# Misallocative Growth

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**Abstract**

Exploiting variation across Swedish local labor markets between 1986 and 2018, I estimate that individuals are less likely to start new firms and switch employers in an older labor market. To account for these patterns, I propose a tractable equilibrium theory of growth with frictional labor markets. Workforce aging raises the level of output by shifting composition toward older individuals, who have had more time to find a good match with existing production technologies. The resulting higher opportunity cost of switching to new technologies, however, reduces incentives to introduce them, lowering the growth rate of output. Due to the offsetting level and growth effects, aging generates an increase in growth through the mid-1990s, followed by lower growth that is expected to continue for another 30 years. A labor market entrant is two percent worse off in 2010 relative to 1965.

Keywords: Start-ups; Worker mobility; Demographics
JEL Codes: O3; M130; E240; J110

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# 1 Introduction

The rapid aging of the labor force over the past 35 years is one of the most profound issues facing advanced economies. Across the OECD, high fertility rates after World War II resulted in the entry to the labor force of the large baby boomer generation between 1965 and 1985. As this inflow subsequently came to an abrupt end, the fraction of the workforce that is aged 45 years and older experienced a sharp reversal, as illustrated by Figure 1 for a selection of countries. For instance, in the country at the focus of this study, Sweden, the share of the workforce that is aged 45 years and older rose by six percentage points between 1986 and 2010, while it increased by a remarkable 11 percentage points in the U.S. and was similarly pronounced in other Western countries. The impact of these dramatic demographic shifts on the performance of the labor market, economic growth and welfare is not yet fully understood.

FIGURE 1. THE SHARE OF THE WORKFORCE THAT IS 45 YEARS AND OLDER

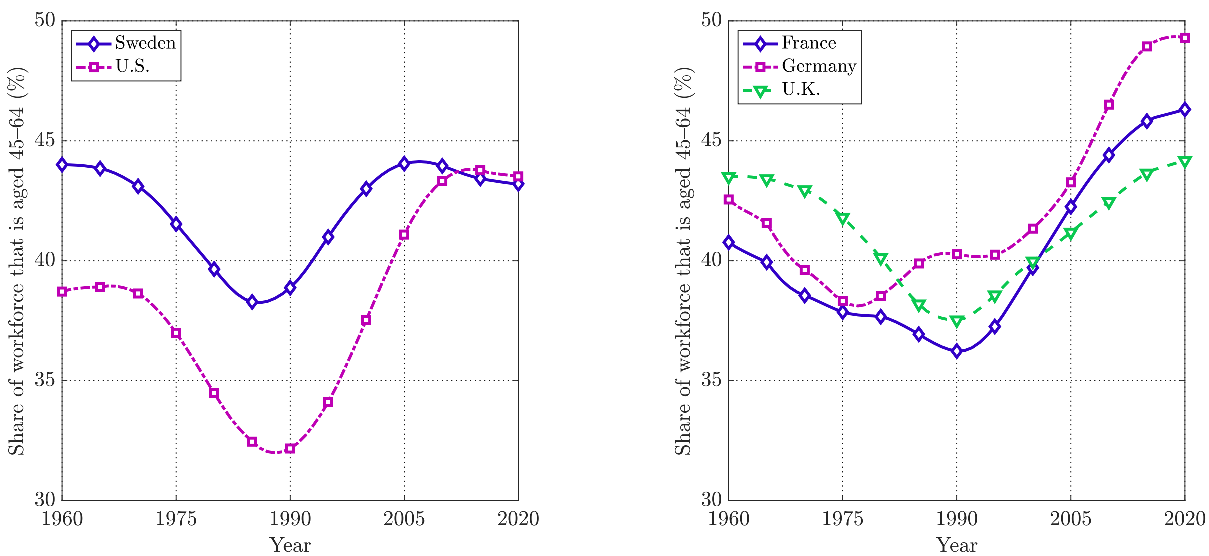



Figure 1 plots the share of all individuals aged 20–64 that is aged 45–64. *Source:* OECD.

This paper assesses the impact of workforce aging on labor market dynamics, growth and welfare using the case of Sweden between 1986 and 2018. Specifically, I make three contributions. First, I provide reduced-form empirical evidence of the impact of aging on labor market dynamics exploiting variation in the timing and magnitude of aging across 68 local labor markets (Shimer, 2001; Skans, 2005). I estimate that a one percent increase in the share of workforce participants younger than 45 raises firm creation by 1.21 percent and worker relocation by 0.99 percent. These estimates would predict (out of sample) that aging reduced firm creation by 12 percent and worker relocation by 10 percent at the national level between 1986 and 2018, relative to 25 and 18 percent declines in the data, respectively. The estimates are similar in the tradable sector and larger when I instrument for the age composition using lagged births.

I proceed to show that older individuals are less likely to start firms and switch employers, although the probability of the former rises early in careers. Given these pronounced life-cycle differences, one may expect aging to impact the aggregate firm creation and worker relocation rates by shifting the relative weight placed on subpopulations that differ in their probability of starting a firm and switching employer. Over and above this *composition effect*, I estimate that individuals in older labor markets are less likely to start firms and switch employers also conditional on their own age.

Holding their own age fixed, why would individuals' likelihood of starting a firm and switching employer fall as people around them age? Moreover, what are the aggregate implications of these reduced-form patterns? The second contribution of this paper is to develop an equilibrium theory of growth with labor market frictions to address these questions.[1] The proposed framework micro-founds the process of worker relocation following Diamond (1982)-Mortensen and Pissarides (1994) in a Schumpeterian model of imitation and selection in the spirit of Luttmer (2007) and Perla and Tonetti (2014). In so doing, I offer a unified perspective on the relationship between micro-level worker dynamics and aggregate economic growth, complementing a literature linking firm dynamics and growth (Klette and Kortum, 2004).

In the model, a unique consumption good is produced by entrepreneurs who differ in the idiosyncratic productivity of their blueprint for transforming labor into output. Entrants may imitate and improve upon the blueprints of existing firms, in the process bidding up prices of factors of production and gradually pricing out incumbent firms. This process requires the continual relocation of workers across heterogeneous firms, whose rank in the productivity distribution changes as they gradually become obsolete. Due to labor market frictions, such relocation takes time. In contrast to the premise of most of the previous literature on growth and labor market dynamics, however (Aghion and Howitt, 1994; Mortensen and Pissarides, 1998; Hornstein et al., 2007), in the current framework it need not be associated with unemployment. Instead, following Michau (2013) I emphasize the role of direct *job-to-job (JJ)* moves in facilitating such relocation, consistent with the empirical ubiquity of such flows.

The framework offers a rich account of labor market dynamics and growth, yet it remains analytically tractable, with key insights derived in closed-form. In particular, I show that under some parameter restrictions, the economy obtains a unique *Balanced Growth Path* (BGP) equilibrium. On this BGP, two opposing forces impact the distribution of employment over firms ranked by productivity—*the job ladder*. On the one hand, individuals gradually relocate across existing technologies so as to become better at

[1] The theory is a richer version of that studied in an earlier paper (Engbom, 2019). In addition to several key theoretical extensions, including the modeling of specific knowledge spillovers, the current paper uses rich Swedish matched employer-employee-entrepreneur data to discipline the theory as well as provide new reduced-form evidence of the impact of aging.

using them—they move up the job ladder. On the other hand, as technological advancements displace incumbent technologies, individuals fall behind in the job ladder. Because older individuals have had more time to relocate up the job ladder, they tend to be better matched to existing technologies.

Whereas semi-endogenous growth theory assigns a key role to the growth rate of the workforce in determining economic growth (Peters and Walsh, 2022; Jones, 2022a,b), I highlight the importance of its *age composition*. The new focus is important in light of commonly discussed policies to mitigate the impact of demographic change. For instance, a gradual raising of the retirement age would increase the growth rate of the workforce relative to counter-factual, but also make it older. Aging tilts the workforce toward older individuals, who tend to be better matched to existing technologies. Consequently, it improves static allocative efficiency, raising aggregate output through a positive *level effect*. The economy's better ability to produce using existing technologies, however, increases the opportunity cost of switching to new technologies, discouraging their introduction. In equilibrium, growth falls, i.e. aging has a negative *growth effect*. As the rate of obsolescence declines, individuals are provided more time to relocate across existing technologies before they become obsolete. As a result, individuals are better matched to incumbent technologies also conditional on age in the older, slower growing economy. The higher opportunity cost of switching to new technologies further disincentivizes their introduction. I term this equilibrium feedback channel the *misallocation effect*, and show that its strength is inverse-U shaped in the intensity of on-the-job search. That is, the equilibrium effects of aging are large when individuals only slowly relocate to take advantage of new ways to produce.

The effects of aging highlighted by the theory arise because JJ mobility facilitates individuals' ability to produce using existing technologies, without similarly improving their innovative capacity. A better match with existing technologies may, however, also make an individual better at coming up with new ideas. Indeed in the data, individuals employed by more productive firms are less likely to start new firms—consistent with the view that they have a higher opportunity cost of trying something new—but conditional on doing so, they tend to start more productive firms. To account for this pattern, I extend the model to incorporate a richer model of knowledge spillovers. Specifically, I assume that prospective entrepreneurs may build on the blueprints of their current employer.

The third contribution of this paper is to quantitatively assess the impact of aging on labor market dynamics, growth and welfare in Sweden over the past 35 years. To that end, I inform the parameters of the extended model using a set of moments in 2014–2018 on firm, worker and entrepreneur dynamics uniquely available in the Swedish matched employer-employee-entrepreneur data. The theory matches well the joint life-cycle dynamics of workers and firms in the data, including the facts that older firms are more productive and more likely to hire (and employ) older individuals (Ouimet and Zarutskie, 2014).

The latter pattern arises in equilibrium because older individuals tend to be better matched to existing technologies, such that they are less inclined to accept an outside offer from a new firm.

With the estimated model at hand, I change the growth rate of labor supply to match the evolution of the age composition of the Swedish labor force between 1930 and 2060 (based on official projections for the future). Across BGPs, aging of the magnitude experienced by Sweden since 1986 reduced firm creation by 13 percent, relative to a 25 percent decline at the national level in Sweden since 1986. It lowered worker relocation by 14 percent, relative to an 18 percent decline in the data. These structural estimates are similar in magnitude to the reduced-form estimates across Swedish local labor markets.

In both the model and data, the lower entry rate in the older economy is primarily accounted for by a smaller probability of starting a firm conditional on age, as opposed to a shift in composition toward age groups with a lower probability of entry. Two forces contribute to this age-specific decline. First, it is harder for new entrepreneurs to poach workers, discouraging entry. Second, potential entrepreneurs are better matched to existing technologies conditional on age in the older, slower growing economy, because they have had more time to relocate across existing technologies before they are replaced. Their higher opportunity cost dissuades them from attempting entrepreneurship. These theoretical predictions are consistent with the view of Salgado (2020) and Jiang and Sohail (2021), who argue that a higher return to wage relative to self employment has reduced entrepreneurship in the U.S.[2]

The shift in composition accounts for just over half of the aggregate decline in worker relocation, with the remainder due to an age-specific decline. Workers in the older economy are less likely to switch employer conditional on age for two reasons. First, firms create fewer jobs, expecting a harder recruiting environment (Shimer, 2001). Second, workers are less likely to accept an outside job offer, because they are better matched to existing technologies conditional on age. The reason is that they have had more time to relocate across these technologies before they are replaced in the older, slower growing economy.

Across BGPs, the annual growth rate of output per worker is 0.52 percentage points lower in the older economy. At the same time, the level of output is higher by over 50 percent, as the older, slower growing economy is better matched to existing technologies. Due to these offsetting level and growth effects, growth is not monotone over the 1970–2020 period, despite the fact that the rate at which new technologies are introduced declines monotonically. Instead, growth is low in the 1970s and early 1980s,

[2] Salgado (2020) and Jiang and Sohail (2021) are more ambitious than the current paper in that they also address why the decline in entrepreneurship in the U.S. was more pronounced among high-skilled individuals. Although the theoretical framework proposed here does not feature ex ante worker heterogeneity, I show that its central force depends crucially on the rate at which employed workers move up the job ladder. Aging induces a fall in the rate of obsolescence, which makes individuals better matched in the labor market. Their better match effectively raises wage employment earnings, discouraging entrepreneurship. To the extent that low-skilled workers move up the job ladder less—for which there is some empirical support—they would be less subject to this force, leading to a smaller decline in entrepreneurship among such workers.

as a large number of poorly matched young individuals enter the labor market. It subsequently rises until the mid-1990s, as the big baby boomer generation gradually finds a good match. After that, it falls again. Although these patterns match well the Swedish growth experience over this period, the annual growth rate varies by over one percentage point over the 1970–2020 period in the data, whereas aging gives rise to variation of less than two-tenths of a percentage point. In other words, other important forces were at work over this period too. Despite the fact that the workforce—as opposed to the overall population—is not expected to age much more over the foreseeable future, growth falls for another 30 years. Relative to a counter-factual scenario without demographic change, a labor market entrant in 1965 is two percent better off than their counterpart in 2010.

The theory offers several additional predictions that find support in the data. For instance, while aging has no effect on employment-unweighted firm productivity, it shifts employment toward more productive firms, thus increasing aggregate productivity. Moreover, it raises firm pay and reduces poaching, and—conditional on a poach taking place—increases the relative productivity of a poached worker's previous employer. These patterns suggest that it is harder to poach in an older labor market. Finally, it raises income in wage employment and lowers it in self-employment, consistent with the view that aging discourages firm creation both by raising pay in wage employment and hence the opportunity cost of entrepreneurship, and by reducing profits in entrepreneurship through higher labor costs.

**Related literature.** This paper makes three contributions to two influential recent papers that study the impact of a decline in labor supply growth on labor market dynamics in the U.S. (Karahan et al., 2022; Hopenhayn et al., 2022).[3] First, these papers are motivated by the observation that firm exit and size changed little conditional on firm age over the past 40 years in the U.S., which I confirm is also the case in Sweden. At the same time, however, both Sweden and the U.S. experienced a pronounced decline in job reallocation conditional on firm age, which the labor supply channel in these papers cannot account for. This observation calls for an assessment also of other potential channels.

Second, I focus on changes in the age composition as distinct from labor supply. Consistent with a role for the age composition, my reduced-form estimates across Swedish local labor markets imply that aging lowers firm creation and worker relocation, controlling for labor supply growth (Karahan et al., 2022, present similar evidence for the U.S.). Bornstein (2021) shows that aging reduced firm creation through a demand channel. Indeed, consistent with his argument, I estimate a smaller effect of aging on entry in the tradable sector. Yet a substantial effect remains, suggesting a role also for a supply channel.

[3] These papers relate to a large literature on changes in U.S. labor market dynamics—see, for instance, Davis and Haltiwanger (2014) and Akcigit and Ates (2021); **?**. This literature has primarily focused on the U.S., with much less work on secular changes in labor market dynamics in other countries.

Third, I estimate the effect of demographic change on growth, which Karahan et al. (2022) and Hopenhayn et al. (2022) treat as exogenous. While less is known theoretically about the impact of the age composition on growth,[4] an active reduced-form literature estimates the effect of aging on growth, with mixed findings. Maestas et al. (2022) show that more rapidly aging U.S. states experienced relative declines in growth, while Acemoglu and Restrepo (2017) estimate a null effect across OECD countries. The offsetting level and growth effects highlighted in the current paper suggest that it may be hard to draw robust inference about the long-run effects of aging on growth even with 20–40 years of data.[5]

This paper also contributes to a theoretical literature on growth and labor market dynamics. Lentz and Mortensen (2012) show that a Klette and Kortum (2004) model of firm-level innovation combined with a micro-founded process of worker relocation is consistent with key empirical patterns of firm productivity, wages and worker flows. Martellini and Menzio (2020) argue that unemployment may remain steady despite long-run improvements in the search technology. Bilal et al. (2021) study how a decline in the economy's ability to come up with great ideas impacts labor market dynamics.

Finally, the misallocation effect I highlight relates to the vintage capital literature (Chari and Hopenhayn, 1991). In Violante (2002), higher growth increases residual wage dispersion. Atkeson and Kehoe (2007) argue that agents are more likely to adopt new technologies when the pace of technological change is fast, because they have accumulated less knowledge about existing technologies. Jovanovic and Nyarko (1996) derive a similar insight within the context of a Bayesian model of learning. I show that gradual JJ mobility across existing technologies generates a related force. A useful feature of the current focus on frictional employment relocation is that it allows me to inform the strength of this mechanism based on measures of worker relocation. In addition, I introduce a demographic structure in this framework to study how aging impacts growth.

**Outline.** This paper is organized as follows. Second 2 introduces the data and documents a set of new facts on secular changes in labor market dynamics in Sweden. Section 3 exploits variation across Swedish local labor markets to present reduced-form evidence on the impact of aging on firm creation and worker relocation. Section 4 outlines an equilibrium theory of growth with labor market frictions, and provides a qualitative analysis of the channels through which aging impacts the labor market and growth. Section 5 estimates a quantitative version of the model, while Section 6 uses it to quantify the impact of aging on the performance of the labor market, growth and welfare. Finally, Section 7 confronts some of the key predictions of the theory with the reduced-form variation. Section 8 concludes.

[4] Acemoglu and Restrepo (2022) presents a theory that highlights that aging incentivizes adoption of industrial robots.

[5] Skans (2008) estimates that a larger share of older workers raises productivity levels across Swedish local labor markets.

## 2 Swedish labor market trends

This section introduces the data sources used in this paper, discusses how I construct my focus population of individuals and firms, and provides a brief overview of Swedish labor market trends.

### 2.1 Data

My analysis relies mainly on three administrative data sources, linked via individual and firm identifiers into a matched employer-employee-entrepreneur data set covering all Swedish individuals and firms. I discuss these three data sources briefly below, and relegate a more detailed discussion to Appendix A.1.

**JOBB.** The *Jobbregistret* (JOBB) contains all employment spells of all individuals aged 16–74 in each year 1985–2021.[6] Employers are required to provide these data for their wage employees and the self-employed are required to report on behalf of themselves to Swedish tax authorities. Up to 2019, these data were collected annually, with information on total annual gross pay, type of employment, as well as start and end month of the employment spell. Starting in 2019, gross pay has instead been reported on a monthly basis for each spell that is active in that month. Although this switch in reporting is arguably an improvement, it introduces a significant time-series break, leading me to stop my analysis in 2018. Because I also require one prior year to correctly construct firm entry rates, I start my analysis in 1986.

Employment spells are classified as wage employment, self-employment in unincorporated firms, or self-employment in incorporated firms (if the incorporation status of a self-employed individual's firm changes in a year, two separate reports are produced). Prior to 1993, self-employment in incorporated firms were instead reported as wage employment, making it impossible to identify incorporated owner-operators prior to 1993. As I discuss in further detail in Appendix A.1, self-employed in incorporated firms are only identified if firm ownership is sufficiently concentrated—for instance, owner-operators of publicly traded firms are not classified as such, but instead are reported as wage employees in the firm.

The age thresholds for who is included in JOBB have changed slightly over time, particularly at the upper age threshold. For this reason, I restrict my analysis throughout to individuals aged 20–64. As I discuss further below, the average age of entry into the labor force has been stable at around 22–23 years old and the average retirement age has been stable at just below 65 over the 1986–2018 period.

**LISA.** The *Longitudinell integrationsdatabas för sjukförsäkrings- och arbetsmarknadsstudier* (LISA) contains demographic information for all Swedish individuals aged 16 and above starting in 1990. I standardize

[6]This data set forms the basis for the more commonly used *Registerbaserad arbetsmarknadsstatistik* (RAMS), which focuses on a "main" employment spell in November each year that is selected from JOBB based on highest income.

gender, year of birth, and highest obtained educational degree to the modal value across all years of available data. I use LISA to assign these outcomes also in 1986–1989 to individuals in JOBB. As a result, I drop from my analysis any individual aged 20–64 in 1985–1989 if they either emigrate from Sweden or pass away before the first year of LISA data in 1990. I do not believe that this is a major source of error.

**FEK.** The *Företagens ekonomi* (FEK) contains annual income and balance sheet data on all private sector firms in Sweden starting in 1997, with only a few limited exceptions. It forms the basis for the Swedish national accounts. Although some information is also provided at the establishment level, the firm-level data are more extensive. For this reason, I use outcomes at the firm-level throughout my analysis.

### 2.2 Sample selection

Table 1 summarizes the data. Panel A includes all individuals aged 20–64, which covers 181 million individual-years. 21 percent of individuals are not employed, 11 percent are self-employed, 41 percent are private sector wage employees and 27 percent work in the public sector. It is not possible to separate not in the labor force from unemployment in the administrative data. Flows between the public and private sector are small, i.e. workers tend to remain in their respective sectors.

Since the public sector is likely subject to a different set of forces than those at the focus of this paper, I drop such observations from my analysis. Panel B summarizes the population of non-public sector individuals (see Appendix A.2 for additional outcomes). The monthly JJ rate is 2.3 percent and the employment-to-nonemployment (EN) rate is 1.1 percent. The low nonemployment-to-employment (NE) rate of 3.8 percent is likely due to the fact that the nonemployed include also those not in the labor force. The monthly firm creation rate is 0.1 percent, i.e. an order of magnitude lower than worker flows.

Panel C summarizes the firm-level data, which cover roughly 23 million firm-years in the private sector. The low average firm size of just over four is due to the fact that the data include also small, non-employer businesses.[7] The firm size distribution is highly skewed, with the 0.15 percent largest firms accounting for 30 percent of employment. The annual job reallocation rate—the sum of jobs created and destroyed in a year relative to employment—is roughly 28 percent, which is comparable to the U.S. (see Appendix A.3 for a further discussion). A majority of job creation and destruction is done by incumbent firms as opposed to entrant and exiting firms. Worker flows are almost three times as high as job flows, and two thirds of hires are poached directly from other firms, as opposed to hired from non-employment.

[7] In the U.S. *Business Dynamic Statistics* (BDS) data, average firm size is around 20, but this number excludes non-employer businesses. According to the U.S. Census Bureau, there were roughly 26.5 million firms in the U.S. in 2018 but only 6.1 million employer firms. Assuming that each non-employer firm consists of only one self-employed, average firm size in the U.S. including non-employer firms is approximately $(20 * 6.1 + 20.4)/26.5 = 5.4$, i.e. broadly consistent with the Swedish data.

TABLE 1. SUMMARY STATISTICS

| A. Overall population aged 20–64 (monthly flows) | | | | | | |
|---|---|---|---|---|---|---|
| Years | Non-empl. (%) | Self empl. (%) | Wage empl. (%) | Public (%) | Entry to public (%) | Exit from public (%) |
| 177,032,547 | 20.97 | 10.76 | 41.13 | 27.14 | 0.400 | 0.812 |
| B. Excluding public sector (monthly flows) | | | | | | |
| Male (%) | College (%) | St.d. earnings | JJ (%) | EN (%) | NE (%) | Entry (%) |
| 58.71 | 28.39 | 0.577 | 2.259 | 1.078 | 3.788 | 0.087 |
| C. Private sector firms (annual flows) | | | | | | |
| Years | Firm size | St.d. va.p.w. | Firms, 250+ (%) | Empl., 250+ (%) | Firms, age 11+ (%) | Empl., age 11+ (%) |
| 22,562,643 | 4.927 | 0.763 | 0.169 | 30.33 | 38.86 | 60.46 |
| Entry (%) | JC (%) | JD (%) | JC, inc. (%) | JD, inc. (%) | Hires from E (%) | Hires from N (%) |
| 14.60 | 14.25 | 13.31 | 7.73 | 7.45 | 27.58 | 13.53 |

Table 1 provides summary statistics on all individuals aged 20–64 between 1986–2018 (panel A), all individuals aged 20–64 between 1986–2018 who are not public sector employees (panel B), and private sector firms between 1997–2018 (panel C). Share college includes those with a bachelor's degree or higher. Earnings are log average monthly real earnings. The JJ rate is the fraction of private sector workers in month $t$ who have a different main employer in month $t+1$ (including switching to a public sector employer). The EN rate is the fraction of private sector workers in month $t$ who are not employed in month $t+1$. The NE rate is the fraction of non-employed individuals in month $t$ who are employed in month $t+1$ (including in the public sector). The entry rate is the fraction of not self-employed individuals in month $t$ who are the owner-operator of a new firm in month $t+1$, where a new firm is a firm founded in the current year with at most 10 identified founders. St.d. of va.p.w. is the employment-unweighted standard deviation of log value added per worker. Firms, 250+ (age 11+), is the share of firms with 250 or more employees (aged 11 or older). Empl, 250+ (age 11+), is the share of workers employed by firms with 250 or more employees (aged 11 or older). Firm entry is the share of all firms with positive employment in year $t$ that had zero employment in $t-1$. JC (JD) is the sum of employment gains (losses) across expanding (contracting) firms in the year divided by average employment. JC (JD), inc., is the sum of employment gains (losses) of expanding (contracting) incumbents in the year divided by average employment of all firms. Hires from E (N), is the sum of hires from employment/non-employment in the year divided by average employment. All moments are constructed by first computing means by year, and subsequently computing means across all years, giving equal weight to each year. Whenever applicable, the moments are computed at the firm level. *Source:* FEK, JOBB, LISA.

## 2.3 Swedish labor market trends

Figure 13 summarizes Swedish labor market trends over the past 35 years (see Appendix A.3 for a comparison with the U.S.). The entry rate fell by 31 percent since 1986 (panel A), with a similarly large decline if I condition on subsequent growth of 25, 50 or 100 percent over a firm's first five years of operation (but note the large differences in levels). Job reallocation also declined, regardless of whether I measure it at the firm or establishment level (panel B).[8] Panel C illustrates further the declines by showing trends

[8] Heyman et al. (2019) document only minor changes in firm dynamics in Sweden over the 1993–2013 period. Three reasons lead me to a different conclusion. First, Heyman et al. (2019) focus on the distribution of overall job creation, employment and firms across firms by firm age, finding relatively small changes in such shares over time. Although these shares are related to outcomes such as the aggregate job reallocation rate, they are not identical. Second, consistent with their findings as well as U.S. evidence (Karahan et al., 2022), I find that job reallocation fell by less conditional on firm age (see Appendix A.7). Third, Heyman et al. (2019) rely on firm and establishment identifiers that are constructed by Sweden's national statistical office—*Statistiska centralbyrån* (SCB)—to be consistent over time, available via a data source known as *Företagens och arbetsställenas dynamik* (FAD) (the firm and establishment identifiers in this data set are referred to as *FAD-F-ID* and *FAD-A-ID*, respectively). FAD abstracts, however, from all firms which no individual has as their "main" employer in November of the year. This concerns almost 40 percent of firms in JOBB, which covers the universe of firms which at least one individual works for at some point in the year (including as self-employed). Although such firms tend to be small, some of them later grow to be large, so it is not clear that one wants to discard such observations. Moreover, the mapping between the firm and establishment identifiers in JOBB (*PeOrgNr* and *Cfar-Nr*, respectively) and *FAD-F-ID* and *FAD-A-ID* in a given year is not unique across vintages of FAD. Possibly for this reason, measures of firm and establishment dynamics based on *FAD-F-ID* and *FAD-A-ID* are very high, more than twice the corresponding numbers in the U.S. (this observation is consistent with Figure 5 in Heyman et al., 2019, which shows that entrant firms based on *FAD-F-ID* create more than 40 percent of all jobs in Sweden or more than twice the corresponding share in the U.S. BDS). Although JOBB lacks a consistent firm identifier—*PeOrgNr* changes whenever the firm changes ownership, organizational form, etc—*Cfar-Nr* only changes when at least two of the following three change

relative to 1986. Both job reallocation of entrant and exiting firms and that of incumbents declined. The worker relocation rate—the sum of hires and separations in a year divided by average employment—is more volatile over the business cycle, but also displays a trend decline. In contrast to the secular declines in firm creation, job reallocation and worker relocation, the aggregate growth rate rose from the mid-1970s through the 1990s, and only subsequently declined (panel D).[9]

One recently emphasized factor behind the decline in firm entry in the U.S. is a fall in the growth rate of labor supply (Karahan et al., 2022; Hopenhayn et al., 2022; Peters and Walsh, 2022). Sweden, however, experienced no pronounced change in the growth rate of the working age population over the past 60 years (panel E). In fact, it modestly *rose* since the mid-1970s. The growth in the number of labor force participants aged 16 or older did decline until the mid-1990s (data on the labor force start in 1968). Since then, however, labor force growth trended up (the divergence in the growth rate of the workforce and labor force in the late 1980s and 1990s coincides with an economic boom followed by a large contraction).

Over the past 60 years, Sweden saw large shifts in the age composition of the workforce (panel F). For instance, the share of the workforce that is 45 years and older first fell by six percentage points from the late 1950s until the mid-1980s, and subsequently rose by six percentage points. Although the levels differ, the trends in the age composition of the workforce and labor force are similar, with the latter rising by nine percentage points since the mid-1980s. Appendix A.5 discusses the difference between the labor and work force further, highlighting no pronounced trends in labor force participation rates by age over this period.[10] This finding is consistent with official publications showing an increase in the average age of labor force entry during the financial crisis of the early 1990s, but a subsequent stabilization around 22–23 years old, as well as only a marginal change in the average age of retirement, with most Swedes retiring at age 65.[11] A substantial share of these changes in the age composition are accounted for by past fertility, as illustrated by the "lagged birth" series. Specifically, it plots the ratio of the sum of births 45–64 years earlier to the sum of births 20–64 years earlier, i.e. the share of the workforce that would be older solely based on past fertility, abstracting from immigration, emigration and mortality.

Appendix A.2 establishes a series of additional facts, which I briefly summarize here. The firm exit

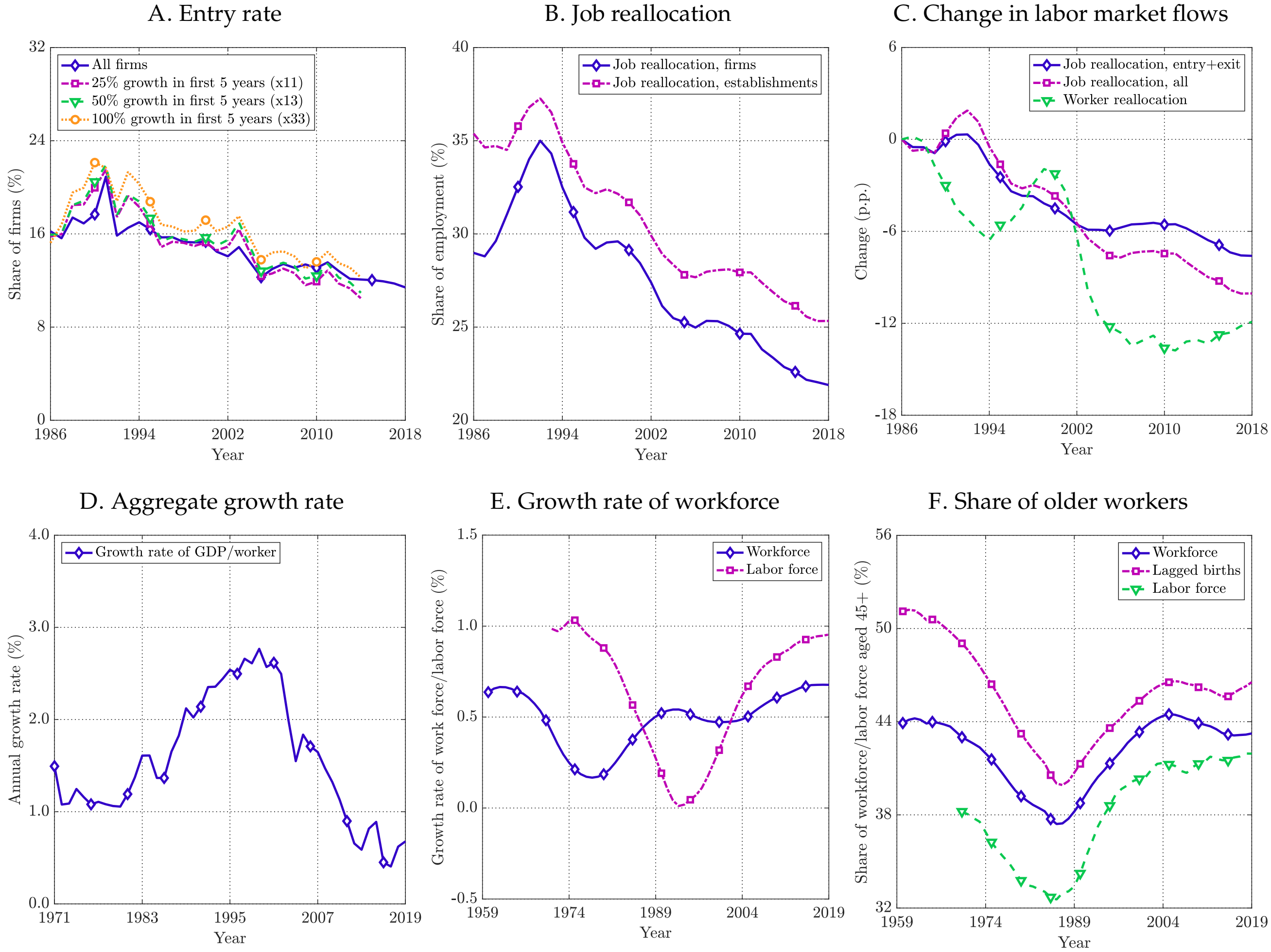
for the establishment: location, sector, or ownership. Hence *Cfar-Nr* should be a reasonably consistent establishment identifier. Indeed, measures of establishment dynamics based on *Cfar-Nr* broadly align with the U.S., i.e. they are much lower than those implied by *FAD-A-ID*. In fact, job reallocation rate based on *PeOrgNr* is only modestly lower than that based on *Cfar-Nr*, suggesting that these issues may be of less concern. Appendix A.4 provides a further discussion.

[9] I plot the 11-year centered moving average of the growth rate of real GDP per worker, available since 1971 from the OECD.

[10] The main exception is an increasing participation rate of women during the 1970s and early 1980s. Since the early 1980s, women's participation rates have remained high and steady in Sweden, with the exception of women aged 55-64, who experienced a secular increase in labor force participation rates over the entire 1968–2019 period.

[11] See Figure 3.6 and Table 3.1, respectively, in `https://www.regeringen.se/4a67b3/contentassets/c1063c03c89247b694cb895aae28741d/hojda-aldersgranser-i-pensionssystemet-och-i-andra-trygghetssystem_ds-2019_2.pdf` (Swedish only).

FIGURE 2. SWEDISH LABOR MARKET TRENDS




Panel A plots the entry rate of firms, either unconditionally or conditional on a minimum growth rate over the first five years of a firm's existence. The entry rates are spliced in 2004 due to a data break; see Appendix A.6 for details. Panel B plots the sum of jobs created and jobs destroyed across firms/establishments in a year divided by average employment in the year. Job creation (destruction) is the sum of employment gains (losses) across all expanding (contracting) firms/establishments. Panel C plots the sum of jobs created and destroyed by entrant/exiting firms and incumbent firms, in both cases divided by total employment in the year. The worker relocation rate is the sum of hires and separations divided by total employment in the year. All series are in percentage point deviations from 1986. Panels A–C include private sector firms and workers aged 20–64. Panel D plots the 11-year centered moving average of annual growth in real GDP per worker. Panel E plots the annual growth rate of the workforce (population aged 20–64) or labor force (all labor force participants older than 15 years). Panel F plots the share of the workforce or labor force who is 45 years and older. The lagged birth series is the sum of births 45–64 years earlier divided by the sum of births 20–64 years earlier. *Source:* FEK, JOBB, LISA, OECD.

rate fell, average firm size trended up modestly, the ratio of workforce participants to firms increased by even less, productivity and wage dispersion rose, while the volatility of productivity changes remained constant. Entrant firms are less productive than all firms, but more productive than exiting firms. In a relative sense, the productivity of entrant firms remained roughly constant, while exiting firms became less productive. Finally, the distributions of both employment and firms shifted toward older firms.

Appendix A.7 documents that conditional on firm age, exit rates and firm size declined modestly, while the fall in job reallocation remains pronounced. Appendix A.7 also shows that these patterns are similar to the U.S., where exit rates fell only modestly and average firm size remained constant conditional on firm age (Karahan et al., 2022), but job reallocation declined substantially conditional on firm age. Appendix A.7 also shows that job reallocation fell across firm size classes. Appendix A.8 highlights that economic activity shifted from less dynamic sectors such as manufacturing toward more dynamic sectors such as services. Consequently, the within-sector decline in job reallocation is *larger* than the aggregate decline. In fact, appendix A.8 finds that job reallocation fell in all major sectors.

## 3 Reduced-form estimates of the effect of aging on the labor market

Several forces are likely behind the secular declines in labor market dynamics in Sweden, including advancements in IT technology (Lashkari et al., 2021), increasing use of intangibles (de Ridder, 2021), falling overhead costs (Aghion et al., 2022), and changes in the nature of knowledge spillovers (Akcigit and Ates, 2019; Olmstead-Rumsey, 2022). To provide more direct evidence on the role of aging, I proceed in this section to highlight how aging impacts labor market dynamics across space in Sweden.

### 3.1 Methodology

The Swedish national statistical office, *Statistiska centralbyrån* (SCB), aggregates Sweden's roughly 290 municipalities into local labor markets—*lokala arbetsmarknader* (LA)—based on commuting flows. The number of LAs declined over time, so to be consistent I use the 2018 delineation, providing 69 LAs. Because *Årjäng* LA forms part of Oslo's local labor market, I drop it from my analysis (but similar results hold including it).[12] I obtain from publicly available data the share of individuals aged 20–64 that are aged 20–44 at the LA-year level,[13] and from the administrative micro data moments on labor market dynamics as well as the share male, the share with a college degree or higher, the share immigrant and the share of immigrants aged 20–64 that are aged 20–44 at the LA-year level for 1986–2018.

I regress labor market outcomes such as the firm creation rate and worker relocation rate in LA $i$ in year $t$ on the share of the local workforce that is 20–44—henceforth the share young—always controlling for LA ($\psi_i$) and year ($\xi_t$) fixed effects, and potentially also other time-varying covariates ($X_{i,t}$),

$$\log y_{i,t} \quad = \quad \alpha \log young_{i,t} + \psi_i + \xi_t + X_{i,t}\beta + \varepsilon_{i,t} \tag{1}$$

[12] Although it is well-known that the Swedish and Finnish parts of a northern border area known as *Tornedalen* see significant cross-border commuting, SCB has not yet been able to determine whether the three border local labor markets that this concerns—*Haparanda*, *Övertorneå* and *Kiruna*—form part of larger Swedish-Finnish local labor markets (see https://www.scb.se/contentassets/c2d754bcaf964bcca33ac7cc2510c765/lokala-arbetsmarknader-over-riksgrans-2003.pdf for a discussion in Swedish). Dropping these three local labor markets makes little difference to the estimates below.

[13] https://www.statistikdatabasen.scb.se/pxweb/sv/ssd/START__BE__BE0101__BE0101A/BefolkningNy/.

The coefficient of interest, $\alpha$, captures the extent to which labor market outcome $y$ changes as a local labor market ages, relative to its own average and other LAs. Because all variables are in logs, the point estimate may be interpreted as an elasticity.[14] Motivated by evidence in Appendix B.1 of both a serial correlation within LAs and a cross-sectional correlation across LAs in the dependent and independent variables, I two-way cluster standard errors at the LA and year levels (Cameron et al., 2011).[15]

Regression (1) leverages the fact that Swedish LAs have aged differentially, both in terms of overall magnitude and timing. In particular, although the inclusion of the LA and year fixed effects shrinks the standard deviation of the log share of young from 0.088 to 0.025, significant variation remains.

A concern with estimation of (1) via OLS is that individuals may move across LAs in response to labor market performance. In particular, results may be biased if young people differentially move to areas that display *temporarily* high labor market dynamics. I address such concerns by instrumenting for the current age composition using lagged births 20–44 years earlier.

To that end, I collect data on births at the LA-year level from 1940 to 2018. Since 1968, annual data on births are available online at SCB at the municipality-year level.[16] From 1958–1967, I digitalize statistical abstracts at the municipality-year level,[17] which I map into the 69 LAs. Prior to 1958, births are only available at the level of Swedish *län*, separately reported for its urban and rural components.[18] *Län* are a higher level of local administration than municipalities, with the number of *län* having fluctuated around 25. Births are also reported separately for larger cities (roughly 110 cities). I digitalize these publications and construct cross-walks between the larger cities, on the one hand, and the LAs and the *län*, on the other. Subsequently, I impute births in an LA by summing births in the larger cities that form part of the LA as well as "residual births" in the LA. To obtain the latter, I first sum rural births in the *län* and the difference between reported urban births in the *län* and the sum of births in the larger cities that belong to the *län*. I apportion this sum across the LAs that form part of a *län* based on each LAs population share of the *län* in 1958. Although I believe that this imputation is accurate since I have data on births in the 110 largest cities, any error in the imputation would simply reduce the power of the first stage.

[14] I prefer logs as I find it easier to interpret an elasticity, but estimating (1) in levels instead yields very similar results.

[15] Appendix B.1 finds no evidence of a correlation between the contemporaneous residual in a LA $i$ and the residuals in neighboring LAs in earlier years, after controlling for the lagged residuals in LA $i$ and the contemporaneous residuals in neighboring LAs. Nevertheless, also accounting for aggregate serial correlation for up to three years following Driscoll and Kraay (1998) does not meaningfully change the standard errors reported below.

[16] https://www.statistikdatabasen.scb.se/pxweb/sv/ssd/START__BE__BE0101__BE0101H/FoddaK/.

[17] These data are publicly available as pdf files via SCB's *Äldre statistik* ("older statistics") online at https://www.scb.se/hitta-statistik/sok/?query=&tab=older. Specifically, these are available in Table 4 (for 1958–1959), Table 5 (for 1960) and Table 6 (for 1961) of the respective yearly publications of *Folkmängden inom administrativa områden*, in Table 1 of the *folkmängdsförändringar kommunvis* as part of *Statistiska meddelanden (SM)* (publications B 1963:16 for 1962, B 1964:9 for 1963, and B 1965:5 for 1964, Be 1966:5 for 1965 and Be 1967:7 for 1966), and Table 1 of the *Befolkningsförändringar del 1* (for 1967).

[18] These data are available in Tables 3–4 of the *Befolkningsrörelsen* publications, available online at https://www.scb.se/hitta-statistik/sok/?query=&tab=older.

I use the log of the sum of births 20–44 years earlier in the LA as an instrument for the current log share of young, conditional on LA and year fixed effects. The identifying assumption is that fertility 20–44 years ago is not based on labor market dynamics today. Births 20–44 years earlier predict well the current share young.[19] A one percent increase in lagged births is associated with a 0.127 percent increase in the share young, conditional on LA and year fixed effects, with a standard error of 0.023. Lagged births explain 26 percent of the residual variation in the current share young after accounting for the fixed effects. Jointly, lagged births and the fixed effects explain 94 percent of the variation in the current share young. The F-test of overall statistical significance is 27 (with standard errors two-way clustered at the LA and year levels). Evidently, while some Swedes do move across LAs, many remain where they are born. Appendix B.2 visualizes this correlation.

Appendix B.3 summarizes key worker and firm level outcomes across Swedish local labor markets, including the size of the workforce, average wages, net wealth, productivity and demographics. Appendix B.4 illustrates some of the identifying variation that drives the regression results below.

### 3.2 Results

According to the OLS estimate in column 1 panel A of Table 2, a one percent increase in the share of young is associated with a 1.21 percent increase in firm creation.[20] Variation in the residual share of young accounts for 6.0 percent of the residual variation in firm creation, conditional on LA and year fixed effects. A one standard deviation change in the residual log share young is associated with roughly a quarter of a standard deviation change in the residual log firm creation rate. At the national level, the share of young fell by roughly 12 percent between 1986 and 2018. Hence, abstracting from national level equilibrium effects, the estimate would imply that aging has reduced firm creation by about 14 percent over this period, corresponding to roughly 40 percent of the aggregate decline.

The IV estimate in column 2 is larger than the OLS estimate, indicating that a one percent increase in the predictable share of young leads to a 2.20 percent increase in firm creation. The fact that the IV estimate is larger than the OLS estimate is a common finding in related work on the impact of aging in U.S.—see, for instance, the discussion in Shimer (2001) or Karahan et al. (2022). At the same time, the underlying variation in the predictable share young is smaller than that in the realized share young. As a result, similar to the OLS estimate, a one standard deviation increase in the predictable share young

[19] A closely related alternative instrument would be the birth *rate*. Although it also predicts well the current share young, it provides a weaker first stage than the lagged sum of births, conditional on LA and year fixed effects.

[20] I do not weigh LAs by their population size. Weighing LAs by their population size in 1986 results in an even more pronounced point estimate, with the OLS specification indicating that a one percent increase in the share of young is associated with a 1.91 percent increase in firm creation and the IV estimate a 2.55 percent increase.

leads to roughly a quarter of a standard deviation rise in residual firm creation.

Columns 3–4 repeat the analysis for the manufacturing sector only, which constitutes about 30 percent of employment in Sweden (Appendix A.2 shows that this share has declined over time). The estimates are smaller than for the overall economy. This observation is consistent with changes in demand induced by aging being behind some of the patterns, which Bornstein (2021) shows is important in the U.S. context. At the same time, the point estimates indicate that aging also reduces firm creation in the tradable sector in Sweden, suggesting that also a supply-based channel is at work.

Recent work stresses that a decline in labor supply growth accounts for some of the decline in firm creation in the U.S. (Karahan et al., 2022; Hopenhayn et al., 2022; Peters and Walsh, 2022). To assess the importance of this channel, column 5 instead projects outcomes on the growth rate of the workforce (all individuals aged 20–64) in regression (1). Higher labor supply growth is associated with higher firm entry, consistent with prior research. Column 6 adds as instrument for the growth rate of the workforce the log number of births in the LA 20 years earlier. The point estimate turns negative, but the first stage is too weak to be able to draw any robust inference.[21]

Columns 7–8 include both the share of young and the growth rate of the workforce, suggesting a role for both (although the point estimate on labor supply growth turns statistically insignificant, it remains economically significant—see Karahan et al., 2022, for complementary evidence to this point across U.S. states). The first stage is too weak to allow any robust inference in the IV specification.

Finally, columns 9–10 include the share of private sector employment that is male, has a college degree or higher and is immigrant, as well as the size of the workforce aged 20–64 and its growth rate. Controlling for changes in these dimensions reduces the point estimate in the OLS specification, but raises it in the IV specification. In the IV specification, however, the first stage weakens and the standard errors grow, such that the point estimate is only weakly statistically significant (p-value of 0.073). The reason is the inclusion of the share with a college degree, which on its own has a highly statistically insignificant effect on firm creation, with a p-value of 0.92. Dropping this statistically insignificant control, the point estimate changes little but it again becomes strongly statistically significant.

Panel B shows that aging reduces also worker mobility. The OLS estimate in column 1 indicates that a one percent increase in the share of young is associated with a 0.99 percent increase in worker relocation. Variation in the residual share of young accounts for 5.1 percent of the residual variation in worker relocation, conditional on LA and year fixed effects. A one standard deviation change in the residual log share young (conditional on LA and year fixed effects) is associated with about a quarter

[21] I have alternatively used the log of the birth rate 20 years earlier, which leads to an even weaker first stage. The point estimate on the growth rate of the workforce in the second stage continues to be negative.

of a standard deviation change in the residual log worker relocation rate. As for firm creation, the IV estimate in column 2 is larger than the OLS estimate.

The point estimates for the manufacturing sector only in columns 3–4 are smaller than the aggregate economy. As for firm creation, this observation is consistent with the view that aging reduces labor market dynamics through a demand channel, which has been shown to be important in the U.S. (Bornstein, 2021). Yet the estimates in the tradable sector remain statistically and economically significant, suggesting that aging also impacts labor market dynamics through a supply channel.

Labor supply growth has no statistically significant effect on worker relocation, either in a univariate specification (columns 5–6) or when also controlling for the share of young (columns 7–8). Moreover, controlling for the growth rate of the workforce does not much impact the estimated impact of aging.

Finally, controlling for changes in the gender, educational and immigrant composition of the workforce as well as its size and growth rate moderates the impact of aging according to the OLS estimate (column 9), but increases it according to the IV estimate (column 10).

TABLE 2. THE IMPACT OF AGING ON LABOR MARKET DYNAMICS

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) |
|---|---|---|---|---|---|---|---|---|---|---|
| | Baseline | | Manufacturing | | Supply growth | | Joint | | Controls | |
| | OLS | IV | OLS | IV | OLS | IV | OLS | IV | OLS | IV |
| | *Panel A. Firm creation rate* | | | | | | | | | |
| Share 20–44 | 1.207*** | 2.196** | 1.012*** | 1.767** | | | 1.200*** | 2.397*** | 0.798** | 2.810* |
| | (0.358) | (0.820) | (0.305) | (0.679) | | | (0.359) | (0.839) | (0.371) | (1.516) |
| Δ 20–64 | | | | | 0.623** | −16.552 | 0.378 | −14.550 | 0.426 | 0.236 |
| | | | | | (0.269) | (23.131) | (0.310) | (18.511) | (0.287) | (0.396) |
| Obs. | 2,244 | 2,244 | 2,241 | 2,241 | 2,244 | 2,244 | 2,244 | 2,244 | 2,244 | 2,244 |
| R-squared | 0.691 | | 0.491 | | 0.673 | | 0.692 | | 0.716 | |
| within | 0.059 | | 0.012 | | 0.002 | | 0.060 | | 0.134 | |
| F-stat | | 27.1 | | 27 | | 2.3 | | 1.2 | | 12.2 |
| | *Panel B. Worker relocation rate* | | | | | | | | | |
| Share 20–44 | 0.988*** | 2.594*** | 0.768** | 2.375*** | | | 0.981*** | 2.756*** | 0.578** | 4.013** |
| | (0.237) | (0.754) | (0.316) | (0.789) | | | (0.237) | (0.763) | (0.246) | (1.618) |
| Δ 20–64 | | | | | 0.571 | −14.069 | 0.370 | −11.767 | 0.462 | 0.137 |
| | | | | | (0.443) | (17.213) | (0.360) | (13.269) | (0.354) | (0.410) |
| Obs. | 2,244 | 2,244 | 2,244 | 2,244 | 2,244 | 2,244 | 2,244 | 2,244 | 2,244 | 2,244 |
| R-squared | 0.791 | | 0.619 | | 0.780 | | 0.791 | | 0.796 | |
| within | 0.051 | | 0.011 | | 0.002 | | 0.051 | | 0.074 | |
| F-stat | | 27.1 | | 27.1 | | 2.3 | | 1.2 | | 12.2 |

Table 2 presents OLS and IV estimates based on regression (1) using annual data from 68 LA between years 1986–2018. The independent variable is the log share of all individuals aged 20–64 that are aged 20–44 in the LA in that year. The outcome variables are for private sector firms and individuals aged 20–64, averaged in levels at the LA-year level and subsequently logged. The instrument is the sum of births 20–44 years earlier in the LA, and subsequently logged. Standard errors are two-way clustered at the LA and year levels. Columns 5–6 regress outcomes on the annual log difference in the number of workforce participants (all individuals aged 20–64). The instrument is the log number of births 20 years earlier in the LA. Columns 7–8 include both the share of young and the growth rate of the workforce, potentially instrumented for using the log sum of births 20–44 years earlier and the 20-year lagged number of births. Columns 9–10 adds to the independent variables in columns 7–8 the share of private sector employment aged 20–64 that is male, has a college degree or higher and is immigrant, as well as the number of individuals aged 20–64. The share of young is potentially instrumented for using the log sum of births 20–44 years earlier. Panel A shows results for the firm creation rate as the dependent variable, defined as the share of firms with positive employment in the current year that had zero employment in the previous year. Panel B shows results for the worker relocation rate as the dependent variable, defined as the sum of hires and separations in a year divided by average employment in the year. *Source:* JOBB, LISA, SCB.

A remaining concern is that these estimates reflect a third factor that is correlated with fertility 20–44 years earlier and contemporaneous labor market dynamics. For instance, urban areas may have disproportionately benefitted from forces such as globalization, and they may have experienced smaller declines in fertility. Although columns 9–10 of Table 2 show that the results hold controlling for the share male, with a college degree and immigrant, as well as the size and growth rate of the workforce, Appendix B.5 contains a battery of additional robustness exercises. For instance, the point estimates are only marginally affected by adding separate linear time trends interacted with the initial share of private sector employment that is male, has a college degree or more, or is immigrant, as well as the initial size of the workforce and initial average net wealth, or separate linear time trends interacted with initial value added per worker, the initial share of manufacturing firms and initial investment per worker. They also hold if I exclude the three largest LAs Stockholm, Gothenburg and Malmö (roughly half of Sweden's employment). Finally, adding LA-decade fixed effects has only a minor effect on the OLS estimate on firm creation and increases the IV estimate. The standard errors grow, however, such that the IV estimate is no longer statistically significant at conventional levels (p-value of 0.117). It reduces the OLS estimate on worker relocation to the point where it is no longer statistically significant (p-value of 0.138), but the IV estimate increases in magnitude and remains statistically significant (p-value of 0.022).

In Appendix B.6, I establish that aging also leads to declines in the entry rate of high growth firms, as measured by subsequent growth of at least 25 or 50 percent in the first five years after entry; has no statistically significant effect on average firm size or investment per worker; and lowers job reallocation. The effects of aging hold controlling for sector (Appendix B.7) and firm age (Appendix B.8).

### 3.3 Age-conditional results

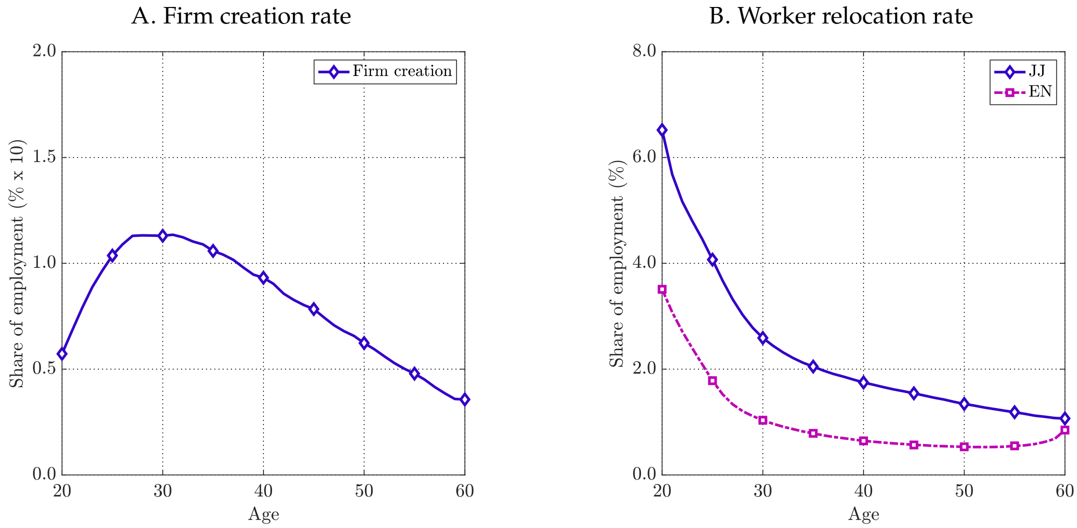
Firm creation and worker relocation display distinct life-cycle patterns (Figure 3). In particular, the probability of starting a firm rises up to age 30, then declines, particularly after around age 40 (panel A).[22] The probability of making a JJ or EN transition decline monotonically with age (panel B).[23]

In light of these distinct life-cycle patterns to firm creation and worker relocation, one may expect

[22] Azoulay et al. (2018) find that in the population of new firms in the U.S., the modal age of founders is about 38 years. Apart from the fact that Sweden and the U.S. are different countries, several factors could be behind the difference. First, they study the age distribution of founders, as opposed to the entry rate by age. These two objects are only the same if the underlying population age distribution is uniform (which is neither the case in the U.S. nor Sweden). Second, due to data limitations, they are forced to impute the founders of a large share of new firms. Third, their data cover primarily firms founded in the Great Recession and its immediate aftermath. It is possible that older, more experienced founders fared relatively better during this turbulent period. In any case, the peak entry age to entrepreneurship in Sweden is consistent with evidence from other countries (Liang et al., 2018). Moreover, Section 6 estimates a relatively small *direct, composition* effect of aging on firm creation, with most of the estimated fall being accounted for by a lower probability of entry conditional on age in the older economy. For this reason, I believe that it less critical for the structural estimates where exactly the peak in entry takes place.

[23] The firm-level worker relocation rate in Table 2 is the sum of twice the JJ rate, the EN and the hiring rate from non-employment (which in steady-state equals the EN rate plus labor supply growth).

FIGURE 3. COMPOSITION EFFECT OF AGING




Panel A plots the share of individuals in month $m$ who are the owner-operator of a newly founded firm in month $m+1$ which they had never previously been the owner-operator for. A newly founded firm is one which was founded in the current year $t$, and includes both unincorporated and incorporated firms (as long as it a *fåmansföretag*—see Appendix A.1 for details). Panel B plots the fraction of individuals in month $m$ who have a different main employer in month $m+1$ (JJ) or are non-employed in month $m+1$ (EN). All panels: Non-public sector firms and individuals aged 20–64 pooling all years 1993–2017. *Source:* FEK, JOBB, LISA.

a change in the age composition to have a mechanical effect on the aggregate firm creation and worker relocation rate, holding fixed age-conditional firm creation and worker relocation rates. Additionally, aging may also reduce *age-conditional* mobility rates through equilibrium forces.

To assess the importance of the direct composition effect versus indirect equilibrium forces, I compute the firm creation and worker relocation rate separately by LA-year-age bins. Because some LAs are small and firm creation is rare, I use three age groups—20–29, 30–44 and 45–64—but I verified that similar results hold with more disaggregated groups. I define the firm creation rate at the level of individuals as the fraction of employment whose first month as self-employed (either unincorporated or incorporated) in firm $f$ was month $m$, and firm $f$ was founded in the current year. Note that this measure corresponds to the employment-weighted entry rate, whereas that in Table 2 is employment-unweighted.[24] The worker relocation rate is the fraction of employment in month $m$ that was either employed by a different employer or non-employed in month $m-1$. I project these outcomes at the LA-year-age level on the share of young in the LA-year, controlling for LA, year and age fixed effects,

$$\log y_{i,t,a} \quad = \quad \alpha \log young_{i,t} + \psi_i + \xi_t + \phi_a + \varepsilon_{i,t,a} \tag{2}$$

I weigh regressions such that each age group receives a weight commensurate with its employment share

[24]The employment-weighted entry rate refers to the fraction of employment that is a firm founder. This is not the same as the fraction of employment that is an employee at new firms, as for instance reported by the U.S. BDS.

in the LA-year, and each LA-year receives the same aggregate weight.

Table 3 presents results.[25] Columns 1–2 show results without age controls, confirming that aging leads to a relative decline also in the employment-weighted firm creation rate. Controlling for a potential founder's own age reduces the point estimate (columns 3–4). That is, part of the impact of aging on firm creation is accounted for by the fact that older individuals are less likely to start firms, and aging increases their share of the population. Over and above this effect, when a local labor market ages, individuals in that market become less likely to start new firms conditional on their own age. Columns 5–10 present results from (2) separately by age group. Aggregate aging particularly reduces firm creation among young and older potential founders, with a smaller impact on middle aged potential founders.

Panel B confirms the findings from Table 2 that aging leads to declines in worker relocation (if the dependent variable had been in levels instead of logs, the point estimates in columns 1–2 would coincide with those in Table 2 panel B columns 1–2). As expected given Figure 3, controlling for the direct effect of age reduces the impact of aging (columns 3–4). Nevertheless, aggregate aging also reduces age-conditional worker mobility. That is, an individual is less mobile in an older labor market, conditional on her own age. According to columns 5–10, the effect is particularly pronounced among older individuals.

TABLE 3. AGING AND AGE-SPECIFIC ENTRY AND WORKER MOBILITY ACROSS SPACE

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) |
|---|---|---|---|---|---|---|---|---|---|---|
| | All ages | | Age controls | | Ages 20–29 | | Ages 30–44 | | Ages 45–64 | |
| | OLS | IV | OLS | IV | OLS | IV | OLS | IV | OLS | IV |
| | *Panel A. Firm creation rate* | | | | | | | | | |
| Share 20–44 | 1.175*** | 3.685*** | 0.863*** | 3.223*** | 1.276*** | 3.819*** | 0.384 | 1.939** | 0.916** | 3.639*** |
| | (0.292) | (1.117) | (0.283) | (1.082) | (0.363) | (1.345) | (0.290) | (0.840) | (0.401) | (1.371) |
| Obs. | 6,711 | 6,711 | 6,711 | 6,711 | 2,230 | 2,230 | 2,243 | 2,243 | 2,238 | 2,238 |
| R-squared | 0.327 | | 0.582 | | 0.362 | | 0.564 | | 0.469 | |
| within | 0.005 | | 0.005 | | 0.009 | | 0.001 | | 0.005 | |
| F-stat | | 30.1 | | 30.0 | | 32.6 | | 31.7 | | 27.3 |
| | *Panel B. Worker relocation rate* | | | | | | | | | |
| Share 20–44 | 1.112*** | 2.714*** | 0.808*** | 2.227*** | 0.355 | 1.676*** | 0.716*** | 1.811*** | 1.021*** | 2.671** |
| | (0.264) | (0.847) | (0.250) | (0.767) | (0.219) | (0.553) | (0.241) | (0.596) | (0.340) | (1.083) |
| Obs. | 6,732 | 6,732 | 6,732 | 6,732 | 2,244 | 2,244 | 2,244 | 2,244 | 2,244 | 2,244 |
| R-squared | 0.166 | | 0.919 | | 0.810 | | 0.769 | | 0.679 | |
| within | 0.003 | | 0.018 | | 0.008 | | 0.022 | | 0.028 | |
| F-stat | | 30.2 | | 30.2 | | 32.3 | | 31.8 | | 27.6 |

Table 3 presents OLS and IV estimates based on regression (2) using annual data from 68 LA from 1986 to 2018 for three age groups (20–29, 30–44 and 45–64). The independent variable is the log share of all individuals aged 20–64 that are aged 20–44 in the LA in that year. Outcome variables are for private sector firms and individuals aged 20–64, averaged in levels at the LA-year-age group level and subsequently logged. The instrument is the sum of births 20–44 years earlier in the LA, and subsequently logged. Standard errors are clustered at the LA level. Columns 1–2 show results pooling all age groups without age controls. Columns 3–4 add age controls. Columns 5–10 estimate (2) separately by age groups. Panel A presents results for the firm creation rate, defined as the fraction of employment in month $m$ whose first month working in firm $f$ as either an unincorporated or incorporated self-employed was month $m$, and firm $f$ was founded in the current year. Panel B presents results for the worker relocation rate, defined as the fraction of employment in month $m$ that was either employed by a different employer or non-employed in month $m-1$. *Source:* FEK, JOBB, LISA, SCB.

[25]The number of observations differs slightly across specifications for firm entry in panel A and relative to Table 2 because the measured firm entry rate is zero in a few, small LA-year-age groups cells. Consequently, these observations are dropped when taking logs. Similar results hold in levels including these cells.

**Road map.** As a labor market ages, firm creation and worker mobility fall. Partly, the decline is accounted for by the fact that aging shifts workforce composition toward older individuals, who are less likely to start firms and switch employers. Over and above this composition effect, individuals are less likely to start firms and switch employers conditional on their own age when everyone around them is older. Why? Moreover, what are the aggregate and welfare implications of these patterns?

To answer these questions, the next section develops an equilibrium theory of growth with labor market frictions. The tractable framework provides a laboratory to assess how worker dynamics and growth interact, as well as the channels through which aging impacts the economy. I use the model to quantify the aggregate and welfare implications of aging in Sweden between 1986 and 2018.

# 4 A model of labor market dynamics and growth

At the core of the framework proposed in this section is the Burdett and Mortensen (1998) view that individuals gradually relocate toward more productive jobs, such that older people are better matched in the labor market and hence less likely to switch employer. Appendix C.1 motivates this view by showing that older individuals are employed by more productive firms in the data, and that individuals employed in more productive firms are less likely to switch employer.

## 4.1 Preferences and technology

Time is continuous and infinite, and there are no aggregate shocks. I focus the exposition on an economy on a long-run BGP, and later turn to its transitional dynamics in response to aging. Three types of individuals—entrepreneurs, managers and workers—constitute the labor force

$$\hat{N}(t) = \underbrace{e^{\lambda t}}_{\text{growth rate of workforce}} \left( \underbrace{\xi}_{\text{entrepreneurs}} + \underbrace{l}_{\text{managers}} + \underbrace{1}_{\text{workers}} \right)$$

Individuals permanently exit the labor force at rate $\kappa$, at which point they are replaced by their offspring. They have one child with probability $\omega$ and $1 + \nu$ children with complementary probability, such that the labor force grows at rate $\lambda = \kappa(1 - \omega)\nu$. Entrepreneurs' offspring get to take over the firm,[26] while workers' offspring enter the labor market as unemployed.

[26]If an entrepreneur has multiple offspring, I assume that the other children may start spin-offs based on the original blueprint, i.e. knowledge of business ideas transfers perfectly across generations of business owners.

Individuals value the discounted flow of a unique, numeraire consumption good $\hat{c}(t)$

$$\mathscr{U}(t) = \int_t^\infty e^{-(\rho+\kappa)\tau} \hat{c}(t+\tau) d\tau$$

Unemployed individuals enjoy consumption-equivalent flow value of leisure $\hat{b}(t)$ and entrepreneurs enjoy flow value of being their own boss $\hat{k}(t) = k\hat{b}(t)$, in addition to any profits made.

A mass $\hat{L}(t)$ of firms produce the unique final good. To operate, a firm requires an entrepreneur with a blueprint, a unit of a manager's time as a fixed factor,[27] and labor as a variable input. Let $\hat{z}(i)$ denote the idiosyncratic productivity of firm $i$'s blueprint, which is drawn at entry and remains fixed thereafter. If a firm with productivity $\hat{z}(i)$ employs a manager and $\hat{n}(i,t)$ workers, it produces

$$Y\Big(\hat{z}(i), \hat{n}(i,t)\Big) = e^{\hat{z}(i)} \hat{n}(i,t)$$

If the firm stops operating, the blueprint is lost, the entrepreneur has to look for a new business idea, and its workers become unemployed.

## 4.2 Markets

The markets for the final good and for managers are frictionless and competitive.[28] Let $\hat{r}(t)$ denote the rental rate of a manager, which is such that demand is weakly less than the available supply

$$\hat{L}(t) \leq l\hat{N}(t) \tag{3}$$

The labor market is characterized by frictions that prevent the immediate relocation of workers to their most productive use. A worker searches for jobs both when unemployed and employed, in the latter case with relative search efficiency $\phi \geq 0$. On other other side of the market are firms, which choose how many jobs $v$ to advertise subject to an iso-elastic flow recruiting cost

$$C_v\Big(v, \hat{z}(i)\Big) = \frac{c_v}{1+\eta_v} e^{\hat{z}(i)} v^{1+\eta_v}, \qquad \eta_v > 0$$

The cost scales in the firm's productivity, reflecting the view that recruiting requires incumbent workers' time, at the cost of foregone goods production.

[27] The fixed factor could alternatively be interpreted as human capital, new material or land (Aghion and Howitt, 1994).

[28] The simplifying assumption that the market for managers is frictionless could be motivated by the presence of recruiting agencies that specialize in matching managers to firms. To the extent that also the market for managers is subject to frictions, I suspect that the forces highlighted below would apply also to this market, leading to even larger effects of aging.

Workers and firms meet at random in a common labor market. Let $\hat{V}(t)$ denote total vacancies and $\hat{S}(t) = \hat{u}(t) + \phi\hat{e}(t)$ aggregate search intensity, where $\hat{u}(t)$ is the number of unemployed and $\hat{e}(t)$ the number of employed. The worker (job) finding rate $\hat{q}(t)$ ($\hat{p}(t)$) is given by the matching function

$$\hat{q}(t) = \frac{\chi\hat{V}(t)^{\theta}\hat{S}(t)^{1-\theta}}{\hat{V}(t)}, \qquad \hat{p}(t) = \frac{\chi\hat{V}(t)^{\theta}\hat{S}(t)^{1-\theta}}{\hat{S}(t)} \tag{4}$$

where $\chi > 0$ is matching efficiency and $\theta \in [0,1]$ the elasticity of the job finding rate to vacancies.

Workers and entrepreneurs bargain over marginal surplus following Bilal et al. (2022). In the interest of space, I describe only the outcome of the game here, and refer to Bilal et al. (2022) for the details. When a worker and firm meet, the entrepreneur makes the worker a take-leave offer to purchase the job. In equilibrium, the entrepreneur successfully poaches an employed worker if the entrepreneur is more productive than the worker's current employer, and offers the worker the option to purchase the new job at a price that equals the full value of the worker's current job. If the worker accepts the offer, she becomes the owner of the job. Ownership includes the right to the job's proceeds as well as the decision of when to terminate it, with one exception. The entrepreneur, who has to bear the fixed cost of operation $\hat{r}(t)$, may always shut down the firm, in which case its workers become unemployed. Workers may, however, make the entrepreneur a take-leave offer to keep the firm alive for another instant.

### 4.3 The value of a firm

Let $m$ denote the growth rate of output per worker on the BGP and $\underline{\hat{z}}(t)$ the least productive firm in operation at time $t$, which also grows at the rate $m$ on the BGP. I assume throughout that parameter values are such that $m > 0$. In addition, to ensure the existence of a BGP, I impose that

**Assumption 1.** *The flow value of leisure $\hat{b}(t)$ grows at the rate of the economy $m$. Specifically, $\hat{b}(t) = e^{\underline{\hat{z}}(t)}$ and the flow value of being one's own boss $k$ is "sufficiently" high (as made more precise by footnote 31).*

Instead of studying the growing economy, it is convenient to analyze a stationary transformation of it. To that end, define the *relative productivity* of firm $i$ as

$$\underbrace{z(i,t)}_{\text{relative productivity of firm } i \text{ at time } t} = \underbrace{\hat{z}(i)}_{\text{productivity of firm } i} - \underbrace{\underline{\hat{z}}(t)}_{\text{productivity of the least productive firm at time } t}$$

$$dz(i,t) = -\underbrace{m}_{\text{rate of obsolescence}} dt$$

Incumbents fall behind the market at the *rate of obsolescence* $m$, as competition from new firms bids up the prices of factors of production, but incumbents' productivity does not improve.

Under the stipulated bargaining protocol, Bilal et al. (2022) show that the allocation can be solved for by focusing on the joint value of a firm to its entrepreneur owner and workers, $\hat{\mathbf{W}}(\hat{z}, n, t)$, where

**Lemma 1.** *The transformed stationary joint value of a firm, $\mathbf{W}(z, n) = e^{-\hat{\underline{z}}(t)}\hat{\mathbf{W}}(\hat{z}, n, t)$, satisfies*

$$(\rho - m)\mathbf{W}(z,n) = \max_{v\geq 0}\Bigg\{ \underbrace{e^z n + k - r - \frac{c_v}{1+\eta_v} e^z v^{1+\eta_v}}_{\text{net output}} - \underbrace{\kappa n \mathbf{W}_n(z,n)}_{\text{worker retirement}} - \underbrace{\kappa \mathbf{W}(z,0)}_{\text{entrepr. retirement}} - \underbrace{m\mathbf{W}_z(z,n)}_{\text{techn. obsolescence}} -$$
$$+ \underbrace{qv\left(\frac{u}{S}\Big(\mathbf{W}_n(z,n) - U\Big)^+ + \frac{e\phi}{S}\int_{n',z'} \Big(\mathbf{W}_n(z,n) - \mathbf{W}_n(\tilde{z},\tilde{n})\Big)^+ d\mathbf{G}(\tilde{z},\tilde{n})\right)}_{\text{expected return to advertising jobs}}\Bigg\} \tag{5}$$

*where $q = \hat{q}(t)$ on the BGP and $x^+ = \max\{x, 0\}$, for $(z, n)$ interior to separation and exit boundaries*

$$\mathbf{W}_n(z,n) \geq U, \qquad \mathbf{W}(z,n) \geq nU + U^f \tag{6}$$

*where $r = e^{-\hat{\underline{z}}(t)}\hat{r}(t)$ is the relative fixed cost, $S = \hat{S}(t)/N(t)$ is average per capita search intensity, $u = \hat{u}(t)/N(t)$ is the unemployment rate, $e = \hat{e}(t)/N(t)$ is the employment rate, $\mathbf{G}(z, n)$ is the distribution of employment over firms, $U^f$ the transformed stationary value of an inactive entrepreneur, and*

$$U = \frac{1}{\rho + \kappa - m} \tag{7}$$

*is the transformed stationary value of unemployment, $U = e^{-\hat{\underline{z}}(t)}\hat{U}(t)$.*

*Proof.* See Appendix C.2. □

Bilal et al. (2022) provide a derivation of the Hamilton-Jacobi-Bellman (HJB) equation (5). The first term is flow output net of the fixed cost and vacancy costs. The second term reflects the fact that each of the firm's $n$ workers retires at rate $\kappa$, reducing the joint value by the marginal value of the worker. Next, the entrepreneur retires at rate $\kappa$, in which case the entrepreneur's offspring takes over the firm. Since the current coalition does not value its offspring, the joint value falls by the value of the entrepreneur, which equals the value of recruiting more workers into the firm, $\mathbf{W}(z, 0)$. The joint value falls as the firm's blueprint gradually becomes obsolete. Finally, the current coalition gains from hiring new workers, as

I discuss in further detail below. The separation boundary (6) shows that the firm separates workers to unemployment if the marginal value of a worker falls below the value of unemployment. It exits if the joint value falls below the value of unemployment to all its workers as well as the entrepreneur.

### 4.4 Entry

At a point in time, an entrepreneur may either have a blueprint $\hat{z}(i)$ or exert effort looking for a business idea. In the latter case, they choose the arrival rate $\pi s$ of business ideas subject to flow cost

$$C_e\Big(s, \hat{b}(t)\Big) = \frac{c_e}{1+\eta_e}\hat{b}(t)s^{1+\eta_e}, \qquad \eta_e > 0$$

The cost of searching for business ideas scales in the flow value of leisure, reflecting the view that potential entrepreneurs have to spend some of their leisure looking for ideas.

Entrants may imitate and improve upon the blueprints of incumbent firms. Specifically, following Luttmer (2012), I assume that an entrant draws an idiosyncratic, proportional improvement $\varepsilon$ on the least productive firm at time $t$ from an innovation distribution $\Gamma(\varepsilon)$ that is exponential with rate $\zeta$[29]

$$\hat{z}(i) = \hat{\underline{z}}(t) + \varepsilon(i), \qquad \Gamma(\varepsilon) = 1 - e^{-\zeta\varepsilon} \tag{8}$$

I later provide a richer model of both the entrepreneurial choice and knowledge spillovers.

**Lemma 2.** *The transformed stationary value of an inactive entrepreneur, $U^f = e^{-\hat{\underline{z}}(t)}\hat{U}^f(t)$, solves*

$$\Big(\rho - m\Big)U^f = 1 - \kappa U^f + \max_s \left\{ \underbrace{s\pi \int_0^\infty \Big(\mathbf{W}(z,0) - U^f\Big)^+ d\Gamma(z)}_{\text{expected return to coming up with idea}} - \underbrace{c_e\frac{s^{1+\eta_e}}{1+\eta_e}}_{\text{cost of searching for ideas}} \right\} \tag{9}$$

*Proof.* See Appendix C.2. □

The current model of imitation and selection hence centers on the role of modest productivity improvements by entrants at the bottom/middle of the distribution, consistent with the empirical observation that most new firms are low-productive. Nevertheless, as emphasized by Perla and Tonetti (2014), the fact that many individuals attempt entrepreneurship implies that this process may still contribute meaningfully to aggregate growth. When I later go to the data, I allow also for a role for incumbent innovation in order to avoid attributing all growth to imitation and selection via the entry/exit margin.

[29]The tail of the innovation distribution, $\zeta$, cannot be too thick or expected values below are not well-defined. I assume throughout that the tail is sufficiently thin, in order to make the analysis meaningful.

### 4.5 Reducing the state-space

Although technology displays constant returns to scale, the fixed cost $r$ implies increasing returns, necessitating keeping the size of the firm as a state and stipulating a multilateral bargaining protocol. In particular, instances may arise where the entrepreneur wants to shut down the firm to avoid having to pay the fixed cost, even though collectively there is a value to keeping the firm alive. A multilateral bargaining protocol is needed to deal with such cases. Yet under assumption 1, one can show that

**Proposition 1.** *The joint surplus of the firm satisfies*

$$\underbrace{\mathbf{W}(z,n) - nU - U^f}_{\text{joint surplus of firm}} = n \underbrace{J(z)}_{\text{surplus of match}} + \underbrace{\frac{\eta_v}{1+\eta_v}\left(\frac{1}{c_v}\right)^{\frac{1}{\eta_v}} \frac{1}{m} e^{-\frac{\rho+\kappa-m}{m}z} \int_0^z e^{\frac{\rho+\kappa-m}{m}\tilde{z} - \frac{1}{\eta_v}\tilde{z}} R(\tilde{z})^{\frac{1+\eta_v}{\eta_v}} d\tilde{z}}_{\text{discounted cumulative return to hiring}} \quad (10)$$

*where the surplus of a match is for $z \geq \underline{z}^w$*

$$J(z) = \frac{1}{\rho+\kappa-m}\left(\frac{m}{\rho+\kappa} e^{-\frac{\rho+\kappa-m}{m}z} - 1\right) + \frac{1}{\rho+\kappa} e^z \quad (11)$$

*the optimal separation threshold of workers is $\underline{z}^w = 0$, optimal job creation $v(z)$ by incumbent firms is*

$$\underbrace{c_v e^z v(z)^{\eta_v}}_{\text{marginal cost of advertising job}} = \underbrace{R(z)}_{\text{return to hiring}} \quad (12)$$

$$\underbrace{R(z)}_{\text{return to hiring}} = \underbrace{q}_{\text{worker contact rate}} \left( \underbrace{\frac{u}{S} J(z)}_{\text{unemployed potential hire}} + \underbrace{\phi \frac{e}{S} \int_0^z J'(\tilde{z}) G(\tilde{z}) d\tilde{z}}_{\text{employed potential hire}} \right) \quad (13)$$

*the optimal exit threshold of entrepreneurs is $\underline{z} = 0$, the optimal search intensity $s$ for business ideas is*

$$\underbrace{c_e s^{\eta_e}}_{\text{marginal cost of search}} = \pi \underbrace{\frac{\eta_v}{1+\eta_v}\left(\frac{1}{c_v}\right)^{\frac{1}{\eta_v}} \frac{1}{m} \int_0^\infty e^{-\frac{\rho+\kappa-m}{m}\tilde{z}} \int_0^{\tilde{z}} e^{\frac{\rho+\kappa-m}{m}z - \frac{1}{\eta_v}z} R(z)^{\frac{1+\eta_v}{\eta_v}} dz d\Gamma(\tilde{z})}_{\text{expected value of new blueprint}} \quad (14)$$

*the equilibrium fixed cost of a manager is*

$$\underbrace{r}_{\text{fixed cost}} = \underbrace{k}_{\text{flow value of being own boss}} - \underbrace{1}_{\text{forgone value of leisure}} - \underbrace{\frac{\eta_e}{1+\eta_e} c_e s^{1+\eta_e}}_{\text{forgone option value of search for new ideas}} \quad (15)$$

*and the equilibrium number of active firms per capita is $L = \hat{L}(t)/N(t) = l$.*

*Proof.* See Appendix C.3. □

The return to hiring (13) shows that recruiting firms may either contact an unemployed or an employed worker, who is distributed over the job ladder according to $G(z)$. An unemployed worker always accepts the offer, and the firm retains the full surplus from forming the match. An employed worker accepts the offer if they currently work for a less productive firm $z < \tilde{z}$, and the poaching firm gets $J(\tilde{z}) - J(z)$ (integrating by parts, $\int_0^{\tilde{z}} J(\tilde{z}) - J(z) dG(z) = \int_0^{\tilde{z}} J'(z)G(z)dz$). Panel A of Figure 4 illustrates.

The first part of assumption 1 ensures that workers prefer to quit to unemployment before (or exactly when) the entrepreneur wants to shut down the firm. As a result, the exit decision of the entrepreneur is without consequence for workers. It follows that there is no need for a multilateral bargaining protocol, a firm's size does not impact its constituent matches, it suffices to solve the surplus of a match and the entrepreneur in order to find the equilibrium allocation, and the job ladder reduces to a one-dimensional ranking of firms in terms of relative productivity, $z$. Yet despite this tractability, the framework features rich worker and firm dynamics, including endogenous firm exit and job-to-job mobility.

The second part of assumption 1 ensures that the market clearing condition (3) holds with equality, i.e. that managers want to provide their services at the going price. Combined with the assumption that a fixed fraction $l$ of individuals provide such services, it implies that the number of firms per capita is fixed. The model is hence in the spirit of the vertical differentiation literature on economic growth, where a fixed number of product lines are improved upon (Klette and Kortum, 2004). This focus is motivated by the available evidence indicating that such growth is quantitatively more important than increases in the number of varieties per capita (Garcia-Macia et al., 2019).[30] Under these assumptions, the flow value of being one's own boss $k$ and the fixed cost $r$ do not enter any other equilibrium conditions than (15), and the number of active firms per capita is fixed. Consequently, the allocation can be determined without solving for the equilibrium fixed cost. Having solved for the allocation, the flow value of being one's own boss can be set high enough based on (15) such that managers prefer to provide their services.[31]

### 4.6 The distribution of entrepreneurs and workers

Let $\hat{x}(z,t)$ be the number of entrepreneurs who own a blueprint with relative productivity $z$ at time $t$, with $x(z)$ its associated probability density function (pdf), and $\hat{y}(t) = yN(t)$ the number of new

[30] It would be interesting to extend the framework to allow for a choice between being a manager and a worker. I believe that such an extension would infer based on the small change in the number of firms to workforce participants in Sweden over this period (Appendix A.2) or the lack of an impact of aging on average firm size across local labor markets (Appendix B.6) that this occupational choice is inelastic, such that it would not change my main results by much. It remains, however, just a hypothesis.

[31] Since managers can always enjoy $b = 1$ by not participating, $k$ must be such that $r = k - 1 - \frac{\eta_e}{1+\eta_e} c_e s^{1+\eta_e} \geq 1$.

blueprints created at time $t$, with $y$ the per capita entry rate. The number of entrepreneurs with productivity $z > 0$ evolves according to a Fokker-Planck (or Kolmogorov Forward) equation

$$\frac{\partial \hat{x}(z,t)}{\partial t} = \underbrace{m\frac{\partial \hat{x}(z,t)}{\partial z}}_{\text{technological obsolescence}} - \underbrace{\kappa \hat{x}(z,t)}_{\text{retirement of founder}} + \underbrace{\kappa(1+(1-\omega)\nu)\hat{x}(z,t)}_{\text{inheritance of offspring}} + \underbrace{\hat{y}(t)\gamma(z)}_{\text{entry of new firms}} \tag{16}$$

Relative productivity drifts at rate $-m$, founders retire at rate $\kappa$, at which point their offspring inherit the blueprint, and $\hat{y}(t)$ new blueprints are created with an initial productivity drawn from the innovation distribution $\Gamma$. For future reference, let $\gamma(z)$ denote the probability density function (pdf) of the innovation distribution, with $\Gamma(z) = \int^z \gamma(\tilde{z})d\tilde{z}$ its corresponding cumulative distribution function (cdf).

On a BGP, the share of entrepreneurs at each point in the distribution is constant. Hence

$$\hat{x}(z,t) = x(z)\hat{L}(t) = x(z)le^{\lambda t}(1+\xi+l) \tag{17}$$

$$\frac{\partial \hat{x}(z,t)}{\partial t} = x(z)l\lambda e^{\lambda t}(1+\xi+l) \tag{18}$$

$$\frac{\partial \hat{x}(z,t)}{\partial z} = x'(z)le^{\lambda t}(1+\xi+l) \tag{19}$$

$$\hat{y}(t) = ye^{\lambda t}(1+\xi+l) \tag{20}$$

Imposing a BGP in (16) using (17)–(20), the stationary distribution of entrepreneurs over relative productivity is given by the linear second-order non-homogenous ordinary differential equation (ODE)

$$0 = mx'(z) + \frac{y}{l}\gamma(z) \tag{21}$$

subject to the boundary conditions that $X(z) = \int_0^z x(\tilde{z})d\tilde{z}$ has no mass point at 0 and integrates to one

$$X(0) = 0 \tag{22}$$

$$\lim_{z\to\infty} X(z) = 1 \tag{23}$$

Integrating (21) from 0 to infinity and noting that $\lim_{z\to\infty} X(z) = 1$ implies $\lim_{z\to\infty} x(z) = 0$ gives

$$x(0) = \frac{y}{lm} \tag{24}$$

Using an identical approach as for the distribution of entrepreneurs, Appendix C.4 shows that the distribution of workers over relative productivity solves the second-order linear non-homogenous ODE

$$\underbrace{\lambda g(z)}_{\text{labor supply growth}} = \underbrace{m g'(z)}_{\text{technological obsolescence}} - \underbrace{\kappa g(z)}_{\text{retirement of worker}} - \underbrace{\phi p \Big(1 - F(z)\Big) g(z)}_{\text{separations up the job ladder}}$$
$$+ \; p f(z) \left( \underbrace{\frac{u}{1-u}}_{\text{hires from unemployment}} + \underbrace{\phi G(z)}_{\text{hires from below in the job ladder}} \right) \tag{25}$$

subject to the boundary conditions $G(0) = 0$ and $\lim_{z \to \infty} G(z) = 1$, as well as

$$u = \frac{\kappa + \lambda + m g(0)}{\kappa + \lambda + m g(0) + p} \tag{26}$$

Integrating (25) from 0 to $z$, using the boundary condition $G(0) = 0$, an integration by parts, and (26) to substitute for $u$, (25) can be simplified to a linear first-order non-homogenous ODE

$$\frac{m}{\kappa + \lambda} g(z) - \Big(1 + \beta(1 - F(z))\Big) G(z) = \frac{m}{\kappa + \lambda} g(0)(1 - F(z)) - F(z), \qquad \beta \equiv \frac{\phi p}{\kappa + \lambda} \tag{27}$$

subject to $G(0) = 0$, where $g(0)$ is such that $\lim_{z \to \infty} G(z) = 1$ and the offer distribution is

$$F(z) = \frac{l}{V} \int_0^z v(\tilde{z}) x(\tilde{z}) d\tilde{z}, \qquad V = l \int_0^\infty v(z) x(z) dz \tag{28}$$

### 4.7 Equilibrium

To characterize the equilibrium, a useful starting point is to consider how exogenous, off-equilibrium variation in growth affects individuals' incentives to exit and enter entrepreneurship. This hypothetical exercise results in an *exit curve* and an *entry curve*. Of course, the growth rate is an equilibrium object, so an equilibrium must be such that exit, entry and growth are all consistent.

The exit curve $\tilde{y}(m)$ gives the exit rate, $\tilde{y}$, that results from a particular growth rate $m$

**Proposition 2.** *There exists a unique stationary distribution of entrepreneurs over relative productivity given by*

$$x(z) = \zeta e^{-\zeta z} \tag{29}$$

*and the optimally chosen exit rate associated with a particular growth rate is given by the exit curve*

$$\tilde{y}(m) = \zeta l m \tag{30}$$

*In particular, an increase in growth raises firm exit.*

*Proof.* See Appendix C.5. □

Higher growth implies that incumbent firms fall behind the market faster. Consequently, more incumbent firms drift below the endogenous exit threshold at any point in time. Note based on (29) that the distribution of entrepreneurs, $x(z)$, is independent of the rate of obsolescence, $m$.

The entry curve $y(m)$ shows how many individuals want to enter entrepreneurship given a rate of obsolescence $m$. Using the fact that the aggregate entry rate is the product of the arrival rate of business ideas per unit of search intensity ($\pi$), the number of non-producing entrepreneurs ($\xi - l$), and the optimally chosen search intensity ($s$) given by (14), the aggregate entry rate writes

$$y(m) \quad = \quad (\xi - l)\pi^{\frac{1+\eta_e}{\eta_e}} c_e^{-\frac{1}{\eta_e}} \left( \frac{\eta_v}{1+\eta_v} \left(\frac{1}{c_v}\right)^{\frac{1}{\eta_v}} \int_0^{\infty} \frac{e^{-\frac{\rho+\kappa-m}{m}\tilde{z}}}{m} \int_0^{\tilde{z}} e^{\frac{\rho+\kappa-m}{m}z - \frac{1}{\eta_v}z} R(z;m)^{\frac{1+\eta_v}{\eta_v}} dz d\Gamma(\tilde{z}) \right)^{\frac{1}{\eta_e}} \tag{31}$$

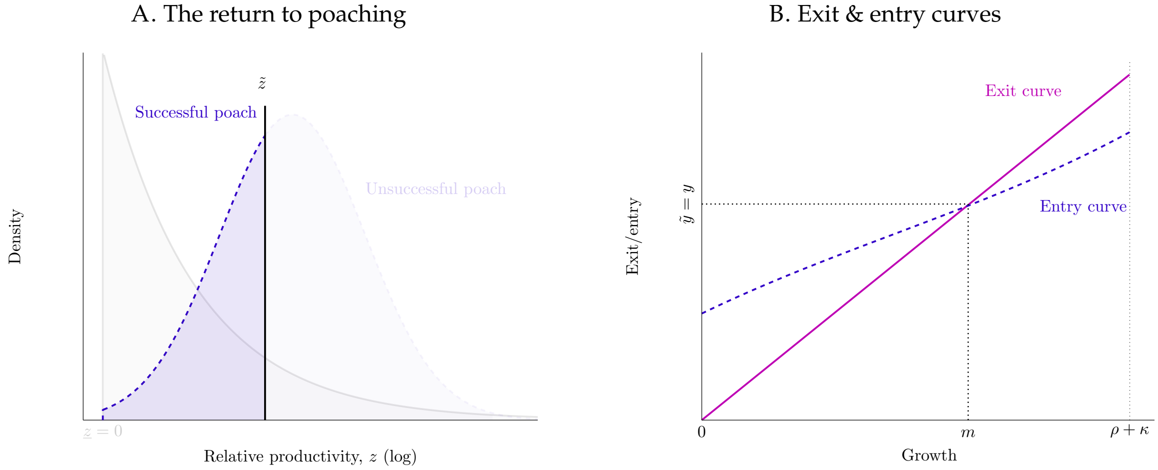
where the return to hiring $R(z;m)$ is given by (13).

**Definition 1.** *A stationary equilibrium with positive growth is $J(z;m)$, $\underline{z}^w(m)$, $v(z;m)$, $\underline{z}(m)$, $s(m)$, $r(m)$, $L(m)$, $p(m)$, $q(m)$, $V(m)$, $S(m)$, $x(z;m)$, $F(z;m)$, $G(z;m)$, $u(m)$, $\tilde{y}(m)$, $y(m)$ and $m \in (0, \rho + \kappa)$ such that: (1) the surplus and reservation threshold of a match solve the problem of a match; (2) the vacancy policy and reservation threshold of a firm solve the problem of an incumbent firm, with $\underline{z}(m) \equiv 0$; (3) the search intensity solves the problem of non-producing entrepreneurs; (4) the fixed cost of a manager is such that demand for managerial services equals supply; (5) the finding rates are consistent with aggregate vacancy creation and search intensity; (6) aggregate vacancies are consistent with firms' optimal vacancy policy and aggregate search intensity is given by $S(m) = u(m) + \phi(1 - u(m))$; (7) the distribution of entrepreneurs is given by* (29)*; (8) the offer distribution is consistent with firms' optimal vacancy policy; (9) the distribution of workers and the unemployment rate solve* (26)–(27)*; (10) the exit rate is consistent with incumbent firms' optimal behavior; (11) the entry rate is consistent with non-producing entrepreneurs' optimal behavior; and (12) the growth rate is such that exit equals entry.*

Panel B of Figure 4 illustrates how the equilibrium is determined in entry/exit-growth space. The exit curve is always upward-sloping, $\tilde{y}'(m) = \zeta l$. A key question is what the slope of the entry curve is.

Incentives to enter are tied to the return to hiring, $R(z;m)$. A key channel through which growth affects the return to hiring is by affecting match surplus, $J(z;m)$. On the one hand, higher growth implies that jobs are not expected to last as long, reducing match surplus through an *obsolescence effect*. On the other hand, it increases the capitalized value of jobs, raising match surplus via a *capitalization effect*.

FIGURE 4. EQUILIBRIUM DETERMINATION




Panel A illustrates who a firm with productivity $z$ may successfully poach. Panel B plots the exit and entry curves (30)–(31). *Source:* Model.

These two forces were highlighted by the seminal paper of Aghion and Howitt (1994). Here, they are unambiguously resolved in favor of the former,[32] such that higher growth *reduces* match surplus

**Proposition 3.** *An increase in growth reduces the surplus of a match*

$$\frac{\partial J(z;m)}{\partial m} \quad = \quad -\frac{e^{-\frac{\rho+\kappa-m}{m}z}}{(\rho+\kappa-m)^2}\left(e^{\frac{\rho+\kappa-m}{m}z}-\left(1+z\frac{\rho+\kappa-m}{m}\right)\right) \quad < \quad 0 \tag{32}$$

*as well as the marginal surplus of a match,* $\partial J'(z;m)/\partial m < 0$.

*Proof.* See Appendix C.6. □

*Ceteris paribus*, the return to hiring (13) falls with growth, discouraging job creation and entry.

In the presence of on-the-job search, however, higher growth also impacts the return to hiring (13) through a third channel, which is new to the literature. To highlight this channel, I henceforth impose

**Assumption 2.** *Vacancy creation is inelastic,* $\eta_v \to \infty$*, and the elasticity of the job finding rate with respect to aggregate search intensity is zero,* $1-\theta \to 0$.

Under assumption 2, all firms create exactly one vacancy, $v(z) \equiv 1$, regardless of idiosyncratic productivity or aggregate equilibrium objects. Consequently, aggregate vacancies equal the mass of firms,

[32] This unambiguity is not surprising, since growth is entirely embodied. The capitalization effect in Aghion and Howitt (1994) arises because technology in their framework is partly disembodied. That is, an entrepreneur pays a set-up cost in return for a research facility that produces a stream of innovations. When an innovation is realized, it starts at the current technological frontier. The research facility is hence subject to disembodied growth, in the sense that it keeps producing innovations at the technological frontier as the economy grows. In particular, there is no need to replace *the research facility* (even though particular innovations produced by the research facility are replaced over time). It is due to this disembodied nature of research facilities that job creation may rise with growth (as made clear by the first part of their second proposition, $\lim_{d\to 0} du^*/dg|_{g=g_0} > 0$). Recent evidence suggests that a large share of growth is embodied (Jorgenson, 2001; Colecchia and Schreyer, 2002).

$V \equiv l$, and the job and worker finding rates are parametric, $p = \chi l \equiv p$ and $q = \chi l \equiv q$. Finally, the offer distribution collapses to the underlying distribution of firms, $F(z) = 1 - e^{-\zeta z}$. Hence, the key economic decisions are reduced to non-producing entrepreneurs' choice of how hard to search for new business ideas, and incumbent firms' choice of when to exit. Section 6 shows quantitatively that the insights below continue to hold also when $\eta_v < \infty$ and $\theta < 1$.

**Proposition 4.** *There exists a unique stationary distribution of workers over relative productivity given by*

$$G(z;m) = \frac{\kappa+\lambda}{m}\int_0^z e^{\frac{\kappa+\lambda}{m}\left(\frac{\beta}{\zeta}\left(e^{-\zeta\tilde{z}}-e^{-\zeta z}\right)+z-\tilde{z}\right)}\left(\frac{\int_0^\infty e^{\frac{\kappa+\lambda}{m}\left(\frac{\beta}{\zeta}e^{-\zeta\hat{z}}-\hat{z}\right)}\left(e^{-\zeta\tilde{z}}-e^{-\zeta\hat{z}}\right)d\hat{z}}{\int_0^\infty e^{\frac{\kappa+\lambda}{m}\left(\frac{\beta}{\zeta}e^{-\zeta\hat{z}}-\hat{z}\right)-\zeta\hat{z}}d\hat{z}}\right)d\tilde{z} \tag{33}$$

*and*

$$u(m) = \frac{\int_0^\infty e^{\frac{\kappa+\lambda}{m}\left(\frac{\beta}{\zeta}e^{-\zeta z}-z\right)}dz}{\int_0^\infty e^{\frac{\kappa+\lambda}{m}\left(\frac{\beta}{\zeta}e^{-\zeta z}-z\right)}\left(\frac{\beta}{\phi}e^{-\zeta z}+1\right)dz} \tag{34}$$

*Proof.* See Appendix C.7. □

**Assumption 3.** *Growth is small relative to the sum of the labor force exit rate and labor supply growth,* $\frac{m}{\kappa+\lambda} \ll 1$.

**Proposition 5.** *Higher growth raises the share of unemployed searchers*

$$\frac{\partial}{\partial m}\left(\frac{u(m)}{S(m)}\right) \approx \frac{\beta}{(1+\beta)^3}\frac{1}{\kappa+\lambda}\zeta > 0 \tag{35}$$

*and shifts employment toward relatively less productive firms*—down the job ladder

$$\frac{\partial G(z;m)}{\partial m} \approx \frac{1}{\kappa+\lambda}\frac{\beta}{1+\beta}\frac{2+\beta(1+e^{-\zeta z})}{(1+\beta e^{-\zeta z})^3}\zeta e^{-\zeta z}(1-e^{-\zeta z}) > 0 \tag{36}$$

*Proof.* See Appendix C.8. □

When new technologies are introduced at a rapid pace, workers are afforded little time to relocate across a given set of technologies before they are replaced with new ones. As a result, individuals are poorly matched to the technologies in existence at any point in time—the labor market is more *misallocated* (in a first best sense). The misallocated labor market raises the return to hiring (13), such that *ceteris paribus* the return to hiring rises with growth, *encouraging* job creation and entry.

What governs the relative strength of the obsolescence, capitalization and novel *misallocation* effects? The impact of growth on the return to hiring is inverse-U shaped in the intensity of on-the-job search

**Proposition 6.** *If the discount rate is not too low, higher growth raises the return to hiring if employed workers search for jobs with intermediate intensity, but lowers it if they search with very low or very high intensity*

$$\exists \rho, \phi_1, \phi_2 \in (0, \infty) : \quad \frac{\partial R(z;m)}{\partial m} \quad = \quad \begin{cases} < \ 0 & \textit{if} \quad \phi < \phi_1 \\ > \ 0 & \textit{if} \quad \phi_1 < \phi < \phi_2 \\ < \ 0 & \textit{if} \quad \phi > \phi_2 \end{cases}$$

*Proof.* See Appendix C.9. □

To understand proposition 6, consider first the limiting case in which workers do not search on the job, $\phi = 0$. In this case, the distribution of workers collapses to the distribution of firms, $G(z) = 1 - e^{-\zeta z}$, with $u(m) = \frac{\kappa+\lambda+m\zeta}{p+\kappa+\lambda+m\zeta}$. Hence, growth does not affect the employment distribution, as illustrated by panel A of Figure 5. Moreover, although higher growth increases unemployment, it does not impact the share of unemployed job searchers, $\partial(u(m)/S(m))/\partial m = 0$, since all searching workers are unemployed. Finally, the worker finding rate $q$ is fixed by assumption. It follows from the fact that the obsolescence effect is stronger than the capitalization effect (proposition 3) that the return to hiring falls with growth.

Consider next the case in which workers very quickly take full advantage of new technologies, $\phi \to \infty$. With an unbounded productivity distribution and constant returns to scale in production, all workers would instantly move to an infinitely productive firm and enjoy unbounded utility, which is not sensible. The intuition, however, carries over to the case in which the offer distribution $F(z)$ has finite upper support $\bar{z} < \infty$. In this case, the economy converges to a situation in which all workers work for the most productive firm $\bar{z}$, as illustrated by panel B. Growth has no effect on the employment distribution, again rendering the misallocation effect inoperative, such that the return to hiring falls with growth.

In contrast, when $\phi$ is positive but moderate, workers gradually become better at using existing technologies by relocating across them. Slow growth affords workers more time to relocate across existing technologies before these technologies are displaced, leading workers to be better matched to them. Panel C illustrates this case. The harder recruiting environment reduces the return to hiring.

A change in growth impacts entry (31) through the return to hiring $R(z;m)$, but also the first term in the integral in (31) (which contains another manifestation of the obsolescence and capitalization effects). When the discount rate is sufficiently high, however, the behavior of the entry curve (31) is determined by how growth affects the return to hiring $R(z;m)$.

**Proposition 7.** *An increase in growth raises entry if the employed search with positive but moderate intensity and the discount rate $\rho$ is sufficiently high.*

FIGURE 5. THE EFFECT OF GROWTH ON THE EMPLOYMENT DISTRIBUTION

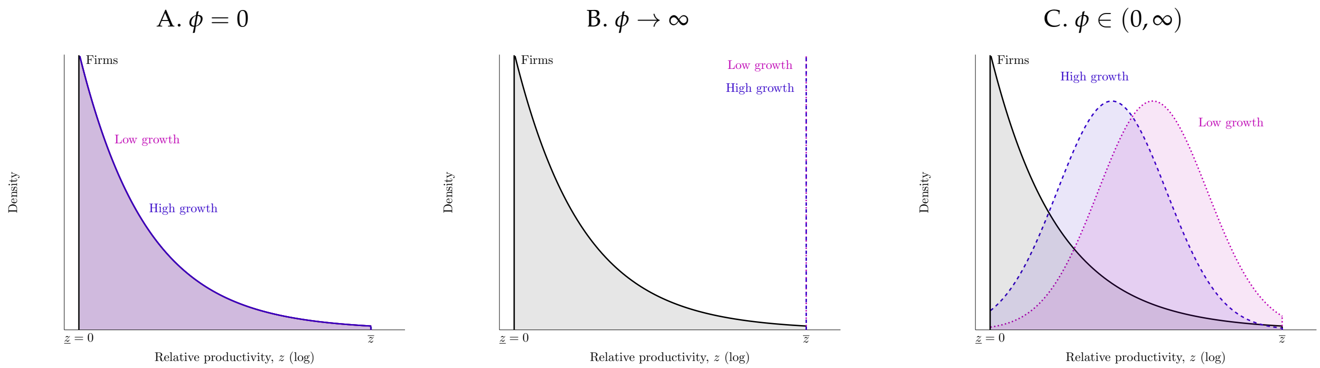



Figure 5 plots the distribution of employment, $g(z; m)$, over relative productivity $z$ for a high and low growth rate, $m$. Panel A shows the case when employed workers do not search for jobs, $\phi = 0$. Panel B shows the case when employed workers immediately move to the top of the job ladder, $\phi \to \infty$. Panel C shows the case when employed workers move up the job ladder at positive but moderate speed, $\phi \in (0, \infty)$. *Source:* Model.

*Proof.* See Appendix C.10. □

JJ mobility serves to lock workers into existing technologies, discouraging the introduction of new technologies. As noted above, lower growth provides workers more time to relocate across existing technologies before these technologies become obsolete, strengthening this lock-in effect.[33]

The discussion so far focused on how exogenous variation in growth affects exit and entry. The growth rate, however, is an endogenous outcome that in equilibrium must be consistent with exit and entry. In particular, for a stationary equilibrium to exist, the exit and entry curves must cross at least once for $m \in (0, \rho + \kappa)$. A negative growth rate would imply negative exit, which is not sensible, while the growth rate must be below the rate of effective discount, $\rho + \kappa$, or the values of being unemployed (7) and a searching entrepreneur (9) explode. Assumption 3 is not required for the following to hold

**Proposition 8.** *If the arrival rate of business ideas per unit of search intensity ($\pi$) and/or the share of entrepreneurs in the population ($\xi$) is sufficiently small, there exists at least one stationary equilibrium. If in addition the curvature in the cost of searching for business ideas ($\eta_e$) is sufficiently high, the stationary equilibrium is unique.*

*Proof.* See Appendix C.11. □

## 4.8 The impact of aging

I now analyze the effect of aging on the equilibrium across BGPs, focusing on the case of a unique BGP equilibrium. To that end, it is useful to track an individual's calendar age $a$, despite the fact that it is not

[33] The discount rate factors into this result because higher growth makes the labor market more misallocated, but it also lowers the expected duration of firms and matches. When the discount rate is sufficiently high, entrepreneurs value more the easier hiring environment today relative to the higher exit rate in the future.

a state. Let $\Lambda(a)$ be the cdf of the age distribution of the workforce, which evolves according to

$$\lambda\Lambda'(a) \quad = \quad -\Lambda''(a) - \kappa\Lambda'(a)$$

subject to $\lim_{a\to\infty}\Lambda(a) = 1$. The solution is $\Lambda(a;\lambda) = 1 - e^{-(\lambda+\kappa)a}$. A fall in labor supply growth, $\lambda$, hence leads to a decline in the share of the workforce younger than $a$, $\partial\Lambda(a;\lambda)/\partial(-\lambda) < 0$. Moreover

**Proposition 9.** *Holding fixed growth, aging lowers unemployment, shifts employment up the job ladder, and reduces the return to hiring* (13)

$$\begin{aligned}
\left.\frac{\partial u(m,\lambda)}{\partial(-\lambda)}\right|_{m\ fixed} &\approx -\frac{\beta}{(1+\beta)^2}\frac{1}{\kappa+\lambda} \\
\left.\frac{\partial G(z;m,\lambda)}{\partial(-\lambda)}\right|_{m\ fixed} &\approx -\frac{\beta e^{-\zeta z}}{(1+\beta e^{-\zeta z})^2}\frac{1}{\kappa+\lambda}\left(1-e^{-\zeta z}\right) \\
\left.\frac{\partial R(z;m,\lambda)}{\partial(-\lambda)}\right|_{m\ fixed} &\approx -q\frac{1}{\kappa+\lambda}\frac{1}{\rho+\kappa}\frac{\beta}{1+\beta}\left(\frac{e^{z(1-\zeta)}}{1+\beta e^{-\zeta z}} + \int_0^z \frac{e^{\tilde{z}(1-\zeta)}\left(\zeta+\beta\left(1-e^{-\zeta\tilde{z}}\right)\right)}{(1+\beta e^{-\zeta\tilde{z}})^2}d\tilde{z}\right)
\end{aligned} \tag{37}$$

*Proof.* See Appendix C.12. □

Panel A of Figure 6 illustrates how aging shifts employment up the job ladder. The harder recruiting environment reduces the return to hiring (13). The magnitude of the impact of aging on the return to hiring is again inverse-U shaped in how intensely employed workers search for jobs, $\phi$

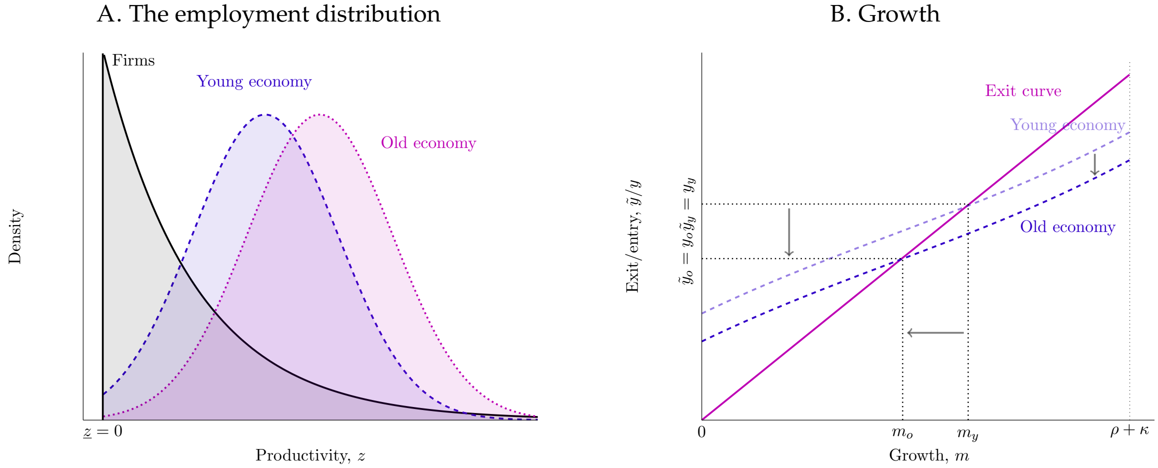
**Lemma 3.** *Holding fixed growth, aging has no effect on the return to hiring* (13) *when workers either do not search on the job, $\phi \to 0$, or search with very high intensity, $\phi \to \infty$.*

*Proof.* It follows directly from taking the limits $\beta = \frac{\phi p}{\kappa+\lambda} \to 0$ and $\beta = \frac{\phi p}{\kappa+\lambda} \to \infty$ of (37). □

As the return to hiring (13) falls, the entry curve (31) shifts down as in panel B. In contrast, aging has no effect on the exit curve (30). In equilibrium, entry and growth decline. The intuition is that an older workforce is better matched to incumbent technologies, because they have had more time to relocate in the labor market. Consequently, aging improves static allocative efficiency. The improved allocation of workers to existing technologies, however, serves to raise the opportunity cost of switching to new technologies. As a result, new technologies are introduced at a slower rate, i.e. growth falls.

Aging reduces worker mobility through both a direct and an indirect channel. First, holding fixed growth, workforce composition shifts toward older, better matched individuals. Since better matched individuals are less likely to accept an outside job offer, JJ mobility falls. Second, as growth declines in

FIGURE 6. THE IMPACT OF AGING ON LABOR MISALLOCATION AND GROWTH




Panel A plots the stationary distribution of employment $g(z)$ under a high and low rate of labor supply growth, $\lambda$, holding all equilibrium objects fixed. Panel B plots the exit curve (30) and entry curve (31) for a high and low rate of labor supply growth, $\lambda$. *Source:* Model.

equilibrium, individuals become better matched to existing technologies also conditional on their age, reducing the probability that they accept an outside job offer. Consequently, JJ mobility falls further.

## 4.9 Model extensions

I end with three extensions that better allow the framework to speak to the rich micro data. Appendix C.15 outlines the value functions and Appendix C.16 defines the equilibrium in the extended model.

**The entrepreneurial choice and knowledge spillovers.** I allow individuals to move between wage and self employment, motivated by the empirical observation that most founders of new firms in Sweden at some point also were wage employed (see Appendix C.13 for details). Panel A of Figure 7 shows that individuals who are currently employed in more productive firms are less likely to quit their job to start a new firm, little of which is accounted for by observable characteristics of the individual.

To account for this pattern, I assume that workers may search for business ideas at cost

$$C_e\left(s, z\right) = \frac{c_e}{1+\eta_e} e^z s^{1+\eta_e}$$

where $z$ is the relative productivity of a worker's current employer, with $z = 0$ for the unemployed. When an individual exits entrepreneurship, they become unemployed.

Although individuals employed in more productive firms are less likely to start new firms, conditional on doing so, they tend to start more productive firms (panel B). Some of this pattern is accounted for by observable individual characteristics, including the wealth of the individual, but even controlling for such factors, the pattern remains. These differences moderate somewhat as the new firm ages, but important differences remain even 10 years after the new firm was founded (panel C).

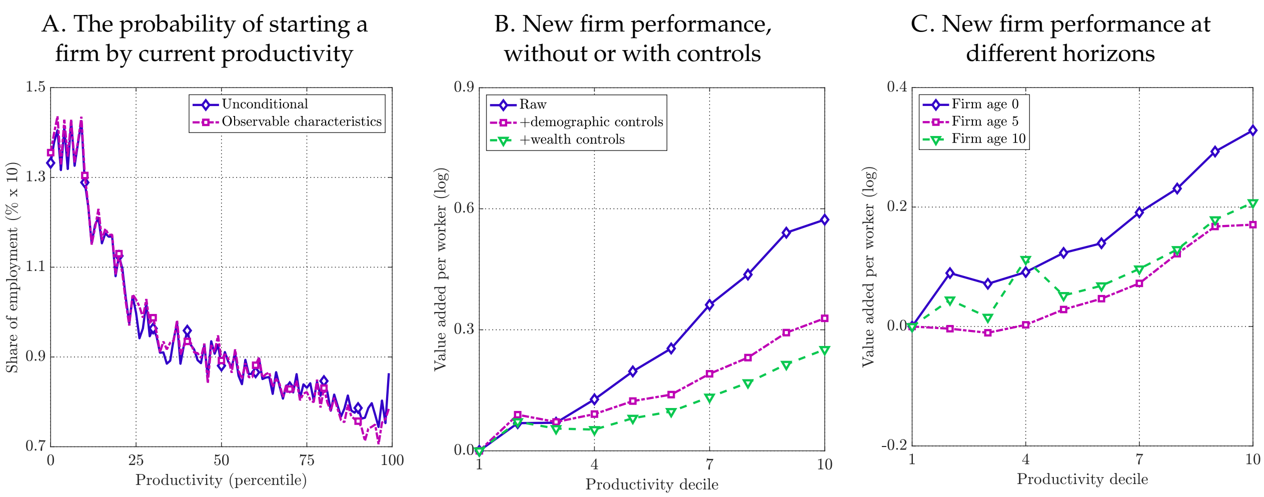
Motivated by these patterns, I assume that entrants draw an initial relative productivity $z$ given by

$$\Gamma(z|\tilde{z}) \quad \sim \quad \mathcal{N}\Big( \alpha_0 + (1-\alpha_1)\bar{z} + \alpha_1 \tilde{z} \, , \, \zeta \Big) \tag{38}$$

where $\bar{z}$ is average relative productivity, $\alpha_0$ governs the extent of *general knowledge spillovers*, $\alpha_1$ captures the degree of *specific knowledge spillovers* from the prior employer $\tilde{z}$, with $\tilde{z} = 0$ for the unemployed.

In the extended model, aging impacts entry and growth through two additional, opposing channels. On the one hand, older individuals are better matched, raising their opportunity cost of attempting entrepreneurship. For this reason, entry falls with age, such that *ceteris paribus* workforce aging *reduces* entry and growth. On the other hand, in the presence of specific knowledge spillovers ($\alpha_1 > 0$), older individuals tend to start more productive firms, since they are on average better matched. For this reason, entry rises with age, such that *ceteris paribus* workforce aging *raises* entry and growth.

FIGURE 7. FIRM CREATION AND SUBSEQUENT FIRM PERFORMANCE BY FIRM PRODUCTIVITY




Panel A plots the share of non-entrepreneurs in month $t$ who are the owner-operator of a new firm in month $t+1$ by the percentile of value added per worker of the individual's current employer, where a new firm is a firm started in the current year and which has at most 10 identified founders in its first year of operation. Residual controls flexibly for gender and education. Panel B plots value added per worker of new firms by decile of the productivity of the founder's prior employer. Demographic controls controls for sector and year of foundation of the new firm, as well as education, gender and age of the founder at time of foundation fully interacted with age of the new firm. Wealth controls also for the net wealth of the founder. Panel C plots value added per worker of new firms by decile of the productivity of the founder's prior employer, controlling for sector and year of foundation of the new firm, as well as education, gender and age of the founder at time of foundation fully interacted with age of the new firm, at firm age 0, 5 and 10. All panels. Private sector firms and individuals aged 20–64 between years 1997–2018. *Source:* FEK, JOBB, LISA.

**Age.** I allow for a direct role for age. Individuals enter the labor market at age $a = 1$ and remain with certainty up to age $\overline{A}$, after which they exit at rate $\kappa > 0$ to be replaced by their offspring. Consequently, older individuals are less likely to start firms both because they are better matched to existing technologies and because they have a shorter expected remaining time to benefit from starting new firms.

The flow value of leisure, $b(a)$, and of being one's own boss, $k(a)$, may depend flexibly on age, subject to the constraint that $\underline{z}^w(a) \geq \underline{z}(a) = 0$ for all $a$. Because all entrepreneurs stay in business at least up to the point where workers prefer to quit to unemployment and offspring keep firms in business in case their parent retires, the age of a firm's entrepreneur is irrelevant for its workers.

At labor market entry, the arrival rate of business ideas is $\pi_0 \geq 0$ per unit of search intensity, which increases to $\pi_1 \geq \pi_0$ after some age $\overline{A}^{\pi}$. This assumption captures in reduced form the idea that some minimum amount of work experience facilitates coming up with successful business ideas.

**Incumbent innovation and idiosyncratic shocks.** I assume that firm productivity evolves according to

$$dZ(i,t) \;=\; \mu dt \,+\, \sigma dW(t)$$

where the drift $\mu$ captures incumbent innovation, $\sigma \geq 0$ is the intensity of idiosyncratic shocks, and $W(t)$ the standard Wiener process. By allowing for incumbent innovation ($\mu > 0$), I let the data flexibly determine the relative importance of entry/exit versus incumbent innovation toward overall economic growth. That being said, I assume that incumbent innovation is unaffected by aging, motivated by the observation that aging does not impact, for instance, investment per worker (Appendix B.6).

Firms also exit at exogenous rate $d$, allowing the model to match the fact that even some high productive firms exit in the data. Separations to unemployment also take place at exogenous rate $\delta(z)$, allowing the model to match a decline in the EN rate with productivity in the data.

Appendix C.14 derives the law of motion for relative productivity and shows that a fall in entry raises productivity dispersion, consistent with Swedish trends over this period.

# 5 Bringing the theory to the data

I calibrate the model targeting 2014–2018, since productivity data are not available prior to 1997.[34] The next section subsequently considers the impact of a change in the age composition, holding all parameters fixed at their estimated values, either across BGPs or over the transition path.

[34] As illustrated by Figure 1, aging of the workforce was largely completed by the mid-2000s. When I later consider a transition experiment, I find that labor market dynamics converge to steady-state relatively quickly. Hence, labor market flows in 2014–2018 are not far off those in the older economy steady-state, justifying the targeting of moments in 2014–2018.

### 5.1 Externally calibrated parameters

I parameterize the model in three steps. First, I pre-set or normalize four parameters, summarized by panel A of Table 4 (whenever applicable, parameter values and moments are expressed at a monthly frequency). This includes the equivalent of an annual discount rate ($\rho$) of four percent and an elasticity of the job finding rate to vacancies of $\alpha = 0.5$ (Moscarini and Postel-Vinay, 2018).

Second, I calibrate offline 10 parameters, summarized in panel B. I fix the overall growth rate of the economy to two percent annually, $m + \mu = 0.002$. The share of managers ($l$) targets a ratio of workers to firms of 4.62 and the exogenous firm exit rate targets the exit rate of high-productive firms ($d = 0.002$).

I construct a grid for age and assume that individuals enter the economy at its lowest point. To match the labor force participation rate by age in the data, I assume that individuals' recorded calendar age at entry is drawn from a bounded exponential distribution with rate parameter 0.029 over ages 18 to 35. I assume that the arrival rate of business ideas jumps at the second grid point for age, corresponding to $\overline{A}^{\pi} = 120$ (10 years into careers), and that individuals start to retire at age $\overline{A} = 480$ (40 years into careers). I calibrate the retirement rate ($\kappa = 0.022$) such that the model matches the labor force participation rate late in careers. Finally, I find the growth rate of labor supply ($\lambda$) such that the model matches the share of the labor force that is 45 years and older. Appendix D.1 shows that the model matches well the empirical labor force participation rate by age and the age composition of the labor force.

### 5.2 Internally calibrated parameters

In the third step, I determine 13 parameters so as to minimize the sum of squared percent deviations between 13 moments in the model and data. For each potential vector of these 13 parameters, I normalize the flow value of being one's own boss $k(a)$ such that all entrepreneurs are indifferent between keeping their firm alive and exiting to non-employment at the lowest point on the discretized grid for productivity, under a normalized fixed cost of $r = 1$. The implied flow value of being one's own boss corresponds to about 20 percent of average flow output per worker (see Appendix D.1).

Panel D summarizes the internally calibrated parameters. Although the estimation is joint, some moments particularly inform some parameters, as I now discuss heuristically (Appendix D.2 offers a more rigorous analysis). I set matching efficiency ($\chi$) to target the NE rate.[35] Relative search efficiency of

[35]The non-employed in the administrative data likely contains some individuals who never participate in the labor market, which the theory abstracts from. I have alternatively targeted an imputed NE rate based on the steady-state flow balance identity that $n = en/(en + ne)$ and a measure of the unemployed plus marginally attached from a separate labor force survey, *Arbetskraftsundersökningarna* (AKU).[36] AKU indicates a stock of unemployed plus marginally attached of 10.94 percent, for an imputed NE rate of 7.87 percent, i.e. roughly twice as high as in the administrative data. The estimated impact of aging in the next section is little changed under this alternative calibration.

the employed ($\phi$) is informed by the JJ rate. A higher $\phi$ raises the JJ rate, holding $\chi$ fixed. The estimated $\phi \approx 2.8$ is high, consistent with recent evidence that the employed are more efficient at searching for jobs (Faberman et al., 2020). Mechanically, the primary reason for the high estimated $\phi$ is the low NE rate, which implies a low matching efficiency, $\chi$. Hence, $\phi$ needs to be high to match the JJ rate.

I assume that workers' reservation threshold is a linear function of their age $\underline{z}^w(a) = \beta a$. Although the reservation threshold is an endogenous equilibrium object, the flow value of leisure $b(a)$ is allowed to vary freely. Appendix C.17 shows how it can be recovered to rationalize any reservation threshold. I set the slope $\beta$ to target the JJ rate at age 50 relative to age 30. Low productive jobs have a high subsequent JJ rate. If older individuals are less likely to accept such jobs, the JJ rate falls more with age. Although I prefer to target the life-cycle behavior of the JJ rate given its central role in the analysis, the model matches well also a decline in the NE rate with age in the data. The implied flow value of leisure corresponds to between 5–25 of average flow output per worker (see Appendix D.1).

I parameterize the separation rate as $\delta(z) = \delta_0 e^{\delta_1 z}$ and inform $\delta_0$ and $\delta_1$ by the overall EN rate as well as the EN rate in the third quintile of firm productivity relative to the first quintile. If $\delta_0$ is higher, the overall EN rate is higher. If $\delta_1$ is more negative, the EN rate falls more with productivity.

I target for the arrival rate of business ideas ($\pi_0, \pi_1$) the overall entry rate as well as the entry rate at age 20 relative to age 30.[37] If $\pi_0$ and $\pi_1$ are both higher, the entry rate is higher, whereas if $\pi_1$ is higher holding $\pi_0$ fixed, entry rises more early in careers. I estimate that the arrival rate of ideas more than doubles during the first 10 years, $\pi_1/\pi_0 \approx 2.3$. The curvature of the cost of searching for ideas ($\eta_e \approx 2.2$) is set to match the entry rate at age 50 relative to age 30. In the estimated model, the net gain from entering declines with age, because older individuals are better matched and hence have a higher opportunity cost of entering as well as a shorter expected time to recoup the cost of entry. The lower is $\eta_e$, the more entry responds to the fall in the net gain, such that entry declines more with age.

General knowledge spillovers ($\alpha_0$) are set to match the roughly eight log point growth in residual log value added per worker between firm age one and 11. If $\alpha_0$ is more negative, productivity grows more over the firm life-cycle. Specific knowledge spillovers ($\alpha_1$) are set to match the difference in residual value added per worker at firm age five between founders who were previously employed in a firm in the third quintile of residual value added per worker relative to the first quintile.

The dispersion in initial productivity ($\zeta$) is set to match the annual autocovariance of residual log value added per worker among firms aged one. A higher initial dispersion implies higher productivity dispersion among young firms. I target the annual autocovariance, assuming that measured productivity

[37] I exclude spinoffs created by offspring from my entry measure in the model, as it is not clear exactly how these should be treated. They constitute a very small share of all entrants.

is the sum of a permanent "true" component and i.i.d. measurement error. The dispersion of productivity shocks ($\sigma$) targets the increase in the autocovariance of residual log value added per worker between firm age one and five. More pronounced shocks lead productivity dispersion to rise more with firm age.

I target for the curvature of the vacancy cost ($\eta_v$) the vacancy share of the top two quintile of firms, ranked by employment-unweighted value added per worker. If $\eta_v$ is higher, it is costlier for firms to scale up hiring, such that high productive firms create fewer jobs. The very high estimated curvature, $\eta_v \approx 18$, is intriguing, driven by the relatively low vacancy share of high productive firms. I stress with respect to this very high estimate that the model under the estimated $\eta_v$ is broadly consistent with average firm size by age as well as levels of job reallocation by firm age, providing some external validity. In any case, the high value for $\eta_v$ implies that job creation is inelastic, such that the estimated impact of aging in the next section is arguably conservative. In practice, however, the results in the next section are not sensitive to the exact value of $\eta_v$ (for instance, I get similar results under $\eta_v = 2$). This surprising result is likely related to Bilal et al. (2022)'s finding that it is difficult to identify $\eta_v$ off presumably informative moments, because the direct effect of changes in $\eta_v$ on these moments is offset by equilibrium forces.

### 5.3 Life-cycle worker and firm dynamics

Figure 8 summarizes the model's ability to match life-cycle worker and firm dynamics. The entry rate initially rises with age and subsequently declines (panel A). As I discuss further in Appendix D.3, the initial increase is accounted for by an estimated increase in the arrival rate of business ideas with experience ($\pi_1 > \pi_0$) and the assumption that individuals enter the labor market at a random age such that the model matches the labor force participation rate by age in the data. The subsequent decline is due to three factors. First, older individuals have a lower expected return to entry, since they have a shorter time remaining in the market. Second, they have a higher opportunity cost of entry, since they are better matched. Third, some of them have exited the labor force. I stress, however, that although the probability of entry is inverse-U shaped in age, the share of individuals who are self-employed rises monotonically with age, in both the model and the data. The JJ rate declines substantially with age (panel B), as older individuals have had more time to climb the job ladder. In addition, I estimate that they have a higher reservation threshold, such that they are less likely to accept low productivity jobs with high subsequent mobility (Appendix D.3 provides a decomposition of the role of these forces).

New firms do not enter at the "technological frontier", in the sense that they are on average low productive (panel C). The current model of imitation and gradual improvements at the middle of the productivity distribution matches this empirical feature very well. Only through a fortunate sequence

TABLE 4. PARAMETER VALUES AND TARGETED MOMENTS

| Parameter | | Value | Moment | Data | Model |
|---|---|---|---|---|---|
| | | | A. Externally set or externally normalized | | |
| $\rho$ | Discount rate | 0.003 | 5% annual real interest rate | | |
| $\theta$ | Elasticity of matches w.r.t. vacancies | 0.5 | Moscarini and Postel-Vinay (2018) | | |
| $c_v$ | Scalar in vacancy cost | 1 | Normalization | | |
| $c_e$ | Scalar in search cost | 1 | Normalization | | |
| | | | B. Calibrated offline | | |
| $m+\mu$ | Aggregate growth rate | 0.002 | Annual growth rate of 2% | 0.020 | 0.020 |
| $l$ | Share of managers | 0.216 | Worker-to-firm ratio | 4.619 | 4.619 |
| $d$ | Exogenous exit rate | 0.002 | Exit rate, top productivity quintile | 0.002 | 0.002 |
| | Calendar age at entry, shape | 0.029 | Labor force participation rate by age | See Appendix D.1 | |
| | Calendar age at entry, minimum | 18 | Labor force participation rate by age | See Appendix D.1 | |
| | Calendar age at entry, maximum | 35 | Labor force participation rate by age | See Appendix D.1 | |
| $\overline{A}^{\pi}$ | Age of high arrival rate | 120 | Peak entry age | 30 | 30 |
| $\overline{A}$ | Retirement age | 480 | Labor force participation rate by age | See Appendix D.1 | |
| $\kappa$ | Retirement rate | 0.022 | Labor force participation rate by age | See Appendix D.1 | |
| $\lambda$ | Labor force growth rate | 0.000 | Age distribution of labor force | See Appendix D.1 | |
| | | | C. Internally normalized | | |
| $k(a)$ | Flow value of being own boss | See Appendix D.1 | Indifference at 1st grid point btw unempl. and entrepreneurship | | |
| | | | D. Minimum distance routine | | |
| $\chi$ | Matching efficiency | 0.343 | NE rate | 0.036 | 0.033 |
| $\phi$ | Relative search efficiency | 2.800 | JJ rate | 0.022 | 0.022 |
| $\beta$ | Reservation productivity, slope | 0.723 | JJ rate, age 50 | 0.560 | 0.589 |
| $\delta_0$ | Separation rate, intercept | 0.020 | EN rate | 0.010 | 0.010 |
| $\delta_1$ | Separation rate, slope | -0.417 | EN rate, productivity quintile 3 to 1 | 0.579 | 0.590 |
| $\pi_0$ | Arrival rate, initial | 0.001 | Entry rate (x100) | 0.118 | 0.093 |
| $\pi_1$ | Arrival rate, later (relative to initial) | 2.318 | Entry rate, age 20 (x100) | 0.580 | 0.591 |
| $\eta_e$ | Curvature of search cost | 2.223 | Entry rate, age 50 | 0.613 | 0.608 |
| $\alpha_0$ | Diffusion, general | -0.323 | Va.p.w., firm age 11 to age 1 | 0.082 | 0.090 |
| $\alpha_1$ | Diffusion, specific | 0.083 | Va.p.w. by va.p.w. of founder at firm age 5 | 0.072 | 0.061 |
| $\zeta$ | Dispersion in entry productivity | 0.548 | Autocovariance of va.p.w., firm age 1 | 0.248 | 0.255 |
| $\sigma$ | St.d. of productivity shocks | 0.046 | Autocovariance of va.p.w., firm age 5 | 0.015 | 0.016 |
| $\eta_v$ | Curvature of vacancy cost | 17.540 | Vacancy share, 40% most productive firms | 0.705 | 0.723 |

Table 4 summarizes the targeted moments in the model and data as well as the estimated parameter values that minimize the (equally weighted) sum of squared percentage deviations between the moments in the model and data. Worker relocation rates are monthly. In the model, they are constructed based on the exact continuous time solution of the model, appropriately time aggregated to the monthly frequency. Firm reallocation rates are annual. In the model, they are constructed based on a simulated discrete-time monthly approximation of the underlying continuous-time model for 500,000 firms, and subsequently aggregated to the annual frequency in an identical manner as the actually data. The empirical moments are for private sector firms and individuals aged 20–64 between years 2014–2018. *Source:* FEK, JOBB, LISA, model.

of shocks does an entrant eventually become high productive. One explanation for the initial jump in productivity—which the model cannot account for—is that new business ideas are partly an experience good. In both the model and data, firm size grows over firms' first 10 years, as productivity rises and firms gradually grow toward their optimal size conditional on productivity.[38]

Young firms have a high job creation rate (panel E) and job destruction rate (panel F), although the decline in the latter with firm age is less pronounced. Young firms have a high hiring rate from other firms (panel G) and non-employment (panel H), as well as high separation rates to other employers (panel I) and non-employment (panel J). Net poaching—the difference between poached hires and poached

[38]Although technology displays constant returns to scale, the curvature in the hiring technology implies that firms have a well-defined optimal size, where optimal hiring equals separations.

FIGURE 8. TARGETED AND NON-TARGETED MOMENTS

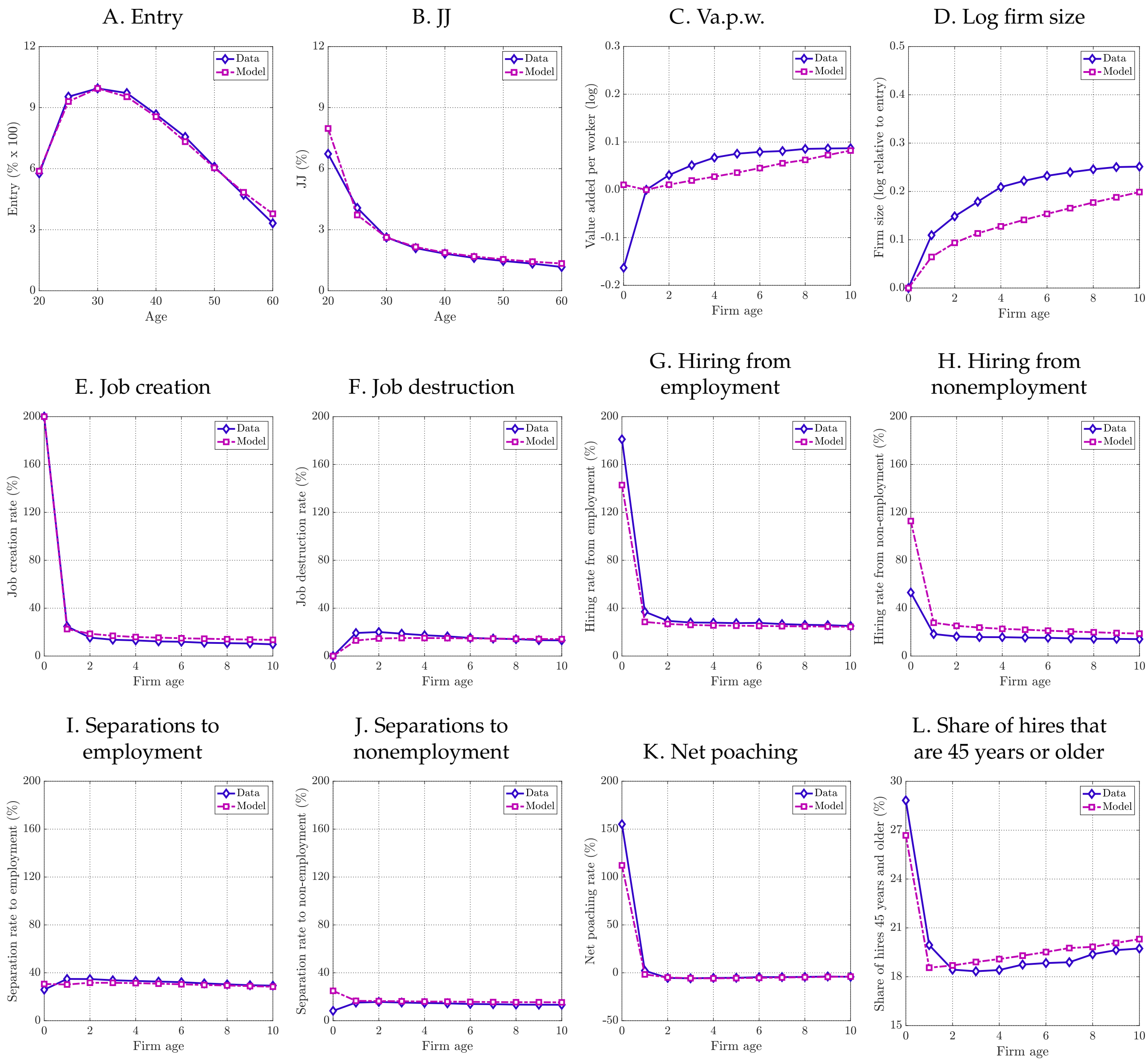



Figure 8 contrasts worker and firm life-cycles outcomes in the model and data. Worker outcomes in panels A–B are monthly. In the model, they are constructed based on the exact continuous time solution of the model, appropriately time aggregated to the monthly frequency. Firm outcomes in panels C–L are annual. In the model, they are constructed based on a simulated discrete-time monthly approximation of the underlying continuous-time model for 500,000 firms, and subsequently aggregated to the annual frequency in an identical manner as the actually data. The empirical moments are for private sector firms and individuals aged 20–64 between years 2014–2018. The model moment for the share of hires that are 45 years or older is normalized such that the mean equals that in the data. *Source:* FEK, JOBB, LISA, model.

separations—at first declines sharply with firm age (panel K). The high net poaching rate of new firms is despite the fact that young firms are low productive (panel C) and hence low in the job ladder. Consequently, they have a low vacancy fill rate and create few vacancies in an absolute sense. Yet they are small, such that their vacancy rate is high and they hire many workers relative to their size. Net poaching eventually becomes positive among very old firms (not shown).

Young firms disproportionately hire (and employ) young workers (panel L). The reason is that they are low-productive (panel C), and hence low in the job ladder. Consequently, they cannot compete for well-matched, older workers (the initial decline is because I include in hires also the founder, both in the model and data, who tends to be older than hires). This pattern is consistent with the view that new firms depend on young workers, who have not yet found a good match with existing technologies.

## 6 Structural estimates of the effect of aging on the labor market

This section uses the estimated model to provide a structural estimate of the impact of aging on the labor market. To do so, I hold all parameters fixed at their estimated values, apart from the growth rate of labor supply, $\lambda$, which I change to target the evolution of the share of the overall labor force aged 16–64 that is aged 45–64 over the 1960–2060 period (using predictions from SCB). I start with a comparative static exercise across BGPs and subsequently turn to a transition experiment.

### 6.1 The impact of aging across BGPs

I start by changing the growth rate of labor supply, $\lambda$, to generate a decrease in the share of the labor force that is aged 45–64 from 39.7 percent in 2014–2018 to 34.2 percent in 1986–1990.[39] This requires $\lambda$ to rise from 0.0003 to 0.0013. Appendix E.1 shows that the model matches reasonably well the age composition of the labor force in 1986–1990 (it matches that in 2014–2018 very well, see Appendix D.1).

Table 5 contrasts the impact of aging with Swedish trends. The entry rate falls by 13 percent, relative to a 25 percent decline in the data, and the JJ rate by 14 percent, relative to an 18 percent decline in the data. I discuss further below the mechanisms behind these declines. Note that the early period for the entry rate refers to 1993–1997, since 1993 is the first year in which it is possible to link incorporated firms to their founders. Aging accounts for little of the decline in the NE rate and generates a counter-factual fall in the EN rate of seven percent, as the fall in the rate of obsolescence implies that matches last longer.

The job creation rate falls by 19 percent, relative to a 25 percent decline in the data.[40] Because total employment has to grow at the same rate as labor supply on the BGP, this fall in job creation is partly mechanical. This, however, is the only direct effect of the change in labor supply growth. In addition, there is less need to reallocate jobs in the older, slower growing economy, leading to a fall also in job

[39] In estimation, I target the share that is 45–74 of the overall labor force age 15–74. Historical data are only available for the share of the labor force that is aged 45–64 of the overall labor force aged 16–64.

[40] The difference between aggregate job creation and destruction is the growth rate of employment, which on the BGP equals the growth rate of labor supply. The job creation and destruction rates in Table 5 do not exactly satisfy this accounting identity, because they rely on a simulated, monthly discrete time approximation to the underlying model.

TABLE 5. EFFECT OF AGING ACROSS BGPS

| | Panel A. Individual outcomes (monthly) | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | Entry rate | | JJ rate | | NE rate | | EN rate | |
| | Data | Model | Data | Model | Data | Model | Data | Model |
| Early period | 0.16% | 0.11% | 2.67% | 2.53% | 4.47% | 3.34% | 0.90% | 1.04% |
| Late period | 0.12% | 0.09% | 2.19% | 2.17% | 3.57% | 3.34% | 0.97% | 0.96% |
| Change | -24.52% | -13.18% | -17.84% | -14.10% | -20.19% | 0.20% | 7.17% | -7.45% |
| | Panel B. Firm outcomes (annual) | | | | | | | |
| | JC rate | | JD rate | | Covariance, va.p.w. | | Empl., firms 11+ | |
| | Data | Model | Data | Model | Data | Model | Data | Model |
| Early period | 15.51% | 12.32% | 14.11% | 10.88% | 0.229 | 0.391 | 54.9% | 76.2% |
| Late period | 11.59% | 10.00% | 10.58% | 9.74% | 0.279 | 0.457 | 65.8% | 85.6% |
| Change | -25.23% | -18.81% | -25.04% | -10.48% | 0.049 | 0.066 | 10.9 p.p. | 9.4 p.p. |

| | Panel C. Gross output | | | |
|---|---|---|---|---|
| | Annual growth (%) | Gross output (log) | Net output (log) | Value of entrant (log) |
| Early period | 2.52% | 3.24 | 3.00 | 8.00 |
| Late period | 2.00% | 3.70 | 3.43 | 7.88 |
| Change | -0.52% | 58.49% | 53.31% | -11.02% |

Table 5 shows the effect of an increase in the share of the labor force aged 16–64 that is aged 45–64 from 34.2 to 39.7 percent across BGPs. It requires a change in the growth rate of labor supply $\lambda$ from 0.0013 to 0.0003 percent (at a monthly frequency). All other parameters are held fixed at their estimated values. The early period refers to 1993–1997 for the entry rate, 1986–1990 for the JJ, NE and EN rates as well as the JC and JD rates, and 1997–2000 for the covariance and employment share of firms aged 11 and older. These are the earliest years for which each series can be consistently constructed. The late period refers to 2014–2018 for all moments. The growth rate is the annual growth rate of output per capita. Gross and net output are relative to the productivity of the least productive firm. Gross output is total output produced by wage employees, and does not net out the time costs of recruiting and search, payments to managers, or the foregone flow values of leisure and of being one's own boss. Net output makes these adjustments. The value of an entrant is the expected discounted value of a labor market entrant. *Source:* FEK, JOBB, LISA, model.

destruction of 10 percent, relative to a 25 percent fall in the data.

Productivity dispersion rises, consistent with the data. I focus on the first annual autocovariance of residual value added per worker controlling for sector-year, with the implicit assumption that labor productivity is the sum of a permanent and transitory part. Note that because productivity data are only available since 1997, the early period refers to 1997–2000. I suspect, however, based on a gradual increase in firm pay dispersion since 1986 that the increase in productivity dispersion started prior to 1997. The reason for the increase in productivity dispersion is that technological obsolescence is a force that holds some firms back from becoming, in relative sense, very productivity, as I demonstrate formally in Appendix C.14. The fall in obsolescence lengthens the firm life-cycle, leading to a shift of employment toward older firms. This pattern is consistent with Swedish trends, although the left-censoring of the data in 1986 implies that I can only measure the employment share of firms older than 11 since 1997.

The older economy grows by 0.52 percentage points less per year. At the same time, the level of output relative to the least productive firm increases by 58 percent. The reason is that older individuals are better matched, and they constitute a larger share of the workforce in the older economy. Moreover, the lower rate of obsolescence affords individuals more time to become better matched to incumbent technologies also conditional on age, before those technologies are replaced with new ones. A young worker is 11 percent worse off entering in the older economy.

### 6.2 The role of direct and equilibrium channels

To highlight the channels through which aging affects firm creation and worker relocation, note that

$$\begin{aligned} y(m,\lambda) &= \int_0^\infty \underbrace{\pi(a)\left(\frac{u(a;m,\lambda)s_u(a;m,\lambda)}{u(a;m,\lambda)+e(a;m,\lambda)} + \frac{e(a;m,\lambda)}{u(a;m,\lambda)+e(a;m,\lambda)}\int_0^\infty s(z,a;m,\lambda)d\tilde{G}(z|a;m,\lambda)\right)}_{\text{age-conditional entry rate}} \\ &\times \underbrace{\frac{u(a;m,\lambda)+e(a;m,\lambda)}{1-l}}_{\text{age group's share of the non-entrepreneurial population}} da \\ JJ(m,\lambda) &= \int_0^\infty \underbrace{p(m,\lambda)\phi\int_0^\infty \Big(1-F(z;m,\lambda)\Big)d\tilde{G}(z|a;m,\lambda)}_{\text{age-conditional mobility rate}} \times \underbrace{\frac{e(a;m,\lambda)}{e(m,\lambda)}}_{\text{age group's share of employment}} da \end{aligned}$$

The change in composition has a *direct effect* on firm creation and worker mobility, since older individuals are less likely to start a firm and switch employer, and aging increases their share of the workforce. I isolate this effect by updating the composition of the labor force to only reflect the direct effect of changes in $\lambda$, holding all equilibrium objects fixed at their values in the estimated economy. According to Table 6, the direct effect contributes to an 1.5 percent decline in entry and a 7.8 percent fall in JJ mobility.[41]

In addition to the direct effect, age-specific mobility rates also change, for two reasons. First, holding fixed growth, potential entrepreneurs are discouraged from entering and incumbent firms are dissuaded from creating jobs by the harder recruiting environment in the older economy. Consequently, entry and JJ mobility fall conditional on the productivity of an individual's current employer in the older economy, as illustrated by panels A and B of Figure 9 (to reduce clutter, I plot outcomes for the youngest age group only, but similar insights hold for the other age groups). According to Table 6, this channel generates a further 3.0 percent decline in entry and 0.8 percent fall in JJ mobility.

Second, as growth falls, age-specific employment shifts toward relatively more productive firms, as highlighted by proposition 5 and illustrated by panel C. The reason is that individuals have more time

[41] Appendix E.1 provides a reduced-form shift-share analysis, finding that most of the decline in entry is accounted for by an age-specific decline in both the model and data, whereas shifts in composition are more important in accounting for the fall in worker relocation. Appendix E.2 shows that the structural estimates match well the reduced-form estimates across Swedish local labor markets.

to relocate across incumbent technologies before these technologies are replaced. The higher opportunity cost of entering entrepreneurship and switching employer generates an additional 8.6 percent fall in entry and 5.5 percent decline in JJ mobility. Section 7 provides reduced reduced-form evidence exactly in line with this prediction: aging raises pay in wage employment by shifting employment toward more productive, higher paying firms, and reduces it in self-employment by making it harder for entrepreneurs to poach workers. This finding is also consistent with Salgado (2020), who documents an increasing relative return to wage employment in the U.S. over the past decades.

FIGURE 9. DISSECTING THE EFFECTS OF AGING ACROSS BGPS

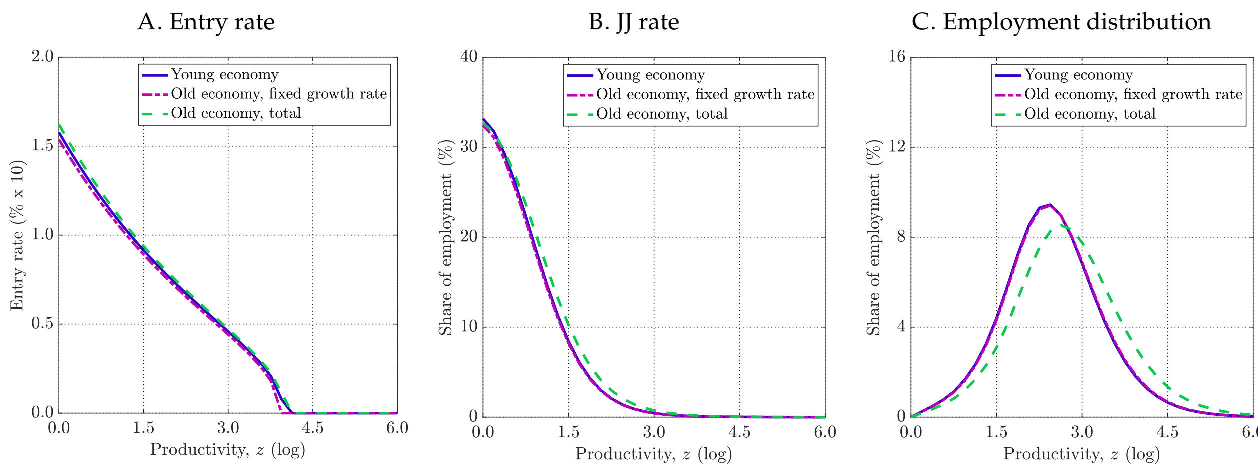



Figure 9 shows the effect of an increase in the share of the labor force aged 16–64 that is aged 45–64 from 34.2 to 39.7 percent across BGPs. It requires a change in the growth rate of labor supply $\lambda$ from 0.0013 to 0.0003 percent (at a monthly frequency). All other parameters are held fixed at their estimated values. Results are for the youngest age group, $a = 1$. Panel A shows the probability of entering entrepreneurship conditional on the productivity of a potential entrepreneur's current employer. Panel B shows the probability of switching employer conditional on the productivity of a worker's current employer. Panel C plots the distribution of employment over firms by productivity. *Source:* Model.

TABLE 6. DECOMPOSITION OF EFFECT OF AGING ACROSS BGPS

| | Entry | JJ |
|---|---|---|
| Composition effect: fixed policies & growth rate | -1.52% | -7.81% |
| Change in age-specific entry & mobility rates | -11.66% | -6.29% |
| *Fixed growth rate* | *-3.02%* | *-0.78%* |
| *Growth rate adjusts* | *-8.64%* | *-5.51%* |
| **Total effect** | **-13.18%** | **-14.10%** |

Table 6 shows the effect of an increase in the share of the labor force aged 16–64 that is aged 45–64 from 34.2 to 39.7 percent across BGPs. It requires a change in the growth rate of labor supply $\lambda$ from 0.0013 to 0.0003 percent (at a monthly frequency). All other parameters are held fixed at their estimated values. The direct effect of aging changes the growth rate of labor supply $\lambda$, holding fixed all policies and the growth rate at their levels in the estimated model. The incumbent effect resolves the model for the new $\lambda$, but counterfactually holding the growth rate fixed at its level in the estimated model. The growth effect allows also the growth rate to adjust, and is computed as the residual between the total change and the sum of the direct and incumbent effects. *Source:* Model.

### 6.3 Aging shifts employment toward high-productive firms

Aging impacts the distribution of employment over firm productivity through three channels. First, older individuals tend to be employed by more productive firms, since they have had more time to relocate in the labor market. Consequently, employment shifts toward more productive firm through a composition effect, holding equilibrium objects fixed. The dashed-green line in panel A of Figure 10 illustrates the impact of this composition effect on the average size of firms conditional on productivity.

Second, holding fixed growth, firms create fewer jobs conditional on productivity in the older economy, since they anticipate a harder time filling them (the dash-dotted pink line in panel B). This force is particularly strong among low-productive firms, because they depend heavily on poorly matched, typically young workers to fill their jobs, and there are fewer of them in the older economy.[42] The smaller share of job creation by low productive firms would, *ceteris paribus*, generate a shift of employment toward high-productive firms. At the same time, however, low-productive firms on net lose workers through poaching, and there is less poaching in the older economy. *Ceteris paribus*, the decline in poaching shifts employment toward low-productive firms. On net, these two forces roughly offset, leaving the distribution of employment largely unaffected (the dash-dotted pink line in panel A). Because individuals are discouraged from entering entrepreneurship by the harder hiring environment, however, wage employment rises and self-employment falls, generating a rise in firm size across the board.

Third, the lower rate of obsolescence in the older economy implies that high-productive firms remain so for longer before they are displaced, and in the process grow larger. Moreover, low-productive firms disproportionately hurt from the harder recruiting environment in the slower growing, better matched economy, such that they cut vacancy creation disproportionately (the solid blue line in panel B). For both reasons, employment shifts further up the job ladder (the solid blue line in panel A).

### 6.4 Level versus growth effects

I end with an analysis of how aging impacts growth and welfare. To that end, I solve a perfect foresight, MIT-style transition experiment. Starting from a BGP in 1930, I assume that individuals suddenly realize that labor supply growth, $\lambda(t)$, will evolve so as to match the evolution of the share of older labor force participants over the past 90 years and the coming 40 years (based on official projections). After that, it will return to its original level, consistent with long-term forecasts. All other parameters are held fixed. Appendix E.3 contrasts the time path for the share of older in the model and data.

[42] This finding is reminiscent of the argument in Moscarini and Postel-Vinay (2018) that low-productive, small firms are better able to compete for workers during economic troughs.

FIGURE 10. IMPACT OF AGING ON FIRMS ACROSS BGPS

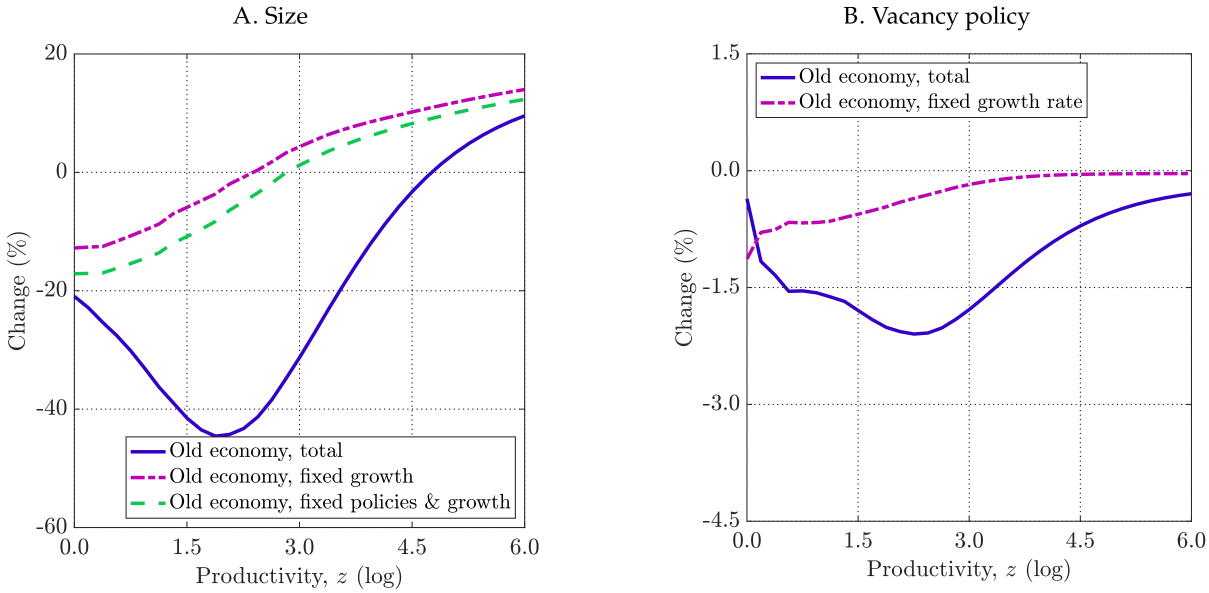



Figure 9 shows the effect of an increase in the share of the labor force aged 16–64 that is aged 45–64 from 34.2 to 39.7 percent across BGPs. It requires a change in the growth rate of labor supply $\lambda$ from 0.0013 to 0.0003 percent (at a monthly frequency). All other parameters are held fixed at their estimated values. Panel A plots the change in size of firms conditional on productivity. The fixed growth rate line shows the effect of letting policies adjust to the old economy, but counter-factually holding the growth rate fixed at that in the young economy. Panel B plots the size of firms conditional on productivity. *Source:* Model.

Over the transition, two forces impact growth. First, the rate at which new technologies are introduced fluctuates—the growth rate $m(t)$ adjusts—which I refer to as the *growth effect*. Second, how well individuals are matched to existing technologies varies, i.e. the employment distribution relative to the exit threshold $\hat{\underline{z}}(t)$ changes. I refer to the latter as the *level effect*. The growth effect gradually declines since the late 1970s (panel A). In contrast, the level effect generates depressed growth in the 1970s and 1980s, as a large number of poorly matched young individuals enter the labor force. As these individuals find a good match, the level effect contributes positively toward aggregate growth through the 2020s. After that, the retirement of the now well-matched baby boomers becomes a drag on growth.

Panel B plots the impact of aging on the annual, demeaned growth rate of output per worker as well as the growth rate of real GDP per worker in the data (smoothed using a 11 year moving average). Note that the former is scaled by a factor of five. The level and growth effects combine to generate a hump-shaped behavior of aggregate growth, which rises up to the mid-1990s and then falls. The correlation between the raw data and the estimated impact of aging is high. That being said, the growth rate rose by over a full percentage point from the early 1970s to 1995, whereas aging only generated an increase in growth of roughly two-tenths of a percentage point according to the model. Evidently, other important forces were at work too over this period. The drag on growth from aging is expected to last another 30 years, despite the fact that the workforce is not expected to age more over this period.

Finally, panel C plots the life-time discounted value of a labor market entrant, expressed as deviations from what it would have been in a counter-factual scenario without demographic changes. Relative to this counter-factual, a labor market entrant is two percent worse off in 2010 versus in 1960 (panel C).

FIGURE 11. EFFECT OF AGING OVER THE TRANSITION PATH

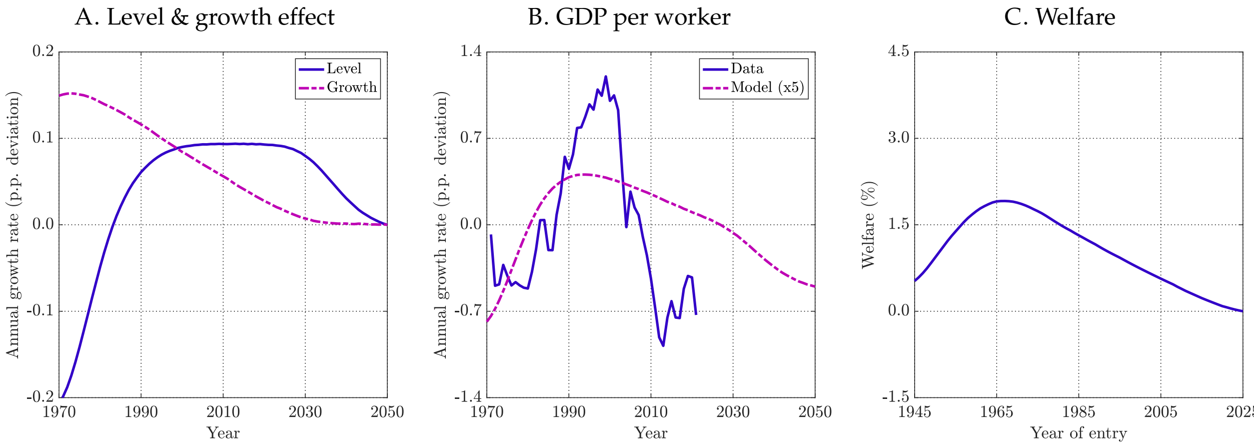



Figure 11 shows the effect of letting the growth rate of labor supply, $\lambda(t)$, vary to match the evolution of the share of the labor force that is aged 45–64 over the 1930–2060 period, using official projections for the future. All other parameters are held fixed. The growth effect in panel A is the annual rate of obsolescence, normalized to its steady-state value. The level effect is the annual growth rate of average gross output per worker relative to the least productive firm. The model and data in panel B are smoothed using an 11 year moving average. The data are annual real growth in output per worker. *Note that the model counterpart is scaled by a factor of 5.* Welfare at time $t$ in panel C is the life-time discounted value of a labor market entrant at time $t$ under demographic change, expressed relative to a counter-factual world without demographic change. *Source:* SCB, model.

## 7 Validating key predictions of the theory

I end by confirming key predictions of the theory in the cross-sectional data. In the interest of space, Table 7 focuses on the IV specification. Aging reduces firm exit (column 1) and shifts the population of firms toward old firms (column 2), consistent with the hypothesis that it reduces the rate of obsolescence.

Aging has no statistically significant effect on employment-unweighted value added per worker (column 3).[43] It raises employment-unweighted pay (column 4), consistent with the view that firms need to pay workers more in an older, better matched labor market. In contrast to the null effect on unweighted productivity, aging raises employment-weighted value added per worker (column 5) as well as firm pay (column 6). This pattern is consistent with aging leading to a shift of employment toward more productive, higher paying firms, although it could also be due to an increase in productivity among the largest firms.[44] More directly, column 7 shows that aging shifts employment toward more productive firms.

[43]In unreported results, I also find that aging has a statistically insignificant positive effect on the productivity of entrants.

[44]Due to the shift toward more productive firms, which tend to have a lower labor share, aging does not necessarily raise the aggregate labor share, despite the fact that it raises pay conditional on productivity.

Aging reduces poaching (column 8) and raises the productivity of poached hires' previous employer (column 9), consistent with the hypothesis that poaching is harder in the older labor market.

Aging raises pay in wage employment (column 10), for two reasons. First, it induces firms to pay better (column 4). Second, it relocates employment toward higher-paying firms (column 6). On the other hand, aging reduces income in self-employment (column 11), although the point estimate is not statistically significant at conventional levels (p-value 0.261). These patterns are consistent with the hypothesis that aging raises the opportunity cost of entry by improving potential entrepreneurs' match in the labor market. Moreover, the return to entry is lower, since entrepreneurs have to pay labor more.

TABLE 7. THE IMPACT OF AGING ON LABOR MARKET DYNAMICS

| | (1) Exit | (2) 11+ | (3) $\frac{y}{n}$ (UW) | (4) $\frac{w}{n}$ (UW) | (5) $\frac{y}{n}$ (W) | (6) $\frac{w}{n}$ (W) | (7) Empl. 10% | (8) Hire (e) | (9) Hire $\frac{y}{n}$ | (10) $y$, wage | (11) $y$, self |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Share 20–44 | 1.915** | –1.152*** | –0.629 | –0.569* | –1.792* | –1.203** | –3.215* | 2.846*** | –0.762** | –1.412*** | 1.216 |
| | (0.744) | (0.370) | (0.666) | (0.296) | (0.937) | (0.510) | (1.751) | (0.910) | (0.323) | (0.446) | (1.062) |
| P-value | 0.015 | 0.005 | 0.356 | 0.068 | 0.069 | 0.028 | 0.080 | 0.004 | 0.028 | 0.003 | 0.261 |
| Obs. | 2,244 | 1,564 | 1,496 | 1,496 | 1,496 | 1,496 | 1,496 | 2,244 | 1,496 | 2,244 | 2,244 |
| F-stat | 27.1 | 16.8 | 14.8 | 14.8 | 14.8 | 14.8 | 14.8 | 27.1 | 14.8 | 27.1 | 27.1 |

Table 7 presents IV estimates based on regression (1) using annual data from 68 LA between years 1986–2018. The independent variable is the log share of all individuals aged 20–64 that are aged 20–44 in the LA in that year. The outcome variables are for private sector firms and individuals aged 20–64. Income of the wage/self employed and the value added per worker of the previous employer are aggregated in logs at the LA-year level; other outcomes are aggregated in levels at the LA-year level and subsequently logged. The instrument is the sum of births 20–44 years earlier in the LA, and subsequently logged. Standard errors are two-way clustered at the LA and year levels. Columns 1 shows the firm exit rate, defined as the fraction of firms with positive employment in year $t$ which have zero employment in year $t+1$. Column 2 shows the share of firms that are 11 years or older. Column 3 shows employment-unweighted value added per worker. Column 4 shows employment-unweighted firm level average wages. Column 5 shows employment-weighted value added per worker. Column 6 shows employment-weighted firm level average wages. Column 7 shows the share of employment of the top decile of firms ranked by employment-unweighted value added per worker. Column 8 shows the fraction of employment that was hired directly from another employer in the year. Column 9 shows the value added per worker of a poached hire's previous employer relative to the value added per worker of the current employer. Column 10 shows annual income of wage employed individuals. Column 11 shows annual income of self- employed individuals. *Source:* FEK, JOBB, LISA, SCB.

## 8 Conclusion

When a local labor market ages, individuals in that labor market become less likely to start a firm and switch employer conditional on their own age. To account for these new empirical patterns and assess their aggregate implications, I develop an equilibrium model of growth with frictional labor markets. Older individuals are better matched to existing technologies, because they have had more time to relocate across them. By shifting workforce composition toward such better matched subpopulations, aging increases the level of output. The better match with existing technologies, however, also serves to raise the economy's opportunity cost of switching to new technologies. As a result, the rate at which new technologies are introduced falls, leading to a decline in the growth rate of output.

A quantitative version of the model estimated on Swedish matched employer-employee-entrepreneur data finds that the growth rate of output per worker rose from the 1970s to the mid-1990s, even though the rate at which new technologies are introduced declined monotonically. The reason is that the baby boomers relocated in the labor market to become better at using existing technologies. Since then, growth declined, and it is expected to remain suppressed for another 30 years, even though the labor force is not expected to age much more. A labor market entrant in 2010 is two percent worse off than an entrant in 1960, relative to a counter-factual scenario without demographic change.

The rich, tractable framework presented in this paper could be extended in several directions to study further how worker dynamics and growth interact. For instance, it would be interesting to study optimal policy when growth causes some workers to become displaced. To properly answer this question may require adding risk aversion and imperfect financial markets. It would also be interesting to endogenize innovation of incumbent firms in order to study how aging, or more broadly the quality of the workforce, affects incumbent firms' incentives to innovate.

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# A Swedish labor market trends

This section contains additional Swedish secular labor market trends (Appendix A.2); a comparison with secular trends in U.S. firm dynamics (Appendix A.3); further details on the data sources and variable definitions (Appendix A.1); a discussion of the *FAD* data base (Appendix A.4); trends in labor force participation rates (Appendix A.5); important time series breaks in the data sources and how I adjust the raw data for them (Appendix A.6); labor market trends by firm age and size (Appendix A.7); and labor market trends by sector (Appendix A.8).

## A.1 Data sources and variable definitions

I discuss in this section the data sources, sample selection and variable construction in further detail.

**JOBB.** Data on wage employees come from information provided by employers via the *Kontrolluppgift* (KU) tax form *KU10*, an English version of which is shown in Figure 12. Prior to 2019, employers provided this information for all employment spells that were active at any point during the previous year. This had to be reported to *Skatteverket* by January 31 of the following year. As I discuss further in Appendix A.6, since 2019 employers have instead been required to submit payroll data on a monthly basis for all ongoing employment spells as part of the *Arbetsgivardeklaration på individnivå* (AGI). Employers report gross salary (box 011), inclusive of taxes leveled on employees, bonus payments, exercised stock options, etc, but exclusive of social security taxes leveled on employers. Such taxes are non-trivial in Sweden, adding roughly 30 percent to reported gross pay. Given that this project does not rely heavily on the wage data and the fact that—with only very limited exceptions—social security taxes are proportional, I do not adjust the reported gross pay to reflect social security payments by the employer. Employers must also report in kind payments (in box 012).

The KU data also include a work site number (box 060), information on whether the payee is a partner in a close company (box 061), and the start (box 008) and end (box 009) month of the employment relationship. The work site number allows the linking of employees to establishments within firms. The information in box 061 allows the identification of incorporated self-employed in so called *fåmansföretag* starting in 1993. A *fåmansföretag* is an incorporated, not publicly listed company in which at most four owner-groups collectively control more than 50 percent of voting shares. An owner-group consists of one or more individuals who are linked through family ties, including spouses, parents, grandparents, children (including spouses' children, foster children and children's spouses), siblings and siblings' spouses and children, and non-married couples who live together who were either previously married or have or have had children together.[45] Individuals who are not connected through family ties but are actively working together in a company are also counted as one owner-group (for instance law firms with multiple partners).[46] For this reason, the number of identified founders can be significantly larger than four. Indeed, the rules are designed exactly so as to classify the great majority of small Swedish companies as *fåmansföretag* in order to limit owners' ability to reclassify labor income as capital income.[47] I stress that because I link firms to founders in the first year of a firm's operation, I am able to identify the founders also of firms that subsequently acquire more owners, for instance through a public listing.

The data on unincorporated self-employed in *JOBB* come from a different underlying data source, the *Inkomst- och taxeringsregistret* (IoT) prior to 2004 and the *Standardiserade räkenskapsutdrag* (SKU) starting in 2004. These data contain information on annual earnings of the non-incorporated self-employed, but lack information on start and end dates of employment spells. Lacking such data, I assume that all non-incorporated self-employment spells last the entire year.

Based on the *JOBB* data, I construct monthly individual and firm level data sets for 1985–2019. For each month, I determine an individual's main employer in the following way. I start by classifying an individual as self-employed if they have an ongoing self-employment spell—either unincorporated or incorporated—in that month. In case they have multiple active self-employment spells, I determine their main firm as that with the highest earnings in the year. If an individual has no active self-employment spells in the month, but an active wage employment spell, I classify them as wage employed. In case they have multiple active wage employment spells, their main firm is that with the highest earnings in the year. If an individual neither has any active self or wage employment spell, they are coded as non-employed.

[45]The rules governing what is classified as a *fåmansföretag* are extensive and complex, designed with the goal of preventing tax evasion (known as the *3:12 rules* in Sweden after the paragraph in the law that outlines them). The law also prevents owners from artificially pooling several independent companies under one parent company, with the goal of avoiding a classification as a *fåmansföretag*. In this case, Swedish tax authorities have broad latitude in classifying each subsidiary as a *fåmansföretag*.

[46]For instance, in court ruling RÅ1993 ref. 99, the *Högsta förvaltningsdomstolen* (the Swedish high court) ruled that a company with 150 owners who were also full-time employed in the company was to be classified as a *fåmansföretag*.

[47]Since a major tax reform in 1990, Sweden has implemented a dual tax system, whereby labor income is taxed at an almost 70 percent effective marginal rate at high incomes, but dividend income is subject to a proportional effective tax rate of about 45 percent. Consequently, many owners have an incentive to reclassify labor income as capital income in order to lower their tax rate. Swedish tax authorities are notoriously zealous in clamping down on such attempts to avoid taxes.

FIGURE 12. TAX FORM KU10

Swedish Tax Agency

**Income statement from employers etc.**

**KU10**

Income year **2015**

Information is available in Swedish in the brochure SKV 304 ("Kontrolluppgifter - lön, förmåner, m.m."). Amounts should be stated as whole numbers.

Samråd enligt SFS 1982:668 har skett med Näringslivets Regelnämnd.

| | |
|---|---|
| 570 | Specification number |

This income statement shall: 210 correct a previously submitted income statement / 205 remove a previously submitted income statement

**Payer/Employer**

| | |
|---|---|
| 201 | Personal/corporate identity number |
| | Name |

**Tax**

| | |
|---|---|
| 001 | Tax deducted |

**Salary and other cash payments**

| | |
|---|---|
| 011 | Gross salary etc. |
| 025 | Remunerations for which the employee pays individual social security contributions |
| 031 | Remunerations for which social security contributions are not paid |
| 093 | Social security agreement exists |

**Benefits in kind etc.**

| | |
|---|---|
| 012 | Taxable benefits exclusive of employer-provided car and free fuel in connection with employer-provided car |
| 041 | Free housing 1- or 2-family house |
| 042 | Free meals |
| 043 | Free housing, other than code 041 |
| 044 | Interest |
| 045 | Free parking |
| 047 | Other benefits |
| 048 | Benefit has been adjusted |
| 049 | Benefit as pension |
| 065 | Specification of other benefits at code 047 |
| 013 | Taxable benefit of employer-provided car exclusive of fuel |
| 018 | Free fuel in connection with employer-provided car |
| 014 | SKV-code of employer-provided car |
| 015 | Number of months with employer-provided car |
| 016 | Number of kilometers with car allowance for employer-provided car |
| 017 | Employee's payment for employer-provided car |

**Payee/Employee**

| | |
|---|---|
| 215 | Personal/corporate identity number |
| | Name |
| | Street address |
| | Postal number |
| | Postal address |
| 061 | Partner etc. in a close company |
| 008 / 009 | Employment time (e.g. 04-12): From / Up to |
| 060 | Work site number allocated by the Central Bureau of Statistics (SCB) |

**Compensation for expenses**

| | |
|---|---|
| 050 | According to fixed standard rates: Car allowance |
| 051 | According to fixed standard rates: Per diem, Sweden |
| 052 | According to fixed standard rates: Per diem, other countries |
| 055 | Equivalent to actual costs etc. for: Business travel expenses |
| 056 | Equivalent to actual costs etc. for: Accomodation, business travels |
| 053 | Business trip lasting more than three months: Within Sweden |
| 054 | Business trip lasting more than three months: Other countries |
| 020 | Compensation for expenses not ticked in boxes by codes 050-056 |

**Occupational pension, other remunerations, certain deductions**

| | |
|---|---|
| 030 | Occupational pension |
| 032 | Remunerations for which social security contributions are not paid and which are not entitled to special job deduction |
| 037 | Certain deductions |
| 070 | Specification of amount at code 037 |
| 035 | Not taxable remunerations to foreign key persons according to a decision from the Swedish Forskarskattenämnden |

**Capital**

| | |
|---|---|
| 039 | Rent |

**Tax reduction for "rut-/rot-work"**

| | |
|---|---|
| 021 | Basis for tax reduction for "rut-work" |
| 022 | Basis for tax reduction for "rot-work" |

SKV-2300-1-25-en-KU10-2015

SKV 2300 25 en web 01

*Note:* Tax form that employers are required to submit on behalf of all employees during the year. *Source:* Skatteverket.

**FEK.** The FEK form the basis of Swedish national accounts (it was referred to as the *FÃ¶retagsstatistik* from 1997–2002). Given its importance, these data are subject to extensive checks by SCB in order to limit measurement error. Among others, SCB cross-checks the collected data with previous years, other data sources and follow-ups with firms in order to achieve as precise a measure as possible of, ultimately, aggregate GDP.

The collection of firm financial data has a long history in Sweden. Since 1950, the *Finansstatistik för företag* has surveyed medium and large manufacturing and trading firms with questions on revenues and costs on an annual basis. Coverage was gradually expanded to other sectors and firm size classes, and contains since 1965 also information on balance sheet items. The *Industristatistik* started in 1913 with questions on quantities produced, prices, costs, employment, investments, energy consumption, etc. It covers medium and large manufacturing firms. The *Industristatistik* in turn builds on earlier firm surveys that stretch as far back as 1830. Given that this survey is still ongoing—now called the *Industrins varuproduktion*—it is one of the longest continuous data sources on firm outcomes in the world. Unfortunately, the historical data prior to 1997 are in long-term storage at *Riksarkivet* (the Swedish National Archives), and not easily accessible.

In 1997, SCB switched to collecting income and balance sheet data from *SKU*, which covers all firms with the exception of holding companies and similar. For the largest roughly 450 companies, however, data are collected via a survey, the so called *fullständiga blanketten* (the "complete survey"). Although the tax registry data from *SKU* are considered of high quality, the purpose of surveying the largest firms is to further reduce measurement error (these firms account for roughly a third of Swedish GDP). The primary object of *FEK* is firms, but some information is also reported at the establishment level.

Even though the tax data are already fairly detailed, SCB additionally surveys a sample of firms every year with more detailed questionnaires. Two such surveys obtain more detailed information on types of investment than what is available via *SKU*: *Investeringsenkäten* and *SpecI*. *Investeringsenkäten* samples about 2,000 firms (in 2020), with complete coverage of the largest firms and (stratified) random sampling of smaller firms (the smallest firms are completely excluded from the survey). It is conducted at the level of establishments (covering roughly 7,500 establishments). *SpecI* samples roughly 5,000 firms (in 2020). A third survey obtains more detailed information on revenues and costs, *SpecRR*, covering roughly 16,500 companies (in 2020).

## A.2 Additional time series trends

Figure 13 presents additional facts on Swedish labor market trends over the past 35 years. Firm exit declined over this period (panel A), while average firm size trended up modestly (panel B).[48] Because average firm size can be decomposed as $\frac{N_w}{N_f} = (1-u)\frac{N_{wf}}{N_f}$, where $N_w$ ($N_f$) is the number of workers (firms), $u$ the non-employment rate, and $N_{wf}$ the size of the workforce, the trend in average firm size can be decomposed into changes in the non-employment rate and the ratio of the size of the workforce to the number of firms. Panel B finds that the ratio of the size of the workforce to the number of firms remained roughly constant. Hence, the increase in average firm size is accounted for by a fall in non-employment.

At the same time as Sweden saw a constant ratio of firms to workforce participants, firms grew older and employment shifted toward older firms (panel C). Because older firms are less dynamic, this shift partly accounts for the aggregate declines in firm dynamics in Sweden, similar to the U.S. (Hopenhayn et al., 2021; Karahan et al., 2022).

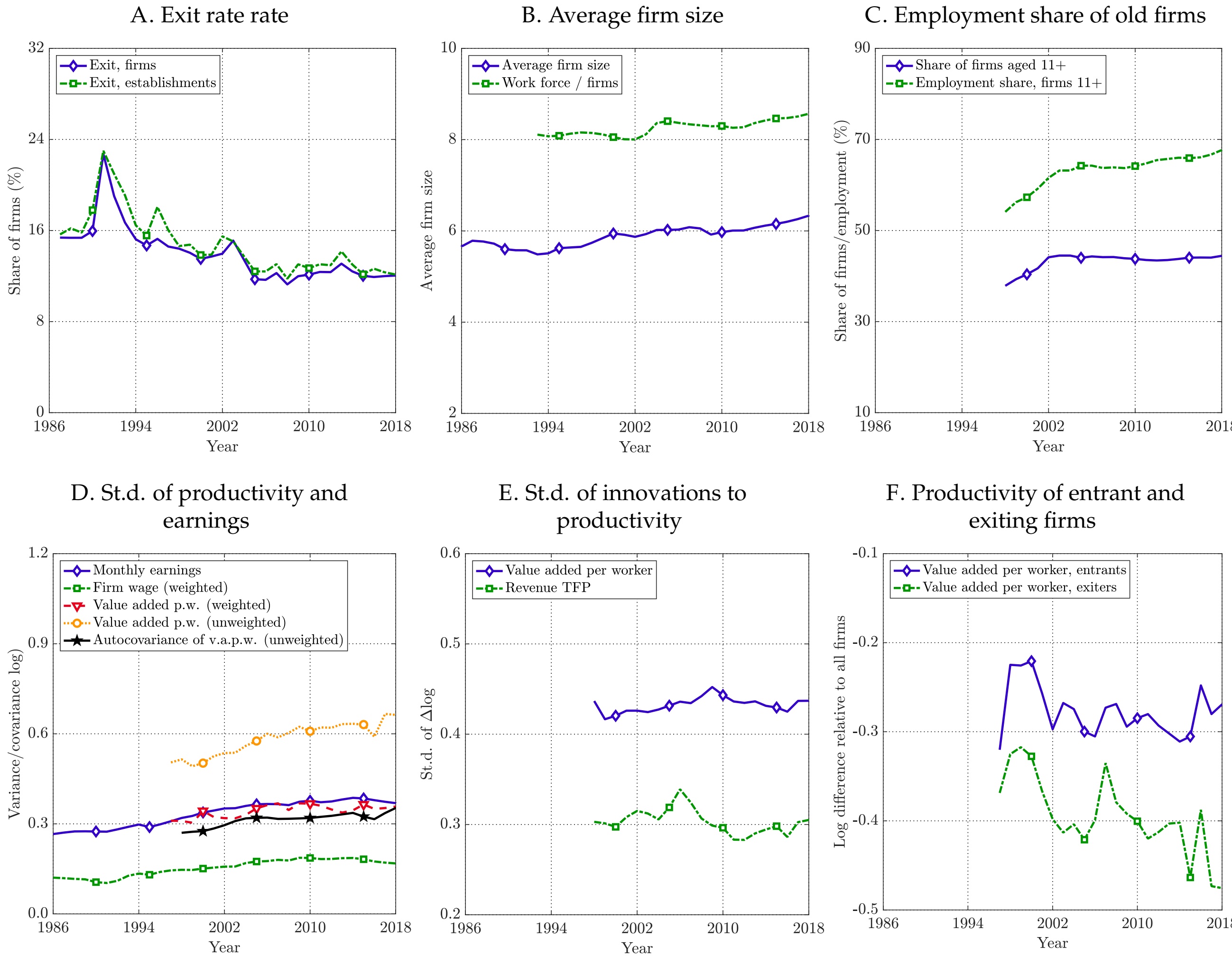
Earnings dispersion widened in Sweden over this period (panel D), mirroring well-known patterns in the U.S. (Barth et al., 2016). In particular, the standard deviation of residual log monthly earnings rose from 0.53 to 0.64. Productivity dispersion also increased, although consistent productivity data are not available prior to 1997. The rise in productivity dispersion is evident in the standard deviation of log value added per worker, as well as firm revenue TFP and the first autocovariance of log value added per worker. The latter suggests that a large share of the increasing dispersion is permanent in nature.

Firm productivity is subject to large idiosyncratic changes, but the volatility of such innovations to firm productivity has not changed much over time (panel E). I compute these innovations as the residual of a regression of productivity on its own lag.

Panel F plots labor productivity of entrant and existing firms relative to all firms. Three observations stand out. First, new firms do not appear to enter at the "technological frontier," in the sense that they are less productive than incumbent firms. This observation motives me to model entry and innovation at the bottom/middle of the productivity distribution, following Perla and Tonetti (2014). Second, there is little evidence of entrants falling increasingly behind incumbent firms over this period. Third, although entrants are not particularly productive, exiting firms are even less productive. Consequently, in an accounting sense replacing an exiting firm with an entrant contributes to growth.

[48] Average firm size is significantly lower in Sweden than in the U.S., likely due to the fact that the Swedish data cover also non-employer firms. These firms are almost exclusively one-person firms. Nevertheless, because job and worker reallocation are employment-weighted outcomes, the inclusion of such firms is likely of less relevance for these outcomes. Indeed, I show below that also large firms experienced declines in job reallocation over time in Sweden.

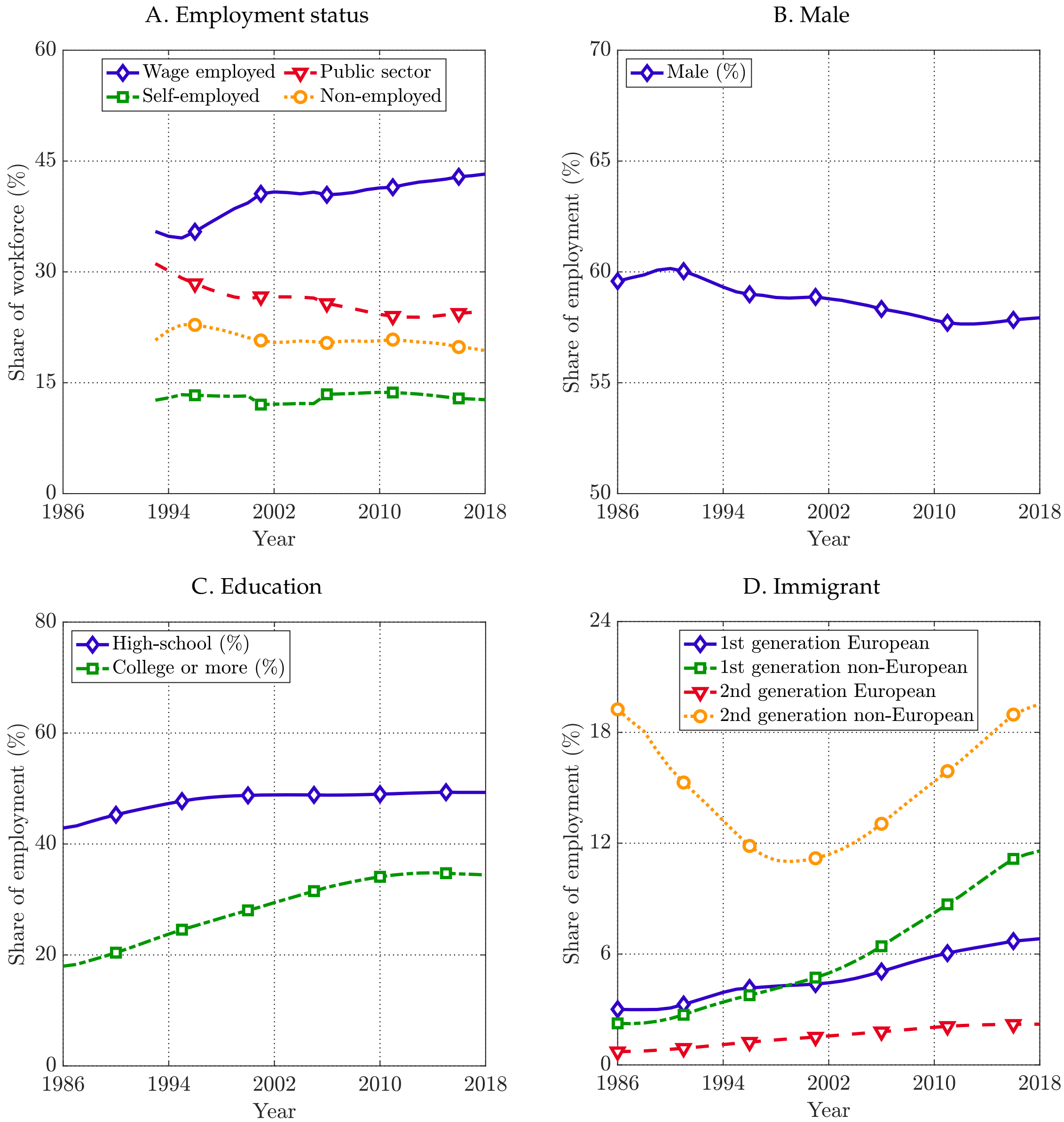
FIGURE 13. SWEDISH LABOR MARKET TRENDS




*Note:* Panel A. Annual firm and establishment exit rate. Panel B. Workforce/firms is the total non-private sector workforce aged 20–64 divided by the total number of private sector firms. The data are spliced in 2004; see Appendix A.6 for details. Panel C. Share of firms that are 11 years and older as well as the are of employment that works for firms that are 11 years and older. The data are spliced in 2004; see Appendix A.6 for details. Panel D. Firm pay is the log of average monthly earnings at the firm. Autocovariance is the first annual autocovariance. Panel F. Average log productivity of entrants/exiting firms in the year minus average log productivity of all firms (all employment-*un*weighted). All panels. Private sector firms and workers aged 20–64. *Source:* FEK, JOBB, LISA.

Figure 14 summarizes other changes in the demographic composition of the Swedish workforce over this period apart from in the age dimension. The employment share of the public sector has declined, as has the non-employment rate (panel A). The wage employment rate has risen, while the self-employment rate has remained roughly constant. Note that the ability to identify self-employed individuals is incomplete prior to 1993, since those running incorporated firms are classified as wage employees prior to 1993. The share of men in the non-public workforce has trended down modestly over this period (panel B), while the share with a college degree or more has risen (panel C). The share of immigrants has gradually risen, to stand at roughly 20 percent of the non-public workforce today (panel D).

FIGURE 14. COMPOSITION OF NON-PUBLIC WORKFORCE

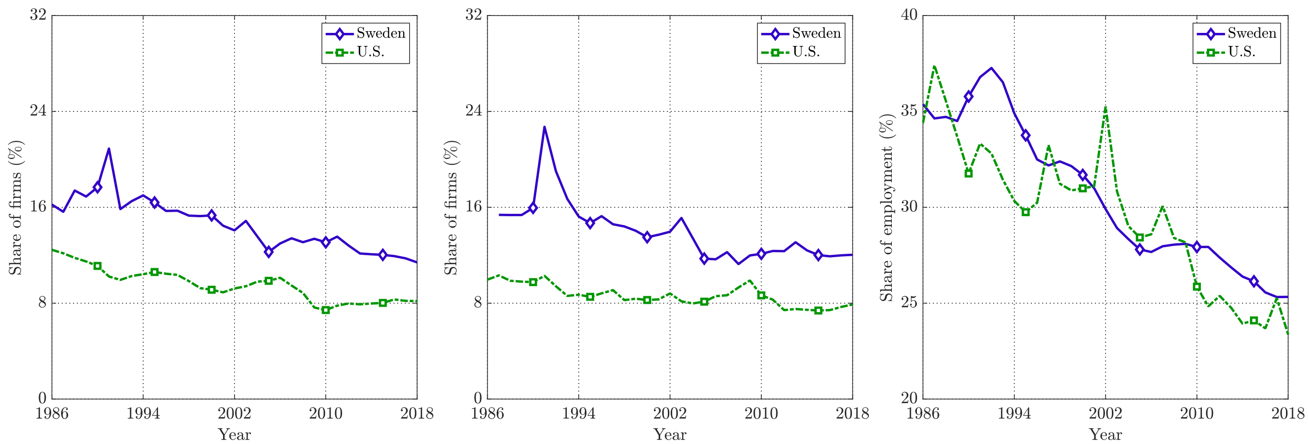



*Note:* Panel A. Share of all individuals aged 20–64 who are private sector wage employees, private sector self-employed (including both unincorporated and incorporated owner-operator of *fåmansaktiebolag*), public sector employees and non-employed. Panel B. Share of non-public sector individuals aged 20–64 who are men. Panel C. Share of non-public sector individuals aged 20–64 who have a college degree or more or a high-school degree. The excluded third education group consists of those with less than a high-school degree. Panel D. Share of non-public sector individuals aged 20–64 who are a first or second generation immigrant by source country. An individual is classified as a second-generation immigrant if they have at least one parent who is born abroad (and a non-European second generation immigrant if at least one parent was born outside Europe). *Source:* JOBB, LISA.

## A.3 Benchmarking against U.S. labor market trends

Figure 15 plots the annual firm entry rate (panel A), firm exit rate (panel B) and job reallocation rate (panel C) in the U.S. The level of entry and exit is higher in Sweden, likely due to the fact that the Swedish data contain also non-employer firms. These firms are typically small and likely have high entry and exit rates. Yet in a relative sense, the fall in entry and exit is very similar in the two countries. Both the level and the decline in job reallocation are of a similar magnitude in the U.S. and Sweden.

FIGURE 15. SWEDISH AND U.S. FIRM DYNAMICS




*Note:* Panel A. Share of all firms that started in the current year. Panel **??**. Share of all firms that exit in the current year. Panel C. Sum of jobs created and destroyed across establishments divided by average employment in the year. *Source:* BDS, JOBB, LISA.

## A.4 The firm and establishment identifiers and FAD data

A general issue with administrative, tax based data sets such as those used in this paper is that firm and establishment identifiers change for reasons such as ownership changes etc. In the Swedish data, this is particularly a concern with the firm identifier, known as the *PeOrgNr*. It is less of a concern with the establishment identifier (*CfarNr*), since it is designed to remain the same when, for instance, ownership changes.

To address such concerns, SCB constructs a data set, the *Företagens och arbetsställenas dynamik* (FAD), which contains firm IDs (*FAD-F-ID*) and establishment IDs (*FAD-A-ID*) that are designed to be unaffected by changes in the underlying firm (*PeOrgNr*) and establishment (*CfarNr*) identifiers in the tax data that are not due to "true" economic events. To this end, SCB exploits the nature of worker flows to link firms over time. In particular, if a firm in the underlying data changes ID across two years, but a majority of its workforce remains the same, *FAD* classifies this as the same firm (with a few exceptions for small employers). Although this adjustment seems reasonable, there are several important issues with *FAD*. First, *FAD* contains only firms and establishments which at least one individual has as their main employer in November of the year. The reason is that it is based on the *Registerbaserad arbetsmarknadsstatistik* (*RAMS*), which is a consolidated version of *JOBB* with information only on the main employer in November of the year (SCB classifies the main employer in November primarily based on highest annual income, with some adjustments such as inflating self-employment income). As a result, *FAD* does not assign a *FAD-F-ID* and *FAD-A-ID* for close to 40 percent of all *PeOrgNr*'s and *CfarNr*'s IDs in *JOBB*. Although these firms are small—typically secondary employment for small independent business owners—it is not clear that one wants to drop these firms, in particular as some of them subsequently grow.

Second, SCB constructs annual vintages of *FAD*, where a given vintage $t$ contains an underlying $PeOrgNr_1$, $CfarNr_1$, $PeOrgNr_2$, $CfarNr_2$, *FAD-F-ID* and *FAD-A-ID*. The $PeOrgNr_1$ ($CfarNr_1$) refers to the *PeOrgNr* (*CfarNr*) in year $t-1$ that maps into the *FAD-F-ID* (*FAD-A-ID*), while the $PeOrgNr_2$ ($CfarNr_2$) refers to the *PeOrgNr* (*CfarNr*) in year $t$ that maps into the unique *FAD-F-ID* (*FAD-A-ID*). Hence in theory, a *PeOrgNr* in *JOBB* in year $t$ can be mapped to a unique *FAD-F-ID* based on either $PeOrgNr_2$ in FAD vintage $t$ or $PeOrgNr_1$ in FAD vintage $t+1$. The issue is that the mapping provided by different vintages of *FAD* is not unique in the same year. In particular, roughly four percent of *PeOrgNr* map to a different *FAD-F-ID* depending on whether $PeOrgNr_1$ in FAD vintage $t+1$ or $PeOrgNr_2$ in FAD vintage $t$ is used. In discussions with the data experts at SCB, we have confirmed that this is an issue, but it remains unclear what causes it (because the underlying code used to generate *FAD* is confidential, I have not been granted the chance to inspect the code that generates *FAD*). The lack of insight into what causes this issue leaves me uncomfortable using *FAD*.

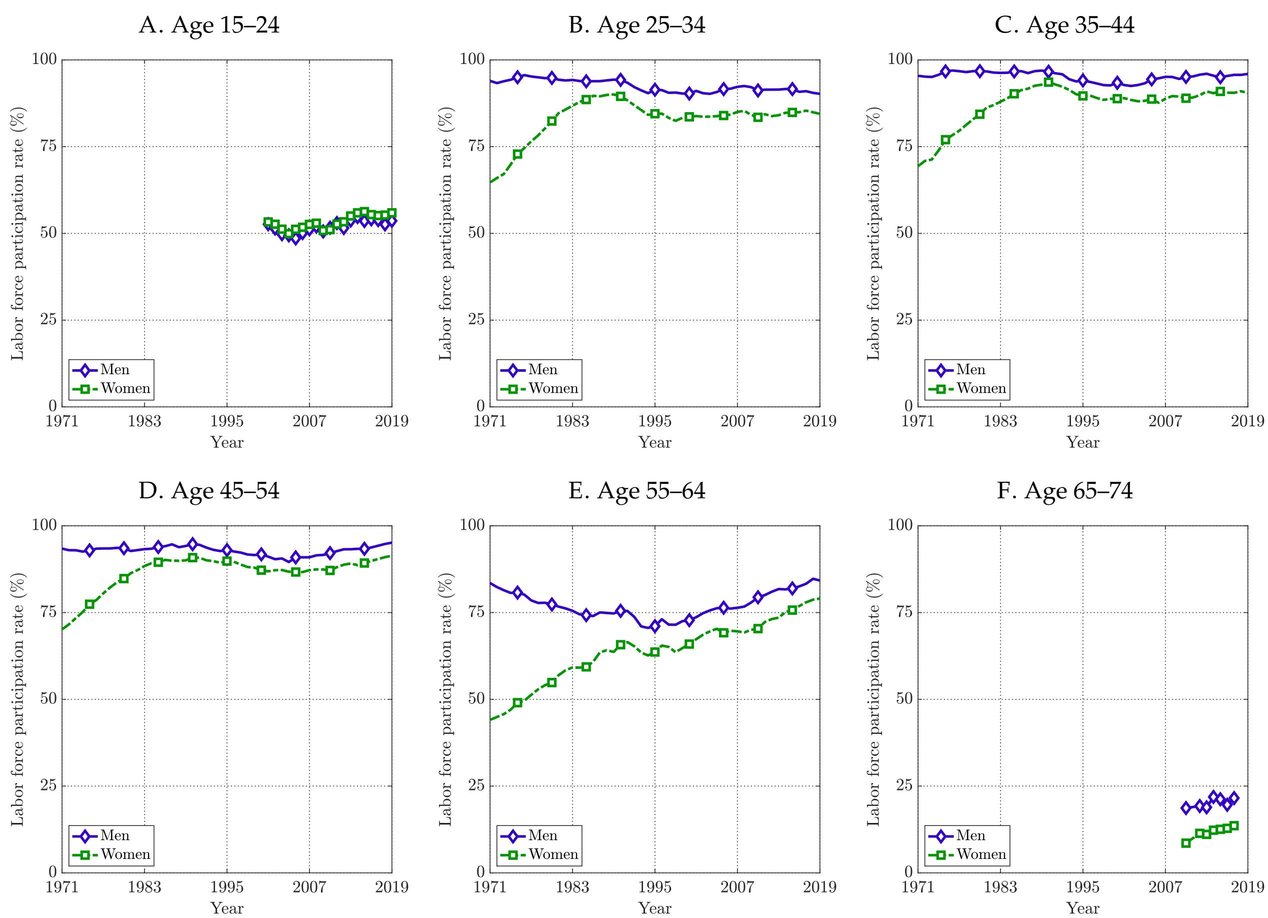
Finally, labor market flows based on the firm and establishment IDs in *FAD* appear to be very high. For instance, the job reallocation rate based on *FAD-A-ID* is more than twice the corresponding rate in the U.S. BDS data. Moreover, the job reallocation rate based on *FAD-A-ID* is more than twice as high as that based on *CfarNr*, even though the latter is supposed to be consistent over time (whereas the firm ID in the tax data, *PeOrgNr*, arguably is not). I view these differences as implausibly large.

For these reasons, I prefer to use the underlying *CfarNr* and *PeOrgNr* in my analysis. That being said, the aggregate annual job reallocation rate based on *FAD-F-ID* fell from 88.5 percent in 1986 to 58.1 percent in 2018, or a 34 percent decline. Although the level of job reallocation is much higher based on the *FAD* identifiers, the relative decline is of a similar magnitude as that using *PeOrgNr* and *CfarNr*.

### A.5 Participation rates by age over time

Figure 16 plots labor force participation rates by age groups over time. Participation rates rise rapidly between age 20 and 25 and fall rapidly after age 65. Between ages 30–55 they are, to a first order, flat in age. Men have not seen any major changes over time in their age-conditional participation rates. The most prominent change was a decline in male participation rates until the mid-1990s, and a subsequent recovery over the past 25 years. Women, on the other hand, experienced significant gains in participation rates throughout the 1970s and early 1980s. After that, female participation rates have largely stabilized at a level somewhat below that of men. The main exception are women aged 55–64, who experienced continuous gains in participation over this period. My reading of these trends is that from the mid-1980s, labor force participation rates have been broadly stable, with the main exception being women aged 55–64.

FIGURE 16. LABOR FORCE PARTICIPATION RATES BY AGE AND TIME




*Note:* Ratio of the sum of employed and unemployed to total population. Data for the youngest age group, 15–24, is not available prior to 2001. Data for the oldest age group, 65–74, is not available prior to 2010 and after 2017. *Source:* SCB.

## A.6 Important breaks in the data

The underlying data sources used in this project have been subject to several adjustments to the way the data are collected and processed, which may carry implications for some of the time series trends documented in this paper. I discuss in more detail below some of the most important breaks. I also caution, however, that slow-moving forces have also been at work over this period. For instance, *Skatteverket* has increasingly allowed employers to submit tax information electronically, which may have impacted the magnitude of measurement error in earnings, start and end dates of employment spells, etc. In fact, one could even imagine that such changes could be correlated with aging across space, to the extent that young individuals are more likely to adopt such new ways of reporting information. As little can be done to assess the implications of such changes, I take to simply note such valid concerns.

**The 2019 switch to AGI.** In 2019, *Skatteverket* started requiring employers to report payroll data on all active employment spells in a month on a monthly basis, as opposed to once a year. Although employers prior to 2019 reported start and end dates of employment spells, this change likely reduced measurement error in the start and end months of spells. Indeed, hiring and separation rates show an increase in the 2018–2010 break. While the increase in the poaching rate is relatively modest of less than 10 percent, the hiring from and separation rates to non-employment show a more pronounced increase of almost 40 percent. This is consistent with some measurement error in start and end dates leading to a general understatement of both poaching flows and flows through non-employment, together with some short non-employment spells being incorrectly classified as poaching flows. Consequently, all flows rise with the change in reporting, but in particular flows through non-employment.

To limit the impact of this break, I end my analysis in 2018. I note that although measurement error in the start and end months may contribute to deflated flows prior to 2019, under the assumption that such measurement error has not changed over time, the time series trends would still be consistent.

**The 2004 switch to SKU.** Starting in 2004, data on the unincorporated self-employed come from the *SKU* instead of the *IoT*. This switch led to the inclusion of also unincorporated self-employed with a negative profit in the year, which were previously excluded since they are not required to pay tax. The change led to a permanent increase in the number of self-employed in 2004 and a temporary spike in the measured entry rate in 2004, as a substantial number of firms that likely were already in existence prior to 2004 started to be recorded in the data. It also led to a discrete drop in average firm size and the share of firms that are old.

Given that these are unincorporated firms with negative profits, they are almost exclusively small and likely of limited macroeconomic importance. Nevertheless, in order to create a consistent time series for high growth entry, the stock of self and wage employed, average firm size, and the share of employment in large and old firms, I splice these aggregate data series in the break.[49] I do so by pooling data for the outcome of interest plus/minus 15 years around the 2004 gap, dropping 2004 for the entry rate due to a large spike in measured entry in this year. I subsequently project the outcome of interest on a constant, a linear time trend and an indicator for whether the year is after 2004. Finally, I adjust the series by adding the estimated coefficient on the post 2004 dummy to the outcome of interest to all years after 2004. Admittedly, this approach is somewhat ad hoc. In any case, because the spatial regressions in Section 3 include a year fixed effect, the data break and the adjustment I do to the series are inconsequential for any of the results in this paper.

**Information on incorporated self-employment.** Data on incorporated self-employment are only available since 1993, when a major tax reform led to the creation of the current Swedish dual income taxation system (these reforms, which were initiated in 1990, are referred to as the "tax reform of a century" in Sweden). As a result, *Skatteverket* started collecting data on owner-operators of *fåmansföretag*. Prior to 1993, the incorporated self-employed are coded as regular wage employees. For this reason, my analysis linking founders to firms starts in 1993. To avoid any mechanical bias in the measured rates of worker relocation over time, I include in my measures of hires and separations also the self-employed. That is, for instance the JJ rate includes also self-employed individuals who switch firm. I note, however, that flows into and out of self-employment are an order of magnitude lower than flows of wage employees across firms. Hence in practice, it makes little difference whether the self-employed are included in the measured worker relocation rates.

Although *Skatteverket* has recorded information on owner-operators of *fåmansföretag* since 1993, this information is mysteriously missing from the *JOBB* data base prior to 2004, even though the information is available in other data products at SCB such as *LISA*. It is not clear what exactly has caused this information to be missing from JOBB, but together with SCB we have designed a workaround that uses information on the top three employers in a year from *LISA* to import an indicator for whether what looks like a wage employment spell in *JOBB* 1993–2003 is in fact an incorporated self-employment spell. Unfortunately, *LISA* only contains information on up to three employers, so this imputation fails to reclassify a wage employee as an incorporated self-employed in 1993–2003 if this employment spell was not one of the top three sources of income for an individual in that year. Because few individuals have

[49] In theory, the overall entry rate would also be affected by the change in methodology. In practice, however, I find that the impact of the change in data collection is too minor to make any notable difference to the aggregate entry rate. For this reason, I do not splice this.

more than three employment spells in a year, I do not think that this is a major source of error.

## A.7 Firm dynamics by firm age and size

Figure 17 plots key firm reallocation rates by firm age groups. In contrast to the modest increase in average firm size in the aggregate, firm size conditional on firm age has not changed much over this period (panel A). The jumps in some of these series are driven by changes in the thresholds for when firms are included in the underlying tax level data. I have tried as well as possible to splice such changes, but the influence of these changes remain a concern. That being said, because these changes exclusively affected low value added, typically small firms, I believe that they are of little concern for employment-weighted measured of reallocation. The exit rate has declined also conditional on firm age, particularly among younger firms (panel B). In contrast, the oldest firms in fact saw a modest increase in their exit rate. The job reallocation rate displays a pronounced secular decline also conditional on firm age (panel C), even among the oldest firms which employ most workers.

FIGURE 17. DYNAMICS BY FIRM AGE

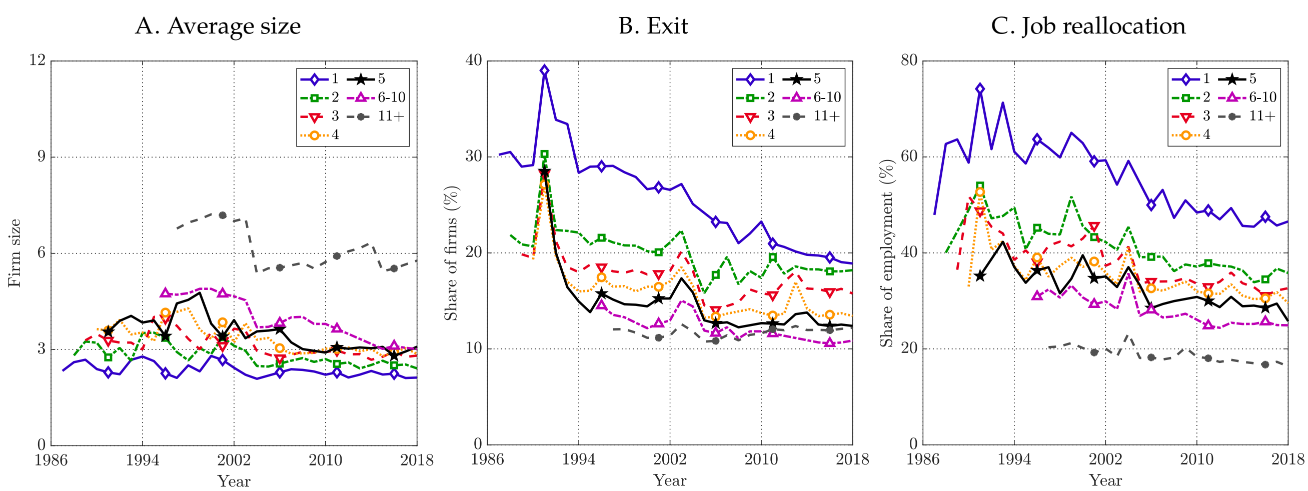



Figure 17 plots average firm size (panel A), the fraction of firms with positive employment in year $t$ that have zero employment in year $t+1$ (panel B), and the job reallocation rate (C), defined as the sum of jobs created and destroyed in a year divided by average employment in the year. *Source:* JOBB, LISA.

Figure 18 shows that the Swedish trends in firm dynamics by firm age are similar to those in the U.S. over the same period. In particular, average firm size conditional on firm age shows little change over the past 35 years in the U.S. (panel A), while firm exit fell among young firms but did not change by much for older firms (Karahan et al., 2022) (panel B). In contrast, also the U.S. displays a pronounced secular decline in job reallocation conditional on firm age over the past 35 years (panel C).

Note also the large differences in average firm size between the U.S. and Sweden. As I discuss in

Section 2, this difference is likely largely due to the fact that the U.S. BDS includes only employer firms, whereas the Swedish data include all firms. The U.S. Census Bureau reports that there were roughly 26.5 million firms in the U.S. in 2018, but only 6.1 million employer firms. Since non-employer firms are mostly small, average firm size of all firms is presumably much lower than that of employer firms only.

FIGURE 18. DYNAMICS BY FIRM AGE IN THE U.S.

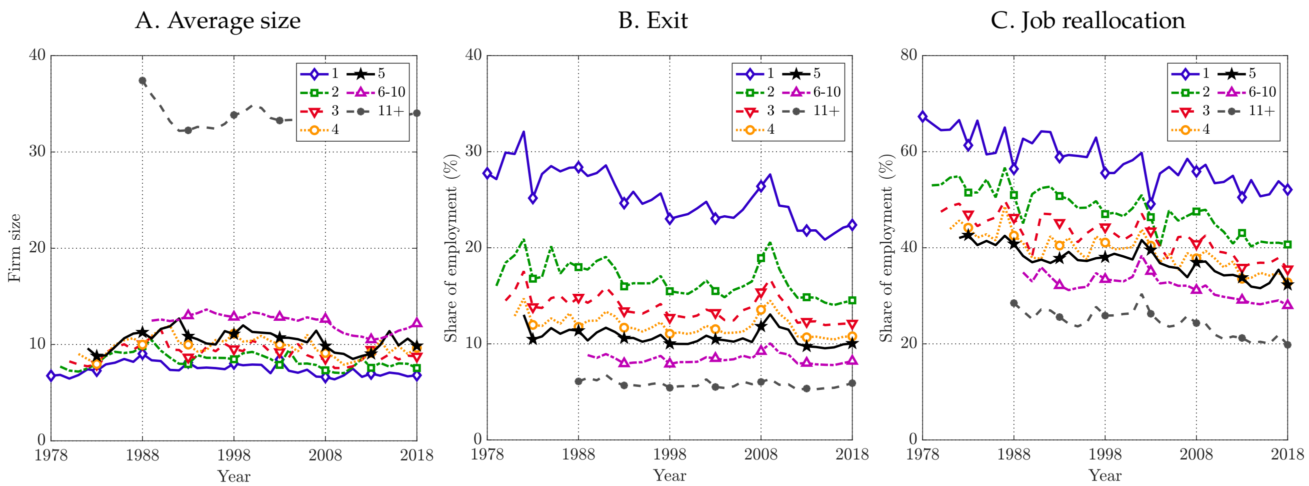



Figure 18 plots average firm size (panel A), the employment-unweighted exit rate of firms (panel B), and the job reallocation rate (C), defined as the sum of jobs created and destroyed in a year divided by average employment in the year. *Source:* BDS.

Figure 19 shows that firm exit (panel A) and job reallocation (panel B) fell within firm size groups over the past 35 years in Sweden.

FIGURE 19. DYNAMICS BY FIRM SIZE GROUPS

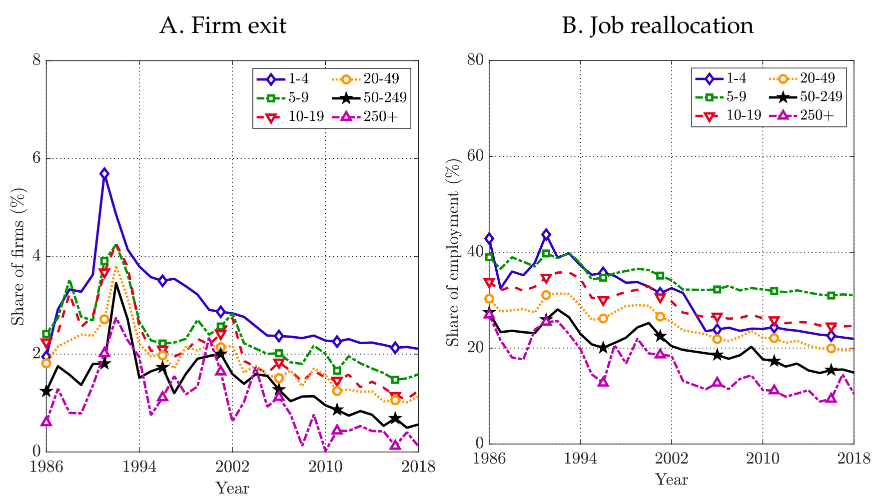



Figure 19 plots the share of firms with positive employment in year $t$ that have zero employment in year $t+1$ (panel A) and the job reallocation rate by firm size groups (panel B), defined as the sum of jobs created and destroyed in a year divided by average employment in the year. *Source:* JOBB, LISA.

### A.8 Trends within one-digit sectors

Figure 20 plots the share of firms (panel A), share of employment (panel B) and job reallocation rate (panel C) by one digit sector. As in many advanced countries over this period, employment has shifted out of manufacturing and into services in Sweden. Because the latter sectors tend to have higher reallocation rates, this shift tends to, *ceteris paribus*, lead to higher reallocation rates. Consequently, the within-sector decline in job reallocation is larger than the aggregate decline. While there are some differences in the timing and magnitude of the fall in job reallocation across sectors, there is also substantial similarity in that all major sectors experienced declining job reallocation over this period.

FIGURE 20. DYNAMICS BY ONE DIGIT SECTOR

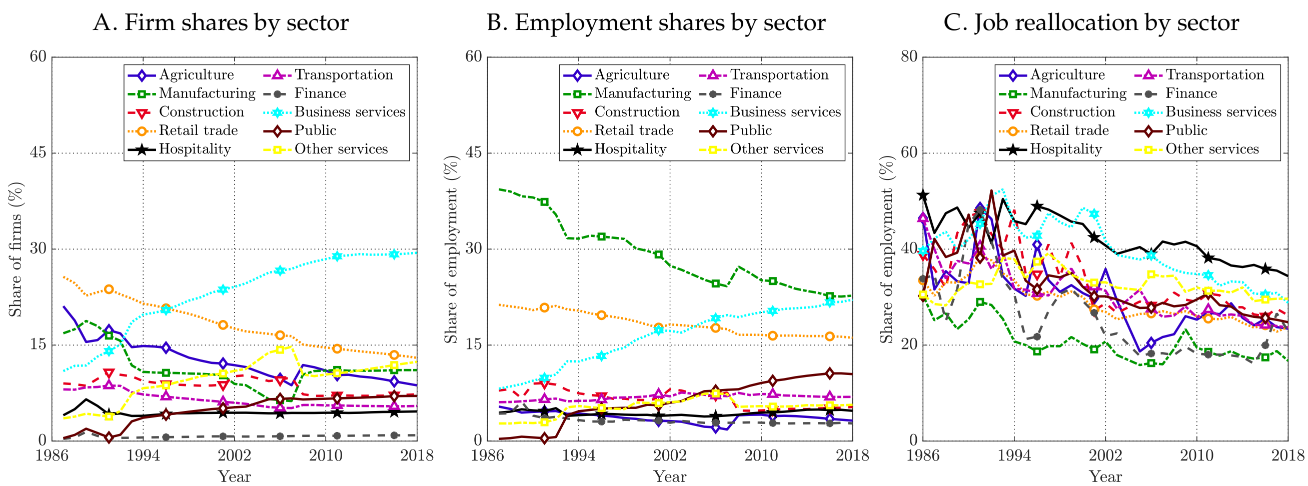



Figure 20 plots the share of firms (panel A) and the share of employment (panel B) by one-digit sector, as well as the job reallocation rate by one-digit sector (panel C), defined as the sum of jobs created and destroyed in a year divided by average employment in the year. *Source:* JOBB, LISA.

## B Aging and labor market dynamics

This section contains an assessment of the correlation structure of the dependent and independent variables (Appendix B.1); further details on the relationship between past births and aging across Swedish LAs (Appendix B.2); an illustration of key characteristics of Swedish local labor markets (Appendix B.3); a graphical illustration of long run changes in aging and key labor market outcomes (Appendix B.4); additional robustness specifications (Appendix B.5); additional results on the relationship between aging and labor market outcomes (Appendix B.6); the relationship between aging and labor market dynamics conditional on sector (Appendix B.7); as well as the relationship between aging and firm age conditional outcomes (Appendix B.8).

## B.1 On the spatial and time series correlation

To assess the prevalence of a time series of errors, I first obtain the residuals $\hat{\varepsilon}_{it}$ from a projection of the dependent and independent variables on LA and time fixed effects. Subsequently, I first project the contemporaneous residuals on up to seven years of their own lags

$$\hat{\varepsilon}_{it} = \sum_{\tau=1}^{7} \beta_{\tau} \hat{\varepsilon}_{it-\tau} + \gamma_{it} \tag{39}$$

Figure 21 plots the point estimates $\hat{\beta}_i$ from regression (39) for the share of young (panel A), firm creation (panel B) and worker relocation (panel C). When the share young, firm creation and worker relocation were above trend in the previous year, it is above trend in the current year. That is, the residuals are autocorrelated, with evidence suggesting that an AR1 correction is not sufficient to capture the structure of residuals. Consequently, I cluster standard errors by LA.

FIGURE 21. AUTOCORRELATION OF RESIDUALS

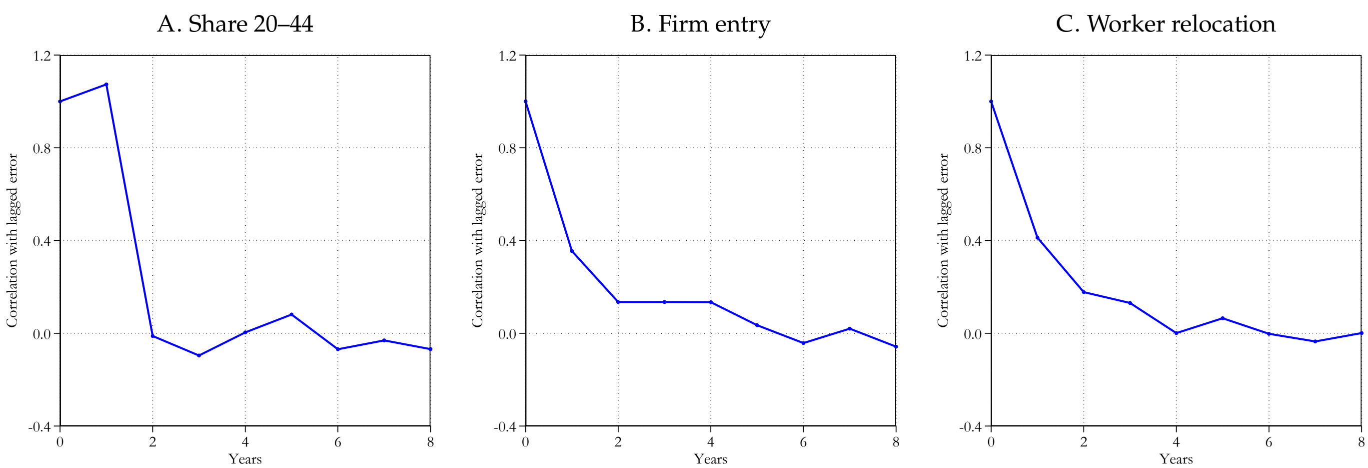



Figure 21 plots the point estimates from regression (39) for the share young (panel A), firm creation (panel B) and worker relocation (panel C). *Source:* JOBB, LISA, SCB.

To assess the prevalence of a spatial structure of errors, I project the contemporaneous residuals $\hat{\varepsilon}_{it}$ on the contemporaneous residuals $\hat{\varepsilon}_{jt}$ in all other labor markets interacted with the distance $d_{ij}$ to labor market $i$ (binned into deciles), as well as two years of lags of the residuals in labor market $i$ and two years of lagged residuals from all other labor markets interacted with their distance to labor market $i$,

$$\hat{\varepsilon}_{it} = \sum_{\tau=1}^{2} \alpha_{\tau} \hat{\varepsilon}_{it-\tau} + \sum_{\tau=0}^{2} \sum_{j \neq i} \beta_{\tau}^{d} \hat{\varepsilon}_{jt-\tau} d_{ij} + \gamma_{it} \tag{40}$$

Figure 22 plots the point estimates $\hat{\beta}_i$ from regression (40) for the share of young (panel A), firm creation (panel B) and worker relocation (panel C). When the share young, firm creation and worker relocation are above trend in neighboring markets, they are also above trend in market $i$. This evidence leads me to also cluster standard errors by year. I note, however, that controlling for the contemporaneous residual in neighboring markets as well as lagged residuals in market $i$, the contemporaneous residual in market $i$ is not systematically correlated to the lagged residuals in neighboring markets. Nevertheless, also allowing for an aggregate autocorrelation for up to two years following Driscoll and Kraay (1998) does not change the main results in this paper.

FIGURE 22. SPATIAL CORRELATION OF RESIDUALS

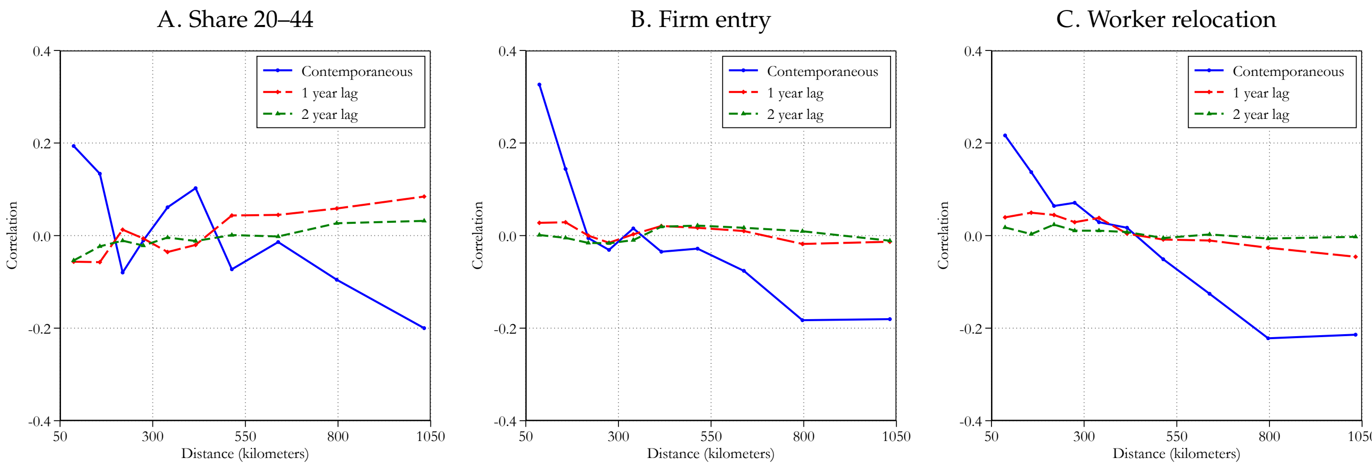



Figure 22 plots the point estimates from regression (40) for the share young (panel A), firm creation (panel B) and worker relocation (panel C).
*Source:* JOBB, LISA, SCB.

## B.2 The relationship between past births and aging across Swedish LAs

Figure 23 illustrates the relationship between past births and aging. Panel A plots the long-run cumulative change between 1986 and 2018 in the residual log share of young against the long-run change in the residual log sum of lagged births. As expected, LAs that experienced larger relative declines in lagged births have aged more. Panel B plots the residual share young against the residual log lagged births pooling all years 1986–2018. Both are constructed as the residual conditional on LA and year fixed effects. As expected, when a LA has a larger number of residual lagged births, it is younger.

FIGURE 23. RESIDUAL SHARE YOUNG AND RESIDUAL LAGGED BIRTHS, 1986–2018

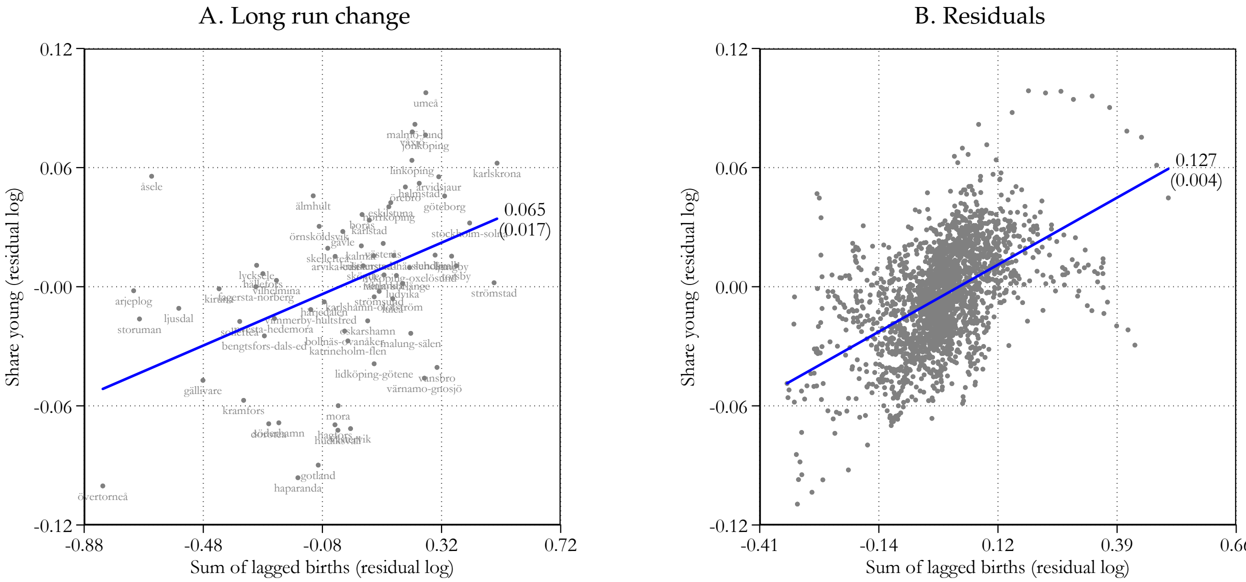



Figure 23 plots the within-LA long difference between 2018 and 1986 in the residual share young (share of all individuals aged 20–64 who are aged 20–44) against the within-LA long difference in the residual log sum of lagged births 20–44 years earlier (panel A) and the residual share young against the residual log lagged births (panel B). Residuals are conditional on LA and year fixed effects. *Source:* SCB.

## B.3 Characteristics of Swedish local labor markets

Figure 24 illustrates characteristics of Swedish LAs based on average outcomes over the 1986–2018 period (1999–2007 for net wealth). The largest areas are Stockholm, Gothenburg and Malmö (panel A). These areas also experienced the greatest growth in their workforces over the past 35 years (panel B), and they are the youngest (panel C).

FIGURE 24. CHARACTERISTICS OF SWEDISH LAS

A. Size of workforce B. Growth rate of workforce C. Share aged 20–44

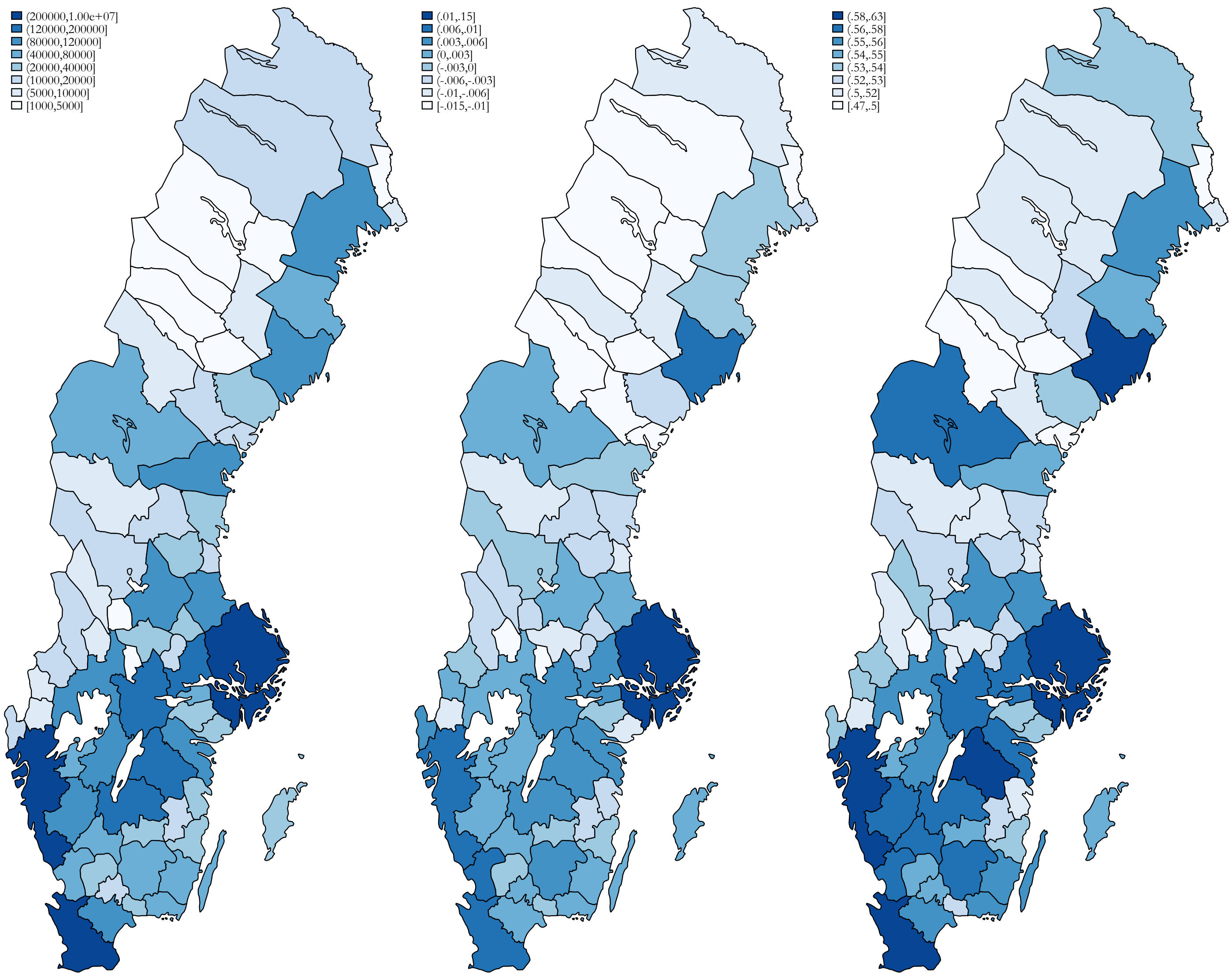



Figure 24 plots characteristics of Swedish LAs based on average outcomes over the 1986–2018 period. Panel A plots the number of individuals aged 20–64. Panel B plots the growth rate of the workforce (all individuals aged 20–64). Panel C plots the share of all individuals aged 20–64 who are aged 20–44. *Source:* JOBB, LISA, SCB.

Figure 25 provides a further illustration of how Swedish LAs differ. Stockholm, *Årjäng* close to Oslo, and areas in the southern Swedish region of *Småland* are the richest in terms of net wealth (panel A). Although it is not entirely clear what explains the latter, this region is known for a strong entrepreneurial spirit (referred to as *Gnosjöandan* in Swedish)—for instance, IKEA was founded in this area. The large metropolitan areas are the highest paying, although pay is also high in several less populous areas (panel B). Public sector employment is more prevalent in the more rural inland areas, in particular in the north (panel C).

FIGURE 25. CHARACTERISTICS OF SWEDISH LAS, CONTINUED

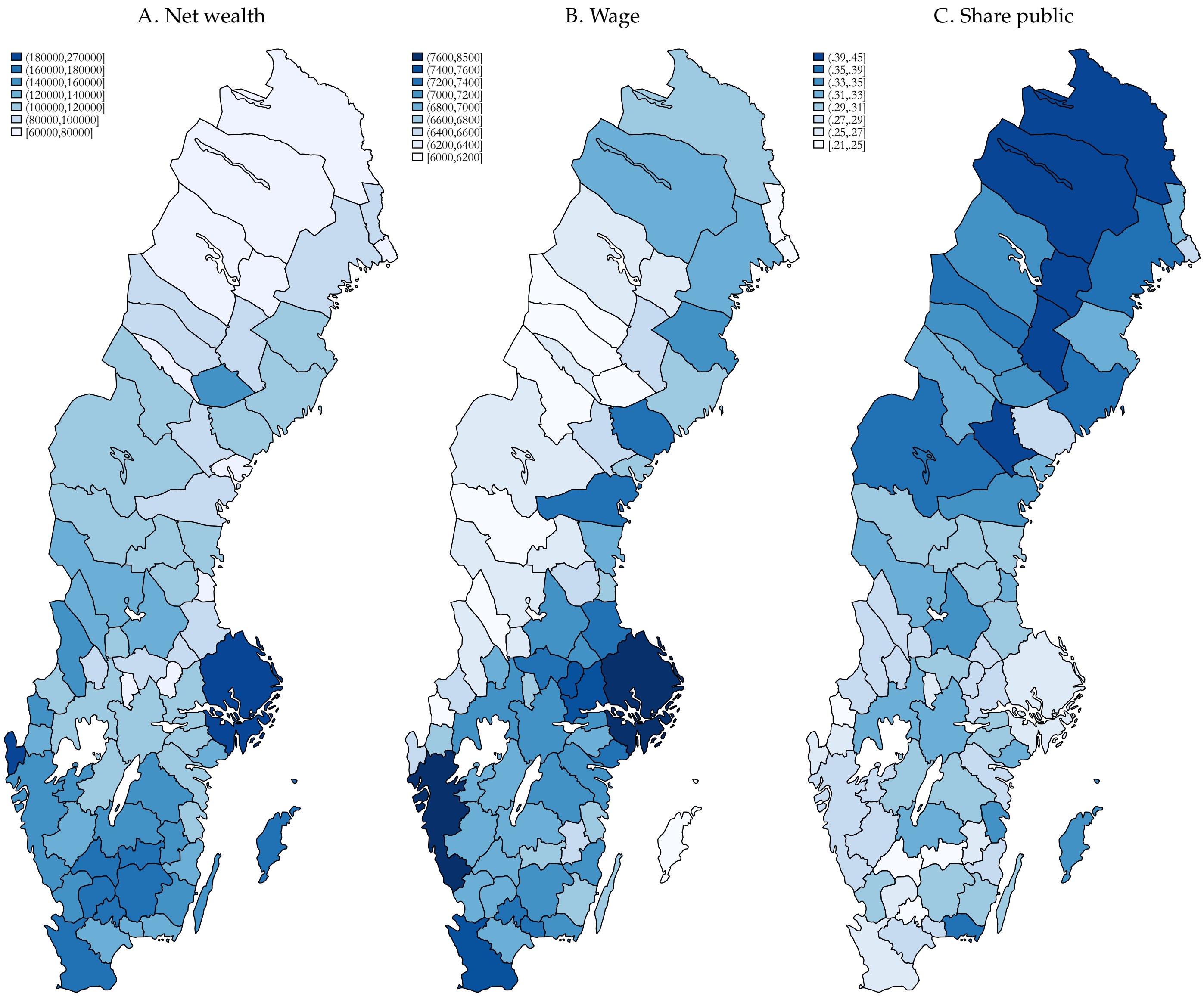



Figure 25 plots characteristics of Swedish LAs based on average outcomes over the 1986–2018 period (1999–2007 for net wealth). Panel A plots the average net wealth in real 1980 Swedish kronor. Panel B plots the average monthly wage of individuals aged 20–64 working in the private sector in real 1980 Swedish kronor. Panel C plots the number of individuals aged 20–64. *Source:* JOBB, LISA, SCB.

Figure 26 illustrates employment outcomes across Swedish local labor markets based on average outcomes over the 1986–2018 period. The likely reason for the high measured non-employment rate in the LAs bordering Finland in the north is cross-border commuting—employment abroad is not recorded in the Swedish administrative data, because a Nordic agreement stipulates that labor taxes are paid in the country where they are earned (panel A). The same reason is likely behind the high measured non-employment in the areas across the border from Oslo. Although there is cross-border commuting to Denmark from the southernmost region of *Skåne*, this is unlikely to be a major factor behind the high non-employment rate, because available data suggest that such flows are a small share of this area's large population. Private sector wage employment is more common in southern Sweden (panel B), while self-employment is more common in the northern inland (panel C).

FIGURE 26. EMPLOYMENT OUTCOMES ACROSS SWEDISH LAS

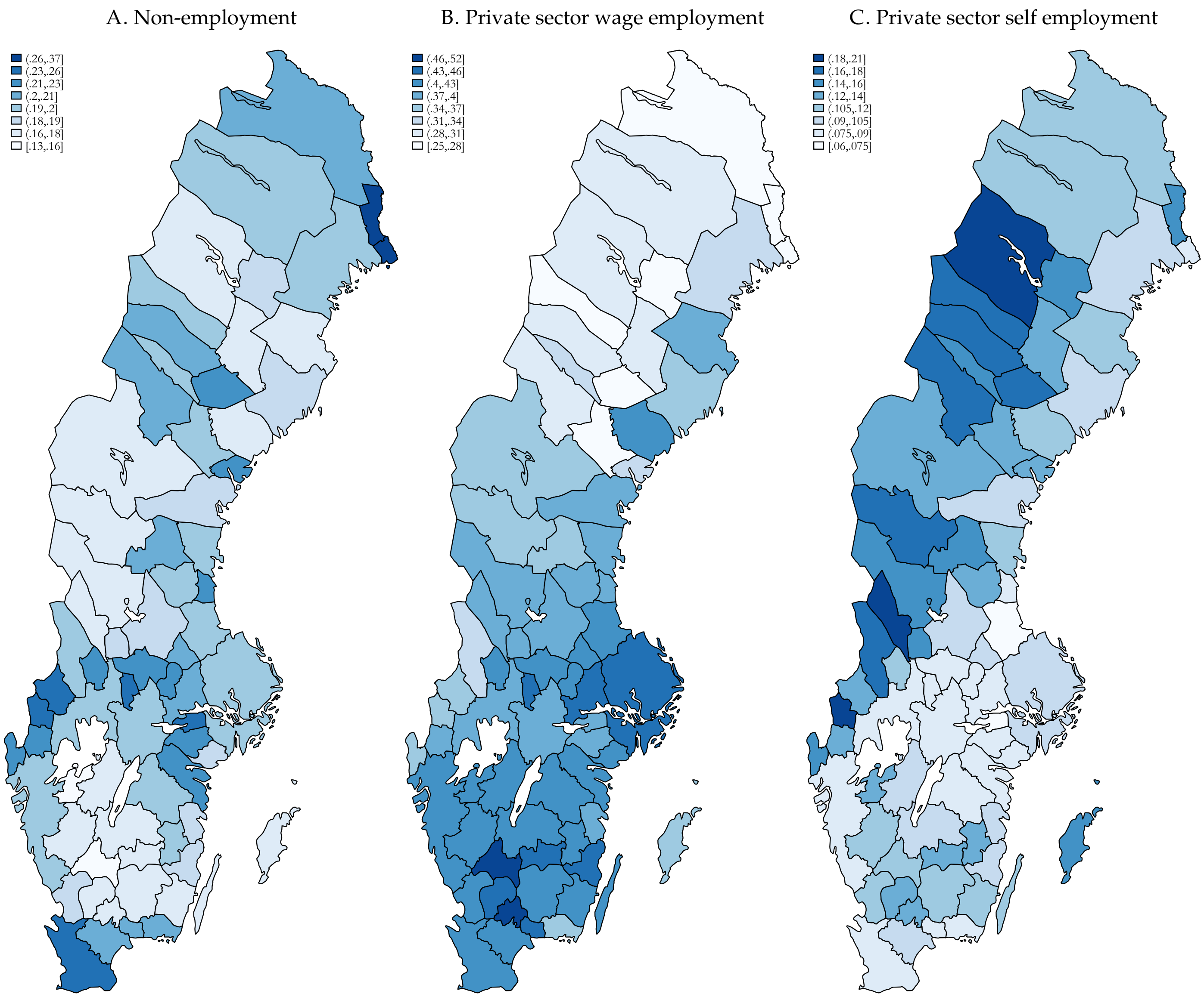



Figure 26 plots employment outcomes of Swedish LAs based on average outcomes over the 1986–2018 period. Panel A shows the share of non-employed individuals aged 20–64. Panel B shows the share of private sector wage employed individuals aged 20–64. Panel C shows the share of self employed individuals aged 20–64. *Source:* JOBB, LISA, SCB.

Figure 27 provides further worker-level outcomes across Swedish LAs based on average outcomes over the 1986–2018 period. The most educated areas include the three largest cities of Stockholm, Gothenburg and Malmö, but also, for instance, the northern area around *Umeå*, which is a major university town (panel A). Women constitute a larger share of private sector employment in the more populous coastal areas, likely due to a smaller role for the public sector in these areas (panel B). Immigrants as concentrated in southern Sweden and in particular Stockholm and Malmö, with the exception of the LAs bordering Finland to the north which has a large number of Finnish immigrants (panel C).

FIGURE 27. CHARACTERISTICS OF PRIVATE SECTOR EMPLOYMENT

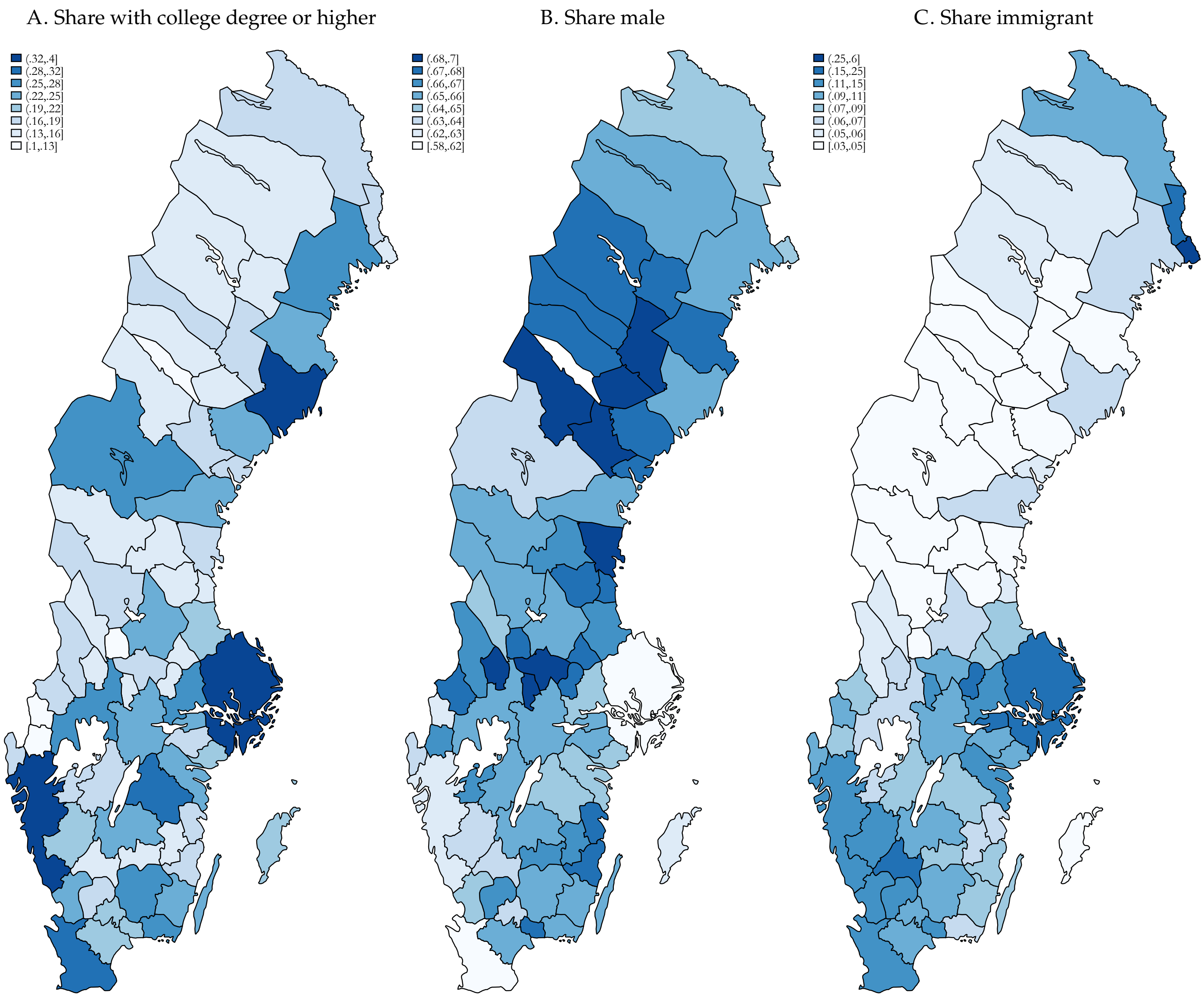



Figure 27 plots characteristics of Swedish LAs based on average outcomes over the 1986–2018 period. Panel A plots the share of private sector employment aged 20–64 with a college degree or higher. Panel B plots the share of private sector employment aged 20–64 that is male. Panel C plots the share of private sector employment aged 20–64 that was born abroad. *Source:* JOBB, LISA, SCB.

Figure 28 illustrates additional firm-level characteristics of Swedish LAs based on on average outcomes over the 1986–2018 period (1997–2018 for value added). Manufacturing is concentrated in the Swedish inland (panel A). The most productive LAs are generally those which have a large share of manufacturing, as well as the metropolitan areas of Stockholm and Gothenburg (panel B). One exception is the south-eastern area of *Oskarshamn*, which is home to a nuclear power plant that produces about 10 percent of Sweden's electricity. Average firm size is highest in Stockholm (panel C).

FIGURE 28. FIRM CHARACTERISTICS

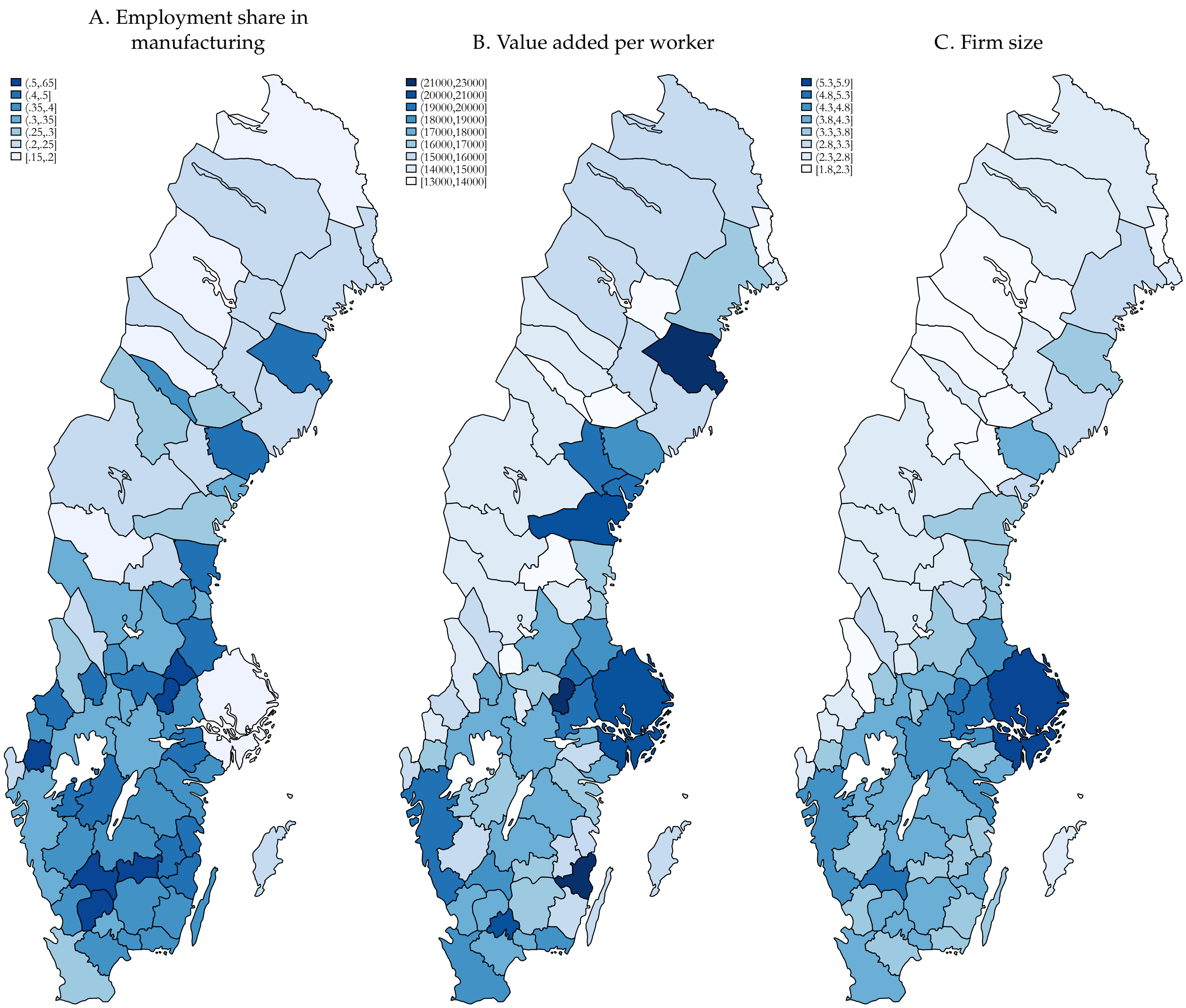



Figure 28 plots characteristics of Swedish local labor markets based on average outcomes over the 1986–2018 period (1997–2018 for value added). Panel A plots the fraction of private sector employment aged 20–64 that is in manufacturing. Panel B plots average value added per worker of private sector employment aged 20–64 in real 1980 Swedish kronor. Panel C plots average firm size of private sector firms (including all employees aged 20–64). *Source:* FEK, JOBB, LISA, SCB.

Figure 29 plots the predicted share of young, firm entry and worker relocation across Swedish LAs based on averages over the 1986–2018 period. The predicted youngest areas based on past fertility are the three largest cities, Stockholm, Gothenburg and Malmö (panel A). The highest firm entry rates are found in the three largest cities (panel B), while worker relocation is the highest in the north (panel C). One likely factor behind the latter is seasonal employment associated with winter tourism. Consistent with this view, the highest worker relocation rates are found in *Kiruna*, home to the *Riksgränsen*, *Björkliden* and *Abisko* ski resorts as well as the *Jukkasjärvi Icehotel*; *Storuman*, which contains the *Hemavan* and *Tärnaby* ski resorts; *Härjedalen*, with several large ski resorts; and *Malung-Sälen*, with the *Sälen* skiing complex.

FIGURE 29. SHARE YOUNG, FIRM CREATION AND WORKER RELOCATION

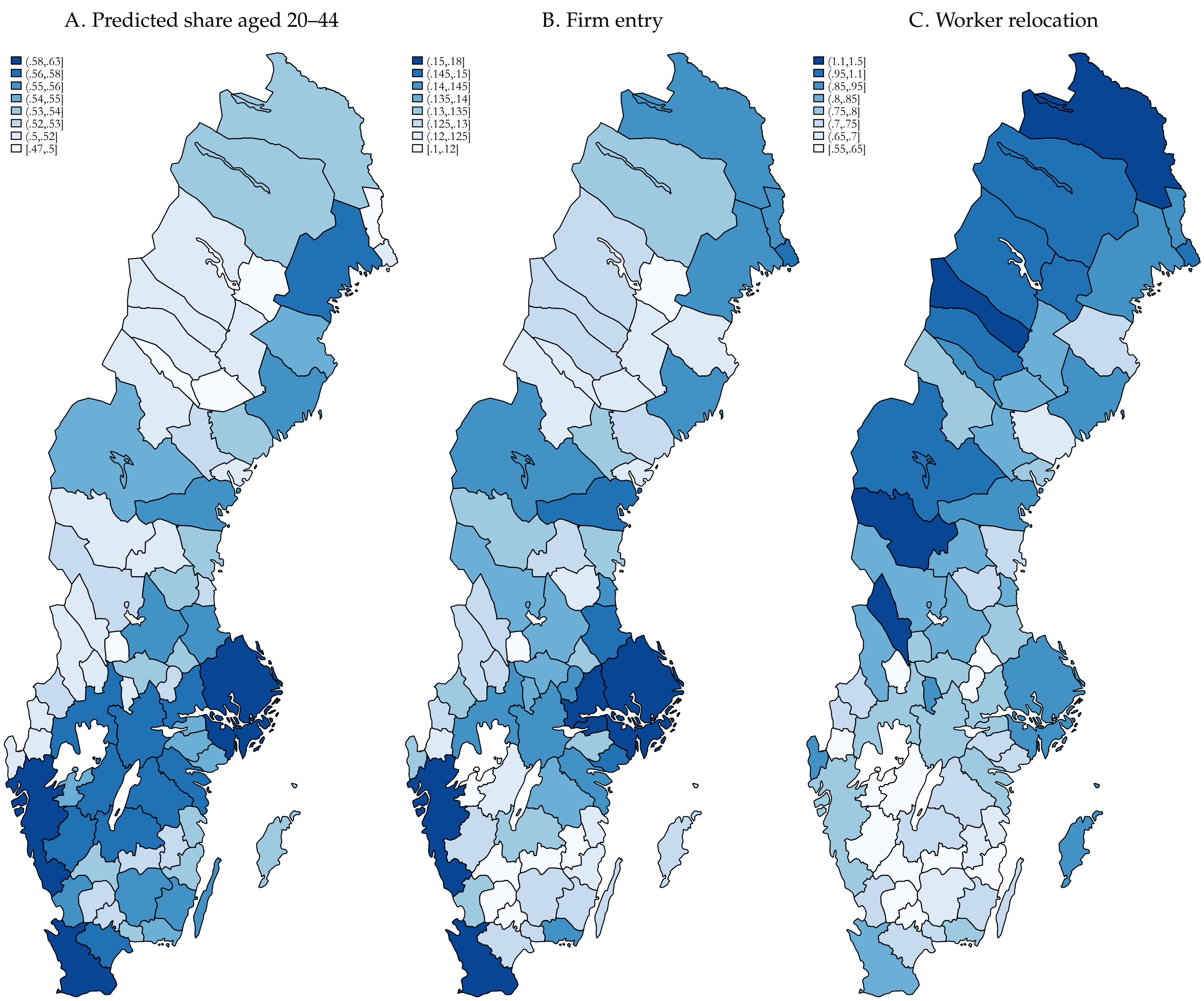



Figure 29 plots the age composition, firm creation and worker relocation across Swedish local labor markets based on average outcomes over the 1986–2018 period. Panel A plots the number of individuals aged 20–64 who are aged 20–44. Panel B plots the fraction of all private sector firms with positive employment in a year that had zero employment in the previous year. Panel C plots the fraction of private sector individuals aged 20–64 who were either hired or separated in the current year. *Source:* JOBB, LISA, SCB.

### B.4 Long run changes in predicted aging and key labor market dynamics outcomes

Figure 30 plots aging as predicted by lagged fertility (panel A), the change in firm creation (panel B) and worker relocation (panel C) between 1986 and 2018 after taking out local labor market and year fixed effects. There is some evidence of spatial correlation in the extent of aging, but important variation also remains within Swedish regions. For instance, Sweden is divided into three broad regions: a southern region (*Götaland*), a middle region (*Svealand*) and a northern region (*Norrland*). Adding region-year fixed effects to the projection of the log share of young on LA and year fixed effects shrinks the standard deviation of the residual log share of young only marginally from 0.025 to 0.023.

FIGURE 30. CHANGE IN INDEPENDENT AND DEPENDENT VARIABLES BETWEEN 1986–2018

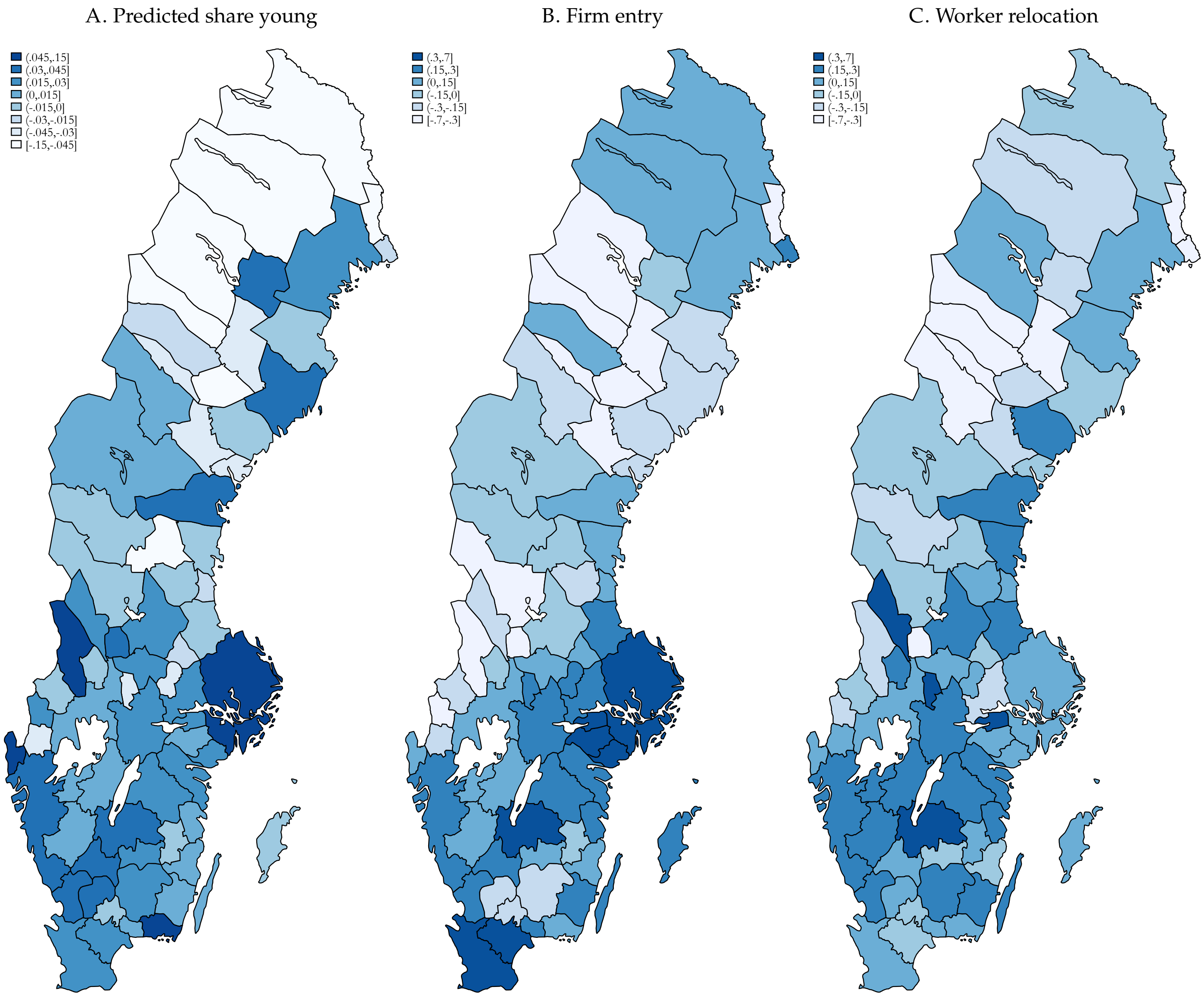



Figure **??** plots the long difference between 2018 and 1986 in the predicted share of young (panel A), firm creation (panel **??**).) and worker relocation (panel C). Panel A first computes the predicted share young by projecting the actual log share on the log sum of lagged births 20–44 years earlier, LA and year fixed effects, obtaining the predicted values based on the log sum of lagged births, and subsequently computing the long difference between 2018 and 1986. Panels **??**–C residualize each outcome by projecting it on LA and year fixed effects, and subsequently compute the long difference between 2018 and 1986. *Source:* SCB.

Figure 31 illustrates some of the patterns that drive the main results. Panel A plots the change in the residual firm entry rate between 1986 and 2018 against the change in the predicted residual log share of young based on lagged births (in both cases conditional on LA and year fixed effects). Panel B shows the change in the residual worker relocation rate relative to the change in the predicted share young. Predicted aging based on changes in lagged births is associated with a statistically significant relative decline in firm entry and worker relocation. Although a useful first way to illustrate the variation, it is important to note that regression (1) also exploits differential timing of aging over these 35 years.

FIGURE 31. LONG-RUN CHANGE IN KEY LABOR MARKET OUTCOMES AGAINST THE CHANGE IN THE PREDICTED SHARE OF YOUNG

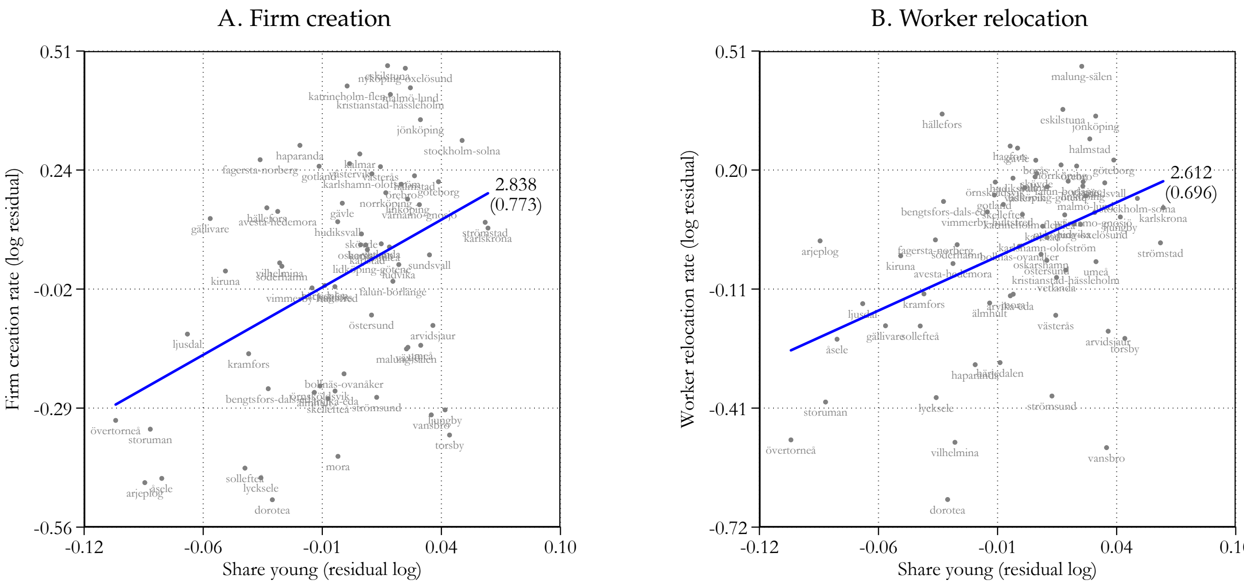



Figure 31 plots the difference between residual outcome in 2018 and 1986. Panel A shows residual log firm creation, based on a projection of actual log firm creation on LA and year fixed effects, against the predicted log share of young, based on a projection of the actual log share of young on the residual log sum of lagged births (conditional on LA and year fixed effects). Panel B shows residual log worker relocation, based on a projection of actual log worker relocation on LA and year fixed effects, against the predicted log share of young, based on a projection of the actual log share of young on the residual log sum of lagged births (conditional on LA and year fixed effects). *Source:* FEK, JOBB, LISA, SCB.

## B.5 Robustness specifications

Table 8 presents a series of robustness specifications. Columns 3–4 add separate linear time trends interacted with the initial share of private sector employment that is male, has a college degree or more, or is immigrant, as well as the initial size of the workforce and initial average net wealth. It changes the point estimates by little, but the standard errors widen significantly in the IV specification such that the estimated impact of aging on firm creation is no longer statistically significant at conventional levels. The weaker first stage and wider standard errors are due to the inclusion of the interaction of time with the initial share with a college degree, which on its own is highly statistically significant (p-value 0.806). Dropping this statistically insignificant control, the point estimate again becomes statistically significant.

Columns 5–6 include separate linear time trends interacted with initial value added per worker, the initial share of manufacturing firms and initial investment per worker. Results are robust. Moreover, results are little changed if I drop the three largest LAs Stockholm, Gothenburg and Malmö, which jointly account for about half of Sweden's population (columns 7–8). Finally, the standard errors rise if I add LA times decade fixed effects, to the point where the IV estimate for firm creation and the OLS estimate for worker relocation are no longer statistically significant at conventional levels (p-values of 0.117 and 0.138, respectively, under two-way clustered standard errors).

TABLE 8. THE IMPACT OF AGING ON LABOR MARKET DYNAMICS, ROBUSTNESS

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) |
|---|---|---|---|---|---|---|---|---|---|---|
| | Baseline | | Controls-time trend | | Size-time trend | | Drop large cities | | LA-decade FEs | |
| | OLS | IV | OLS | IV | OLS | IV | OLS | IV | OLS | IV |
| | *Panel A. Firm creation rate* | | | | | | | | | |
| Share 20–44 | 1.207*** | 2.196** | 0.757** | 2.106 | 1.232*** | 1.916** | 1.029*** | 2.054** | 1.044** | 2.536 |
| | (0.358) | (0.820) | (0.338) | (1.262) | (0.320) | (0.822) | (0.361) | (0.876) | (0.427) | (1.574) |
| P-value | 0.002 | 0.012 | 0.032 | 0.105 | 0.001 | 0.026 | 0.008 | 0.025 | 0.020 | 0.117 |
| Obs. | 2,244 | 2,244 | 2,244 | 2,244 | 2,244 | 2,244 | 2,145 | 2,145 | 2,244 | 2,244 |
| R-squared | 0.691 | | 0.732 | | 0.694 | | 0.684 | | 0.808 | |
| within | 0.059 | | 0.184 | | 0.066 | | 0.041 | | 0.013 | |
| F-stat | | 27.1 | | 16.3 | | 24.2 | | 23.4 | | 17 |
| | *Panel B. Worker relocation rate* | | | | | | | | | |
| Share 20–44 | 0.988*** | 2.594*** | 0.646** | 3.051** | 0.938*** | 2.490*** | 1.022*** | 2.772*** | 0.471 | 3.251** |
| | (0.237) | (0.754) | (0.245) | (1.300) | (0.246) | (0.851) | (0.241) | (0.829) | (0.310) | (1.347) |
| P-value | 0.000 | 0.002 | 0.013 | 0.025 | 0.001 | 0.006 | 0.000 | 0.002 | 0.138 | 0.022 |
| Obs. | 2,244 | 2,244 | 2,244 | 2,244 | 2,244 | 2,244 | 2,145 | 2,145 | 2,244 | 2,244 |
| R-squared | 0.791 | | 0.804 | | 0.793 | | 0.791 | | 0.886 | |
| within | 0.051 | | 0.108 | | 0.059 | | 0.050 | | 0.004 | |
| F-stat | | 27.1 | | 16.3 | | 24.2 | | 23.4 | | 17 |

Table 8 presents OLS and IV estimates based on regression (1) using annual data from 68 LA between years 1986–2018. The independent variable is the log share of all individuals aged 20–64 that are aged 20–44 in the LA in that year. Outcome variables are for private sector firms and individuals aged 20–64, averaged in levels at the LA-year level and subsequently logged. The instrument is the sum of births 20–44 years earlier in the LA, and subsequently logged. Standard errors are two-way clustered at the LA and year levels. Columns 1–2 reproduce the baseline results from Table 2. Columns 3–4 include separate linear time trends interacted with the share college, male, immigrant and population size, all measured in 1986. Columns 5–6 include separate linear time trends interacted with average value added per worker, the share of firms in manufacturing, and average investment per worker, all measured in the earliest year of data (1997 for value added and investment, 1986 for the share in manufacturing). Columns 7–8 exclude the largest three metro areas—Stockholm, Gothenburg and Malmö—which constitute roughly half of the Swedish population. Columns 9–10 add LA-decade fixed effects. Panel A shows results for the firm creation rate as the dependent variable, defined as the share of firms with positive employment in the current year that had zero employment in the previous year. Panel B shows results for the worker relocation rate as the dependent variable, defined as the sum of hires and separations in a year divided by average employment in the year. *Source:* FEK, JOBB, LISA, SCB.

## B.6 The impact of aging: additional outcomes

Table 9 summarizes the impact of aging on a range of additional outcomes. Aging reduces the entry rate also of high growth firms, measured by either at least 25 or 50 percent growth in employment during the first five years of a firm's operation. It has no statistically significant effect on firm size or investment per worker. Finally, it lowers job reallocation.

## B.7 Aging and within-sector outcomes

The impact of aging on labor market dynamics could partly arise through a shift of sectoral activity if aging shifts economic activity toward intrinsically less dynamic sectors. To assess the importance of such shifts, I collect outcome variables at the LA-year-sector level and project them on the share of young in

TABLE 9. THE IMPACT OF AGING ON ADDITIONAL LABOR MARKET DYNAMICS

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) |
|---|---|---|---|---|---|---|---|---|---|---|
| | Entry, 25% | | Entry, 50% | | Firm size | | Investment p.w. | | JR | |
| | OLS | IV | OLS | IV | OLS | IV | OLS | IV | OLS | IV |
| Share 20–44 | 0.998** | 2.057** | 0.835* | 1.970* | 0.281 | –0.157 | 0.453 | 1.042 | 0.526** | 1.372* |
| | (0.390) | (0.925) | (0.435) | (1.040) | (0.291) | (0.661) | (0.561) | (1.552) | (0.238) | (0.687) |
| p value | 0.016 | 0.034 | 0.065 | 0.069 | 0.342 | 0.813 | 0.428 | 0.509 | 0.034 | 0.054 |
| Obs. | 1,963 | 1,963 | 1,958 | 1,958 | 2,244 | 2,244 | 1,496 | 1,496 | 2,244 | 2,244 |
| R-squared | 0.510 | | 0.466 | | 0.862 | | 0.501 | | 0.639 | |
| within | 0.009 | | 0.005 | | 0.004 | | 0.001 | | 0.006 | |
| F-stat | | 30.4 | | 29.9 | | 27.1 | | 14.8 | | 27.1 |

Table 9 presents OLS and IV estimates based on regression (1) using annual data from 68 LA between years 1986–2018. The independent variable is the log share of all individuals aged 20–64 that are aged 20–44 in the LA in that year. The outcome variables are for private sector firms and individuals aged 20–64, averaged in levels at the LA-year level and subsequently logged. The instrument is the sum of births 20–44 years earlier in the LA, and subsequently logged. Standard errors are two-way clustered at the LA and year levels. *Source:* JOBB, LISA, SCB.

the LA-year, controlling for LA, year and sector fixed effects,

$$\log y_{i,t,s} \quad = \quad \alpha \log young_{i,t} + \psi_i + \xi_t + \phi_s + \varepsilon_{i,t,a} \tag{41}$$

I weigh (42) such that each sector gets a weight corresponding to its share of firms (firm entry) or employment (worker and job reallocation) in the LA-year, and each LA-year gets the same aggregate weight.

Table 10 presents results from (41). Aging reduces firm entry, job and worker reallocation conditional on sector. This is broadly consistent with the time series evidence, which also shows that the majority of the decline in labor market dynamics has taken place within sectors (Appendix A.8).

TABLE 10. AGING AND SECTOR-CONDITIONAL OUTCOMES ACROSS SPACE

| | (1) | (2) | (3) | (4) | (9) | (10) |
|---|---|---|---|---|---|---|
| | Firm entry | | WR | | JR | |
| | OLS | IV | OLS | IV | OLS | IV |
| Share 20–44 | 1.038*** | 1.901** | 0.791*** | 2.079*** | 0.467** | 1.263* |
| | (0.346) | (0.784) | (0.213) | (0.728) | (0.226) | (0.674) |
| Obs. | 21,260 | 21,260 | 22,018 | 22,018 | 21,988 | 21,988 |
| Clusters | 68 | 68 | 68 | 68 | 68 | 68 |
| R-squared | 0.536 | | 0.679 | | 0.447 | |
| within | 0.007 | | 0.006 | | 0.001 | |
| F-stat | | 30.5 | | 30.2 | | 30.2 |

Table 10 presents OLS and IV estimates based on regression (42) using annual data from 68 LA between 1997–2018. Share 20–44 is the log share of all individuals aged 20–64 that are aged 20–44 in the LA in that year. Outcome variables are for private sector firms and individuals aged 20–64. All dependent variables are first averaged in levels at the LA-year-firm age level and subsequently logged. The instrument is the sum of births 20–44 years earlier in the LA, and subsequently logged. Regressions are weighed such that each firm age bin gets a weight corresponding to its share of firms (firm exit and firm size) or employment (worker and job reallocation) in the LA-year, and each LA-year gets the same aggregate weight. Standard errors are clustered at the LA level. *Source:* FEK, JOBB, LISA, SCB.

## B.8 Aging and firm age conditional outcomes

Appendix B.6 finds that aging leads to an increase in the share of firms that are 11 years and older. Since older firms are on average less dynamic, this shift may hence account for some or all of the overall impact of aging on labor market dynamics. To assess the importance of this shift toward older firms, I project labor market dynamics outcomes at the LA-year-firm age level on the share of young in the LA-year, controlling for LA, year and firm age fixed effects,

$$\log y_{i,t,a} \quad = \quad \alpha \log young_{i,t} + \psi_i + \xi_t + \phi_a + \varepsilon_{i,t,a} \tag{42}$$

I weigh (42) such that each firm age bin gets a weight corresponding to its share of firms (firm exit and firm size) or employment (worker and job reallocation) in the LA-year, and each LA-year gets the same aggregate weight. Due to the left-censoring of the data, I focus on the 1997–2018 period.

Table 11 presents results from (42). Aging reduces worker and job reallocation as well as firm exit conditional on firm age, but has no statistically significant effect on average size. These findings are consistent with the time series evidence, which also shows only a small change in firm size conditional on firm age, but a substantial decline in job reallocation conditional on firm age (Appendix A.7).

TABLE 11. AGING AND FIRM AGE CONDITIONAL OUTCOMES ACROSS SPACE

| | (1) | (2) | (3) | (4) | (9) | (10) | (11) | (12) |
|---|---|---|---|---|---|---|---|---|
| | WR | | JR | | Exit | | Size | |
| | OLS | IV | OLS | IV | OLS | IV | OLS | IV |
| Share 20–44 | 1.130*** | 3.441*** | 0.734*** | 1.533* | 1.073*** | 2.111** | -0.337 | -2.905 |
| | (0.270) | (1.077) | (0.246) | (0.830) | (0.399) | (0.945) | (1.493) | (3.473) |
| Obs. | 19,498 | 19,498 | 19,491 | 19,491 | 19,041 | 19,041 | 19,513 | 19,513 |
| Clusters | 68 | 68 | 68 | 68 | 68 | 68 | 68 | 68 |
| R-squared | 0.546 | | 0.625 | | 0.661 | | 0.426 | |
| within | 0.008 | | 0.002 | | 0.005 | | 0.000 | |
| F-stat | | 23 | | 23 | | 23.373 | | 23 |

Table 11 presents OLS and IV estimates based on regression (42) using annual data from 68 LA between 1997–2018. Share 20–44 is the log share of all individuals aged 20–64 that are aged 20–44 in the LA in that year. Outcome variables are for private sector firms and individuals aged 20–64. All dependent variables are first averaged in levels at the LA-year-firm age level and subsequently logged. The instrument is the sum of births 20–44 years earlier in the LA, and subsequently logged. Regressions are weighed such that each firm age bin gets a weight corresponding to its share of firms (firm exit and firm size) or employment (worker and job reallocation) in the LA-year, and each LA-year gets the same aggregate weight. Standard errors are clustered at the LA level. *Source:* FEK, JOBB, LISA, SCB.

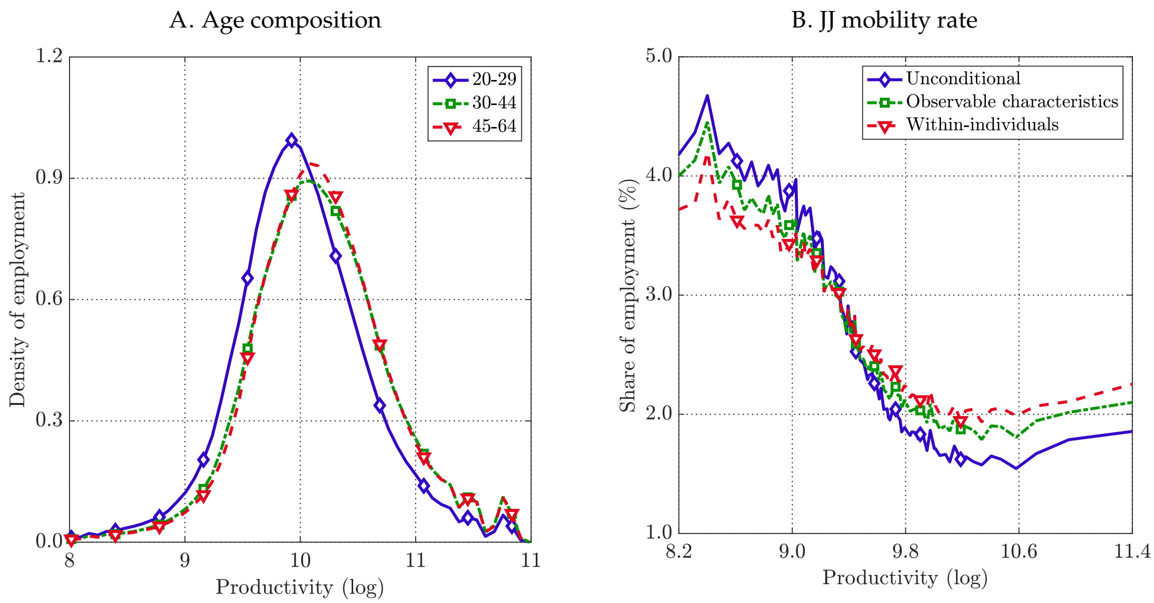
# C Model

This section contains motivating evidence on the role of the job ladder in careers (Appendix C.1); a proof of Lemmas 1–2 (Appendix C.2); a proof of Proposition 1 (Appendix C.3); a derivation of the FP equation for the distribution of employment (Appendix C.4); a proof of Proposition 2 (Appendix C.5); a proof of Proposition 4 (Appendix C.7); a proof of Proposition 3 (Appendix C.6); a proof of Proposition 5 (Appendix C.8); a proof of Proposition 6 (Appendix C.9); a proof of Proposition 7 (Appendix C.10); a proof of Proposition 8 (Appendix C.11); a proof of Proposition 9 (Appendix C.12); a reduced-form assessment of the joint dynamics of entrepreneurship and search in the data (Appendix C.13); the law of motion for relative productivity as well as an illustration of the impact of entry on inequality (Appendix C.14); the value functions in the full model (Appendix C.15); an equilibrium definition for the full model (Appendix C.16); and an outline of the algorithm used to solve the model (Appendix C.17).

### C.1 Motivating evidence on the role of the job ladder in careers

To motivate the view of careers adopted by this paper, panel A of Figure 32 plots the distribution of employment by age over log value added per worker. Older individuals tend to be employed by more productive firms. Panel B shows that workers who are employed by more productive firms on average are less likely to switch employer, both unconditionally and conditional on observable and unobservable characteristics.[50] These differences are large: when an individual is employed at the bottom of the productivity distribution, they are roughly twice as likely to make a JJ move relative to when they are employed at the top.

FIGURE 32. AGE COMPOSITION AND WORKER RELOCATION RATE BY FIRM PRODUCTIVITY




Panel A plots the distribution of individuals by age over log value added per worker. Panel B plots the share of workers in month $t$ who have a different main employer in month $t+1$, where the main employer in a month is that paying the most in the year. "Observable characteristics" controls flexibly for gender, education and age. "Within-individuals" controls for individual-fixed effects. All panels: Private sector firms and individuals aged 20–64 between years 1997–2018. *Source:* FEK, JOBB, LISA

[50] An alternative interpretation of panel A is that some firms are more productive because they hire older, more productive workers. Bagger et al. (2014) find, however, that workforce composition accounts for little of firm productivity differences.

### C.2 Lemmas 1–2: Stationary transformation

The value of unemployment $\hat{U}(t)$ at time $t$ is given by the HJB equation

$$\rho\hat{U}(t) = \underbrace{be^{\hat{z}(t)}}_{\text{flow value of leisure}} + \underbrace{\dot{\hat{U}}(t)}_{\text{time drift}} - \underbrace{\kappa\hat{U}(t)}_{\text{retirement of worker}} \tag{43}$$

The joint value of a coalition between an entrepreneur with productivity $\hat{z}$ and their $n$ workers at time $t$, $\hat{\mathbf{W}}(\hat{z}, n, t)$, is given by

$$\begin{aligned}
\rho\hat{\mathbf{W}}(\hat{z}, n, t) &= \underbrace{e^{\hat{z}}n + e^{\hat{z}(t)}kb - \hat{r}(t)}_{\text{net flow output}} \\
&+ \underbrace{\hat{\mathbf{W}}_t(\hat{z}, n, t)}_{\text{time drift}} \\
&- \underbrace{\kappa n\hat{\mathbf{W}}_n(\hat{z}, n, t)}_{\text{retirement of a worker}} \\
&- \underbrace{\kappa\hat{\mathbf{W}}(\hat{z}, 0, t)}_{\text{retirement of entrepreneur}} \\
&+ \max_v \Bigg\{ \hat{q}(t)\Bigg( \underbrace{\frac{\hat{u}(t)}{\hat{S}(t)}\Big(\hat{\mathbf{W}}_n(\hat{z}, n, t) - \hat{U}(t)\Big)^+}_{\text{contact with unemployed potential hire}} \\
&+ \underbrace{\frac{\phi\hat{e}(t)}{\hat{S}(t)}\int_0^\infty\int_0^\infty \left(\hat{\mathbf{W}}_n(\hat{z}, n, t) - \hat{\mathbf{W}}_n(\hat{z}', n', t)\right)^+ \tilde{\mathbf{g}}(\hat{z}', n', t)d\hat{z}'dn'}_{\text{contact with employed potential hire}} \Bigg) \\
&- \underbrace{\frac{c_v}{1+\eta_v}e^{\hat{z}}v^{1+\eta_v}}_{\text{cost of recruiting}} \Bigg\}
\end{aligned} \tag{44}$$

subject to

$$\hat{\mathbf{W}}(\hat{z}, n, t) \geq n\hat{U}(t) + \hat{U}^f(t), \quad \text{and} \quad \hat{\mathbf{W}}_n(\hat{z}, n, t) \geq \hat{U}(t) \tag{45}$$

where $\hat{\mathbf{W}}_i(\cdot) = \partial\hat{\mathbf{W}}(\cdot)/\partial i$ is the derivative of $\hat{\mathbf{W}}$ with respect to $i$.

The value of a non-producing entrepreneur is

$$\begin{aligned} \rho\hat{U}^f(t) &= be^{\underline{\hat{z}}(t)} - \kappa\hat{U}^f(t) + \dot{\hat{U}}^f(t) \\ &+ \max_s \left\{ s\pi \int_0^\infty \left(\hat{\mathbf{W}}(\underline{\hat{z}}(t)+z,0,t) - \hat{U}^f(t)\right)^+ d\Gamma(z) - c_e e^{\underline{\hat{z}}(t)} \frac{s^{1+\eta_e}}{1+\eta_e} \right\} \end{aligned} \tag{46}$$

Guess that $\hat{\mathbf{W}}(\hat{z},n,t) = \mathbf{W}(z,n)e^{\underline{\hat{z}}(t)}$, $\hat{U}(t) = Ue^{\underline{\hat{z}}(t)}$, and $\hat{U}^f(t) = U^f e^{\underline{\hat{z}}(t)}$, where $\hat{z} = \underline{\hat{z}}(t) + z$ and $\underline{\hat{z}}(t) = \underline{\hat{z}}(0) + mt$. Hence, $\hat{\mathbf{W}}(\hat{z},n,t) = \mathbf{W}(\hat{z} - mt, n)e^{\underline{\hat{z}}(t)}$. Differentiating

$$\begin{aligned} \hat{\mathbf{W}}_{\hat{z}}(\hat{z},n,t) &= \mathbf{W}_z(z,n)e^{\underline{\hat{z}}(t)} \\ \hat{\mathbf{W}}_n(\hat{z},n,t) &= \mathbf{W}_n(z,n)e^{\underline{\hat{z}}(t)} \\ \hat{\mathbf{W}}_t(\hat{z},n,t) &= -\mathbf{W}_z(z,n)e^{\underline{\hat{z}}(t)}m + \mathbf{W}(z,n)e^{\underline{\hat{z}}(t)}m \end{aligned}$$

Since $z = \hat{z} - \underline{\hat{z}}(t)$, it follows from a change of variables that

$$\mathbf{g}(z,n) = \hat{\mathbf{g}}(\hat{z},n,t)$$

Substituting these observations into the value function (44), and imposing the BGP condition that $q = \hat{q}(t)$, $u = \hat{u}(t)$, $e = \hat{e}(t)$ and $\hat{r}(t) = e^{\underline{\hat{z}}(t)}r$

$$\begin{aligned} \rho e^{\underline{\hat{z}}(t)}\mathbf{W}(z,n) &= e^{\underline{\hat{z}}(t)}e^z n + e^{\underline{\hat{z}}(t)}kb - e^{\underline{\hat{z}}(t)}r \\ &- \mathbf{W}_z(z,n)e^{\underline{\hat{z}}(t)}m + \mathbf{W}(z,n)e^{\underline{\hat{z}}(t)}m \\ &- n\kappa e^{\underline{\hat{z}}(t)}\mathbf{W}_n(z,n) \\ &- \kappa e^{\underline{\hat{z}}(t)}\mathbf{W}(z,0) \\ &+ \max_v \left\{ q\left(\frac{u}{S}\left(\mathbf{W}_n(z,n)e^{\underline{\hat{z}}(t)} - Ue^{\underline{\hat{z}}(t)}\right)^+ \right.\right. \\ &+ \left. \frac{\phi e}{S}\int_0^\infty\int_0^\infty \left(\mathbf{W}_n(z,n)e^{\underline{\hat{z}}(t)} - \mathbf{W}_n(z',n')e^{\underline{\hat{z}}(t)}\right)^+ \mathbf{g}(z',n')dz'dn'\right) \\ &- \frac{c_v}{1+\eta_v}e^z v^{1+\eta_v} \end{aligned}$$

subject to

$$\mathbf{W}(z,n)e^{\underline{\hat{z}}(t)} \geq nUe^{\underline{\hat{z}}(t)} + U^f e^{\underline{\hat{z}}(t)}, \quad \text{and} \quad \mathbf{W}_n(z,n)e^{\underline{\hat{z}}(t)} \geq Ue^{\underline{\hat{z}}(t)}$$

Cancelling the $e^{\underline{\hat{z}}(t)}$ term on all sides and moving the $\mathbf{W}(z,n)m$ term to the other side gives (5)–(6). A straightforward exercise along the same lines yields (7)–(9).

### C.3 Proposition 1: Constant returns to scale

Guess that the joint value $\mathbf{W}(z,n)$ given by (10) can be written as

$$\mathbf{W}(z,n) \quad = \quad n(J(z)+U)+O(z)+U^f \tag{47}$$

where the surplus of a match solves

$$(\rho-m)\,J(z) \quad = \quad e^z-1-mJ'(z)-\kappa J(z) \tag{48}$$

subject to $J(\underline{z}^w)=0$ and $J'(\underline{z}^w)=0$, where $\underline{z}^w=0$, and the surplus of an entrepreneur solves

$$\begin{aligned}(\rho-m)\,O(z) \quad &= \quad k-1-r-mO'(z)-\kappa O(z) \\ &- \quad \max_s\left\{s\pi\int_0^\infty O(\tilde{z})^+ d\Gamma(\tilde{z})-\frac{c_e}{1+\eta_e}s^{1+\eta_e}\right\} \\ &+ \quad \max_v\left\{qv\left(\frac{u}{S}J(z)+\phi\frac{e}{S}\int_0^z J'(\tilde{z})G(\tilde{z})d\tilde{z}\right)-\frac{c_v}{1+\eta_v}e^z v^{1+\eta_v}\right\}\end{aligned} \tag{49}$$

subject to $O(\underline{z})=0$ and $O'(\underline{z})=0$, where $\underline{z}=0$.

Under the guess (47)

$$\begin{aligned}\mathbf{W}_n(z,n) \quad &= \quad J(z)+U \\ \mathbf{W}_z(z,n) \quad &= \quad nJ'(z)+O'(z)\end{aligned}$$

Substituting (48)–(49) into (5) and simplifying (using (9))

$$\begin{aligned}(\rho-m)\mathbf{W}(z,n) \quad &= \quad n\Big(\Big(e^z-1-mJ'(z)-\kappa J(z)\Big)+\Big(1-\kappa U\Big)\Big) \\ &+ \quad k-1-r-mO'(z)-\kappa O(z)-\max_s\left\{s\pi\int_0^\infty O(\tilde{z})^+ d\Gamma(\tilde{z})-\frac{c_e}{1+\eta_e}s^{1+\eta_e}\right\} \\ &+ \quad \max_v\left\{vq\left(\frac{u}{S}J(z)^+ +\frac{e\phi}{S}\int_{n',z'}\Big(J(z)-J(\tilde{z})\Big)^+ dG(\tilde{z})\right)-\frac{c_v}{1+\eta_v}e^z v^{1+\eta_v}\right\} \\ &+ \quad 1-\kappa U^f+\max_s\left\{s\pi\int_0^\infty O(\tilde{z})^+ d\Gamma(\tilde{z})-\frac{c_e}{1+\eta_e}s^{1+\eta_e}\right\} \\ &= \quad (\rho-m)\Big(n\Big(J(z)+U\Big)+\Big(O(z)+U^f\Big)\Big)\end{aligned}$$

which confirms the guess (47).

The surplus of a match is given by (48) for $z > 0$ and is $J(z) = 0$ otherwise. Hence, for $z > 0$

$$\mathbf{W}_n(z, n) \quad = \quad J(z) + U \quad > \quad U$$

since $J(z) > 0$, while for $z \leq 0$

$$\mathbf{W}_n(z, n) \quad = \quad J(z) + U \quad = \quad U$$

since $J(z) = 0$. Hence the separation boundary of the coalition and the match coincides.

The exit decision is the potentially problematic decision. Because the fixed cost can be split over many workers, size may matter for exit, thus requiring keeping track of size and stipulating a multilateral bargaining protocol. Under the assumption that the flow value of leisure satisfies $b = 1$, however, workers want to separate to unemployment at the same time as the entrepreneur wants to exit. Consequently, the exit threshold chosen by the entrepreneur trivially coincides with that which maximizes the joint value of a firm, since the firm consists of only the entrepreneur at the point of exit.

The surplus of an entrepreneur is given by (49) for $z > 0$ and $O(z) = 0$ otherwise. Hence, for $z > 0$

$$\mathbf{W}(z, n) \quad = \quad n\Big(J(z) + U\Big) + O(z) + U^f \quad > \quad nU + U^f$$

since $J(z) > 0$ and $O(z) > 0$, while for $z \leq 0$

$$\mathbf{W}(z, n) \quad = \quad n\Big(J(z) + U\Big) + O(z) + U^f \quad = \quad nU + U^f$$

since $J(z) = 0$ and $O(z) = 0$. Hence, individual matches and the entrepreneur behave in a way that also maximizes coalition value, verifying that the joint surplus can be written as (47).

Differentiating (11)

$$\begin{aligned} J'(z) &= -\frac{1}{\rho + \kappa - m} \frac{m}{\rho + \kappa} \frac{\rho + \kappa - m}{m} e^{-\frac{\rho+\kappa-m}{m} z} + \frac{1}{\rho + \kappa} e^{z} \\ &= \frac{1}{\rho + \kappa} \left( e^{z} - e^{-\frac{\rho+\kappa-m}{m} z} \right) \end{aligned} \tag{50}$$

Substituting for $J'(z)$ in (48)

$$\begin{aligned}
(\rho+\kappa-m)\,J(z) &= e^{z}-1-m\left(-\frac{1}{\rho+\kappa-m}\frac{m}{\rho+\kappa}\frac{\rho+\kappa-m}{m}e^{-\frac{\rho+\kappa-m}{m}z}+\frac{1}{\rho+\kappa}e^{z}\right)\\
&= e^{z}-1-\frac{m}{\rho+\kappa}\left(e^{z}-e^{-\frac{\rho+\kappa-m}{m}z}\right)\\
&= \frac{\rho+\kappa-m}{\rho+\kappa}e^{z}-1+\frac{m}{\rho+\kappa}e^{-\frac{\rho+\kappa-m}{m}z}\\
J(z) &= \frac{1}{\rho+\kappa-m}\left(\frac{m}{\rho+\kappa}e^{-\frac{\rho+\kappa-m}{m}z}-1\right)+\frac{1}{\rho+\kappa}e^{z}
\end{aligned}$$

which verifies (11). Imposing $\underline{z}^{w}=0$ in (11)

$$\begin{aligned}
J(0) &= \frac{1}{\rho+\kappa-m}\left(\frac{m}{\rho+\kappa}-1\right)+\frac{1}{\rho+\kappa}\\
&= -\frac{1}{\rho+\kappa-m}\frac{\rho+\kappa-m}{\rho+\kappa}+\frac{1}{\rho+\kappa} \;=\; 0
\end{aligned}$$

verifying the value matching condition. Substituting $\underline{z}^{w}=0$ into (50)

$$J'(0) \;=\; \frac{1}{\rho+\kappa}(1-1) \;=\; 0$$

verifying the smooth pasting condition. Hence, the surplus of a match (11) together with $\underline{z}^{w}=0$ solve the stopping time problem of a match.

The first-order condition for optimal vacancy creation in (49) is

$$\begin{aligned}
c_{v}e^{z}v(z)^{\eta_{v}} &= q\left(\frac{u}{S}J(z)+\phi\frac{e}{S}\int_{0}^{z}J'(\tilde{z})G(\tilde{z})d\tilde{z}\right)\\
v(z) &= \left(\frac{R(z)}{c_{v}e^{z}}\right)^{\frac{1}{\eta_{v}}} \qquad (51)
\end{aligned}$$

where

$$R(z) \;=\; q\left(\frac{u}{S}J(z)+\phi\frac{e}{S}\int_{0}^{z}J'(\tilde{z})G(\tilde{z})d\tilde{z}\right)$$

Since $v(z)=0$ for $z\leq 0$, imposing the boundary conditions $O(\underline{z})=0$ and $O'(\underline{z})=0$ in (49)

$$0 \;=\; k-1-r-\max_{s}\left\{s\pi\int_{0}^{\infty}O(\tilde{z})^{+}d\Gamma(\tilde{z})-\frac{c_{e}}{1+\eta_{e}}s^{1+\eta_{e}}\right\} \qquad (52)$$

Substituting (51) and (52) into (49)

$$
\begin{aligned}
(\rho - m)\, O(z) &= -mO'(z) - \kappa O(z) + \left(\frac{R(z)}{c_v e^z}\right)^{\frac{1}{\eta_v}} R(z) - \frac{c_v}{1+\eta_v} e^z \left(\frac{R(z)}{c_v e^z}\right)^{\frac{1+\eta_v}{\eta_v}} \\
(\rho + \kappa - m)\, O(z) &= -mO'(z) + \left(\frac{1}{c_v}\right)^{\frac{1}{\eta_v}} e^{-z\frac{1}{\eta_v}} R(z)^{\frac{1+\eta_v}{\eta_v}} - \frac{1}{1+\eta_v} R(z)^{\frac{1+\eta_v}{\eta_v}} \left(\frac{1}{c_v}\right)^{\frac{1}{\eta_v}} e^{-\frac{1}{\eta_v}} \\
&= -mO'(z) + \frac{\eta_v}{1+\eta_v} \left(\frac{1}{c_v}\right)^{\frac{1}{\eta_v}} e^{-z\frac{1}{\eta_v}} R(z)^{\frac{1+\eta_v}{\eta_v}} \qquad (53)
\end{aligned}
$$

One can verify that

$$
O(z) = \frac{\eta_v}{1+\eta_v} \left(\frac{1}{c_v}\right)^{\frac{1}{\eta_v}} \frac{1}{m} e^{-\frac{\rho+\kappa-m}{m} z} \int_0^z e^{\frac{\rho+\kappa-m}{m}\tilde{z} - \frac{1}{\eta_v}\tilde{z}} R(\tilde{z})^{\frac{1+\eta_v}{\eta_v}} d\tilde{z}
$$

and hence

$$
O'(z) = \frac{\eta_v}{1+\eta_v} \left(\frac{1}{c_v}\right)^{\frac{1}{\eta_v}} \frac{1}{m} \left( -\frac{\rho+\kappa-m}{m} e^{-\frac{\rho+\kappa-m}{m} z} \int_0^z e^{\frac{\rho+\kappa-m}{m}\tilde{z} - \frac{1}{\eta_v}\tilde{z}} R(\tilde{z})^{\frac{1+\eta_v}{\eta_v}} d\tilde{z} + e^{-\frac{1}{\eta_v}\tilde{z}} R(\tilde{z})^{\frac{1+\eta_v}{\eta_v}} \right)
$$

and $\underline{z} = 0$ solves this first-order ODE subject to the initial value conditions $O(0) = 0$ and $O'(0) = 0$.

The optimal search intensity is given by the first-order condition

$$
s = \left( \frac{\pi \int_0^\infty O(\tilde{z}) d\Gamma(\tilde{z})}{c_e} \right)^{\frac{1}{\eta_e}} \qquad (54)
$$

Substituting this into (52)

$$
\begin{aligned}
r &= k - 1 - \left( \left( \frac{\pi \int_0^\infty O(\tilde{z}) d\Gamma(\tilde{z})}{c_e} \right)^{\frac{1}{\eta_e}} \pi \int_0^\infty O(\tilde{z}) d\Gamma(\tilde{z}) - \frac{c_e}{1+\eta_e} \left( \frac{\pi \int_0^\infty O(\tilde{z}) d\Gamma(\tilde{z})}{c_e} \right)^{\frac{1+\eta_e}{\eta_e}} \right) \\
&= k - 1 - \frac{\eta_e}{1+\eta_e} c_e \left( \frac{\pi \int_0^\infty O(\tilde{z}) d\Gamma(\tilde{z})}{c_e} \right)^{\frac{1+\eta_e}{\eta_e}} \\
&= k - 1 - \frac{\eta_e}{1+\eta_e} c_e s^{1+\eta_e}
\end{aligned}
$$

which verifies (15). As long as the fixed cost is high enough that managers prefer to provide their services over enjoying leisure, $r \geq b = 1$, all managers participate in the market, $L = l$.

### C.4 Derivation of the FP equation for employment

Let $\hat{g}(z,t)$ denote the number of workers employed in firms with relative productivity $z$ at time $t$ (suppressing the dependence on $m$ and $\lambda$ to reduce clutter). Its evolution is characterized by the FP equation

$$\begin{aligned} \frac{\partial \hat{g}(z,t)}{\partial t} &= \underbrace{m\frac{\partial \hat{g}(z,t)}{\partial z}}_{\text{technological obsolescence}} - \underbrace{\kappa \hat{g}(z,t)}_{\text{retirement of worker}} - \underbrace{\phi \hat{p}(t)\Big(1-\hat{F}(z,t)\Big)\hat{g}(z,t)}_{\text{separations up the job ladder}} \\ &+ \hat{p}(t)\hat{f}(z,t)\left(\underbrace{\hat{u}(t)}_{\text{hires from unemployment}} + \underbrace{\phi\hat{G}(z,t)}_{\text{hires from below in the job ladder}}\right) \end{aligned} \tag{55}$$

where $\hat{f}(z,t)$ is the distribution of recruiting firms at time $t$

$$\hat{f}(z,t) = \frac{1}{\hat{V}(t)}\hat{v}(z,t)\hat{x}(z,t), \qquad \hat{V}(t) = \int_0^\infty \hat{v}(z,t)\hat{x}(z,t)dz$$

On the BGP, $\hat{v}(z,t) = v(z)$ and $\hat{x}(z,t) = x(z)L(t)$ such that $\hat{f}(z,t) = f(z)$ is stationary. Moreover, the job finding rate is constant,

$$\hat{p}(t) = \chi \hat{S}(t)^{1-\theta}\hat{V}(t)^{\theta-1} = \chi\Big(SN(t)\Big)^{1-\theta}\Big(VN(t)\Big)^{\theta-1} = \chi S^{1-\theta}V^{\theta-1}$$

where $S$ is total per capita search efficiency and $V$ total per capita vacancies. Finally, on a BGP, the number of unemployed workers as well as the number of workers at each point in the distribution must grow at the same rate as aggregate labor supply—$\hat{u}(t) = uN(t)$ and $\hat{g}(z,t) = g(z)eN(t)$—as must the number of workers employed below $z$—$\hat{G}(z,t) = G(z)eN(t)$. Since $N(t) = e^{\lambda t}(1+\xi+l)$,

$$\begin{aligned} \frac{\partial \hat{g}(z,t)}{\partial t} &= g(z)e\lambda N(t) \\ \frac{\partial \hat{g}(z,t)}{\partial z} &= g'(z)eN(t) \end{aligned}$$

Imposing these conditions in (55),

$$\begin{aligned} g(z)e\lambda N(t) &= mg'(z)eN(t) - \kappa g(z)eN(t) - \phi p\Big(1-F(z)\Big)g(z)eN(t) + pf(z)\Big(uN(t) + \phi G(z)eN(t)\Big) \\ g(z)\lambda &= mg'(z) - \kappa g(z) - \phi p\Big(1-F(z)\Big)g(z) + pf(z)\left(\frac{u}{e} + \phi G(z)\right) \end{aligned}$$

Note first that $\lim_{z\to\infty} G(z;m) = 1$ implies $\lim_{z\to\infty} g(z;m) = 0$. Integrating (25) from zero to infinity

$$\kappa + \lambda = -mg(0;m) - \phi p(m) \int_0^\infty \Big(1 - F(z;m)\Big) g(z;m)dz + p(m)\frac{u(m)}{e(m)} + \phi p(m) \int_0^\infty f(z;m)G(z;m)dz$$

Integrating the third term by parts and cancelling terms

$$\begin{aligned}
\kappa + \lambda &= -mg(0;m) - \phi p(m)\Big(1 - \int_0^\infty F(z;m)g(z;m)dz\Big) + p(m)\frac{u(m)}{e(m)} \\
&+ \phi p(m)\left(1 - \int_0^\infty F(z;m)g(z;m)dz\right) \\
\kappa + \lambda &= -mg(0;m) + p(m)\frac{u(m)}{e(m)}
\end{aligned}$$

Finally, using the fact that $u(m) + e(m) + \xi + l = 1 + \xi + l$

$$\begin{aligned}
p(m)u(m) &= \Big(\kappa + \lambda + mg(0;m)\Big)\Big(1 - u(m)\Big) \\
u(m)\Big(p(m) + \kappa + \lambda + mg(0;m)\Big) &= \kappa + \lambda + mg(0;m) \\
u(m) &= \frac{\kappa + \lambda + mg(0;m)}{p(m) + \kappa + \lambda + mg(0;m)}
\end{aligned}$$

### C.5 Proposition 2: The exit curve

Recall the non-homogenous *boundary value problem (BVP)* (21)–(24)

$$\begin{aligned}
0 &= mx'(z) + \frac{y}{l}\zeta e^{-\zeta z} \\
X(0) &= 0 \\
\lim_{z\to\infty} X(z) &= 1 \\
x(0) &= \frac{y}{ml}
\end{aligned}$$

Integrating the ODE (21) from 0 to $z$

$$0 = c_1 + x(z) - \frac{y}{ml}e^{-\zeta z} \tag{56}$$

Imposing the condition that $x(0) = \frac{y}{ml}$ in (56)

$$\begin{aligned} 0 &= c_1 + x(0) - \frac{y}{ml} \\ c_1 &= 0 \end{aligned}$$

Substituting $c_1 = 0$ in (56) and again integrating from 0 to $z$

$$\begin{aligned} 0 &= c_2 + X(z) + \frac{y}{\zeta ml} e^{-\zeta z} \\ X(z) &= -\left( c_2 + \frac{y}{\zeta ml} e^{-\zeta z} \right) \end{aligned}$$

The first boundary condition (22) implies that

$$\begin{aligned} X(0) &= -\left( c_2 + \frac{y}{\zeta ml} \right) &= 0 \\ c_2 &= -\frac{y}{\zeta ml} \end{aligned}$$

The second boundary condition (23) implies that

$$\begin{aligned} \lim_{x \to \infty} X(z) &= \frac{y}{\zeta ml} &= 1 \\ y &= \zeta ml \end{aligned}$$

Combining these insights

$$\begin{aligned} X(z) &= \frac{y}{\zeta ml} \left( 1 - e^{-\zeta z} \right) \\ X(z) &= 1 - e^{-\zeta z} \\ x(z) &= \zeta e^{-\zeta z} \end{aligned}$$

### C.6 Proposition 3: Obsolescence vs capitalization effects

Differentiating the surplus of a match (11) with respect to $m$, for all $m \in [0, \rho + \kappa)$, gives (32)

$$\begin{aligned}
\frac{\partial J(z;m)}{\partial m} &= -\frac{e^{-\frac{\rho+\kappa-m}{m}z}}{(\rho+\kappa-m)^2}\left(e^{\frac{\rho+\kappa-m}{m}z} - \left(1 + z\frac{\rho+\kappa-m}{m}\right)\right) \\
&= -\frac{e^{-\frac{\rho+\kappa-m}{m}z}}{(\rho+\kappa-m)^2}\left(1 + \frac{\rho+\kappa-m}{m}z + \frac{\left(\frac{\rho+\kappa-m}{m}z\right)^2}{2!} + \cdots - \left(1 + z\frac{\rho+\kappa-m}{m}\right)\right) \\
&= -\frac{e^{-\frac{\rho+\kappa-m}{m}z}}{(\rho+\kappa-m)^2}\left(\frac{\left(\frac{\rho+\kappa-m}{m}z\right)^2}{2!} + \frac{\left(\frac{\rho+\kappa-m}{m}z\right)^3}{3!} + \cdots\right) \quad < \quad 0
\end{aligned}$$

I assume throughout that $\rho + \kappa > m$ holds, since otherwise utility is infinite. Hence, $\partial J(z;m)/\partial m < 0$ for all $z > 0$. Differentiating the surplus of a match (11), the marginal surplus of a match writes

$$J'(z;m) \quad = \quad \frac{1}{\rho+\kappa}e^{z}\left(1 - e^{-\frac{\rho+\kappa}{m}z}\right) \tag{57}$$

Differentiating the marginal surplus of a match (57) with respect to $m$

$$\frac{\partial J'(z;m)}{\partial m} \quad = \quad -\frac{1}{m^2}e^{-\frac{\rho+\kappa-m}{m}z}z \tag{58}$$

which clearly is negative for all $z > 0$.

### C.7 Proposition 4: Employment distribution

When the offer distribution is exponential, (27) becomes (omitting the dependence on $m$ to reduce clutter)

$$g(z) \underbrace{-\frac{\kappa + \lambda + \phi p e^{-\zeta z}}{m}}_{\equiv x(z)} G(z) \quad = \quad \underbrace{-\frac{\kappa + \lambda - \frac{pue^{-\zeta z}}{1-u}}{m}}_{\equiv y(x)}$$

Let us first solve the homogenous equation

$$\begin{aligned} g(z)+x(z)G(z) &= 0 \\ \frac{g(z)}{G(z)} &= -x(z) \\ \log G(z) &= \log C + \int_0^z -x(z)dz \\ G(z) &= C\underbrace{e^{-\int_0^z x(\tilde{z})d\tilde{z}}}_{\equiv h(z)} \\ h'(z) &= Ce^{-\int_0^z x(\tilde{z})d\tilde{z}}(-x(z)) \\ &= -h(z)x(z) \end{aligned}$$

For the non-homogenous equation, apply variation of parameters

$$\begin{aligned} \tilde{G}(z) &= v(z)h(z) \\ \tilde{g}(z) &= v'(z)h(z)+v(z)h'(z) \\ \tilde{g}(z) &= v'(z)h(z)-v(z)h(z)x(z) \\ \tilde{g}(z) &= v'(z)h(z)-\tilde{G}(z)x(z) \\ \tilde{g}(z)+x(z)\tilde{G}(z) &= v'(z)h(z) \end{aligned}$$

For $\tilde{G}(z)$ to solve the non-homogenous equation, we need

$$\begin{aligned} \tilde{g}(z)+x(z)\tilde{G}(z) &= y(x) \\ v'(z)h(z) &= y(x) \\ v'(z) &= \frac{y(x)}{h(z)} \\ v(z) &= K+\int_0^z \frac{y(\tilde{z})}{h(\tilde{z})}d\tilde{z} \end{aligned}$$

Hence, the general solution is

$$G(z) = \left(K+\int_0^z \frac{y(\tilde{z})}{h(\tilde{z})}d\tilde{z}\right)e^{\frac{\kappa+\lambda}{m}z+\frac{\phi p}{m\zeta}\left(1-e^{-\zeta z}\right)}$$

Since the initial value condition $G(0) = 0$ requires $K = 0$

$$\begin{aligned}
G(z) &= e^{\frac{\kappa+\lambda}{m}z+\frac{\phi p}{m\zeta}\left(1-e^{-\zeta z}\right)}\int_0^z \frac{-\frac{\kappa+\lambda-\frac{pue^{-\zeta\tilde{z}}}{1-u}}{m}}{e^{\frac{\kappa+\lambda}{m}\tilde{z}+\frac{\phi p}{m\zeta}(1-e^{-\zeta\tilde{z}})}}d\tilde{z} \\
&= \frac{1}{m}\int_0^z e^{\frac{\phi p}{m\zeta}e^{-\zeta\tilde{z}}-\frac{\kappa+\lambda}{m}\tilde{z}-\left(\frac{\phi p}{m\zeta}e^{-\zeta z}-\frac{\kappa+\lambda}{m}z\right)}\left(\frac{pue^{-\zeta\tilde{z}}}{1-u}-(\kappa+\lambda)\right)d\tilde{z}
\end{aligned}$$

where $u$ is such that

$$\begin{aligned}
\lim_{z\to\infty} G(z) &= 1 \\
\frac{1}{m}\lim_{z\to\infty} e^{-\frac{\phi pe^{-\zeta z}}{m\zeta}+\frac{\kappa+\lambda}{m}z}\int_0^z e^{\frac{\phi pe^{-\zeta\tilde{z}}}{m\zeta}-\frac{\kappa+\lambda}{m}\tilde{z}-\zeta\tilde{z}}\left(\frac{pu}{1-u}-(\kappa+\lambda)e^{\zeta\tilde{z}}\right)d\tilde{z} &= 1
\end{aligned} \tag{59}$$

Is there always a $\frac{u}{1-u}$ such that (59) holds? Consider the limit

$$\frac{1}{m}\lim_{z\to\infty}\underbrace{e^{-\frac{\phi pe^{-\zeta z}}{m\zeta}+\frac{\kappa+\lambda}{m}z}}_{\equiv h(z)}\underbrace{\int_0^z e^{\frac{\phi pe^{-\zeta\tilde{z}}}{m\zeta}-\frac{\kappa+\lambda}{m}\tilde{z}-\zeta\tilde{z}}\left(\frac{pu}{1-u}-(\kappa+\lambda)e^{\zeta\tilde{z}}\right)d\tilde{z}}_{\equiv A(z)}$$

Since $\lim_{z\to\infty} h(z) = \infty$, such a $u$ must at the very least ensure that

$$\begin{aligned}
\lim_{z\to\infty} A(z) &= 0 \\
\lim_{z\to\infty}\int_0^z e^{\frac{\phi pe^{-\zeta\tilde{z}}}{m\zeta}-\frac{\kappa+\lambda}{m}\tilde{z}-\zeta\tilde{z}}\left(\frac{pu}{1-u}-(\kappa+\lambda)e^{\zeta\tilde{z}}\right)d\tilde{z} &= 0 \\
\frac{pu}{1-u}\lim_{z\to\infty}\int_0^z e^{\frac{\phi pe^{-\zeta\tilde{z}}}{m\zeta}-\frac{\kappa+\lambda}{m}\tilde{z}-\zeta\tilde{z}}d\tilde{z} &= (\kappa+\lambda)\lim_{z\to\infty}\int_0^z e^{\frac{\phi pe^{-\zeta\tilde{z}}}{m\zeta}-\frac{\kappa+\lambda}{m}\tilde{z}}d\tilde{z} \\
\frac{pu}{1-u} &= (\kappa+\lambda)\frac{\int_0^\infty e^{\frac{\phi pe^{-\zeta\tilde{z}}}{m\zeta}-\frac{\kappa+\lambda}{m}\tilde{z}}d\tilde{z}}{\int_0^\infty e^{\frac{\phi pe^{-\zeta\tilde{z}}}{m\zeta}-\frac{\kappa+\lambda}{m}\tilde{z}-\zeta\tilde{z}}d\tilde{z}}
\end{aligned}$$

Not only, however, does $u$ have to be such that $\lim_{z\to\infty} A(z) = 0$, it must be such that $A(z)$ goes to zero as exactly the rate that $h(z)$ goes to infinity as $z \to \infty$, such that the two forces offset. Moreover, the product must equal exactly $m$, such that $\lim_{z\to\infty}\frac{1}{m}h(z)A(z) = 1$.

To see that this choice of $u$ ensures this more stringent condition, note that with some abuse of terminology, we can write

$$A(z) = A(\infty) - \underbrace{\int_z^\infty e^{\frac{\phi pe^{-\zeta\tilde{z}}}{m\zeta}-\frac{\kappa+\lambda}{m}\tilde{z}-\zeta\tilde{z}}\left(\frac{pu}{1-u}-(\kappa+\lambda)e^{\zeta\tilde{z}}\right)d\tilde{z}}_{\equiv a(z)}$$

where under our particular choice of $u$

$$A(\infty) \quad = \quad 0$$

Consider the behavior of

$$a(z) \quad = \quad \int_z^\infty e^{\frac{\phi p e^{-\zeta \tilde{z}}}{m\zeta} - \frac{\kappa+\lambda}{m}\tilde{z} - \zeta\tilde{z}} \left( \frac{pu}{1-u} - (\kappa+\lambda) e^{\zeta \tilde{z}} \right) d\tilde{z}$$

as $z \to \infty$. For sufficiently large $z$, the second term in the parenthesis dominates the first, such that $a(z)$ is well-approximated as

$$a(z) \quad \approx \quad -(\kappa+\lambda) \int_z^\infty e^{\frac{\phi p e^{-\zeta \tilde{z}}}{m\zeta} - \frac{\kappa+\lambda}{m}\tilde{z}} d\tilde{z}$$

Moreover, for sufficiently large $z$, the contribution of the $\frac{\phi p e^{-\zeta \tilde{z}}}{m\zeta}$ term becomes small, so that $a(z)$ can be well-approximated as

$$a(z) \quad \approx \quad -(\kappa+\lambda) \int_z^\infty e^{-\frac{\kappa+\lambda}{m}\tilde{z}} d\tilde{z} \quad = \quad -(\kappa+\lambda) \left[ -\frac{m}{\kappa+\lambda} e^{-\frac{\kappa+\lambda}{m}\tilde{z}} \right]_{\tilde{z}=z}^\infty \quad = \quad -m e^{-\frac{\kappa+\lambda}{m} z}$$

Combining these insights, the product $\lim_{z\to\infty} \frac{1}{m} h(z) A(z)$ is hence

$$\frac{1}{m} \lim_{z\to\infty} h(z) \left( \underbrace{A(\infty)}_{\equiv 0} - A(z) \right) \quad = \quad \frac{1}{m} \lim_{z\to\infty} e^{-\frac{\phi p e^{-\zeta z}}{m\zeta} + \frac{\kappa+\lambda}{m} z} m e^{-\frac{\kappa+\lambda}{m} z} \quad = \quad 1$$

Hence, the choice of $u$ (59) ensures that $\lim_{z\to\infty} G(z) = 1$.

## C.8 Proposition 5: The misallocation effect

Suppose for small $m$ that $G(z)$ can be well-approximated as

$$\begin{aligned} G(z) \quad &= \quad \frac{\kappa+\lambda}{\kappa+\lambda+\phi p e^{-\zeta z}} \left(1 - e^{-\zeta z}\right) \\ &+ \quad m \frac{(\kappa+\lambda)(\kappa+\lambda+\phi p)}{\kappa+\lambda+\phi p e^{-\zeta z}} \zeta e^{-\zeta z} \left( \frac{1}{\left(\kappa+\lambda+\phi p e^{-\zeta z}\right)^2} - \frac{1}{\left(\kappa+\lambda+\phi p\right)^2} \right) \\ &+ \quad O(m^2) \end{aligned} \tag{60}$$

where $O(m^2)$ are second and higher-order terms in $m$. Then

$$g(z) = \frac{(\kappa+\lambda)(\kappa+\lambda+\phi p)}{(\kappa+\lambda+\phi p e^{-\zeta z})^2}\zeta e^{-\zeta z} + O(m^1)$$

where $O(m^1)$ are first and higher-order terms in $m$. Under the guess

$$\begin{aligned} g(z) - g(0)e^{-\zeta z} &= \frac{(\kappa+\lambda)(\kappa+\lambda+\phi p)}{(\kappa+\lambda+\phi p e^{-\zeta z})^2}\zeta e^{-\zeta z} + O(m^1) - e^{-\zeta z}\left(\frac{(\kappa+\lambda)(\kappa+\lambda+\phi p)}{(\kappa+\lambda+\phi p)^2}\zeta + O(m^1)\right) \\ &= (\kappa+\lambda)(\kappa+\lambda+\phi p)\,\zeta e^{-\zeta z}\left(\frac{1}{(\kappa+\lambda+\phi p e^{-\zeta z})^2} - \frac{1}{(\kappa+\lambda+\phi p)^2}\right) + O(m^1) \end{aligned}$$

Furthermore

$$\begin{aligned} &\left(\kappa+\lambda+\phi p e^{-\zeta z}\right) G(z) \\ = \;& (\kappa+\lambda)\left(1 - e^{-\zeta z} + m(\kappa+\lambda+\phi p)\,\zeta e^{-\zeta z}\left(\frac{1}{(\kappa+\lambda+\phi p e^{-\zeta z})^2} - \frac{1}{(\kappa+\lambda+\phi p)^2}\right)\right) + O(m^2) \end{aligned}$$

Substituting this into (27)

$$\begin{aligned} & m\left((\kappa+\lambda)(\kappa+\lambda+\phi p)\,\zeta e^{-\zeta z}\left(\frac{1}{(\kappa+\lambda+\phi p e^{-\zeta z})^2} - \frac{1}{(\kappa+\lambda+\phi p)^2}\right)\right) + O(m^2) \\ = \;& (\kappa+\lambda)\left(1-e^{-\zeta z}\right) + m(\kappa+\lambda)(\kappa+\lambda+\phi p)\,\zeta e^{-\zeta z}\left(\frac{1}{(\kappa+\lambda+\phi p e^{-\zeta z})^2} - \frac{1}{(\kappa+\lambda+\phi p)^2}\right) \\ - \;& (\kappa+\lambda)\left(1-e^{-\zeta z}\right) + O(m^2) \end{aligned}$$

verifying the guess. Note that the guess also satisfies the boundary condition $G(0) = 0$ as well as $\lim_{z\to\infty} G(z) = 1$. Hence for small $m$, the employment distribution can be approximated to a first-order as

$$\begin{aligned} G(z) &\approx \frac{\kappa+\lambda}{\kappa+\lambda+\phi p e^{-\zeta z}}\left(1 - e^{-\zeta z} + m(\kappa+\lambda+\phi p)\,\zeta e^{-\zeta z}\left(\frac{1}{(\kappa+\lambda+\phi p e^{-\zeta z})^2} - \frac{1}{(\kappa+\lambda+\phi p)^2}\right)\right) \\ &\approx \frac{1}{1+\beta e^{-\zeta z}}\left(1 - e^{-\zeta z} + \frac{m}{\kappa+\lambda}(1+\beta)\,\zeta e^{-\zeta z}\left(\frac{1}{(1+\beta e^{-\zeta z})^2} - \frac{1}{(1+\beta)^2}\right)\right) \end{aligned}$$

Since the unemployment rate is given by (26), the share of unemployed among all job seekers $S =$

$u + \phi(1-u)$ is

$$\frac{u(m)}{S(m)} = \frac{\frac{\kappa+\lambda+mg(0)}{\kappa+\lambda+mg(0)+p}}{\frac{\kappa+\lambda+mg(0)}{\kappa+\lambda+mg(0)+p} + \phi\left(1 - \frac{\kappa+\lambda+mg(0)}{\kappa+\lambda+mg(0)+p}\right)} = \frac{\kappa+\lambda+mg(0)}{\kappa+\lambda+mg(0)+\phi p}$$

Differentiating this with respect to $m$ and evaluating the derivative at $\frac{m}{\kappa+\lambda} \ll 1$ small

$$\frac{\partial \frac{u(m)}{S(m)}}{\partial m} \approx \frac{\beta}{(1+\beta)^3} \frac{1}{\kappa+\lambda} \zeta$$

Differentiating (60) with respect to $m$ and evaluating the derivative at $\frac{m}{\kappa+\lambda} \ll 1$ small

$$\begin{aligned}
\frac{\partial G(z)}{\partial m} &\approx \frac{1}{1+\beta e^{-\zeta z}} \frac{1}{\kappa+\lambda} (1+\beta)\, \zeta e^{-\zeta z} \left( \frac{1}{(1+\beta e^{-\zeta z})^2} - \frac{1}{(1+\beta)^2} \right) \\
&\approx \frac{1}{1+\beta e^{-\zeta z}} \frac{1}{\kappa+\lambda} (1+\beta)\, \zeta e^{-\zeta z} \left( \frac{(1+\beta)^2 - (1+\beta e^{-\zeta z})^2}{(1+\beta e^{-\zeta z})^2 (1+\beta)^2} \right) \\
&\approx \frac{1}{1+\beta e^{-\zeta z}} \frac{1}{\kappa+\lambda} \zeta e^{-\zeta z} \left( \frac{1 + 2\beta + \beta^2 - 1 - 2\beta e^{-\zeta z} - \beta^2 e^{-2\zeta z}}{(1+\beta e^{-\zeta z})^2 (1+\beta)} \right) \\
&\approx \frac{1}{1+\beta e^{-\zeta z}} \frac{1}{\kappa+\lambda} \zeta e^{-\zeta z} \beta \left( \frac{2(1-e^{-\zeta z}) + \beta(1-e^{-2\zeta z})}{(1+\beta e^{-\zeta z})^2 (1+\beta)} \right) \\
&\approx \frac{1}{\kappa+\lambda} \zeta e^{-\zeta z} \frac{\beta}{1+\beta} \frac{2(1-e^{-\zeta z}) + \beta(1-e^{-2\zeta z})}{(1+\beta e^{-\zeta z})^3} \\
&\approx \frac{1}{\kappa+\lambda} \zeta e^{-\zeta z} \frac{\beta}{1+\beta} \frac{2(1-e^{-\zeta z}) + \beta(1+e^{-\zeta z})(1-e^{-\zeta z})}{(1+\beta e^{-\zeta z})^3} \\
&\approx \frac{1}{\kappa+\lambda} \frac{\beta}{1+\beta} \frac{2 + \beta(1+e^{-\zeta z})}{(1+\beta e^{-\zeta z})^3} \zeta e^{-\zeta z} (1 - e^{-\zeta z})
\end{aligned}$$

## C.9 Proposition 6: Growth and the return to hiring

Recall the return to job creation (13)

$$R(z;m) = q \left( \frac{u(m)}{S(m)} J(z;m) + \left(1 - \frac{u(m)}{S(m)}\right) \int_0^z J'(\tilde{z};m) G(\tilde{z};m) d\tilde{z} \right)$$

where the worker finding rate $q$ is independent of equilibrium objects under the simplifying assumptions that $\eta_v \to \infty$ and $\theta \to 1$. Differentiating with respect to the growth rate $m$

$$\begin{aligned}
\frac{\partial R(z;m)}{\partial m} &= q\frac{\partial}{\partial m}\left(\frac{u(m)}{S(m)}\right)\left(J(z;m) - \int_0^z J'(\tilde{z};m)G(\tilde{z};m)d\tilde{z}\right) \\
&+ q\left(\frac{u(m)}{S(m)}\frac{\partial J(z;m)}{\partial m} + \left(1-\frac{u(m)}{S(m)}\right)\int_0^z\left(\frac{\partial J'(\tilde{z};m)}{\partial m}G(\tilde{z};m) + J'(\tilde{z};m)\frac{\partial G(\tilde{z};m)}{\partial m}\right)d\tilde{z}\right) \\
&= q\left(\frac{\partial}{\partial m}\left(\frac{u(m)}{S(m)}\right)\left(J(z;m)(1-G(z;m)) + \int_0^z J(\tilde{z};m)g(\tilde{z};m)d\tilde{z}\right)\right. \qquad (61)\\
&+ \left.\left(\frac{u(m)}{S(m)}\frac{\partial J(z;m)}{\partial m} + \left(1-\frac{u(m)}{S(m)}\right)\int_0^z\left(\frac{\partial J'(\tilde{z};m)}{\partial m}G(\tilde{z};m) + J'(\tilde{z};m)\frac{\partial G(\tilde{z};m)}{\partial m}\right)d\tilde{z}\right)\right)
\end{aligned}$$

where the second equality follows from integration by parts.

For small $m$, we have

$$\lim_{m\to 0} J(z;m) = \frac{1}{\rho+\kappa}e^z \qquad (62)$$

$$\lim_{m\to 0} J'(z;m) = \frac{1}{\rho+\kappa}e^z \qquad (63)$$

$$\lim_{m\to 0} \frac{\partial J(z;m)}{\partial m} = -\frac{1}{(\rho+\kappa)^2} \qquad (64)$$

$$\lim_{m\to 0} \frac{\partial J'(z;m)}{\partial m} = 0 \qquad (65)$$

$$\lim_{m\to 0} \frac{u(m)}{S(m)} = \frac{\kappa+\lambda}{\kappa+\lambda+\phi p} \qquad (66)$$

$$\lim_{m\to 0} G(z;m) = \frac{(\kappa+\lambda)\left(1-e^{-\zeta z}\right)}{\kappa+\lambda+\phi p e^{-\zeta z}} \qquad (67)$$

$$\lim_{m\to 0} g(z;m) = \frac{(\kappa+\lambda+\phi p)(\kappa+\lambda)}{\left(\kappa+\lambda+\phi p e^{-\zeta z}\right)^2}\zeta e^{-\zeta z} \qquad (68)$$

Using these observations as well as (35)–(36) in (61)

$$\begin{aligned}
\lim_{m\to 0}\frac{\partial R(z;m)}{\partial m} &= q\frac{1}{\rho+\kappa}\frac{1}{1+\beta}\left(-\frac{1}{\rho+\kappa} + \frac{\beta}{1+\beta}\frac{\zeta}{\kappa+\lambda}\right. \qquad (69)\\
&\times \left.\left(\frac{e^{z(1-\zeta)}}{1+\beta e^{-\zeta z}} + \int_0^z \frac{e^{\tilde{z}(1-\zeta)}\left(\zeta\left(1+\beta e^{-\zeta\tilde{z}}\right) + \beta\left(2+\beta\left(1+e^{-\zeta\tilde{z}}\right)\right)\left(1-e^{-\zeta\tilde{z}}\right)\right)}{\left(1+\beta e^{-\zeta\tilde{z}}\right)^3}d\tilde{z}\right)\right)
\end{aligned}$$

The term multiplying the parenthesis in (69) is a positive constant. Hence, for small $m$ the sign of the derivative $\partial R(z;m)/\partial m$ is determined by the sign of the term in the parenthesis in (69)

$$-\frac{1}{\rho+\kappa}+\frac{\beta}{1+\beta}\frac{\zeta}{\kappa+\lambda}\left(\frac{e^{z(1-\zeta)}}{1+\beta e^{-\zeta z}}+\int_0^z \frac{e^{\tilde{z}(1-\zeta)}\left(\zeta\left(1+\beta e^{-\zeta\tilde{z}}\right)+\beta\left(2+\beta\left(1-e^{-\zeta\tilde{z}}\right)\right)\left(1-e^{-\zeta\tilde{z}}\right)\right)}{\left(1+\beta e^{-\zeta\tilde{z}}\right)^3}d\tilde{z}\right) \tag{70}$$

The first term in (70) is clearly negative, while the second term is positive. Note that as $\phi\to 0$, $\beta = p\phi/(\kappa+\lambda)\to 0$ and hence the second term in (70) tends to 0 as $\phi\to 0$. It follows from continuity that

$$\exists\phi_1\in(0,\infty): \qquad \lim_{m\to 0}\frac{\partial R(z;m)}{\partial m} \;<\; 0, \qquad \forall\phi<\phi_1$$

Note next that as $\phi\to\infty$, $\beta = p\phi/(\kappa+\lambda)\to\infty$. Moreover

$$\lim_{\beta\to\infty}\underbrace{\frac{\beta}{1+\beta}}_{\to 1}\frac{\zeta}{\kappa+\lambda}\left(\underbrace{\frac{e^{z(1-\zeta)}}{1+\beta e^{-\zeta z}}}_{\to 0}+\int_0^z \underbrace{\frac{e^{\tilde{z}(1-\zeta)}\left(\zeta\left(1+\beta e^{-\zeta\tilde{z}}\right)+\beta\left(2+\beta\left(1+e^{-\zeta\tilde{z}}\right)\right)\left(1-e^{-\zeta\tilde{z}}\right)\right)}{\left(1+\beta e^{-\zeta\tilde{z}}\right)^3}}_{\to 0}d\tilde{z}\right)$$

Hence, the second term in (70) tends to 0 as $\phi\to\infty$. It follows from continuity that

$$\exists\phi_2\in(0,\infty): \qquad \lim_{m\to 0}\frac{\partial R(z;m)}{\partial m} \;<\; 0, \qquad \forall\phi>\phi_2$$

Finally, note that the discount rate $\rho$ only enters the first term in (70), which falls in absolute terms as $\rho$ grows and becomes arbitrarily small as $\rho$ increases without bound. Consequently, for any positive but finite $\phi\in(0,\infty)$, there always exists $\hat{\rho}<\infty$ such that the second term in (70) is larger than the first

$$\exists\tilde{\rho}\in(0,\infty): \qquad \lim_{m\to 0}\frac{\partial R(z;m)}{\partial m} \;>\; 0, \qquad \forall\rho>\tilde{\rho}$$

### C.10 Proposition 7: Growth and firm creation

Imposing $\eta_v\to\infty$ in the ODE characterizing the surplus of an entrepreneur (53)

$$(\rho+\kappa-m)\,O(z) \;=\; -mO'(z)+R(z;m) \tag{71}$$

For small $m$

$$\lim_{m\to 0}(\rho+\kappa)\,O(z;m) \;=\; \lim_{m\to 0}R(z;m) \tag{72}$$

$$\lim_{m\to 0}(\rho+\kappa)\,O'(z;m) \;=\; \lim_{m\to 0}R'(z;m) \tag{73}$$

Differentiating both sides of (71) with respect to $m$ and evaluating it for small $m$

$$(\rho+\kappa)\lim_{m\to 0}\frac{\partial O(z;m)}{\partial m}-\lim_{m\to 0}O(z;m) \quad = \quad -\lim_{m\to 0}O'(z;m)+\lim_{m\to 0}\frac{\partial R(z;m)}{\partial m}$$

Rearranging, using (72)–(73), and dropping the small $m$ notation to reduce clutter

$$(\rho+\kappa)\frac{\partial O(z)}{\partial m} \quad = \quad \frac{1}{\rho+\kappa}\left(R(z)-R'(z)\right)+\frac{\partial R(z)}{\partial m} \tag{74}$$

The derivative of the return to hiring (13) is

$$R'(z) \quad = \quad q\left(\frac{u}{S}J'(z)+\frac{\phi e}{S}J'(z)G(z)\right) \tag{75}$$

Consequently,

$$R(z)-R'(z) \quad = \quad q\left(\frac{u}{S}J(z)+\frac{\phi e}{S}\int_0^z J'(\tilde{z})G(\tilde{z})d\tilde{z}-\frac{u}{S}J'(z)-\frac{\phi e}{S}J'(z)G(z)\right)$$

Using (62)–(63) to substitute for the surplus and marginal surplus of a match, and simplifying

$$R(z)-R'(z) \quad = \quad q\frac{\phi e}{S}\left(\int_0^z J'(\tilde{z})G(\tilde{z})d\tilde{z}-J'(z)G(z)\right)$$

Integrating by parts

$$R(z)-R'(z) \quad = \quad q\frac{\phi e}{S}\left(J(z)G(z)-\int_0^z J(\tilde{z})g(\tilde{z})d\tilde{z}-J'(z)G(z)\right)$$

and again using (62)–(63)

$$R(z)-R'(z) \quad = \quad -q\frac{\phi e}{S}\frac{1}{\rho+\kappa}\int_0^z e^{\tilde{z}}g(\tilde{z})d\tilde{z}$$

Substituting this in (74), using (66) to substitute for $\lim_{m\to 0}(e(m)/S(m))$, (68) to substitute for $\lim_{m\to 0}g(\tilde{z};m)$

and (69) to substitute for $\lim_{m\to 0} \partial R(z;m)/\partial m$

$$
\begin{aligned}
\frac{\partial O(z)}{\partial m} &= \frac{q}{(\rho+\kappa)^2}\left(-\frac{1}{\rho+\kappa}\left(\beta\zeta\int_0^z \frac{e^{\tilde{z}(1-\zeta)}}{\left(1+\beta e^{-\zeta\tilde{z}}\right)^2}d\tilde{z}+\frac{1}{1+\beta}\right)\right.\\
&+ \left.\frac{\beta}{(1+\beta)^2}\frac{\zeta}{\kappa+\lambda}\left(\frac{e^{z(1-\zeta)}}{1+\beta e^{-\zeta z}}+\int_0^z \frac{e^{\tilde{z}(1-\zeta)}\left(\zeta\left(1+\beta e^{-\zeta\tilde{z}}\right)+\beta\left(2+\beta\left(1+e^{-\zeta\tilde{z}}\right)\right)\left(1-e^{-\zeta\tilde{z}}\right)\right)}{\left(1+\beta e^{-\zeta\tilde{z}}\right)^3}d\tilde{z}\right)\right)\\
&= \frac{q}{(\rho+\kappa)^2}\frac{1}{1+\beta}\left(-\frac{1}{\rho+\kappa}\left(\beta(1+\beta)\zeta\int_0^z \frac{e^{\tilde{z}(1-\zeta)}}{\left(1+\beta e^{-\zeta\tilde{z}}\right)^2}d\tilde{z}+1\right)\right.\\
&+ \left.\frac{\beta}{1+\beta}\frac{\zeta}{\kappa+\lambda}\left(\frac{e^{z(1-\zeta)}}{1+\beta e^{-\zeta z}}+\int_0^z \frac{e^{\tilde{z}(1-\zeta)}\left(\zeta\left(1+\beta e^{-\zeta\tilde{z}}\right)+\beta\left(2+\beta\left(1+e^{-\zeta\tilde{z}}\right)\right)\left(1-e^{-\zeta\tilde{z}}\right)\right)}{\left(1+\beta e^{-\zeta\tilde{z}}\right)^3}d\tilde{z}\right)\right)
\end{aligned}
$$

The term multiplying the parenthesis is strictly positive. For sufficiently high $\rho$, the first term inside the parenthesis is smaller than the second, such that the derivative is positive. For small $m$, the slope of the entry curve is

$$
y'(m) = \pi(\xi - l)\left(\frac{\pi}{c_e}\right)^{\frac{1}{\eta_e}}\frac{1}{\eta_e}\left(\int_0^\infty O(z;m)d\Gamma(z)\right)^{\frac{1}{\eta_e}-1}\int_0^\infty \frac{\partial O(z;m)}{\partial m}d\Gamma(z)
$$

which hence is positive for sufficiently high $\rho$.

### C.11 Proposition 8: Existence and uniqueness of the equilibrium

The analysis above has established that for a given growth rate $m \in (0, \rho+\kappa)$, there exists a unique surplus of a match, distribution of entrepreneurs and distribution of workers. Hence, to show that there exists at least one equilibrium, it suffices to show that the exit curve (30) and entry curve (31) cross at least once for $m \in (0, \rho+\kappa)$.

The limits of the exit curve (30) are

$$
\begin{aligned}
\lim_{m\to 0}\hat{y}(m) &= 0\\
\lim_{m\to\rho+\kappa}\hat{y}(m) &= \zeta l(\rho+\kappa) > 0
\end{aligned}
$$

Using the fact that the finding rates are fixed when $\eta_v \to \infty$ and $\theta \to 1$, the return to hiring (13) simplifies to

$$
R(z;m) = q\left(\frac{u(m)}{S(m)}J(z;m)+\frac{1-u(m)}{S(m)}\int_0^z J'(\tilde{z};m)G(\tilde{z};m)d\tilde{z}\right)
$$

Since $J(z;m)$ and $J'(z;m)$ are both strictly positive and bounded for $z \in (0,\infty)$ and $m \in [0, \rho+\kappa]$,

$u(m)/S(m) \in (0,1)$ and $G(z;m) \in [0,1]$, it follows that $R(z;m)$ is strictly positive and bounded for $m \in [0,\rho+\kappa]$ and $z \in (0,\infty)$.

When $\eta_v \to \infty$, the entry curve (31) reduces to

$$y(m) = \pi(\xi - l)\left(\frac{\pi}{c_e}\right)^{\frac{1}{\eta_e}}\left(\int_0^\infty \int_0^z \frac{e^{\frac{\rho+\kappa}{m}(\tilde{z}-z)}}{m} R(\tilde{z};m)d\tilde{z}d\Gamma(z)\right)^{\frac{1}{\eta_e}}$$

Since $R(z;m)$ is strictly positive and bounded for $z \in (0,\infty)$ and by assumption the tail of the innovation distribution is sufficiently thin, the entry rate is strictly positive and bounded for $m \in (0,\rho+\kappa)$. In particular,

$$\begin{aligned}
\lim_{m\to 0} y(m) &= \pi(\xi - l)\left(\frac{\pi}{c_e}\right)^{\frac{1}{\eta_e}}\left(\frac{1}{\rho+\kappa}\int_0^\infty \lim_{m\to 0} R(z;m)d\Gamma(z)\right)^{\frac{1}{\eta_e}} \in (0,\infty) \\
\lim_{m\to\rho+\kappa} y(m) &= \pi(\xi - l)\left(\frac{\pi}{c_e}\right)^{\frac{1}{\eta_e}}\left(\frac{1}{\rho+\kappa}\int_0^\infty \int_0^z e^{-(z-\tilde{z})} \lim_{m\to\rho+\kappa} R(\tilde{z};m)d\tilde{z}d\Gamma(z)\right)^{\frac{1}{\eta_e}} \in (0,\infty)
\end{aligned}$$

As $\pi \to 0$ and/or $\xi \to l$, $\lim_{m\to\rho+\kappa} y(m)$ becomes arbitrarily small (but strictly positive).

It follows from the observations above that $\lim_{m\to 0} y(m) > \lim_{m\to 0} \hat{y}(m) = 0$ and that, for sufficiently small $\pi$ and/or $\xi$ sufficiently close to $l$, $\lim_{m\to\rho+\kappa} y(m) < \lim_{m\to\rho+\kappa} \hat{y}(m) = \zeta l(\rho+\kappa)$. It follows from continuity that the exit and entry curves intersect at least once in $m \in (0,\rho+\kappa)$.

In the limit $\lim_{\eta_e\to\infty} y(m) = \pi(\xi - l)$ for all $m \in (0,\rho+\kappa)$. That is, the entry curve (31) becomes arbitrarily flat at a strictly positive number that is independent of any equilibrium objects. As long as

$$\lim_{m\to 0} \hat{y}(m) = 0 < \pi(\xi - l) < \lim_{m\to\rho+\kappa} \hat{y}(m) = \zeta l(\rho+\kappa)$$

there is a unique intersection between the exit and entry curves. It follows from continuity of the entry curve in $\eta_e$ that a unique intersection exists for sufficiently high $\eta_e$.

### C.12 Proposition 9: The impact of aging

For small growth

$$\frac{u(m,\lambda)}{S(m,\lambda)} \approx \frac{\kappa+\lambda}{\kappa+\lambda+\phi p}$$

and hence

$$\frac{\partial}{\partial \lambda}\left(\frac{u(m,\lambda)}{S(m,\lambda)}\right)\Big|_{m \text{ fixed}} \approx \frac{\beta}{(1+\beta)^2}\frac{1}{\kappa+\lambda}$$

Recall that for small growth, the employment distribution is to a first-order given by (60). Differentiating this with respect to $\lambda$ holding fixed $m$

$$\frac{\partial G(z;m,\lambda)}{\partial \lambda}\Big|_{m \text{ fixed}} \approx \frac{\beta e^{-\zeta z}}{(1+\beta e^{-\zeta z})^2}\frac{1}{\kappa+\lambda}\left(1-e^{-\zeta z}\right)$$

Recall that under the assumption that $\eta_v \to \infty$ and $\theta \to 1$, $q$ is parametric such that the return to hiring (13) is

$$R(z;m,\lambda) = q\left(\frac{u(m,\lambda)}{S(m,\lambda)}J(z;m)+\left(1-\frac{u(m,\lambda)}{S(m,\lambda)}\right)\int_0^z J'(\tilde{z};m)G(\tilde{z};m,\lambda)d\tilde{z}\right)$$

Differentiating with respect to $\lambda$ holding fixed the growth rate

$$\begin{aligned}\frac{\partial R(z;m,\lambda)}{\partial \lambda}\Big|_{m \text{ fixed}} &= q\left(\frac{\partial}{\partial \lambda}\left(\frac{u(m,\lambda)}{S(m,\lambda)}\right)\Big|_{m \text{ fixed}}\left(J(z;m)(1-G(z;m,\lambda))+\int_0^z J(\tilde{z};m)g(\tilde{z};m,\lambda)d\tilde{z}\right)\right.\\ &+ \left.\left(1-\frac{u(m,\lambda)}{S(m,\lambda)}\right)\int_0^z J'(\tilde{z};m)\ \frac{\partial G(\tilde{z};m,\lambda)}{\partial \lambda}\Big|_{m \text{ fixed}} d\tilde{z}\right)\end{aligned}$$

Taking $m$ to be small, this simplifies to

$$\frac{\partial R(z;m,\lambda)}{\partial \lambda}\Big|_{m \text{ fixed}} \approx q\frac{1}{\kappa+\lambda}\frac{1}{\rho+\kappa}\frac{\beta}{1+\beta}\left(\frac{e^{z(1-\zeta)}}{1+\beta e^{-\zeta z}}+\int_0^z \frac{e^{\tilde{z}(1-\zeta)}\left(\zeta+\beta\left(1-e^{-\zeta z}\right)\right)}{(1+\beta e^{-\zeta \tilde{z}})^2}d\tilde{z}\right)$$

## C.13 The joint dynamics of entrepreneurship and search

To motivate the focus on h joint theory of labor market mobility and entrepreneurship, Table 12 shows the distribution of individuals born between 1953 and 1973 across number of jobs held and number of new firms started between 1993 and 2017.[51] Because attrition is limited—only if an individual moves to or from Sweden or dies does she leave the sample—I have 24 years of close to complete data for the 1953–1973 cohorts. The vast majority of individuals who founded h firm over this period also at some point worked as a wage employee. For instance, only 0.9 percent of all individuals started at least one

[51]Because an individual could later become a wage employee in their own firm—for instance through h public listing—I standardize employment status within an individual-firm match to that when the match was first formed. I drop 2004 due to h time series break in the underlying data. Specifically, Statistics Sweden switched to h different source of data to identify the self-employed—the *Standardiserade rikenskapsutdrag*—and they started recording also companies with negative profits. This lead to h one time jump (consisting of almost exclusively small firms) in the measured entry rate.

firm over this 24 year period without ever holding a wage employment job. For comparison, 24.9 percent of individuals started at least one firm. In fact, even among the less than 0.1 percent of individuals who started five or more firms, most of them held at least one wage employment job over h 24 year period.

TABLE 12. DISTRIBUTION OF EMPLOYMENT AND FIRM CREATION SPELLS

| | # jobs held | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| # firms created | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10+ | **Marginal** |
| 0 | 8.37 | 14.36 | 11.75 | 9.93 | 8.02 | 6.27 | 4.75 | 3.51 | 2.54 | 1.76 | 3.83 | **75.09** |
| 1 | 0.72 | 3.36 | 3.02 | 2.60 | 2.15 | 1.70 | 1.29 | 0.96 | 0.70 | 0.49 | 1.04 | **18.01** |
| 2 | 0.13 | 0.73 | 0.86 | 0.78 | 0.67 | 0.52 | 0.41 | 0.30 | 0.22 | 0.15 | 0.31 | **5.08** |
| 3 | 0.03 | 0.17 | 0.22 | 0.22 | 0.19 | 0.16 | 0.12 | 0.09 | 0.06 | 0.04 | 0.08 | **1.39** |
| 4 | 0.01 | 0.04 | 0.05 | 0.06 | 0.05 | 0.04 | 0.03 | 0.02 | 0.02 | 0.01 | 0.02 | **0.34** |
| 5+ | 0.00 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.00 | 0.00 | 0.00 | **0.08** |
| **Marginal** | **9.26** | **18.66** | **15.91** | **13.60** | **11.08** | **8.70** | **6.62** | **4.89** | **3.53** | **2.46** | **5.28** | **100.00** |

Table 12 shows the distribution of individuals born between 1953 and 1973 across number of different jobs and firms started between 1993–2017. The data exclude year 2004 due to h break in the sample. All individuals aged 20–64. *Source:* FEK, JOBB, LISA.

## C.14 Incorporating productivity shocks

With incumbent innovation at rate $\mu$ and stochastic shocks to productivity at intensity $\sigma$, relative productivity $z = Z(i,t) - \underline{Z}(t)$ evolves according to

$$dz = -mdt + \sigma dW(t)$$

and the overall growth rate is $m + \mu$. Apart from the fact that the economy now grows at rate $m + \mu$, the HJB and FP equations remain essentially the same as above with the addition of the standard term for productivity shocks. In particular, the evolution of the distribution of entrepreneurs is now

$$0 = mx'(z) + \frac{\sigma^2}{2}x''(z) + \frac{y}{L}\gamma(z)$$

subject to $X(0) = 0$, $\lim_{z\to\infty} X(z) = 1$ and[52]

$$\frac{y}{l} = \frac{\sigma^2}{2}x'(0)$$

[52]This follows from the fact that $0 = m\int_0^\infty x'(z)dz + \frac{\sigma^2}{2}\int_0^\infty x''(z)dz + \frac{y}{L}\int_{\underline{z}}^\infty \gamma(z)dz = -\frac{\sigma^2}{2}x'(0) + y$ since in order for the density to integrate to one, $\lim_{z\to\infty} x(z) = 0$ and $\lim_{z\to\infty} x'(z) = 0$, and $x(0) = 0$.

Given an aggregate entry rate, this second-order ODE determines the distribution, $x$, and the rate of obsolescence, $m$. The solution is[53]

$$x(z) \;=\; \frac{\frac{y}{l}}{-m \,+\, \frac{\sigma^2}{2}\zeta}\left(e^{\frac{2(-m)}{\sigma^2}z} \,-\, e^{-\zeta z}\right)$$

and the exit curve continues to be given by

$$\hat{y}(m) \;=\; l\zeta m$$

Define log productivity $z$ as following a power law if constants $a, \zeta^* > 0$ exist such that $Pr(Z > z) = ae^{-\zeta^* z}$. Productivity follows an asymptotic power law if constants $a, \zeta^* > 0$ exist such that $Pr(Z > z) \sim ae^{-\zeta^* z}$ as $z \to \infty$, where for any $f, g$, $f(z) \sim g(z)$ means $\lim_{z\to\infty} f(z)/g(z) = 1$. Then the endogenous stationary distribution of productivity follows an asymptotic power law with tail parameter $\zeta^*$ given by

$$\zeta^* \;=\; \begin{cases} \zeta & \text{if } \frac{(\sigma\zeta)^2}{2} \leq \frac{y}{l} \\ \frac{2\frac{y}{l}}{\sigma^2\zeta} & \text{if } \frac{(\sigma\zeta)^2}{2} > \frac{y}{l} \end{cases}$$

Hence, if the initial entry rate is sufficiently low, $\frac{(\sigma\zeta)^2}{2} > \frac{y}{l}$—the empirically relevant case—a decline in entry increases inequality. Creative destruction is a force that holds some firms back from becoming in a relative sense very productive. If the entry rate is high, $\frac{y}{l} > (\sigma\zeta)^2/2$, this force is so strong that the right tail of the productivity distribution is entirely driven by the tail of the entry distribution. It is as though firms fall behind the market so fast that no firm manages to move into the right tail of the distribution after entry. If, on the other hand, the entry rate is lower than this (the empirically relevant case), the tail of the productivity distribution is driven by endogenous productivity dynamics. In this case, lower entry is associated with greater tail dispersion in productivity across firms.

[53] Parameter restrictions have to be imposed to ensure that h BGP equilibrium exists. As before, the resulting growth rate must be below $\rho + \kappa$ or values explode. Moreover with productivity shocks, it cannot be too small, because then a stationary distribution of firms may not exist. I assume throughout that these parameter restrictions hold.

## C.15 Value functions in the full model

In the extended model, the value of unemployment writes

$$
\begin{aligned}
(\rho - m)U(a) \quad = \quad & \underbrace{b(a)}_{\text{flow value of leisure}} \\
+ \quad & \underbrace{U'(a)}_{\text{aging}} \\
- \quad & \underbrace{\mathbb{1}\{a \geq \overline{A}\}\kappa U(a)}_{\text{labor force exit}} \\
+ \quad & \underbrace{\frac{\eta_e}{1+\eta_e} c_e e^{\underline{z}^w(a)} s_u(a)^{1+\eta_e}}_{\text{net return to search for business ideas}}
\end{aligned}
\tag{76}
$$

where $\mathbb{1}\{x \geq X\}$ is an indicator taking value one if $x \geq X$ and zero otherwise, and optimal search is

$$
s_u(a) \quad = \quad \left( \frac{\pi(a)}{c_e e^{\underline{z}^w(a)}} \int_0^{\infty} \Big( V^e(\tilde{z}, a) - U(a) \Big) d\Gamma(\tilde{z} | \underline{z}^w(a)) \right)^{\frac{1}{\eta_e}} \tag{77}
$$

where

$$
\pi(a) \quad = \quad \begin{cases} \pi^l & \textit{if } a \leq \overline{A}^{\pi} \\ \pi^h & \textit{if } a \geq \overline{A}^{\pi} \end{cases}
$$

Note that I assume that the cost of search scales in the reservation threshold, $e^{\underline{z}(a)}$, and not the flow value of leisure, $b(a)$, as in the analytical model in Section 4. The reason for this minor change is that it allows me to solve for the entire equilibrium allocation without solving for the flow value of leisure. The latter can be recovered ex post, substantially speeding up the solution of the model. I note that although the two formulations are isomorphic in estimation, they do differ when I change the age composition.

Let $V^w(z, a)$ be the value of a match, which now depends on both productivity, $z$, and the age of the worker, $a$. Importantly, however, it does not depend on the age of the founder, as long as $b(a)$ is high enough that all workers want to terminate a match before an entrepreneur of each age wants to shut down the firm. This property simplifies the solution of the problem substantially by reducing the need to keep track of the entrepreneur's age when solving the problem of the match. The value of a match solves for $z \geq \underline{z}^w(a)$ the stopping time problem

$$
\begin{aligned}
(\rho - m)V^w(z,a) \quad = \quad & \underbrace{e^z}_{\text{flow output}} \\
+ \quad & \underbrace{V^w_a(z,a)}_{\text{aging}} \\
- \quad & \underbrace{\mathbb{1}\{a \geq \overline{A}\}\kappa V^w(z,a)}_{\text{labor force exit}} \\
- \quad & \underbrace{mV^w_z(z,a)}_{\text{technological obsolescence}} \\
+ \quad & \underbrace{\frac{\sigma^2}{2}V^w_{zz}(z,a)}_{\text{productivity shocks}} \\
+ \quad & \underbrace{\frac{\eta_e}{1+\eta_e}c_e e^z s(z,a)^{1+\eta_e}}_{\text{net return to search for business ideas}} \\
+ \quad & \underbrace{\delta(z)\Big(U(a) - V^w(z,a)\Big)}_{\text{exogenous match separation}} \\
+ \quad & \underbrace{d\Big(U(a) - V^w(z,a)\Big)}_{\text{exogenous firm exit}}
\end{aligned} \tag{78}
$$

where $X_i = \partial X / \partial i$ is short hand for the partial derivative of $X$ with respect to $i$, subject to the value matching and smooth pasting conditions $V^w(\underline{z}^w(a), a) = U(a)$ and $V^w_z(\underline{z}^w(a), a) = 0$, where optimal search is

$$
s(z,a) \quad = \quad \left( \frac{\pi(a)}{c_e e^z} \int_0^\infty \Big( V^e(\tilde{z}, a) - V^w(z,a) \Big) d\Gamma(\tilde{z}|z) \right)^{\frac{1}{\eta_e}} \tag{79}
$$

Subtracting the value of unemployment (76) from the value of a match (78); evaluating the difference at $z = \underline{z}^w(a)$; using the fact that $V^w(\underline{z}^w(a), a) = U(a)$ for all $a$ implies that $V^w_a(\underline{z}^w(a), a) = U'(a)$; noting that by (77)–(79), $s_u(a) = s(\underline{z}^w(a), a)$; and imposing the optimal stopping time conditions $V^w(\underline{z}^w(a), a) = U(a)$ and $V^w_z(\underline{z}^w(a), a) = 0$ for all $a$ gives that the reservation threshold is characterized by

$$
e^{\underline{z}^w(a)} \quad = \quad b(a) - \frac{\sigma^2}{2} V^w_{zz}\Big(\underline{z}^w(a), a\Big) \tag{80}
$$

The value of entrepreneurship $V^e(z,a)$ solves for $z \geq \underline{z}(a)$ the stopping time problem

$$
\begin{aligned}
(\rho - m)V^e(z,a) \quad = \quad & \underbrace{k(a)}_{\text{flow value of being one's own boss}} \\
- \quad & \underbrace{r}_{\text{fixed cost}} \\
+ \quad & \underbrace{V^e_a(z,a)}_{\text{aging}} \\
- \quad & \underbrace{\mathbb{1}\{a \geq \overline{A}\}\kappa V^e(z,a)}_{\text{labor force exit}} \\
- \quad & \underbrace{m V^e_z(z,a)}_{\text{technological obsolescence}} \\
+ \quad & \underbrace{\frac{\sigma^2}{2} V^e_{zz}(z,a)}_{\text{productivity shocks}} \\
+ \quad & \underbrace{d\Big(U(a) - V^e(z,a)\Big)}_{\text{exogenous firm exit}} \\
+ \quad & \underbrace{\frac{\eta_v}{1+\eta_v} c_v e^z v(z,a)^{1+\eta_v}}_{\text{net return from hiring}}
\end{aligned}
\tag{81}
$$

subject to $V^e(\underline{z}(a),a) = U(a)$ and $V^e_z(\underline{z}(a),a) = 0$, where optimal vacancy creation is

$$
v(z,a) \quad = \quad \left( \frac{q}{c_v e^z} \int_0^\infty \left( \frac{u(\tilde{a})}{S} \Big( V^w(z,\tilde{a}) - U(\tilde{a}) \Big)^+ + \frac{\phi e}{S} \int_0^\infty \Big( V^w(z,\tilde{a}) - V^w(\tilde{z},\tilde{a}) \Big)^+ g(\tilde{z},\tilde{a}) d\tilde{z} \right) d\tilde{a} \right)^{\frac{1}{\eta_v}} \tag{82}
$$

where $u(a)$ is the number of age $a$ unemployed and $e$ the total number of employed.

Subtracting the value of unemployment (76) from that of entrepreneurship (81), and imposing the optimal stopping time conditions $V^e(\underline{z}(a),a) = U(a)$ and $V^e_z(\underline{z}(a),a) = 0$ for all $a$ implies that

$$
b(a) + \frac{\eta_e}{1+\eta_e} c_e e^{\underline{z}^w(a)} s_u(a)^{1+\eta_e} \quad = \quad k(a) - r + \frac{\sigma^2}{2} V^e_{zz}\Big(\underline{z}(a), a\Big)
$$

Using (80), the fixed cost is

$$
r \quad = \quad k(a) + \frac{\sigma^2}{2} V^e_{zz}\Big(\underline{z}(a),a\Big) - e^{\underline{z}^w(a)} - \frac{\sigma^2}{2} V^w_{zz}\Big(\underline{z}^w(a),a\Big) - \frac{\eta_e}{1+\eta_e} c_e e^{\underline{z}^w(a)} s_u(a)^{1+\eta_e} \tag{83}
$$

where since the economy is normalized to the least productive firm,

$$\min_a \underline{z}(a) = 0 \tag{84}$$

### C.16 Equilibrium in the full model

The distribution of workers over productivity and age, $g(z,a)$, is given by the FP equation

$$\begin{aligned}
\underbrace{\lambda g(z,a)}_{\text{labor supply growth}} &= -\underbrace{g_a(z,a)}_{\text{aging}} \\
&- \underbrace{\mathbb{1}\{a \geq \overline{A}\}\kappa g(z,a)}_{\text{labor force exit}} \\
&+ \underbrace{m g_z(z,a)}_{\text{technological obsolescence at rate -}m} \\
&+ \underbrace{\frac{\sigma^2}{2} g_{zz}(z,a)}_{\text{productivity shocks}} \\
&- \underbrace{\delta(z) g(z,a)}_{\text{exogenous match separation}} \\
&- \underbrace{d g(z,a)}_{\text{exogenous firm exit}} \\
&- \underbrace{\pi(a) s(z,a) g(z,a)}_{\text{exit to entrepreneurship}} \\
&- \underbrace{\phi p \Big(1 - F(z)\Big) g(z,a)}_{\text{separations to employment}} \\
&+ \underbrace{f(z) p \frac{u(a)}{e}}_{\text{hires from unemployment}} \\
&+ \underbrace{\phi p f(z) \int_{\underline{z}^w(a)}^{z} g(\tilde{z},a) d\tilde{z}}_{\text{hires from employment}}
\end{aligned} \tag{85}$$

where $e = 1 - l - \int_0^\infty u(a)da$, subject to the boundary conditions

$$g(\underline{z}^w(a), a) = 0 \tag{86}$$

$$\int_0^\infty \int_0^\infty g(z,a) dz da = 1 \tag{87}$$

The first boundary condition (86) imposes that the density is zero at the boundary, which holds in the presence of shocks, $\sigma > 0$. The intuition is that at $\underline{z}^w(a)$, the outflow due to the shocks is an order of magnitude larger than the inflow, because there is no inflow of workers from below in the distribution (since workers exit whenever their productivity falls below $\underline{z}^w(a)$). The second boundary condition (87) is implied by the fact that $g(z,a)$ is a density.

The number of unemployed of age $a$ are given by

$$\begin{aligned}
\underbrace{\lambda u(a)}_{\text{labor supply growth}} &= -\underbrace{u'(a)}_{\text{aging}} \\
&- \underbrace{\mathbb{1}\{a \geq \overline{A}\}\kappa u(a)}_{\text{labor force exit}} \\
&- \underbrace{pu(a)}_{\text{outflow to employment}} \\
&- \underbrace{\pi(a)s_u(a)u(a)}_{\text{outflow to entrepreneurship}} \\
&+ \underbrace{\frac{\sigma^2}{2}g_z\left(\underline{z}^w(a),a\right)e}_{\text{endogenous separation (workers)}} \\
&+ \underbrace{e\int_{\underline{z}^w(a)}^{\infty}\delta(z)g(z,a)dz}_{\text{exogenous match separation}} \\
&+ \underbrace{ed\int_{\underline{z}^w(a)}^{\infty}g(z,a)dz}_{\text{exogenous firm exit (workers)}} \\
&+ \underbrace{\frac{\sigma^2}{2}x_z\left(\underline{z}(a),a\right)l}_{\text{endogenous separation (entrepreneurs)}} \\
&+ \underbrace{ld\int_{\underline{z}(a)}^{\infty}x(z,a)dz}_{\text{exogenous firm exit (entrepreneurs)}}
\end{aligned} \tag{88}$$

where

$$u(0) = \kappa(1+(1-\omega)\nu)\int_{\overline{A}}^{\infty}\left(u(a)+(1-l)\int_{\underline{z}^w(a)}^{\infty}g(z,a)dz\right)da \tag{89}$$

The evolution of the share of entrepreneurs with productivity $z$ and age $a$ is given by

$$\begin{aligned}
\lambda x(z,a) &= -\underbrace{x_a(z,a)}_{\text{aging}} \\
&- \underbrace{\mathbb{1}\{a \geq \overline{A}\}\kappa x(z,a)}_{\text{labor force exit}} \\
&+ \underbrace{m x_z(z,a)}_{\text{technological obsolescence at rate -}m} \\
&+ \underbrace{\frac{\sigma^2}{2} x_{zz}(z,a)}_{\text{productivity shocks}} \\
&+ \underbrace{\frac{\pi(a)}{l} u(a)\gamma\left(z|\underline{z}^w(a)\right) s_u(a)}_{\text{entry from unemployment}} \\
&+ \underbrace{\frac{\pi(a)}{l} e \int_{\underline{z}^w(a)}^{\infty} \gamma(z|\tilde{z}) s(\tilde{z},a) g(\tilde{z},a) d\tilde{z}}_{\text{entry from employment}}
\end{aligned} \tag{90}$$

subject to the boundary conditions

$$x(\underline{z}(a),a) = 0 \tag{91}$$

$$\int_0^\infty \int_0^\infty x(z,a) dz da = 1 \tag{92}$$

$$x(z,0) = \kappa(1+(1-\omega)\nu) \int_{\overline{A}}^{\infty} x(z,a) da \tag{93}$$

**Definition 2.** *A stationary equilibrium consists of value functions $U(a)$, $V^w(z,a)$ and $V^e(z,a)$; reservation thresholds $\underline{z}^w(a)$ and $\underline{z}(a)$; search policies $s_u(a)$ and $s(z,a)$; a vacancy policy $v(z,a)$; a fixed cost, $r$; finding rates $p$ and $q$; a distribution of workers $g(z,a)$, number of employed and unemployed $\{e(a), u(a)\}$, and a distribution of entrepreneurs $x(z,a)$; an offer distribution $f(z)$, an aggregate mass of vacancies $V$, and aggregate search intensity $S$; an aggregate exit rate $\hat{y}(m)$; an aggregate entry rate $y(m)$; and a rate of obsolescence, $m$, such that:*

1. *The value function $U(a)$ is given by (76) and the optimal search intensity $s_u(a)$ by (77);*

2. *The value function $V^w(z,a)$ is given by (78), the optimal search intensity $s(z,a)$ by (79), and the optimal reservation threshold $\underline{z}^w(a)$ by (80);*

3. *The value function $V^e(z,a)$ is given by (81), the optimal vacancy policy by (82), and the optimal reservation policy and fixed cost by the system (83)–(84);*

4. *The finding rates $p$ and $q$ are given by (4) given aggregate vacancies $V$ and an aggregate search intensity $S$;*

5. *The distribution of workers, $g(z,a)$, the number of unemployed, $u(a)$, and the distribution of entrepreneurs, $x(z,a)$, are given by (85)–(93).*

6. *The vacancy-weighted distribution of firms $f(z)$ is given by*

$$f(z) \quad = \quad \frac{l}{V}\int_0^\infty v(z,a)x(z,a)da$$

*where the aggregate number of vacancies $V$ are given by*

$$V \quad = \quad l\int_0^\infty \int_0^\infty v(z,a)x(z,a)dzda$$

*and aggregate search intensity is $S = u + \phi e$, where $u = \int_0^\infty u(a)da$ and $e = 1-u$;*

7. *The aggregate exit rate is*

$$\hat{y}(m) \quad = \quad l\left(d + \frac{\sigma^2}{2}\int_0^\infty x_z(\underline{z}(a),a)da\right)$$

8. *The aggregate entry rate is*

$$y(m) \quad = \quad \int_0^\infty \pi(a)\left(s_u(a)u(a) + e\int_{\underline{z}^w(a)}^\infty s(z,a)g(z,a)dz\right)da$$

9. *The aggregate exit rate equals the aggregate entry rate, $\hat{y}(m) = y(m)$.*

## C.17 Algorithm

Equation (80) can be rewritten as

$$b(a) \quad = \quad e^{\underline{z}^w(a)} + \frac{\sigma^2}{2}V^w_{zz}(\underline{z}^w(a),a)$$

while equation (83) can be rearranged as

$$k(a) - r \quad = \quad e^{\underline{z}^w(a)} - \frac{\sigma^2}{2}V^e_{zz}(\underline{z}(a),a) + \frac{\sigma^2}{2}V^w_{zz}(\underline{z}^w(a),a) + \frac{\eta_e}{1+\eta_e}c_e e^{\underline{z}^w(a)}s_u(a)^{1+\eta_e}$$

Substituting for $b(a)$ and $k(a)-r$ using these equations, optimal search intensity in unemployment using (77) and employment using (79), and optimal vacancy creation using (82), the value functions can be written as (suppressing the arguments $z$ and $a$ whenever possible to simplify the notation)

$$
\begin{aligned}
(\rho+\kappa-m)U &= e^{\underline{z}^w} \\
&+ \frac{\sigma^2}{2}V^w_{zz}(\underline{z}^w,a) \\
&+ U_a \\
&+ \pi(a)s_u(a)\frac{\eta_e}{1+\eta_e}\int_0^\infty \Big(V^e(\tilde{z},a)-U\Big)\gamma(\tilde{z}|\underline{z}^w)d\tilde{z} \\
(\rho+\kappa-m)V^w &= e^z \\
&+ V^w_a \\
&- mV^w_z \\
&+ \frac{\sigma^2}{2}V^w_{zz} \\
&+ \delta(z)\Big(U-V^w\Big) \\
&+ d\Big(U-V^w\Big) \\
&+ \pi(a)s(z,a)\frac{\eta_e}{1+\eta_e}\int_0^\infty \Big(V^e(\tilde{z},a)-V^w\Big)\gamma(\tilde{z}|z)d\tilde{z} \\
(\rho+\kappa-m)V^e &= e^{\underline{z}^w} \\
&+ \frac{\sigma^2}{2}V^w_{zz}(\underline{z}^w,a)-\frac{\sigma^2}{2}V^e_{zz}(\underline{z},a)+\pi(a)s_u(a)\frac{\eta_e}{1+\eta_e}\int_0^\infty \Big(V^e(\tilde{z},a)-U\Big)\gamma(\tilde{z}|\underline{z}^w)d\tilde{z} \\
&+ V^e_a \\
&- mV^e_z \\
&+ \frac{\sigma^2}{2}V^e_{zz} \\
&+ d(U-V^e) \\
&+ qv(z,a)\frac{\eta_v}{1+\eta_v}\int_0^\infty \left(\frac{u(\tilde{a})}{S}\Big(V^w-U\Big)^+ + \frac{\phi e}{S}\int_{\underline{z}^w}^\infty \Big(V^w(z,\tilde{a})-V^w(\tilde{z},\tilde{a})\Big)^+ g(\tilde{z},\tilde{a})d\tilde{z}\right)d\tilde{a}
\end{aligned} \tag{94}
$$

In estimation, I normalize $\underline{z}(a)=0$ for all $a$ and $\underline{z}^w(a)=\underline{z}^w$ for some constant $\underline{z}^w \geq 0$—in practice, I set it to the second point on the grid for productivity. Under the normalization, I solve the value functions based on (94). Having solved for the equilibrium allocation, I recover the flow values of leisure and of being one's own boss based on (80) and (83).

As the economy ages, the reservation threshold $\underline{z}^w(a)$ will in general change, which imposes an additional computational burden. I note based on (80), however, that the only reason $\underline{z}^w(a)$ changes in response to aging is through changes in the second order term, $\frac{\sigma^2}{2}V^w_{zz}$. Since the value function turns out to not be very concave, to simplify the numerical solution, I assume that $b(a)$ adjusts in response to aging such that $\underline{z}^w(a)$ remains fixed, i.e. I set updated flow values of leisure based on

$$\hat{b}(a) \quad = \quad e^{\underline{z}^w(a)} + \frac{\sigma^2}{2}\hat{V}^w_{zz}(\underline{z}^w(a), a)$$

where $\hat{V}^w(z, a)$ is the value of a match under a different age composition. In practice given the estimated volatility of productivity, the required changes in $b(a)$ in response to the empirically relevant amount of aging are small.

In response to aging, the reservation threshold $\underline{z}(a)$ will also in general change. Although I set $k(a)$ such that $\underline{z}(a) = 0$ for all $a$ in estimation and even though the fixed cost $r$ adjusts in response to aging based on (84) such that $\min_a \underline{z}(a) = 0$, it may be that $\underline{z}(a) > 0$ for some $a$ as the economy ages. I simplify by assuming that also $k(a)$ adjusts based on (83) such that $\underline{z}(a) = 0$ for all $a$. That is, I first compute the new fixed cost based on

$$\hat{r} \quad = \quad \max_a \left\{ k(a) + \frac{\sigma^2}{2}\hat{V}^e_{zz}(\underline{z}(a), a) - e^{\underline{z}^w(a)} - \frac{\sigma^2}{2}\hat{V}^w_{zz}(\underline{z}^w(a), a) - \frac{\eta_e}{1+\eta_e} c_e e^{\underline{z}^w(a)} \hat{s}_u(a)^{1+\eta_e} \right\}$$

Setting the fixed cost to the highest valuation of land across age groups ensures that all age groups want to exit at $\underline{z}(a) \geq 0$. Given a new fixed cost $\hat{r}$, I recover updated flow values of being one's own boss based on

$$\hat{k}(a) \quad = \quad \hat{r} - \frac{\sigma^2}{2}\hat{V}^e_{zz}(\underline{z}(a), a) + e^{\underline{z}^w(a)} + \frac{\sigma^2}{2}\hat{V}^w_{zz}(\underline{z}^w(a), a) + \frac{\eta_e}{1+\eta_e} c_e e^{\underline{z}^w(a)} \hat{s}_u(a)^{1+\eta_e}$$

In practice, the required adjustments to $k(a)$ are small.

These two simplifying assumptions imply that the system (94) can be used to also solve for the value functions across BGPs and over the transition path.

Define the stacked vector of values

$$W = \begin{bmatrix} U(a_1) \\ V^w(z_1, a_1) \\ \vdots \\ V^w(z_{N_z}, a_1) \\ V^e(z_1, a_1) \\ \vdots \\ V^e(z_{N_z}, a_1) \\ \vdots \\ U(a_{N_a}) \\ V^w(z_1, a_{N_a}) \\ \vdots \\ V^w(z_{N_z}, a_{N_a}) \\ V^e(z_1, a_{N_a}) \\ \vdots \\ V^e(z_{N_z}, a_{N_a}) \end{bmatrix}$$

and the flow payoff vector

$$Y = \mathbb{1}_{N_a \times 1} \otimes \begin{bmatrix} e^{z_1} \\ e^{z_1} \\ e^{z_2} \\ \vdots \\ e^{z_{N_z}} \\ e^{z_1} \\ \vdots \\ e^{z_1} \end{bmatrix}$$

where $\mathbb{1}_{n \times 1}$ is an $n \times 1$ vector of ones. Note that I set the flow value of leisure such that the reservation threshold of workers is the first point on the productivity grid.

Define the stacked distribution of individuals

$$g = \begin{bmatrix} u(a_1) \\ e * g(z_1, a_1) \\ \vdots \\ e * g(z_{N_z}, a_1) \\ l * x(z_1, a_1) \\ \vdots \\ l * x(z_{N_z}, a_1) \\ \vdots \\ u(a_{N_a}) \\ e * g(z_1, a_{N_a}) \\ \vdots \\ e * g(z_{N_z}, a_{N_a}) \\ l * x(z_1, a_{N_a}) \\ \vdots \\ l * x(z_{N_z}, a_{N_a}) \end{bmatrix}$$

and the entry vector

$$B = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

**Exogenous matrices.** Define the following matrices which do not change with decisions:

1. The aging matrix

$$\mathbf{A} = \frac{1}{da}\texttt{spdiags}\left( \begin{bmatrix} -\mathbb{1}_{((N_a-1)(2N_z+1))\times 1} & 0_{(2N_z+1)\times 1} \\ 0_{(2N_z+1)\times 1} & \mathbb{1}_{((N_a-1)(2N_z+1))\times 1} \end{bmatrix}, [0, 2N_z+1], N_a(2N_z+1), N_a(2N_z+1) \right)$$

2. The exit matrices

$$\begin{aligned}
\mathbf{X}^v &= -\kappa \times \texttt{spdiags}\left(\begin{bmatrix} 0_{(N_a-1)(2N_z+1)\times 1} \\ \mathbb{1}_{(2N_z+1)\times 1} \end{bmatrix}, 0, N_a(2N_z+1), N_a(2N_z+1)\right) \\
\mathbf{X}_1 &= -\kappa \times \texttt{spdiags}\left(\begin{bmatrix} 0_{(N_z+1)\times 1} \\ \mathbb{1}_{N_z\times 1} \end{bmatrix}, 0, 2N_z+1, 2N_z+1\right) \\
\mathbf{X}_2 &= \texttt{spdiags}\Big(N_a, 1, 1, N_a, N_a\Big) \\
\mathbf{X}^d &= \mathbf{X}^v + \mathbf{X}_2 \otimes \mathbf{X}_1
\end{aligned}$$

3. The labor supply matrix

$$\begin{aligned}
\mathbf{L}_1 &= -\lambda \times \texttt{speye}(N_a) \otimes \texttt{spdiags}\left(\begin{bmatrix} \mathbb{1}_{(N_z+1)\times 1} \\ 0_{N_z\times 1} \end{bmatrix}, 0, 2Nz+1, 2Nz+1\right) \\
\mathbf{L}_2 &= -\lambda \times \texttt{speye}(N_a) \otimes \texttt{spdiags}\left(\begin{bmatrix} 0_{(N_z+1)\times 1} \\ \mathbb{1}_{N_z\times 1} \end{bmatrix}, 0, 2Nz+1, 2Nz+1\right) \\
\mathbf{L}_2 &= \mathbf{L}_2 - \texttt{sum}\Big(\mathbf{L}_2, 2\Big) \\
\mathbf{L} &= \mathbf{L}_1 + \mathbf{L}_2
\end{aligned}$$

4. The drift matrix

$$\mathbf{D} = \frac{1}{dz}\texttt{speye}(N_a) \otimes \texttt{spdiags}\left(\begin{bmatrix} 0 & 0 \\ 1 & 0 \\ \mathbb{1}_{(N_z-2)\times 1} & -\mathbb{1}_{(N_z-2)\times 1} \\ 0 & -1 \\ 1 & 0 \\ \mathbb{1}_{(N_z-2)\times 1} & -\mathbb{1}_{(N_z-2)\times 1} \\ 0 & -1 \end{bmatrix}, [-1, 0], 2N_z+1, 2N_z+1\right)$$

5. The shock matrix

$$
\begin{aligned}
\mathbf{S} &= \frac{1}{dz^2}\texttt{speye}(N_a) \\
&\otimes \quad \texttt{spdiags}\left(\begin{bmatrix} 0 & 0 & 0 \\ 1 & -1 & 0 \\ \mathbb{1}_{(N_z-2)\times 1} & -2_{(N_z-2)\times 1} & \mathbb{1}_{(N_z-2)\times 1} \\ 0 & -1 & 1 \\ 1 & -1 & 0 \\ \mathbb{1}_{(N_z-2)\times 1} & -2_{(N_z-2)\times 1} & \mathbb{1}_{(N_z-2)\times 1} \\ 0 & -1 & 1 \end{bmatrix}, -1:1, 2N_z+1, 2N_z+1\right)
\end{aligned}
$$

6. The exogenous separation matrix

$$
\mathbf{Q}_1 = \texttt{speye}(N_a) \otimes \begin{bmatrix} \begin{matrix} 0 & \cdots & \cdots & \cdots & 0 \\ \delta(z_1) & -\delta(z_1) & 0 & \cdots & \vdots \\ \vdots & 0 & \ddots & \ddots & \vdots \\ \vdots & & \ddots & \ddots & 0 \\ \delta(z_{N_z}) & 0 & \cdots & 0 & -\delta(z_{N_z}) \end{matrix} & 0_{(N_z+1)\times N_z} \\ \multicolumn{2}{c}{0_{(N_z)\times(2N_z+1)}} \end{bmatrix}
$$

7. The exogenous exit matrix

$$
\mathbf{E}_1 = d \times \texttt{speye}(N_a) \otimes \begin{bmatrix} 0 & & 0_{1\times 2N_z} & & \\ 1 & -1 & & 0_{1\times(2N_z-1)} & \\ 1 & 0 & -1 & 0_{1\times(2N_z-2)} & \\ \vdots & \vdots & & \ddots & \ddots \\ 1 & 0 & \dots & 0 & -1 \end{bmatrix}
$$

I combine these matrices into the exogenous transition matrices

$$
\begin{aligned}
\mathbf{T}^v &= \mathbf{A} + \mathbf{X}^v + m\mathbf{D} + \frac{\sigma^2}{2}\mathbf{S} + \mathbf{Q}_1 + \mathbf{E}_1 \\
\mathbf{T}^d &= \mathbf{A} + \mathbf{X}^d + \mathbf{L} + m\mathbf{D} + \frac{\sigma^2}{2}\mathbf{S} + \mathbf{Q}_1 + \mathbf{E}_1
\end{aligned}
$$

Note that $\mathbf{T}^v$ and $\mathbf{T}^d$ can be constructed without solving for individuals' optimal behavior, i.e. they do not have to be updated.

Next, define the flow value of leisure matrix

$$\mathbf{B} = \frac{\sigma^2}{2dz^2}\texttt{speye}(N_a) \otimes \begin{bmatrix} 0 & 1 & -2 & 1 & 0_{1\times 2N_z-3} \\ \multicolumn{5}{c}{0_{N_z\times(2N_z+1)}} \\ 0 & 1 & -2 & 1 & 0_{1\times 2N_z-3} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 1 & -2 & 1 & 0_{1\times 2N_z-3} \end{bmatrix}$$

and the flow value of being one's own boss matrix

$$\mathbf{K} = -\frac{\sigma^2}{2dz^2}\texttt{speye}(N_a) \otimes \begin{bmatrix} \multicolumn{5}{c}{0_{N_z+1\times(2N_z+1)}} \\ 0_{1\times N_z+1} & 1 & -2 & 1 & 0_{1\times N_z-3} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0_{1\times N_z+1} & 1 & -2 & 1 & 0_{1\times N_z-3} \end{bmatrix}$$

These matrices can also be constructed without solving for individuals' optimal behavior.

**Endogenous matrices.** It remains to construct the matrices that change across iterations as individuals' optimal behavior is updated. To that end, define first the following useful vectors

1. The arrival of ideas vector

$$\Pi = \begin{bmatrix} \pi(a_1) \\ \vdots \\ \pi(a_{N_a}) \end{bmatrix} \otimes \mathbb{1}_{(2N_z+1)\times 1}$$

where $\pi(a_1) = \pi^l$ if $a_1 < \overline{A}$ and $\pi^h$ otherwise.

2. The cost of search vectors

$$
\begin{aligned}
Z &= \mathbb{1}_{N_a \times 1} \otimes \begin{bmatrix} e^{-z_1} \\ e^{-z_1} \\ \vdots \\ e^{-z_{N_z}} \\ e^{-z_1} \\ \vdots \\ e^{-z_{N_z}} \end{bmatrix} \\
ZE &= \mathbb{1}_{N_a \times 1} \otimes \begin{bmatrix} e^{-z_1} \\ e^{-z_1} \\ \vdots \\ e^{-z_1} \\ e^{-z_1} \\ \vdots \\ e^{-z_1} \end{bmatrix}
\end{aligned}
$$

Then define the following matrices

1. The endogenous separation matrix

$$
\mathbf{Q}_2 = v \times \texttt{speye}(N_a) \otimes \begin{bmatrix} 0 & & 0_{1\times 2N_z} & \\ 1 & -1 & 0_{1\times(2N_z-1)} & \\ 1 & 0 & -1 & 0_{1\times(2N_z-2)} \\ & & 0_{(2N_z-2)\times(2N_z+1)} & \end{bmatrix}
$$

   where $v$ is a large number.

2. The endogenous exit matrix

$$
\mathbf{E}_2 = v \times \texttt{speye}(N_a) \otimes \begin{bmatrix} & & 0_{(N_z+1)\times(2N_z+1)} & \\ 1 & 0_{1\times N_z} & -1 & 0_{1\times(N_z-1)} \\ & & 0_{(N_z-1)\times(2N_z+1)} & \end{bmatrix}
$$

   where $v$ is a large number.

3. The innovation matrix

$$\mathbf{\Gamma} \quad = \quad \texttt{speye}(N_a) \otimes \begin{bmatrix} -1 & 0 & \cdots & 0 & \gamma(z_1|z_2) & \cdots & \gamma(z_{N_z}|z_2) \\ 0 & \ddots & \ddots & \vdots & \gamma(z_1|z_1) & \cdots & \gamma(z_{N_z}|z_1) \\ \vdots & \ddots & \ddots & 0 & & & \\ 0 & \cdots & 0 & -1 & \gamma(z_1|z_{N_z}) & \cdots & \gamma(z_{N_z}|z_{N_z}) \\ \multicolumn{7}{c}{0_{N_z \times (2N_z+1)}} \end{bmatrix}$$

and

$$\mathbf{\Gamma}_e \quad = \quad \texttt{speye}(N_a) \otimes \begin{bmatrix} \multicolumn{5}{c}{0_{(N_z+1)\times(2N_z+1)}} \\ -1 & 0_{1\times 2N} & \gamma(z_1|z_2) & \cdots & \gamma(z_{N_z}|z_2) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ -1 & 0_{1\times 2N} & \gamma(z_1|z_2) & \cdots & \gamma(z_{N_z}|z_2) \end{bmatrix}$$

These objects need to be updated in each iteration, because they depend on average productivity of incumbents through general knowledge spillovers. The mobility into entrepreneurship vector is

$$P \quad = \quad \Pi. * \left( \frac{1}{c_e} Z. * \Pi. * (\mathbf{\Gamma} * \mathbf{W}) \right)^{\frac{1}{\eta_e}}$$

where the power is element by element, and the forgone option of search vector is

$$P_e \quad = \quad \Pi. * \left( \frac{1}{c_e e^{z_2}}. * \Pi. * (\mathbf{\Gamma}_e * \mathbf{W}) \right)^{\frac{1}{\eta_e}}$$

Define the matrices

$$\begin{aligned} \mathbf{P}_1^v &= \frac{\eta_e}{1+\eta_e} * P. * \mathbf{\Gamma} \\ \mathbf{P}_2^v &= \frac{\eta_e}{1+\eta_e} * P_e. * \mathbf{\Gamma}_e \\ \mathbf{P}^d &= P. * \mathbf{\Gamma} \end{aligned}$$

4. The search-weighted distribution of hires matrix needs to be defined iteratively, looping over age.

To that end, let $\mathbf{g} = \texttt{reshape}(g, 2N_z+1, N_a)$. Then for $i = 1, \cdots, N_a$, define

$$\mathbf{G}_i = \left[\begin{array}{cccccc} & & 0_{(N_z+1)\times(N_z+1)} & & & \\ -\mathbf{g}(u,i) & \mathbf{g}(u,i) & 0 & \cdots & 0 & \\ -\mathbf{g}(u,i) & -\phi * \mathbf{g}(z_1,i) & \mathbf{g}(u,i) + \phi\sum_{j=1}^{1}\mathbf{g}(z_j,i) & \ddots & \vdots & \\ -\mathbf{g}(u,i) & -\phi * \mathbf{g}(z_1,i) & \ddots & \ddots & \vdots & 0_{(2N_z+1)\times N_z} \\ \vdots & \vdots & \ddots & \ddots & 0 & \\ -\mathbf{g}(u,i) & -\phi * \mathbf{g}(z_1,i) & \cdots & \cdots & \mathbf{g}(u,i) + \phi\sum_{j=1}^{N_z-1}\mathbf{g}(z_j,i) & \end{array}\right]$$

then the stacked matrix

$$\mathbf{G} = \frac{1}{S}\mathbb{1}_{N_a\times 1} \otimes \left[\mathbf{G}_1, \ldots, \mathbf{G}_{N_z}\right]$$

and the return to vacancy creation vector

$$R = \frac{1}{c_v} Z. * (\mathbf{G} * \mathbf{W})$$

Then

$$\begin{aligned} q &= \chi V^{\theta-1} S^{1-\theta} \\ q &= \chi \left( l \int_0^\infty \int_0^\infty v(z,a) x(z,a) dz da \right)^{\theta-1} S^{1-\theta} \\ q &= \chi \left( l \int_0^\infty \int_0^\infty (qR(z,a))^{\frac{1}{\eta v}} x(z,a) dz da \right)^{\theta-1} S^{1-\theta} \\ q^{\frac{1}{\theta-1}} &= \frac{\chi^{\frac{1}{\theta-1}}}{S} l q^{\frac{1}{\eta v}} \int_0^\infty \int_0^\infty R(z,a)^{\frac{1}{\eta v}} x(z,a) dz da \\ q^{\frac{1}{\theta-1}-\frac{1}{\eta v}} &= \frac{\chi^{\frac{1}{\theta-1}}}{S} l \Big( \left(R^{\frac{1}{\eta v}}\right)^T * \left( \left( \mathbb{1}_{N_a\times 1} \otimes \begin{bmatrix} 0_{(N_z+1)\times 1} \\ \mathbb{1}_{N_z\times 1} \end{bmatrix} \right) . * g \right) \end{aligned} \tag{95}$$

Once $q$ is pinned down by (95), aggregate vacancies $V$, the job finding rate $p$ and firm vacancies are

$$\begin{aligned} V &= \left(\frac{q}{\chi}\right)^{\frac{1}{\theta-1}} S \\ p &= \chi \left(\frac{V}{S}\right)^{\theta} \\ v &= \left(qR\right)^{\frac{1}{\eta v}} \end{aligned}$$

The hiring matrix is

$$\mathbf{V}^v = qv\frac{\eta_v}{1+\eta_v}.*\mathbf{G}$$

Define $\mathbf{v} = \texttt{reshape}\,(v, 2N_z+1, N_a)$ and the vacancy shares

$$f = \texttt{sum}\Big(\mathbf{v}(N_z+2:\texttt{end},:).*\mathbf{g}(N_z+2:\texttt{end},:),2\Big)/V$$

Then the mobility matrix is

$$\mathbf{M} = p\begin{bmatrix} \begin{matrix} 0 & f_1 & f_2 & \cdots & f_{N_z} \\ 0 & 0 & \phi f_2 & \cdots & \phi f_{N_z} \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & \cdots & 0 & \phi f_{N_z} \\ 0 & \cdots & \cdots & \cdots & 0 \end{matrix} & 0_{N_z+1,N_z} \\ 0_{N_z\times(2N_z+1)} & \end{bmatrix}$$

$$\mathbf{V}^d = \texttt{speye}(N_a)\otimes\Big(\mathbf{M} - \texttt{spdiags}\ \Big(\texttt{sum}\Big(\mathbf{M},2\Big),0,2N_z+1,2N+1\Big)\Big)$$

Using these matrices, the value function is updated according to

$$\mathbf{W}' = \Big(\Big(\rho+\kappa-m+1/dt\Big)\texttt{speye}(N_a(2Nz+1)) - \Big(\mathbf{T}+\mathbf{B}+\mathbf{K}+\mathbf{P}^v+\mathbf{V}^v\Big)\Big)\setminus\Big(F+1/dt\mathbf{W}\Big)$$

and the distribution of individuals is updated according to

$$g = -\Big(\Big(\mathbf{T}+\mathbf{P}^d+\mathbf{V}^d+\mathbf{L}\Big)\Big)^T\setminus B$$

# D Appendix: Estimation

This section contains additional targeted moments and parameters (Appendix D.1); details on how the model is estimated and identification (Appendix D.2); and a decomposition of life-cycle dynamics in the estimated model (Appendix D.3).

## D.1 Additional moments and parameters

Panel A of Figure 33 contrasts the labor force participation rate by age groups in the model with the data. Panel B plots the share of the labor force by age in 2014–2018. The model matches these outcomes well. Panel C plots the estimated flow values of leisure, $b(a)$, and of being one's own boss, $k(a)$, relative to average age-conditional match output. On average, the flow value of leisure corresponds to 5–25 percent of match output, while the flow value of being one's own boss is about 20 percent of average match output.

FIGURE 33. ADDITIONAL MOMENTS AND PARAMETERS

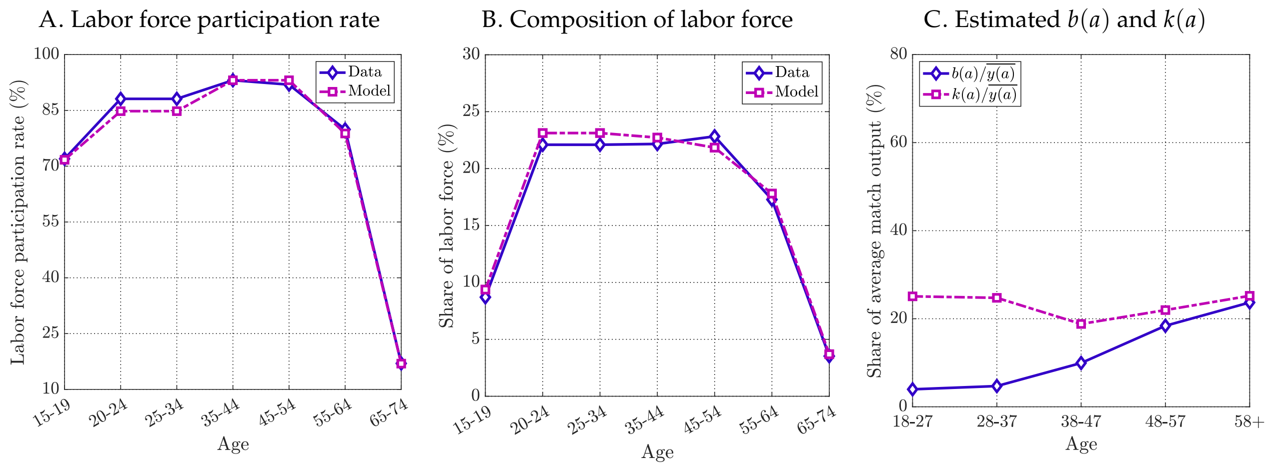



Figure 33 shows additional moments and parameters in the model and data. Panel A plots the share of the population that participates in the labor market by age groups. Panel B plots each age group's share of the labor force (all labor force participants aged 15–74). Panel C plots the estimated flow values of leisure, $b(a)$, and of being one's own boss, $k(a)$, relative to average flow output per employed worker of that age group, $\overline{y}(a)$. All data moments refer to 2014–2018 averages. *Source:* AKU, model.

## D.2 Identification

To determine the internally estimated parameters, I use a two-step algorithm. First, I search for an approximate global solution for the 13 internally estimated parameters. To that end, I sample parameter vectors randomly across a wide grid of potential values using Sobol sequences. This approach is beneficial because it avoids getting stuck when a particular parameter vector is not associated with an equilibrium. For each draw of a parameter vector, I determine internally the flow value of leisure such that workers' reservation threshold equals the reduced form specification $\underline{z}^w(a) = \beta a$ and the flow value of being one's own boss such that entrepreneurs of each age are indifferent between keeping their firm in business and exiting to unemployment at the lowest grid point for productivity. I solve the model, compute a set of moments, and store these. Subsequently, I pick the parameter vector that minimizes the sum of squared percentage deviations between the 13 targeted moments in the model and data.

In the second step, I perfect the global solution by constructing a new grid for each of the 13 parameters that spans a smaller ball around the approximate global solution. I again solve the model a large number of times for parameter vectors in this more narrow space, and record a set of moments. I pick as the estimated parameter vector that which minimizes the objective function.

Although the estimation is joint in the sense that all moments inform all parameters, some moments are more informative of some parameters. To highlight the heuristic identification argument made in Section 5, Figure 34 plots how each parameter affects its designated moment as it varies around its estimated value, holding all other parameters fixed at their estimated values. Each parameter induces a distinct movement in its chosen targeted moment, in the expected direction. That is, a greater efficiency of the matching function ($\chi$) raises the NE rate. A higher relative search intensity in employment ($\phi$) raises JJ mobility, although the impact is surprisingly modest. A larger rise in the reservation threshold with age ($\beta$) leads to a smaller ratio of JJ mobility at age 50 to age 30. The reason is that it implies that older workers are pickier in terms of what offers they accept out of non-employment, such that their subsequent probability of accepting an outside offer is smaller.

A higher exogenous separation rate ($\delta_0$) raises the EN rate. A more negative slope with productivity ($\delta_1$)—going to the right in the graph—is associated with a larger fall in the EN rate with productivity. A higher arrival rate of business ideas per unit of search intensity ($\pi_0$) is associated with a higher entry rate. A larger increase in the arrival rate with experience ($\pi_1$) is associated with a lower entry rate at age 20 to age 30. A higher curvature of the cost of searching for ideas ($\eta_e$) leads to a smaller decline in entry with productivity.

Smaller general knowledge spillovers ($\alpha_0$)—going to the right in the graph since the estimated $\alpha_0$ is negative—is associated with a larger productivity gap between firms of age 1 and 10. A higher $\alpha_1$ leads to relatively higher productivity of firms started by individuals who were previously employed in better jobs. Larger dispersion in the innovation distribution ($\zeta$) is associated with higher productivity dispersion among entrant firms. Higher dispersion of subsequent shocks ($\sigma$) leads to greater productivity dispersion among older firms. A higher curvature of the vacancy cost ($\eta_v$) is associated with a smaller vacancy share of the most productive firms.

## FIGURE 34. CHANGE IN TARGETED MOMENT IN RESPONSE TO EACH PARAMETER

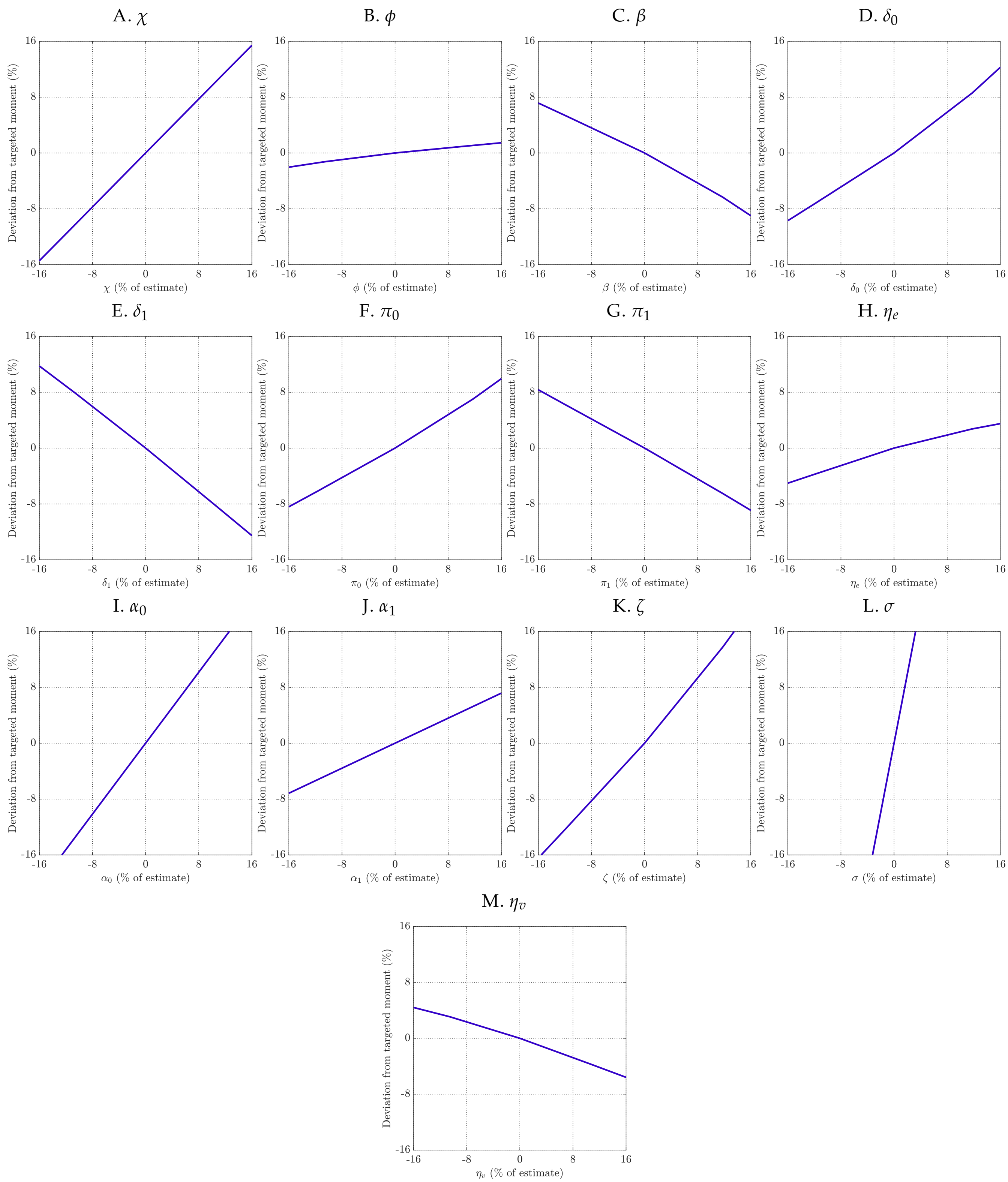



Figure 34 plots how each moment (in percent deviations from that at the estimated parameter vector) varies as each highlighted parameter varies, holding all other parameters fixed at their estimated values. *Source:* Model.

Figure 35 instead plots how the overall minimum distance moves with each parameter, holding all other parameters fixed at their estimated values. The parameters appear to be well-informed by the targeted moments, with one main caveat. The model wants $\eta_v$ to be at the very end of the pre-set global grid. Because in practice the estimated $\eta_v \approx 18$ implies that job creation of incumbents is close to completely inelastic, I opt to terminate the estimation at this high value. I have, however, confirmed that letting $\eta_v$ take even higher values makes practically no difference to the results (because $\eta_v$ hits the end of the grid, it is not meaningful to include the minimum distance plot for this parameter).

FIGURE 35. CHANGE IN MINIMUM DISTANCE IN RESPONSE TO EACH PARAMETER

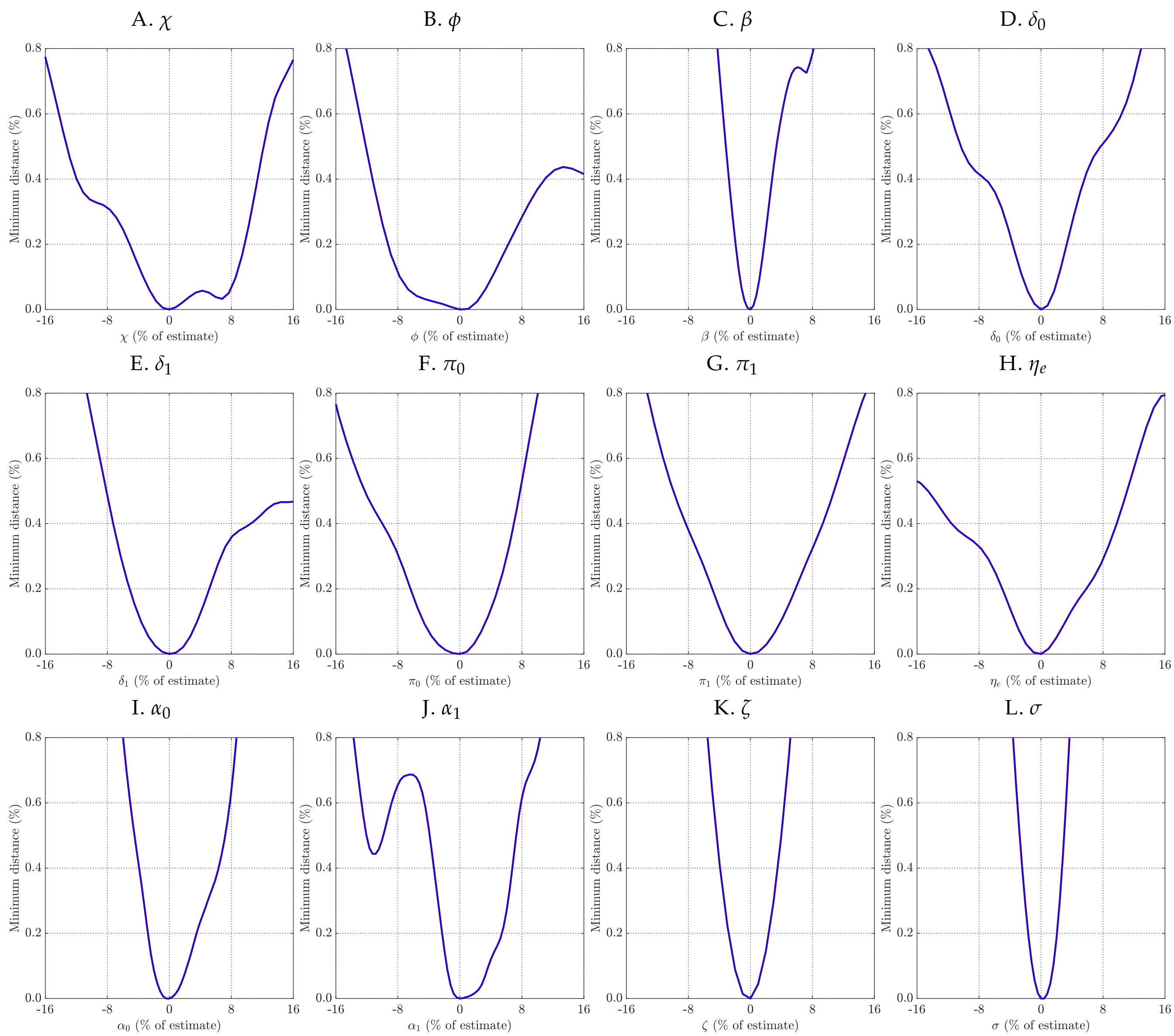



Figure 35 plots how the minimum distance criterium varies as each highlighted parameter varies around its estimated value, holding all other parameters fixed at their estimated values. *Source:* Model.

### D.3 A decomposition of life-cycle dynamics

Figure 36 provides a decomposition of life-cycle dynamics in the estimated model into the role of participating in the labor market, climbing the job ladder, and aging. According to panel A, the initial increase in firm creation is accounted for by two forces. First, individuals enter the labor market at a random age such that the model matches the labor force participation rate by age in the data, and by assumption individuals cannot start firms before they have entered the labor market. Second, the arrival rate of ideas is estimated to increase during the first 10 years of careers. The subsequent decline in firm creation with age is accounted for by three forces. First, at some point individuals start exiting the labor force, and by assumption individuals cannot create firms when they do not participate in the labor market. Second, older individuals have a shorter expected time remaining in the market, and are hence less likely to enter. Third, older individuals are better matched and hence have a higher opportunity cost of entry.

Most of the decline in JJ mobility with age is accounted for by individuals moving up the job ladder, with some role also for a higher reservation threshold for older individuals (panel B).

FIGURE 36. LIFE-CYCLE DECOMPOSITION

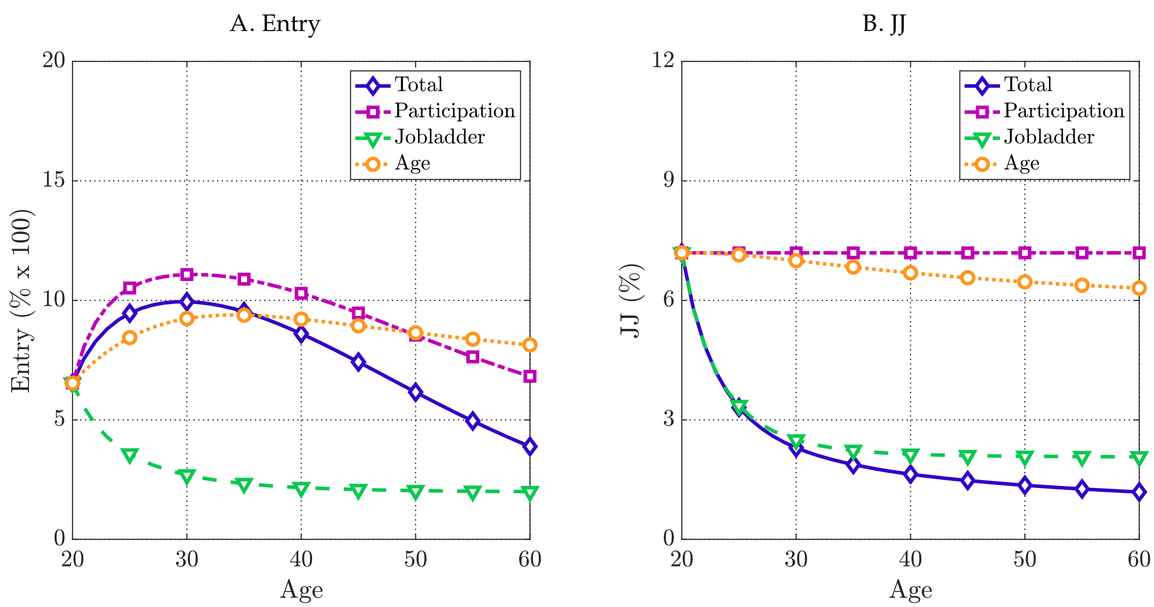



Figure 36 provides a decomposition of life-cycle dynamics in the estimated model. The participation effect in panel A arises because individuals enter the labor market at an age calibrated such that the model matches the empirical labor force participation rate by age, and by assumption individuals cannot start firms before they have entered the labor market. The age effect is due to the fact the arrival rate of business ideas jumps with some minimum labor market experience, and later in life incentives to create firms are lower because individuals have a shorter expected time remaining in the market. The age effect in panel B arises because older individuals are estimated to have a higher reservation threshold ($\beta > 0$). Consequently, they are less likely to accept a low productive job with a high subsequent mobility rate. *Source:* Model.

# E Appendix: Aging

This section contains a shift-share analysis of the effects of aging on entry and JJ mobility (Appendix E.1).

## E.1 Shift-share analysis of the effect of aging

Table 13 provides a shift-share analysis of the impact of aging in the data and model across BGPs.[54] Holding fixed age-specific mobility, shifts in the age composition generates a five percent fall in entry in the data and a four percent decline in the model. Mechanically, shifts in the age composition play a relatively minor role in accounting for the aggregate decline in firm creation, because firm creation is non-monotone in age. Hence, most of the aggregate decline in firm creation is accounted for by an age-specific decline in the probability of entry. Shifts in composition generates an eight percent fall in JJ mobility in the data versus a nine percent fall in the model. Changes in age-specific mobility accounts for another eight percent fall in JJ mobility in the data and a five percent decline in the model. Hence, both shifts in composition and age-specific declines in mobility play an important role in accounting for the declines in JJ mobility. Appendix E.1 provides a further discussion of these effects.

TABLE 13. SHIFT-SHARE ANALYSIS OF EFFECT OF AGING ACROSS BGPS

| | Entry | | | | JJ | | | |
|---|---|---|---|---|---|---|---|---|
| | Data | | Model | | Data | | Model | |
| | p.p. | % | p.p. | % | p.p. | % | p.p. | % |
| Composition | -0.005 | -5.1% | -0.003 | -3.5% | -0.227 | -8.3% | -0.242 | -8.8% |
| Return | -0.021 | -22.0% | -0.009 | -11.2% | -0.229 | -8.3% | -0.124 | -4.5% |
| Total change | -0.025 | -25.4% | -0.013 | -14.8% | -0.446 | -16.2% | -0.360 | -13.1% |

Table 5 shows the effect of an increase in the share of the overall labor force aged 16–64 that is aged 45–64 from 34.2 to 39.7 percent across BGPs, driven by a change in the growth rate of labor supply $\lambda$ from 0.0013 to 0.0003 percent. All other parameters are held fixed at their estimated values. The composition effect constructs age conditional mobility rates in age bins 16-24, 25-34, 35-44, 45-54 and 55-64, holds these fixed at their level in the early period (1993–1997 for entry and 1986–1990 for JJ mobility), and shifts only share of each age group to match the change in their share of the labor force. In the data, the latter is constructed as the change between 1986–1990 and 2014–2018. In the model, it is constructed as the change in response to a change in $\lambda$ from 0.0013 to 0.0003 percent. The return effect instead holds each age group's share of the labor force fixed, and shifts age conditional mobility rates. In the data, the latter is constructed as the change between 1993–1997 (entry) and 1986–1990 (JJ), and 2014–2018. In the model, it is constructed as the change in response to a change in $\lambda$ from 0.0013 to 0.0003 percent. *Source:* AKU, FEK, JOBB, LISA, model.

## E.2 Structural versus reduced-form estimates of age-specific impact

Panel B of Figure 37 shows that aging reduces the age-conditional probability of starting a firm, consistent with Swedish trends over this period. The estimated effect of aging on the age-conditional entry rate also matches well the cross-sectional evidence in Section 3. In particular, I plot in Figure 37 the change in age-conditional mobility implied by the OLS and IV estimates in Table 3 in response to the observed

[54] Table 13 aggregates ages to 10 year age bins in both the model and data, which accounts for the slight difference to Table 5.

change in the share of older labor force participants. Panel C shows that the structural estimate is also consistent with both the time series trend in JJ by age in Sweden as well as the cross-sectional estimates in Section 3.

FIGURE 37. EFFECT OF AGING ACROSS BGPS

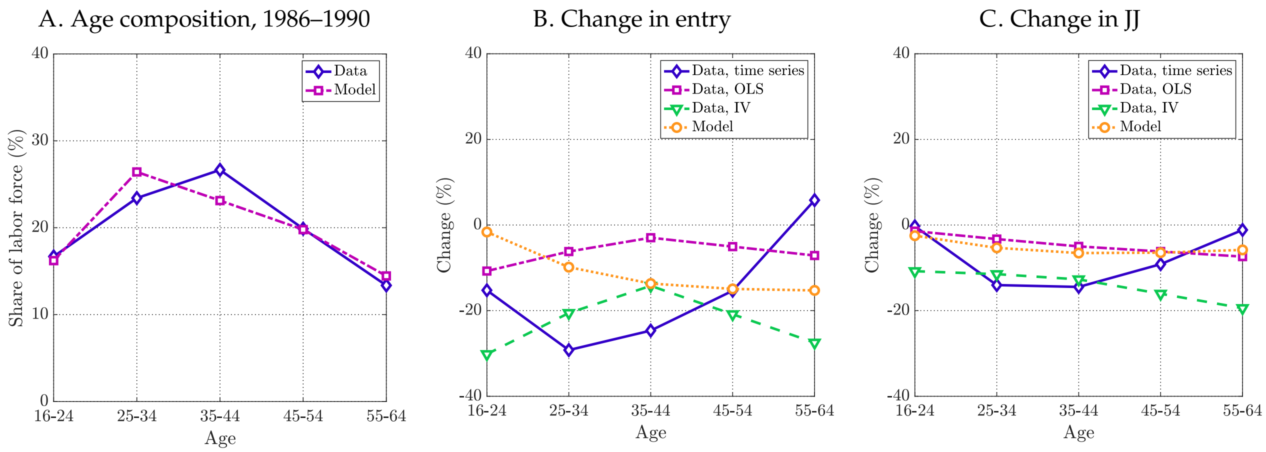



Figure 37 shows the effect of an increase in the share of the overall labor force aged 16–64 that is aged 45–64 from 34.2 to 39.7 percent across BGPs, driven by a change in the growth rate of labor supply $\lambda$ from 0.0013 to 0.0003 percent. All other parameters are held fixed at their estimated values. Panel A plots each age group's share of the overall labor force aged 16–64. The time series change in panels B–C plots the change in the entry and JJ rate by age groups between years 1993–1997 (entry) and 1986–1990 (JJ rate) and 2014–2018. The entry rate can only be constructed at the individual level since 1993. The cross-sectional change is that predicted by the cross-sectional OLS estimate by age in Section 3 in response to a 6.6 log point change in the share of older individuals. I linearly interpolate between the three aggregate age groups in Table 3. *Source:* AKU, FEK, JOBB, LISA, model.

## E.3 Aging over the transition

Figure 38 plots the share of older individuals in the labor force over time in the data and model over the transition. The model is able to match pretty well the evolution of the age composition of the labor force.

FIGURE 38. AGING OVER THE TRANSITION PATH

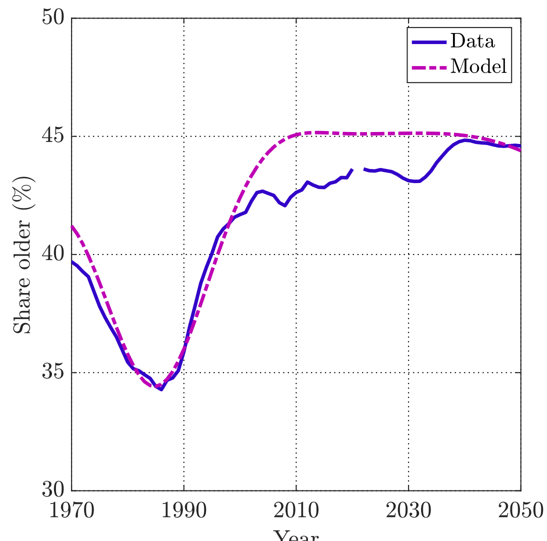



Figure 11 shows the effect of letting the growth rate of labor supply, $\lambda(t)$, vary to match the evolution of the share of the labor force that is aged 45–64 over the 1930–2060 period, using official projections for the future. *Source:* SCB, model.