks, were driven by one random factor, the interest rate that converged randomly to the long-run stable value that enables the government to convert the 4% debt.

A one factor model of the price of the 4% annuity in terms of the price of the 3% annuity is estimated in Section 4, for the period from April 1748 to May 1749, six months before the policy announcement of Pelham. The estimation shows that the terms of the interest reduction were well anticipated by the market, that is an interest reduction that was equivalent to a redemption of the 4% annuity with a end payment of 4 when the 3% annuity was stable at par. The model also shows that the point of redemption was reached much earlier than anticipated by the market: the interest rate fell after the war much faster than expected.

The 1749 interest rate reduction applied the same terms to both the 14 millions pound of the new debt issued in the war and the 43 millions of the pre-existing debt. Dickson in his description (1967) gives much credit to Pelham for the implementation of the interest reduction that was opposed by institutional investors (the Bank of England and the South Sea Company) that acted as a cartel on behalf of the annuity holders. The market prices put some strong light on the contempories' expectations. For the debt that had been issued during the war, the market took Pelham's plan as a *fait accompli* two months before its formal announcement to the parliament. For the old debt that existed before the war, expectations were slightly different, measured by one more year of 4% payment, but by the end of December, the market put zero expectation on any gain from the further bargaining that took place at the beginning of 1750.

In Section 5, the attempt of the 1737 interest reduction is reconsidered in view of the later success and remarks on policy are provided. The callable feature of the debt remarkably enabled the government to turn the excess pessimism of the market ot its advantage. The 4% debt was *de facto* the sum of a perpetual 3% bond and

[3] All dates in this paper refer to the Julian calendar, used in England until 1752 and lagging the Gregorian calendar by 11 days.

an annuity paying 1 per year until the long-run interest would return to about 3%. As the market believed this date to be distant in the future, more than 10 years, it bought the contingent annuity from the government at a price that grossly exceeded the *ex post* payments by the government. The government won a large bet against the people who paid 12 in 1747 to receive a total of merely 7. If only Terray could have dealt with such contingent annuities.

2 Public loans before and during the war of the Austrian succession

The debt before the war

The public debt in England took off after the 1788 revolution and was a consequence of the wars against Louis XIV. From a negligible level, it rose to more than 50 M in 1720 (Dickson, 1967, Table 9), while total government revenues grew gradually from 2 M to 5.8 M (O'Brien, 1988, Table 2). In the period of peace that followed, interest rates decreased steadily from 8% in 1710 (Homer and Sylla, 1991) to 3% in the thirties. After the South Sea bubble in 1720, the debt was restructured mainly in South Sea annuities[4]. The stylized features that matter for the context of this paper are the decline of interest rates during the 1715-1743 peace and the magnitude of the public debt. Its composition in 1739 is presented in Table 1. Most of the assets paid a coupon at or above 4 percent. Some annuities at 3 percent had been issued in 1731 and 1736, but their total amount was small at 1.4 M (Dickson 1967, Table 22).

Table 1: The composition of the public debt (December 31, 1739)

Liability	Rate (%)	Amount (millions)
Bank of England stock	6	1.6
Exchequer bills	4	0.5
4% annuities	4	7.0
East-India Company	4	3.2
South Sea stock and annuities	4	27.3
Various annuities	4.8	6.5
Navy debt	4	0.8
SUM	4.5	46.95

Source : Grellier (1810).

[4]See Carlos, Neal and Wandschneider (2007), Quinn (2008).

The main part of the public debt was redeemable at par. When the interest rate fell and the market value of the debt would increase, the government could reduce it by running a surplus or by issuing new loans at lower interest. We will see in the next section that a more effective method was to lower the interest rate on the debt. In 1737, the 3% annuities had been above par for a year and an attempt was made to reduce the interest rate on 4% annuities. This attempt which failed will be discussed after the analysis of the successful interest reduction of 1749.

Loans during the war of the Austrian succession

The war of the Austrian succession provides a textbook case of debt financing. It set the stage for the next two wars before 1792. The amounts borrowed are presented graphically in Figure 1 together with the price of 3% annuity (to be called consol in 1752). Government borrowing took place only during the war years and the bulk of it was done when the long-term interest rate was at its highest level. The loan amounts are the amounts actually raised, but since loans were issued at a discounted, they are smaller than the increases of the face value of the government debt.

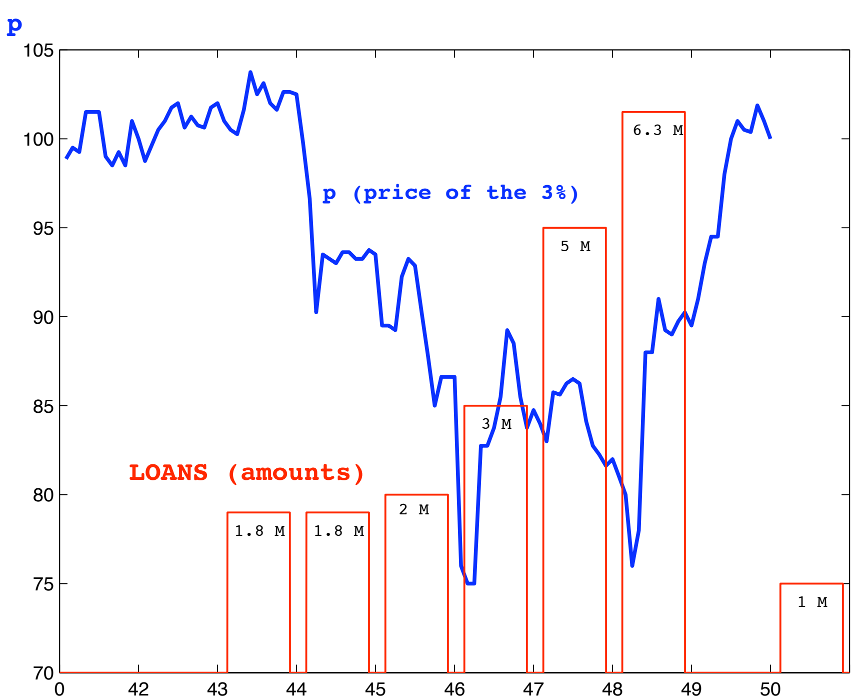


Figure 1: Prices of the 3 % annuity and amounts of loans (1741 - 1749)

The floating of a new war loan was an operation similar to the privatization today of a large state company or an IPO, and was at the time more important relative to the overall size of the market. As today, and it entailed some fixed costs and the contemporary accounts emphasize that for the government it was essential that each single issue should be "successfull", which meant at the time significantly oversubscribed, as it would be today with investment banks and IPOs.

The process for a new issue began in the late Fall of each war year with the parliamentary session that assessed the amount and the broad terms of the loan. Preparations continued during the winter as more discussions with the "moneyed men" took place for the fine-tuning of the terms of issue to contemporary market conditions. The subscription was paid in monthly installments of 10 to 20 percent beginning at various times from December to May. Investors seem to have appreciated gambles and a significant fraction of the loans was raised by lottery tickets with government bonds as prizes. The total value of the prizes was equal to the value of the tickets and could exceed it when special prizes were added in the fine-tuning to secure success of the issue.

The amounts and types of loans[5] are presented in Column 2 and 3 of Table 2. For example in 1743 and in 1744, a payment of $\mathcal{L}$ 100 would get a 3% annuity with a face value of 100 (Column 3). As interest rates were higher in 1745 (Figure 1), a life-time annuity of $\mathcal{L}$ 1.125 per year (to be written on any person of choice with no distinction for age) was added to the 3% annuity[6] for the same payment of $\mathcal{L}$ 100. The debt financing of the war of the Austrian succession relied mainly on 3% and 4% annuities. Life-time annuities represented less than 5 percent of the borrowings. Since the loans were issued when the prices were below par (Figure 1), the amounts raised were smaller than the increments of the face value of the public debt.

The borrowing cost

One measure of the borrowing cost is the coupon rate, with an adjustment for the premium at the time of issuance or the equivalent rate of a life annuity. That value is reported in Column 4 from Grellier (1812) who probably had information on the

[5] The financial instruments of all the loans are easy to trace thanks to a commission of the British Parliament in the late 19th century. Grellier (1812), (1810), provides a number of details on the specifics of the loans, and consolidated data on the British debt in some years.

[6] As reported Column 8, the loan was partially issued through a lottery: for each say $\mathcal{L}$ 400, the investor would receive a 300 face value of 3% bonds and 10 lottery tickets (at $\mathcal{L}$ 10 each). These tickets would entitle the investor to a life-time annuity of $\mathcal{L}$ 4 and 10 shillings in addition to the prize of the lottery. The prizes were in 3% bonds with fair odds. A similar scheme was used in 1746.

Table 2: The war of the Austrian succession (1743-50)

1	2	3	4	5	6	7	8
Date	Amount £ M	Instruments	Rate (%)	Yield (%)	*Ex post* rate	Market prices	Remarks
1743	1.8	100(3%)	3.42	3.42	3.42	3%: 100	1M by subscription, 0.8M by lottery.
1744	1.8	100(3%)	3.33	3.33	3.33	3%: 93	1.2M by subscription, 0.6M by lottery, premium of 3%.
1745	2.0	100(3%), L(1.125)	4.02	4.02	4.02	3%: 89	1.5 M. subscription, 0.5 M lottery, life ann. 4£ 10 for £ 100 in lott.
1746	3.0	100(4%), L(1.5)	5.4	4.81	4.72	3%: 75-83 4%: 91-94	2.5 M subscription, 0.5M lottery, life ann. 9£ for £ 100 in lott.
1747	4.0	110(4%)	4.4	3.73	3.54	3%: 85 4%: 96	plus a 10% premium in bonds (effective rate 4.4 %).
1747	1.0	100(4%)					Lottery
1748	6.3	110(4%)	4.4	3.71	3.51	3%: 80 4%: 90	As the price fell, payment dates were delayed.
1750	1.0	100(3%)				3%: 100	Conversion Navy bills
SUM	20.7						

Source : United Kingom, Parliamentary Papers (1898), Grellier (1810), (1812).

For definitions of the items and descriptions of the computations, see the text.

terms of issuance and used an implicit maturity of 60 years for the life annuities[7]. This accounting method that uses the coupon rate overstates the cost of borrowing because it cannot take into account the redemption features of the loans and it must assume that coupons are paid forever, contrary to market expectations.

The proper measure of the borrowing cost is the long-term internal rate of return of the loan using market prices. We consider the equivalent financing through a 3% annuity. In 1747, investors should be willing to receive for $\mathcal{L}$ 100 an amount equal to $110 \times 96/85$ of 3% annuities, where 96 and 85 are the prices of the 4% and the 3% annuities (Column 7), which generates a yield of 3.73. Assuming this coupon to be perpetual, (if not we have an upper-bound), the computed value is equal to the yield reported in Column 5. A similar method is used to compute the yield of the 1746 loan[8].

The yields in Column 5 illustrate the high cost of life annuities: in 1747 and 1748, when the level of borrowing was at its highest (at 5 and 6 millions), and the long-term rate was as high as at any other time in the war (Table 2 and Figure 1), the yield was significantly lower than in 1745-46 when the amount of the loans was only half. Raising loans through financial instruments actively traded in the market and with a contingent redemption date was much more effective than life annuities *ex ante*. The *ex post* cost (the rate the government has ultimately paid) will be considered in a later section. It turned out to be lower because the 4% annuity were recalled earlier than expected, as will be shown later.

3 The interest reduction

In the Fall of 1749, the consol (the 3% annuity) had been around par for a few months. The long-term rate was therefore back to 3%, and was not expected to increase in the near future. The price of the 4% had reached 105. The king in his opening speech to the session of the Parliament, made it official that interest payments should be lowered on the entire 4% debt which greatly exceeded the debt incurred in the last war. The government of Pelham ruled out an immediate reduction of the rate to 3%

[7] The maturity could be reconstructed by simulation. Under the assumption of a 50 years maturity, the rate is 3.96%.

[8] In 1746, we have a mix of a callable 4% and a life-annuity. The 4% component is equivalent to a perpetual payment of $3 \times 94/83 = 3.3976 = c$, where 94 and 83 are the prices of the 4% and the 3% annuities in the Spring of 1746, (which are more relevant than the prices at the beginning of the year because the payments for the subscription were made in the spring and the summer). The life-annuity is taken as a payment of 1.5 for 60 years. The yield R is such that $100 = \sum_{1 \leq i \leq 60}(1.5 + c)/(1+R)^i + (c/R)/(1+R)^{61}$, which gives a solution of 4.81 %.

and fixed the terms at the end of November[9]: the 4% bonds would receive a coupon of 4% for the year of 1750, and then 3.5% for the following 6 years during which they were not redeemable. After 7 years, there would be no distinction between these bonds and the 3% bonds. For a holder of a 4% bond, the interest reduction was equivalent to conversion into a 3% bond with a payment of about $\mathcal{L}$ 4 per bond, paid in installments. That plan was implemented with minor variations for the total debt of 57.7M (millions $\mathcal{L}$) at 4%. By May 1750, only 7M from the 57.7M were not converted. The holders of these bonds were paid off by a new loan.

According to Sutherland (1946, pp. 22-29), and Dickson (1967, pp. 228-245), the plan was strongly resisted, in particular by the large institutional investors, and it was apparently in jeopardy during the winter. These descriptions will be compared below with the evidence from the market prices, but they illustrate, together with the 1737 failed attempt, that the calling of bonds in 18th century England was subject to constraints that were different from those of a modern corporation operating in a large financial market.

Why not redeem at par?

For U.S. callable Treasury bonds (available up to 1995), the call had to be announced 120 days before. Since the interest rate fluctuates randomly during this period, the optimal price at which the bond should be called is strictly above par, and depends on the volatility of the interest rate)[10]. There was no formal delay for implementation in 18th century England, but a delay and a special compensation at the time of redemption had to take place because the following constraints:

1. The redemption of the 4% debt could be done only by issuing new debt at 3%. However, issuing costs were large. Since the new bond holders would be the same people as for the old bonds, transaction costs would be saved by reducing the interest of the debt but the reduction of the interest rate was not in the debt contract. Hence the government and bond holders were competing to capture the rent of the saved costs. Not surprisingly, the three large companies that represented a dominant fraction of bond holders and had more bargaining power, the South Sea and the East India Company, and the Bank of England, opposed the plan at the beginning. Standard arguments were made on behalf of the widows who would suffer from the reduction. The institutions eventually went along but the discussions imposed a delay between announcement and implementation.

[9] See the accounts of Grellier (1810), p. 215-221, and Dickson (1967), p. 231-241.

[10] Bliss and Ronn (1995), (1997), Grau, Forsyth and Vetzal (2003).

2. Bond holders could always reject the conversion at a lower rate and despite the good terms offered by Pelham, some did. Because the government had to provide an interval of time to convert their holding, it had to to provide a premium that would dominate possible price losses of the 3% bond in the short-term. The payment of $\mathcal{L}$ 4 per face value of $\mathcal{L}$ 100 provided some guarantee[11].

3. In 18th century England, the interest rate depended mainly on the government fiscal policy which itself depended on the military policy. Reputation was essential during this period of growth of the public debt. The redemption of callable bonds raised issues of asymmetric information, moral hazard and fairness. An increase of interest rates and a fall of the bond prices soon after the conversion would have raised the suspicion of "inside trading" and a government taking advantage of private information about the future evolution of interest rates. The government needed to give some compensation against a possible capital loss on the newly converted debt, at least for the near future after the time of the conversion. Contemporary discussions of the policy emphasize that the 3% annuity had been around par for a few months and were likely to stay at that level in the future. The issue of moral hazard is similar: the government had to avoid any suspicion that it would take advantage of the interest reduction and the lighter burden of the debt to start on new ventures. These arguments strengthen the case in item 2 for a conversion rate above par.

4. The interest reduction also raises an issue of asymmetric information and moral hazard *ex post* with respect to the terms of issuance of the bonds. The 4% bond was called when the government had reached a stable fiscal condition, an event about which there may have been some asymmetric information with bond holders. We will see that the Pelham government had reached this favorable situation much earlier than what the market had anticipated. Did the government issue such bonds with more information than the market? A compensation more generous than the strict terms could alleviate that concern.

5. Finally, there may also have been a perception of fairness. The dividends of peace had come much sooner than expected. Should the goverment capture all these dividends by a strict application of the contract? In the environment of contingent payments without completely specified contracts, the government spent great efforts on good relations with the financial community (the monied men). The government

[11] The variance of the price changes of the 3% was around 0.2 when the price was around par. One can verify that the variance for a month was around 4 times that value, as expected from uncorrelated changes. A variance of 4 months would be 3.2 which means a standard deviation of 1.8. Twice that deviation means 3.6 which fits with the actual payment of 4 for a probability of a loss smaller than 2%.

may have wanted to share some of the "peace dividends" with the bond holders (who, it should not be forgotten, had made a fast and large capital gain on their holdings).

Redemption, swap or interest reduction?

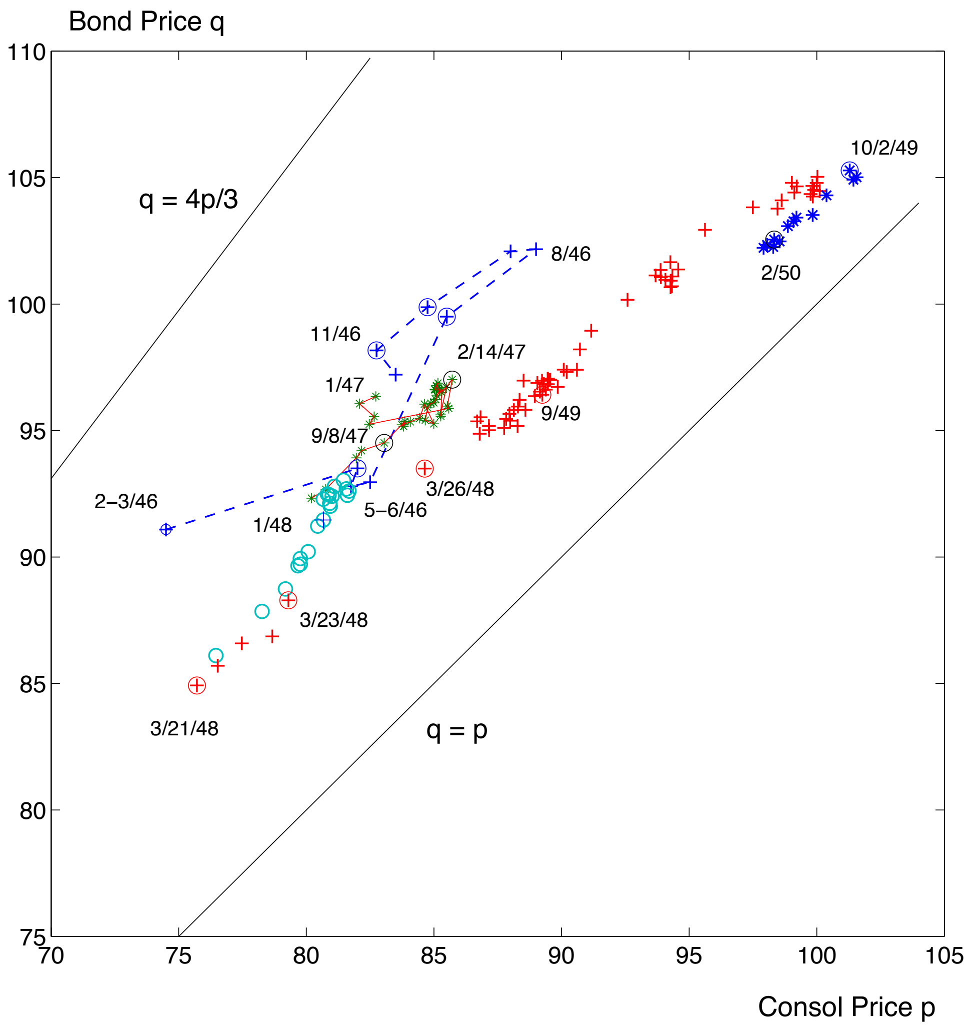
Instead of a swap where each 4% bond is converted into a multiple of 3% bond, Pelham implemented a schedule of gradual interest reductions to the level of 3%. Both methods may have the same present value, but two differences should be mentioned.

In the interest reduction, the face value of the loan is not changed, while it increases in the swap. Hence, once the interest reduction has been phased in (seven years here), the service of the debt is smaller after the interest reduction than after the swap. The interest reduction is thus a method to repay some of the debt and saves of the fixed costs of any operation of debt reduction in the future.

Second, agents may have different perceptions in the two methods. The interest reduction may have looked more "natural" than the conversion in multiple of 3% annuities: in November 49, the long-term rate was stable at 3%. It the context of unspecified contingent agreements, perceptions of *ex post* fairness matter greatly and may rest on idiosyncratic and subtle details. Bond holders at 4% seemed to have a special advantage, with some justification, but given the conditions of the time, they "should be satisfied" to enjoy this advantage for a limited time. Under the swap that provides an amount of 3% bonds with a face value higher than 100, the premium may be viewed as an "advantage" gained by holders of 4% redeemables. Furthermore, the rate of the swap may end as a benchmark that could constrain similar redemptions in the future.

The 4% annuity as a derivative

The previous discussion of the historical context and events shows that rational bond holders anticipated that the 4 % annuities would be redeemed if the interest rate would decrease to 3 %, that is if the 3% annuity would reach par. At any time, the future payoffs of the 4 % annuity and therefore its current price depends on the future evolution of price of the 3% annuity. In today's terms, the 4% annuity is a derivative of the 3% annuity. In order to simplify the notation, from now on we will call "consol" the 3% annuity, although it formally became consol only in 1752, and "bond" the 4% annuities that were issued during the 1743-48 war. The prices of the two assets are represented for the years 1746-50 in Figure 2. We can make the following remarks.



Prices in different time intervals are represented by different symbols (with the indicated dates). The consol is the 3% annuity that was issued during the war and the bond is the 4% annuity that was issued in 1746. The data for 1746 is monthly. All other points are weekly averages of daily prices (when available). Data points in 1746 and in 1747 are joined with a line to highlight the evolution of the point (p, q) over time. Note the price jump at the beginning of the peace negotiations at Aix-la-Chapelle (April 24, 1749). All prices are from the Gentleman's Magazine, adjusted ex-coupon.

Figure 2: The price of the bond (4%) in relation to the price of the consol (3%) (February 1746 to February 1750)

(i) The prices p and q of the bond and the consol are such that the points of coordinates (p, q) are between the two lines $q = p$ and $q = (4/3)p$, and

$$p < q < \frac{4}{3}p.$$

The bond is priced higher than the consol price since it pays a higher coupon and is eventually redeemed into a consol with a conversion ratio not smaller than one. The main observation in the figure is that the bond is priced much below $(4/3)p$, which is the price when the interest rate is never lowered: expectations about redemption played a major role in the determination of the bond price, at all times during the existence of these bonds.

(ii) Prices after October 2, 1749 are represented by blue stars. There is no apparent discontinuity of the *level* of q in relation to p at the time of the redemption, but there is a discontinuity in the *schedule* between the two prices. The new schedule is a line $q = p + h$, with h equal to about 4: after October 1, two months before the official announcement of the interest reduction plan, the market treated that plan as a *fait accompli* for the debt issued during the war: from that date on[12], the premium of the bond over the consol is constant and equal to the value that will hold after February 1750. The relation between the two prices after April 1748 will be shown to satisfy a model of rational expectations.

(iii) The price difference $q - p$ is the value of a annuity paying 1 per year until the consol reaches its par value (the time of the interest reduction), with a final payment at the time of redemption. The difference evolves randomly following the events of the war and decreases when the end is nearer. But throughout the war, this difference is high and exceeds 10: people expected the interest rate to stay above 3% much longer than it actually did. This point will be reexamined below.

(iv) On Sunday April 24 1748, the peace conference began at Aix-la-Chapelle. Between the previous Friday and the following Tuesday, in the largest jump of the war (except for the days around Culloden, April 27, 1746), the consol rose from 80 to 85 with further gains immediately after (see the figure). April 1748 marks a threshold between two separate regimes.

(v) The beginning of the peace negotiations did not mean the end of uncertainty. The guns of Maurice de Saxe[13] conducting the siege of Maastricht could be heard a

[12] The plan was certainly discussed publicly before November 1749. Pelham was against secrecy in the determination of the terms of the interest reduction. We will see in Section 4 that the market treated differently the debt that had been issued before the war.

[13] In a letter to the Comte de Maurepas, Minister of the Marine, the ablest and most successfull

dozen miles away[14]. Long lists of ships captured at sea were listed each month in the Gentleman's Magazine until the signing of the peace treaty, October 18, 1748. Nevertheless, April 1748 had simplified the issues that had to be addressed for the computation of expected future prices: the trend for interest rates was definitely downwards and the main question was how fast they would come down. We will see in the next section that this relation can be explained by a model of derivative asset pricing where one factor explains both prices and moves randomly with a trend toward the situation where the consol is back to its par level. Such a model will provide a tool for the measurement of the expectations of contemporaries.

(v) Before 1748, no simple relation appears between the asset prices in this very complex war (Browning, 1993), between England, France, Holland, Austria, Prussia, Spain and Russia, with shifting alliance. Observe for example in Figure 2 that for a consol price of 85 we have at least three different levels of the bond price, each with a specific local schedule. The expectation of longer hostilities could increase the premium of the bond price over the consol price with a relatively small effect on the latter.

4 Expectations after April 1748

In 18th century England before the industrial revolution, the credit market was dominated by the government and all shocks to the interest rate were caused by the borrowing policy that was a consequence of wars. There cannot be one simple model that applies to all years of the century. Investors obviously used their knowledge of political and military events to establish their expectations about the future and the prices of the assets, day by day. During the ten years of peace that preceded the war of the Austrian succession, the interest rate was hovering in a narrow band around 3%. Following the onset of the war, the interest rate followed a random path that was subject to the fortunes of war. After the beginning of the peace negotiations in April 1748, we may assume that the main forecasting issue was the random decline

general of the time wrote: "In matters political I am only a chatterbox; and if the military situation compels me to discuss them betimes, I don't quote my own opinion as being particularly sound. What I know, and what you ought to know, is that the enemy, however numerous they may be, cannot again penetrate these territories, which I should be very sorry to give up. They are, indeed, a dainty morsel, and when our present woes are forgotten we shall regret having abandoned them. I am ignorant of finance and of our national resources, but I am aware that the rate of interest in England at the close of the last war was only four percent. It has lately reached the unprecedented pitch of twelve percent! As commercial credit is the backbone of England and Holland, I conclude that both are on their last legs. This is not the case with ourselves." (Skrine, 1906, p. 347).

[14] Today, Aachen (Aix-la-Chapelle) in Germany and Maastricht in the Netherlands share the same airport.

of the rate of interest rate toward conditions similar to the pre-war situation.

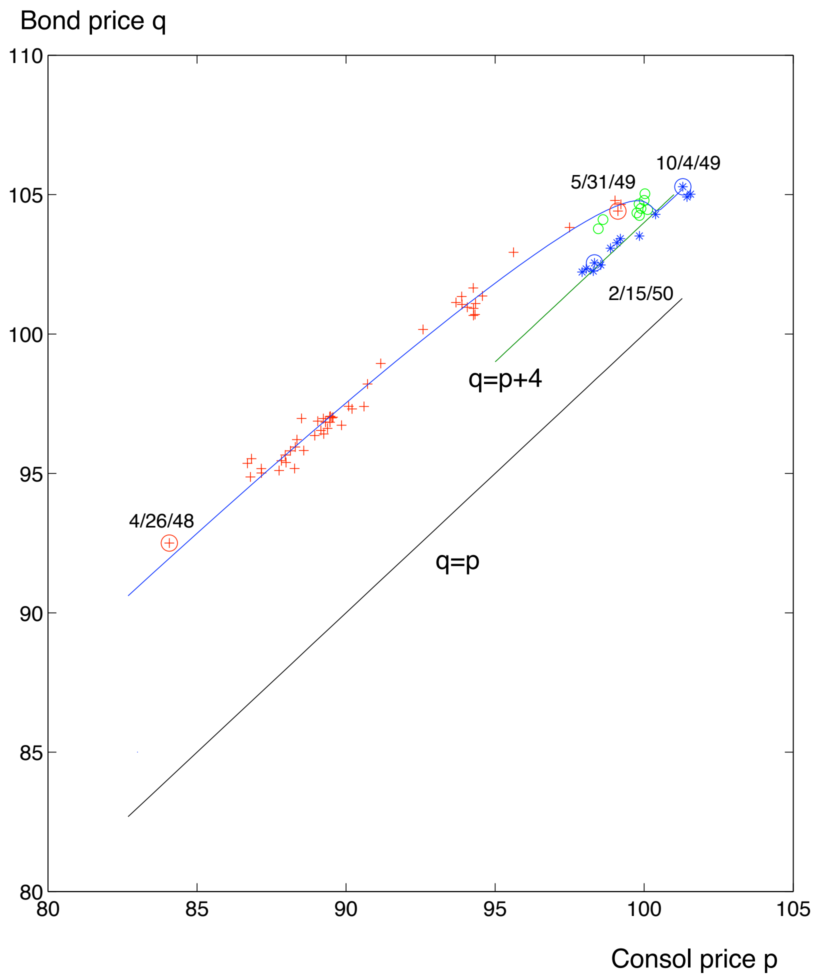
A given price of the consol is compatible with different probability distributions on future interest rates, and in particular with different (stochastic) rates of convergence towards a peace level. The combination of both prices of the consol and the bond provides some additional information on that convergence. The price differences between the bond and the consol (Figure 2), indicate that the market did not anticipate that the interest reduction (which required at least the par for the consol) would come so soon and therefore underestimated the speed of the interest rate decline. In order to have some measurement on these expectations, the prices after April 1748 are used to estimate a model of derivative pricing.

A one factor model

In the model (detailed in the Appendix), the short-term interest rate follows a Markov process with a random convergence to a long-run level according to an Orenstein-Uhlenbeck type: the evolution depends only on the current value of the interest rate and parameters that define the rate of convergence and the variance in the short-term. At any time, for given parameters of the model, the distribution of future interest rates depends on its current value. In this one-factor model, the parameters of the model (including the redemption rule) determine a functional relation from the bond to the consol price. Call it $f(p;z)$ where z is the vector of parameters of the model. These parameters are estimated by minimizing the sum of the squares of $q - f(p;z)$, using the price observations in some interval of time. That interval is chosen between April 1748 and the end of May 1749. The end of the estimation period is taken to be 6 months before Pelham first announced his plan to the parliament. The 6 months interval is chosen to illustrate how the terms of the plan were anticipated by the market before they were announced.

Following the historical events, the model assumes a redemption rule such that the bond is converted to a consol with an additional payment of h when the consol reaches par. The actual premium In Pelham's plan is 3.85 which is the present value of the remaining coupons higher than 3 during a 7 years period. The value of h is one of three parameters to be estimated. The next two parameters determine the rate of convergence of the interest rate toward its long-term value[15], and the short-term variance of the interest rate. Agents are risk-neutral, a reasonable assumption in view of their taste for lotteries. The case of risk-aversion will be discussed briefly later.

[15]The long-run value corresponds to the par value of the consol and is a little lower than 3% in this stochastic model. It is endogenous to the other parameters of the model.



The derivative asset pricing of the bond in the estimated model is represented by the plain curve. The points used in the estimation (up to May 1749) are marked by vertical crosses. (The observations after May 1749 are marked by circles and are not part of the estimation sample). The starred points are the observations after October 2, 1749. The short straight line is the consol price plus a premium of 4.

Figure 3: Prices between April 1748 and February 1750

The expected terms of the interest reduction

The model predicts well the premium h at 3.96 (Table 3 in the Appendix), and therefore the terms of the interest reduction. The strong correlation between the three parameters (see the table) is to be expected: the premium of the bond over the consol $f(p; z) - p$ is positively related to the value of h, negatively related to the convergence rate α and also negatively related to the value of Var because of the concavity of $f(p; z)$. Note however, that the estimated value of Var at 0.41 is near the actual value of 0.45 when the consol price is in the range $88 - 93$.

Table 4 presents the standard errors when Var is fixed at its previously estimated value. There remains the high correlation between the rate of convergence, α, and the premium, h, as a higher premium is equivalent to a delay of the redemption with a smaller end payment. Despite the strong correlation, the value of h is estimated with some precision with a standard error of 0.2.

The expected time to implementation

The rate of convergence is inversely related to the expected time to redemption which can be computed for any consol price from the estimated model (see the Appendix). For a consol value equal to 89, which is the actual value in June 1748, the expected time is equal to 6 years with a standard deviation of 3.5 months. The redemption took place 1 year and a half later.

In discussing the expected time to the interest reduction, one should note that the model does not predict the expected time to the interest reduction but the expected time to the price configuration in which the interest reduction took place. We have seen that the announcement by Pelham produced no impact on the prices. At the time, the consol was at 100 and the bond at 104. These prices indicate that the interest was low at the time and the model provides a strong indication that during the war and after the beginning of the peace negotiations, investors did not expect the interest rate to come down so fast.

The expected value does not give an indication on the probability estimate of the actual event. The model of derivative asset pricing provides a tool to evaluate the tail of this distribution. Let us ask the following: if the price of the consol is 88.95, as it was in November 1748, ($p_{34} = 88.96$ for the estimated model), what is the probability that the interest reduction will be announced, with the additional payment of 4, within time T? That probability was computed for a value $\alpha = 0.024$ (an upper-bound at the 1% confidence level[16]), by a large number of simulations of the model.

[16] This confidence should be computed with fixed h. When h is allowed to vary in the estimation,

The probability that the redemption would take place within a year is about 0.3 percent. For interval of times of 1.5 and 2 years, these probabilities are ** and **, respectively.

The short-term interest rate

The one-factor model generates a relation between the short-term interest rate and the price of the consol. According to the model, the rate of interest is equal to about 9.5% when the consol is at 75, and to 7% when the consol is at 90. The model can be extended to include a risk-aversion in the form of a marginal utility of wealth that decreases wit the price of the consol. The introduction of risk-aversion hardly modifies the estimates of the terms of the interest reduction or the expected time to that reduction, but it lowers the schedule of short-term interest rates. For some reasonable values of the parameters that defines the risk-aversion, the short-term interest rate is equal to 7% and 5.5% when the consol is at 75 and 90, respectively.

The impact of a possible default

A positive probability of partial default obviously lowers the price of government liabilities. The possibility of default lowers the price of the consol when it is low, say at 75, not because people expect a partial default at that price but because they are aware of the random walk of prices and that a lower price today entails a higher probability of reaching an even lower price in the near future with some possible default. A positive probability of default can be introduced in the model for the high values of the interest rate r_i. The impact on the properties of the model is intuitive: it lowers the short-term interest rate for a given level of the consol price, and it increases the variance of the consol (since it increases the price differences between grid points). Numerical experimentations shows that the probability of default on the consol did not add much to the results and this possibility will be ignored here.

A more interesting issue perhaps is a possible default on the redeemable debt. The higher coupon rate may be singled out for a selective reduction of the payments. When a government implements a partial default, the higher interest rate is reduced first, as in 1770 France. The selective default would be a default on the 1 percent premium paid by the bond over the consol. Hence, such a probability would decrease the observed price difference $q - p$. The most robust result of the model is that the market overpriced the bond over the consol in view of the actual redemption date. A probability of default on the bond would strengthen that result.

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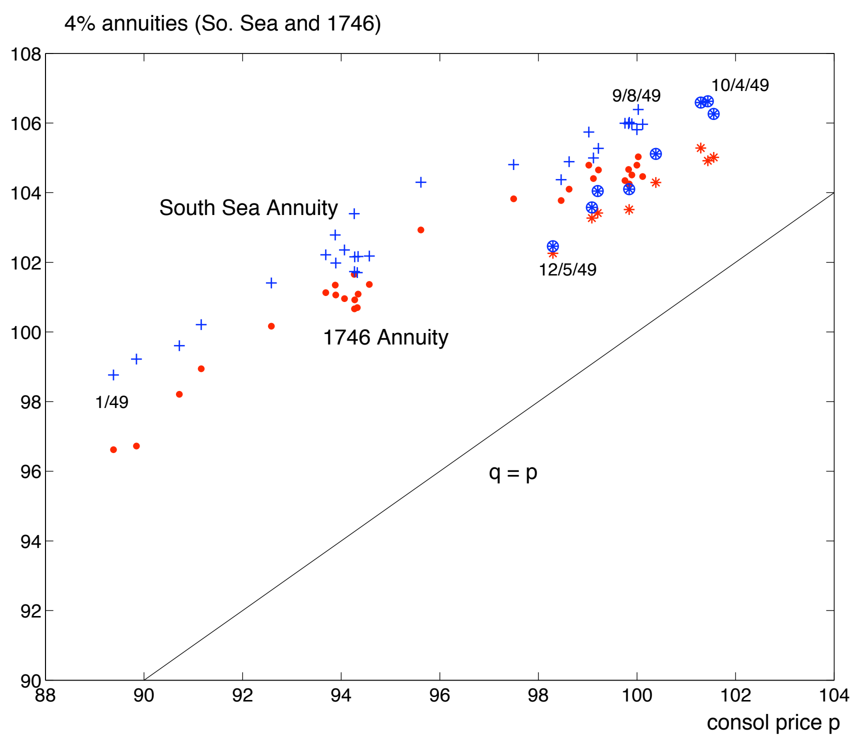
the value of α may be higher, but a higher h is equivalent to a postponement of the interest reduction. (An increase of h by 1 is equivalent ot a postponement of one year).

Other assets

The amount of 4% annuities issued during the war was 15.4 M, but the interest reduction applied to all 4% annuities which amounted to 58M (Dickson, Table 29). The bulk of the old 4% annuities was in South Sea annuities (24M). After Pelham presented his plan to the parliament in December 1749, he faced serious opposition from institutional investors especially the Bank of England and the South Sea Company who represented the interest of the debt holders, that is endow them with market power. There was an obvious advantage in prolonging the 4% coupons payments as much as possible, and according to Sutherland (1946) and Dickson (1967), these institutions put the plan in jeopardy in early 1750. The events are less dramatic when we look at the market prices as represented in Figure 4.



The South Sea annuity is represented by crosses until the beginning of September. After the beginning of October, the 1746 annuity becomes perfect substitute to the consol plus a premium (stars), while the evolution of South Sea annuity is gradual and completed only in December (circled stars).

Figure 4: The redemption of the South Sea Annuities

The figure shows that the South Sea annuities did carry a premium until the beginning of October 1749. (This premium appears for most but not al the war years). At the beginning of October when agents had integrated completely the Pelham plan in the price of the new 4% debt, the South Sea annuities were at a premium of about 1. In early December that premium had vanished and from now on, there was no price difference between the old and the new debt. Investors realized already in the Fall of 1749 that their bargaining position was weak.

5 Policy evaluation

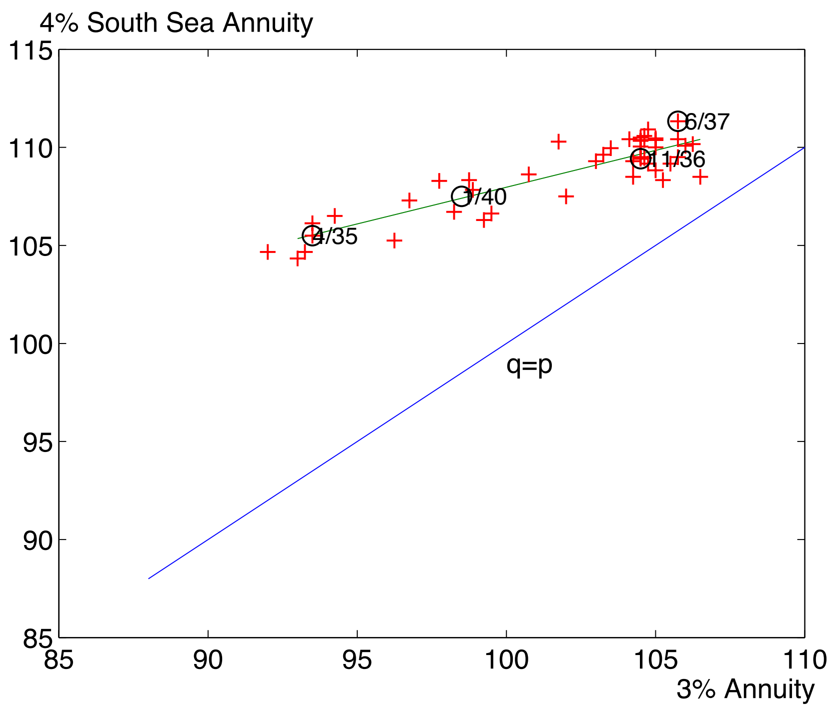
The attempt of 1737

A reduction of the interest rate on government liabilities had been discussed in 1737 when the long-term interest was around 3% (Dickson, 1967, pp. 212-214). A parliamentary committee was set in March but the project was abandoned by the end of April. The reasons for the failures are not very clear and should be investigated in more details. According to Dickson, the project was attached to some other policies that were not politically acceptable by a large fraction of the parliament such as a commitment to reduce the tax burden.

We have seen that market expectations should play a critical role for the implementation of an interest reduction and the price information on these expectations is ignored in the works of Sutherland and Dickson. The prices of the 3% annuity and the 4% South Sea annuity are presented in Figure 5 for the years 1735-40. In 1737, the price of the 3% was stable around 105-106, and the 4% annuity commanded, at that price, a premium of about 4 or 5, not much different from the premium in 1749. The figure also shows a stable relation between the prices of the two annuities, with a slope smaller than one, such that when the 3% was at par the premium of the 4% was about 7. It seems that a well prepared policy with a schedule of gradual reductions (equivalent to the payment of a conversion with a premium), could have had a chance.

Lobbies could work against such a reduction in an environment that was more favorable for them in 1737 than in 1749. The government did not seem well prepared as it attempted initially to reduced the interest only on some annuities, thus stimulating the lobbying by the affected bond holders. The 3% annuities had been introduced for the first time 6 years before and were only a small fraction of the total debt. In 1749, 3% annuities had been the main instruments of debt financing during the previous war and the new 4% annuities had been associated closely to them. When the 3% reached par, it was perhaps more "natural" to reduce the interest on the new

4% annuities. Once this was accepted, the path was set for the application of the reduction to the entire debt, as described in the previous section. Finally, annuity holders did not benefit as in 1749 from a very large windfall gain due to a decrease of the interest rate that was much faster than expected.



The South Sea annuity is the "New Annuity". The line is obtained by linear regression.

Figure 5: South Sea 4% Annuities 1735-40

Policy against pessimism

Did the prices of the government bonds drop "too much" during the wars in view of the subsequent evolutions of the short-term rates? The volatility of long-term interest rates in expectations models of the term structure has been analyzed by Shiller (1989). His tests have been applied by Weiller and Mirowski (1990) to long-term bonds in 18th century England, with mixed results[17]. The methodology rests

[17] Note that the fluctuation of the bonds can only be significant downwards because of the callable features of the bonds (which is ignored by Weiller and Mirowski). In the context of the 18th century, the 3% bond could be assumed to be non redeemable, but the previous discussion of the redemption policy shows that if the consol had increased to a stable level above say 105, then the reedemable

essentially on a comparison between the variance of the short-term rates with that of the long-term rate and the method requires a sufficient number of events. In the 18th century, the sample generates three large fluctuations between 1715 and 1792, one for each war.

This study provides a different type of test about an excess fluctuation. The main issue at the end of war, as attested by the policy of issuing redeemable bonds was the speed of convergence of the bond to its par value. If people expected a slow convergence, then the consol price did not overreact. The information contained in the two prices of the 3% and 4% bonds with the option feature of the latter provides a strong test about two issues.

First, the market was remarkably rational in pricing financial assets consistently, as shown by the good fit of the derivative asset pricing model of the 3% and the 4% annuities. Moreover, the terms of the payoff at redemption were accurately predicted and taken into account long before the redemption.

Second, the market showed an excess pessimism about the recovery of bond prices after the war which is measured in two different ways: the expected time for the bond reaching its par value was strongly overestimated; second the subset of events such that the consol would return to par within a year (which it did) was given a subjective probability of less than 1/4 percent by the market. The price observations provides therefore strong evidence that in the case of the war of the Austrian succession succession, the market overreacted and showed considerable excess pessimism.

In this context, the debt policy during the war of the Austrian succession offers a spectacular illustration of the power of marketed contingent financial instruments. The loans of 1745 and 1746 included non marketed life-time annuities and were more expensive *ex ante* than later loans although the long-term rate was not smaller in 1747-1748 and the amount of the loans more than twice as high.

The contigent feature of the callable bonds enabled the government to take advantage of the excess-pessimism. Simply put, the market paid in April 1746 during the subscription of the 4% bond, a price of $\mathcal{L}$ 12 for the contingent annuity of $\mathcal{L}$ 1 per year until the call of the bond. It was willing to pay that price because it was pessimistic about future interest rate. Its expectation was that the annuity would pay for about 10 years with a final payment of 4. In fact, the annuity lasted only for three years. That price of 12 was much more than the total amount collected on

feature would have depressed it price. The consol still exists today with an interest of 2.5% since 1903, down from 2.75 since 1889, (Miller, 1890, Harley, 1976, Klovland, 1994).

the annuity, ignoring discounting, since the government paid a total of 7 (1 per year from 1746 to 1748, and a total of 4 after). The government was therefore able[18] to bet with great success against the pessimism of the market.

The success of the contingent policy can also be measured by the *ex post* rate of return that the government paid on its loans. This rate (the long-term internal rate of return) is presented in Column 6 of Table 2. Once the government abandoned the inefficient life annuities for the redeemable 4%, it was able to finance the war a long-term rate that even during the worst years, turned out to be only 3.5%.

Finally, the successful betting by the government against pessimistic investors was repeated recently and in a similar context. In the early 1980s, the British government faced adverse expectations of private investors who were pessimistic about the government's conduct and the evolution of interest rates. This time, the ennemy was not France, but inflation. Margaret Thatcher was more confident than the market that she would prevail and her government issued inflation indexed bonds, with coupons linked to the inflation rate. Expecting high coupons for a long time, investors paid high prices to the government, like the buyers of 4% annuities in 1747. Inflation came down much sooner than expected (with some help from Paul Volcker). In the 1980s as in the 1740s, the government won against excessive market pessimism[19].

[18]The extent of the bet was limited however, perhaps by institutional factor since the government could not issue solely annuities contigent on the consol below par: to each such annuity it had to attach one consol which had indeed a low price because of the pessimism. The government could also have even done better if the loans had been issued with a coupon of 4.5% instead of a 10% premium over par.

[19]If the government had used standard (non inflation linked) bonds, it would have had to pay an average *ex post* real return of 7.7% in the 15 years after 1982 (while the average inflation rate had been 4.3%). On the inflation linked bonds the government paid only 2.8%.

APPENDIX

The one-factor model of the government bond prices: 48-49

The model of interest rate is based on the Vasicek model with specific parameters for a regime of war time[20]. Some specifications in that class of models generate a close form solution in continuous time. But these special cases are too constraining, and to have more freedom for the fitting of the data, we use a discrete numerical model.

The random variable that drives the prices is short-term rate, r. It is assumed to take its values on a grid $\{r_i\}_{1\leq i\leq N}$, where the function r_i of i is increasing. The rate moves randomly on the grid, one step at a time, according to a Markov process. The transition probabilities to move from r_i to r_{i+k}, $k \in \{-1, 0, 1\}$ are defined by $m_{i,i+k}$ with

$$\begin{cases} m_{i,i-1} = \alpha + \beta, \quad m_{i,i+1} = \beta, \text{ if } i > L, \\ \\ m_{i,i-1} = \beta, \quad m_{i,i+1} = \alpha + \beta, \text{ if } i < L \\ \\ m_{L,L-1} = \beta, \quad m_{L,L+1} = \beta. \end{cases} \tag{1}$$

The values are adjusted at the boundaries $i = 1$ and $i = N$, and for all i, $m_{i,i} = 1 - m_{i,i-1} - m_{i,i+1}$. The transition probabilities in equation (1) define the matrix of transition probabilities $M = [m_{i,j}]$.

The price of the consol

A consol is the sum of an infinite number of zero coupons bonds of different maturities. Each zero coupon bond is priced using the probabilities of future rates and the price of the bond is the sum of the values of all the zero coupon bonds imbedded in it. Agents are risk-neutral, an assumption that is not contradicted by their revealed preference for lotteries.

Coupons were paid twice a year, around the 5th day of January and July. Because the bond price depends on the interest rate which follows a Markov process, the price of the consol in any period depends only on the interest rate in that period and the time to the next coupon payment. Let p be the vector or prices of the consol p_i when the interest is r_i immediately after the payment of the coupon, and $p(t)$ the price t days before the payment of the coupon. Assuming the year to have 366 days, for simplicity, $p(183) = p$. Let u be the column vector of dimension N with all elements equal to 1, and D the diagonal matrix where i-diagonal element is equal to $1/(1+r_i)$.

[20] Such a model could not be applied with constant coefficients for large intervals of time that include war and peace. For example, after 1749, the variance of the consol was much smaller than before.

In each period where no coupon is paid, assuming risk-neutrality in a first step, the price of the consol satisfies the discounting relation

$$p_i = \frac{1}{1+r_i}\Big(m_{i,i-1}p_{i-1} + m_{i,i}p_i + m_{i,i+1}p_{i+1}\Big). \tag{2}$$

This equation is generalized in matricial form for the price in any period with t days before the payment of a coupon, $1 \leq t \leq 183$:

$$p(t) = DMp(t-1), \quad \text{with} \quad p(0) = 1.5u + p. \tag{3}$$

The matrix $A = DM$ is the matrix of discounted probabilities of the values of an asset in the next period. Let $J = A^{183}$ define the discounted probabilities over the six months time interval between coupon payments.

By iteration of (3), $p(t) = A^t(1.5u + p)$. Since $p(183) = p$, $\quad p = A^{183}(1.5u + p)$, and

$$p = 1.5(I - J)^{-1}Ju, \tag{4}$$

where I is the identity matrix of dimension N.

From the value of p we have for any $t \geq 1$,

$$p(t) = 1.5A^t(I - J)^{-1}Ju. \tag{5}$$

On the L grid point, the rate r_L is by definition equal to the long-run value r^* to which the rate converges (if there were no shock). That grid point is adjusted such that when the rate is equal to $r_L = r^*$, the consol is at par[21]: $p_L = 100$. When the process is deterministic, the value of $r_L = r^*$ is obviously equal to 3%. When the process of the interest rate is random, the long-run value r^* is a little smaller.

The price of the callable bond

Following the historical policy of Pelham, the redemption of the bond is determined by the following rule: in any period with a consol price p_i, the probability of redemption is equal to π_i: if the redemption takes place, the government is committed to redemption at the date of the next coupon payment; at that date, the bond is transformed into a consol plus an amount h that is equal to the present value of difference between the coupon payments and 1.5 during the phasing in of the interest reduction. (The value of h is the one that fits best the data after the redemption: $h = 4$).

[21]This modeling choice is validated by the evolution of the consol after the interest reduction of 1749: during the year 1750, the consol always stayed in the interval $[98.5, 101.5]$.

Let Π be the diagonal matrix with elements π_i. If t is the number of days until the next coupon payment, let $q(t)$ and $v(t)$ be the prices of a bond with no announcement of a redemption and with a commitment to redemption (with the next coupon), respectively. Recall that $p(t)$ and $q(t)$ are vectors.

Using the standard backward induction,

$$\begin{cases} q(1) = A\Big((I-\Pi)(2u+q)+\Pi w\Big), \quad \text{with} \quad w=(1.5+h)u+p, \\ \\ v(1) = Aw, \end{cases} \tag{6}$$

where p is the price of the bond immediately after the payment of the coupon. For $1 \leq t \leq 182$,

$$\begin{cases} q(t+1) = A\Big((I-\Pi)q(t)+\Pi v(t)\Big), \\ \\ v(t+1) = Av(t). \end{cases} \tag{7}$$

Let $\tilde{A} = A(I-\Pi)$ and $\tilde{J} = \tilde{A}^{183}$. The price of the bond immediately after the payment of the coupon is q and since $q = q(183)$,

$$q = \tilde{J}(2u+q) + \sum_{k=0}^{182} \tilde{A}^{182-k} A\Pi A^k((1.5+h)u+p).$$

The vector-price of the bond is therefore determined[22] by the equation

$$q = \Big(I-\tilde{J}\Big)^{-1}\Big(2\tilde{J}u + \sum_{k=0}^{182} \tilde{A}^{182-k} A\Pi A^k((1.5+h)u+p)\Big). \tag{8}$$

The expected time to redemption

Let Θ be the vector $\Theta = \{\theta_i\}_{1\leq i\leq N}$. The equation (**??**) in the text can be written in matricial form

$$\Theta = u + M(I-\Pi)\Theta,$$

where u is a column-vector of N ones, M is the matrix of elements $m_{i,j}$, I the identity matrix, and Π is the diagonal matrix with diagonal elements π_i. The expected time to redemption is a function of the realization of the interest rate and is expressed by

$$\Theta = (I - M(I-\Pi))^{-1}u. \tag{9}$$

3. Estimation

[22] Since the matrix $I - \tilde{J}$ may be difficult to invert, it is easier to determine the value of q by iteration of the system (7).

The model depends on vector of 4 parameters $\zeta = (\alpha, h, \beta, \gamma)$ which are ranked by decreasing order of importance. Before the estimation, it is useful to review their impact on the shape of the derivative price schedule $\phi(p; \zeta)$.

1. The rate of convergence of the consol's price to the par is determined by α, perhaps the most important parameter of the model, (equation (1)). A higher α generates a shorter time, on average, to the redemption, thus reducing the value of the contingent annuity, $q - p$, and lowering the schedule of $q = \phi(p; \zeta)$.

2. The premium h paid at the time of the redemption defines the market's expectations about the terms of redemption. A higher premium raises the value of the annuity $q - p$, and has an impact that is similar to a lower rate of convergence. The estimates of α and h will be therefore be positively correlated and this positive correlation is specific to the investors' problem: a higher premium h is roughly equivalent to a delay of the redemption, *i.e.*, a lower value of α.

3. The variance of the consol prices that is generated by the model, over a week and around some arbitrary value $\hat{p}$, is approximated by the equation

$$Var = 7(\alpha - \alpha^2 + 2\beta)\Delta^2 p, \tag{10}$$

   where $\Delta^2 p$ is an average of the terms $(p_{k+1} - p_k)^2$ around $\hat{p}$, and which are generated by the model. We replace $\Delta^2 p$ by its historical average in the interval of prices $(88, 94)$. Given α, equation (10) implies an equivalence between the parameters β and Var. The estimation will be made directly with respect to the paramer Var. Its comparison with the actual variance of the consol prices in the range $(88, 94)$ will provide an independent check of the model.

   An increase of the parameter Var lowers the schedule $q = \phi(p; \zeta)$ because of the concavity of that schedule. We should therefore anticipate a negative correlation between the estimates of Var and the parameter of convergence α.

The vector ζ of parameters to be estimated is $\zeta = (\alpha, h, \beta, \gamma)$. , ζ the vector of parameters of the model to be estimated. For a given value of ζ, the vector-price p is determined by (4) and the vector-price q by (8). These vector-price determine points a the function $q = \phi(p; \zeta)$. Given the set actual prices[23] $\{p_t, q_t\}$, $(1 \leq t \leq n)$,

[23] Points before May 1747 are omitted. They fit well the curve however. Similarly points after September 1749 are omitted from the sample of estimation since they reflect additional information, as will be discussed later.

the estimated parameter ζ^* minimizes the sum of the squares[24]

$$\mathcal{S} = \sum_{t=1}^{n} (q_t - \phi(p_t;\zeta))^2.$$

The previous formulae for the prices p and q were established for the the prices the day after the payment of the coupon. For the estimation, all observed prices (from various issues of the Gentleman's Magazine), are adjusted ex-coupon: the pro-rated accumulation of the next coupon, from the date of the last payment, is deducted from the price.

Table 3: Standard errors and correlations

	Estimate	α	h	$Var(p)$
α	0.022	0.012	-0.97	-0.97
h	3.96		13.83	0.999
Var	0.41			5.84

The rate of convergence to the long-term values is measured by α, the last payment at the time of interest reduction is measured by h and Var correspond to the variance of the price of the 3% annuity when that price is about 90. The standard errors are on the diagonal, and the coefficients of correlation are above the diagonal. The table on the right presents the standard errors and the coefficient of correlation when $Var(p)$ is fixed at its estimated value on the left (0.45). When h is constrained at its estimated value, the standard error of α is 0.001.

Table 4: Standard errors and correlations with fixed variance of prices

	α	h
α	0.003	0.93
h		0.21

The table presents the standard errors and the coefficient of correlation when $Var(p)$ is fixed at its estimated value on the left (0.45). When h is constrained at its estimated value, the standard error of α is 0.001.

[24]Since the function ϕ is determined by a grid, the value of $\phi(p_t;\zeta)$ is determined by linear interpolation with the two nearest grid points

Risk-aversion

If agents are risk averse, the previous discounting equation (2) has to be modified. In this model where all the uncertainty arises in one factor, it is assumed that portfolio holders have a marginal utility of wealth that depends only on the state, and that is increases with the current value of the interest rate r_i. (For example, consumption could be smaller at the time of large government borrowings and higher interest rates). This marginal utility λ_i will be assumed to be normalized to 1 for $i \leq L$, that is for $r < r^*$, and to be an increasing function of r_i for $i > L$ such that

$$\begin{cases} \lambda_i = 1 & \text{if } i \leq L, \\ \lambda_i = 1 + \gamma\Big(\dfrac{i-L}{N-L}\Big)^2 & \text{if } i > L, \end{cases} \tag{11}$$

where the parameter γ will be ajusted to fit the observations.

Equation (2) is replaced now by

$$p_i = \frac{1}{1+r_i}\Big(m_{i,i-1}\frac{\lambda_{i-1}}{\lambda_i}p_{i-1} + m_{i,i+1}\frac{\lambda_{i+1}}{\lambda_i}p_{i+1} + m_{i,i}p_i\Big), \tag{12}$$

which has the same form as the equation (2) provided that M replaced by $\tilde{M}$ with

$$\tilde{m}_{i,i-1} = m_{i,i-1}\frac{\lambda_{i-1}}{\lambda_i}, \qquad \tilde{m}_{i,i+1} = m_{i,i+1}\frac{\lambda_{i+1}}{\lambda_i}, \qquad \tilde{m}_{i,i} = m_{i,i}. \tag{13}$$

The matrix M is the matrix of true probabilities and the matrix $\tilde{M}$ is the matrix of risk-neutral probabilities. When agents are risk-neutral, $\gamma = 0$ and the two matrices are identical.

The risk-aversion lowers the value of the short-term interest rate for any given value of the consol. Consider the arbitrage equation (12) between the consol prices in consecutive periods. Risk-aversion lowers the next period value of the consol price if it higher (because of the declining marginal utility of wealth), and increases it if the price falls. In the regime where the bond had not been converted yet, the trend of the asset prices is up. Hence the first effect is stronger than the second and the term in parenthesis in the right-hand-side of (12) is smaller. To keep the equation balance, the interest rate r_i has to decrease.

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