A Model of the Fed's View on Inflation

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Abstract

A view often expressed by the Fed is that three components matter in inflation dynamics: a trend anchored by long run inflation expectations; a cycle connecting nominal and real variables; and oil prices. This paper proposes an econometric structural model of inflation formalising this view. Our findings point to a stable expectational trend, a sizeable and well identified Phillips curve and an oil cycle which, contrary to the standard rational expectations model, affects inflation via expectations without being reflected in the output gap. The latter often overpowers the Phillips curve. In fact, the joint dynamics of the Phillips curve cycle and the oil cycle explain the inflation puzzles of the last ten years.

Keywords: Phillips curve, output gap, unobserved components, Bayesian estimation.

JEL Classification: C11, C32, C53, E31, E32, E52.

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> Inflation is characterized by an underlying trend that has been essentially constant since the mid-1990s; [. . .]. Theory and evidence suggest that this trend is strongly influenced by inflation expectations that, in turn, depend on monetary policy. In particular, the remarkable stability of various measures of expected inflation in recent years presumably represents the fruits of the Federal Reserve's sustained effort since the early 1980s to bring down and stabilize inflation at a low level. The anchoring of inflation expectations [. . .] does not, however, prevent actual inflation from fluctuating from year to year in response to the temporary influence of movements in energy prices and other disturbances. In addition, inflation will tend to run above or below its underlying trend to the extent that resource utilization – which may serve as an indicator of firms' marginal costs – is persistently high or low.
>
> Yellen (2016), 'Macroeconomic Research After the Crisis'
> Speech for the 60th Boston Fed Conference

In the Federal Reserve's view – and, more generally, for central bankers in advanced countries – three components matter to inflation dynamics: trend-expectations, oil prices and the Phillips curve (PC). This simple representation, however, clashes with results in the academic literature. First, many have found the Phillips curve to be unstable, hard to identify and not a good forecasting model. Indeed, Phillips curve based forecasting models have been shown to perform poorly with respect to naive benchmarks (see, Atkeson and Ohanian, 2001 and Stock and Watson, 2007 for more recent evidence and relevant discussion). Second, an increasingly popular literature has challenged the idea that expectations are fully anchored and forward looking. For example, since the 2008 crisis, papers have connected the 'missing disinflation puzzle' to the partial dis-anchoring of consumers' inflation expectations that in turn can be accounted for by the evolution of oil prices (see, e.g., Coibion and Gorodnichenko, 2015).

This paper proposes an econometric representation of the central bankers' view and tries to reconcile it with the apparent wide range of contrasting findings in the academic literature.[1] Our strategy is to specify a trend-cycle model for inflation dynamics that captures the three components the Fed is focusing on – trend-expectations, oil prices and the Phillips curve – and to assess how well this model fits the data, how it relates to the 'narratives' identified by the literature, and how it performs in an out-of-sample forecasting exercise.

[1] A survey of the extensive empirical literature on the PC is beyond the scope of this paper. For a recent survey of the New Keynesian Phillips curve focusing on univariate limited-information methods, see Mavroeidis et al. (2014). For a review of results using full-information methods to estimate dynamic stochastic general equilibrium (DSGE) models, see An and Schorfheide (2007). Nakamura and Steinsson (2013) review the use of microeconomic data to study price dynamics. Coibion et al. (2017) discuss the incorporation of survey data on inflation expectations in models of inflation dynamics. Other surveys, providing complementary approaches, include Henry and Pagan (2004), Ólafsson (2006), Rudd and Whelan (2007), Nason and Smith (2008), Gordon (2011), and Tsoukis et al. (2011).

The baseline model includes CPI inflation, oil prices, and GDP and the unemployment rate as proxies for real economic conditions. Further, we include the median forecasts for 1-year-ahead CPI inflation from the University of Michigan consumers survey (UoM) and the Philadelphia Fed Survey of Professional Forecaster (SPF), as measures of agents' expectations. We impose minimal restrictions on the data to identify three orthogonal components of inflation: (i) a drifting unit root trend common to inflation and inflation expectations;[2] (ii) a stationary stochastic cycle connecting real variables, nominal variables and expectations, that we think of as the Phillips curve and that relates to the output gap; (iii) a stationary stochastic cycle capturing the common dynamics between oil prices, inflation expectations and CPI inflation but not affecting real variables.

Our identification remains agnostic about the nature of the trend and only requires inflation deviations from the trend to be stationary, yet it allows for a meaningful economic interpretation of the three components. First, the unit root trend common to inflation and inflation forecasts can be related to agents' long term expectations, under the assumption that the 'law of iterated expectations' holds (see Beveridge and Nelson, 1981 and Mertens, 2016). In fact, the impact of all transitory components has to be zero in the long run.[3] Second, and importantly, our specification nests several potentially different forward and backward looking Phillips curve models, including the standard New-Keynesian Phillips curve (NKPC) in which inflation is a pure forward-looking process, driven by expectations of future real economic activity. Third, the second cycle, a key and novel feature of our model, allows for disturbances in expectations due to energy price movements to directly impact inflation without affecting the measures of real activity, in line with the findings of Coibion and Gorodnichenko (2015). In other words, in contrast to a standard rational expectations model where the effect of oil shocks on price inflation is fully captured by the Phillips curve and the output gap, we allow energy price disturbances to temporarily 'disanchor' inflation expectations generating price movements not transmitted through the measure of slack in the economy. Finally, we also allow survey data to depart from the full-information rational expectations model without imposing any specific form of information friction. We do not require either of the two surveys to be an efficient and unbiased predictor of future inflation, and allow for temporary and permanent deviations from a rational forecast, potentially capturing measurement and observational errors, as well as a time-dependent bias in inflation expectations.

Figure 1 shows the three components as they are understood by our model and portrays a synthetic view of our findings. The upper panel describes the stationary component of inflation and splits it into the contribution of the Phillips curve (blue area), an 'energy price cycle' (red area) and an idiosyncratic stationary component uncorrelated to real variables and oil prices (yellow area). The bottom panel describes the expectation trend. Through the lens of our model, it appears that both oil prices and real activity matter in explaining

[2] The choice of modelling inflation as non-stationary is supported by findings in the forecasting literature which suggests that models perform better when they allow for trend inflation (Faust and Wright, 2013).

[3] A discussion on the conditions under which survey data can be employed to study the PC is Adam and Padula (2011).

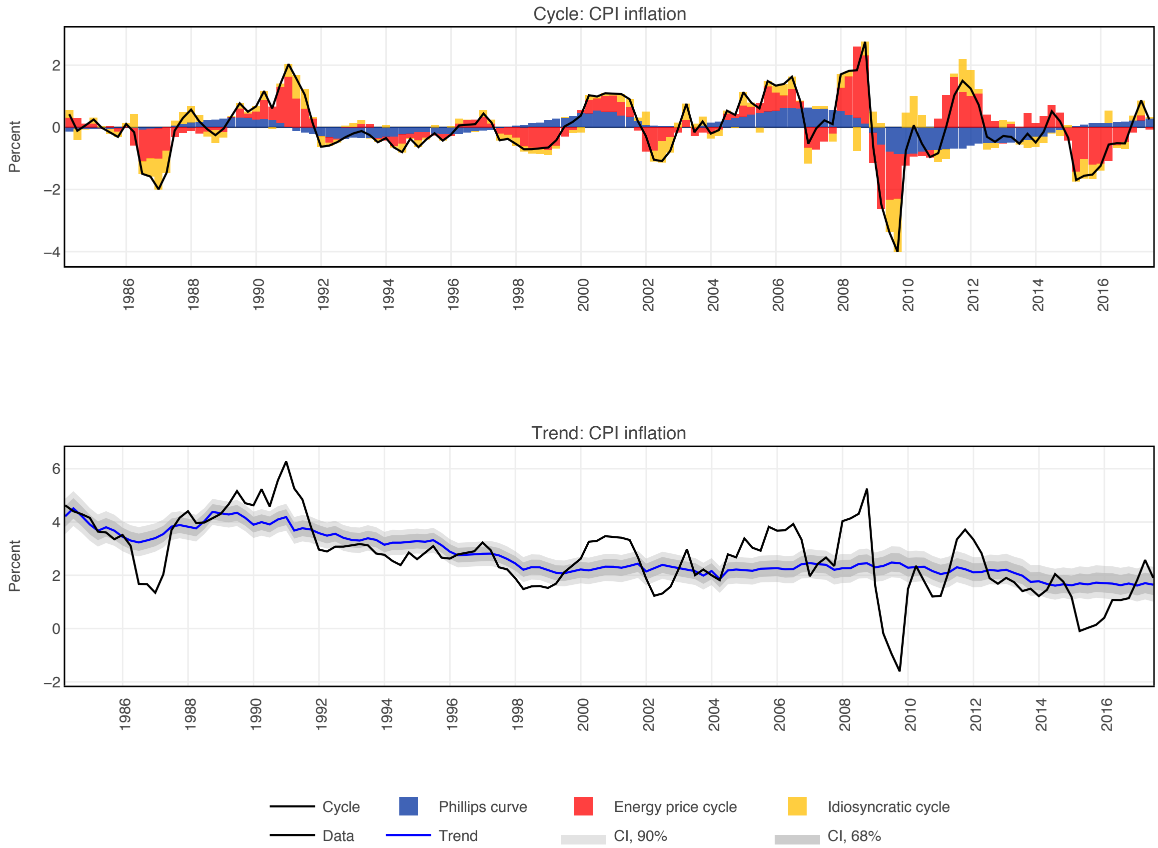

Figure 1: Trend and Cycle of inflation

temporary fluctuations in inflation and that the two cycles have different characteristics in terms of volatility and persistence. In the bottom panel we can see that the the trend in inflation has been slowly drifting downwards – quite stable from 2000 until 2010, and slightly declining since.

Results reported indicate that the Phillips curve – better understood as a relationship connecting nominal variables with real variables and inflation expectations – is alive and well, and has been fairly stable since the early '80s.[4] Importantly, our cycle decomposition shows that the Phillips curve is not always the dominant component. Large oil price fluctuations can move consumers' expectations away from the real-nominal relationship – 'disanchor' them – and induce expectation driven fluctuations in prices. This result confirms the intuition of Coibion and Gorodnichenko (2015) in a methodology which, in contrast to their approach, allows the Phillips curve to be recovered as orthogonal to oil-driven movements in the expectations and prices that are not transmitted to real variables. We provide confirmation of the importance of using expectational data to identify both trend inflation and the Phillips

[4]While we observe that a fixed parameter model is able to capture a stable Phillips curve from the '80s, it is possible that time-variation in the parameters (for example of the drift in the unit root trend inflation) or stochastic volatility may be important over a longer sample (see Stock and Watson, 2007; Mertens and Nason, 2017). We do not explore this possibility in this paper. Indeed, estimation uncertainty is likely to obfuscate all gains coming from a more sophisticated model.

curve, while dealing with disturbances to expectations that, albeit reflected in inflation, are unrelated to real variables and fundamentals. From a policy perspective, the stable inflation trend is an indication of the Fed's success in anchoring expectations. However, our results also point to the challenges that policymakers have to overcome in guiding expectations and stabilising the economy in the presence of large energy price disturbances.

From the econometric point of view, our work builds on the tradition of structural time series models (see Harvey, 1985) where observed time series are modelled as the sum of unobserved components: common and idiosyncratic trends and common and idiosyncratic cycles. In doing this and by focussing on inflation dynamics, this paper relates to the literature on the output gap, the Phillips curve and trend inflation estimation with unobserved components models, started by Kuttner (1994). Similarly to Baştürk et al. (2014) and Lenza and Jarociński (2016), we do not pre-filter data to stationarity; we model their low frequency behaviour by allowing for trends in GDP and inflation variables. As in Gordon (1982) and Basistha and Startz (2008) we use multiple real activity indicators to increase the reliability of the output gap estimates. Also, we relate to a number of papers which have studied trend inflation in unobserved component models augmented with data on medium-/long-term inflation expectations, as for example, Clark and Doh (2014), and Mertens (2016).

Our econometric representation can be understood as a restricted Vector Autoregressive (VAR) model where, by adopting minimal economic restrictions to identify the potentially different dynamic components of inflation, we induce 'informed' parsimony thereby helping with signal extraction and forecasting. The proposed decomposition leads to a rather complex state space form. In order to deal with this complexity, we employ Bayesian methods. A Bayesian approach in the context of a similar but simpler model has been proposed by Planas et al. (2008) who implement a Bayesian version of the work of Kuttner (1994) and, more recently, by Lenza and Jarociński (2016). The latter paper is the closest to our work but it focuses on estimating measures of the output gap in the Euro Area rather than on providing a decomposition to be used for understanding of the drivers of inflation dynamics.

2. A trend-cycle model for inflation

2.1. A stylised model for inflation dynamics

To provide intuition for our empirical strategy, we consider a stylised textbook rational expectations model for inflation and output. We assume that inflation and output can be decomposed into three components:[5] (i) an independent trend μ_t^j, (ii) a common stationary cycle relating nominal and real variables ψ_t^{PC}, and (iii) some idiosyncratic white noise

[5] In the text, the superscript j identifies either the related variable for idiosyncratic components – e.g. $j = \{\pi, y, \dots\}$ –, or one of the common components. The Phillips curve and the energy price cycles are indicated as PC and EP, respectively. In the text we use the same same mnemonics as in Table 1.

disturbance that can be thought of as classic measurement error ψ_t^j

$$\begin{aligned} y_t &= \mu_t^y + \psi_t^{PC} + \psi_t^y \ , \\ \pi_t &= \mu_t^\pi + \delta_\pi \psi_t^{PC} + \psi_t^\pi \ , \end{aligned}$$

where the independent trends are assumed to be unit-root processes with drifts

$$\mu_t^j = \tau^i + \mu_{t-1}^j + u_t^j \ .$$

The economic interpretation of the different trend and cycle components is standard. While the output trend is usually thought of as driven by technological innovation, the inflation trend is often linked to the behaviour of monetary policy (for example, policymakers may have targets that change over time). Conversely, the short-run cyclical fluctuations of inflation around the trend are often thought of as reflecting the effect of temporary shocks in the presence of price rigidity. Therefore, we can interpret the component of GDP explained by the common cycle with inflation variables as the output gap (this is the Phillips curve component of GDP).

In line with the econometric literature on the output gap, we assume that ψ_t^{PC} is a stationary process with stochastic cyclical behaviour. The simplest process allowing for such a stochastic cycle is an AR(2) process with complex roots of the form

$$\psi_t^{PC} = \alpha_1 \psi_{t-1}^{PC} + \alpha_2 \psi_{t-2}^{PC} + v_t \ .$$

It is worth noting that the AR(2) ψ_t^{PC} is a solution to a hybrid New Keynesian Phillips Curve connecting the cyclical components of output, inflation and inflation expectations, of the form:

$$\hat{\pi}_t = \sum_{i=1}^{2} \hat{\pi}_{t-i} + \mathbb{E}_t \left[\hat{\pi}_{t+1}\right] + \hat{y}_t + v_t \ ,$$

where hats indicate deviations from trends.[6] In this model, rational expectation agents correctly form model-consistent expectations about inflation, that is

$$\begin{aligned} \mathbb{E}_t \left[\pi_{t+1}\right] = \mathbb{E}_t \left[\mu_{t+1}^\pi + \delta_\pi \psi_{t+1}^{PC} + \psi_{t+1}^\pi\right] &= \tau^\pi + \mu_t^\pi + \delta_\pi (\alpha_1 \psi_t^{PC} + \alpha_2 \psi_{t-1}^{PC}) \\ &= \tau^\pi + \mu_t^\pi + \delta_{exp,1} \psi_t^{PC} + \delta_{exp,2} \psi_{t-1}^{PC} \ . \end{aligned}$$

It is also worth noting that trend inflation corresponds to the long-run forecast for inflation, which implies

$$\lim_{h\to\infty} \mathbb{E}_t \{\pi_{t+h}\} = \lim_{h\to\infty} \{h\tau^\pi + \mu_t^\pi\} \ ,$$

[6] Empirical studies often feature hybrid Phillips curves to account for inflation persistence (a recent survey is in Tsoukis et al., 2011). Several different mechanisms have been proposed in the literature to introduce hybrid Phillips curves such as indexation assumptions (e.g. Gali and Gertler, 1999), state-contingent pricing (e.g. Dotsey et al., 1999), or deviations from rational expectations assumption (e.g. Erceg and Levin, 2003; Milani, 2007).

in the spirit of Beveridge and Nelson (1981).

This stylised model admits a compact reduced form representation in terms of a common cycle and a common trend

$$\begin{pmatrix} y_t \\ \pi_t \\ \mathbb{E}_t\left[\hat{\pi}_{t+1}\right] - \tau \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \delta_\pi & 1 \\ \delta_{exp,1} + \delta_{exp,2}L & 1 \end{pmatrix} \begin{pmatrix} \psi_t^{PC} \\ \mu_t^C \end{pmatrix} + \begin{pmatrix} \mu_t^y \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} \psi_t^y \\ \psi_t^\pi \\ 0 \end{pmatrix}, \tag{1}$$

that in principle can also accommodate different specifications for the Phillips Curve, under suitable parameter restrictions. For example, an AR(1) ψ_t^{PC} would be the solution to a purely forward looking New-Keynesian Phillips Curve. It also nests the backwards looking 'Old-Keynesian' Phillips curve connecting output gap and prices – as in the 'triangle model of inflation' (see Gordon, 1982, 1990).

While such a stylised rational expectations model can provide the gist of the intuition and of the interpretation of our econometric model, it is likely to be too simplistic a representation of inflation dynamics. For example, it has been argued in the literature that, once inflation expectations are admitted to a forward- or backward-looking Phillips curve equation, it is also possible that economic disturbances impact prices without any intermediating transmission through the output gap or other measures of slackness in the economy (see, for example, Sims, 2008). In this spirit, Coibion and Gorodnichenko (2015) argue that the absence of disinflation during the Great Recession can be explained by the rise of consumers' inflation expectations between 2009 and 2011 due to the increase in oil prices in this period. Also, while macro-variables are likely to be affected by non-classical measurement error, agents' expectations, as captured by consumers' and professional forecasters' surveys, are likely to be only partially in line with national accounting definitions of aggregate prices, and can introduce measurement errors and biases of a different nature.[7] For these reasons, the empirical model which we will bring to the data and describe in the next section allows for these additional features.

2.2. The empirical model

In designing our empirical model, we generalise the functional form of the model in Equation 1 to allow for deviations from the perfect rationality in the stylised model. In particular, we want the model to be able to account for several potential deviations from the rational expectations benchmark. We allow for (i) oil price disturbances to affect prices directly via economic agents' expectations, with a stationary cycle connecting oil prices, expectations and inflation but not the measure of slack in the economy, and hence inducing a transitory disanchoring of expectations; (ii) a time varying bias i.e. a permanent disanchoring of expectations in the form of unit root processes; (iii) non-classic measurement error in the variables and other sources of coloured noise.

[7]For example, especially in consumer surveys the forecast horizon may be loosely defined while the relevant price index may be left unspecified. Also, projections are often reported at different frequencies and can have different forecasting points.

Table 1: Data and transformations

Variable	Mnemonic	Transformation
Unemployment rate	u	Levels
Real GDP	y	Levels
Oil price	oil	Levels
CPI inflation	π	YoY
UoM: Expected inflation	uom	Levels
SPF: Expected CPI	spf	Levels

Note: The table lists the macroeconomic variables used in the empirical model. "UoM: Expected inflation" is the University of Michigan, 12-months ahead expected inflation rate. "SPF: Expected CPI" is the Survey of Professional Forecasters, 4-quarters ahead expected CPI inflation rate. The oil price is the West Texas Intermediate Spot oil price.

We summarise these modelling choices in the following assumptions.

Assumption 1 CPI inflation and agents' (consumers and professional forecasters) inflation expectations share a common random walk trend with drift.

Assumption 2 Inflation, inflation expectations and real variables are connected by a Phillips curve in the form of a common ARMA(2,1) stationary cycle with complex roots.

Assumption 3 Inflation expectations, inflation and oil prices are connected by a common ARMA(2,1) cycle (Oil Price cycle), orthogonal to the Phillips curve cycle (output gap).

Assumption 4 All variables can have an idiosyncratic cycle component, capturing non-classic measurement error, differences in definitions and other sources of noise.

Assumption 5 Real variables have independent trends modelled with unit roots with drifts.

Assumption 6 Agents' (consumers and professional forecasters) expectations have independent idiosyncratic unit roots without drift, capturing time varying bias in the forecast.

Assumption 7 All components are mutually orthogonal.

Our benchmark empirical model expands on the core rational expectations model in Equation 1 by incorporating a fairly rich information set including the unemployment rate and output as measures of real activity, oil prices to proxy for energy prices, CPI inflation, and the median of the UoM consumer survey and of the Philadelphia Fed SPF one year ahead inflation forecast as proxies for consumers' and professionals' expectations (see Table 1 for details). Data are quarterly, with the sample starting in Q1 1984 and ending in Q2 2017. All variables enter the model in levels, except for CPI, which is transformed to the year-on-year inflation rate. The series are standardised so that the standard deviations of their first differences are equal to one.

For the purpose of this analysis the UoM's survey of consumers, and the Federal Reserve Bank of Philadelphia's SPF were chosen because they both have relatively long histories and are available at quarterly frequency. Both target CPI inflation, either explicitly as is the case for the SPF, or implicitly by surveying consumers, as is the case for UoM. From both surveys, we use the median expected price change in the four quarters following the date of the survey, which is consistent with our use of year-on-year inflation.

The maintained assumptions allow the identification of the model in $x_t \equiv \{u_t, y_t, oil_t, \pi_t, uom_t, spf_t\}$ (see, e.g. Bai and Wang, 2015), that can be written as

$$\begin{pmatrix} u_t \\ y_t \\ oil_t \\ \pi_t \\ uom_t \\ spf_t \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ \delta_y & 0 & 0 \\ 0 & 1 & 0 \\ \delta_\pi & \gamma_\pi & \phi_\pi \\ \delta_{uom,1} + \delta_{uom,2} L & \gamma_{uom} & \phi_{uom} \\ \delta_{spf,1} + \delta_{spf,2} L & \gamma_{spf} & \phi_{spf} \end{pmatrix} \begin{pmatrix} \psi_t^{PC} \\ \psi_t^{EP} \\ \mu_t^{C} \end{pmatrix} + \begin{pmatrix} \psi_t^u \\ \psi_t^y \\ \psi_t^{oil} \\ \psi_t^\pi \\ \psi_t^{uom} \\ \psi_t^{spf} \end{pmatrix} + \begin{pmatrix} \mu_t^u \\ \mu_t^y \\ \mu_t^{oil} \\ 0 \\ \mu_t^{uom} \\ \mu_t^{spf} \end{pmatrix} \tag{2}$$

We normalise ϕ_π, ϕ_{uom}, and ϕ_{spf} to the reciprocal of the standard deviation of the first difference of the respective variable.

It is worth noting that our empirical specification in Equation 2 would reduce to the stylised rational expectations model in Equation 1, under suitable parametric restrictions.

As discussed in Harvey (1990), the common and idiosyncratic ARMA(2,1) stationary cycles can be written in the following equivalent form:

$$\begin{bmatrix} \psi_t^j \\ \bar{\psi}_t^j \end{bmatrix} = \rho^j \begin{bmatrix} \cos(\lambda^j) & \sin(\lambda^j) \\ -\sin(\lambda^j) & \cos(\lambda^j) \end{bmatrix} \begin{bmatrix} \psi_{t-1}^j \\ \bar{\psi}_{t-1}^j \end{bmatrix} + \begin{bmatrix} v_t^j \\ \bar{v}_t^j \end{bmatrix}, \quad \begin{bmatrix} v_t^j \\ \bar{v}_t^j \end{bmatrix} \sim N(0, \varsigma_j^2 I_2) \tag{3}$$
$$\text{where } j \in \{PC, EP, x_1, \dots, x_n\}$$

where $\bar{\psi}^j$ is just a modelling device and the stationarity requirement imposes $0 < \lambda^j \leq \pi$ and $0 < \rho^j < 1$.[8] As discussed, the common and idiosyncratic trends are random walks (with/without drifts) that can be written as

$$\mu_t^j = \tau^j + \mu_{t-1}^j + u_t^j, \quad u_t^j \sim N(0, \sigma_j^2) \; .$$

It is also worth noting that the common and idiosyncratic trends in inflation and inflation expectations are identified up to a constant. For the sake of interpretation, we attribute the constant to the common trend so that it is on the same scale as the observed inflation variables.

Our estimation strategy builds on the approach recently suggested by Harvey et al. (2007), that combines 'structural' trend-cycle models á la Harvey (1985) with modern Bayesian techniques. Indeed, the frequentist maximum likelihood estimation of unobserved component models delivers inaccurate estimates and – as a result – implausible cycles and trends. Conversely, Bayesian estimation methods can effectively deal with these estimation issues by the

[8] The intuition for this is closely related to the standard multivariate AR(1) representation of univariate AR(p) processes.

use of priors.

We impose diffuse or not very informative priors and then maximise and simulate the posterior with a Metropolis-Within-Gibbs algorithm that is structured in two blocks. In the first block, we estimate the state space parameters by the Metropolis algorithm and in the second block we use the Gibbs algorithm to draw unobserved states conditional on model parameters. Relevant details and references are in the text and Appendix A. Section 4.1 reports our diagnostic analysis showing the role of priors in the estimates.[9]

3. Cycles and Trends in Inflation

3.1. Cycles

We first illustrate the Phillips curve and oil price cycles as identified and estimated by our model. Figure 2 shows results in both the time and frequency domains. The upper charts report the median of the posterior distribution of the Phillips curve and energy price cycles with relative confidence intervals at 68% coverage (dark shade) and 90% coverage (light shade). The lower charts show the associated spectral densities and coverage. The charts indicate that the Phillips curve cycle is less volatile than the oil price cycle. It is also more persistent with a cycle between 7 and 8 years periodicity, about twice as long as that of the oil price cycle.

Figure 3 shows the output gap estimate produced by the model. This is obtained by rescaling the Phillips curve cycle to match the GDP scale, and summing to it its idiosyncratic cycle to make it comparable with the output gap produced by the Congressional Budget Office (CBO). Since the new millennium our estimate and that of the CBO have given a different view of the degree of slack in the economy. At the time of the slowdown of 2001-2002, our model indicates that the economy went from over-capacity to trend growth but, unlike the CBO's, does not identify a protracted period of slack. Moreover, we estimate a milder recession since 2008 and find the economy to have been above full capacity since 2015. Our model, which, as we have seen, has a variable random trend, attributes the slowdown since the early millennium to the trend rather than the cycle. This is a nice feature since research has pointed to a slowdown in productivity growth since that date (see, for example, Hall et al. 2017).

Let us now turn to the contributions of the different cyclical components to the total cycles of the six variables considered in the model. Figure 4 shows the results. There are a number of interesting features which we would like to highlight.

1. The CPI inflation cycle is dominated by the oil price components although the Phillips curve captures lower frequency dynamics. Notice that the two cycles are not in any sense 'synchronised'. For example, from 2009 to 2014 the Phillips curve drags inflation down while oil prices push it up.

[9]The lags for the survey variables in Equation 2 can easily be implemented by including the auxiliary cycle $\bar{\psi}_t^j$ from Equation 3.

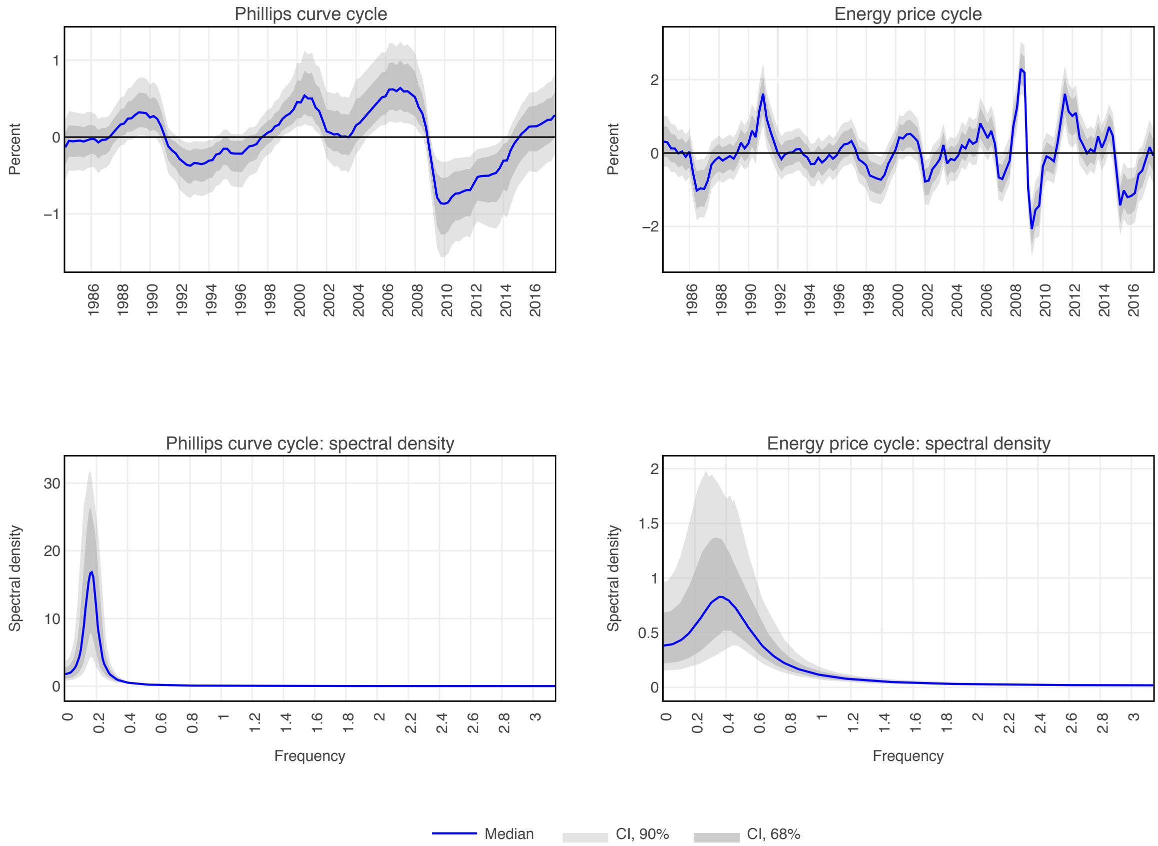

Figure 2: Common cycles in time and frequency domain

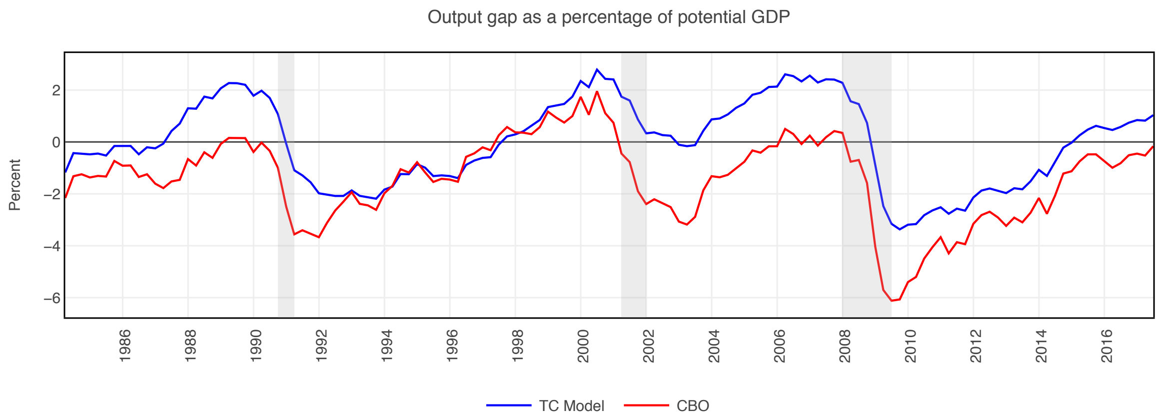

Figure 3: Output gap from trend-cycle model and CBO output gap

2. There is a striking difference between the cycle of SPF expectations and that of UoM consumer expectations. While the latter show a decomposition which is similar to that of CPI inflation, with a sizeable contribution from the oil price cycle, the SPF cyclical dynamics are dominated by the Phillips curve which accounts for most of their cyclical component.

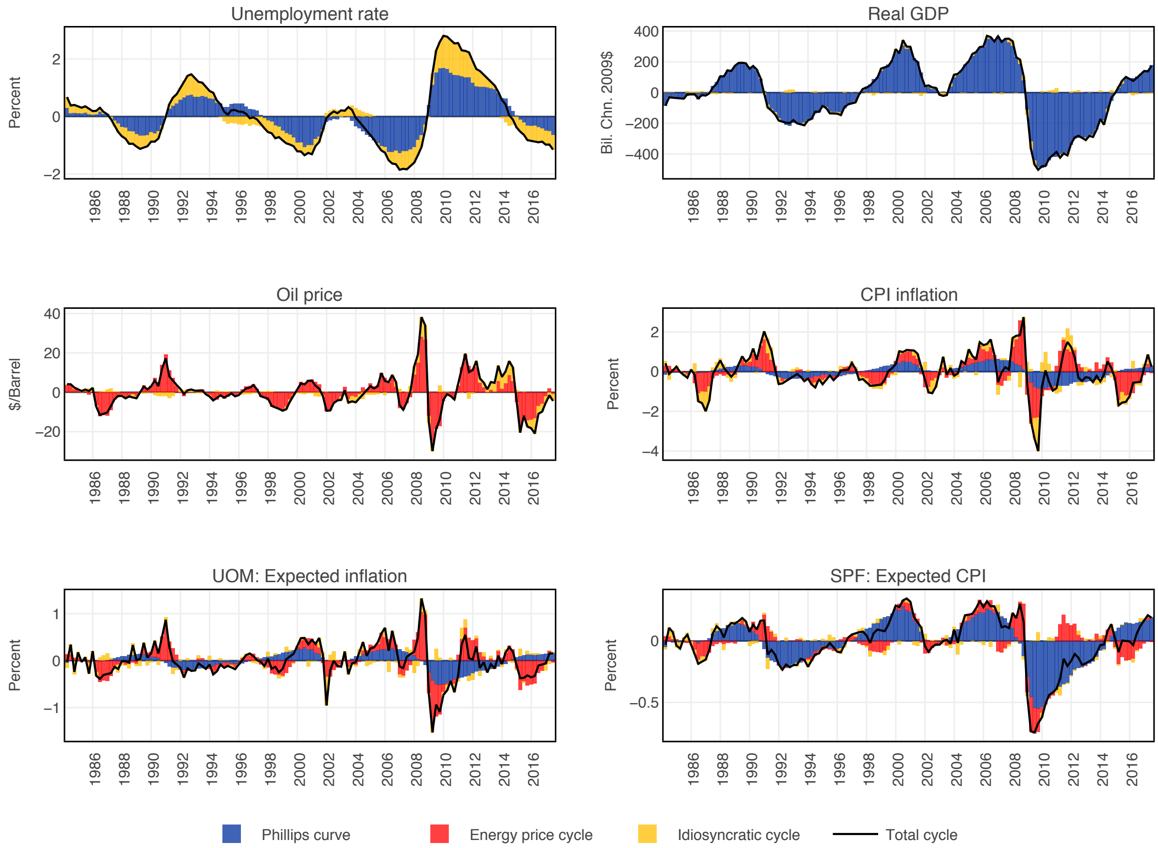

Figure 4: Historical decomposition of the cycles

3. Unemployment has a larger idiosyncratic cycle than GDP which is likely to reflect its lagging dynamics with respect to the GDP cycle.

The historical decomposition sheds light on some of the puzzling behaviour of inflation since 2008. From 2011 to mid 2012 the inflation cycle is supported by oil prices while the Phillips curve exerts negative pressure. The opposite is true from 2015 to the end of 2016 when oil prices drag inflation down while the Phillips curve exerts a small upward pressure. The cyclical part of inflation is well captured by these two components and little is left to idiosyncratic forces. Not surprisingly, consumer expectations follow a similar pattern to that of CPI while the SPF cycle is dominated by the Phillips curve. Notice that the Phillips curve cycle is quite small since 2014. This is likely to reflect the fact that the recovery in output was weak relative to that of employment. Indeed, since 2014, unemployment shows a large idiosyncratic component.

3.2. Trends

Figure 5 shows CPI inflation and the two inflation expectation variables against the median of the estimated common trend. We also report confidence bands with coverage at 68% (darker shade) and at 90% (lighter shade). The common trend is roughly stable from 2000 to

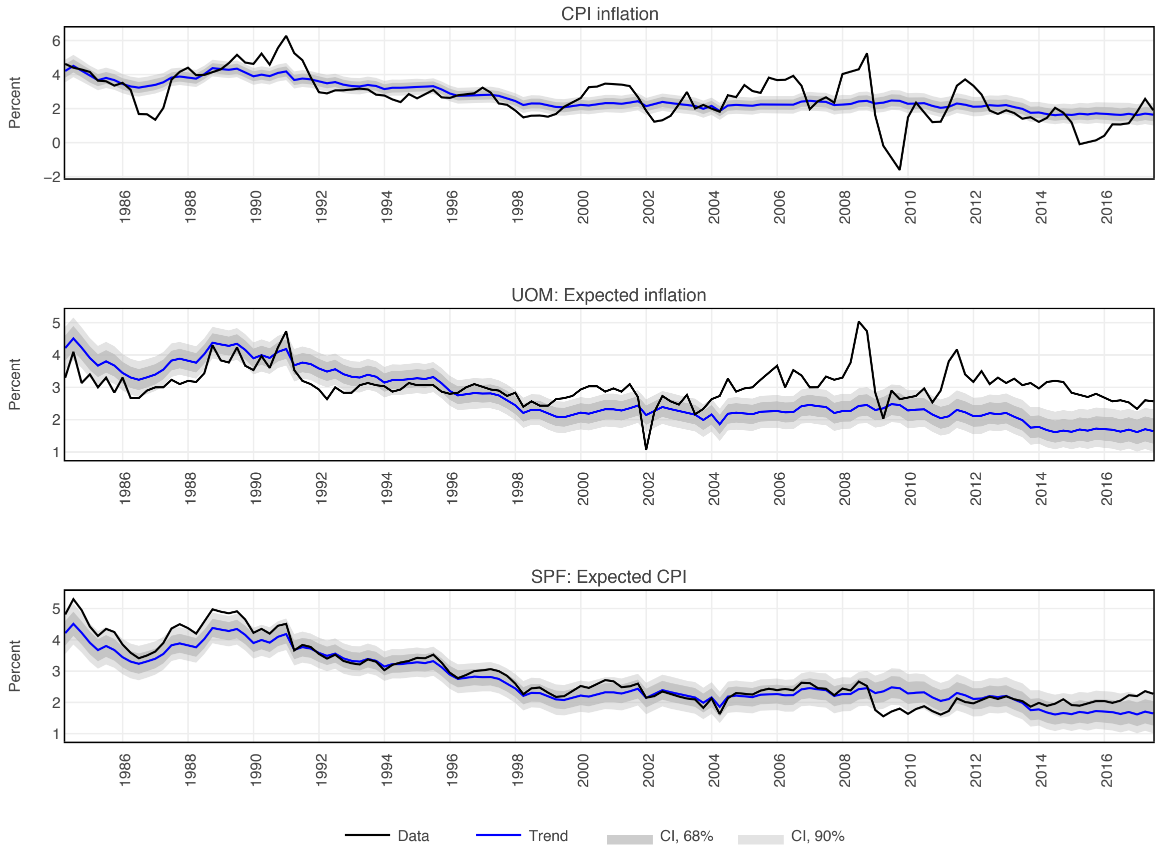

Figure 5: Common trend and CPI inflation, expectations

2010 and closely correlated with the SPF. Since the common trend corresponds to long-term inflation expectations plus a common drift, this suggests that the Fed has been able to keep expectations anchored, although a slight decline can be detected since 2010. Figure 6 also shows that the one-year ahead SPF trend is highly correlated with the 10-year ahead SPF confirming the interpretation that the model trend estimate captures long-term expectations. Indeed the unit root component in the inflation dynamics can be interpreted as agents' long term expectations, under the assumption that the 'law of iterated expectations' holds (see Beveridge and Nelson, 1981 and Mertens, 2016).

The behaviour of UoM expectations, on the other hand, shows large and persistent deviations from the common trend (long-term inflation expectations) since 2004. We interpret this sizeable time-varying idiosyncratic trend as a bias in consumer expectations. The latter, as indicated by Figure 7, suggests that consumer price expectations have a persistent component related to a persistent component in oil prices.

The result indicates that, as suggested by Coibion and Gorodnichenko (2015), household expectations are not fully anchored and respond strongly to oil price changes. In our model the oil price cycle can be seen to contribute to the cyclical component of the UoM survey but can also spill over into in its idiosyncratic trend as a modelled bias component. In line with our results, Coibion and Gorodnichenko (2015) document that more than half of the

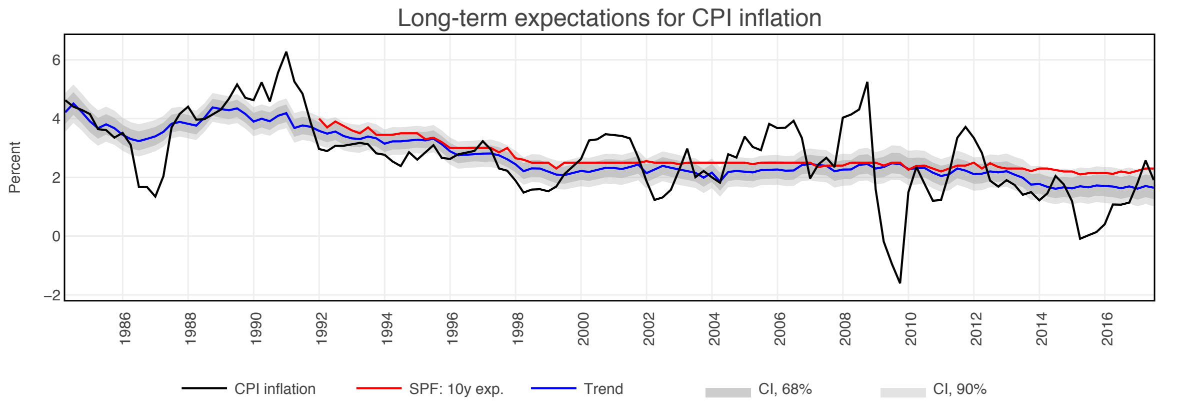

Figure 6: Common trend and 10 year expectations for CPI inflation

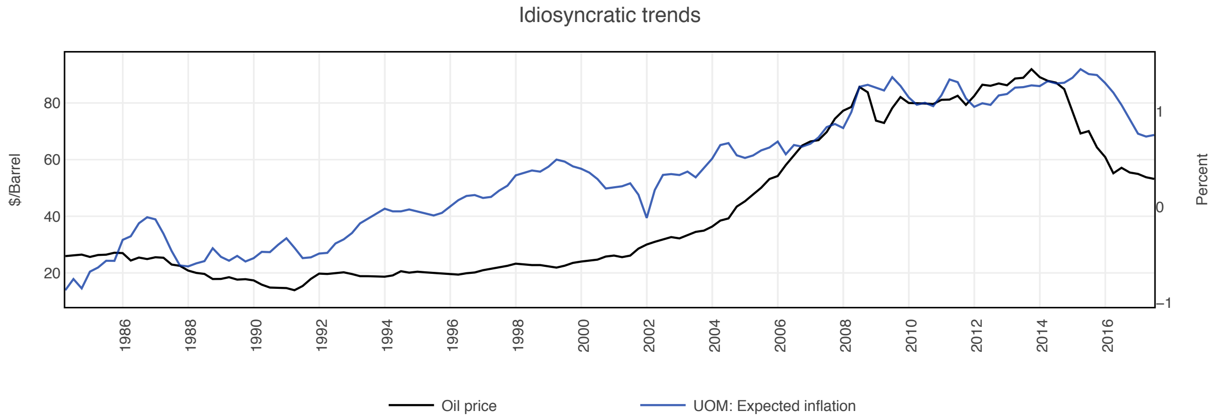

Figure 7: Idiosyncratic trend of oil price (left) and UOM Expected Inflation (right)

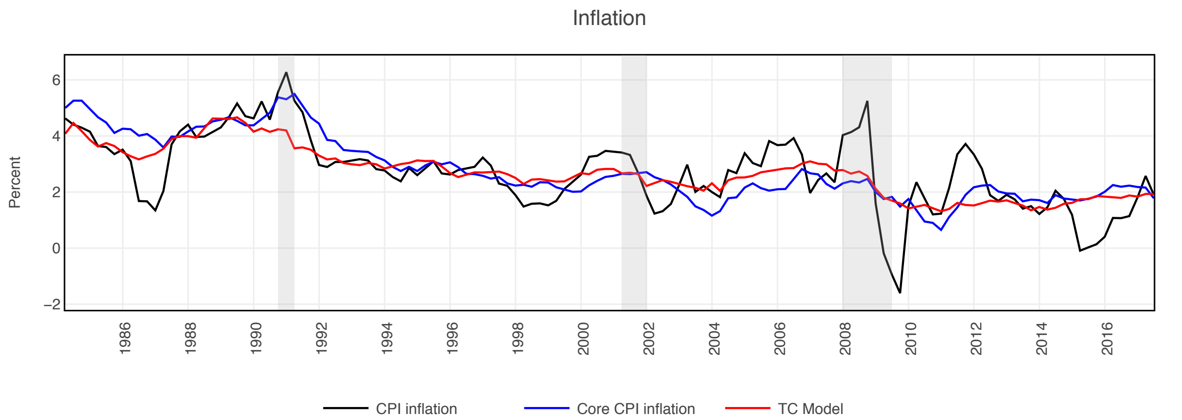

Figure 8: Core CPI inflation versus model implied core CPI inflation

historical differences in inflation forecasts between households and professional forecasters can be accounted for by the level of oil prices.

The model does not include a measure of core inflation. However, since we identify the stationary and non-stationary components of CPI inflation that are due to oil prices, we can

Table 2: Prior distributions

Name	Support	Density	Parameter 1	Parameter 2
δ, γ, ϕ and τ	$\mathbb{R}$	Normal	0	1000
σ^2 and ς^2	$(0, \infty)$	Inverse-Gamma	3	1
ρ	$[0.001, 0.970]$	Uniform	0.001	0.970
λ	$[0.001, \pi]$	Uniform	0.001	π

Note: The prior scheme includes only diffuse priors, and refers both to common and idiosyncratic components. The uniform priors ensure that the stochastic cycles are stationary and correctly specified according to the restrictions described in Harvey (1990).

create a 'model-implied' measure of core inflation by removing the oil component from CPI. Figure 8 compares core CPI inflation with this 'model-implied' measure.

4. Model Diagnostics

In the next section we will assess the robustness of our results. We will do it in different ways: comparing priors and posteriors, presenting the distribution of draws on the coefficients and performing an out-of-sample exercise.

4.1. Priors and posteriors

Table 2 reports the parameters of our priors while Figure 9 and Figure 10 illustrate the priors against the posteriors for the variance of the error terms of the unobserved components, the frequency and the persistence of the two common cycles. The charts provide an indicator of the degree of 'informativeness' of the data. Results indicate that the data are informative in estimating the many parameters of the model – in particular the variance of the shocks of the common components and the frequencies of the cycles.

Importantly, the posterior distributions of the coefficients for both the Phillips curve and the energy price cycles (Figure 11) indicate that coefficients equal to zero have negligible probability to be drawn in both cases.

4.2. Out-of-sample results

The forecasting exercise is conducted with quarterly data on a sample from Q1 1984 to Q2 2017. The period from Q1 1984 to Q4 1998 serves as our presample, the evaluation sample starts in Q1 1999 and ends in Q2 2017. We use an expanding window and recursively forecast up to 8 quarters ahead. Every quarter we reestimate the model, including the unobserved components and the coefficients. Apart from our model (TC), we consider (i) a BVAR where priors are set as in Giannone et al. (2015) and (ii) an univariate unobserved components IMA(1,1) with stochastic volatility model as suggested by Stock and Watson (2007) to be tough benchmarks for inflation forecasts. For all models we report the root mean squared

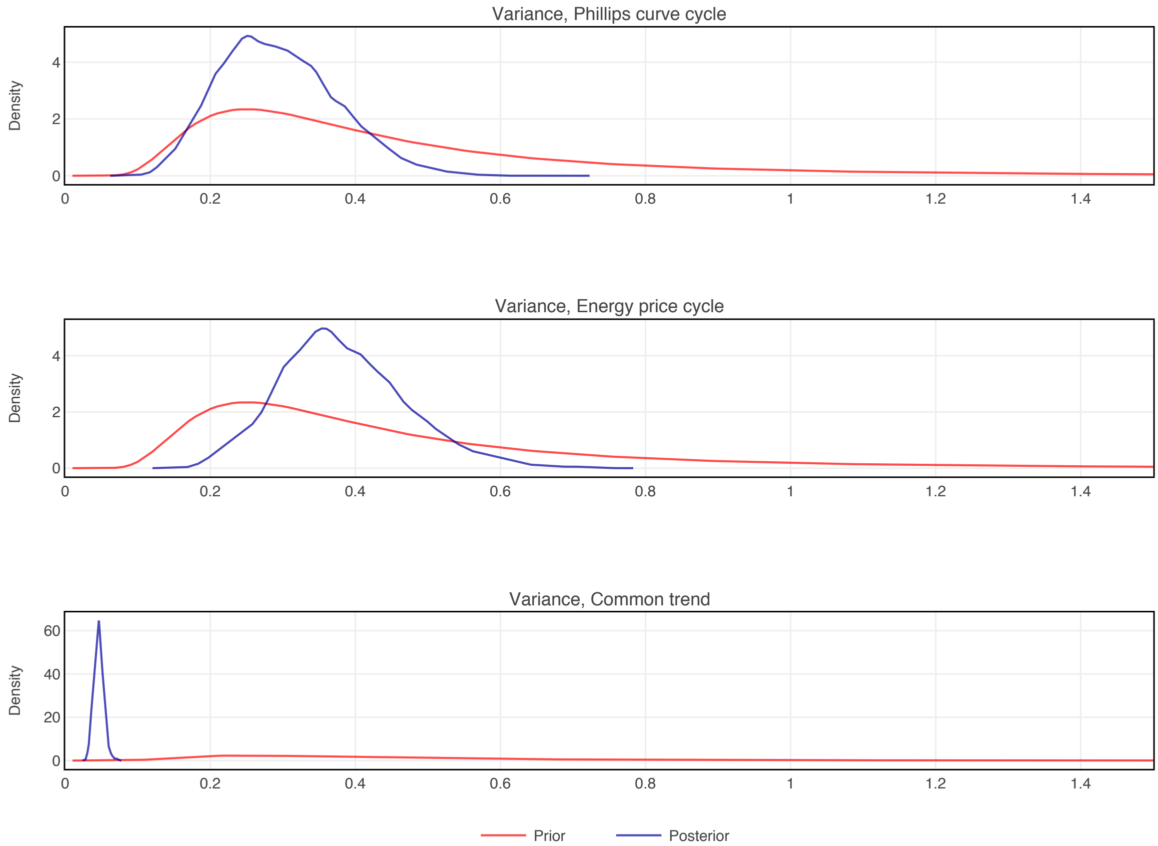

Figure 9: Priors and posteriors for the variance of the shocks of the common components

forecast errors relatice to those of a random walk with drift for forecasting horizons of one, two, four, and eight quarters.

Our TC model outperforms all others for CPI inflation and does particularly well at the two years horizon. Our conjecture is that our advantage with respect to the BVAR is driven by the random walk trend which captures the slow-moving, low frequency component while the advantage with respect to the UC-SV models is explained by the Phillips curve which captures cyclical co-movements. The TC model and the BVAR have similar performance on the other variables with the exception of the unemployment rate one and two quarters ahead where the BVAR outperforms us.

We consider these results as strong support for a model with both a variable trend for inflation captured by inflation expectations and a stationary Phillips curve component.

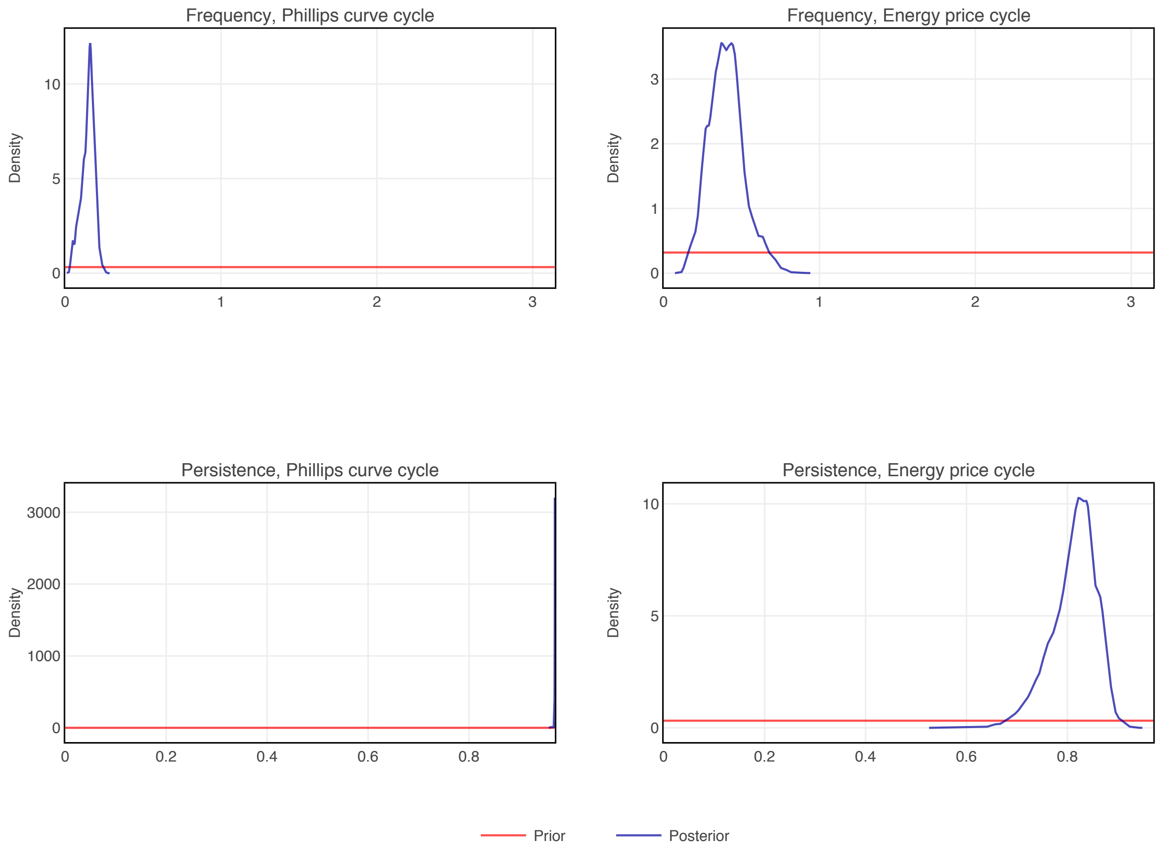

Figure 10: Priors and posteriors for the frequency and persistence of the common cycles

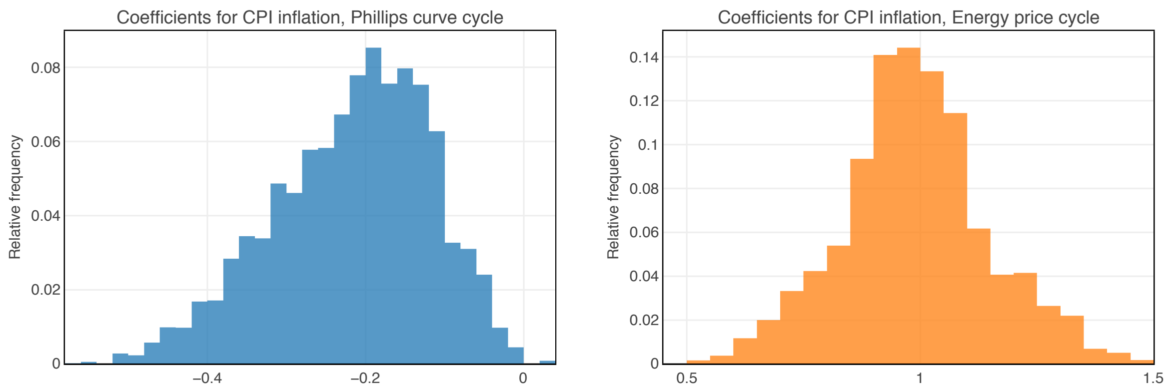

Figure 11: Posterior for the coefficients for CPI inflation

Table 3: Relative Root Mean Squared Errors

Horizon	Variable	TC Model	BVAR	UC-SV
h=1	Unemployment rate	0.83	**0.65**	x
	Real GDP	1.00	**0.92**	x
	Oil price	**1.02**	1.08	x
	CPI Inflation	0.92	**0.91**	1.00
	UOM: Expected inflation	**0.97**	1.03	x
	SPF: Expected CPI	**0.95**	1.10	x
h=2	Unemployment rate	0.85	**0.68**	x
	Real GDP	1.03	**0.91**	x
	Oil price	**1.04**	1.18	x
	CPI Inflation	**0.87**	1.00	0.99
	UOM: Expected inflation	**0.95**	1.09	x
	SPF: Expected CPI	**0.95**	1.24	x
h=4	Unemployment rate	0.89	**0.79**	x
	Real GDP	1.09	**0.97**	x
	Oil price	**1.04**	1.26	x
	CPI Inflation	**0.81**	1.13	0.98
	UOM: Expected inflation	**0.93**	1.14	x
	SPF: Expected CPI	**0.87**	1.35	x
h=8	Unemployment rate	**0.93**	0.97	x
	Real GDP	**1.17**	1.18	x
	Oil price	**1.04**	1.39	x
	CPI Inflation	**0.79**	1.07	0.96
	UOM: Expected inflation	**0.92**	1.30	x
	SPF: Expected CPI	**0.84**	1.39	x

Note: This table shows the RMSEs relative to the random walk with drift. The BVAR was estimated using Giannone et al. (2015). The UC-SV model was first proposed in Stock and Watson (2007).

5. Summary of our findings and implications

We have presented an econometric model which provides a formalisation of the 'Fed's view' according to which the three main drivers of inflation are: (i) a trend, reflecting expectations; (ii) the Phillips curve, relating economic slack to prices; and (iii) an oil price component unrelated to real variables. Our econometric model encompasses various specifications of the New Keynesian Phillips curve but also allows for deviations from it.

Our findings point to a very well identified Phillips curve which captures a cyclical component of CPI inflation with maximum power at around eight years periodicity. The latter is defined as the common cycle between real variables, inflation and (survey) inflation expectations. Contrary to a large body of evidence, the Phillips curve is well captured by a linear and constant parameter model such as ours.

CPI inflation, however, also has a sizeable cycle which is unrelated to real variables and captures the correlation between inflation expectations and oil prices. This cycle, which is of slightly shorter periodicity than the Phillips curve cycle and is more volatile, points to a channel through which oil price developments temporarily affect consumer price expectations away from the nominal-real relationship captured by the Phillips curve. In the presence of large oil price shocks this component may dominate. The finding of such a sizeable component suggests a deviation from the rational expectations model of the Phillips curve whereby oil prices affect prices via the output gap and it is in line with the core intuition of the pioneering work of Coibion and Gorodnichenko (2015). However, unlike those authors, we identify a relation between real variables, inflation and inflation expectations which is independent of the oil cycle.

Trend inflation is well captured by SPF expectations and indeed their role is crucial to identify trend inflation as suggested by the related literature (see Lenza and Jarociński (2016) and Mertens (2016)). The SPF are found not to be biased while UoM consumer expectations are where the bias is defined as an idiosyncratic unit root component. This suggests only a partial success by the Central Bank in anchoring expectations.

From the policy perspective our findings indicate that the central bank can exploit the Phillips curve trade-off but only in a limited way since the latter, although well identified, is a small component of inflation dynamics. Indeed, some of the so-called puzzles of inflation behaviour of the last decade can be explained by disentangling the Phillips curve from the oil cycle.

Managing expectations, we find, has a key role. A problematic issue for the central bank is that, facing volatile and persistent oil price dynamics, consumer expectations can affect price dynamics producing large and persistent deviations from a stable trend.

Our conclusions are therefore quite open-ended. The Fed's view that inflation is dominated by three components is supported by the data. However, the ability of the Central Bank to anchor expectations is limited especially because oil affects consumer expectations persistently and independently from the state of the real economy.

A. Metropolis-Within-Gibbs

Starting with the work of Beveridge and Nelson (1981), Harvey (1985), and Clark (1987) several methodologies have been suggested in the literature to estimate tend-cycle models with unobserved components. As discussed in Harvey et al. (2007), frequentist techniques tend to deliver inaccurate estimates and – as a result – implausible cycles and trends, due to large estimation uncertainty. Conversely, Bayesian methods, which allow for the incorporation of a-priori knowledge into the model estimation, make it possible to consistently estimate both univariate and multivariate trend-cycle decompositions via efficient numerical methods.

In estimating our model we adopt a Metropolis-Within-Gibbs algorithm. However, since this approach tend to have slow performances in large dimensions, we run a simulation smoother only after the burn-in period to gain computational speed. In fact, during the burn-in period, we only employ a Kalman filter with exact diffuse initial conditions to estimate the likelihood function, as described in Koopman and Durbin (2000) and Durbin and Koopman (2012). This significantly increases the speed of the estimation, give the large state-space of our model.

A.1. Algorithm

The algorithm is structured in two blocks: (1) a Partially Adaptive Metropolis (e.g., Herbst and Schorfheide, 2015) step for the estimation of the state-space parameters, (2) a Gibbs sampler to draw the unobserved states conditional on the model parameters. In a Partially Adaptive Metropolis the variance covariance matrix, Σ, of the candidate distribution is generated in an initialisation step.

Algorithm: Metropolis-Within-Gibbs

Initialisation

For $s = 1, \dots, n_s$ ($n_s = 40000$)

1. *Metropolis Algorithm*

   i. Draw a candidate vector for the unbounded parameters (θ_*), from a multivariate normal distribution with mean θ_{s-1} and variance $\omega\mathbb{I}$, where ω is a scaling constant used to get an acceptance rate between 25% and 35%

   ii. Set

$$\theta_s = \begin{cases} \theta_* & \text{with probability } \eta \\ \theta_{s-1} & \text{with probability } 1-\eta \end{cases} \tag{4}$$

   for

$$\eta = min\left(1, \frac{p(y \mid f(\theta_*)^{-1})\, p(f(\theta_*)^{-1})\, J(\theta_*)}{p(y \mid f(\theta_{s-1})^{-1})\, p(f(\theta_{s-1})^{-1})\, J(\theta_{s-1})}\right) \tag{5}$$

2. Discard the first $s = 1, \dots, n_0$ ($n_0 = 20000$) draws of θ_s.

Recursion

1. *Metropolis Algorithm*

   Set Σ to the sample covariance of the chain of θ_s, $(s = \{n_0, \ldots, n_s\})$, from the Initialisation step.

   For $q = 1, \ldots, n_q$ $(n_q = 20000)$

   i. Draw a candidate vector for the parameters (θ_*), from a multivariate normal distribution with mean θ_{q-1} and variance $\omega\Sigma$, where ω is set to have an acceptance rate between 25% and 35%

   ii. Set

   $$\theta_q = \begin{cases} \theta_* & \text{with probability } \eta \\ \theta_{q-1} & \text{with probability } 1-\eta \end{cases} \tag{6}$$

   where η is defined as in the Initialisation step.

2. *Gibbs sampling*

   For $n_q > n_\emptyset$ for $n_\emptyset = 10000$ (burn-in period), apply the univariate approach for multivariate time series of Koopman and Durbin (2000) to the simulation smoother proposed in Durbin and Koopman (2002) to sample the unobserved states, conditional on the parameters. In doing so, we follow the refinement proposed in Jarociński (2015).

3. Discard the first $q = 1, \ldots, n_\emptyset$ draws of θ_q.

Jacobian Most of these parameters are constrained (or bounded) in their support (e.g. the variances of the shocks are greater than zero). The standard approach used to tackle this problem is to transform the bounded parameters (Θ) so that the support of the transformed parameters (θ) is unbounded. Our Metropolis algorithm draws the model parameters in the unbounded space, in order to avoid a-priori rejections and to obtain a more efficient estimation routine.[10] The following transformations have been applied to parameters with Normal, Inverse-Gamma and Uniform priors respectively:

$$\begin{aligned} \theta_j^N &= \Theta_j^N \\ \theta_j^{IG} &= \ln(\Theta_j^{IG} - a_j) \\ \theta_j^U &= \ln\left(\frac{\Theta_j^U - a_j}{b_j - \Theta_j^U}\right) \end{aligned}$$

Where a_j and b_j are the lower and the upper bounds for the j-th parameter. These transformations are functions $f(\Theta) = \theta$, with inverses $f(\theta)^{-1} = \Theta$ given by:

[10]This description uses the same notation and a similar approach to the one described in Warne (2008)

$$
\begin{aligned}
\Theta_j^N &= \theta_j^N \\
\Theta_j^{IG} &= \exp(\theta_j^{IG}) + a_j \\
\Theta_j^U &= \frac{a_j + b_j \, \exp(\theta_j^U)}{1 + \exp(\theta_j^U)}
\end{aligned}
$$

These transformations must be taken into account when evaluating the natural logarithm of the prior densities in (5), by adding the Jacobians of the transformations of the variables:

$$
\begin{aligned}
\ln\left(\frac{d\Theta_j^N}{d\theta_j^N}\right) &= 0 \\
\ln\left(\frac{d\Theta_j^{IG}}{d\theta_j^{IG}}\right) &= \theta_j^{IG} \\
\ln\left(\frac{d\Theta_j^U}{d\theta_j^U}\right) &= \ln(b_j - a_j) + \theta_j^U - 2\ln(1 + \exp(\theta_j^U))
\end{aligned}
$$

Code The code is written in Julia (http://julialang.org) and it has been parallelised since it is computationally expensive.

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