Sticky Spending, Sequestration, and Government Debt*

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Abstract

Once established, government spending programs tend to continue. Spending inertia can lead to unsustainable debt levels that require fiscal stabilization, such as “sequestration.” We develop a political economy model of debt with sticky spending by assuming that the government must maintain a fraction of past spending. We show that inertia insures against the risk of political turnover, which may reduce politicians’ incentives to accumulate debt. However, if preexisting commitments are large, as in the current U.S. context, inertia exacerbates incentives to increase debt; faced with the prospect of stabilization, the government overspends to “dilute” the spending commitments of past administrations.


Keywords: Government Debt, Mandatory and Discretionary Spending, Entitlement Spending Reform, Delayed Stabilization, Budgetary Inertia.

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1 Introduction

Public spending is sticky: once new programs are established, they tend to persist over time. For instance, entitlement spending programs such as Social Security and Medicare are mandated by existing laws and continue on autopilot unless a new law is passed that alters them. Moreover, due to political and bureaucratic pressures, discretionary programs such as education and defense are often maintained despite automatically expiring at the end of the budget year, without any legal obligation to be renewed.\footnote{Budgetary inertia might arise because spending creates entrenched constituencies by changing voters’ reference point (\cite{Alesina2019} and \cite{Charite2015}). Stickiness could also be due to labor laws that make it difficult to transfer public employees across spending agencies. Finally, inertia may be the result of supermajority thresholds or veto power (\cite{Krehbiel1998}).}

A commonly held view is that budgetary inertia mechanically leads to high debt buildup, compromising fiscal sustainability. For instance, many U.S. commentators have been warning that entitlements make spending “uncontrollable,” sending the economy on an unsustainable fiscal trajectory.\footnote{As argued by \cite{Schick2003}, when budgets are sticky, new governments “accommodate fresh demands by spending more, not by substituting new priorities for old ones,” which also mechanically increases spending and debt. On budgetary inertia see also \cite{Wildavsky1964}.} This unsustainability appears to be the “fiscal problem of the 21st century” (\cite{Jones2003}).\footnote{See \cite{CBO2020}, \cite{Bohn2005}, \cite{Hamilton2013}, \cite{Auerbach1994}, and \cite{Auerbach2014} evaluate the present value of implicit commitments (e.g., Social Security and Medicare) to assess the sustainability of U.S. fiscal policy.} So far, calls for fiscal adjustments have fallen on deaf ears. This can’t go on forever; policymakers know that sooner or later, fiscal stabilization must occur.

This paper shows that spending persistence and the prospect of future stabilization generate novel strategic interactions with important implications for the expected outcomes. Contrary to the common view, we show that with functioning checks and balances, they can reduce overaccumulation of debt and even restore efficiency. However, with weaker checks and balances, high budgetary inertia could exacerbate underlying political frictions, speeding debt accumulation and making future cuts even more painful.

We derive these findings by introducing spending inertia into a standard model of strategic debt. The current political economy paradigm, such as \cite{Tabellini1990}, \cite{Battaglini2008} and many others, has shown that politicians tend to overaccumulate debt. Typically, however, debt sustainability is not a concern. This is because the standard assumption is that budgets are not inertial: when each program must be justified afresh each year, there are no, strictly speaking, “fiscal adjustments” to be made to ensure sustainability. Introducing budgetary stickiness enables us to study not only whether the
debt is inefficiently high, but also whether the debt is on an unsustainable path.

We consider a simple setting to make our point clearly. There are two goods that are supplied by the government and financed by nondistortionary taxes and debt. Two parties stochastically alternate in power and disagree as to how to allocate the budget. As is well known, this setting predicts a “deficit bias” wherein the incumbent overspends on her preferred good and issues debt to “force” the subsequent government to spend less on the good she does not value. The standard approach is to assume that the incumbent can cut spending programs put in place by previous administrations. In reality, this may not be possible due to legal and political costs. We thus assume that the incumbent must maintain an exogenous fraction $\alpha \geq 0$ of past spending. The higher $\alpha$, the stickier the budget. We start by assuming that the same $\alpha$ applies to all spending, and then in Section 4.3 we relax this assumption introducing mandatory programs that are stickier than discretionary ones.

Budgetary inertia brings out the sustainability issue and the implied timing of adjustment. We begin by assuming that the budget must always be sustainable. As soon as the present value of spending commitments is larger than the present value of government revenues, a “sequestration” occurs. The incumbent executes the sequestration by cutting all programs proportionally and has no discretion to choose which entitlements to curtail. Later, we will relax the assumption that the sequestration is triggered automatically: in Section 4.1 the incumbent knows that an adjustment would eventually happen, but she may choose to delay it by running additional debt.

Because of budgetary inertia, spending becomes an additional strategic variable used by the government to directly influence the choices of her successors. By allocating funding to her preferred programs, the incumbent locks in future spending in case she is voted out of office, attenuating the spending fluctuations due to political turnover. This insurance motive has an ambiguous effect on overspending and debt accumulation. On the one hand, for small and strictly positive $\alpha$, as $\alpha$ increases the present government can more effectively bind the choices of future governments. This strengthens the incentive to spend, validating the common view that budgetary inertia leads to large debt buildup. On the other hand, when $\alpha$ is sufficiently large the incumbent attains full insurance. Then, as the stickiness increases, the cost of locking in subsequent government decreases, reducing the incentives to overspend and accumulate debt. As a result, as long as previous entitlements are not large, there is a

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4Sequestration is a provision of U.S. law, introduced in 1985, that is designed to keep federal deficits below an upper limit, establishing automatic, across-the-board reductions of nonexempt mandatory spending programs. We will use the term sequestration to indicate, more generally, a fiscal stabilization.
hump-shaped relationship between spending persistence and debt accumulation.

When preexisting entitlements are sufficiently large, another mechanism comes into play: faced with the prospect of future stabilization, there is a \textit{dilution} motive to spend. One might expect that the “threat” of fiscal cuts would discipline the current incumbent. We find instead that it could aggravate the incentives to overspend: because in a sequestration each program will be reduced \textit{proportionally}, the incumbent allocates more funding to her preferred programs to dilute the opposition’s entitlements in a future sequestration.\footnote{This echoes the debt-dilution channel in the sovereign-default literature. When a country approaches a financial crisis, its government may have an incentive to issue new bonds, which dilutes existing creditors \cite{Bolton2009, Hatchondo2016, Chatterjee2012}.} The dilution effect is stronger when the incumbent inherits large commitments on spending programs that she does not value. Moreover, this motive arises only when budgets are sticky: when $\alpha$ is zero, there are no government commitments to begin with, and politicians have no incentive to dilute past commitments.

Taking both motives into account, we show that in some cases, higher budgetary stickiness reduces the need to strategically use debt. However, if preexisting entitlements are large, as in the current U.S. context, the dilution motive operates in full and budgetary stickiness worsens the deficit bias, even beyond the standard “strategic-manipulation” effect.

An implication of this theory is that budgetary stickiness and the expectations of future cuts may fuel a spending spiral, in which loose fiscal policy by past governments triggers even looser policies by subsequent ones. This may be a good description of how the two major American parties have been flirting with fiscal unsustainability. During the Obama administration, Democrats approved a steep rise in spending on Social Security and Medicare. Facing the sharp rise in entitlements and frustrated by their failure to repeal Obamacare, Republicans pushed for a loose fiscal stance: funding the border wall, spending more on defense, and cutting taxes. When the opposition’s entitlements cannot be cut directly, the dilution channel predicts that the incumbent will do it indirectly through sequestrations.

The previous results hinge on the assumption that spending programs are downwardly rigid, but the incumbent government has full discretion to increase spending. Can further checks and balances prevent inefficient outcomes? To answer this question, we assume that any spending increase must be negotiated with the opposition. This captures supermajority requirements needed to pass new legislation such as in the U.S. Senate. In contrast to the environment without checks and balances, we show that stickiness can eliminate debt accumulation, even when the opposition’s entitlements are large. In addition, higher oppo-
osition entitlements raise the opposition’s bargaining power and implement a more equitable spending mix. In recent years, many important U.S. reforms (e.g., the Bush and Trump tax cuts, parts of Obamacare) were approved by bypassing the Senate filibuster through so-called budget reconciliation. Our results show that debt would have been lower without these simple-majority-expedited measures.

Next, we allow politicians to delay sequestration. We show that when the opposition’s entitlements are large, the incumbent postpones fiscal adjustments, making future cuts even more painful. An incumbent that inherits the opposition’s large commitments takes advantage of being in power to boost spending. Besides the positive effect on current utility, the additional spending increases the relative size of her preferred programs, which dilutes the opposition’s entitlements in a future sequestration. Still, both parties prefer a sequestration if total entitlements are large enough and eventually the sequestration occurs.

Finally, one may wonder what would happen when the government can cut less-persistent programs rather than forcing a sequestration. We extend the model by assuming that some programs are not sticky: this captures discretionary spending, which requires annual appropriation bills and, thus, are less sticky than mandatory spending. We show that the presence of discretionary spending does not qualitatively change our results, but does expose new dimensions of inefficiency. Since mandatory spending can be used to manipulate future governments, it is overprovided compared to the first-best.

The rest of the paper is as follows. In Section 2 we review the literature. In Section 3 we present the benchmark model. In Section 4.1, we analyze the case with delayed sequestration. In Section 4.2, we consider a bargaining model in which the incumbent needs the opposition’s approval to increase spending. In Section 4.3, we allow for two types of spending with different degrees of stickiness. Section 5 concludes. All proofs are in the Appendix.

2 Literature Review

Our paper builds on the political economy literature on strategic debt. A key difference between that literature and our paper is that we introduce budgetary inertia, which allows

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6 A fiscal adjustment must eventually happen when default is not possible. Notice that even if the real interest rate were below the growth rate of the economy, debt rollovers (Ball et al. 1995, Blanchard 2019) can only be used to finance a one-time spending increase, not a persistent one (see Reis 2021).

us to investigate debt sustainability and delayed stabilization. 

Bouton et al. (2020) were the first to study how entitlements affect debt accumulation. Their paper is related to ours, but our approaches differ. They model entitlements as transfers that are decided one period in advance: the incumbent chooses the current public good and the entitlements that her constituency will receive in the next period. We assume instead that current spending and entitlements are not two separate decisions: future entitlements can be created only by spending in the current period, thus capturing the idea that budgets are inertial. An important consequence of this assumption is that entitlements are potentially not sustainable in our model. In their model, instead, the incumbent must satisfy an intertemporal budget constraint when choosing future entitlements. Moreover, in their paper, entitlements and debt are substitutes: entitlements allow the incumbent to constrain future governments, which weakens the incentive to use debt. In our model, entitlements can be created only by spending and, potentially, running debt. Because of this, budgetary stickiness may contribute to high debt buildups.

Our work is also related to a growing political economy literature that studies policy inertia. In the “endogenous status quo” literature, policy inertia arises endogenously in a dynamic legislative-bargaining model. The key assumption is that the default option in case of disagreement coincides with the previous period’s policy (i.e., the status quo). A result of this literature (e.g., Baron (1996), Bowen et al. (2014) and Azzimonti et al. (2020)) is that the endogenous status quo exerts an important insurance effect by raising the bargaining power of the nonproposing players. Similarly to these papers, we find that sticky spending provides insurance. The key difference between the prior literature and our paper is that the former does not study debt, which is our focus. Moreover, by introducing debt-sustainability issues, we highlight the dilution channel, which has not yet been studied by this literature.

Another strand of the political economy literature studies policy inertia in models without bargaining but with a cost of changing policy (e.g., Gersbach et al. (2019), Gersbach et al. (2020), Eraslan and Piazza (2020), and Chatterjee and Eyigungor (2020)). These papers, which also abstain from studying debt, find conditions under which costly policy changes lead to lower policy fluctuations. Our approach differs, in that we assume an asymmetric

\[8\] Early contributions to this literature include Baron (1996) and Kalandrakis (2004). For a comprehensive overview, see Eraslan et al. (2020).

\[9\] Bowen et al. (2014) study legislative bargaining over public spending; they find that the public good is efficiently provided in the long run when its status quo level is endogenous. Azzimonti et al. (2020) assume a fixed status quo for the public good and an endogenous status quo level for tax rates and transfers. They find that politicians “overuse” the inertial policy instruments to constrain future governments.
cost: in our baseline model, spending is downwardly rigid, but can be increased at no cost.

Finally, this paper is related to the influential work by Alesina and Drazen (1991) on delays in stabilizations. In their model, two groups must agree on how to share the cost of fiscal adjustments; stabilization is delayed until one group concedes and bears a disproportionate share of the burden. Our work differs along three dimensions. First, in their model stabilization is needed because an exogenous shock lowers tax revenues. In our paper, debt unsustainability is an endogenous outcome. Second, in their model, the burden of the stabilization is shared according to fixed proportions, while we assume that cuts are proportional and depend on past expenditures. Third, in their model, political gridlock leads to inaction. In our model, the dilution channel implies that the incumbent delays stabilization by spending even more.

3 The Model

We consider a two-period model: \( t = 1, 2 \). In the first period, public spending is financed by current taxes and debt. In the second period, any remaining debt must be paid. The two-period assumption is stark, but it captures the two phases that would be observed in the infinite horizon: an early phase when the government runs deficits and a later one in which spending must be cut.

Public policies are decided by two parties (denoted by \( I \) and \( O \)) that stochastically alternate in power. We assume that party \( I \) is the incumbent government in the first period. The opposition at time 1 is party \( O \). At the end of the first period, there is an election, and party \( I \) stays in power with exogenous probability \( q \). With a complementary probability \((1 - q)\) party \( O \) seizes power and \( I \) becomes the opposition party. The two parties have opposite preferences over the desired composition of public spending.\(^{10}\) There are two types of goods, and each party would like to allocate the whole budget to one of them. For example, the two goods could represent partisan expenditures that target each party’s constituency. In American politics, the traditional division is between Democrats, who favor spending on Social Security and education, and Republicans, who favor defense spending, border protection, and tax cuts. For most practical purposes, tax breaks can be considered equivalent to spending: they favor specific constituencies, raise debt, and cannot be easily undone, because they run

\(^{10}\)If we assume that the two parties are less than fully polarized, this would change the results in the intuitive sense: it would lower the deficit bias.
through the tax code\textsuperscript{11}

We denote by $g^I$ and $g^O$, the good that is valued by party $I$ and $O$, respectively. The per-period utility of the two parties is:

$$u_j(g^j, g^{-j}) = u(g^j) = \frac{(g^j)^{1-\sigma} - 1}{1 - \sigma}$$

with $j = I, O$ and $\sigma \in (0, 1]$. Each party discounts the future using the factor $\beta = 1$.

We assume that the tax revenue is exogenous and equal to $\tau$ in both periods\textsuperscript{12}. To avoid cluttered notation, we also assume that the interest rate and initial debt are both zero. The incumbent faces two sets of constraints. First, there are two standard budget constraints. Second, there are two sustainability constraints, determining whether it is feasible to maintain the prior government’s commitments. The government budget constraints at $t = 1$ and $t = 2$ are, respectively,

$$g^I_1 + g^O_1 \leq \tau + b_1$$

and

$$g^I_2 + g^O_2 \leq \tau - b_1$$

where $b_1$ is the stock of debt issued at $t = 1$. To ensure solvency, assume $b_1 \leq \tau$ so that it is always feasible to pay the outstanding debt in the final period.

Since the discount rate is equal to the interest rate ($\beta = 1$ and zero interest rate) and the tax revenue is constant, it is easy to show that a social planner who values both goods would choose zero debt. In turn, by varying the planner’s social weights, we can trace out the entire Pareto frontier. For example, if both parties are equally weighted, $g^i_t = \tau/2$ for $t = 1, 2$ and $j = I, O$. More generally, for arbitrary weights in the planner’s problem, the ratio $g^i_t/g^o_t$ must be constant over time.

Our first key departure from the literature is that, whenever fiscally sustainable, the incumbent must maintain a proportion $\alpha \in [0, \bar{\alpha}]$ of the previous year’s spending programs:

$$g^i_t \geq \alpha g^i_{t-1}$$

with $j = I, O$ and $t = 1, 2$. This specification captures, in a reduced form, the political and legal costs of cutting existing programs. The parameter $\alpha$ summarizes the extent of

\textsuperscript{11}The concept of “tax expenditure” is discussed in Tanzi (2008) and OECD (2010).

\textsuperscript{12}The assumption that taxes are exogenous is not important for the main point of our paper, but it greatly simplifies the analysis.
budgetary stickiness. We assume $\bar{\alpha} \geq 1$. If $\alpha = 1$ budgets are highly rigid, meaning that the incumbent cannot cut the existing spending programs. When $\alpha = 0$, we obtain the canonical model of Tabellini and Alesina (1990). We assume here that all spending is equally sticky. We will later relax this assumption by assuming that some spending is stickier than others, capturing the distinction between mandatory and discretionary spending.

Our second departure, which is required by the existence of persistence, is the explicit consideration of budget sustainability. To assess whether maintaining previous spending levels is sustainable, notice that at $t = 2$, $g^I_1$ and $g^O_1$ define the implicit commitments that the second-period government must uphold. When

$$\alpha g^I_1 + \alpha g^O_1 > \tau - b_1$$

such commitments are not sustainable, in which case we impose a sequestration. Each program is cut down proportionally to restore feasibility. Specifically, each spending program receives a share of available resources in proportion to its relative size:

$$g^I_2 = (\tau - b_1) \left( \frac{\alpha g^I_1}{\alpha g^I_1 + \alpha g^O_1} \right) \equiv \psi$$

and

$$g^O_2 = (\tau - b_1) \left( \frac{\alpha g^O_1}{\alpha g^I_1 + \alpha g^O_1} \right) \equiv 1 - \psi$$

The proportions $\psi$ and $1 - \psi$ determine how the two parties share the sequestration burden. These shares are endogenous to the model, but they are predetermined at time $t = 2$. Upon sequestration, the incumbent government has no fiscal space, because all available resources have already been committed. The party in office is not free to choose which programs to curtail. This assumption is coherent with the sequestration rule in U.S. budgeting, which prescribes that the executive cannot alter the spending priorities previously established by Congress (Stith, 1988). The thrust of our results would be maintained if we assume that the ability of the incumbent to selectively cut spending is less than full.

At $t = 1$, the entitlement’s feasibility considers that future entitlements constitute a liability for the government. Let $g^O_0$ and $g^I_0$ be the spending levels that were chosen in

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13When $\alpha > 1$, spending programs expand over time. For example, Medicare spending might increase because the elderly population is growing.

14For clarity, we develop the main results assuming that government revenue is exogenous, and thus sequestration is the only way to keep the budget sustainable. In Sections 4.1 and 4.3 we allow the incumbent to avoid sequestration either by running more debt or by cutting discretionary spending.

15If the incumbent government were able to selectively cut spending upon sequestration, we would introduce an artificial discontinuity: when (5) holds with equality, the incumbent would have no fiscal space, but it would be able to allocate spending freely as soon as that inequality becomes strict.
(unmodeled) period 0. We assume that these entitlements are not sustainable if the present value of “committed” spending is higher than the present value of taxes over the two periods:

\[(\alpha + \alpha^2)g^I_0 + (\alpha + \alpha^2)g^O_0 > 2\tau\]  

(7)

To understand this expression, recall that the interest rate is zero and that by (4) entitlements depreciate (when \(\alpha < 1\)) or grow (when \(\alpha > 1\)) geometrically. When (7) holds, past commitments are not sustainable in the long run and spending programs must be cut to restore sustainability. There is a key difference between (5) and (7) regarding the urgency of a sequestration. In the final period, a sequestration cannot be postponed. If instead (7) holds, politicians might be tempted to ignore the constraint, run debt to pay current obligations, and postpone the sequestration. We start by analyzing a benchmark where we abstract from these considerations by assuming that obligations from \(t = 0\) are sustainable.

Assumption 1: \(g^I_0\) and \(g^O_0\) are sufficiently small so that equation (7) does not hold for all \(\alpha \in [0, \bar{\alpha}]\).

We will remove this assumption later on to study delayed sequestration. Figure 1 summarizes the timeline. Given \(g^I_0\) and \(g^O_0\), at \(t = 1\) party \(I\) selects \(g^I_1\) and \(g^O_1\) subject to (2) and (4). At \(t = 2\), party \(I\) stays in power with probability \(q\) and party \(O\) becomes the new incumbent with probability \(1 - q\). If (5) holds, a sequestration occurs. If (5) does not hold, the party in office chooses policies \(g^I_2\) and \(g^O_2\) subject to (3) and (4).
3.1 Solution

We solve the model backwards. At $t = 2$, using (2) and (5) we obtain that a sequestration occurs if:

$$b_1 \geq \tilde{b} \equiv \frac{\tau (1 - \alpha)}{(1 + \alpha)}$$

where $\tilde{b}$ is the sequestration threshold. When $b_1 \in (\tilde{b}, \tau]$ previous governments’ commitments are not sustainable and must be cut down. The threshold is decreasing in $\alpha$. This is intuitive: when $\alpha$ is small, sequestration occurs only for very high levels of past spending.

Define the indicator function $\Phi$, which takes the value one when a sequestration occurs at $t = 2$ and the value zero otherwise.

At $t = 2$, available resources are $\tau - b_1$. It is straightforward to compute the consumption of party $I$ in the final period. If there is no sequestration and $I$ stays in power, she will have to (partially) maintain the opposition’s entitlements and keeps the remaining resources for herself, then $g'_2 = \tau - b_1 - \alpha g^O_1$. If, instead, $I$ loses power, $g'_2 = \alpha g'_1$. When there is sequestration, spending programs are downsized using the sequestration rule in (6). Summarizing:

$$g'_2 = \begin{cases} 
\tau - b_1 - \alpha g^O_1, & \text{if } \Phi = 0 \text{ and } I \text{ stays in power} \\
\alpha g'_1, & \text{if } \Phi = 0 \text{ and } I \text{ loses power} \\
\psi(\tau - b_1), & \text{if } \Phi = 1 
\end{cases}$$

Moving to the first period, Party $I$’s dynamic problem can be written as:

$$\max_{\{g^O_1, g'_1, b_1\}} \left\{ u(g'_1) + (1 - \Phi) \left[ q u \left( \tau - b_1 - \alpha g^O_1 \right) + (1 - q) u(\alpha g'_1) \right] + \Phi u(\psi(\tau - b_1)) \right\}$$

subject to (2), (4) and $b_1 \leq \tau$.

There are two areas to consider: when $b_1$ is above and below the sequestration threshold. After noticing that party $I$ optimally allocates $\alpha g^O_0$ to the opposition, when $b_1 < \tilde{b}$, so that $\Phi = 0$, the first-order condition equalizing the marginal benefit and cost of debt is:

\[\text{IMF (2002) considers that public debt is sustainable “if it satisfies the present value budget constraint without a major correction in the balance of income and expenditure given the costs of financing it faces in the market.”}\]
The term (a) indicates that higher $\alpha$ raises the marginal utility of consumption today. If the incumbent wants to achieve a given target for current consumption, she needs to raise more debt. The term (b) reflects the “insurance” effect: higher spending today secures higher spending in the next period in case party $I$ is voted out of office. The term (c) is the future marginal cost of debt. This term captures the standard channel emphasized by the strategic models of debt: a higher probability of losing power (lower $q$) makes the incumbent more myopic, raising the deficit bias.

When $b_1 \geq \tilde{b}$, so that $\Phi = 1$, the first-order condition is given by:

$$
(\tau + b_1 - \alpha g_0^O)^{-\sigma} + (1 - q)\alpha(\tau + b_1 - \alpha g_0^O)^{-\sigma} = q(\tau - b_1 - \alpha^2 g_0^O)^{-\sigma}
$$

(10)

The term (a) indicates that higher $\alpha$ raises the marginal utility of consumption today. If the incumbent wants to achieve a given target for current consumption, she needs to raise more debt. The term (b) reflects the “insurance” effect: higher spending today secures higher spending in the next period in case party $I$ is voted out of office. The term (c) is the future marginal cost of debt. This term captures the standard channel emphasized by the strategic models of debt: a higher probability of losing power (lower $q$) makes the incumbent more myopic, raising the deficit bias.

As in (10), the term (d) is the marginal utility of consumption today. The right-hand side in (11) does not include $q$ anymore. In a sequestration, political risk is eliminated because regardless of the electoral outcome, party $I$ obtains a proportion $\psi = g_1^I/(g_1^I + g_0^O)$ of available resources. The right-hand side in (11) depends on $g_0^O$ both directly and indirectly through $\psi$. There are two cases to consider: (1) when $g_0^O = 0$ and (2) when $g_0^O > 0$. When the opposition has no initial entitlements, upon sequestration all available resources will be allocated to $I$’s preferred good. Then, because $\psi = 1$, the right-hand side becomes $\psi(\tau - b_1)^{-\sigma}$ and, consequently, the incumbent fully internalizes the cost of debt. When instead $g_0^O > 0$, and hence $\psi < 1$, the incumbent no longer internalizes the whole cost of debt because saved resources will partly benefit the opposition. Moreover, there is a dilution effect, which lowers the marginal cost of debt: because $d\psi/db_1 \geq 0$, more spending today dilutes the opposition’s spending share upon sequestration. The dilution effect is larger when the opposition’s entitlements are more important (i.e., $\alpha g_0^O$ is high) or when more resources will be distributed upon sequestration (i.e., $\tau - b_1$ is large).

A widely held view is that debt would be smaller if budgets were less inertial. To assess this view, we analyze how $\alpha$ affects the insurance and dilution channels. First, because

$$
(\tau + b_1 - \alpha g_0^O)^{-\sigma} = (\psi(\tau - b_1)^{-\sigma} \left[ \psi - \alpha g_0^O \right] 
$$

(11)
the opposition’s entitlements at \( t = 2 \) are increasing in \( \alpha \), lower budgetary inertia weakens the dilution channel. This first effect validates the view that less inertia would lead to less spending. The effect of \( \alpha \) on the insurance channel is less straightforward because there are, roughly speaking, income and substitution effects pushing in opposite directions. On the one hand, looking at term (b) in (10), lower stickiness makes spending less effective in constraining future governments, inducing less spending and debt. On the other hand, with a lower \( \alpha \), a given target level of future spending is achieved with more debt. Lower stickiness increases the sequestration threshold: more overspending is needed to fully lock in the subsequent government and eliminate political risk, increasing the incentives to overspend.

Taking stock, we find that the combined effect of the insurance and dilution channels implies a nonmonotonic relation between \( \alpha \) and debt. We state the following result:

**Proposition 1** Benchmark characterization:

(i) Equilibrium debt when \( \alpha = 0 \) is lower than debt when \( \alpha = 1 \) if preexisting opposition’s entitlements are sufficiently large:

\[
g_0^O > \frac{2\tau(1-q)}{(1+q^{\frac{1}{2}})^2} \tag{12}
\]

(ii) Debt is increasing in \( \alpha \) if stickiness is sufficiently close to zero. Debt is a nonmonotone function of \( \alpha \).

*Proof:* See Appendix A.

The above proposition provides a simple condition for when stickiness aggravates the debt problem: when political turnover is low and/or when the opposition’s entitlements are high. This result is obtained because a lower \( q \) makes the incumbent more myopic when \( \alpha = 0 \), while a higher \( g_0^O \) raises debt when \( \alpha = 1 \) due to the dilution channel.\(^{18}\)

Figure 2 illustrates the relation between stickiness (x-axis) and debt (y-axis).\(^{19}\) The dashed red line is the sequestration threshold, which from (8) is decreasing in \( \alpha \). In Panel A, we assume that \( g_0^O = 0 \) and thus abstract from the dilution channel. The insurance channel implies a hump-shaped relation between \( \alpha \) and debt. When \( \alpha \) is low, triggering a sequestration would require implausibly high spending levels. In this range, the relevant

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\(^{18}\)Lower \( \sigma \) reduces the cost of debt when \( \alpha = 0 \), lowering the threshold in (12).

\(^{19}\)We set \( q = \sigma = 0.5 \). In Panel B of Figure 2, \( g_0^O = 0.6 \).
The first-order condition is (10), which explains why debt is increasing in $\alpha$. The positive slope confirms the common view that higher stickiness leads to more debt accumulation.

As stickiness increases, however, the sequestration threshold declines, and it becomes optimal to trigger a sequestration. The incumbent eliminates political risk by choosing a debt level exactly equal to the sequestration threshold. Since this threshold is decreasing in $\alpha$, this explains why equilibrium debt is decreasing when $\alpha$ is sufficiently high. When $\alpha \geq 1$, stickiness allows party $I$ to implement the commitment solution: as a result, debt is zero.\footnote{However, this allocation strongly favors party $I$. In Section 4.2 we will introduce a supermajority requirement and show that when $\alpha = 1$ we achieve a more equitable allocation on the Pareto frontier.}

In Panel B, we assume that $g^O_o > 0$. We show that for large $\alpha$ a new effect dominates the other channel: as $\alpha$ increases, the opposition’s entitlements (a combination of $g^O_o$ and $\alpha$) become more important, giving the incumbent the incentive to raise debt above the sequestration threshold to dilute the opposition’s entitlements.

It is interesting to notice the different roles played by the sequestration when there are no preexisting commitments (Panel A) vs. when they are high (Panel B). In Panel A, when $\alpha \leq 1$ cuts never happen in equilibrium, sequestration acts as an off-equilibrium path threat that reduces debt’s over-accumulation. In Panel B, sequestration is actively used on the equilibrium path to dilute past entitlements and, consequently, further fuels the incentives to overaccumulate debt.

The dilution motive is a novel channel not present in the related literature. In the canon-
ical strategic model of debt with $\alpha = 0$, the incumbent is constrained only by the inherited stock of debt; how spending was allocated by past governments is completely irrelevant. In our model, instead, fixing initial debt, the higher $g_0^O$ is, the stronger the incentive to accumulate debt. The dilution channel implies a spending spiral in which higher opposition’s entitlements lead to overspending and more debt. To see this more clearly, using (11) we provide a simple closed-form solution for debt when $\alpha \simeq 1$ and $\sigma \to 1$:

$$b_1^* = \frac{-\tau + \sqrt{\tau^2 + 4\tau \alpha g_0^O}}{2}$$

which shows that debt is increasing in $g_0^O$ when $\alpha$ is close to one.\footnote{If initial debt $b_0$ were positive, the dilution motive would still be present: expression (13) would be decreasing in $b_0$ and increasing in $g_0^O$.} In other words, with a looming fiscal stabilization, a loose fiscal policy leads to even looser policies by subsequent governments. One could view Obama’s and Trump’s fiscal policies through this lens: i.e., the surge in Social Security and health-program entitlements under the Obama administration was followed by tax cuts and other spending measures that further bloated U.S. debt.

4 Extensions

In this section, we extend the basic model along three dimensions. We allow politicians to avoid sequestration either by running more debt (Section 4.1) or by cutting less inertial spending (Section 4.3). In Section 4.2 we introduce a supermajority requirement for increasing spending.

4.1 Delayed Sequestration

Politicians are known for delaying fiscal adjustments until no further delay is possible. The benchmark setting abstracts from these considerations by assuming that entitlements can become unsustainable only in the second period, when delaying a stabilization is not possible. In this subsection, we drop Assumption 1 and assume that entitlements are unsustainable already at $t = 1$, by considering the case in which condition (7) holds. We endow the incumbent with the choice of whether to sequester or not. We find conditions under which delay occurs.

To analyze the incumbent’s trade-off, it is necessary to determine not only how resources
are shared in case of sequestration at $t = 1$, but also how much debt is allowed contingent on the fiscal stabilization. Let $b^*$ be the level of debt allowed during a sequestration. Consistent with the $t = 2$ rule, we assume that upon sequestration current resources are shared in proportion to preexisting entitlements. Thus, at time 1, available resources are given by $\tau + b^*$. Therefore, upon sequestration, spending at time 1 is given by:

$$
g_O^1 = \frac{g_O^0(\tau + b^*)}{g_O^0 + g_I^0}, \quad \text{and} \quad g_I^1 = \frac{g_I^0(\tau + b^*)}{g_O^0 + g_I^0}
$$ (14)

We could make different assumptions about $b^*$, without changing the main thrust of the results. In what follows, for tractability we assume that debt is allowed so that in the second period entitlements are exactly sustainable:

$$
\alpha \left( \tau + b^* \right) g_I^0 + \alpha \left( \tau + b^* \right) g_O^0 = \tau - b^*
$$ (15)

Using the first-period budget constraint, equation (15) generates $b^* = (1 - \alpha)/(1 + \alpha)$. For example, $\alpha = 1$ implies $b^* = 0$: upon sequestration $\tau$ is shared in each period.

If sequestration occurs, the incumbent cuts spending as discussed above. If, instead, sequestration is delayed, the incumbent must pay existing commitments, and she accumulates debt to spend more on the goods that she values. Besides increasing her current utility, higher spending improves the terms of a future sequestration: future cuts will be more drastic, but they will be relatively more favorable to $I$. 22

Denote by $V_I^D$ and $V_I^S$ the incumbent’s value from delaying and sequestering, respectively. Ignoring the constant in the utility function, and using the time-1 sequestration rule, we obtain:

$$
V_I^S = (1 + \alpha^{1-\sigma}) \left( \frac{g_I^0}{g_O^0 + g_I^0} \right)^{1-\sigma} \left( \tau + \frac{1 - \alpha}{1 + \alpha} \right)^{1-\sigma} \frac{1}{1 - \sigma}
$$ (16)

$$
V_I^D = \left[ (\tau + b_1 - \alpha g_O^0)^{1-\sigma} + \left( \frac{\tau + b_1 - \alpha g_O^0}{\tau + b_1}(\tau - b_1) \right)^{1-\sigma} \right] \frac{1}{1 - \sigma}
$$ (17)

where $b_1$ solves the first-order condition (11). Both value functions are decreasing in $g_O^0$, while $V_I^S$ is increasing in $g_I^0$. A sequestration is triggered at $t = 1$ when $V_I^D \leq V_I^S$.

---

22To streamline the analysis we assume that avoiding a sequestration by running more debt is not penalized by financial markets. If we assumed instead that the interest rate increases, the threat of higher borrowing cost would make delay less appealing.
Assume, for simplicity, that $\alpha = 1$. When $g_0^O + g_0^I \leq \tau$, inequality (7) does not hold: pre-commitments are sustainable, and no sequestration is needed. When, instead, $g_0^O + g_0^I \geq 2\tau$, the government has a solvency problem: a sequestration is unavoidable, because the present value of taxes is larger than existing commitments. Finally, when $\tau < g_0^O + g_0^I < 2\tau$, the incumbent faces a meaningful trade-off. The government is solvent, but the fiscal adjustment must take place, in which case the incumbent can choose whether to do it immediately or leave it to the next government.

We find that a higher ratio $g_0^O / g_0^I$ generally leads to delay. Intuitively, delay is more likely to occur if the party in power has an unfavorable entitlements ratio, which she will try to improve by spending more. As total commitments increase, however, the fiscal space of the incumbent is reduced: after paying existing entitlements, the incumbent would have a small margin for improving the ratio, making delay a less profitable strategy.

In the next proposition, we assume $\alpha = 1$ and characterize the outcome focusing on the region $\mathcal{I} = \{(g_0^O, g_0^I) : \tau \leq g_0^O + g_0^I \leq 2\tau\}$. Figure 3 illustrates the incentives to delay assuming $\alpha = \tau = 1$ and $\sigma \to 1$. 

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{delay图}
\caption{Delay and Sequestration Regions}
\end{figure}
Proposition 2 (delayed sequestration) Let $\alpha = 1$.

(i) If $g^O_0 + g^I_0 = 2\tau$. Then $V^S_i \geq V^D_i$, with strict inequality whenever $g^I_0 > 0$.

(ii) If $g^O_0 + g^I_0 = \tau$. Then $V^S_i \leq V^D_i$, with strict inequality whenever $g^O_0 > 0$.

(iii) Finally, if $(g^O_0, g^I_0)$ is such that $\tau < g^O_0 + g^I_0 < 2\tau$. As $g^I_0/g^O_0 \to \infty$ delay never occurs. As $g^I_0/g^O_0 \to 0$ delay always occurs.

Proof: See Appendix B.

4.2 Checks and Balances

In the previous analysis, we have assumed that spending is downwardly rigid and that the party in office is able to unilaterally increase spending. In practice, however, executives may need the opposition’s approval to establish new spending programs or to expand the current ones. For example, in the United States it is common to have divided governments, implying that both parties must approve a policy change. To analyze this issue, we reinstate Assumption 1 and we modify the decision process. We introduce a supermajority requirement for increasing spending: the executive has the power to propose a policy change, but any such change must also be approved by the opposition. The timing of events is as follows:

1) Period $t = 1$. The executive (party $I$) makes a take-it-or-leave-it offer $(g^O_1, g^I_1, b_1)$ to party $O$, which must satisfy $g^O_1 \geq \alpha g^O_0$ and $g^I_1 \geq \alpha g^I_0$. If party $I$’s proposal is rejected, spending is equal to $(\alpha g^I_0, \alpha g^O_0)$ and debt is residually determined by the budget constraint (2).

2) Period $t = 2$. Party $I$ stays in power with probability $q$ and with probability $1-q$ party $O$ becomes the new incumbent government. Whenever condition (5) holds, past entitlements are not sustainable, and there is a sequestration as discussed in (6). If past entitlements are instead sustainable, the incumbent makes a proposal $(g^O_2, g^I_2)$ that satisfies (3) and (4). If the proposal is rejected, spending is equal to $(\alpha g^I_1, \alpha g^O_1)$.

We solve the model backwards. In the second period, the bargaining outcome is straightforward. Suppose, for instance, that party $I$ is still in power at $t = 2$. If there is no sequestration, party $I$ gives the opposition her outside option $(\alpha g^O_1)$ and keeps the remaining resources for herself.
In the first period, the incumbent chooses \((g_1^O, g_1^I, b_1)\) by solving problem (9) with the additional constraint that her proposal must be accepted:

\[
\begin{align*}
    u(g_1^O) + (1 - \Phi) \left[ qu(\alpha g_1^O) + (1 - q)u(\tau - b_1 - \alpha g_1^I) \right] + \Phi u((\tau - b_1)(1 - \psi)) \\
    \geq u(\alpha g_0^O) + \left[ qu(\alpha^2 g_0^O) + (1 - q)u(\tau - b_1^* - \alpha^2 g_0^I) \right] \tag{AC}
\end{align*}
\]

The acceptance constraint (AC) implies that the opposition accepts the incumbent’s proposal if the utility of accepting is greater than or equal to the outside option. If negotiations break down, the opposition obtains \(\alpha g_0^O\) and moves to the next period with a level of debt equal to \(b_1^*\), where \(b_1^* = \alpha g_0^O + \alpha g_0^I - \tau\). The next proposition states a stark result: budgetary stickiness, together with checks and balances, eliminates the deficit bias.

**Proposition 3** When \(\alpha = 1\), a supermajority requirement leads to zero debt for any \((g_0^O, g_0^I)\).

**Proof:** See Appendix C.

In the baseline model, we showed that when \(g_0^O > 0\) and \(\alpha = 1\), party I selects an inefficient allocation for dilution purposes. With a supermajority requirement, the dilution motive is not present anymore because party I cannot obtain the opposition’s approval to increase \(g_1^I\). When \(\alpha = 1\), efficiency is obtained: debt is exactly zero and the parties’ ratio of marginal utilities across the two periods is equalized. The initial level of preexisting commitments \((g_0^O, g_0^I)\) will determine how equitable the final allocation will be. Without a
supermajority requirement, a higher $g_0^O$ would lead to more inefficiencies, by strengthening the dilution effect. Conversely, when a supermajority is needed, a higher $g_0^O$ would preserve the zero-debt result and, by raising the opposition’s bargaining power, would lead to a more equitable spending allocation between the two parties. Figure illustrates debt accumulation as a function of $\alpha$ in the model under supermajority rule (“bargaining”) and in the baseline model (“no bargaining”). When $\alpha$ is smaller than 1, debt is still positive, but lower than without bargaining. Debt is lower because the acceptance constraint makes it more costly for party $I$ to raise debt: party $O$ accepts to increase debt only if she is given higher $g_1^O$.  

4.3 Mandatory and Discretionary Spending

We return to the baseline model: Assumption 1 holds, and the incumbent does not need to negotiate policies. In the previous analysis, we have assumed that all spending is equally sticky. We now relax this assumption to model the distinction between discretionary and mandatory spending. Since discretionary programs require annual appropriation bills, they are often assumed to be less inertial than entitlements, which are mandated by law and continue from year to year until a new law is passed that alters them. The extended utility becomes:

$$u_j(g_j, g_{-j}, d_j, d_{-j}, \ldots) = \frac{g_j^{1-\sigma} - 1}{1 - \sigma} + \frac{\theta d_j^{1-\sigma} - 1}{1 - \sigma}$$

where $\theta > 0$. Goods $d_I$ and $d_O$ represent discretionary spending. Party $I$ (party $O$) values good $d_I$ ($d_O$). To keep the analysis tractable, we assume that spending for $d_I$ and $d_O$ is not sticky and can be cut at no cost. This implies that the party in power will not provide the discretionary good that is valued by the opposition. Goods $g_O$ and $g_I$ can now be interpreted as mandatory spending. As before, the stickiness constraint (4) holds and the parameter $\alpha$ denotes the persistence of mandatory spending. The budget constraints are:

$$g_1^O + g_1^I + d_1^I + d_1^O \leq \tau + b_1$$

$$g_2^O + g_2^I + d_2^I + d_2^O \leq \tau - b_1$$

Piguillem and Riboni (2020) also find that deficit bias is reduced when the party in office negotiates policies with the opposition. In that paper, we abstract from budgetary inertia, assuming that past spending can be cut at no cost, and focus on a very different question: how fiscal rules affect debt accumulation.

The thrust of the analysis would be maintained if we assume that spending for $d_I$ and $d_O$ is also sticky due to political and bureaucratic costs, but less sticky than mandatory spending $g_I$ and $g_O$.  

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20
We assume that mandatory spending is prioritized over discretionary spending so that a sequestration is triggered only if, after reducing discretionary spending to zero, remaining resources are not sufficient to pay existing entitlements. As before, at \( t = 2 \) a sequestration occurs if \( \alpha(g_1^O + g_1^I) \geq \tau - b_1 \). Using (18) and since party \( I \) optimally sets \( d_1^O = 0 \), a sequestration occurs if

\[
b_1 \geq \frac{\tau(1 - \alpha)}{(1 + \alpha)} + \frac{\alpha d_1^I}{(1 + \alpha)}
\]

Note that the sequestration threshold now depends on the (endogenous) amount of time-1 discretionary spending. Clearly, the planner’s solution is again zero debt and a smooth path of spending. Moreover, for \( j = I, O \) and \( t = 1, 2 \), intratemporal efficiency requires

\[
\frac{d_j^t}{g_t} = \theta^{1/\sigma}.
\]

This section shows that party \( I \) will deviate from the planner’s solution by selecting a strictly positive level of debt and by overproviding mandatory spending.

We solve the equilibrium backwards. If there is a sequestration at \( t = 2 \), discretionary spending will be zero for both parties and entitlements will be cut down. Party \( I \) will obtain a share \( \psi = g_1^I/(g_1^I + g_1^O) \) of available resources. If there is no sequestration, the incumbent at \( t = 2 \) retains some discretion to allocate resources after satisfying past commitments. In this case, let \( V_2^I(g_1^I, b_1) \) be the time-2 value function of party \( I \) if she stays in power:

\[
V_2^I(g_1^I, b_1) = \max_{\{g_2^I, d_2^I\}} u(g_2^I) + \theta u(d_2^I)
\]

s.t. \( g_2^I + d_2^I + \alpha^2 g_0^O \leq \tau - b_1 \)

\[
g_2^I \geq \alpha g_1^I
\]

Because the constraint (21) might be binding, we obtain:

\[
g_2^I = \max \left\{ \frac{1}{1 + \theta^{1/\sigma}}(\tau - b_1 - \alpha^2 g_0^O), \alpha g_1^I \right\}
\]

\[
d_2^I = \tau - b_1 - \alpha^2 g_0^O - g_2^I
\]

Let \( \lambda \) denote the multiplier associated to the constraint (21), which will be binding when \( \alpha \)

\[25\] We could also add a required minimum level of discretionary spending, without changing the results.
is sufficiently large. We now move to the first period.

\[
\max_{\{g_l^1, d_l^1, b_1\}} \left\{ u(g_l^1) + \theta u(d_l^1) + (1 - \Phi) \left[ q V_2^I (g_l^1, b_1) + (1 - q) u(\alpha g_l^1) \right] + \Phi u((\tau - b_1) \psi) \right\}
\]

\[
s.t. \quad \tau + b_1 \geq g_l^1 + d_l^1 + \alpha g_0^O
\]

We consider the case in which the incumbent chooses allocations in the nonsequestration region. In this section, we provide the main equations to help intuition; we refer to Appendix D for more details. Knowing that \( \partial V_2^I / \partial g_l^1 = -\alpha \lambda \), the intratemporal and intertemporal allocations are characterized by

\[
[1 + (1 - q)\alpha^{1 - \sigma}] (g_l^1)^{-\sigma} = q \alpha \lambda + \theta (d_l^1)^{-\sigma},
\]

\[
d_l^1 = q^{1/\sigma} d_l^1. \tag{24}
\]

We obtain several results. First, condition (24) implies that when \( I \) stays in power (and there is no sequestration) discretionary spending decreases over time. Intuitively, a higher probability of losing power implies that the incumbent will frontload discretionary spending more. Condition (24) resembles the dynamics in the standard model à la Alesina and Tabellini (1990), with only discretionary spending. Surprisingly, equation (24) implies that the ratio does not vary with \( \alpha \). As it is clear from equation (23), budgetary persistence does affect the intratemporal allocation between mandatory and discretionary spending, but given this “level” distortion the dynamics of discretionary spending is unaffected.

Second, mandatory spending is overprovided compared to efficiency. Two mechanisms are affecting the intratemporal allocation. The first one, captured by the term \( (1 - q)\alpha^{1 - \sigma} \) in equation (23), is the insurance effect due to the presence of turnover risk that can be hedged with mandatory spending. The second one is the term \( q \alpha \lambda \) in the same equation, which captures the possibility that if the incumbent remains in power, she will not be able to provide her desired level of spending. The equilibrium spending ratio satisfies:

\[
\frac{d_l^1}{g_l^1} = \begin{cases} \frac{\theta^2}{(1 + (1 - q)\alpha^{1 - \sigma})^{1/\sigma}}, & \text{if } \lambda = 0 \\ \frac{\theta^2 (1 + \alpha)^{1/\sigma}}{(1 + \alpha^{1 - \sigma})^{1/\sigma}}, & \text{if } \lambda > 0 \end{cases} \tag{25}
\]

To interpret the above expression recall that the optimal ratio is \( \theta^{1/\sigma} \) and that \( \lambda \) starts being positive when \( \alpha \) is above a certain threshold, denoted by \( \bar{\alpha} > 0 \). Start by considering
values of $\alpha < \tilde{\alpha}$. Because $\lambda = 0$, the future level of mandatory spending remains uncertain. When $\alpha = 0$, the equilibrium ratio coincides with the efficient one because mandatory spending cannot be used to smooth spending. As $\alpha$ increases, $I_1^T$ becomes more valuable as an insurance tool, so that the incumbent tilts spending towards it. When $\alpha \geq \tilde{\alpha}$, the uncertainty about $I_2^T$ disappears. Because $\lambda > 0$, independently of who will be in power, the current incumbent enjoys the same level of mandatory spending $\alpha g_1^T$ in the second period. For this reason, $q$ vanishes in the second line of (25). As long as $\alpha < 1$, the incumbent is still left with a dynamic inefficiency, because to obtain one unit of consumption tomorrow, it must spend $1/\alpha$ units today. Thus, the underprovision of discretionary spending remains. But as $\alpha \to 1$, the incumbent can obtain full insurance and perfect intertemporal smoothing: the distortive effect of persistence vanishes and the intratemporal spending ratio converges to the optimal one.

The underprovision of discretionary spending is shown in Figure 5, which illustrates mandatory and discretionary spending in the first period as a function of $\alpha$. In Panel A we assume $g_0^O = 0$ and $\theta = 1$, which implies that the ratio $d_1^T/g_1^T$ should be one. When $\alpha = 0$ the ratio $d_1^T/g_1^T$ coincides with the efficient one. When $\alpha \in (0,1)$ the ratio is less than one, decreasing for small $\alpha$’s and then increasing again. In Panel B, we show that when $g_0^O > 0$, there is an additional effect due to the possibility of a sequestration on the equilibrium path. We will discuss sequestration further below.

Third, as in Section 3, debt is a nonmonotone function of $\alpha$ and the dilution effect
is present when $g_0^O > 0$ (see Figure 6). There are, however, some qualitative differences. To understand them, we present in (26) the closed-form solution for debt accumulation (conditional on no future sequestration), which is illustrated in Panel A of Figure 6:

$$
\begin{align*}
\Xi_0 &= \frac{q^{\frac{1}{\sigma}}(1 + \theta^{1/\sigma})}{(1 + (1 - q)\alpha^{1-\sigma})^{\frac{1}{\sigma}} + \theta^{1/\sigma}} \leq 1, \quad \text{and} \quad \Xi_1 = \left( \frac{\theta(1 + \alpha)}{(1 + \alpha^{1-\sigma})} \right)^{\frac{1}{\sigma}}.
\end{align*}
$$

From the first line of (26), debt is increasing in $\alpha$ when persistence is low and $g_0^O = 0$. This follows because $\Xi_0$ is decreasing in $\alpha$ as long as $\sigma < 1$. As in Section 3, budgetary persistence makes mandatory spending valuable, leading to over-accumulation of debt. When $\alpha$ is sufficiently large, the multiplier $\lambda$ becomes positive: the incumbent realizes that, despite whoever will be in power, she will enjoy the same level of mandatory spending. As a result, the incumbent internalizes the value of future resources and debt starts decreasing.

Another important difference compared to Section 3 is that in Panel A debt is strictly positive for all $\alpha \in [0, 1]$. We obtain this result because party I does not fully commit period-
2’s budget. Debt is strictly below the sequestration threshold because party $I$ wants to have some fiscal space to provide discretionary spending in case she is reelected. Since some risk remains, the cost of debt is not fully internalized even when $g_0^O = 0$ and $\alpha = 1$.

As in the benchmark setting, Panel B of Figure 6 shows that sequestration occurs when $g_0^O > 0$ and $\alpha$ is large enough. Note that there is now a discontinuous jump. Since triggering a sequestration has a sizable welfare cost (because discretionary spending is zero), triggering a “small” sequestration would be inefficient. The first-order conditions under sequestration are discussed in Appendix D.

5 Conclusions

The unsustainability of current policies is the “fiscal problem of the 21st century” (Jones (2003)). In the past two decades, bipartisan commissions have called for drastic entitlement-spending reforms. Politicians are not only not taking action, they seem to be following the opposite path: in recent years, their spending decisions have been worsening the U.S. fiscal balance. It is possible that politicians underestimate the magnitude of the problem, but we provide another, possibly complementary, explanation. We show that when budgets are sticky, the prospect of stabilization makes politicians fiscally irresponsible: when debt is on an unsustainable path under current policies, politicians have the incentive to worsen the fiscal balance by spending even more to “dilute” existing entitlements.

Our model provides several insights on how to avoid this spending spiral. A widely held view is that budgetary inertia is responsible for high debt buildups. Along these lines, for instance, the OECD has recently recommended sunset clauses (specifying a date beyond which spending programs automatically expires) to reduce spending inertia. In addition to mechanically affecting debt, we show that inertia changes the incentives to spend. Because of these strategic channels, our results show that reducing budgetary inertia may worsen the debt problem. We show that when political institutions combine budgetary stickiness with a supermajority requirement for enacting policy changes, debt is reduced and, in some cases, eliminated. In the United States, Republicans and Democrats alike have passed several bills and tax reforms by using the so-called budget-reconciliation process, which allows the Senate to bypass the supermajority requirement. One conclusion of our analysis is that enforcing the Senate’s 60% rule would lower U.S. debt.

See OECD (2012).
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APPENDIX

A Proof of Proposition 1

To prove part (i) of the proposition, we compute equilibrium debt when $\alpha = 0$ and when $\alpha = 1$. When $\alpha = 0$, a sequestration never occurs because (5) never holds. The first-order condition is then

$$(\tau + b_1)^{-\sigma} = (\tau - b_1)^{-\sigma} q,$$  \hspace{1cm} (27)

which can be easily solved as follows:

$$b_{\alpha=0}^* = \frac{\tau(1 - q^2)}{(1 + q^2)}.$$  \hspace{1cm} (28)

We first show that party I does not choose a level of debt strictly below the sequestration threshold $\tilde{b}$. When $\alpha = 1$, recall that $\tilde{b} = 0$. To show that $b_1 < 0$ is not optimal, take the first-order condition (10) when $\alpha = 1$ under no sequestration:

$$(\tau + b_1 - g_0^O)^{-\sigma} = \frac{q}{2 - q}(\tau - b_1 - g_0^O)^{-\sigma}.$$  \hspace{1cm} (29)

Since $q/(2 - q) \leq 1$ we have that $b_1 < 0$ is not optimal. The first-order condition under sequestration (11) can be written as:

$$(\tau + b_1 - \alpha g_0^O)^{-\sigma} = (\frac{\tau - b_1}{\tau + b_1})(\tau + b_1 - \alpha g_0^O)^{-\sigma}(1 - \frac{2\tau}{(\tau + b_1)^2 \alpha g_0^O})$$  \hspace{1cm} (30)

After simplifying both sides, rewrite the above condition as

$$(\tau + b_1)^{-\sigma} = (\tau - b_1)^{-\sigma}(1 - \frac{2\tau}{(\tau + b_1)^2 \alpha g_0^O})$$  \hspace{1cm} (31)

Since the right-hand side in (31) is increasing in $b_1$, while the left-hand side is decreasing, the solution is unique. For example, as $\sigma \to 1$ and $\alpha = 1$, we obtain a simple closed-form solution for $b_1$, which is given by (13).

To show that $b_{\alpha=0}^* < b_{\alpha=1}^*$ we need to show that when (31) is evaluated at $b_{\alpha=0}^*$ we have

$$(\tau + b_{\alpha=0}^*)^{-\sigma} > (\tau - b_{\alpha=0}^*)^{-\sigma}(1 - \frac{2\tau}{(\tau + b_{\alpha=0}^*)^2 \alpha g_0^O})$$  \hspace{1cm} (32)

That is, the marginal benefit of debt is higher than the marginal cost of debt, implying that
optimal debt is higher than \( b_{\alpha=0}^* \). Focusing on \( \alpha = 1 \), and using the first-order condition \( \alpha = 1 \), and using the first-order condition [28], the inequality above can be written as:

\[
(1 - \frac{2\tau}{(\tau + b_{\alpha=0}^*)^2} g_0^O) < q
\]  

(33)

Plugging (28) we obtain

\[
(1 - \frac{(1 + q^{1/\sigma})^2}{2\tau} g_0^O) < q
\]  

(34)

or

\[
g_0^O > \frac{2\tau (1 - q)}{(1 + q^{1/\sigma})^2}
\]  

(35)

This condition implies that \( b_{\alpha=0}^* < b_{\alpha=1}^* \).

We now prove the second part of Proposition 1. We first show that the partial derivative of equilibrium debt is positive when evaluated at \( \alpha = 0 \). When \( \alpha \) is close to zero, sequestration is not triggered. One can thus compute the interior solution for debt as follows:

\[
b_1^* = \frac{(1 + \alpha^{-\sigma}(1-q))^{1/\sigma} - 1}{\alpha g_0^O}
\]  

(36)

We let \( D \) and \( N \) denote, respectively, the denominator and the numerator in (36). Moreover, we define \( \Delta \equiv (1 + \alpha^{1-\sigma}(1-q)) \).

\[
\frac{\partial b_1^*}{\partial \alpha} = \frac{(\tau - N)\alpha^{-\sigma}(1-q)}{\Delta(1/\sigma - 1)} + g_0 q^{1/\sigma} D - D2\alpha g_0^O \Delta(1/\sigma) - \alpha^{-\sigma} g_0^{1/\sigma}(\Delta^{1/\sigma})(1/\sigma - 1)(1-q)D
\]  

(37)

When \( \alpha \) is close to zero, we have \( \tau D > N \). Then, the limit of (37) is \(+\infty\) as \( \alpha \to 0 \) (provided that \( \sigma < 1 \)). Therefore, equilibrium debt is increasing in \( \alpha \) when \( \alpha \) is sufficiently close to zero. When instead \( \alpha \) is close to one (hence, sequestration is triggered), from (31) equilibrium debt is increasing in \( \alpha \) when \( g_0^O > 0 \). When \( \alpha = 1 \) and \( g_0^O = 0 \), equilibrium debt is zero. This discussion proves that equilibrium debt is a nonmonotone function of \( \alpha \). \( \Box \)

## B Proposition of Proposition 2

Assume \( \alpha = 1 \). Since Proposition 2 focuses on the region \( \mathcal{I} = \{(g_0^O, g_0^I) : \tau \leq g_0^O + g_0^I \leq 2\tau \} \), there are three cases to consider.
(i) Assume $g_0^O + g_0^I = 2\tau$. From (16) the value of sequestering is:

$$V_i^S = \frac{2}{1-\sigma} \left( \frac{g_0^I}{g_0^O + g_0^I} \right)^{1-\sigma} (\tau)^{1-\sigma}$$  (38)

If, instead, sequestration is delayed, party $I$ pays existing commitments by choosing the maximum possible amount of debt, $b_1 = \tau$. As a result, in the second period, spending will be zero. Therefore, from (17) the value of delaying the sequestering is:

$$V_i^D = \frac{(g_0^I)^{1-\sigma}}{1-\sigma}$$  (39)

When $g_0^I = 0$, we have $V_i^S = V_i^D$. When $g_0^I > 0$ we have $V_i^S > V_i^D$ if and only if

$$2\tau^{1-\sigma} > (g_0^O + g_0^I)^{1-\sigma}$$  (40)

From $g_0^O + g_0^I = 2\tau$, it follows that $V_i^S > V_i^D$ when $\sigma > 0$. By continuity, $V_i^S > V_i^D$ for $(g_0^O, g_0^I) \in I$ close to $g_0^O + g_0^I = 2\tau$ and $g_0^I > 0$.

(ii) Assume now $g_0^O + g_0^I = \tau$. We compute the values of sequestering and delaying:

$$V_i^S = 2(g_0^I)^{1-\sigma}$$  (41)

$$V_i^D = (g_0^I + b_1)^{1-\sigma} + \left( \frac{g_0^I + b_1}{\tau + b_1} (\tau - b_1) \right)^{1-\sigma}$$  (42)

where $b_1$ solves the first-order condition under sequestration. Note that the two expressions coincide if $b_1$ is zero. When $g_0^O = 0$, optimal debt is indeed zero, and thus we obtain $V_i^S = V_i^D$. When $g_0^O > 0$ it is immediate to verify that optimal debt is strictly positive, implying that $V_i^S < V_i^D$. By continuity, $V_i^S < V_i^D$ for $(g_0^O, g_0^I) \in I$ close to $g_0^O + g_0^I = \tau$ and $g_0^O > 0$.

(iii) Finally, assume $g_0^O + g_0^I < 2\tau$. The values of sequestering and delaying are:

$$V_i^S = 2 \left( \frac{g_0^I}{g_0^O + g_0^I} \right)^{1-\sigma} (\tau)^{1-\sigma}$$  (43)

$$V_i^D = (\tau + b_1 - g_0^O)^{1-\sigma} + \left( \frac{\tau + b_1 - g_0^O}{\tau + b_1} (\tau - b_1) \right)^{1-\sigma}$$  (44)

When $(g_0^O, g_0^I) \in I$, the ratio $g_0^I/g_0^O$ goes to $+\infty$ when $g_0^O$ goes to zero. In this case, $V_i^S >
$V^D_I$ since party $I$ can implement her preferred allocation under commitment by triggering a sequestration at $t = 1$. Finally, $(g^O_0, g^I_0) \in \mathcal{I}$ the ratio $g^I_0/g^O_0$ goes to zero, when $g^I_0$ goes to zero. In this case, $V^S_I$ converges to zero and, consequently, $V^D_I > V^S_I$. □

C Proof of Proposition 3.

Assume $\alpha = 1$ and let the initial entitlements $(g^O_0, g^I_0)$ be given.

**Step 1:** Assume that a sequestration is triggered at $t = 2$. We show that zero debt is optimal.

Assume that party $I$ chooses $b_1 \geq \tilde{b}$. We will verify this claim later in Step 2. Since $\alpha = 1$, a sequestration occurs when $b_1 \geq 0$. By triggering a sequestration political risk vanishes in the second period. We define the variable $\chi \equiv \frac{\tau - b_1}{\tau + b_1} \leq 1$. The agenda setter’s problem can be written as:

$$\max_{\{b_1, g^O_1, g^I_1\}} u(\tau + b_1 - g^O_1) + u(\chi g^I_1)$$

subject to

$$u(g^O_1) + u(\chi g^I_1) \geq u(g^O_0) + qu(g^O_0) + (1 - q)u(2\tau - 2g^I_0 - g^O_0)$$

and

$$g^I_1 + g^O_1 = \tau + b_1$$

Let $\lambda$ be the multiplier associated to the acceptance constraint. Taking derivatives with respect to $g^O_1$ we obtain

$$u'(\tau + b_1 - g^O_1) + \chi u'(\chi g^I_1) = \lambda [u'(g^O_1) + \chi u'(\chi g^O_1)]$$

$$u'(g^I_1)(1 + \chi^{1-\sigma}) = \lambda [u'(g^O_1)(1 + \chi^{1-\sigma})]$$

which gives us

$$\lambda = \frac{u'(\tau + b_1 - g^O_1)}{u'(g^O_1)}$$

(45)

Next, we take the derivative with respect to $b_1$ and obtain
\[ u'(\tau + b_1 - g_1^O) + u'(\chi(\tau + b_1 - g_1^O)) \left[ \frac{-2\tau}{(\tau + b_1)^2} (\tau + b_1 - g_1^O) + \frac{(\tau - b_1)}{\tau + b_1} \right] \\
\quad - \lambda \left[ u'(\chi g_1^O) g_1^O \frac{2\tau}{(\tau + b_1)^2} \right] = 0 \] 

(46)

After inserting (45) we verify that when \( b_1 = 0 \), the above condition is satisfied. In fact,

\[ u'(\tau - g_1^O) + u'(\tau - g_1^O) \left[ -\frac{2}{\tau} (\tau - g_1^O) + 1 \right] - \frac{2u'(\tau - g_1^O)}{\tau} g_1^O = 0 \] 

(47)

or

\[ u'(\tau - g_1^O) = u'(\tau - g_1^O) \] 

(48)

Zero debt is therefore optimal. Note that \( g_1^O \) solves the acceptance constraint with equality, for given initial default option \((g_0^O, g_0^I)\). That is,

\[ 2u(g_1^O) = u(g_0^O) + qu(g_0^O) + (1 - q)u(2\tau - 2g_0^I - g_0^O) \] 

(49)

**Step 2:** When \( \alpha = 1 \), \( \tilde{b} = 0 \). We show that choosing \( b_1 < 0 \) is not optimal.

When there is no sequestration, uncertainty in the second period is maintained. The agenda setter’s problem is written as follows:

\[
\max_{\{b_1, g_1^O, g_1^I\}} u(\tau + b_1 - g_1^O) + qu(\tau - b_1 - g_0^O) + (1 - q)u(\tau + b_1 - g_1^O)
\]

subject to

\[ u(g_1^O) + qu(g_1^O) + (1 - q)u(g_1^O - 2b_1) \geq u(g_0^O) + qu(g_0^O) + (1 - q)u(2\tau - 2g_0^I - g_0^O) \]

We take the derivatives with respect to \( g_1^O \):

\[-u'(\tau + b_1 - g_1^O)(2 - q) - qu'(\tau - b_1 - g_1^O) \]
\[+ \lambda \left[ u'(g_1^O)(1 + q) + u'(g_1^O - 2b_1)(1 - q) \right] = 0. \] 

(50)
We take the derivatives with respect to $b$:

\[
u'((\tau + b_1 - g_1^0)(2 - q) - qu'(\tau - b_1 - g_1^0) - \lambda [(1 - q)u'(g_1^O - 2b_1)2] = 0.\]

(51)

We evaluate the first-order condition (50) at $b_1 = 0$:

\[
\lambda = \frac{u'((\tau - g_1^O)(1 - q))}{2u'(g_1^O)}. \tag{52}
\]

The first-order condition (51) evaluated at $b_1 = 0$ does not hold. In fact,

\[
u'((\tau - g_1^O)(2 - q) - qu'(\tau - g_1^O) - \frac{u'((\tau - g_1^O)(1 - q))[2(1 - q)u'(g_1^O)]}{2u'(g_1^O)} > 0. \tag{53}\]

Thus, the two parties fully commit all the resources in the second period: $\chi \leq 1$. □

D Derivation of Equations in Section 4.3.

D.1 Second period without sequestration.

The second period problem can be written as:

\[
V_I^I(g_1^I, b_1) = \max_{g_2^I \geq \alpha g_1^I} \frac{(g_2^I)^{1-\sigma} + \theta(\tau - b_1 - \alpha^2 g_0^O - g_2^I)^{1-\sigma}}{1 - \sigma}.
\]

Letting $\lambda$ be the multiplier of the constraint, the first-order conditions are:

\[
g_2^I : (g_2^I)^{-\sigma} + \lambda = \theta(d_2^I)^{-\sigma} = \theta(\tau - b_1 - \alpha^2 g_0^O - g_2^I)^{-\sigma},
\]

\[
\lambda : \lambda(g_2^I - \alpha g_1^I) = 0.
\]

When $\lambda = 0$, it is immediate to compute that:

\[
g_2^I = \frac{1}{1 + \theta^{1/\sigma}(\tau - b_1 - \alpha^2 g_0^O)}
\]
which leads to equation (22). If this satisfies constraint (21), it is the solution, otherwise:

\[
g_2^I = \alpha g_1^I, \\
\lambda = \theta (d_2^I)^{-\sigma} - (g_2^I)^{-\sigma}.
\]  

(54)

For any \( \lambda \) the solution implies \( d_2^I = \tau - b_1 - \alpha^2 g_0^O - \alpha g_1^I \). Note that by the envelope theorem:

\[
\frac{\partial V'_2(g_1^I, b_1)}{\partial g_1^I} = -\alpha \lambda, \quad \frac{\partial V'_2(g_1^I, b_1)}{\partial b_1} = -\theta (d_2^I)^{-\sigma}.
\]  

(55)

Note that the second derivative is the same whether the constraint binds or not.

D.2 First period without sequestration.

Denoting by \( \mu \) the multiplier of the budget constraint, the first-order necessary conditions are as follows:

\[
g_1^I : (g_1^I)^{-\sigma} + q \frac{\partial V'_2(g_1^I, b_1)}{\partial g_1^I} + (1 - q) \alpha^{1-\sigma} (g_1^I)^{-\sigma} = \mu, \\
d_1^I : \theta (d_1^I)^{-\sigma} = \mu, \\
b_1 : q \frac{\partial V'_2(g_1^I, b_1)}{\partial b_1} = -\mu.
\]

Note that using the last two equations together with (55), generates the Euler equation:

\[
d_2^I = q^{\frac{1}{\sigma}} d_1^I
\]  

(56)

which is independent on whether \( \lambda \) is positive in the second period.

If in the second period \( \lambda = 0 \), because of equation (55), the first two optimality conditions deliver equations (57) and (58):

\[
d_1^I* = \left( \frac{\theta^{1/\sigma}}{(1 + (1 - q)\alpha^{1-\sigma})^{\frac{1}{\sigma}} + \theta^{1/\sigma}} \right) (\tau + b_1^* - \alpha g_0^O) \\
g_1^I* = \left( \frac{(1 + (1 - q)\alpha^{1-\sigma})^{\frac{1}{\sigma}}}{(1 + (1 - q)\alpha^{1-\sigma})^{\frac{1}{\sigma}} + \theta^{1/\sigma}} \right) (\tau + b_1^* - \alpha g_0^O)
\]  

(57)

(58)

Computing the ratio between the two types of spending delivers the first line of expression
Replacing $d_1^I$ and $d_2^I$ in equation (56), the level of debt must satisfy:

$$
\frac{\tau - b_1^* - \alpha^2 g_0^O}{1 + \theta^{1/\sigma}} = q^{\frac{\sigma}{2}} \left( \frac{\tau + b_1^* - \alpha g_0^O}{(1 + (1 - q)\alpha^{1-\sigma})^{\frac{\sigma}{2}} + \theta^{1/\sigma}} \right)
$$

Defining $\Xi_0 = \frac{q^{\frac{\sigma}{2}(1+\theta^{1/\sigma})}}{(1+(1-q)\alpha^{1-\sigma})^{\frac{\sigma}{2}} + \theta^{1/\sigma}} \leq 1$, by simple algebra it follows that:

$$
b_1^* = \frac{\tau(1 - \Xi_0) - \alpha g_0^O(\alpha - \Xi_0)}{1 + \Xi_0}, \text{ if } \lambda = 0 \quad (59)
$$

When $\lambda > 0$, using (55), we can combine the two first order conditions into:

$$
[1 + (1 - q)\alpha^{1-\sigma}](g_1^I)^{-\sigma} = q\alpha \lambda + \theta(d_1^I)^{-\sigma}
$$

Considering that when $\lambda > 0$, it must be that $g_2^I = \alpha g_1^I$, and replacing equation (54) in the last one:

$$
[1 + (1 - q)\alpha^{1-\sigma}](g_1^I)^{-\sigma} = q\alpha[\theta(d_2^I)^{-\sigma} - (\alpha g_1^I)^{-\sigma}] + \theta(d_1^I)^{-\sigma}
$$

$$
[1 + \alpha^{1-\sigma}](g_1^I)^{-\sigma} = \theta[\alpha q(d_2^I)^{-\sigma} + (d_1^I)^{-\sigma}]
$$

Replacing the Euler equation (56), one obtains:

$$
[1 + \alpha^{1-\sigma}](g_1^I)^{-\sigma} = \theta(1 + \alpha)(d_1^I)^{-\sigma}
$$

Computing the ratio $d_1^I/g_1^I$ delivers the second line of expression (25) in Section 4.3. Using the budget constraint, it is straightforward to show that:

$$
d_1^I = \frac{\Xi_1}{1 + \Xi_1}(\tau + b_1^* - \alpha g_0^O) \quad (60)
$$

$$
g_1^I = \frac{1}{1 + \Xi_1}(\tau + b_1^* - \alpha g_0^O) \quad (61)
$$

where $\Xi_1 = \left(\frac{\theta(1+\alpha)}{(1+\alpha^{1-\sigma})}\right)^{\frac{1}{\sigma}}$. Here the insurance effect makes the allocations independent of $q$. Equation (61) implies that in the second period discretionary spending must be:

$$
d_2^I = \tau - b_1^* - \alpha^2 g_0^O - \alpha g_1^I = \tau - b_1^* - \alpha^2 g_0^O - \alpha \frac{1}{1 + \Xi_1}(\tau + b_1^* - \alpha g_0^O)
$$
As a result:

\[ d_I^* = \frac{(\tau - \alpha^2 g_0^O)(1 - \alpha + \Xi_1) - b_1^*(1 + \alpha + \Xi_1)}{1 + \Xi_1} \]

Since the Euler equation is still valid, using equations (60) and the last expression, the optimal level of debt solves:

\[ (\tau - \alpha^2 g_0^O)(1 - \alpha + \Xi_1) - b_1^*(1 + \alpha + \Xi_1) = q^1\Xi_1(\tau + b_1^* - \alpha g_0^O) \]

As a result:

\[ b_1^* = \frac{\tau((1 - q^1)\Xi_1 + 1 - \alpha) - \alpha g_0^O(\alpha(1 - \alpha + \Xi_1) - q^1\Xi_1)\psi}{(1 + q^1)\Xi_1 + 1 + \alpha} \]

(62)

D.3 First period with sequestration.

If the incumbent chooses a level of debt in which sequestration is triggered, the optimal allocation of spending and debt solves:

\[
\max_{\{g_1^I, d_1^I, b_1^I\}} \left\{ u(g_1^I) + \theta u(d_1^I) + u(\tau - b_1^I)\psi \right\} \\
\text{s.t.} \quad \tau + b_1^I \geq g_1^I + d_1^I + \alpha g_0^O \\
\psi = g_1^I / (g_1^I + \alpha g_0^O) 
\]

Letting \( \mu \) be the multiplier in the budget constraint, the first order necessary conditions are:

\[ g_1^I : (g_1^I)^{-\sigma} + ((\tau - b_1^I)\psi)^{-\sigma} \frac{\alpha g_0^O(\tau - b_1^I)}{(g_1^I + \alpha g_0^O)^2} = \mu, \]

\[ d_1^I : \theta(d_1^I)^{-\sigma} = \mu, \]

\[ b_1 : \psi((\tau - b_1^I)\psi)^{-\sigma} = \mu. \]

Combining the first and the last equations, and using the budget constraint we obtain:

\[ (g_1^I)^{-\sigma} = ((\tau - b_1^I)\psi)^{-\sigma} \left[ \psi - \frac{\alpha g_0^O(\tau - b_1^I)}{(\tau + b_1^I - d_1^I)^2} \right] \]

Note that if either \( g_0^O = 0 \) or \( \alpha = 0 \) then we have \( \psi = 1 \) and \( d_1^I = \theta^\frac{1}{\sigma} g_1^I \). Then, the Euler
equation becomes:

\[
\left( \frac{\tau + b_1}{1 + \theta \tau} \right)^{-\sigma} = (\tau - b_1)^{-\sigma}
\]

Replacing \(\psi\), one obtains

\[
(g^I_1 + \alpha g^O_0)^{-\sigma} = (\tau - b_1)^{-\sigma}
\left[ \frac{g^I_1}{g^I_1 + \alpha g^O_0} - \frac{\alpha g^O_0 (\tau - b_1)}{(g^I_1 + \alpha g^O_0)^2} \right].
\]