Financial sector origins of economic growth delusion*

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Abstract

After financial crises, GDP is typically persistently weak compared to pre-crisis trends. We build a simple competitive general equilibrium model to highlight role that the financial sector may have in boosting GDP to unsustainable, undesirable levels before financial crises. Allowing banks to freely trade in financial securities exacerbates the problem. Because loans generate collateral, banks are willing to make lending losses in equilibrium in order to generate trading profit. Our analysis suggests that economists that forecast growth on the basis of time-series trends could be deluded into thinking that the inefficient boost to GDP that derives from an increased ability by the financial sector to exploit such a mechanism is sustainable potential output capacity.

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1 Introduction

Eight years after the onset of the financial crisis, there has only been a modest recovery of GDP in affected countries. The cause of this long-lasting weakness of GDP is not clear. It could be that aggregate demand remains highly deficient (Hall 2011, Krugman 2012, Mian and Sufi 2014, Summers 2015), but it could also be that supply has weakened, for instance because of hysteresis effects (Ball 2014, DeLong and Summers 2012) or excessive fiscal consolidation (Fatás and Summers 2015).



Figure 1: The Persistent Weakness of US Real GDP since 2008

Notes: This figure shows three vintages of the US potential Real GDP estimates from the Congressional Budget Office (CBO). The figure shows that the CBO has been revising down the estimated level of trend GDP since the financial crisis. The black dashed line is Bureau of Economic Analysis Real GDP ("measured GDP"). The CBO projections data is available on that CBO website (https://www.cbo.gov/about/products/budget-economic-data).

An assumption, explicit or implicit, of these approaches is that they take the precrisis behavior of GDP as normal. In this paper we challenge this assumption. In particular, we argue that ex-ante incentives in the financial sector can have strong effects on real economic activity and that the pre-crisis trend in GDP may have been neither sustainable, nor desirable. Our reference to GDP delusion refers to the (deluded) belief that the behavior of GDP in the run up to the crisis was the normal level to which we should expect (and wish) to return.

We are not aware of previous research formally studying a similar hypothesis. In a sense, it is related to Larry Summers' hypothesis that a financial bubble has, for a while, masked the process of secular stagnation (Summers (2014)). However, our hypothesis is not one of secularly low long-term interest rates.¹ What we propose is a finance-induced, supply-side explanation for the relative weakness of GDP since the financial crisis. While there are many supply-side narratives for the weakness, a novel feature of our analysis is that it focuses on factors affecting the run-up, not the aftermath.

¹Nor is it a story of GDP mismeasurement as in Oulton (2013) or Haldane, Brennan, and Madouros (2010).

Our baseline model comprises a banking sector, a firm sector that operates a constantreturn-to-scale production function, a household sector that provides labor inelastically, and the rest of the world, populated by risk-neutral agents that provide intertemporal substitution and insurance. All these agents act competitively. The key ingredients are: (i) investment is risky because it must be committed before the realization of a shock to Total Factor Productivity (TFP); (ii) firms need to borrow from banks, which exposes banks to investment risk; (iii) banks issue liabilities that are guaranteed by the government (for simplicity, we can think of insured deposits).

The main mechanism works as follows. When banks default, they do not repay depositors in full. Because they are insured, depositors do not need to be compensated by a higher return when the bank does not default.² Hence, banks borrow from depositors at artificially low rates. But banks compete for firm lending, which means that they pass this artificially low interest rate onto the firms. Firms, in turn, equate the capital marginal rate of return to the borrowing rate, which leads to over investment in aggregate. Given that firms compete for workers, this means that the equilibrium wage is inflated (if we include structures as a form of capital in the production function, their price is inflated too). Now, the key point is that GDP is also inflated. For all realizations of TFP, GDP is higher than what it would have been absent the distortion from government guarantees. However, whereas GDP would be a reasonable measure of welfare in our economy without the distortions due to government guarantees, it is not when they are present. Instead, a better measure is Net Domestic Product (GDP adjusted for capital depreciation) because it captures the negative effect of the distortions on consumption. In equilibrium, while expected GDP is inflated, expected NDP is below its first-best value, which reflects unsustainability.

This inefficiency is greatly magnified if we allow banks to engage in a form of proprietary trading (namely, trading in securities backed by the assets of their loan book). The reason is that, from a bank's private point of view, any repayment it makes to depositors when it defaults corresponds to money left on the table. Hence, the bank has strong incentives to find ways to divert these resources. Imagine that it can use loans as collateral to fund derivative positions that pay off in the states where it does not default. In such a context, more collateral means more trades, hence potentially more profit in the good states. Thus, banks have incentives to issue more loans, which may lead to a decrease in lending standards and, ultimately, to further over-investment by firms. This is what happens in the model if we allow banks to trade in a set of Arrow securities and assume that trades must be collateralized for repayments to be credible. In equilibrium, banks make strictly positive expected profits from these trades, but this is compensated by expected losses on lending, which reflect a further deterioration of

²For simplicity, the government does not charge the banks for this insurance. What matters is that the insurance premium does not make the bank fully internalize the loss given default.

the real allocation.

Our theoretical methodology is relatively straightforward, but nonetheless original. We want to derive analytical results, in a general equilibrium model that includes a financial sector, which we allow to engage in collateralized trade of Arrow securities. To achieve this, we do not analyse the transitional dynamics. Even though propagation mechanisms play a central part in modern macro, they are in fact not essential to the points we want to make. Hence we first make our main points in a single period model. Our approach, for most of our analysis, can be interpreted as a steady-state comparison between different regulatory regimes. We believe this is appropriate because our objective is to understand differences in trends rather than fluctuations around a single trend (Summers 2015, for instance, stresses the need for new models using such an approach).

In order to link our result to our initial motivation, we must address why the mechanism would have been at play in the run up to the crisis and not in the aftermath. Focusing on the US, claiming that government guarantees were present before the crisis and then suddenly remove may not be very credible.³ However, nor is such a removal of guarantees vital for our story. What really matters is the ability for banks to exploit such guarantees. Hence, if this ability increased before the crisis but has been impaired in the aftermath, our story remains relevant.

We argue that several factors point in this direction. Deregulation (e.g. the repeal of the Glass-Steagall Act in 1999, the removal of capital requirement for Broker Dealers in 2004) is likely to have played a role. But financial innovation may have been important too. First, engineering trades corresponding to the one we describe above, and making sure that they are bankruptcy remote is not that straightforward. Second, one should also be able to generate the collateral. Securitization comes to mind, but also the use of subprime mortgages and the associated lowering of lending standards. Finally, general equilibrium effects can be important: in the early years, due to limited competition, it may have been the case that proprietary trading profit was not offset by expected losses on lending.⁴ In the aftermath of the financial crisis, there has undeniably been a regulatory crackdown. Government guarantees are, of course, still present, but banks are arguably less able to exploit them. The Volcker Rule limits proprietary trading and stress-tests aim to make sure that, even in the most adverse scenario, banks would not fail. If we take stress tests' recent positive results at face value, this implies that our mechanism is no longer at play, which could explain a reversion to a lower, but, in our analysis, efficient and sustainable trend. We do not want to argue there is no residual

³In countries where bank bailouts tested the limits of the government's fiscal capacity, this may however not be so far-fetched. However, what really matters is the ability for banks to exploit the guarantees.

⁴Martinez-Miera and Suarez (2014) and Malherbe (2016) additionally show how the accumulation of banking capital erodes scarcity rents in the banking sector and endogenously increases the distortions from government guarantees.

effect of guarantees, but rather we believe that these elements contribute to a decrease in banks' ability to exploit the guarantees and, therefore, to mitigate the impact of our mechanism.

Although the model explores only a single source of the apparent weakness of output, we of course acknowledge that many different channels could simultaneously be at work. While a full quantitative assessment of all possible channels is beyond the scope of this paper, we show that a number of empirical developments in the US macroeconomic time-series are consistent with the predictions of our model.

To do so, we use an extended version of our model (with two different type of capital and wage downward rigidities).⁵ We show that the period before the financial crisis was a period characterised by a divergence between GDP and NDP, indicating welfare rose by less than is suggested looking at GDP. There was significant asset price growth for investment goods that are, or need to be combined with goods that are, inelastically supplied. There is evidence that lending standards and profit margins on lending activities fell. Real wages grew strongly before stagnating once the financial crisis occurred.

We also quantify the distortion by calibrating changes in the capital-output ratio not accounted for other driving variables; in our model these are driven by the distortion. Armed with estimates of the size of the distortion over time, we calculate an adjusted GDP series which, if used to extrapolate trend GDP, is less susceptible to having to revise the estimated trend as more information from the crisis period becomes available. In this sense, our adjusted GDP series would have been less misleading. We also show that, using the adjusted GDP series, there was less of a great recession and more of a reversal of a great distortion.

While our analysis focuses on the US economy, this mechanism is likely to have much broader relevance. Reinhart and Rogoff (2014), after examining 100 financial crises, highlight that financial crisis episodes are typically followed by protracted recoveries. They find that the mean time to recover to pre-crisis levels of GDP per capita is about 8 years (the median is 6.5 years). Others too find that financial crises give rise to recessions that are longer and deeper than other recessions (Jordà, Schularick, and Taylor 2013). Notably, Ball (2014) contrasts the effect of loss of output (hysteresis) with the even more serious lost growth capacity (super hysteresis). Blanchard, Cerutti, and Summers (2015) show empirically that both of these effects are present in broad panel of countries. In our model, the slower growth is actually a reversal of earlier inefficiently high growth. In that sense, the banking sector magnifies the business cycle, which is one of the broad insights form the financial cycles literature (see e.g. Borio

⁵This is where we allow the study of interesting transitional dynamics. The modeling trick we use is to shut down further TFP shocks and do comparative dynamics on the path of recovery to full employment.

2013, Schularick and Taylor 2012, and Jordà, Schularick, and Taylor 2016).

We stress that it is not our intention to claim (or appear to claim) that our story is the only explanation for the period of protracted weakness that has been identified in many countries. Rather, we simply wish to highlight that the empirical observation of these phenomena which have been labeled as hysteresis is consistent with our story in which the post-crisis level of activity may be more appropriate and sustainable than the pre-crisis level.

2 A baseline model of inflated GDP

In this section we introduce the most simple version of our model. There is a single period, in which there is a single output good. The model economy comprises a household sector, a production sector, a banking sector, a government, and the rest of the world. We shall discuss each in turn as well as the links between them.

2.1 Model Setup

Households A risk-averse representative household maximizes expected utility:

$$E\left[u\left(c
ight)
ight]$$
,

where *c* is its consumption at the end of the period, and u(.) is increasing and concave. Households are endowed with *e* unit of the consumption good at the beginning of the period, which they can use to buy securities. Available securities are indexed by $i \in \{1, 2...\}$. The beginning of period budget constraint is:

$$\sum_i \theta_i p_i \leq e,$$

where θ_i denotes the holding of security *i*, whose price is p_i . Households are also endowed with one unit of human capital which they supply inelastically (aggregate labor is N = 1) in return for a wage *w*. Households pay lump-sum taxes τ to the government. Hence, realized consumption is given by:

$$c = w - \tau + \sum_{i} \theta_i R_i,$$

where R_i denotes security *i*'s payoff.

Firms There is a continuum of firms that each borrow capital from banks and hire workers to maximize expected profit. Each production firm is penniless and has limited liability. Firms competitively borrow capital from banks and hire workers. Physi-

cal capital fully depreciates in production. The firms operate a constant returns to scale production function which combines physical capital (k) and labor (n) to produce the output good:

$$Ak^{\alpha}n^{1-\alpha}$$

where *A* is an aggregate productivity shock that takes the value A_H with probability *p* and a strictly smaller value A_L with probability (1 - p), and $\alpha \in (0, 1)$. Thus, total output in the economy is:

$$Y = AK^{\alpha}N^{1-\alpha},\tag{1}$$

where upper case variables are the aggregate counterparts of the lower cases.

Banks There is a banking sector where banks intermediate lending between households and the production sector. Banks raise funds (consumption goods) competitively by issuing liabilities to households and the rest of the world. They transform them, one to one, into capital which they provide to firms.

For the sake of concreteness, we assume banks only issue two types of securities -deposits and equity- and that they lend capital to the firms according to a standard debt contract. This is without loss of generality.⁶

Rest of the world The rest of the world consists in a large number of risk-neutral agents that can buy and sell securities. Their opportunity cost of funds is 1.

Government The main purpose of this paper is to study the implications for economic activity of government guarantees. We will compare to different regimes. One with, and one without, government guarantees. While we could think of this as a formal deposit insurance scheme (which may cover deposits only up to a limit), we rather interpret this more broadly reflecting the implicit (and then explicit) guarantee that governments typically provide for banking systems during financial crises.⁷ When deposits are guaranteed, they pay the world's safe interest rate (i.e. 0) and depositors are compensated even if the bank has a shortfall when production returns are realized.

⁶There are only two states, hence the state-space can be spanned with two securities. On the lending side, a standard debt contract replicates the allocation that would obtain under an optimal contract offered to production firms by the banks specifying a production plan $\{n, k\}$ and contingent repayments per unit of capital $\{\rho_H, \rho_L\}$. Under such a contract, banks maximize expected profit given *w* and ensure that producers break even in all states. There is no possible profitable deviation for a couple bank-firm. Through this contract, banks effectively compete for workers and end up making zero expected profit in equilibrium.

⁷Even if it were a simple deposit insurance scheme, then if the institutional creditors were able to run faster than the household depositors (as we saw in the case of Northern Rock in the UK), then the lack of guarantee is not a problem for non-insured creditors.

In the absence of guarantees, depositors risk the loss of their deposits if the bank has a shortfall. The government collects lump-sum taxes from the household sector in order to cover any payments made through the bank guarantee scheme. For analytical simplicity, there is no other government spending or taxation and we assume that the government must balance its budget in all states.

Equilibrium definition All private agents are price taking and enjoy limited liability. In an equilibrium: (i) households maximize expected utility; (ii) banks and firms maximize expected profits; (iii) expected profits are nil (iv); the rest of the world breaks even in expectation; and (v) the government breaks even in all states.

2.2 Timing

- 1. Financial market activity takes place: households, banks, and the rest of the world trade in securities. Banks produce capital and lend to firms.
- 2. Firms hire workers competitively, for a non-contingent wage.
- 3. A realizes, production takes place, factors are paid
- 4. Banks repay deposits (in full if they can, or pro-rata if they are insolvent)
- 5. Other financial claims settle.
- 6. The government compensates the depositors and taxes households.
- 7. Consumption takes place.

The essential aspect of the time-line is that investment decisions are taken before productivity realizes. Finally, note that given this is a single period model, the capital stock is equal to investment.

2.3 Benchmark: no guarantees

Real investment (changing consumption goods into capital) is risky because it is done before *A* realizes. Hence, banking is risky too. Given that agents from the rest of the world are risk neutral, they are willing to hold any security that yields an expected return equal to the risk free rate. Households, on the other hand, are risk averse. Hence, they would require a risk premium to hold risky securities. As a consequence, the rest of the world bears all the risk in equilibrium. That is, households hold deposits only if there is a sufficiently large buffer of junior securities (such as equity) to absorb the losses in the bad state. Except for this, the composition of liabilities is indeterminate. **Lemma 1.** In the economy without guarantees, the Modigliani and Miller irrelevance theorem applies. The equilibrium level of capital is $K_B = (\alpha \bar{A})^{\frac{1}{1-\alpha}}$, where $\bar{A} \equiv E[A]$.

Proof. See Appendix A.⁸

Note that the subscript *B* stands for benchmark. Given that there is no risk premium in equilibrium, the expected marginal cost of funds for the bank is 1. Banks compete to lend to firms. So the expected marginal cost of capital for firms is also 1, and so must be the equilibrium expected marginal return to capital. The condition that pins down the equilibrium is thus:

$$\alpha \bar{A} \left(K_B \right)^{\alpha - 1} = 1.$$

2.4 Equilibrium with government guarantees

When the government guarantees deposits, households are willing to hold them irrespective of the buffer offered by equity. In particular, this will be the case if the bank promises a zero interest rate. This is because the government makes the depositors whole in the case the bank cannot repay in full. On the other hand, the government does not compensate junior creditors or shareholders. Hence, to be willing to buy equity, agents must be compensated by a higher return in the states where the bank does not default. To sum up, all securities yield an expected payoff of one to their holder in equilibrium. However, for deposits, if the bank defaults with positive probability, part of this payoff comes from the taxpayer. Hence, from the bank point of view, deposits are cheaper.⁹

Lemma 2. *In the economy with government guarantees, the bank only issues deposits in equilibrium.*

Thus, to lend an amount *k* of capital, the bank must promise to repay *k* to depositors. The bank therefore solves:

$$\max_{k\geq 0} E\left[\tilde{\rho}k-k\right]^+,$$

where $\tilde{\rho}$ is the realized unit repayment on loans. That is, $\tilde{\rho}$ is the minimum between the firm promised repayment to the bank and what it can repay if it does not have enough resources to meet the promise. That is: $\tilde{\rho}k = \min \{\rho k, Ak^{\alpha}n^{1-\alpha} - wn\}$.

⁸All the proofs that are omitted in the text are in Appendix A.

⁹For simplicity, the government does not charge the banks for this insurance. What matters is that the insurance premium does not make the bank fully internalize the loss given default. Likewise, imposing a sufficiently high capital requirement on banks would prevent the bank from defaulting and there would be no implicit subsidy. Our analysis would apply to economies where capital requirements are not sufficiently high and banks default with positive probability in equilibrium. See for instance Bahaj and Malherbe (2016) and Malherbe (2016).

Without a sufficient equity buffer, the bank is vulnerable to an adverse shock.¹⁰ As a result:

Lemma 3. *In the economy with government guarantees, in equilibrium, the bank fails in state* A_L .

As a result, in effect, the bank ends up ignoring the marginal effect of its decisions on the bad state (a marginal change in the negative profit does not affect its payoff in that state) and maximizing profits in the good state.

$$\max_{k\geq 0}\rho_H k - k.$$

The first order condition is

$$p_H \le 1$$
 (2)

Hence, in equilibrium we must have: $\rho_H = 1$.

Lemma 4. In the economy with government guarantees, the equilibrium level of capital is $K^* = (\alpha A_H)^{\frac{1}{1-\alpha}} > K_B.$

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Banks borrow from depositors at artificially low rates. But banks compete for firm lending, which means that they pass this artificially low rate onto the firms. Firms, in turn, equate the capital marginal rate of return to the borrowing rate, which leads to more investment in the aggregate.

The relevant equilibrium conditions become.

$$\begin{cases} \alpha A_H (K^*)^{\alpha - 1} = 1\\ (1 - \alpha) A_H (K^*)^{\alpha} = w^* \end{cases}$$

where we use the superscript* to denote equilibrium variables.

2.5 Efficiency

We first make a very simple but important point.

Proposition 1. *In the economy with guarantees, investment is inefficiently high. That is, while output is higher than in the benchmark economy, expected net output is lower.*

¹⁰Adding a capital requirement to the model does not qualitatively affect our results as long as the bank still fails with strictly positive probability in equilibrium. However, the stringency of bank prudential regulation and supervision will affect the magnitude of the distortion, which is what we exploit in Section 5.

Output is AK^{α} . For any realization of A, it is increasing in K. That is, more investment (which translates in a higher capital stock) always means more output. However, in terms of investment efficiency, the relevant concept is expected net output: $E[AK^{\alpha} - K]$. The difference is that between *gross domestic product* (GDP) and *net domestic product* (NDP), which takes capital depreciation into account.¹¹ For that very simple reason, GDP growth is not necessarily a good thing. However, normative statements cannot be based on investment efficiency only. As we show below, it turns out that welfare is unambiguously lower in the economy with guarantees.

2.6 Welfare

Given that the rest of the world is risk neutral and always breaks even in equilibrium, the relevant notion of welfare is the representative agent expected utility.

Proposition 2. *In the economy with guarantees, the increase in wage is more than offset by the expected increase in tax. It follows that welfare must be strictly lower than in the benchmark economy.*

Proof. Because the rest of the world is risk neutral and must break even in expectation, we have $p_i = E[R_i]$ and, therefore, $E[\sum_i \theta_i^* R_i] = e$. Hence, household expected wealth is given by:

$$v \equiv w - E[\tau] + e.$$

In the economy with guarantees, we have

$$\begin{cases} w^* = (1 - \alpha) A_H (K^*)^{\alpha} \\ \tau_H = 0 \\ \tau_L = A_L (K^*)^{\alpha} - (1 - \alpha) A_H (K^*)^{\alpha} \end{cases}$$

Put together, these give:

$$v^* = \underbrace{\bar{A}(K^*)^{\alpha} - K^*}_{E[NDP]} + e.$$

We have shown that: since the rest of the world must break even in expectation, households expected wealth equals the sum of their endowment and expected NDP (which is the economic surplus from production). Since expected NDP is lower than in the benchmark economy (Proposition 1), and in the benchmark economy households

¹¹For simplicity, we have assumed full depreciation. Note that if capital could be transformed into consumption goods, at the end of the period, at a rate $1 - \delta$ (where, $\delta \in (0, 1)$ would capture the depreciation rate), the conclusions would be identical.

are perfectly insured, welfare must be strictly lower in the economy with guarantees.¹²

Let us consider two examples. First, assume that households can trade a full complete set of securities with the rest of the world. In that case, they will also get full insurance in the economy with guarantees. As a result they will consume their expected net wealth in all states. The welfare loss comes here only from investment inefficiency. Now, imagine that households can only buy equity or deposits of domestic banks. In that case, they will prefer to hold deposits only (equity as a low payoff in the bad state). As a result, they do not get any insurance against taxation risk, and it is easy to verify that their realized wealth corresponds to their endowment plus *realized* NDP.¹³ Hence, guarantees imply additional welfare losses.

That NDP can be a good indicator of welfare is not new, nor is it a specific feature of our model. For instance, Weitzman (1976) shows that in a dynamic economy NDP is a proxy for the present discounted value of consumption.¹⁴ However, what is clear and more novel from Proposition 2 is that when investment decisions are distorted, GDP growth is unlikely to be a good indicator of welfare growth. As we will highlight in Section 5, in these circumstances, current GDP trend is also unlikely to be a good indicator of the path for future GDP.

3 Deregulation and financial innovation

In the economy with guarantees, from the bank's point of view, any repayment made to depositors can be seen as money left on the table. To see this, imagine that the promised repayment is 10 and realized proceeds from lending in state A_L are 5. If, for some exogenous reasons, it were less than 5, this would not affect the bank payoff (it would still be 0), and depositors would not require compensation in the other state since they are insured.

This gives banks an incentive to find ways to divert such resources from the low state towards the high state. One way to do this is to pick riskier or more correlated borrowers (we study this in Section 5.3). Here, however, inspired by financial innovation before the recent crisis, we consider the case where loans can be used as collateral to fund financial trades.

¹²It should now be clear that the benchmark economy is efficient in the sense that it maximizes social welfare (as it maximizes expected economic surplus and ensures perfect insurance).

¹³If households can short sell bank equity, this restores perfect risk sharing.

¹⁴Interestingly, Weitzman point was not about whether one should account or not for depreciation, which was seen as uncontroversial, rather it was about about whether and how one should include current capital stock.

The trading environment Assume the representative bank can trade in a set of Arrow securities, denoted *H* and *L*. These securities trade competitively in financial markets. As a consequence, they are actuarially fairly priced. For simplicity, we assume that the bank ex-ante trading position must be self-financed. That is, the bank trading position cannot be ex-ante financed by deposits.¹⁵ Formally: $ph + (1 - p)l \le 0$, where *h* and *l* denote the bank's net holding of the two securities (a negative number corresponds to the bank selling the security).

We still assume that wages are always senior to capital (alternatively, they are paid upfront). But, in effect, Arrow securities are senior to deposits (either because they are bankruptcy remote, like repo's, or because institutional investors run faster than insured depositors). Finally, we assume $A_L - (1 - \alpha)A_H > 0$. If this condition is not satisfied, the bank cannot credibly issue Arrow securities in equilibrium.

Equilibrium

Lemma 5. The equilibrium trade is given by $l^* = -\rho_L k^*$ and $h^* = \frac{(1-p)\rho_L k^*}{p}$.

Proof. By definition, one cannot default on an Arrow security in equilibrium. For a given amount of lending k^* , the maximum amount the representative bank can credibly commit to repay in state L is $\rho_L k^*$. Selling security L increases the losses of the bank in state A_L . But this does not affect expected profits because of limited liability. Hence, from the bank's perspective, the expected marginal cost of selling the L security is nil (this corresponds to selling promises that will, ultimately, be honored by taxpayers). The strategy that maximizes trading profit follows: sell as much as possible of the L security (hence, the constraint must be binding), and use the proceeds to buy the H security. Since the securities are fairly priced, this allows the bank to buy $\frac{(1-p)\rho_L k^*}{p}$ of the H security.

With this strategy, the bank extracts rent from the taxpayer. This materializes through an extra term, equal to h, in the bank profits in the good state. Hence the bank lending problem becomes

$$\max_{k\geq 0}\rho_H k - k + \frac{1-p}{p}\rho_L k.$$

And, from the first order condition, we get in equilibrium:

$$\rho_H + \frac{1-p}{p}\rho_L = 1 \tag{3}$$

¹⁵Otherwise, banks could make infinite profits by issuing deposits and buying one of the security (because, in practice, the government would then guarantee the short selling of securities). Alternatively, we could assume that banking supervision limits direct gambling with insured deposits. As long as there is such a limit, our analysis remains directly valid. Otherwise, we would have to explicitly model the limits of government fiscal capacity.

This reflects that more lending helps relax the trading constraint and allows the bank to buy more of the security that pays in the good state. As a result, over-investment becomes worse in equilibrium.

Proposition 3. *In the economy with government guarantees, output is higher with proprietary trading but over-investment is worse, and welfare lower.*

Denoting K_T^* the equilibrium level of capital (the subscript *T* stands for trading), we have:

$$K_T^* = \left(\alpha A_H + \frac{1-p}{p} \left(A_L - (1-\alpha) A_H \right) \right)^{\frac{1}{1-\alpha}},$$

where the second term in the parenthesis captures the extra kick from financial trading.¹⁶

Since there is even more capital, this means that banks make lending losses, even in the good state. In equilibrium, these losses are just compensated by financial trading profit:

$$\underbrace{\rho_H(K_T^*)K_T^* - K_T^*}_{\text{lending losses}} + \underbrace{\frac{1-p}{p}\rho_L K_T^*}_{\text{trading profit}} = 0.$$

This provides a novel interpretation to the well documented decrease in lending standards in the run up to the crisis in the US (see Section 5) and other countries, and for the sharp increase in the importance of trading activities for bank profits. In what follows, we refer to this version of the model as the economy with proprietary trading.

4 An extended model

In the two previous sections, we have highlighted our mechanisms in the most transparent way. A drawback is that the model is perhaps too simple too draw a full set of testable empirical predictions. In this section, we outline a more elaborate version of the model. This model can then provide the basis of a comparison of the empirical predictions with recent developments in the US (section 5).

Our strategy will be to interpret our economy as two different regimes of the same economy; one with guarantees and one without guarantees. As we motivate below, we will first think (in the context of the US) of a gradual, relatively smooth transition from the former to the latter in the run up to the 2008 crisis. Then, we will consider a more abrupt change of regime, where, however, transition dynamics play a role due to wage downward rigidities. To facilitate this exercise, in this section we first highlight

¹⁶Note that if $A_L < (1 - \alpha)A_H$, financial trading is not possible in equilibrium.

the empirical predictions across regimes and then consider the predictions related to a back-to-benchmark transition period.

4.1 Modification to the environment

The first modification to the baseline model of Section 2 is to introduce a second type of capital. The production function in the economy is still the Cobb-Douglas function given by (1), but the stock of capital itself is given by the following constant elasticity of substitution aggregator:

$$K = (Q^{\gamma} + S^{\gamma})^{rac{1}{\gamma}}$$
 ,

where $\gamma \in (0, 1)$ is the elasticity of substitution between Q and S, Q denotes equipment and S stands for structures. For simplicity, equipment is imported, at an exogenous unit price $q_e > 0$, and structures are in perfectly-inelastic supply.¹⁷ The firm buys structures from households at the beginning of the period at an endogenous competitive price q_s .

Because we want to be able to account for possible secular trends in the economy (which are potential confounding factors), we now parametrize the world's interest rate (r > 0) and capital depreciation. In particular, we assume that, at the end of the period, equipment and structures can be converted into consumption goods at rates $q_e (1 - \delta)$ and $(1 - \delta)$, respectively, with $\delta \in (0, 1]$.¹⁸ Finally, in a reduced form interpretation of the proprietary trading model, we assume that the bank can pledge up to a share $\lambda \in [0, 1]$ of the structures to outside investors.¹⁹

4.2 Equilibrium

Let us turn directly to the equilibrium with guarantees. As above, the representative bank will maximize rent extraction, which gives:

$$h^* = \lambda \frac{1-p}{p} (1-\delta) S^*.$$

Accordingly, the lending problem of the bank can formalized as:

$$\max_{N,Q,S} p\left(A_H N^{1-\alpha} \left(Q^{\gamma} + S^{\gamma}\right)^{\frac{\alpha}{\gamma}} - Nw - q_e(\delta + r)Q - q_s(\delta + r)S + \lambda \frac{1-p}{p}(1-\delta)S\right).$$

¹⁷An alternative would be to have structures built with a combination of land, labor, and equipment. This would complicate the analysis without adding much further insight.

¹⁸Assuming that structures depreciate at a rate $q_s (1 - \delta)$ does not substantially affect the analysis.

¹⁹Put differently, structure ownership is put into bankruptcy remote vehicles so that, in case the bank fails, the claims backed by such collateral are senior to deposits.

From the first order conditions for *Q* and *S*, we get:

$$Q^* = S^* \left(\frac{q_s(\delta + r) - \lambda \frac{1-p}{p}(1-\delta)}{q_e(\delta + r)} \right)^{\frac{1}{1-\gamma}}.$$
(4)

Substituting for *Q* in the first order conditions for *N* and *S*, and using N = S = 1 yields:

$$\begin{cases} (1-\alpha) A_H \left(\left(\frac{q_s(\delta+r) - \lambda \frac{1-p}{p}(1-\delta)}{q_e(\delta+r)} \right)^{\frac{\gamma}{1-\gamma}} + 1 \right)^{\frac{\alpha}{\gamma}} = w \\ \alpha A_H \left(\left(\frac{q_s(\delta+r) - \lambda \frac{1-p}{p}(1-\delta)}{q_e(\delta+r)} \right)^{\frac{\gamma}{1-\gamma}} + 1 \right)^{\frac{\alpha}{\gamma}} = q_s(\delta+r) - \lambda \frac{1-p}{p}(1-\delta) \quad , \end{cases}$$
(5)

which is a system of two equations in q_s and w that pins down the equilibrium.

It is easy to check that to obtain the equilibrium conditions of the corresponding benchmark economy of the extended model, one must substitute \bar{A} for A_H and set λ to 0 in System 5 and Equation 4 above.

4.3 **Regime comparison**

Comparing the two regimes gives then the following empirical predictions. In the extended model, compared to the benchmark economy, the economy with guarantees exhibits:²⁰

- Higher GDP and lower expected NDP;
- Inflated wage;
- Inflated asset (i.e. structure, or fixed asset) prices. In particular, *q*^{*}_s is increasing in *A_H* and *λ*;
- Over investment in capital (materializing through an increase in *Q**);
- A higher capital to output ratio;
- Ambiguous changes in the share of fixed asset income in GDP $\sigma^* \equiv \frac{Q^*q_e}{S^*q_s}$. In particular, σ is increasing in A_H , but decreasing in λ . Total gross income of all factors benefit from the increase in GDP, but, for instance, if the latter effect dominates, the increase is greater for fixed assets (structures), then labor, then equipment.²¹

²⁰Matlab code and full details available on request.

²¹Studying the consequences of our mechanism for the factors of productions' share of income is beyond the scope of this paper. However, the way this prediction directly speak to the literature on the medium to long term trends for these indicators (see e.g. Karabarbounis and Neiman 2013, Piketty 2014, and Piketty 2014) only illustrates the potential for ramifications of our analysis.

4.4 Back-to-benchmark transition

The motivation of the paper is intimately linked to the question of whether we are at a "new normal" or still in a transition period returning to the old trend. Hence, instead of thinking of a simple regime-change reversion to the benchmark economy, we can use our extended model to explicitly model the transition. The main idea behind treating the aftermath transition differently than the run up is the well established fact that wages exhibit downward rigidity.

Since the wage is inflated in the economy with guarantees, an easy way to think about back-to-benchmark transition dynamics is to consider an economy whose steady state is the benchmark economy of the extended model, but that starts from a situation where the wage is above steady state level and can only adjust downward at some decay rate ξ . Denote w_0 the initial wage (which we will interpret as the wage just before the crisis) and w_B^* the relevant steady state wage, we have:

$$w_t = w^* + \xi^t (w_0 - w_B^*).$$

Then, we can solve for $q_{s,t}^*$, Q_t^* , and the employment rate *N* from the corresponding system:

$$\begin{cases} (1-\alpha)\,\bar{A}\left(\left(\frac{q_{s,t}^*}{q_e}\right)^{\frac{\gamma}{1-\gamma}}+1\right)^{\frac{\alpha}{\gamma}}(N_t^*)^{-\alpha}=w_t\\ \alpha\bar{A}\left(\left(\frac{q_{s,t}^*}{q_e}\right)^{\frac{\gamma}{1-\gamma}}+1\right)^{\frac{\alpha}{\gamma}}(N_t^*)^{1-\alpha}=q_{s,t}^*(\delta+r)\\ Q_t^*=\left(\frac{q_{s,t}^*}{q_e}\right)^{\frac{1}{1-\gamma}}\end{cases}\end{cases}$$

to obtain the following predictions. In a back-to-benchmark transition, if wages are sticky, the economy exhibits:

- Unemployment $(N_t^* < 1)$;
- Depressed structure prices (that is, q^{*}_{s,t} instantly drops from a value that was inflated by the guarantees to a value below the benchmark value; an overshooting phenomenon);
- Depressed investment (same story for *Q*^{*}_t);
- A gradual decrease in the capital to output ratio (unemployment contributes to keeping it above its steady state level).

5 Empirical Relevance

5.1 Time varying distortion: the US since the 1990s

In what follows, we argue that our mechanism may be relevant to the recent US history. In our analysis, we made our main points comparing two regimes. One without guarantees, and one with guarantees (possibly combined with proprietary trading). However, we abstracted from important aspects of bank regulation (such as capital regulation), that could mitigate the extent of the distortion. An interpretation is that what really matters for the extent of the realized distortion is not only the presence of guarantees, but also the ability of banks to exploit them. Along these lines, we formulate the hypothesis that, in the two decades leading to the 2008 crisis, the US has seen a significant increase in such ability. But that a tightening of regulation has dramatically reduced it in the crisis aftermath.

We argue that several factors points in this direction. Deregulation (examples below) is likely to have played a role. But financial innovation may have been important too. First, engineering trades corresponding to the one we describe above, and making sure that they are bankruptcy remote is not that straightforward. Second, one should also be able to generate the collateral. Securitization comes to mind, but also the use of subprime mortgages. Finally, general equilibrium effects can be important: in the early years, due to limited competition, it may have been the case that proprietary trading profit was not offset by expected losses on lending.

As an illustration, we provide below a selected list of potentially important milestones which increase the exploitability of guarantees:

- **1984** Repurchase agreements are confirmed to be bankruptcy remote (extended in mid-1990s and 2005).
- **1996** The Glass-Steagall Act is reinterpreted to allow banks to have up to 25% of revenue from their investment banking activities.
- **1997** Bear Sterns securitizes the first loans under the Community Reinvestment Act (these, potentially problematic, loans are guaranteed by Fannie Mae).
- 1999 The Glass-Steagall Act is repealed.
- 2000 The FDIC grants safe harbor protection for securitization.

2004 The SEC removes leverage restriction on investment banks.

2004 The OCC removes anti-predatory lending restrictions on national banks.

In the aftermath of the financial crisis, there has undeniably been a regulatory crackdown. Government guarantees are, of course, still present, but banks are arguably less



Notes: This figure shows the ratio of nominal GDP to nominal NDP using BEA data.

able to exploit them. The Volcker Rule limits proprietary trading and stress-tests aim to make sure that, even in the most adverse scenario, banks would not fail. If we take stress tests' recent positive results at face value, this implies that our mechanism is no longer at play, which could explain a reversion to a lower, but, in our analysis, efficient and sustainable trend. We do not want to argue there is no residual effect of guarantees, but rather we believe that these elements contribute to a decrease in banks' ability to exploit the guarantees and, therefore, to mitigate the impact of our mechanism.

5.2 Stylized facts over the pre- and post-crisis periods

In this section we examine the broad developments in the empirical counterparts to our model predictions.

GDP and NDP The pre-crisis period was marked by a divergence between gross and net domestic product. Figure 2 shows that the ratio of annual nominal GDP (Billions of Dollars) to NDP (equivalent basis) increased from 1.17 in 1997 to 1.2 at the start of 2009; while this may not seem like a huge change, it represents a 1.3 standard deviation shift based on changes over the post-war period. It is driven by GDP growth of 67.5% over the period, while NDP grew by only 63.5%.

Asset prices The extended model predicts that when banks become better able to exploit guarantees, this inflates the price of collateralizable assets. Figure 3 shows that the relative price of investment, which has been on a well-documented secular downward trend since the 1970s actually increased slightly before the financial crisis. While equipment goods prices continued to decline, the prices of residential and particularly non-residential structure prices grew strongly for many years before the crisis. These trends are consistent with our model predictions.



Figure 3: US Relative Price of Capital Investment (Index, 1990=100) Diverging paths of the prices of different types of capital goods

Notes: This figure shows Bureau of Economic Analysis (BEA) data on the investment good deflators as a ratio to the GDP deflator series. The evolution of the index describes the behaviour of the *relative* price of that type of investment good.

Trading book profits and lending standards Another implication of the model is that, as banks become better able to exploit the guarantees, trading book profits should grow in importance relative to profits from the loan book, despite an increase in lending volume.²² Haldane and Alessandri (2009) document the growth of the trading book as a source of bank profits. They describe the period before the financial crisis as "an Alice in Wonderland world in which everybody had won and all had prizes." When the financial crisis came, they highlight that trading book losses were sizeable.

In order to expand their trading book activities, banks in the model loosen lending standards. The Senior Loan Officer Opinion Survey (SLOOS), provided quarterly since 1990 by the Federal Reserve Board of Governors, provides an empirical assessment of lending standards. Senior loan officers are very senior bank officials (from a cross section of anonymous lending institutions in the US) who indicate in the survey if lending standards are becoming tighter that quarter relative to the previous one. We can also measure the percentage that are increasing their margins on lending (spread over funding cost). Figure 4 shows that the period from 2002 until the onset of the financial crisis in 2008 was first marked by a reduced proportions of banks tightening conditions, and then a period during which more banks were loosening both lending standards as well as the spread over bank funding (the profit margin). In early 2005, over 70% of respondents were reducing their profit margin. Given that the survey measures a flow of weakening (rather than the level of the standards), this period of persistent reducing of standards is likely associated with a large reduction in standard that was reversed only during the financial crisis.²³ Keys, Mukherjee, Seru, and Vig

²²Philippon (2015) documents the surge in the volume of intermediation and the overall increase in the income share of the finance industry.

²³One weakness of this survey is that it does not measure the intensity of the reductions in margins or standards; more than 70% of banks could reduce standards a lot or a little with no difference in reported SLOOS indicator.

Figure 4: US Lending Standards from Senior Loan Officer Opinion Survey



Notes: This figure shows data from the Federal Reserve Board of Governors' Senior Loan Officer Survey. The lines represent, respectively, the net percentage of domestic banks tightening standards for commercial and industrial loans to large and middle-market firms; the net percentage of domestic banks tightening standards for commercial and industrial loans to small firms; and the net percentage of domestic banks increasing spreads of loan rates over banks' cost of funds to large and middle-market firms.



Real wages accelerated before, and then stagnated after, the financial crisis



Notes: This figure shows the index of nonfarm business sector real compensation per hour. The index is set to 100 in 2009.

(2008) and Bassett, Chosak, Driscoll, and Zakrajšek (2014) document the decline in standards in more detail.

Real wage The prediction from the models presented above is that the period in advance of the crash would also have been marked by significant real wage growth. Figure (5) shows the index of real compensation per hour. This begins to accelerate from around 1998 until the financial crisis. Between 2009 and 2014, real wages were virtually stagnant. Clymo (2016) provides evidence that real wages were gradually adjusted down in the US (as well as in the UK) after the crash. The persistent increase in the unemployment rate over this period is well-documented.

5.3 Quantifying the magnitude of the distortion

To quantify the effect, consider the basic predictions of the model under a benchmark economy (still indicated by subscript B) and an economy subject to our financial distortion (subscript FD). With *A* as realized TFP, \bar{A} as expected TFP and A^+ as the inflated conditional expectation that is central to the FD distortion (including, where appropriate, the effects of propriety trading as described in section 3)²⁴, we can write the equilibrium level of capital in each regime as:

$$K_{t,B} = \left(\frac{\alpha \bar{A}_t}{\delta_t + r_t}\right)^{\frac{1}{1-\alpha}}; \qquad K_{t,FD} = \left(\frac{\alpha A_t^+}{\delta_t + r_t}\right)^{\frac{1}{1-\alpha}}$$

The level of output is given by:

$$Y_{t,B} = A_t \left(\frac{\alpha \bar{A}_t}{\delta_t + r_t}\right)^{\frac{\alpha}{1-\alpha}}; \qquad Y_{t,FD} = A_t \left(\frac{\alpha A^+}{\delta_t + r_t}\right)^{\frac{\alpha}{1-\alpha}}$$

In an economy with the distortion, the level of GDP would be higher by:

$$\frac{Y_{t,FD}}{Y_{t,B}} = \frac{A_t \left(\frac{\alpha A_t^+}{\delta_t + r_t}\right)^{\frac{\alpha}{1-\alpha}}}{A_t \left(\frac{\alpha \bar{A}_t}{\delta_t + r_t}\right)^{\frac{\alpha}{1-\alpha}}} = \left(\frac{A_t^+}{\bar{A}_t}\right)^{\frac{\alpha}{1-\alpha}}$$

Therefore, in order to get a sense of the size of this distortion in the data, we need to assess $\left(\frac{A_t^+}{A_t}\right)^{\frac{\alpha}{1-\alpha}}$. Expressing this distortion, or wedge, as a % of the benchmark level of output, we define:

$$m_t \equiv \ln\left(\frac{A_t^+}{\bar{A}_t}\right)^{\frac{\alpha}{1-\alpha}}$$

In the model, this ratio also drives the behaviour of the capital-output ratio. Comparing a regime subject to the distortion with a baseline regime facing the same series of shocks, the K-Y ratio is higher in the economy subject to distortions.

$$\frac{K_{t,B}}{Y_{t,B}} = \frac{\alpha}{(\delta_t + r_t)} \frac{\bar{A}_t}{A_t}; \qquad \frac{K_{t,FD}}{Y_{t,FD}} = \frac{\alpha}{(\delta_t + r_t)} \frac{A_t^+}{A_t}$$
$$\Rightarrow \frac{\frac{K_{t,FD}}{Y_{t,FD}}}{\frac{K_{t,B}}{Y_{t,B}}} = \frac{A_t^+}{\bar{A}_t}$$

²⁴That is, without proprietary trading $A^+ = A_H$, and with proprietary trading $A^+ = A_H + \frac{1-p}{\alpha p} (A_L - (1-\alpha)A_H)$.

Figure 6: US Capital-Output Ratio





Notes: This figure shows the nominal non-consumer durables capital stock to nominal GDP ratio using BEA data. The dashed line is the average capital-output ratio during our benchmark years of 1989-1993.

5.3.1 Baseline back of the envelope calculation

As described above, we consider the period from the late 1990s until the financial crisis to be a period in which there were both guarantees and also an ability to exploit them; we call this the FD regime (t=1). We compare it to a benchmark period of 1989-1993 (t = 0). We think that while the guarantees certainly existed at this time, they were less exploitable.

Figure 6 shows the nominal capital-output ratio between 1985, the beginning of the period that became known as the Great Moderation, and 2015. These data come from the Bureau of Economic Analysis and the measure of capital stock includes both public and private fixed assets, but excludes consumer durables. In the baseline period, the capital output ratio was 2.8 $\left(\frac{K_{0,B}}{Y_{0,B}} = 2.8\right)$ and by 2008 it had risen to 3.1 $\left(\frac{K_{1,FD}}{Y_{1,FD}} = 3.1\right)$.

As a baseline back of the envelope calculation, we calibrate the scale of the distortion based on the observed changes in the ratios. Using $\alpha = 0.35$, we have $m_1 = \frac{\alpha}{1-\alpha} \ln\left(\frac{3.1}{2.8}\right) = 5.5\%$. This should be interpreted as GDP being 5.5% higher than the benchmark economy in 2008. If the post-crisis period was characterized by a reduced distortion of this magnitude, we should not expect to recover this portion of the (apparently lost) output.

5.3.2 A more complete calibration

This baseline calculation assumes that the earlier period is, apart from the ability to exploit the guarantees, otherwise identical to the later period. If, however, there are other things also changing, the earlier period is a less good control for this baseline calculation. For example, if there are secular trends in δ and r, there will be other drivers of the capital-output ratio. Using the observed change in the ratios could wrongly at-

tribute to $\left(\frac{A_t^+}{A_t}\right)$ the changes in the other drivers. Allowing *r*, δ , and actual TFP draws to vary:

$$\frac{\frac{K_{1,FD}}{Y_{1,FD}}}{\frac{K_{0,B}}{Y_{0,B}}} = \frac{\frac{\alpha}{(\delta_1 + r_1)} \frac{A_1^+}{A_1}}{\frac{\alpha}{(\delta_0 + r_0)} \frac{A_0}{A_0}} = \frac{(\delta_0 + r_0)}{(\delta_1 + r_1)} \frac{A_1^+}{\frac{A_0}{A_0}} = \frac{(\delta_0 + r_0)}{(\delta_1 + r_1)} \left(\frac{A_0}{A_1}\right) \left(\frac{A_1^+}{\bar{A}_0}\right)$$

This means we need to purge the change in the capital output ratio for these other drivers:

$$ln\left(\frac{A_1^+}{\bar{A}_0}\right) = ln\left(\frac{\frac{K_{1,FD}}{Y_{1,FD}}}{\frac{K_{0,B}}{Y_{0,B}}}\right) + ln\left(\frac{(\delta_1 + r_1)}{(\delta_0 + r_0)}\right) + ln\left(\frac{A_1}{A_0}\right)$$
(6)

The estimated $\left(\frac{A_t^+}{A_t}\right)$ ratio remains the key driver of the effect of the distortion on output. The m_1 calculation will simply use the ratio estimated in (6).²⁵

Depreciation Using BEA data on nominal capital consumption ($\delta_t K_t$), we can derive an official estimate of depreciation in each period. Figure 7 plots the time-series of this depreciation rate. We extend the series in this figure back to 1970 (rather than 1985 as in other figures) to show that the trend in the implied depreciation rate continued only until the late 1990s. The depreciation rate actually fell back somewhat in the early 2000s. During the benchmark period, the rate was 5.3%.

Real cost of capital The calculation above assumes that there is no change in the real cost of obtaining funds. Given that there is a large literature that suggests there has been a secular decline in interest rates over the last 25 years, this assumption may also be problematic. Rognlie (2015) backs out an implied measure of the real cost of funds from financial markets. He compares the difference between firms' market value and the value of their fixed assets which captures the discounted value of expected future pure profits which can then be used to infer an implied *r*. This approach helps to overcome the difficulty, caused by the fact that reliance on debt versus equity finance differ across firms, of backing the real cost of funds out of bond and equity prices.

The Rognlie calculation yields a very high value of the real return; the estimates are in the range 12-15%. This is driven by the fact that in the US Financial Accounts the market value is below the book value for much of the sample and firms are assumed

$$\frac{Y_{1,MM}}{Y_{0,FB}} = \frac{A_1 \left(\frac{\alpha A_1^+}{\delta_1 + r_1}\right)^{\frac{\alpha}{1-\alpha}}}{A_0 \left(\frac{\alpha \bar{A}_0}{\delta_0 + r_0}\right)^{\frac{\alpha}{1-\alpha}}} = \frac{A_1}{A_0} \left(\frac{\delta_0 + r_0}{\delta_1 + r_1}\right)^{\frac{\alpha}{1-\alpha}} \left(\frac{A_1^+}{\bar{A}_0}\right)^{\frac{\alpha}{1-\alpha}}$$

²⁵The difference in levels of output will be given by:



Figure 7: US Depreciation Rate Implied by BEA Capital Consumption The trend increase in δ_t happened up to 2000 before it fell back somewhat

Notes: This figure shows the implied nominal depreciation rate obtained by dividing nominal consumption of fixed capital by the capital stock using BEA data. The dashed line is the average capital-output ratio during our benchmark years of 1989-1993.

Figure 8: Estimates of the secular trends in real costs of funds by Rognlie (2015)



Notes: This figure shows the trends for the real cost of funds provided in Rognlie (2015). Owing to concerns expressed about the mean level of these estimates, we adjust all estimates by 5pp.

to make no pure profits on average. In his discussion of Rognlie's paper, Robert Solow questioned whether level was biased and so was too high saying: "It is hard to believe that the discount rate was this high from 1950 to 2010. (Household saving was available at an interest cost of 4 to 5 percent; one would have expected more investment to have taken place)" (Solow, 2015). Given this criticism, and because declines in r from a higher level would attribute more of the rise in capital-output ratios to our distortion, we adjust the level of the Rognlie estimates by a fixed 5pp across the entire sample.

Rognlie estimates the trends over the period 1947 to 2013, as reported in Figure 7 of Rognlie (2015); we update the linear and quadratic trends to get estimates for 2014 and 2015. Over the whole sample r has declined overall but when using a quadratic trend, he finds that r was slightly rising in the 2000s. Figure 8 shows the adjusted trend values for r from 1985 to 2015. We calibrate the distortion using both of these estimates in order to compare the effects of different estimates for r.

An additional concern may be the endogeneity of r. In the model, r is exogenous whereas the distortion may cause it to change endogenously. However, the direction of the endogenous reaction is not clear cut. One channel of the effect would be that, because households' future tax liabilities increase on average, saving may increase meaning that in a closed economy r would decline. A second channel comes from increased investment demand which would cause r to increase. We therefore proceed without further adjustment of the cost of funds to account for endogenous reaction to the distortion.

TFP The final driver of the capital output ratio that may differ across time are the realised productivity draws. Assuming that $\frac{A_1}{A_0}$ is constant when in fact the later period (t = 1) was marked by above trend productivity draws would bias down our estimate of $\left(\frac{A_1}{A_0}\right)$ since $\left(\frac{A_0}{A_1}\right) < 1$ contaminates the empirical measure of the capital output ratio. In order to adjust for this, we can use capacity-utilisation-adjusted TFP estimates provided by Fernald (2012). Panel A of figure 9 shows an index of this adjusted TFP series, as well as a quadratic trend estimated between 1947 and 2015. Our estimates of non-trend TFP shock at time t is the deviation from the estimated trend; the time-series of these deviations from trend are shown in panel B.

Counterfactuals Armed with estimates of δ_t , r_t and A_t , we use equation (6) to derive a more precise estimate of m_t . Using the same baseline as earlier, we can produce a time-series estimate of the relative effect of the distortion over time which allows us to plot a counterfactual path for GDP under the different assumptions in figure 10; as well as two different paths for r_t described above, we consider the naive assumption of unchanged δ_t , r_t and A_t that we used for the baseline back of the envelope calculation.

The gap between the measured GDP series and any of the counterfactual lines cap-



Notes: This figure shows the time series of Fernald's capacity-utilisation adjusted TFP index. Panel A shows the trend in this series. Panel B shows the deviation of the series from



Figure 10: Counterfactual paths for US GDP

Notes: This figure shows that our adjusted paths for GDP are much below measured GDP in the pre-financial crisis period. This reflects our analysis that the distortion is meaningful in this period.

tures the effect of the distortion under the assumptions of that counterfactual. This chart shows that during the late 1990s the size of this estimated distortion grew; the estimates come from increases in the capital-output ratio that cannot be explained by the movements in other drivers. The size of the distortion is particularly large when we use the quadratic trend in r_t because those assumptions actually suggest that the real return term was increasing which would have pushed the capital-output ratio down leaving more of an increase to explain with the distortion. Under all the assumptions the distortion pushed up GDP in the early 2000s. To the extent that we delude ourselves that this was sustainable production levels, this distortion has exacerbated the perceived loss of output since the financial crisis.

5.4 The distortion makes extrapolating current GDP trends misleading

As discussed in the introduction, many forecasters have been surprised by the extent of the weakness of US GDP since the financial crisis and have adjusted their estimates of the medium-term trend. We argue that the distortion means that the current GDP trend is unlikely to be a good indicator of the path for future GDP.

We now exploit the adjusted series and ask how misleading the measured GDP series is. In other words, if we had access to adjusted GDP, would we have been less surprised by the the slowdown following the financial crisis? We estimate a simple statistical measure of trend; we use linear and quadratic trend terms to adjust the log of the GDP series we are using. There are two estimates of the trend that we calculate. First we use data until the end of 2007 (a 2008 trend) which is then extrapolated to the end of 2015. The second estimates a trend using all data to the end of 2015. A large difference between the lines captures the fact that the pre-crisis evolution of the series was misleading (and would, with hindsight, have been ignored).

In the left panel of figure 11, we use the measured GDP index. Using this series in our exercise, we would, after eight years of the financial crisis and weak recovery, have revised the level 2015 trend GDP down by 20% of measured GDP. The right panel of Figure 11 runs the same trend analysis on our adjusted GDP series. The downward revision to trend GDP would have only been 1% suggesting that our adjusted series would have been much less misleading.

An alternative exercise is to run a Hodrick-Prescott filter (with $\lambda = 100$) through the two series and to calculate the deviation of measured GDP from the extracted trend measure. Of course is subject to end point problems which would affect the level of trend in recent years, but it may still highlight the potential effect of the distortion on perceptions of how far the US economy was from trend at the height of the financial crisis in 2009 and 2010. Figure 12 shows the estimated deviations from trend using this



Notes: These figures display a statistical measure of trend GDP estimated using a quadratic trend through the log of the GDP series. The blue line is the trend estimated using data through 2007 (as if the econometrician is sitting in 2008) and then extrapolated to 2015. The red line is the trend estimated through all of the data to 2015. The left panel (A) use the measured GDP series from the BEA which is plotted in both panels as the black dashed line. The right panel uses our adjusted GDP series which is plotted only in that panel as the grey line.

Figure 12: Deviation of US GDP from trends extracted using a Hodrick-Prescott filter



Notes: This figure shows the estimated deviation from trend derived by passing measured GDP (blue line) and, separately, our adjusted GDP (yellow line) through an HP filter.

approach. For example, with the measured GDP series the financial crisis represents a swing from 2.8% above trend in 2007 to 3.2% below trend in 2009. Strikingly, measured GDP (which includes the distortion) never falls below the trend estimated through adjusted GDP. Using the deviation of the measured GDP from the trend estimated using the adjusted GDP series, the same swing is +6.4% to +0.1%; the great recession looks more like the reversal of a great distortion!

Of course, many economic forecasters and analysts do not rely directly on statistical measures but rather estimate production functions. This is true of the CBO estimates of potential presented in 1. Although such production function approaches purport to estimate potential GDP, as opposed to simply a statistical trend, these methods typically need to estimate the evolution of the production inputs and statistical approaches typically feature in generating these estimates. As such, they could also be misled.

Figure 13: Beliefs about US longer term potential growth A. Consensus Long-Term growth expectations B. What if trend GDP growth didn't increase?



Notes: The left panel of this figure (A) shows the Consensus Economics long-term real GDP growth (6-10 years ahead) for the US. The right panel (B) shows BEA-measured real GDP (black dashed line), a counterfactual path if GDP growth had evolved in line with consensus long-term beliefs as displayed in panel A (red line) and a second counterfactual path if GDP trend was 2.5%, the consensus level for most of the 1990s.

5.5 Growth Delusion Effects

Given that we think that measured GDP may have contributed to misheld beliefs that the old trend is the level to which we should expect and wish to return, we use information from a survey of professional forecasters. At regular intervals since 1991, Consensus Economics have asked their survey respondents to give their view of US GDP growth 6-10 years ahead. These data are plotted in the left panel of Figure 13.²⁶

As is apparent in this figure, until around 1998 the forecasters consistently expected longer-term growth to be around 2.5% per annum for the US. After this point, they seem to have become convinced that the US could sustain much higher rates of growth going forward. The average across respondents was then over 3% per annum. While there are a number of factors that may drive these beliefs, in the interests of illustrating a point we interpret this series as beliefs about the longer-term trend at the time surveyed.

With this interpretation, we can make very simply calculations on the evolution of trend output under two alternative scenarios. The first is that beliefs about trend evolved according to the series in the left panel of Figure 13. The second counterfactual assumes that the trend continued to grow at 2.5% as respondents never experienced the temporary increase in their beliefs about long-term growth. The figure in the right panel of 13 shows the resulting indices for the two alternative projections of trend growth, together with the index of measured real GDP. Consistent with the earlier exercises, GDP in 2010 looks almost exactly in line with the counterfactual trend growth that evolved at the rate consistent with beliefs before the increases in the late 1990s.

²⁶This question was initially asked semi-annually, and later quarterly. To produce our quarterly timeseries measure, we linearly-interpolate the semi-annual measure until we have a full quarterly series.

5.6 Other evidence consistent with our channel

5.6.1 Effects of deregulation-induced credit shocks in the US

Within the model, we emphasize a channel that operates through the financial sector. There are alternative approaches taken in the literature to examine the effects of identified credit shocks on the economy. Here, we discuss two separate approaches to identifying the effects of credit shocks on the economy to examine whether the identified effects are consistent with the predictions of our theoretical model.

As our main narrative considers the expansion of credit in the economy as driven by financial innovation and (implicit or explicit) financial guarantees, we start with the closest possible match to such credit shocks. DiMaggio and Kermani (2016) use county-level US data together with an identification approach that exploits heterogeneity in national bank presence across counties as well as state-level differences in anti-predatory laws. Specifically, they use the fact that since 1999 a number of states adopted anti-predatory lending laws. Designed to protect borrowers, these laws restricted lending by banks and led to riskier borrowers. But in 2004 national banks were exempted from these restrictions. This represented a credit supply shock in counties in those states subject to the anti-predatory lending laws that have a large number of national banks active; suddenly these large banks in the area are released from a restricted supply of credit, particularly toward riskier borrowers. It is also worth pointing out that lending at favorable rates to riskier borrowers may be exactly a way to expand lending at low or even negative profits as would be optimal under our proprietary trading model.

Comparing the effects within a given anti-predatory lending state, but comparing across counties differentiated by the prevalence of national banks, the authors get an estimate of the increase in lending from the credit shock. They find that the relaxed regulation gives rise to a boost in annual lending of around 11%.

They then trace the effects of this credit shock onto other aspects of the real economy. They find that it is associated with a 3.3% rise in annual house price growth rate and a 2.2% expansion of employment in the non-tradable sectors. They also show that the lending was not necessarily to the best borrowers; the lending leads to a decline in mortgage delinquency rates during the boom years which is reversed in after the financial crisis. Such evidence is consistent with the effects of our model as discussed above. However, the authors do not explore the effects on real wages or on beliefs about economic growth.

Bassett, Chosak, Driscoll, and Zakrajšek (2014) take a VAR-identification approach to examine the effects of credit supply shocks. Their innovation is to use the Senior Loan Officer Opinion Survey (SLOOS) as a measure of credit supply and to embed it in a VAR with other standard macro time series such as the Federal Funds Rate, CPI inflation and GDP, as well as a measure of lending. Senior loan officers are very senior bank officials (across a cross section of anonymous lending institutions in the US) who indicate if lending is becoming more or less plentiful that quarter relative to before. The identification in Bassett, Chosak, Driscoll, and Zakrajšek (2014) is based on a Choleski ordering with the SLOOS credit supply variable ordered first meaning that shocks to it can affect all other variables contemporaneously but credit supply takes time to adjust to shocks to other variables. They find that credit shocks via this identification channel boost economic activity and lead to a tightening of monetary policy.

These authors, like DiMaggio and Kermani (2016), do not examine the effects of on real wages, growth expectations or asset prices. Therefore, in order to explore the effects on these variable, we follow their work but add real wages, and the Case-Shiller House Price Index. Moreover, to explore the effects on beliefs, we add our measure of long-term growth expectations as presented in figure 13 - this is to capture the effects of credit shocks on longer term growth beliefs (as a delusion indicator).

The results, discussed more fully in the appendix, indicate that credit supply shocks boost real wages, albeit that they adjust with a lag of about 2 years (8 quarters). Also after a lag of about 12 quarters, asset prices tend to be boosted, although this is only significant at the 90% level. Moreover, long-term growth expectations increase initially. These predictions are consistent with the predictions of our model.

5.6.2 The effect in other countries

While we have motivated the discussion and analysis with the experience of the US, it is not simply the US experience that gives rise to these concerns. Reinhart and Rogoff (2014), after examining 100 financial crises, highlight that financial crisis episodes are typically followed by protracted recoveries. They find that the mean time to recover to pre-crisis levels of GDP per capita is about 8 years (the median is 6.5 years). Others too find that financial crises give rise to recessions that are longer and deeper than other recessions. Notably, Ball (2014) contrasts the effect of lost of output (hysteresis) with the even more serious lost growth capacity (super hysteresis). Blanchard, Cerutti, and Summers (2015) empirically show that both of these effects are present in broad panel of countries.

In fact, there are some European countries where the size of the effects are even larger than in the US and the effects of the distortion in our model may be even larger. Figure 14 shows, trend estimates for Ireland and Spain that is similar to the exercise in figure 11. On each chart, one line captures the log-linear trend from 1980 to 2014 and the other line captures a log-linear trend estimated to 2007 and then extrapolated to 2014. In both countries the period just before the financial crisis was marked by an even larger increase in trend followed by and even larger fall. For both countries,



Figure 14: Trend Estimates for Ireland and Spain

Notes: These figure show trend extrapolation exercises, in the spirit of figure 11, for two European countries; Ireland (left panel) and Spain (right panel). The extrapolation of pre-crisis trends is carried out using a log-linear trend estimated from 1980 to 2007 and a trend estimation through the entire financial crisis.

using a quadratic trend (as we did above for the US) yields even larger over-estimation of GDP when the pre-crisis trend is extrapolated to 2014 and so here we show the more conservative log-linear trend.

6 Conclusion

We provide a novel explanation for the apparent decline in economic potential that typically follows banking crises. Our theoretical model highlights a finance-induced, supply-side mechanism by which the banking sector passes the implicit subsidy from government guarantees onto the real economy. It also highlights that a rise of banks' trading activities can magnify the problem. When we examine data for the US economy, our model is able the account for series of stylized facts associated with the run up to the 2008 crisis. Our analysis suggests a substantial role for our mechanism during that period. From that viewpoint, the great recession can look more like a reversing of a great distortion.

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Appendix

A Proofs

Proof of Lemma 1

Proof. First, note that no arbitrage condition implied by the presence of rest of the world agents implies: $p_i = E[R_i]$. This means that no security can offer a risk premium. Hence, in equilibrium, irrespective of its liability structure, the expected marginal cost of funds for the representative bank is 1, and so is the economy's expected marginal return to capital.

Proof of Lemma 2

Proof. The lemma directly follows from the reasoning in the text. \Box

Proof of Lemma 3

Proof. Imagine the bank does not fail in equilibrium. Then, deposits are always fully repaid. Hence, the expected marginal cost of funds is 1. This means that the expected marginal return to capital cannot be greater than 1 in equilibrium. But since $A_L < \overline{A}$, the bank must make losses in the bad state. From Lemma 2, the bank only issues deposits in equilibrium. Hence, there is no equity buffer and the bank defaults; a contradiction.

Proof of Lemma 4

Proof. Since the bank makes losses in state A_L , it must be that the representative firm repays less than what it has borrowed. Given the firm has limited liability, its decisions at the margin will only be based on state A_L . From, first order condition (2), we must have $\rho_H = 1$ in equilibrium. Hence, the relevant maximization problem for the firm is $\max_{k,n} A_H k^{\alpha} n^{1-\alpha} - wn - k$. The result follows.

Proof of Proposition 1

Proof. First, AK^{α} is strictly increasing in *K*. Second, expected output, which is given by $E[AK^{\alpha} - K]$, reaches a global maximum in $K_B < K^*$.

Proof of Proposition 3

Proof. From first order condition (3), we get the equilibrium level of capital (the subscript *T* stands for trading):

$$K_T^* = \left(\alpha A_H + \frac{1-p}{p} \left(A_L - (1-\alpha)A_H\right)\right)^{\frac{1}{1-\alpha}}$$

Since $A_L > (1 - \alpha)A_H$, we have that $K_T^* > K^*$. The result directly follows.

B Correlated investment

B.1 An extension with idiosyncratic risk and diversification in a closed economy

In the baseline model above, we only consider aggregate risk. In macroeconomic models, such an assumption is often justified by the idea that idiosyncratic risk can be diversified, and that this will be the case in the presence of risk aversion. However, government guarantees can make banks risk loving. In this section, we provide an extension of the model, where the bank can use a safe storage technology (that yields a 0 net return) and lend to two sectors, which exhibit different levels of firm correlation. In the diversified sector, we assume, for simplicity, that firm risk is *i.i.d.* The correlated sector corresponds to the production sector we have considered so far.

In practice, we assume that firms in the diversified sector operate the following CRS production function:

$$Ak^{\alpha}n^{1-\alpha}$$
,

where we use bold equivalent letters for the notation. The key difference here is that *A* is a constant.

Proposition 4. Assume $1 > A > A_H$. In the economy without guarantees, production only takes place in the diversified sector (and GDP is a constant). However, under government guarantees, production only takes places in the correlated sector. Considering $A \rightarrow 1$, compared to the baseline comparison,

- (i) The inefficiency in terms of net output is identical
- (ii) The inefficiency in terms of welfare corresponds to the incomplete market case.

The logic is the following:

• We solve for the optimal contract between banks and firms

- First, we show that both sectors cannot operate at the same time. Then, if this is the diversified sector that is operated in equilibrium, there is a profitable deviation for a couple bank-firm: since the wage is the first-best. Operating at the high level of capital generates extra profit.
- In equilibrium, we get that not only the risky technology is chosen, but it is operated at an (risk-neutral) inefficient level.
- If there was only the correlated sector, since we are in a closed economy, the first best would be to operate at a level below the risk-neutral level. In the regime without government guarantees, this would ensue. Hence, again, the guarantees boost investment and wages compared to first best.

C Effects of VAR-identified credit shocks

Bassett, Chosak, Driscoll, and Zakrajšek (2014) take a VAR-identification approach to examine the effects of credit supply shocks. Their innovation is to use the Senior Loan Officer Opinion Survey (SLOOS) as a measure of credit supply and to embed it in a VAR with other standard macro time series such as the Federal Funds Rate, CPI inflation and GDP, as well as a measure of lending. Senior loan officers are very senior bank officials (across a cross section of anonymous lending institutions in the US) who indicate if lending is becoming more or less plentiful that quarter relative to before. The identification in Bassett, Chosak, Driscoll, and Zakrajšek (2014) is based on a Choleski ordering with the SLOOS credit supply variable ordered first meaning that shocks to it can affect all other variables contemporaneously but credit supply takes time to adjust to shocks to other variables. They find that credit shocks via this identification channel boost economic activity and lead to a tightening of monetary policy.

However, these authors, like DiMaggio and Kermani (2016), do not examine the effects of on real wages, growth expectations or asset prices. Therefore, in order to explore the effects on these variable, we follow their work but add real wages, and the Case-Shiller House Price Index. Moreover, to explore the effects on beliefs, we add the measure of long-term growth expectations as presented in figure 13 - this is to capture the effects of credit shocks on longer term growth beliefs (as a delusion indicator). Our expanded data set is ordered as follows:



Figure A.1: Impulse Response Function to a shock to credit supply

The results of a shock to credit supply are shown in figure A.1. We present results from a Bayesian VAR estimated in levels but with all variables detrended using a quadratic trend; the VAR estimated in first differences is similar. We find that credit supply shocks boost real wages, albeit that they adjust with a lag of about 2 years (8 quarters). Also after a lag of about 12 quarters, house prices tend to be boosted, although this is only significant at the 90% level. Moreover, long-term growth expectations increase initially.

D Extended model

$$K = (Q^{\gamma} + S^{\gamma})^{\frac{1}{\gamma}}$$

$$S = (L^{\mu} + M^{\mu})^{\frac{1}{\mu}}$$
$$h^* = \lambda \frac{1-p}{p} S^*.$$

where *L* is land in fixed supply and *M* is labour in the construction sector. In this case here, λ also includes depreciation.

Accordingly, the lending problem of the bank can formalized as:

$$\max_{N,Q,M,L} p\left(A_{H}N^{1-\alpha}\left(Q^{\gamma} + (L^{\mu} + M^{\mu})^{\frac{\gamma}{\mu}}\right)^{\frac{\alpha}{\gamma}} - wN - wM - c_{e}Q - c_{l}L + \lambda_{s}\left(L^{\mu} + M^{\mu}\right)^{\frac{1}{\mu}}\right)$$

where

$$c_e \equiv q_e(\delta + r)$$

$$c_l \equiv (q_0 r + q_0 - q_1)$$

where q_0 is the equilibrium price of land, taking future price (q_1) as given.

$$\lambda_s \equiv \lambda q_1 \frac{1-p}{p}$$

From the first order conditions for *L* and *M*, we get:

$$M = L \left(\frac{w}{c_l}\right)^{\frac{1}{\mu-1}}$$
$$S = L \left(1 + \left(\frac{w}{c_l}\right)^{\frac{\mu}{\mu-1}}\right)^{\frac{1}{\mu}}$$
(7)

Expressing *M* and *L* as a function of *S*:

$$L = S\left(1 + \left(\frac{w}{c_l}\right)^{\frac{\mu}{\mu-1}}\right)^{\frac{-1}{\mu}}$$
(8)

$$M = S\left(1 + \left(\frac{w}{c_l}\right)^{\frac{\mu}{\mu-1}}\right)^{\frac{-1}{\mu}} \left(\frac{w}{c_l}\right)^{\frac{1}{\mu-1}},\tag{9}$$

we can reduce the dimension of the problem:

$$\max_{N,Q,S} A_H N^{1-\alpha} \left(Q^{\gamma} + S^{\gamma} \right)^{\frac{\alpha}{\gamma}} - Nw - c_e Q - c_s S + \lambda_s S$$

where

$$c_s \equiv \left(1 + \left(\frac{w}{c_l}\right)^{\frac{\mu}{\mu-1}}\right)^{\frac{-1}{\mu}} \left[w\left(\frac{w}{c_l}\right)^{\frac{1}{\mu-1}} + c_l\right]$$
(10)

From the first order conditions for *Q* and *S*, we have:

$$Q = S\left(\frac{c_e}{c_s - \lambda_s}\right)^{\frac{1}{\gamma - 1}}.$$

$$K = S\left(\left(\frac{c_e}{c_s - \lambda_s}\right)^{\frac{\gamma}{\gamma - 1}} + 1\right)^{\frac{1}{\gamma}}$$
(11)

Expressing *S* and *Q* as a function of *K*:

$$S = K \left(\left(\frac{c_e}{c_s - \lambda_s} \right)^{\frac{\gamma}{\gamma - 1}} + 1 \right)^{\frac{-1}{\gamma}}$$

$$Q = K \left(\left(\frac{c_e}{c_s - \lambda_s} \right)^{\frac{\gamma}{\gamma - 1}} + 1 \right)^{\frac{-1}{\gamma}} \left(\frac{c_e}{c_s - \lambda_s} \right)^{\frac{1}{\gamma - 1}},$$
(12)

we can reduce further

$$\max_{N,Q,S} A_H N^{1-\alpha} K^{\alpha} - Nw - c_k K$$

where

$$c_k \equiv \left(\left(\frac{c_e}{c_s - \lambda_s} \right)^{\frac{\gamma}{\gamma - 1}} + 1 \right)^{\frac{-1}{\gamma}} \left[c_e \left(\frac{c_e}{c_s - \lambda_s} \right)^{\frac{1}{\gamma - 1}} + (c_s - \lambda_s) \right]$$

The first order conditions for *N* and *K* are:

$$\begin{cases} (1-\alpha) A_H N^{-\alpha} K^{\alpha} = w \\ \alpha A_H N^{1-\alpha} K^{\alpha-1} = c_k \end{cases}$$
(13)

D.1 Collect

$$M = L \left(\frac{w}{c_l}\right)^{\frac{1}{\mu-1}}$$
$$S = L \left(1 + \left(\frac{w}{c_l}\right)^{\frac{\mu}{\mu-1}}\right)^{\frac{1}{\mu}}$$

$$c_{s} \equiv \left(1 + \left(\frac{w}{c_{l}}\right)^{\frac{\mu}{\mu-1}}\right)^{\frac{-1}{\mu}} \left[w\left(\frac{w}{c_{l}}\right)^{\frac{1}{\mu-1}} + c_{l}\right]$$
$$Q = S\left(\frac{c_{e}}{c_{s} - \lambda_{s}}\right)^{\frac{1}{\gamma-1}}$$
$$K = S\left(\left(\frac{c_{e}}{c_{s} - \lambda_{s}}\right)^{\frac{\gamma}{\gamma-1}} + 1\right)^{\frac{1}{\gamma}}$$
$$\begin{cases} (1 - \alpha) A_{H} N^{-\alpha} K^{\alpha} = w\\ \alpha A_{H} N^{1-\alpha} K^{\alpha-1} = c_{k} \end{cases}$$
$$c_{k} \equiv \left(\left(\frac{c_{e}}{c_{s} - \lambda_{s}}\right)^{\frac{\gamma}{\gamma-1}} + 1\right)^{\frac{-1}{\gamma}} \left[c_{e} \left(\frac{c_{e}}{c_{s} - \lambda_{s}}\right)^{\frac{1}{\gamma-1}} + (c_{s} - \lambda_{s})\right]$$
$$L = 1$$

M + N = 1

which is 10 equations in $\{K, Q, S, L, M, w, c_l, c_s, K, c_k\}$ 10 unknowns.

D.2 Reduce

Using the last 3, we have:

$$N = 1 - \left(\frac{w}{c_l}\right)^{\frac{1}{\mu-1}}$$
$$S = \left(1 + \left(\frac{w}{c_l}\right)^{\frac{\mu}{\mu-1}}\right)^{\frac{1}{\mu}}$$
$$c_s = \left(1 + \left(\frac{w}{c_l}\right)^{\frac{\mu}{\mu-1}}\right)^{\frac{-1}{\mu}} \left[w\left(\frac{w}{c_l}\right)^{\frac{1}{\mu-1}} + c_l\right]$$
$$Q = S\left(\frac{c_e}{c_s - \lambda_s}\right)^{\frac{1}{\gamma-1}}$$
$$K = S\left(\left(\frac{c_e}{c_s - \lambda_s}\right)^{\frac{\gamma}{\gamma-1}} + 1\right)^{\frac{1}{\gamma}}$$

$$(1-\alpha) A_H N^{-\alpha} K^{\alpha} = w$$
$$\alpha A_H N^{1-\alpha} K^{\alpha-1} = \left(\left(\frac{c_e}{c_s - \lambda_s} \right)^{\frac{\gamma}{\gamma-1}} + 1 \right)^{\frac{-1}{\gamma}} \left[c_e \left(\frac{c_e}{c_s - \lambda_s} \right)^{\frac{1}{\gamma-1}} + (c_s - \lambda_s) \right]$$

D.3 Final system

Getting rid of Q and S gives a system of 5 equations in 5 unknowns $\{N, K, c_s, w, c_l\}$

$$N = 1 - \left(\frac{w}{c_l}\right)^{\frac{1}{\mu-1}}$$

$$c_s = \left(1 + \left(\frac{w}{c_l}\right)^{\frac{\mu}{\mu-1}}\right)^{\frac{-1}{\mu}} \left[w\left(\frac{w}{c_l}\right)^{\frac{1}{\mu-1}} + c_l\right]$$

$$K = \left(1 + \left(\frac{w}{c_l}\right)^{\frac{\mu}{\mu-1}}\right)^{\frac{1}{\mu}} \left(\left(\frac{c_e}{c_s - \lambda_s}\right)^{\frac{\gamma}{\gamma-1}} + 1\right)^{\frac{1}{\gamma}}$$

$$w = (1 - \alpha) A_H N^{-\alpha} K^{\alpha}$$

$$\alpha A_H N^{1-\alpha} K^{\alpha-1} = \left(\left(\frac{c_e}{c_s - \lambda_s} \right)^{\frac{\gamma}{\gamma-1}} + 1 \right)^{\frac{-1}{\gamma}} \left[c_e \left(\frac{c_e}{c_s - \lambda_s} \right)^{\frac{1}{\gamma-1}} + (c_s - \lambda_s) \right]$$

It could still be reduced, but things become hard to read. See note. Then, to back out the remaining variables, we use:

L = 1

$$M = 1 - N$$

$$c_{k} \equiv \left(\left(\frac{c_{e}}{c_{s} - \lambda_{s}} \right)^{\frac{\gamma}{\gamma-1}} + 1 \right)^{\frac{-1}{\gamma}} \left[c_{e} \left(\frac{c_{e}}{c_{s} - \lambda_{s}} \right)^{\frac{1}{\gamma-1}} + (c_{s} - \lambda_{s}) \right]$$
$$M = \left(\frac{w}{c_{l}} \right)^{\frac{1}{\mu-1}}$$
$$S = \left(1 + \left(\frac{w}{c_{l}} \right)^{\frac{\mu}{\mu-1}} \right)^{\frac{1}{\mu}}$$

$$Q = \left(1 + \left(\frac{w}{c_l}\right)^{\frac{\mu}{\mu-1}}\right)^{\frac{1}{\mu}} \left(\frac{c_e}{c_s - \lambda_s}\right)^{\frac{1}{\gamma-1}}$$

D.4 Rearrange

$$N = 1 - \left(\frac{w}{c_l}\right)^{\frac{1}{\mu-1}}$$

$$c_s = \left(1 + (1-N)^{\mu}\right)^{\frac{-1}{\mu}} \left[w\left(1-N\right) + c_l\right]$$

$$K = \left(1 + (1-N)^{\mu}\right)^{\frac{1}{\mu}} \left(\left(\frac{c_e}{c_s - \lambda_s}\right)^{\frac{\gamma}{\gamma-1}} + 1\right)^{\frac{1}{\gamma}}$$

$$w = (1-\alpha) A_H N^{-\alpha} K^{\alpha}$$

$$N = \left(\frac{\left(\left(\frac{c_e}{c_s - \lambda_s}\right)^{\frac{\gamma}{\gamma - 1}} + 1\right)^{\frac{-1}{\gamma}} \left[c_e \left(\frac{c_e}{c_s - \lambda_s}\right)^{\frac{1}{\gamma - 1}} + (c_s - \lambda_s)\right]}{\alpha A_H K^{\alpha - 1}}\right)^{\frac{1}{1 - \alpha}}$$