Tasks, Occupations, and Wage Inequality in an Open Economy*

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Abstract

This paper documents and theoretically explains a nexus between globalization and residual wage inequality through internal labor market reorganization. Combining time-varying within-occupation task information from representative German labor force surveys with linked plant–worker data for Germany, we establish three interrelated facts: (1) larger plants and exporters organize production into more occupations, and (2) workers at larger plants and exporters perform fewer tasks within occupations, while (3) overall and residual wages are more dispersed at larger plants. To explain these facts, we build a model in which the plant endogenously bundles tasks into occupations and workers match to occupations. By splitting the task range into more occupations, the plant can assign workers to a narrower task range per occupation, reducing worker mismatch and raising worker efficiency as well as the within-plant dispersion of wages. Embedding this rationale into a Melitz (Econometrica 2003) model, where fixed span-of-control costs increase with occupation counts, we show that inherently more productive (exporter) plants exhibit higher worker efficiency and wider wage dispersion and that economy-wide wage inequality is higher in the open economy for an empirically confirmed parametrization. Estimation of our model shows that a worker's average number of tasks is inversely related to plant revenues and that the within-plant wage dispersion is positively related to plant size.

Keywords: International trade; firm-internal labor allocation; heterogeneity

JEL Classification: F12, F16, J3, L23

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> "It is the great multiplication of the productions of all the different arts, in consequence of the division of labour, which occasions, in a well-governed society, that universal opulence which extends itself to the lowest ranks of the people."
>
> — Adam Smith (1776): The Wealth Of Nations, Book I, Chapter I

1 Introduction

Recent theories of international trade at the firm level have opened new insights into a nexus between globalization and wage inequality within sectors and occupations. Much of the emphasis to date has been on the wage dispersion between firms, given the wage premia that exporters pay to otherwise similar workers within sectors and occupations (Helpman, Itskhoki and Redding 2010, Egger and Kreickemeier 2009, Davis and Harrigan 2011, Amiti and Davis 2012). The empirical importance of the between-firm or between-plant dispersion of wages for changes in overall wage inequality has been documented for labor markets in general (Card, Heining and Kline 2013, Lopes de Melo 2013, Song et al. 2015) and for labor market outcomes in open economies in particular (Egger, Egger and Kreickemeier 2013, Coşar, Guner and Tybout 2015, Helpman et al. 2017, Eaton, Kortum and Kramarz 2015). In the cross section of workers, however, the commanding component of wage variation is within firms: studies such as Abowd et al. (2001) and Menezes-Filho, Muendler and Ramey (2008), for instance, control for worker and employer characteristics, as well as firm effects, in Mincer regressions and show a dominance of the residual wage component; Lemieux (2006) documents the response of the residual wage component to economic change. In this paper we relate back to the basic principle of the division of labor, within plants and within occupations across workers as well as across countries in the global economy. We explore how the large within-plant part of wage inequality responds to trade—through internal labor-market reorganization—and show that the size of a plant's global product market translates into its internal division of labor, so that global specialization affects inequality across the "ranks of the people."

In his foundational analysis of the division of labor, Adam Smith (1776, Book I, Chapter I) described tasks that a single worker could perform cumulatively or that the employer could alternatively assign to several workers:

> "[M]aking a pin is . . . divided into about eighteen distinct operations. . . . [T]en persons . . . could make among them upwards of forty-eight thousand pins in a day. But if they had all wrought separately and independently . . . they certainly could not each of them have made twenty, perhaps not one pin in a day."

To elicit information on cumulative tasks in Adam Smith's operational sense, and on the organization of the workplace in today's economy, we use the six German Qualifications and Career Surveys (BIBB-BAuA surveys) conducted over the years 1979 through 2012 in six-to-eight-year intervals and build time-consistent measures of workplace activities.[1] The survey data allow us to discern 15 cumulative workplace activities that are consistently reported over the three decades (examples are: Produce Goods; Develop, Research, Construct; Organize, Plan, Prepare; and Oversee, Control Machinery or Processes).[2] Importantly for an understanding of the evolving division of labor between employers, the BIBB-BAuA surveys also allow us to quantify how many tasks workers perform over time and across plants but *within* their occupations (jobs). We combine the task information by occupation, industry, location and plant size with German linked plant–worker data (LIAB). The employer data comes at the plant level and from annual surveys, which are available since the 1990s. We therefore mostly use the four BIBB-BAuA waves since 1992 to map task information at the worker level to LIAB.

Three striking facts emerge. First, larger plants and plants that are predicted to become exporters adopt a wider count of occupations. Second, however, workers at these larger plants perform, on average, narrower ranges of tasks within occupations. In other words, increasing plant-level trade openness promotes the internal division of labor. Third, both overall and residual wages are more dispersed within larger plants, conditional on the occupations of workers. Our hypothesis is that workers differ in their unobserved ability to conduct the tasks in an occupation, so that mismatches result in varying labor efficiencies within an occupation. To the extent that wages are linked to worker efficiency, the ability mismatches generate wage inequality—in accordance with the empirical observation that the major part of residual wage inequality (that is wage dispersion not explained by observable differences of workers) materializes within plants, within layers of hierarchy, and within occupations.

To explain these facts, we propose a model of endogenous occupation choice and task assignments by the employer. Employers can organize the full range of tasks that need to be performed for production into fewer or into more occupations. A smaller count of occupations at a plant implies that the workers in those occupations have to carry out a wider range of tasks. Conversely, in plants with a larger count of occupations, each job only requires a narrower range of tasks to be performed. In other words, the finer the task space is divided into jobs, the more specialized in fewer tasks a worker in each job can become. We postulate that workers have a core ability that makes them most efficient at one particular task in the full task range and monotonically less efficient at tasks that are more distant from their core ability. Workers assortatively match to task ranges that include their core ability. As a consequence, when a plant's task ranges are narrower, then the degree of mismatch between a worker and the tasks is smaller because all of an occupation's tasks are closer to a worker's core ability. Workers are therefore more efficient at plants with more occupations and a finer division of labor.

[1]We combine all six BIBB-BAuA surveys for the first time in this paper, constructing time-consistent task measures. Select survey questions and years have been used in earlier research, for example by Acemoglu and Pischke (1998), Spitz-Oener (2006), Gathmann and Schönberg (2010) and Becker and Muendler (2015).

[2]The BIBB-BAuA surveys also report descriptive task aspects, such as whether work is intensive in cognitive or routine work steps. Those task properties are not necessarily cumulative in Adam Smith's operational sense (see e.g. Becker and Muendler 2015, documenting a roughly constant frequency of those task aspects over the period 1979-2006).

In such a world, all plants would opt to divide the task range into possibly many jobs if this division were costless. To explain finite occupation counts at plants, and the observed larger count of occupations at larger plants, we posit that there is a plant-level fixed cost of operation that increases with the count of occupations. Managing additional occupations and coordinating workers across more specifically defined jobs causes a span of control problem whose cost we assume to increase in the count of occupations. As is well established in Melitz (2003) models with heterogeneous producers, more productive plants recover the fixed costs with their operating profits, and in our framework this implies that the equilibrium outcome results in the adoption of a larger count of occupations at plants that are inherently more productive. In particular, relatively productive plants that also select into exporting will choose more occupations with narrower task ranges compared to non-exporters.

The model has a number of testable features and predictions. Chief among the features is the inverse relationship between a plant's count of occupations and the width of its average task range per occupation. We document that this inverse relationship between occupation counts and task ranges robustly holds, using both linear predictions and instrumental-variable approaches in the spirit of Autor, Dorn and Hanson (2013) and Dauth, Findeisen and Suedekum (2014) by instrumenting endogenous plant-level regressors, such as (export) sales, the occupation counts, and interactions of those two, with foreign market shocks from China and Eastern Europe. The model also offers concrete predictions for the within-plant wage inequality across workers—a dominant component of wage inequality. If a basic coefficient of performance is positive, and thus adds to worker efficiency from specialization on a narrow task range, then more productive, larger plants exhibit higher within-plant wage inequality than less productive, smaller plants. We document in this paper for both overall wages and the residual wage component in German manufacturing data that the within-plant wage dispersion is higher at more productive plants.[3]

The relationship between globalization, technical change and earnings inequality is arguably a crucial concern for policy and economic analysis. Our model lends itself to log-linear relationships that can be estimated structurally, using maximum likelihood and GMM approaches. In this draft, we state the estimable relationship but have to defer the inclusion of structural estimation results, and policy simulations on their basis, to future versions of this paper. Some relationships are directly estimable with individual linear regressions, however. In particular, we find that the semi-elasticity of operational fixed costs with respect to the count of occupations is significant, though not excessively large in magnitude, suggesting that employers have scope to raise occupation

[3]Instead of considering total wages or wage residuals that exclude the part predicted by worker characteristics, as we do here, Bombardini, Orefice and Tito (2015) focus on a permanent worker-specific and time invariant wage component, measured alternatively with average observed lifetime earnings or with a worker fixed effect following Abowd, Creecy and Kramarz (2002). They find that the long-term wage component is less dispersed at larger, more productive French manufacturing firms. In light of our model, the use of a permanent worker-specific and time invariant wage component to proxy worker efficiency is akin to working with a negative coefficient of performance.

counts and narrow task ranges per occupation. We also find that the estimated coefficient of performance, which regulates the relationship between plant size and within-plant wage dispersion in the model, is strictly positive for both overall wages and residual wages.[4] This result reconfirms, in the light of our model, our reduced-form regression results that more productive plants exhibit higher within-plant wage inequality.

The positive estimate for the model's coefficient of performance has direct implications for welfare and wage inequality. In the model, opening up to trade leads to a selection of the most productive plants into exporting, raising welfare, but results in an asymmetric response in plant-level wage inequality. The variance of wages increases in exporting plants, if wage inequality was already high at these producers under autarky, while within-plant wage inequality declines at non-exporters. Given the asymmetry in plant-level implications, access to foreign trade exerts counteracting effects on economy-wide wage inequality. We can show, however, that economy-wide wage inequality is higher in the open than in the closed economy if and only if the coefficient of performance is positive. Our preliminary positive estimate for the coefficient of performance therefore suggests that globalization aggravates economy-wide wage inequality in all open economies around the world.

Workplace tasks are an important, employer-driven characteristic of the labor market, and have been documented to relate closely to recent labor market changes including wage polarization (Autor, Katz and Kearney 2006, Goos, Manning and Salomons 2009) and the offshorability of jobs (Leamer and Storper 2001, Levy and Murnane 2004, Blinder 2006). The assignment of tasks in an open economy, and the implications for welfare and wage inequality, have been studied from a theoretical perspective in industry-level models, including the Heckscher-Ohlin (Grossman and Rossi-Hansberg 2008, 2010) and the Ricardian framework (Rodríguez-Clare 2010, Acemoglu and Autor 2011). Our model complements the industry-level perspective with a plant-level view. Beyond considerations of offshorability, our treatment of tasks emphasizes the quality of the worker-task match as a key determinant of plant performance and in this regard relates closely to studies of internal labor markets by Barron, Black and Loewenstein (1989), who show that the quality of worker-task matches reduces on-the-job training costs, as well as Meyer (1994) and Burgess et al. (2010), who document that the quality of worker-task matches raises team efficiency and the effectiveness of incentives.

Human resource management practices have been found to be an important determinant of the variation in plant and firm productivity within and across countries (Bloom and Van Reenen 2011). Yet, aspects of the internal labor market and residual wage inequality are difficult to observe directly. Recent studies of the firm's internal labor market have turned to the importance of observable hierarchies (Caliendo and Rossi-Hansberg 2012, Caliendo, Monte and Rossi-Hansberg 2015) and their response to firm-level trade. Our model complements

[4] The lack of full longitudinal worker data in the IAB-LIAB random sample of plants does not allow us to replicate the permanent wage component measures from Bombardini, Orefice and Tito (2015) in our context.

the hierarchical approach to a firm's internal organization with a perspective on the horizontal differentiation of worker abilities and their tasks within hierarchical layers. In fact, we find that most employer-level residual wage inequality in the German data is also within hierarchies (and within occupation categories), suggesting that an important horizontal wage differentiation component acts within hierarchies. The internal organization of plants and firms also involves the motivation of workers to exert effort. Related studies analyze the response of employers' incentives for workers, and observable incentive pay in particular, when global competition changes (Guadalupe 2007, Cunat and Guadalupe 2009). Our paper complements the view on incentives for worker effort with a perspective on management responses to product-market opportunities, as employers adjust the observable count of occupations they offer and coordinate the observable range of tasks they assign within jobs.

An alternative approach to modelling worker-level wage dispersion within and between employers considers the employer-worker matching process (see e.g. Legros and Newman 2002, Eeckhout and Kircher 2011).[5] The potential efficiency gains from improved assortative matching have received according attention in the trade literature (Costinot and Vogel 2010, Sampson 2014). Several studies highlight trade-induced changes in match quality as a key aspect of trade in terms of welfare, employment and wage inequality (Amiti and Pissarides 2005, Davidson, Matusz and Shevchenko 2008, Davidson et al. 2014). More recent studies have started to complement the analysis of cross-industry and cross-firm matches with an analysis of within-firm matches.[6] Larch and Lechthaler (2011) study the assignment of workers across plants within multinational firms, and Bombardini, Orefice and Tito (2015) investigate the permissible ability ranges of workers at firms when worker-firm matches are formed. Our model highlights that an additional source of efficiency gains for employers is to improve match quality by narrowly assigning tasks to workers with the best fit to those tasks (a core ability within the occupation's task range).[7] Our worker-reported task frequencies within occupations characterize empirically the assignment of workplace activities to workers at different plants.

In our model relatively more productive plants choose to augment their elemental productivity with narrower task ranges that make their workers more efficient, thus concentrating the firm size distribution beyond the inherent productivity dispersion and introducing a feedback effect on the market entry and export participation decisions. While the principal selection of more productive firms into exporting remains a basic force in the model (as documented empirically by Clerides, Lach and Tybout 1998, e.g., and others), the feedback of ex-

[5]The literature estimating search models of the labor market more generally includes Burdett and Mortensen (1998), Cahuc, Postel-Vinay and Robin (2006), Postel-Vinay and Robin (2002), and Postel-Vinay and Turon (2010).

[6]In a review of the literature on the structure of wages within and across firms, Lazear and Shaw (2009) conclude that the wage structure appears to be more dependent on firm- or within-plant sorting of workers to occupations than on sorting of workers to firms or plants.

[7]An interpretation related to the core ability of workers, most suitable for specific tasks, is that human capital is occupation specific. Kambourov and Manovskii (2009) and Sullivan (2010) provide empirical evidence on occupation-specific human capital.

porting into worker efficiency through narrower task ranges at exporters is akin to a learning-by-exporting effect (for direct evidence on learning-by-exporting see, e.g., Crespi, Criscuolo and Haskel 2008). The labor market feedback effect in our model is similar to the outcome of screening in Helpman, Itskhoki and Redding (2010), the effect of investment into innovations in Aw, Roberts and Xu (2011), and the effect on team production in Chaney and Ossa (2013). In the Helpman, Itskhoki and Redding (2010) model, screening for higher ability workers raises the returns to exporting and vice versa; in the Aw, Roberts and Xu (2011) model, R&D investments raise the returns to exporting and vice versa; in the Chaney and Ossa (2013) model a larger number of more specialize teams increases the returns to exporting and vice versa; in our model, improving the worker-task match quality raises the returns to exporting and vice versa. The Chaney and Ossa (2013) model considers a similar mechanism to ours in explaining how market size affects a firm's incentives to reduce worker-task mismatches, but employers in their model are symmetric and workers within a team homogeneous. We introduce employer and worker heterogeneity to establish a link between labor efficiency across employers and wage inequality within their occupations.

The remainder of this paper proceeds as follows. In Section 2, we present our data, explain the construction of time-varying task measures and their combination with longitudinal plant–worker data, and collect descriptive evidence in three main facts that motivate our model. In Section 3, we build a model of production with task assignment to occupations. We derive the equilibrium for a closed economy in Section 4. We extend the model to two symmetric open economies that trade final goods in Section 5. In Section 6 we estimate key relationships of the model and subject it to empirical tests. There, we also provide preliminary evidence in support of the formal structure of our model but defer a detailed structural estimation of our model to future versions of our manuscript. Section 7 concludes.

2 Data and Descriptives

The two main sources for our novel micro-level data on employer-level task assignments are (i) the German Qualifications and Career Surveys (BIBB-BAuA surveys), and (ii) the Linked Plant–Worker Data provided by IAB (LIAB). In this section, we elicit three empirical facts from these two datasets to motivate a theory that can explain the division of labor at employers and the resulting wage dispersion within occupations. Additionally we use sector-level bilateral merchandise trade data from the United Nations Commodity Trade Statistics Database (Comtrade) and service trade from the trade in services database (TSD) from the World Bank to construct instrumental variables related to globalization shocks that are exogenous to the employers. We consolidate varying sector definitions and construct 39 longitudinally consistent industries for all data sources. Our industry definition is based on an aggregation of NACE 1.1 for the European Communities, which is equivalent to the German *Klassifikation der Wirtschaftszweige* WZ 2003 at the 2-digit level (see Becker and Muendler 2015).

2.1 Linked plant–worker data

To link workers to their employers, we use data at the German Federal Employment Office's Institute for Employment Research (IAB): the matched plant–worker data LIAB. The LIAB data combine detailed administrative records on workers from the German social security system with the IAB plant panel data.[8] On the employer side, LIAB provides detailed plant information from surveys on an annual basis since 1993. Information on plants in East Germany is only available since 1996. We therefore restrict the sample period to the years 1996-2014 to cover the German economy as a whole. At the plant level we use information on revenues, export status, export revenues and employment as well as region and industry categories. At the individual worker level, LIAB offers a comprehensive set of characteristics. We use demographic, tenure and education indicators, occupation characteristics, and data on daily wages.[9] Larger plants are over-represented in the plant panel. We therefore use the weighting factors provided by IAB and make our plant-level data representative for the German economy as a whole.

LIAB allows us to quantify sources of wage variation in the German labor market. To assess the dispersion in daily wages, we first remove observed demographic, education and tenure information together with time, industry and region effects from log daily wages in a Mincer regression, and obtain residual log daily wages. We remove observed worker characteristics because they are well explained by existing labor-market theories and have been addressed with classic trade theory and its extension to offshoring (see, e.g., Katz and Murphy 1992, Feenstra and Hanson 1999). Observed worker characteristics explain about 53 percent of the log wage variation (42 percent if we omit industry effects). Similar to other studies for both industrialized and developing countries (see e.g. Abowd et al. 2001, Menezes-Filho, Muendler and Ramey 2008), this finding implies that almost half of the wage dispersion remains unexplained at this level of analysis.

Table 1 follows up with further decompositions of the variance of the (exponentiated) log daily wage residual. Variation between plants explains about 24 percent of residual daily wages (exponentiated Mincer residuals) in 1996-2014 (column 2), still leaving 76 percent of residual wages unexplained. Recent trade theories such as Helpman, Itskhoki and Redding (2010), Egger and Kreickemeier (2009), Davis and Harrigan (2011) and Amiti and Davis (2012) address ways in which globalization can affect the between-employer variation in wages, and

[8] See `http://fdz.iab.de/en/Integrated_Establishment_and_Individual_Data/LIAB.aspx`.

[9] Wage information in the social security records is right-censored, so we replace censored wages by imputed wages, following the procedure proposed by Baumgarten (2013). Hourly wages cannot be constructed. We therefore use daily wages as the most precise measure of earnings.

Table 1: Decompositions of Residual Wage Inequality in Linked Plant–Worker Data

		Exponentiated Mincer residual			
Contribution of component (%)	1996-2014	1996-2014	1999	2006	2012
within industry[a]	88	.	.	.	.
within occupation	84	87	91	85	85
within plant	71	76	83	73	72
within plant-layer	65	69	77	65	63
within plant-occupation[b]	54	58	61	52	53

[a]Mincer regression excludes industry effects ($R^2 = 0.42$).

[b]The within plant-layer-occupation decomposition is identical to the within plant-occupation decomposition because occupations at the 3-digit KldB-88 level are nested strictly within layers of hierarchy.

Source: LIAB 1996-2014.

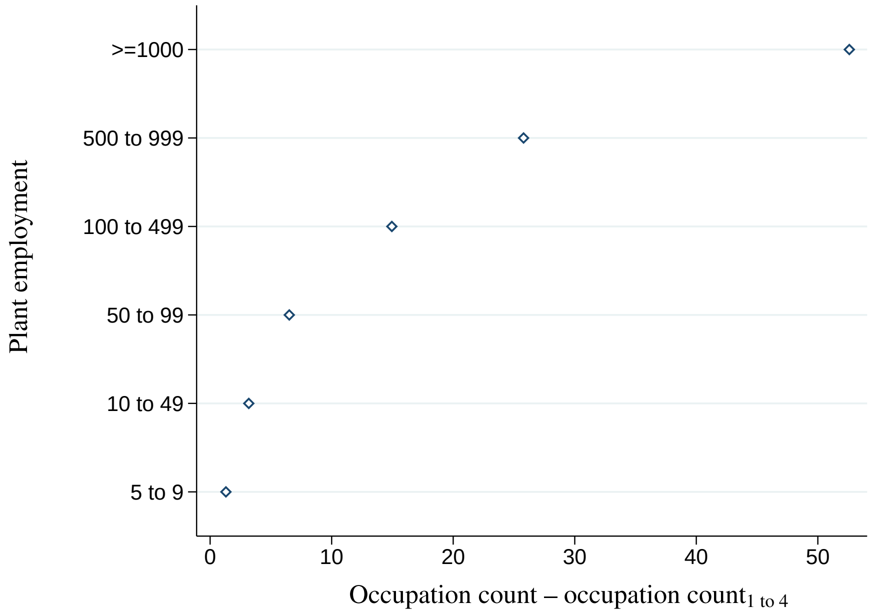
Notes: Residual log daily wage from standard Mincer regression (exponentiated in four latter columns), taking out demographic, education and tenure information as well as time, industry and region effects ($R^2 = 53\%$). 357 occupations at the 3-digit KldB-88 level. The variance of the log hourly wage w_{it} is linearly decomposed into a within and a between part. The reported percentages are the contribution of the within component to the total. Layers of hierarchy based on a mapping of the Caliendo, Monte and Rossi-Hansberg (2015) hierarchies to KldB-88 using ISCO-88.

Egger, Egger and Kreickemeier (2013), Coşar, Guner and Tybout (2015) Helpman et al. (2017) and Eaton, Kortum and Kramarz (2015) provide according empirical evidence. Looking at the variation between plants and between their managerial hierarchies brings the unexplained part of residual wage dispersion down by another 7 percentage points in 1996-2014 (to 69 percent). Recent models of the firm's internal labor market have turned to the importance of hierarchies (Caliendo and Rossi-Hansberg 2012) and Caliendo, Monte and Rossi-Hansberg (2015) provide evidence on earnings responses across hierarchies to firm-level trade. Considering the residual daily wage variation between plants and between their occupations (357 occupations at the 3-digit KldB-88 level) pushes the unexplained part further down by another 11 percentage points. Occupations are perfectly nested within hierarchies (using the occupation-to-hierarchy mapping from Caliendo, Monte and Rossi-Hansberg 2015). We are not aware of theory or empirical work on within-employer reallocations across occupations in response to globalization shocks. Importantly, 58 percent of the residual daily wage variation remain unexplained even at the plant-occupation level. In other words, much wage variation (58 percent of the 47 percent not explained with Mincer regressions) occurs within plant-occupations. That is the wage variation we take on in this paper. This finding also suggests that the wage gaps between senior management and the median worker at an employer are less relevant for overall wage inequality than is the wage dispersion within (mainly horizontally differentiated) occupations.

The LIAB data allow us to establish

Fact 1. *The count of occupations at a plant increases with plant employment.*

Figure 1: Count of Occupations by Plant Employment



Source: LIAB 1996-2014.
Notes: Prediction of occupation count n by plant employment category, controlling for sector, region, occupation and worker characteristics. Results are differences to smallest plant-size category (1 to 4 workers). Thick, medium, and thin lines represent the 99, 95, and 90 percent confidence intervals.

We project a plant's observed count of occupations n on sector, region, occupation and worker characteristics and then plot, in Figure 1, the so normalized count of occupations n (on the horizontal axis) against the plant's employment by size category (on the vertical axis). We normalize the occupation count (on the horizontal axis) by subtracting the count at the smallest plants (with 1 to 4 workers). We choose the depicted size categories (on the vertical axis) because they are the ones reported in our other data source. The figure shows that the occupations count n increases monotonically with plant size. Around the average occupation count per plant-size category, the figure draws thick, medium, and thin lines that represent the 99, 95, and 90 percent confidence intervals, but those lines are largely invisible given only minor dispersions of the normalized occupation counts within size categories. Of course, the monotonic increase of the occupation count in plant size is not necessarily evidence in favor of a finer division of labor in larger plants. Trivially, the simple fact that bigger plant have more workers to a assign to a larger count of occupations is also consistent with Fact 1. To establish relevant facts about the varying division of labor across plants of different sizes we therefore need to look within occupations and into the tasks per occupation. German labor force survey data allow us just that look.

2.2 Labor force survey data

A meaningful analysis of the within-plant-occupation component requires measurable properties of occupations. We take information on the organization of the workplace from three German Qualifications and Career Surveys conducted over the years 1999 through 2012 by Germany's Federal Institute for Vocational Education and Training BIBB (most recently in collaboration with the think tank BAuA). Each wave is based on a frame that selects a random sample of around one-tenth of a percent of the German labor force with more than 20 hours of work during the survey week. The BIBB-BAuA data report detailed information on workplace properties, worker characteristics, the industry, occupation and earnings, as well as rudimentary information on the employer, such as the size of a worker's plant in seven size categories.[10] Most importantly, we observe workers' responses to survey questions that regard the tasks they perform in their occupation. Following the time consistent definitions in Becker and Muendler (2015), who used German Qualifications and Career Surveys conducted over the years 1979 through 2006, we append the 2012 survey data and make use of the questions that elicit *what* activity workers carry out on the job. A worker may report these activities as performed or not. We can discern 15 such workplace activities, surveyed in a time consistent manner throughout the three BIBB-BAuA waves: 1. Manufacture, Produce Goods; 2. Repair, Maintain; 3. Entertain, Accommodate, Prepare Foods; 4. Transport, Store, Dispatch; 5. Measure, Inspect, Control Quality; 6. Gather Information, Develop, Research, Construct; 7. Purchase, Procure, Sell; 8. Program a Computer; 9. Apply Legal Knowledge; 10. Consult and Inform; 11. Train, Teach, Instruct, Educate; 12. Nurse, Look After, Cure; 13. Advertise, Promote, Conduct Marketing and PR; 14. Organize, Plan, Prepare (others' work); 15. Oversee, Control Machinery and Technical Processes. These workplace activities (*what* tasks) are cumulative and exhibit a pronounced change towards more multitasking over time until the early 2000s (Becker and Muendler 2015), with a relatively stable level of multitasking from then on.[11]

As Table 2 documents, German workers perform on average 7.3 workplace activities (tasks) in their respective occupation in 2006 and 2012, up from 5.3 tasks in 1999. The most frequent number of tasks is 4 to 7 in 1999, but a shift in the mass follows so that 8 or more tasks dominate in subsequent years. The frequency of tasks

[10] Plant size categories are 1-4; 5-9; 10-49; 50-99; 100-499; 500-999; 1000 or more workers.

[11] Research into wage polarization (e.g. Autor, Katz and Kearney 2006, Goos, Manning and Salomons 2009) and offshorability (Leamer and Storper 2001, Levy and Murnane 2004, Blinder 2006, e.g.) frequently considers a different dimension of "tasks," including the routineness of work steps and codifiability of job descriptions, which are also reported in the BIBB-BAuA surveys. Becker and Muendler (2015) call those tasks, which are related to *how* workers conduct their work, performance requirements and document that those tasks exhibit little time variation even though they are not mutually exclusive tasks. In contrast to the (*what*) activities here, German workers do not report more simultaneous performance requirements over time (Becker and Muendler 2015, Table 2). In the spirit of Adam Smith's division of labor, and for the purposes of our model of a plants' internal labor markets, we are most interested in tasks that are empirically found to be cumulated at the workplace into multitasking occupations. We therefore restrict our attention to the 15 (*what*) activities in the BIBB-BAuA data.

Table 2: Multitasking in Simultaneous Workplace Activities

Nr. of tasks	1999	2006	2012
up to 3	.298	.101	.096
4 to 7	.477	.431	.425
8 or more	.225	.469	.479
Average	5.250	7.261	7.316

Source: BIBB-BAuA 1999-2012.
Notes: Shares of worker observations per wave with given number of reported activities. Missing entries are less than one-tenth percent of a percent of observations. Workplace activities are: 1. Manufacture, Produce Goods; 2. Repair, Maintain; 3. Entertain, Accommodate, Prepare Foods; 4. Transport, Store, Dispatch; 5. Measure, Inspect, Control Quality; 6. Gather Information, Develop, Research, Construct; 7. Purchase, Procure, Sell; 8. Program a Computer; 9. Apply Legal Knowledge; 10. Consult and Inform; 11. Train, Teach, Instruct, Educate; 12. Nurse, Look After, Cure; 13. Advertise, Promote, Conduct Marketing and PR; 14. Organize, Plan, Prepare (others' work); 15. Oversee, Control Machinery and Technical Processes.

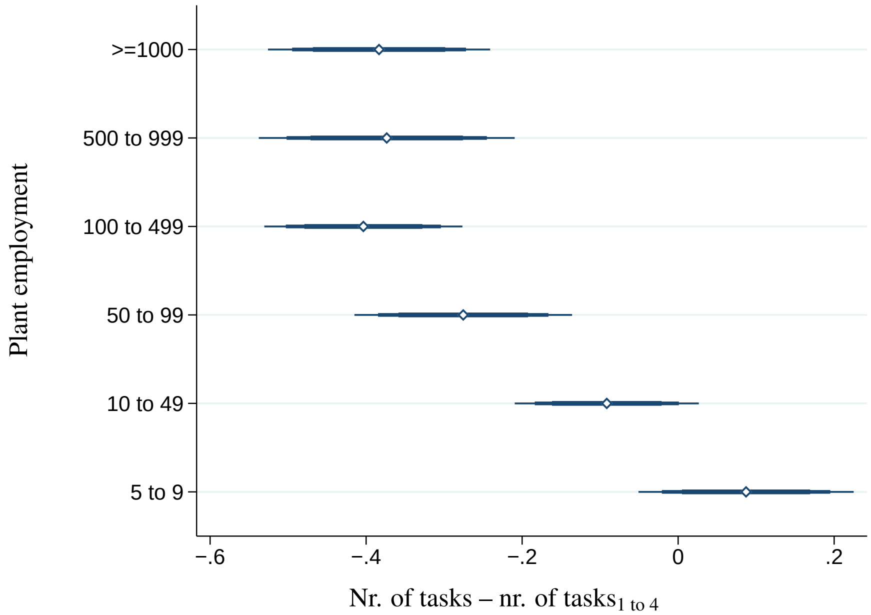
between 2006 and 2012 remains relatively similar. To control for the time variation in multitasking regimes, in future empirical work we will estimate structural relationships by year and query their robustness. We compare task frequencies by layer of hierarchy in Table A1 in the Appendix and find that multitasking occurs to a relatively similar extent across layers of hierarchy, with workers in any occupation performing 6.7 tasks on average and managers reporting about 7.6 tasks in their occupations—not quite one additional task in higher layers of hierarchy compared to workers in the average occupation.

The BIBB-BAuA data allow us to revisit evidence on the division of labor and establish

Fact 2. *The number of tasks within an occupation at a plant decreases with plant employment.*

We compute the number of tasks that workers in their respective occupations report in the BIBB-BAuA data. We then project the reported number of tasks per occupations b on the same sector, region, occupation and worker characteristics as before. Figure 2 plots the so normalized number of tasks b per occupation (on the horizontal axis) against the plant's employment by size category (on the vertical axis). We normalize the occupation count (on the horizontal axis) by subtracting the count at the smallest plants (with 1 to 4 workers). The depicted size categories (on the vertical axis) are the ones reported in the BIBB-BAuA data. The figure shows that the number of tasks b strictly decreases with plant size, relative to the smallest plants, up to a plant employment of 500 workers and then remains constant. In other words, larger plants choose a finer division of labor and assign narrower task ranges to their workers (who fill more occupations by Fact 1). Similar to Adam Smith's tenet, workers engage in less multitasking at more pin-factory like larger plants. The differences in the numbers of tasks are not statistically significant. Around the average number of tasks per occupation in a plant-size category, the figure shows thick, medium, and thin lines that represent the 99, 95, and 90 percent confidence intervals. But the differences in the numbers of tasks are not large—a change of less than half a task between a plant with 5

Figure 2: Number of Tasks per Occupation by Plant Employment



Source: BIBB-BAuA 1999, 2006 and 2012.
Notes: Prediction of number of tasks b within plant-occupation by plant employment category, controlling for sector, region, occupation and worker characteristics. Results are differences to smallest plant-size category (1 to 4 workers). Thick, medium, and thin lines represent the 99, 95, and 90 percent confidence intervals.

to 9 workers and a plant with 500 or more workers. (In comparison, multitasking increased by more than one task between 1999 and 2006 as shown in Table 1, though it remained relatively constant thereafter, and managers perform close to one additional task compared to the average worker as shown in Appendix Table A1.) From the threshold of about 500 workers on, plants assign roughly similar task ranges to their workers.

2.3 Data combination

Task information is not available in the linked plant–worker data LIAB. To conduct an employer-level analysis, we therefore need to combine the BIBB-BAuA labor force survey information with the LIAB linked plant–worker records through imputation. A large set of worker characteristics and plant attributes overlaps between the BIBB-BAuA survey and the LIAB records. We use these common variables to conduct imputations in both possible directions: task information from the BIBB-BAuA survey into LIAB in one direction, and plant-level information from LIAB into BIBB-BAuA in the alternate reverse direction.

For much of our plant-level analysis, the imputation of BIBB-BAuA task information into LIAB is most important. To combine BIBB-BAuA task information with the LIAB plant-worker data and preserve within-occupation and time variation with possibly much precision, we opt for regression-based imputation. Note that

Table 3: Descriptive Statistics for Combined Data

	Obs.	Mean	Median	StDev.	Min.	Max.
log Revenues	116,931	13.98	13.76	0.01	8.88	24.63
log Export revenues	36,473	17.48	17.33	0.03	10.92	29.01
Export indicator	116,933	0.17	0	.	0	1
Employment	116,933	18.48	6	0.12	3	44,419
log Daily wage	116,933	4.13	4.14	.	1.96	5.76
CV Daily wage	116,933	0.32	0.31	.	.	4.02
StDev Residual daily wage	116,933	23.07	19.82	0.19	.	1,167.85
Count 2-digit occupations n	116,933	3.5	2	0.01	1	63
Average number of tasks b	116,933	3.96	3.91	0.01	0.32	8.87
Normalized number of tasks $b/\tilde{z}$	116,933	0.36	0.36	.	0.03	0.70

Sources: LIAB 1996-2014 and BIBB-BAuA 1992-2012. Sample restricted to plants with more than 2 full-time workers.
Notes: Descriptive statistics based on annual plant observations, using inverse probability weights to make plant sample representative of Germany economy, as suggested by the Research Data Centre at the IAB. *CV* is coefficient of variation of daily wage within a plant-occupation. *StDev Residual daily wage* measures the standard deviation of the (exponentiated) daily (log) wage residual from a Mincer regression (in logs), including demographic, education and tenure information as well as time, sector and region fixed effects and plant revenues.

the imputation is based on the empirical covariation between common worker variables in both data sets and the tasks that the workers report in BIBB-BAuA, and this covariation preserves the statistically relevant task-related information from BIBB-BAuA in the LIAB data. We first run a linear (OLS) model on the BIBB-BAuA data, regressing the number of tasks (the sum over the 15 activity task indicators) on a set of worker, occupation and plant attributes that are jointly observed in the BIBB-BAuA and in the LIAB data.[12] With the estimated coefficients at hand we perform an out-of-sample linear prediction in the LIAB data using all common variables. Under this procedure we obtain, for 76% of the LIAB observations, an individual-specific number of tasks. Finally, by computing the mean over all individuals within a plant, we end up with a measure of the (mean) number of tasks b that workers perform per occupation within a plant.

As Table 3 shows, the average number of tasks per occupation at the plant level varies between 0.32 and 8.87, with a mean of 3.96 and a standard deviation of 0.01.[13] The LIAB data also allow us to compute the coefficient of variation *CV* of the daily wages within plant-occupations. This coefficient of variation disregards the dispersion of wages across occupations, across plants, and across sectors. It instead isolates the within plant-occupation component in the wage variance, which was shown to be the dominant component of the residual

[12]The independent variables used in the regression are log daily wage, job experience, squared job experience together with indicators for (i) gender, (ii) 7 schooling and vocational training indicators, (iii) 16 regions, (iv) 34 sectors, (v) 7 plant-size categories, and (vi) 335 occupations. In the baseline regression we pool over the years 1992, 1999, 2006 and 2012. In an alternative specification we estimate the number of tasks separately for these four years and compute year-specific predictions from a moving average.

[13]In the BIBB-BAuA data the average number of tasks for 7 different plant-size categories varies between 4.77 and 5.32, with a mean of 4.92 and standard deviation 0.2. The differences in the task number intervals are mainly due to differences in the wage levels and perhaps the fact the BIBB-BAuA only covers workers with more than 20 hours of work per week.

wage in Table 1. The coefficient of variation is 0.3 at the mean (and the median) but is more than 4 at the most unequal plants in the combined sample.

In addition to mapping information on the number of tasks from BIBB-BAuA to LIAB, we can also estimate the probability of performing a specific task in BIBB-BAuA and make an out-of-sample prediction regarding the probability that an individual worker perform this specific task in the LIAB data. For this purpose, we run 15 probit regressions (one for each task) with the same set of explanatory variables as in the regression for the number of tasks outlined above. With these out-of-sample predictions at hand, we can then construct a measure for the overall number of distinct tasks performed at a plant in LIAB. Due to the chosen estimation approach, the total number of distinct tasks must be smaller than 15 and it is larger than zero if our mapping was successful for at least one worker at the plant.[14] We then divide the average number of tasks (b) by the full number of distinct tasks observed (denoted with $\tilde{z}$ in the model below), to obtain a normalized measure of the number of tasks—a real number on the unit interval: $b/\tilde{z} \in (0, 1]$. As shown in Table 3, the normalized number of tasks $b/\tilde{z}$ varies between 0.03 and 0.7 with a mean of 0.36 and a tight standard deviation.

Table 3 also reports summary statistics on revenues and other relevant plant attributes from the combined LIAB and BIBB-BAuA data. Excluding plants for which we lack relevant information as well as plants with employment of two or fewer workers (for which we cannot compute meaningful measures of wage dispersion), our sample covers 116,931 plant-year observations, with 36,473 of these observations referring to exporters.

The imputation of BIBB-BAuA task information into the LIAB linked plant–worker data allows us to establish

Fact 3. *The coefficient of variation* CV *of daily wages within a plant-occupation decreases with the number of tasks within a plant-occupation.*

We project the coefficient of variation *CV* of the (exponentiated) residual daily wages within a plant-occupation on sector, region, occupation and worker characteristics, as before. Figure 3 plots the so normalized *CV* of daily wages within a plant-occupation (on the horizontal axis after subtracting the coefficient of daily wage variation in the range of less than one imputed task) against numbers of tasks (on the vertical axis), for plants with at least two workers. There is a clear inverse relationship with an S-like shape: wage variability drops strongly as the number of tasks per plant-occupation increases from one task to about six tasks, then it drops less pronounced, and drops again more sensitively in the upper ranges of more than nine tasks. Workers within the same occupation are subject to more wage inequality within their occupation at the same employer if they

[14] The total number of distinct tasks varies between a minimum of 3.58 and a maximum of 15, with a mean of 11.2 and standard deviation 0.02.

Figure 3: Residual Wage Inequality per Plant-Occupation by Number of Tasks

Sources: LIAB 1996-2014, with imputed task information per plant-occupation from BIBB-BAuA 1999, 2006 and 2012.
Notes: Prediction of coefficient of variation of daily wage residual (exponentiated Mincer residual) CV within plant-occupation by task number, controlling for sector, region, occupation and worker characteristics. Results are differences to smallest task-number category (0 to 1 tasks). Thick, medium, and thin lines represent the 99, 95, and 90 percent confidence intervals.

are assigned narrower task ranges. We can also relate the normalized coefficient of variation CV of daily wages within a plant-occupation to plant size, similar to Figures 1 and 2. As Appendix Figure A1 shows, using LIAB 1996-2014 only, wage variability within plant-occupations increases strongly with plant employment. Workers within the same occupation are subject to more wage inequality within their occupation at larger employers.

One consistent hypothesis is that workers who perform only a few tasks have a strong impact on the surplus that they generate at the employer. If workers who specialize in a narrow task range make mistakes, those mistakes weigh down surplus heavily, and their wages are lower. Conversely, workers who specialize in a narrow task range and perform strongly generate large surplus and receive high wage compensation. Under this hypothesis, surplus and wage payments will be particularly sensitive to worker mismatches in occupations with narrow task ranges. In other words, plants that behave more like Adam Smith's pin factory also exhibit more wage dispersion within plant-occupations. Our theory is devised to relate the more pronounced within plant-occupation wage dispersion back to the plant's internal division of labor, that is the plant's internal labor market organization.

2.4 Trade data

To link the plant-internal division of labor, and wage inequality, back to the plant's globalization status and predicted export sales, we need trade data. Our information on Germany's sector-level imports and exports with China and Eastern Europe comes from the United Nations Commodity Trade Statistics Database (Comtrade) and the trade in services database (TSD) at the World Bank. To construct instruments for German exports and imports we follow Autor, Dorn and Hanson (2013) and Dauth, Findeisen and Suedekum (2014) and use shipments between Australia, Canada, Japan, Norway, New Zealand, Sweden, Singapore, and the United Kingdom on the one hand and China and Eastern Europe on the other hand as the instrument group.[15] We map the SITC Rev. 2 sector information to a common sector definition across all waves of the German data. To create a concordance from SITC Rev. 2 to the 39 longitudinally consistent industries, we rely on existing mappings from SITC Rev. 2 to ISIC Rev. 3.1 and from ISIC Rev. 3.1 to WZ 2003.

3 A Model of Production with Task Assignment

3.1 Consumers

We consider an economy with a population of L individuals, who are risk neutral. As consumers, the individuals have homothetic preferences over a continuum of differentiated goods labelled $\omega \in \Omega$. The representative consumer maximizes utility

$$U = \left[\int_{\omega \in \Omega} x(\omega)^{\frac{\sigma-1}{\sigma}} \, \mathrm{d}\omega \right]^{\frac{\sigma}{\sigma-1}} \tag{1}$$

subject to the economy-wide budget constraint $\int_{\omega \in \Omega} p(\omega) x(\omega) \, \mathrm{d}\omega = Y$, where $p(\omega)$ is the price of variety ω, Y is aggregate income, and $\sigma > 1$ is the elasticity of substitution between varieties. The resulting economy-wide demand for variety ω of the consumption good is:

$$x(\omega) = \left(\frac{p(\omega)}{P} \right)^{-\sigma} \frac{Y}{P}, \tag{2}$$

where $P \equiv \left[\int_{\omega \in \Omega} p(\omega)^{1-\sigma} \, \mathrm{d}\omega \right]^{1/(1-\sigma)}$ is the CES price index. A producer of variety ω faces total demand $x(\omega)$ for its product. We introduce heterogeneity in individual consumers' budget sets given differentiated individual wages below.

[15] Trade flows are converted into Euros using annual exchange rates from the German Bundesbank.

3.2 Production

A plant ω is fully characterized with a tupel of three properties in our baseline model. We assume that the plant receives an *elemental productivity* draw $\tilde{\varphi}(\omega)$ from a lottery, as in Melitz (2003). In the baseline model, as in Chaney (2008), plants draw their $\tilde{\varphi}(\omega)$ from a common Pareto distribution $G(\tilde{\varphi}) = 1-\tilde{\varphi}^{-\theta}$ with shape parameter θ, where $\theta > 1$ to ensure a finite mean of productivity. To participate in the lottery, plants hire f_e workers at the going wage rate w. After the lottery, the cost is sunk and, depending on its elemental productivity draw and other properties in its tupel, a plant decides on whether to start production. Production requires the additional fixed employment of $f(\omega)$ workers for overhead services in operation, including the division of tasks into occupations.

The plant also receives from the lottery a draw of its plant-specific *full task range* $\tilde{z}(\omega)$, which is required to produce its outputs. All conceivable tasks are line up around a unit circle, but plants need to cover different segments $\tilde{z}(\omega) < 1$ of the unit circle. We consider the plant-specific full task range $\tilde{z}(\omega)$ to be the product of two parts

$$\tilde{z}(\omega) = \tilde{\zeta}(\omega) \cdot z(\omega),$$

where $\tilde{\zeta}(\omega)$ is a plant's unobserved task range variability and $z(\omega) < 1$ is the observed total number of tasks at the plant. From imputation of the BIBB-BAuA task data into the LIAB plant–worker data, we can measure a plant's total number of tasks $z(\omega)$—a fraction of the 15 possibly observable tasks. However, it is unobserved what range of tasks a plant outsources to domestic suppliers and what range it offshores to foreign suppliers, in-house or at arm's length. We therefore treat $\tilde{\zeta}(\omega)$ as an unobserved property of the plant and will allow $\tilde{\zeta}(\omega)$ to covary with the other properties in the plant's tupel in future structural estimation, including with its elemental productivity $\tilde{\varphi}(\omega)$.

In addition, the plant receives a draw of its individual *coefficient of performance* $\tilde{\eta}(\omega)$ that characterizes how sensitive the plant's surplus is to the mismatch of workers to its tasks. By allowing the coefficient of performance to vary across plants, we can accommodate heterogeneity in the link between a plant's average number of tasks per occupation and its wage variability within occupations. The coefficient of performance $\tilde{\eta}(\omega)$ is not known but, under the structural relationships that we will derive, it can be recovered from observed variables.

Instead of carrying around the plant identifier ω, we will soon describe a plant's decisions given this tupel of three properties $(\tilde{\varphi}, \tilde{z}, \tilde{\eta})$. In a future version of our theoretical model, currently under elaboration, we will also allow plants to differ in one additional property: plants will draw a stochastic fixed cost of exporting $\tilde{f}_x(\omega)$ as in Helpman et al. (2017) to break the deterministic link between elemental productivity and export-market participation, which exhibits variation in the data. In the general model under elaboration, a plant's decisions will ultimately depend on its tupel of four properties $(\tilde{\varphi}, \tilde{z}, \tilde{\eta}, \tilde{f}_x)$. (To simplify exposition and to derive intuitive general-equilibrium relationships in closed form for the closed and open economy in this draft of our paper, however, in Sections 4 and 5 we will restrict the plant's tupel of properties to just the two stochastic components $(\tilde{\varphi}, \tilde{z})$.) For now, consider the plant to be a tupel of three properties in the baseline model $(\tilde{\varphi}, \tilde{z}, \tilde{\eta})$.

Each variety of the consumption good is produced by a unique plant ω. When it comes to market structure, we assume that plants are monopolistic competitors. Labor is the only input. As workers, individuals are endowed with one unit of labor, which they supply inelastically to plants. Individual individuals differ in their core ability as workers.

Production requires that workers perform tasks in their respective occupations. A plant ω decides about three types of employment outcomes. First, the plant chooses the total number of occupations $n(\omega) + 1$ that it wants to offer (a plant's count of occupations in the data). We consider the possible count of occupations $[n(\omega) + 1] = 1, 2, \ldots$ to be countable and require a plant to offer at least one occupation—when $n(\omega) = 0$. Second, the plant assigns an occupation-invariant measure of tasks $b(\omega)$ that need to be performed within each occupation at the plant. For tractability, we make the total measure of tasks $b(\omega)$ a real number. By the technology we propose, the first choice of the total count of occupations $n(\omega) + 1$ will inversely determine the measure of tasks $b(\omega)$ as an outcome at the plant level. And third, the plant chooses a measure of workers $\ell(\omega)$ to hire into the occupations that it offers.

A plant ω with elemental productivity $\tilde{\varphi}(\omega)$ produces quantity $q(\omega)$ of its variety by combining the individual outputs $q_j(\omega)$ of its occupations $j = 1, \ldots, n(\omega) + 1$ into a Cobb-Douglas production function:

$$q(\omega) = \tilde{\varphi}(\omega)\, \tilde{z}(\omega)\, [n(\omega) + 1]\, \exp\left[\frac{1}{n(\omega) + 1} \sum_{j=1}^{n(\omega)+1} \ln q_j(\omega) \right], \tag{3}$$

where $q_j(\omega)$ is the output of occupation j and $n(\omega) + 1$ is the count of distinct occupations at the plant.[16] The plant-specific draw $\tilde{z}(\omega) < 1$ is the plant ω's full task range required to produce its outputs $q_j(\omega)$. The way in which the term $n(\omega) + 1$ enters the production function implies that, in the case of symmetric occupations, plants can raise their output by creating additional occupations, with an elasticity of one. Therefore, worker efficiency does not change in our model just because a plant adds new occupations. Only if workers get to specialize on a smaller range of tasks when new occupations are added, then worker efficiency increases with the addition of these occupations (see below).[17] To simplify notation for now, we suppress the variety label ω and consider a

[16]Output in equation (3) corresponds to to a Cobb-Douglas production function of the form $q = \tilde{\varphi}\tilde{z} \prod_j (q_j/\alpha_j)^{\alpha_j}$, with $\sum_j \alpha_j = 1$ and $\alpha_j = 1/(n+1)$ under symmetry.

[17]A generalization to a non-unitary elasticity does not result in substantively different model predictions but would make the model

single plant.

3.3 Task assignments to occupations and labor efficiency

Workers have innate abilities to carry out tasks more or less efficiently. Complementing the knowledge view of the firm, there are no organizational hierarchies in our model. Instead, the worker abilities are *horizontally* differentiated and uniformly distributed over a circle with circumference 1. This ability circle simultaneously represents the technology space and characterizes the set of distinct possible tasks, which are also uniformly distributed with measure one. The location of a worker on the circle indicates the task that corresponds to his or her core ability.

Plants cannot use the full circle of tasks for production. Instead they are restricted to select tasks from a subinterval with maximal length $\tilde{z} < 1$. We do not observe a plant that is predicted to conduct all 15 tasks in our data (see Table 3 in Section 2.3). The length of the maximally feasible task range $\tilde{z} < 1$ is plant-specific and exogenously given to the plant. However, plants have a choice to bundle adjacent tasks into occupations, which are then executed by the workers hired for these occupations. Plants must choose a common number (measure) of tasks $b < \tilde{z}$ for all occupations that they offer. In other words, we impose that plants choose symmetric divisions of the segment of the task circle on which they operate. We choose this restriction to reduce the number of parameters to estimate, so our focus in this paper rests on the average number tasks performed per occupation within a plant. We leave potential plant-and-occupation-specific task range choices for future work and start our analysis with only plant-specific task range choices. The measure of tasks b that a plant adopts is thus the same across all of the plant's occupations.

Suppose for a moment that tasks on the plant's segment of the unit circle are mutually exclusively assigned to n separate occupations with no overlap. The plant's chosen count of occupations n and its chosen measure of tasks b would then be linked to each other according to

$$b = \frac{\tilde{z}}{n+1}.$$

However, in practice and in our data, the same activities (*what* tasks) are typically performed across multiple occupations. We therefore introduce an exogenous degree of overlap ν—a common parameter beyond a plant's control. With a common degree of overlap, a plant's measure of tasks per occupation becomes

$$b = \frac{\tilde{z}}{\nu n+1}, \qquad \text{with} \quad \nu \in (0,1]. \tag{4}$$

more complicated and would require an additional parameter to discipline with data.

If $\nu = 1$, eq. (4) collapses to the simple case above with no overlap. If $\nu < 1$, the mapping of tasks to occupations is not unique and the task intervals overlap. In the limiting case of $\nu = 0$, each occupation uses the whole range of tasks of the plant, irrespective of n.

Workers have to allocate the same amount of time to all tasks specified by the occupation, with worker efficiency falling in the distance of their core ability to a task. We can therefore interpret the average distance of a worker at location i to the various tasks in interval $[0, b]$ within each occupation as a measure of mismatch. Provided that labor is not misallocated so that workers have their core ability in one of the tasks spanned by the occupation (see below), we can compute a linear measure of mismatch $m(i, b)$ of a worker i with the b tasks in an occupation according to

$$m(i, b) = \frac{1}{b} \left\{ \int_0^i (i - t)\,\mathrm{d}t + \int_i^b (t - i)\,\mathrm{d}t \right\} = \frac{b^2 + 2i(i - b)}{2b}, \tag{5}$$

where t is the running index of the task location. The mismatch depends on the worker's position in the interval and is lowest (highest) if the worker is located in the middle (at the boundaries) of the task interval. Intuitively, there is an inverse link between mismatch $m(i, b)$ and a worker i's efficiency $\lambda(i, b)$, which we define as

$$\lambda(i, b) \equiv \frac{\tilde{\eta}}{\tilde{z}} + \frac{1}{m(i, b)} = \frac{\tilde{\eta}}{\tilde{z}} + \frac{2b}{b^2 + 2i(i - b)}. \tag{6}$$

For worker efficiency to be well defined, we impose that the coefficient of performance $\tilde{\eta}$ satisfies $\tilde{\eta} > -2z$, so that all workers from interval $[0, b]$ have positive efficiency for all possible outcomes $b \leq \tilde{z} < 1$. The coefficient of performance $\tilde{\eta}$ will play an important role below when it comes to the intra-plant dispersion of wages and how that wage dispersion varies between plants with different productivities. Note that we do not restrict $\tilde{\eta}$ to be positive.

The plant can choose to hire a measure $\ell_j(i, b)$ of workers with core ability i into occupation j given a task range b per occupation. Occupation-level employment is therefore $\ell_j(b) \equiv \int_0^b \ell_j(i, b)\,\mathrm{d}i$ at a plant with task range b per occupation. Average worker efficiency in occupation j is then

$$\lambda_j(b) = \frac{1}{\ell_j(b)} \int_0^b \lambda(i, b) \ell_j(i, b)\,\mathrm{d}i, \qquad \text{where} \qquad \ell_j(b) \equiv \int_0^b \ell_j(i, b)\,\mathrm{d}i \tag{7}$$

denotes the total amount of labor hired for occupation j. Occupation-level output is then

$$q_j = \lambda_j(b) \ell_j(b). \tag{8}$$

Note that, if $\ell_j(i,b)$ is the same for all workers i in occupation j then, by (7), $\ell_j(b) = \int_0^b \ell_j(i,b)\,\mathrm{d}i$ is the same across occupations j because b is occupation invariant, and $\lambda_j(b)$ is the same across all occupations j. Then q_j is the same at all occupations j of a plant by (8). (We will show below that $\ell_j(i,b)$ is the same for all workers i in occupation j.)

Differentiation of eq. (7) with respect to the task range b per occupation yields

$$\lambda_j'(b) = \frac{\ell_j(b,b)}{\ell_j(b)}[\lambda(b,b) - \lambda_j(b)] < 0. \tag{9}$$

This negative relationship between the average worker efficiency in a plant-occupation j and the task range b per plant-occupation is a characterization of the benefits from the division of labor. The relationship provides the theoretical rationale for our Fact 2 (Figure 2), by which larger plants adopt narrower task ranges per occupation. As Adam Smith's pin factory tenet suggests, the efficiency of workers employed in a plant's occupations is higher when occupations are more specialized in narrower task ranges, because workers are more efficient on average (closer to their core ability) when conducting fewer tasks in their occupation.

3.4 Hiring, production, and wage setting

Labor is employed in three different roles: for the sunk cost to make the productivity draw f_e, for the fixed input into overhead services $f(\omega)$ to manage and coordinate the occupations, and for the variable input into production. In the first two roles, workers have an efficiency of one, whereas in the third role their efficiency is given by $\lambda(i,b)$ and thus match specific. To hire workers for production, plants post occupations in a competitive labor market at the going wage w. The occupation posting provides a binary signal that informs workers about whether their core ability is within the occupations's task interval, or not, but not on their specific location within this interval. One way to think about this is that the location of the occupation's task interval on the unit circle is not part of the occupation description but that workers can receive a costless test report that reveals with certainty whether or not their ability is within the occupation's task range. The occupation posting does specify what the wage schedule will be for the worker upon accepting the occupation offer. Given their risk neutrality, workers will accept any wage schedule that pays an expected wage rate w.

In Appendix C, we show how Stole and Zwiebel (1996) wage bargaining in the presence of equilibrium unemployment can be embedded into our production model. For our baseline framework and its equilibrium relationships in this draft of the paper, we want to set aside unemployment and introduce an equivalent wage schedule to the one that would arise under Stole-Zwiebel wage bargaining by allowing for workers' endogenous effort choice. Production workers can choose an effort level e from interval $[0,1]$ and thus the time productively used in their occupation, so the output of worker i in occupation j is given by

$$q_j(i) = e(i)\lambda(i,b) \qquad \text{in every occupation } j = 1,\dots,n(\omega)+1.$$

Suppose for a moment that full effort were enforceable through monitoring. Then a plant could not do better than offering a constant wage w to all workers that equals the going wage in the economy.

Now suppose the utility of workers is reduced by a constant factor $\varepsilon > 0$ per unit of effort. Then plants will link wage payments to the ex post output if the effort is unobservable for outsiders and hence not contractible. The lacking contractibility of effort rules out a uniform wage for all production workers. In fact, plants cannot do better than setting

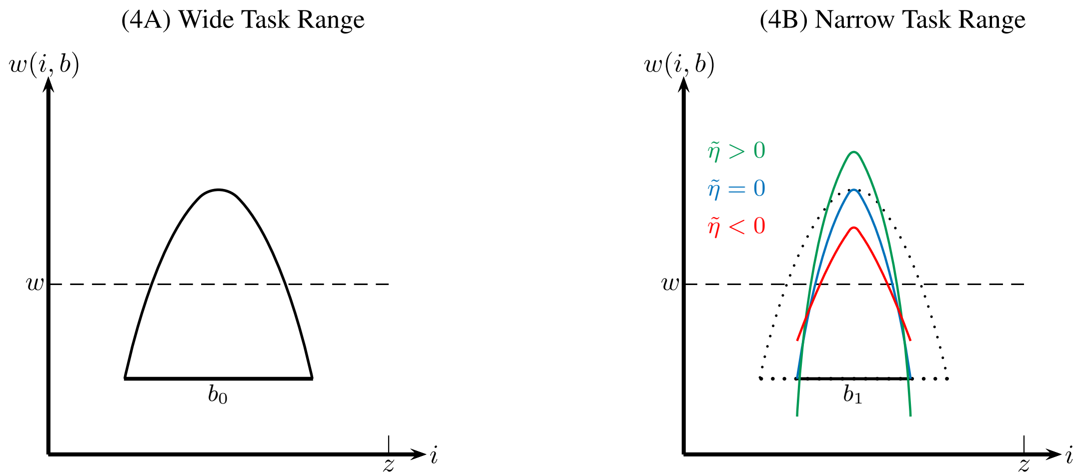
$$w(i,b) = w\frac{\lambda(i,b)}{\lambda(b)} \tag{10}$$

for a constant going wage w, prompting workers to provide full effort $e = 1$ if ε is sufficiently small. To see that the going wage is occupation independent, note that all occupations inside a plant are symmetric in that they require the same task range b, so $w(i,b)/\lambda(i,b) = w_j/\lambda_j(b)$ for all occupations j, hence $w_j/\lambda_j(b) = w/\lambda(b)$ because workers of type i and of type $i+b$ have an equivalent degree of mismatch in their respective occupations. Following this reasoning, plants pay a constant wage per efficiency unit of $w/\lambda(b)$ to all of their production workers (and w to workers providing fixed inputs), implying that the efficiency differences of workers in the occupation translate one-to-one into wage differences between workers.[18]

Plants pay the same wage per efficiency unit of labor. Plants are therefore indifferent between all applicants. Furthermore, workers are ex ante indifferent between all occupations that correspond to their qualification, that is all occupations for which their core ability lies within the covered task interval. Plants therefore end up hiring workers whose abilities are uniformly distributed over the task intervals covered by their occupations, and $\ell_j(i,b)$ is the same for all workers i in occupation j. As a result, average worker efficiency is the same for all occupations

[18]Under the wage schedule (10), there are workers who earn less than the going wage rate w. These workers would benefit from quitting and searching for a new occupation elsewhere because, in expectation, a new occupation that covers their core ability would offer a payment w. By design, endogenous quits are not a problem in a static setting. However, if worker efficiency is revealed ex post, one can extend the model to a variant with involuntary unemployment under search frictions to ensure that quitting remains unattractive for workers even after their efficiency has been revealed. In Appendix C, we provide such an extension and consider Stole-Zwiebel bargaining instead of efficiency wages as a modelling strategy for worker-specific wages. There we show that the main results on wage dispersion within occupations remain unaffected by this modification, and that the extended model leads to a setting with involuntary unemployment in equilibrium.

Figure 4: Within-occupation Wage Schedule and Plant Choice of the Task Range



Notes: A plant receives a stochastic draw of its full task range $\tilde{z}$, which is required to produce its outputs. The index i denotes the core ability of a worker (the worker type) and designates the ideal task to which the worker is matched without any mismatch. The plant chooses its task range b. A worker with a core ability in the interior of a given task range b has a lower degree of mismatch the narrower the task range (mismatch is the average distance of a worker i from all the tasks that fall within the range b). It depends on the plant's coefficient of performance $\tilde{\eta}$ how sensitive its surplus is to the mismatch of workers to tasks: $w(i,b) = w \cdot \lambda(i,b)/\lambda(b)$ by eq. (10) and $\lambda(i,b)/\lambda(b)$ is a function of $\tilde{\eta}$ and given by the ratio of eqs. (6) and (11a).

in the plant and given by

$$\lambda(b) = \frac{1}{b}\int_0^b \lambda(i,b)\,\mathrm{d}i = \frac{1}{b}\left[\frac{i\tilde{\eta}}{\tilde{z}} + 2\arctan\left(\frac{2i-b}{b}\right)\right]_0^b = \frac{\tilde{\eta}}{\tilde{z}} + \frac{\pi}{b} \quad \text{or equivalently} \tag{11a}$$

$$\lambda(\omega) = \frac{1}{\tilde{z}}\left[\tilde{\eta} + \pi(\nu n(\omega)+1)\right], \tag{11b}$$

where the final equality follows when substituting $\tilde{z}/b(\omega) = \nu n(\omega) + 1$ from eq. (4). It is a consequence of constant $\lambda_j(b) = \lambda(b)$, as noted before, that q_j is the same at all occupations j of a plant by equations (7) and (8).

It follows from these insights that the wage dispersion is the same in all occupations of a plant and linked to the plant's chosen task range per occupation with

$$var(\omega) = \frac{w^2}{b}\int_0^b \left(\frac{\lambda(i,b)}{\lambda(b)}\right)^2 \mathrm{d}i - w^2 = w^2\frac{[4-\pi(\pi-2)](\nu n(\omega)+1)^2}{[\tilde{\eta}+\pi(\nu n(\omega)+1)]^2}. \tag{12}$$

Graph A of Figure 4 illustrates the wage dispersion within a plant-occupation that spans a task range b_0, where a worker i's wage $w(i,b) = w \cdot \lambda(i,b)/\lambda(b)$ under eqs. (6) and (7). Now suppose the plant optimally adopts a narrower task range $b_1 < b_0$ as depicted in Graph B of Figure 4. The wage schedule will still vary around the unchanged economy-wide wage w, but it depends on the coefficient of performance $\tilde{\eta}$ whether the worker efficiency dispersion, and hence the wage dispersion around the economy-wide mean, stays constant, rises, or falls. For a positive parameter $\tilde{\eta} > 0$, a narrower task range $b_1 < b_0$ magnifies the worker efficiency dispersion in that it induces more vertical variation in any worker i's wage (hence our label coefficient of performance for $\tilde{\eta}$). Larger plants with narrower task ranges will exhibit a wider wage dispersion within plant-occupation for $\tilde{\eta} > 0$. We consider it an empirical matter how task ranges should relate to wage outcomes across workers within a plant-occupation and therefore introduce the parameter $\tilde{\eta}$ for estimation. In practice, workers with badly matched abilities near the boundary of a narrow task range might exhibit a more than proportionally diminished efficiency, if their mistakes on the job can result in heavier losses to the employer than in wider task ranges. A priori, it is equally conceivable that badly matched workers in narrow task ranges suffer only a less than proportional reduction in efficiency, compared to their efficiency in wide task ranges, if their mistakes matter little to the employer, because narrower task ranges may have a lesser impact on overall production.

We will return to plant-level optimality conditions in our outline for structural estimation in Section 6.2 below, where we also recover the coefficient of performance $\tilde{\eta}(\omega)$ from the model's structural relationships.

4 Division of Labor in the Closed Economy

To derive equilibrium relationship in closed from, we now simplify our model and make the coefficient of performance plant-invariant: $\tilde{\eta}(\omega) = \eta$. We maintain the mild condition that $\eta > -2z$ from above, but do not require η to have a specific sign so that the adoption of narrower task ranges by larger plants may result in reduced or heightened within-occupation wage variability. In this section, a plant ω is a tupel of two properties $(\tilde{\varphi}, \tilde{z})$. (In a future version of our theoretical model, currently under elaboration, we will allow plants to differ in the coefficient of performance.)

4.1 Profit maximization in the closed economy

Plants decide about entry and production in three stages. On stage one, a plant ω decides on paying the sunk cost of f_e units of labor for entering the elemental productivity draw. On stage two, the plant decides on starting production conditional on its productivity draw. Prior to production on stage three, the plant must also determine on stage two the count of occupations $n(\omega)$ and pay a fixed cost of $f(\omega)$ units of labor to operate. We set the plant's fixed cost of operation to

$$f(\omega) = f_0 + \{\eta + \pi([\nu n(\omega) + 1]\}^{\gamma}$$

with a semi-elasticity of the fixed cost with respect to occupation counts $\gamma > 0$, so that the overhead costs are positively linked to the count of occupations $n(\omega)$ at the plant. It is costly to the plants to create additional occupations (and have a narrower task range per occupations). This span-of-control cost for a plant is more convex for larger γ. Figure 2 (Fact 2) above documents that the number of tasks within an occupation at a plant decreases with plant employment, and the number of tasks becomes largely insensitive to further size increases of the plant above a threshold of about 500 workers. Our model can capture the insensitivity of task ranges to plant size beyond a threshold with highly convex fixed costs. To keep the model parsimonious, we choose a constant semi-elasticity γ of the span-of-control cost with respect to occupation counts. We will test this parametrization in future structural work through size interactions with the variables that identify γ.

On stage three, plants hire production workers $\ell(\omega)$, manufacture output $q(\omega)$ and sell this output to consumers. We solve the three-stage decision problem by backward induction. On stage three, a plant sets $\ell_j(\omega)$ to maximize its profits

$$\psi(\omega) = p(\omega)q(\omega) - w \sum_{j=1}^{n(\omega)+1} \ell_j(b(\omega)) - w \left\{\eta + \pi[\nu n(\omega) + 1]\right\}^{\gamma} - w f_0, \tag{13}$$

subject to aggregate consumer demand for their variety (2), the market clearing condition $x(\omega) = q(\omega)$ for their variety, and the plant's production function

$$q(\omega) = \tilde{\varphi}(\omega)[n(\omega) + 1] \left\{\eta + \pi[\nu n(\omega) + 1]\right\} \exp\left[\frac{1}{n(\omega) + 1} \sum_{j=1}^{n(\omega)+1} \ln \ell_j(b(\omega))\right], \tag{14}$$

under a set of common non-negativity constraints. Profit maximization on stage three results in the first-order condition for revenues and employment

$$r(\omega)[(\sigma - 1)/\sigma] = [n(\omega) + 1] w \ell_j(b(\omega)),$$

with $r(\omega) \equiv p(\omega)q(\omega)$. This first-order condition establishes the intuitive result that plant ω chooses the same employment level for all occupations j: $\ell_j(b(\omega)) = \hat{\ell}(\omega)$. Furthermore, the profit-maximizing price can be expressed as a constant markup over the plant's marginal cost $p(\omega) = [\sigma/(\sigma - 1)]c(\omega)$, given CES demand, with

$$c(\omega) \equiv \frac{w}{\tilde{\varphi}(\omega) \left\{\eta + \pi[\nu n(\omega) + 1]\right\}}. \tag{15}$$

We now turn to stage two. Plants rationally anticipate the profit value on stage three as a function of their entry decisions and choice of the count of occupations. Substituting equation (15) into equation (13) and accounting for equation (2) yields profits of plant ω as a function of the count of occupations chosen by the plant $n(\omega)$:

$$\psi(\omega)=\frac{Y}{P^{1-\sigma}}\frac{1}{\sigma}\left[\frac{\sigma}{\sigma-1}\frac{w}{\tilde{\varphi}(\omega)\left\{\eta+\pi[\nu n(\omega)+1]\right\}}\right]^{1-\sigma}-w\left\{\eta+\pi[\nu n(\omega)+1]\right\}^{\gamma}-wf_0. \tag{16}$$

Plants face the trade-off that increasing the count of occupations lowers marginal costs with a positive effect on profits, but at the same time raises the overhead costs with a negative effect on profits. This trade-off is similar to the one in Eckel (2009) and Bustos (2011), where producers can pay a fixed cost to reduce variable production costs.

Treating $n(\omega)$ as a continuous variable for purposes of exposition, the first-order condition for the profit-maximization problem at stage two is given by

$$r(\omega)\frac{\sigma-1}{\sigma}=\gamma w\left\{\eta+\pi[\nu n(\omega)+1]\right\}^{\gamma}. \tag{17}$$

We assume that $\gamma>\sigma-1$, a necessary condition for an interior solution to be a maximum. In addition, we assume that parameters are such that every plant benefits from specifying more than one occupation, i.e. from setting $n(\omega)>0$. Think of plants with at least one worker in a more organizational (senior) role and another worker in a more operational (junior) role. The plant that gains least from increasing $n(\omega)$ is the plant with the lowest $\tilde{\varphi}(\omega)$. This is the plant that makes zero profits from production $\hat{\psi}(\omega)=0$, provided that not all plants find it attractive to start production (see below). In an interior maximum, this zero-profit condition can be expressed as

$$\frac{f_0(\sigma-1)}{\gamma-\sigma+1}=\{\eta+\pi[\nu n(\omega)+1]\}^{\gamma}, \tag{18}$$

and hence we can safely conclude that the maximization problem has an interior solution if equation (18) holds for a strictly positive $n(\omega)$, that is for $f_0(\sigma-1)/[\gamma-\sigma+1]>(\eta+\pi)^{\gamma}$. This latter inequality characterizes the parameter domain to which we restrict ourselves because, in combination with $\gamma>\sigma-1$, it is sufficient for a unique maximum at stage two, with $n(\omega)>0$ for all plants.

Eqs. (2) and (15) together with market clearing condition $x(\omega)=q(\omega)$ establish a first relationship between relative revenues of two plants and the relative count of distinct occupations in these plants, while equation (17) establishes a second relationship between these variables. We have:

$$\frac{r(\omega_1)}{r(\omega_2)}=\left(\frac{\tilde{\varphi}(\omega_1)[\eta+\pi[\nu n(\omega_1)+1]]}{\tilde{\varphi}(\omega_2)[\eta+\pi[\nu n(\omega_2)+1]]}\right)^{\sigma-1}, \qquad \frac{r(\omega_1)}{r(\omega_2)}=\left(\frac{\eta+\pi[\nu n(\omega_1)+1]}{\eta+\pi[\nu n(\omega_2)+1]}\right)^{\gamma} \tag{19}$$

respectively. These expressions allow us to express relative revenues and the relative count of occupations of two plants as a function of these plants' relative differences in their elemental productivity parameter $\tilde{\varphi}$:

$$\frac{r(\omega_1)}{r(\omega_2)} = \left(\frac{\tilde{\varphi}(\omega_1)}{\tilde{\varphi}(\omega_2)}\right)^{\xi}, \qquad \frac{\eta + \pi[\nu n(\omega_1) + 1]}{\eta + \pi[\nu n(\omega_2) + 1]} = \left(\frac{\tilde{\varphi}(\omega_1)}{\tilde{\varphi}(\omega_2)}\right)^{\frac{\xi}{\gamma}}, \tag{20}$$

where $\xi \equiv \gamma(\sigma - 1)/(\gamma - \sigma + 1)$ denotes the elasticity of revenues with respect to productivity parameter $\tilde{\varphi}$. Using eqs. (2) and (14), we can also determine relative output and relative plant-level employment of production workers as a function of the productivity differential between these plants:

$$\frac{q(\omega_1)}{q(\omega_2)} = \left(\frac{\tilde{\varphi}(\omega_1)}{\tilde{\varphi}(\omega_2)}\right)^{\frac{\xi\sigma}{\sigma-1}}, \qquad \frac{\ell(\omega_1)}{\ell(\omega_2)} = \left(\frac{\tilde{\varphi}(\omega_1)}{\tilde{\varphi}(\omega_2)}\right)^{\xi}, \tag{21}$$

where $\ell(\omega) \equiv [n(\omega) + 1]\hat{\ell}(\omega)$ is the employment of production workers at plant ω. Our framework shares with other models of heterogeneous employers the empirically well-documented property that workers in larger firms are more productive (see Idson and Oi 1999). However, in contrast to other contributions, the productivity differences are further scaled up under the plants' profit maximizing choice of the count of occupations, which raises worker efficiency.

It is an important insight from eqs. (20) and (21) that plant outcomes are fully characterized by exogenous differences in $\tilde{\varphi}$. Hence, we can drop ω and index plants by their elemental productivity parameter from now on.

Denoting productivity of the marginal plant by φ^*, revenues of and the count of occupations in the marginal plant are given by

$$r(\varphi^*) = \frac{\sigma\xi w f_0}{\sigma - 1}, \qquad \nu n(\varphi^*) + 1 = \frac{1}{\pi}\left[\left(\frac{\xi f_0}{\gamma}\right)^{\frac{1}{\gamma}} - \eta\right], \tag{22}$$

respectively, according to eqs. (17) and (18). Furthermore, the variance of wages within occupations for the marginal producer can be expressed as

$$var(\varphi^*) = [4 - \pi(\pi - 2)]\left(\frac{w}{\pi}\right)^2 \left[\frac{(\xi f_0/\gamma)^{1/\gamma} - \eta}{(\xi f_0/\gamma)^{1/\gamma}}\right]^2, \tag{23}$$

according to eqs. (12), (18), and (22). The variance of wages in plants with $\tilde{\varphi} > \varphi^*$ is then given by

$$var(\tilde{\varphi}) = var(\varphi^*)\left[\frac{(\tilde{\varphi}/\varphi^*)^{\xi/\gamma}(\xi f_0/\gamma)^{1/\gamma} - \eta}{(\tilde{\varphi}/\varphi^*)^{\xi/\gamma}[(\xi f_0/\gamma)^{1/\gamma} - \eta]}\right]^2, \tag{24}$$

according to eqs. (17), (20), and (23). It is easily confirmed from equation (24) that the variance of wages is the same for all producers and thus independent of productivity parameter $\tilde{\varphi}$ if $\eta = 0$. In contrast, the variance of wages strictly increases in $\tilde{\varphi}$ if $\eta > 0$, and weakly decreases otherwise.

To solve for the plants' problem at stage one, we note that free entry is consistent with profit maximization if and only if the expected profit from participating in the productivity draw, $\int_{\varphi^*}^{\infty} \psi(\tilde{\varphi})\, \mathrm{d}G(\tilde{\varphi})$, just compensates plants for the sunk costs of participation, wf_e. As shown in the Appendix, we have $\int_{\varphi^*}^{\infty} \psi(\tilde{\varphi})\, \mathrm{d}G(\tilde{\varphi}) = [1 - G(\varphi^*)]wf_0\xi/(\theta - \xi)$, which allows us to solve for the productivity of the marginal plant:

$$\varphi^* = \left(\frac{f_0}{f_e}\frac{\xi}{\theta - \xi}\right)^{\frac{1}{\theta}} \tag{25}$$

that participates in the productivity draw, where we assume $g > \xi$ to ensure a positive and finite value of aggregate revenues and profits, and we assume $f_0/f_e > g/\xi - 1$ to ensure $\varphi^* > 1$ and hence an outcome by which only relatively more productive plants start production at stage two.

4.2 The autarky equilibrium

With the insights from the previous subsection, we can now solve for the general equilibrium. For this purpose, we choose labor as the numéraire and set $w = 1$. Since profit income is used to pay for entrance into the productivity lottery, the mass of producers is determined by the condition that economy-wide labor income, L, equals total consumption expenditures, Y, and thus aggregate revenues $R = Mr(\varphi^*)g/(\theta - \xi)$. Accounting for equation (22), we obtain

$$M = \frac{L(\sigma - 1)}{\sigma\xi f_0}\frac{\theta - \xi}{\theta}. \tag{26}$$

Welfare of the representative agent is (proportional to) the real wage and thus given by the inverse of the CES price index: $W = P^{-1}$. The price index can be expressed as $P = [gM/(\theta - \xi)]^{1/(1-\sigma)}p(\varphi^*)$ and it therefore follows from eqs. (15), (25), (26) and constant markup pricing that welfare is given by

$$W = \left(\frac{L}{\gamma}\right)^{\frac{1}{\sigma-1}} \left(\frac{\gamma}{f_0\xi}\right)^{\frac{1}{\xi}} \left(\frac{\sigma - 1}{\sigma}\right)^{\frac{\sigma}{\sigma-1}} \left(\frac{f_0}{f_e}\frac{\xi}{\theta - \xi}\right)^{\frac{1}{\theta}}. \tag{27}$$

To complete the discussion of the closed economy, we compute economy-wide wage inequality as the employment-share weighted average of the variances of wages at the plant level:

$$Var = \frac{\theta - \xi}{gl(\varphi^*)} \int_{\varphi^*}^{\infty} var(w, \tilde{\varphi})\ell(\tilde{\varphi})\, \mathrm{d}G(\tilde{\varphi}), \tag{28}$$

where $\ell(\varphi^*)g/(\theta-\xi)$ is the average employment of production workers per plant. Solving for the integral gives

$$Var = var(\varphi^*)\left\{1+\frac{2\xi/\gamma}{\theta-\xi+2\xi/\gamma}\frac{\eta}{(f_0\xi/\gamma)^{1/\gamma}-\eta}\left[1+\frac{(f_0\xi/\gamma)^{1/\gamma}}{(f_0\xi/\gamma)^{1/\gamma}-\eta}\frac{\xi/\gamma}{\theta-\xi+2\xi/\gamma}\right]\right\}, \qquad (29)$$

according to equation (24). This implies that $Var > var(\varphi^*)$ if and only if $\eta > 0$.

5 Division of Labor in the Open Economy

To derive global equilibrium relationship in closed from under free trade, we maintain the simplifying assumption that the coefficient of performance is plant-invariant: $\tilde{\eta}(\omega) = \eta > -2z$. We do not require η to have a specific sign so that the adoption of narrower task ranges by larger plants may result in reduced or heightened within-occupation wage variability. In this section, a plant ω is a tupel of two properties $(\tilde{\varphi}, \tilde{z})$. (In a future version of our theoretical model, currently under elaboration, we will allow plants to differ in the coefficient of performance. In addition, we will allow plants to draw a stochastic fixed cost of exporting $\tilde{f}_x(\omega)$ to break the deterministic link between elemental productivity and export-market participation.)

5.1 Fundamentals

In this section, we consider trade between two symmetric countries with consumption and production as in the previous section. There are two types of trade costs: fixed costs $f_x > 0$ (in units of labor) for setting up a foreign distribution network; and variable iceberg transport costs $\tau > 1$ with the usual interpretation that τ units of the consumption good must be shipped in order for one unit to arrive in the foreign economy. Both of these costs are also present in the Melitz (2003) framework and – in combination with the heterogeneity of plants in their elemental productivity parameter $\tilde{\varphi}$ – they generate self-selection of only the most productive producers into exporting, provided that the trade costs are sufficiently high. However, compared to other studies the decision to start exporting is more sophisticated in our model, because it influences a plant's optimal choice of $n(\tilde{\varphi})$ and thus exerts a feedback effect on profits attainable in the domestic market. Due to this feedback effect, we have to distinguish between variables referring to exporters (denoted by super-/subscript e) and non-exporters (denoted by super-/subscript d). Furthermore, we use subscript t to refer to variables associated with total (domestic and foreign) market activities.

Holding economy-wide variables constant, access to exporting does not affect the maximization problem of non-exporters. Things are different, however, for exporters, who make revenues $\tau^{1-\sigma}r^e(\tilde{\varphi})$ in the foreign market in addition to their revenues $r^e(\tilde{\varphi})$ in the domestic market, implying that in the open economy this plants' profit-maximizing choice of $n(\tilde{\varphi})$ is given by

$$\left(1+\tau^{1-\sigma}\right) r^e(\tilde{\varphi})\frac{\sigma-1}{\sigma}=\gamma\left[\eta+\pi(1+\nu n(\tilde{\varphi}))\right]^\gamma \tag{30}$$

instead of equation (17). Since equation (30) is structurally the same for all exporters, we can conclude that the ratios depicted by eqs. (20) and (21) remain unaffected in the open economy, provided that the two plants have the same export status (d or e). In contrast, when comparing two plants with the same productivity parameter $\tilde{\varphi}$ but differing export status, we obtain

$$\frac{r^e(\tilde{\varphi})}{r^d(\tilde{\varphi})}=\left(1+\tau^{1-\sigma}\right)^{\frac{\xi}{\gamma}}, \qquad \frac{\eta+\pi(1+\nu n^e(\omega))}{\eta+\pi(1+\nu n^d(\omega))}=\left(1+\tau^{1-\sigma}\right)^{\frac{\xi}{\gamma(\sigma-1)}}, \tag{31}$$

$$\frac{q^e(\tilde{\varphi})}{q^d(\tilde{\varphi})}=\left(1+\tau^{1-\sigma}\right)^{\frac{\sigma\xi}{\gamma(\sigma-1)}}, \qquad \frac{\ell_t^e(\tilde{\varphi})}{\ell_t^d(\tilde{\varphi})}=\left(1+\tau^{1-\sigma}\right)^{\frac{\xi}{\sigma-1}}, \tag{32}$$

where $\ell_t^e(\tilde{\varphi})$ and $\ell_t^d(\tilde{\varphi})$ denote total labor input of plant $\tilde{\varphi}$ under exporting and non-exporting, respectively. From the analysis of the closed economy we know that larger plants have more occupations. Since, all other things equal, exporting generates additional revenues from sales to foreign consumers, it leads to a larger count of occupations: $n^e(\tilde{\varphi})>n^d(\tilde{\varphi})$. The stronger division of labor makes exporters more productive and lowers their unit production costs. This stimulates sales in both the domestic and the foreign market, establishing $r^e(\tilde{\varphi})>r^d(\tilde{\varphi})$ and $q^e(\tilde{\varphi})>q^d(\tilde{\varphi})$ in eqs. (31) and (32), respectively. Hence, there is a positive feedback effect of exporting on domestic revenues and this raises the incentives of plants to export ceteris paribus. From equation (32) we can also infer that exporting changes the hiring decision. Whereas exporters install more occupations and therefore increases productivity of their workforce, the associated rise in efficiency units of labor does not fully compensate for the higher demand of labor from higher sales at home and abroad. Hence, a plant that starts to export also hires additional workers.

Despite the additional complexity arising from the feedback effect that a plant's exporting decision exerts on its domestic profits, our model preserves key properties of the Melitz (2003) model, regarding the partitioning of plants by export status. To see this, we can make use of (17), (20), (22), (30), and (31) and write a plant's profit gain from exporting, $\Delta\psi_t(\tilde{\varphi})\equiv\psi_t^e(\tilde{\varphi})-\psi_t^d(\tilde{\varphi})$, as follows:

$$\Delta\psi_t(\tilde{\varphi})=\left[\left(1+\tau^{1-\sigma}\right)^{\frac{\xi}{\sigma-1}}-1\right]\left(\frac{\tilde{\varphi}}{\varphi^*}\right)^\xi f_0-f_x. \tag{33}$$

The profit differential in (33) increases in the elemental productivity parameter $\tilde{\varphi}$, and hence there is selection of the most productive plants into exporting as in other applications of the Melitz model if the two trade cost parameters f_x and τ are sufficiently high. This is the case we are focussing on in this paper, and we can therefore characterize the plant that is indifferent between exporting and non-exporting by $\Delta\psi_t(\tilde{\varphi}) = 0$. We denote the (cutoff) productivity of this plant by $\tilde{\varphi}_x^*$, implying that plants with $\tilde{\varphi} \geq \tilde{\varphi}_x^*$ are exporters, while plants with $\tilde{\varphi} < \tilde{\varphi}_x^*$ are non-exporters. Solving $\Delta\psi_t(\tilde{\varphi}_x^*) = 0$ for the ratio of the two productivity cutoffs $\tilde{\varphi}_x^*$ and φ^* and noting that the share of exporters is given by $\chi \equiv [1 - G(\varphi_x^*)]/[1 - G(\varphi^*)]$, we can compute

$$\chi = \left\{ \frac{f_0}{f_x} \left[\left(1 + \tau^{1-\sigma}\right)^{\frac{\xi}{\sigma-1}} - 1 \right] \right\}^{\frac{\theta}{\xi}}, \tag{34}$$

with $\chi < 1$ if there is partitioning of plants by their export status. From equation (34), we can conclude that higher trade costs, i.e. a higher f_x or a higher τ, raise the minimum productivity level that is necessary to render exporting an attractive choice, thereby lowering the share of exporters in the total population of active plants χ. With this insights at hand, we are now equipped to solve for the open economy equilibrium.

5.2 The open economy equilibrium

Profit maximization in the open economy is described by a three-stage decision problem that is similar to the closed economy, but additionally involves the decision to start exporting or to sell exclusively to the domestic market (at stage 2). Access to the export market raises profits of the most productive plants, and thus the expected profit prior to entry into the productivity lottery, which as shown in the Appendix is given by

$$\int_{\varphi^*}^{\infty} \psi_t(\tilde{\varphi})\, \mathrm{d}G(\tilde{\varphi}) = [1 - G(\varphi^*)]\, \frac{\xi f_0}{\theta - \xi} \left(1 + \frac{\chi f_x}{f_0}\right) \tag{35}$$

in the open economy. Free entry into the productivity lottery establishes $\int_{\varphi^*}^{\infty} \psi_t(\tilde{\varphi})\, \mathrm{d}G(\tilde{\varphi}) = f_e$ and thus $\varphi^*/\varphi_a^* = (1 + \chi f_x/f_0)^{1/\theta}$, where index a is used to indicate an autarky variable. Access to exporting increases expected profits from production, and hence the probability to start production $1 - G(\varphi^*)$ must decrease in order to restore the condition of zero expected profits from entry, which is achieved by a higher cutoff productivity level. This mechanism is well understood from Melitz (2003) and points to asymmetric effects of openness at the plant level. Whereas the most productive plants see their profits soaring due to access to the foreign market, low-productivity plants experience a profit loss due to stronger competition (for scarce labor), with the least productive ones being forced to cease production.

To shed further light on the asymmetry in plant-level responses to trade, we can study how producers adjust their assignment of workers to tasks in the open economy. We start with a close look at non-exporting plants. The fixed overhead costs of the marginal producer $f(\varphi^*)$ remain to be determined by equation (18). However, the marginal producer in the open economy must have a higher elemental productivity parameter than the marginal producer in the closed economy, and that plant's fixed costs are therefore lower than under autarky. In view of equation (20) fixed costs are lower for all non-exporting plants, implying that these plants reduce their count of occupations in response to trade. This is intuitive because non-exporting plants lose market shares in the open economy, and hence find it more difficult to bear the overhead costs associated with the creation of distinct occupations. Contrasting the overhead costs for the division of tasks into occupations in the closed and the open economy, we can compute in the case of non-exporting plants:

$$\frac{\eta + \pi(1 + \nu n^d(\tilde{\varphi}))}{\eta + \pi[\nu n^a(\tilde{\varphi}) + 1]} = \left(\frac{1}{1 + \chi f_x/f_0}\right)^{\frac{\xi}{\theta\gamma}} \equiv \rho_d(\tilde{\varphi}) < 1, \tag{36}$$

according to equation (20) and our previous insight that $\varphi^*/\varphi_a^* = (1 + \chi f_x/f_0)^{1/\theta}$. For exporting plants, we can compute

$$\frac{\eta + \pi(1 + \nu n^e(\tilde{\varphi}))}{\eta + \pi[\nu n^a(\tilde{\varphi}) + 1]} = \left(\frac{\left(1 + \tau^{1-\sigma}\right)^{\frac{\theta}{\sigma-1}}}{1 + \chi f_x/f_0}\right)^{\frac{\xi}{\gamma g}} = \left(\frac{\left(1 + \chi^{\xi/\theta} f_x/f_0\right)^{\frac{\theta}{\xi}}}{1 + \chi f_x/f_0}\right)^{\frac{\xi}{\gamma g}} \equiv \rho_e(\tilde{\varphi}), \tag{37}$$

where the first equality sign follows from eqs. (31) and (36), whereas the second one follows from equation (34). Note that $g > \xi$ holds by assumption, so it is straightforward to show that $n^e(\tilde{\varphi}) > n^a(\tilde{\varphi})$ and thus $\rho_e(\tilde{\varphi}) > 1$. A plant that starts exporting in the open economy realizes higher revenues and thus raises the count of distinct occupations. It is notable that the asymmetric response of plants in their internal division of labor is the consequence of an asymmetric exposure of this plants to exporting. If all plants would start exporting ($\chi = 1$), the marginal plant would be the same as in the closed economy, $\varphi^* = \varphi_a^*$, implying $n^e(\tilde{\varphi}) = n^a(\tilde{\varphi})$ for all active producers. It is therefore the asymmetric exposure to exporting rather than the market size increase *per se* that is responsible for plant-level adjustments in the division of tasks into occupations. The following proposition summarizes the insights from the previous analysis.

Proposition 1. *Opening up to trade with selection of the most productive plants into exporting leads to an asymmetric response in the within-plant assignment of workers to tasks. Whereas exporter lower the number of tasks covered by occupations, resulting in lower mismatch and higher worker efficiency, non-exporters increase the task coverage of occupations, resulting in larger mismatch and lower worker efficiency.*

The asymmetric response of plants to trade regarding the division of tasks into occupations has consequences for wage differences in those occupations. Following the derivation steps of the closed economy, we can express wage inequality of non-exporters, $var_d(w,\tilde{\varphi})$, as a function of wage inequality in the marginal plant, $var_d(w,\varphi^*)$, by equation (24). Thereby, wage inequality in the marginal plant is the same as in the closed economy. However, in the open economy this is a plant with higher productivity. Accounting for $\varphi^*/\varphi_a^* = (1+\chi f_x/f_0)^{1/\theta}$ the effect of openness on wage inequality at the plant-level for non-exporters is therefore determined by

$$var_d(w,\tilde{\varphi}) = var_a(w,\tilde{\varphi}) \left[\frac{(\tilde{\varphi}/\varphi_a^*)^{\xi/\theta}(\xi f_0/\gamma)^{1/\gamma} - \eta\rho_d^{-1}(\tilde{\varphi})}{(\tilde{\varphi}/\varphi_a^*)^{\xi/\theta}(\xi f_0/\gamma)^{1/\gamma} - \eta} \right]^2, \tag{38}$$

according to equation (36). Accounting for $r_d(\tilde{\varphi}) < 1$, it follows from equation (38) that $var_d(w,\tilde{\varphi}) > var_a(w,\tilde{\varphi})$ if and only if $\eta < 0$. Looking at exporting plants, we can compute

$$var_e(w,\tilde{\varphi}) = var_a(w,\tilde{\varphi}) \left[\frac{(\tilde{\varphi}/\varphi_a^*)^{\xi/\theta}(\xi f_0/\gamma)^{1/\gamma} - \eta\rho_e^{-1}(\tilde{\varphi})}{(\tilde{\varphi}/\varphi_a^*)^{\xi/\theta}(\xi f_0/\gamma)^{1/\gamma} - \eta} \right]^2, \tag{39}$$

according to eqs. (24) and (36). Accounting for $r_e(\tilde{\varphi}) > 1$ it therefore follows that $var_e(w,\tilde{\varphi}) > var_a(w,\tilde{\varphi})$ if and only if $\eta > 0$. The following proposition summarizes the effects of trade on plant-level wage inequality.

Proposition 2. *Opening up to trade with selection of the most productive plants into exporting leads to an asymmetric response in plant-level wage inequality. The variance of wages increases (decreases) in exporting plants if wage inequality was already exceptionally high (low) for these producers under autarky. The opposite is true for non-exporters.*

Given the asymmetry in the plant-level implications, access to trade exerts counteracting effects on the general equilibrium variables of interest: welfare W and economy-wide wage inequality Var. Similar to autarky, welfare in the open economy is given by the real wage and hence inversely related to the CES price index $P = [gM(1+\chi f_x/f_0)/(\theta-\xi)]^{1/(1-\sigma)}p^d(\varphi^*)$. The mass of producers in the open economy is given by $M = M_a/(1+\chi f_x/f_0)$ and thus smaller than in the closed economy. Noting further that $p(\varphi^*) = p^a(\varphi^*)(1+\chi f_x/f_0)^{-1/\theta}$, we can relate welfare in the open economy to welfare in the closed economy, according to

$$W = W_a \left(1 + \frac{\chi f_x}{f_0}\right)^{\frac{1}{\theta}}. \tag{40}$$

For plant entry in our model is allocationally efficient (similar to the case in Dhingra and Morrow 2016), a movement from autarky to trade is akin to lifting a technology barrier, which must be welfare enhancing.

As shown in the Appendix, economy-wide wage inequality in the open economy is given by

$$Var = Var_a + \frac{\eta\, var_d(w,\varphi^*)}{[(\xi f_0/\gamma)^{1/\gamma}-\eta]^2}\frac{\theta-\xi}{\theta-\xi+2\xi/\gamma}\frac{\chi^{1-\frac{\xi}{\theta}}}{1+\chi f_x/f_0}V(\chi), \tag{41}$$

with

$$V(\chi) \equiv 2\left(\frac{\xi f_0}{\gamma}\right)^{1/\gamma}\frac{\theta-\xi+2\xi/\gamma}{\theta-\xi+\xi/\gamma}\left\{\chi^{\frac{\xi}{\theta\gamma}}-1+\left(1+\chi^{\frac{\xi}{\theta}}\frac{f_x}{f_0}\right)\left[1-\left(1+\chi^{\frac{\xi}{\theta}}\frac{f_x}{f_0}\right)^{-\frac{1}{\gamma}}\chi^{\frac{\xi}{\theta\gamma}}\right]\right\}$$
$$-\eta\left\{\chi^{\frac{2\xi}{\theta\gamma}}-1+\left(1+\chi^{\frac{\xi}{\theta}}\frac{f_x}{f_0}\right)\left[1-\left(1+\chi^{\frac{\xi}{\theta}}\frac{f_x}{f_0}\right)^{-\frac{2}{\gamma}}\chi^{\frac{2\xi}{\theta\gamma}}\right]\right\} > 0. \tag{42}$$

Economy-wide wage inequality is therefore higher (lower) in the open than the closed economy if and only if $\eta > 0$. If $\eta > 0$, then wage inequality within high-productivity plants increases while wage inequality within low-productivity plants falls. As a consequence, there are counteracting effects on economy-wide wage inequality. However, the combined effect is unambiguous for two reasons. On the one hand, aggregate overhead expenditures associated with the division of tasks into occupations increase, which raises wage inequality if $\eta > 0$. On the other hand, exporters expand production and hire new workers, whereas non-exporters contract production and release part of their workforce. Hence, plants with a larger wage inequality get a higher weight in the computation of Var, which contributes to an increase in economy-wide wage inequality. We summarize the effects of trade on welfare and economy-wide wage inequality in the following proposition.

Proposition 3. *Opening up to trade with selection of the most productive plants into exporting increases welfare. Trade increases (decreases) economy-wide wage inequality if it widens (compresses) wage differences within exporters.*

We complete the analysis in this section by shedding light on the consequences of a marginal reduction in transport cost parameter τ. Such a decline increases the expected income from exporting, and thus raises χ, according to (34). From eqs. (36) and (37), we can infer $\mathrm{d}\rho_d(\tilde{\varphi})/\mathrm{d}\chi < 0$ and $\mathrm{d}\rho_e(\tilde{\varphi})/\mathrm{d}\chi > 0$, and can hence conclude that the effect of declining transport costs on the division of tasks into occupations and its consequences for plant-level worker efficiency and within-plant wage inequality are monotonic. Welfare increases monotonically in χ, according to equation (40). However, the impact of a higher χ on economy-wide wage inequality needs not be monotonic. The reason is that plants that start to export adjust their count of distinct occupations discretely, and hence the effect on plant-level employment is stronger for new than for incumbent exporters. At high levels of χ, new exporters are low-productivity plants and, if $\eta > 0$, these are plants with relatively low wage inequality. If employment in these plants increases disproportionately, we conjecture that this change may dominate the overall increase in the count of distinct occupations across plants, thereby lowering economy-wide wage inequality at high levels of χ. We will explore this possibility with numerical simulations in future drafts and revisit the question after structurally estimating parameters for the German economy.

6 Empirics

6.1 Empirical Tests of the Model

The theoretical model outlined in Sections 4 and 5 takes a plant as a tupel of two properties: its elemental productivity $\tilde{\varphi}$ and its specific task range $\tilde{z}$ required to produce output. The model gives rise to two testable hypotheses at the plant level:

Hypothesis 1. The number of tasks and revenues are inversely related.

Hypothesis 2. The within-plant wage dispersion is positively related to plant revenues iff the coefficient of performance is positive, $\eta \geq 0$.

To test these hypotheses and to see whether their patterns are robust, we run a series of regressions, in which we vary the set of explanatory variables and instrument those regressors, whose endogeneity is suggested by our theoretical model. We consider the variables in logs to reduce sensitivity with respect to outliers and to make the results of our analysis more easily accessible to economic interpretation. To guard our estimates against an omitted variable bias, we control in all specification for time, region and sector fixed effects in addition to the explanatory variables listed in the tables. Standard errors are clustered at the plant level, because we observe a large fraction of plants repeatedly.

Test of hypothesis 1: Table 4 reports the results from estimating the link between the normalized number of tasks $b/\tilde{z}$ and plant-level revenues. The first three columns of the table present the outcome of OLS regressions, which support our theoretical hypothesis of a negative link between plant-level revenues and the number of tasks. The baseline specification in Column 1 suggests that a ten percent increase in plant-level revenues is associated with a one percent decline in the (normalized) number of tasks $b/\tilde{z}$. This effect gets smaller when we add the log count of distinct occupations in a plant and the interaction term of the log count of occupations and log revenues as further explanatory variables. A negative impact of the count of distinct occupations on the number of tasks is in line with our theoretical model. However, from our model one may expect that the count of occupations and

Table 4: Predictors of the Number of Tasks

	Dependent variable: log Normalized number of tasks $\ln b/\tilde{z}$						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	OLS	OLS	OLS	IV	IV	IV	IV
log revenues	-0.091***	-0.057***	-0.051***	-0.021	-0.021*	-0.259***	-0.257***
	(0.003)	(0.007)	(0.009)	(0.013)	(0.013)	(0.077)	(0.076)
log count of occupations		-0.257***	-0.328***			4.363**	4.428**
		(0.037)	(0.075)			(1.975)	(2.010)
log revenues		0.009***	0.013**			-0.226**	-0.230**
$\times$ log count of occupations		(0.003)	(0.005)			(0.110)	(0.112)
Plant FE	no	no	yes	no	no	no	no
R^2	0.234	0.244	0.845				
Adj. R^2	0.234	0.243	0.793				
Hansen J (p-val.)	n.a.	n.a.	n.a.	n.a.	0.288	n.a.	0.872
Observations	126,488	126,488	126,488	64,907	64,616	64,777	64,563

Sources: LIAB 1996-2014 and BIBB-BAuA 1992-2012. Sample restricted to plants with more than 2 full-time workers.
Notes: The panel covers all industries and regions of Germany between 1996-2014. All regressions include time, region, and sector fixed effects. IV estimation is based on GMM. Standard errors in parentheses. Significance levels are indicated by * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

revenues are perfectly correlated, which is not the case.[19] But this should not be interpreted as evidence against the formal structure of our model because the model does not predict a linear relationship between log revenues (or the log count of occupations) and the log normalized number of tasks $b/\tilde{z}$, and hence the fact that we are able to estimate significant effects of all three explanatory variables in Column 2 could simply reflect non-linearities in the relationship between these variables and the log normalized number of tasks. Overall, the marginal effect of an increase in log revenues on the log normalized number of tasks is still negative and amounts to -0.042, when evaluated at the mean of the log count of occupations, 1.648. In Column 3 we see that the negative relationship between revenues and the count of occupations is also robust to adding fixed plant effects.

Our theoretical model implies that plant-level revenues, the count of occupations, and the number of tasks carried out by workers, are jointly endogenous to the plant's market conditions. The OLS estimates in Columns 1 through 3 therefore do not have a causal interpretation. We use an instrumental variable (IV) approach and estimate the relationship between revenues and the number of tasks (in logs), using GMM. The second-stage results of the respective regressions are reported in Columns 4-7 of Table 4, and Table 5 shows the first-stage results are.

Our choice of instruments is guided by insights from our theoretical model, which predicts that globalization in in the form of more industry exports and more industry imports affects plant-level revenues, the count of dis-

[19]The correlation coefficient of these two variables amounts to 0.671.

Table 5: PREDICTORS OF THE NUMBER OF TASKS: FIRST STAGE

Dep. Variable:	log rev.	log rev.	log rev.	log occupations	interac.	log rev.	log occupations	interac.
	(4.1)	(5.1)	(6.1)	(6.2)	(6.3)	(7.1)	(7.2)	(7.3)
export dum$_{t-1}$	0.116***	0.116***	0.219***	2.428***	0.129***	0.212***	2.373***	0.126***
$\times$ log exports$_{CHN}$	(0.007)	(0.007)	(0.020)	(0.243)	(0.015)	(0.018)	(0.239)	(0.015)
log imports$_{CHN}$		-0.017				-0.193***	-1.201***	-0.069***
		(0.010)				(0.007)	(0.088)	(0.006)
export dum$_{t-1}$			-0.201***	-2.007***	-0.104***	-0.191***	-1.935***	-0.101***
$\times$ log exports$_{EE}$			(0.021)	(0.259)	(0.016)	(0.019)	(0.253)	(0.016)
mill. rev. dis.$_{t-1}$			0.001***	0.007***	0.0004***	0.001***	0.007***	0.0004***
$\times$log imports$_{EE}$			(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
F-stat.	312.3	160.5	6,290.4	1,416.8	1,074.1	4674.5	1044.3	800.5
Observations	64,907	64,616	64,563	64,563	64.563	64,563	64,563	64,563

Sources: LIAB 1996-2014 and BIBB-BAuA 1992-2012. Sample restricted to plants with more than 2 full-time workers.
Notes: The panel covers all industries and regions of Germany between 1996-2014. All regressions include time, region, and sector fixed effects. IV estimation is based on GMM. Standard errors in parentheses. Significance levels are indicated by * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

tinct occupations, and thus the number of tasks carried out by workers. This suggests using exports and imports at the industry level as instruments. However, these industry aggregates themselves depend on the plants' decision regarding the distinct number of occupation, and they are likely not exogenous. Therefore, we follow the reasoning of Autor, Dorn and Hanson (2013) and use other high-income countries' exports to and imports from China (CHN) as instruments for German exports and imports at the industry level. In the selection of other high-income countries, we follow Dauth, Findeisen and Suedekum (2014) and use Australia, Canada, Japan, Norway, New Zealand, Sweden, Singapore, and the United Kingdom as instrument group. Since shipments to China affect exporters differently from non-exporters, we interact the log exports to China with a dummy capturing the export status of a plant in the previous business year. With three potentially endogenous regressors in Columns 2 and 3, it is not sufficient to specify just two instruments. We therefore add log exports to Eastern Europe (EE) interacted with a plant's export status in the previous business year and the log of imports from Eastern Europe interacted with a plant's millentile position in the revenue distribution from the previous business year as additional instruments, when necessary (and consider the same instrument group as for China). Accounting for exports to and imports from Eastern Europe as an additional set of instruments is motivated by the work of Dauth, Findeisen and Suedekum (2014), who show that trade exposure to China and trade exposure to Eastern Europe tend to have opposite consequences for the German economy.[20]

Column 4 reports the IV results when considering only log revenues as an explanatory variable (in addition to time, region, and industry fixed effects). In this case, we only need one instrument, for which we choose the interaction of log exports to China and the lagged exporter dummy. From the first stage regression in Table 5, we can conclude that this interaction term has indeed a positive and significant effect on log revenues, and the high F-test of excluded instruments shows that we need not be concerned about weak instrumentation. However, for this specification we do not find a statistically significant impact of log revenues on the log normalized number of tasks on the second stage at conventional levels of significance. In an additional specification we add log imports from China as a second instrument. As we can see from Column 2 of Table 5, this second instrument itself does not exert a significant impact on log revenues. However, the F-statistic shows that our instruments are not weak and the overidentification test (Hansen's J) does not reject validity of our instruments. More importantly, with the additional instrument, the estimated impact of log revenues on the log of normalized tasks on the second stage becomes statistically significant. In a further regressions, we add the log count of distinct occupations and its interaction with log revenues as additional explanatory variables and instrument the now three endogenous

[20] As in their study, we associate Eastern Europe with Bulgaria, the Czech Republic, Hungary, Poland, Romania, Slovakia, Slovenia, the former USSR and its successor Russian Federation, Belarus, Estonia, Latvia, Lithuania, Moldova, Ukraine, Azerbaijan, Georgia, Kazakhstan, Kyrgyzstan, Tajikistan, Turkmenistan, and Uzbekistan.

regressors by the interaction of log exports to CHN with the lagged exporter dummy, the interaction of log exports to EE with the lagged exporter dummy, and log imports from EE with the lagged millentile position of a plant in the revenue distribution. From Columns 3-5 of Table 5, we see that the three instruments are significant in all three regressions, and the F-tests of excluded instruments are high in all three first-stage regressions.[21] on the second stage we find a (now larger) negative and statistically significant effect of log revenues on the log normalized number of tasks, whereas the coefficients of the log count of distinct occupations and its interaction with log revenues change their signs when using an IV approach. In a final specification, we add the log of imports as additional instrument. This allows us to test for overidentification and the high p-value reported in the Table 4 shows that we cannot reject validity of our instruments. Adding the additional instrument has only minor effects on the parameter estimates on the second stage (see Column 7). (IV estimates under plant fixed effects not result in statistically significant results, so we do not report the respective results here.)

Test of hypothesis 2: To test for the sign of the correlation between wage differences inside a plant and its occupation count, depending on η, we proceed in two steps: in a first step, we estimate the relationship between within-plant residual wage differences and the log occupation count as a measure of the intra-plant division of labor, conditional on log revenues, and test whether it is positive; in a second step we check whether the positive relationship between the occupation count and within-plant wage differences reported is in line with a positive estimate of η.

For the first step, as a measure of wage dispersion we use the standard deviation of residual daily wages within plant-occupations. Results are reported in Table 6. (In Appendix Table A2 we repeat the exercises for the dispersion of the total daily wage.) We use the same empirical specifications and the same instruments as in our estimation for the relationship between the log normalized number of tasks and the log revenues. We do not report the first-stage results for the IV specifications since they are closely similar to those reported in Table 5—except for minor changes in the number of observations. The results in Table 6 indicate a clear positive relationship between revenues and residual wage dispersion within plant-occupations for OLS, as well as a clearly positive relationship between residual wage dispersion within plant-occupations and the occupation count. In the long specification of Column 2, a 10 percent increase in the occupation count predicts a more than ten percentage-point increase in the standard deviation of residual wages within plant-occupations. After controlling for plant fixed effects, this positive association is further strengthened; in Column 3, a 10 percent increase in the occupation count predicts a more than twelve percentage-point increase in the standard deviation of residual wages within

[21] With multiple endogenous regressors the F-tests on the first stage are not sufficient for rejecting the null that instruments are weak. Unfortunately, clustered standard errors do not offer a straightforward alternative to testing for weak instruments. The Kleibergen-Paap LM test rejects the null of underidentification.

Table 6: Predictors of Within-Plant Residual Daily Wage Dispersion

	Dependent variable: log StDev Residual daily wage						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	OLS	OLS	OLS	IV	IV	IV	IV
log revenues	0.185***	0.174***	0.104***	0.295***	0.293***	0.065	0.059
	(0.005)	(0.013)	(0.019)	(0.025)	(0.025)	(0.093)	(0.093)
log count of occupations		1.042***	1.232***			5.783***	6.006**
		(0.078)	(0.173)			(2.429)	(2.458)
log revenues		-0.055***	-0.067***			-0.309***	-0.321**
$\times$ log count of occupations		(0.006)	(0.012)			(0.135)	(0.137)
Plant FE	no	no	yes	no	no	no	no
Hansen J (p-val.)	n.a.	n.a.	n.a.	n.a.	0.165	n.a.	0.685
R^2	0.293	0.345	0.836				
Adj. R^2	0.292	0.345	0.781				
Observations	126,483	126,483	126,483	64,905	64,614	64,775	64,561

Sources: LIAB 1996-2014 and BIBB-BAuA 1992-2012. Sample restricted to plants with more than 2 full-time workers.
Notes: The log StDev Residual daily wage is computed within plant-occupations. The panel covers all industries and regions of Germany between 1996-2014. All regressions include time, region, and sector fixed effects. IV estimation is based on GMM. Standard errors in parentheses. Significance levels are indicated by * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

plant-occupations.

In Columns 4 and 5 of Table 6 we report estimates when instrumenting log revenues as the single endogenous regressor. Test statistics are consistent with the hypothesis that the interaction of log exports to China with the plant's lagged exporter dummy and the log of imports from China provide valid instruments, and the positive effect of revenues on wage variation within plant-occupations remains robust to the change in estimation strategy. In Columns 6 and 7 we treat all three regressors—log revenues, the log count of occupations and the interaction term of these two variables—as endogenous variables and instrument them with the variables reported in Table 5. Log revenues lose statistical significance at conventional levels, whereas the association between the occupation count and residual wage dispersion within plant-occupations becomes stronger than under OLS. We interpret this evidence as suggestive of a direct reorganization channel in the plant's internal labor market, by which product-market expansions in the wake of globalization trigger a more specialized division of labor, which in turn leads to more residual wage inequality within plant-occupations.

In a second step of our test of hypothesis 2, we check whether the positive relationship between revenues and within-plant wage differences reported in Table 6 is in line with a positive estimate of η. For this purpose, we can

Table 7: Coefficient of Performance η and Semi-elasticity of Control Costs γ

	Dep. var.: $\sqrt{4-\pi(\pi-2)}/cvw-\pi$			Dep. var.: logarithm of $cvw \times b/\tilde{z}$		
	(1)	(2)	(3)	(4)	(5)	(6)
$b/\tilde{z}$	3.302**	4.258***	5.338***			
	(0.820)	(1.136)	(1.605)			
log employment				-0.067***	-0.061***	0.016
				(0.007)	(0.006)	(0.025)
constant	-0.996***	-1.813***	n.a.	-2.0378***	-2.2543***	n.a.
	(-0.350)	(0.601)	n.a.	(0.022)	(0.054)	n.a.
Sector and Region FE	no	yes	yes	no	yes	yes
Plant FE	no	no	yes	no	no	yes
R^2	0.001	0.007	0.626	0.031	0.201	0.787
Adj. R^2	0.001	0.006	0.500	0.030	0.201	0.716
Observations	126,483	126,483	126,483	126,483	126,483	126,483

Sources: LIAB 1996-2014 and BIBB-BAuA 1992-2012. Sample restricted to plants with more than 2 full-time workers.
Notes: The panel covers all industries and regions of Germany between 1996-2014. All regressions include time fixed effects. Standard errors in parentheses. Significance levels are indicated by * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

first combine eqs. (4) and (12) to solve for

$$\left[\frac{\sqrt{4-\pi(\pi-2)}}{cvw}-\pi\right]_{t,\omega}=\eta\left(\frac{b}{\tilde{z}}\right)_{t,\omega}, \tag{43}$$

where t and ω are year and plant indices, respectively. Eq. (43) offers an empirically testable link between cvw and $b/\tilde{z}$. Following the reasoning in our model, we use the coefficient of variation of wages to construct the left-hand side variable in eq. (43), estimate the model with OLS, and report the results of this estimation in Columns 1-3 of Table 7. The estimate of η is positive in all specifications and, together with the estimation results on the relationship between the within-plant wage dispersion and revenues in Table 6, we take this as strong supportive evidence for hypothesis 2.

For a final test of the structure of theoretical model in Section 4 and 5, we note that constant markup pricing establishes $\ell(\omega) = [(\sigma-1)/\sigma]r(\omega)/w$ in our model. Substitution into equation (17) yields $\ell(\omega) = \gamma[\eta + \pi(1+\nu n(\omega))]$, which in view of eqs. (4) and (12) establishes a log-linear relationship between the coefficient of variation of wages within plants, the normalized number of tasks, and the employment of production workers at the plant:

$$\log\left[cvw\frac{b}{\tilde{z}}\right]_{t,\omega}=\log\left[\frac{\sqrt{4-\pi(\pi-2)}}{\gamma^{1/\gamma}}\right]-\frac{1}{\gamma}\ln\ell_{t,\omega}. \tag{44}$$

We can construct the left-hand side variable and estimate eq. (44), using OLS. Columns 4-6 of Table 7 report the results of this specification. They broadly support a positive sign of γ (associated with a negative coefficient), except for the case with plant fixed effects when the coefficient on log employment loses significance at conventional level. The small point estimates of the coefficient of log employment suggest a strong convexity of the span-of-control cost in the occupation count.

6.2 Towards structural estimation

We now turn to establishing a structural estimation model of plant-level outcomes. For this purpose, we return to the general model specification in Section 3, where a plant with identifier ω is an elemental tupel of four stochastic parameters that may change during the sample period 1996-2012: the plant's elemental productivity $\tilde{\varphi}(\omega)_t$, the plant's fixed cost draw for exporting $\tilde{f}_x(\omega)_t$, the plant's coefficient of performance $\tilde{\eta}(\omega)_t$ (which relates the plant's number of tasks per occupation to its within-occupation wage inequality), and a plant's full task range $\tilde{z}(\omega)_t$. The plant's full task range is not observed because we can only partially infer the plant's overall task range out of 15 possible (and time consistent) workplace activities ("what" tasks). To link theory to data, we therefore consider $\tilde{z}(\omega)_t = \tilde{\zeta}(\omega)_t \cdot z(\omega)_t$, where $z(\omega)_t$ is a plant's observed (imputed) overall task range out of 15 possible tasks and $\tilde{\zeta}(\omega)_t$ is a plant-specific task range variability that can be smaller or larger than one, depending on whether the plant exhausts the full 15 possible tasks (then $\tilde{\zeta}(\omega)_t < 1$), such as through outsourcing and offshoring, or exceeds the 15 maximally observed tasks (then $\tilde{\zeta}(\omega)_t > 1$).

The plant's profit-maximizing conditions from the four-stage optimization problem in Section 4 imply a set of estimable equations. One characterization is a three-equation system that involves plant revenues $r(\omega)_t$, the plant's coefficient of variation of the daily wage residual within its occupations $cvw(\omega)_t$, the normalized number of tasks per occupation $b(\omega)_t/\tilde{z}(\omega)_t$ at the plant (the theoretical counterpart to the empirically imputed total number of tasks performed at a plant out of the maximum of 15 in the data), and an export indicator $\mathbf{1_x}(\omega)_t$ as the observed variables $\mathbf{x}(\omega)_t = [r(\omega)_t, CV(\omega)_t, b(\omega)_t/z(\omega)_t, \mathbf{1_x}(\omega)_t]$:

$$\ln r(\omega)_t = \alpha_{0,t} + \alpha_{1,t}\mathbf{1_x}(\omega)_t + \xi \ln \tilde{\varphi}(\omega)_t, \tag{45a}$$

$$\ln CV(\omega)_t + \ln b(\omega)_t/z(\omega)_t = \beta_{0,t} - (1/\gamma)\, \ln r(\omega)_t + \ln \tilde{\zeta}(\omega)_t, \tag{45b}$$

$$\mathbf{1_x}(\omega)_t = 1 \Leftrightarrow \gamma_{0,t} \geq \ln \tilde{f}_x(\omega)_t - \xi \ln \tilde{\varphi}(\omega)_t, \tag{45c}$$

where the estimation parameter γ is the (constant) elasticity of the span-of-control fixed cost and the remaining parameters $\alpha_{0,t}$, $\alpha_{1,t}$, $\beta_{0,t}$ and $\gamma_{0,t}$ are composites of model fundamentals including time-varying trade costs τ_t, the time-varying common part of export fixed cost $f_{0,t}$, and the time-invariant elasticity of substitution σ, as well as the time-varying domestic and foreign market sizes. This equation system involves only three of the four stochastic terms that characterize a plant: $\ln \tilde{\varphi}(\omega)_t$, $\ln \tilde{\zeta}(\omega)_t$ and $\ln \tilde{f}_x(\omega)_t$.

This model is estimable with maximum likelihood. Once estimated, the model can be simulated to quantify the importance of globalization for the intra-plant division of labor.

Importantly, the estimation model can set aside the plant's coefficient of performance $\tilde{\eta}(\omega)_t$. The reason is that the plant's profit-maximizing conditions imply that the plant-specific $\tilde{\eta}(\omega)_t$ can be recovered directly from the data without estimation:

$$\ln\left[\sqrt{4-\pi(\pi-2)}/CV(\omega)_t - \pi\right] - \ln b(\omega)_t/z(\omega)_t = \ln[\tilde{\eta}(\omega)_t] \qquad \text{if } \tilde{\eta}(\omega)_t \geq 0, \tag{46a}$$

$$\ln\left[-\sqrt{4-\pi(\pi-2)}/CV(\omega)_t + \pi\right] - \ln b(\omega)_t/z(\omega)_t = \ln[-\tilde{\eta}(\omega)_t] \quad \text{if } \tilde{\eta}(\omega)_t < 0. \tag{46b}$$

From the data, we can compute $\tilde{\eta}(\omega)_t$ since the right-hand side of eq. (46) is observed per plant.

The questionnaire of the BIBB-BAuA surveys also includes the question whether a worker's small mistakes in his or her occupation cause the employer financial losses ("Financial losses by small mistake," see Becker and Muendler 2015). With measures of the plant's coefficients of performance at hand by (46), we can test our fundamental tenet that an employer's surplus is more sensitive to worker performance when the division of labor is more specialized. Then narrower task ranges are associated with higher coefficients of performance $\tilde{\eta}$, and thus more wage variability within plant-occupations (by Figure 4). Answers to the question "Financial losses by small mistake" come in four degrees: "never," "seldom," "occasionally," and "frequently or almost always." We can run a regression of $\tilde{\eta}(\omega)_t$ on the three categories other than "never" in the BIBB-BAuA on the 44,733 observations of workers who report answers to the "loss" question, where $\tilde{\eta}(\omega(i))_t$ is the coefficient of performance at the worker's plant. We control for the full set of plant size categories and, relative the omitted loss category "never," find the result

$$\tilde{\eta}(\omega(i))_t = \underset{(0.026)}{0.536} \cdot \mathbb{1}(\text{seldom}, i)_t + \underset{(0.028)}{0.704} \cdot \mathbb{1}(\text{occasionally}, i)_t + \underset{(0.026)}{1.084} \cdot \mathbb{1}(\text{frequently}, i)_t$$

with an adjusted goodness of fit of $R^2 = 0.1749$.

The result suggests that plants have significantly higher coefficients of performance $\tilde{\eta}$ if their workers report more frequent financial losses for the employer when they make small mistakes on the job. At the plants that are more sensitive to worker performance, worker mismatches tend to translate task specialization into higher wage dispersion through a higher coefficient of performance $\tilde{\eta}$.

7 Concluding Remarks

We document empirically that workers in larger plants perform fewer tasks and that a dominant part of residual wage inequality materializes within plants, layers of hierarchies, and occupations.

Based on these observations, we augment the Melitz (2003) heterogeneous-firm model with a production technology for the internal labor market that allows plants to assign the range of tasks that need to be performed to an endogenous count of occupations. The within-plant wage dispersion changes with the plant's choice of the count of occupations because workers in our model have a core ability that makes them most efficient at performing one specific task and monotonically less efficient in tasks that are farther from their core ability. A plant can therefore achieve a labor efficiency gain from narrowing the range of tasks performed by workers when raising the count of occupations to which it assigns tasks. More specialized occupations allow the plant to narrow the range of tasks performed by workers within each job and, because workers' core abilities are distributed over the task range, raise average labor efficiency. Extending the count of occupations, however, comes at an overhead cost because a larger count of occupations aggravates the span of control and generates coordination frictions that increase overhead costs. In equilibrium, inherently more productive plants and exporters adopt a larger count of occupations and thus boost their elemental productivity further with higher labor efficiency, by concentrating occupations on narrower task ranges. Under certain magnitudes of the baseline labor efficiency of workers, which we find confirmed in our estimation of the model, more productive plants end up with a wider dispersion of worker efficiencies. A worker's wage is linked to the individual labor efficiency on the job, so that more productive plants exhibit higher wage dispersion in equilibrium.

For the open economy case, we show that trade between symmetric countries exerts opposing effects on exporters and non-exporters regarding their decisions on expanding or contracting the count of occupations. Exporters benefit from foreign market access, find it easier to cover their fixed costs, and therefore add new occupations to achieve higher labor efficiency by concentrating occupations on narrower task ranges. Non-exporters, in contrast, face stronger competition, due to the employment expansion of local exporters and the market entry of foreign firms, and therefore reduce the count of occupations—accepting lower labor efficiency in order to economize on fixed overhead costs. The asymmetric responses of exporting and non-exporting plants regarding the operation of occupations widens the gap in the within-plant dispersion of wages between plants and leads to an increase in economy-wide wage inequality, provided that larger plants experience higher wage dispersion prior to the globalization shock. As a consequence, gains from trade are accompanied by higher wage inequality in all open economies.

Combining matched plant–worker data from Germany with time-varying worker survey information on the tasks that workers perform on their jobs in Germany, we can test two main hypotheses from our theoretical model. First, we confirm a negative link between plant-level revenues and the number of tasks, using OLS. This result extends to a more robust empirical specification, in which we instrument plant-level revenues with sector-level trade shocks that mimic changes in German trade exposure with trade shocks between China or Eastern Europe on the one side and other high-income countries on the other side. Second, our model suggests a positive relationship between revenues and wage dispersion at the plant level under specific conditions. To test this hypothesis, we specify similar OLS and IV regressions as for the former hypothesis and establish a robust positive relationship between plant size and within-plant wage dispersion. Finally, we infer from our theoretical model structural relationships that allow us to test the required magnitudes of underlying parameters, including a crucial coefficient of performance, and we find estimates that establish internal consistency.

In future drafts of this paper, we plan to conduct full-scale structural estimation to infer the relevant model parameters. We will then establish in simulations to what extent the internal labor markets at German plants responded to globalization shocks over time.

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Appendix

A Mathematical Appendix

A.1 Derivation of $\int_{\varphi^*}^{\infty} \psi(\tilde{\varphi})\,\mathrm{d}G(\tilde{\varphi})$ in the closed economy

Using Eqs. (16), (17), and (22) we can write profits in the closed economy as follows

$$\psi(\tilde{\varphi}) = \frac{r(\tilde{\varphi})}{\sigma}\frac{\gamma - \sigma + 1}{\gamma} - wf_0 = \frac{r(\tilde{\varphi})}{r(\varphi^*)}wf_0 - wf_0. \tag{A.1}$$

Substituting $r(\tilde{\varphi})/r(\varphi^*) = (\tilde{\varphi}/\varphi^*)^{\xi}$ from equation (20), we can then compute

$$\int_{\varphi^*}^{\infty} \psi(\tilde{\varphi})\,\mathrm{d}G(\tilde{\varphi}) = wf_0(\varphi^*)^{\xi}\theta \int_{\varphi^*}^{\infty} \tilde{\varphi}^{\xi-\theta-1}\,\mathrm{d}\tilde{\varphi} - wf_0\theta \int_{\varphi^*}^{\infty} \tilde{\varphi}^{-\theta-1}\,\mathrm{d}\tilde{\varphi} = (\varphi^*)^{-\theta} wf_0 \frac{\xi}{\theta - \xi}. \tag{A.2}$$

Accounting for $1 - G(\varphi^*) = (\varphi^*)^{-\theta}$ then gives the respective expression in the main text.

A.2 Derivation of eqs. (28) and (29)

The average employment of production workers per plant in the closed economy can be computed according to

$$\int_{\varphi^*}^{\infty} \ell(\tilde{\varphi})\ frac \mathrm{d}G(\tilde{\varphi})1 - G(\varphi^*) = \ell(\varphi^*)(\varphi^*)^{\theta-\xi}\theta \int_{\varphi^*}^{\infty} \tilde{\varphi}^{\xi-\theta-1}\,\mathrm{d}\tilde{\varphi} = \ell(\varphi^*)\frac{\theta}{\theta - \xi}, \tag{A.3}$$

according to equation (21). The employment-share weighted average of the variances of wages at the plant-level is then given by equation (28). Substitution of equation (24) for $var(\tilde{\varphi})$ and $\ell(\tilde{\varphi}) = (\tilde{\varphi}/\varphi^*)^{\xi}\ell(\varphi^*)$ from equation (21) further establishes

$$\begin{aligned} Var = var(\varphi^*)\frac{\theta - \xi}{(b - \eta)^2}\Bigg\{ & b^2(\varphi^*)^{\theta-\xi}\int_{\tilde{\varphi}}^{\infty} \tilde{\varphi}^{\xi-\theta-1}\,\mathrm{d}\tilde{\varphi} - 2b\eta(\varphi^*)^{\theta+\frac{\xi}{\gamma}-\xi}\int_{\varphi^*}^{\infty} \tilde{\varphi}^{\xi-\frac{\xi}{\gamma}-\theta-1}\,\mathrm{d}\tilde{\varphi} \\ & + \eta^2(\tilde{\varphi}*)^{\theta+\frac{2\xi}{\gamma}-\xi}\int_{\varphi^*}^{\infty} \tilde{\varphi}^{\xi-\frac{2\xi}{\gamma}-\theta-1}\,\mathrm{d}\tilde{\varphi}\Bigg\}, \end{aligned} \tag{A.4}$$

where $b \equiv (\xi f_0/\gamma)^{1/\gamma}$ has been used. Solving for the integral gives

$$Var = \frac{var(\varphi^*)}{[(\xi f_0/\gamma)^{1/\gamma} - \eta]^2}\left[\left(\frac{\xi f_0}{\gamma}\right)^{\frac{2}{\gamma}} - 2\eta\left(\frac{\xi f_0}{\gamma}\right)^{\frac{1}{\gamma}}\frac{\theta - \xi}{\theta - \xi + \xi/\gamma} + \eta^2\frac{\theta - \xi}{\theta - \xi + 2\xi/\gamma}\right] \tag{A.5}$$

and finally equation (29).

A.3 Derivation of $\int_{\varphi^*}^{\infty} \psi_t(\tilde{\varphi})\, \mathrm{d}G(\tilde{\varphi})$ in the open economy

Using eqs. (16), (17), and (22), we can write profits of non-exporters as follows $\psi_t^d(\tilde{\varphi}) = [r^d(\tilde{\varphi})/r^d(\varphi^*)]f_0 - f_0$, where $w = 1$ because of our choice of numéraire. Total profits of exporters are given by

$$\psi_t^e(\tilde{\varphi}) = \left(1+\tau^{1-\sigma}\right)^{\frac{\xi}{\sigma-1}} \frac{r^d(\tilde{\varphi})}{\sigma}\frac{\gamma-\sigma+1}{\gamma} - f_0 - f_x = \left(1+\tau^{1-\sigma}\right)^{\frac{\xi}{\sigma-1}} \frac{r^d(\tilde{\varphi})}{r^d(\varphi^*)} f_0 - f_0 - f_x, \tag{A.6}$$

according to eqs. (16), (22), (30), and (31). Accounting for $r^d(\tilde{\varphi})/r^d(\varphi^*) = (\tilde{\varphi}/\varphi^*)^{\xi}$ from equation (20) then allows us to compute

$$\begin{aligned}
\int_{\varphi^*}^{\infty} \psi_t(\tilde{\varphi})\, \mathrm{d}G(\tilde{\varphi}) &= \int_{\varphi^*}^{\tilde{\varphi}_x^*} \psi_t^d(\tilde{\varphi})\, \mathrm{d}G(\tilde{\varphi}) + \int_{\tilde{\varphi}_x^*}^{\infty} \psi_t^e(\tilde{\varphi})\, \mathrm{d}G(\tilde{\varphi}) \\
&= f_0(\varphi^*)^{-\xi}\theta \int_{\varphi^*}^{\tilde{\varphi}_x^*} \tilde{\varphi}^{\xi-\theta-1}\, \mathrm{d}\tilde{\varphi} + \left(1+\tau^{1-\sigma}\right)^{\frac{\xi}{\sigma-1}} f_0(\varphi^*)^{-\xi}\theta \int_{\tilde{\varphi}_x^*}^{\infty} \tilde{\varphi}^{\xi-\theta-1}\, \mathrm{d}\tilde{\varphi} \\
&\qquad - f_0\theta \int_{\varphi^*}^{\infty} \tilde{\varphi}^{-\theta-1}\, \mathrm{d}\tilde{\varphi} - f_x\theta \int_{\tilde{\varphi}_x^*}^{\infty} \tilde{\varphi}^{-\theta-1}\, \mathrm{d}\tilde{\varphi}
\end{aligned} \tag{A.7}$$

Solving for the integrals yields

$$\begin{aligned}
\int_{\varphi^*}^{\infty} \psi_t(\tilde{\varphi})\, \mathrm{d}G(\tilde{\varphi}) &= f_0(\varphi^*)^{-\theta}\frac{\theta}{\theta-\xi}\left\{1 + \left(\frac{\tilde{\varphi}_x^*}{\varphi^*}\right)^{\xi-\theta}\left[\left(1+\tau^{1-\sigma}\right)^{\frac{\xi}{\sigma-1}} - 1\right]\right\} \\
&\qquad - (\varphi^*)^{-\theta} f_0 \left[1 + \left(\frac{\tilde{\varphi}_x^*}{\varphi^*}\right)^{-\theta} \frac{f_x}{f_0}\right],
\end{aligned} \tag{A.8}$$

which in view of $1 - G(\varphi^*) = (\varphi^*)^{-\theta}$, $\chi = (\tilde{\varphi}_x^*/\varphi^*)^{-\theta}$, and equation (34) can be rewritten as equation (35).

A.4 Derivation of eqs. (41) and (42)

First of all, the average employment of production workers per plant in the open economy can be computed according to

$$\begin{aligned}
\int_{\varphi^*}^{\infty} \ell_t(\tilde{\varphi}) \frac{\mathrm{d}G(\tilde{\varphi})}{1-G(\varphi^*)} &= \int_{\varphi^*}^{\tilde{\varphi}_x^*} \ell_t^d(\tilde{\varphi}) \frac{\mathrm{d}G(\tilde{\varphi})}{1-G(\varphi^*)} + \int_{\tilde{\varphi}_x^*}^{\infty} \ell_t^e(\tilde{\varphi}) \frac{\mathrm{d}G(\tilde{\varphi})}{1-G(\varphi^*)} \\
&= \ell_t^d(\varphi^*)(\varphi^*)^{\theta-\xi}\theta \left[\int_{\varphi^*}^{\tilde{\varphi}_x^*} \tilde{\varphi}^{\xi-\theta-1}\, \mathrm{d}\tilde{\varphi} + (1+\tau^{1-\sigma})^{\frac{\xi}{\sigma-1}} \int_{\tilde{\varphi}_x^*}^{\infty} \tilde{\varphi}^{\xi-\theta-1}\, \mathrm{d}\tilde{\varphi}\right] \\
&= \ell_t^d(\varphi^*)\frac{\theta}{\theta-\xi}\left(1 + \frac{\chi f_x}{f_0}\right),
\end{aligned} \tag{A.9}$$

where the second equality sign follows from equation (32) and the third equality sign follows from equation (34). The economy-share weighted average of the variances of wages at the plant-level can then be computed in analogy to the closed economy:

$$Var = \frac{\theta-\xi}{\theta \ell_t^d(\varphi^*)}\left(1+\frac{\chi f_x}{f_0}\right)^{-1}\left[\int_{\varphi^*}^{\tilde{\varphi}_x^*} var_d(\tilde{\varphi})\ell_t^d(\tilde{\varphi})\frac{\mathrm{d}G(\tilde{\varphi})}{1-G(\varphi^*)} + \int_{\tilde{\varphi}_x^*}^{\infty} var_e(\tilde{\varphi})\ell_t^e(\tilde{\varphi})\frac{\mathrm{d}G(\tilde{\varphi})}{1-G(\varphi^*)}\right]. \tag{A.10}$$

We look at the integrals on the right-side separately. Following the derivation steps of the closed economy and accounting for $b = (\xi f_0/\gamma)^{1/\gamma}$, we compute

$$\begin{aligned}
\int_{\varphi^*}^{\tilde{\varphi}_x^*} var_d(\tilde{\varphi})\ell_t^d(\tilde{\varphi})\frac{\mathrm{d}G(\tilde{\varphi})}{1-G(\varphi^*)} &= \ell_t^d(\varphi^*)\frac{var_d(\varphi^*)}{(b-\eta)^2}\theta\left\{b^2(\varphi^*)^{\theta-\xi}\int_{\varphi^*}^{\tilde{\varphi}_x^*}\tilde{\varphi}^{\xi-\theta-1}\,\mathrm{d}\tilde{\varphi}\right.\\
&\left.\quad -2b\eta(\varphi^*)^{\theta+\frac{\xi}{\gamma}-\xi}\int_{\varphi^*}^{\tilde{\varphi}_x^*}\tilde{\varphi}^{\xi-\theta-\frac{\xi}{\gamma}-1}\,\mathrm{d}\tilde{\varphi} + \eta^2(\varphi^*)^{\theta-\xi-\frac{2\xi}{\gamma}}\int_{\varphi^*}^{\tilde{\varphi}_x^*}\tilde{\varphi}^{\xi-\theta-\frac{2\xi}{\gamma}-1}\,\mathrm{d}\tilde{\varphi}\right\}\\
&= \ell_t^d(\varphi^*)\frac{var_d(\varphi^*)}{(b-\eta)^2}\frac{\theta}{\theta-\xi}\left\{\left[b^2+2b\eta\frac{\theta-\xi}{\theta-\xi+\xi/\gamma}+\eta^2\frac{\theta-\xi}{\theta-\xi+2\xi/\gamma}\right]\left[1-\left(\frac{\tilde{\varphi}_x^*}{\varphi^*}\right)^{\xi-\theta}\right]\right.\\
&\quad +2b\eta\frac{\theta-\xi}{\theta-\xi+\xi/\gamma}\left(\frac{\tilde{\varphi}_x^*}{\varphi^*}\right)^{\xi-\theta}\left[\left(\frac{\tilde{\varphi}_x^*}{\varphi^*}\right)^{-\frac{\xi}{\gamma}}-1\right]\\
&\left.\quad -\eta^2\frac{\theta-\xi}{\theta-\xi+2\xi/\gamma}\left(\frac{\tilde{\varphi}_x^*}{\varphi^*}\right)^{\xi-\theta}\left[\left(\frac{\tilde{\varphi}_x^*}{\varphi^*}\right)^{-\frac{2\xi}{\gamma}}-1\right]\right\}.
\end{aligned} \tag{A.11}$$

For the second integral, we obtain

$$\begin{aligned}
\int_{\tilde{\varphi}_x^*}^{\infty} var_e(\tilde{\varphi})\ell_t^e(\tilde{\varphi})\frac{\mathrm{d}G(\tilde{\varphi})}{1-G(\varphi^*)} &= \ell_t^d(\varphi^*)\frac{var_d(\varphi^*)}{(b-\eta)^2}\left(1+\tau^{1-\sigma}\right)^{\frac{\xi}{\sigma-1}}\theta\left\{b^2(\varphi^*)^{\theta-\xi}\int_{\tilde{\varphi}_x^*}^{\infty}\tilde{\varphi}^{\xi-\theta-1}\,\mathrm{d}\tilde{\varphi}\right.\\
&\left.\quad -2b\eta(\varphi^*)^{\theta+\frac{\xi}{\gamma}-\xi}\int_{\tilde{\varphi}_x^*}^{\infty}\tilde{\varphi}^{\xi-\theta-\frac{\xi}{\gamma}-1}\,\mathrm{d}\tilde{\varphi} + \eta^2(\varphi^*)^{\theta-\xi-\frac{2\xi}{\gamma}}\int_{\tilde{\varphi}_x^*}^{\infty}\tilde{\varphi}^{\xi-\theta-\frac{2\xi}{\gamma}-1}\,\mathrm{d}\tilde{\varphi}\right\}\\
&= \ell_t^d(\varphi^*)\frac{var_d(\varphi^*)}{(b-\eta)^2}\frac{\theta}{\theta-\xi}\left(1+\tau^{1-\sigma}\right)^{\frac{\xi}{\sigma-1}}\left\{\left[b^2+2b\eta\frac{\theta-\xi}{\theta-\xi+\xi/\gamma}+\eta^2\frac{\theta-\xi}{\theta-\xi+2\xi/\gamma}\right]\left(\frac{\tilde{\varphi}_x^*}{\varphi^*}\right)^{\xi-\theta}\right.\\
&\quad +2b\eta\frac{\theta-\xi}{\theta-\xi+\xi/\gamma}\left(\frac{\tilde{\varphi}_x^*}{\varphi^*}\right)^{\xi-\theta}\left[\left(1+\tau^{1-\sigma}\right)^{-\frac{\xi}{\gamma(\sigma-1)}}\left(\frac{\tilde{\varphi}_x^*}{\varphi^*}\right)^{-\frac{\xi}{\gamma}}-1\right]\\
&\left.\quad -\eta^2\frac{\theta-\xi}{\theta-\xi+2\xi/\gamma}\left(\frac{\tilde{\varphi}_x^*}{\varphi^*}\right)^{\xi-\theta}\left[\left(1+\tau^{1-\sigma}\right)^{-\frac{2\xi}{\gamma(\sigma-1)}}\left(\frac{\tilde{\varphi}_x^*}{\varphi^*}\right)^{-\frac{2\xi}{\gamma}}-1\right]\right\}.
\end{aligned} \tag{A.12}$$

Substituting eqs. (A.11) and (A.12) into (A.9) and accounting for $\tilde{\varphi}_x^*/\varphi^* = \chi^{-1/\theta}$ and eqs. (29) and (34), we arrive at equation (41), with $V(\chi)$ given by equation (42). From $\chi < 1$ it follows that $\chi^{\frac{\xi}{\theta\gamma}} - 1 > \chi^{\frac{2\xi}{\theta\gamma}} - 1$. Noting further that $\left(\frac{\xi f_0}{\gamma}\right)^{1/\gamma} > \eta$, it follow from equation (42) that

$$2\left[1-\left(1+\chi^{\frac{\xi}{\theta}}\frac{f_x}{f_0}\right)^{-\frac{1}{\gamma}}\chi^{\frac{\xi}{\theta\gamma}}\right] > \left[1-\left(1+\chi^{\frac{\xi}{\theta}}\frac{f_x}{f_0}\right)^{-\frac{2}{\gamma}}\chi^{\frac{2\xi}{\theta\gamma}}\right] \tag{A.13}$$

or, equivalently,

$$2 > \left[1 + \left(1 + \chi^{\frac{\xi}{\theta}} \frac{f_x}{f_0}\right)^{-\frac{1}{\gamma}} \chi^{\frac{\xi}{\theta\gamma}}\right] \tag{A.14}$$

is sufficient for $V(\chi) > 0$. In view of $[1 + \chi^{\xi/\theta} f_x/f_0]^{-1/\gamma}\chi^{\xi/(\theta\gamma)} < 1$, this establishes the positive sign of $V(\chi)$.

A.5 Marginal effect of a change in τ on Var

In the main text, we argue that the impact of a change in τ on the economy-wide variance of wages Var is not necessarily monotonic. To show this result formally, we use the following definitions:

$$v(\chi) \equiv \frac{\chi^{1-\frac{\xi}{\theta}}}{1 + \chi f_x/f_0}, \qquad \text{and} \qquad \hat{V}(\chi) \equiv v(\chi)V(\chi). \tag{A.15}$$

From equation (41), we can conclude that monotonicity of Var in τ requires monotonicity $\hat{V}(\chi)$. Furthermore, it follows from $\hat{V}(0) = 0$ and $\hat{V}(\chi) > 0$ for all $\chi > 0$ (see Appendix A.4) that $V'(\chi) > 0$ at low levels of χ. To see whether $V'(\chi)$ also extends to high levels of χ, we can differentiate $\hat{V}(\chi)$. This gives

$$\hat{V}'(\chi) = v'(\chi)V(\chi) + v(\chi)V'(\chi), \tag{A.16}$$

with

$$v'(\chi) = \frac{v(\chi)}{\chi} \frac{1 - (\xi/\theta)[1 + \chi f_x/f_0]}{1 + \chi f_x/f_0} \tag{A.17}$$

and

$$V'(\chi) \equiv 2\left(\frac{\xi f_0}{\gamma}\right)^{1/\gamma} \frac{\theta - \xi + 2\xi/\gamma}{\theta - \xi + \xi/\gamma} \frac{1}{\chi} \left\{ \frac{\xi}{\theta\gamma} \chi^{\frac{\xi}{\theta\gamma}} \left[1 - \left(1 + \chi^{\frac{\xi}{\theta}} \frac{f_x}{f_0}\right)^{-\frac{1}{\gamma}}\right] + \frac{\xi}{\theta} \chi^{\frac{\xi}{\theta}} \frac{f_x}{f_0} \left[1 - \left(1 + \chi^{\frac{\xi}{\theta}} \frac{f_x}{f_0}\right)^{-\frac{1}{\gamma}} \chi^{\frac{\xi}{\theta\gamma}}\right]\right\} \tag{A.18}$$

according to equation (42). Rearranging terms, we get $\hat{V}\prime(\chi) = [v(\chi)/\chi]\left[\hat{V}_0(\chi) - \hat{V}_1(\chi)\right]$, with

$$\begin{aligned}\hat{V}_0(\chi) \equiv 2\left(\frac{\xi f_0}{\gamma}\right)^{\frac{1}{\gamma}} \frac{\theta - \xi + 2\xi/\gamma}{\theta - \xi + \xi/\gamma} \Bigg\{ &\chi^{\frac{\xi}{\theta\gamma}} \left[1 - \left(1 + \chi^{\frac{\xi}{\theta}} \frac{f_x}{f_0}\right)^{-\frac{1}{\gamma}}\right] \left[\frac{1}{1 + \chi f_x/f_0} + \frac{\xi}{\theta\gamma} - \frac{\xi}{\theta}\right] \\ &+ \frac{\chi^{\frac{\xi}{\theta}} f_x/f_0}{1 + \chi f_x/f_0} \left[1 - \left(1 + \chi^{\frac{\xi}{\theta}} \frac{f_x}{f_0}\right)^{-\frac{1}{\gamma}} \chi^{\frac{\xi}{\theta\gamma}}\right] \Bigg\}\end{aligned} \tag{A.19}$$

and

$$\hat{V}_1(\chi) \equiv \frac{\eta}{\chi}\Bigg\{\chi^{\frac{\xi}{\theta\gamma}}\left[1-\left(1+\chi^{\frac{\xi}{\theta}}\frac{f_x}{f_0}\right)^{-\frac{1}{\gamma}}\right]\left[1+\left(1+\chi^{\frac{\xi}{\theta}}\frac{f_x}{f_0}\right)^{-\frac{1}{\gamma}}\right]\left[\frac{1}{1+\chi f_x/f_0}+\frac{\xi}{\theta\gamma}-\frac{\xi}{\theta}\right]$$
$$+\frac{\chi^{\frac{\xi}{\theta}}f_x/f_0}{1+\chi f_x/f_0}\left[1-\left(1+\chi^{\frac{\xi}{\theta}}\frac{f_x}{f_0}\right)^{-\frac{1}{\gamma}}\chi^{\frac{\xi}{\theta\gamma}}\right]\left[1+\left(1+\chi^{\frac{\xi}{\theta}}\frac{f_x}{f_0}\right)^{-\frac{1}{\gamma}}\chi^{\frac{2\xi}{\theta\gamma}}\right]\Bigg\} \tag{A.20}$$

Recollecting from equation (22) that $n^* > 0$ requires $(\xi f_0/\gamma)^{1/\gamma} > \eta$ and noting that ${}^chi < 1$ establishes

$$2\frac{\theta-\xi+2\xi/\gamma}{\theta-\xi+\xi/\gamma}\frac{\chi^{\frac{\xi}{\theta}}f_x/f_0}{1+\chi f_x/f_0}\left[1-\left(1+\chi^{\frac{\xi}{\theta}}\frac{f_x}{f_0}\right)^{-\frac{1}{\gamma}}\chi^{\frac{\xi}{\theta\gamma}}\right]-\frac{\chi^{\frac{\xi}{\theta}}f_x/f_0}{1+\chi f_x/f_0}\left[1-\left(1+\chi^{\frac{\xi}{\theta}}\frac{f_x}{f_0}\right)^{-\frac{1}{\gamma}}\chi^{\frac{2\xi}{\theta\gamma}}\right]$$
$$>\frac{\theta-\xi+2\xi/\gamma}{\theta-\xi+\xi/\gamma}\frac{\chi f_x/f_0}{1+\chi f_x/f_0}\left[1-\left(1+\chi^{\frac{\xi}{\theta}}\frac{f_x}{f_0}\right)^{-\frac{1}{\gamma}}\right]-\frac{\chi f_x/f_0}{1+\chi f_x/f_0}\left[1-\left(1+\chi^{\frac{\xi}{\theta}}\frac{f_x}{f_0}\right)^{-\frac{2}{\gamma}}\right]$$

we can safely conclude that

$$\begin{aligned}\bar{V}(\chi) &\equiv 2\frac{\theta-\xi 2+\xi/\gamma}{\theta-\xi+\xi/\gamma}\left[\frac{1-\xi/\theta}{1+\chi f_x/f_0}+\frac{\xi}{\theta\gamma}\right]-\left[1+\left(1+\chi^{\frac{\xi}{\theta}}\frac{f_x}{f_0}\right)^{-\frac{1}{\gamma}}\right]\left[\frac{1-\xi/\theta}{1+\chi f_x/f_0}+\frac{\xi}{\theta\gamma}\right]\\ &=\frac{\theta-\xi+2\xi/\gamma}{1+\chi f_x/f_0}\left[1-\left(1+\chi^{\frac{\xi}{\theta}}\frac{f_x}{f_0}\right)^{-\frac{1}{\gamma}}\right]\\ &\qquad-\frac{2\xi}{\theta\gamma}\frac{\chi f_x/f_0}{1+\chi f_x/f_0}\left[\frac{\theta-\xi+2\xi/\gamma}{\theta-\xi+\xi/\gamma}+\left(1+\chi^{\frac{\xi}{\theta}}\frac{f_x}{f_0}\right)^{-\frac{1}{\gamma}}\right]\geq 0\end{aligned} \tag{A.21}$$

is sufficient for $\hat{V}'(\chi) < 0$. We can distinguish two cases regarding the ranking of

$$\frac{\theta-\xi+2\xi/\gamma}{\theta-\xi+\xi/\gamma} \lesseqgtr \left(1+\chi^{\frac{\xi}{\theta}}\frac{f_x}{f_0}\right)^{-\frac{1}{\gamma}}. \tag{A.22}$$

If the left-hand side of (A.22) is strictly smaller than the right-hand side, $\bar{V}(\chi) > 0$ follows from the first line in equation (A.21). If however, the left-hand side of (A.22) is strictly smaller than the right-hand side, $\bar{V}(\chi) > 0$ follows from the second line of equation (A.21). This proves that $\hat{V}'(\chi) > 0$ holds for all possible realizations of χ.

Table A1: Frequency of Workplace Activities by Layer of Hierarchy

Task	All Layers	Managers
Manufacture, Produce Goods	.19	.15
Repair, Maintain	.36	.31
Entertain, Accommodate, Prepare Foods	.28	.30
Transport, Store, Dispatch	.45	.34
Measure, Inspect, Control Quality	.60	.63
Gather Information, Develop, Research, Construct	.75	.92
Purchase, Procure, Sell	.47	.50
Program a Computer	.10	.18
Apply Legal Knowledge	.52	.69
Consult and Inform	.84	.95
Train, Teach, Instruct, Educate	.51	.70
Nurse, Look After, Cure	.27	.35
Advertise, Promote, Conduct Marketing and PR	.40	.55
Organize, Plan, Prepare (others' work)	.65	.80
Oversee, Control Machinery and Techn. Processes	.35	.28
Total Number of Tasks (Sum)	6.66	7.58

Source: BIBB-BAuA 1999, 2006 and 2012.
Note: Frequencies at the worker level (inverse sampling weights).

B Empirical Appendix

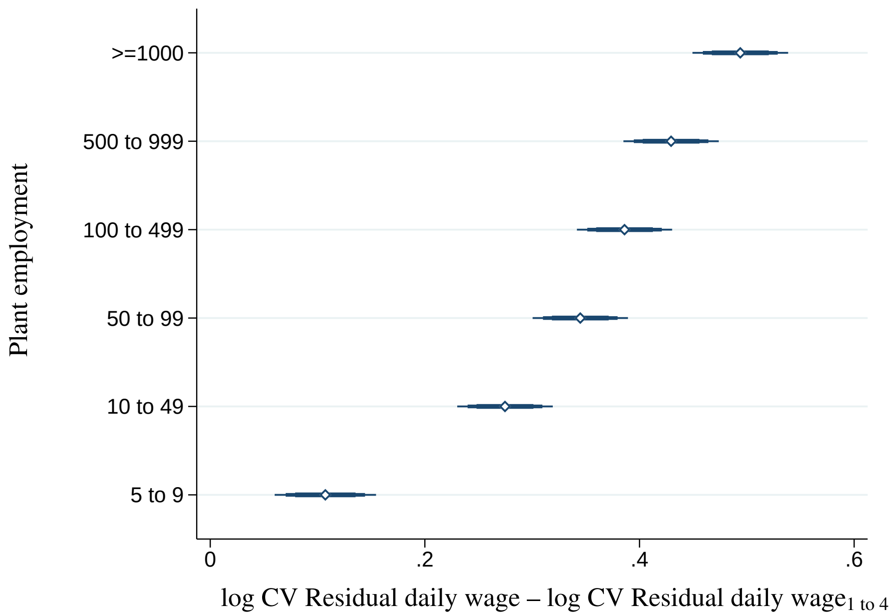
B.1 Frequency of workplace activities by layer of hierarchy

Using the BIBB-BAuA labor force survey data for the three waves 1999, 2006 and 2012, Table A1 shows the frequency of workplace activities (tasks) by layer of hierarchy, reporting workers in any layer of hierarchy in the first column and managers in the second column. A comparison across the two columns suggests that German workers engage in multitasking to a relatively similar extent across layers of hierarchy, performing 6.7 tasks on average in any occupation and about 7.6 tasks in managerial occupations—not quite one additional task in the higher layers of hierarchy. The most salient differences in task frequencies are observed for tasks such as "Train, Teach, Instruct, Educate" or "Apply Legal Knowledge" and "Gather Information, Develop, Research, Construct.' Managers perform those activities with a higher frequency of 17 additional percentage points or more. Managers exhibit higher frequencies in a majority of tasks, with the notable exception of typically more manual-work intensive activities such as "Manufacture, Produce Goods" as well as "Repair, Maintain" and "Transport, Store, Dispatch."

B.2 Residual wage inequality per plant-occupation by plant employment

We project the coefficient of variation *CV* of the (exponentiated) residual daily wages within a plant-occupation on sector, region, occupation and worker characteristics. Figure A1 plots the so normalized *CV* of daily wages within a plant-occupation

Figure A1: Residual Wage Inequality per Plant-Occupation by Plant Employment



Source: LIAB 1996-2014.
Notes: Prediction of (log) coefficient of variation of daily wage residual (exponentiated Mincer residual) CV within plant-occupation by plant employment category, controlling for sector, region, occupation and worker characteristics. Results are differences to smallest plant-size category (1 to 4 workers). Thick, medium, and thin lines represent the 99, 95, and 90 percent confidence intervals.

in logs (on the horizontal axis after subtracting the coefficient of daily wage variation at plants with up to four workers) against numbers of tasks (on the vertical axis). We use the logarithm on the horizontal axis to treat idiosyncratic variability and to align the graph with structural estimation of eq. (45b). There is a clearly positive relationship: wage variability within plant-occupations increases strongly with plant employment. Workers within the same occupation are subject to more wage inequality within their occupation at larger employers.

B.3 Predictors of within-plant daily wage dispersion

In Table A2, we repeat the empirical exercises from Table 6 in the text but now consider total rather than residual wages and specify as the dependent variable the coefficient of variation of wages as a measure of within-plant-occupation wage dispersion. OLS estimates in Columns 1 and 3 are closely comparable to those in Table 6 in the text. In Column 3, a 10 percent increase in the occupation count predicts a more than 14 percentage-point increase in the coefficient of variation of wages within plants. In Columns 4 and 5 of Table A2 we report estimates when instrumenting log revenues as the single endogenous regressor. Similar to Table 6 in the text, test statistics are consistent with the hypothesis that the interaction of log exports to China with the plant's lagged exporter dummy and the log of imports from China provide valid instruments, and the positive effect of revenues on wage variation within plant-occupations remains robust to the change in estimation strategy. In Columns 6 and 7 we treat all three regressors—log revenues, the log count of occupations and the interaction

Table A2: Predictors of Within-Plant Daily Wage Dispersion

	Dependent variable: log CV Daily wage						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	OLS	OLS	OLS	IV	IV	IV	IV
log revenues	0.086***	0.056***	0.067***	0.129***	0.127***	0.038	0.026
	(0.005)	(0.013)	(0.021)	(0.027)	(0.027)	(0.067)	(0.067)
log count of occupations		-0.827***	1.425***			0.118	0.221
		(0.083)	(0.208)			(2.111)	(2.148)
log revenues × log count of occupations		-0.040***	-0.075**			0.003	-0.001
		(0.006)	(0.014)			(0.121)	(0.123)
Plant FE	no	no	yes	no	no	no	no
Hansen J (p-val.)	n.a.	n.a.	n.a.	n.a.	0.172	n.a.	0.196
R^2	0.156	0.195	0.767				
Adj. R^2	0.156	0.195	0.688				
Observations	126,483	126,483	126,483	64,905	64,614	64,775	64,561

Sources: LIAB 1996-2014 and BIBB-BAuA 1992-2012. Sample restricted to plants with more than 2 full-time workers.
Notes: The coefficient of variation of the daily wage is computed within plant-occupations. The panel covers all industries and regions of Germany between 1996-2014. All regressions include time, region, and sector fixed effects. IV estimation is based on GMM. Standard errors in parentheses. Significance levels are indicated by * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

term of these two variables—as endogenous variables and instrument them with the variables reported in Table 5. In contrast with Table 6 in the text, all regressors now lose statistical significance.

C Extension to Stole-Zwiebel Bargaining

A plant ω's revenues are

$$r(\omega) = A^{\frac{1}{\sigma}} \left\{ \tilde{\varphi}(\omega)\tilde{z}(\omega)[n(\omega)+1] \exp\left[\frac{1}{n(\omega)+1} \sum_{j=1}^{n(\omega)+1} \ln \left(\int_0^{b(\omega)} \ell_j(i, b(\omega))\, \lambda(i, b(\omega))\, \mathrm{d}i \right) \right] \right\}^{1-\frac{1}{\sigma}} \tag{C.1}$$

where $n(\omega)+1$ is the plant's occupation count, $b(\omega)$ is the plant's task range per occupation, $\tilde{z}(\omega)$ is plant's full task range required for production, $\ell_j(i, b(\omega))$ is employment of workers of type (core ability) i in the task interval of job j, $\lambda(i, b)$ is the labor efficiency of type-i workers in a task interval with range $b(\omega)$, $\tilde{\varphi}(\omega)$ is plant-specific elemental productivity, and A is a constant that captures demand shifters. We assume that hiring is subject to search frictions and wage setting is the result of individual bargaining of the employer with a continuum of workers as derived by Stole and Zwiebel (1996). We can distinguish $n(\omega)+1$ groups of workers by their occupation j and characterize the bargaining outcome at the employer with two equations of the following form:[22]

$$\psi(\omega) = \frac{1}{\ell(\omega)} \int_0^{\ell(\omega)} r\left[k\, \mathbf{s}(\omega)\right]\, \mathrm{d}k, \tag{C.2}$$

$$\frac{\partial \psi(\omega)}{\partial \ell_j(i, b)} = w_j(i, b), \tag{C.3}$$

where $\psi(\omega)$ is the plant's operating profit (and equal to each worker's share in revenues under Stole and Zwiebel (1996) bargaining), k denotes a proportional increase in employment symmetrically over all the plant's occupations $n(\omega)+1$, $r[\cdot]$ are the plant's revenues as a function of its occupational employment-shares vector $\mathbf{s}(\omega)$, $\ell_j(\omega) \equiv \int_0^b \ell_j(k, b)\mathrm{d}k$ is employment in a task interval with range b, $\ell(\omega) \equiv \sum_{j=1}^{n(\omega)+1} \ell_j(\omega)$ is the plant's total employment, $w_j(i, b)$ is type-i worker's wage in an occupation j with task range b, and each occupation j's employment share at the plant $s_j(\omega) \equiv \ell_j(\omega)/\ell(\omega)$ enters the occupational employment-share vector

$$\mathbf{s}(\omega) \equiv (s_1(\omega), ..., s_{n(\omega)+1}(\omega)).$$

The first equation (C.2) links the result of the employer-worker bargaining outcome to the Aumann-Shapley value (Aumann and Shapley 1974).[23] Intuitively, equation (C.2) assures that the employer's entire revenues are fully exhausted through bargaining. By equation (C.3), the employer and every worker split the surplus equally so that revenues are divided by the mass of all workers and the employer but, since the employer is non-atomic, it does not affect the mass $\ell(\omega)$ and

[22] Existence and uniqueness of this solution follow from Theorem 9 in Stole and Zwiebel (1996).

[23] As put forward in a recent paper by Brugemann, Gautier and Menzio (2015), there is a conceptual problem with Stole and Zwiebel bargaining because, unlike the argument in the original paper, the order in which workers bargain with the employer does matters for the payoff they receive. As a result, the outcome of the Stole and Zwiebel game differs from the equilibrium prescribed by Aumann-Shapley values. As a remedy, Brugemann, Gautier and Menzio (2015) propose to replace the Stole and Zwiebel game with by a Rolodex game, by which workers are randomly picked to bargain from a Rolodex shuffle, so as to root the bargaining outcome of Stole and Zwiebel (1996) in non-cooperative game theory. The outcome of the Rolodex game remains the same as the one posited in Stole and Zwiebel (1996), so we acknowledge the correction but refer to Stole and Zwiebel (1996) when discussing the solution concept.

revenues are divided by $\ell(\omega)$. The plant's operating profit is therefore $\psi(\omega)$.

Employers allocate workers symmetrically over the task range of jobs, so $\ell_j(i, b(\omega)) = \ell_j(0, b(\omega)) = \ell_j(b(\omega), b(\omega))$ for all $i \in (0, b(\omega))$, we obtain

$$\psi(\omega) = \frac{\sigma}{2\sigma - 1} r(\omega), \tag{C.4}$$

where revenues $r(\omega)$ are restated in the notation from (C.1). Substitution into equation (C.3) yields

$$w_j(i,b) = \frac{\sigma - 1}{2\sigma - 1} \frac{r(\omega)\lambda(i,b)}{\lambda_j(\omega)\ell_j(\omega)} \frac{1}{n(\omega)+1}, \tag{C.5}$$

where occupation-level labor efficiency is

$$\lambda_j(\omega) \equiv \frac{1}{\ell_j(\omega)} \int_0^b \ell_j(k,b)\lambda(k,b)\, \mathrm{d}k.$$

Combining eqs. (C.4) and (C.5) establishes

$$\frac{w_j(i,b)}{\lambda_j(i,b)} \lambda_j(\omega)\ell_j(\omega) = \frac{\sigma - 1}{\sigma} \frac{\psi(\omega)}{n(\omega)+1}. \tag{C.6}$$

Every worker in occupation j therefore receives the same wage per efficiency unit of labor: $w_j^e(\omega) \equiv w_j(i,b)/\lambda_j(i,b)$, and this condition is sufficient to guarantee a symmetric allocation of workers over their task range, if worker types are uniformly distributed over the employer's full task range $\tilde{z}(\omega)$ and an employer gets a random draw of the workers.

With the bargaining solution at hand, we can turn to hiring. We assume that hiring takes place prior to the wage negotiation and involves the costs of advertising jobs for employers. Risk-neutral workers apply for those jobs that promise the highest expected return given the imperfect signal they receive regarding their suitability for executing the tasks required in an occupation, according to a posted vacancy. We assume that the signal the workers receive through a vacancy posting only informs them about whether their core ability i falls within the respective task range, but does not provide further details regarding their core ability's exact position within the task interval. Vacancy posting costs are given by sb, where s is a service fee equal to the return on labor used for providing services. Following Helpman, Itskhoki and Redding (2010), we propose that vacancy posting costs are positively related to labor market tightness, and decrease in the unemployment rate u. The ex ante probability of workers to be matched with an employer is $(1-u)$. Vacancy posting costs are specified to equal $sb = sB(1-u)^{\varepsilon}$, where $B > 1$ is a constant parameter and $\varepsilon > 0$ is the elasticity of vacancy posting costs with respect to the employment rate. The hiring problem of the employer can therefore be stated as follows:

$$\max_{\ell_j(\omega)} \psi(\omega) - \sum_{j=1}^{n(\omega)+1} sB(1-u)^{\varepsilon}\ell_j(\omega) - s\lambda(\omega)^{\gamma} - sf_0. \tag{C.7}$$

The first-order condition of this optimization problem is equivalent to

$$[n(\omega)+1]\ell_j(\omega) = \frac{\sigma-1}{\sigma}\frac{\tilde{\psi}(\omega)}{sB(1-u)^{\varepsilon}} = \ell(\omega), \tag{C.8}$$

so that employers hire the same number of workers for all of their (symmetric) jobs. Combining the results yields

$$r(\omega) = A\left[c(\omega)\right]^{1-\sigma}, \qquad c(\omega) \equiv \frac{w}{\tilde{\varphi}(\omega)\left\{\eta+\pi[\nu n(\omega)+1]\right\}}, \tag{C.9}$$

$$\lambda(\omega) = \frac{1}{b(\omega)}\int_0^{b(\omega)} \lambda(k,b(\omega))\mathrm{d}k = \frac{\eta}{\tilde{z}} + \frac{\pi}{b(\omega)} = \frac{1}{\tilde{z}}\left\{\eta+\pi[\nu n(\omega)+1]\right\}, \tag{C.10}$$

$$\psi(\omega) = \frac{r(\omega)}{2\sigma-1} - s\left\{\eta+\pi[\nu n(\omega)+1]\right\}^{\gamma} - sf_0, \quad \text{and} \quad \lambda(\omega)w^e(\omega) = sB(1-u)^{\varepsilon} = \frac{\sigma-1}{2\sigma-1}\frac{r(\omega)}{l(\omega)} \equiv w. \tag{C.11}$$

The optimal count of occupations is then determined by maximizing $\psi(\omega)$ with respect to $n(\omega)$, which yields

$$r(\omega)\frac{\sigma-1}{\gamma(2\sigma-1)} = s\left\{\eta+\pi[\nu n(\omega)+1]\right\}^{\gamma}. \tag{C.12}$$

The zero-cutoff profit condition then establishes

$$r(\omega) = sf_0\frac{\gamma(2\sigma-1)}{\gamma-\sigma+1} \qquad \Longleftrightarrow \qquad \frac{f_0(\sigma-1)}{\gamma-\sigma+1} = \left\{\eta+\pi[\nu n(\omega)+1]\right\}^{\gamma}. \tag{C.13}$$

The rest of the analysis follows as in the main text in Section 3.

However, the derivations of equilibrium in the closed (Section 4) and open economy (Section 5) differ because, under Stole-Zwiebel bargaining, there is unemployment in equilibrium. Risk-neutral workers must be indifferent between applying for jobs in the production sector (with an ex-ante expected wage w) or providing service inputs at a pay s (which is associated with self-employment so that production workers do not switch to the service sector ex post). The unemployment rate (of production workers) is then given by the requirement that $s = (1-u)w$, establishing $B(1-u)^{1+\varepsilon}$ from equation (C.11). This equal-pay condition implies for the employment rate $1-u = B^{-1/(1-\varepsilon)} < 1$, which is a constant in our model because labor is used for production as well as services provision.[24] Finally, we need to check that the wages paid to production workers are (weakly) higher than their expected income outside the job $(1-u)w$. The wage of the least productive worker at employer ω is given by

$$w(0,b(\omega)) = \frac{w\lambda(0,b(\omega))}{\lambda(\omega)} = w\frac{\eta+2[\nu n(\omega)+1]}{\eta+\pi[\nu n(\omega)+1]} \equiv \underline{w}(n(\omega)). \tag{C.14}$$

Note that $\underline{w}'(n(\omega)) < 0$ and that $\lim_{n(\omega)\to\infty}\underline{w}((n(\omega)) = 2w/\pi$. It follows that $\underline{w}(n(\omega)) > (1-u)w$ is satisfied for all

[24] Alternatively, we could use final output as a services input. However, in that case, we would need to constrain the external economies of scale in order to ensure a stable interior solution (see Felbermayr and Prat 2011).

employers if $B < (\pi/2)^{1+\varepsilon}$. In this case, no workers who is matched to a production job will quit ex post. Therefore, we can maintain the parameter constraint $B \in (1, (\pi/2)^{1+\varepsilon})$ throughout our extended analysis.