Robust Mechanisms for the Regulation of Bank Risk

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Abstract

This paper aims at designing mechanisms for the regulation of bank risk that are robust to large misspecifications of the regulators information regarding the bank's assets. Assuming that banks can (on average) discern the true level of risk of the other regulated banks, we construct a robust mechanism that allows the regulator to bound the worst case probability of bank failure by any arbitrary amount. Importantly, we show that any informationally robust mechanism that meets the regulators budget constraint must necessarily require banks to issue subordinated debt to other regulated banks *and* must guarantee that the probability of joint failure between at least two of the banks is strictly less than 1. We show that the only way the regulator can achieve the latter objective is by providing a single bank with an explicit guarantee against losses in the event that another bank fails and how, when coupled with a minimum subordinated debt requirement and interest rate ceiling, such a guarantee can ensure an arbitrarily low probability of bank failure.

1 Introduction

It is well known that there exists a large asymmetry of information between bank regulators and the banks that they regulate. Moreover, current banking regulation (Basel I-III) relies heavily on both

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the accurate risk weighting of broad asset classes (via the standardized approach) and the ability to check that the banks own internal risk models are consistent (via the internal models approach). In light of this, it is natural to desire an understanding of how well current mechanisms may fair when the asymmetry of information between the regulator and the banks becomes large.

Given that it is easy to construct examples where the existing mechanisms of Basel I-III are not robust to increases in the asymmetry of information (see e.g., Jones (2000)), the literature has turned to the possibility of using *market discipline* to help curb the level of bank risk (see Evanoff and Wall (2000) for a survey of proposals). The idea is that if markets can discern the true risk of the bank better than the regulator, then by forcing the bank to regularly issue a minimum level of subordinated debt (i.e. unsecured junior debt), the regulator would be imposing on the bank a cost that increases with the bank's risk (probability of default).

The purpose of this paper is to understand, in a general banking model, to what extent market discipline can help to curb bank risk as regulators become more uninformed about the bank's portfolio characteristics. We consider a general model whereby banks, with limited liability, choose a distribution of returns and financing mix to maximize the value of their equity. The regulator's goal is to minimize the cost that the regulation imposes on the bank *subject to* the constraint that the bank's probability of failure does not exceed a certain threshold and that the regulator does not exceed his budget (in expectation). We model the asymmetry of information between the regulator and the bank, in the spirit of Carroll (2015), by assuming that while the regulator has some idea of the distributions of returns that the bank's assets can generate, they may also generate other distributions unknown to the regulator. In this case, the regulator aims to bound the worst case level of risk that any incentive compatible mechanism can guarantee no matter the size of the asymmetry of information between the bank and the regulator. In doing so, we characterize robust mechanisms that will allow the regulator to achieve his objective without making any informational assumptions about asset returns.

Our main result is the construction of an *informationally robust mechanism* that allows the regulator to guarantee an arbitrarily small bound on the level of risk in the banking system. The key insight developed is that when creditors can (on average) discern the true level of risk of the bank, then for any budget of the regulator, any informationally robust mechanism that can bound the worst case level of bank risk *and* satisfy the regulators budget constraint must satisfy three conditions: 1.) capital requirements should be uniform across all banks; 2.) the mechanism must require a minimum amount of subordinated debt to be issued by each regulated bank *to some other regulated bank* at an interest rate below a predefined, leverage dependent, interest rate ceiling; 3.) the regulator must have the ability to provide an explicit guarantee to at least one of the banks to insure its solvency *conditional on its subordinated debtors failure*. Once the regulator has the ability to implement such a mechanism, then by carefully constructing the network of subordinated debt exposures, we can guarantee an arbitrarily low bound on the level of bank risk. Importantly, we show that these three conditions are necessary for any informationally robust mechanism to satisfy incentive compatibility, to meet the regulators budget constraint, and to bound the worst case level of risk of the bank. Furthermore, we show that in general a minimum subordinated debt requirement *per se* will provide banks with incentives to increase the correlation of their portfolio's failure with their creditors, which can give them incentives to take more risk than without the requirement. We then show how our optimal mechanism avoids creating these perverse incentives through the aforementioned explicit guarantee. Finally, we turn to the case where creditors may have a bias in determining bank risk[1] (*i.e.* they may consistently underestimate the bank's true probability of failure) and provide a bound on the worst case level of bank risk that any mechanism can guarantee as a function of the magnitude of the biases.

To see why our three conditions are necessary, first note that if the regulator has little information regarding the bank's portfolio characteristics, then any mechanism that is not uniform in its treatment of capital requirements will require the bank to report its true characteristics (e.g. distribution of returns) which will determine its capital requirements.[2] In order to provide the bank

[1]Our results hold when there is noise in the creditors evaluation of the bank's risk, as long as the noise is mean zero.

[2]This is due to the revelation principle (Forges (1986) and Myerson (1986)) which states that any indirect mech-

with the correct incentives to report truthfully the regulator therefore must be able to exploit some trade off between higher capital requirements and, for example, higher penalties (see e.g., Kupiec and O'Brian (1995) and Prescott (1997)). While there is hope for separation when the regulator is confident making certain assumptions about the distribution of returns of the bank given some observable characteristics of its assets, once the regulator becomes less confident in his information, capital requirements intuitively must converge to a uniform requirement.[3]

Once we show that capital requirements must be uniform, the next question we ask is how well market discipline can perform in curbing the risk of the bank. We show that a minimum subordinated debt requirement + interest rate ceiling[4] can help to the bound level of risk of the bank, but that in general, the best bound on the bank's probability of default that such a mechanism can guarantee is *the worst case probability of default of the creditor of subordinated debt to the bank*. To understand why this is the case, we note that even if a creditor can perfectly infer the banks true level of risk, the interest rate at which the bank lends to the creditor may still fall below the fair value interest rate on the debt. This is due to the fact that if the creditor has limited liability (either as a manager or an institution), then the required rate of return depends crucially on the probability that the bank fails *conditional on the creditor surviving*. For example, if in any state of the world where the bank fails the creditor also fails, then the creditor will never internalize the loss incurred by the failure of the bank to repay its subordinated debt. In this extreme case the creditors required return on the banks subordinated debt is simply the risk free rate. Hence, even if markets can perfectly discern the true risk of the bank, this need not translate into direct market discipline. Finally, we show that, intuitively, this high level of correlation between the failure of the bank and its creditor can only be sustained when the probability of the bank's failure is less than or equal to the probability of the creditor's failure. To the author's knowledge, this is the first

anism can be implemented with a direct mechanism whereby agents report their private information to the principal who then suggests to each agent an action to take.

[3]It has been well documented how bank's can exploit this asymmetry of information when capital requirements are not uniform as illustrated by, among others, Acarya et. al. (2013), Calomiris and Mason (2003), and Yorulmazer (2013).

[4]Namely, banks must issue a minimum amount of subordinated debt at a rate below the interest rate ceiling or face regulatory inspection/nationalization.

paper to point out this flaw in the use of subordinated debt for regulating risk.

Due to the fact that the price of the bank's subordinated debt may be influenced by the correlation between the bank's portfolio and the creditor's portfolio implies that if the regulator does not know the characteristics of the bank's creditors then it will be unable to bound the level of bank risk. Furthermore, *internalizing* the subordinated debt minimum + interest rate ceiling mechanism by forcing banks to issue subordinated debt to other regulated banks can only guarantee that each bank's probability of failure falls below the riskiest regulated bank's probability of failure. What we show then, is that the regulator can resolve this issue by providing an explicit guarantee to one of the banks to ensure its solvency conditional on the failure of one of the other banks to which it is a subordinated creditor. In doing so, the regulator will guarantee that the bank internalizes the losses on the subordinated debt whenever the borrowing bank fails. Therefore, no matter the level of correlation between the portfolios of the two banks, the crediting bank will always price the borrowing bank's subordinated debt at or above its fair value. In this case, the subordinated debt minimum + interest rate ceiling will guarantee that the borrowing bank's level of risk falls below the level associated with the interest rate ceiling (which can be made arbitrarily small). We then show that by properly coordinating the exposures of subordinated debt amongst the regulated banks, the regulator can guarantee an arbitrarily small level of system wide bank risk. The intuition here is that if we can guarantee that one bank's risk is arbitrarily small, then we can also guarantee that the risk of any bank that issues subordinated debt to this bank is also arbitrarily small — in the worst case the two bank's failures are perfectly correlated, but this can only be the case when the borrowing bank's risk is less than or equal to the lending bank's risk, which is arbitrarily small. Hence, by using a single guarantee and properly coordinating the subordinated debt exposures (e.g. in a cycle) we can guarantee that all banks have an arbitrarily small probability of default.

Finally, we show that while the regulator could always provide the explicit guarantee to any one of the bank's non-bank creditors to ensure that the bank's debt is appropriately priced, in doing so he may face an unbounded expected cost of the guarantee. In contrast, by appropriately designing a leverage specific interest rate ceiling in our robust mechanism, we show the regulator can arbitrarily bound the expected cost of providing the guarantee to a regulated bank and therefore meet his budget constraint. Here we highlight a tradeoff that the regulator faces between allowing banks to take a certain amount of risk (e.g. to provide better credit conditions) and the expected subsidy the regulator must provide to the banks. Namely, while we can guarantee expected budget balance (i.e. the banks expected subsidy is equal to zero), this will either require banks to be risk free (i.e. have zero probability of default), or to finance their assets with 100% capital.

1.1 Related Literature

Our motivation for robust mechanisms begins with the introduction of deposit insurance and subsequent implementation of Basel I and Basel II capital requirements. Many papers since (e.g., Koehn and Santomero (1980), Kahane (1977), and Gennotte and Pyle (1991), Blum (1999)) have shown how inefficiently priced deposit insurance can lead to higher incentives for bank risk taking and how the introduction of a leverage ratio can potentially exacerbate these incentives. Kim and Santomero (1988) and Rochet (1992) show that for this reason capital requirements should be weighted by the risk of the bank's assets and construct the *theoretically optimal* risk weights under differing assumptions. In line with this reasoning, the standardized approach of Basel I-III defines capital requirements by associating with each asset a risk weight and then determines the banks capital requirements as a percentage of *risk weighted assets*. In light of this, Chan et. al. (1992) show that when depository institutions are perfectly competitive, then an incentive compatible and risk sensitive deposit insurance pricing scheme may be infeasible. Similarly, Giammarino et. al. (1993) extend the results of Chan et. al. (1992) to show that in general the regulator can discriminate among banks on the basis of their level of risk, but that any mechanism that does so will give banks an incentive to lower their asset quality.[5] For a summary on how the current arbitrary risk

[5]Chan et. al. (1992) assumes, unlike this paper, that the regulator maximizes social welfare and therefore internalizes the profits of the bank in his objectives. They further make an assumption that banks can exert effort to lower the risk of their distribution of returns in a sense modeled by first or second order stochastic dominance.

weighting of assets is unsatisfactory for the regulation of bank risk see U.S. Shadow Regulatory Committee (2000).

In light of this, the *internal ratings based* (IRB) approach to capital regulation, largely influenced by Gordy (2003), was introduced in the Basel II accord.[6] In this approach, banks use their own internal risk models to determine a value-at-risk (VAR) statistic — VAR measures the maximum potential losses the bank may face over a certain period of time (normally 10-days) with a certain level of confidence (usually 99%)— which determines the banks capital requirements.

Since the introduction of the IRB approach the literature has provided significant evidence that IRB capital regulation produces incentives for banks to underreport their true risk. Empirical studies of the under reporting of bank risks includes Begley et. al. (2017), Behn et. al. (2014), and Plosser and Santos (2014) among others. Notably, Plosser and Santos (2014) show that risk estimates produced by different banks for the probability of default of the same syndicated loan have varied by as much as 100 basis points which can result in a decrease of up to 33% of required regulatory capital for the loan. Furthermore, Begley et. al. (2017) show that banks consistently underreport their risk following periods of poor stock returns when raising capital is more costly. Finally, strategic underreporting of bank risk has been studied theoretically in papers such as Lucas (2001), Prescott (2004), and Colliard (2017). In particular, Colliard (2017) shows that when the bank's internal risk estimates are private information, costly auditing leads to less risk-sensitive capital requirements in order to counteract the bank's incentives to underreport their true risk.

Another motivating line of research for robust mechanisms comes from the literature on regulatory arbitrage through financial innovation. The idea of regulatory capital arbitrage, first introduced by Jones (2000) and documented empirically in Acharya et. al. (2013), highlights how securitization has produced the ability for banks to restructure their portfolios to lower the burden of capital regulation while effectively maintaining the same risk characteristics. Namely, Acharya

[6]Note that the IRB approach is not sound theoretically in the context of informational robustness as Gordy (2003) shows that the value at risk statistic can be a sufficient measure of capital adequacy assuming normally distributed returns and necessarily requiring a single systemic macroeconomic risk factor (see Dewatripont, Rochet, and Tirole (2010) for a discussion).

et. al. (2013) show how prior to the financial crisis, banks' would commonly securitize assets which would then be sold to *conduits* that finance the purchase of the securitized assets by issuing asset backed commercial paper (ABCP). This would allow the banks to remove loans off of their balance sheet, substantially lowering their capital requirements, while retaining the underlying risk by providing guarantees to the ABCP issued by the conduits.[7][8] The key issue here was the regulators inability to foresee that the banks were retaining such risk on their balance sheets (i.e. a misspecification of the risk weights for guarantees on ABCP).

Given the above issues with the current banking regulation, the literature has turned to *market discipline* as a potential regulatory tool. The majority of papers that study the use of subordinated debt for bank regulation aim at empirically testing whether creditors and markets actually provide discipline to riskier banks through higher primary and secondary market interest rate spreads. For a summary of the empirical results on the existence of market discipline see Study Group on Subordinated Notes and Debentures (1999). While the empirical evidence on the existence of market discipline through subordinated debt interest rates is mixed, we note that, to the author's knowledge, the theoretical effect of bank and creditor portfolio correlations on subordinated debt pricing has not been taken into account in these studies. Therefore, one would expect a downward bias in the estimated disciplinary effects in both primary and secondary market subordinated debt yields without controlling for such correlation.

The key proposals for the use of subordinated debt for bank capital regulation are summarized in Evanoff and Wall (2001a) and The Federal Reserve Board of the U.S. Treasury (2000). Of such proposals Calomiris (1999) suggests the most similar mechanism to ours. Namely, he proposes a minimum subordinated debt requirement coupled with an interest rate ceiling as in our paper. The key insight that we add to this proposal is that we show, theoretically, that while such a mechanism

[7]In fact, Acharya et. al. (2013) estimate losses on subprime mortgages and their associated collateralized debt obligations after the housing crash to be between $68 and $204 billion yet only $1.8-5.2 billion of these losses were borne by outside investors not involved in originating or securitizing the loans.

[8]Similar types of *securitization without risk transfer* for the purpose of regulatory arbitrage has further been documented by Calomiris and Mason (2003) in the case of securitization of credit card receivables, and Yorulmazer (2013) in the case of credit default swaps. The key issue highlighted in these papers is how financial innovation can lead to regulatory arbitrage due to the regulators inability to understand or correctly estimate the underlying risk of the banks.

can help curb bank risk, is not sufficient to limit the bank's risk by itself. In fact, we show that such a mechanism *per se* can lead to an increase in bank risk. This insight highlights the danger of proposals stating that banks should be allowed to meet their capital requirements with an unlimited amount of subordinated debt as proposed by U.S. Shadow Regulatory Committee (2000).

Of the few theoretical papers on subordinated debt and bank risk taking incentives, Levonian (2001) shows, in a theoretical model, that subordinated debt may be inferior to equity when the regulator has concerns regarding both the probability of bank default and the liability of the deposit insurance fund. Important in his model is the fact that returns are normally distributed so that an increase in equity strictly decreases bank risk. This is precisely the departure of the current paper from the previous literature. If the regulator does not have information regarding the distribution of returns, which may have fat tails or be discontinuous, then such a statement cannot be made. Blum (2002) further shows that unless the bank can commit to not increasing risk in the future, subordinated debt pricing may not lead to a reduction in bank risk. One way around this dynamic problem, as proposed in Calomiris (1999) is to have banks issue subordinated debt on a regular basis with overlapping maturities which would limit the scope for banks to increase their risk in the interim before the next subordinated debt issuance.

Finally, this paper draws a large influence from the current game theoretic motivation for designing mechanisms that are robust to underlying informational assumptions as initiated by the Wilson (1987) doctrine. Similar theoretical approaches include papers such as Morris and Bergemann (2005) who characterize mechanisms robust to the informational assumption that agents have common prior beliefs, and a series of papers Carroll (2015), Carroll (2016a), and Carroll (2016b). In fact we take direct influence from Carroll (2015) who looks at procurement problems when the principal knows only a subset of the agent's potential actions and characterizes the highest level of surplus that the principal can achieve under a worst case criteria (with respect to the true set of the agents actions), and shows that this surplus can be achieved with linear contracts. In our definition of robustness we assume, in a similar fashion, that the regulator may know some true distributions of returns of the bank, but that there may exist other distributions of returns that the bank's assets can generate unknown to the regulator. While Carroll (2015) shows how the principal can guarantee a minimum amount of surplus subject to the known set of agents actions, we show that the worst case level of risk that the regulator can guarantee will always be independent of the known set of distributions and how this leads to uniform capital requirements across the regulators information. Finally, we note that Carroll (2015) was not the first to utilize a worst case objective in a setting where the principal may misspecify the underlying problem. There is a long line of literature in macroeconomics, micro founded on the minmax expected utility model of Gilboa and Schmeidler (1989), on the calibration of dynamic models when the underlying model may be misspecified and how to robustly calibrate these models in the presence of such misspecification (see e.g., Hansen and Sargent (2003)).

2 The Model

To start we consider the case of a single regulator and a single bank. The bank at time $t = 0$ chooses a portfolio of assets and issues equity and debt to finance its portfolio. At time $t = 1$ the assets generate a return x distributed according to some Borel measurable distribution $f \in \Delta(\mathbb{R})$. We assume that the bank is risk neutral (a discussion of risk aversion is provided below), maximizes the value of its equity, and has limited liability.

The regulator's objective is minimize the cost of the regulation to the banks (i.e. provide minimal interference in the banking sector) subject to the probability of default of the bank being below some threshold. Namely, whenever the bank with a balance sheet of size A chooses a distribution of returns f, and a level of equity financing K, the probability that the bank fails is

$$\alpha(f, K) = \int_{-A}^{-K} f(x)dx$$

and therefore, the regulator's would like ensure $\alpha(f, K) \leq \alpha^\star$ where $\alpha^\star$ is a predetermined *accept-*

able level of risk.[9] Important to note here is that we are implicitly assuming that banks cannot short sell assets implying that the largest loss of the bank is A. As we will see, the only time that this assumption will be important is when considering the regulators budget balance constraint (introduced below). We therefore assume the regulators problem is to design a mechanism $\mathcal{M}$ to align the incentives of the bank and the deposit insurance institution in the spirit of the Representation Hypothesis of Dewatripont and Tirole (1994).

We assume the regulator can identify any composition of assets as lying within a particular class $l = 1, ..., L$. Further, for any portfolio of assets in the class l, the regulator can identify a set of distributions $C_l^0 \subseteq \Delta(\mathbb{R})$ consisting of the possible distributions of returns f that any portfolio in class l can generate.[10] The description of which portfolios fall in which classes can be thought of as endogenously chosen by the reguater in a broad sense similar to the way portfolios are treated in the standardized approach of Basel I-III. For example, one asset class could be portfolios of US treasury bonds with varying maturities while another can be portfolios with a makeup of 50% residential mortgages, 30% AAA rated corporate bonds, and 20% of a trading book of various financial instruments. We would like to think of the asset classes as a crude representation of the limitations of any regulatory policy that maps assets to a particular class and then assigns capital (and other) requirements based on the asset class.

In order to model the asymmetric information between the regulator and the bank (without making common prior assumptions) we assume that the regulator knows of potential distributions of returns, C_l^0, of portfolios falling within each class l but that there may be other distributions of returns that he is unaware of (in the spirit of Carroll (2015)). Denoting by C_l the true set of distributions of returns generated by portfolios in the class l, this implies that $C_l^0 \subseteq C_l$. In order to proceed, we assume that the regulator takes a worst case, or *minmax*, approach in designing the

[9]In general the regulator may set $\alpha^\star > 0$, if it believes that banks must be allowed to take some minimal amount of risk, for example, to ensure the proper function of financial markets.

[10]The fact that C_l^0 is not a singleton represents the fact that the regulator cannot perfectly distinguish between the distribution of returns of assets in a particular class.

mechanism $\mathcal{M}$ to solve the following problem

$$\min_{\mathcal{M}} \; c(K, D) + \mathbf{T}$$

$$\text{Subject to:} \quad (1) \max_{\substack{f \in C_l \supset C_l^0 \\ l=1,\ldots,L}} \alpha(f, K) \leq \alpha^\star$$

$$(2) \; \mathbb{E}_{\mathcal{M}}^{WC}[-\mathbf{T}] \leq \hat{T}$$

$$(3) \; \mathcal{M} \text{ is incentive compatible and feasible}$$

where $c(K, D)$ is the cost of financing for the bank and $\mathbf{T}$ any possible transfers from the banks to the regulator required by the mechanism. The constraint (2) states that the regulator would like to limit the *worst case* expected transfer to the bank, $\mathbb{E}_{\mathcal{M}}^{WC}(-\mathbf{T})$, by some amount $\hat{T}$, equivalent to a budget balance requirement for the regulator. We use the worst case expected cost due the fact that the mechanism must allow the bank to choose any distribution of returns and financing decision and therefore once the regulator largely misspecifies the problem, he will be unable foresee the optimal decision of the bank given the mechanism (see section 2.1 for our definition of misspecification). One simple interpretation for this constraint is that the regulator has some expectation of the worst case cost of *ex-post* intervention, $\hat{T}$, and therefore whenever the expected cost of ex-ante intervention, $\mathbb{E}_{\mathcal{M}}^{WC}(-\mathbf{T})$, is greater than the cost of ex-post intervention, the regulator would rather let the banks operate under a *laissez faire* mechanism and intervene ex-post in the case of bank failure. Finally, the regulator requires that the mechanism $\mathcal{M}$ satisfy incentive compatibility — banks to report truthfully and take the correct actions suggested to them— and feasibility which requires the mechanism have a feasible solution. Each of these components of the regulators constraints will be defined precisely after we define the general class of mechanisms that we study in Subsection 2.2. Important to note here is the fact that the regulator requires the bank's level of risk to fall below the threshold $\alpha^\star$ as a constraint, rather than the main objective, in order to illustrate that the regulators incentive is to keep bank risk below a certain threshold and once that threshold is met, then the regulator would like to minimize the regulatory cost he imposes on the bank.

The problem for the banks is to maximize expected returns (i.e. the value of equity) by choosing a portfolio which generates a distribution of returns f in a particular class.[11] The bank finances their assets of size A with equity K and debt debt $D := (D_\psi)_{\psi=0,1,...,J}$ of various priority ψ. Namely, debt will be characterized by its priority $\psi = 0, 1, ..., J$ where $\psi = J$ is the lowest priority debt (i.e. the lowest priority to be paid back in the event of insolvency) otherwise known as subordinated debt, $\psi = 0$ is the highest priority debt which we equate to insured deposits, and 1 is the highest priority uninsured debt, sometimes referred to as *senior debt*. For each priority ψ, we denote by D_ψ the dollar value of debt with priority ψ issued by the bank, and by r_ψ the interest rate paid on the debt D_ψ. Therefore, if the face value of the bank's assets is A and the return (i.e. gain or loss) on the assets is x, then the return on one dollar of debt with priority ψ is[12]

$$\begin{cases} 1 + r_\psi & \text{if } x + A \geq \sum_{j \leq \psi} D_j(1 + r_j) \\ \frac{1}{D_\psi}(x + A - \sum_{j<\psi} D_j(1 + r_j)) & \text{if } \sum_{j<\psi} D_j(1 + r_j) \leq x + A < \sum_{j\leq\psi} D_j(1 + r_j) \\ 0 & \text{if } x + A < \sum_{j<\psi} D_j(1 + r_j) \end{cases}$$

Then, using this notation we can denote by

$$\mathcal{K}(\psi, K) := K + \sum_{j>\psi} D_j - \sum_{j\leq\psi} D_j r_j$$

the maximal loss that the bank can sustain while still being able meet its priority ψ debt obligations. In what follows, for simplicity we will assume that priority ψ debt contracts are binary and repay $1 + r_\psi$ if $x \geq \mathcal{K}(\psi, K)$ and 0 otherwise.[13] In this case, whenever the bank makes the investment

[11] Note that we do not impose a cost on a bank for investing in a particular class, but note that this is without loss as we can restrict our attention to *feasible* asset classes for the bank. In this sense, if it would be very costly for a bank that specializes in residential mortgages to start investing in loans to small to medium sized enterprises (SME's), then this will be reflected in the opportunity sets C_l; the cost would be reflected in the set of feasible return distributions in any asset class that invests heavily in SME's via lower expected returns.

[12] We implicitly assume here that no interest is paid if $x < -K$.

[13] As we will see the optimal debt contracts in any regulatory mechanism that utilizes market discipline will satisfy this property.

decision (f, K), the required rate of return on such a debt contract is given by the equation

$$(1 + r_\psi) \int_{-\mathcal{K}(\psi,K)}^{+\infty} f(x)dx - \int_{-A}^{-\mathcal{K}(\psi,K)} f(x)dx = 1 + r_0$$

where r_0 is the risk free rate. This yields a required return of at least

$$r_\psi \geq \frac{r_0 + 2\int_{-A}^{-\mathcal{K}(\psi,K)} f(x)dx}{1 - \int_{-A}^{-\mathcal{K}(\psi,K)} f(x)dx}. \tag{1}$$

Furthermore, it may be the case that the bank has access to a limited amount of deposits D_0 (which we refer to as priority 0 debt), in which case we add the constraint $D_0 \leq \bar{D}_0$.[14] Denoting by $\mathcal{C} = \cup_{l=1}^{L} C_l$, the bank's maximization problem becomes

$$\max_{\substack{f \in \mathcal{C} \\ K,D}} \int_{-K}^{+\infty} x f(x)dx - c(K, D)$$

$$\text{subject to} \quad K + \sum_{\psi} D_\psi = A$$

$$K \geq 0, D_\psi \geq 0 \text{ for all } \psi = 0, 1, ..., J, \text{ and } D_0 \leq \bar{D}$$

Here limited liability shows up in the conditional expectation as the bank is only concerned with maximizing expected returns conditional on solvency (i.e. losses less than the value of capital).

2.1 Informationally Robust Mechanisms

Given that the regulator may be misinformed regarding the worst case distribution of returns, he may be unable to bound the risk taken by the bank using conventional regulatory tools. Namely, letting $\mathcal{M}$ be a general regulatory mechanism, and $\alpha^\star$ the maximal level of risk the regulator is willing to allow the bank to take, it may be the case that the worst case level of risk under $\mathcal{M}$,

[14] Once this is the case we can show that the bank may find it optimal to issue subordinated debt without any regulatory requirements (as is seen in reality) as it will lower the cost of its more senior debt (which it may issue due to lack of deposits). We elaborate on this in Section 3.1.

denoted by $\alpha_{\mathcal{M}}$, is such that $\alpha_{\mathcal{M}} > \alpha^{\star}$. The question we will be interested in answering here is, for a given mechanism $\mathcal{M}$, does there exists a $\delta \in (\alpha^{\star}, 1)$ such that $\alpha_{\mathcal{M}} \leq \delta$, no matter the level of asymmetry of information between the regulator and the bank (as captured by the sets C_l^0 and C_l). To model this asymmetry we will introduce the following definition of misspecification.

Definition : (1) Let K be the amount of equity issued by the bank. The regulator misspecifies the regulatory problem by ϵ if

$$\max_{l=1,\dots,L} \max_{f \in C_l} \min_{f_0 \in C_l^0} \int_{-A}^{+\infty} |f(x) - f_0(x)| dq = \epsilon. \tag{2}$$

While many definitions would suffice for misspecification, our sole intention is to represent the fact that if $C_l = C_l^0$, then the regulator is perfectly informed, and the level of misspecification $\epsilon = 0$. Otherwise, there exists $f \in C_l$ such that $\int |f(x) - f_0(x)| dx > 0$ for all $f_0 \in C_l^0$. Therefore, in the best case, the regulator misspecifes the bank's returns by $\min_{f_0} \int |f(x) - f_0(x)| dx > 0$ and thus the worst case level of best case misspecification over any distribution $f \in C_l$ in any asset class $l = 1, ..., L$ is given by (2). We are now ready to introduce our definition of robustness.

Definition : The mechanism $\mathcal{M}$ is δ-informationally robust if the worst case level of risk taken by the bank under the mechanism $\mathcal{M}$ is bounded by δ for any level of misspecification of the regulatory problem: $\alpha_{\mathcal{M}} \leq \delta$ for all $\epsilon > 0$.

Given our definition of robustness, we can now describe the regulators goal as designing a budget balanced and incentive comaptible $\alpha^{\star}$-informationally robust mechanism.

2.2 General Mechanisms for the Bank Regulation Problem

We will now describe in detail what we refer to as general mechanisms in the bank regulation problem. The observables over which the mechanism operates are the asset class l, future returns x, interest rates on debt $\mathbf{r} := (r_{\psi})_{\psi=0,..,J}$, the funding decision of the bank K and $D := (D_{\psi})_{\psi=0,..,J}$, and the size of the banks balance sheet A. When convenient we will refer to the bank's observable type as $(\mathbf{r}, l, A)$. In fact, given that our results will not be specific to the observable class l and the size of the bank's balance sheet A, we will simply assume the only relevant observable parameter is $\mathbf{r}$. We will now define the general class of mechanisms that we will consider throughout.

Definition : (1) A regulatory capital mechanism $\mathcal{M} = (\mathbf{T}, \mathbf{K}, \mathbf{D})$ is a mapping that takes the bank's reported distribution of returns f and observable type $\mathbf{r}$ and maps it to a system of transfers and liability requirements:

$$(f, \mathbf{r}) \mapsto (\mathbf{T}(f, \mathbf{r}), \mathbf{K}(f), (\mathbf{D}_\psi(f))_{\psi=1,\dots,J})$$

where

• $\mathbf{T}(f|\mathbf{r}) = (T_R(\cdot|f, \mathbf{r}, K, D), T(f, \mathbf{r}, K, D))$ is a system of transfers consisting of $T_R(\cdot|f, \mathbf{r}, K, D)$: $\mathbb{R} \to \mathbb{R}$, a function that for any report f and observable type $(\mathbf{r}, K, D)$, maps future returns $x \in \mathbb{R}$ to transfers $T_R(x|f, \mathbf{r}, K, D) \in \mathbb{R}$ from the bank to the regulator and $T(f, \mathbf{r}, K, D)$ is an $(f, \mathbf{r})$ specific transfer independent from returns.

• $\mathbf{K}(\cdot) : [0, 1] \to \mathcal{P}([0, A])$ is a correspondence that maps reported types to required levels of capital such that for any report f, the bank is required to finance its assets with some amount of equity $K \in \mathbf{K}(f) \subset [0, A]$.

• $\mathbf{D} := (\mathbf{D}_\psi(\cdot) : [0, 1] \to \mathcal{P}([0, A]))_{\psi=0,\dots,J}$ is tuple of ψ-specific correspondences each which maps reported types to required levels of debt such that for any report f, the bank is required to finance its assets such that $D_\psi \in \mathbf{D}_\psi(f) \subset [0, A]$ for each $\psi = 0, ..., J$. When this condition is satisfied we will use the notation that $D \in \mathbf{D}(f)$.

Finally, we assume that the mechanism may allow for subsidies to creditors, even if they are not regulated banks. Naturally, in such a case we should require that the mechanism allows the creditor to choose any financing mix, must not discriminated based on the creditors profile $(f, \mathbf{r}, K, D)$ (given that this may not be known to the regulator), and that the regulator can only provide subsidies to the unregulated creditors, not penalties. In this case we impose the following restrictions on mechanism

• The mechanism is restricted for unregulated creditors to subsidies only. Therefore the mechanism

(1) is invariant to the profile of the creditor $(f_C, \mathbf{r}_C, K_C, D_C)$

(2) sets no capital requirements $\mathbf{K}^C = [0,1]$, $\mathbf{D}^C = [0,1]$

(3) cannot penalize the creditor $T_R^C(x|f, \mathbf{r}, K, D) \leq 0$ for all $x \in \mathbb{R}$ and $T^C(f, \mathbf{r}, K, D) \leq 0$.

In our general definition of the regulators available mechanisms we have implicitly assumed a timing structure. Namely, at time $t = 0$ bank chooses their distribution of returns f and financing decision (K, D). Then, at time $t = \frac{1}{2}$ the bank makes a report of its distribution of returns $\tilde{f}$ to the regulator and makes the non-return specific transfer $T(\tilde{f}, \mathbf{r}, K, D)$. Finally, at time $t = 1$, returns x are generated according to the true distribution f and the transfer $T_R(x|\tilde{f}, \mathbf{r}, K, D)$ is made. In line with our motivation, the expected transfer (to either the regulated bank or its creditor) will be measured in the worst case

$$\mathbb{E}_{\mathcal{M}}^{WC}(-\mathbf{T}) := \min_{\substack{f \in \mathcal{C}, \mathbf{r} \in [0,1]^{J+1} \\ K, D}} T(f, \mathbf{r}, K, D) + \int_{-A}^{+\infty} f(x) T_R(x|f, \mathbf{r}, K, D) dx$$

Further note that under this definition, when the regulator regulates multiple banks, the regulators budget balance constraint simply requires, without loss, that the worst case expected transfer to any bank *or* their creditors fall below the threshold $\hat{T}$ as we have chosen $\hat{T}$ arbitrarily.

Finally, we will present our definition of a feasible mechanism.

Definition : (1) A regulatory capital mechanism $\mathcal{M} = (\mathbf{T}, \mathbf{K}, \mathbf{D})$ is *consistent* if for all $f \in \Delta(\mathbb{R})$ and $\mathbf{r} \in [0,1]^{J+1}$, there exists $K \in \mathbf{K}(f)$ and $D_\psi \in \mathbf{D}_\psi(f)$ for all $\psi = 0, 1, ..., J$ satisfying

$$K + \sum_{\psi} D_\psi = A \qquad \text{and} \qquad D_0 \leq \bar{D}$$

(2) A mechanism $\mathcal{M} = (\mathbf{T}, \mathbf{K}, \mathbf{D})$ satisfies *limited liability* if for all $f \in \Delta(\mathbb{R})$, $\mathbf{r} \in [0,1]^{J+1}$, and

$K \in \mathbf{K}(f)$, the transfer scheme satisfies

$$T_R(x|f, \mathbf{r}, K, D) \leq x + K \qquad \text{for all } x \in \mathbb{R}$$

When the mechanism $\mathcal{M}$ is consistent and satisfies limited liability then we say that it is *feasible*.

We note that consistency is necessary for the mechanism to allow for a solution to the banks constrained optimization problem, and limited liability is required due to limited liability of the bank; the bank cannot be fined more than the value of its equity plus current returns (positive or negative). Hence, in what follows we will assume that all mechanisms under consideration are feasible.

3 Main Results

In this section we will start by stating our main result, necessary conditions for a mechanism to be $\alpha^{\star}$-informationally robust. In what follows, we suppose that the bank issues subordinated debt in over the counter markets, thereby bargaining with its creditor over the interest rate on the loan, and that the creditor may have limited liability either as an institution or a manager (see Section 4 for the complete model of subordinated debt issuance). Thus, the level of creditor's risk is just the probability of the creditor's limited liability being invoked and therefore them not internalizing losses past that threshold. In the case of the creditor being another bank, the creditor's risk is simply the probability of failure (i.e. losses exceeding the value of equity), while in the case of the creditor being a fund, an example of the creditor's risk could be the probability of the funds closure or failure to meet margin payments on borrowed assets due to substantial losses.

Before stating our main results, we will first introduce a particular class of *internal subordinated debt mechanisms*.

Definition : A conditional failure (CF) guarantee $g(S + \eta)$ is an explicit guarantee from the regulator to the bank's creditor to reimburse any time $t = 1$ losses $x < 0$ of the creditor, excluding losses incurred from the failure of repayment of the bank's subordinated debt, up to $\max\{S + \eta - (K + x), 0\}$, *conditional* on the bank's failure. Namely, the regulator sets the subsidy $T^C_R(\cdot)$ as[15]

$$T^C_R(x) = \begin{cases} \eta + S - (K + x) & \text{if } x \leq -K + \eta + S \\ 0 & \text{otherwise.} \end{cases}$$

If the CF guarantee is made to Bank i in the event of the failure of Bank j, then we refer to it as a Bank j conditional failure guarantee to Bank i.

Theorem 1 *If the mechanism $\mathcal{M} = (\mathbf{T}, \mathbf{K}, \mathbf{D})$ is $\alpha^\star$-informationally robust, then*

(1) $\mathcal{M}$ must guarantee that the joint probability of failure of at least one bank and its creditor is strictly less than one.

(2) If the regulator has no information regarding the bank's creditor's portfolio, then he must provide the banks creditor with a CF guarantee in order to ensure (1).

(3) $\mathcal{M}$ must set uniform capital and debt requirements: for any two distributions f and $\tilde{f}$, $\mathbf{K}(f) = \mathbf{K}(\tilde{f}) = [0, 1]$ and $\mathbf{D}(f) = \mathbf{D}(\tilde{f}) = [0, 1]$.

(4) The worst case expected cost of the CF guarantee is unbounded whenever the creditor is not a regulated bank.

Theorem 1 states that a necessary condition for budget balance and $\alpha^\star$-robustness is that the mechanism requires banks to issue subordinated debt to other regulated banks, that it ensures the probability joint failure of at least two of the banks is strictly less than 1, and that when the latter condition cannot be satisfied, then no $\alpha^\star$-informationally robust mechanism exists. Important to note here is that $\alpha^\star$-robustness can be achieve without requiring all banks to necessarily issue subordinated debt to each other, but that any mechanism that achieves this will violate the regulators budget balance constraint. In what follows we will sketch the proof of Theorem 1 and then con-

[15] We have left out the profile $(f, \mathbf{r}, K, D)$ as the subsidy from the regulator to the bank's creditor is invariant to the creditors profile as assumed in section 2.1

struct a mechanism that is $\alpha^\star$-informationally robust and budget balanced for any arbitrary bound $\hat{T}$.

Sketch of proof of Theorem 1. We start by showing that any $\alpha^\star$-informationally robust mechanism must set the capital and debt constraints equally for any two *profiles* $(f, \mathbf{r}, K, D)$ and $(\tilde{f}, \tilde{\mathbf{r}}, \tilde{K}, \tilde{D})$ of the bank that are *indistinguishable* through the interest rates $\mathbf{r}$ and $\tilde{\mathbf{r}}$. Namely, two banks are distinguishable whenever a bank with probability of default $\alpha(f, K)$ issues debt at rate $\mathbf{r}$, then it may be the case that a bank with probability of default $\alpha(\tilde{f}, \tilde{K})$ would be unable to issue debt at the same rate $\mathbf{r}$. For example, if $\alpha(\tilde{f}, \tilde{K}) > \alpha(f, K)$ and the bank's creditor's returns are independent from the bank's returns, then the creditor will require a rate of return of at least $r_S \geq \frac{2\alpha(f,K)}{1-\alpha(f,K)}$ on the subordinated debt of the $\alpha(f, K)$ bank while it would require instead $\tilde{r}_S \geq \frac{2\alpha(\tilde{f},\tilde{K})}{1-\alpha(\tilde{f},\tilde{K})} > r_S$ for the $\alpha(\tilde{f}, \tilde{K})$ type. Denoting by $\mathcal{I}(f, K, D)$ the set of interest rates *compatible* with the profile (f, K, D) so that $r_S \in \mathcal{I}(f, K, D)$ implies $r_S \geq \frac{2\alpha(f,K)}{1-\alpha(f,K)}$, then we say two profiles $(f, \mathbf{r}, K, D)$ and $(\tilde{f}, \tilde{\mathbf{r}}, \tilde{K}, \tilde{D})$ are indistinguishable if $\tilde{\mathbf{r}} \in \mathcal{I}(f, K, D)$ and $\mathbf{r} \in \mathcal{I}(\tilde{f}, \tilde{K}, \tilde{D})$.

We first show that for any two indistinguishable profiles $(f, \mathbf{r}, K, D)$ and $(\tilde{f}, \tilde{\mathbf{r}}, \tilde{K}, \tilde{D})$, any $\alpha^\star$-informationally robust mechanism that is incentive compatible treats both profiles identically so that $\mathbf{K}(f) = \mathbf{K}(\tilde{f})$, $\mathbf{D}(f) = \mathbf{D}(\tilde{f})$, $T_R(\cdot|f, \mathbf{r}, K, D) = T_R(\cdot|\tilde{f}, \tilde{\mathbf{r}}, \tilde{K}, \tilde{D})$, and $T(f, \mathbf{r}, K, D) = T(\tilde{f}, \tilde{\mathbf{r}}, \tilde{K}, \tilde{D})$. The intuition is that if the regulator has very poor information regarding the distribution of returns of the bank then whenever the return specific transfers $T_R(\cdot|f, \mathbf{r}, K, D) \neq T_R(\cdot|\tilde{f}, \tilde{\mathbf{r}}, \tilde{K}, \tilde{D})$ are not equal, then there always exists a distribution of returns of the bank f' that either will find it optimal to report that he is type f and finance according to (K, D) at rates $\mathbf{r}$ or to report that he is type $\tilde{f}$ and finance according to $(\tilde{K}, \tilde{D})$ at rates $\tilde{\mathbf{r}}$ whenever $(f', \mathbf{r}, K, D)$ is indistinguishable from $(f, \mathbf{r}, K, D)$ and $(f', \tilde{\mathbf{r}}, \tilde{K}, \tilde{D})$ is indistinguishable from $(\tilde{f}, \tilde{\mathbf{r}}, \tilde{K}, \tilde{D})$. Hence, return specific transfers must be equal across all indistinguishable profiles. Then, we show that when return specific transfers are equal across indistinguishable profiles, then whenever $T(f, \mathbf{r}, K, D) > T(\tilde{f}, \tilde{\mathbf{r}}, \tilde{K}, \tilde{D})$, it must be the case that the cost of financing with $(\tilde{\mathbf{r}}, \tilde{K}, \tilde{D})$ is strictly less than the cost of financing with $(\mathbf{r}, K, D)$. But, in this case, the type f bank will always optimally report that he is the $\tilde{f}$ type and the fact that $(f, \mathbf{r}, K, D)$ and $(\tilde{f}, \tilde{\mathbf{r}}, \tilde{K}, \tilde{D})$ are indistinguishable guarantees that the f type can finance is assets with $(\tilde{K}, \tilde{D})$ at a rate $\tilde{r}$. Hence, incentive compatibility requires $T(f, \mathbf{r}, K, D) = T(\tilde{f}, \tilde{\mathbf{r}}, \tilde{K}, \tilde{D})$ and therefore $\mathbf{K}(f) = \mathbf{K}(\tilde{f})$ and $\mathbf{D}(f) = \mathbf{K}(\tilde{f})$ as now there is no cost for a bank of type f to report he is of type $\tilde{f}$ and vice-versa.

To prove (1) we show that unless the regulator can guarantee that the joint probability of failure between the bank and its creditor is strictly less than 1, then all profiles of the bank $(f, \mathbf{r}, K, D)$ and $(\tilde{f}, \tilde{\mathbf{r}}, \tilde{K}, \tilde{D})$ are indistinguishable. This comes from the fact that if the probability of failure of the bank and the creditor is equal to 1, then $0 \in \mathcal{I}(f, K, D)$ and $0 \in \mathcal{I}(\tilde{f}, \tilde{K}, \tilde{D})$ for all (f, K, D) and $(\tilde{f}, \tilde{K}, \tilde{D})$. In Section 4 we develop this idea and show that denoting by ρ^F the probability of the creditor's failure conditional on the bank's failure, then in general the creditor's required rate of return on the bank's debt, as a function of ρ^F is given by

$$\frac{\alpha(f, K)(1 - \rho^F)}{1 - \alpha_C - \alpha(f, K)(1 - \rho^F)}$$

Hence, if $\rho^F = 1$ then the bank can potentially issue its subordinated debt at the risk free rate. Then, given that in order for ρ^F to be equal to 1, it must be the case that $\alpha(f, K) \leq \alpha_C$, the probability of the creditor's failure. Hence, any mechanism that cannot guarantee that the joint probability of failure between the bank and its creditor is strictly less than 1, cannot bound the level of risk of the bank as in general α_C can be unbounded. Given this, we note that the only way for the regulator to bound the level of risk for the bank is to either bound the level of risk of its creditor (which in general he cannot do) or to bound the probability of failure of the creditor conditional on the bank's failure. Hence, given that the regulator cannot control the level of correlation between the bank and its creditor, the only way to guarantee the creditor survives when the bank fails is by issuing a CF guarantee to the creditor, thus proving (2).

Next, to prove (3), we show that in addition to uniform capital requirements for indistinguishable bank profiles, even if the regulator can distinguish between two profiles through the aforementioned guarantee, the CF guarantee plus interest rate ceiling mechanism is necessary and sufficient to limit the bank's risk below any threshold. Therefore, imposing non-uniform capital requirements for banks whose creditors have a guarantee will impose unnecessary costs on the bank, in which case the regulator can be made strictly better off setting capital requirements across all bank profiles uniformly and unconstrained: $\mathbf{K}(f) = [0, 1]$ and $\mathbf{D}(f) = [0, 1]$ for all f.

Finally, to prove (4) we note that if the creditor is not a regulated bank, then the worst case expected cost of the CF guarantee to the creditor is given by the creditor's worst case expected losses conditional on the bank's failure. Given that the regulator cannot regulate the creditors portfolio (most importantly it's leverage), we show that is easy for the creditor to arbitrage the guarantee leading to an unbounded expected cost. □

Now we will proceed to construct a mechanism that is $\alpha^\star$-informationally robust and satisfies the banks budget constraint.

Definition : An internal subordinated debt mechanism $(\mathcal{N}, S, \bar{r})$ consists of a directed network of exposures $\mathcal{N} := (I, E(I))$ with vertices I and edge set $E(I) \subset I \times I$, a minimum amount of subordinated debt S, and a maximal interest rate $\bar{r}$ such that:

(1) If $ij \in E(I)$, then Bank i must issue at least S dollars of subordinated debt to bank j at a rate $r \leq \bar{r}$.

(2) If the Bank i cannot issue S to Bank j at a rate $r \leq \bar{r}$, then Bank i is nationalized by the regulator effectively setting $T(f, \bar{r}, K, D) = \mathbb{E}_{f_i}(x|x \geq -K_i)$ whenever $r_{ij} > \bar{r}$, where f_i is the true distribution of returns of Bank i and K_i its chosen level of equity.

We will also assume that the interest rate ceiling can depend on the observable characteristics of the bank. Most importantly, it will depend on the leverage of the bank $A - K$ for which we introduce the following definition.

Definition : A *leverage based interest ceiling* $\bar{r} : \mathbb{R}_+ \to [0, 1]$ is a function that maps the leverage of the bank $A - K$ to a require rate of interest $\bar{r}(A - K)$.

Theorem 2 For any $i, j \in I$, if the regulator makes a CF guarantee to Bank i conditional on the failure of Bank j, then there exists a network of exposures $\mathcal{N}$ and a leverage based interest ceiling $\bar{r}$ such that for any $S > 0$, the internal subordinated debt mechanism $(\mathcal{N}, \bar{r}, S)$ is $\alpha^\star$-informationally robust and satisfies the regulators budget constraint for any $\hat{T} \geq 0$.

Proof. We will now construct a mechanism for the proof of Theorem 2 that utilizes only a single CF guarantee.

Without loss suppose that $\mathcal{N} = (I, E(I))$ is a cycle such that $(i+1)i \in E(I)$ (mod n) for

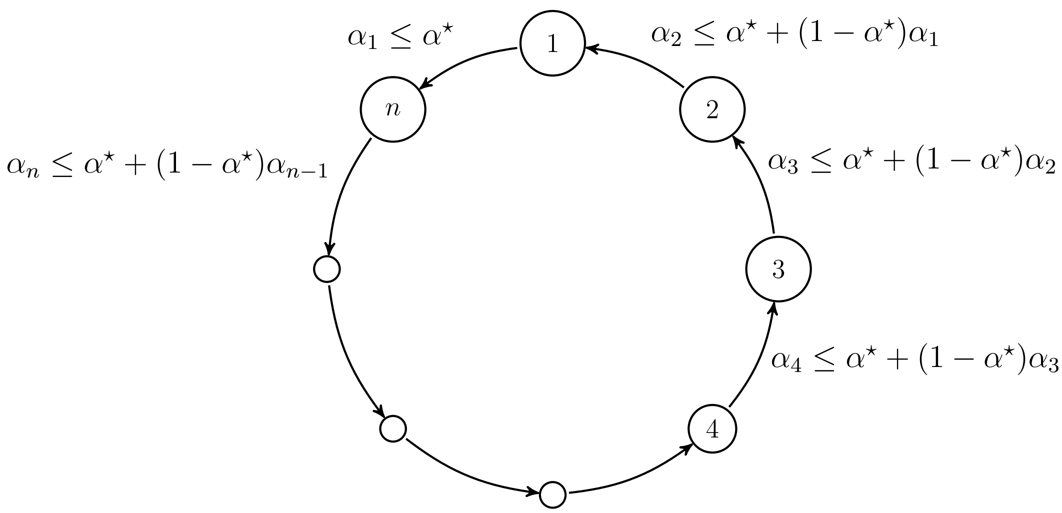

Figure 1: Cyclic Network of Exposures

all $i = 1, ..., n$ as illustrated in Figure 1. Further, without loss, suppose that the regulator makes a single CF guarantee to Bank n, conditional on Bank 1's failure. Now, fix $\bar{\alpha}$ and denote by $b_n := \frac{\hat{T}}{A_n - K_n}$ and define the interest ceiling for any Bank $i \in I$ as

$$\bar{r} = \min\{ \frac{2\bar{\alpha}}{1-\bar{\alpha}} , \frac{2b_n}{1-b_n} \}$$

What we claim is that the internal subordinated debt mechanism $(\mathcal{N}, \bar{r}, S)$ is $\alpha^\star$-informationally robust and satisfies the budget constraint of the regulator when $\bar{\alpha} = 1 - (1 - \min\{b_n, \alpha^\star\})^{\frac{1}{n}}$.

In order to do so, we first note that given that Bank 1 must issue S_{min} to Bank n, the bank with the explicit guarantee, at a rate $r \leq \bar{r}_1(A_1 - K_1) \leq \frac{2\bar{\alpha}}{1-\bar{\alpha}}$, implies that $\alpha_1 \leq \bar{\alpha}$. Next, given that Bank

2 must issue subordinated debt to Bank 1, we claim that it must be the case that $\alpha_2 \leq \bar{\alpha}+(1-\bar{\alpha})\alpha_1$. To see why this is the case, we note that if Bank 2 has a probability of default α_2, the the only way to issue subordinated debt to Bank 1 at a rate less than or equal to $\bar{r}$ is if

$$\frac{2\alpha_2(1-\rho_{12}^F)}{1-\alpha_1-\alpha_2(1-\rho_{12}^F)} \leq \frac{2\bar{\alpha}}{1-\bar{\alpha}}$$

where ρ_{12}^F is the conditional probability of Banks 1's failure given Bank 2's failure. Then rearranging, we see that this implies

$$\rho_{12}^F \geq 1-\frac{\bar{\alpha}}{\alpha_2}(1-\alpha_1)$$

Therefore, given that $\rho_{12}^F \leq \min\{\frac{\alpha_1}{\alpha_2}, 1\}$ implies that either $\alpha_2 \leq \alpha_1$ or

$$\frac{\alpha_1}{\alpha_2} \geq \rho_{12}^F \geq 1-\frac{\bar{\alpha}}{\alpha_2}(1-\alpha_1)$$

and rearranging we obtain that this implies $\alpha_2 \leq \bar{\alpha}+(1-\bar{\alpha})\alpha_1$. Therefore, continuing in this fashion we see that our mechanism guarantees for any $\bar{\alpha}$ and bank $k \in I$, that $\alpha_k \leq \bar{\alpha}+(1-\bar{\alpha})\alpha_{k-1}$ (mod n) as illustrated in Figure 1. Hence, given that the right hand side of this inequality is an increasing sequence we can see that whenever $\alpha_n \leq \min\{\alpha^\star, b_n\}$, then $\alpha_k \leq \min\{\alpha^\star, b_n\}$ for all $k = 1, 2, ..., n-1$. Now, solving $\alpha_n \leq \bar{\alpha}+(1-\bar{\alpha})\alpha_{n-1}$ recursively, we obtain

$$\alpha_n \leq \bar{\alpha}+(1-\bar{\alpha})\bar{\alpha}\sum_{j=0}^{n-2}(1-\bar{\alpha})^j = 1-(1-\bar{\alpha})^n$$

and therefore, whenever $\bar{\alpha} \leq 1-(1-\min\{\alpha^\star, b_n\})^{\frac{1}{n}}$, then $\alpha_n \leq 1-(1-\bar{\alpha})^n \leq \min\{\alpha^\star, b_n\}$ and therefore the mechanism guarantees $\alpha_k \leq \alpha^\star$ for all $k = 1, 2, ..., n$.

To see that the mechanism guarantees budget balance, we simply note that the worst case cost of the guarantee to Bank n is $A_n - K_n$ and therefore the worst case expected cost of the guarantee is $\alpha_n(A_n - K_n)$, but the mechanism guarantees that $\alpha_n \leq \min\{\alpha^\star, b_n\}$ which implies that the worst case expected cost $\alpha_n(A_n - K_n) \leq b_n(A_n - K_n) = \hat{T}$, by construction. □

4 Subordinated Debt Pricing

In this section we will discuss the implication of adding a subordinated debt minimum to the mechanism $\mathcal{M}$ when creditors are perfectly informed regarding the bank's risk. We assume that the bank issues subordinated debt in an over the counter fashion by soliciting potential creditors and bargaining over the interest rate. In what follows we will consider the general case whereby the bank's creditor may have limited liability. For simplicity we will assume this other creditor to be a bank, but stress that this need not be the case. For example, some non-bank institutions such as insurance funds and corporations would have limited liability due to their financing structure (i.e. the amount of outstanding private debt and public bonds) but this does not preclude managers of 100% equity financed institutions from having limited liability. For example, while hedge/mutual funds are 100% equity financed, they are prone to failure (just like banks) in the face of large losses. Of course the mechanism for the failure of a fund is not the failure to repay debts, but rather the inability to retain equity as investors withdraw their funds given losses, creating a liquidity shock and subsequent fire sale of the funds assets. Equivalently, even though hedge funds are equity financed they are generally highly leveraged through borrowing of assets via short sales. In this case, a hedge fund can collapse when facing losses by failing to make margin payments on its borrowed assets (e.g. the failure of Long Term Capital Management). In this sense we could take the *capital* of a hedge fund to be a certain threshold such that whenever losses exceed this threshold then it is almost certain that the fund's investors would demand their money back and force an untimely liquidation of the portfolio. Finally, compounding this idea of limited liability via the financing structure of the creditor, limited liability naturally shows up in the incentive contracts of fund managers who in general are not forced to bear large losses on the portfolios in which they manage (see e.g. Rajan and Srivastava (2000)). As we will see, such incentives will lead managers naturally to *double down* on risky investments if they can be relatively sure that such investments go sour only in states of the world where the fund would fail anyway.

In light of this, we will now show that when creditors have limited liability, the resulting interest rate after the bargaining process is in general a function of the correlation between the bank's failure and its creditor's failure. After we introduce some notation we will explain how this correlation between failures can also be interpreted as an implicit guarantee to the creditor (e.g. a bailout to the creditor in the face of the bank's failure). Let us first note that the distribution of returns of the bank, f, and the returns of its creditor's portfolio, denoted by f_C, in general follow a joint distribution h such that

$$\int_{-\infty}^{+\infty} h(x, x_C)dx = f_C(x_C)$$

and

$$\int_{-\infty}^{+\infty} h(x, x_C)dx_C = f(x)$$

Now, given this notation, we can revisit the required rate of return on the bank's subordinated debt. Namely the creditor is indifferent between lending S dollars of subordinated debt to the bank and investing S dollars in the risk free asset whenever

$$P(x \geq -K | x_C \geq -K_C)(1 + r) - P(x \leq -K | x_C \geq -K_C) = 1 + r_0 \tag{3}$$

where K_C is either the equity capital of the creditor if it is a debt financed institution, or the above mentioned threshold if it is another institution.[16] We will proceed to analyze the correlation of "failure" of the bank and its creditor by assuming, without loss, that for any level of risk α, it has the structure induced by h represented in Table 1 where "Failure B (C)" and "Success B (C)" represent the event that the bank (creditor) fails or succeeds respectively. Here $\rho^F \in [0, 1]$ and $\rho^S \in [0, 1]$ (the true domains are subsets of $[0, 1]$ as will be formulated below) represent respectively the correlation of failure and success of the bank and its creditor. In this case, if $\rho^F = 1$, then the probability that the creditor fails conditional on the bank failing is equal to 1. One explanation for this could be a scenario where the set of states of the world where the bank fails is a subset of the set of the states of the world where the creditor fails. On the other hand,

[16] Naturally, we can also interpret K_C as a combination of the two situations where a creditor is both debt financed and faces a run on its equity.

	$Failure\ C$	$Survival\ C$
$Failure\ B$	$\alpha\rho^F$	$\alpha(1-\rho^F)$
$Survival\ B$	$(1-\alpha)(1-\rho^S)$	$(1-\alpha)\rho^S$

Table 1: Joint probability $P(E \cap E_C)$ of event $E \in \{Failure\ B, Survival\ B\}$ and event $E_C \in \{Failure\ C, Survival\ C\}$.

if $\rho^F = 0$, then the probability that the creditor fails, conditional on the bank failing is equal to 0; the set of states in which the bank fails is disjoint from the set of states where the creditor fails. Similarly $\rho^S = 1$ represents perfectly correlated conditional success and $\rho^S = 0$ implies perfectly diversified conditional success. Lastly, if $\rho^F = \alpha_C$ and $\rho^S = 1 - \alpha_C$, then we are in the case of independent success/failure of the creditor conditional on the success/failure of the bank: $P(Failure\ C \mid Failure\ B) = P(Failure\ C \mid Success\ B) = \alpha_C$ which is equivalent to the case where creditors have limited liability.

Now, we can continue our analysis in this light. To start, using the parameterization of Table 1, the required rate of return given by equation (3) can now be formulated as

$$\frac{(1-\alpha)\rho^S}{\alpha(1-\rho^F)+(1-\alpha)\rho^S}(1+r) - \frac{\alpha(1-\rho^F)}{\alpha(1-\rho^F)+(1-\alpha)\rho^S} = 1$$

or equivalently

$$r = \frac{2\alpha}{1-\alpha}\frac{1-\rho^F}{\rho^S}$$

therefore whenever $1-\rho^F < \rho^S$ the required rate of return on the bank's subordinated debt is less when the creditor has limited liability. What this condition states is that the more correlated are the success and failure of the banks portfolio with its creditors portfolio, the lower the required rate of return is on the banks subordinated debt. Furthermore, the more correlated are the assets of the bank and its creditor, the less the probability of default of the bank, α, affects the required rate of return.

In order to model the over the counter (OTC) process by which subordinated debt (and other debt) is issued we borrow from Afonso and Lagos (2015) who model interbank OTC lending as a Nash bargaining problem. Namely, we assume the bank solicits creditors with which it bargains over the interest rate on the subordinated debt issuance with dollar value $S := D(J)$. We assume that debt contracts only pay principal plus interest at maturity and therefore coupon payments are zero. If the creditor (C) is perfectly informed and fully liable, then their surplus from the bargaining is

$$V_C := r - \frac{2\alpha(K)}{1 - \alpha(K)}$$

where r is the rate at which it lends to the bank. Namely, given that the subordinated bond pays $(1 + r)S$ in the even of solvency and 0 (hence a payoff of $-S$ given that the creditor loses his principal) the break even condition for a risk neutral creditor is

$$(1 - \alpha(K))(1 + r) - \alpha(K) \geq (1 + r_0)$$

and therefore the creditors surplus is the rate they receive minus the required rate of return $\frac{2\alpha(K)}{1-\alpha(K)}$.

Similarly, the surplus for the bank from lending at a rate r, is

$$V := \bar{r} - r$$

where $\bar{r}$ is the bank's outside option. Given that this is difficult to quantify in a setting where there are no centralized markets for subordinated bank debt, we will introduce a parameter here $\Delta(S, K, f) \geq 0$ which represents the risk/liquidity premium that the bank faces when forced to issue subordinated debt in a short period of time (e.g. to meet the minimum requirement). In this case, given that $\Delta(S, K, f)$ is an arbitrary cost, dependent on the amount of debt S to be issued, the level of risk (deduced from f and K), and potentially the asset class $C_l \ni f$, then we will write

$$\bar{r} := \frac{2\alpha(K)}{1 - \alpha(K)} + \Delta(S, K, f)$$

namely, the bank expects to pay at least the fair value rate $\frac{2\alpha(f,K)}{1-\alpha(f,K)}$ plus some risk/liquidity premium.

Now that we have defined the surplus from trade, we assume that the bank bargains with some creditor with the solution being the Nash bargaining solution (see Rubinstein and Wolinsky (1985) and Duffie et. al (2007) for a justification of this solution) whereby the interest rate maximizes the joint surplus

$$V_C^{\theta} \cdot V^{1-\theta} = (r - \frac{2\alpha(K)}{1-\alpha(K)})^{\theta}(\frac{2\alpha(K)}{1-\alpha(K)} + \Delta(S, K, f) - r)^{1-\theta}$$

where θ is the bargaining power of the creditor (Rubinstein and Wolinsky (1985) explicitly calculate θ as a result of the timing and conditions of the bargaining game). Which leads to a solution of

$$r_S := \frac{2\alpha(K)}{1-\alpha(K)} + \theta \cdot \Delta(S, K, f)$$

Now, under our OTC bargaining assumptions we now arrive to a rate of interest

$$r_S := \frac{2\alpha}{1-\alpha}\frac{1-\rho^F}{\rho^S} + \theta \cdot \Delta(S, K, f)$$

Therefore, if the bank has some expectation of the correlation between its return and it's creditors portfolio returns, i.e. ρ^F and ρ^S, then it expects to pay r_S. Furthermore, given that r_S is decreasing in both ρ^F and ρ^S, the bank, prior to bargaining over the terms of the loan with its creditor will have an incentive to adjust its portfolio provided that it is not too costly to do so. [Namely, we will now show that if the cost for the bank to adjust its portfolio to increase ρ^F and ρ^S is small, then not only would the bank prefer to do so, but it would subsequently prefer to increase its default risk α as well.]

First, we note that for a fixed α and α_C, we can pin down ρ^S as a function of ρ^F. Namely, due to the fact that $P(F_i|S_j) + P(S_i|S_j) = 1$, then we obtain

$$\frac{\alpha}{1-\alpha_C}(1-\rho^F) + \frac{1-\alpha}{1-\alpha_C}\rho^S = 1$$

and therefore

$$\rho^S = \frac{1-\alpha_C}{1-\alpha} - \frac{\alpha}{1-\alpha}(1-\rho^F)$$

Finally, if we pin down ρ^S, then using the fact that $\rho^S \in [0,1]$ we obtain that

$$max\{\frac{\alpha+\alpha_C-1}{\alpha}, 0\} \leq \rho^F \leq min\{\frac{\alpha_C}{\alpha}, 1\} =: \bar{\rho}^F$$

Therefore, for any fixed α, α_C, and ρ^F in the above range,

$$r_S(\alpha; \alpha_C, \rho^F) = \frac{2\alpha(1-\rho^F)}{1-\alpha_C-\alpha(1-\rho^F)} + \theta \cdot \Delta(S, K, f) \tag{4}$$

Furthermore, utilizing the same technique we can express the interest rate on priority ψ debt as

$$r_\psi(f; \alpha_C, \rho^F_\psi) = \frac{2(1-\rho^F_\psi)\int_{-\infty}^{-\mathcal{K}(\psi,K)} f(x)dx}{1-\alpha_C-(1-\rho^F_\psi)\int_{-\infty}^{-\mathcal{K}(\psi,K)} f(x)dx} + \theta \cdot \Delta_\psi(S, K, f).$$

There are two important distinctions to be made here. First, while in general priority ψ debt depends on the distribution of returns f, subordinated debt only depends on the probability of default $\alpha(f, K)$.[17] Second, $\rho^F_\psi \geq \rho^F$ for all $\psi = 0, 1, ..., J-1$. To see why this second point is true, note that ρ^F is defined as the probability of failure of the creditor (losses greater than equity), conditional on the bank's failure. In contrast, ρ^F_ψ is the probability of failure of the creditor conditional on the bank's losses greater than $\mathcal{K}(\psi, K) \geq K$. Therefore, the event "losses greater than $\mathcal{K}(\psi, K)$" is a subset of the event "losses greater than K."

[17]Namely, you can have two distribution of returns with the same probability of failure that command different interest rates on priority ψ debt while the only difference in the interest rates on subordinated debt would be due to the differences in the risk/liquidity premium.

4.1 Robust Bounds When Banks Have Biased Risk Estimates

Now that we have constructed an $\alpha^\star$-informationally robust mechanism, a natural question to ask is how this mechanism is affected if the bank's do not perfectly evaluate each others risks. First, we note that the above mechanism remains robust to imperfect risk evaluation as long as each bank believes that its crediting bank will be able to correctly evaluate its risk *on average*. What this means is that there is noise in Bank i's evaluation of the risk of Bank j so that whenever Bank j's true risk is α_j, then Bank i believes its risk to be $\alpha_j + \beta_{ij}$ where β_{ij} is some random error, then if $\mathbb{E}(\beta_{ij}) = 0$, our mechanism is still robust, no matter the variance of β_{ij}. Still, it may be the case that Bank j receives some signal regarding its true level of risk and therefore knows that Bank i's estimate of its risk will biased. In this case, $\mathbb{E}(\beta_j) = -\epsilon_i$ and Bank j believes that Bank i will under estimate the true risk of Bank j which will lead to a violation of our $\alpha^\star$ bound. Our next result shows that for any number of guarantees g the regulator is willing to make and any number e of subordinated debt exposures the regulator is willing to impose on each bank, we can construct the optimal worst case level of risk $\delta(g, e)$ for any vector of under evaluation biases $(-\epsilon_1, ..., -\epsilon_n)$, and the network of exposures that reaches that bound.

In what follows, it will be useful to use the notation $\epsilon^1, ..., \epsilon^n$, where ϵ^k is the k^{th} largest bias. Given that the regulator is unaware of which Bank i has the k^{th} largest bias (i.e. $\epsilon_i = \epsilon^k$) we will frame our results in terms of ϵ^k instead of ϵ_i.

Theorem 3 Suppose the regulator is willing to provide g failure guarantees and require each bank to issue a minimum level of subordinated debt to at most e other banks. Then the optimal bound $\delta(g, e)$ satisfies

$$\delta(\frac{n(n-1)}{2}, n-1) = \epsilon^2 \leq \delta(g, e) \leq \epsilon^n + \epsilon^{n-1} = \delta(1, 1)$$

5 Incentives for Banks to Increase Failure Correlation

In this section, we will examine the tradeoff between robustness and the incentives for banks to increase the correlation of failure between their portfolio's and those of their perspective subordinated debt creditors created by robust mechanisms.

In order to proceed, we will first translate the bank's maximization problem to a version that will be easier to work with. Namely, we have shown above that δ-robust mechanisms do not require return specific transfers and can rely solely on capital and subordinated debt requirements (as opposed to other higher priority debt requirements). Therefore, in what follows, the only relevant statistic of the bank's distribution of returns f that matter to the regulator when utilizing a δ-robust mechanism is the probability of default $\alpha(f, K)$. Therefore, if the bank optimally has a probability of default of $\hat{\alpha}$ resulting from the chosen distribution of returns f and capital level K (i.e. $\alpha(f, K) = \hat{\alpha}$) then for all f' with $\alpha(f', K) = \hat{\alpha}$, optimality requires that the expected returns conditional on solvency of f must be weakly greater than the expected conditional returns of f': $\mathbb{E}_f[x|x \geq -K] \geq \mathbb{E}_{f'}[x|x \geq -K]$. Hence, if we denote by

$$f_{\hat{\alpha}}[K] := \underset{\substack{f \in \mathcal{C} \\ \alpha(f,K) := \hat{\alpha}}}{\operatorname{argmax}} \; E_f[x|x \geq -K]$$

the distribution that maximizes expected conditional returns with probability of default exactly equal to $\hat{\alpha}$ and equity capital K, we can simply assume that *ceteris paribus* a bank with probability of default $\hat{\alpha}$ has a distribution of returns $f_{\hat{\alpha}}[K]$. Therefore, denoting by $\alpha(K) := \alpha(K, f_{\alpha(K)}[K])$ and letting $R(\alpha(K))$ denote the maximal conditional returns for any level of risk $\alpha(K)$ such that

$$R(\alpha(K)) := \mathbb{E}_{f_{\alpha(K)}[K]}[x|x \geq -K]$$

we can rewrite the bank's optimization problem in terms of maximizing the *level of risk* $\alpha(K)$ (as opposed to the distribution of returns) in the form of [18]

$$\max_{\substack{\alpha(K)\\K,D}} \; R(\alpha(K) - c(K)) - \sum_{\psi} r_\psi D_\psi$$

$$\text{subject to} \;\; K + \sum_{\psi} D_\psi = A$$

Finally, before proceeding we will make one necessary technical assumption regarding $R(\alpha(K))$.

Assumption 1 For all $K \geq 0$, $R(\alpha(K))$ is piecewise continuous and concave increasing in $\alpha(K)$.

Namely, Assumption 1 states that for a given level of equity, higher levels of risk lead to weakly higher returns and that this relationship between risk and return is concave. Now, if there is no cost for the bank to adjust its portfolio and therefore ρ^F, the bank would optimally set $\rho^F = \bar{\rho}^F$ (or as close as possible). More realistically, there is a cost to the bank associated with adjusting ρ^F as in general bank's specialize over time in investing in a particular industry. Therefore, we introduce a cost $\gamma(\rho'; \rho)$ to the bank for altering its portfolio's failure correlation with a specific creditor from ρ to ρ' and assume it satisfies the following natural assumptions:

Assumption 2 $\gamma(\rho'|\rho)$ is concave decreasing on the interval $[\underline{\rho}^F, \rho)$, convex increasing on the interval $[\rho, \bar{\rho}^F]$, and $\gamma(\rho|\rho) = 0$.

$$\max_{\substack{\alpha(K),\rho\\K,S,D_1,D_0}} \; R(\alpha(K)) - c(K) - r_S(\alpha(K), \rho^F)S - \sum_{\psi=0}^{J-1} r_\psi \cdot D_\psi - \gamma(\rho|\rho^0)$$

$$subject\ to\ K + \sum_{\psi=0}^{J-1} D_\psi + S = 1$$

$$K \geq 0, S \geq 0, D_1 \geq 0$$

$$0 \leq D_0 \leq \bar{D}$$

[18] $c(K, D)$ or $(1 - \alpha(K))c(K, D)$?

The following proposition states how adding a subordinated debt requirement $\bar{S}$ to the bank's optimization problem will affect its incentives for risk taking and correlation.

Proposition 1. *Let* $(\alpha^0(K^0), \rho^0, S^0, K^0, D_1^0, D_0^0)$ *be the solution to Bank* i*'s investment problem without a mandatory subordinated debt requirement. If the requirement* $\underline{S}$ *is binding (*$S^0 < \underline{S}$*) and the correlation between the bank's and creditor's failure is not perfect (*$\rho^0 < \bar{\rho}^F$*), then*
(1) The bank optimally increases its correlation of failure with its creditor, ρ^F*, after the mandatory subordinated debt requirement is implemented.*
(2) If the change in ρ^F *in response to the mandatory subordinated debt requirement is large, or the difference between* S^* *and* $\underline{S}$ *is small, then Bank* i *optimally increases its probability of failure* $\alpha(K)$.

6 Conclusion

In this paper we have studied the use of informationally robust mechanisms for the use of banking regulation. We have shown that when the regulator can rely on market discipline, then there exists an informationally robust mechanism that can limit the probability of default the bank by any arbitrary amount. Our key insight is that market discipline may not be effective in limiting the risk taking of the banks and crucially depends on the correlation of the portfolio's of the bank and its creditor. We show how this issue can be overcome with a guarantee made to the bank's creditor to ensure that they internalize the loss on the bank's subordinated debt (in the event of the bank's failure). When providing such a guarantee, the regulator can ensure that the creditor appropriately prices the bank's subordinated debt, in which case, a minimum subordinated debt requirement + leverage specific interest rate ceiling can guarantee the bank's risk falls below any arbitrarily chosen threshold. Finally we show that when the regulator internalizes the subordinated debt minimum by requiring banks to issue subordinated debt to other regulated banks, then he can arbitrarily bound the level of risk in the banking system while satisfying his budget constraint.

7 Appendix

7.1 Proof of Theorem 1

In what follows, it will be very important for us to know what the possible reports of the bank can be given its true type. As we will see, the observable $\mathbf{r}$ will play a role in this. In what follows, we will denote by $\mathcal{I}(f, K, D)$ the set of interest rates *compatible* with (f, K, D). Namely,

$$\mathcal{I}(f, K, D) := \{(\mathbf{r} \in [0,1]^{J+1} \mid r_\psi \leq \frac{2\int_{\infty}^{-\mathcal{K}(\psi,K)} f(x)dx}{1 - \int_{\infty}^{-\mathcal{K}(\psi,K)} f(x)dx} \text{ for all } \psi = 0, 1, ..., J\}$$

is the set of interest rates $\mathbf{r}$ such that each r_ψ is less than or equal to the fair value of the bank's priority ψ debt given its report f and capital K. The question then, is to understand to what extent the regulator can use this information whenever it is unaware of α_C and ρ. Namely, for large misspecification ϵ, the regulators only observable information about the true distribution of the banks returns, f, given a report $\tilde{f}$ is that $f \in \{f' : \mathbf{r} \in \mathcal{I}(f', K)\}$. This leads us to the following definition.

Definition : We say $(f, \mathbf{r}, K, D)$ and $(\tilde{f}, \tilde{\mathbf{r}}, \tilde{K}, \tilde{D})$ are *indistinguishable* under $\mathcal{M} = (\mathbf{T}, \mathbf{K}, \mathbf{D})$ if the following two conditions are satisfied:

(1) $K \in \mathbf{K}(f)$, $D \in \mathbf{D}(f)$, $\tilde{K} \in \mathbf{K}(\tilde{f})$, and $\tilde{D} \in \mathbf{D}(\tilde{f})$.

(2) $\tilde{\mathbf{r}} \in \mathcal{I}(f, \tilde{K}, \tilde{D})$ and $\mathbf{r} \in \mathcal{I}(\tilde{f}, K, D)$.

Proof. As a first step we will prove the following claim:

Claim 1 If the mechanism $\mathcal{M} = (\mathbf{T}, \mathbf{K}, \mathbf{D})$ is δ-informationally robust and optimal, then it must be the case that $\mathbf{K}(f) = \mathbf{K}(\tilde{f})$, $\mathbf{D}_\psi(f) = \mathbf{D}_\psi(\tilde{f})$ for all $\psi = 0, ..., J$, $T_R(\cdot|f, \mathbf{r}, K, D) = T_R(\cdot|\tilde{f}, \tilde{\mathbf{r}}, \tilde{K}, \tilde{D})$, and $T(f, \mathbf{r}, K, D) = T(\tilde{f}, \tilde{\mathbf{r}}, \tilde{K}, \tilde{D})$ whenever $(f, \mathbf{r}, K, D)$ and $(\tilde{f}, \tilde{\mathbf{r}}, \tilde{K}, \tilde{D})$ are indistinguishable under $\mathcal{M}$.

Proof. Let us suppose that $\mathcal{M}$ is δ-robust for some δ. In order to prove Claim 1, we need to first describe the bank's optimization problem under $\mathcal{M}$.

The bank's optimization problem under $\mathcal{M}$

$$\max_{\substack{f,\tilde{f}\\K,D}} (1-\alpha(f,K))\int_{-K}^{+\infty} f(x)(x-T_R(x|\tilde{f},\mathbf{r},K,D))dx - c(K) - \sum_{\psi} D_\psi r_\psi - T(\tilde{f},\mathbf{r},K,D) \quad (5)$$

$$\text{subject to } \; K \in \mathbf{K}(\tilde{f}), D \in \mathbf{D}(\tilde{f})$$
$$K + \sum_{\psi} D_\psi = A, \text{ and } D_0 \leq \bar{D}$$

Further, if $\mathcal{M}$ is δ robust, then it must satisfy the following incentive compatibility constraint

$$(1-\alpha(f,K))\int_{-K}^{+\infty} f(x)(x-T_R(x|f,\mathbf{r},K,D))dx - c(K) - \sum_{\psi} D_\psi r_\psi - T(f,\mathbf{r},K,D) \geq$$

$$(1-\alpha(f,K))\int_{-K}^{+\infty} f(x)(x-T_R(x|\tilde{f},\tilde{\mathbf{r}},\tilde{K},\tilde{D}))dx - c(\tilde{K}) - \sum_{\psi} \tilde{D}_\psi \tilde{r}_\psi - T(\tilde{f},\tilde{\mathbf{r}},\tilde{K},\tilde{D}) \quad (6)$$

for all $\tilde{f} \in \Delta(\mathbb{R})$, $\tilde{K} \in \mathbf{K}(\tilde{f})$, $\tilde{D} \in \mathbf{D}(\tilde{f})$, $\mathbf{r} \in \mathcal{I}(f,K,D)$ and $\tilde{\mathbf{r}} \in \mathcal{I}(f,\tilde{K},\tilde{D})$. Important to note here is that we restrict $\tilde{\mathbf{r}} \in \mathcal{I}(f,\tilde{K},\tilde{D})$ instead of $\mathcal{I}(\tilde{f},\tilde{K},\tilde{D})$ because the bank's true distribution of returns is f. Then, rearranging Inequality 6 we obtain

$$(1-\alpha(f,K))\left(\int_{-K}^{+\infty} f(x)(T_R(x|\tilde{f},\tilde{\mathbf{r}},\tilde{K},\tilde{D}) - T_R(x|f,\mathbf{r},K,D))dx + \int_{-K}^{-\tilde{K}} f(x)T_R(x|\tilde{f},\tilde{\mathbf{r}},\tilde{K},\tilde{D})dx\right) \geq$$

$$c(K) - c(\tilde{K}) + \sum_{\psi}(D_\psi r_\psi - \tilde{D}_\psi \tilde{r}_\psi) + T(f,\mathbf{r},K,D) - T(\tilde{f},\tilde{\mathbf{r}},\tilde{K},\tilde{D}) \quad (7)$$

As a first step to showing that these incentive constraints cannot be satisfied for all $\epsilon > 0$ unless the mechanism satisfies the conditions of our claim, we will introduce the notation that for any $(\tilde{f}, \tilde{\mathbf{r}}, \tilde{K}, \tilde{D})$ we denote by $X(\tilde{f}, \tilde{\mathbf{r}}, \tilde{K}, \tilde{D}) \subseteq \mathbb{R}$ the set of returns such that $T_R(x|f, \mathbf{r}, K, D) \geq T_R(x|\tilde{f}, \tilde{\mathbf{r}}, \tilde{K}, \tilde{D})$ and $Y(\tilde{f}, \tilde{\mathbf{r}}, \tilde{K}, \tilde{D}) \subseteq \mathbb{R}$ such that $T_R(x|f, \mathbf{r}, K, D) < T_R(x|\tilde{f}, \tilde{\mathbf{r}}, \tilde{K}, \tilde{D})$. Now, let $f_0 \in C_l^0$ be the distribution of returns in the class l (assumed to be the same class as f) that minimizes the LHS of (7).

Now, assume for the moment that $f_0(Y(\tilde{f}, \tilde{\mathbf{r}}, \tilde{K}, \tilde{D})) > 0$ and let us introduce the following function $f' \in \Delta(\mathbb{R})$ such that

$$f'(x) = \begin{cases} f_0(x)(1 + \frac{\epsilon}{f_0(Y(\tilde{f}, \tilde{\mathbf{r}}, \tilde{K}, \tilde{D}))}) & \text{if } x \in X(\tilde{f}, \tilde{\mathbf{r}}, \tilde{K}, \tilde{D}) \\ f_0(x)(1 - \frac{\epsilon}{f_0(Y(\tilde{f}, \tilde{\mathbf{r}}, \tilde{K}, \tilde{D}))}) & \text{if } x \in Y(\tilde{f}, \tilde{\mathbf{r}}, \tilde{K}, \tilde{D}) \end{cases}$$

It is easy to check that f' is a well defined pdf for small ϵ, that $\int_x |f'(x) - f^0(x)| dq = \epsilon$, and that the LHS of (7) evaluated at f' is

$$(1-\alpha(f_0, K))(\int_{-K}^{+\infty} f_0(x)(T_R(x|\tilde{f}, \tilde{\mathbf{r}}, \tilde{K}, \tilde{D}) - T_R(x|f, \mathbf{r}, K, D))dx + \int_{-K}^{-\tilde{K}} f_0(x) T_R(x|\tilde{f}, \tilde{\mathbf{r}}, \tilde{K}, \tilde{D})dx)$$

$$-\epsilon_f \int_{-K}^{+\infty} f_0(x) |T_R(x|\tilde{f}, \tilde{\mathbf{r}}, \tilde{K}, \tilde{D}) - T_R(x|f, \mathbf{r}, K, D)| dx$$

which shows that as $-\epsilon_f$ increases, an upper bound for the LHS of (7) is

$$(1 - \alpha(f_0, K)) \int_{-K}^{-\tilde{K}} f_0(x) T_R(x|\tilde{f}, \tilde{\mathbf{r}}, \tilde{K}, \tilde{D}) dx. \tag{8}$$

Technically all we need to show is that once $\epsilon_f = Y(\tilde{f}, \tilde{r}, \tilde{K}, \tilde{D})$, then

$$(1 - \alpha(f_0, K)) \int_{-K}^{+\infty} f_0(x)(T_R(x|\tilde{f}, \tilde{\mathbf{r}}, \tilde{K}, \tilde{D}) - T_R(x|f, \mathbf{r}, K, D))dx < 0$$

but f' may not be properly defined for large ϵ_f. Whenever this is the case, it implies that when shifting the mass f_0 puts on $Y(\tilde{f}, \tilde{\mathbf{r}}, \tilde{K}, \tilde{D})$ to $X(\tilde{f}, \tilde{\mathbf{r}}, \tilde{K}, \tilde{D})$ in the fashion defined above, there is some subset $B \subset X(\tilde{f}, \tilde{\mathbf{r}}, \tilde{K}, \tilde{D})$ such that $f'(B) > 1$. In order to resolve this we note that we can always shift the mass away from x while keeping it in the range $X(\tilde{f}, \tilde{\mathbf{r}}, \tilde{K}, \tilde{D})$ to produce a well defined pdf f' that puts zero mass on $Y(\tilde{f}, \tilde{\mathbf{r}}, \tilde{K}, \tilde{D})$. This is due to the fact that $f_0(X(\tilde{f}, \tilde{\mathbf{r}}, \tilde{K}, \tilde{D})) = 1 - f_0(Y(\tilde{f}, \tilde{\mathbf{r}}, \tilde{K}, \tilde{D}))$. Therefore, in what follows, we will assume without loss that f' is constructed in a well defined way for all $0 < \epsilon \leq f_0(Y(\tilde{f}, \tilde{\mathbf{r}}, \tilde{K}, \tilde{D}))$.

What we would like to show is that there exists $\hat{\epsilon}$ such that for all $\epsilon \geq \hat{\epsilon}$ the quantity given by (8) is less than or equal to zero. To do so, we first assume that $K > \tilde{K}$ as otherwise our claim is satisfied for all ϵ. Next, consider $f'' \in \Delta(\mathbb{R})$ constructed from f_0 as

$$f''(x) = \begin{cases} f_0(x)(1 + \frac{\epsilon}{f_0(Y(\tilde{f}, \tilde{\mathbf{r}}, \tilde{K}, \tilde{D}) \backslash [-K, -\tilde{K}])} + \frac{f_0([-K, -\tilde{K}])}{f_0(Y(\tilde{f}, \tilde{\mathbf{r}}, \tilde{K}, \tilde{D}) \backslash [-K, -\tilde{K}])}) & \text{if } x \in X(\tilde{f}, \tilde{\mathbf{r}}, \tilde{K}, \tilde{D}) \backslash [-K, -\tilde{K}] \\ f_0(x)_{(}1 - \frac{\epsilon}{f_0(Y(\tilde{f}, \tilde{\mathbf{r}}, \tilde{K}, \tilde{D}) \backslash [-K, -\tilde{K}])}) & \text{if } x \in Y(\tilde{f}, \tilde{\mathbf{r}}, \tilde{K}, \tilde{D}) \backslash [-K, -\tilde{K}] \\ 0 & \text{if } x \in [-K, -\tilde{K}] \end{cases}$$

Hence, $f''(x)$ mimics what $f'(x)$ does by starting with f_0 and shifting mass from $Y(\tilde{f}, \tilde{\mathbf{r}}, \tilde{K}, \tilde{D})$ to $X(\tilde{f}, \tilde{\mathbf{r}}, \tilde{K}, \tilde{D})$ but then also shifts all mass from the region $[-K, -\tilde{K}]$ to the region $X(\tilde{f}, \tilde{\mathbf{r}}, \tilde{K}, \tilde{D}) \backslash [-K, -\tilde{K}]$.[19] Therefore, under $f''(x)$ there exists $\hat{\epsilon}$ such that for all $\epsilon > \hat{\epsilon}$, the LHS of (7) is bounded above by 0. What this implies is that in order for (7) to be satisfied, it must be the case that

$$c(K) - c(\tilde{K}) + \sum_{\psi} (D_\psi r_\psi - \tilde{D}_\psi \tilde{r}_\psi) + T(f, \mathbf{r}, K, D) - T(\tilde{f}, \tilde{\mathbf{r}}, \tilde{K}, \tilde{D}) \leq 0 \tag{9}$$

for all $\tilde{f} \in \Delta(\mathbb{R})$, $\tilde{K} \in \mathbf{K}(\tilde{f})$, $\tilde{D} \in \mathbf{D}(\tilde{f})$, $\tilde{\mathbf{r}} \in \mathcal{I}(f, \tilde{K}, \tilde{D})$, and $\mathbf{r} \in \mathcal{I}(f, K, D)$. Otherwise, there exists an ϵ and a PDF $f \in C_l$ such that $\int_x |f'(x) - f^0(x)| dq = \epsilon$ and the incentive compatibility constraint for f is not satisfied.

Now, we note that if (9) holds for all $\tilde{f} \in \Delta(\mathbb{R})$, $\tilde{K} \in \mathbf{K}(\tilde{f})$, $\tilde{D} \in \mathbf{D}(\tilde{f})$, and $\tilde{\mathbf{r}} \in \mathcal{I}(f, \tilde{K}, \tilde{D})$

[19] Again we can guarantee the existence of f'' that is well defined and puts zero mass on $Y(\tilde{f}, \tilde{\mathbf{r}}, \tilde{K}, \tilde{D}) \cup [-K, \tilde{K}]$ by properly shifting all of the mass from $Y(\tilde{f}, \tilde{\mathbf{r}}, \tilde{K}, \tilde{D}) \cup [-K, \tilde{K}]$ to $X(\tilde{f}, \tilde{\mathbf{r}}, \tilde{K}, \tilde{D}) \backslash [-K, \tilde{K}]$ and then appropriately shifting mass within $X(\tilde{f}, \tilde{\mathbf{r}}, \tilde{K}, \tilde{D}) \backslash [-K, \tilde{K}]$.

then looking at the incentive compatibility constraint of a bank with type $(\tilde{f}, \tilde{\mathbf{r}}, \tilde{K}, \tilde{D})$ we obtain

$$(1-\alpha(\tilde{f}, \tilde{K})) \int_{-\tilde{K}}^{+\infty} \tilde{f}(x)(T_R(x|f, \mathbf{r}, K, D) - T_R(x|\tilde{f}, \tilde{\mathbf{r}}, \tilde{K}, \tilde{D}))dx + \int_{-\tilde{K}}^{\tilde{K}} \tilde{f}(x)T_R(x|f, \mathbf{r}, K, D) \geq$$

$$c(\tilde{K}) - c(K) + \sum_{\psi}(\tilde{D}_{\psi}\tilde{r}_{\psi} - D_{\psi}r_{\psi}) + T(\tilde{f}, \tilde{\mathbf{r}}, \tilde{K}, \tilde{D}) - T(f, \mathbf{r}, K, D) \tag{10}$$

for all $f \in \Delta(\mathbb{R})$, $K \in \mathbf{K}(f)$, $D \in \mathbf{D}(f)$, $\mathbf{r} \in \mathcal{I}(\tilde{f}, K, D)$ and $\tilde{\mathbf{r}} \in \mathcal{I}(\tilde{f}, \tilde{K}, \tilde{D})$.

Now, note that using the same method in the construction of f' and f'' (only exchanging the sets $X(\tilde{f}, \tilde{\mathbf{r}}, \tilde{K}, \tilde{D})$ and $Y(\tilde{f}, \tilde{\mathbf{r}}, \tilde{K}, \tilde{D})$), there exists $\tilde{f}$ such that the LHS of (10) is bounded above by zero. Finally, if $(f, \mathbf{r}, K, D)$ and $(\tilde{f}, \tilde{\mathbf{r}}, \tilde{K}, \tilde{D})$ are indistinguishable, then this implies that a type f bank can finance its assets with $\tilde{K}$ and $\tilde{D}$ at a rate $\tilde{\mathbf{r}} \in \mathcal{I}(\tilde{f}, \tilde{K}, \tilde{D}) \cap \mathcal{I}(f, \tilde{K}, \tilde{D})$ and a type $\tilde{f}$ bank can finance its assets with K and D at a rate $\mathbf{r} \in \mathcal{I}(f, K, D) \cap \mathcal{I}(\tilde{f}, K, D)$. Therefore, a necessary condition for both incentive constraints (7) and (10) to be satisfied when $(f, \mathbf{r}, K, D)$ and $(\tilde{f}, \tilde{\mathbf{r}}, \tilde{K}, \tilde{D})$ are indistinguishable is if $T_R(\cdot|f, \mathbf{r}, K, D) = T_R(\cdot|\tilde{f}, \tilde{\mathbf{r}}, \tilde{K}, \tilde{D})$. Further, whenever this is the case, then it must be that

$$\int_{-\tilde{K}}^{\tilde{K}} \tilde{f}(x)T_R(x|f, \mathbf{r}, K, D) = \int_{-K}^{-\tilde{K}} f(x)T_R(x|\tilde{f}, \tilde{\mathbf{r}}, \tilde{K}, \tilde{D})dx) = 0$$

and therefore, incentive compatibility requires

$$c(K) - c(\tilde{K}) + \sum_{\psi}(D_{\psi}r_{\psi} - \tilde{D}_{\psi}\tilde{r}_{\psi}) + T(f, \mathbf{r}, K, D) - T(\tilde{f}, \tilde{\mathbf{r}}, \tilde{K}, \tilde{D}) = 0 \tag{11}$$

Now, we claim that if the regulator sets $T(f, \mathbf{r}, K, D) = T(\tilde{f}, \tilde{\mathbf{r}}, \tilde{K}, \tilde{D})$, and capital requirements $\mathbf{K}'$ and $\mathbf{D}'$ such that $\mathbf{K}'(f) = \mathbf{K}'(\tilde{f}) = \mathbf{K}(f) \cup \mathbf{K}(\tilde{f})$, and $\mathbf{D}'_{\psi}(f) = \mathbf{D}'_{\psi}(\tilde{f}) = \mathbf{D}(f) \cup \mathbf{D}(\tilde{f})$, then whether the bank is of type f or $\tilde{f}$, it optimally chooses $K = \tilde{K}$ and $D = \tilde{D}$ so that (11) is guaranteed to hold. To see why this is true, we simply note that when $T(\cdot|f, \mathbf{r}, K, D) = T(\cdot|\tilde{f}, \tilde{\mathbf{r}}, \tilde{K}, \tilde{D})$ and $T(f, \mathbf{r}, K, D) = T(\tilde{f}, \tilde{\mathbf{r}}, \tilde{K}, \tilde{D})$, then if the type f bank prefers $(\tilde{\mathbf{r}}, \tilde{K}, \tilde{D})$ financing then it will always report $\tilde{f}$ to obtain the opportunity to finance its assets with $(\tilde{\mathbf{r}}, \tilde{K}, \tilde{D})$

instead of f given that there is no longer any cost to reporting $\tilde{f}$. Similarly, the $\tilde{f}$ type will always prefer reporting f if it prefers to finance its assets with $(\mathbf{r}, K, D)$. Therefore, the only way to incentivize the bank to report truthfully is if (K, D) is available to the $\tilde{f}$ type and $(\tilde{K}, \tilde{D})$ is available to the f type: Equation (11) guarantees that the cost of an f type, reporting $\tilde{f}$ and financing with $\tilde{K}$,$\tilde{D}$ is the same as the cost of an $\tilde{f}$ type reporting f and financing with K, D.

Finally, we note that any mechanism that does not set $\mathbf{K}(f) = \mathbf{K}(\tilde{f})$ and $\mathbf{D}(f) = \mathbf{D}(\tilde{f})$ must set either $T(f, \mathbf{r}, K, D) > 0$ or $T(\tilde{f}, \tilde{\mathbf{r}}, \tilde{K}, \tilde{D}) > 0$ in order to retain incentive compatibility. Further, given that (11) holds implies that types $(f, \mathbf{r}, K, D)$ and type $(\tilde{f}, \tilde{\mathbf{r}}, \tilde{K}, \tilde{D})$ face the same financing costs and the same non-return specific penalties regardless of their report, hence setting $T(f, \mathbf{r}, K, D) > 0$ or $T(\tilde{f}, \tilde{\mathbf{r}}, \tilde{K}, \tilde{D}) > 0$ is purely wasteful due to the fact that it does not change the f nor $\tilde{f}$ type bank's incentive to take risk. □

The next step in the proof of Theorem 1 is to prove the following claim:

Claim 2 For any mechanism $\mathcal{M}$ and any $f, \tilde{f} \in \Delta(\mathbb{R})$, $K \in \mathbf{K}(f)$, $\tilde{K} \in \mathbf{K}(\tilde{f})$, $D \in \mathbf{D}(f)$, and $D \in \mathbf{D}(\tilde{f})$ such that $\alpha(f, \tilde{K}) \leq \alpha_C^{max}$ and $\alpha(\tilde{f}, K) \leq \alpha_C^{max}$, there exists $\mathbf{r}$ and $\tilde{\mathbf{r}}$ such that $(f, \mathbf{r}, K, D)$ and $(\tilde{f}, \tilde{\mathbf{r}}, \tilde{K}, \tilde{D})$ are indistinguishable.

Hence, once we prove Claim 2, we have proven that any δ-robust mechanism must be invariant to all types $(f, \mathbf{r}, K, D)$ and $(\tilde{f}, \tilde{\mathbf{r}}, \tilde{K}, \tilde{D})$ with $\alpha(\tilde{f}, K) \leq \alpha_C^{max}$ and $\alpha(\tilde{f}, K) \leq \alpha_C^{max}$ and therefore, in the worst case, allows a bank to take risk up to $\alpha_C^{\max}$.

Proof. In order to prove Claim 2, we will show that for any $f, \tilde{f} \in \Delta(\mathbb{R})$, $K \in \mathbf{K}(f)$, $\tilde{K} \in \mathbf{K}(\tilde{f})$, $D \in \mathbf{D}(f)$, and $D \in \mathbf{D}(\tilde{f})$ such that $\alpha(f, \tilde{K}) \leq \alpha_C^{max}$ and $\alpha(\tilde{f}, K) \leq \alpha_C^{max}$, then $0 \in \mathcal{I}(f, \tilde{K}, \tilde{D})$ and $0 \in \mathcal{I}(\tilde{f}, K, D)$. To show this we simply note that whenever $\alpha(\tilde{f}, K) \leq \alpha_C^{max}$ and $\alpha(f, \tilde{K}) \leq \alpha_C^{max}$, then both types can find a creditor with $\rho^F = 1$. Namely, in order for the banks to finance at a rate 0 it must be the case that

$$\frac{2(\int_{\infty}^{-\mathcal{K}(\psi,\tilde{K})} f(x)dx)(1-\rho^F)}{1-\alpha_C - (\int_{\infty}^{-\mathcal{K}(\psi,\tilde{K})} f(x)dx)(1-\rho^F)} = 0 \quad \text{and} \quad \frac{2(\int_{\infty}^{-\mathcal{K}(\psi,K)} \tilde{f}(x)dx)(1-\rho^F)}{1-\alpha_C - (\int_{\infty}^{-\mathcal{K}(\psi,K)} \tilde{f}(x)dx)(1-\rho^F)} = 0$$

therefore whenever $\rho^F = 1$ this is satisfied. To see that this condition cannot be satisfied whenever $\alpha(\tilde{f}, K) > \alpha_C^{max}$ we simply note that under our parameterization we have

$$\rho^F \leq \min\{\frac{\alpha_C}{\alpha}, 1\}$$

therefore whenever $\alpha(\tilde{f}, K) > \alpha_C^{max}$ the bank cannot guarantee $\rho^F = 1$. □

Now, what we have proven in Claim 2 is that whenever $\alpha(\tilde{f}, K) \leq \alpha_C^{\max}$ and $\alpha(\tilde{f}, K) \leq \alpha_C^{\max}$ then a robust mechanism does not differentiate between a $(f, 0, K, D)$ and $(\tilde{f}, 0, \tilde{K}, \tilde{D})$ type (while satisfying incentive compatibility). Now, while there may be other types that the mechanism does not differentiate between, we have shown that when the mechanism does not differentiate between $(f, 0, K, D)$ and $(\tilde{f}, 0, \tilde{K}, \tilde{D})$, then $\delta \geq \alpha_C^{\max}$. Now, given that *external* creditors are not regulated, there is no way to bound the level of risk $\alpha_C^{\max}$. Therefore, the only way to guarantee any level of robustness is for the regulator to be able to guarantee $\rho^F < 1$. Hence, if the mechanism is $\alpha^\star$-robust then it must guarantee the joint probability of failure between the bank and its creditor is strictly less than 1. What we will now show is that if the creditor is not another regulated bank, then any $\alpha^\star$-informationally robust mechanism has an unbounded worst case expected cost thereby proving that any budget balanced, $\alpha^\star$-informationally robust mechanism must require banks to issue subordinated debt to other banks, in which case the above result implies that the regulator must guarantee the probability of failure between two banks is strictly less than 1.

Now, noting that $\rho^F = P(F_C|F_B)$ where F_B is the event where the bank fails and F_C is the event where the creditor fails, given that the regulator cannot control the returns of the portfolio's of both the bank and the creditor, then the only way to guarantee $\rho^F < 1$ is to guarantee that the creditor does not fail whenever the bank fails. The only way to guarantee this, without being able to control the bank's and creditor's distribution of returns, is to provide an explicit guarantee to the creditor that the regulator will reimburse the creditor's losses to ensure the value of equity of the creditor is strictly positive. Furthermore, if the guarantee ensures the creditor the value of its equity the is strictly larger than the amount of subordinated debt issued S whenever the bank fails, then the regulator can guarantee that the creditor correctly prices the bank's subordinated debt, as if their portfolio returns were independent.

We can now formally state our above claim that such a guarantee will force the creditor to price the bank's debt as if their portfolio returns were independent.

Claim 3 If the regulator offers a $g(S+\eta)$ CF guarantee to the bank's creditor, for any $\eta > 0$, the creditor will optimally require a rate of return r_S on the bank's debt issue of value S equal to the fair value of the banks debt:

$$r_S = \frac{2\alpha(f,K)}{1-\alpha(f,K)}$$

Proof. To prove this claim, we simply note that the guarantee $g(S+\eta)$ ensures that conditional on the bank failing, the creditor succeeds; $P(S_C|F_B) = 1$, and therefore$P(F_B, S_C) = P(F_B)$. Thus, denoting by $\pi(E,r)$ the payoff of the bank's subordinated debt with interest rate r to the creditor in the event $E \in \{(F_B,F_C),(S_B,F_C),(F_B,S_C),(S_B,S_C)\}$, we note that

$$\pi(E,r) = \begin{cases} -S & \text{if } E = (F_B,S_C) \\ (1+r) & \text{if } E = (S_B,S_C) \\ 0 & \text{if } E \in \{(F_B,F_C),(S_B,F_C)\} \end{cases}$$

Further, given that we have normalized $r_0 = 0$, given that the bank's payoff of holding the risk free asset is 1 in state $E \in \{(F_B,S_C),(F_B,S_C)\}$ and 0 in state $E \in \{(F_B,F_C),(S_B,F_C)$ we see that the required rate of return on the banks subordinated debt r_S must satisfy

$$P(S_B,S_C)(1+r_S) - P(F_B,S_C) = P(S_B,S_C) + P(F_B,S_C)$$

Finally, using the fact that $P(F_B,S_C) = P(F_B)$ and therefore $P(S_B,S_C) = 1 - P(F_B)$ and plugging into the above equation we obtain

$$(1-P(F_B)(1+r_S) - P(F_B) = 1$$

Hence, noting that $P(F_B) = \alpha(f, K)$ and rearranging the expression we obtain our result. $\square$

The key point to make here is that the proof of Claim 3 relies on the fact that regardless of the correlation between the bank and its creditor's portfolio, the CF guarantee by the regulator can set the joint probability of failure of the two banks to 0.

What we will now show is that while a CF guarantee can allow the regulator to construct a mechanism that is $\alpha^\star$-informationally robust, the worst case expected cost of any such mechanism will be unbounded. To see why this is the case, we note that the expected cost of the CF guarantee to a creditor with capital K_C and returns f_C is

$$\int_{-\infty}^{-K_C} T_R^C(x) f(x_C|F_B)dx = (\eta + S - K_C)\rho^F - \int_{-\infty}^{-K} x_C f_C(x_C|F_B)dx$$

where $f_C(\cdot|F_B)$ is the distribution of the creditor's returns conditional on the bank's failure. To see that this expression may be unbounded, we note that if the regulator cannot control the size of the creditor's balance sheet, then

$$-\int_{-\infty}^{-K} x_C f_C(x_C|F_B)dx$$

will be unbounded. For example, suppose that the creditor invests X dollars in an asset that pays $X(1+r)$ in the even of the bank's success, and pays 0 otherwise.[20] In this case, the expected increase in the cost of the CF guarantee after the creditor makes this investment would be $\alpha(f, K) \cdot X$ whenever $X \geq K - \eta - S$ is large. More importantly, whenever $X \geq K - \eta - S$, the benefit to the creditor would be $X(1+r)$ so that whenever $r > 0$, the creditor will want to choose X as high as possible. Given that the regulator cannot regulate the size of creditor's balance sheet, this implies that the worst case expected cost of the CF guarantee to unregulated creditors is unbounded and we have proven Theorem 1.

$\square$

[20] For example, if the creditor were an insurance company, then the creditor could achieve this return by selling under priced insurance against the bank's failure.

7.2 Proof of Theorem 3

Proof. Throughout, we assume the regulator sets $\bar{\alpha} = 0$. Then, in the case where $(g, e) = (1, 1)$ we note that the best the regulator can do is to make any single guarantee g_{ij} and then have j issue its subordinated debt to i, and all remaining banks (including i) issue to j. This results in the following network where the bold edge represents the guarantee g_{ij}.

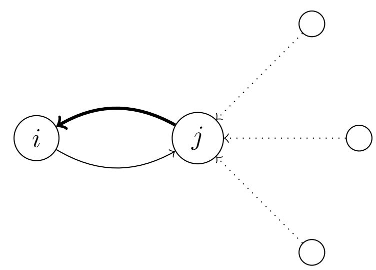

Figure 2: Network of Exposures

Now, when the exposures satisfy the network structure of Figure 2, we know from the guarantee g_{ij} that $\alpha_j \leq \epsilon_i$. Therefore, the risk of any bank l that issues to bank j must satisfy $\alpha_i \leq \epsilon_1 + \epsilon_2$. Finally, given that the regulator does not know which banks are more biased than the others, he might as well randomly assign banks to the nodes in the network of Figure 2, in which case the worst case level of risk is

$$\max_{\substack{i,j,k \in I \\ i \neq j}} \{\epsilon_i + \epsilon_j\} = \epsilon^n + \epsilon^{n-1}$$

To show that the regulator can do no better when $(g, e) = (1, 1)$, we first note that if some bank is not required to issue their debt to Bank j or some other Bank k that issues to Bank j, then their level of risk is potentially unbounded as the guarantee g_{ij} is the only thing that bounds the level of risk in this network. Next, we note that if the regulator had any Bank l issue their subordinated debt to Bank k (who issues to Bank j) instead of Bank j, then the level of risk will satisfy $\alpha_l \leq \epsilon_k + \epsilon_j$ in which case we still achieve our bound. Finally, if Bank l issues to Bank k who issues to Bank k' but not j, and Bank k' issues to Bank j, then the risk of Bank l would satisfy $\alpha_l \leq \epsilon_k + \epsilon_{k'} + \epsilon_j$

which is strictly worse than our bound.

Now, to obtain the best case bound of $\min_{i \in I} \ \epsilon_i$ we will use the complete directed graph $\mathcal{N}^\star = (I, A^\star(I))$ with $A^\star(I) := \{ij : i \in I, j \in I, i \neq j\}$ which has each bank issue to the remaining $n-1$ banks. Further, the regulator will provide a guarantee g_{ij} for each $i, j \in I$ such that $i \neq j$. This amounts to $\frac{n(n-1)}{2}$ guarantees. Now in this case, given that Bank i issues to each Bank j and there is a guarantee g_{ji} for each $j \neq i$, then it must be the case that $\alpha_i \leq \min_{j \neq i} \ \epsilon_j$. Finally, aggregating among all banks, it must be the case that the worst case level if risk is the second lowest bias ϵ^2. Finally, to prove that this is the best bound the regulator can achieve we simply note that any additional subordinated debt issues or guarantees would be redundant given that we already have every bank issue subordinated debt to every other bank, and each bank is guaranteed.

□

7.3 Proof of Proposition 1

Proof. (1) Suppose that the mandatory subordinated debt requirement is binding so that the bank optimally chooses $S^0 < \underline{S}$ prior to the subordinated debt issuance. Then, the FOC for ρ^F of the optimization problem without the minimum subordinated debt requirement is

$$\frac{2\alpha^0(K)(1-\alpha_C)}{(1-\alpha_C-\alpha^0(K)(1-\rho^0))^2} S^0 = \gamma'(\rho^0|\rho) \tag{12}$$

and given that the LHS of (12) is strictly greater than zero implies that if γ is concave decreasing on $[\underline{\rho}^F, \rho)$, then the RHS is negative in that interval. Thus, in order to satisfy the FOC, the solution ρ^0 must be in the interval $(\rho, \bar{\rho}^F]$. Now, if the subordinated debt requirement is binding, it implies that $S^0 < \underline{S}$ and therefore at the unconstrained solution

$$\frac{2\alpha^0(K)(1-\alpha_C)}{(1-\alpha_C-\alpha^0(K)(1-\rho^0))^2} \underline{S} > \gamma'(\rho^0|\rho)$$

and given that $\rho^F \in (\rho, \bar{\rho}^F]$ implies that the firm optimally increases ρ as the LHS is decreasing in ρ and the RHS increasing.

(2) Now, given that the bank optimally increases ρ once the minimum is imposed, we need to check the first order conditions for $\alpha(K)$ as they are a function of ρ. Namely the FOC for $\alpha(K)$ in the unconstrained problem is

$$R_\alpha(\alpha^0(K)) - [r_\alpha^S(\alpha^0(K), \rho^0) + \Delta(f, K, l)]S^0 = 0 \tag{13}$$

therefore, given that $r_\alpha^S(\alpha, \rho)$ is decreasing in ρ, and the solution to $\hat{\rho}$ to the FOC satisfying

$$-\frac{2\alpha^0(K)(1-\alpha_C)}{(1-\alpha_C-\alpha^0(K)(1-\hat{\rho}))^2}\underline{S} = \gamma'(\hat{\rho}|\rho)$$

is strictly greater than ρ^0, then

$$R_\alpha(\alpha^0(K)) - [r_\alpha^S(\alpha^0(K), \hat{\rho}) + \Delta(S, K, f)]S^0 > 0.$$

Finally, if

$$R_\alpha(\alpha^0(K)) - [r_\alpha^S(\alpha^0(K), \hat{\rho}) + \Delta(S, K, f)]\underline{S} > 0 \tag{14}$$

then the bank optimally increases $\alpha(K)$. Specifically, subtracting (13) from (14), we see that this is the case whenever

$$[r_\alpha^S(\alpha^0(K), \rho^0) + \Delta(S, K, f)]S^0 > [r_\alpha^S(\alpha^0(K), \hat{\rho}) + \Delta(S, K, f)]\underline{S}$$

$\square$

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[31] Hett and Schmidt (2016): Bank Rescues and Bailout Expectations.

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Both Hett and Schmidt (2016) and Acarya, Anginer, Warburton (2016) show empirical evidence towards lower market discipline for larger and more systemic banks. Hett and Schmidt (2016) look at bailout expectations as an explanation (both do?).