Killer Acquisitions and Beyond: Policy Effects on Innovation Strategies

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Abstract

This paper provides a theory of strategic innovation project choice by incumbents and start-ups. We show that prohibiting killer acquisitions strictly reduces the variety of innovation projects. By contrast, we find that prohibiting other acquisitions only has a weakly negative innovation effect, and we provide conditions under which the effect is zero. Furthermore, for both killer and other acquisitions, we identify market conditions under which the innovation effect is small, so that prohibiting acquisitions to enhance competition would be justified.

Keywords: innovation, killer acquisitions, merger policy, potential competition, start-ups.

JEL: O31, L41, G34

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1 Introduction

Mergers rarely trigger interventions by competition authorities unless they involve substantial additions of incumbent market shares. Recently, many competition policy practitioners and academics have argued that this approach to merger control may be flawed. There is an increasing concern that mergers between firms that are not currently competing might be problematic as well, because they may eliminate potential competition.[1] Such worries even arise when "the target firm has no explicit or immediate plans to challenge the incumbent firm on its home turf, but is one of several firms that is best placed to do so in the next several years" (Shapiro, 2018). The issue becomes more pressing when the acquiree is working on a technology that would enable it to compete against the incumbent in the near future.

Such concerns arise in various sectors. For instance, in the digital economy, *Alphabet*, *Amazon*, *Apple*, *Facebook* and *Microsoft* bought start-ups worth a total of 31.6 billion USD in 2017.[2] *Google* acquired about one firm per month between 2001 and 2018.[3] There are several conceivable motives for such behavior. For instance, the acquiring firms may be better at commercializing the ideas of the start-ups, so that an acquisition may be efficient. Recent evidence suggests, however, that anti-competitive motives may also be important. The work of Cunningham et al. (2020) for the pharmaceutical industry is a compelling case in point. The authors show that incumbent firms often engage in so-called *killer acquisitions* by purchasing start-ups with the sole purpose of eliminating potential competition without intending to commercialize the entrant's innovation.[4] Even when incumbents do commercialize the innovation, acquisitions need not be innocuous, as they may widen the technological lead of a dominant incumbent, making entry ever harder (e.g. Bryan and Hovenkamp, 2020b).

These considerations suggest rethinking the predominant practice in most jurisdictions, which is to wave through acquisitions of small innovative start-ups by incumbent firms.[5] Indeed, there appears to be a broad consensus among economists that this approach is excessively lenient. That said, a per-se prohibition of start-up acquisitions would not

[1] This concern is reflected in policy reports such as Crémer et al. (2019) ("EU Report"), Furman et al. (2019) ("Furman Report") or Scott Morton et al. (2019) ("Stigler Report"); see also Salop (2016), Salop and Shapiro (2017), Hovenkamp and Shapiro (2017), Bryan and Hovenkamp (2020b).

[2] See `https://en.wikipedia.org/wiki/List_of_mergers_and_acquisitions_by_Alphabet`

[3] See *The Economist* 26/10/2018 "American tech giants are making life tough for start-ups". For more descriptive statistics on start-up acquisitions, see Gautier and Lamesch (2020). Examples include Facebook's takeovers of *WhatsApp*, *Instagram* and *Oculus CR*, *Google*'s acquisition of *DoubleClick*, *Waze* and *YouTube*, and *Microsoft*'s purchases of *GitHub* and *LinkedIn*.

[4] The use of the "killer" metaphor in the literature is not uniform. For instance, by contrast with Cunningham et al. (2020), other authors apply the expression "kill zone" to start-up activities that are so close to those of dominant incumbents that they may trigger acquisitions or hostile behavior towards the entrant, without implying that the incumbent would not commercialize the start-up's technologies.

[5] A rare early exception was the FTC's intervention against the acquisition of *HeartWare* by *Thoratec*, a maker of left ventricular assist devices, in 2009 on the grounds that "HeartWare alone represents a significant threat to Thoratec's LVAD monopoly;" see `https://www.ftc.gov/sites/default/files/documents/cases/2009/07/090730thorateadminccmpt.pdf`. More recently, there have been further interventions (see OECD, 2020). The biotech firm *Illumina* abandoned its proposed acquisition of the small rival *Pacific Biosciences* following opposition of the U.S. FTC and the U.K. CMA. The former explicitly referred to the extinction of *Pacific Biosciences* as a "nascent competitive threat". For similar reasons, the FTC imposed a divestiture before approving the acquisition of *College Park* by *Ossur*, both producers of prosthetic devices.

be desirable either: For instance, as many observers have pointed out, the prospect of selling the shop should increase the entrant's incentive to engage in innovation in the first place, no matter whether the acquirer commercializes the entrant's product or not.[6] Going back at least to Rasmussen (1988), several academic papers have made this point in formal models (see Section 2). However, the extent to which prohibiting acquisitions will decrease the entrant's incentive to innovate (as well as the extent of the anti-competitive harm) should be expected to depend on the characteristics of the market under consideration. This suggests a market-by-market approach towards treating start-up acquisitions, where the competition authority intervenes only in markets where the benefits from preserving potential competition outweigh any possible negative effects on innovation.

The purpose of our paper is to provide guidance for such an approach. Our analysis is based on a novel theory of R&D project choice, which enables us to study *variety* and *duplication* of R&D projects in which incumbents and start-ups invest. We characterize the innovation effect of prohibiting acquisitions. We show that it is weakly negative and describe how its size depends on market characteristics. We use our theory to analyze the effects of acquisition policy and other interventions on innovation. In particular, our analysis can help to identify industries where prohibiting acquisitions is more appropriate than elsewhere.

With this goal in mind, we provide a model that is generic rather than specifically tailored to any single industry. In this model, an incumbent monopolist possesses a technology that allows her to operate in a product market without incurring any innovation cost. By contrast, an entrant has to innovate in order to produce. Contrary to most papers in the innovation literature, which only analyze the overall level of R&D spending, we allow firms to strategically choose *in which* innovation projects to invest as well as *how much* to invest in each project. Such a representation captures important aspects of real-world innovation decisions.[7] We assume that there is a continuum of projects and that firms choose a subset of projects to invest in. Ex ante, projects exclusively differ with respect to investment costs; ex post, only one project will lead to an innovation. This innovation can be drastic or non-drastic, with exogenous probability of each case. We assume that, even when both firms discover an innovation, only one of them gets a patent. A patent holder who commercializes a drastic innovation earns monopoly profits (which are higher than what the incumbent previously obtained), and the other firm cannot compete. A non-drastic innovation allows the entrant to compete, while it may or may not allow the incumbent to increase her profits. In a *laissez-faire* setting without policy interventions, the incumbent can acquire the entrant once the innovation outcomes become common knowledge. We assume that an acquisition takes place if and only if it

[6] See for instance Bourreau and de Streel (2019), Crémer et al. (2019), Furman et al. (2019) and, most recently, Cabral (2020). It should be noted that the prospect of buying an innovative entrant could have a negative effect on the incumbent's incentives to innovate, since the incumbent can protect her monopoly by acquiring the entrant. Therefore, the overall effect that a prohibition of start-up acquisitions would have on innovation is unclear ex ante.

[7] In the pharmaceutical industry, development of new vaccines typically involves exploring various approaches simultaneously, such as using the attenuated or deactivated whole virus, or only DNA or virus-like particles, among others. Often, it is not clear ex-ante which approach will work; see for instance Le et al. (2020) for Covid-19 vaccine development. A prominent example for different approaches to an innovation in the digital industry is the development of the internet. While there were multiple competing methods to connect different networks and transmit data, the packet switching method turned out to be the one efficient enough to build the internet as we know it today (Leiner et al., 2009).

increases joint payoffs. In case an acquisition takes place, the trading surplus is split according to exogenously given shares reflecting bargaining power.[8] The firm possessing the innovation technology then decides whether to commercialize it at some fixed cost or not. We compare this *laissez-faire* setting with an alternative policy regime where acquisitions are prohibited.

We provide a full characterization of the equilibrium structure, which enables us to analyze policy effects on innovation strategies. Our main focus is on the effects of prohibiting start-up acquisitions on innovation.[9] The analysis turns out to be non-trivial because incumbents and entrants react differently to such a policy. Nevertheless, we obtain clear results. We distinguish between two parameter regimes according to whether the non-drastic innovation is sufficiently attractive that the incumbent would want to commercialize it or not.[10] Our analysis reveals a critical and surprising difference between these two cases. While prohibiting acquisitions always has a strictly negative innovation effect in the case without commercialization (i.e. for killer acquisitions), this is not necessarily true for acquisitions with commercialization. Thus, even though killer acquisitions may appear to be particularly problematic, the case for prohibiting them is not necessarily stronger than for acquisitions with commercialization if one takes ex-ante innovation incentives into account.

Crucially, in all equilibria in the killer acquisition case, the entrant's incentives determine the variety of innovation projects pursued. As the absence of the acquisition option reduces his investment incentives, overall variety declines when acquisitions are prohibited. By contrast, when non-drastic innovations are sufficiently valuable for the incumbent to commercialize, her incentives to innovate may be higher than those of the entrant. In this case, the incumbent's incentives (rather than the entrant's) will be decisive for the variety of innovation, and it will turn out that they are not affected by the policy regime. Without an adverse innovation effect, the prohibition of acquisitions is welfare-improving because it exclusively enhances competition.

In all other cases, however, policy has to trade off the positive competition effect of preventing acquisitions against the negative innovation effect. To this end, it is useful to understand for which market characteristics the innovation effect is likely to be small. We show that, from a consumer surplus perspective, the pro-competitive effects of prohibiting acquisitions are likely to dominate the adverse innovation effects in markets in which the entrant's bargaining power is low and potential competition between entrants and incumbents is not too intense. Thus, innovation effects should not be seen as a *carte blanche* for allowing acquisitions. Rather, whether or not acquisitions should be allowed depends on the specifics of the industry.

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[8]This assumption is in line with several related papers, e.g., Phillips and Zhdanov (2013), Cabral (2018) and Kamepalli et al. (2020).

[9]An outright prohibition is not the only way to handle acquisitions. Alternatively, firms acquiring innovative targets may be put under particular scrutiny ex post. For instance, after *Mallinckrodt*'s subsidiary *Questcor* acquired the rights for Synacthen from *Novartis*, the FTC successfully took the firm to court for anti-competitive behavior, which was manifest in excessive prices (see `https://www.ftc.gov/system/files/documents/cases/170118mallinckrodt\_complaint\_public.pdf`). For a broader discussion of conceivable policy responses, see OECD (2020).

[10]This distinction mirrors the contrast between killer acquisitions and nascent potential competitor theory of harms. As to the latter case, it arises if "the acquired product might grow into a rival product, and hence ... controlling that product (but not killing it), removes the competitive threat that it poses" (OECD, 2020, p.7).

Apart from variety, the acquisition policy affects other aspects of innovation strategies. Since firms can select between R&D projects rather than merely choose overall R&D effort, we can separate the effects of acquisitions on innovation probability from those on innovation efforts. When acquiring the entrant is not allowed, the incumbent has a stronger incentive to invest in the same R&D projects as the entrant because this is now the only strategy to prevent competition. Due to the potential increase in R&D duplication, the prohibition of start-up acquisitions may increase the overall R&D investments, while nevertheless resulting in a lower probability of discovering the innovation.

In spite of our focus on acquisition policy, our analysis also provides some insights on other policy measures. We show that the variety of pursued projects is weakly increasing in the entrant's bargaining power and in his stand-alone duopoly profits. By contrast, variety is weakly decreasing in the incumbent's stand-alone duopoly profits. Thus, any policy which improves the market position of start-ups relative to incumbents tends to increase the variety of equilibrium innovation projects and thereby the probability of a successful innovation. While innovation policies targeting small firms are usually justified as a way to alleviate financial constraints of those firms (see Bloom et al., 2019, p. 178), our analysis suggests that such policies have a positive innovation effect even in the absence of such constraints.

Section 2 reviews the literature. Section 3 introduces the model. Section 4 characterizes innovation behavior in the laissez-faire case. Section 5 deals with the effects of prohibiting acquisitions. Section 6 presents additional results. It analyzes the welfare trade-offs. Further, it provides a comparison to a one-dimensional model where firms only choose innovation efforts. Finally, it shows the robustness of the conclusions to modifications in the assumptions (uncertainty about innovation outcomes at the acquisition stage, technological asymmetries and multiple entrants). Section 7 concludes. All proofs of the formal results are in Appendix A. Appendix B provides additional formal results and proofs which support our claims in Section 6.

2 Relation to the Literature

Cunningham et al. (2020) not only provide empirical evidence for the existence of killer acquisitions, but they also develop a theoretical model to explain the rationale behind discontinuing development. The main difference between their model and ours is that we emphasize the initial innovation decisions, which they do not analyze.

Recent theoretical literature on mergers and innovation has mainly focused on mergers between incumbents, analyzing how product market characteristics, the nature of innovation and the innovation technology determine whether mergers reduce or increase (one-dimensional) innovation efforts. Federico et al. (2017, 2018) and Motta and Tarantino (2018) identify negative effects, whereas Denicolò and Polo (2018) find positive effects. In Bourreau et al. (2019), both possibilities arise.[11] In models with multiple research approaches, Letina (2016) and Gilbert (2019) obtain negative effects on R&D diversity; Letina also finds that mergers reduce research duplication. Moraga-González et al. (2019)

[11] A related literature investigates the effects of the number of firms on innovation, see e.g. Yi (1999), Norbäck and Persson (2012) and Marshall and Parra (2019). More broadly related, many papers discuss the relation between other measures of competitive intensity and innovation; see Vives (2008) and Schmutzler (2013) for unifying approaches.

show that mergers can potentially increase welfare by alleviating biases in the direction of innovation.[12]

While maintaining the emphasis on multiple research approaches, we address a fundamentally different question, namely how the possibility of acquiring entrants affects the innovations of incumbents and entrants. The literature on this topic goes back at least to Rasmussen (1988) who identified an incentive to enter a market to get bought by the current incumbent, suggesting that a lenient acquisition policy can increase welfare by incentivizing entry; see Mason and Weeds (2013) for similar reasoning. In Phillips and Zhdanov (2013) a laissez-faire policy not only fosters the entrant's innovation, but also the incumbent's.[13] Mermelstein et al. (2020) and Hollenbeck (2020) use computational methods to study the long-run effects of merger policy in dynamic oligopoly models with entry-for-buyout incentives; the latter finds that prohibiting mergers can lead to a lower rate of innovation and lower long-run consumer welfare. By contrast, Kamepalli et al. (2020) and Katz (2020) argue that, in the tech industry, a laissez-faire policy may have negative effects on start-up innovations.[14] Fumagalli et al. (2020) focus on acquisitions of financially constrained start-ups. They identify a novel benefit of acquisitions, which in this setting enable the incumbent to bankroll the development of innovations beyond the capabilities of the start-up, and they characterize the optimal competition policy. Unlike our paper, these papers do not analyze the strategic choices of innovation projects.

In related papers, Gans and Stern (2000) and Gans et al. (2002) focus on the endogenous decision of start-ups to sell their technology or enter the product market, while Bryan and Hovenkamp (2020a) consider distortions in the innovation decisions of start-ups who produce inputs for competing incumbents, without considering entry into this competition. In Cabral (2018) asymmetric competitors can pay to acquire each others' knowledge (a technology transfer rather than an acquisition).

Compared with the above literature, the goal of our paper is to identify market characteristics driving the size of the innovation effect and justifying intervention. On a closely related note, we also show how the case for intervention differs between killer acquisitions and others. Our emphasis on innovation portfolios allows us to analyze policy effects on project variety and duplication rather than merely on overall innovation efforts.

3 The Model

We will consider two variants of a multi-stage game, corresponding to different policy regimes. We will first describe the game capturing a *laissez-faire policy (A)* which tolerates acquisitions, then we will consider a *no-acquisition policy (N)*. We capture the laissez-faire policy in a multi-stage game between two firms, an entrant ($i = E$) and an incumbent

[12]More broadly related are Bryan and Lemus (2017) who study the direction of innovation, Letina and Schmutzler (2019) who consider research variety in innovation contests, Bardey et al. (2016) who analyze the effect of health insurance policy on diversity of treatment options and Bavly et al. (2020) who introduce asymmetric beliefs about the success of different projects.

[13]This difference to our work arises because the authors allow large firms to sell their own product and the target's product after the acquisition, so that there is an additional value from applying an innovation to the target's product as well as the own product.

[14]While the results of the two papers are similar, the central mechanisms differ. In Kamepalli et al. (2020), expectations of "techies" (potential early adopters of a new technology) drive the result. In Katz (2020), the key assumption is that potential entrants can choose innovation quality.

($i = I$). The entrant has to invest in R&D before he can produce. The incumbent owns a technology with which she can produce goods. In addition, she can invest in R&D as well.

In the first stage of the game, the *investment stage*, the firms choose in which research projects θ from a continuum $\Theta = [0, 1)$ to invest, and, for each project, how much to invest. Only one project, $\hat{\theta} \in \Theta$, will result in an innovation (be the *correct* project). All other projects will lead to a dead end and produce no valuable output. We assume that each project is equally likely to be correct. For all $\theta \in [0, 1)$, each firm chooses a research intensity $r_i(\theta) \in [0, 1]$. If $\hat{\theta}$ is the correct project, then $r_i(\hat{\theta})$ is the probability that firm i will discover the innovation. We restrict the firms' choices to the set $\mathcal{R}$ of measurable functions $r : [0, 1) \to [0, 1]$. The cost of investing with intensity r_i in project θ is given as $r_i(\theta)C(\theta)$, where the cost function $C : [0, 1) \to \mathbb{R}_+$ is continuous, differentiable, strictly increasing and convex. Moreover, we assume that $\lim_{\theta \to 1} C(\theta) = \infty$ and that $C(0) = 0$. The total investment cost of firm i is thus $\int_0^1 r_i(\theta)C(\theta)d\theta$.

The correct research project $\hat{\theta}$ can lead to two levels of innovation. With exogeneously given probability p, the correct project results in a high technological state (H), corresponding to a drastic innovation compared to the incumbent's current technology. With probability $1 - p$, the correct project results in a low technological state (L). L corresponds to a non-drastic innovation, which would allow the entrant to compete with the incumbent and obtain positive profits from the product market. If a single firm discovers the innovation, it receives a patent. If both firms discover the innovation, only one firm receives the patent, which is allocated randomly with equal probability.[15] We assume that only the patent holder can use the new technology. Once the correct project has been realized, both firms learn the resulting technology level, summarized in the *interim technology states* $(t_I^{int}, t_E^{int}) \in \mathcal{T} := \{(\ell, 0), (\ell, L), (\ell, H), (L, 0), (H, 0)\}$, where ℓ corresponds to the incumbent's initial technology and 0 corresponds to the entrant's initial technology.

In the second stage of the game under laissez-faire, the *acquisition stage*, the incumbent can acquire the entrant by paying the profits that the latter could achieve by competing on the market plus a share of the (bargaining) surplus $\beta \in (0, 1)$. We will assume that the acquisition takes place if and only if the bargaining surplus is strictly positive. If the entrant is acquired, then any patent held by the entrant is transferred to the incumbent.

In the third stage, the *commercialization stage*, the patent holder can bring the new technology to the market at some commercialization cost $\kappa > 0$.[16] We denote the technology states resulting after the acquisition and commercialization stages as *final technology states* $(t_I^{fin}, t_E^{fin}) \in \mathcal{T}$. Finally, in the *product market stage*, the firms collect product market profits which depend on the technology available to the firms. Denote the profit of firm $i \in \{I, E\}$, when it has technology t_i and its competitor has technology t_j, as $\pi(t_i, t_j)$.[17] We introduce the following assumptions.

Assumption 1 (Market profits)**.**

(i) Profits are non-negative, so that $\pi(t_i, t_j) \geq 0$ for any t_i and t_j. Monopoly profits are strictly positive, that is, $\pi(t_i, 0) > 0$ for any t_i.

[15]We consider asymmetric chances of receiving patents in Section 6.3.2.

[16]Implicit is the assumption that the commercialization is equally costly for both firms. As we show in Section 6.3.3, none of our main insights depends on this assumption.

[17]It may be helpful (but is not necessary) to think of technology states as real numbers corresponding to product quality or the (inverse) cost level.

(ii) Without an innovation, the entrant cannot compete. Thus, $\pi(0, t_j) = 0$ *for* $t_j \in \{\ell, L, H\}$.

(iii) Technology H *corresponds to a drastic innovation, so that the owner gets the monopoly profit* $\pi(H) := \pi(H, \ell) = \pi(H, 0) > max\{\pi(L, 0), \pi(\ell, 0)\}$ *and* $\pi(\ell, H) = 0$.

(iv) Competition decreases total profits, that is, $\max\{\pi(L, 0), \pi(\ell, 0)\} > \pi(\ell, L) + \pi(L, \ell)$.

We do not assume that technology L is necessarily an improvement over the status-quo technology ℓ for the incumbent: $\pi(L, 0) > \pi(\ell, 0)$ and $\pi(L, 0) \leq \pi(\ell, 0)$ are both possible.

Assumption 2. *Commercialization costs satisfy*

$$(i)\ \pi(L, \ell) \geq \kappa;$$
$$(ii)\ \pi(H) - \pi(\ell, 0) \geq \kappa.$$

Thus, for the entrant, even the duopoly profit obtained thanks to a non-drastic innovation is at least as high as the commercialization cost. For the incumbent, the increase in the monopoly profit obtained by using the drastic innovation outweighs the commercialization cost. For the non-drastic innovation, this may or may not be the case.

We refer to the firms' continuation payoffs at the beginning of the acquisition stage, conditional on the realization of the interim states t_I^{int} and t_E^{int}, as their *values* $v_I(t_I^{int}, t_E^{int})$ and $v_E(t_E^{int}, t_I^{int})$, respectively. These values depend on the policy regime (laissez-faire or no-acquisition). When either firm has state H, the values are independent of the competitor state; thus, we simply write $v_I(H)$ and $v_E(H)$. The expected total payoff of the incumbent who chooses an investment function $r_I(\theta)$ when facing an entrant who chooses $r_E(\theta)$ is

$$\begin{aligned}
\mathbb{E}\Pi_I(r_I, r_E) = &- \int_0^1 r_I(\theta)C(\theta)d\theta + \int_0^1 r_I(\theta)(1 - r_E(\theta)) \left[pv_I(H) + (1 - p)v_I(L, 0)\right] d\theta \\
&+ \int_0^1 (1 - r_I(\theta))r_E(\theta) \left[(1 - p)v_I(\ell, L)\right] d\theta + \int_0^1 (1 - r_I(\theta))(1 - r_E(\theta))v_I(\ell, 0)d\theta \\
&+ \int_0^1 r_I(\theta)r_E(\theta) \left[p\left(\frac{1}{2}v_I(H)\right) + (1 - p)\left(\frac{1}{2}v_I(L, 0) + \frac{1}{2}v_I(\ell, L)\right)\right] d\theta.
\end{aligned}$$

The first integral captures the innovation costs that the incumbent incurs by using the innovation strategy r_I. The second integral represents the incumbent's continuation payoff when she discovers an innovation and the entrant does not. The third integral captures her continuation payoff in the opposite case, when she does not discover an innovation but the entrant does. The fourth integral represents the continuation payoff when neither firm innovates, and the fifth is for the case when both firms innovate.

The expected total payoff of the entrant is given analogously as

$$\begin{aligned}
\mathbb{E}\Pi_E(r_E, r_I) = &- \int_0^1 r_E(\theta)C(\theta)d\theta + \int_0^1 r_E(\theta)(1 - r_I(\theta)) \left[pv_E(H) + (1 - p)v_E(L, \ell)\right] d\theta \\
&+ \int_0^1 r_E(\theta)r_I(\theta) \left[\frac{p}{2}v_E(H) + \frac{1 - p}{2}v_E(L, \ell)\right] d\theta.
\end{aligned}$$

We will characterize subgame-perfect equilibria of the game. For the investment stage, this amounts to finding functions $r_I, r_E \in \mathcal{R}$ such that for any $r_I', r_E' \in \mathcal{R}$

$$\mathbb{E}\Pi_I(r_I, r_E) \geq \mathbb{E}\Pi_I(r_I', r_E)$$
$$\mathbb{E}\Pi_E(r_E, r_I) \geq \mathbb{E}\Pi_E(r_E', r_I).$$ [18]

The characterization of the equilibrium investment will rely on *critical projects* θ_E^1, θ_E^2, θ_I^1 and θ_I^2, which are defined implicitly by:

$$
\begin{aligned}
C(\theta_E^1) &= pv_E(H) + (1-p)v_E(L,\ell) \\
C(\theta_E^2) &= \frac{1}{2}\left(pv_E(H) + (1-p)v_E(L,\ell)\right) \\
C(\theta_I^1) &= pv_I(H) + (1-p)v_I(L,0) - v_I(\ell,0) \\
C(\theta_I^2) &= \frac{p}{2}v_I(H) + (1-p)\left(\frac{1}{2}v_I(L,0) + \frac{1}{2}v_I(\ell,L)\right) - (1-p)v_I(\ell,L).
\end{aligned}
$$

Roughly speaking, the critical projects are those for which the innovation cost equals the expected future profit increases they generate. To make this notion precise, it is necessary to distinguish between incumbents and entrants; moreover, we differentiate between cases when firms are expecting the competitor to invest in the same project and when they are not. Accordingly, project θ_i^1 is defined by the requirement that its cost equals the expected value increase to firm i if it invests in the correct project when the other firm does not. Since project costs are increasing in θ, this implies that firm i would want to invest in any $\theta \in [0, \theta_i^1)$ for which it assumes that the competitor does not invest in, and it would not want to invest in any $\theta \in (\theta_i^1, 1)$ in which it believes the competitor is not investing. Similarly, θ_i^2 is defined by the requirement that its cost equals the expected value increase to firm i if it invests in a correct project in which the other firm invests as well. If firm i believes that the competitor is going to invest in some project in $\theta \in [0, \theta_i^2)$, then it wants to invest in this project as well; similarly, it does not want to invest in any $\theta \in (\theta_i^2, 1)$ if it believes the competitor invests in this project. These observations will help us to determine the best reply of firm i to $r_j(\theta)$ at any project θ, based on the location of θ relative to θ_i^1 and θ_i^2. This will be a crucial ingredient of the equilibrium analysis.

To sum up, the incumbent and the entrant play a multi-stage game, which has the following stages under a *laissez-faire policy* (A).

1. **Investment stage:** Nature determines the correct project, and whether the innovation is drastic (H) or non-drastic (L). Simultaneously with the move of nature, firms invest in research projects. Thereafter all uncertainty is resolved. If only one firm discovers the innovation, it receives the patent on the underlying technology (L or H). If both firms discover the innovation, the patent is allocated randomly with equal probability. If neither firm discovers the innovation, neither firm receives the patent. Interim technology states t_I^{int} and t_E^{int} are realized.

2. **Acquisition stage:** The firms negotiate an acquisition, which takes place if and only if it strictly increases total payoffs. If there is an acquisition, the incumbent pays the entrant the foregone market profits less the commercialization costs, plus a share β of the bargaining surplus.

3. **Commercialization stage:** The firm holding the patent (if any) decides whether to commercialize the technology, thereby incurring costs κ. At the end of the commercialization stage, the firms' final technology states t_I^{fin} and t_E^{fin} are realized.

[18] Obviously, for any equilibrium (r_I, r_E), any pair of functions $(\tilde{r}_I, \tilde{r}_E)$ which only differ from (r_I, r_E) on a set of measure zero also is an equilibrium. We omit the necessary "almost everywhere" qualifications from the statements of our formal results for ease of exposition.

4. **Market stage:** The incumbent and the entrant receive profits $\pi(t_I^{fin}, t_E^{fin})$ and $\pi(t_E^{fin}, t_I^{fin})$, respectively. Total payoffs result from subtracting potential investment and commercialization costs and adding/subtracting potential acquisition payments.

We compare the outcome of this game with a set-up corresponding to a *no-acquisition policy (N)* that prevents the incumbent from acquiring the entrant. This alternative does not contain the acquisition stage, whereas all other stages remain as before.

4 Investments under the Laissez-Faire Policy

We now analyze investments in the laissez-faire case. In Section 4.1, we provide some auxiliary results. In Section 4.2, we characterize the equilibrium investment. Section 4.3 discusses how policy influences innovation behavior.

4.1 Auxilliary Results

We begin by summarizing the result of the acquisition subgame emerging after the realization of the interim technology states.

Lemma 1 (Acquisitions)**.** *Under the laissez-faire policy, the incumbent acquires the entrant if and only if the latter holds a patent for technology L. In the commercialization subgame, if the entrant holds the patent to either technology L or technology H, he commercializes it. The incumbent always commercializes technology H, while she commercializes technology L if and only if* $\pi(L, 0) - \pi(\ell, 0) \geq \kappa$.

Intuitively, if the entrant has access to technology L, an acquisition increases total profits by eliminating competition, whereas it leaves profits unaffected otherwise. The incumbent's commercialization decision depends on the value of the non-drastic innovation. If $\pi(L, 0) - \pi(\ell, 0) < \kappa$, commercialization is not worthwhile — the only motive for an acquisition is the elimination of competition. If $\pi(L, 0) - \pi(\ell, 0) \geq \kappa$, the incumbent additionally benefits from a better technology.

Using Lemma 1, we obtain firm values after the realization of innovation outcomes.

Lemma 2 (Values)**.** *Consider the laissez-faire policy:*
(i) The entrant's values after the realization of the innovation outcomes are

$$v_E(H) = \pi(H) - \kappa$$
$$v_E(L, \ell) = \pi(L, \ell) - \kappa + \beta\big(\max\{\pi(L, 0) - \kappa, \pi(\ell, 0)\} - \pi(L, \ell) - \pi(\ell, L) + \kappa\big)$$
$$v_E(0, t_I) = 0 \text{ for } t_I \in \{\ell, L, H\}.$$

(ii) The incumbent's values after the realization of the innovation outcomes are

$$v_I(H) = \pi(H) - \kappa$$
$$v_I(L, 0) = \max\{\pi(L, 0) - \kappa, \pi(\ell, 0)\}$$
$$v_I(\ell, L) = v_I(L, 0) - v_E(L, \ell)$$
$$v_I(\ell, 0) = \pi(\ell, 0)$$
$$v_I(\ell, H) = 0.$$

The values involving technology L require an explanation. After a non-drastic innovation by the entrant, $(t_I^{int}, t_E^{int}) = (\ell, L)$. The incumbent then acquires the entrant, so that $v_E(L, \ell)$ is the acquisition price, which consists of the entrant's stand-alone profit and his share of the acquisition surplus. $v_I(\ell, L)$ is the monopolist's stand-alone profit, net of the acquisition price. Finally, the *max*-operators take into account the difference between the commercialization and non-commercialization case. On the basis of Lemma 2, we can now discuss the ordering of critical projects (introduced in Section 3), which is essential for the equilibrium properties.

Lemma 3. *Under laissez-faire, the only possible relations between the critical projects are:*

(i) $\theta_I^1 \leq \theta_I^2 = \theta_E^2 < \theta_E^1$*;*

(ii) $\theta_I^2 = \theta_E^2 < \theta_I^1 < \theta_E^1$*;*

(iii) $\theta_I^2 = \theta_E^2 < \theta_E^1 \leq \theta_I^1$*.*

Relation (iii) can only arise in the case with commercialization.

Lemma 3 reveals some common properties of all equilibria. First, the projects which the incumbent is willing to duplicate (i.e., invest in if the entrant also does) are exactly those which the entrant is willing to duplicate as well; we thus write $\theta^2 := \theta_I^2 = \theta_E^2$.[19] Second, $\theta_E^2 < \theta_E^1$, so that the entrant is always willing to invest in a larger range of projects if he is the sole innovator than if the incumbent also invests in these projects. Intuitively, the incumbent's investment reduces the entrant's probability of receiving a patent.

All orderings of the critical values that are compatible with these two conditions are consistent with Lemma 3. However, there is a crucial difference between the cases with and without commercialization. While all three orderings can arise in the former case, $\theta_I^1 < \theta_E^1$ must hold in the no-commercialization case, so that only the first two orderings are possible. Intuitively, conditional on the other firm not investing, the entrant is willing to invest in more expensive projects than the incumbent. This is due to the well-known Arrow replacement effect: An L innovation does not increase incumbent profits, and her profit increase from the H innovation is lower than the entrant's, since without the innovation the entrant receives zero profits. Hence, the entrant's willingness to pay to be the sole innovator is greater than the incumbent's. This will be important for our result that prohibiting acquisitions has a negative effect on equilibrium investments.

Contrary to the no-commercialization case, the incumbent's critical project θ_I^1 may lie above the entrant's critical project θ_E^1 in the commercialization case, as in ordering (iii). This requires technology L to be sufficiently lucrative for the incumbent, so that the prospect of discovering it provides a large investment incentive. Moreover, an entrant's gain from competing with technology L has to be small and his bargaining power low.[20]

[19] To understand why, note that if a project in which both firms invest delivers an H technology, both firms receive the same expected net payoff from investing, because not investing means losing the high innovation to the rival and receiving 0 for sure rather than obtaining the high monopoly profit with probability 1/2. If a project delivers an L technology instead, the entrant gains the acquisition price with probability 1/2 by investing, while the incumbent saves the acquisition price with probability 1/2 by investing. Thus, the expected benefits of investing (conditional on the other firm investing) are the same for entrants and incumbents.

[20] A simple comparison of the definition of $C(\theta_I^1)$ and $C(\theta_E^1)$ shows that the case in Lemma 3(iii) occurs if and only if $(1-p)v_I(L,0) - v_I(\ell,0) \geq (1-p)v_E(L,\ell)$. Using Lemma 2, this expression can be rewritten as $(1-p)\left[(1-\beta)(\pi(L,0) - \pi(L,\ell)) + \beta\pi(\ell,L)\right] \geq \pi(\ell,0)$ in the case with commercialization. Hence, this case occurs when p and β are small (that is, an L-innovation is likely and the incumbent captures most of the bargaining surplus), $\pi(L,0)$ is large and $\pi(\ell,0)$ is small (so that L constitutes a significant innovation, even if it is a non-drastic one) and when $\pi(L,\ell)$ is small (that is, competition is intense).

4.2 Equilibrium Investments

We now provide a full characterization of the equilibrium R&D investments. The result will show that both firms invest in all sufficiently cheap projects, but none of the firms invests in the most expensive projects. Moreover, it will describe how the investment functions for intermediate cost levels depend on which of the three orderings in Lemma 3 applies.

Proposition 1 (Equilibrium R&D investment). *In any equilibrium under laissez-faire,*

$$\begin{aligned}&(a)\ \ r_E(\theta)=1 \text{ and } r_I(\theta)=1 \text{ for } \theta\in[0,\theta^2],\\ &(b)\ \ r_E(\theta)=0 \text{ and } r_I(\theta)=0 \text{ for } \theta\in(\max\{\theta_E^1,\theta_I^1\},1).\end{aligned}$$

(i) If $\theta_I^1 \le \theta^2 < \theta_E^1$, then there exists a unique equilibrium. In addition to (a) and (b), this equilibrium satisfies $r_E(\theta)=1$ and $r_I(\theta)=0$ for $\theta\in(\theta^2,\theta_E^1]$.

(ii) If $\theta^2 < \theta_I^1 < \theta_E^1$, the equilibrium is not unique. A strategy profile is an equilibrium if and only if it satisfies (a) and (b) as well as (c) and (d) below:

$$\begin{aligned}&(c)\ \ r_E(\theta)=1 \text{ and } r_I(\theta)=0 \text{ for } \theta\in(\theta_I^1,\theta_E^1]\\ &(d)\ \ \text{for any } \theta\in(\theta^2,\theta_I^1] \text{ either:}\\ &\quad r_E(\theta)=1 \text{ and } r_I(\theta)=0, \text{ or}\\ &\quad r_E(\theta)=0 \text{ and } r_I(\theta)=1, \text{ or}\\ &\quad r_E(\theta)=\tfrac{C(\theta_I^1)-C(\theta)}{C(\theta_I^1)-C(\theta^2)} \text{ and } r_I(\theta)=\tfrac{C(\theta_E^1)-C(\theta)}{C(\theta_E^1)-C(\theta^2)}.\end{aligned}$$

(iii) If $\theta^2 < \theta_E^1 \le \theta_I^1$, the equilibrium is not unique. A strategy profile is an equilibrium if and only if it satisfies (a) and (b) as well as (e) and (f) below:

$$\begin{aligned}&(e)\ \ r_E(\theta)=0 \text{ and } r_I(\theta)=1, \text{ for } \theta\in(\theta_E^1,\theta_I^1]\\ &(f)\ \ \text{for any } \theta\in(\theta^2,\theta_E^1] \text{ either:}\\ &\quad r_E(\theta)=1 \text{ and } r_I(\theta)=0, \text{ or}\\ &\quad r_E(\theta)=0 \text{ and } r_I(\theta)=1, \text{ or}\\ &\quad r_E(\theta)=\tfrac{C(\theta_I^1)-C(\theta)}{C(\theta_I^1)-C(\theta^2)} \text{ and } r_I(\theta)=\tfrac{C(\theta_E^1)-C(\theta)}{C(\theta_E^1)-C(\theta^2)}.\end{aligned}$$

We will refer to equilibria with $r_E(\theta)\in\{0,1\}$ and $r_I(\theta)\in\{0,1\}\ \forall\theta\in[0,1)$ as *simple equilibria*. Proposition 1 implies that a simple equilibrium exists for any choice of parameters. In particular, it arises in the cases with and without commercialization. In case (i), which is depicted in the left plot of Figure 1, both firms invest fully (that is, with $r_i=1$) in all projects in the interval $[0,\theta^2]$, while only the entrant invests in the projects in $(\theta^2,\theta_E^1]$. Neither firm invests in projects in $(\theta_E^1,1)$. In case (ii), this simple equilibrium coexists with infinitely many other (simple and non-simple) equilibria, reflecting the fact that in any project in $[\theta^2,\theta_I^1)$ each firm only wants to invest if the other one does not. As a result, in any equilibrium either only one of the firms invests fully in the project whereas the other one does not invest at all, or both firms invest with intensity between 0 and 1. The middle plot of Figure 1 shows an equilibrium where both choose intermediate investment intensities in the interval $(\theta^2,\theta_I^1]$.

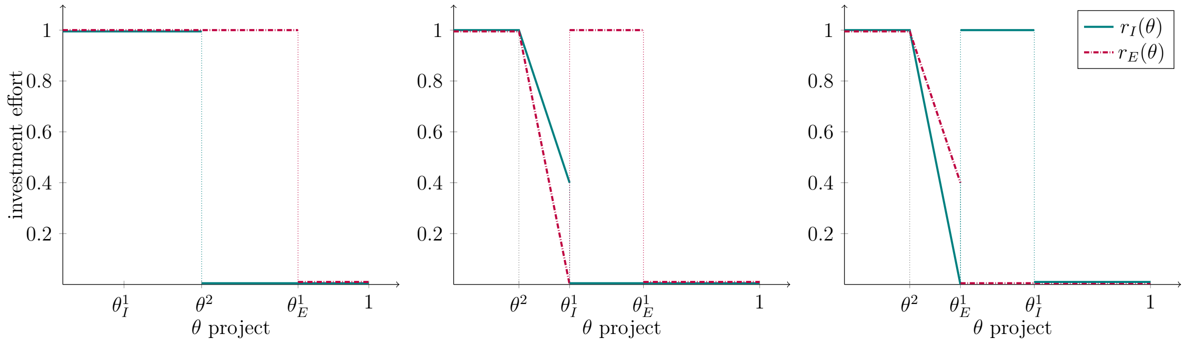

Figure 1: Equilibrium portfolio of entrant and incumbent for the three cases of Proposition 1: Case (i) in the left, case (ii) in the middle and case (iii) in the right plot.

In the no-commercialization case, only the equilibrium constellations described under (i) and (ii) can arise. In the commercialization case, the incumbent has additional investment incentives coming from the possibility of increasing the monopoly profit with a non-drastic innovation. Nevertheless, if $\theta_I^1 < \theta_E^1$ as without commercialization, the equilibrium structure is the same. In particular, the entrant's critical project θ_E^1 is the most costly one that is pursued in equilibrium. However, the possibility that the critical project θ_I^1 of the incumbent lies above the critical project θ_E^1 of the entrant has repercussions for the equilibrium structure. The right plot of Figure 1, which corresponds to Proposition 1(iii), shows one potential equilibrium when $\theta_I^1 \geq \theta_E^1$. As depicted in the figure, in all equilibria in this last case, the incumbent's critical project is the most costly one pursued.

To understand the role of Proposition 1, note that in any equilibrium there exists a set of projects in which only the firm with higher θ_i^1 invests. Even if this firm decided not to invest in these projects, which are the most costly among those pursued, the competitor would not replace the rival's investments. In the no-commercialization case (where (i) or (ii) applies), decreasing the entrant's innovation incentives will cause him to reduce investment *in exactly the projects which cost the most and which only he would pursue*. In the case with commercialization, this logic no longer applies in case (iii), as the incumbent now is the one whose critical project θ_I^1 is the most costly one pursued.

4.3 The Determinants of Innovation Strategies

We now investigate the drivers of equilibrium R&D strategies in the laissez-faire situation, which arguably corresponds to the status quo in most jurisdictions. This helps to identify various policy levers that can influence innovation behavior and outcomes.

We are primarily interested in the policy effects on innovation probability, but this probability is sensitive to equilibrium selection. For this reason, we introduce a closely related proxy, namely the variety of research projects pursued in equilibrium. Formally, given any two research strategies r_I and r_E, we define variety as

$$\mathcal{V}(r_I, r_E) = \int_0^1 \mathbf{1}(r_I(\theta) + r_E(\theta) > 0) d\theta.$$

Thus, variety captures the size of the set of projects in which at least one firm invests a positive amount. The probability that at least one firm discovers an innovation is

$$\mathcal{P}(r_I, r_E) = \int_0^1 \big(r_I(\theta) + r_E(\theta) - r_I(\theta) r_E(\theta)\big) d\theta.$$

The next result, which follows immediately from Proposition 1, shows that variety is a useful proxy for probability: it is invariant to equilibrium selection, it provides an upper bound to probability of innovation in any equilibrium and it is actually equal to the probability in any simple equilibrium.[21]

Corollary 1. *Under a laissez-faire policy, if (r_I, r_E) is an equilibrium, then $\mathcal{V}(r_I, r_E) = \max\{\theta_E^1, \theta_I^1\} \geq \mathcal{P}(r_I, r_E)$. If (r_I, r_E) is a simple equilibrium, then $\mathcal{V}(r_I, r_E) = \mathcal{P}(r_I, r_E)$.*

Thus, we can use Proposition 1 to understand how a marginal parameter change affects variety and innovation probability:

Proposition 2 (Comparative statics)**.** *Consider any equilibrium (r_I, r_E) under a laissez-faire policy.*

(i) Variety $\mathcal{V}(r_I, r_E)$ is (a) weakly increasing in the bargaining power of the entrant β; it is (b) weakly decreasing in the incumbent's profits $\pi(\ell, L)$, but (c) weakly increasing in the entrant's profits $\pi(L, \ell)$ under competition.

(ii) The effects in (i) are strict if $\theta_I^1 < \theta_E^1$ and they are zero if $\theta_I^1 > \theta_E^1$.

To see the intuition, first consider the no-commercialization case. There, we know that $\theta_I^1 < \theta_E^1$ and thus the entrant's innovation incentives determine variety. Result (a) follows strictly because an increase in β makes the innovation more valuable to the entrant. According to (b) and (c), the firms' duopoly profits affect variety in opposite directions. An increase in the incumbent's duopoly profit decreases the acquisition surplus, but leaves the entrant's outside option unaffected, so that variety decreases when the incumbent's duopoly profit increases. By contrast, the entrant's competition profit increases his outside option, but decreases the acquisition surplus. Since he only receives a share β of the acquisition surplus, the former effect dominates. In the commercialization case, the intuition is analogous, except that the parameters do not affect variety if $\theta_E^1 < \theta_I^1$, as they do not influence the incumbent's critical project θ_I^1.

For simple equilibria, Proposition 2 directly shows how the parameters affect the probability of discovering an innovation. The result suggests several policy levers which can be used to promote innovation. First, by (a), strengthening the bargaining position of the entrant fosters variety. One practical way to achieve this is to make it easier or less costly for the entrant to enforce his IP rights. Second, by (b) and (c), any policy which increases the entrant's duopoly profits and lowers those of the incumbent has a positive effect on innovation. This suggests that stricter competition policy, which makes it harder for incumbents to abuse dominance, can foster innovation. Moreover, this result provides support for innovation policies targeting small firms such as preferential R&D tax credits and subsidized loans (see Bloom et al., 2019). While the usual rationale for such policies is that small firms are more likely to face financial constraints, our analysis shows that, even in the absence of such constraints, targeting small firms can foster innovation by increasing the variety of R&D projects which are pursued in equilibrium.

[21] Recall that simple equilibria always exist under laissez-faire. Furthermore, one can show that *only* simple equilibria exist with an alternative time structure where the incumbent moves first.

5 Prohibiting Acquisitions

We now analyze the effects of prohibiting start-up acquisitions. In Section 5.1, we show that such a policy reduces the equilibrium project variety and the probability of innovation. Section 5.2 analyzes how the size of this negative effect depends on the market environment. In Section 5.3, we discuss R&D duplication. Throughout the section, we denote the critical values under the laissez-faire and no-acquisition policies as $\theta_i^k(A)$ and $\theta_i^k(N)$, $k \in \{1, 2\}$, respectively.

5.1 The Effects on Variety

Firm behavior in the commercialization and market stages remains unchanged when acquisitions are prohibited. By Lemma 1, such a policy affects the outcome only when the entrant has a non-drastic innovation. Analogously to the laissez-faire case, in any equilibrium (r_I^N, r_E^N) of the no-acquisition regime the firms invest in all projects below $\max\{\theta_E^1(N), \theta_I^1(N)\}$, but in no other projects.[22] Hence, variety in this regime is given by $\mathcal{V}^N = \max\{\theta_E^1(N), \theta_I^1(N)\}$. Since by Corollary 1 variety in any laissez-faire equilibrium is $\mathcal{V}^A = \max\{\theta_E^1(A), \theta_I^1(A)\}$, the size of the policy effect on variety is $\Delta_{\mathcal{V}} := \mathcal{V}^A - \mathcal{V}^N = \max\{\theta_E^1(A), \theta_I^1(A)\} - \max\{\theta_E^1(N), \theta_I^1(N)\}$. Our next result characterizes the sign of this effect.

Proposition 3. *Consider the no-acquisition policy.*

(i) In any equilibrium, (a) the variety of research projects is weakly smaller than in any equilibrium under laissez-faire and (b) the probability of an innovation is weakly smaller than in any simple equilibrium under laissez-faire.

(ii) The inequalities in (i) are strict, except that there is no effect on variety in the case with commercialization if $\theta_E^1(A) \leq \theta_I^1(A)$.

Proposition 3 shows that a restrictive acquisition policy never increases variety. However, (ii) highlights a crucial difference between the cases with and without commercialization. While the policy effect is strictly negative in the killer acquisitions case, it may be zero in the case with commercialization.

Two simple observations are critical for the intuition. First, $\theta_E^1(N) < \theta_E^1(A)$: Intuitively, prohibiting acquisitions reduces the entrant's expected payoff from R&D investments, since he cannot sell the firm when it would be profitable to do so. Second, $\theta_I^1(A) = \theta_I^1(N) =: \theta_I^1$: If the entrant does not invest in the correct project, there will be no reason to acquire him, so that the policy regime is irrelevant for θ_I^1. Only three possible orderings for θ_I^1 and the entrant's critical projects $\theta_E^1(A)$ and $\theta_E^1(N)$ are compatible with these two observations:

(I) $\theta_I^1 < \theta_E^1(N) < \theta_E^1(A)$

(II) $\theta_E^1(N) \leq \theta_I^1 < \theta_E^1(A)$

(III) $\theta_E^1(N) < \theta_E^1(A) \leq \theta_I^1$.

[22] We provide a full characterization of the equilibria under the no-acquisition policy in Propositions A.2 and A.3 in Appendix A.4.

When (I) or (II) applies, $\theta_E^1(A)$, which reflects the entrant's incentives, determines the equilibrium variety under laissez-faire. A ban on acquisitions weakens these incentives and therefore reduces variety to $\theta_E^1(N)$ under ordering (I) or to θ_I^1 under (II). Figure 2(I) and 2(II) illustrate these two cases, respectively. When (III) applies, θ_I^1 determines the equilibrium variety in both policy regimes. Hence, as illustrated in Figure 2(III), a prohibition of acquisitions has no effect. Importantly, in the case without commercialization, ordering (I) always applies, so that the policy effect is strict in this case.

Furthermore, since in any equilibrium $\mathcal{P}(r_I, r_E) \leq \mathcal{V}(r_I, r_E)$, while in any *simple* equilibrium $\mathcal{P}(r_I, r_E) = \mathcal{V}(r_I, r_E)$, the statement in Proposition 3 on innovation probabilities immediately follows from the effect on variety.

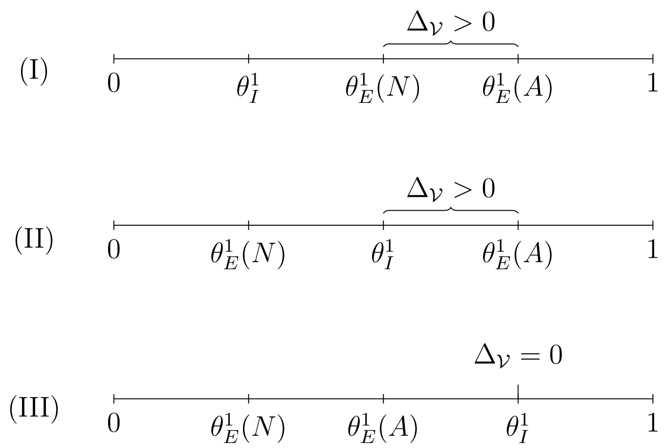

Figure 2: The effect of prohibiting acquisitions on project variety.

5.2 The Size of the Effect on Variety

As an input into our subsequent welfare analysis, we analyze how the market environment determines the size of the innovation-reducing effect of restricting acquisitions. In particular, our results highlight the importance of bargaining power and the intensity of competition in the market.

Proposition 4. *Consider any equilibrium under a laissez-faire policy (r_I^A, r_E^A) and any equilibrium under the no-acquisition policy (r_I^N, r_E^N).*

(i) The size of the policy effect $\Delta_{\mathcal{V}}$ is (a) weakly increasing in entrant bargaining power β, (b) weakly decreasing in the incumbent's profits under competition $\pi(\ell, L)$ and (c) strictly decreasing in the entrant's profits under competition $\pi(L, \ell)$ if $\theta_I^1 < \theta_E^1(N)$, but weakly increasing if $\theta_E^1(N) < \theta_I^1$.

(ii) The effects in (i) are strict if $\theta_I^1 < \theta_E^1(A)$ and they are zero if $\theta_I^1 > \theta_E^1(A)$.

This central result identifies the circumstances under which the innovation effect is important. To understand it, recall that in both policy regimes the variety of research projects is determined by the most expensive project some firm is willing to invest in, so that $\Delta_{\mathcal{V}} = \max\{\theta_E^1(A), \theta_I^1\} - \max\{\theta_E^1(N), \theta_I^1\}$. Thus, the effect of a parameter on the loss of variety is equivalent to its effect on the difference between these critical projects.

An increase in the entrant's bargaining power β increases his share of the acquisition surplus and thus his payoff in case of an acquisition. Hence, it increases $\theta_E^1(A)$. However, the change affects neither $\theta_E^1(N)$ (since acquisitions are not allowed) nor θ_I^1 (since there is no acquisition if the entrant does not innovate). Combining these observations, for orderings (I) and (II), an increase in β strictly increases $\Delta_{\mathcal{V}}$, as it increases $\theta_E^1(A)$ without affecting $\theta_E^1(N)$ and θ_I^1. For ordering (III), an increase in β has no effect, as it does not change θ_I^1.[23]

Next, an increase in the incumbent's profits under competition $\pi(\ell, L)$ neither affects $\theta_E^1(N)$ nor θ_I^1, but it reduces the acquisition surplus and therefore decreases $\theta_E^1(A)$. The overall effect is a strict reduction in $\Delta_{\mathcal{V}}$ for orderings (I) and (II), and no effect for ordering (III). Finally, the effect of an increase in the entrant's duopoly profit $\pi(L, \ell)$ is more subtle. $\pi(L, \ell)$ increases both $\theta_E^1(A)$ and $\theta_E^1(N)$, but the increase is greater for $\theta_E^1(N)$.[24] For ordering (I), the overall effect is therefore a strict decrease in $\Delta_{\mathcal{V}}$. Since $\pi(L, \ell)$ does not affect θ_I^1, this implies a strict increase in $\Delta_{\mathcal{V}}$ for ordering (II) and no effect for (III).

As the case without commercialization satisfies ordering (I), where the effects are strict, whereas ordering (III) may arise only with commercialization, these arguments again highlight the importance of distinguishing these two cases.

To summarize, Proposition 4 shows how the loss of variety depends on bargaining power and the intensity of potential competition as captured by duopoly profits. This result is a useful ingredient in the welfare analysis, as it identifies circumstances in which competition authorities can implement a more restrictive acquisition policy without substantial negative effects on innovation. However, the welfare analysis remains incomplete without discussing the effects of the market environment on consumer surplus, an issue to which we return in Section 6.1.

5.3 The Effect on Duplication

The acquisition policy not only affects variety and thereby the probability of innovation, but also the firms' incentives to duplicate research projects. Contrary to the laissez-faire case, duopolistic competition arises after a non-drastic innovation of the entrant. This affects the critical values θ_I^2 and θ_E^2.

Corollary 2. *(i)* $\theta_I^2(N) > \theta^2(A)$ *and (ii)* $\theta^2(A) > \theta_E^2(N)$.

Thus, prohibiting acquisitions increases the incumbent's duplication incentives and decreases those of the entrant. Intuitively, (i) if the entrant invests in a project, the incumbent gains more from duplicating it under a no-acquisition policy than under laissez-faire: Without the acquisition option, own investments that duplicate entrant's research are the only means of preventing competitive entry. As to the entrant, (ii) duplicating the incumbent's investments is less attractive under the no-acquisition policy than under laissez-faire because of the absence of prospective gains from selling the firm. We discuss the complex net effects of these policy reactions in Proposition B.1 in Online Appendix

[23] This argument, and the one in the next paragraph, applies when orderings (II) and (III) are strict. When θ_I^1 is equal to one of the entrant's critical projects, the matters are more subtle, but the intuition is similar. See the proof for details.

[24] The reason for this is that when acquisitions are allowed, an increase in $\pi(L, \ell)$ increases the entrant's outside option, but decreases the acquisition surplus. This countervailing effect is absent when acquisitions are not allowed, leading to a larger overall increase in $\theta_E^1(N)$.

B.1. If $\theta_I^2(N) \leq \theta_I^1$, then the negative policy effect on the entrant's incentives dominates and there is less duplication under the no-acquisition policy. If $\theta_I^1 < \theta_I^2(N)$, this conclusion only holds if the entrant's bargaining power β is sufficiently high. The reason is that his reaction only dominates if the entrant is relatively more affected by the policy compared to the incumbent. In turn, if β is low, the incumbent is relatively more affected and thus overall duplication increases.

To summarize, the fact that we are investigating investment portfolios rather than just overall investment efforts allow us to identify the effects of a ban on start-up acquisitions on the duplication incentives of incumbents and entrants. Because the no-acquisition policy affects duplication, a negative effect of the policy on innovation probability may go hand in hand with a positive effect on R&D effort.

6 Discussion and Further Results

We now provide additional results and discuss the robustness of our findings. Section 6.1 deals with consumer surplus effects. In Section 6.2, we compare our analysis with a more standard model where firms cannot target specific innovation projects. Finally, in Section 6.3 we show that our analysis is robust to various changes in the modelling assumptions.

6.1 Consumer Surplus Effects

We now ask under which circumstances the well-known positive competition effect of prohibiting acquisitions dominates the negative innovation effect from a consumer perspective. We focus on the case without commercialization.[25]

We denote consumer surplus when the entrant competes with technology L against the incumbent as $S(\ell, L)$, and as $S(t)$ for a monopoly with technology $t \in \{\ell, H\}$.[26] We assume that $S(H) > S(\ell, L) > S(\ell)$. Thus, consumers prefer the high-state monopoly to the duopoly, which they prefer to the low-state monopoly in turn. We denote the probability of a duopoly in policy regime R as $prob^R(\ell, L)$ and the probability of a monopoly with technology $t \in \{\ell, H\}$ as $prob^R(t)$.[27] Then, the expected consumer surplus under laissez-faire is:

$$prob^A\left(H\right)S\left(H\right) + prob^A\left(\ell\right)S\left(\ell\right).$$

Under the no-acquisition policy, the expected consumer surplus is:

$$prob^N\left(H\right)S\left(H\right) + prob^N\left(\ell, L\right)S\left(\ell, L\right) + prob^N\left(\ell\right)S\left(\ell\right).$$

The following result gives a simple condition under which the competition effect dominates the innovation effect from a consumer perspective.

[25]For the case with commercialization, such an analysis is not necessary for $\theta_E^1(A) \leq \theta_I^1(A)$, because then there is no innovation effect by Proposition 3. If $\theta_E^1(A) > \theta_I^1(A)$, the analysis and the insights for the cases with and without commercialization are similar. However, since the decomposition of the welfare effect is more involved in the former case, we focus on the killer acquisition case.

[26]Note that, while only the incumbent can be a monopolist with technology ℓ, both incumbent and entrant may end up with an H monopoly in both regimes.

[27]Note that these probabilities follow directly from the equilibrium innovation strategies (r_I, r_E), characterized in Propositions 1, A.2 and A.3.

Proposition 5. *Suppose the no-commercialization case applies. Prohibiting start-up acquisitions increases the expected consumer surplus if and only if*

$$prob^N(\ell, L)\left[S(\ell, L) - S(\ell)\right] > \left[prob^A(H) - prob^N(H)\right]\left[S(H) - S(\ell)\right].$$

The proposition illustrates the countervailing effects of prohibiting acquisitions. On the one hand, the policy measure introduces desirable competition (and potentially better technology) with probability $prob^N(\ell, L)$, leading to a competitive surplus $S(\ell, L)$ rather than the non-competitive surplus $S(\ell)$. On the other hand, the measure reduces the probability of a drastic innovation (which would increase consumer surplus from $S(\ell)$ to $S(H)$) by $prob^A(H) - prob^N(H)$. Note that $S(H) - S(\ell)$ depends on the size of the drastic innovation and, closely related, on its effect on demand, whereas $S(\ell, L) - S(\ell)$ captures the consumer value of duopolistic competition. Both terms are independent of the firms' investment decisions. By contrast, $prob^N(\ell, L)$ is the product of the entrant's endogenous innovation probability under the no-acquisition policy and the conditional probability $1-p$ that this innovation is non-drastic. $prob^A(H) - prob^N(H)$ is the product of the effect of the acquisition policy on the probability of an innovation success (see Section 4) and the conditional probability p that an innovation is drastic.

These general considerations lead to some insights into the determinants of the consumer surplus effect. Assuming that the effect on probability corresponds to the effect on variety (see the discussion of Proposition 3(b)), an increase in the entrant's bargaining power β increases $prob^A(H) - prob^N(H)$ and thus the adverse innovation effect of a restrictive acquisition policy; there is no such effect when $\beta = 0$.[28] Therefore, a restrictive acquisition policy will always be justified for sufficiently low bargaining power of the entrant, but not necessarily when this bargaining power increases.

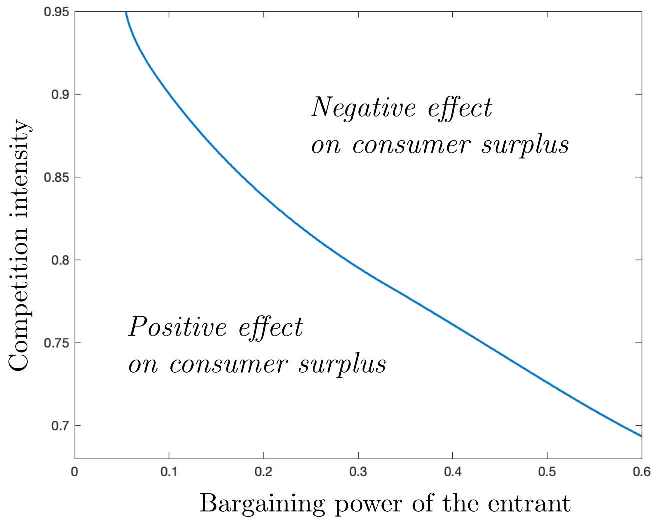

Figure 3: Effect of prohibiting acquisitions on consumer surplus based on a parameterized example of Bertrand competition with heterogeneous goods (See Online Appendix B.4).

By contrast, whether prohibiting acquisitions increases or decreases consumer surplus depends on product market competition in an ambiguous way. According to Proposition

[28] Remember that the extent to which the policy induces desirable competition only depends on the entrant's innovation probability under no-acquisition, which is independent of β.

4, in the case without commercialization an exogenous reduction in the entrant's duopoly profits $\pi(L,\ell)$ tends to increase the size of the adverse innovation effect. However, such a change in the market environment may reflect more intense competitive interaction between the firms and therefore a higher consumer surplus $S(\ell, L)$ relative to the monopoly case. Thus whether a reduction in the entrant's duopoly profits makes a positive consumer surplus effect of prohibiting acquisitions more or less likely is not clear without considering special parameterized models. Similar arguments apply to the incumbent's duopoly profits.

We analyze these ambiguities in a standard heterogeneous Bertrand model with linear demand (à la Shapley-Shubik). Figure 3 shows that, in line with our comparative statics result for the killer acquisition case (see Proposition 4), prohibiting acquisitions has a positive effect on consumer surplus only if the bargaining power of the entrant is small and competition intensity on the product market is not too intense.[29]

Our focus on consumer surplus in this welfare discussion reflects the common practice of many competition agencies. That said, extending the analysis beyond this welfare standard may well be interesting. For instance, the discussion of duplication in Section 5.3 suggests further channels by which the acquisition policy can affect welfare.

6.2 One-dimensional Innovation Efforts

We now briefly discuss an alternative setting where firms can only choose total innovation efforts rather than which projects to invest in (see Appendix B.2 for details). This one-dimensional model differs from our main model only in how innovation probabilities are determined. We assume that both firms exert R&D effort, which determines the probability of innovation success independently across firms. The analysis of the acquisition and commercialization stages applies as before. As the incumbent acquires the entrant only if the latter has discovered a non-drastic innovation, the effect of prohibiting acquisitions is driven by the differences in firm values in this situation.

For similar reasons as in our main model, prohibiting acquisitions reduces the entrant's investment incentives. Conversely, prohibiting acquisitions increases the incumbent's investment incentives, as she can no longer use acquisitions to avoid competition. Thus, she has a higher incentive to block the entrant by own investments. The effect of prohibiting acquisitions on innovation probability thus depends on the relative magnitudes of changes in the firms' incentives and can either be positive or negative.

This ambiguous effect results from the restrictive model structure and in particular the assumption that firms cannot affect the correlation between the outcomes of their R&D activities. The only way the incumbent can decrease the probability that the entrant receives the patent in this simplified model is to increase overall R&D spending. This, in turn, *by assumption* leads to an increase in the overall probability that the innovation is discovered. As our previous analysis shows, this relationship does not necessarily have to hold. If firms increase their investments by duplicating the efforts of other firms, then the probability that the innovation is discovered does not necessarily increase.

[29] Here, the intensity of competition corresponds to the degree of substitution between the goods, with higher intensity (i.e. higher substitutability between goods) leading to lower duopoly profits. The details of the model and our calculations can be found in Appendix B.4.

6.3 Robustness

We now show the robustness of our results with respect to uncertainty about the entrant's innovation level, asymmetries between firms as well as multiple entrants. Appendix B.3 contains formal results and proofs.

6.3.1 Innovation Uncertainty at the Time of Acquisition

In our model, before entering acquisition negotiations, both firms know whether the innovation is drastic or not. In practice, it may often be difficult to evaluate the start-up's technology level. Extensive testing may be necessary to identify cost savings or quality improvements. In this section, we show that the policy effects remain similar if the technology level of an innovation is uncertain at the time of the acquisition.

We maintain the setting of Section 3, but assume that only the correct project is revealed at the end of the investment stage, not its technology level. Thus, before the acquisition stage, interim technology states $(t_I^{int}, t_E^{int}) \in \{(0,0),(0,1),(1,0)\}$ are realized, where 1 indicates that the firm received a patent and 0 indicates that it did not. After the acquisition stage, the technology level of the correct project is realized as L or H. Thereafter, firms decide on commercialization, before the final technology states $(t_I^{fin}, t_E^{fin}) \in \mathcal{T}$ are realized. Everything else remains as before. Proposition B.2 in Appendix B.3.1 shows that, irrespective of the policy regime, uncertainty does not affect equilibrium investments and thus does not change the policy effect. However, uncertainty does influence the frequency of acquisitions. The incumbent will acquire the entrant irrespective of the technology level of the latter's innovation because the expected surplus at the time is positive, since it is a convex combination of a positive acquisition surplus in case of the L technology and no acquisition surplus in case of the H technology.

6.3.2 Asymmetric Chances of Receiving Patents

We now show that the variety of pursued investment projects is invariant to the assumption that, if both firms discover an innovation, they each have an equal chance to receive the patent. Let the probability of receiving the patent (when both firms discover an innovation) be $\rho_I \in (0,1)$ for the incumbent and thus $(1-\rho_I)$ for the entrant.[30] Proposition B.3 in Appendix B.3.2 shows that, regardless of ρ_I, banning acquisitions reduces the variety of pursued research projects and thereby the probability that an innovation will be discovered. Furthermore, the size of the policy effect is independent of ρ_I. Therefore, the results on the relation between parameters and the size of the policy effect identified in Proposition 4 are also robust to changes in ρ_I. This result holds because ρ_I matters only when both firms discover an innovation. Thus, it affects duplication incentives, but not the incentives to invest in projects in which the competitor is not investing. Since variety is given by $\max\{\theta_E^1, \theta_I^1\}$, it is not affected by ρ_I in either policy regime, so that the size of the policy effect does not depend on ρ_I.

6.3.3 Heterogeneous Commercialization Costs

Throughout the paper, we have assumed that both firms would face the same commercialization cost κ. However, due to a better infrastructure or a more developed sales network,

[30] The main model corresponds to $\rho_I = 1/2$.

the incumbent might be able to commercialize the innovation at a lower cost. To capture this possibility, we denote the commercialization costs of the incumbent and the entrant with κ_I and κ_E, respectively, where $\kappa_I < \kappa_E$. Adjusting Assumption 2, we assume that (i) $\pi(L,\ell) \geq \kappa_E$ and $\pi(H) - \pi(\ell,0) \geq \kappa_I$. We focus on the no-commercialization case, so that $\pi(L,0) - \pi(\ell,0) < \kappa_I$. We add the innocuous assumption that $\pi(L,\ell) \leq \pi(L,0)$, which requires that a monopolist with an L technology obtains market profits at least as high as a firm with L technology which competes with a firm with technology ℓ.[31] Generalizing Proposition 3, Proposition B.4 in Appendix B.3.3 shows that banning acquisitions reduces the variety of research projects, which tends to reduce the innovation probability. Moreover, in this setting, a prohibition of acquisitions results in an additional inefficiency, as it forces the entrant to commercialize the technology using the cost κ_E instead of letting the incumbent commercialize it at the lower cost κ_I.

6.3.4 Multiple Entrants

We argue briefly, without going into details of equilibrium existence and characterization, that the effects of a restrictive acquisition policy on innovation do not change substantially when there are multiple entrants, even though the analysis becomes more complex. We focus on the case without commercialization, assuming there are two entrants.

Compared with the main model, the analysis changes mainly because the firms need to take into account the possibility that two (potential) competitors invest in some project, which reduces the probability of obtaining a patent. To capture the willingness to invest in such projects, we define critical projects θ_i^3 in a similar way as θ_i^1 and θ_i^2. Clearly, $\theta_i^3 < \theta_i^2$, reflecting the lower probability of obtaining a patent when three rather than two firms invest. Crucially, however, the number of entrants does not affect the critical values θ_i^1 and θ_i^2. Therefore, the highest critical value is still θ_E^1, no matter which policy regime applies. Moreover, in any equilibrium of the game, for any project $\theta \leq \theta_E^1$ there must exist at least one firm investing a positive amount in this project: Otherwise one of the entrants could profitably deviate by investing a positive amount. Thus, as in the main model, the entrants' critical projects determine variety. Therefore, the policy effect on variety remains the same with multiple entrants as with a single entrant.[32]

7 Conclusion

Recently, there has been an intense debate on the interactions between mergers and innovation, with particular emphasis on start-up acquisitions. Motivated by this debate, our paper provides a theory of the strategic choice of innovation projects by incumbents and start-ups which allows for endogenous acquisition and commercialization decisions.

Very generally, both firms invest as much as possible into low-cost projects, whereas neither invests at all in high-cost projects. For projects with intermediate costs, at least one firm invests. This structure is independent of whether acquisitions are allowed or not and whether the incumbent commercializes the entrant's innovation after an acquisition or not. We find that prohibiting start-up acquisitions weakly reduces the variety of research

[31] We do not rely on this natural assumption in the main model, which is why we only add it here.

[32] One difference is that, with multiple entrants, equilibria cannot be unique: For projects just below θ_E^1, both entrants will want to invest if and only if no other firm has invested.

projects pursued and thereby the probability of discovering innovations, and it may induce the incumbent to strategically duplicate projects of the entrant to prevent competition.

Our analysis reveals conditions under which a restrictive acquisition policy is called for. It turns out that the negative innovation effect of prohibiting acquisitions may well be absent for innovations with sufficient commercialization potential. Even for less attractive innovations that the incumbent would not want to commercialize, the adverse innovation effects may be negligible if the entrant has low bargaining power and the incumbent's duopoly profits are high, so that the competition-enhancing effect of prohibiting acquisitions is likely to dominate in this case.

While our analysis covers several interesting aspects of start-up acquisitions, it leaves some issues untouched. For instance, we focus on incumbents' takeovers of would-be entrants into a market where the incumbent is already present. Our analysis does not directly apply to the equally interesting case where an incumbent in one market acquires a start-up that has recently entered a related market which the incumbent cannot serve with his existing technology.

Moreover, our approach focuses on the short run policy effects. Going beyond our static model, acquisitions with commercialization might give rise to concerns that arise only in the longer term. For instance, rather than merely killing a potential entrant, the incumbent can combine the knowledge of the two firms to expand its technological lead. This is likely to restrict potential competition in the long term by reducing incentives for innovation.[33] It would be interesting to analyze how incumbents and potential entrants target their innovation activities when entry can take place repeatedly and the incumbent's technology improves as a result of acquisitions. Is increasing dominance of the incumbent an inevitable outcome? Will the innovation process eventually slow down because it becomes too hard for entrants to compete? While these questions are beyond the scope of the current paper, our analysis suggests that to answer them it would be expedient to take the policy effects on project choice into account, rather than only the effects on the overall innovation level.

[33] This argument is reminiscent of Cabral (2018).

A Appendix

This appendix provides proofs of our formal results as well as a full description of equilibria under the policy (Propositions A.2 and A.3). It is organized as follows. Section A.1 provides the proof of Lemma 1, which describes the equilibrium of the acquisition game. Section A.2 collects the results on the order of critical projects. It contains the proof of Lemma 3, as well as the statement and proof of Lemma A.1, which characterizes the order of critical projects under the policy. With these results in place, we then characterize every equilibrium for each conceivable constellation of critical projects (see Section A.3). Section A.4 provides the statement of Propositions A.2 and A.3. The proofs of Propositions 1, A.2 and A.3 can be found in Section A.5. These proofs are straightforward implications of the results in Section A.3. Finally, Section A.6 collects all the remaining proofs (of Propositions 2, 3, 4 and 5).

A.1 Proof of Lemma 1

Consider first the commercialization subgame. The entrant commercializes a technology if the payoff from doing so is at least zero. Since $\pi(L,\ell) \geq \kappa$ by Assumption 2(i) and $\pi(H) \geq \kappa$ by Assumptions 1(i) and 2(ii), the entrant commercializes both technologies. The incumbent commercializes a technology if the payoff of doing so is at least $\pi(\ell, 0)$. Since $\pi(H) - \kappa \geq \pi(\ell, 0)$ by Assumption 2(ii), the incumbent always commercializes the H technology. The incumbent commercializes the L technology if and only if $\pi(L, 0) - \pi(\ell, 0) \geq \kappa$.

Now consider the acquisitions subgame. There are three possible cases. Either the entrant holds no patent, or he holds the H patent or the L patent. We will examine the three cases in turn. First, suppose that the entrant holds no patent. Then, since the entrant cannot compete without an innovation, the incumbent's profits are the same with or without the acquisition. Thus, the incumbent has no reason to acquire the entrant. Second, suppose the entrant holds a patent on the H technology. Without an acquisition, the entrant commercializes the technology and obtains the payoff $\pi(H) - \kappa$ while the incumbent obtains $\pi(\ell, H) = 0$. With the acquisition, the incumbent commercializes the technology and obtains the payoff $\pi(H) - \kappa$. Thus the total payoffs are equal with or without the acquisition. Since the acquisition (by assumption) only goes through if the total payoffs strictly increase, the incumbent does not acquire the entrant. Third, consider the case when the entrant has a patent for the L technology. If there is no acquisition, the entrant commercializes the technology and obtains payoffs $\pi(L, \ell) - \kappa$, while the incumbent's payoffs are $\pi(\ell, L)$. If the incumbent acquires the entrant and commercializes the technology, she obtains $\pi(L, 0) - \kappa$, while without commercialization she obtains $\pi(\ell, 0)$. Thus she will choose to commercialize only if $\pi(L, 0) - \kappa \geq \pi(\ell, 0)$. The incumbent's payoff is $\max\{\pi(L, 0) - \kappa, \pi(\ell, 0)\}$, while the entrant obtains a payoff of zero. Consequently, the acquisition surplus is positive if and only if $\max\{\pi(L, 0) - \kappa, \pi(\ell, 0)\} > \pi(L, \ell) + \pi(\ell, L) - \kappa$. We can add κ to both sides of the inequality and use Assumption 1(iv) to show that this inequality indeed holds:

$$\max\{\pi(L, 0), \pi(\ell, 0) + \kappa\} \geq \max\{\pi(L, 0), \pi(\ell, 0)\} > \pi(L, \ell) + \pi(\ell, L).$$

A.2 Characterization of the order of critical projects

A.2.1 Proof of Lemma 3

The result will follow immediately from Steps 1 and 2 below.
Step 1: (a) $\theta_I^2 = \theta_E^2$ *and (b)* $\theta_E^2 < \theta_E^1$.

(a) To prove this statement, note that $v_I(H) = v_E(H)$. Thus

$$\begin{aligned} C(\theta_I^2) &= \frac{1}{2}\left[pv_E(H) + (1-p)\left(v_I(L,0) - v_I(\ell,L)\right)\right] \\ &= \frac{1}{2}\left[pv_E(H) + (1-p)\left(v_I(L,0) - \left(v_I(L,0) - v_E(L,\ell)\right)\right)\right] \\ &= \frac{1}{2}\left[pv_E(H) + (1-p)v_E(L,\ell)\right] = C(\theta_E^2) \end{aligned}$$

(b) Since $C(\theta_E^2) < C(\theta_E^1)$, part (b) of Step 1 follows immediately.
Step 2: *In the case without commercialization, only* $\theta_I^1 < \theta_E^1$ *is possible.*

To see this, note that in cases without commercialization, $\max\{\pi(L,0) - \kappa, \pi(\ell,0)\} = \pi(\ell,0)$ has to hold, so that $v_I(L,0) = v_I(\ell,0)$. Then $\theta_I^1 < \theta_E^1$ if and only if:

$$\begin{aligned} C(\theta_I^1) &< C(\theta_E^1) \\ pv_I(H) + (1-p)v_I(L,0) - v_I(\ell,0) &< pv_E(H) + (1-p)v_E(L,\ell) \\ (1-p)v_I(L,0) - v_I(\ell,0) &< (1-p)v_E(L,\ell) \\ -pv_I(\ell,0) &< (1-p)v_E(L,\ell), \end{aligned}$$

which always holds.

A.2.2 Critical Projects under a No-acquisition Policy

Lemma A.1. *Consider the no-acquisition policy.*

(i) In the case without commercialization, the following relations hold:
(a) $\theta_E^2 < \theta_I^2$*; (b)* $\theta_E^2 < \theta_E^1$*; (c)* $\theta_I^1 < \theta_E^1$.

(ii) In the case with commercialization, the following relations hold:
(a) $\theta_E^2 < \theta_I^2$*; (b)* $\theta_E^2 < \theta_E^1$.

Proof. Note that (a) and (b) are the same in the case with and without commercialization. We prove them without distinguishing between the cases.

(a): Note that $v_E(H) = v_I(H)$. $\theta_E^2 < \theta_I^2$ will hold if and only if:

$$\begin{aligned} C(\theta_E^2) &< C(\theta_I^2) \\ \frac{1}{2}\left(pv_E(H) + (1-p)v_E(L,\ell)\right) &< \frac{1}{2}\left(pv_I(H) + (1-p)(v_I(L,0) - v_I(\ell,L))\right) \\ v_E(L,\ell) &< v_I(L,0) - v_I(\ell,L) \\ \pi(L,\ell) - \kappa &< \max\{\pi(\ell,0), \pi(L,0) - \kappa\} - \pi(\ell,L) \end{aligned}$$

which is satisfied by Assumption 1(iv).

(b): Since $C(\theta_E^2) < C(\theta_E^1)$, it follows immediately that $\theta_E^2 < \theta_E^1$.

For (c), we restrict attention to the case without commercialization.

(c): Consider the case without commercialization. The claim will hold if and only if

$$\begin{aligned} C(\theta_I^1) &< C(\theta_E^1) \\ pv_I(H) + (1-p)v_I(L,0) - v_I(\ell,0) &< pv_E(H) + (1-p)v_E(L,\ell) \\ -pv_I(\ell,0) &< (1-p)v_E(L,\ell) \end{aligned}$$

where the equivalence of the last two lines follows from $v_I(L,0) = v_I(\ell,0)$, which always holds in the case without commercialization, and Assumption 1(i). □

A.3 Characterization of Equilibrium Behavior

As an immediate implication of Lemmas 3 and A.1, we find two relations which hold in all cases we consider.

Corollary A.1. *Irrespective of policy, the following relations hold:*

(i) $\theta_E^1 > \theta_E^2$

(ii) $\theta_I^2 \geq \theta_E^2$.

Next, for all orderings which satisfy the conditions of Corollary A.1, we characterize equilibrium choices $r_i(\theta)$ on the basis of the relation of that project θ to the critical projects $\theta_E^1, \theta_I^1, \theta_E^2$ and θ_I^2.

Proposition A.1. *Any equilibrium must satisfy (a)-(f) below. If (a)-(f) all hold, the investment functions* $r_E(\theta)$ *and* $r_I(\theta)$ *can be sustained as an equilibrium.*

(a) $r_E(\theta) = 1$ *and* $r_I(\theta) = 1$ *whenever* $\theta \in [0, \theta_E^2]$

(b) $r_E(\theta) = 0$ *and* $r_I(\theta) = 0$ *whenever* $\theta \in (\max\{\theta_I^1, \theta_E^1\}, 1)$

(c) $r_E(\theta) = 1$ *and* $r_I(\theta) = 0$ *whenever* $\theta \in (\max\{\theta_I^2, \theta_I^1\}, \theta_E^1]$

(d) $r_E(\theta) = \frac{C(\theta_I^1)-C(\theta)}{C(\theta_I^1)-C(\theta_I^2)}$ *and* $r_I(\theta) = \frac{C(\theta_E^1)-C(\theta)}{C(\theta_E^1)-C(\theta_E^2)}$ *whenever* $\theta \in (\max\{\theta_I^1, \theta_E^2\}, \min\{\theta_E^1, \theta_I^2\}]$.

(e) Either (i) $r_E(\theta) = 1$ *and* $r_I(\theta) = 0$ *or (ii)* $r_E(\theta) = 0$ *and* $r_I(\theta) = 1$ *or (iii)* $r_E(\theta) = \frac{C(\theta_I^1)-C(\theta)}{C(\theta_I^1)-C(\theta_I^2)}$ *and* $r_I(\theta) = \frac{C(\theta_E^1)-C(\theta)}{C(\theta_E^1)-C(\theta_E^2)}$ *holds whenever* $\theta \in (\theta_I^2, \min\{\theta_I^1, \theta_E^1\}]$.

(f) The equilibrium satisfies $r_E(\theta) = 0$ *and* $r_I(\theta) = 1$ *in all remaining cases.*

Proof. (a) Projects in this interval are (weakly) profitable for the entrant irrespective of the behavior of the incumbent since $\theta \leq \theta_E^2 < \theta_E^1$ by Corollary A.1(i). Given that the entrant invests, investing is also profitable for the incumbent, as $\theta \leq \theta_I^2$ by Corollary A.1(ii). Consequently, investment behavior on this interval is consistent with an equilibrium if and only if $r_E(\theta) = 1$ and $r_I(\theta) = 1$.

(b) Projects in this interval are never profitable for the entrant irrespective of the behavior of the incumbent since $\theta_E^2 < \theta_E^1 < \theta$ by Corollary A.1(i). As the entrant does not invest, investment is not profitable for the incumbent as $\theta > \theta_I^1$.

(c) In this interval, it is a unique best response of the incumbent not to invest irrespective of the investment of the entrant. Therefore, using $\theta \leq \theta_E^1$, it is always a unique best response of the entrant to choose $r_I(\theta) = 1$.

(d) By now straightforward considerations, there can be no equilibrium where firms choose zero or maximal investment (on a positive measure of projects). Thus, any equilibrium must involve firms choosing strictly interior equilibrium efforts. However, since project payoffs are linear in effort, interior equilibrium efforts can only be sustained when each firm makes the rival exactly indifferent between investing and not investing into a given project. The incumbent is indifferent if and only if

$$\begin{aligned}(1 - r_E(\theta))v_I(\ell, 0) + r_E(\theta)(1 - p)v_I(\ell, L) = \\ -C(\theta) + (1 - r_E(\theta))\left[pv_I(H) + (1 - p)v_I(L, 0)\right] \\ + r_E(\theta)\frac{1}{2}\left[pv_I(H) + (1 - p)\left(v_I(L, 0) + v_I(\ell, L)\right)\right]\end{aligned}$$

where the LHS represents the incumbent's project payoff from investing 0 and the RHS represents the incumbent's project payoff from investing 1.

Using the definitions of $C(\theta_I^1)$ and $C(\theta_I^2)$ and solving for $r_E(\theta)$, we arrive at $r_E(\theta) = \frac{C(\theta) - C(\theta_I^1)}{C(\theta_I^2) - C(\theta_I^1)}$, which is the unique solution to the above equation. We can proceed analogously to arrive at $r_I(\theta) = \frac{C(\theta_E^1) - C(\theta)}{C(\theta_E^1) - C(\theta_E^2)}$. Moreover, these investment levels are feasible, since they are between 0 and 1 for every $\theta \in (\max\{\theta_I^1, \theta_E^2\}, \min\{\theta_I^2, \theta_E^1\}]$. Thus, effort levels r_E and r_I for each project θ in this interval are consistent with equilibrium behavior if and only if they are defined as in (d).

(e) It is simple to show that the strategies delineated in (i) and (ii) constitute equilibrium behavior and that no other equilibrium with minimal or maximal effort choices exists. Proceeding as in (d), we find that the strategy in (iii) is also consistent with equilibrium behavior for θ in this interval, and there is no other strategy with interior effort levels for which this is the case.

(f) In (a)-(e), we have shown that, if θ lies in the given interval for each of the cases, we arrive at the respective equilibrium behavior for project θ.

We now show that in all remaining cases one of the following must hold:

(i) $\theta \in (\theta_E^2, \min\{\theta_I^1, \theta_I^2\}]$ and $\min\{\theta_I^1, \theta_I^2\} < \theta_E^1$

(ii) $\theta \in (\max\{\theta_I^2, \theta_E^1\}, \theta_I^1]$

(iii) $\theta \in (\theta_E^2, \theta_I^1]$ and $\min\{\theta_I^1, \theta_I^2\} \geq \theta_E^1$

All equilibria satisfy (a) and (b), but which ones of the remaining cases apply in the interval $(\theta_E^2, \max\{\theta_I^1, \theta_E^1\}$ depends on the exact order of critical projects. We will thus consider each case (c)-(f) in turn and show that, if there are still intervals not covered, they fall into at least one of the listed cases:

Assuming case (c) occurs, we need to characterize the possible constellations in the interval $(\theta_E^2, \max\{\theta_I^2, \theta_I^1\}]$. $(\min\{\theta_I^2, \theta_I^1\}, \max\{\theta_I^2, \theta_I^1\}]$ corresponds to case (d) if $\theta_I^1 \leq \theta_I^2$ and to case (e) if $\theta_I^1 > \theta_I^2$. Thus, we are left with the interval $(\theta_E^2, \min\{\theta_I^1, \theta_I^2\}]$, which is case (i) above.

Assuming case (d) occurs, we need to characterize the possible constellations in the intervals $(\theta_E^2, \max\{\theta_I^1, \theta_E^2\}]$ and $(\min\{\theta_E^1, \theta_I^2\}, \theta_E^1]$. Since $(\min\{\theta_E^1, \theta_I^2\}, \theta_E^1]$ only has positive measure if $\theta_I^2 < \theta_E^1$, the interval falls into case (c). $(\theta_E^2, \max\{\theta_I^1, \theta_E^2\}]$ only has positive measure if $\theta_E^2 < \theta_I^1$, and then the interval corresponds to case (i) above.

Assuming case (e) occurs, we need to characterize the possible constellations in the intervals $(\theta_E^2, \theta_I^2]$ and $(\min\{\theta_I^1, \theta_E^1\}, (\max\{\theta_I^1, \theta_E^1\}]$. For the second interval, if $\theta_I^1 < \theta_E^1$, we are in case (c) and if $\theta_I^1 \geq \theta_E^1$, we are in case (ii) above. $(\theta_E^2, \theta_I^2]$ corresponds to case (i) above.

Cases (c), (d) and (e) all require $\min\{\theta_I^1, \theta_I^2\} < \theta_E^1$. Assuming that $\min\{\theta_I^1, \theta_I^2\} \geq \theta_E^1$ implies that neither (c), (d) or (e) occurs. Case (iii) above therefore covers the whole interval $(\theta_E^2, \theta_I^1]$. Moreover, if $\min\{\theta_I^1, \theta_I^2\} < \theta_E^1$, at least one of the three cases, (c), (d) or (e), occurs and thus there are no cases left to consider.

Having established that we identified the remaining cases, we can use arguments that are standard by now to show that efforts in each of those cases are consistent with equilibrium behavior if and only if $r_E(\theta) = 0$ and $r_I(\theta) = 1$. □

A.4 Equilibria under a No-acquisition Policy

We now state equilibrium characterizations for the no-acquisition policy (first for the case without commercialization, then for the case with commercialization). The proofs are in Section A.5.

Proposition A.2 (Without commercialization)**.** *Consider the case without commercialization under a no-acquisition policy.*

(a) If $\theta_I^2 \geq \theta_I^1$, then $\max\{\theta_I^1, \theta_E^2\} \leq \min\{\theta_I^2, \theta_E^1\}$. The equilibrium is unique. Functions r_I and r_E constitute an equilibrium if and only if conditions (i)-(v) hold:

(i) $r_E(\theta) = 1$ *and* $r_I(\theta) = 1$, *for* $\theta \in [0, \theta_E^2]$

(ii) $r_E(\theta) = 0$ *and* $r_I(\theta) = 1$, *for* $\theta \in (\theta_E^2, \max\{\theta_I^1, \theta_E^2\}]$

(iii) $r_E(\theta) = \frac{C(\theta)-C(\theta_I^1)}{C(\theta_I^2)-C(\theta_I^1)}$ *and* $r_I(\theta) = \frac{C(\theta_E^1)-C(\theta)}{C(\theta_E^1)-C(\theta_E^2)}$, *for* $\theta \in (\max\{\theta_I^1, \theta_E^2\}, \min\{\theta_I^2, \theta_E^1\}]$,

(iv) $r_E(\theta) = 1$ *and* $r_I(\theta) = 0$, *for* $\theta \in (\min\{\theta_I^2, \theta_E^1\}, \theta_E^1]$,

(v) $r_E(\theta) = 0$ *and* $r_I(\theta) = 0$, *for* $\theta \in (\theta_E^1, 1)$.

(b) If $\theta_I^2 < \theta_I^1$ then $\theta_E^2 < \theta_I^2 < \theta_I^1 < \theta_E^1$. The equilibrium is not unique. Functions r_I and r_E constitute an equilibrium if and only if conditions (i)-(v) hold:

(i) $r_E(\theta) = 1$ *and* $r_I(\theta) = 1$, *for* $\theta \in [0, \theta_E^2]$

(ii) $r_E(\theta) = 0$ *and* $r_I(\theta) = 1$, *for* $\theta \in (\theta_E^2, \theta_I^2]$

(iii) $r_E(\theta) = 1$ *and* $r_I(\theta) = 0$, *for* $\theta \in (\theta_I^1, \theta_E^1]$,

(iv) $r_E(\theta) = 0$ *and* $r_I(\theta) = 0$, *for* $\theta \in (\theta_E^1, 1)$.

(v) for any $\theta \in (\theta_I^2, \theta_I^1]$ *either:*

$r_E(\theta) = 1$ *and* $r_I(\theta) = 0$, *or*

$r_E(\theta) = 0$ *and* $r_I(\theta) = 1$, *or*

$r_E(\theta) = \frac{C(\theta_I^1)-C(\theta)}{C(\theta_I^1)-C(\theta_I^2)}$ *and* $r_I(\theta) = \frac{C(\theta_E^1)-C(\theta)}{C(\theta_E^1)-C(\theta_E^2)}$.

Proposition A.3 (With commercialization)**.** *Consider the case with commercialization under a no-acquisition policy.*

(a) *If $\theta_I^2 \geq \theta_I^1$, then $\max\{\theta_I^1, \theta_E^2\} \leq \min\{\theta_I^2, \theta_E^1\}$. The equilibrium is unique. Functions r_I and r_E constitute an equilibrium if and only if conditions (i)-(v) hold:*

(i) $r_E(\theta) = 1$ *and* $r_I(\theta) = 1$, *for* $\theta \in [0, \theta_E^2]$

(ii) $r_E(\theta) = 0$ *and* $r_I(\theta) = 1$, *for* $\theta \in (\theta_E^2, \max\{\theta_I^1, \theta_E^2\}]$

(iii) $r_E(\theta) = \frac{C(\theta)-C(\theta_I^1)}{C(\theta_I^2)-C(\theta_I^1)}$ *and* $r_I(\theta) = \frac{C(\theta_E^1)-C(\theta)}{C(\theta_E^1)-C(\theta_E^2)}$, *for* $\theta \in (\max\{\theta_I^1, \theta_E^2\}, \min\{\theta_I^2, \theta_E^1\}]$,

(iv) $r_E(\theta) = 1$ *and* $r_I(\theta) = 0$, *for* $\theta \in (\min\{\theta_I^2, \theta_E^1\}, \theta_E^1]$,

(v) $r_E(\theta) = 0$ *and* $r_I(\theta) = 0$, *for* $\theta \in (\theta_E^1, 1)$.

(b) *If $\theta_I^2 < \theta_I^1 < \theta_E^1$, then $\theta_E^2 < \theta_I^2 < \theta_I^1 < \theta_E^1$. The equilibrium is not unique. Functions r_I and r_E constitute an equilibrium if and only if conditions (i)-(v) hold:*

(i) $r_E(\theta) = 1$ *and* $r_I(\theta) = 1$, *for* $\theta \in [0, \theta_E^2]$

(ii) $r_E(\theta) = 0$ *and* $r_I(\theta) = 1$, *for* $\theta \in (\theta_E^2, \theta_I^2]$

(iii) $r_E(\theta) = 1$ *and* $r_I(\theta) = 0$, *for* $\theta \in (\theta_I^1, \theta_E^1]$,

(iv) $r_E(\theta) = 0$ *and* $r_I(\theta) = 0$, *for* $\theta \in (\theta_E^1, 1)$.

(v) *for any* $\theta \in (\theta_I^2, \theta_I^1]$ *either:*

$r_E(\theta) = 1$ *and* $r_I(\theta) = 0$, *or*

$r_E(\theta) = 0$ *and* $r_I(\theta) = 1$, *or*

$r_E(\theta) = \frac{C(\theta_I^1)-C(\theta)}{C(\theta_I^1)-C(\theta_I^2)}$ *and* $r_I(\theta) = \frac{C(\theta_E^1)-C(\theta)}{C(\theta_E^1)-C(\theta_E^2)}$.

(c) *If $\theta_I^2 < \theta_E^1 \leq \theta_I^1$, then $\theta_E^2 < \theta_I^2 < \theta_E^1 \leq \theta_I^1$. The equilibrium is not unique. Functions r_I and r_E constitute an equilibrium if and only if conditions (i)-(v) hold:*

(i) $r_E(\theta) = 1$ *and* $r_I(\theta) = 1$, *for* $\theta \in [0, \theta_E^2]$

(ii) $r_E(\theta) = 0$ *and* $r_I(\theta) = 1$, *for* $\theta \in (\theta_E^2, \theta_I^2]$

(iii) $r_E(\theta) = 0$ *and* $r_I(\theta) = 1$, *for* $\theta \in (\theta_E^1, \theta_I^1]$,

(iv) $r_E(\theta) = 0$ *and* $r_I(\theta) = 0$, *for* $\theta \in (\theta_I^1, 1)$.

(v) *for any* $\theta \in (\theta_I^2, \theta_E^1]$ *either:*

$r_E(\theta) = 1$ *and* $r_I(\theta) = 0$, *or*

$r_E(\theta) = 0$ *and* $r_I(\theta) = 1$, *or*

$r_E(\theta) = \frac{C(\theta_I^1)-C(\theta)}{C(\theta_I^1)-C(\theta_I^2)}$ *and* $r_I(\theta) = \frac{C(\theta_E^1)-C(\theta)}{C(\theta_E^1)-C(\theta_E^2)}$.

(d) *If $\theta_E^1 \leq \min\{\theta_I^2, \theta_I^1\}$, then $\theta_E^2 < \theta_E^1 \leq \min\{\theta_I^1, \theta_I^2\}$. The equilibrium is unique. Functions r_I and r_E constitute an equilibrium if and only if conditions (i)-(iii) hold:*

(i) $r_E(\theta) = 1$ *and* $r_I(\theta) = 1$, *for* $\theta \in [0, \theta_E^2]$

(ii) $r_E(\theta) = 0$ *and* $r_I(\theta) = 1$, *for* $\theta \in (\theta_E^2, \theta_I^1]$

(iii) $r_E(\theta) = 0$ *and* $r_I(\theta) = 0$, *for* $\theta \in (\theta_I^1, 1)$.

A.5 Proofs of Equilibrium Characterizations

A.5.1 Proof of Proposition 1

According to Lemma 3, in the laissez-faire regime one of the following constellations applies:

$$\text{(i) } \theta_I^1 \leq \theta^2 < \theta_E^1 \qquad \text{(ii) } \theta^2 < \theta_I^1 < \theta_E^1 \qquad \text{(iii) } \theta^2 < \theta_E^1 \leq \theta_I^1.$$

Applying Proposition A.1 to each constellation immediately gives the result.

A.5.2 Proof of Proposition A.2

According to Lemma A.1(a), under the no-acquisition policy in the case without commercialization, one of the following five constellations applies:

$$\begin{array}{lll} \text{(i) } \theta_I^1 \leq \theta_E^2 < \theta_I^2 \leq \theta_E^1 & \text{(ii) } \theta_I^1 \leq \theta_E^2 < \theta_E^1 < \theta_I^2 & \text{(iii) } \theta_E^2 < \theta_I^1 \leq \theta_I^2 \leq \theta_E^1 \\ \text{(iv) } \theta_E^2 < \theta_I^1 < \theta_E^1 < \theta_I^2 & \text{(v) } \theta_E^2 < \theta_I^2 < \theta_I^1 < \theta_E^1. & \end{array}$$

Applying Proposition A.1 to each constellation immediately gives the result.

A.5.3 Proof of Proposition A.3

According to Lemma A.1(b), under the no-acquisitions policy in the case with commercialization, one of the following constellations applies:

$$\begin{array}{lll} \text{(i) } \theta_I^1 \leq \theta_E^2 < \theta_I^2 \leq \theta_E^1 & \text{(ii) } \theta_I^1 \leq \theta_E^2 < \theta_E^1 < \theta_I^2 & \text{(iii) } \theta_E^2 < \theta_I^1 \leq \theta_I^2 \leq \theta_E^1 \\ \text{(iv) } \theta_E^2 < \theta_I^1 \leq \theta_E^1 < \theta_I^2 & \text{(v) } \theta_E^2 < \theta_I^2 < \theta_I^1 \leq \theta_E^1 & \text{(vi) } \theta_E^2 < \theta_E^1 < \theta_I^1 \leq \theta_I^2 \\ \text{(vii) } \theta_E^2 < \theta_E^1 \leq \theta_I^2 < \theta_I^1 & \text{(viii) } \theta_E^2 < \theta_I^2 < \theta_E^1 < \theta_I^1. & \end{array}$$

Applying Proposition A.1 to each constellation immediately gives the result.

A.6 Other Proofs

A.6.1 Proof of Proposition 2

By Corollary 1, $\mathcal{V}(r_I, r_E) = \max\{\theta_E^1, \theta_I^1\}$ and since C is a strictly increasing function, the sign of the effect of any parameter x on $\mathcal{V}(r_I, r_E)$ will be equal to the sign of its effect on $\max\{C(\theta_E^1), C(\theta_I^1)\}$.

Case 1: Suppose $C(\theta_I^1) > C(\theta_E^1)$ (and hence $\theta_I^1 > \theta_E^1$). Since $C(\theta_I^1) = p(\pi(H) - \kappa) + (1-p)\max\{\pi(L,0) - \kappa, \pi(\ell,0)\} - \pi(\ell,0)$, which does not depend on β, $\pi(\ell, L)$ or $\pi(L,\ell)$, it follows immediately that

$$\partial C(\theta_I^1)/\partial\beta = \partial C(\theta_I^1)/\partial\pi(\ell, L) = \partial C(\theta_I^1)/\partial\pi(L,\ell) = 0.$$

Case 2: Now suppose $C(\theta_I^1) < C(\theta_E^1)$ (and hence $\theta_I^1 < \theta_E^1$). By definition $C(\theta_E^1) = pv_E(H) + (1-p)v_E(L,\ell)$ and by Lemma 2, we can substitute $v_E(H)$ and $v_E(L,\ell)$ so that

$$C(\theta_E^1) = p(\pi(H)-\kappa)+(1-p)(\beta(\max\{\pi(L,0)-\kappa, \pi(\ell,0)\}-\pi(\ell,L))+(1-\beta)(\pi(L,\ell)-\kappa)).$$

We examine how $C(\theta_E^1)$ changes with respect to parameters β, $\pi(L,\ell)$ and $\pi(\ell, L)$ as follows:

(a)

$$\partial C(\theta_E^1)/\partial\beta =$$
$$\frac{\partial}{\partial\beta}\Big(p(\pi(H)-\kappa)+(1-p)(\beta(\max\{\pi(L,0)-\kappa,\pi(\ell,0)\}-\pi(\ell,L))+$$
$$(1-\beta)(\pi(L,\ell)-\kappa))\Big) =$$
$$(1-p)(\max\{\pi(L,0)-\kappa,\pi(\ell,0)\}-\pi(\ell,L)-\pi(L,\ell)+\kappa) > 0,$$

where the inequality follows from Assumption 1(iv).

(b)

$$\partial C(\theta_E^1)/\partial\pi(\ell,L) =$$
$$\frac{\partial}{\partial\pi(\ell,L)}\Big(p(\pi(H)-\kappa)+(1-p)(\beta(\max\{\pi(L,0)-\kappa,\pi(\ell,0)\}-\pi(\ell,L))+$$
$$(1-\beta)(\pi(L,\ell)-\kappa))\Big) =$$
$$-(1-p)\beta < 0.$$

(c)

$$\partial C(\theta_E^1)/\partial\pi(L,\ell) =$$
$$\frac{\partial}{\partial\pi(L,\ell)}\Big(p(\pi(H)-\kappa)+(1-p)(\beta(\max\{\pi(L,0)-\kappa,\pi(\ell,0)\}-\pi(\ell,L))+$$
$$(1-\beta)(\pi(L,\ell)-\kappa))\Big) =$$
$$(1-p)(1-\beta) > 0.$$

Case 3: Finally, suppose $C(\theta_I^1) = C(\theta_E^1)$.

(a) Take β_0, such that $C(\theta_I^1(\beta_0)) = C(\theta_E^1(\beta_0))$ (and hence $\theta_I^1(\beta_0) = \theta_E^1(\beta_0)$). Denoting with ∂_- and ∂_+ the left and the right derivative (at β_0), respectively, as well as using Step 1 and 2 above we get:

$$\frac{\partial_-\mathcal{V}(r_I,r_E)}{\partial\beta} = \frac{\partial C(\theta_I^1)}{\partial\beta} = 0 \quad\text{and}\quad \frac{\partial_+\mathcal{V}(r_I,r_E)}{\partial\beta} = \frac{\partial C(\theta_E^1)}{\partial\beta} > 0.$$

(b) Analogously for $\pi(\ell,L)$:

$$\frac{\partial_-\mathcal{V}(r_I,r_E)}{\partial\pi(\ell,L)} = \frac{\partial C(\theta_E^1)}{\partial\pi(\ell,L)} < 0 \quad\text{and}\quad \frac{\partial_+\mathcal{V}(r_I,r_E)}{\partial\pi(\ell,L)} = \frac{\partial C(\theta_I^1)}{\partial\pi(\ell,L)} = 0.$$

(c) And $\pi(L,\ell)$:

$$\frac{\partial_-\mathcal{V}(r_I,r_E)}{\partial\pi(L,\ell)} = \frac{\partial C(\theta_I^1)}{\partial\pi(L,\ell)} = 0 \quad\text{and}\quad \frac{\partial_+\mathcal{V}(r_I,r_E)}{\partial\pi(L,\ell)} = \frac{\partial C(\theta_E^1)}{\partial\pi(L,\ell)} > 0.$$

A.6.2 Proof of Proposition 3

Denote the equilibrium strategies under laissez-faire and the no-acquisition policy as (r_I^A, r_E^A) and (r_I^N, r_E^N), respectively. The result follows from Steps 1-5.

Step 1: $\mathcal{V}^A = \max\{\theta_E^1(A), \theta_I^1(A)\}$ *and* $\mathcal{V}^N = \max\{\theta_E^1(N), \theta_I^1(N)\}$.

The first claim holds because, by Proposition 1, $r_I^A(\theta) + r_E^A(\theta) = 0$ if and only if $\theta \in (\max\{\theta_E^1(A), \theta_I^1(A)\}, 1)$. Hence, $\mathcal{V}^A = \max\{\theta_E^1(A), \theta_I^1(A)\}$. The second claim holds because, by Propositions A.2 and A.3, $r_I^N(\theta)+r_E^N(\theta) = 0$ if and only if $\theta \in (\max\{\theta_E^1(N), \theta_I^1(N)\}, 1)$.

Step 2: $\theta_I^1(A) = \theta_I^1(N)$.

To show this, it is sufficient that $C(\theta_I^1(A)) = C(\theta_I^1(N))$, or equivalently

$$pv_I^A(H) + (1-p)v_I^A(L, 0) - v_I^A(\ell, 0) = pv_I^N(H) + (1-p)v_I^N(L, 0) - v_I^N(\ell, 0).$$

This holds since $v_I^A(t, 0) = v_I^N(t, 0)$ for all $t \in \{\ell, L, H\}$.

Step 3: $\theta_E^1(N) < \theta_E^1(A)$.

To show this, it is sufficient that $C(\theta_E^1(N)) < C(\theta_E^1(A))$ The claim requires that

$$pv_E^N(H) + (1-p)v_E^N(L, \ell) < pv_E^A(H) + (1-p)v_E^A(L, \ell).$$

This holds because

$$\begin{aligned}
&p(\pi(H) - \kappa) + (1-p)(\pi(L, \ell) - \kappa) < p(\pi(H) - \kappa) + (1-p)v_E^A(L, \ell) \\
&\pi(L, \ell) - \kappa < v_E^A(L, \ell) \\
&\pi(L, \ell) - \kappa < \beta(\max\{\pi(L, 0) - \kappa, \pi(\ell, 0)\} - \pi(\ell, L)) + (1-\beta)(\pi(L, \ell) - \kappa) \\
&\pi(L, \ell) - \kappa < \max\{\pi(L, 0) - \kappa, \pi(\ell, 0)\} - \pi(\ell, L)
\end{aligned}$$

where simple algebra leads to the last inequality, which holds by Assumption 1(iv).

Step 4: *If* $\theta_E^1(A) > \theta_I^1(A)$, *then* $\mathcal{V}^A > \mathcal{V}^N$ *and if* (r_I^A, r_E^A) *is a simple equilibrium then* $\mathcal{P}(r_I^A, r_E^A) > \mathcal{P}(r_I^N, r_E^N)$.

Since $\theta_E^1(A) > \theta_E^1(N)$ by Step 3 and $\theta_I^1(A) = \theta_I^1(N)$ by Step 2, we obtain $\theta_E^1(A) > \max\{\theta_E^1(N), \theta_I^1(N)\}$. Hence, $\mathcal{V}^A > \mathcal{V}^N$. Since $\mathcal{P}(r_I, r_E) \le \mathcal{V}(r_I, r_E)$ for any (r_I, r_E) and $\mathcal{P}(r_I, r_E) = \mathcal{V}(r_I, r_E)$ for simple equilibria, then also $\mathcal{P}(r_I^A, r_E^A) > \mathcal{P}(r_I^N, r_E^N)$ if (r_I^A, r_E^A) is a simple equilibrium.

Step 5: *If* $\theta_E^1(A) \le \theta_I^1(A)$, *then* $\mathcal{V}^A = \mathcal{V}^N$ *and if* (r_I^A, r_E^A) *is a simple equilibrium then* $\mathcal{P}(r_I^A, r_E^A) \ge \mathcal{P}(r_I^N, r_E^N)$.

If $\theta_E^1(A) \le \theta_I^1(A)$, then by Steps 2 and 3, $\theta_E^1(N) < \theta_I^1(N)$. Then $\mathcal{V}^A = \theta_I^1(A) = \theta_I^1(N) = \mathcal{V}^N$. Since $\mathcal{P}(r_I, r_E) \le \mathcal{V}(r_I, r_E)$ for any (r_I, r_E) and $\mathcal{P}(r_I, r_E) = \mathcal{V}(r_I, r_E)$ for simple equilibria, then also $\mathcal{P}(r_I^A, r_E^A) \ge \mathcal{P}(r_I^N, r_E^N)$.

A.6.3 Proof of Proposition 4

Proposition 3(i) implies $\Delta_{\mathcal{V}} = \mathcal{V}^A - \mathcal{V}^N = \max\{\theta_E^1(A), \theta_I^1(A)\} - \max\{\theta_E^1(N), \theta_I^1(N)\} \ge 0$, where $\theta_I^1(A) = \theta_I^1(N) = \theta_I^1$ and $\theta_E^1(A) > \theta_E^1(N)$. We will analyze the change of $\Delta_{\mathcal{V}}$ as a result of a change in β, $\pi(\ell, L)$ and $\pi(L, \ell)$ for all orderings of $\theta_I^1, \theta_E^1(A)$ and $\theta_E^1(N)$. This gives us the following five cases which we will examine in turn:

Case 1: $\theta_I^1 < \theta_E^1(N) < \theta_E^1(A)$,
Case 2: $\theta_E^1(N) < \theta_I^1 < \theta_E^1(A)$,
Case 3: $\theta_E^1(N) < \theta_E^1(A) < \theta_I^1$,
Case 4: $\theta_E^1(N) = \theta_I^1 < \theta_E^1(A)$,
Case 5: $\theta_E^1(N) < \theta_I^1 = \theta_E^1(A)$.

The statement of the proposition aggregates the effects in these five cases.

Case 1: If $\theta_I^1 < \theta_E^1(N) < \theta_E^1(A)$, then $\Delta_{\mathcal{V}} = \theta_E^1(A) - \theta_E^1(N)$. For any parameter x, applying the inverse function theorem gives

$$\frac{\partial(\theta_E^1(A) - \theta_E^1(N))}{\partial x} =$$
$$\frac{\frac{\partial}{\partial x}(p(\pi(H) - \kappa) + (1-p)(\beta(\max\{\pi(L,0) - \kappa, \pi(\ell,0)\} - \pi(\ell,L)) + (1-\beta)(\pi(L,\ell) - \kappa)))}{C'(\theta_E^1(A))}$$
$$- \frac{\frac{\partial}{\partial x}(p(\pi(H) - \kappa) + (1-p)(\pi(L,\ell) - \kappa))}{C'(\theta_E^1(N))}.$$

(*a*) $\partial\Delta_{\mathcal{V}}/\partial\beta > 0$

$$\frac{\partial(\theta_E^1(A) - \theta_E^1(N))}{\partial\beta} = \frac{(1-p)(\max\{\pi(L,0) - \kappa, \pi(\ell,0)\} - \pi(\ell,L) - \pi(L,\ell) + \kappa)}{C'(\theta_E^1(A))} > 0$$

which follows from Assumption 1(iv).

(*b*) $\partial\Delta_{\mathcal{V}}/\partial\pi(\ell,L) < 0$

$$\frac{\partial(\theta_E^1(A) - \theta_E^1(N))}{\partial\pi(\ell,L)} = \frac{-(1-p)\beta}{C'(\theta_E^1(A))} < 0.$$

(*c*) $\partial\Delta_{\mathcal{V}}/\partial\pi(L,\ell) < 0$

$$\frac{\partial(\theta_E^1(A) - \theta_E^1(N))}{\partial\pi(L,\ell)} = \frac{(1-p)(1-\beta)}{C'(\theta_E^1(A))} - \frac{(1-p)}{C'(\theta_E^1(N))} < 0$$
$$\Leftrightarrow (1-\beta) < \frac{C'(\theta_E^1(A))}{C'(\theta_E^1(N))}$$

where the inequality follows from the convexity of C.

Case 2: If $\theta_E^1(N) < \theta_I^1 < \theta_E^1(A)$, then $\Delta_{\mathcal{V}} = \theta_E^1(A) - \theta_I^1$, hence the effect on variety is

$$\frac{\partial(\theta_E^1(A) - \theta_I^1)}{\partial x} =$$
$$\frac{\frac{\partial}{\partial x}(p(\pi(H) - \kappa) + (1-p)(\beta(\max\{\pi(L,0) - \kappa, \pi(\ell,0)\} - \pi(\ell,L)) + (1-\beta)(\pi(L,\ell) - \kappa)))}{C'(\theta_E^1(A))}$$
$$- \frac{\frac{\partial}{\partial x}(p(\pi(H) - \kappa) + (1-p)\max\{\pi(L,0) - \kappa, \pi(\ell,0)\} - \pi(\ell,0))}{C'(\theta_I^1)}.$$

(*a*) $\partial\Delta_{\mathcal{V}}/\partial\beta > 0$

$$\frac{\partial(\theta_E^1(A) - \theta_I^1)}{\partial\beta} = \frac{(1-p)(\max\{\pi(L,0) - \kappa, \pi(\ell,0)\} - \pi(\ell,L) - \pi(L,\ell) + \kappa)}{C'(\theta_E^1(A))} > 0$$

which follows from Assumption 1(iv).

(*b*) $\partial\Delta_{\mathcal{V}}/\partial\pi(\ell, L) < 0$

$$\frac{\partial(\theta_E^1(A) - \theta_I^1)}{\partial\pi(\ell, L)} = \frac{-(1-p)\beta}{C'(\theta_E^1(A))} < 0.$$

(*c*) $\partial\Delta_{\mathcal{V}}/\partial\pi(L, \ell) > 0$

$$\frac{\partial(\theta_E^1(A) - \theta_I^1)}{\partial\pi(L, \ell)} = \frac{(1-p)(1-\beta)}{C'(\theta_E^1(A))} > 0.$$

Case 3: If $\theta_E^1(N) < \theta_E^1(A) < \theta_I^1$, $\Delta_{\mathcal{V}} = 0$ and $\partial\Delta_{\mathcal{V}}/\partial x = 0$ for $x \in \{\beta, \pi(\ell, L), \pi(L, \ell)\}$.

Case 4: If $\theta_E^1(N) = \theta_I^1 < \theta_E^1(A)$, then $\Delta_{\mathcal{V}} = \theta_E^1(A) - \max\{\theta_I^1, \theta_E^1(N)\}$. Provided that the derivative exists, the effect on variety is

$$\frac{\partial(\theta_E^1(A) - \max\{\theta_I^1, \theta_E^1(N)\})}{\partial x}.$$

Note that $\partial\theta_I^1/\partial x = 0$ and $\partial\theta_E^1(N)/\partial x = 0$ for $x \in \{\beta, \pi(\ell, L)\}$, which implies that the derivative exists and $\partial \max\{\theta_I^1, \theta_E^1(N)\}/\partial x = 0$. Therefore, $\partial\Delta_{\mathcal{V}}/\partial\beta = \partial\theta_E^1(A)/\partial\beta > 0$ and $\partial\Delta_{\mathcal{V}}/\partial\pi(\ell, L) = \partial\theta_E^1(A)/\partial\pi(\ell, L) < 0$.

Case 5: If $\theta_E^1(N) < \theta_I^1 = \theta_E^1(A)$, then $\Delta_{\mathcal{V}} = \max\{\theta_I^1, \theta_E^1(A)\} - \theta_I^1$. Since $\partial\theta_I^1/\partial x = 0$ for $x \in \{\beta, \pi(\ell, L), \pi(L, \ell)\}$, the effect on variety is equal to the effect on $\max\{\theta_I^1, \theta_E^1(A)\}$.

First, consider the effect of β. Note that $\partial\theta_E^1(A)/\partial\beta > 0$ and fix β_0 such that $\theta_I^1(\beta_0) = \theta_E^1(A, \beta_0)$. Then for all β' and β'' such that $\beta' < \beta_0 < \beta''$, we have $\theta_I^1(\beta') > \theta_E^1(A, \beta')$ and $\theta_I^1(\beta'') < \theta_E^1(A, \beta'')$. Denoting with ∂_- and ∂_+ the left and the right derivative (at β_0) respectively, the argument above implies

$$\frac{\partial_-\Delta_{\mathcal{V}}}{\partial\beta} = \frac{\partial\theta_I^1}{\partial\beta} = 0 \quad \text{and} \quad \frac{\partial_+\Delta_{\mathcal{V}}}{\partial\beta} = \frac{\partial\theta_E^1(A)}{\partial\beta} > 0.$$

Next, noting that $\partial\theta_E^1(A)/\partial\pi(\ell, L) < 0$ and $\partial\theta_E^1(A)/\partial\pi(L, \ell) > 0$, we analogously obtain

$$\frac{\partial_-\Delta_{\mathcal{V}}}{\partial\pi(\ell, L)} = \frac{\partial\theta_E^1(A)}{\partial\pi(\ell, L)} < 0 \quad \text{and} \quad \frac{\partial_+\Delta_{\mathcal{V}}}{\partial\pi(\ell, L)} = \frac{\partial\theta_I^1}{\partial\pi(\ell, L)} = 0$$

and

$$\frac{\partial_-\Delta_{\mathcal{V}}}{\partial\pi(L, \ell)} = \frac{\partial\theta_I^1}{\partial\pi(L, \ell)} = 0 \quad \text{and} \quad \frac{\partial_+\Delta_{\mathcal{V}}}{\partial\pi(L, \ell)} = \frac{\partial\theta_E^1(A)}{\partial\pi(L, \ell)} > 0.$$

A.6.4 Proof of Proposition 5

Subtracting the two expressions for expected consumer surplus gives the welfare difference

$$\begin{aligned} &prob^N(\ell, L)\, S(\ell, L) + \left[prob^N(H) - prob^A(H)\right] S(H) + \\ &\qquad\qquad \left[prob^N(\ell) - prob^A(\ell)\right] S(\ell) = \\ &prob^N(\ell, L)\left[S(\ell, L) - S(\ell)\right] + \left[prob^N(H) - prob^A(H)\right] S(H) + \\ &\qquad\qquad \left[prob^N(\ell) + prob^N(\ell, L) - prob^A(\ell)\right] S(\ell) \end{aligned}$$

The result then follows because

$$prob^N(\ell) + prob^N(\ell, L) - prob^A(\ell) = prob^A(H) - prob^N(H).$$

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B Online Appendix

In this Appendix, we collect additional formal results and present parametric examples for the curious reader. Specifically, in Section B.1, we evaluate the effect of prohibiting acquisitions on duplication of innovation investment, as discussed towards the end of Section 5. Section B.2 considers the one-dimensional model mentioned in Section 6.2, where firms can choose total investment level. Formal results supporting the robustness claims of Section 6.3 can be found in Section B.3. Finally, in Section B.4 we use a parameterized model to depict the policy effects captured in Figure 3.

B.1 The Effect on Duplication

Duplication is measured by the probability that both firms discover the innovation:

$$\mathcal{D}(r_I, r_E) = \int_0^1 r_I(\theta) r_E(\theta) d\theta.$$

We distinguish between equilibria where $\theta_I^2(N) \leq \theta_I^1(N)$ and equilibria where $\theta_I^2(N) > \theta_I^1(N)$.

Proposition B.1 (The effect of prohibiting start-up acquisitions on duplication)**.**

(i) When $\theta_I^2(N) \leq \theta_I^1(N)$, duplication is strictly smaller in any simple equilibrium under the no-acquisition policy than in any simple equilibrium under laissez-faire.

(ii) When $\theta_I^2(N) > \theta_I^1(N)$, there exists a threshold bargaining power $\tilde{\beta} \in [0, 1)$ such that in any equilibrium under the no-acquisition policy duplication is

(a) larger than in any simple equilibrium under laissez-faire if $\beta < \tilde{\beta}$, and

(b) smaller than in any simple equilibrium under laissez-faire if $\beta > \tilde{\beta}$.

Proof. (i) First note that, under the no-acquisition policy, simple equilibria only exist when $\theta_I^2(N) \leq \theta_I^1(N)$. In any simple equilibrium under laissez-faire, $\mathcal{D}(r_I^A, r_E^A) = \theta^2(A)$ by Proposition 1 and in any simple equilibrium under the no-acquisition policy $\mathcal{D}(r_I^N, r_E^N) = \theta_E^2(N)$ by Proposition A.2. For $\theta_E^2(N) < \theta^2(A)$, we need

$$\begin{aligned}
C(\theta_E^2(N)) &< C(\theta^2(A)) \\
v_E^N(L, \ell) &< v_E^A(L, \ell) \\
\pi(L, \ell) - \kappa &< \beta(\max\{\pi(L, 0) - \kappa, \pi(\ell, 0)\} - \pi(\ell, L)) + (1 - \beta)(\pi(L, \ell) - \kappa) \\
\pi(L, \ell) - \kappa &< \max\{\pi(L, 0) - \kappa, \pi(\ell, 0)\} - \pi(\ell, L)
\end{aligned}$$

which holds by Assumption 1(iv). Hence $\mathcal{D}(r_I^N, r_E^N) < \mathcal{D}(r_I^A, r_E^A)$, which establishes part (i) of the Proposition.

(ii) We need to consider duplication in any equilibrium under the no-acquisition policy. When $\theta_I^2(N) > \theta_I^1(N)$, only non-simple equilibria exist under the no-acquisition policy. In these cases, by Proposition A.2, duplication is given by:

$$\mathcal{D}(r_I^N, r_E^N) = \theta_E^2(N) + \int_{\max\{\theta_E^2(N), \theta_I^1(N)\}}^{\min\{\theta_I^2(N), \theta_E^1(N)\}} r_I^N(\theta) r_E^N(\theta) d\theta.$$

Now we show that there exists a threshold $\tilde{\beta}$ which determines the sign of the effect of acquisitions on duplication. When $\beta = 0$, then $\theta^2(A) = \theta_E^2(N)$, thus

$$\mathcal{D}(r_I^A, r_E^A; \beta = 0) - \mathcal{D}(r_I^N, r_E^N; \beta = 0) = -\int_{\max\{\theta_E^2(N), \theta_I^1(N)\}}^{\min\{\theta_I^2(N), \theta_E^1(N)\}} r_I^N(\theta) r_E^N(\theta) d\theta \leq 0.$$

When $\beta = 1$, then $\theta^2(A) = \theta_I^2(N)$, thus

$$\mathcal{D}(r_I^A, r_E^A; \beta = 1) - \mathcal{D}(r_I^N, r_E^N; \beta = 1) = \\ \theta_I^2(N) - \theta_E^2(N) - \int_{\max\{\theta_E^2(N), \theta_I^1(N)\}}^{\min\{\theta_I^2(N), \theta_E^1(N)\}} r_I^N(\theta) r_E^N(\theta) d\theta > 0.$$

The last inequality follows from the following two observations, which are implied by Proposition A.2: (i) $0 \leq \min\{\theta_I^2(N), \theta_E^1(N)\} - \max\{\theta_E^2(N), \theta_I^1(N)\} \leq \theta_I^2(N) - \theta_E^2(N)$, and (ii) $r_I^N(\theta) r_E^N(\theta) \leq 1$ for all θ and $r_I^N(\theta) r_E^N(\theta) < 1$ for some θ. Finally, the effect of β on the change in duplication is monotone:

$$\frac{\partial(\mathcal{D}(r_I^A, r_E^A) - \mathcal{D}(r_I^N, r_E^N))}{\partial \beta} = \frac{\partial \theta^2(A)}{\partial \beta} \\ = \frac{(1-p)(\max\{\pi(L,0) - \kappa, \pi(\ell,0)\} - \pi(\ell,L) - \pi(L,\ell) + \kappa)}{C'(\theta^2(A))} > 0.$$

□

The intuition for the result is the following: If $\theta_I^2(N) \leq \theta_I^1(N)$, and we only consider simple equilibria in both regimes, then it is only the change in the entrant's incentive to invest in duplicate projects $\theta_E^2(N)$ which will determine the result. Similar to the effect on variety, banning acquisitions also decreases the profitability of duplicate innovations for the entrant, which leads to lower $\theta_E^2(N)$ and thus less duplication.

In the complementary case, $\theta_I^2(N) > \theta_I^1(N)$, there exists a positive measure of projects in which the incumbent invests only if this reduces the entrant's probability of receiving a patent on an L innovation. This creates additional duplication for more costly projects. Allowing for acquisitions will reduce this duplication, while still increasing the duplication for rather cheap projects as discussed in the previous paragraph. The overall effect of allowing acquisitions then depends on the relative size of those countervailing effects, which is determined by the bargaining power.

The above analysis identifies two distinct reasons for duplication: Duplication because of high relative payoff of innovation, such that both investing in the same project still pays off, and duplication due to the blocking incentives of the incumbent. The latter is stronger when acquisitions are not allowed and, while duplication by the incumbent has the negative side-effect of preventing the commercialization of a new L innovation and thereby competition, conversely, duplication by the entrant has the positive side-effect of increasing competition and can thus not only be considered as wasteful. However, even if duplication might be decreased by allowing acquisitions, the positive side-effect of duplication by the entrant is also shut down because his L innovation will just be acquired and will not lead to more product market competition.

B.2 One-dimensional Innovation Model

In this section we show that, in a model where firms only choose the amount of resources they invest in research, banning acquisitions will have an ambiguous effect on innovations.

Let x_i be the probability that the firm $i \in \{I, E\}$ discovers the innovation, with the associated cost given by $K(\cdot)$, where K is strictly increasing and convex. Apart from the investment stage, the model is unchanged.

Profits and Best Responses The expected profit of the incumbent and the entrant, given x_I and x_E, can be written as

$$\begin{aligned}
\mathbb{E}\Pi_I(x_I, x_E) &= x_I(1 - \frac{1}{2}x_E)\,[pv_I(H) + (1-p)v_I(L,0)] \\
&\quad + x_E(1 - \frac{1}{2}x_I)(1-p)v_I(\ell, L) + (1-x_I)(1-x_E)v_I(\ell, 0) - K(x_I) \\
\mathbb{E}\Pi_E(x_E, x_I) &= x_E(1 - \frac{1}{2}x_I)\,[pv_E(H) + (1-p)v_E(L,\ell)] - K(x_E).
\end{aligned}$$

Consequently, the first-order conditions and, implicitly, the best responses of the firms are

$$\begin{aligned}
K'(x_I(x_E)) &= (1-x_E)\,[pv_I(H) + (1-p)v_I(L,0) - v_I(\ell,0)] \\
&\quad + \frac{1}{2}x_E\,[pv_I(H) + (1-p)(v_I(L,0) - v_I(\ell, L))] \\
K'(x_E(x_I)) &= (1-\frac{1}{2}x_I)\,[pv_E(H) + (1-p)v_E(L,\ell)]
\end{aligned}$$

The Nash equilibrium solves the above system of equations and is denoted by (x_I^*, x_E^*).[34]

Note that the values $v_I(t_I^{int}, t_E^{int})$ and $v_E(t_E^{int}, t_I^{int})$ are exactly the same as in the main model and thus given by Lemma 2 for the laissez-faire regime. If acquisitions are prohibited, the only terms changing in the above first-order conditions are $v_I(\ell, L)$ and $v_E(L, \ell)$. Thus, we use superscripts to disentangle the different regimes: $v_I^A(\ell, L)$ and $v_E^A(L, \ell)$ in laissez-faire and $v_I^N(\ell, L)$ and $v_E^N(L, \ell)$ in the no-acquisition policy, where $v_I^N(\ell, L) = \pi(\ell, L)$ and $v_E^N(L, \ell) = \pi(L, \ell) - \kappa$.

If acquisitions are prohibited, the incumbent's payoff is lower when the entrant discovers an innovation (compared to the case when acquisitions are allowed), *increasing* her incentives to invest into R&D in order to drive out the entrant. However, the entrant also receives lower profits when he obtains a non-drastic innovation, which *reduces* his overall innovation incentives. Due to these counteracting effects, the net effect of a ban on acquisitions on the sum of investment levels is not clear ex-ante.

Effect of Acquisitions on Innovation Probability We assume $\pi(\ell, 0) > \pi(L, 0) - \kappa$, so that $v_I(L, 0) = v_I(\ell, 0)$. To simplify the comparison between policy regimes, we introduce a new parameter μ, where μ represents the probability that the acquisition will be allowed. The first order conditions of the entrant and incumbent for a given regime μ

[34] Second order conditions are satisfied due to convexity of $K(x)$.

are given by:

$$\begin{aligned}
K'(x_I(x_E);\mu) =&(1-x_E)p(v_I(H)-v_I(\ell,0)) \\
&+\frac{1}{2}x_E(pv_I(H)+(1-p)\left[\mu v_E^A(L,\ell)+(1-\mu)(v_I(\ell,0)-v_I^N(\ell,L))\right]) \\
K'(x_E(x_I);\mu) =&(1-\frac{1}{2}x_I)(pv_E(H)+(1-p)\left[\mu v_E^A(L,\ell)+(1-\mu)v_E^N(L,\ell)\right]).
\end{aligned}$$

The probability of an innovation, and its change when μ increases are given by:

$$\begin{aligned}
Pr(\text{Innovation}) &= x_I^*(\mu)+x_E^*(\mu)-x_I^*(\mu)x_E^*(\mu) \\
\Rightarrow \frac{dPr(\text{Innovation})}{d\mu} &= (1-x_E^*(\mu))\frac{dx_I^*(\mu)}{d\mu}+(1-x_I^*(\mu))\frac{dx_E^*(\mu)}{d\mu}.
\end{aligned}$$

We use the implicit function theorem on the first order conditions of the incumbent and entrant to evaluate the effect on the innovation efforts, $\frac{dx_I^*(\mu)}{d\mu}$ and $\frac{dx_E^*(\mu)}{d\mu}$. Inserting these expressions into the above derivative of the innovation probability, we get:

$$\begin{aligned}
\frac{dPr(\text{Innovation})}{d\mu} &= \frac{\frac{1}{2}x_E^*(\mu)(1-p)(v_I^A(\ell,L)-v_I^N(\ell,L))*\mathcal{I}}{|J|} \\
&+\frac{(1-\frac{1}{2}x_I^*(\mu))(1-p)(v_E^A(L,\ell)-v_E^N(L,\ell))*\mathcal{E}}{|J|}
\end{aligned}$$

where

$$\begin{aligned}
\mathcal{I} =& \frac{1}{2}(1-x_I^*(\mu))(pv_E(H)+(1-p)(\mu v_E^A(L,\ell)+(1-\mu)v_E^N(L,\ell))) \\
&-(1-x_E^*(\mu))K''(x_E^*(\mu))
\end{aligned}$$

and

$$\begin{aligned}
\mathcal{E} =& \frac{1}{2}(1-x_E^*(\mu))\left[pv_I(H)+(1-p)(v_I(\ell,0)-\mu v_I^A(\ell,L)-(1-\mu)v_I^N(\ell,L))\right. \\
&\left.-2p(v_I(H)-v_I(\ell,0))\right]+(1-x_I^*(\mu))K''(x_I^*(\mu)).
\end{aligned}$$

Note that the Jacobian matrix J is the collection of second-order partial derivatives and is negative definite assuming strict convexity of the cost function $K(x)$. Hence the determinant of the Jacobian matrix $|J|$ is positive and the sign of the effect of acquisitions on innovation probability is the same as the sign of weighted sum of $\mathcal{I}$ and $\mathcal{E}$.

This sign is not clear ex-ante. If $\beta=0$, so that $v_E^A(L,\ell)=v_E^N(L,\ell)$, then the sign of the effect on innovation probability is determined by

$$\frac{dPr(\text{Innovation})}{d\mu}|_{\beta=0} \gtrless 0 \Leftrightarrow (pv_E(H)+(1-p)v_E^N(L,\ell)) \gtrless 2\frac{(1-x_E^*(\mu))K''(x_E^*(\mu))}{1-x_I^*(\mu)}.$$

This effect is likely to be positive for large competition intensity in a duopoly, i.e. relatively small $\pi(L,\ell)=v_E^N(L,\ell)+\kappa$.

If the entrant has all bargaining power, i.e. $\beta=1$ and $v_I^A(\ell,L)=v_I^N(\ell,L)$, we get a similar expression for the sign of the effect:

$$\begin{aligned}
&\frac{dPr(\text{Innovation})}{d\mu}|_{\beta=1} \gtrless 0 \\
&\quad\Leftrightarrow (1-p)(v_I(L,0)-v_I^N(\ell,L))+p(2v_I(\ell,0)-v_I(H)) \gtrless -2\frac{(1-x_I^*(\mu))K''(x_I^*(\mu))}{1-x_E^*(\mu)}
\end{aligned}$$

If drastic innovation is not too profitable, i.e. $v_I(H) < 2v_I(\ell, 0)$, a more lenient regime towards acquisitions will increase innovation probability, irrespective of product market competition intensity when both firms are active.

The above analysis shows that, in a model where firms cannot target their innovation efforts towards specific R&D projects, the innovation effect of a more restrictive policy towards acquisition of start-ups will in general be ambiguous.

B.3 Robustness Results

B.3.1 Innovation Uncertainty at the Time of Acquisition

The new timeline leads to the following result in the acquisition subgame:

Lemma B.1 (Acquisitions)**.** *Suppose at the time of the acquisition the technology level of the innovation is uncertain. The incumbent acquires the entrant if and only if the entrant holds a patent for the innovation. After the acquisition, the incumbent always commercializes the H technology. She commercializes the L technology if and only if $\pi(L, 0) - \pi(\ell, 0) \geq \kappa$.*

Proof. First, suppose that the entrant holds no patent. Then, since the entrant cannot compete without an innovation, the incumbent's profits are the same with or without the acquisition. Thus, the incumbent has no reason to acquire the entrant.

Second, suppose the entrant holds a patent. Without an acquisition, the entrant commercializes the technology irrespective of the realized technology level according to Assumption 2. He thus obtains the expected payoff $p\,(\pi(H) - \kappa) + (1-p)\,(\pi(L, \ell) - \kappa)$ while the incumbent obtains $(1-p)\pi(\ell, L)$. With the acquisition, the incumbent commercializes the H technology according to Assumption 2, but only commercializes the L technology if $\pi(L, 0) - \kappa \geq \pi(\ell, 0)$. Thus, the incumbent's expected payoff is $p\,(\pi(H) - \kappa) + (1 - p)\,(\max\{\pi(L, 0) - \kappa, \pi(\ell, 0)\})$. The entrant obtains a payoff of zero. Consequently, the expected acquisition surplus is

$$(1 - p)\,[\max\{\pi(L, 0) - \kappa, \pi(\ell, 0)\} - \pi(L, \ell) - \pi(\ell, L) + \kappa]$$

The acquisition surplus is positive if and only if $\max\{\pi(L, 0) - \kappa, \pi(\ell, 0)\} > \pi(L, \ell) + \pi(\ell, L) - \kappa$, which holds by Assumption 1(iv). □

Comparing Lemma B.1 to Lemma 1, we can see that acquisitions happen more frequently. Not only does the incumbent acquire the entrant if his innovation turns out to be non-drastic, but also if the entrant's innovation turns out to be drastic. Thus, the entrant will never enter the market, neither as competitor nor as new monopolist. However, he will be compensated for the possibility that his innovation may turn out to be drastic.

Proposition B.2. *With uncertainty at the time of acquisition, any investment equilibrium under the alternative timeline with uncertainty is an investment equilibrium under the original timeline without uncertainty and vice versa.*

Proof. As in the main model, the equilibrium investment behavior will depend on the critical projects, for which the respective firm E or I is just indifferent between investing and not investing conditional on the behavior of the rival. Since, to be indifferent, payoffs need to equal investment costs, we will first introduce the new values $\tilde{v}_i$ for each firm $i \in \{I, E\}$ at the beginning of the acquisition stage in the laissez-faire regime, depending on whether the firm owns a patent, $t_i^{int} \in \{0, 1\}$:

Lemma B.2 (Payoffs).
In the case with uncertainty at the time of acquisition, consider the laissez-faire policy.
(i) The entrant's values after the realization of the innovation results are
$\tilde{v}_E(1,0) = p\pi(H) + (1-p)\pi(L,\ell) - \kappa + \beta(1-p)(\max\{\pi(L,0) - \kappa, \pi(\ell,0)\} - \pi(L,\ell) - \pi(\ell,L) + \kappa)$
$\tilde{v}_E(0, t_I^{int}) = 0$ *for* $t_I^{int} \in \{0,1\}$.
(ii) The incumbent's values after the realization of the innovation results are
$\tilde{v}_I(1,0) = p(\pi(H) - \kappa) + (1-p)\max\{\pi(L,0) - \kappa, \pi(\ell,0)\}$
$\tilde{v}_I(0,1) = p(\pi(H) - \kappa) + (1-p)\max\{\pi(L,0) - \kappa, \pi(\ell,0)\} - \tilde{v}_E(1,0)$
$\tilde{v}_I(0,0) = \pi(\ell,0)$.

We will refer to the critical thresholds under the alternative timeline, and thus new values, as $\tilde{\theta}_i^1, \tilde{\theta}_i^2$, $i \in \{E, I\}$. It turns out that these critical projects are identical to their counterparts in the original timeline without uncertainty:

$$\begin{aligned}
C(\tilde{\theta}_E^1) &= \tilde{v}_E(1,0) \\
&= p\pi(H) + (1-p)\pi(L,\ell) - \kappa + \\
&\qquad \beta(1-p)(\max\{\pi(L,0) - \kappa, \pi(\ell,0)\} - \pi(L,\ell) - \pi(\ell,L) + \kappa) \\
&= p(\pi(H) - \kappa) + (1-p)\Big(\pi(L,\ell) - \kappa + \\
&\qquad \beta(\max\{\pi(L,0) - \kappa, \pi(\ell,0)\} - \pi(L,\ell) - \pi(\ell,L) + \kappa)\Big) \\
&= pv_E(H) + (1-p)v_E(L,\ell) = C(\theta_E^1)
\end{aligned}$$

$$C(\tilde{\theta}_E^2) = \frac{1}{2}\tilde{v}_E(1,0) = \frac{1}{2}pv_E(H) + (1-p)v_E(L,\ell) = \frac{1}{2}C(\theta_E^1) = C(\theta_E^2)$$

$$\begin{aligned}
C(\tilde{\theta}_I^1) &= \tilde{v}_I(1,0) - \tilde{v}_I(0,0) \\
&= p(\pi(H) - \kappa) + (1-p)\max\{\pi(L,0) - \kappa, \pi(\ell,0)\} - \pi(\ell,0) \\
&= pv_I(H) + (1-p)v_I(L,0) - v_I(\ell,0) = C(\theta_I^1)
\end{aligned}$$

$$\begin{aligned}
C(\tilde{\theta}_I^2) &= \frac{1}{2}\tilde{v}_I(1,0) + \frac{1}{2}\tilde{v}_I(0,1) - \tilde{v}_I(0,1) = \frac{1}{2}v_E(1,0) \\
&= \frac{1}{2}(p\pi(H) + (1-p)\pi(L,\ell) - \kappa + \\
&\qquad \beta(1-p)(\max\{\pi(L,0) - \kappa, \pi(\ell,0)\} - \pi(L,\ell) - \pi(\ell,L) + \kappa)) \\
&= \frac{1}{2}(pv_I(H) + (1-p)v_E(L,\ell)) = C(\theta_I^2).
\end{aligned}$$

Since projects costs are strictly increasing in θ, equality of costs establishes equality of the values themselves, i.e. $\tilde{\theta}_i^1 = \theta_i^1$ and $\tilde{\theta}_i^2 = \theta_i^2$ for $i \in \{I, E\}$.

Again, under the no-acquisition policy, only two values change, $v_E(1,0) = p\pi(H) + (1-p)\pi(L,\ell) - \kappa$ and $v_I(0,1) = (1-p)\pi(\ell, L)$. Moreover, it is easy to see that the critical values are identical irrespective of which timeline we assume. Recall that according to Proposition A.1, the relative position of critical values is sufficient for the construction of equilibrium research strategies. □

Proposition B.2 implies that equilibrium research strategies in the two policy regimes do not depend on whether there is uncertainty at the time of acquisition. Moreover, we can apply Propositions 3 to evaluate the effect of prohibiting acquisitions. Since the effect is solely based on the research strategies, it is not affected by the amount of the uncertainty at the time of acquisition.

B.3.2 Asymmetric Chances of Receiving Patents

We now prove the following result.

Proposition B.3. *Consider the case with asymmetric patenting probabilities $\rho_I \in (0,1)$ and $\rho_E = 1 - \rho_I$.*

(a) In any equilibrium (r_I^N, r_E^N) under the no-acquisition policy, (i) the variety of research projects is weakly smaller than in any equilibrium (r_I^A, r_E^A) under the laissez-faire policy; and (ii) the probability of an innovation is weakly smaller than in any simple equilibrium (r_I^A, r_E^A) under the laissez-faire policy.

(b) The policy effect $\Delta_{\mathcal{V}} = \mathcal{V}^A - \mathcal{V}^N$ is independent of $\rho_I \in (0,1)$.

The subgames after the end of the investment stage are the same as in the main model, so that the continuation values under the laissez-faire policy are given by Lemma 2, and under the no-acquisition policy they are the same as in Lemma 2 except that $v_I^N(\ell, L) = \pi(\ell, L)$ and $v_E^N(L, \ell) = \pi(L, \ell) - \kappa$ (as in the main model). In addition, the critical projects θ_I^1 and θ_E^1 do not depend on ρ; so that their definition in Section 3 still applies. However, ρ_I affects the critical projects θ_I^2 and θ_E^2 and thus the equilibrium investments r_i. Denote the critical project θ_I^2 under the policy A for the given ρ_I as $\theta_E^2(A, \rho_I)$, and similarly for the other critical projects. Under laissez-faire,

$$C(\theta_E^2(A,\rho_I)) = (1-\rho_I)\left(pv_E(H) + (1-p)v_E(L,\ell)\right)$$
$$C(\theta_I^2(A,\rho_I)) = p\rho_I v_I(H) + (1-p)\left(\rho_I v_I(L,0) + (1-\rho_I)v_I(\ell,L)\right) - (1-p)v_I(\ell,L).$$

First, note that $\theta_E^1(A) > \theta_E^2(A,\rho_I)$ for all $\rho_I \in (0,1)$. Furthermore, since $\theta_E^1(A)$ and $\theta_I^1(A)$ do not depend on ρ_I, the following result follows directly (by arguments which are standard by now).

Lemma B.3. *Fix any $\rho_I \in (0,1)$. Under the laissez-faire policy, in any equilibrium, $\mathcal{V}^A = \max\{\theta_E^1(A), \theta_I^1(A)\}$.*

The critical projects under the no-acquisition policy, $\theta_I^1(N)$ and $\theta_E^1(N)$, are given as in Section 3 and thus are independent of ρ_I. $\theta_I^2(N,\rho_I)$ and $\theta_E^2(N,\rho_I)$ are defined implicitly as follows:

$$C(\theta_E^2(N,\rho_I)) = (1-\rho_I)\left(pv_E(H) + (1-p)v_E^N(L,\ell)\right)$$
$$C(\theta_I^2(N,\rho_I)) = p\rho_I v_I(H) + (1-p)\left(\rho_I v_I(L,0) + (1-\rho_I)v_I^N(\ell,L)\right) - (1-p)v_I^N(\ell,L).$$

Again, note that $\theta_E^1(N) > \theta_E^2(N,\rho_I)$ for all $\rho_I \in (0,1)$ and thus, since $\theta_E^1(N)$ and $\theta_I^1(N)$ do not depend on ρ_I, it follows directly that:

Lemma B.4. *Fix any $\rho_I \in (0,1)$. Under the laissez-faire policy, in any equilibrium, $\mathcal{V}^N = \max\{\theta_E^1(N), \theta_I^1(N)\}$.*

Therefore, neither $\mathcal{V}^A$ nor $\mathcal{V}^N$ depend on ρ_I, proving Proposition B.3.

B.3.3 Heterogeneous Commercialization Costs

We now prove the following result.

Proposition B.4. *Suppose $\kappa_I < \kappa_E \leq \pi(L,\ell)$ and $\pi(L,0)-\pi(\ell,0) < \kappa_I \leq \pi(H)-\pi(\ell,0)$. In any equilibrium (r_I^N, r_E^N) under the no-acquisition policy,*
(i) the variety of research projects is strictly smaller than in any equilibrium (r_I^A, r_E^A) under the laissez-faire policy;
(ii) the probability of an innovation is strictly smaller than in any simple equilibrium (r_I^A, r_E^A) under the laissez-faire policy.

Solving the game backwards, we first characterize the behavior of the firms in the commercialization and acquisition subgames.

Lemma B.5. *In the model with heterogeneous commercialization costs, the incumbent acquires the entrant whenever the entrant holds a patent for any technology. The incumbent commercializes only the technology H. The entrant commercializes both technologies.*

Proof. Since by assumption $\pi(L,0)-\pi(\ell,0) < \kappa_I \leq \pi(H)-\pi(\ell,0)$, the incumbent commercializes only the H technology. Since $\pi(H) > \pi(L,\ell) \geq \kappa_E$, the entrant commercializes both technologies. In the acquisition stage, if the entrant does not hold a patent, there is no reason for the acquisition. If the entrant holds a patent for the H technology, joint profits strictly increase after the acquisition, since $\pi(H)-\kappa_I > \pi(H)-\kappa_E$. Hence, the incumbent acquires the entrant. If the entrant holds a patent for the L technology, joint profits strictly increase after the acquisition, since $\pi(\ell,0) > \pi(L,\ell)-\kappa_E+\pi(\ell,L)$, which holds by Assumption 1. Hence, the incumbent acquires the entrant. □

Under the laissez-faire policy, the continuation payoffs are given below.
(i) The entrant's values after the realization of the innovation results are
$v_E(H) = \pi(H)-\kappa_E+\beta(\kappa_E-\kappa_I)$
$v_E(L,\ell) = \pi(L,\ell)-\kappa_E+\beta(\pi(\ell,0)-\pi(L,\ell)-\pi(\ell,L)+\kappa_E)$
$v_E(0,t_I) = 0$ for $t_I \in \{\ell, L, H\}$.
(ii) The incumbent's values after the realization of the innovation results are
$v_I(H) = \pi(H)-\kappa_I$
$v_I(L,0) = v_I(\ell,0) = \pi(\ell,0)$
$v_I(\ell,L) = \pi(\ell,0)-v_E(L,\ell)$
$v_I(\ell,H) = \pi(H)-\kappa_I-v_E(H) = (1-\beta)(\kappa_E-\kappa_I)$.
Using these continuation values to calculate the critical values, we immediately obtain that $\theta_E^2(A) < \theta_E^1(A)$. Next, $\theta_I^1(A) < \theta_E^1(A)$ if and only if

$$\begin{aligned} C(\theta_I^1(A)) &< C(\theta_E^1(A)) \\ pv_I(H)+(1-p)v_I(L,0)-v_I(\ell,0) &< pv_E(H)+(1-p)v_E(L,\ell) \\ p(\pi(H)-\kappa_I)-p\pi(\ell,0) &< p\left(\pi(H)-\kappa_E+\beta(\kappa_E-\kappa_I)\right)+(1-p)v_E(L,\ell) \\ p\left((1-\beta)(\kappa_E-\kappa_I)-\pi(\ell,0)\right) &< (1-p)v_E(L,\ell). \end{aligned}$$

Since $v_E(L,\ell) > 0$, for the above to hold it is sufficient that $\kappa_E-\kappa_I \leq \pi(\ell,0)$. Since $\pi(L,\ell) \geq \kappa_E$ and $\pi(L,0)-\pi(\ell,0) < \kappa_I$ by assumption, then $\kappa_E-\kappa_I < \pi(L,\ell)-(\pi(L,0)-\pi(\ell,0))$. Furthermore, $\pi(L,\ell) \leq \pi(L,0)$ implies that $\pi(L,\ell)-(\pi(L,0)-\pi(\ell,0)) \leq \pi(\ell,0)$, so that $\kappa_E-\kappa_I < \pi(\ell,0)$ always holds. Therefore, $\theta_I^1(A) < \theta_E^1(A)$ is always satisfied.

Together, this implies that $\theta_E^1(A) > \max\{\theta_E^2(A), \theta_I^1(A)\}$, which leads (by arguments which are standard by now) to the following result.

Lemma B.6. *Suppose that $\kappa_I < \kappa_E \leq \pi(L,\ell)$ and $\pi(L,0)-\pi(\ell,0) < \kappa_I \leq \pi(H)-\pi(\ell,0)$. Then, in any equilibrium under the laissez-faire policy, $\mathcal{V}^A = \theta_E^1(A)$.*

Next, we analyze the no-acquisition policy. The continuation payoffs are given below.
(i) The entrant's values after the realization of the innovation results are
$v_E(H) = \pi(H) - \kappa_E$
$v_E(L,\ell) = \pi(L,\ell) - \kappa_E$
$v_E(0,t_I) = 0$ for $t_I \in \{\ell, L, H\}$.
(ii) The incumbent's values after the realization of the innovation results are
$v_I(H) = \pi(H) - \kappa_I$
$v_I(L,0) = v_I(\ell,0) = \pi(\ell,0)$
$v_I(\ell,L) = \pi(\ell,L)$
$v_I(\ell,H) = 0$.
As before, it is immediate that $\theta_E^2(N) < \theta_E^1(N)$. Next, $\theta_I^1(N) \leq \theta_E^1(N)$ if and only if

$$\begin{aligned}
C(\theta_I^1(N)) &\leq C(\theta_E^1(N)) \\
pv_I(H) + (1-p)v_I(L,0) - v_I(\ell,0) &\leq pv_E(H) + (1-p)v_E(L,\ell) \\
p(\pi(H)-\kappa_I) - p\pi(\ell,0) &\leq p\left(\pi(H)-\kappa_E\right) + (1-p)(\pi(L,\ell)-\kappa_E) \\
-p\kappa_I - p\pi(\ell,0) &\leq (1-p)\pi(L,\ell) - \kappa_E.
\end{aligned}$$

For this inequality to hold, it is sufficient that

$$\begin{aligned}
-p(\pi(L,0)-\pi(\ell,0)) - p\pi(\ell,0) &\leq (1-p)\pi(L,\ell) - \pi(L,\ell) \\
-p\pi(L,0) &\leq -p\pi(L,\ell) \\
\pi(L,0) &\geq \pi(L,\ell)
\end{aligned}$$

which is satisfied by assumption. Therefore, $\theta_E^1(N) \geq \max\{\theta_E^2(N), \theta_I^1(N)\}$, which leads (by arguments which are standard by now) to the following result.

Lemma B.7. *Suppose $\kappa_I < \kappa_E \leq \pi(L,\ell)$ and $\pi(L,0)-\pi(\ell,0) < \kappa_I \leq \pi(H)-\pi(\ell,0)$. Then, in any equilibrium under the no-acquisition policy, $\mathcal{V}^N = \theta_E^1(N)$.*

Since $\theta_E^1(N) < \theta_E^1(A)$, the two lemmas in this section prove Proposition B.4.

B.4 Consumer Surplus Effects in Figure 3

Product Market We consider a heterogenous Bertrand model with linear demand (Shapley-Shubik). The utility of the representative consumer is given by:

$$U(q_I, q_E)) = \alpha_I q_I + \alpha_E q_E - \frac{1}{2}\left[(q_I^2 + q_E^2) + 2\gamma q_I q_E\right]$$

where q_i is the quantity consumed from firm $i \in \{I, E\}$, α_i is a quality parameter and γ governs substitutability. If $\gamma = 0$, both products are independent. As γ approaches 1, competition becomes more intense as the consumer prefers to consume more of only one product, instead of balanced quantities of both products. Hence, when both firms are active, the demand functions are:

$$q_i(p_i, p_j) = \frac{\alpha_i - \alpha_j\gamma - p_i + \gamma p_j}{1-\gamma^2}$$

Define $t_i = a_i - c_i$ as the net surplus firm i can provide to the consumer. The incumbent produces with technology $t_L \in \mathbb{R}^+$, which is also the technology level of the entrant under a non-drastic innovation, i.e. $L = \ell$. The minimum level of a drastic innovation is then given by the condition that, even if the firm owning the drastic technology t_H sets a monopoly price, the rival firm cannot profitably compete in the market, which can be derived as $t_H \geq \frac{2}{\gamma} t_L$.

Assumptions 1(i), (ii) and (iii) are satisfied by construction. For suitable parameter spaces 1 (iv) and 2 are satisfied as well.

Innovation Effect We assume that $C(\theta) = \frac{s\theta}{1-\theta}$ (where $s > 0$) to calculate the equilibrium investments. Remember that for some critical value constellations, equilibria are not unique. Therefore, we calculate bounds on the innovation and competition effects. Using Proposition 1, the upper and lower bound innovation probabilities in the laissez-faire regime are

$$\begin{aligned}
\overline{prob^A(H)}/p &= \theta_E^1(A) \\
\underline{prob^A(H)}/p &= \theta_E^1(A) - \max\{\theta_i^1(A) - \theta^2(A), 0\} \\
&\qquad + \max\{\int_{\theta^2}^{\theta_I^1} r_E(A) + r_I(A) - r_E(A) r_I(A) d\theta, 0\}
\end{aligned}$$

Using Proposition A.2, the upper and lower bound in the no-acquisition regime are:

$$\begin{aligned}
\overline{prob^N(H)}/p &= \theta_E^1(N) \\
&\quad - \max\{\min\{\theta_I^2(N), \theta_E^1(N)\} - \max\{\theta_I^1(N), \theta_E^2(N)\}, 0\} \\
&\quad + \max\{\int_{\max\{\theta_I^1, \theta_E^2\}}^{\min\{\theta_I^2, \theta_E^1\}} r_E(N) + r_I(N) - r_E(N) r_I(N) d\theta, 0\} \\
\underline{prob^N(H)}/p &= \theta_E^1(N) \\
&\quad - \left|\min\{\theta_I^2(N), \theta_E^1(N)\} - \max\{\theta_I^1(N), \theta_E^2(N)\}\right| \\
&\quad + \left| \int_{\max\{\theta_I^1, \theta_E^2\}}^{\min\{\theta_I^2, \theta_E^1\}} r_E(N) + r_I(N) - r_E(N) r_I(N) d\theta \right|
\end{aligned}$$

We obtain the upper bound on the effect on drastic innovation by selecting equilibria in the two regimes, such that the policy has the least negative effect on the probability of drastic innovation, which is $\overline{prob^N(H)} - \underline{prob^A(H)}$. Similarly, the lower bound is $\underline{prob^N(H)} - \overline{prob^A(H)}$.

Competition Effect The competition effect is given by the reduction in the entry probability. Since there is no competition in the laissez-faire regime, we only need to consider the probability of an L innovation by the entrant in the no-acquisition regime. We again calculate upper and lower bounds using Proposition A.2:

$$\begin{aligned}
\overline{prob^N(L,L)}/(1-p) &= \frac{1}{2}\theta_E^2(N) + \theta_E^1(N) - \min\{\theta_I^2(N), \theta_E^1(N)\} \\
&\quad + \max\{\int_{\max\{\theta_E^2(N),\theta_I^1(N)\}}^{\min\{\theta_I^2(N),\theta_E^1(N)\}} r_E(N)(1-r_I(N)) + \frac{1}{2} r_E(N) r_I(N) d\theta, 0\} \\
\underline{prob^N(L,L)}/(1-p) &= \frac{1}{2}\theta_E^2(N) + \theta_E^1(N) - \min\{\max\{\theta_I^1(N), \theta_I^2(N)\}, \theta_E^1(N)\} \\
&\quad + \max\{\int_{\max\{\theta_E^2(N),\theta_I^1(N)\}}^{\min\{\theta_I^2(N),\theta_E^1(N)\}} r_E(N)(1-r_I(N)) + \frac{1}{2} r_E(N) r_I(N) d\theta, 0\}
\end{aligned}$$

Overall Consumer Surplus Effect Note that consumer surplus differences $S(L,L) - S(L)$ and $S(H) - S(L)$ are calculated by the utility difference of the representative consumer for the respective technological states of the firms. The upper bound on the consumer surplus effect $\overline{\Delta S}$ represents the effect of banning acquisitions when selecting equilibria which are most preferable to the policy change, thus considering the upper bound on the competition and the innovation effect; vice versa for the lower bound on the consumer surplus effect $\underline{\Delta S}$ (see Proposition 5):

$$\begin{aligned}
\overline{\Delta S} &= \overline{prob^N(L,L)}\,[S(L,L) - S(L)] + (\overline{prob^N(H)} - \underline{prob^A(H)})\,[S(H) - S(L)] \\
\underline{\Delta S} &= \underline{prob^N(L,L)}\,[S(L,L) - S(L)] + (\underline{prob^N(H)} - \overline{prob^A(H)})\,[S(H) - S(L)]
\end{aligned}$$

Parameter Values Figure 3 is then constructed considering product innovation with identical costs c, and taking the following values for the quality parameters: $a_L = 1.85$, $a_H = 3.5$. The scaling parameter in the investment cost function is taken to be 0.5 and the commercialization costs are given by 0.01. Finally we consider an exogenous probability of drastic innovation of 1.5%, which enables us to present unique consumer surplus effects, that is effects where $\overline{\Delta S} = \underline{\Delta S}$. Making sure Assumption 1(iv) is satisfied in the model requires $\gamma \geq 0.65$, while Assumption 2 requires $\gamma \leq 0.95$. Hence, Figure 3 is depicted in the parameter space $\gamma \in [0.65, 0.95]$ and $\beta \in [0, 0.6]$.