FISCAL LIMITS AND THE PRICING OF EUROBONDS

KEVIN PALLARA AND JEAN-PAUL RENNE

ABSTRACT. This paper proposes a methodology to price bonds jointly issued by a group of countries—Eurobonds in the euro-area context. We consider two types of bonds: the first is backed by several and joint (SJG) guarantees, the second features several but not joint (SNJG) guarantees. The pricing of SJG and SNJG bonds reflects different assumptions regarding the pooling of debtors' fiscal resources. We estimate fiscal limits for the four largest euro-area economies over 2008-2021 and deduce counterfactual Eurobond prices. Amid the euro-debt crisis, 5-year SNJG bond yield spreads would have been about three times larger than SJG ones. Hence, issuing SJG bonds would result in gains at the aggregate level. We finally show that (i) the gains stemming from the issuance of SJG bonds could be shared among countries through post-issuance redistribution schemes and that (ii) these schemes may alleviate the reduction in market discipline resulting from common bonds issuances.

JEL: C58, E43, F34, G12.

Keywords: Sovereign Credit Risk, Eurobonds, Public Debt Management, Joint Liability, Term Structure of Interest Rates.

This version: November 22, 2021.

Renne: University of Lausanne, Quartier Chamberonne, CH-1015 Lausanne, Switzerland (email: jean-paul.renne@unil.ch); Pallara: University of Lausanne (email: kevin.pallara@unil.ch). This work is supported by the "Fiscal Limits and the Pricing of Sovereign Debt" project funded by the Swiss National Science Foundation (SNSF) under Grant No 182293. The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper. We are grateful to Marielle de Jong and Irina Zviadadze for having discussed this paper, as well as participants at the AFSE 2021 annual meeting, the 2021 annual meeting of the International Association for Applied Econometrics, the 2021 annual meeting of the Society for Financial Econometrics, the 2021 spring meeting of the French Finance Association, the 14th Financial Risks International Forum, the 2021 North American Summer Meeting of the Econometric Society, the 2021 European Summer Meeting of the Econometric Society, seminar at LUISS university, the CEPR-Banque de France conference on "Monetary Policy, Fiscal Policy and Public Debt in a Post COVID World". Complete R codes are available upon request.

1. INTRODUCTION

Following the last financial crisis and the COVID-19 pandemic, sovereign debts across the euro area have risen to levels unprecedented since the Second World War. In this context, the sustainability of fiscal positions—especially in the peripheral Member States—has been called into question. Against this backdrop, numerous academics, policymakers, and analysts have discussed proposals for issuing common bonds—often referred to as Eurobonds. The rationale behind such common bonds is most often, and more or less explicitly, a debt service relief for peripheral member states (Beetsma and Mavromatis, 2014; Favero and Missale, 2012). An ulterior motive backing common issuances is to ensure financial stability, notably by addressing the demand of financial institutions for safe assets (Brunnermeier et al., 2017).[1] Moreover, if issued on a large scale, a joint debt instrument is advocated as a useful device to increase bond market liquidity in the euro area (Hellwig and Philippon, 2011).

Surprisingly, the different proposals for common debt issuance seldom come with pricing attempts.[2] Arguably, this shortage of quantitative analysis may have contributed to the lack of support for common bond issuances. This paper offers a way to explore the pricing of joint sovereign debt instruments.

Guarantees play a significant role in the pricing of joint debt instruments. Our analysis focuses on two polar cases: (a) the case of several and joint guarantees (SJG) whereby all countries are jointly liable for each other's default through the common debt instrument and (b) the case of several but not joint guarantees (SNJG) whereby each debtor is responsible only for a percentage contribution to each redemption. In the former case, participating European countries are responsible not only for their own percentage contribution to the bond, but also for covering any other state's unpaid contributions. In the latter case (SNJG bonds), each participant is liable only for the debt service and principal redemption corresponding to its share of the bond. In both cases, the joint debt instrument would trade as a single bond; it could be issued by an independent debt agency, with funds raised, and obligations divided

[1] Although Eurobonds may constitute a way to guide the euro area towards financial stability, the objectives of Eurobond proposals do not fully overlap with those of the European Financial Stability Facility's (EFSF) and the European Stability Mechanism's (ESM) programs. Typically, the objective of the ESM is to provide financial assistance to euro-area countries experiencing, or threatened by, severe financing problems; this would complement joint issuance in times of financial distress, but goes beyond the preventive intention of a common euro-area bond.

[2] The evaluation of price effects remains merely speculative in this literature (Claessens, Mody, and Vallée, 2012, end of Section IV.B).

between participating issuers in fixed shares. The obligations of each participating country would depend on the size of the funds obtained (see, e.g., De Grauwe and Moesen, 2009; European Commission, 2011; Delivorias and Stamegna, 2020).

In the present paper, we propose a multi-country credit-risk model where both standard and common sovereign bonds—featuring one of the two polar types of guarantees discussed above—can be priced. The model estimation relies on national bond prices; the sample covers the period from 2008Q1 to 2021Q2.[3] The estimation . We focus on the four largest euro-area economies: Germany, France, Italy, and Spain (making up 75% of the entire euro-area GDP). Once the model is estimated, we compute counterfactual Eurobond prices over the same period.

In the model, the probability of default depends on the considered entity's fiscal space, which can be a single country or a group of countries. The fiscal space is the distance between public debt and the so-called fiscal limit; this limit, in turn, represents the maximum outstanding debt that can credibly be covered by future primary budget surpluses (Bi, 2012; Bi and Leeper, 2013). The probability of default gets strictly positive only if public debt breaks the fiscal limit, that is, if the fiscal space is negative. In this framework, a natural way to conceive a SJG bond is to consider that it is issued by an entity for which both underlying debtors' fiscal revenues and debts are pooled. By contrast, a SNJG bond is equivalent to a combination of national bonds weighted by their participation share in the debt instrument.

Estimating the model involves the estimation of both the model parameters and the time series of national fiscal limits. These two tasks are jointly carried out by resorting to an adaptation of the so-called "inversion technique" *à la* Chen and Scott (1993). For a given model parameterization, formulas for the sovereign bond yield spreads are inverted to get fiscal limit estimates.[4] The maximum likelihood function can then be computed, and it is maximized to yield the estimated model parameterization.

[3]Some bonds issued by European institutions can be seen as proxies for Eurobonds (see end of Subsection 6.1, where we compare our model-implied SJG prices with the latter). However, for the time being, there are not enough of these bonds to determine constant-maturity interest rates on a sufficiently long sample.

[4]We posit reduced-form dynamics for national debts and fiscal limits and derive resulting bond pricing. Our approach shares some similarities with the Black-Scholes-Merton model (Black and Scholes, 1973; Merton, 1974, and its numerous extensions) in that it also features a default threshold. As noted by Duffie and Singleton (2003, Subsection 3.2.2), the tractability of the Black-Scholes-Merton model rapidly declines as one allows for a time-varying default threshold. Although our framework features a time-varying debt threshold, tractability is preserved thanks to approximation formulas—presented in Appendix D—that build on the literature on shadow-rate term-structure models (see, e.g., Krippner, 2015; Wu and Xia, 2016).

Our model features a good fit of the observed term-structure of bond yield spreads across all countries; this fit is comparable to the one obtained in term-structure studies where default intensities are purely latent and have no macro-finance interpretation. We also obtain sizeable estimates of sovereign credit risk premiums, defined as those components of sovereign spreads that would not exist if agents were risk-neutral. Moreover, to the best of our knowledge, this paper is the first to provide time-varying estimates of fiscal limits for different euro-area countries.[5]

Our counterfactual analysis results highlight the importance of guarantees on Eurobond pricing. By design, yield spreads associated with Eurobonds featuring several but not joint guarantees (SNJG) are close to the (issuance-weighted) average of country-specific spreads. By contrast, common bonds with several and joint guarantees (SJG) benefit from fiscal diversification effects resulting in a sizeable credit spread compression: during the height of the euro-area sovereign debt crisis, the SNJG bond yield spread was three times larger than the SJG one (5-year maturity). Hence, raising funds through a joint liability debt instrument—the SJG bond—may substantially reduce *aggregate* debt service in the presence of heterogenous fiscal conditions. Interestingly, depending on the parameterization, and for shorter maturities, the yield spread associated with the SJG bond is, at times, lower than the German bond one. (The German bonds, called Bunds, are considered the safest bond in the euro area.) Even when this is not the case, i.e. when SJG yields are higher than those of the bonds issued by the best-rated countries, one can envision post-issuance redistribution schemes under which all countries eventually benefit from common issuances. One such scheme is to distribute the overall gains in such a way as to achieve a reduction in "post-redistribution yields" that is the same in all countries (w.r.t. the yield on their respective national bonds). For the 10-year maturity, this reduction would have been about 30 basis points over the estimation sample.

The main concern associated with common debt issuance usually pertains to moral hazard. Under several and joint guarantees issuance schemes, some countries might be tempted to issue more debt given that the interest rate on jointly guaranteed debt is less sensitive to an individual debt increase than non-guaranteed debt. Although our reduced-form modeling framework does not deal with moral hazard in a structural way, our findings remain valid

[5] Pallara and Renne (2020) provide time-varying estimates of fiscal limits for eight economies, but none of them is in the euro area. Moreover, in Pallara and Renne (2020), each country is considered independently from the others.

under the conditions that (i) the amount of debt issued under the SJG scheme is relatively small or that (ii) some form of ex-post redistribution of the yield gains applies. First, as long as a sizable share of countries' funding needs are met with the issuance of national bonds, the overall debt service remains sensitive to countries' indebtedness. Thus, in the absence of redistribution schemes (case (i)), a necessary condition for market discipline to remain effective is to limit the issuance of Eurobonds. In our calculation, we typically envision that jointly issued debt does not exceed 5% of total consolidated GDP. Second, we show that some ex-post yield gains' redistribution schemes may dampen moral hazard effects (case (ii)). For instance, considering the above-mentioned scheme—in which the issuance of SJG bonds ultimately translates into the same yield reduction for all involved countries—the funding costs of the different countries remain sensitive to the national fiscal conditions, thereby alleviating concerns of reduced fiscal discipline stemming from the issuance of common bonds. More precisely, for this scheme, we obtain that the slope of the curve relating post-redistribution yields to indebtedness is similar to that associated with national bonds (but, for each country, the former curve is about 30 basis points below the latter, which reflects aggregate gains).

The rest of this paper is organized as follows. Section 2 reviews related literature. The model is developed in Sections 3 (stylized version) and 4 (full-fledged version). Section 5 describes the estimation strategy. Section 6 discusses the results. Section 7 summarizes our findings and makes concluding remarks. The appendix gathers technical results; an online appendix provides additional details, proofs and results.

2. Related Literature

This paper contributes to the growing literature on sovereign credit risk and its pricing. Specifically, this paper is among the first to provide a quantitative assessment of Eurobonds' pricing. To do so, we develop a novel credit risk model where default intensities explicitly depend on fiscal variables.

2.1. **Eurobonds.**

Various policy-oriented papers discuss pros and cons of common bond issuance in the euro area, and propose different forms of common bonds. Several of these studies stress that, if issued in large scale, a joint debt instrument could reduce market fragmentation and compete, in terms of size and liquidity, with the US bond market (Giovannini, 2000; Hellwig and Philippon, 2011). De Grauwe and Moesen (2009) and Claessens et al. (2012) argue that joint debt issuance can reduce borrowing costs for stressed sovereigns, allowing for gains at the aggregate level. Following the Great Financial crisis and the euro-debt crisis, common debt issuance has been advocated by several policy-oriented studies as a device to enhance financial stability, notably because such a safe asset could break the "bank-sovereign doom loop" (European Commission, 2011; Brunnermeier et al., 2017; Delivorias and Stamegna, 2020). The challenges associated with joint debt issuances include coordination issues, political hurdle in transferring sovereignty to the EU level, and the removal of incentives for sound budgetary policies under the current fiscal discipline methods (Issing, 2009; Claessens et al., 2012). According, among others, to Delpla and Von Weizsacker (2010), Bofinger et al. (2011), and Claessens et al. (2012), common debt issuance calls for enhanced institutional frameworks and ex-ante surveillance to strengthen fiscal discipline.

In Table 1, we review the features of some prominent proposals for a European joint debt instrument. Three proposals involve joint guarantees, but with varying proportions: the "Stability bond" approach no. 1 of the European Commission (2011) considers a full replacement of standard national issuances by those of an SJG bond; only short-term debt instruments, amounting to 10% of GDP, would benefit from joint guarantees under the "Eurobills" scheme proposed by Hellwig and Philippon (2011); under the blue/red scheme of Delpla and Von Weizsacker (2010), European countries would pool their public debt up to the Maastricht Treaty threshold—60% of GDP—under joint and several liability as senior ("blue") debt, while debt above this threshold would be issued as junior ("red") debt.

Other schemes depart from joint liability and consist of the partial substitution of European Member States' national issuance with several but not joint guarantees (SNJG) bonds. This is for instance the case of the "Stabilitity bond" approach no. 3 of the European Commission (2011).[6] In this scheme, Member States would retain liability for their respective share of "Stabilitity bond" issuance—as well as for their national issuances, naturally.[7] Due to the several but not joint guarantees, moral hazard would be mitigated. The continued issuance of national bonds would indeed expose Member States to market judgement.

[6] Issuance of bonds with several but not joint guarantees can be centralized (e.g., joint debt agency, European Commission, 2011; Delivorias and Stamegna, 2020) or left decentralized (De Grauwe and Moesen, 2009).

[7] The credit quality of a "Stabilitity bond" underpinned by several but not joint guarantees would be close to the weighted average of the credit qualities of the euro-area Member States.

TABLE 1. Eurobond proposals: main features

Features	**Joint bond denomination**				
	Stability bonds[a]		**Euro-bills**[b]	**Blue/Red bonds**[c]	**ESBies/EJBies**[d]
	Approach no. 1	**Approach no. 3**			
Guarantees	SJG[e]	SNJG[f]	SJG (10% of GDP)	Only blue: SJG (60% of GDP)	
Tranching		✓		✓	✓
Pooling[g]	✓	✓	✓	✓	✓
New issuance[h]	✓	✓ (partial)	✓ (partial)	✓ (partial)	
Risk of moral hazard	✓	✓	✓	✓	
Coordinated revenue management	✓			✓	
Coordinated debt management	✓	✓	✓	✓	
Pricing attempt in the study					✓ (partial and incomplete)

Notes: This table shows key features of some prominent euro-area joint debt instrument proposals in the literature. *a*: European Commission (2011); *b*: Hellwig and Philippon (2011); *c*: Delpla and Von Weizsacker (2010); *d*: Brunnermeier et al. (2017); *e*: Joint and several guarantees; *f*: Several but not joint guarantees; *g*: with "Pooling" we mean the pooling or common issuance of sovereign debts (either *ex ante* or *ex post* via pooling a portfolio of sovereign debts); *h*: with "New Issuance" we mean the issuance of a new debt instrument replacing totally or partially national bond issuance.

The absence of joint guarantees also underlies Brunnermeier et al. (2017) proposal. Differently from the "Stability bond" approach no. 3 (European Commission, 2011), their proposal does not imply any substitution of national issuance. In their scheme, two synthetic tranches would be created out of a portfolio of (standard) national sovereign bonds, the senior and the junior tranche being respectively dubbed "European Safe Bonds" (ESBies) and "European junior bonds" (EJBies). As safe and liquid assets, ESBies would help limit financial institutions' exposure to sovereign credit risk, and thereby break the sovereign-bank loop. Brunnermeier et al. (2017) simulate the loss given default of ESBies and EJBies under different tranching scenarios, thereby providing a partial pricing attempt for their instruments.

A few theoretical studies focus on Eurobonds. Tirole (2015) studies the effect of common bonds' issuance, focusing on the moral hazard implications. He distinguishes between two forms of solidarity in a finite-horizon two-country setup: ex-post (spontaneous), e.g., bailouts, and ex-ante (contractual), e.g., joint-bond issuance. Given that one country's default imposes collateral damage on the other country, Tirole (2015) finds that ex-ante (ex-post) solidarity is optimal when both countries exhibit a similar (different) risk profile. Tsiropoulos (2019) builds a two-country general-equilibrium model of sovereign default and finds that welfare consequences of introducing SJG bonds hinge critically on the timing of their introduction. Lastly, Dávila and Weymuller (2016) study the optimal design of flexible joint borrowing agreements between a safe and a risky country. Their results point to higher welfare gains under joint liability schemes.

2.2. **Reduced-form approaches and sovereign risk premiums.** The present study draws extensively from the reduced-form approaches for pricing sovereign credit risk. Ang and Longstaff (2013) consider multi-factor affine models allowing for both systemic and sovereign-specific credit shocks to price the term structures of US states and Eurozone Member States. Estimating the default intensities for 26 countries, Longstaff, Pan, Pedersen, and Singleton (2011) find that the risk premium represents about a third of credit spreads on average. Monfort and Renne (2014) also estimate substantial sovereign risk premiums in euro-area sovereign spreads, employing a model allowing for both credit and liquidity effects. These studies show a close fit of sovereign bond yields and spreads and provide useful estimates of sovereign risk premiums. However, they do not explicitly account for the economic forces driving the movements of the sovereign default probabilities. By contrast, Borgy, Laubach, Mésonnier, and Renne (2011) and Hördahl and Tristani (2013) propose sovereign credit risk frameworks where default intensities explicitly depend on fiscal variables, and demonstrate that the fiscal environment is able to capture part of the fluctuations of sovereign credit spreads.

2.3. **Theory of fiscal limits.** Our paper relates to the literature studying the concept of *fiscal limit*, namely the maximum outstanding debt that a country could credibly sustain. In Bi (2012), Leeper (2013), Bi and Leeper (2013), Bi and Traum (2012), Bi and Traum (2014), the concept of fiscal limit corresponds to the net present value of future maximum primary surpluses.[8] These maximum surpluses represent those surpluses implicit in the peak of the Laffer curve (Trabandt and Uhlig, 2011). After having introduced an estimated parametric reaction function of primary surpluses in a model of debt accumulation, Ghosh et al. (2013) show that there is a point—akin to the fiscal limit—where the primary balance cannot keep pace with the rising interest burden as debt increases. Beyond this point, debt dynamics becomes explosive and the government becomes unable to fully meet its obligations. Collard, Habib, and Rochet (2015) also exploit the idea of a maximum primary surplus to derive a measure of debt limit. Contrary to the previous studies, Collard et al. (2015)'s approach is not based on the computation of the discounted present value of future maximum primary surpluses; instead, their notion of maximum sustainable debt derives from the maximum amount that can be issued on each date (that itself depends on the maximum budget surplus). More recently, Mehrotra and Sergeyev (2020) combine disaster risk and fiscal fatigue. In their framework, as in Lorenzoni and Werning (2013), debt dynamics are subject to a tipping point situation: in some instances, the public debt can be on an unsustainable path without immediately triggering default.

In the present paper, we do not make the maximum surplus explicit; we rely instead on a reduced-form approach and, under the assumption that the default intensity becomes strictly positive when the effective (observed) debt is higher than the (unobserved) fiscal limit, we estimate the latter from bond prices.

3. Stylized model

As mentioned above, a crucial ingredient of our modelling framework is the relationship between the fiscal space—the difference between the fiscal limit and debt—and the sovereign probability of default. The parametric function we retain to model this relationship is presented in Subsection 3.1. Before incorporating this ingredient in a standard asset pricing model (in Section 4), we present a stylized model in Subsection 3.2. In Subsection 3.3, we elaborate on the pricing of SJG and SNJG common bonds in this simplified framework; and we discuss resulting asset-pricing mechanisms in Subsection 3.4.

[8] We refer to Aguiar and Amador (2014) or Yue and Wei (2019) for a general presentation of the theory of sovereign debt.

3.1. **Sovereign default probability.** On each date t, we assume that the default probability of country j ($j = A, B$) is given by

$$1 - \exp(-\underline{\lambda}_{j,t}), \tag{1}$$

where the default intensity $\underline{\lambda}_{j,t}$ is assumed to negatively depend on the fiscal space, defined as the distance between fiscal limit-to-GDP ($\ell_{j,t}$) and debt-to-GDP ($d_{j,t}$). Specifically:[9]

$$\underline{\lambda}_{j,t} = \alpha \max(0, d_{j,t} - \ell_{j,t}). \tag{2}$$

The previous formulation implies that the probability of default is strictly positive only if the fiscal space is negative, i.e. if debt stands above the fiscal limit. Parameter α characterizes the nature of the fiscal limit: if α is large, the fiscal limit is "strict", as the probability of default becomes large as soon as debt breaches the fiscal limit; for lower values of α, the fiscal limit is "soft", as negative fiscal spaces then do not necessarily trigger default. In this section, we consider the case of $\alpha = 1$, implying a relatively soft concept of fiscal limit: if the fiscal space is equal to -1% of GDP, the probability of default is of 1%.[10]

3.2. **Assumptions of the stylized model.** Investors are risk-neutral and risk-free interest rates are zero. In this context, the date-t price of a one-period zero-coupon zero-recovery-rate bond issued by j is simply given by:

$$P_{t,1}^{(j)} = \mathbb{E}_t \exp(-\max[0, d_{j,t+1} - \ell_{j,t+1}]), \tag{3}$$

where $\mathbb{E}_t$ denotes the expectation conditional on the information available to the investor as of date t.

[9]It can be seen that we have $\underline{\lambda}_{j,t} = \max(0, \lambda_{j,t})$, with $\lambda_{j,t} = \alpha \times (d_{j,t} - \ell_{j,t})$. Using the vocabulary introduced by Black (1995), $\lambda_{j,t}$ can be interpreted as a "shadow default intensity." Alternatively, to have a non-negative intensity, $\underline{\lambda}_{j,t}$ could be modeled as a quadratic function of the fiscal space (see, e.g., Doshi et al., 2013). However, it is impossible to have a monotonous relationship between the (non-negative) default intensity and the covariates in a quadratic framework (while such a monotonous relationship is expected to hold in the present context). Coroneo and Pastorello (2020) also employ the shadow-rate approach to price sovereign bonds issued by different countries; contrary to the present paper though, sovereign default probabilities (or default intensities) are not explicitly modeled in their yields-only reduced-form framework. Therefore, the framework of Coroneo and Pastorello (2020) does not allow to recover sovereign probabilities of default, and cannot preclude negative default probabilities.

[10]Low values of α allow for approximate pricing formulas (Appendix D) that are intensively used in our empirical analysis (Section 4). As shown by Footnote 11, these approximate formulas are not needed in the context of the stylized model.

For each country, the fiscal limit-to-GDP ($\ell_{j,t}$) is constant, fixed at $\overline{\ell}_j$, and the debt-to-GDP ratios are i.i.d. Gaussian:

$$\begin{bmatrix} d_{A,t} \\ d_{B,t} \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \overline{d}_A \\ \overline{d}_B \end{bmatrix}, \begin{bmatrix} \sigma^2 & \rho\sigma^2 \\ \rho\sigma^2 & \sigma^2 \end{bmatrix} \right). \tag{4}$$

In this context, the prices of zero-coupon bonds (see eq. 3) admit closed-form solutions deduced from standard results on truncated normal distributions.[11]

3.3. **Common bonds.** We consider two types of common bonds: the first is backed by several and joint (SJG) guarantees, whereby each issuing country guarantees the totality of the obligations, and the second features several but not joint (SNJG) guarantees, whereby each issuing country guarantees only its share of the joint instrument.

A natural way to conceive the SJG bond is to consider that it is issued by a synthetic area where both fiscal revenues and debts are pooled, and to assume that this area also features a probability of default of the form of (1). Denoting by ω the vector of GDP weights, the price of SJG bond is given by:[12]

$$P_{t,1}^{(SJG)} = \mathbb{E}_t \exp(-\max[0, \omega \cdot d_{t+1} - \omega \cdot \overline{\ell}]), \tag{5}$$

where $\omega \cdot d_{t+1} = \omega_A d_{A,t+1} + \omega_B d_{B,t+1}$ and $\omega \cdot \overline{\ell} = \omega_A \overline{\ell}_{A,t+1} + \omega_B \overline{\ell}_{B,t+1}$ are, respectively, the GDP-weighted debt-to-GDP ratio and the GDP-weighted fiscal limit.

Regarding the SNJG bond, the absence of joint guarantee implies that the payoff of this bond is of the form $\omega \cdot (1 - \mathcal{D}_{t+1})$, where $\mathcal{D}_{t+1} = [\mathcal{D}_{A,t+1}, \mathcal{D}_{B,t+1}]$ is the vector of default indicators—a default indicator being equal to 1 in the case of default, and to 0 otherwise. In other words, the payoff is equal to 1 if none of the countries default on date $t+1$, ω_A (respectively ω_B) if only B (resp. A) defaults on date $t+1$, and 0 if both countries default. This implies that the price of a SNJG bond is given by:

$$P_{t,1}^{(SNJG)} = \omega_A \mathbb{E}_t(1 - \mathcal{D}_{A,t+1}) + \omega_B \mathbb{E}_t(1 - \mathcal{D}_{B,t+1}) = \omega \cdot P_{t,1}, \tag{6}$$

[11] Formally, $P_{t,1}^{(j)}$ is given by:

$$\Phi\left(\frac{\overline{\ell}_j - \overline{d}_j}{\sigma}\right) + \left(1 - \Phi\left(\frac{\overline{\ell}_j - \overline{d}_j}{\sigma}\right)\right) \exp\left(\alpha(\overline{\ell}_j - \overline{d}_j) + \frac{\alpha^2\sigma^2}{2}\right) \left\{1 - \Phi\left(\frac{\overline{\ell}_j - \overline{d}_j}{\sigma} + \alpha\sigma\right)\right\} \Big/ \left\{1 - \Phi\left(\frac{\overline{\ell}_j - \overline{d}_j}{\sigma}\right)\right\}.$$

[12] ω is such that $\omega = [\omega_A, 1 - \omega_A]$, with $\omega_A = Y_A/(Y_A + Y_B)$, where Y_j is country j's GDP.

with $P_{t,1} = [P_{t,1}^{(A)}, P_{t,1}^{(B)}]$.

3.4. **Calibration and resulting yields.** The different calibrations used in this section are summarized in Table 2. In our baseline case, we set the average fiscal spaces of both countries to 20% ($= \bar{\ell}_j - \bar{d}_j = 100\% - 80\%$), and the two countries are alike in all respects. In particular, they have the same (GDP) size, i.e. $\omega_A = \omega_B = 50\%$, and the correlation between debts is set to 50%. In this baseline case, the yields on one-year national bonds are equal to 28 basis points.[13] In this baseline context, where both countries are similar, it also comes that SNJG bond prices are equal to those of country-specific bonds (see eq. 6, with $P_{t,1}^{(A)} = P_{t,1}^{(B)}$); the SNJG bond yield is therefore also equal to 28 basis points. By contrast, the price of the SJG bond is higher, the SJG bond yield being of 13 basis points. This results from the fact that, for the synthetic "pooled" area, the probability to have an (average) debt-to-GDP larger than the (average) fiscal limit is lower than for a single country. Formally:

$$\mathbb{P}(\omega_A d_{A,t} + \omega_B d_{B,t} \geq \bar{\ell}) < \mathbb{P}(d_{j,t} \geq \bar{\ell}), \quad j = A, B,$$

which is true as long as the correlation between the two debt-to-GDP ratios is strictly lower than 1. The fact that the SJG bond yield is lower than national bond yields implies that both countries would reduce their debt service through the issuance of joint-liability bonds.

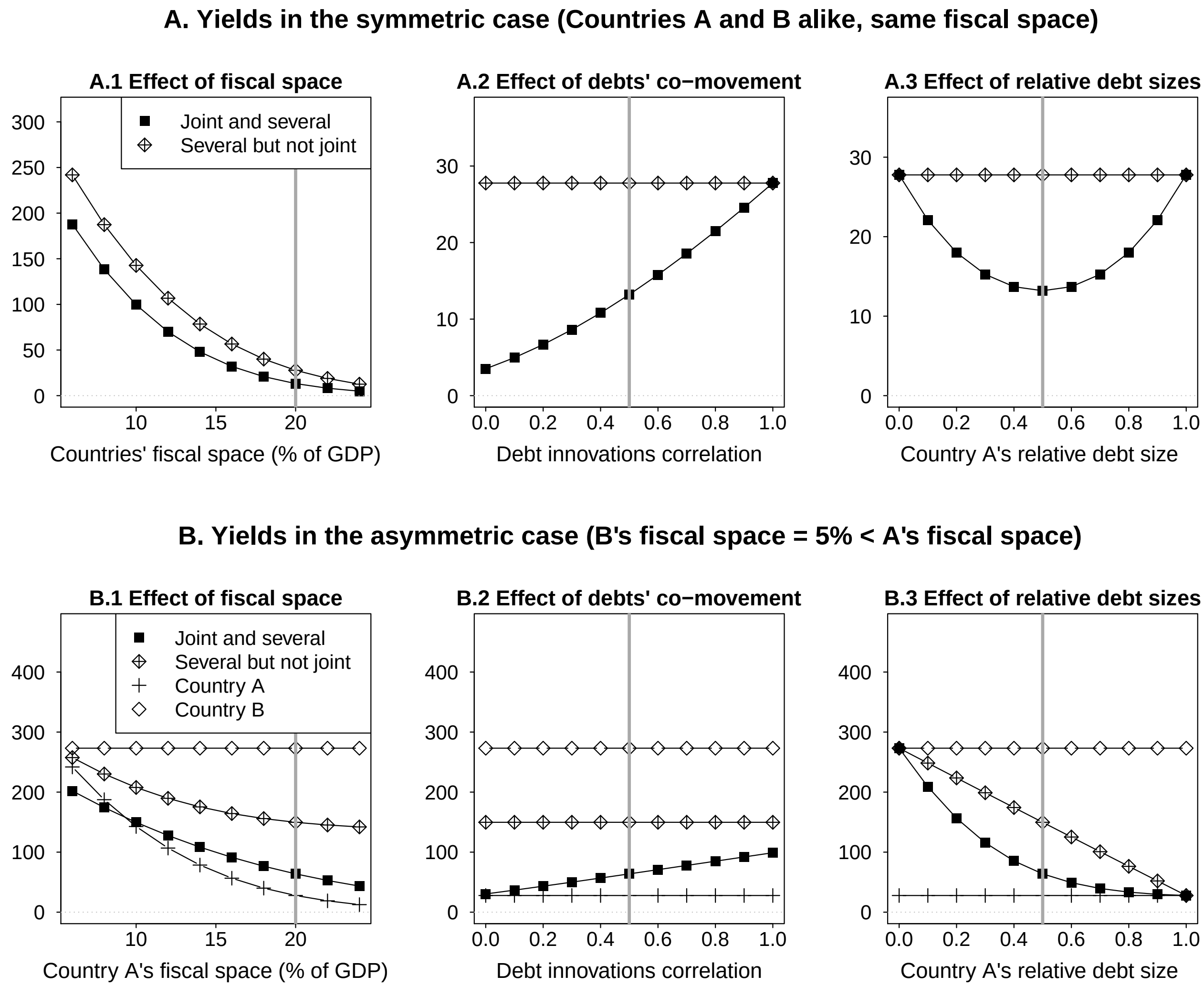
TABLE 2. Calibrations of stylized models

	Baseline	Symmetric case (A and B are alike)			Asymmetric case (B's fiscal space $\leq$ A's fiscal space)		
		A.1	**A.2**	**A.3**	**B.1**	**B.2**	**B.3**
σ	12.5%						
$\bar{\ell}_{A,B}$	100%						
$\bar{d}_B$	80%	75%→95%			95%	95%	95%
$\bar{d}_A$	80%	75%→95%			75%→95%		
ρ	50%		0%→100%			0%→100%	
ω_A	50%			0%→100%			0%→100%

Notes: This table summarizes the calibrations used in our stylized model. The first column shows the calibration of the baseline case (represented by a vertical grey line in the first row of plots of Figure 1). The average fiscal space of country j corresponds to $\bar{\ell}_j - \bar{d}_j$, and ω_A denotes the relative GDP size of country A (such that $\omega_B = 1 - \omega_A$).

[13] Since, in the stylized model described in this section, risk-free yields are taken equal to zero, bond yields essentially correspond to credit spreads. In addition, since the recovery rate is also zero, yields here coincide with probabilities of default. These restrictions are relaxed in the extended model (Section 4).

FIGURE 1. Two-country stylized model mechanisms



Notes: These plots show the yields-to-maturity, expressed in basis points, of different types of one-period bonds; it also shows how these yields are affected by changes in the calibration of the stylized model (see Table 2 for details regarding the baseline calibration and the alternative calibrations underlying Panels A.1 to B.3 of this figure). Three types of bonds are considered: national, or country-specific, bonds issued by countries A and B; a bond with several and joint guarantees (SJG); and a bond with several but not joint guarantees (SNJG). See Subsection 3.4 for more details. On each row of plots, the vertical grey line represents the same situation—the "baseline" case of Table 2.

The baseline situation discussed above is represented by a vertical grey bar in the first row of plots in Figure 1. These plots further show how the SNJG and SJG yields are affected with respect to: (Panel A.1) changes in the fiscal spaces of the two countries, (Panel A.2) changes in the correlation across debts, and (Panel A.3) changes in the relative size of country A (in terms of GDP).

Panel A.1 shows that both SJG and SNJG bond yields nonlinearly decrease when fiscal spaces increase. It also shows that SJG bond yields are consistently lower than those of SNJG bonds. Panel A.2 illustrates the importance of debt co-movements to account for the yield reduction resulting from joint guarantees: while the SNJG bond yield is not affected by changes in debts' correlations, the yield of a SJG bond is reduced by a factor of 8 when the correlation decreases from 100%—in which case all bonds are equivalent—to 0%. Panel A.3 focuses on the effect of the two countries' relative sizes. In the extremes, when the relative size of country A is either 0 or 1, there is no difference between SJG and SNJG bonds. As in the case of debt co-movement, and because we consider two equally-risky countries for the time being, the relative size of country A has no effect on the SNJG yield. But it has on the SJG yield; the effect is maximum when the two countries are equally large, corresponding to a situation where diversification effects are maximum.[14]

The second row of plots in Figure 1 displays results obtained in an asymmetric situation, where country B is riskier than country A. We fix the fiscal space of country B to 5%, keeping A's one at 20%. National bond yields are now different for the two countries, and we add them to each plot. Up to very small convexity effects, it can be checked that SNJG yields are equal to the GDP-weighted averages of the two national bond yields. In particular, in Panel B.3, where we modify the relative size of country A from 0 to 1, the SNJG bond yield goes from the (higher) country-B yield to the (lower) country-A yield. Regarding the difference between SNJG and SNJ yields, an interesting situation is captured by Panel B.1: for low values of country A's fiscal space, not only is the SJG bond yield below the SNJG one (i.e. the average of the two national bond yields), it is also lower than the safer country's bond yields. Finally, Panel B.2 shows that when the two countries do not have the same average fiscal space, a correlation of 1 across debts does not imply that the SJG and the SNJG bonds are equivalent. In this extreme case, and contrary to the symmetric case, diversification effects are still at play in the SJG bond pricing: the SJG bond yield is 1.5 times lower than the SNJG one.

[14]Formally, this is because the variance of the aggregate debt-to-GDP ratio—that is $\sigma^2(\omega_A^2 + (1-\omega_A^2) + 2\rho\omega_A(1-\omega_A))$—admits a minimum for $\omega_A = \frac{1}{2}$.

4. MODEL

In this section, we enrich the stylized model to make it amenable to the data. We consider N countries. While the conditional probabilities of default remain as in Subsection 3.1—with default intensities that depend on fiscal spaces—debt-to-GDP ratios and fiscal limits are now time-varying; in addition, the state vector is augmented with a stochastic short term interest rate (Subsection 4.1). The representative investor is now risk averse, her risk preferences being captured by a reduced-form stochastic discount factor (Subsection 4.2). After having derived prices of zero-coupon risk-free bonds, we discuss the pricing of zero-coupon bonds with non-zero recovery rates and bond yield spreads (Subsection 4.3). The ability to swiftly price risk-free bonds and yield spreads is crucial to estimate the model (Section 5).

4.1. **Dynamics of the state vector.**

Debts and fiscal limits follow autoregressive processes:

$$\ell_{j,t} = (1-\rho_\ell)\bar{\ell}_j + \rho_\ell \ell_{j,t-1} + \varepsilon_{\ell,j,t} \quad (7)$$

$$d_{j,t} = (1-\rho_d)\bar{d}_j + \rho_d d_{j,t-1} + \varepsilon_{d,j,t}, \quad (8)$$

where $\varepsilon_{d,i,t}$ and $\varepsilon_{\ell,i,t}$ are correlated across countries. More precisely, each of these innovations is a linear combinations of i.i.d. standard Gaussian shocks: $\eta_t^{(d)}$ and $\eta_t^{(\ell)}$, that are respectively debt and fiscal-limit shocks common to all countries, as well as country-variable-specific shocks $\eta_{j,t}^{(d)}$ and $\eta_{j,t}^{(\ell)}$. Formally:

$$\varepsilon_{d,j,t} = \alpha_d \eta_t^{(d)} + \gamma_d \eta_{j,t}^{(d)} \quad (9)$$

$$\varepsilon_{\ell,j,t} = \alpha_\ell \eta_t^{(\ell)} + \gamma_\ell \eta_{j,t}^{(\ell)}. \quad (10)$$

Moreover, we introduce a stochastic short-term risk-free interest rate. This variable also follows an auto-regressive process:

$$i_t = (1-\rho_i)\bar{i} + \rho_i i_{t-1} + \sigma_i \eta_{i,t}, \quad \eta_{i,t} \sim i.i.d.\, \mathcal{N}(0,1). \quad (11)$$

Let us denote by d_t and ℓ_t two N-dimensional vectors gathering countries' debt-to-GDP ratios and fiscal limits, respectively. Under the previous assumptions, it is easily seen that the state vector $X_t = [i_t, i_{t-1}, d_t', \ell_t']'$ follows a vector autoregressive process of order one. That is:

$$X_t = \mu + \Phi X_{t-1} + \Sigma \eta_t, \quad (12)$$

where

$$\eta_t = [\eta_{i,t}, \eta_{1,t}^{(d)}, \dots, \eta_{N,t}^{(d)}, \eta_{1,t}^{(\ell)}, \dots, \eta_{N,t}^{(\ell)}, \eta_t^{(d)}, \eta_t^{(\ell)}]' \sim \mathcal{N}(0, I), \tag{13}$$

μ, Φ and Σ being expanded in Appendix A.

4.2. **Stochastic discount factor and the term structure of risk-free rates.** We assume that arbitrage opportunities do not exist, which ensures the existence of a positive stochastic discount factor (s.d.f.). Following Ang and Piazzesi (2003), we posit a reduced-form exponential affine s.d.f. between dates t and $t+1$:

$$\mathcal{M}_{t,t+1} = \exp(-i_t)\frac{\xi_{t+1}}{\xi_t}, \tag{14}$$

where ξ_{t+1} follows:

$$\xi_{t+1} = \xi_t \exp\left(-\frac{1}{2}\psi'\psi - \psi'\eta_{t+1}\right), \tag{15}$$

ψ being a vector of prices of risk.[15]

In this context, it is well-known that risk-free bond prices admit closed-form recursive solutions. Specifically, the date-t price of a risk-free zero-coupon bond of maturity h is given by (proof in Appendix C):

$$B_{t,h} = \exp(A_h + B_h X_t), \tag{16}$$

where, for $h > 1$:

$$\left\{\begin{array}{lcl} A_h & = & A_{h-1} + B'_{h-1}(\mu - \Sigma\psi) + \frac{1}{2}B'_{h-1}\Sigma\Sigma' B_{h-1} \\ B_h & = & B_1 + \Phi' B_{h-1}, \end{array}\right. \tag{17}$$

with $A_1 = 0$ and $B_1 = [-1, 0, \dots]'$.

Equivalently, the yield of a risk-free zero-coupon bond of maturity h is given by:

$$i_{t,h}^0 = -\frac{A_h}{h} - \frac{B'_h}{h}X_t. \tag{18}$$

(Note that $i_{t,1}^0 = i_t$.)

4.3. **Zero-coupon bonds with non-zero recovery rates and sovereign bond yield spreads.** Consider a zero-coupon bond of maturity h issued by country j. Our recovery payoff assumption is based on the "Recovery of Treasury" (RT) convention of Duffie and Singleton

[15]While we could sophisticate the model by considering a time-varying vector of prices or risk (making it affine in X_t, as in Ang and Piazzesi, 2003), we use a constant vector ψ for the sake of parsimony.

(1999): on date $t+k$, with $0 < k < h$, the payoff of the considered bond is zero, unless the country defaults on date $t+k$, in which case the bond payoff is assumed to be the fraction RR (recovery rate) of the price of a risk-free zero-coupon bond of equivalent residual maturity, i.e. $\exp[-(h-k)i^0_{t+k,h-k}]$. Hence, the payoffs of this bond are of the form:

$$\begin{cases} RR \times \exp(-(h-k)i^0_{t+k,h-k}) \times (\mathcal{D}_{j,t+k} - \mathcal{D}_{j,t+k-1}) & \text{if} \quad 0 < k < h, \\ 1 - \mathcal{D}_{j,t+k} + RR \times (\mathcal{D}_{j,t+k} - \mathcal{D}_{j,t+k-1}) & \text{if} \quad k = h. \end{cases}$$

Denoting by $\mathcal{M}_{t,t+k}$ the stochastic discount factor between dates t and $t+k$ (i.e., $\mathcal{M}_{t,t+k} = \mathcal{M}_{t,t+1} \times \cdots \times \mathcal{M}_{t+k-1,t+k}$) and after some algebra (Appendix E), the price of this bond is given by:

$$\mathcal{P}^{(j)}_{t,h} \quad = \quad (1-RR) \times \mathbb{E}_t\left(\mathcal{M}_{t,t+h}(1-\mathcal{D}_{j,t+h})\right) + RR \times B_{t,h}. \tag{19}$$

where $B_{t,h}$, the price of the risk-free bond (Subsection 4.2), is equal to $\mathbb{E}_t(\mathcal{M}_{t,t+h})$, and the conditional expectation $\mathbb{E}_t\left(\mathcal{M}_{t,t+h}(1-\mathcal{D}_{j,t+h})\right)$ corresponds to the date-t price of a zero-coupon zero-recovery-rate bond of maturity h providing a payoff of 1 on date $t+h$ if country j has not defaulted before $t+h$, and zero otherwise (see Appendix D for the derivation of eq. 19). Sovereign bond yields for country j are given by:

$$i^{(j)}_{t,h} = -\frac{\log(\mathcal{P}^{(j)}_{t,h})}{h}, \tag{20}$$

and sovereign spreads are computed as follows:

$$s^{(j)}_{t,h} = i^{(j)}_{t,h} - i^0_{t,h}, \tag{21}$$

where $i^0_{t,h}$ is given by eq. (18).

5. Estimation

5.1. **Data.** We consider four European countries: Germany, France, Italy, and Spain. These countries' GDPs make up three-quarters of the entire Euro-area's GDP. The data are quarterly and span the period from 2008Q1 to 2021Q2. Sovereign yields and the 3-month Overnight Indexed Swap (OIS) interest rate—our short-term risk-free rate—are extracted from Thomson Reuters Datastream. Following Monfort and Renne (2014), risk-free yields of maturities of 2, 3, 5, and 10 years are proxied for by the difference between German bond yields and German

CDSs of matching maturities. Observations of sovereign spreads ($s_{t,h}^{(j)}$'s in eq. 21) are computed as the difference between national bond yields and these risk-free yields. We consider three maturities of bond yield spreads: 3, 5, and 10 years. Time series of gross government debts and GDPs are collected from the Eurostat ESA2010 database.

5.2. **Estimation approach.** The model can be cast into a state-space form, with (i) transition equations describing the dynamics of the state variables (this is eq. 12) and (ii) measurement equations describing the relationships between observed financial market data—prices and yield spreads—and the state vector. Let us denote by Θ the set of model parameters,[16] the state-space model is of the form:

$$\begin{aligned} (i) \quad X_t &= \mathcal{F}(X_{t-1}, \eta_t; \Theta), \quad \text{(reformulation of eq. 12)} \\ (ii) \quad Y_t &= \mathcal{G}(X_t; \Theta) + \xi_t, \end{aligned}$$

where $X_t = [i_t, i_{t-1}, d_t', \ell_t']'$ is the state vector, Y_t denotes the vector of financial market data (gathering risk-free yields and sovereign spreads), and ξ_t is a vector of i.i.d. Gaussian measurement errors. Function $\mathcal{G}$ stands for pricing formulas, associating the state X_t to risk-free yields and sovereign spreads. While the risk-free rates are affine in X_t (see eq. 18), this is not the case for sovereign spreads because of the nonlinearity of the default intensity (resulting from the "max" operator in eq. 2).

The vector of state variables X_t is only partially observed by the econometrician since the N national fiscal limits (ℓ_t) are latent. We therefore face two types of unknowns: the model parameters and the fiscal limits. We address this problem by employing "inversion techniques". These techniques, originally introduced by Chen and Scott (1993) in the term structure literature, consist in estimating the latent pricing factors by inverting a non-singular system relating prices to latent factors. This system results from the assumption that some of the observed prices are modeled without errors. In the present case, we assume that, for each country, the averages of the three sovereign spreads (with maturities 3, 5, and 10 years) are perfectly priced. Under this assumption, we can recover the fiscal limits and, simultaneously, compute the likelihood function associated with the considered model parametrization.[17] This opens

[16]We have $\Theta = \{\bar{i}, \rho_i, \sigma_i, \bar{d}, \rho_d, \bar{\ell}, \rho_\ell, \alpha_d, \gamma_d, \alpha_\ell, \gamma_\ell, \psi\}$.

[17]The likelihood then involves an adjustment term corresponding to the determinant of the Jacobian matrix associated with the non-singular system; this adjustment results from the transformation of the observables to

the door to maximum-likelihood estimation. Online Appendix III details the computation of the log-likelihood.

In order to discipline the estimation, we adopt a parsimonious specification for the prices of risk (ψ in eq. 15) by assuming that $\psi = [\psi_i, \nu\omega', -\nu\omega', \nu, -\nu]'$, where ψ_i therefore is the price of (risk-free) interest-rate risk, and where ν is a parameter that completely specifies the prices of risk associated with those shocks that affect debts and fiscal limits—the η shocks appearing in eqs. (9) and (10).[18] Given the specification of the shock vector η_t (eq. 13), this structure implies that the shocks $\eta^{(d)}_{j,t}$, $\eta^{(\ell)}_{j,t}$, $j = 1, \ldots, N$, $\eta^{(d)}_t$ and $\eta^{(\ell)}_t$ are priced.

Moreover, in the spirit of Cochrane and Saa-Requejo (2000), we impose an upper bound for the maximum Sharpe ratio (see Appendix F for the computation of the maximum Sharpe ratio). Following Cochrane and Saa-Requejo (2000), this bound is set to 1.

To facilitate the estimation and ensure plausible fiscal limit estimates, some parameters are calibrated or restricted to lay in pre-specified intervals. For all countries, we set the stationary debt-to-GDP ($\bar{d}$) to 90%, which is the observed sample average (across time and countries). The unconditional mean of fiscal limit ($\bar{\ell}$) is set to 150%.[19] The mean of the risk-free short-term rate ($\bar{i}$) is taken equal to 44 basis points, that is the 3-month OIS sample average. The autoregressive parameter for $d_{j,t}$ is restricted to be between 0.9 and 0.99, this range reflecting the observed dispersion of country-specific OLS estimates of autoregressive parameters. To favor numerical stability, we impose upper bounds, of 0.99, to the autoregressive parameters of $\ell_{j,t}$ and i_t. The unconditional standard deviation for $d_{j,t}$ is set to the observed cross-country average of debt-to-GDP standard deviations, that is 11.6%.[20] The correlation across debt innovations is taken equal to 50%, which is the cross-country average of sample correlations between changes in debt-to-GDP ratios; the correlation across fiscal limit innovations is set to the same value.[21] We use a minimal value of 0.1 for α, the elasticity of the default probability

the latent components (see e.g. Ang and Piazzesi, 2003, Appendix B; Liu, Longstaff, and Mandell, 2006, eq. 19). Computational details are given in the Online Appendix III.

[18]Vector ω has not to be estimated, it is the vector of relative GDP weights.

[19]This unconditional mean of fiscal limits is based on the average of estimates obtained by Ghosh et al. (2013) and Collard et al. (2015). These (static) estimates are reported in Table 5 in Appendix G.

[20]From eq. (9), note that the 1-step ahead conditional standard deviation for $d_{j,t}$ is equal to $\sqrt{\alpha_d^2 + \gamma_d^2}$ and the unconditional standard deviation is equal to $\sqrt{(\alpha_d^2 + \gamma_d^2)/(1 - \rho_d^2)}$.

[21]The former correlation is $\alpha_d^2/(\alpha_d^2 + \gamma_d^2)$; the latter is $\alpha_\ell^2/(\alpha_\ell^2 + \gamma_\ell^2)$.

to the fiscal space (see eq. 2).[22] The standard deviations of the measurement errors associated with yields and sovereign spreads are respectively set to 10 basis points and to 10% of the country-wise sample standard errors of sovereign spreads.

The resulting model parametrization is given in Table 3. Several of the restrictions described above turn out to be binding, which we indicate by "†" in the table.

TABLE 3. Model parametrization

Param.	**Value**	**Param.**	**Value**	**Param.**	**Value**
$\bar{i}$	0.27†	$\bar{d}$	0.90†	$\bar{\ell}$	1.50†
ρ_i	0.99†	ρ_d	0.90†	ρ_ℓ	0.98
σ_i	0.21	α_d	3.58†	α_ℓ	1.45
		γ_d	3.58†	γ_ℓ	1.45
σ_i^ξ	0.13†	$\sigma_{YS,DE}^\xi$	0.02†	$\sigma_{YS,FR}^\xi$	0.04†
		$\sigma_{YS,IT}^\xi$	0.12†	$\sigma_{YS,ES}^\xi$	0.13†
ν	0.51	$\psi_{0,i}$	0.18	α	0.10†

Notes: Parameters ψ_i and ν determine the vector of prices of risk ψ (see eq. 15); specifically, $\psi = [\psi_i, \nu\omega', -\nu\omega', \nu, -\nu]'$, where ω is the observed vector of GDP weights. The standard deviation of the innovation of the short-term risk rate (σ_i), is expressed in percentage points, as well as the standard deviations of the measurement errors (σ_i^ξ for risk-free yields, and $\sigma_{YS,j}^\xi$ for sovereign bond yield spreads). The subscript † indicates calibrated parameters, or parameters for which the restrictions described in 5.2 turn out to be binding. Parameters α_d, α_ℓ, γ_d and γ_ℓ (that specify the variances/covariances of debts and fiscal limits, see eqs. 9 and 10) are multiplied by 100.

5.3. **Sovereign spreads fit and credit risk premiums.** Figure 2 shows the fit of sovereign spreads. The fit is comparable to the one obtained in term-structure studies where default intensities are purely latent and have no macro-finance interpretation. Figure 2 also displays credit risk premiums, that are by-products of our estimation approach. Risk premiums are defined as those components of returns that would not exist if investors were not risk averse.

On Figure 2, model-implied spreads (dotted black lines) result from eq. (21), which involves formulas using the stochastic discount factor $\mathcal{M}_{t,t+1}$ that itself depends on prices of risk ψ (eq. 15). The black solid lines represent the (model-implied) spreads that would be observed

[22]When $d_{j,t} > \ell_{j,t}$, the larger α, the higher the default probability. In other words, for a given level of default probability, the larger α, the higher the fiscal limit estimate. Values of α lower than 0.1 would imply too "soft" a concept of fiscal limit, which would be difficult to interpret and would not be in line with standard concepts of the fiscal limit defined in previous literature. Values of α above or equal to 0.1 offer a good compromise between data fitting performances and interpretability of the fiscal limit.

if agents were not risk averse; these spreads are obtained by implementing the formulas implicit in eq. (21) after having set the prices of risk to zero. The differences between the two types of model-implied spreads correspond to credit-risk premiums. Our results indicate that these risk premiums are sizeable. The ratio between the two types of spreads, which reflects the importance of risk premiums, is broadly comparable to the ones found in sovereign credit-risk studies based on reduced-form intensity approaches (e.g. Pan and Singleton, 2008; Longstaff et al., 2011; Monfort and Renne, 2014; Monfort et al., 2020). Lastly, Figure 3 shows that the model captures a substantial share of the fluctuations of risk-free rates across all maturities.

5.4. **Fiscal limit estimates.** To the best of our knowledge, the present paper is among the first studies to propose time-varying estimates of fiscal limits.[23] These estimates, expressed in percent of GDP, are displayed in Figure 4. On a given quarter, if debt-to-GDP ($d_{j,t}$, black solid line) is higher than the fiscal limit ($\ell_{j,t}$, grey solid line), then, the probability of default is strictly positive (see eq. 1). Everything else equal, if debt-to-GDP stays above the black dotted line (respectively in the grey-shaded area) for four quarters in a row, then the annual default probability of the considered country would be larger than 10% (respectively in $]0\%, 10\%]$). For what follows, and unless differently specified, our numbers refer to the threshold fiscal limit estimates, namely the grey solid lines in Figure 4. According to our estimates, the global financial crisis of 2008 translated into a decrease of the fiscal limits. On average, fiscal limits decreased by 10 percent of GDP from 2008Q2 to 2010Q1.[24] From early 2010 to mid-2012, amid the European sovereign debt crisis, fiscal limits recorded an average decrease close to 20 percent of GDP.[25] Notably, the "*whatever it takes*" statement by Draghi (2012, July) and the European Central Bank (ECB) announcement of the Outright Monetary Transactions (OMT)

[23]See Footnote 5.

[24]This may be seen as a consequence of transfers from private to public debts through explicit channels (bank bailouts) or implicit ones (debt and deposit guarantees), along the logic of the so-called sovereign-bank nexus (see e.g. Acharya, Drechsler, and Schnabl, 2014; Jordà, Schularick, and Taylor, 2016).

[25]In Spain, the fiscal limit dropped below the debt-to-GDP ratio only by the end of 2011, before rising again with the inception of the ESM programme for the banking sector recapitalisation in 2012; fiscal space—the difference between fiscal limit and debt—for Spain returned positive only in 2014. The French and German fiscal limits were at their minima in the midst of the euro-area debt crisis, in late 2011, respectively amounting to 106% and 91% of GDP. An exhaustion of the Italian fiscal space is notable during the Great Financial crisis. The fiscal constraint for Italy tightens even more at the fall of Silvio Berlusconi's government in autumn 2011, reaching a minimal value of 73% of GDP at the end of 2011. The Italian fiscal space remains negative until the end of the estimation sample. The government election of a populist political alliance, in mid-2018, caused a drop of 5% in the fiscal limit for Italy.

were followed by a 10 p.p. jump in the average fiscal limit (from 2012Q21 to 2012Q4).[26] From 2014 until the onset of the COVID-19 pandemic, fiscal limits across countries show an increasing trend, translating into widening fiscal spaces in Europe. Fiscal limits decrease by 10 p.p. on average across countries during the 2020 due to the pandemic.

6. Results

6.1. **Pricing Eurobonds.** In Figure 5, we compare counterfactual yield spreads associated with common bonds benefitting from several and joint guarantees (SJG) and bonds with several but not joint guarantees (SNJG). By design, the latter is close to the debt-weighted average of country-specific observed sovereign spreads. The difference between SNJG and SJG is positive and sizeable across the estimation sample. This result suggests that raising funds through a joint liability debt instrument—the SJG bond—may substantially reduce debt service in the presence of heterogenous fiscal conditions. This is due to the associated diversification of fiscal risks across countries: as long as the fiscal positions across countries are not perfectly correlated, one can expect gains from common bond issuance in the presence of joint and several guarantees (SJG) w.r.t. several but not joint guarantees (SNJG). The maximum gains, in terms of debt service relief associated with the SJG bond, are obtained during the euro-debt crisis. Notably, in this case, the ratio of SNJG bond yield spread on the SJG one is approximately equal to 10, 3.5 and 1.5 for the 3-, 5- and 10-year maturities, respectively. Over the estimation sample, the distance between SJG and SNJG bond yields is equal, on average, to 55, 45 and 30 basis points for the same three respective maturities. The grey shaded-area shows the range of SJG spreads obtained when varying the correlation across debt and fiscal limit innovations (respectively the $\varepsilon_{d,j,t}$'s of eq. 9 and the $\varepsilon_{\ell,j,t}$'s of eq. 10) from 30% to 99%, the baseline case (in black) corresponding to a 50% correlation.

For the sake of comparison, we add the German bond yield spreads in Figure 5 (black circles). Interestingly, during the great financial crisis, the baseline SJG spread lays below the German yield spread for the 3-year maturity. Hence, diversification effects underyling the SJG bond pricing might, at times, prove beneficial also for fiscally virtuous countries in the euro area—and not only for the peripheral Member States. Notwithstanding, even in the

[26]The OMT represents a mechanism aimed to "safeguard an appropriate monetary policy transmission and the singleness of the monetary policy" (2012, August).

scenarios under which SJG bond yields are higher than Bunds' ones, one can design post-issuance redistribution schemes translating into gains to all countries. This is discussed in the next two subsections (6.2 and 6.3).

The magnitudes of our model-implied SJG and SNJG bond spreads are broadly in line with those pertaining to observed proxies of (SJG) Eurobonds. We consider as Eurobond proxies those bonds issued by the following European institutions: the European Investment Bank (EIB), the European Financial Stability Facility (EFSF), the European Stability Mechanism (ESM), and the European Commission itself, which, against the backdrop of the COVID-19 crisis, has initiated large-scale issuance programs.[27] These bonds benefit from various types of guarantees, which makes them close to SJG bonds.[28] Figure 6 shows the spreads between such 10-year bonds and the German benchmark bond (the Bund). It also displays, in grey, proxies of SNJG spreads, computed as GDP-weighted averages of national spreads versus the Bund. It appears that the prices of the different SJG Eurobond proxies are close to each other. The red dots indicate the model-implied SJG and SNJG bond spreads (versus Germany). The plot shows that the model captures a substantial amount of the fluctuations of observed spreads.

6.2. **Aggregate gains and redistribution.** In Subsection 6.1, we have seen that the price of a common debt instrument might be lower than the German one (equivalently, Eurobond yields are higher than Bund ones). However SJG bond prices are higher than SNJG ones. Since the latter correspond to a weighted average of national bond prices, replacing national bonds with SJG bonds results in aggregate gains. These gains could be redistributed ex-post—i.e. after issuance—across all countries. In that case, and considering only strictly positive redistribution weights, the issuance of SJG bonds would eventually result in a reduction

[27]These programs notably include the SURE program (for "Support to mitigate Unemployment Risks in an Emergency") and the Next-Generation-EU program. See, e.g., the investor presentation of the European Commission (12 March 2021), available at https://ec.europa.eu/info/sites/default/files/about_the_european_commission/eu_budget/ip_07.2021.pdf. The EU already had issued some bonds before 2020, in particular in the context of the Euratom loans.

[28]To justify Moody's top rating (Aaa) for the EU's bond programs, the rating agency points out, for example, that "the multiple layers of debt service protection, including explicit recourse to extraordinary support [...] creates the equivalent of a joint and several undertaking and obligation on the part of EU member states to provide financial support to the EU" (https://www.moodys.com/research/Moodys-affirms-the-European-Unions-Aaa-rating-outlook-stable--PR_430731).

in funding costs for all countries (w.r.t. the issuance of national bonds).[29] Naturally, the number of redistribution schemes is infinite. In this subsection, we focus on three situations. In the first one (Scheme A), countries pay the same yield (i.e., there is no redistribution); in the second one (Scheme B), gains are distributed in proportion to GDP; in the third one (Scheme C), gains are distributed in such a way that the interest rate reduction—relative to the respective national bond rates—is the same for all countries. Formulas used to perform these exercises are detailed in Online Appendix IV.[30]

Table 4 shows the results of these counterfactual exercises. We focus on 5-year bonds (5 years roughly being to the average issuance maturity in the euro area), and three periods: beginning of the estimation sample (2008Q1), midst of the euro debt crisis (2011Q3), and end of the estimation sample (2021Q2). The three upper panels (A, B and C) of Table 4 correspond to the three SJG-based schemes described above. For the sake of comparison, the lower panel (Panel D) shows results for the SNJG case, for which there are no aggregated gains. For this latter case (Scheme D), we consider only the situation in which all countries pay the same interest rate (i.e. the SNJG issuance yield). Table 4 also reports post-redistribution yields, which are the differences between national bond yields and reductions in the funding costs (or "yield gains") resulting from the considered schemes. In addition, we show redistribution weights; these weights indicate how aggregate gains are shared across countries.

Let us stress that the reported reduction in the funding cost (or yield gain) pertains to one given bond, and not to the whole debt outstanding. To be sure: a yield gain of 100 basis points (say) would effectively translate into a reduction of yearly aggregate funding costs of €1bn if an outstanding amount of €100bn of SJG bonds was issued. This being said, to give an idea of the amounts potentially involved, the top part of Table 4 indicates the aggregate gains that would have resulted from the issuance of the equivalent of 5% of the euro-area GDP (€522bn) during the three considered quarters. For instance, for the same face value (€522bn), issuing SJB bonds instead of SNJG bonds in 2008Q1 would have increased the issuance proceeds by €2.92bn. For 2011Q3 and 2021Q2, the gains would have been €8.68bn and €6.21bn, respectively. Not surprisingly, the highest aggregate gains are obtained during the euro debt crisis.

[29] In some sense, any scheme involving strictly positive weights can be seen as Pareto-improving.

[30] This online appendix also reports results of schemes where the funding costs of Germany and France are left unchanged (see Online Appendix IV.5).

Panel A of Table 4 characterizes the scheme where there is no redistribution of the aggregated gains (Scheme A). As illustrated by our results, this scheme can result in negative "gains" for some countries: German funding cost gets higher for the three considered periods, same goes for France in 2011Q3. Italy and Spain are the countries that benefit the most out of the SJG issuance scheme in 2011Q3: the spread between post-redistribution and national yields is equal to 300 basis points for Italy and 257 basis points for Spain.

By contrast, Schemes B and C are such that all countries mechanically benefit from the issuance of SJG bonds. These two schemes deliver similar results (see Panels B and C of Table 4). While yield reductions are modest in the pre-sovereign debt crisis period (14 basis points in 2008Q1), they become sizeable during the euro-debt crisis (about 40 basis points in 2011Q3), and remain substantial at the end of the estimation sample (about 25 basis points in 2021Q2).

Figure 7 displays the time series of yield gains associated with Scheme C. We consider three maturities: 3, 5 and 10 years. For the three maturities, yield gains peak towards the end of the euro-debt crisis, in 2012Q2, reaching approximately 120 basis points for the 3-year maturity. For the 10-year maturity, post-redistribution yield gains revolve around 25 basis points over the estimation sample, reaching a maximum of about 40 basis points in 2013Q1.

6.3. **Moral hazard and redistribution schemes.** Usual concerns associated with common debt issuance pertains to moral hazard (see, e.g., Issing, 2009; Claessens et al., 2012; Favero and Missale, 2012; Tirole, 2015; Dávila and Weymuller, 2016): knowing that part of their debt is guaranteed by other countries, some countries may be tempted to increase their spending—and start issuing more debt—since the interest rate on jointly guaranteed debt is less sensitive to an individual debt increase than non-guaranteed debt.

Although our reduced-form modeling framework does not allow to explore such mechanisms in a structural way, it can illustrate how market discipline would be impaired by massive issuance of SJG bonds. Specifically, we perform counterfactual exercises in which Italy and Spain decide to deviate from their current debt level, all else being equal. We then observe the changes in spreads induced by these modifications. We consider two dates: 2011Q4 (euro-area debt crisis) and 2021Q2 (end of the estimation sample). Figure 8 shows the results. For each date and each country, large increases in the debt-to-GDP ratio result in modest increases in SJG and SNJG Eurobond spreads (see, respectively, the grey and black solid lines).

TABLE 4. Effect of redistribution schemes on funding costs

	2008-06-30			2011-12-31			2021-06-30		
SJG									
Aggr. gains	€2.92 bn			€8.68 bn			€6.21 bn		
Panel A: SJG, Same funding costs (i.e. no ex-post redistribution)									
	redist. weigth[a]	post redist. yield[b]	yield gain[c]	redist. weigth	post redist. yield	yield gain	redist. weigth	post redist. yield	yield gain
DE	-11%	409	-4	-111%	282	-106	-3%	-3	-2
FR	26%	409	13	-24%	282	-33	18%	-3	16
IT	62%	409	41	153%	282	300	60%	-3	68
ES	22%	409	24	82%	282	257	25%	-3	47
Panel B: SJG, Redistribution based on GDP weights									
	redist. weigth	post redist. yield	yield gain	redist. weigth	post redist. yield	yield gain	redist. weigth	post redist. yield	yield gain
DE	39%	392	14	39%	140	36	39%	-29	24
FR	27%	409	14	27%	211	37	27%	-11	24
IT	21%	436	14	21%	538	44	21%	40	24
ES	13%	419	14	13%	495	43	13%	19	24
Panel C: SJG, Same yield gains across countries									
	redist. weigth	post redist. yield	yield gain	redist. weigth	post redist. yield	yield gain	redist. weigth	post redist. yield	yield gain
DE	39%	392	14	42%	137	39	40%	-29	24
FR	27%	409	14	28%	210	39	27%	-11	24
IT	21%	436	14	18%	544	39	21%	41	24
ES	13%	419	14	12%	500	39	13%	19	24
SNJG									
Aggr. gains	€0 bn			€0 bn			€0 bn		
Panel D: SNJG, Same funding costs									
	redist. weigth	post redist. yield	yield gain	redist. weigth	post redist. yield	yield gain	redist. weigth	post redist. yield	yield gain
DE	–	423	-18	–	320	-145	–	20	-26
FR	–	423	0	–	320	-72	–	20	-8
IT	–	423	27	–	320	262	–	20	45
ES	–	423	10	–	320	218	–	20	23

Notes: This table compares post-redistribution funding costs across countries under the two issuance schemes (SJG and SNJG) and under different redistribution schemes described in Subsection 6.2. We focus on the 5-year maturity and on three periods: beginning of the estimation sample (2008Q1), midst of the euro debt crisis (2011Q3) and end of the estimation sample (2021Q2). Yields are expressed in basis points. Aggregate gains (reported at the top of the table) are computed under the assumption that total issuance is equal to 5% of aggregate GDP. In each panel, for all countries and dates, we show the redistribution weights, the post-redistribution yields, and the spread between national yields and the post-redistribution yields (that are the yield gains). Under SNJG (Panel D), redistribution weights are unnecessary since there are no aggregated gains. See Online Appendix IV for computational details.

These increases are far lower than those of national bond yields (grey dashed line). This illustrates the moral hazard issue: under the issuance of common bonds, and if the each country pays the issuance SJG/SNJG yield (i.e., under Schemes A or D), then the ability of financial markets to restore fiscal discipline via rising interest rates is hampered. Let us stress that the strength of this hampering effect depends on the extent to which national issuances would be replaced with eurobonds: as long as a sizable share of countries' funding needs are met with the issuance of national bonds, the overall debt service remains sensitive to countries' indebtedness. In other words, under Schemes A or D, a necessary condition for market discipline to remain effective is to limit the issuance of eurobonds (as suggested by Delpla and Weizsacker, 2010; Hellwig and Philippon, 2011). The simulation results suggest that moral hazard effects are dampened under Schemes B and C (see black dashed lines in Figure 8); these schemes indeed imply that post-redistribution funding costs, although lower than national bond yields (grey dashed line), remain sensitive to countries' indebtedness.

7. Concluding remarks

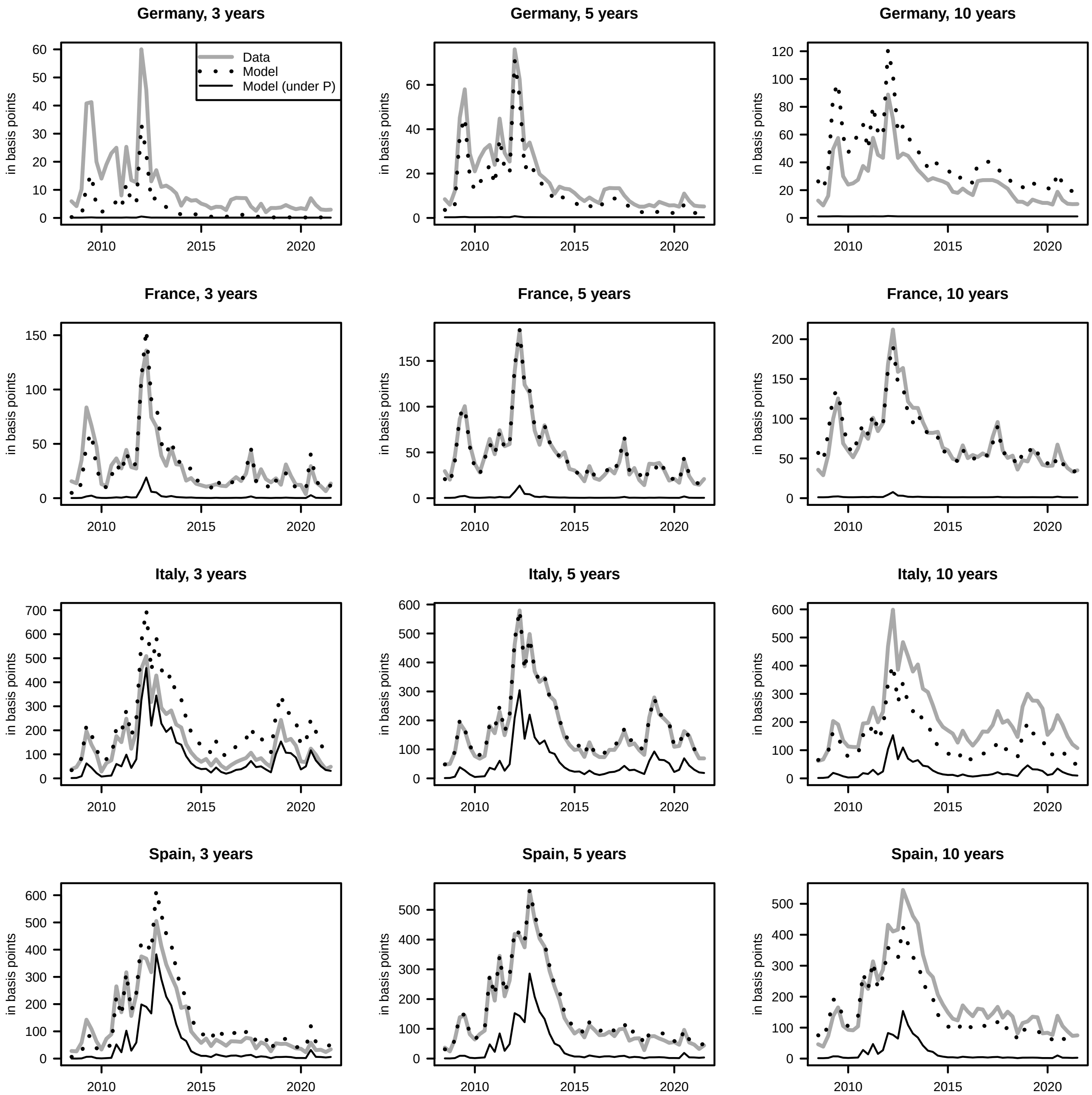
This paper aims at pricing bonds jointly issued by a group of countries. Our focus is on Eurobonds, which are debt instruments jointly issued by euro-area countries. We consider two types of common bonds: the first features joint and several guarantees (SJG bond); the second is characterized by several but not joint guarantees (SNJG bond). To price these two types of common bonds, we develop a novel multi-country sovereign credit risk framework. Our model captures the joint dynamics of national bond prices, sovereign debt, and the fiscal limit—the level of debt beyond which the risk of default is no longer zero.

Estimating the model involves both determining the model parameterization and countries' fiscal limits. Thanks to the tractability of our asset-pricing framework, these two tasks are operated jointly. Our estimation sample comprises data associated with the four largest euro-area economies (Germany, France, Italy, and Spain) and covers the period from 2008$Q1$ to 2021$Q2$. The estimated model fits observed sovereign spreads across maturities and countries. To the best of our knowledge, this paper is the first to provide time-varying estimates of fiscal limits for the euro area.

The estimated model is exploited to examine the pricing of (counterfactual) SJG and SNJG bonds. Yields associated with SNJG bonds are always higher than those associated with their

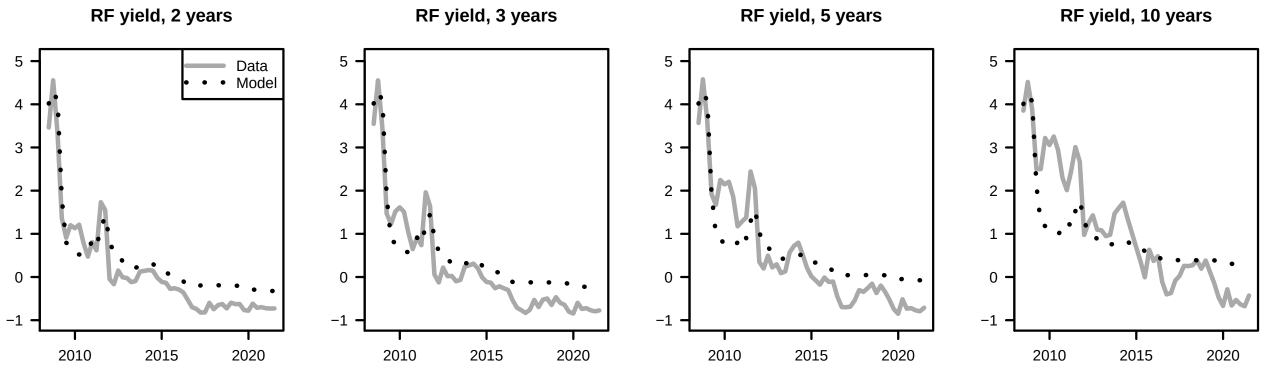
SJG equivalents. Notably, during the euro-debt crisis, the 5-year SNJG bond yield spread, w.r.t. a risk-free rate, is three times larger than the SJG one. Therefore, in the presence of heterogenous fiscal conditions, raising funds through SJG bonds may lower aggregate debt service (w.r.t. situations where only national bonds and/or SNJG bonds are issued). We discuss potential ex-post redistributions of such aggregate gains, and we show that some of these redistribution schemes may alleviate the reduction in market discipline resulting from joint bond issuances.

FIGURE 2. Model fit of sovereign bond yield spreads



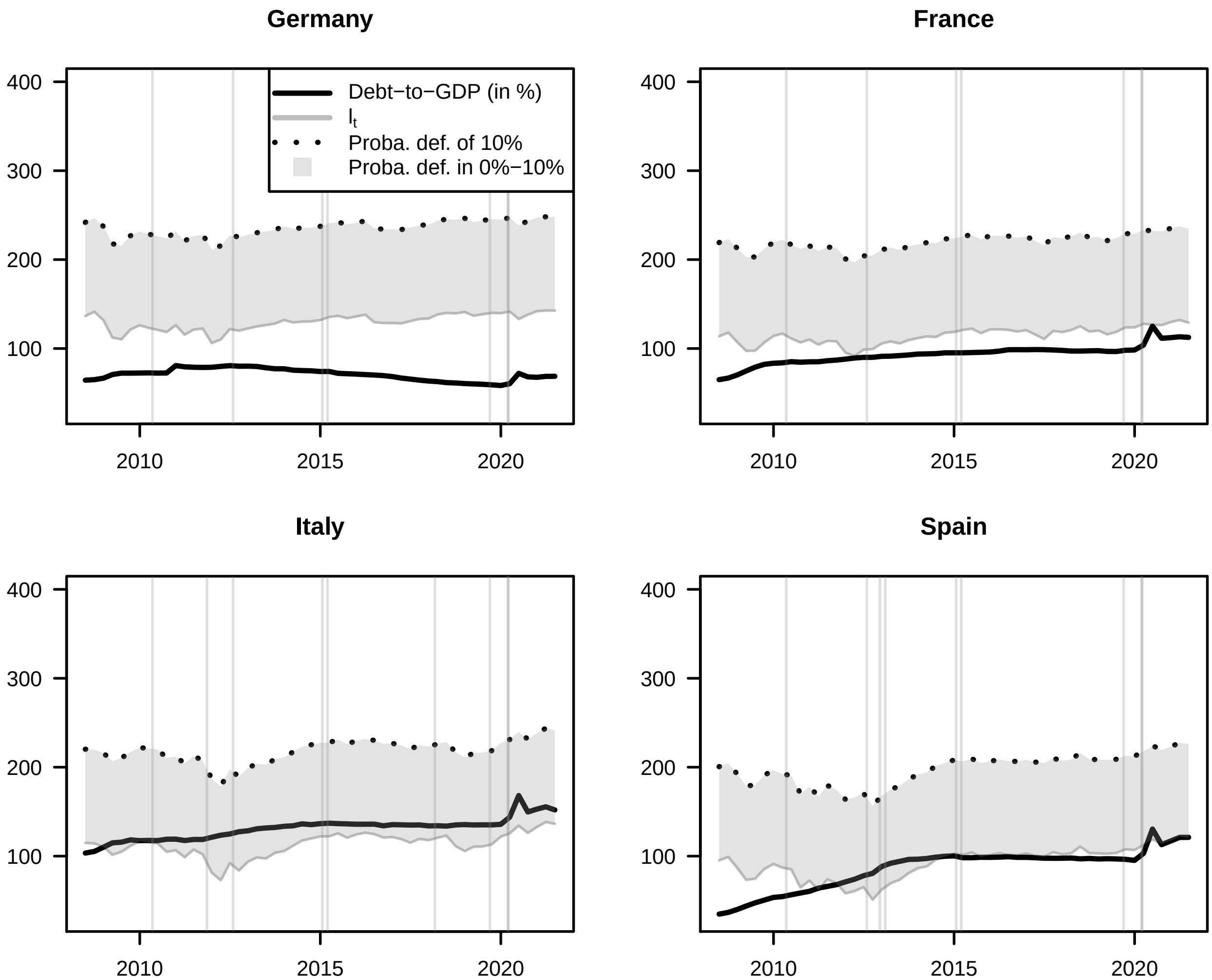
Notes: Model-implied sovereign spreads result from eq. (21). Dashed lines represent the (model-implied) spreads that would be observed if agents were not risk averse (obtained also by eq. 21, but after having set the prices of risk, that are the components of ψ, to zero). The differences between the two types of model-implied spreads (dotted and solid lines) correspond to credit-risk premiums.

FIGURE 3. Model fit of risk-free yields



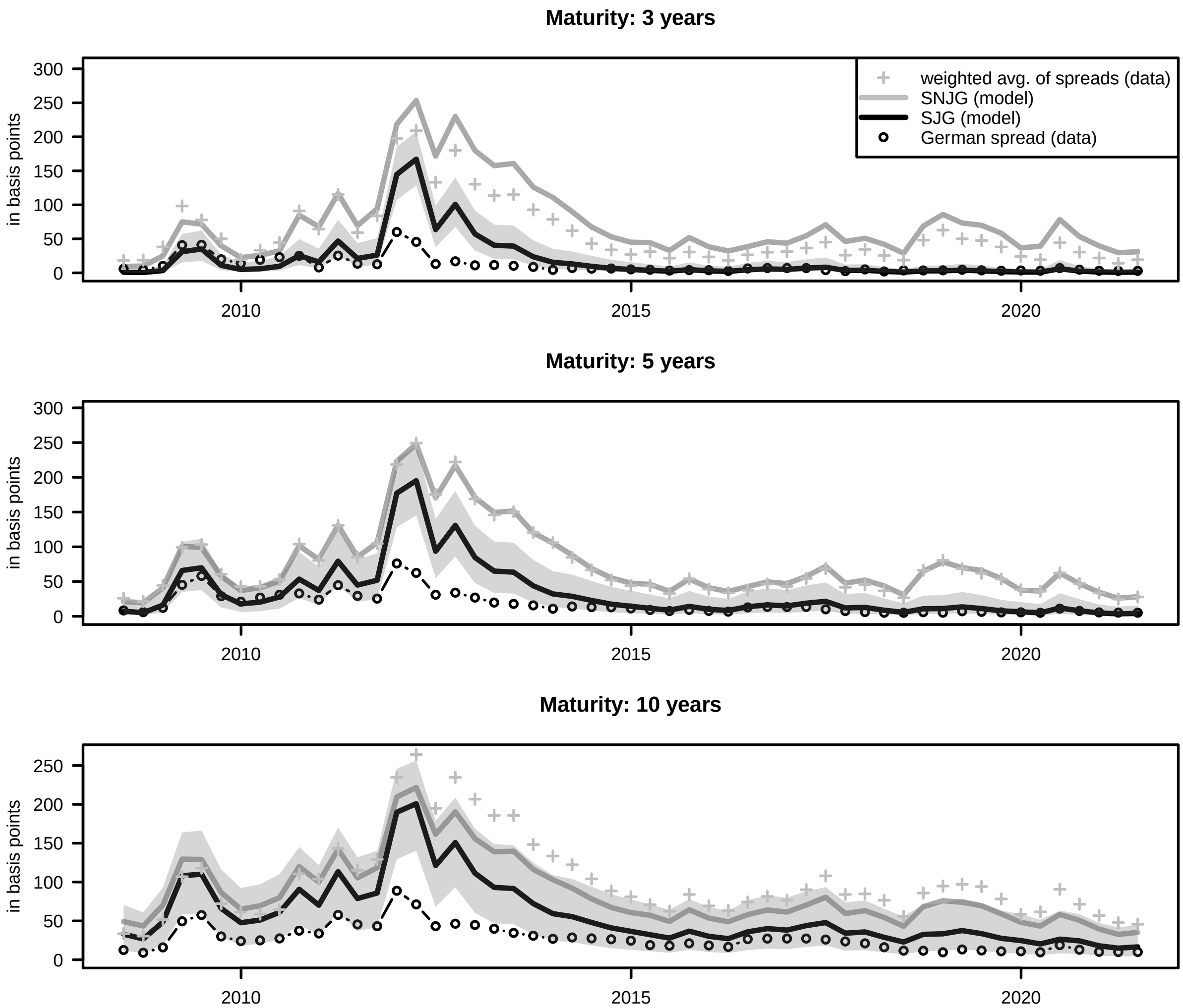
Notes: The model implied risk-free yields (grey solid line) result from eq. (18). Interest rates are annualized, and expressed in percentage points.

FIGURE 4. Estimated fiscal limits



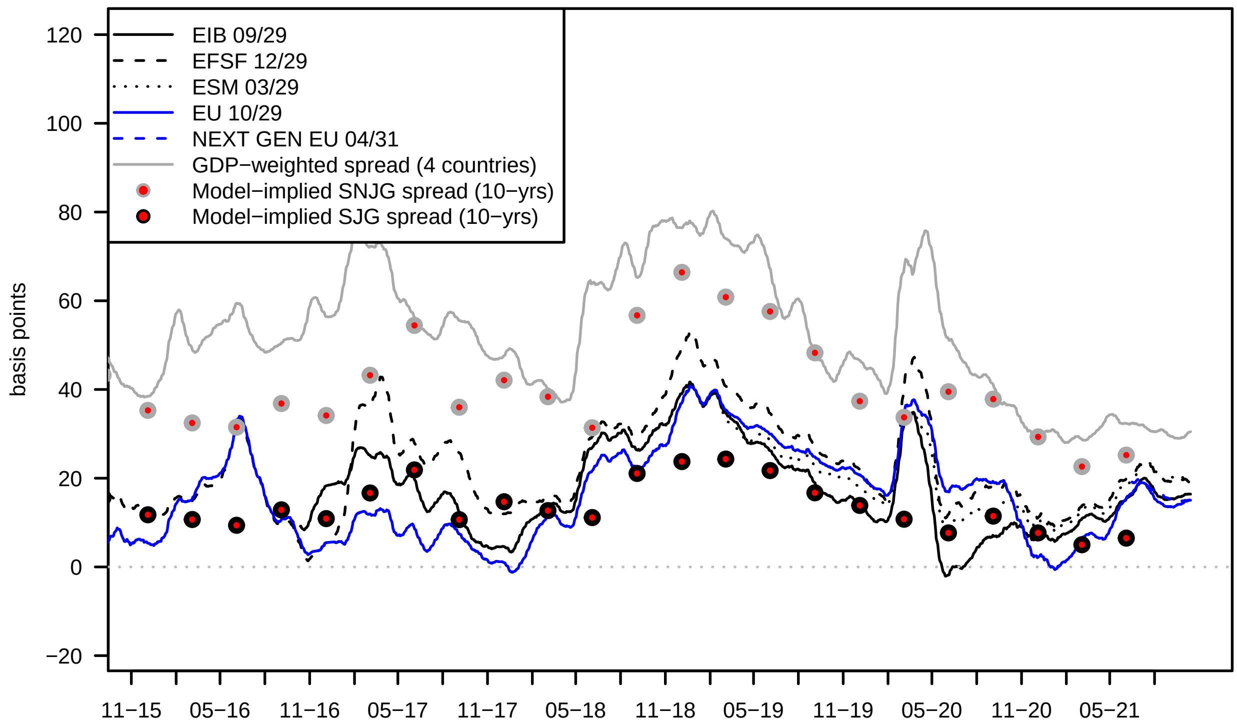
Notes: These plots display estimated fiscal limits ($\ell_{j,t}$) and observed public debts ($d_{j,t}$), both expressed in % of GDP. On a given quarter, if debt-to-GDP is higher than the fiscal limit (grey solid line), then the probability of default is strictly positive (see eq. 1). Everything else equal, if debt-to-GDP (black solid line) stayed above the black dotted line (respectively in the grey-shaded area) for four quarters in a row, then the annual default probability of the considered country is larger than 10% (respectively in $]0\%, 10\%]$). On each plot, the vertical bars indicate important dates (monetary-policy decisions and/or noteworthy pivotal economic events): **All countries**—10/05/2010: Announcement of Securities Market Program (SMP); 02/08/2012: ECB announces it may undertake outright transactions in sovereign bond markets (OMT); 22/01/2015: ECB announces expanded asset purchase programme to include bonds issued by euro area central governments, agencies and European institutions (combined monthly asset purchases to amount to €60bn); 04/03/2015: Announcement of the Public Sector Purchase Programme (PSPP); 12/09/2019: Announcement that net purchases will be restarted under the Governing Council's asset purchase programme (APP) at a monthly pace of €20bn as from 1 November 2019. **Italy**—12/11/2011: Berlusconi resigns from office (BTP/Bund spread is over 550 bps); 04/03/2018: Populist parties (M5S and Lega) win the majority of votes in Italian government elections. **Spain**—11/12/2012: ESM (European Stability Mechanism) disburses €39.5bn for recapitalisation of banking sector; 05/03/2013: ESM disburses €1.9bn.

FIGURE 5. Counterfactual bond yield spreads



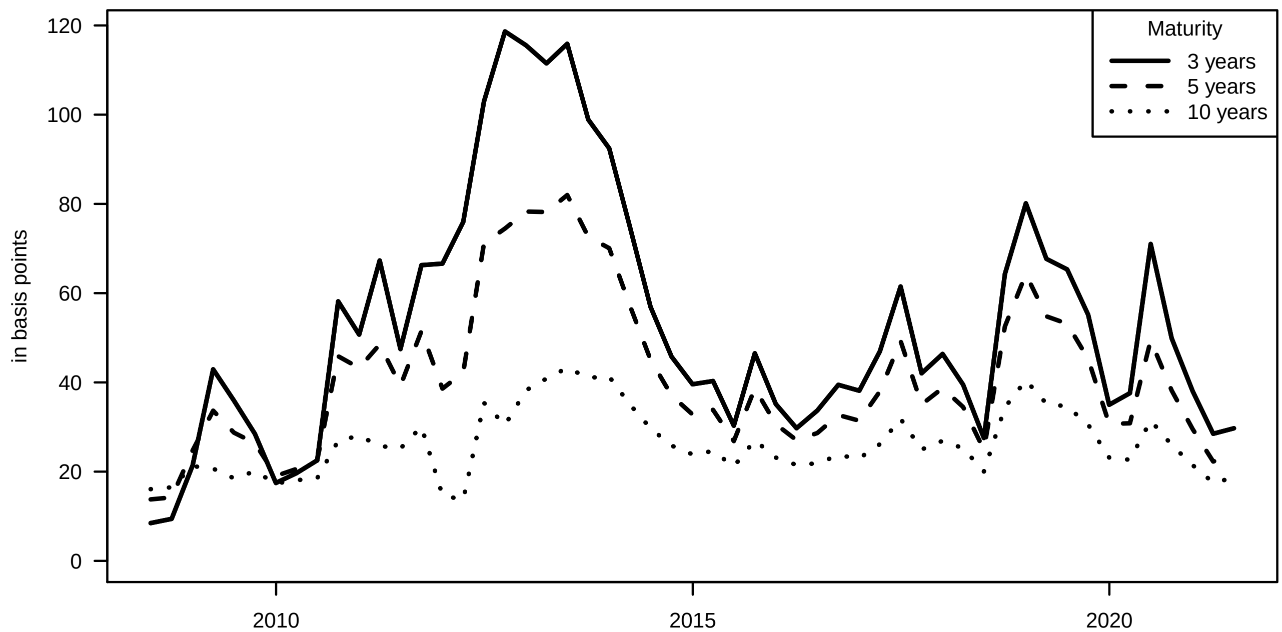
Notes: This figure compares counterfactual yield spreads (versus risk-free interest rates) associated with common bonds benefitting from several and joint guarantees (SJG) and bonds with several but not joint guarantees (SNJG). For the sake of comparison, we also add German bond yield spreads (circles). The grey shaded-area shows the range of SJG spreads obtained when varying the correlation across debt and fiscal limit innovations (respectively the $\varepsilon_{d,j,t}$s of eq. 9 and the $\varepsilon_{\ell,j,t}$s of eq. 10) from 30% to 99%; the baseline case (in black) is for a 50% correlation.

FIGURE 6. Observed proxies of common bond spreads versus 10-year German benchmark



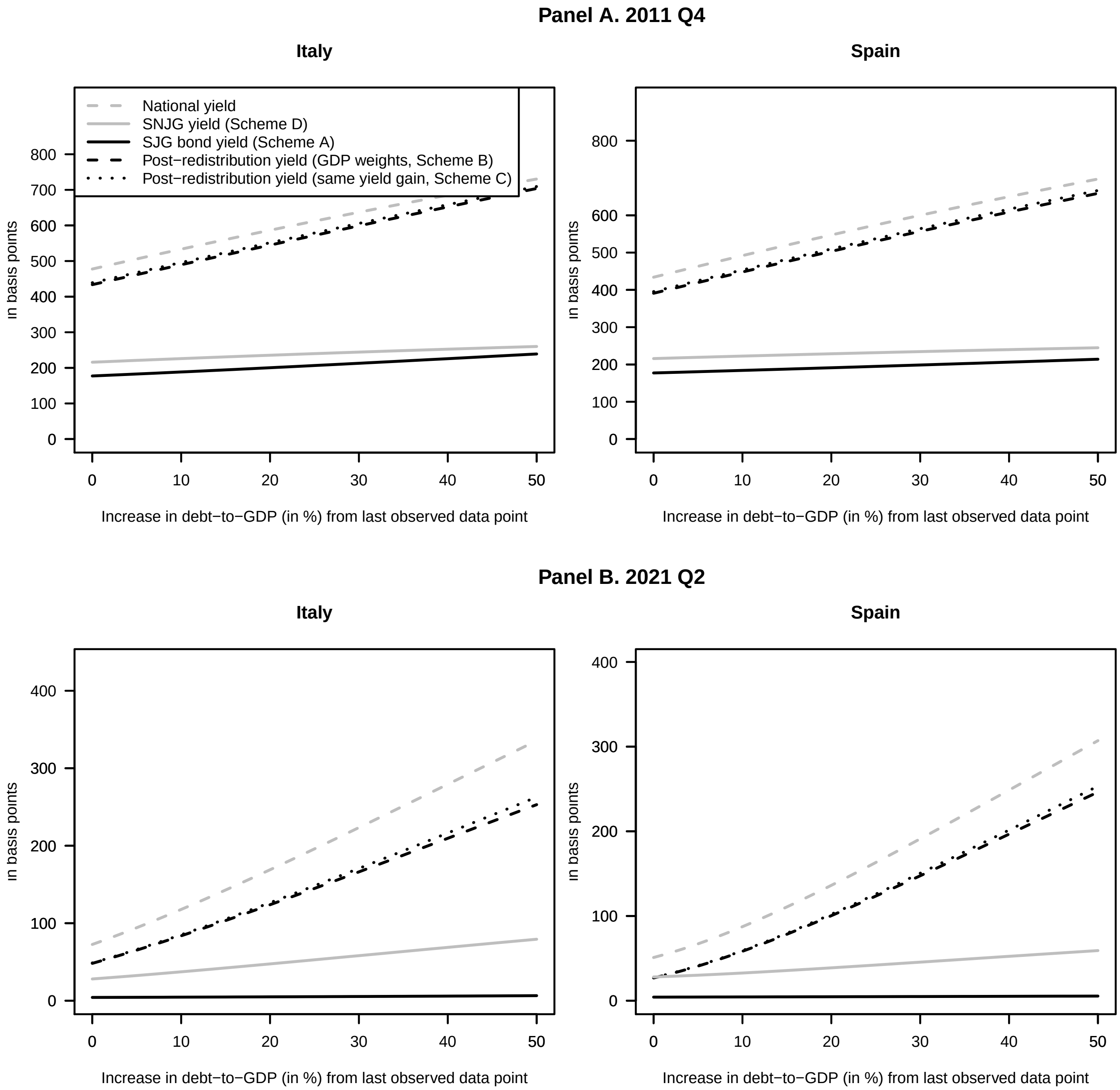
Notes: This figure shows bond yield spreads w.r.t. the German 10-year benchmark bond. Black and blue (respectively grey) lines correspond to proxies for SJG bonds (resp. SNJG bonds). We consider bonds issued by the European Investment Bank (EIB), the European Financial Stability Facility (EFSF), the European Stability Mechanism (ESM), the European Union (EU, NEXT GEN EU). The SNJG proxy (grey lines) is computed as a GDP-weighted average of national-bond spreads (versus Germany). The data are at the daily frequency; they span the period from February, 26 20219 to June 17, 2021. The dates reported in the legend of the figure correspond to maturity dates (2029 or 2031) of the specific bonds. The spreads are computed as the differences in asset swap spreads w.r.t. to the Bund; (see Online Appendix V for more details). As of November 2021, the credit ratings of the considered European institutions were as follows (Moody's/S&P/Fitch): EIB (Aaa/AAA/AAA), EFSF (Aa1/AA/AA+), ESM (Aa1, AAA/AAA), and EU (Aaa/AA/AAA).

FIGURE 7. Yield gains associated with redistribution scheme with same yield gains across countries



Notes: This figure shows yield gains associated with redistribution Scheme C (same yield gains across countries) throughout the whole estimation sample and for different maturities. See Subsection 6.2 for details regarding this redistribution scheme. Yield gains are expressed in basis points.

FIGURE 8. Moral hazard risk and redistribution: counterfactual exercise



Notes: This figure shows the increase in different bond spreads (w.r.t. to risk-free rates) resulting from counterfactual increases in Italian indebtedness (left column of plots) or Spanish indebtedness (right column of plots), all else being equal. The two rows correspond to different periods, namely 2011Q4 (euro-area sovereign debt crisis) and 2021Q2 (end of the estimation sample). The different schemes (A to D) are described in Subsection 6.2.

References

Acharya, V., I. Drechsler, and P. Schnabl (2014). A Pyrrhic Victory? Bank Bailouts and Sovereign Credit Risk. *Journal of Finance 69*(6), 2689–2739.

Aguiar, M. and M. Amador (2014). *Sovereign Debt*, pp. 647–87. North-Holland.

Ang, A. and F. A. Longstaff (2013). Systemic Sovereign Credit Risk: Lessons from the U.S. and Europe. *Journal of Monetary Economics 60*(5), 493–510.

Ang, A. and M. Piazzesi (2003). A No-Arbitrage Vector Autoregression of Term Structure Dynamics with Macroeconomic and Latent Variables. *Journal of Monetary Economics 50*(4), 745–787.

Beetsma, R. and K. Mavromatis (2014). An Analysis of Eurobonds. *Journal of international money and finance 45*, 91–111.

Bi, H. (2012). Sovereign Default Risk Premia, Fiscal Limits, and Fiscal Policy. *European Economic Review 56*(3), 389–410.

Bi, H. and E. M. Leeper (2013). Analyzing Fiscal Sustainability. Staff working papers, Bank of Canada.

Bi, H. and N. Traum (2012). Estimating Sovereign Default Risk. *American Economic Review, Papers & Proceedings 102*(3), 161–166.

Bi, H. and N. Traum (2014). Estimating Fiscal Limits: The Case of Greece. *Journal of Applied Econometrics 29*(7), 1053–1072.

Black, F. (1995). Interest Rates as Options. *Journal of Finance 50*(5), 1371–76.

Black, F. and M. S. Scholes (1973). The Pricing of Options and Corporate Liabilities. *Journal of Political Economy 81*(3), 637–54.

Bofinger, P., L. P. Feld, W. Franz, C. M. Schmidt, and B. W. di Mauro (2011). A European Redemption Pact. *VoxEU, November 9*.

Borgy, V., T. Laubach, J.-S. Mésonnier, and J.-P. Renne (2011). Fiscal Sustainability, Default Risk and Euro Area Sovereign Bond Spreads Markets. Technical report.

Brunnermeier, M. K., S. Langfield, M. Pagano, R. Reis, S. V. Nieuwerburgh, and D. Vayanos (2017). ESBies: Safety in the Tranches. *Economic Policy 32*(90), 175–219.

Chen, R.-R. and L. Scott (1993). Maximum Likelihood Estimation for a Multifactor Equilibrium Model of the Term Structure of Interest Rates. *Journal of Fixed Income 3*, 14–31.

Claessens, S., A. Mody, and S. Vallée (2012). Paths to Eurobonds. IMF Working Papers 12/172, International Monetary Fund.

Cochrane, J. H. and J. Saa-Requejo (2000). Beyond Arbitrage: Good-Deal Asset Price Bounds in Incomplete Markets. *Journal of Political Economy 108*(1), 79–119.

Collard, F., M. Habib, and J.-C. Rochet (2015). Sovereign Debt Sustainability In Advanced Economies. *Journal of the European Economic Association 13*(3), 381–420.

Coroneo, L. and S. Pastorello (2020). European Spreads at the Interest Rate Lower Bound. *Journal of Economic Dynamics and Control 119*(C).

Dávila, E. and C.-H. Weymuller (2016). Optimal Joint Bond Design. 2016 Meeting Papers 1447, Society for Economic Dynamics.

De Grauwe, P. and W. Moesen (2009). Gains for All: a Proposal for a Common Euro Bond. *Intereconomics 44*(3), 132–135.

Delivorias, A. and C. Stamegna (2020). Joint Debt Instruments: a Recurrent Proposal to Strengthen Economic and Monetary Union. Briefing PE 649.361, European Parliamentary Research Service.

Delpla, J. and J. Von Weizsacker (2010). The Blue Bond Proposal. *Bruegel Policy Brief 3*(6).

Delpla, J. and J. V. Weizsacker (2010, May). The Blue Bond Proposal. Policy Briefs 403, Bruegel.

Doshi, H., J. Ericsson, K. Jacobs, and S. M. Turnbull (2013). Pricing Credit Default Swaps with Observable Covariates. *The Review of Financial Studies 26*(8), 2049–2094.

Draghi, M. (2012). Introductory Statement to the Press Conference (following The Governing Council On 6 September 2012). Speech by the President of the European Central Bank, European Central Bank.

Duffie, D. and K. Singleton (2003). *Credit Risk: Pricing, Measurement, and Management*. Princeton Series in Finance. Princeton University Press.

Duffie, D. and K. J. Singleton (1999). Modeling Term Structures of Defaultable Bonds. *Review of Financial Studies 12*(4), 687–720.

European Commission (2011). Green Paper on the Feasibility of Introducing Stability Bonds.

Favero, C. and A. Missale (2012). Sovereign Spreads in the Eurozone: Which Prospects for a Eurobond? *Economic Policy 27*(70), 231–273.

Ghosh, A., J. I. Kim, E. Mendoza, J. Ostry, and M. Qureshi (2013). Fiscal Fatigue, Fiscal Space and Debt Sustainability in Advanced Economies. *Economic Journal*, F4–F30.

Ghosh, A. R., J. D. Ostry, and M. S. Qureshi (2013). Fiscal Space and Sovereign Risk Pricing in a Currency Union. *Journal of International Money and Finance 34*, 131 – 163.

Giovannini, A. (2000). Co-ordinated Public Debt Issuance in the Euro Area. *Report of the Giovannini Group*.

Hansen, L. P. and R. Jagannathan (1991). Implications of Security Market Data for Models of Dynamic Economies. *Journal of Political Economy 99*(2), 225–262.

Hellwig, C. and T. Philippon (2011). Eurobills, not Eurobonds. *VoxEU. org, December 2*, 2011.

Hördahl, P. and O. Tristani (2013). Macro Factors and Sovereign Bond Spreads: A Quadratic No-Arbitrage Model. In *HKIMR Seminar, August 2013, Hong Kong, China*. Citeseer.

Issing, O. (2009). Why a Common Eurozone Bond Isn't Such a Good Idea. *Center for Financial Studies, Frankfurt*.

Jordà, Ò., M. Schularick, and A. M. Taylor (2016). Sovereigns versus Banks: Credit, Crises, and Consequences. *Journal of the European Economic Association 14*(1), 45–79.

Krippner, L. (2015). *Zero Lower Bound Term Structure Modeling*. Palgrave Macmillan, New York.

Leeper, E. M. (2013). Fiscal Limits and Monetary Policy. NBER Working Papers 18877, National Bureau of Economic Research, Inc.

Liu, J., F. A. Longstaff, and R. E. Mandell (2006). The Market Price of Risk in Interest Rate Swaps: The Roles of Default and Liquidity Risks. *The Journal of Business 79*(5), 2337–2360.

Longstaff, F. A., J. Pan, L. H. Pedersen, and K. J. Singleton (2011). How Sovereign Is Sovereign Credit Risk? *American Economic Journal: Macroeconomics 3*(2), 75–103.

Lorenzoni, G. and I. Werning (2013). Slow Moving Debt Crises. Technical report, National Bureau of Economic Research.

Mehrotra, N. and D. Sergeyev (2020). Debt Sustainability in a Low Interest Rate World. CEPR Discussion Papers 15282, C.E.P.R. Discussion Papers.

Merton, R. C. (1974). On the Pricing of Corporate Debt: The Risk Structure of Interest Rates. *Journal of Finance 29*(2), 449–70.

Monfort, A., F. Pegoraro, J.-P. Renne, and G. Roussellet (2020). Affine Modeling of Credit Risk, Pricing of Credit Events and Contagion. *Management Science forthcoming*.

Monfort, A. and J.-P. Renne (2013). Default, Liquidity and Crises: An Econometric Framework. *Journal of Financial Econometrics 11*(2), 221–262.

Monfort, A. and J.-P. Renne (2014). Decomposing Euro-Area Sovereign Spreads: Credit and Liquidity Risks. *Review of Finance 18*(6), 2103–2151.

Pallara, K. and J.-P. Renne (2020). Fiscal Limits and Sovereign Credit Spreads. Technical report.

Pan, J. and K. Singleton (2008). Default and Recovery Implicit in the Term Structure of Sovereign CDS Spreads. *Journal of Finance 63*(5), 2345–84.

Tirole, J. (2015). Country Solidarity in Sovereign Crises. *American Economic Review 105*(8), 2333–63.

Trabandt, M. and H. Uhlig (2011). The Laffer Curve Revisited. *Journal of Monetary Economics 58*(4), 305–327.

Tsiropoulos, V. (2019). Eurobonds : A Quantitative Analysis of Joint-Liability Debt. Policy Research Working Paper Series 9017, The World Bank.

Wu, J. C. and F. D. Xia (2016). Measuring the Macroeconomic Impact of Monetary Policy at the Zero Lower Bound. *Journal of Money, Credit and Banking 48*(2-3), 253–291.

Yue, V. Z. and B. Wei (2019). Sovereign Debt: Theory. *Oxford Research Encyclopedias of Economics and Finance*.

APPENDIX A. X_t'S DYNAMICS

The state vector X_t follows the vector autoregressive process of order one given in eq. (12), with:

$$\mu = \begin{bmatrix} (1-\rho_i)\bar{i} \\ 0 \\ (1-\rho_d)\bar{d}_1 \\ \vdots \\ (1-\rho_d)\bar{d}_N \\ (1-\rho_\ell)\bar{\ell}_1 \\ \vdots \\ (1-\rho_\ell)\bar{\ell}_N \end{bmatrix}, \quad \Phi = \begin{bmatrix} \rho_i & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & \rho_d \mathbf{Id}_{N\times N} & 0 \\ 0 & 0 & 0 & \rho_\ell \mathbf{Id}_{N\times N} \end{bmatrix},$$

and

$$\Sigma = \begin{bmatrix} \sigma_i & \cdots & & & \\ 0 & \cdots & & & \\ \mathbf{0}_{N\times 1} & \gamma_d \mathbf{Id}_{N\times N} & \mathbf{0}_{N\times N} & \alpha_d \mathbf{1}_{N\times 1} & \mathbf{0}_{N\times 1} \\ \mathbf{0}_{N\times 1} & \mathbf{0}_{N\times N} & \gamma_\ell \mathbf{Id}_{N\times N} & \mathbf{0}_{N\times 1} & \alpha_\ell \mathbf{1}_{N\times 1} \end{bmatrix}.$$

APPENDIX B. $\mathbb{P}$ TO $\mathbb{Q}$ DYNAMICS

Let us introduce the risk-neutral measure, defined with respect to the physical measure through the following Radon-Nikodym derivative:

$$\left.\frac{d\mathbb{Q}}{d\mathbb{P}}\right|_{t,t+1} = \frac{\mathcal{M}_{t,t+1}}{\mathbb{E}_t(\mathcal{M}_{t,t+1})} = \exp\left(-\frac{1}{2}\psi'\psi - \psi'\eta_{t+1}\right).$$

Under the physical measure, the conditional Laplace transform of X_t is given by

$$\mathbb{E}_t(\exp(u'X_{t+1})) = \exp\left(u'\mu + u'\Phi X_t + \frac{1}{2}u'\Sigma'\Sigma u\right). \tag{a.1}$$

Let us now compute the conditional Laplace transform of X_t under the risk-neutral measure:

$$\begin{aligned} \mathbb{E}_t^{\mathbb{Q}}(\exp(u'X_{t+1})) &= \mathbb{E}_t\left(\exp\left(-\frac{1}{2}\psi'\psi - \psi'\eta_{t+1}\right)\exp(u'X_{t+1})\right) \\ &= \mathbb{E}_t\left(\exp\left(-\frac{1}{2}\psi'\psi + u'\mu + u'\Phi X_t + (\Sigma'u - \psi)'\eta_{t+1}\right)\right) \\ &= \exp\left(u'(\mu - \Sigma\psi) + u'\Phi X_t + \frac{1}{2}u'\Sigma'\Sigma u\right). \end{aligned}$$

By analogy with (a.1), it comes that the risk-neutral dynamics of X_t reads:

$$X_t = \mu^{\mathbb{Q}} + \Phi^{\mathbb{Q}}X_{t-1} + \Sigma\eta_t^{\mathbb{Q}}, \quad \eta_t^{\mathbb{Q}} \sim i.i.d.\mathcal{N}(0, I), \tag{a.2}$$

where $\mu^{\mathbb{Q}} = \mu - \Sigma\psi$, and $\Phi^{\mathbb{Q}} = \Phi$.

APPENDIX C. PRICING OF RISK-FREE BONDS

By definition of the state vector $X_t = [i_t, i_{t-1}, d_t, \ell_t]$, eq. (18) is satisfied for $h = 1$, with:

$$A_1 = 0, \quad \text{and} \quad B_1 = -[1, 0, \dots]'.$$

Let us assume that eq. (18) holds for a maturity $h-1$, with $h > 1$. Then, the price of a risk-free zero-coupon bond of maturity $h-1$ is given by

$$P_{t,h-1} = \exp(A_{h-1} + B'_{h-1}X_t). \tag{a.3}$$

Let us then express the price of a risk-free zero-coupon bond of maturity h:

$$\begin{aligned} P_{t,h} &= \mathbb{E}_t(\mathcal{M}_{t,t+1}P_{t,h-1}) = \exp(-i_t)\mathbb{E}_t^{\mathbb{Q}}(\exp(A_{h-1} + B'_{h-1}X_{t+1})) \quad \text{using (a.3)} \\ &= \exp(B'_1X_t)\mathbb{E}_t^{\mathbb{Q}}(\exp(A_{h-1} + B'_{h-1}[\mu^{\mathbb{Q}} + \Phi^{\mathbb{Q}}X_t + \Sigma\eta_{t+1}])) \\ &= \exp\left(A_{h-1} + B'_{h-1}\mu^{\mathbb{Q}} + \frac{1}{2}B'_{h-1}\Sigma\Sigma'B_{h-1} + [B_1 + \Phi^{\mathbb{Q}'}B_{h-1}]'X_t\right), \end{aligned}$$

which leads to eq. (17), using the definitions of $\mu^{\mathbb{Q}}$ and $\Phi^{\mathbb{Q}}$ given in (a.2).

APPENDIX D. PRICING OF ZERO-COUPON ZERO-RECOVERY RISKY BONDS

Denote $P^{(j)}_{t,h}$ the date-t price of a zero-coupon bond providing a payoff of 1 on date $t+h$ if country j has not defaulted before $t+h$, and zero otherwise (see Appendix I). Thus, we have:

$$\begin{aligned} P^{(j)}_{t,h} &= \mathbb{E}_t^{\mathbb{Q}}(\Lambda_{t,t+h}(1-\mathcal{D}_{j,t+h})) = \mathbb{E}_t^{\mathbb{Q}}\left\{\exp(-i_t - \dots - i_{t+h-1})(1-\mathcal{D}_{j,t+h})\right\} \\ &= \mathbb{E}_t^{\mathbb{Q}}\left\{\mathbb{E}_t^{\mathbb{Q}}\left\{\exp(-i_t - \dots - i_{t+h-1})(1-\mathcal{D}_{j,t+h})|X_{t+h}, X_{t+h-1}, \dots\right\}\right\} \\ &= \mathbb{E}_t^{\mathbb{Q}}\left\{\exp(-i_t - \dots - i_{t+h-1} - \underline{\lambda}_{j,t+1} - \dots - \underline{\lambda}_{j,t+h})\right\}, \end{aligned} \tag{a.4}$$

where the last equality is obtained under the assumption that $\mathcal{D}_t$ does not cause X_t in the Granger's or Sims' sense (Monfort and Renne, 2013, Proposition 3).

Because the default intensities $\underline{\lambda}_{j,t}$ involve a max operator (see eq. 2), eq. (a.4) does not admit closed-form solutions. We follow Wu and Xia (2016) and look for an approximation for the following "forward" rate:

$$p_{j,h-1,h} = -\log(P^{(j)}_{t,h}) + \log(P^{(j)}_{t,h-1}) \tag{a.5}$$

Then, we get an approximation to $P^{(j)}_{t,h}$ by taking the exponential of the cumulated forward rates. The approximation is essentially based on $\log \mathbb{E}[\exp(Z)] \approx \mathbb{E}(Z) + \frac{1}{2}\mathbb{V}(Z)$, which is exact when Z is Gaussian, but not if it is truncated Gaussian, as is the case here.

As detailed in the Online Appendix I, we get:

$$\begin{aligned} p_{j,k-1,k} \approx\ & \delta'\mu^{\mathbb{Q}}_{t,k} + \Phi\left(\frac{\mu^{\mathbb{Q}}_{j,t,k}}{\sigma^{\mathbb{Q}}_{j,k}}\right)\mu^{\mathbb{Q}}_{j,t,k} + \phi\left(-\frac{\mu^{\mathbb{Q}}_{j,t,k}}{\sigma^{\mathbb{Q}}_{j,k}}\right)\sigma^{\mathbb{Q}}_{j,k} - \frac{1}{2}\left(\mathfrak{q}_{j,t,k}(\delta+\delta_j)'\Gamma^{\mathbb{Q}}_{k,0}(\delta+a_j) + (1-\mathfrak{q}_{j,t,k})\delta'\Gamma^{\mathbb{Q}}_{k,0}\delta\right) \\ & - \sum_{i=1}^{k-1}\left(\mathfrak{q}_{j,t,k-i}(\delta+\delta_j)'\Gamma^{\mathbb{Q}}_{k,i}(\delta+\delta_j) + (1-\mathfrak{q}_{j,t,k-i})\delta'\Gamma^{\mathbb{Q}}_{k,i}\delta\right), \end{aligned} \tag{a.6}$$

where $\delta_j = [0, 0, \alpha e_j', -\alpha e_j']'$ (e_j denoting the j^{th} column vector of the $N \times N$ identity matrix), $\mathfrak{q}_{j,t,k} = \Phi\left(\mu^{\mathbb{Q}}_{t,k}/\sigma^{\mathbb{Q}}_{j,k}\right)$, and

$$\begin{cases} \mu^{\mathbb{Q}}_{t,k} & = & \mathbb{E}^{\mathbb{Q}}_t(X_{t+k}) & = & (Id - \Phi^{\mathbb{Q}})^{-1}(Id - \Phi^{\mathbb{Q}^k})\mu^{\mathbb{Q}} + \Phi^{\mathbb{Q}^k}X_t, \\ \Gamma^{\mathbb{Q}}_{k,0} & = & \mathbb{V}^{\mathbb{Q}}_t(X_{t+k}) & = & \Omega + \Phi^{\mathbb{Q}}\Gamma^{\mathbb{Q}}_{k-1,0}\Phi^{\mathbb{Q}'}, \quad \text{with} \quad \Gamma^{\mathbb{Q}}_{1,0} = \Omega \\ & & & = & \Omega + \Phi^{\mathbb{Q}}\Omega\Phi^{\mathbb{Q}'} + \dots + \Phi^{\mathbb{Q}^{k-1}}\Omega\Phi^{\mathbb{Q}^{k-1}\prime}, \\ \Gamma^{\mathbb{Q}}_{k,i} & = & \mathbb{C}ov^{\mathbb{Q}}_t(X_{t+k}, X_{t+k-i}) & = & \Phi^{\mathbb{Q}^i}\Gamma^{\mathbb{Q}}_{k-i,0} \quad \text{if} \quad k - i > 0, \end{cases}$$

where $\mu^{\mathbb{Q}}$ and $\Phi^{\mathbb{Q}}$ are given in Appendix B.

Appendix E. Pricing zero-coupon bonds with non-zero recovery rates

Consider a zero-coupon bond of maturity h issued by country j. Assume the recovery rate is RR. On date $t+k$, with $0 < k < h$, the payoff of this bond is zero, unless the country defaults on date $t+k$, in which case the bond payoff is assumed to be the fraction RR of the price of a risk-free zero-coupon bond of equivalent residual maturity, i.e. $\exp[-(h-k)i_{t+k,h-k}]$ (this is the Recovery of Treasury convention—RT—of Duffie and Singleton, 1999). Hence, the payoffs of this bond are of the form:

$$\begin{cases} RR \times \exp(-(h-k)i_{t+k,h-k}) \times (\mathcal{D}_{j,t+k} - \mathcal{D}_{j,t+k-1}) & \text{if} \quad 0 < k < h, \\ 1 - \mathcal{D}_{j,t+k} + RR \times (\mathcal{D}_{j,t+k} - \mathcal{D}_{j,t+k-1}) & \text{if} \quad k = h. \end{cases}$$

As a result, denoting by $\Lambda_{t,t+k}$ the (non stochastic) discount factor $\exp(-i_t - \dots - i_{t+k-1})$, the price of this bond is given by:

$$\begin{aligned} \mathcal{P}^{(j)}_{t,h} &= \mathbb{E}^{\mathbb{Q}}_t\left(\Lambda_{t,t+h}(1 - \mathcal{D}_{j,t+h}) + RR\sum_{k=1}^{h}\Lambda_{t,t+k}\exp(-(h-k)i_{t+k,h-k})(\mathcal{D}_{j,t+k} - \mathcal{D}_{j,t+k-1})\right) \\ &= \mathbb{E}^{\mathbb{Q}}_t(\Lambda_{t,t+h}(1 - \mathcal{D}_{j,t+h})) + RR\sum_{k=1}^{h}\mathbb{E}^{\mathbb{Q}}_t\left[\Lambda_{t,t+k}\mathbb{E}^{\mathbb{Q}}_{t+k}\left\{\exp(-i_{t+k} - \dots - i_{t+h-1})\right\}(\mathcal{D}_{j,t+k} - \mathcal{D}_{j,t+k-1})\right] \\ &= \mathbb{E}^{\mathbb{Q}}_t(\Lambda_{t,t+h}(1 - \mathcal{D}_{j,t+h})) + RR\sum_{k=1}^{h}\mathbb{E}^{\mathbb{Q}}_t\left[\Lambda_{t,t+h}(\mathcal{D}_{j,t+k} - \mathcal{D}_{j,t+k-1})\right] \quad \text{(by the law of iterated expectations)} \\ &= \mathbb{E}^{\mathbb{Q}}_t(\Lambda_{t,t+h}(1 - \mathcal{D}_{j,t+h})) + RR\,\mathbb{E}^{\mathbb{Q}}_t(\Lambda_{t,t+h})\sum_{k=1}^{h}\mathbb{E}^{\mathbb{Q}}_t\left[(\mathcal{D}_{j,t+k} - \mathcal{D}_{j,t+k-1})\right], \end{aligned}$$

where the conditional expectation $\mathbb{E}^{\mathbb{Q}}_t(\Lambda_{t,t+h}(1 - \mathcal{D}_{j,t+h}))$ represents the date-t price of a zero-coupon zero-recovery risky bond of maturity h providing a payoff of 1 on date $t+h$ if country j has not defaulted before $t+h$, and zero otherwise (see Appendices D for an approximation of this price). Moreover, $\mathbb{E}^{\mathbb{Q}}_t(\Lambda_{t,t+h}\mathcal{D}_{j,t+k}) = \mathbb{E}^{\mathbb{Q}}_t(\Lambda_{t,t+h})\mathbb{E}^{\mathbb{Q}}_t(\mathcal{D}_{j,t+h})$ results from the fact that, under our assumptions regarding the s.d.f., $\mathcal{D}_t$ and i_t are independent under the risk-neutral measure $\mathbb{Q}$ (as they are under $\mathbb{P}$). Therefore:

$$\begin{aligned} \mathcal{P}^{(j)}_{t,h} &= \mathbb{E}^{\mathbb{Q}}_t(\Lambda_{t,t+h}(1 - \mathcal{D}_{j,t+h})) + RR\,\mathbb{E}^{\mathbb{Q}}_t(\Lambda_{t,t+h}\mathcal{D}_{j,t+h}) \\ &= \mathbb{E}^{\mathbb{Q}}_t(\Lambda_{t,t+h}(1 - \mathcal{D}_{j,t+h})) - RR\,\mathbb{E}^{\mathbb{Q}}_t(\Lambda_{t,t+h}(1 - \mathcal{D}_{j,t+h})) + RR\,\mathbb{E}^{\mathbb{Q}}_t(\Lambda_{t,t+h}) \\ &= (1 - RR)P^{(j)}_{t,h} + RR\exp(-hi^0_{t,h}), \end{aligned}$$

where approximation formulas for $P_{t,h}^{(j)}$ are given in Appendix D.

Appendix F. Maximum Sharpe Ratio

Hansen and Jagannathan (1991) show that the maximum Sharpe ratio for a one-period investment is given by:

$$maxSR_t \quad = \quad \frac{\sqrt{\mathbb{V}ar_t(\mathcal{M}_{t,t+1})}}{\mathbb{E}_t(\mathcal{M}_{t,t+1})}.$$

In the present context, the exponential affine form of our s.d.f. (14) implies that:

$$maxSR_t = \frac{\sqrt{\mathbb{V}ar_t \exp(-\psi' \varepsilon_{t+1})}}{\mathbb{E}_t \exp(-\psi' \varepsilon_{t+1})} = \sqrt{\exp(\psi'\psi) - 1},$$

which does not depend on time. Since $\psi = [\psi_i, \nu\omega', -\nu\omega', \nu, -\nu]'$ (where ω is the vector of GDP weights), we obtain:

$$maxSR = \sqrt{\exp(\psi_i^2 + 2\nu^2\omega'\omega + \nu^2) - 1}.$$

Appendix G. Alternative (static) fiscal limit estimates

Table 5. Fiscal limit estimates comparison

Ctry	Ghosh et al. (2013)		Collard et al. (2015)					
	Hist.	**Proj.**	5% **MPS**	**MRR**	**TVR**	**CATA**	4% **MPS**	**h. MPS**
DE	154.1	175.8	130.1	132.3	114.6	85.5	104.1	112.9
FR	170.9	176.1	146.6	148.6	119.8	97.8	117.2	40
IT	–	–	113.2	115.6	106.8	74.2	90.6	147.5
ES	218.3	153.9	144.2	146.2	119.3	95.8	115.3	115.6

Note: All estimates are reported in percent of GDP. **This paper – $\overline{\textbf{FL}}$**: Sample mean of the fiscal limit estimates. **SD**(**FL**): Standard deviation of the fiscal limit estimates. min(**FL**): absolute minimum for fiscal limit estimates. max(**FL**): absolute maximum for fiscal limit estimates. **Estimates of Ghosh et al. (2013) –** Debt limits (fiscal limits in our terminology) are statically estimated through the interest payment schedule for the period 1985-2007. **Hist.**: Estimates are based on the average interest rate / growth differential of 1998-2007, using the implied interest rate on public debt; **Proj.**: The interest rate / growth differential is based on the long term government bond yield (average for 2010-2014, IMF projections as of 2010). **Estimates of Collard et al. (2015) –** The computation of maximum sustainable debts (fiscal limits in our terminology) exploits the idea of a maximum primary surplus (MPS). In the model, there is a maximum amount that can be issued on each date (that itself depends on the MPS). 5% **MPS**: Case where the MPS is set to to 5%; **MRR**: The computation involves a maximum recovery rate; **TVR**: The model features a time-varying interest rate; **CATA**: The model features catastrophes; 4% **MPS**: The MPS is set to 5%; **h. MPS**: The MPS is set to the historical peak of primary surplus-to-GDP.

—Online Appendix –

Fiscal Limits and the Pricing of Eurobonds

Kevin PALLARA and Jean-Paul RENNE

APPENDIX I. APPROXIMATE FORMULA FOR ZERO-COUPON RISKY BOND

This appendix details the approximation to the price $P^{(j)}_{t,h}$ (this price being defined though eq. a.4); the resulting formula is given in Appendix D.

Since $X_t = [i_t, i_{t-1}, d_{1,t}, \cdots, d_{n,t}, \ell_{1,t}, \cdots, \ell_{n,t}]'$, we have

$$i_{t-1} = \delta' X_t, \tag{I.1}$$

where $\delta = [0, 1, 0, \dots, 0]'$. Moreover, we also introduce the following notation:

$$\lambda_{j,t} = \delta_j' X_t,$$

where $\delta_j = [0, 0, \alpha e_j', -\alpha e_j']'$, e_j denoting the j^{th} column vector of the $N \times N$ identity matrix. With these notations, eq. (2) rewrites:

$$\underline{\lambda}_{j,t} = \max(0, \lambda_{j,t}),$$

that is, $\lambda_{j,t}$ can be seen as a "shadow intensity". With these notations, we can rewrite eq. (a.4) as:

$$P^{(j)}_{t,h} \quad = \quad \mathbb{E}^{\mathbb{Q}}_t[\exp(-\delta' X_{t+1} - \max(0, \lambda_{j,t+1}) - \cdots - \delta' X_{t+h} - \max(0, \lambda_{j,t+h}))]. \tag{I.2}$$

Let us recall the notation introduced in Appendix D:

$$p_{j,h-1,h} \quad = \quad -\log(P^{(j)}_{t,h}) + \log(P^{(j)}_{t,h-1}). \tag{I.3}$$

In the spirit of Wu and Xia (2016), we determine approximations to $p_{j,h-1,h}$ that we further use to get approximations to $P^{(j)}_{t,h}$, using:

$$P^{(j)}_{t,h} \quad = \quad \exp(p_{j,0,1} + p_{j,1,2} + \cdots + p_{j,h-1,h}). \tag{I.4}$$

The approximation to $p_{j,h-1,h}$ is essentially based on $\log \mathbb{E}[\exp(Z)] \approx \mathbb{E}(Z) + \frac{1}{2}\mathbb{V}(Z)$, which is exact when Z is Gaussian, but not if it is truncated Gaussian, as is the case here. This gives:

$$\begin{aligned} p_{j,k-1,k} \quad = \quad & \mathbb{E}^{\mathbb{Q}}_t(\delta' X_{t+k} + \underline{\lambda}_{j,t+k}) - \frac{1}{2}\mathbb{V}^{\mathbb{Q}}_t(\delta' X_{t+k} + \underline{\lambda}_{j,t+k}) - \\ & -\mathbb{C}ov^{\mathbb{Q}}_t\left(\delta' X_{t+k} + \underline{\lambda}_{j,t+k}, \sum_{i=1}^{k-1}(\delta' X_{t+i} + \underline{\lambda}_{j,t+i})\right) \end{aligned} \tag{I.5}$$

Following Wu and Xia (2016), considering that $\lambda_{j,t}$ is a persistent process and introducing the following notation:

$$\mathfrak{q}_{j,t,k} = \mathbb{P}^{\mathbb{Q}}_t(d_{j,t+k} > \ell_{j,t+k}),$$

we have, for $k > 0$ and $0 \leq i \leq k$:

$$\mathbb{C}ov_t^{\mathbb{Q}}(\underbrace{i_{t-1+k}}_{\delta' X_{t+k}}, \underline{\lambda}_{j,t+k-i}) \approx \mathfrak{q}_{j,t,k-i}\mathbb{C}ov_t^{\mathbb{Q}}(\underbrace{i_{t-1+k}}_{\delta' X_{t+k}}, \lambda_{j,t+k-i}) \quad \text{(I.6)}$$

$$\mathbb{C}ov_t^{\mathbb{Q}}(\underline{\lambda}_{j,t+k}, \underline{\lambda}_{j,t+k-i}) \approx \mathfrak{q}_{j,t,k-i}\mathbb{C}ov_t^{\mathbb{Q}}(\lambda_{j,t+k}, \lambda_{j,t+k-i}) \quad \text{(I.7)}$$

Using the last two equations, we can rewrite eq. (I.5) as follows:

$$\begin{aligned} p_{j,k-1,k} \approx\ & \mathbb{E}_t^{\mathbb{Q}}(\delta' X_{t+k} + \underline{\lambda}_{j,t+k}) - \\ & -\frac{1}{2}\Big(\mathfrak{q}_{j,t,k}\mathbb{V}_t^{\mathbb{Q}}(\delta' X_{t+k} + \lambda_{j,t+k}) + (1-\mathfrak{q}_{j,t,k})\mathbb{V}_t^{\mathbb{Q}}(\delta' X_{t+k})\Big) - \\ & -\sum_{i=1}^{k-1}\Big(\mathfrak{q}_{j,t,i}\mathbb{C}ov_t^{\mathbb{Q}}\Big(\delta' X_{t+k} + \lambda_{j,t+k}, \delta' X_{t+i} + \lambda_{j,t+i}\Big) + \\ & +(1-\mathfrak{q}_{j,t,i})\mathbb{C}ov_t^{\mathbb{Q}}\Big(\delta' X_{t+k}, \delta' X_{t+i}\Big)\Big). \end{aligned} \quad \text{(I.8)}$$

Posing

$$\begin{aligned} \mu_{t,k}^{\mathbb{Q}} &= \mathbb{E}_t^{\mathbb{Q}}(X_{t+k}), & \mu_{j,t,k}^{\mathbb{Q}} &= \mathbb{E}_t^{\mathbb{Q}}(\lambda_{j,t+k}), \\ \sigma_{j,k}^{\mathbb{Q}} &= \sqrt{\mathbb{V}_t^{\mathbb{Q}}(\lambda_{j,t+k})}, & \Gamma_{k,i}^{\mathbb{Q}} &= \mathbb{C}ov_t^{\mathbb{Q}}(X_{t+k}, X_{t+k-i}), \end{aligned}$$

and using $\lambda_{j,t} = \delta_j' X_t$, we finally obtain

$$\begin{aligned} p_{j,k-1,k} \approx\ & \delta' \mu_{t,k}^{\mathbb{Q}} + \Phi(\mu_{j,t,k}^{\mathbb{Q}}/\sigma_{j,k}^{\mathbb{Q}})\mu_{j,t,k}^{\mathbb{Q}} + \phi(-\mu_{j,t,k}^{\mathbb{Q}}/\sigma_{j,k}^{\mathbb{Q}})\sigma_{j,k}^{\mathbb{Q}} - \\ & -\frac{1}{2}\Big(\mathfrak{q}_{j,t,k}(\delta+\delta_j)'\Gamma_{k,0}^{\mathbb{Q}}(\delta + a_j) + (1-\mathfrak{q}_{j,t,k})\delta'\Gamma_{k,0}^{\mathbb{Q}}\delta\Big) - \\ & -\sum_{i=1}^{k-1}\Big(\mathfrak{q}_{j,t,k-i}(\delta+\delta_j)'\Gamma_{k,i}^{\mathbb{Q}}(\delta+\delta_j) + (1-\mathfrak{q}_{j,t,k-i})\delta'\Gamma_{k,i}^{\mathbb{Q}}\delta\Big), \end{aligned} \quad \text{(I.9)}$$

with

$$\mathfrak{q}_{j,t,k} = \Phi\left(\mu_{t,k}^{\mathbb{Q}}/\sigma_{j,k}^{\mathbb{Q}}\right).$$

The next appendix details a fast (coding-oriented) approach to compute the $\mu_{t,k}^{\mathbb{Q}}$s and $\Gamma_{k,j}^{\mathbb{Q}}$s.

Appendix II. Computation of $\mu_{t,k}^{\mathbb{Q}}$ and $\Gamma_{k,j}^{\mathbb{Q}}$

Recall X_t's law of motion (eq. 12):

$$X_t = \mu^{\mathbb{Q}} + \Phi^{\mathbb{Q}} x_{t-1}^{\mathbb{Q}} + \Sigma\varepsilon_{x,t}^{\mathbb{Q}}, \quad \varepsilon_{x,t} \sim i.i.d.\mathcal{N}(0, Id).$$

Using the notation $\Omega = \Sigma\Sigma'$, we have:

$$\left\{ \begin{array}{lllll} \mu_{t,k}^{\mathbb{Q}} & = & \mathbb{E}_t^{\mathbb{Q}}(X_{t+k}) & = & (Id - \Phi^{\mathbb{Q}})^{-1}(Id - \Phi^{\mathbb{Q}^k})\mu^{\mathbb{Q}} + \Phi^{\mathbb{Q}^k} X_t, \\ \Gamma_{k,0}^{\mathbb{Q}} & = & \mathbb{V}_t^{\mathbb{Q}}(X_{t+k}) & = & \Omega + \Phi^{\mathbb{Q}}\Gamma_{k-1,0}^{\mathbb{Q}}\Phi^{\mathbb{Q}'}, \quad \text{with} \quad \Gamma_{1,0}^{\mathbb{Q}} = \Omega \\ & & & = & \Omega + \Phi^{\mathbb{Q}}\Omega\Phi^{\mathbb{Q}'} + \cdots + \Phi^{\mathbb{Q}^{k-1}}\Omega\Phi^{\mathbb{Q}^{k-1'}}, \\ \Gamma_{k,i}^{\mathbb{Q}} & = & \mathbb{C}ov_t^{\mathbb{Q}}(X_{t+k}, X_{t+k-i}) & = & \Phi^{\mathbb{Q}^i}\Gamma_{k-i,0}^{\mathbb{Q}} \quad \text{if} \quad k-i > 0. \end{array} \right.$$

The estimation involves a large number of computations of the $\Gamma^{\mathbb{Q}}_{k,j}$'s. In order to speed up the computation, one can employ the following approach.

Consider a vector β of dimension n_x, that is the dimension of X_t, and let us denote by ξ_i^β the vector defined by $\xi_i^\beta = (\Phi_x^{\mathbb{Q}^i})'\beta$ (β will typically be δ_j, or $(\delta_j+\delta)$). Because we have $\Gamma^{\mathbb{Q}}_{k,i} = \Phi_x^{\mathbb{Q}^i}\Omega + \Phi_x^{\mathbb{Q}^{i+1}}\Omega\Phi_x' + \dots + \Phi_x^{k-1}\Omega\Phi_x^{k-1-i'}$, it comes that:

$$\beta'\Gamma_{k,j}\beta = \xi_i^{\beta'}\Omega\xi_0^\beta + {\xi_{i+1}^\beta}'\Omega\xi_1^\beta + \dots + {\xi_{k-1}^\beta}'\Omega\xi_{k-1-i}^\beta. \tag{II.10}$$

Let us consider a maximal value for k, say H, and let us denote by Ξ_β the $n_x \times (H+1)$ matrix whose w^{th} column is ξ_{w-1}^β. It can then be seen that the (i,k) entry of $\Psi^\beta := {\Xi_\beta}'\Omega\Xi_\beta$ – which is a matrix of dimension $(H+1)\times(H+1)$ – is equal to ${\xi_{i-1}^\beta}'\Omega\xi_{k-1}^\beta$. The sum of the entries $(i+1,1)$, $(i+2,2)$, ..., $(i+k,k)$ of Ψ^β therefore is

$$\xi_j^{\beta'}\Omega\xi_0^\beta + {\xi_{i+1}^\beta}'\Omega\xi_1^\beta + \dots + {\xi_{i+k-1}^\beta}'\Omega\xi_{k-1}^\beta,$$

which is equal to $\beta'\Gamma^{\mathbb{Q}}_{i+k,i}\beta$ according to (II.10). Equivalently, $\beta'\Gamma^{\mathbb{Q}}_{k,i}\beta$ is the sum of the entries $(i+1,1)$, $(i+2,2)$, ..., $(k,k-i)$ of Ψ^β.

In particular, the entry $(1,1)$ of Ψ^β is equal to $\beta'\Gamma_{1,0}\beta$, the sum of the entries $(1,1)$ and $(2,2)$ is equal to $\beta'\Omega\beta + \beta'\Phi_x\Omega\Phi_x'\beta = \beta'\Gamma_{2,0}\beta$, and, more generally, the sum of the entries $(1,1),\dots,(n-1,n-1)$ of Ψ^β is equal to $\beta'\Gamma_{n,0}\beta$.

Appendix III. Inversion technique

This appendix describes the computation of the likelihood function (see Subsection 5.2 for a general description of our estimation approach).

We consider the following decomposition of the state vector $X_t = [i_t, i_{t-1}, d_t', \ell_t']'$:

$$\underbrace{X_t}_{m\times 1} = \begin{bmatrix} \underbrace{\widetilde{X}_t}_{(m-N)\times 1} \\ \underbrace{\ell_t}_{N\times 1} \end{bmatrix},$$

where $\widetilde{X}_t$ are the observable components of X_t.

The state vector follows a vector autoregressive process of order one (eq. 12).

The vector of observed financial data is organized as follows:

$$Y_t = \begin{bmatrix} \underbrace{Y_t^{(y)}}_{n_y\times 1} \\ \underbrace{Y_{1,t}^{(YS)}}_{n_1\times 1} \\ \underbrace{Y_{2,t}^{(YS)}}_{N\times 1} \end{bmatrix},$$

where $Y_t^{(y)}$ is a vector of risk-free yields (of maturities 2, 3, 5 and 10 years), $Y_{1,t}^{(YS)}$ is a vector of imperfectly-fitted bond yield spreads (e.g. maturities 2 and 10 yrs) and $Y_{2,t}^{(YS)}$ is a $N\times 1$ vector of perfectly-fitted bond yield spreads (in our case, the average of bond yield spreads of maturities 2, 5 and 10 years). These yields and spreads are given by:

$$\left\{\begin{array}{rcl} Y_t^{(y)} &=& A_y + B_y' \widetilde{X}_t + \xi_t^{(y)} \\ Y_{1,t}^{(YS)} &=& f_1(\widetilde{X}_t, \ell_t) + \xi_t^{(YS)} \\ Y_{2,t}^{(YS)} &=& f_2(\widetilde{X}_t, \ell_t) \quad \text{(these spreads are perfectly fitted).} \end{array}\right. \tag{III.11}$$

We assume that the components of $\xi_t^{(y)}$ and $\xi_t^{(YS)}$ are i.i.d. normally-distributed measurement errors. The variance of each component of $\xi_t^{(y)}$ is σ_y^2. The variance of the i^{th} component of $\xi_t^{(YS)}$ is $\sigma_{YS,i}^2$.

System (III.11) can be rewritten:

$$\left\{\begin{array}{rcl} Y_t^{(y)} &=& A_y + B_y' \widetilde{X}_t + \xi_t^{(y)} \\ Y_{1,t}^{(YS)} &=& f_1(\widetilde{X}_t, f_2^*(\widetilde{X}_t, Y_{2,t}^{(YS)})) + \xi_t^{(YS)}, \end{array}\right. \tag{III.12}$$

where function f_2^* represents the inversion of the pricing of $Y_{2,t}^{(YS)}$, i.e.:

$$Y_{2,t}^{(YS)} = f_2\left(\widetilde{X}_t, \ell_t\right) \Leftrightarrow \ell_t = f_2^*\left(\widetilde{X}_t, Y_{2,t}^{(YS)}\right).$$

Let us use the following notations:

$$W_t = \left[\begin{array}{c} Y_t^{(y)} \\ Y_{1,t}^{(YS)} \\ \widetilde{X}_t \\ Y_{2,t}^{(YS)} \end{array}\right] \quad \text{and} \quad Z_t = \left[\begin{array}{c} Y_t^{(y)} \\ Y_{1,t}^{(YS)} \\ \widetilde{X}_t \\ \ell_t \end{array}\right] = \left[\begin{array}{c} Y_t^{(y)} \\ Y_{1,t}^{(YS)} \\ X_t \end{array}\right].$$

Under the assumption that $Y_{2,t}^{(YS)}$ is perfectly fitted by the model, the information contained in Z_t is the same as that contained in W_t. But the p.d.f. of Z_t, conditional on W_{t-1} (or, equivalently, conditional on Z_{t-1}), is easier tom derive than that of W_t.

Indeed, we have:

$$\begin{aligned} & \log f_{Z_t|Z_{t-1}}(Z_t) = \\ & -\frac{n_y}{2}\log(2\pi) - n_y \log\sigma_y - \frac{1}{2\sigma_y^2}\left(Y_t^{(y)} - A_y - B_y'\widetilde{X}_t\right)'\left(Y_t^{(y)} - A_y - B_y'\widetilde{X}_t\right) \\ & -\frac{n_1}{2}\log(2\pi) - \sum_{i=1}^{n_1}\log\sigma_{YS,i} - \frac{1}{2}\left(Y_{1,t}^{(YS)} - f_1(\widetilde{X}_t, \ell_t)\right)'\mathrm{diag}(1/\sigma_{YS}^2)\left(Y_{1,t}^{(YS)} - f_1(\widetilde{X}_t, \ell_t)\right) \\ & -\frac{m}{2}\log(2\pi) - \frac{1}{2}\log(|\Sigma\Sigma'|) - \frac{1}{2}\left(X_t - \mu - \Phi X_{t-1}\right)'(\Sigma\Sigma')^{-1}\left(X_t - \mu - \Phi X_{t-1}\right), \end{aligned} \tag{III.13}$$

where $\mathrm{diag}(1/\sigma_{YS}^2)$ is a diagonal matrix whose i^{th} diagonal entry is $1/\sigma_{YS,i}^2$.

Remark that this does not provide us with the likelihood associated with observed data since ℓ_t is not directly observed.

We have:

$$W_t = g(Z_t),$$

with

$$g\left(\begin{bmatrix} Y_t^{(y)} \\ Y_{1,t}^{(YS)} \\ \widetilde{X}_t \\ \ell_t \end{bmatrix}\right) = \begin{bmatrix} Y_t^{(y)} \\ Y_{1,t}^{(YS)} \\ \widetilde{X}_t \\ Y_{2,t}^{(YS)} \end{bmatrix} = \begin{bmatrix} Y_t^{(y)} \\ Y_{1,t}^{(YS)} \\ \widetilde{X}_t \\ f_2(\widetilde{X}_t, \ell_t) \end{bmatrix}.$$

In general, we have:

$$f_{W_t|W_{t-1}}(W_t) = \left|\frac{\partial g^{-1}(W_t)}{\partial W'}\right| f_{Z_t|Z_{t-1}}(g^{-1}(W_t)), \tag{III.14}$$

and, therefore:

$$\log f_{W_t|W_{t-1}}(W_t) = \underbrace{\log\left|\frac{\partial g^{-1}(W_t)}{\partial W'}\right|}_{\text{calculated using eq. (III.16)}} + \underbrace{\log f_{Z_t|Z_{t-1}}(g^{-1}(W_t))}_{\text{calculated using eq. (III.13)}}, \tag{III.15}$$

where, using the inverse function theorem and the fact that $\left|\frac{\partial g^{-1}(W_t)}{\partial W'}\right|$ is diagonal:

$$\log\left|\frac{\partial g^{-1}(W_t)}{\partial W'}\right| = -\sum_{i=1}^{N} \log \frac{\partial f_2(\widetilde{X}_t, \ell_t)}{\partial \ell_{i,t}}. \tag{III.16}$$

In practice, in (III.13) and (III.16), we replace ℓ_t by $f_2^*\left(\widetilde{X}_t, Y_{2,t}^{(YS)}\right)$ —that is the fiscal limit recovered by the inversion technique.

The vector of observed variables can be extended to include $\mathcal{D}_t$. Using the notation $W_t^* = [W_t', \mathcal{D}_t']'$ and exploiting the fact that $\mathcal{D}_t$ does not Granger-cause W_t, we have:

$$\log f_{W_t^*|W_{t-1}^*}(W_t, \mathcal{D}_t) = \underbrace{\log f_{W_t|W_{t-1}}(W_t)}_{\text{calculated using eq. (III.15)}} + \underbrace{\log f_{\mathcal{D}_t|W_t}(\mathcal{D}_t).}_{\text{calculated using eq. (III.18)}} \tag{III.17}$$

In particular, if all the components of $\mathcal{D}_t$ are zero (absence of default), we have:

$$\log f_{W_t^*|W_{t-1}^*}(W_t, \mathcal{D}_t = 0) = \log f_{W_t|W_{t-1}}(W_t) + \sum_{j=1}^{N} \log\left[1 - \mathcal{F}(d_{j,t} - \ell_{j,t})\right]. \tag{III.18}$$

Appendix IV. Redistribution schemes: formulas and additional schemes

This appendix details the formulas underlying Section 6.2 of the paper.

IV.1. **General formulas.** Assume that, on date t, a European debt agency issues common bonds with maturity h and face value F (it repays F at date $t+h$). The proceeds of the issuance are $P_{t,h}^e F$, with $e \in \{SJG, SNJG\}$, depending on the type of common bond that is issued. The proceeds are allocated across countries proportionally to GDPs. Recalling that GDP weights are denoted by ω_j, country j receives $\omega_j P_{t,h}^e F$. If country j had issued national bonds with the same face value ($\omega_j F$), it would have obtained $P_{t,h}^{(j)} \omega_j F$ on date t. Therefore, at the euro-area level, the gains are:

$$G_{t,h}F = P_{t,h}^e F - \left(\omega' \mathbf{P}_{t,h} F\right), \tag{IV.19}$$

where $\mathbf{P}_{t,h}$ represent the N-dimensional vector of national prices and ω stands for the N-dimensional vector of GDP weights. (It can be seen from the previous formula that the aggregate gains are null when $e = SNJG$.)

Now, denote by ω_G the redistribution weights of the gains (with $\sum_j \omega_{G,j} = 1$). The after-gain-redistribution proceeds are:

$$\omega' \mathbf{P}_{t,h} F + G_{t,h} \omega_G F,$$

which is of the form $\omega' \mathbf{P}_{e,t,h}(\omega_G) F$, with

$$\mathbf{P}_{e,t,h}(\omega_G) = \mathbf{P}_{t,h} + G_{t,h} \frac{\omega_G}{\omega}, \tag{IV.20}$$

where, by abuse of notation, $\frac{\omega_G}{\omega}$ denotes the vector whose j^{th} entry is $\omega_{G,j}/\omega_j$. $\mathbf{P}_{e,t,h}(\omega_G)$ can be interpreted as the pseudo issuance N-dimensional vector of prices after redistribution. The post-redistribution yields faced by the different countries are given by the following N-dimensional vector:

$$\mathbf{i}_{e,t,h}(\omega_G) = -\frac{1}{h} \log \mathbf{P}_{e,t,h}(\omega_G), \tag{IV.21}$$

where, by abuse of notation, the log operator is applied element-wise.

Below, we describe the different after-gain redistribution schemes that we propose. Given that aggregate gains for the SNJG bond issuance scheme are nil, for the latter, we only focus on the scheme in which all countries face the same funding costs.

IV.2. **Scheme where countries face the same funding costs.** In this scheme, the after-redistribution issuance price faced by all countries is the eurobond price. That is:

$$\mathbf{P}_{e,t,h}(\omega_G) = P^e_{t,h}.$$

Using $G_{t,h} = P^e_{t,h} - \omega' \mathbf{P}_{t,h}$ together with (IV.20) then gives:

$$\omega_G = \omega \odot \frac{P^e_{t,h} \mathbf{1} - \mathbf{P}_{t,h}}{P^e_{t,h} - \omega' \mathbf{P}_{t,h}},$$

where $\odot$ is the element-wise product. Note that the sign of each country's redistribution weight $\omega_{G,j}$ depends on $P^e_{t,h} - P^{(j)}_{t,h}$. Therefore, this scheme implies negative "gains" for those countries j whose national bond prices are higher than that of the considered eurobond.

IV.3. **Scheme with GDP weights.** In this case, the redistribution weights (ω_G) are equal to the GDP weights (ω). Using $G_{t,h} = P^e_{t,h} - \omega' \mathbf{P}_{t,h}$ (i.e., Eq. (IV.19)), Eq. (IV.20) gives:

$$\mathbf{P}_{e,t,h}(\omega_G) = \mathbf{P}_{t,h} + (P^e_{t,h} - \omega' \mathbf{P}_{t,h}) \mathbf{1}.$$

IV.4. **Scheme with the same yield gains across countries.** Under this scheme, all countries benefit from the same yield gain, denoted by Δi_t. Denote by $\mathbf{i}_{t,h}$ the N-dimensional vector of national bond yields. We want to have $\mathbf{P}_{e,t,h}(\omega_G) = \exp(-h(\mathbf{i}_{t,h} - \Delta i_t))$. Using (IV.20), we get:

$$\mathbf{P}_{t,h} + G_{t,h} \frac{\omega_G}{\omega} = \exp(-h(\mathbf{i}_{t,h} - \Delta i_t)),$$

where, by abuse of notation, $\frac{\omega_G}{\omega}$ denotes the vector whose j^{th} entry is $\omega_{G,j}/\omega_j$. This gives:

$$\omega_G = \frac{1}{G_{t,h}} \omega \odot \left[\exp(-h(\mathbf{i}_{t,h} - \Delta i_t)) - \mathbf{P}_{t,h} \right],$$

where $\odot$ is the element-wise product. Since the components of ω_G have to sum to one, we have:

$$1 = \mathbf{1}' \left(\frac{1}{G_{t,h}} \omega \odot \left[\exp(-h(\mathbf{i}_{t,h} - \Delta i_t)) - \mathbf{P}_{t,h} \right] \right),$$

or, using that $\exp(-h\mathbf{i}_{t,h}) = \mathbf{P}_{t,h}$:

$$G_{t,h} = (\exp(h\Delta i_t) - 1)\mathbf{1}'(\omega \odot \mathbf{P}_{t,h}).$$

This further gives:

$$1 + \frac{G_{t,h}}{\mathbf{1}'\omega \odot \mathbf{P}_{t,h}} = \exp(h\Delta i_t),$$

and, finally:

$$\Delta i_t = \frac{1}{h} \log \left(1 + \frac{G_{t,h}}{\mathbf{1}'(\omega \odot \mathbf{P}_{t,h})} \right).$$

IV.5. **Scheme with no change in funding costs for Germany and France.** Table 6 complements the analysis developed in Subsection 6.2 with two additional schemes. In the first scheme (respectively second scheme), Germany (resp. both Germany and France) faces the same funding costs it would have faced under national issuance. Moreover, the aggregate gains are shared among the other countries on the base of their relative GDP size.

TABLE 6. Effect of redistribution schemes on funding costs (additional schemes)

	2008-06-30			2011-12-31			2021-06-30		
				SJG					
Panel A: No change in German funding cost									
	redist. weigth	post redist. yield	yield gain	redist. weigth	post redist. yield	yield gain	redist. weigth	post redist. yield	yield gain
DE	0%	405	0	0%	175	0	0%	-6	0
FR	44%	400	23	44%	188	61	44%	-26	39
IT	34%	427	23	34%	511	72	34%	25	40
ES	21%	410	23	21%	468	70	21%	4	39
Panel B: No change in German and French funding cost									
	redist. weigth	post redist. yield	yield gain	redist. weigth	post redist. yield	yield gain	redist. weigth	post redist. yield	yield gain
DE	0%	405	0	0%	175	0	0%	-6	0
FR	0%	423	0	0%	248	0	0%	13	0
IT	62%	409	41	62%	456	127	62%	-6	71
ES	38%	392	40	38%	415	124	38%	-27	70

Notes: This table compares post-redistribution funding costs across countries under the two issuance schemes (SJG and SNJG) and under the redistribution schemes described in IV.5. We focus on the 5-year maturity and on three periods: beginning of the estimation sample (2008Q1), midst of the euro debt crisis (2011Q3) and end of the estimation sample (2021Q2). Yields are expressed in basis points. Aggregate gains are computed under the assumption that total issuance is equal to 5% of aggregate GDP. In each panel, for all countries and dates, we show the redistribution weights $\omega_{G,j}$, the post-redistribution yields and the yield gains, that are the differences between national bond yields and post-redistribution yields.

Appendix V. Bonds used in Figure 6

Table 7. Bonds used in Figure 6

Issuer	Eikon ticker	Coupon (in percent)	Maturity date
France	FRGV	2.50	25-May-2030
Belgium	BEGV	0.55	04-Mar-2029
Portugal	PTGV	3.875	15-Feb-2030
ESM	ESM	0.50	05-Mar-2029
Spain	ESGV	1.95	30-Jul-2030
Netherlands	NLGV	0.25	15-Jul-2029
Germany	DEGV IO Str	0	04-Jul-2030
NEXTGENEU	EUUNI	0	04-Jul-2031
EFSF	EFSFC	2.75	03-Dec-2029
EU	EUUNI	1.375	04-Oct-2029
Italy	ITGV	3.50	01-Mar-2030
EIB	EIB	0.25	14-Sep-2029

Notes: This table lists the bonds used in Figure 6. Asset swap spreads (ASW) are computed by Refinitiv Eikon.