Evaluating Policy Institutions*

–150 Years of US Monetary Policy–

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Abstract

How can we evaluate the performance of a policy institution? An evaluation based on realized outcomes over time is flawed, since different time periods can witness both different economic environments and different economic or geopolitical shocks. In this work, we show that it is possible to quantitatively evaluate and rank policy performance using only two estimable statistics: (i) the impulse responses of the policy objectives to policy shocks, and (ii) the impulse responses of the same policy objectives to non-policy shocks, e.g., aggregate demand shocks, energy price shocks, productivity shocks, or war shocks. For a large class of models, the correlation between these two sets of impulse responses directly captures the performance of the policy institution: A correlation of zero indicates best performance —the policy institution could not have reacted any better to the shocks that affected the economy—, while a correlation of one (in absolute value) indicates worst performance —the institution could have (but did not) perfectly met its objectives by undoing the effects of the non-policy shocks—. We use our methodology to evaluate US monetary policy over the past 150 years; from the Gold standard period, the early Fed years and the Great Depression to the post World War II period and the post-Volcker regime.

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1 Introduction

How can we evaluate and compare the performance of policy institutions over time? A naive approach would be to evaluate performance based on realized economic or social outcome variables. For instance, we could compare central bank chairs based on average inflation and unemployment outcomes, or presidents of countries based on average growth and change in inequality. While such approach is commonly adopted in the popular press, it suffers a number of major problems: (i) different policy makers face different initial conditions, e.g. a president can inherit a strong or weak economy from her predecessor, (ii) different policy makers face different economic shocks, e.g. oil price shocks can affect the ability of central banks to control inflation but their occurrence is not controlled by the central bank, and (iii) different policy makers face different structural economic environments, e.g. a decline in the slope of the Phillips curve creates a new environment which requires a different approach to offsetting adverse shocks.

This triplet of confounding factors coming from different initial conditions, different structural shocks and different economic environments severely complicates the evaluation of policy makers based on realized outcomes.[1]

To make progress it is instructive to consider an ideal, yet infeasible, approach for comparing policy makers: an experimental setting. Consider setting up a laboratory, in which different policy makers enter and are subjected to the same initial conditions and economic environment. Subsequently they are exposed to the same sequence of shocks and can make decisions that aim to achieve their mandates. Afterwards, we can compare their performance based on any desired outcome variables and conclude which policy maker performed better. Since the only variation in outcomes comes from the decisions of the policy makers, such comparison would be on equal grounds.

The objective of this paper is to develop econometric methods that can bring us closer to mimicking this ideal experiment using observational data.

To set up our approach, we formalize the actions that the policy maker could take in terms of a generic reaction function which captures how the policy maker responds to all endogenous and exogenous variables in the economy. Importantly, we will not assume that we know or can estimate this function, we merely pose its existence. Further, we aggregate the different outcome variables of interest in a quadratic loss function. The choice for the variables in the loss function is determined by the researcher conducting the comparison.[2] Finally, we assume that the underlying economic model falls in a general class of linear forward looking models, which includes a large number of models that are commonly used for policy making, such as log linearized New Keynesian models, structural VARs, linear

[1]See Fair (1978) for an early discussion of these points.

[2]Note that it is not our objective to evaluate policy makers based on their own, often unknown, objectives.

rational expectation models, etc.

For this set-up our evaluation of policy performance is based on measuring the distance between the policy maker's reaction function and the optimal reaction function —the systematic response to all endogenous and exogenous variables that minimizes the expected loss function—. We formalize this distance as a multi-dimensional object with elements that measure how the response to any non-policy shock should have been adjusted for each policy instrument. We compute these individual distances using *Optimal Reaction Function Adjustment* (ORA) statistics, which exactly measure how the reaction function should have been adjusted in order to minimize the loss function. ORA statistics are thus defined for any combination of policy instrument and non-policy shock and they can be aggregated to obtain an overall measure of performance.

There are three main benefits of using ORA statistics for evaluating and comparing policy makers. First, by defining optimality in the direction of identified non-policy shocks we avoid confounding from initial conditions and other structural shocks. Second, by considering the adjustment to the same non-policy shock and policy instrument we measure distance in the same units and make the ORA statistic comparable across different economic environments. Third, and most practically relevant, the ORA statistic depends only on the impulse responses to policy and non-policy shocks implying that we do not need to know the reaction function of the policy maker in order to compute the ORA.

In fact, the ORA is the projection of the impulse responses of the objectives to non-policy shocks on the impulses responses of the objectives to policy shocks with a minus sign. Intuitively, this follows as in equilibrium the effect of any change in the *response to a non-policy shock* on the policy objectives is proportional to the effect of an identified *policy shock* on those same objectives; both are innovations to the policy rule. This implies that the response to a non-policy shock in equilibrium can be written as a sum of the original policy maker's response (i.e. the impulse response to the non-policy shock) plus an adjustment that may lower the loss function, which is proportional to the impulse response to the policy shock. The optimal adjustment, which minimizes the expected loss function, then uses the impulse responses to the policy shocks to minimize the impulse responses to the non-policy shocks; leading to the ORA statistic. Further, if the policy maker's reaction function was optimal the impulse responses to the non-policy shocks should be orthogonal to the impulse responses to the policy shocks; there is no adjustment that the policy maker could have made that could have further lowered the loss function and the ORA is zero.

In a nutshell, the key insight underlying our work is that the effect of an (unknown) reaction function is *already* encoded in the impulse responses to (non-)policy shocks, capturing what the policy maker did and what could have been done. Thus estimating impulse responses to policy and non-policy shocks is enough to assess (a) the optimality of a policy maker's reaction function, (b) compute the distance of the adopted reaction function to the optimal reaction function and (c) compare different policy makers based on their reaction function.

In practice any desired identification and estimation method can be used to estimate the impulse responses. Inevitably, this requires specifying an econometric model, e.g. choosing instruments and control variables, but an advantage is that multiple structural models can imply the same reduced form econometric model. A recent discussion of structural shock identification can be found in Ramey (2016) and modern impulse response estimation methods are discussed and compared in, among others, Stock and Watson (2016), Kilian and Lütkepohl (2017) and Li, Plagborg-Møller and Wolf (2022).

In addition to highlighting how the existing macro econometric literature can be leveraged for estimating the ORA, we provide provide a convenient one-step GMM approach that (a) avoids the estimation of the impulse response to the non-policy shocks by a generalized least squares transformation and (b) yields closed form expressions for the asymptotic variance of the ORA. The GMM approach is shown compatible with several existing macroeconomic identification strategies.

To illustrate our methodology, we evaluate and compare the performance of the different Fed chairs between 1914 and 2018. Over this period the Fed had different chairs who all inherited a different economy, faced different shocks and economic conditions.

Perhaps surprising, the literature has produced only a few types of methods for evaluating and comparing macroeconomic policy makers.

An early contribution is Fair (1978) who highlights the distortions stemming from different initial conditions and economic environments. His solution was to adopt optimal control methods to compare policy makers. This approach amounts to specifying a structural model, calculating what would have been the optimal policy based on the model and comparing the loss under such optimal policy to the loss under the implemented policy. Clearly such recipe is general and has been used within the context of other structural models, including New Keynesian models and larger dsge models (e.g. Gali and Gertler, 2007).

If the proposed model is correctly specified, using such model-based approach for policy evaluation is appropriate. Unfortunately, specifying the correct model for (i) the economy and (ii) the behavior of the policy maker is a difficult task. Using our approach we can reduce the risk of model misspecification as we only require the estimation of impulse responses for which more robust reduced form econometric methods can be adopted (e.g. Ramey, 2016; Stock and Watson, 2016).

Moreover, related to (ii) we note that in practice policy makers do not mechanically follow simple policy rules, and they respond to much more information than captured by a few variables.[3] In fact, in many practical policy settings, e.g. fiscal or climate, the reaction function is rarely explicitly modeled, discussed, let alone known, yet it is clear that decisions are based on some information set. As such, the reaction function can be viewed as a complex function that incorporates a large information set, but is unlikely that it can be written down explicitly.[4]

Outside of specific "model-based" approaches to policy evaluation there exists few methods for evaluating and ranking policy makers. A notable exception is Blinder and Watson (2016) who improve on the naive approach to policy evaluation by projecting out specific non-policy shocks to assess the contribution of such shocks to explain the unconditional difference between policy makers. While their approach captures the average loss controlling for the identified non-policy shocks, it does not answer whether a particular policy maker responded better or worse to any given non-policy shock. The latter is the evaluation criteria proposed in this paper.[5]

The remainder of this paper is organized as follows. The next section illustrates our method for a simple New Keynesian model. Section 3 presents the general environment. Section 4 provide the key population results for evaluating and ranking policy makers. Details for the econometric implementation are given in Section 5. The results from the empirical study for monetary policy are discussed in Section 6. Section 7 concludes.

2 Illustrative example

To provide the intuition for our approach we informally present an example to illustrate how we can evaluate and compare policy makers based on their reaction function without having access to the specification of the underlying economic model nor access to their reaction function. We take the baseline New Keynesian (NK) model as the underlying economy and postulate that the researcher is interested in evaluating a central bank based on its ability to control inflation and the output gap under discretion, see Galí (2015, Section 5.1.1). In

[3]As Svensson puts it, "An optimal policy responds to all relevant state variables (including all relevant information), and there are many more relevant state variables and much more relevant information than current inflation and output" (e.g. Svensson, 2003).

[4]Policy makers repeatedly voice their concern that simple algebraic instrument rules are too simple to capture the complexity of the underlying economy. For instance, in the context of monetary policy Svensson (2017) writes "Taylor-type rules are too restrictive and mechanical, not taking into account all relevant information, and the ability to handle the complex and changing situations faced by policy makers". Algebraic rules cannot capture ex-ante all relevant contingencies, and a lot of information may simply be "non-rulable" (Kocherlakota, 2016; Blinder, 2016). See Blanchard (2018); Blanchard, Leandro and Zettelmeyer (2020) for similar arguments in the context of fiscal policy.

[5]To be more specific, the question in Blinder and Watson (2016) is whether the difference in average GDP growth between republican and democratic presidents survives after projecting out non-policy shocks. In this context, projecting out the non-policy shocks is clearly appropriate, however, when the interest is in whether the policy maker responded optimally to non-policy shocks it is not.

this setting the optimal policy is well understood, and analytically tractable, allowing us to highlight the main mechanisms of our approach and to link back to the broad NK literature (e.g. Galí, 2015).[6]

Specifically, we consider evaluating a central bank based on the loss function

$$\mathcal{L}_t = \frac{1}{2}(\pi_t^2 + x_t^2) \ , \tag{1}$$

with π_t the inflation gap and x_t the output gap.

The log-linearized Phillips curve and intertemporal (IS) curve of the baseline New-Keynesian model (Galí, 2015) are given by

$$\begin{aligned} \pi_t &= \mathbb{E}_t \pi_{t+1} + \kappa x_t + \xi_t \ , & (2) \\ x_t &= \mathbb{E}_t x_{t+1} - \frac{1}{\sigma}(i_t - \mathrm{E}_t \pi_{t+1}) \ , & (3) \end{aligned}$$

with i_t the nominal interest rate set by the central bank and ξ_t an iid cost-push shock. We denote by θ the parameter vector that includes all parameters of the Phillips and IS curves; that is the economic environment in this simple example.

To illustrate our approach suppose that the policy maker decides on the interest rate by responding to the economy according to

$$i_t = \phi_\pi \pi_t + \phi_\xi \xi_t + \epsilon_t \ , \tag{4}$$

where $\phi = (\phi_\pi, \phi_\xi)$ is the reaction function which captures the systematic response of the central bank and ϵ_t is a policy shock. We assume that the structural shocks ξ_t and ϵ_t have mean zero, variances σ_ξ^2 and σ_ϵ^2, and are mutually uncorrelated. As we will see below, the benefit of including ξ_t in (4) is that it ensures the existence of a unique equilibrium under an optimal policy rule, irrespective of the parameter values θ (e.g. Galí, 2015, page 133).

Equilibrium allocation

For any $\phi_\pi > 1$ we can write the model for $Y_t = (\pi_t, x_t)'$ and i_t as

$$Y_t = \Gamma \xi_t + \mathcal{R} \epsilon_t \qquad \text{and} \qquad i_t = \Theta_\xi \xi_t + \Theta_\epsilon \epsilon_t \ , \tag{5}$$

[6] In Appendix A we discuss two other examples. First, we consider the case where the economy is specified by the same baseline NK model, but now the loss function of the researcher is specified for the entire future path of inflation and output gaps, i.e. under commitment Galí (2015, Section 5.1.2). Second, we consider the case where the economy is specified by a general structural VAR model.

with

$$\Gamma = \begin{bmatrix} \frac{1-\kappa\phi_\xi/\sigma}{1+\kappa\phi_\pi/\sigma} \\ \frac{-\phi_\pi/\sigma-\phi_\xi/\sigma}{1+\kappa\phi_\pi/\sigma} \end{bmatrix} , \quad \mathcal{R} = \begin{bmatrix} \frac{-\kappa/\sigma}{1+\kappa\phi_\pi/\sigma} \\ \frac{-1/\sigma}{1+\kappa\phi_\pi/\sigma} \end{bmatrix} , \quad \Theta_\xi = \frac{\phi_\pi + \phi_\xi}{1+\kappa\phi/\sigma} \quad \text{and} \quad \Theta_\epsilon = \frac{1}{1+\kappa\phi_\pi/\sigma} ,$$

where Γ and $\mathcal{R}$ capture the responses of the structural shocks ξ_t and ϵ_t on the policy objectives. Similarly, Θ_ξ and Θ_ϵ capture the effect of the structural shocks on the interest rate. We stress that each term depends on the reaction function ϕ.

For future reference it is useful to make explicit that representation (5) arises from premultiplying the display below by $\mathcal{A}^{-1}$.

$$\underbrace{\begin{bmatrix} 1 & -\kappa & 0 \\ 0 & 1 & 1/\sigma \\ -\phi_\pi & 0 & 1 \end{bmatrix}}_{=\mathcal{A}} \begin{bmatrix} \pi_t \\ x_t \\ i_t \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ \phi_\xi \end{bmatrix} \xi_t + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \epsilon_t \ . \tag{6}$$

The point to note is that innovations to the reaction function, here ϵ_t, enter the equilibrium representation (5) via the expression $\mathcal{A}^{-1}(0,0,1)' = (\mathcal{R}', \Theta_\epsilon)'$.

Optimal reaction function

An optimal reaction function is defined as any $\phi = (\phi_\pi, \phi_\xi)$ that minimizes the expected loss subject to the equations that describe the economy. Formally, let Φ denote the subset of reaction functions that lead to a unique equilibrium. The set of optimal reaction functions is given by

$$\Phi^{\text{opt}} = \left\{ \phi \in \Phi : \phi \in \operatorname*{argmin}_{\phi\in\Phi} \mathbb{E}\mathcal{L}_t \quad \text{s.t.} \quad (2)-(4) \right\} \ .$$

We note that since the economy (2)-(3) is linear and the reaction function (4) allows the central bank to respond to all available information (i.e. all endogenous variables and structural shocks) the set Φ^{opt} is not a priori constrained.[7] Moreover, by virtue of the rule (4) we have Φ^{opt} is non-empty for any θ, see Galí (2015, page 133).

In general Φ^{opt} will include infinitely many reaction functions as optimal responses to structural shocks can often be interchanged with optimal responses to the endogenous variables. Note that we purposely allow for this possibility as we do not wish to impose any knowledge of the variables or shocks to which the policy maker responds.

[7] For instance, if the output gap x_t was affected by an additional shock, say a productivity shock, we would include the output gap or the productivity shock in the policy rule as well.

Testing the reaction function

As a first step towards evaluating the central bank we consider testing whether the reaction function of the central bank was optimal.

Let ϕ^0 denote the central bank's reaction function which we assume is unknown to the researcher. We are interested in testing whether $\phi^0 \in \Phi^{\text{opt}}$, i.e. assessing whether the policy maker minimized the expected loss. To construct a test statistic, we consider a thought experiment where the proposed policy rule (4) under ϕ^0 is modified with some change τ in response to the cost-push shock:

$$i_t = \phi^0_\pi \pi_t + (\phi^0_\xi + \tau)\xi_t + \epsilon_t \ . \tag{7}$$

If for any $\tau \neq 0$ we are able to lower the expected loss we may conclude that ϕ^0 was not optimal.

To verify whether this is the case, suppose that ϕ^0 leads to a unique equilibrium, i.e. $\phi^0 \in \Phi$. Then, following the same steps that led to (5) we have that under (7)

$$Y_t = (\Gamma^0 + \mathcal{R}^0 \tau)\xi_t + \mathcal{R}^0 \epsilon_t \ , \tag{8}$$

where Γ^0 and $\mathcal{R}^0$ denote the responses to the structural shocks under the rule ϕ^0 and are defined in (5). To see how this expression arises, consider the modified version of (6) under the augmented reaction function (7):

$$\underbrace{\begin{bmatrix} 1 & -\kappa & 0 \\ 0 & 1 & 1/\sigma \\ -\phi^0_\pi & 0 & 1 \end{bmatrix}}_{=\mathcal{A}} \begin{bmatrix} \pi_t \\ x_t \\ i_t \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ \phi^0_\xi \end{bmatrix} \xi_t + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tau\xi_t + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \epsilon_t \ . \tag{9}$$

The key insight is that $\tau\xi_t$ enters the equilibrium allocation for $Y_t = (\pi_t, x_t)'$ in exactly the same way as ϵ_t, i.e. via $\mathcal{A}^{-1}(0,0,1)'$. This is understandable when viewing $\tau\xi_t$ as an innovation to the policy rule, which in equilibrium has effect $\mathcal{R}^0$ on the policy objectives. Indeed, any innovation to the instrument has an effect proportional to $\mathcal{R}^0$ in equilibrium, the monetary shock is only a special case, which is usually normalized to have a unit effect on i_t.

Now, as mentioned, if $\phi^0 \in \Phi^{\text{opt}}$ there should not be any $\tau \neq 0$ that is able to lower the loss function. A necessary condition for this is that the gradient of the loss function with respect to τ evaluated at $\tau = 0$ should be zero. This leads to the following equivalence

$$\phi^0 \in \Phi^{\text{opt}} \qquad \Longleftrightarrow \qquad \nabla_\tau \mathbb{E}\mathcal{L}_t\big|_{\tau=0} \propto \mathcal{R}^{0'}\Gamma^0 = 0 \ . \tag{10}$$

This is a key result of this paper: if and only if the reaction function is optimal the impulse responses of the cost-push shock on the policy objectives Γ^0 should be orthogonal to the impulse responses of the policy shock on the policy objectives $\mathcal{R}^0$. To see that $\mathcal{R}^{0\prime}\Gamma^0 = 0$ implies $\phi^0 \in \Phi^{\text{opt}}$, we note that the second derivative is given by $\nabla^2_\tau \mathbb{E}\mathcal{L}_t = \sigma^2_\xi \mathcal{R}'\mathcal{R} > 0$ and hence a global (non-unique) minimum is obtained at ϕ^0 and thus $\phi^0 \in \Phi^{\text{opt}}$.

Intuitively, the inner product $\mathcal{R}^{0\prime}\Gamma^0$ captures exactly (i) what the central bank did on average to offset the cost-push shock, i.e. Γ^0, and (ii) what the central bank could have done to offset the cost-push shock, i.e. $\mathcal{R}^0$. Note that (ii) follows from our previous observation that changes in the response to the cost-push shock, i.e. changes in ϕ_ξ, have an equilibrium effect on Y_t that is proportional to $\mathcal{R}^0$.

In practice, we can use different macro econometric methods and identification approaches to estimate the impulse responses Γ^0 and $\mathcal{R}^0$ (e.g. Ramey, 2016). We emphasize that these methods typically only require the estimation of a reduced form econometric model in combination with some identification strategy, e.g. short-run, long-run, or sign restrictions, or external instruments, heteroskedasticity, non-Gaussianity, etc. This implies that for testing whether $\mathcal{R}^{0\prime}\Gamma^0 = 0$ we do not need to know the specific underlying structural model nor the reaction function ϕ^0.

That said, in this example we can easily verify the optimality condition as we know that the optimal reaction function takes the form $\phi^{\text{opt}}_\xi = (\kappa\sigma - \phi^{\text{opt}}_\pi)/(1+\kappa^2)$ for any $\phi^{\text{opt}}_\pi > 1$, see Galí (2015, page 133). Writing out $\mathcal{R}^{0\prime}\Gamma^0$ under any $\phi^0 = \phi^{\text{opt}}$ gives

$$
\begin{aligned}
\Gamma^{0\prime}\mathcal{R}^0 &= (1+\kappa\phi^0_\pi/\sigma)^{-2}[-\kappa/\sigma \ , \ -1/\sigma] \begin{bmatrix} 1-\kappa\phi^0_\xi/\sigma \\ -\phi^0_\pi/\sigma - \phi^0_\xi/\sigma \end{bmatrix} \\
&= \frac{-\kappa/\sigma + \kappa/\sigma + \phi^0_\pi/\sigma^2 - \phi^0_\pi/\sigma^2}{(1+\kappa\phi^0_\pi/\sigma)^2} = 0 \ .
\end{aligned}
$$

This mechanically shows that under an optimal rule the gradient condition holds.

Optimal Reaction Function Adjustment

Besides testing whether the given reaction function is optimal we may also compute the distance to the optimal reaction function. This provides a continuous measure for evaluating policy makers and allows to assess how the reaction function should have been adjusted.

Specifically, we measure how far the policy maker's choice ϕ^0 is from the set of optimal policies by

$$
\begin{aligned}
\tau^* &= \underset{\tau}{\operatorname{argmin}} \ \mathbb{E}\mathcal{L}_t \qquad \text{s.t.} \qquad Y_t = (\Gamma^0 + \mathcal{R}^0\tau)\xi_t + \mathcal{R}^0\epsilon_t \\
&= \underset{\tau}{\operatorname{argmin}} \ \sigma^2_\xi(\Gamma^0 + \mathcal{R}^0\tau)'(\Gamma^0 + \mathcal{R}^0\tau) \ , \qquad (11)
\end{aligned}
$$

where the second equality uses that the structural shocks have mean zero and are uncorrelated. We refer to the statistic τ^* as the *Optimal Reaction Function Adjustment* or ORA as it measures exactly how much more or less the policy maker should have responded to the cost-push shock in order to minimize the loss function. It is easy to verify that $\phi^* = (\phi_\pi^0, \phi_\xi^0 + \tau^*) \in \Phi^{\text{opt}}$.

A closed form solution for the ORA statistic is given by

$$\tau^* = -(\mathcal{R}^{0\prime}\mathcal{R}^0)^{-1}\mathcal{R}^{0\prime}\Gamma^0 \ , \tag{12}$$

which is equal to the projection of Γ^0 —what the policy maker did— on $\mathcal{R}^0$ —what the policy maker could have done—. This projection coefficient can be obtained from the policy problem in impulse response space:

$$\Gamma^0 = -\mathcal{R}^0\tau + U \ , \tag{13}$$

where U is some remainder term. The minus sign in (13) is because the objective of the central bank is to offset the effect of the cost-push shock, i.e. Γ^0.The ORA is equal to the least squares projection coefficient τ^* which minimizes Γ^0 using $-\mathcal{R}^0$.

Comparing reaction functions

The ORA statistic can be used to compare the reaction functions of different policy makers. To avoid excessive notation at this stage, consider comparing two policy makers that used reaction functions ϕ^0 and ϕ^1, respectively, and let the economic environment that they faced by captured by the parameter vectors θ^0 and θ^1, respectively, which include all coefficients in the Phillips and IS curves.

For each policy maker we compute the distance to the set of optimal reaction functions, noting that since the economic environments are different these sets are not the same. We have

$$\tau_0^* = -(\mathcal{R}^{0\prime}\mathcal{R}^0)^{-1}\mathcal{R}^{0\prime}\Gamma^0 \qquad \text{and} \qquad \tau_1^* = -(\mathcal{R}^{1\prime}\mathcal{R}^1)^{-1}\mathcal{R}^{1\prime}\Gamma^1 \ .$$

We rank the policy maker with ϕ^0 above policy maker with ϕ^1 if $|\tau^0| < |\tau^1|$.

The key insight is that while the environments are different, the ORA statistics τ_0^* and τ_1^* measure the same quantity: the distance to the optimal reaction function in units of the policy instrument: e.g. a one unit average increase in the cost push shock the policy maker should have increased the interest rate by τ units. Moreover, by explicitly measuring the distance to optimality in the direction of the identified cost-push shock we avoid confounding from initial conditions or other shocks.

In sum, this example illustrates how we can evaluate and compare policy makers based on their reaction function without knowing the reaction function. The next sections show that these findings continue to hold for a general linear macro model and discuss the econometric implementation.

3 Environment

We consider a researcher who is interested in evaluating a policy maker based on the $M_y \times 1$ vector of gaps y_{t+h} —deviations of some variables from the researcher's desired targets— for the current period $h = 0$ and possibly future periods $h = 1, 2, \ldots$. We denote by $\mathbf{Y}_t = (y_t', y_{t+1}', \ldots)'$ the path of the gaps that starts at a given time period t and we refer to this path as the objectives. We aggregate the objectives in the loss function

$$\mathcal{L}_t = \mathbb{E}_t \mathbf{Y}_t' \mathcal{W} \mathbf{Y}_t \ , \tag{14}$$

where $\mathbb{E}_t(\cdot) = \mathbb{E}(\cdot|\mathcal{F}_t)$, with $\mathcal{F}_t$ the time t information set. The weighting matrix $\mathcal{W}$ is assumed to be diagonal allowing the researcher to place more or less weight on certain objectives and horizons.

Suppose that the policy maker has M_p instruments denoted by $p_t = (p_{1,t}, \ldots, p_{M_p,t})'$. At time t the policy maker can set the time t value of the instruments as well as their expected future path. We denote the path by $\mathbf{P}_t = (p_t', p_{t+1}', \ldots)'$ and the time t expected path is given by $\mathbb{E}_t \mathbf{P}_t$. A generic model for $\mathbf{Y}_t$ is completed by the path for the additional endogenous variables $\mathbf{W}_t = (w_t', w_{t+1}', \ldots)'$, the path of the structural shocks $\mathbf{\Xi}_t = (\xi_t', \xi_{t+1}', \cdots)'$ and the vector $\mathbf{X}_{-t} = (y_{t-1}', w_{t-1}', p_{t-1}', y_{t-2}', \ldots)'$ which captures the initial conditions. Combining we have that the economy at time t can be written as

$$\begin{cases} \mathcal{A}_{yy}\mathbb{E}_t\mathbf{Y}_t - \mathcal{A}_{yp}\mathbb{E}_t\mathbf{P}_t - \mathcal{A}_{yw}\mathbb{E}_t\mathbf{W}_t &= \mathcal{B}_{yx}\mathbf{X}_{-t} + \mathcal{B}_{y\xi}\mathbb{E}_t\mathbf{\Xi}_t \\ \mathcal{A}_{ww}\mathbb{E}_t\mathbf{W}_t - \mathcal{A}_{wp}\mathbb{E}_t\mathbf{P}_t - \mathcal{A}_{wy}\mathbb{E}_t\mathbf{Y}_t &= \mathcal{B}_{wx}\mathbf{X}_{-t} + \mathcal{B}_{w\xi}\mathbb{E}_t\mathbf{\Xi}_t \end{cases} , \tag{15}$$

where $\mathcal{A}_{..}$ and $\mathcal{B}_{..}$ denote conformable linear maps. This model is general and allows expected future policy decisions to affect current and expected future outcomes. Many models found the literature can be written in this form; prominent examples include New Keynesian models and more modern heterogeneous agents models, see Barnichon and Mesters (2022) and McKay and Wolf (2022) for a more elaborate discussion of this model class. We normalize the non-policy news shocks $\mathbb{E}_t\mathbf{\Xi}_t$ —exogenous shocks about the future state of the economy that are released at time t— to have unconditional mean zero, unit variance and to be uncorrelated with the initial conditions. We collect all parameters of the non-policy block (15) in $\theta = \{\mathcal{A}_{yy}, \mathcal{A}_{yp}, \mathcal{A}_{yw}, \mathcal{A}_{ww}, \mathcal{A}_{wp}, \mathcal{A}_{wy}, \mathcal{B}_{yx}, \mathcal{B}_{wx}, \mathcal{B}_{y\xi}, \mathcal{B}_{w\xi}\}$, which can be thought of as describing the economic environment that the policy maker faces.

Turning to the policy block; we postulate that policy decisions can be written as

$$\mathcal{A}_{pp}\mathbb{E}_t\mathbf{P}_t - \mathcal{A}_{py}\mathbb{E}_t\mathbf{Y}_t - \mathcal{A}_{pw}\mathbb{E}_t\mathbf{W}_t = \mathcal{B}_{px}\mathbf{X}_{-t} + \mathcal{B}_{p\xi}\mathbb{E}_t\mathbf{\Xi}_t + \mathbb{E}_t\boldsymbol{\epsilon}_t \ , \tag{16}$$

where $\boldsymbol{\epsilon}_t = (\epsilon_t', \epsilon_{t+1}', \ldots)'$ a sequence of policy shocks and $\mathbb{E}_t\boldsymbol{\epsilon}_t$ transforms the policy shocks into policy news shocks, i.e. exogenous shocks to the future path of $\mathbf{P}_t$ that are released at time t. The policy rule (16) imposes no restrictions as it allows the policy maker to respond to all available variables and shocks in the economy. We normalize all policy news shocks $\mathbb{E}_t\boldsymbol{\epsilon}_t$ to have unconditional mean zero with unit variance, and to be uncorrelated with all non-policy news shocks and initial conditions. We collect the coefficients of the policy rule in $\phi = \{\mathcal{A}_{pp}, \mathcal{A}_{py}, \mathcal{A}_{pw}, \mathcal{B}_{px}, \mathcal{B}_{p\xi}\}$ and refer to these as the reaction function.

We denote by Φ the set of all reaction functions ϕ for which the model (15)-(16) implies a unique equilibrium. We define the set of optimal reaction functions as follows

$$\Phi^{\text{opt}} = \left\{ \phi \in \Phi : \phi \in \underset{\phi \in \Phi}{\operatorname{argmin}}\, \mathbb{E}\mathcal{L}_t \quad \text{s.t} \quad (15) - (16) \right\} \ . \tag{17}$$

The definition implies that we only consider optimal reaction functions that lie in Φ, i.e. the set of reaction functions which imply a unique equilibrium. In addition, optimal reaction functions are defined as minimizing the unconditional loss function.

We observe that for any $\phi \in \Phi$ we can write the expected path of the objectives of the researcher as a linear function of the contemporaneous and expected future shocks as well as the initial conditions.

$$\mathbb{E}_t\mathbf{Y}_t = \underbrace{\mathcal{K}(\phi)\mathbb{E}_t\mathbf{\Xi}_t + \Delta(\phi)\mathbf{X}_{-t}}_{=\Gamma(\phi)\mathbb{E}_t\mathbf{V}_t} + \mathcal{R}(\phi)\mathbb{E}_t\boldsymbol{\epsilon}_t \ , \tag{18}$$

where the maps $\mathcal{K}(\phi)$, $\mathcal{R}(\phi)$ and $\Delta(\phi)$ capture the causal effects of the structural shocks $\mathbb{E}_t\mathbf{\Xi}_t$ and $\mathbb{E}_t\boldsymbol{\epsilon}_t$ and the initial conditions $\mathbf{X}_{-t}$ under ϕ, respectively. For convenience we have defined $\Gamma(\phi) = [\mathcal{K}(\phi), \Delta(\phi)]$ and $\mathbb{E}_t\mathbf{V}_t = (\mathbb{E}_t\mathbf{\Xi}_t', \mathbf{X}_{-t}')'$, such that $\mathbb{E}_t\mathbf{V}_t$ includes all non-policy inputs that (i) determine the objectives and (ii) are orthogonal to the policy shocks.

Clearly, the maps $\Gamma(\phi)$ and $\mathcal{R}(\phi)$ also depend on the environment as summarized by θ, but since θ is not under the control of the policy maker we omit this from the notation. The precise mapping from the model coefficients $\mathcal{A}_{..}$ and $\mathcal{B}_{..}$ to $\Gamma(\phi)$ and $\mathcal{R}(\phi)$ is provided in Appendix B, but we will not require knowledge of this mapping in the main text.

4 Measuring reaction function optimality

We take the position of a researcher who is interested in evaluating and ranking one or more policy makers based on their reaction function, i.e. how they responded on average to the non-policy shocks and initial conditions during their term. We postulate that each policy maker faces an economy that can be represented by the generic model (15)-(16) where the parameters θ and ϕ may vary across policy makers.[8] For ease of notation we postulate that the parameters are constant within the term of each policy maker, but our econometric implementation in Section 5 can allow for parameter variation within each policy maker's term.

We first develop the methodology for evaluating the reaction function of a single policy maker in population. We denote the reaction function of this policy maker by ϕ^0 and work in a setting where ϕ^0 is unknown to the researcher. Our evaluation of ϕ^0 is based on measuring how far ϕ^0 is from the set of optimal reaction functions Φ^{opt} as defined in (17) using the ORA statistic. We then formalize how the ORA can be used as a comparative metric for ranking the performance of multiple policy makers.

4.1 Orthogonality conditions

We start by discussing some necessary conditions that must hold if $\phi^0 \in \Phi^{\text{opt}}$. These conditions are generally useful as they provide the basis for evaluating policy makers and can be used to derive different tests and measures of optimality.

The following proposition establishes a key result.

Proposition 1. *Given the generic model* (15)-(16)*, with* Φ *non-empty, we have that*

$$\phi^0 \in \Phi^{\text{opt}} \qquad \Longleftrightarrow \qquad \mathcal{R}^{0\prime}\mathcal{W}\Gamma^0 = 0 \ , \tag{19}$$

where $\mathcal{R}^0 \equiv \mathcal{R}(\phi^0)$*,* $\Gamma^0 \equiv \Gamma(\phi^0)$ *and* Φ^{opt} *is defined in* (17)*.*

The proof is shown in the appendix and follows similar steps that led to (10) in the New Keynesian example above.

Expression (19) shows that under an optimal reaction function, $\phi^0 \in \Phi^{\text{opt}}$, the dynamic causal effects of the non-policy inputs $\mathbb{E}_t\mathbf{V}_t$ on the objectives (i.e. Γ^0) should be orthogonal to the dynamic causal effects of the policy shocks on the objectives (i.e. $\mathcal{R}^0$). Intuitively, if a policy maker follows an optimal reaction function, this function should have transformed the effects of the non-policy inputs such that there is no more the policy maker can do to lower

[8] Note that the set of included variables is arbitrary in model (15)-(16), and by allowing θ and ϕ to vary across policy makers we can also capture that specific variables perhaps played no role for certain policy makers.

the loss: the impulse responses to non-policy inputs should be orthogonal to the impulse responses to changes in policy.

The key reason we do not need to know or estimate the functional form of the reaction function is that the effect of that reaction function is *already* encoded in the impulse responses Γ^0 and $\mathcal{R}^0$. Thus knowledge of Γ^0 and $\mathcal{R}^0$ is enough to evaluate the optimality of the reaction function ϕ^0.

Subset orthogonality conditions

One difficulty in practice is that there often exists insufficient exogenous variation to identify all impulse responses Γ^0 and $\mathcal{R}^0$. For instance, estimating $\mathcal{R}^0$ would require being able to identify the contemporaneous shocks to all policy instruments as well as all the policy news shocks that create exogenous variation in the expected policy paths.

Fortunately, this is not necessary as we can derive necessary conditions for the optimality of the reaction function by considering any subset of causal effects that can be estimated. Obviously such subset criteria cannot detect all deviations from optimality, but they do allow to leverage all available identified shocks to evaluate the reaction function.

To set this up, we denote by $\mathcal{R}_a^0$ the effects of $\mathbb{E}_t\boldsymbol{\epsilon}_{a,t}$ on $\mathbb{E}_t\mathbf{Y}_t$ under ϕ^0, where $\mathbb{E}_t\boldsymbol{\epsilon}_{a,t}$ is a vector formed from any subset or linear combination of the policy news shocks $\mathbb{E}_t\boldsymbol{\epsilon}_t$. Similarly, denote by Γ_b^0 the impulse responses of subsets or linear combinations $\mathbb{E}_t\mathbf{V}_{b,t}$ on $\mathbb{E}_t\mathbf{Y}_t$. The key requirement for the shocks in $\mathbb{E}_t\mathbf{V}_{b,t}$ is that they are orthogonal to the policy shocks $\mathbb{E}_t\boldsymbol{\epsilon}_t$, but it is not strictly necessary for them to have a specific interpretation.[9]

With this notation the following corollary summarizes the implication of a given subset orthogonality condition.

Corollary 1. *Given the generic model* (15)-(16) *we have that*

$$\mathcal{R}_a^{0\prime}\mathcal{W}\Gamma_b^0 \neq 0 \qquad \Longrightarrow \qquad \phi^0 \notin \Phi^{\text{opt}} \ . \tag{20}$$

The result is of great practically relevant as it shows that researchers never have to recover the entire causal maps Γ^0 and $\mathcal{R}^0$ to evaluate the reaction function. For instance, suppose a researcher is interested in testing the central bank's reaction to an oil price shock when setting the short term interest rate, then only the impulse responses of the researcher's objectives, say inflation and unemployment, to interest rate shocks and oil price shocks are needed. Ideally, in this scenario we would include all interest rate news shocks, but in practice

[9] For instance, if a researcher is sure that a particular sequence of residuals does not depend on the policy shocks such sequence can be used to form $\mathbb{E}_t\mathbf{V}_{b,t}$, see Gali and Gambetti (2020) for an identification approach along these lines.

we can conduct a subset test after only identifying, for instance, the contemporaneous policy shock.

4.2 Optimal reaction adjustment

Suppose that ϕ^0 is not optimal and the orthogonality conditions in Proposition 1 do not hold. In such settings we are interested in computing how far ϕ^0 is from the set of optimal reaction functions to obtain a continuous measure for evaluating and comparing policy makers. We compute this distance in the direction of the response to the non-policy shocks directly. Specifically, we look for a map $\mathcal{T} = [\mathcal{T}_\Xi, \mathcal{T}_X]$ which alters the response to the non-policy shocks and initial conditions to ensure that the orthogonality conditions holds. Consider the augmented policy rule

$$\mathcal{A}^0_{pp}\mathbb{E}_t\mathbf{P}_t - \mathcal{A}^0_{py}\mathbb{E}_t\mathbf{Y}_t - \mathcal{A}^0_{pw}\mathbb{E}_t\mathbf{W}_t = (\mathcal{B}^0_{px} + \mathcal{T}_X)\mathbf{X}_{-t} + (\mathcal{B}^0_{p\xi} + \mathcal{T}_\Xi)\mathbb{E}_t\mathbf{\Xi}_t + \mathbb{E}_t\boldsymbol{\epsilon}_t \ .$$

We can think about the augmented rule as a perturbation to the original rule (16) that was based on the reaction function ϕ^0. Given that $\phi^0 \in \Phi$ we can combine this policy rule with the general model (15) to obtain the equilibrium representation

$$\mathbb{E}_t\mathbf{Y}_t = (\Gamma^0 + \mathcal{R}^0\mathcal{T})\mathbb{E}_t\mathbf{V}_t + \mathcal{R}^0\mathbb{E}_t\boldsymbol{\epsilon}_t \ ,$$

where the equilibrium effect of $\mathcal{T} \neq 0$ is found to be equal to $\mathcal{R}^0\mathcal{T}\mathbb{E}_t\mathbf{V}_t$ on the objectives.

The *Optimal Reaction Adjustment* (ORA) for measuring the overall distance to the optimal reaction function is defined as the $\mathcal{T}$ that minimizes the expected loss function. Formally,

$$\mathcal{T}^* = \underset{\mathcal{T}}{\operatorname{argmin}} \ \mathbb{E}\mathcal{L}_t \qquad \text{s.t.} \quad \mathbb{E}_t\mathbf{Y}_t = (\Gamma^0 + \mathcal{R}^0\mathcal{T})\mathbb{E}_t\mathbf{V}_t + \mathcal{R}^0\mathbb{E}_t\boldsymbol{\epsilon}_t \ . \tag{21}$$

Similar as in the NK example the ORA determines how the policy maker should have responded differently to the non-policy inputs $\mathbb{E}_t\mathbf{V}_t$ to obtain an optimal reaction function for all policy instruments. A given (i, j) entry of $\mathcal{T}$ measures how the policy maker should have responded differently to the jth non-policy input when setting the ith policy instrument.

An explicit expression is given by

$$\mathcal{T}^* = -(\mathcal{R}^{0'}\mathcal{W}\mathcal{R}^0)^{-1}\mathcal{R}^{0'}\mathcal{W}\Gamma^0 \ , \tag{22}$$

which exists provided that the inverse exists, and highlights that the ORA can be viewed as the projection of the causal effects of non-policy inputs on causal effects of policy shocks. The weighting matrix $\mathcal{W}$ incorporates the preferences of the researcher who aims to determine how the reaction function should have been adjusted to achieve the objective of minimizing the loss function (14).

The following corollary establishes a key property of the ORA.

Corollary 2. *Given the generic model* (15)-(16) *we have that*

$$\phi^* \in \Phi^{\text{opt}} \ , \quad \textit{where} \quad \phi^* = \{\mathcal{A}^0_{pp}, \mathcal{A}^0_{py}, \mathcal{A}^0_{pw}, \mathcal{B}^0_{px} + \mathcal{T}^*_X, \mathcal{B}^0_{p\xi} + \mathcal{T}^*_\Xi\} \ ,$$

where $\mathcal{T}^* = [\mathcal{T}^*_\Xi, \mathcal{T}^*_X]$ *is defined in* (21).

The intuition for this result is as follows. The vector $\mathbb{E}_t \mathbf{V}_t$ captures all non-policy inputs that enter the economy at time t. These shocks and initial conditions affect the current and expected future path of the objectives as well as the other endogenous variables in the economy. By adjusting the response of the policy maker to these inputs, i.e. $\mathcal{B}^0_{px} \to \mathcal{B}^0_{px} + \mathcal{T}^*_X$ and $\mathcal{B}^0_{p\xi} \to \mathcal{B}^0_{p\xi} + \mathcal{T}^*_\Xi$, we can attain the minimum of the loss function as there are no other distortions in the economy. Since this change does not affect the coefficients in the non-policy block of the economy (15) there are no second round affects. To clarify, suppose that the coefficients in (15) did depend on say $\mathcal{B}_{p\xi}$ then changing these coefficients would change the economic environment and hence adjusting $\mathcal{B}^0_{p\xi} \to \mathcal{B}^0_{p\xi} + \mathcal{T}^*_\Xi$ may not minimize the loss given the new environment, i.e. the Lucas (1976) critique applies. By assuming that the coefficients in (15) are invariant to changes in $\mathcal{B}^0_{p\xi}$ and $\mathcal{B}^0_{px}$ we exclude this possibility. Clearly, this restriction may not be reasonable for all applications, but it is satisfied for a broad class of macro models that are commonly used for policy making.

The entries of the ORA can be individually studied — how should the response to a specific non-policy shock be adjusted for the reaction function of a specific policy instrument —, but we can also aggregate the entries to construct an overall measure of accuracy. Given that preferences for different evaluation criteria are already included in the loss function, we may simply sum the absolute or squared entries of $\mathcal{T}^*$ to obtain an overall measure of distance.

Subset reaction adjustments

Similar as above we note that in practice we will often not be able to compute $\mathcal{T}^*$ as it requires identifying the effects of all structural shocks and initial conditions. In such cases we may still compute the distance to the optimal reaction function in the direction of the non-policy inputs that can be identified for the policy instruments for which we can identify the effect of an exogenous change. This allows to evaluate policy makers based on the specific structural shocks that the researcher is able to identify. In other words, it allows to leverage existing evidence on the effects of policy and non-policy variables on the objectives to evaluate policy makers.

Specifically, as in the previous section, suppose that we can identify the policy shocks $\mathbb{E}_t\boldsymbol{\epsilon}_{a,t}$ and the non-policy inputs $\mathbb{E}_t\mathbf{V}_{b,t}$. The subset ORA is defined as

$$\mathcal{T}^*_{ab} = \underset{\mathcal{T}_{ab}\in\mathbb{R}^{d_a\times d_b}}{\text{argmin}}\ \mathbb{E}\mathcal{L}_t \qquad \text{s.t.} \quad \mathbb{E}_t\mathbf{Y}_t = (\Gamma^0_b + \mathcal{R}^0_a\mathcal{T}_{ab})\mathbb{E}_t\mathbf{V}_{b,t} + \Gamma^0_{-b}\mathbb{E}_t\mathbf{V}_{-b,t} + \mathcal{R}^0\mathbb{E}_t\boldsymbol{\epsilon}_t \ , \tag{23}$$

where $\mathcal{R}$ separates as $[\mathcal{R}_a : \mathcal{R}_{-a}]$ and Γ as $[\Gamma_b : \Gamma_{-b}]$. The details for the decomposition are provided in Appendix B.

Using that the structural shocks are mean zero and uncorrelated we can derive the closed form solution

$$\mathcal{T}^*_{ab} = -(\mathcal{R}^{0\prime}_a\mathcal{W}\mathcal{R}^0_a)^{-1}\mathcal{R}^{0\prime}_a\mathcal{W}\Gamma^0_b \ , \tag{24}$$

which shows that the subset ORA is equal to the projection of the non-policy impulse responses Γ^0_b on the policy impulse responses $\mathcal{R}^0_a$.

We have the following result.

Corollary 3. *Given the generic model* (15)-(16) *we have that the adjusted reaction function*

$$\phi^*_{ab} = \{\mathcal{A}^0_{pp}, \mathcal{A}^0_{py}, \mathcal{A}^0_{pw}, \mathcal{B}^0_{ab} + \mathcal{T}^*_{ab}, \mathcal{B}^0_{-a-b}\}$$

satisfies

$$\mathbb{E}\mathcal{L}_t(\phi^*_{ab}) \leq \mathbb{E}\mathcal{L}_t(\phi^0) \ ,$$

where $\mathcal{B}^0_{ab}$ captures the entries of $[\mathcal{B}^0_{px}, \mathcal{B}^0_{p\xi}]$ corresponding to the responses of $\mathbb{E}_t\mathbf{P}_{a,t}$ to $\mathbb{E}_t\mathbf{V}_{b,t}$ and $\mathcal{B}^0_{-a-b}$ denotes the remaining entries.

The corollary ensures that adjusting the given reaction function for the policy instruments in the direction of the identifiable non-policy shocks lowers the expected loss function. The intuition here is the same as in the previous section: adjusting $\mathcal{B}^0_{p_a\xi_b}$ does not change the coefficients in the economy and hence for the class of models (15) such change is robust to the Lucas critique.

4.3 Comparing policy makers

With the ORA and its properties established we now discuss how the ORA can be used to compare policy makers. As examples we can think of evaluating different central banks chairs based on their ability to control inflation and output gaps, or different presidents of a country based on their ability to keep output close to potential. Our comparisons are based on evaluating policy makers on their use of the same policy instruments for offsetting the same type of non-policy shocks. As such we may generally compare policy makers from the same institution across different time periods or policy maker from different but comparable institutions from different countries.

Suppose that there are p policy makers that the researcher aims to compare. Each policy maker faces an economy that can be described by the general model (15), but the parameters θ that govern the model may vary across policy makers, say θ_j for $j = 1, \ldots, p$. Similarly, each policy maker is assumed to set policy according to the generic rule (16), but may use a different reaction function ϕ_j. Following the notation defined above, let ϕ_j^0 denote the chosen reaction function of policy maker j. We note that while here we treat the parameters as fixed within the term of each policy maker, in our econometric implementation section below we show how we can extend the approach to allow the parameters to change within terms.

Using the methodology established above we can compute for any given policy maker the distance to the optimal reaction function in any identifiable direction using the (subset) ORA statistic. Specifically, let

$$\mathcal{T}_{ab}^{j*} = -(\mathcal{R}_a^{j,0\prime}\mathcal{W}\mathcal{R}_a^{j,0})^{-1}\mathcal{R}_a^{j,0\prime}\mathcal{W}\Gamma_b^{j,0}$$

denote the subset ORA statistic for policy maker j in the direction of responding to non-policy inputs $\mathbb{E}_t\mathbf{V}_{b,t}$ using the policy instruments moved by the policy shocks $\mathbb{E}_t\boldsymbol{\epsilon}_{a,t}$. We recall that $\mathcal{R}_a^{j,0}$ and $\Gamma_b^{j,0}$ are the impulse responses of the objectives with respect to the policy and non-policy inputs computed under the reaction function ϕ_j^0 and given the economic environment θ_j.

The ORA statistics take into account the preferences of the researcher over the different objectives or ranking criteria. As such if the researcher has no further preferences over the types of shocks we may simply aggregate the entries of $\mathcal{T}_{ab}^{j*}$, i.e.

$$t_{ab}^{j*} = \|\text{vec}\mathcal{T}_{ab}^{j*}\| \ , \tag{25}$$

where any desired norm $\|\cdot\|$ can be used. We rank policy makers based on t_{ab}^{j*}, for $j = 1, \ldots, p$, where the smallest value corresponds to the best performing policy maker. For interpretation purposes it is generally useful to present the ranking separately for each combination of instrument and non-policy input as each ranking is informative about a specific dimension of policy.

In practice there could be cases where the researcher has preferences over the types of non-policy inputs. For instance, when evaluating a central bank one may find it more important that the central bank offset oil price shocks as opposed to TFP shocks. In such cases a weighted norm can be used in (25) to aggregate the entries of $\mathcal{T}_{ab}^{j*}$.

5 Econometrics of the ORA

In this section we discuss we discuss the computation of the optimal reaction function adjustment using observation data. Without loss of generality we will consider the subset ORA statistic defined in (24). Corollaries 1 and 3 show that if $\mathcal{T}_{ab}^* \neq 0$ then ϕ^0 is not optimal and adjusting by $\mathcal{T}_{ab}^*$ brings the reaction function closer to the set of optimal reaction functions. We first consider estimating $\mathcal{T}_{ab}^*$ for a single policy maker, after which we consider estimating multiple $\mathcal{T}_{ab}^{j*}$'s simultaneously.

5.1 Inference for the ORA

The starting point is the equilibrium representation (18), restated here for convenience.

$$\mathbb{E}_t \mathbf{Y}_t = \Gamma^0 \mathbb{E}_t \mathbf{V}_t + \mathcal{R}^0 \mathbb{E}_t \boldsymbol{\epsilon}_t \ ,$$

where we recall that $\mathbb{E}_t \mathbf{V}_t$ includes the non-policy shocks $\mathbb{E}_t \boldsymbol{\Xi}_t$ and the initial conditions $\mathbf{X}_{-t}$.

Following empirical practice, we truncate the relevant horizon at H and define $\mathbf{Y}_{t:t+H} = (y_t', \dots, y_{t+H}')'$ as the evaluation criteria of interest at time t. Further, suppose that the policy maker under consideration was active for periods $t = 1, \dots, n$ during which the reaction function ϕ^0 was used.

The causal effects $\mathcal{R}_a^0$ and Γ_b^0 can be estimated by considering

$$\mathbf{Y}_{t:t+H} = \Gamma_b^0 \mathbb{E}_t \mathbf{V}_{b,t} + \mathcal{R}_a^0 \mathbb{E}_t \boldsymbol{\epsilon}_{a,t} + \mathbf{U}_{t:t+H} \ , \qquad t = 1, \dots, n, \tag{26}$$

where $\mathbf{U}_{t:t+H}$ includes all other structural shocks, both policy and non-policy inputs that are not included in the selections a and b, respectively, as well as the forecast errors $\mathbf{Y}_{t:t+H} - \mathbb{E}_t \mathbf{Y}_{t:t+H}$.

In practice, we may not observe the structural shocks $\mathbb{E}_t \boldsymbol{\epsilon}_t$ and $\mathbb{E}_t \boldsymbol{\Xi}_t$ directly. In such cases we can replace the shocks by suitable endogenous variables on which the structural shocks have a unit effect, i.e. a unit effect normalization (e.g. Stock and Watson, 2018).

After such possible substitutions we can recognize (26) as a system of stacked local projections (Jordà, 2005). This implies that given (i) an appropriate identification strategy and (ii) an accompanying estimation method, we can estimate the impulse responses $\mathcal{R}_a^0$ and Γ_b^0 using standard local projection methods. Any identification strategy — short run, long run, sign, external instruments, etc — can be used, based on which an appropriate estimation method — OLS or IV, with or without shrinkage, etc — can be selected, see Ramey (2016) and Stock and Watson (2018) for different options. Moreover, we recall from

Plagborg-Møller and Wolf (2021) that in population local projections and structural VARs estimate the same impulse responses; therefore all SVAR methods discussed in Kilian and Lütkepohl (2017), for instance, can also be adopted for estimating the impulse responses Γ_b^0 and $\mathcal{R}_a^0$.

In sum, given (26), observational data that covers the term of the policy maker, and by making use of the existing literature we can recover estimates for $\mathcal{R}_a^0$ and Γ_b^0. Here we will not discuss any specific approach but instead directly postulate that the researcher is able to obtain estimates, say $\widehat{\mathcal{R}}_a$ and $\widehat{\Gamma}_b$, which can be approximated by

$$\text{vec}\left(\begin{bmatrix} \widehat{\mathcal{R}}_a \\ \widehat{\Gamma}_b \end{bmatrix} - \begin{bmatrix} \mathcal{R}_a^0 \\ \Gamma_b^0 \end{bmatrix}\right) \overset{a}{\sim} F \ ,$$

where F is some known distribution function that can be estimated consistently by $\widehat{F}$. Such approximation can be obtained for many impulse response estimators using both frequentist (asymptotic and bootstrap) and Bayesian estimators.

Using the approximating distribution $\widehat{F}$ can simulate draws for $\mathcal{R}_a^0$ and Γ_b^0, and compute $\mathcal{T}_{ab}^* = -(\mathcal{R}_a^{0\prime}\mathcal{W}\mathcal{R}_a^0)^{-1}\mathcal{R}_a^{0\prime}\mathcal{W}\Gamma_b^0$ for each draw. Given the sequence of draws we can construct a confidence set for $\mathcal{T}_{ab}^*$, or any of its individual entries at any desired level of confidence. We note that if the distribution F is normal we can use the delta method to analytically compute the distribution of $\mathcal{T}_{ab}^*$.

GMM formulation

Besides simulating from the distribution of the impulse responses there exists an alternative, more direct, GMM approach for estimating $\mathcal{T}_{ab}^*$. This approach effectively replaces estimating Γ_b^0 by estimating $\mathcal{T}_{ab}^*$ directly and has the benefit that it immediately gives the asymptotic distribution of $\mathcal{T}_{ab}^*$ in closed form.

The basis is the following generalized least squares transformation of the truncated equilibrium representation (26).

$$\mathbf{Y}_{t:t+H}^L = -(\mathcal{R}_a^{0\prime}\mathcal{W}\mathcal{R}_a^0)^{-1}\mathcal{R}_a^{0\prime}\mathcal{W}\mathbf{Y}_{t:t+H} \ ,$$

where $\mathbf{Y}_{t:t+H}^L$ has dimension equal to the number of policy shocks considered which is typically much lower then the dimension of $\mathbf{Y}_{t:t+H}^L$. The low-dimensional model for $\mathbf{Y}_t^L$ is given by

$$\mathbf{Y}_t^L = \mathcal{T}_{ab}^*\mathbb{E}_t\mathbf{V}_{b,t} + \mathbf{U}_t^\tau \qquad \text{where} \quad \mathbf{U}_t^\tau = \mathbb{E}_t\boldsymbol{\epsilon}_{a,t} - (\mathcal{R}_a^{0\prime}\mathcal{W}\mathcal{R}_a^0)^{-1}\mathcal{R}_a^{0\prime}\mathcal{W}\mathbf{U}_{t:t+H} \ . \tag{27}$$

We note that the error term $\tilde{\mathbf{V}}_t^\tau$ is a function of all shocks except the non-policy shocks

$\mathbb{E}_t \boldsymbol{\Xi}_{b,t}$.

For any non-policy input $\mathbb{E}_t \mathbf{V}_{b,t,j}$ that is not be observed, say an oil price shock, we set $\mathbf{W}_{b,t,j} = \mathbb{E}_t \mathbf{V}_{b,t,j}$ + other shocks where $\mathbf{W}_{b,t,j}$ is an observable variable on which the non-policy input has a unit effect, e.g. the oil price. When the non-policy input is observed we simply define $\mathbf{W}_{b,t,j} = \mathbb{E}_t \mathbf{V}_{b,t,j}$. Similarly, suppose that the policy shocks $\mathbb{E}_t \boldsymbol{\epsilon}_{a,t}$ have a unit effect on $\mathbf{P}^e_{a,t} = \mathbb{E}_t \mathbf{P}_{a,t}$. Then we can also substitute these shocks with observable variables.

We obtain the models

$$\mathbf{Y}^L_{t:t+H} = \mathcal{T}^*_{ab} \mathbf{W}_{b,t} + \mathbf{U}^\tau_t \qquad \text{and} \qquad \mathbf{Y}_{t:t+H} = \mathcal{R}^0_a \mathbf{P}^e_{a,t} + \mathbf{U}^{\mathcal{R}}_t \ ,$$

where $\mathbf{U}^\tau_t$ is a function of all shocks except again of the non-policy inputs $\mathbb{E}_t \mathbf{V}_{b,t}$. Similarly, $\mathbf{U}^{\mathcal{R}}_t$ is a function of all shocks except of the policy shocks $\mathbb{E}_t \boldsymbol{\epsilon}_{a,t}$.

Given the availability of instruments $\mathbf{Z}_{a,t}$ and $\mathbf{Z}_{b,t}$ that are only correlated with $\mathbb{E}_t \boldsymbol{\epsilon}_{a,t}$ or $\mathbb{E}_t \boldsymbol{\Xi}_{b,t}$, but not with $\mathbf{U}^{\mathcal{R}}_t$ or $\mathbf{U}^\tau_t$, we can estimate $\mathcal{T}^*_{ab}$ and $\mathcal{R}^0_a$ jointly based on the moment conditions

$$\mathbb{E}\left[(\mathbf{Y}^L_{t:t+H} - \mathcal{T}^*_{ab} \mathbf{W}_{b,t}) \mathbf{Z}'_{b,t}\right] = 0 \quad \text{and} \quad \mathbb{E}\left[(\mathbf{Y}_{t:t+H} - \mathcal{R}^0_a \mathbf{P}^e_{a,t}) \mathbf{Z}'_{a,t}\right] = 0 \ .$$

The instruments $\mathbf{Z}_{a,t}$ and $\mathbf{Z}_{b,t}$ can be both internal or external instruments pending the identifying assumptions for the structural shocks. As an example, suppose that the researcher is interested in testing whether the the response of the central bank to an oil price shock was appropriate on average when setting the short term interest rate. The instruments $\mathbf{Z}_{a,t}$ could be taken as the high frequency identified monetary policy shocks from Gürkaynak, Sack and Swanson (2005), whereas $\mathbf{Z}_{b,t}$ may be the oil price shock from Hamilton (2003). The corresponding policy rate $\mathbf{P}_{a,t}$ would be the Fed funds rate and the endogenous variable $\mathbf{W}_{b,t}$ could be the price of crude oil.

Estimates for $\mathcal{T}^*_{ab}$ and $\mathcal{R}^0_a$ can be simultaneously obtained using standard nonlinear GMM methods. We define the estimator

$$\{\widehat{\mathcal{T}}_{ab}, \widehat{\mathcal{R}}^0_a\} = \underset{\mathcal{R}_a, \mathcal{T}_{ab}}{\operatorname{argmin}} \left(\frac{1}{n} \sum_{t=1}^{n} f_t(\mathcal{R}_a, \mathcal{T}_{ab}) \right)' \Omega_n \left(\frac{1}{n} \sum_{t=1}^{n} f_t(\mathcal{R}_a, \mathcal{T}_{ab}) \right) \ , \tag{28}$$

where Ω_n is a positive semi-definite weighting matrix and

$$f_t(\mathcal{R}_a, \mathcal{T}_{ab}) = \begin{bmatrix} (\mathbf{Z}_{b,t} \otimes \mathbf{I}_a)(\mathbf{Y}^L_{t:t+H} - \mathcal{T}_{ab} \mathbf{W}_{b,t}) \\ (\mathbf{Z}_{a,t} \otimes \mathbf{I}_b)(\mathbf{Y}_{t:t+H} - \mathcal{R}_a \mathbf{P}^e_{a,t}) \end{bmatrix} \ .$$

We emphasize that $\mathbf{Y}^L_{t:t+H} = -(\mathcal{R}^{0\prime}_a \mathcal{W} \mathcal{R}^0_a)^{-1} \mathcal{R}^{0\prime}_a \mathcal{W} \mathbf{Y}_{t:t+H}$, i.e a nonlinear function of $\mathcal{R}^0_a$ and a numerical minimizer is used in practice to find the minimum. Further, the efficient weighting matrix based on an estimate for the inverse of the variance $\lim_{n\to\infty} \mathrm{Var}(n^{-1/2}\sum_{t=1}^{n} f_t)$ can be used to improve efficiency.

The asymptotic properties of (28) are well understood and a detailed account can be found in Hall (2005). From the general theory we can deduce mild conditions under which $\widehat{\mathcal{T}}_{ab}$ is consistent for $\mathcal{T}^*_{ab}$ and under which it is asymptotically normal. We provide the necessary details for computing the GMM estimates and constructing confidence sets in the Appendix. We emphasize that standard statistical software can be used to estimate $\widehat{\mathcal{T}}_{ab}$ in this way.

5.2 Joint inference on ORAs

While the terms of policy makers may sometimes appear long, from the perspective of estimating impulse responses and ORA statistics terms are often quite short. For instance, a four year presidency produces only 16 quarters of quarterly data, which is typically insufficient to estimate all impulse responses. Therefore it can be attractive to either (i) pool some parameters across policy makers (e.g. Blinder and Watson, 2016) or (ii) impose some dynamics by which the parameters fluctuate over time (e.g. Primiceri, 2005). Additionally, the joint estimation of different ORA statistics facilitates implementing formal tests for comparing different policy makers.

We discuss simultaneous estimation of ORA using both (i) breaks and (ii) smooth time-varying parameter specifications.

Breaks

Consider the scenario where there are p policy makers and policy maker j operated in periods $t \in \{n_j, \dots, n_{j+1}\}$ using reaction function ϕ_j^0 in environment θ^j, for $j = 1, \dots, p$. The truncated equilibrium model (26) can be stated for all policy makers simultaneously as

$$\mathbf{Y}_{t:t+H} = \sum_{j=1}^{p} \mathbf{1}\{t_j \le t \le n_{j+1}\} \left[\mathcal{R}_a^{j,0}\mathbb{E}_t\boldsymbol{\epsilon}_{a,t} + \Gamma_b^{j,0}\mathbb{E}_t\mathbf{V}_{b,t}\right] + \mathbf{U}_{t:t+H} \ , \qquad t = 1, \dots, n \ , \quad (29)$$

where $\mathbf{1}\{\}$ is the indicator function.

Given this representation estimates for the impulse responses can be obtained using effectively the same GMM approach as outlined in the previous section. Or any other local projection or structural VAR approach, for that matter. We note that for implementation comparison tests it easier to estimate the jointly as this incorporates the correlation among the estimates. Also, it allows for pooling of the effects of any control variables that may be included.

We refer to Antoine and Boldea (2018) for a modern treatment of GMM models with structural breaks. Note that we need not assume that the break dates are known a priori, instead we could let the data estimate the break points and hereby determine whether the impulse responses, or preferably the ORA directly, has changed over time.

Time-varying parameters

There exists a broad literature that allows for time-varying parameters in reduced form econometric models. Common ways of modeling time variation include specifying a random walk model for the individual parameters (e.g. Primiceri, 2005) or allowing the parameters to switch across different regimes (e.g. Sims and Zha, 2006). Most of these specifications have been implemented in the context of structural vector autoregressive models, but there also exists various papers in the GMM context that allow for time-varying parameters, Cui, Feng and Hong (2022) for a recent contribution and further references.

The truncated equilibrium model with time varying parameters is given by

$$\mathbf{Y}_{t:t+H} = \Gamma^0_{b,t}\mathbb{E}_t\boldsymbol{\Xi}_{b,t} + \mathcal{R}^0_{a,t}\mathbb{E}_t\boldsymbol{\epsilon}_{a,t} + \mathbf{U}_{t:t+H} \ , \qquad t = 1, \ldots, n \ , \tag{30}$$

where $\Gamma^0_{b,t}$ and $\mathcal{R}^0_{a,t}$ depend on t and can be modeled in different ways. Common assumptions include specifying an independent random walk for each parameter, allowing for switching across regimes, or fixed smooth function approximations.

Each particular choice allows to estimate a path of the impulse responses, or after a transformation like (27) directly the ORA, which can be aggregated over the terms of the different policy makers for comparison. Clearly, this approach is attractive when one believes that reactions functions, or the economic environment may have changed within the term of the policy maker.

6 Ranking Fed chairs

7 Conclusion

In this paper, we showed that it is possible to evaluate and compare policy makers based on the distance-to-optimality of their reaction function. We introduced ORA statistics to measure the distance and showed that these could be computed from only the impulse responses to policy and non-policy shocks. Moreover, explicit knowledge of the policy maker's reaction function was not necessary. Intuitively, because the effect of an (unspecified) reaction function is already encoded in the impulse responses to (non-)policy shocks, which are estimable.

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Appendix A: Additional examples

Section 2 explained the workings of our methodology for the baseline New Keynesian model where the loss function was taken as $\mathcal{L}_t = (\pi_t^2 + x_t^2)/2$. This set-up had the advantage that all computations could be done in closed form. At the same time it did not highlight how dynamics in either the model or the loss function would affect the intuition as presented. To this extent, in this section we aim to bridge the gap between the simple example and the general framework by providing two additional examples that aim to clarify how the intuition for our methodology works in dynamic settings. We first consider the same baseline NK model, but change the loss function of the researcher to be forward looking. Second, we discuss the workings of our method when the underlying model is assumed to be given by a structural VAR model. Both examples are of independent interest.

NK model with forward looking loss function

Consider the baseline New Keynesian model

$$\begin{aligned}
\pi_t &= \beta \mathbb{E}_t \pi_{t+1} + \kappa x_t + \xi_t \ , \\
x_t &= \mathbb{E}_t x_{t+1} - \frac{1}{\sigma}(i_t - \mathrm{E}_t \pi_{t+1}) \ ,
\end{aligned}$$

where apart from the discount factor β the model is identical as in the main text. The loss function that the researcher considers for evaluating the policy maker is given by

$$\mathcal{L}_0 = \frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (\pi_t^2 + x_t^2) \ ,$$

and the researcher aims to check whether the path for the nominal interest rate was set such that $\mathbb{E}\mathcal{L}_0$ was minimized. In this form the set-up is often referred to as the optimal policy problem under commitment and optimal reaction functions are discussed in (e.g. Galí, 2015, Section 5.1.2). Note that we can view this loss function as case a special case of our general loss function (14) when taking $\mathbb{E}_t$ as $\mathbb{E}_0$, i.e. starting at $t = 0$.

As a first step, we show how the NK model can be written in the general notation of Section 2. Let $\mathbf{Y}_0 = (\pi_0, x_0, \pi_1, x_1, \ldots)'$, $\mathbf{P}_0 = (i_0, i_1, \ldots)'$, $\mathbf{\Xi}_0 = (\xi_0, \xi_1, \ldots)'$ and denote by $\boldsymbol{\epsilon}_0 = (\epsilon_0, \epsilon_1, \ldots)'$ the sequence of policy shocks (note that $\mathbf{W}_t$ does not exist in this application). The general model (15) becomes

$$\mathcal{A}_{yy}\mathbb{E}_0\mathbf{Y}_0 - \mathcal{A}_{yp}\mathbb{E}_0\mathbf{P}_0 = \mathcal{B}_{y\xi}\mathbb{E}_0\mathbf{\Xi}_0$$

where the coefficient maps are given by

$$\mathcal{A}_{yy} = \begin{bmatrix} 1 & -\kappa & -\beta & 0 & \dots & \dots \\ 0 & 1 & -1/\sigma & -1 & 0 & \dots \\ 0 & 0 & 1 & -\kappa & -\beta & \ddots \\ 0 & 0 & 0 & 1 & -1/\sigma & \ddots \\ \vdots & \vdots & \ddots & \ddots & \ddots & \ddots \end{bmatrix} \qquad \mathcal{A}_{yp} = \begin{bmatrix} 0 & 0 & 0 & \dots \\ 1/\sigma & 0 & 0 & \dots \\ 0 & 0 & 0 & \dots \\ 0 & 1/\sigma & 0 & \dots \\ 0 & 0 & 0 & \ddots \\ 0 & 0 & 1/\sigma & \ddots \\ \vdots & \vdots & \ddots & \ddots \end{bmatrix}$$

and

$$\mathcal{B}_{y\xi} = \begin{bmatrix} 1 & 0 & 0 & \dots \\ 0 & 0 & 0 & \dots \\ 0 & 1 & 0 & \dots \\ 0 & 0 & 0 & \dots \\ 0 & 0 & 1 & \ddots \\ 0 & 0 & 0 & \ddots \\ \vdots & \vdots & \ddots & \ddots \end{bmatrix} .$$

The general form of the policy rule is given by

$$\mathbb{E}_0\mathbf{P}_0 - \mathcal{A}_{py}\mathbb{E}_0\mathbf{Y}_0 = \mathcal{B}_{p\xi}\mathbb{E}_0\mathbf{\Xi}_0 + \mathbb{E}_0\boldsymbol{\epsilon}_0 \ ,$$

where the expected path of the interest rate can be set as a function of any variable or shock in economy.

The optimality conditions for this problem are given by

$$x_0 = -\kappa\pi_0 \qquad \text{and} \qquad x_t = x_{t-1} - \kappa\pi_t \ , \quad \forall\ t = 1, 2, \dots \ , \tag{31}$$

or

$$x_t = -\kappa\hat{\mathrm{p}}_t \quad \forall\ t = 0, 1, 2, \dots \ ,$$

where $\hat{\mathrm{p}}_t = \mathrm{p}_t - \mathrm{p}_{-1}$ denotes the (log) deviation between the price level and an "implicit target" given by the price level prevailing one period before the central bank chooses its optimal plan (Galí, 2015, page 135).

A possible interest rate rule that (a) implements this optimal allocation and (b) leads to a unique equilibrium is given by

$$i_t = -[\phi_p + (1-\delta)(1-\kappa\sigma)]\sum_{k=0}^{t}\delta^k\xi_{t-k} - (\phi_p/\kappa)x_t$$

for any $\phi_p > 0$ (Galí, 2015, page 138). Note that this instrument rule is a special case of the generic policy rule (**??**). The coefficients in the rule are given by

$$\delta \equiv \frac{1-\sqrt{1-4\beta a^2}}{2a\beta} \ , \qquad \text{with} \qquad a \equiv \frac{1}{1+\beta+\kappa^2} \ .$$

The forecasts under the optimal allocation can be written as

$$\mathbb{E}_0\pi_0 = \delta\xi_0 \qquad \mathbb{E}_0 x_0 = -\kappa\delta\xi_0 \qquad \mathbb{E}_0\pi_t = (\delta^{t+1} - \delta^t)\xi_0 \qquad \mathbb{E}_0 x_t = -\kappa\delta^{t+1}\xi_0 \tag{32}$$

for $t \geq 1$ (Galí, 2015, page 136).

Next, we rewrite this example in our general notation. It follows that $\mathcal{R}$, after some tedious manipulations, can be written —under the optimal policy rule— as

$$\mathcal{R} = \begin{bmatrix} \kappa/(\sigma v) & \kappa^2/(\sigma^2 v^2) + \kappa/\sigma v^2 + \kappa/(\sigma v) & \dots \\ 1/(\sigma v) & \kappa/(\sigma^2 v^2) + 1/(\sigma v^2) & \dots \\ 0 & \kappa/(\sigma v) & \dots \\ 0 & 1/(\sigma v) & \\ 0 & 0 & \dots \\ \vdots & \vdots & \ddots \end{bmatrix}$$

where $v = 1 - \phi_p/(\kappa/\sigma)$. Note that we only show the first two columns for ease of exposition. Given $\mathcal{R}$ and the forecasts (32) we can verify the equivalence condition, similar as shown in equation (**??**) for the problem under discretion we have

$$\begin{aligned} \left.\frac{\partial \mathcal{L}_0}{\partial \boldsymbol{\epsilon}_0}\right|_{i_t} &= \mathcal{R}'\mathbb{E}_0\mathbf{Y}_0 \\ &= \begin{bmatrix} \kappa/(\sigma v) & \kappa^2/(\sigma^2 v^2) + \kappa/\sigma v^2 + \kappa/(\sigma v) & \dots \\ 1/(\sigma v) & \kappa/(\sigma^2 v^2) + 1/(\sigma v^2) & \dots \\ 0 & \kappa/(\sigma v) & \dots \\ 0 & 1/(\sigma v) & \\ 0 & 0 & \dots \\ \vdots & \vdots & \ddots \end{bmatrix}' \begin{bmatrix} \delta\xi_0 \\ -\kappa\delta\xi_0 \\ (\delta^2 - \delta)\xi_0 \\ -\kappa\delta^2\xi_0 \\ \vdots \\ \vdots \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ \vdots \end{bmatrix}. \end{aligned}$$

This shows that $\mathcal{R}'\mathbb{E}_0\mathbf{Y}_0 = 0$ must also hold under optimality when considering problems with commitment.

In addition, note that as in the case under discretion, the impulse response matrix $\mathcal{R}$ is sufficient to characterize the optimal targeting rule. Working under perfect foresight, the condition $\mathcal{R}'\mathbf{Y}_0 = 0$ corresponds exactly to the optimal targeting rule (31).

SVAR model

The next example that we consider is the structural VAR model (e.g. Sims, 1982). While SVAR models have been criticized for not being robust to the Lucas (1976) critique, they remain a useful, and often used, tool for macro policy making (e.g. Antolin-Diaz, Petrella and Rubio-Ramírez, 2021). Moreover, they are commonly used for impulse response estimation under different identification strategies (e.g. Kilian and Lütkepohl, 2017). As such we view it useful to illustrate how our reaction function evaluation methodology works out when the underlying model is a general SVAR model. We will not repeat all intuition obtained from the New Keynesian examples, but constrain ourselves to highlighting the implications of dynamic responses and the presence of multiple policy and non-policy shocks.

Let the M-dimensional vector for objectives y_t be aggregated in the static loss function

$$\mathcal{L}_t = y_t' y_t \ . \tag{33}$$

Suppose the policy maker has $K < M$ policy instruments p_t available to minimize the loss function. The economy for $x_t = (y_t', p_t')'$ is given by the SVAR model[10]

$$A(L)x_t = e_t \ , \qquad \text{where} \qquad A(L) = A_0 - A_1 L - \ldots - A_q L^q \ , \tag{34}$$

where L denotes the lag operator and $e_t = (\xi_t', \epsilon_t')'$ is the vector of structural shocks; $\xi_t \in \mathbb{R}^M$ capturing the non-policy shocks and $\epsilon_t \in \mathbb{R}^K$ the policy shocks. We assume that these shocks have mean zero, unit variance and are mutually uncorrelated.

For exposition purposes we will assume that the researcher is only able to identify one non-policy shock $\xi_{1,t}$ and one policy shock $\epsilon_{1,t}$. The equation for the first policy instrument is given by

$$a_{p_1 p_1} p_{1,t} + a_{p_1 p_{2:K}} p_{2:K,t} + a_{p_1 y} y_t = a_{p_1 x,1} x_{t-1} + \ldots + a_{p_1 x,q} x_{t-q} + \epsilon_{1,t} \ , \tag{35}$$

Let $\phi = \{a_{pp}, a_{py}, a_{px,1}, a_{px,q}\}$ denote the reaction function of the policy maker of which the coefficients in (35) are a subset. Since, not all shocks can be identified we cannot estimate ϕ nor the subset in (35) as, for instance, $a_{p_1 y}$ cannot be recovered.

Let Φ be the set of reaction functions for which (34) can be inverted. For any $\phi \in \Phi$ we get

$$x_t = C(L)e_t \qquad \text{with} \quad C(L) = \begin{bmatrix} \Gamma_1(L) & \Gamma_{2:M}(L) & \mathcal{R}_1(L) & \mathcal{R}_{2:K}(L) \\ \Theta_{\xi_1}(L) & \Theta_{\xi_{2:M}}(L) & \Theta_{\epsilon_1}(L) & \Theta_{\epsilon_{2:K}}(L) \end{bmatrix} \ , \tag{36}$$

where $C(L)$ is the moving-average polynomial. The set of optimal reaction function is defined similar as before $\Phi^{\text{opt}} = \{\phi \in \Phi : \phi \in \arg\min_{\phi \in \Phi} \mathbb{E}\mathcal{L}_t\}$.

Next, consider a policy maker who proposes $\phi^0 \in \Phi$, we aim to measure the distance of ϕ^0 to Φ^{opt} in the direction of the response to $\xi_{1,t}$.[11] To measure this distance we consider the augmented policy reaction function for $p_{1,t}$

$$a^0_{p_1 p_1} p_{1,t} + a^0_{p_1 p_{2:K}} p_{2:K,t} + a^0_{p_1 y} y_t = a^0_{p_1 x,1} x_{t-1} + \ldots + a^0_{p_1 x,q} x_{t-q} + \tau \xi_{1,t} + \epsilon_{1,t} \ , \tag{37}$$

where τ is a constant that measures the additional response to $\xi_{1,t}$. The moving average representation for the objectives y_t under (37) is given by

$$y_t = \left(\Gamma_1^0(L) + \tau \mathcal{R}_1^0(L)\right) \xi_{1,t} + \Gamma^0_{2:M}(L) \xi_{2:M,t} + \mathcal{R}^0(L)\epsilon_t \ , \tag{38}$$

where again we see that the equilibrium effect of adjusting by τ is proportional to the, now dynamic, effect of the policy shock $\mathcal{R}_1^0(L)$.

To know whether ϕ^0 is optimal we compute the gradient of the loss function with respect

[10] We omit any other endogenous variables w_t for ease of exposition.

[11] Note that in the SVAR model there is no coefficient that explicitly capture the response to $\xi_{1,t}$, instead the policy maker's response to $\xi_{1,t}$ follows from her responses to the endogenous variables Y_t. Obviously this does not prevent us from looking in the direction $\tau\xi_{1,t}$ directly.

to τ. We have

$$\nabla_\tau \mathbb{E}\mathcal{L}_t = \mathcal{R}_1^{0\prime}(\Gamma_1^0 + \mathcal{R}_1^0 \tau)$$

where $\mathcal{R}^0$ and Γ_1^0 are the impulse response coefficients in the polynomials $\mathcal{R}_1(L)$ and $\Gamma_1(L)$ that capture the effect of $\epsilon_{1,t}$ and $\xi_{1,t}$ on $y_t, y_{t+1}, \ldots$ under ϕ^0. It follows that at if $\phi^0 \in \Phi^{\text{opt}}$, the gradient with respect to τ evaluated at $\tau = 0$ should be zero, and we have that

$$\mathcal{R}_1^{0\prime}\Gamma_1^0 = 0 \ .$$

The impulse responses of policy and non-policy shocks are orthogonal to each other. Moreover, we may set the gradient to zero to compute the distance to Φ^{opt} in the direction of $\xi_{1,t}$. We have the ORA statistic

$$\tau^* = -(\mathcal{R}_1^{0\prime}\mathcal{R}_1^0)^{-1}\mathcal{R}_1^{0\prime}\Gamma_1^0 \ .$$

This measure can again be compared across policy makers who have different reaction functions and operate in different environments.

We conclude that if the underlying economy can be written as a structural vector autoregressive model, we can evaluate and compare policy makers based on their reaction function by simply evaluating τ^* — a simple function of the impulse responses to policy and non-policy shocks —. The benefit of this approach is that it can be adopted even in scenarios where A cannot be entirely identified and the reaction function cannot be estimated. Moreover, any type of identification and estimation strategy can be used to recover $\mathcal{R}^0$ and Γ^0. Finally, it is easy to verify that the same approach holds for all time-varying parameter SVAR models (e.g. Primiceri, 2005), regime switching SVAR models (e.g. Sims and Zha, 2006) and state dependent SVAR models (e.g. Barnichon, Debortoli and Matthes, 2021).

Appendix B: Equilibrium relationships

We briefly discuss how the general model (15)-(16) can be written as (18). Define

$$\mathcal{A} = \begin{bmatrix} \mathcal{A}_{yy} & \mathcal{A}_{yw} & \mathcal{A}_{yp} \\ \mathcal{A}_{wy} & \mathcal{A}_{ww} & \mathcal{A}_{wp} \\ \mathcal{A}_{py} & \mathcal{A}_{pw} & \mathcal{A}_{pp} \end{bmatrix} \ , \quad \mathcal{B} = \begin{bmatrix} \mathcal{B}_{y\xi} \\ \mathcal{B}_{w\xi} \\ \mathcal{B}_{p\xi} \end{bmatrix} \ , \quad \mathbf{J} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{I} \end{bmatrix} \quad \text{and} \quad \mathbf{Z}_t = \begin{bmatrix} \mathbf{Y}_t \\ \mathbf{W}_t \\ \mathbf{P}_t \end{bmatrix} \ . \quad (39)$$

The model (15)-(16) is equivalent to

$$\mathcal{A}\mathbb{E}_t\mathbf{Z}_t = \mathcal{B}\mathbb{E}_t\mathbf{\Xi}_t + \mathbf{J}\mathbb{E}_t\boldsymbol{\epsilon}_t \ .$$

For any $\phi \in \Phi$ we have that there exists unique equilibrium representation. This implies that $\mathcal{A}$ is invertible and we obtain

$$\mathbb{E}_t\mathbf{Z}_t = \underbrace{\mathcal{A}^{-1}\mathcal{B}}_{=\mathcal{D}_1}\mathbb{E}_t\mathbf{\Xi}_t + \underbrace{\mathcal{A}^{-1}\mathbf{J}}_{=\mathcal{D}_2}\mathbb{E}_t\boldsymbol{\epsilon}_t \ .$$

The block structure of $\mathcal{D}_1$ and $\mathcal{D}_2$ is given by

$$\mathcal{D}_1 = \begin{bmatrix} \Gamma \\ \mathcal{D}_{1w} \\ \Theta_\xi \end{bmatrix} \quad \text{and} \quad \mathcal{D}_1 = \begin{bmatrix} \mathcal{R} \\ \mathcal{D}_{2w} \\ \Theta_\epsilon \end{bmatrix} ,$$

where the maps Γ and $\mathcal{R}$ appear in the first position as they capture the effects of the shocks on $\mathbb{E}_t\mathbf{Y}_t$. The other maps capture the effects of the shocks on the endogenous variables $\mathbb{E}_t\mathbf{W}_t$ and $\mathbb{E}_t\mathbf{P}_t$.

C: Proofs

Proof of Proposition 1. Let $\mathcal{T}$ be a linear map, sufficiently small such that $\phi = \{\mathcal{A}^0_{pp}, \mathcal{A}^0_{py}, \mathcal{A}^0_{pw}, \mathcal{B}^0_{p\xi} + \mathcal{T}\} \in \Phi$. If $\phi^0 \in \Phi^{\text{opt}}$, $\mathbb{E}\mathcal{L}_t$ cannot be lowered by any $\mathcal{T} \neq 0$. Similar as in (18) we obtain the equilibrium representation

$$\mathbf{Y}_t = (\Gamma^0 + \mathcal{R}^0\mathcal{T})\mathbb{E}_t\mathbf{\Xi}_t + \mathbf{V}_t \ ,$$

where $\mathbf{V}_t = \mathcal{R}^0\mathbb{E}_t\boldsymbol{\epsilon}_t + \mathbf{Y}_t - \mathbb{E}_t\mathbf{Y}_t$ and note that $\mathbb{E}[\mathbb{E}_t\mathbf{\Xi}_t\mathbf{V}_t'] = 0$. The expected loss $\mathbb{E}\mathcal{L}_t$ becomes

$$\begin{aligned} \mathbb{E}\mathcal{L}_t &= \frac{1}{2}\mathbb{E}\left([\Gamma^0 + \mathcal{R}^0\mathcal{T}]\mathbb{E}_t\mathbf{\Xi}_t + \mathbf{V}_t\right)' \mathcal{W} \left([\Gamma^0 + \mathcal{R}^0\mathcal{T}]\mathbb{E}_t\mathbf{\Xi}_t + \mathbf{V}_t\right) \\ &= \frac{1}{2}\mathrm{Tr}\left\{[\Gamma^0 + \mathcal{R}^0\mathcal{T}]'\mathcal{W}[\Gamma^0 + \mathcal{R}^0\mathcal{T}]\mathbf{\Sigma}_\Xi\right\} + \frac{1}{2}\mathbb{E}\left(\mathbf{V}_t'\mathcal{W}\mathbf{V}_t\right) \end{aligned}$$

The derivative of the map $\mathcal{T} \to \mathbb{E}\mathcal{L}_t$ is given by

$$\mathcal{R}^{0'}\mathcal{W}(\Gamma^0 + \mathcal{R}^0\mathcal{T})\mathbf{\Sigma}_\Xi \ . \tag{40}$$

Evaluating at $\mathcal{T} = 0$ and setting the derivative to zero implies

$$\mathcal{R}^{0'}\mathcal{W}\Gamma^0 = 0 \ ,$$

is a necessary condition for optimality. Noting that $\mathcal{T} \to \mathbb{E}\mathcal{L}_t$ is a convex map, it follows that $\mathcal{R}^0\mathcal{W}\Gamma^0 = 0$ is also sufficient for $\mathcal{T} = 0$ being a global minimizer, and thus $\phi^0 \in \Phi^{\text{opt}}$. □

Proof of Corollary 1. The right implication of proposition 1 implies the claim. □

Proof of Corollary 3. The left hand side of (**??**) is equivalent to the subset of (40) corresponding to a, b and after post-multiplying (**??**) by $\mathbf{\Sigma}_\Xi^{-1}$. The right hand side follows by direct calculation. The second part follows directly by noting $\mathcal{T}^*_{a,b}$ sets the local gradient to zero, hence it lowers the loss convex loss function. □