# A Model of Financial Crises in Open Economies*

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**Abstract**

We study a small open economy with flexible exchange rates and a financial sector that faces a potentially binding collateral constraint. Financial crises in the model are self-fulfilling, and they are associated to drops in real economic activity, real exchange rate depreciations, and current account reversals. The presence of dollarized liabilities in the financial sector makes these crises more likely. These currency mismatches arise endogenously because households have a precautionary motive to save in foreign currency when they expect a confidence crisis with sufficiently high probability. In this framework, we analyze the role of a domestic lender of last resort. Precautionary reserve accumulation by the monetary authority facilitates effective lending of last resort, and can lead to a less dollarized financial sector and to a more stable exchange rate.

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# 1 Introduction

Banking panics are a common feature of financial crises, both in emerging and in developed economies. The main feature of a banking panic is that investors lose confidence in the short term liabilities of financial institutions, leading to a loss of funding for these institutions, to depressed asset values, and, eventually, to a contraction in credit and investment. In an open economy, with an open capital account, the problem is typically associated to a generalized flight from all domestic assets, not just banks' liabilities, leading to a current account adjustment and to a depreciation of the domestic currency.

Fixed exchange rate regimes offers stark examples of situations in which investors lose confidence in domestic-currency bank liabilities, i.e., in domestic money, and demand to convert them into foreign currency assets. These situations often lead to a joint banking and currency crisis, i.e., a twin crisis.[1] An open question is whether a regime of flexible exchange rates insulates the domestic banking sector from these tensions. The idea is that in a fixed exchange rate regime selling domestic currency assets becomes a one-way bet as investors expect the currency eventually to devalue. In a flexible exchange rate regime an instantaneous devaluation eliminates this force, dampening the incentive to run from domestic-currency denominated assets. At least since the Asian crisis of 1997-1998, economists have recognized that balance-sheet effects can undermine this argument, as a sudden devaluation can harm institutions exposed to currency risk (Krugman, 1999; Aghion, Bacchetta, and Banerjee, 2001).

The model features multiple equilibria which have the typical features of a banking panic with capital flight. In the bad equilibrium, all investors, domestic and foreign, are less willing to extend credit to banks and, at the same time, the economy overall experiences a current account reversal. We consider interventions by a central bank with limited resources, namely, foreign exchange reserves and a fixed amount of domestic fiscal revenue. We show that the presence of a fully flexible exchange rate and an inflation targeting regime does not eliminate the possibility of multiple equilibria.

Our analysis proceeds in two stages. First, we focus on the period in which the panic occurs, taking as given the economy's initial conditions, including the assets and liabilities of the banks. We show that a flexible exchange rate regime is especially exposed to panics when three weaknesses are present: high leverage of domestic banks; high levels of foreign denominated liabilities; the fact that the fiscal resources that back up central bank interventions are in domestic currency. The ingredients behind these results are not new, leverage plays a similar role as in Gertler and Kiyotaki (2015) in a closed economy context,

[1]See for instance Kaminsky and Reinhart (1999).

and the role of foreign liabilities is in line with the literature following the However, the connection between banks' weak balance sheets and the real exchange rate is derived in a novel (and, we think, realistic) way and the role of the fiscal backing of the central bank is new.

In the second stage of the analysis, we take a step back and analyze the determinants of the banks' balance sheets. Here, we focus on the endogenous choice of domestic consumers between domestic-denominated and dollar-denominated deposits. Our crucial result here is that this endogenous choice does not eliminate multiplicity, but actually adds a new layer to it. Consumers who anticipate banking panics, associated to large fluctuations of the exchange rate, tend to prefer dollar deposits, since they give them some protection against a devaluation. However, their preference for dollar deposits is exactly what pushes banks towards greater degrees of mismatch and thus to a higher probability of a panic. On the other hand, if consumers do not expect banking panics to occur, they have a natural preference for domestic deposits, because, due to the central bank's inflation targeting, domestic deposits provide more stability in terms of domestic purchasing power. So the presence or absence of panics feeds back into the asset choices of consumers, because it changes the nature of exchange rate fluctuations. Without panics, exchange rate risk is just an unwanted additional source of risk for domestic agents. With panics, foreign assets become a good hedge, because the currency tends to depreciate exactly when the country is in a crisis. The endogenous nature of exchange rate risk in presence of financial crises is the main innovative contribution of our paper.

**Literature**. (To be added. Here is a partial list of closely related papers) Twin crises and the exchange rate regime: Jeanne and Wyplosz (2003), Céspedes, Chang, and Velasco (2000). Financial intermediation and open economy macro: Gertler and Karadi (2011), Maggiori and Gabaix (2015), Bruno and Shin (2014). Asset pricing with collateral constraints: Aiyagari and Gertler (1999), Bocola (2016). Foreign reserves: Obstfeld, Shambaugh, and Taylor (2010), Aizenman and Lee (2007), Ranciere and Jeanne (2006). Multiple equilibria through precautionary motives: Broner and Ventura (2016), Fajgelbaum, Schaal, and Taschereau-Dumouchel (2014)

Céspedes, Chang, and Velasco (2017) Three main differences: our focus on domestic asset prices, capital flows not just from banks, ex ante analysis.

# 2 Model

We consider a small open economy that lasts three periods, $t = 0, 1, 2$, populated by two groups of domestic agents, households and bankers, who trade with a large number of foreign investors.

There are two goods: a tradable good and a non-tradable good. We assume that monetary policy keeps the domestic price level stable, so adjustments in the relative price of tradables vs non-tradables lead to fluctuations in the nominal exchange rate. The model features flexible prices, but movements in the nominal exchange rate matter because agents trade financial claims denominated both in domestic and in foreign currency.

The bankers act as intermediaries: they hold all the capital goods in the economy and issue liabilities denominated in domestic and foreign currency. Therefore the price of capital goods and the exchange rate affect bankers' net worth and, due to collateral constraints, bankers' net worth affect real investment in the economy. To allow for the endogenous determination of the price of capital goods, we assume an upward sloping supply of new capital coming from firms producing capital goods subject to convex costs.

We now turn to a detailed description of the environment and to the definition of an equilibrium. Along the way, we make a number of simplifying assumptions. Their role is discussed in detail at the end of the section.

## 2.1 Agents and their decision problems

### 2.1.1 Households

Households enter period $t$ with financial claims on domestic banks and foreigners. Let $a_t$ and $a_t^*$ denote the households' total claims, respectively, in domestic and foreign currency.[2] The nominal exchange rate is $s_t$, expressed as units of domestic currency per unit of foreign currency. Except if otherwise noted, all prices are in domestic currency. Households earn the wage $w_t$ from supplying a unit of labor, inelastically, to the tradable sector. They also receive every period an endowment of non-tradable goods, $e^N$, and the profits of the firms producing capital goods, $\Pi_t$. Households use these resources to buy tradable and non-tradable consumption, and to buy one-period claims in domestic and foreign currency. Accordingly, their period $t$ budget constraint is

$$q_t a_{t+1} + s_t q_t^* a_{t+1}^* + p_t^T C_t^T + p_t^N C_t^N \leq w_t + p_t^N e^N + \Pi_t + a_t + s_t a_t^*, \tag{1}$$

[2] Both $a_t$ and $a_t^*$ are allowed to be negative, thus denoting a debtor position for the household.

where $q_t$ and $q_t^*$ are the prices of one-period claims denominated in domestic and foreign currency, $C_t^T$ and $C_t^N$ are consumption of tradable and non-tradable goods, and $p_t^T$ and $p_t^N$ are their prices.

The household flow utility function is $U(C_t)$, where $C_t$ is the consumption aggregator

$$C_t = (C_t^T)^\omega (C_t^N)^{1-\omega}.$$

Households choose state-contingent plans for assets and consumption levels in order to maximize expected lifetime utility

$$\mathbb{E}\left[\sum_t \beta^t U(C_t)\right]$$

subject to the budget constraints (1) and the terminal conditions $a_3 = a_3^* = 0$.

We can simplify the households' problem by separating the dynamic problem of choosing sequences for $C_t, a_{t+1}, a_{t+1}^*$ from the static problem of allocating consumption expenditure to tradables and non-tradables. Standard steps imply that consumption expenditure can be expressed as

$$p_t^T C_t^T + p_t^N C_t^N = P_t C_t,$$

where

$$P_t \equiv \omega^{-\omega} (1-\omega)^{-(1-\omega)} (p_t^T)^\omega (p_t^N)^{1-\omega} \tag{2}$$

is the domestic CPI. Given the consumption level $C_t$, the optimal demands of tradables and non-tradables are:

$$C_t^T = \omega \frac{P_t C_t}{p_t^T}, \quad C_t^N = (1-\omega)\frac{P_t C_t}{p_t^N}.$$

### 2.1.2 Bankers

Bankers run banks that hold the following assets and liabilities.

On the asset side, banks hold two types of capital goods, $k_t^T$ and $k_t^N$. The first, called $T$ capital, is used as an input in the production function

$$y_t^T = (k_t^T)^\alpha (l_t)^{1-\alpha},$$

and so it earns the rental rate

$$R_t^T = p_t^T \alpha (k_t^T)^{\alpha-1}, \tag{3}$$

since labor supply is 1 in equilibrium. The second, called $N$ capital, is used to produce non-tradable goods according to the linear production function $y_t^N = k_t^N$, so its rental rate is simply

$$R_t^N = p_t^N. \tag{4}$$

The two capital goods trade at prices $\Psi_t^T$ and $\Psi_t^N$. They do not depreciate in periods $t = 0$ and $t = 1$ and fully depreciate after production at $t = 2$.

On the liability side, banks issue one-period claims in domestic and foreign currency, denoted $b_t$ and $b_t^*$.[3] Therefore, the banks' net worth in domestic currency, is

$$n_t = (\Psi_t^T + R_t^T)k_t^T + (\Psi_t^N + R_t^N)k_t^N - b_t - s_t b_t^*. \tag{5}$$

and the banks' budget constraint is

$$\Psi_t^T k_{t+1}^T + \Psi_t^N k_{t+1}^N = n_t + q_t b_{t+1} + s_t q_t^* b_{t+1}^*, \tag{6}$$

as banks use their net worth and newly issued claims to purchase the two capital goods.

There are two important sources of financial frictions in our model. First, only banks can hold capital goods. Second, banks face limits in their ability to raise outside finance. Namely, banks have to satisfy the following collateral constraint, which requires total liabilities to be bounded by a fraction of the $T$ capital held by the bank:

$$q_t b_{t+1} + s_t q_t^* b_{t+1}^* \leq \theta \Psi_t^T k_{t+1}^T, \tag{7}$$

where $\theta$ is a parameter in $[0, 1]$.

We assume that bankers only consume in $t = 2$, are risk neutral, and only consume tradable goods. Therefore, the bankers' problem is to choose state-contingent plans for $\{k_{t+1}^T, k_{t+1}^N, b_{t+1}, b_{t+1}^*\}_{t=0,1,2}$ in order to maximize the expected value of $n_2/_2^T$, subject to the law of motion for net worth (5), the budget constraint (6), the collateral constraint (7), and the terminal condition $b_3 = b_3^* = 0$.

### 2.1.3 Capital goods production

The $N$ capital is in fixed supply, so in equilibrium we have $k_t^N = k^N$. This implies that there is a fixed supply of non-tradable goods denoted by $y^N = k^N + e^N$.

For the production of $T$ capital, there are competitive firms owned by the households, that transform tradable goods into $T$ capital in periods $t = 0, 1$. In order to produce $i_t \geq 0$

[3] Both $b_t$ and $b_t^*$ are allowed to be negative, thus denoting a creditor position for the banks.

units of capital, the producers need $G(i_t)$ units of tradable goods. The function $G$ takes the form

$$G(i_t) = \phi_0 i_t + \frac{\phi_1}{1+\eta} i_t^{1+\eta}.$$

The profits of the capital producing firms are

$$\Pi_t = \max_{i_t \geq 0} \Psi_t^T i_t - p_t^T G(i_t). \tag{8}$$

Market clearing in the capital goods market in periods $t = 0, 1$ is given by

$$k_{t+1}^T = k_t^T + i_t,$$

as the capital inherited from past periods plus the newly produced capital is accumulated by banks for future production. In period $t = 2$ all capital fully depreciates after production has taken place and the capital goods market is not active.

### 2.1.4 Foreign investors

Foreign investors are risk neutral, consume only tradable goods, and discount the future with discount factor $\beta$. We assume that foreign investors can only buy claims denominated in foreign currency. Therefore, equilibrium in the domestic claims market requires $a_t = b_t$. On the other hand, in the foreign claims market the difference $a_t^* - b_t^*$ can be positive or negative, as foreign investors will absorb the difference.

Let $p_t^{T*}$ denote the price of tradable goods in foreign currency, which is exogenous. The law of one price implies:

$$p_t^T = s_t p_t^{T*}. \tag{9}$$

This price $p_t^{T*}$ is normalized to 1 at $t = 0$ and is subject to random shocks at $t = 1, 2$. Specifically, at $t = 1$ the random variable $\varepsilon$ is realized and the price of non-tradables is permanently affected and equal to

$$p_1^{T*} = p_2^{T*} = 1/\varepsilon.$$

The variable $\varepsilon$ is lognormally distributed with mean 1 and variance $\sigma_\varepsilon^2$. This nominal disturbance will generate fluctuations in the nominal exchange rate in equilibrium, and it is introduced to get exchange rate movements that are orthogonal to the fundamentals of the domestic economy.

The price of foreign-denominated bonds is pinned down by the Euler equation of foreign investors

$$q_t^* = \beta \mathbb{E}_t \left[ \frac{p_t^{T*}}{p_{t+1}^{T*}} \right] = \beta,$$

where the last equality follows from the stochastic properties of $p_t^{T*}$.

#### 2.1.5 Monetary regime and the nominal exchange rate

Our economy features flexible prices, so the only role of monetary policy is to determine nominal prices and the nominal exchange rate. The reason why these prices matter for the real allocation is that assets and liabilities are denominated in different currencies, so flucutations in the nominal exchange rate reallocate wealth across agents.

For most of the analysis, we assume that the monetary authority is only concerned with price stability. Namely, we assume that the monetary authority successfully targets a constant CPI:

$$P_t = \bar{P} = \omega^{-\omega}(1-\omega)^{-(1-\omega)}. \tag{10}$$

Combining this rule with the CPI definition (2) and the law of one price (9), we obtain the nominal exchange rate

$$s_t = \frac{1}{p_t^{T*}} \times \left( \frac{p_t^T}{p_t^N} \right)^{1-\omega}. \tag{11}$$

Two forces drive the nominal exchange rate: nominal fluctuations in the price level in the rest of the world and movements in the relative price of tradables and non-tradables. Both forces will be relevant for our analysis.

### 2.2 Equilibrium

There are two sources of uncertainty in this economy, both realized at date $t = 1$. The nominal shock $\varepsilon$ introduced above, and a sunspot variable $\eta$ uniformly distributed in $[0, 1]$. The sunspot will determine which equilibrium is played at $t = 1$ when multiple equilibria are possible. Note that we are leaving implicit in our notation that all variables dated 1 and 2 are function of the state of the world $(\eta, \varepsilon)$.

A competitive equilibrium is a vector of capital prices $\{\Psi_t^T, \Psi_t^N\}$, bond prices $\{q_t, q_t^*\}$, factor prices $\{R_t^T, R_t^N, w_t\}$, good prices $\{p_t^T, p_t^N, p_t^{T*}\}$, nominal exchange rates $\{s_t\}$, asset and consumption choices for households $\{a_{t+1}, a_{t+1}^*, C_t^N, C_t^T\}$, portfolio choices for bankers $\{k_{t+1}^T, k_{t+1}^N, b_{t+1}, b_{t+1}^*\}$, and investment choices for the capital good producers $\{i_t\}$, such that: **(i)** the choices of households, banks, and capital good producers solve their respective decision problem; **(ii)** all domestic markets clear; **(iii)** $q_t^*$ and $p_t^{T*}$ are determined abroad; **(iv)** the law of one price holds; **(v)** and the price level $P_t$ is constant.

## 2.3 Discussion of assumptions

Let us briefly discuss the main simplifying assumptions made in the model.

First, we are making simplifying assumptions on the non-tradable sector: the non-tradable sector does not employ labor, the $N$ capital is in fixed supply, and the $N$ capital cannot be used as collateral. The first assumption simplifies the analysis as we don't have to determine how labor is allocated among the two sectors and the real wage will be immediately derived from the level of capital invested in the $T$ sector. The other two assumptions are convenient since they allow us to characterize all remaining equilibrium prices and quantities without solving for the price of the $N$ capital $\Psi_t^N$.

Second, we are assuming that foreign investors cannot trade domestic-currency claims. We could have a less stark form of segmentation, by allowing foreign investors to accept domestic-currency claims subject to some friction, as long as we don't have an infinitely elastic demand for domestic claims. Ruling out foreign investors' participation altogether is just a useful simplification.

Third, we are representing monetary policy purely as a choice of numeraire and we are assuming the monetary authority can commit to perfect price stability. This is a simple way to model a floating exchange rate regime, where nominal exchange rate volatility is not driven by inflationary choices of the central bank. As we shall see, our main mechanism is based on the relation between the country's real wealth and the real exchange rate, so it is useful to mute other, policy-driven channels of exchange rate instability. In Section X, we will explore a version of the model that captures a pegged exchange rate regime, to study how our mechanism plays out in that case.

## 2.4 Roadmap

In the coming sections we analyze the model in two steps, moving backwards in time. First, we look at the economy starting in period 1, taking as given the capital stocks and the financial claims inherited from period 0, and analyze how the equilibrium is determined in the last two periods. We call this a "continuation equilibrium", and we show that for a subset of initial conditions there can be multiple continuation equilibria in the model. Next, we go back to period $t = 0$ and study the equilibrium determination of investment and financial claims in that period. We then show examples in which equilibrium choices ex ante can lead to equilibrium multiplicity in the following periods.

# 3 Continuation Equilibria

In this section, we focus on the economy at date $t = 1$, taking as given the balance sheets inherited from the past—i.e., taking as given the values of the state variables $k_1^T, a_1, b_1, a_1^*, b_1^*$—and we explore the possibility of multiple equilibria. For now, we do not need to introduce explicitly the sunspot variable $\eta$ and we simply ask whether, for given initial conditions, multiple equilibria are possible.

Our objective is to study continuation equilibria using a simple diagram that plots the demand of tradable capital by banks $k_2^T$ and the supply of tradable capital by capital producing firms as functions of the price of capital in terms of tradables,

$$\psi_1^T \equiv \frac{\Psi_1^T}{p_1^T}.$$

The supply of capital is easily derived from the optimization problem of capital producing firms (8). Rearranging the first order condition gives

$$i_1 = \frac{1}{\phi_1}\left(\psi_1^T - \phi_0\right)^{1/\eta}, \tag{12}$$

if $\psi_1^T \geq \phi_0$. If $\psi_1^T < \phi_0$, the solution is at the corner $i_1 = 0$. The supply of capital goods is then $k_1^T + i_1$.

To derive the demand curve, we need first to obtain a relation between the the equilibrium exchange rate and the price of capital. This relation will be used to determine the value of the banks' liabilities in foreign currency.

## 3.1 Equilibrium exchange rate

The following lemma derives some useful properties of a continuation equilibrium from the household Euler equations and the market clearing condition for non-tradable goods at $t = 1, 2$,

$$(1 - \omega)\frac{P_t C_t}{p_t^N} = y^N. \tag{13}$$

**Lemma 1.** *All continuation equilibria satisfy the following conditions:*

i. *consumption is constant over time,* $C_1 = C_2$*;*

ii. *the relative price of non-tradable goods in terms of tradables and the prices of tradable and non-tradable goods in terms of the CPI are all constant over time,*

$$\frac{p_1^N}{p_1^T} = \frac{p_2^N}{p_2^T}, \quad \frac{p_1^T}{P_1} = \frac{p_2^T}{P_2}, \quad \frac{p_1^N}{P_1} = \frac{p_2^N}{P_2};$$

iii. *the domestic real interest rate is*

$$\frac{1}{q_1}\frac{P_1}{P_2} = \frac{1}{\beta}.$$

The logic of the lemma is simple. Tradable consumption is perfectly smoothed by trading with foreign investors. Non-tradable consumption is constant because the non-tradable endowment is constant. So the relative price of tradables and non-tradables must be constant. The result for tradable and non-tradable prices in CPI terms follows from the definition of the CPI. The result for the domestic real interest rate comes from the Euler equation for domestic bonds.

Substituting constant consumption, constant relative prices, and the real bond prices $q_1^* = q_1 P_2 / P_1 = \beta$ in the household intertemporal budget constraint, we get:

$$C_1 = \frac{1}{1+\beta}\left(\frac{w_1}{P_1} + \beta\frac{w_2}{P_2} + (1+\beta)\frac{p_1^N}{P_1}e^N + \frac{\Pi_1}{P_1} + \frac{a_1 + s_1 a_1^*}{P_1}\right). \tag{14}$$

Consumption is proportional to household total wealth, which is equal to the present value of labor income and endowments, plus the profits of the capital producers, and the financial assets inherited from period 0. We can then substitute for real wages and for the profits of the capital producing firms, expressing both in terms of tradables. Real wages in tradables are equal to the marginal product of labor,

$$\frac{w_t}{p_t^T} = (1-\alpha)(k_t^T)^\alpha.$$

Real profits in tradables can be rewritten defining the profit function $\pi(\psi_1^T)$ as follows

$$\frac{\Pi_1}{p_1^T} = \pi(\psi_1^T) = \frac{\eta}{1+\eta}\frac{1}{\phi_1^\eta}\left(\psi_t^T - \phi_0\right)^{\frac{1+\eta}{\eta}},$$

if $\psi_1^T \geq \phi_0$ and $\pi(\psi_1^T) = 0$ otherwise. This expression comes from solving (8). The market clearing condition for non-tradables (13) can then be rewritten, after rearranging, as

$$\frac{1-\omega}{1+\beta}\left\{\frac{p_1^T}{p_1^N}\left[(1-\alpha)\left(k_1^T\right)^\alpha+\beta(1-\alpha)\left(k_2^T\right)^\alpha+\pi(\psi_1^T)+\varepsilon a_1^*\right]+\left(\frac{p_1^T}{p_1^N}\right)^\omega a_1\right\}+ \\ +(1-\omega)\,e^N=y^N, \quad (15)$$

where the law of one price was used to substitute $s_1=\varepsilon p_1^T$ and the monetary rule was used to express $p_1^N$ in terms of $p_1^N/p_1^T$.[4]

Equation (15) identifies the first crucial mechanism in our model: more capital invested in the tradable sector $k_2^T$ increases labor productivity and real wages; this shifts up the demand for non-tradables and leads to a real exchange rate appreciation (a higher value of $p_1^N/p_1^T$). This is just a version of the Balassa-Samuelson effect. But it plays an important role here because the capital invested in the tradable sector depends on the health of the banks' balance sheets which, in turn, depends on the real exchange rate. This mechanism creates a feedback between banks' balance sheets and real exchange rates.

From now on, we restrict attention to continuation equilibria with $a_1^*\geq 0$ to ensure that the demand for non-tradable goods in (15) is everywhere decreasing and there is a unique value of $p_1^N/p_1^T$ that solves (15). Given $p_1^N/p_1^T$, we can fully determine household consumption, goods prices, and bond prices in the continuation equilibrium. These results are summarized in Lemma 2 below. The lemma uses the supply of capital (12) and equilibrium in the capital markets to obtain a relation between $p_1^N/p_1^T$ and $\psi_1^T$ and show that the relation is non-decreasing.

**Lemma 2.** *Given a vector of initial conditions $(k_1^T,a_1,b_1,a_1^*,b_1^*)$ with $a_1^*\geq 0$, a realization of $\varepsilon$, and a value of $\psi_1^T$, there exists a unique vector of prices and quantities*

$$(q_1,s_1,s_2,p_1^T,p_2^T,p_1^N,p_2^N,C_1,C_2)$$

*consistent with a continuation equilibrium. Let*

$$\frac{p_1^N}{p_1^T}=h(\psi_1^T) \quad (16)$$

*be the relation between $\psi_1^T$ and the relative price of non-tradables $p_1^N/p_1^T$ from the mapping above. The function $h$ is non-decreasing in $\psi_1^T$.*

[4] A constant CPI requires $(p_t^T)^\omega(p_t^N)^{1-\omega}=1$ which yields $p_t^N=(p_t^N/p_t^T)^\omega$.

## 3.2 The demand of capital goods

Turning to the demand of capital goods, we need to use bankers' optimality conditions. The rate of return to tradable capital is $R_2^T/\Psi_1^T$ because capital costs $\Psi_1^T$, earns the dividend $R_2^T$ at $t = 2$, and then fully depreciates. The interest rate at which banks can borrow is $1/q_1$. Comparing these two rates of return, two cases are possible in equilibrium:

1. *Unconstrained banks.* The marginal gain from borrowing an extra unit of domestic currency and investing it in tradable capital is zero and the collateral constraint is slack,
$$\frac{R_2^T}{\Psi_1^T} = \frac{1}{q_1}, \quad (1-\theta)\Psi_1^T k_2^T \leq [(\Psi_1^T + R_1^T)k_1^T + p_1^N k^N - b_1 - s_1 b_1^*].$$

2. *Constrained banks.* The marginal gain from borrowing an extra unit of domestic currency and investing it in tradable capital is positive and the collateral constraint is binding,
$$\frac{R_2^T}{\Psi_1^T} > \frac{1}{q_1}, \quad (1-\theta)\Psi_1^T k_2^T = [(\Psi_1^T + R_1^T)k_1^T + p_1^N k^N - b_1 - s_1 b_1^*].$$

In the conditions above, we used equations (4)-(6) and the equilibrium in the non-tradable capital market $k_t^N = k^N$ to write the collateral constraint compactly. These set of conditions implicitly define a demand schedule for capital goods.

In the unconstrained case, we can substitute the rental rate from (3) and $q_1 = \beta$, to obtain the unconstrained demand for capital:

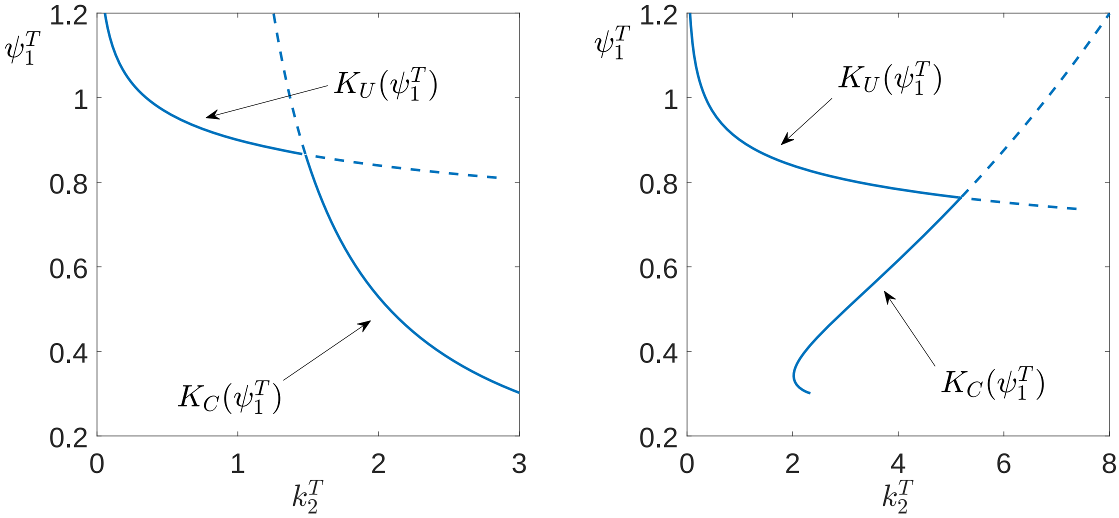
$$K_U\left(\psi_1^T\right) = \left(\frac{\alpha\beta}{\psi_1^T}\right)^{\frac{1}{1-\alpha}}. \tag{17}$$

This relation defines a downward demand schedule for capital goods.

In the constrained case, we can rewrite the binding collateral constraint in terms of tradables and obtain the constrained demand for capital:

$$K_C\left(\psi_1^T\right) = \frac{1}{(1-\theta)\,\psi_1^T}\left[\psi_1^T k_1^T + \alpha\left(k_1^T\right)^{\alpha} + h(\psi_1^T)k^N - \left(h(\psi_1^T)\right)^{1-\omega} b_1 - \varepsilon b_1^*\right], \tag{18}$$

where we used the monetary rule to obtain $p_1^T = (p_1^T/p_1^N)^{1-\omega}$. Differently from the unconstrained portion, the demand for capital by a constrained banker is not necessarily downward sloping. When the price of capital increases we have two effects. First, capital




(a) Downward sloping constrained demand

(b) Uward sloping constrained demand

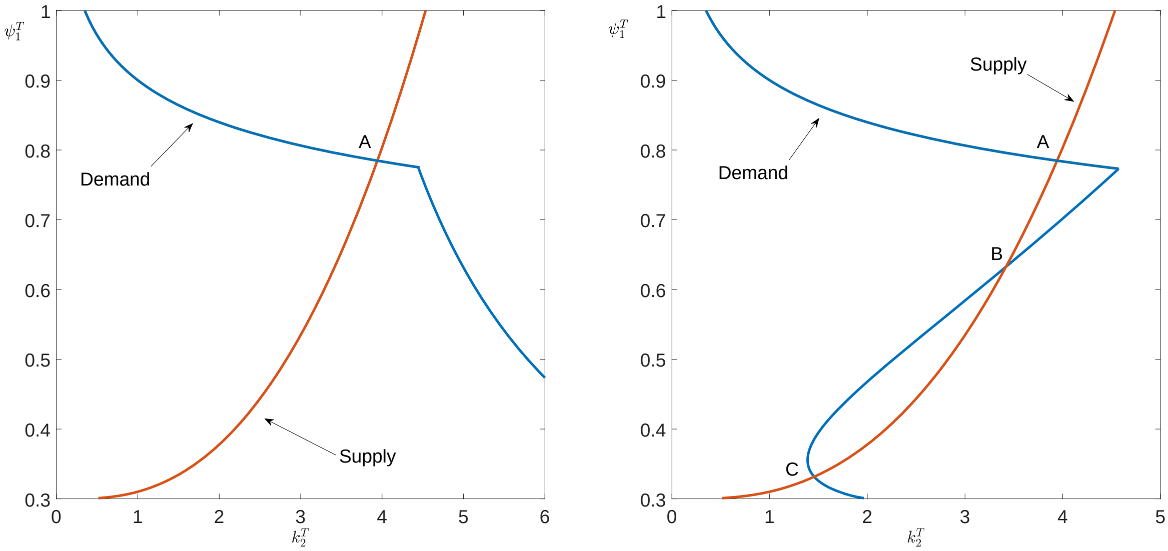
Figure 1: The demand for capital goods

becomes more expensive from the point of view of bankers. Second, the increase in the price of capital affects the value of assets and liabilities of the bankers, influencing their net worth. This variation in bankers' net worth influence capital demand because of the binding collateral constraint. While the first effect unambiguously depresses capital demand, the net worth effect might in principle lead the bankers to demand more capital goods, and to upward sloping portions of the demand schedule.

The demand curve for capital is obtained by taking the lowest value between the constrained and the unconstrained demand at each price $\psi_1^T$,

$$K_D\left(\psi_1^T\right) = \min\left\{K_U\left(\psi_1^T\right), K_C\left(\psi_1^T\right)\right\}.$$

Figure 1 presents some numerical examples for the demand curve of capital goods. In panel (a) we describe a situation where the demand of capital in the constrained region is everywhere downward sloping. As a result, the minimum between the two schedules, represented by the solid line in the figure, is everywhere downward sloping. In panel (b), instead, the constrained portion of the demand curve is upward sloping. This implies that the demand curve is, in some region, upward sloping. We now discuss the conditions under which this might happen, and show that these features of capital demand schedule might lead to multiple continuation equilibria.




(a) Unique equilibrium in the capital market



(b) Multiple equilibria in the capital market

Figure 2: Equilibria in the capital market

### 3.3 Equilibrium in the capital goods market

We are now ready to combine the supply and demand relations to determine the equilibrium price $\psi_1^T$. Lemma 2 then gives all other equilibrium variables. First, we establish a sufficient condition for the existence of a continuation equilibrium.

**Proposition 1.** *Assume the following inequalities are satisfied:*

$$\alpha\left(k_1^T\right)^{\alpha-1} > \phi_0, \quad \alpha\left(k_1^T\right)^{\alpha} + h(\phi_0)k^N + \theta\phi_0 k_1^T > (h(\phi_0))^{1-\omega}b_1 + \bar{\varepsilon}b_1^*. \tag{A1}$$

*Then there exists a continuation equilibrium with $\psi_1^T > \phi_0$ for all realizations of $\varepsilon$.*

From now on, we will focus on economies that satisfy (A1) and restrict attention to continuation equilibria with $\psi_1^T > \phi_0$. The main advantage of these restrictions is that we do not need to worry about the possibility that banks have negative net worth and so we don't need to specify how banks' bankruptcy is resolved for bond holders. Of course, individual banks' bankruptcies are commonplace in financial crises, but since here we are capturing the entire financial system in a single representative bank, it is easier to model a crisis as a severe reduction in the total net worth of the financial sector.

In Figure 2 we plot demand and supply for two numerical examples. In panel (a) there is an example of uniqueness of equilibrium in the capital market where bankers are unconstrained, represented by point A. In this example, the demand schedule is downward sloping everywhere, and this is a sufficient condition for uniqueness of the equilibrium in the capital market. Panel (b), instead, depicts an example of multiplicity. We can see that there are three equilibria in the capital markets, corresponding to points A, B and C. Point A is an equilibrium in which banks are unconstrained. Because the unconstrained demand curve is downward sloping, there can be at most one equilibrium of this type. Points B and C, instead, are equilibria in which the collateral constraint is binding.

What economic mechanisms expose the economy to equilibrium multiplicity? A necessary, although not sufficient, condition is that the demand for capital is upward sloping in some region. We have seen earlier that this might occur in the model only when bankers' net worth responds positively to an increase in the price of capital goods. When the net-worth elasticity to the asset price is sufficiently positive, the economy might experience self-fulfilling financial crises where agents expect low asset prices, the financial wealth of bankers plummets, and their low demand for capital validate the initial pessimistic expectation.

If we interpret a financial crisis as a switch from a high price to a low price equilibrium in the situation depicted in panel (b) of Figure 2, we can easily obtain a number of predictions about the country's consumption, investment, current account and welfare.

**Proposition 2.** *When switching from a high $\psi_1^T$ to a low $\psi_1^T$ equilibrium, the following happens:*

- ***i.*** *Domestic asset prices and the exchange rate drop;*
- ***ii.*** *Consumption and investment are lower;*
- ***iii.*** *The current account balance improves;*
- ***iv.*** *The utility of both consumers and bankers is lower.*

The improvement in the current account shows that the domestic financial crisis is associated to a capital flight from the entire country. The capital flight has a double nature: the contraction in investment is driven by the binding collateral constraints of the banks, while the contraction in consumption is driven by the reduction in the country's wealth due to lower future wages. The recent literature has split between papers that emphasize uncertainty about future income growth (Aguiar and Gopinath (2007)) and binding financial constraints (Mendoza (2010)) as sources of fluctuations in emerging markets.[5] Here both channels are operative. The interesting observation is that even though some agents in the economy are not forced to borrow less from the rest of the world due to the financial contraction, the spillovers from the financially constrained agents induce them to move in the same direction. That is, unconstrained agents act as amplifiers instead of

[5] See Chang and Fernández (2013) for an empirical comparison of the two channels.

shock-absorbers.[6]

The proposition also shows that the equilibria are Pareto ranked. Notice that the foreign investors' supply of funds is perfectly elastic at the rate $1/q_t^*$, so their welfare is unaffected by the equilibrium selected. On the other hand, both consumers and bankers are hurt by a low asset price equilibrium. On the consumers' side, welfare is lower due to lower capital accumulation and hence lower real wages. On the banks' side, the effects are subtler, as the rate of return on banks' net worth is actually higher in a low $\psi_1^T$ equilibrium, because asset prices are lower and future rental rates are higher (due to lower capital). However, low asset prices reduce the banks' initial net worth. The proof of the proposition shows that this second effect always dominate.

The slope of the demand schedule in the constrained case is a function of the assets and liabilities position of bankers, and of model parameters. To further understand what set of initial condition exposes our economy to equilibrium multiplicity we can differentiate (18) and rearranging we can write the elasticity of the constrained demand curve as:

$$K_C'\left(\psi_1^T\right)\frac{\psi_1^T}{k_2^T} = -\frac{\alpha\left(k_1^T\right)^{\alpha} + \frac{p_1^N}{p_1^T}k^N - \left(\frac{p_1^N}{p_1^T}\right)^{1-\omega} b_1 - \varepsilon b_1^*}{\left(1-\theta\right)\psi_1^T k_2^T} + \eta_H \frac{\frac{p_1^N}{p_1^T}k^N - \left(1-\omega\right)\left(\frac{p_1^N}{p_1^T}\right)^{1-\omega} b_1}{\left(1-\theta\right)\psi_1^T k_2^T}.$$

where

$$\eta_H = \frac{H'\left(\psi_1^T\right)}{H\left(\psi_1^T\right)}\psi_1^T,$$

is the elasticity of the real exchange rate to the capital price. Two expressions determine the sign of this elasticity. First, the expression

$$\alpha\left(k_1^T\right)^{\alpha} + \frac{p_1^N}{p_1^T}k^N - \left(\frac{p_1^N}{p_1^T}\right)^{1-\omega} b_1 - \varepsilon b_1^*$$

captures the financing needs of the banks: if the expression is negative it means that the current revenue from tradable and non-tradable capital is not sufficient to cover bond repayments due in domestic and foreign currency. These financing needs are covered by fresh borrowing in excess of new investment, that is, by the difference

$$\psi_1^T[\theta k_2^T - (k_2^T - k_1^T)],$$

[6]The fact that the unconstrained agents here are identified with the household sector is clearly due to specific modeling assumptions. It would be easy to extend the model to a case where constrained and unconstrained agents are present both in the household and in the business sector.

where fresh borrowing is equal to $\theta k_2^T$, as the bank is constrained. Notice that for given financing needs, a higher price of capital translates into a lower value of the expression in square brackets, and thus into higher new investment, i.e., a higher demand for capital. This is the first mechanism that tends to make the demand for capital upward sloping. This mechanism is well known from closed economy financial accelerator models as, e.g., Lorenzoni (2008), and is used to generate equilibrium multiplicity in Gai, Kapadia, Millard, and Perez (2008) and Gertler and Kiyotaki (2015). The novel effect here is captured by the expression

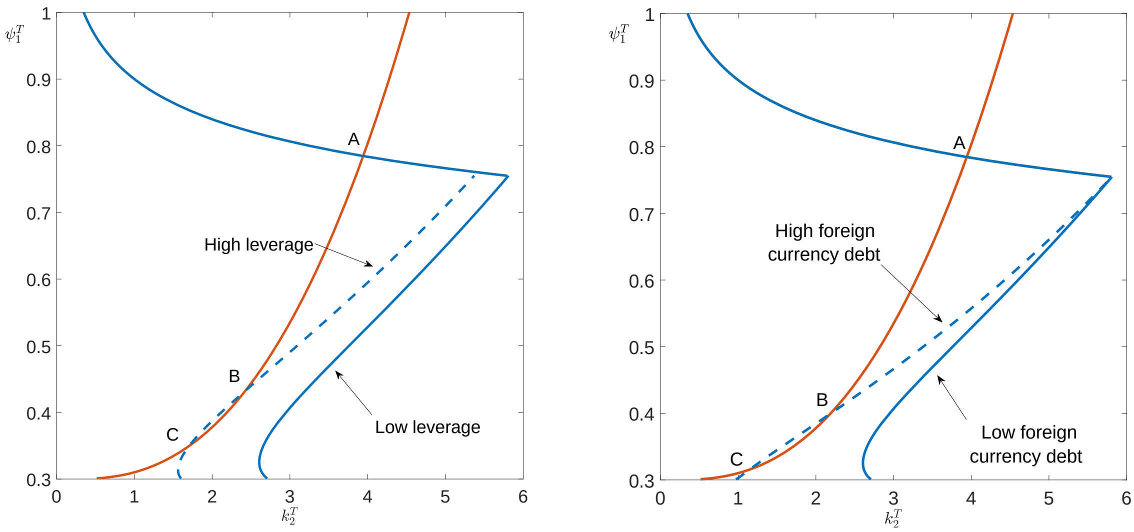
$$\frac{p_1^N}{p_1^T}k^N - (1-\omega)\left(\frac{p_1^N}{p_1^T}\right)^{1-\omega} b_1, \tag{19}$$

which captures the effect of an exchange rate appreciation on the balance sheet of the banks. When this expression is positive, banks have a currency mismatch and an appreciation improves their balance sheet. The sign and magnitude of this expression depends on the currency composition of the bank's liabilities.

Figure 3 explores the role of the bankers balance sheet in determining whether the economy is exposed or not to multiple equilibria. In panel (a) we consider an increase in the leverage of bankers, holding constant their currency exposure. This can be achieved with an increase in $b^\star$, as this shift increases the debt that bankers have to pay in period 1 without altering the expression in equation (19). Ceteris paribus, an increase in leverage raises the sensitivity of net worth to asset prices, increasing the elasticity of the demand function in the constrained portion. Comparing the solid and dotted line, we can verify that an increase in bankers' leverage makes the economy more prone to multiple equilibria.

A similar result is obtained when considering an increase in the currency mismatch in the bankers' balance sheet, holding their leverage constant. Panel (b) of Figure 3 shows how the demand for capital changes when we increase their foreign currency debt $b^\star$, and offset such increase by a corresponding reduction in debt denominated in domestic currency. Because of the positive relation between the real exchange rate and the price of capital, this shift increases the elasticity of the demand function.

The economy might thus exhibit confidence crises characterized by sudden drops in asset prices, exchange rate depreciations, and capital flights, and these are more likely to arise when the financial sector is sufficiently levered and exposed to currency mismatches. Those balance sheet positions, however, are determined endogenously at $t = 0$. We now turn to the households and bankers decisions at $t = 0$ and discuss under what conditions these choices expose the economy to multiple continuation equilibria.




(a) Financial leverage (b) Currency mismatches

Figure 3: The role of financial leverage and currency mismatches

# 4 The determination of leverage and currency mismatches

We now go back to $t = 0$ and illustrate the economic forces that determine the leverage and the currency mismatches of the financial sector.

We start by characterizing a class of equilibria of the model where the collateral constraint of the bankers is slack for every $t$. In those equilibria, households have an incentive to denominate their savings in local currencies, and domestic banks can thus borrow in local currency to finance their operations. The fact that local currency asset markets are well developed limits the currency mismatches of the financial sector and guarantees that the economy is not exposed to confidence crises at $t = 1$. We will label these the "good equilibria".

We next present numerical examples of equilibria where the economy is exposed to equilibrium multiplicity at $t = 1$. In those equilibria, households at $t = 0$ have an incentive to denominate their savings in foreign currency, because foreign currency assets act as an insurance when the bad equilibrium is played at $t = 1$. Bankers thus need to issue foreign currency liabilities, and the resulting currency mismatches expose the economy to confidence crises at $t = 1$. These will be the "bad equilibria".

Before continuing, though, we must adopt a rule for selecting among continuation equilibria when we have more than one, as agents at $t = 0$ need to form expectations over future outcomes. First, we consider only stable continuation equilibria, and we will assume, without loss of generality, that there are at most two stable continuation equilibria. As the equilibria are ranked in terms of welfare, we will refer to the one with higher investment and asset prices as the “good" continuation equilibrium, and the other one as the “bad" continuation equilibrium. Going back to Figure 2, for example, these would correspond to point A and point C respectively. When multiple equilibria are possible at $t = 1$, we assume that agents coordinate to the bad equilibrium if $\eta \leq \pi$. Hence, $\pi$ is the probability at $t = 0$ that the economy plays the bad continuation equilibrium from $t = 1$ onward when the latter is possible.

## 4.1 The good equilibrium

To focus on the role that the currency denomination of assets and liabilities plays in the model, we further simplify our analysis by assuming that at $t = 0$ capital good producing firms are not operative. We can then use equation (5) and (6), along with market clearing, to write the bankers' budget constraint as

$$q_0 b_1 + s_0 q_0^\star b_1^\star = R_0^T k_0^T + R_0^N k_0^N - (b_0 + s_0 b_0^\star). \tag{20}$$

Thus, total liabilities for the bankers are exogenously given, and their only choice is their currency denomination. The households at $t = 0$ face a portfolio problem and decides how much to consume and save, an in which currency to denominate their savings.

We now state a result that characterizes an interesting class of equilibria.

**Proposition 3.** *Suppose that in an equilibrium of the model the collateral constraint of the bankers is slack in period 0, and almost surely in period 1. Then, the equilibrium has the following properties*

- ***i.*** *The equilibrium in the capital market at t $= 1$ is unique, with $(k_2^{T^{eq}}, \psi_1^{T^{eq}})$ being independent on the realization of $p_1^{T\star}$;*
- ***ii.*** *Households at $t = 0$ set $a_1^\star = 0$, and they achieve perfect consumption smoothing, $C_0 = C_1 = C_2$;*
- ***iii.*** *The real exchange rate is constant over time, and equal to $\frac{p^N}{p^T} = H(\psi_1^{T^{eq}})$;*
- ***iv.*** *The balance sheet of the bankers chosen at t $= 0$ equals*

$$b_1 = \frac{1}{\beta}\left[\left(\frac{p^N}{p^T}\right)^{\omega-1}(1-\alpha)(k_0^T)^\alpha + \left(\frac{p^N}{p^T}\right)^{\omega} e^N + a_0 + \left(\frac{p^N}{p^T}\right) a_0^\star - C_0\right], \tag{21}$$

$$b_1^\star = \frac{1}{\beta\left(\frac{p^N}{p^T}\right)}\left[\left(\frac{p^N}{p^T}\right)^{\omega-1}\alpha(k_0^T)^\alpha + \left(\frac{p^N}{p^T}\right)^{\omega} k^N - \left(b_0 + \left(\frac{p^N}{p^T}\right) b_0^\star\right) - b_1\right]. \tag{22}$$

Because the bankers are unconstrained in period 1 by assumption, the equilibrium in the capital market at $t = 1$ is unique. The households decision not to save in foreign currency implies, by equation (15), that the real exchange rate does not depend on the realization of $\varepsilon$. As a result, households wealth is independent on the realization of $\varepsilon$ in period 1, and the households perfectly smooth their consumption over time. Because the consumption and the endowment of non-tradables is constant over time, so does the real exchange rate. The positions of the bankers in equation (21) and (22) are obtained from the market clearing condition $a_1 = b_1$ and the bankers' balance sheet (20).[7]

An important characteristic of the good equilibrium is that households' saving and borrowing is conducted only in domestic currency. To understand this property and its implications, it is instructive to analyze more at depth the portfolio choices of the household. Rearranging the households' Euler equations for local and foreign currency bonds we obtain

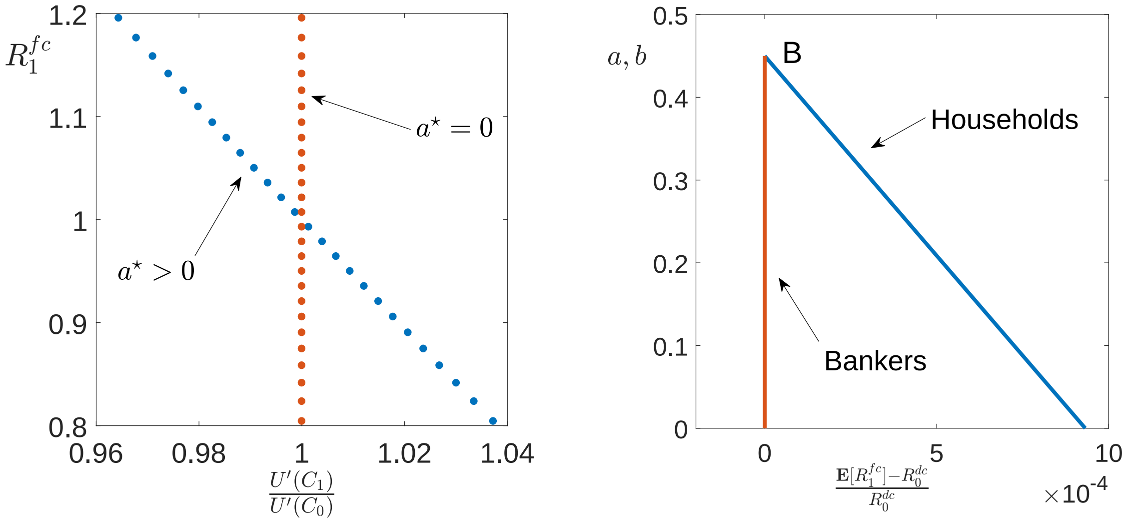
$$\frac{\mathbb{E}_0[R_1^{\text{fc}}] - R_0^{\text{dc}}}{R_0^{\text{dc}}} = -\text{Cov}_0\left[R_1^{\text{fc}}, \beta\frac{U'(C_1)}{U'(C_0)}\right], \tag{23}$$

where $R_1^{\text{fc}} = s_1/(s_0 q_0^*)$ are the $t = 1$ realized returns for bonds denominated in foreign currency, and $R_0^{\text{dc}} = 1/q_0$ are the returns for bonds denominated in domestic currency.

We can use this relation to study the optimal portfolio problem of an household that takes as given the prices of the good equilibrium. Figure 4 plots some key feature of this decision problem for a numerical example. In panel (a) we plot the joint distribution of $U'(C_1)/U'(C_0)$ and the realized return on foreign currency bonds, $R_1^{\text{fc}}$ for different $a_1^\star$ choices for the household at $t = 0$ in a numerical example of our model. Each point represents a different realization of $\varepsilon$ at $t = 1$ and the associated marginal rate of substitution of the household. When $a^\star > 0$, a nominal exchange rate depreciation reduces the returns on foreign currency bonds, and it implies a reduction in households wealth and consumption. When $a^\star = 0$, instead, the two are independent because the household is not exposed to fluctuations in the nominal exchange rate.

In panel (b) of the figure we plot the demand schedule for domestic currency bonds

[7] Equations (21) and (22) can be used to obtain the set of initial conditions $(a_0, a_0^\star, b_0, b_0^\star)$ that guarantees the existence of this good equilibrium. One needs to make sure that the collateral constraint is slack in period 0, and it is slack in period 1 almost surely.




(a) Households' sdf and $R_1^{\text{fc}}$

(b) The local currency asset market

Figure 4: The local currency asset market in the good equilibrium

as a function of the expected excess returns on foreign currency bonds. This function is downward sloping: the higher the expected excess returns that foreign currency bonds offer, the more household save in foreign currency and reduce their demand for local currency bonds. Point $B$ in the figure denotes the optimal asset holdings in domestic currency when expected returns are equalized. From equation (23) and the discussion of panel (a) of Figure 4, we know that at $B$ the households do not save in foreign currency, $a^\star = 0$.

Thus, the households save in foreign currency only when they expect some positive excess returns. This is, however, not possible in the equilibrium described in Proposition 3. To see why, we can rearrange the bankers' Euler equations and obtain

$$\frac{\mathbb{E}_0[R_1^{\text{fc}}] - R_0^{\text{dc}}}{R_0^{\text{dc}}} = -\text{Cov}_0\left[R_1^{\text{fc}}, \frac{\lambda_1}{\lambda_0}\right], \tag{24}$$

where $\lambda_t$ is the banks' marginal value of wealth at time $t$.

In equilibrium, equation (23) and (24) must hold at the same time. Because we are in the good equilibrium, the collateral constraint of the bankers does not bind, and the marginal value of wealth is constant. Therefore, the demand for domestic currency debt by the bankers is perfectly inelastic, see the dotted line in Figure 4. Because of the market clearing condition $a_1 = b_1$, the equilibrium is achieved at point B, where $a_1^\star = 0$.

## 4.2 Bad equilibria

[to be added]

# 5 Applications

## 5.1 Lending of Last Resort

We consider a government that at $t = 0$ levies a lump sum tax $\tau_0$ on the households. The government can use these resources to accumulate reserves in foreign and in domestic currency,

$$q_0 h_1 + s_0 q_0^* h_1^* = \tau_0.$$

To make the problem interesting, we assume that the government faces some upper bound in the taxes that it can collect from households, $\tau_0 \leq \overline{\tau}$.

The reserves can be used in period 1 to conduct operations of lending of last resort. Specifically, we assume that the government can commit to a demand schedule for capital goods at $t = 1$.[8] We denote by $z_1(\Psi^T)$ the demand of capital goods by the government given the price $\Psi^T$,

$$\Psi_1^T z_1(\Psi_1^T) \leq h_1 + s_1 h_1^*.$$

The government holds the capital stock for one period and operates an alternative linear technology that returns $\underline{A} z_1$ units of tradable goods in period 2. These resources are then rebated back to the consumers as a transfer in period 2.

These operations can potentially eliminate the bad continuation equilibria at $t = 1$. By adding to private capital demand, the government can curb the fall in asset prices and the depreciation of the exchange rate that characterizes the bad equilibrium, limiting in this fashion the decline in the banks' net worth that sustains the bad equilibria. However, to be effective, they must also be credible. That is, the government needs to have enough fiscal resources to shift out the demand for capital and eliminate the bad equilibria.

In order to formalize this last point, we can define $Z(\Psi^T)$ to be the difference between the supply of capital and its private demand when the price is $\Psi^T$. The government can credibly eliminate the bad continuation equilibria if

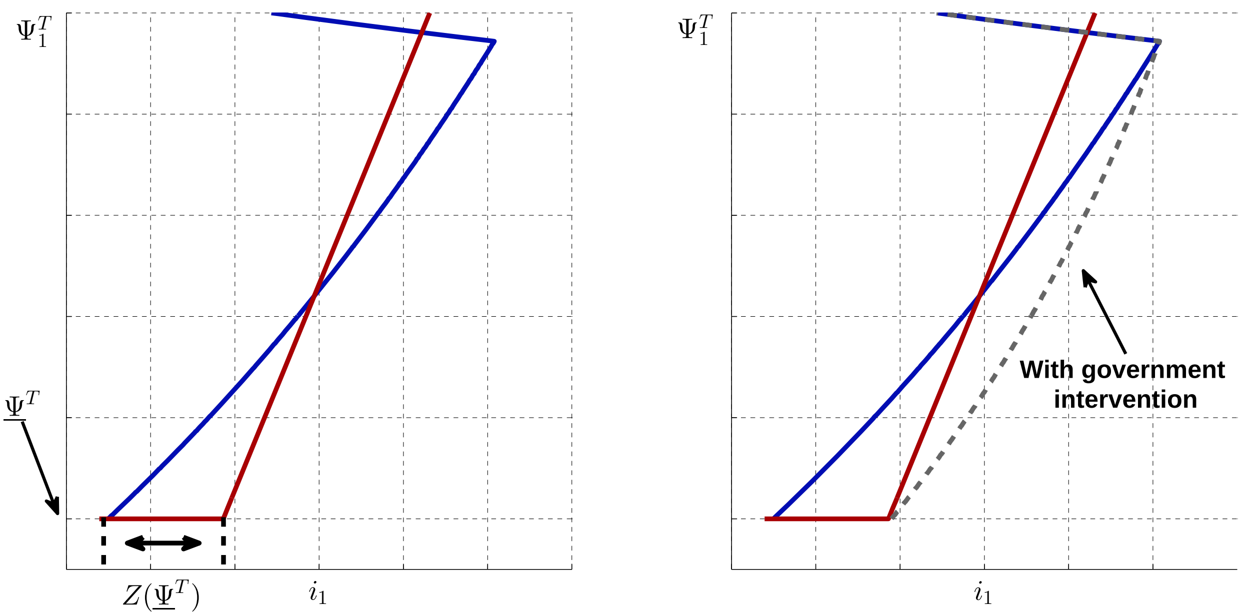
$$\Psi^T Z(\Psi^T) \leq h_1 + s_1 h_1^*,$$

[8] Equivalently, we could have modeled operation of lending of last resort as a direct loan that the government extends to the bank at some penalty rate.

for all $\Psi^T$ that are below the good equilibrium price.

Figure 5 clarifies this discussion. In the left panel we can see that $Z(\underline{\Psi}_1^T)$ is the excess supply of capital in the bad equilibrium of the model. If the government has enough reserves to purchase $Z(\underline{\Psi}_1^T)$ at the market prices $\underline{\Psi}_1^T$, it could prevent the confidence crisis in period 1: by committing to purchase capital goods, the government can shift the demand schedule to the right and implement uniquely the good equilibrium, see the right panel in Figure 5 for an example of a successful operation. If the government does not have enough fiscal resources, however, the bad equilibrium is unavoidable and lending of last resort is less effective.

Figure 5: **Effective lending of last resort**




While this section of the paper is still largely a work in progress, we wish to point out two results that naturally emerge in our environment. First, the possibility of the capital flights makes it harder for the government to credibly avert a confidence crisis in period 1 through lending of last resort. As we have seen earlier, the presence of foreign currency liabilities for banks flattens the capital demand schedule in its upward sloping portion, see Figure **??**. *Ceteris paribus*, this implies an increase in the amount of fiscal resources necessary to avert the bad equilibrium, $\underline{\Psi}_1^T Z(\underline{\Psi}_1^T)$. This is the sense in which an open economy with flexible exchange rates makes effective lending of last resort more difficult.

Second, the model provides a rationale for the ex-ante accumulation of foreign reserves. Foreign reserves have the property of appreciating in the bad equilibrium. As such, they increase the fiscal resources that the government has at its disposal to avert the bad equilibrium, and to effectively conduct lending of last resort. This last result can rationalize the findings of Obstfeld, Shambaugh, and Taylor (2010) and Aizenman and Lee (2007) that the large accumulation of foreign reserves among emerging markets over the last 20 years is strongly associated to the extent of financial openness and financial depth of these economies, and the informal arguments brought in the literature that such accumulation was done in order to facilitate effective lending of last resort.

# 6 Conclusion

[to be completed]

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# Appendix to "A Model of Financial Crises in Open Economies"

by Luigi Bocola and Guido Lorenzoni

# A Proofs

## A.1 Proof of Lemma 1

Since there is no uncertainty left after period 1 the Euler equations for domestic and foreign bonds are

$$q_1 U'(C_1) = \frac{P_1}{P_2}\beta U'(C_2), \tag{A.1}$$

$$q_1^* U'(C_1) = \frac{s_2}{s_1}\frac{P_1}{P_2}\beta U'(C_2). \tag{A.2}$$

Using the market clearing condition in the non-tradable market (13), the definition of CPI (2), and the constant endowment of non-tradables, we get

$$(1-\omega)\left(\frac{p_1^T}{p_1^N}\right)^{\omega} C_1 = (1-\omega)\left(\frac{p_2^T}{p_2^N}\right)^{\omega} C_2$$

Using $q^* = \beta$, the law of one price (9), the assumption of constant foreign prices, and the definition of CPI (2), the Euler equation for foreign bonds (A.2) can be rewritten as

$$U'\left(C_1\right) = \frac{\left(p_1^N/p_1^T\right)^{1-\omega}}{\left(p_2^N/p_2^T\right)^{1-\omega}} U'\left(C_2\right).$$

Combining these two conditions yields

$$\left(U'\left(C_1\right)\right)^{-\frac{\omega}{1-\omega}} C_1 = \left(U'\left(C_2\right)\right)^{-\frac{\omega}{1-\omega}} C_2,$$

which, given the concavity of $U$, implies $C_1 = C_2$. The preceding equation implies a constant relative price of non-tradables. The goods prices in terms of CPI are monotone transformations of $p_t^N/p_t^T$, so they are also constant. The domestic bond price comes from (A.1).

### A.2 Proof of Lemma 2

All the steps are in the text except for the comparative statics with respect to $psi_1^T$, which is straightforward.