New Trade Models, Same Old Optimal Policies?

Gabriel Felbermayr*, Benjamin Jung†, and Mario Larch‡

October 2, 2012

Abstract

Do new trade models featuring imperfect competition and extensive margins have novel implications for optimal policies? We introduce revenue-generating *ad valorem* tariffs into the general framework of Arkolakis, Costinot and Rodriguez-Clare (2012). We find an optimal tariff formula that is isomorphic across models with perfect competition and no extensive margin and models with imperfect competition and/or extensive margins. The optimal import tariff (or, equivalently, an export tax or a subsidy on the consumption of domestic goods) always implements the first-best although different model variants feature different numbers and different types of distortions. As a corollary, conditional on openness and relative to autarky, welfare gains of trade liberalization are always larger when driven by lower tariffs than by reduced iceberg trade costs, and the difference can be quantitatively significant.

JEL-Classification: F12, R12.

Keywords: Gravity Equation; Monopolistic Competition; Heterogeneous Firms; Armington Model; International Trade; Trade Policy; Gains from Trade

---

*ifo Institute for Economic Research, Poschingerstraße 5, 81679 Munich, Germany; LMU Munich; CESifo; GEP; felbermayr@ifo.de.

†University of Tübingen, Nauklerstraße 47, 72074 Tübingen, Germany; CESifo; benjamin.jung@uni-tuebingen.de.

‡University of Bayreuth, Universitätsstraße 30, 95447 Bayreuth, Germany; ifo Institute; CESifo; GEP; mario.larch@uni-bayreuth.de.

1 Introduction

In the last 25 years or so, international trade theory has emphasized the roles of imperfect competition and firm-level heterogeneity (Krugman, (1979 & 1980), Melitz (2003)). This research agenda has been extremely successful, particularly in rationalizing important empirical facts such as intra-industry trade or within sector resource reallocation. On normative grounds, the new trade models have come under attack by an influential paper by Arkolakis, Costinot and Rodriguez-Clare (2012, henceforth ACR). These authors show that the new models have exactly the same welfare implications as the very simple Armington trade model with perfectly competitive and identical firms. In the present paper, we investigate the trade policy implications of these models.

ACR demonstrate that one-sector, one-factor trade models yield a representation of welfare for a specific country that depends only on that country's share of expenditure on domestically produced goods and on a single 'gravity' elasticity under macro-restrictions met by the most popular working-horse models.[1] Imperfect competition and the presence of an extensive margin–the novel features stressed in the recent literature–have no extra role to play for the *ex-post* analysis of trade liberalization scenarios. So, the richer micro-level detail contained in new trade models "has not added much" to the gains from trade.[2]

The ACR paper models trade frictions as iceberg trade costs. It does not analyze trade reform scenarios that consist in the dismantling of classical trade policy instruments such as tariffs or subsidies that affect government budgets but have no direct resource-consuming effects. In this paper, we introduce these instruments into the class of models discussed by ACR and study the link between openness and welfare when variation in openness is due to changes in commercial policy of the home country. We derive the following key results.

(i) In the class of models considered, the general optimal tariff formula does not depend on market structure (perfect versus monopolistic competition) or on the presence of an extensive

[1] Krugman (1980), Anderson and van Wincoop (2003), Melitz (2003) with Pareto-distributed productivity draws, Eaton and Kortum (2002).

[2] Applying their formula to the US, ACR show that the gains from trade obtained from the class of models encompassed by their analysis, are quantitatively rather small (going from autarky to the status quo leads to welfare gains of 0.7 to 1.4% of GDP). This quantitative result is due to a very low measure of observed openness.

margin besides the intensive one. The presence of markups and entry margins notwithstanding, the new trade models yield the same optimal policy than the simple Armington model where these presumed distortions are not present and the only motivation for trade policy relies on terms-of-trade considerations. Analytically, this point is most transparent in the case of a small economy (i.e., if foreign expenditure on Home's goods goes to zero) where the tariff depends only on gravity elasticities. The optimal tariff actually suffices to implement the first-best allocation. Hence, despite their important differences, different new trade models still require only one policy instrument to achieve the first-best. Demidova and Rodriguez-Clare (2009) discuss 'new' distortions associated to imperfect competition (the markup-distortion) or to the extensive margin (the consumer-surplus distortion). In turns out that these distortions involve the violation of the same planner first-order condition that is also violated when firms or consumers do not internalize that terms-of-trade are endogenous to their decisions. Empirically, there is strong support for the role of country-level market power in determining optimal tariffs, see Broda, Limao and Weinstein (2008).

(ii) As a corollary to the existence of an optimal tariff, it follows that, for a given level of observed openness, modeling episodes of trade liberalization by lower iceberg trade costs or disregarding existing tariffs when iceberg costs fall necessarily leads to underestimation of the true gains from trade relative to autarky. Using a very standard calibration of the model, we show that the amount of underestimation is by no means trivial: for the U.S., comparing observed openness as of the year 2000 to the hypothetical case of autarky, the welfare gains from tariff reform are twice as big as those from an equivalent reduction of iceberg costs that yields the same observed level of openness.

A byproduct of the derivations of (i) and (ii), we characterize the exact relation between welfare and openness in the presence of revenue-generating tariffs. This equation can be used for an *ex post* analysis of (marginal) trade liberalization scenarios. The elasticity linking Home's openness and welfare is no longer a constant: it depends on the gravity coefficients of the tariff rate and relative wages, but also on the levels of domestic and foreign openness as well as on the level of the tariff.

Our analysis therefore generalizes the isomorphism derived ACR to the case of tariff reforms.

Our generalization shows that their simple original formula can be a bad guide for the true welfare gains from trade in the presence of tariffs. ACR acknowledge the issue in their paper, but they neither generalize their welfare formula to encompass tariffs, nor do they discuss *optimal* tariffs or the size and sign of the error made by interpreting trade reform as being uniquely driven by lower trade costs.[3]

A couple of authors discuss cases under which the strong equivalence result–identical welfare effects independent of selection effects and endogenous entry–fails. In their paper ACR discuss two important cases. In the presence of *multiple sectors* some sectors have higher gains under monopolistic competition than under perfect competition and other sectors have lower gains. The aggregate welfare effect depends on sectoral weights and is ambiguous.[4] In the presence of *intermediate goods* the gains from trade are always larger under monopolistic competition than under perfect competition.[5] ACR do not, however, discuss optimal policy.

Other authors have hinted towards strong structural similarities between, e.g., the Krugman (1980) and the Melitz (2003) models. Feenstra (2010) also discusses the welfare gains from trade in monopolistic competition trade models and discusses the (absence of) fundamental differences between the two frameworks. Chaney (2008) shows that the gravity equation derived from a Melitz-type model without free entry is structurally similar to the equation based on the Armington model as explained by Anderson and van Wincoop (2003). While Arkolakis, Costinot, Donaldson, and Rodriguez-Clare (2012) show for a Melitz (2003) model with Pareto distributed productivities and variable markups that pro-competitive gains from trade are negative, Edmond, Midrigian and Xi (2012) find the opposite due to their assumption of oligopolistic competition. Dinghra and Morrow (2012) study the welfare properties of general models with firm heterogeneity, but consider the world equilibrium.

---

[3] In their footnote 33 ACR write: "*... our main welfare formula would need to be modified to cover the case of tariffs. In particular, the results derived ... ignore changes in tariff revenues, which may affect real income both directly and indirectly (through the entry and exit of firms).*" In their analysis of tariff reform in Costa Rica, Arkolakis et al. (2008) model trade reform as lower iceberg costs. They write "*One drawback of the model we present here is that we treat tariffs as transportation costs*".

[4] This point is related to Balistreri et al. (2010) who show that equivalence of Armington and Melitz breaks in the presence of a second sector in their case, the second sector competing for labor is leisure). They, too, abstract from tariffs.

[5] ACR also show that the gravity elasticity of trade cost is not sufficient to compute welfare gains in the presence of multiple sectors or intermediate goods.

The ACR result has also prompted a growing discussion in the CGE literature. Balistreri, Hillberry, and Rutherford (2011) argue that "*[revenue-generating tariffs rather than iceberg trade costs] can generate differences in the Melitz formulation relative to a perfect competition model*" (p. 96). In a related paper, Balistreri and Markusen (2009) claim that "*removing rent-generating tariffs have different effects in monopolistic competition versus Armington models, because optimal tariffs are different*". These findings are based on simulation exercises. However, our analytical results suggest these assertions need qualification: the optimal tariff formula actually is isomorphic across these models. So, the CGE differences must derive on other features of the modeling strategy.

While the CGE literature relies on simulation, there is a third strand of research that provides analytical results on the contrast between iceberg trade costs and tariffs. Using a model with heterogeneous firms, Cole (2011a) illustrates that profit for an exporter is more elastic in response to tariffs than iceberg transport costs, which affects the entry/exit decision of firms. In a related paper, Cole (2011b) investigates the roles of different types of trade costs in a gravity equation of the type derived by Chaney (2008). He shows that the trade flow elasticity of tariffs is larger than that of iceberg trade costs. So, estimates derived from variables such as distance may underestimate the trade enhancing effects of tariff reforms.

Our paper also relates to recent research on the effects of policy in new trade theory models with heterogenous firms. Demidova and Rodriguez-Clare (2009) derive the optimal tariff in a small-economy Melitz (2003) model with Pareto distributed productivity. They argue that the tariff remedies two distortions: a consumer-surplus distortion and a markup distortion. Felbermayr, Jung, and Larch (2012) extend their analysis to the case of two large countries, which adds the traditional terms-of-trade distortion to the picture. They provide comparative static results on the equilibrium tariff obtained in a non-cooperative policy game. In contrast to these papers, the present exercise proves that these different distortions can be internalized by one optimal tariff that is isomorphic to the one obtained in the Armington model to achieve the first-best. Compared to existing work on optimal policy in new trade theory models, our paper is more general and also has a different focus: it is not so much interested in the comparative statics of optimal tariffs but more in optimal policy and in the comparison of welfare effects across different liberalization scenarios (tariff reforms versus lower iceberg trade costs).

Alvarez and Lucas (2007) derive the optimal tariff for the case of a small economy in the Eaton and Kortum (2002) model. Their analysis is related to Opp (2010) who discusses optimal import tariffs and trade wars in the continuum of goods Ricardian model of Dornbusch et al. (1977). He argues that "the optimum tariff rate trades off the terms-of-trade improvements with the inefficient expansion of domestic production and the costly reduction in trade volume".

The remainder of the paper is structured as follows. Section 2 introduces the model setup and explains how the introduction of tariffs alters the ACR framework. Section 3 derives the general optimal tariff formula for the ACR class of models and shows that the calculated optimal tariff is the first-best instrument. Section 4 compares welfare effects of tariff reforms to those of lower iceberg costs. It also compares welfare effects of lower iceberg costs in the presence and absence of tariffs. A calibrated version of the model shows that the gains from trade are severely underestimated when the variation in openness is assumed to be due only to changes iceberg costs or in the absence of tariffs. Finally, Section 5 concludes.

2 Theoretical Framework

2.1 Preferences, technology, trade costs, and market structure

Our model is identical to the one used by ACR except for the introduction of tariffs. Moreover, without much loss of generality, we focus on two countries, Home and Foreign, indexed by $i \in \{H, F\}$, that may differ with respect to the size of their endowments and with respect to technology.[6]

Preferences. Representative households in both countries have symmetric CES preferences (Dixit-Stiglitz) over differentiated varieties of final consumption goods **or goods of countries** produced in a single sector,

$$U_i = \left(\int_{\omega \in \Omega_i} q\left[\omega\right]^{\rho} d\omega \right)^{1/\rho}, i \in \{H, F\}, \tag{1}$$

[6] ACR allow for an arbitrary number of countries. One key insight in their analysis is that each country's welfare depends only on its own level of 'autarkiness', and not on the possibly complicated structure of the rest of the world. Therefore, restricting the analysis to two countries comes at little loss of generality.

where Ω_i is the set of varieties available in country i, $q[\omega]$ is the quantity of variety ω consumed and $\sigma = 1/(1-\rho) > 1$ is the constant elasticity of substitution.[7] The price index dual to (1) is $P_i^{1-\sigma} = \int_{\omega \in \Omega_i} p[\omega]^{1-\sigma} d\omega$.

Technology and trade costs. Labor is the only factor of production and is supplied inelastically at quantity L_i and price w_i. Output is linear in labor, and productivity may or may not differ across firms. International trade is subject to frictions while intranational trade is frictionless. In all models considered, exporting from i to j involves iceberg trade costs τ_{ij}, where $\tau_{ii} = 1$. Moreover, there are fixed market access costs that have to be paid to **serve** the home or the domestic market. As in Arkolakis (2008), these access costs can be either in terms of domestic or foreign labor.

Structure of product markets. As ACR, we allow for two types of market structures: perfect competition and monopolistic competition with free entry. In both situations, firms take wages and aggregate variables as given. With perfect competition, fixed innovation and market access costs are zero. With monopolistic competition, in contrast, firms have to pay to obtain blueprints for production, their allocation being random across firms.

Tariffs. The key difference to ACR is that each country j may impose an *ad valorem tariff* $t_{ji} \geq 1$ on its imports from country i, where $t_{ii} = 1$. We assume that tariff revenue is redistributed lump sum. As opposed to iceberg trade costs, a tariff distorts consumption decisions towards domestic goods but does not generate loss in transit.

2.2 Macro-level restrictions

In their welfare analysis ACR impose three restrictions whose key role is to ensure that the framework described above gives rise to a gravity equation, i.a., a representation of bilateral trade flows where elasticities are constant. The first restriction, R1, requires that trade is balanced on a multilateral level; the second (R2) mandates that aggregate gross profits are

[7] We use square brackets to denote functional relationships.

proportional to aggregate revenue, and the third (R3') ensures that the import demand system gives rise to a gravity equation with constant elasticities.

R2 needs no further qualification in the context of our exercise. R1, the balanced trade condition, warrants a comment. In the presence of tariffs, it does not imply that aggregate firm revenue and aggregate consumer spending are the same. To see this, let X_{ij} denote the value of country $j's$ total imports from country i in *domestic* prices (i.e., gross of tariff, gross of iceberg trade costs), then balanced trade requires

$$\sum_{i=1}^{n} \frac{X_{ji}}{t_{ij}} = \sum_{i=1}^{n} \frac{X_{ij}}{t_{ji}}, \text{ for all } j, \tag{2}$$

where $t_{ji} > 0$ is the ad valorem tariff (subsidy) factor that country j levies on goods from country i. Aggregate revenue *accruing to firms* is given by $R_j \equiv \sum_{i=1}^{n} X_{ji}/t_{ij}$, while, with tariff income, *consumer's* aggregate expenditure is $X_j \equiv \sum_{i=1}^{n} X_{ij}$. Hence, balanced trade does not imply $X_j = R_j$, a restriction heavily employed by ACR.

ACR's third macro-level restriction (R3) requires that the import demand system exhibits constant elasticity of substitution (CES). This is, however, not sufficient for welfare analysis (the focus of the present paper), where ACR make a restriction on functional forms so that trade flow equations resulting from the model are similar to known gravity model forms. We employ a similar, albeit slightly more general restriction of the form

$$\frac{X_{ij}}{X_{jj}} = \frac{\chi_{ij}}{\chi_{jj}} \frac{N_i}{N_j} \left(\frac{f^x}{f^d}\right)^{\varepsilon_f} \left(\frac{w_i}{w_j}\right)^{\varepsilon_w} \tau_{ij}^{\varepsilon_\tau} t_{ji}^{\varepsilon_t}, \tag{3}$$

where dividing by X_{jj} eliminates income Y_j and the multilateral resistance term (Anderson and van Wincoop, 2003). χ_{ij} collects constants different from trade costs (τ and t). N_i is the mass of firms potentially active in country i. In the model, that mass is endogenously determined. However, due to R1 and R2, N_i is proportional to exogenous labor endowment L_i so that N_i does not change in a comparative statics exercise on τ or t. The elasticities $\varepsilon_\tau, \varepsilon_t$ and ε_w are constants with negative signs. In ACR, the term $t_{ji}^{\varepsilon_t}$ is not present and the further restriction $\varepsilon_w = \varepsilon_\tau = \varepsilon$ is imposed. We call economies satisfying restrictions R1, R2 and R3' ACR-class economies.

ACR show that the Krugman (1980) and the Armington model by Anderson and van Wincoop (2003) satisfy R1 to R3' without further restrictions. However, the Melitz (2003) and Eaton-Kortum (2002) models satisfy R2 and R3 only under strong functional form assumptions on the distribution governing within country heterogeneity that make sure that there is a unique trade elasticity despite the presence of two margins of adjustment (intensive/extensive). The same functional form restrictions are required in the presence of tariffs (i.e., the Pareto distribution in the Melitz (2003) case and the Fréchet distribution in the Eaton and Kortum (2002) framework).

In the Appendix, for the four special models mentioned above, we show that the elasticities are given by

	Armington/ Krugman (1980)	Eaton&Kortum (2002)	Melitz (2003)
ε_τ	$1-\sigma$	$-\gamma$	$-\theta$
ε_t	$1-\sigma$	$-\gamma$	$1-\theta/\rho$
ε_w	$1-\sigma$	$-\gamma$	$-\theta-\mu\frac{\theta-(\sigma-1)}{\sigma-1}$
ε_f	0	0	$-\frac{\theta-(\sigma-1)}{\sigma-1}$

(4)

where $\gamma > \sigma - 1$ is the (unique, positive) parameter of the Fréchet distribution governing unit labor requirements in the Eaton&Kortum (2002) Ricardian model, and $\theta > \sigma - 1$ is the (unique, positive) parameter of the Pareto distribution governing firm-level productivity draws in a parameterized version of Melitz (2003). Note that in models without an extensive margin we have $\varepsilon_t = \varepsilon_\tau$ since we define export flows as inclusive of tariffs.[8] Moreover, the wage rate bears the same elasticity. In Melitz, the elasticity of the wage rate depends on the parameter $\mu \in \{0, 1\}$, which measures the share of domestic relative to foreign labor in export market access costs $w_i^\mu w_j^{1-\mu} f_{ij}$, where j denotes the export market.[9] It turns out that $\varepsilon_w = \varepsilon_\tau$ for $\mu = 0$ and $\varepsilon_w = \varepsilon_t$ for $\mu = 1$.

[8] This is just a convention that somewhat eases notation in our framework. Writing trade flows net of tariffs yields a gravity coefficient η_t on tariffs of $\varepsilon_t - 1$, and requires rewriting the balanced trade condition. None of our results would change.

[9] The original Melitz (2003) model sets $\mu = 1$.

2.3 Identities

Expenditure and income. *Aggregate expenditure* X_j equals aggregate income. It is made up by labor income plus the government's tariff income, which is rebated lump-sum to the representative household:

$$X_j = w_j L_j + \sum_i \frac{t_{ji} - 1}{t_{ji}} X_{ij}. \tag{5}$$

There are no other sources of income as profits are fully competed away by free entry. With labor the only factor of production, and in the absence of profits, all firm revenue needs to be paid out to workers as *labor income* $w_j L_j$. Hence,

$$w_j L_j = \sum_i \frac{X_{ji}}{t_{ij}}. \tag{6}$$

Expenditure and revenue shares. ACR express country j´s welfare as a function of the share of *expenditure* that falls on its own (domestically produced) goods, i.e.,

$$\lambda_{jj} \equiv \frac{X_{jj}}{X_j} = \frac{X_{jj}}{X_{jj} + X_{ij}}. \tag{7}$$

That share is an inverse measure of $j's$ openness also referred to as its degree of "autarkiness"; $1 - \lambda_{jj}$ would then be its openness.[10] The simplicity of ACR's analysis very much hinges on the fact that λ_{jj} summarizes the country's stance relative to the rest of the world (consisting, potentially, of many countries). Clearly, in the presence of tariffs, that expenditure share differs from the *revenue* share

$$\tilde{\lambda}_{jj} \equiv \frac{X_{jj}}{R_j} = \frac{X_{jj}}{X_{jj} + X_{ji}/t_{ij}} = \frac{X_{jj}}{X_{jj} + X_{ij}/t_{ji}}, \tag{8}$$

where the last equality uses balanced trade, $X_{ij}/t_{ji} = X_{ji}/t_{ij}$. Hence, as long as $t_{ji} \geq 1$, we have $\lambda_{jj} \geq \tilde{\lambda}_{jj}$. In the absence of tariffs, of course, the two shares perfectly coincide. The intuition is that a tariff drives a wedge between domestic expenditure for imports and export sales generated abroad. Balanced trade ties together export sales (net of the tariff), which, in turn, implies that

[10] In the following, with some abuse of wording, we similarly refer to changes in autarkiness or openness when describing changes in λ_{ii}.

income spent on imports is larger than export sales. Given that there is no tax on domestic goods, expenditure for domestic goods equals revenues earned on the domestic market.

3 Optimal policy in ACR-class economies

In this section, we derive the optimal tariff in the class of models investigated by ACR. We show that the optimal tariff implements the first-best allocation and that it is equivalent to an export tax or a subsidy on consumption of domestically produced varieties.

Welfare is given by the per capita value of real income accruing to consumers, hence

$$W_i = \frac{X_i}{L_i P_i} = \frac{w_i L_i + \frac{t_{ij}-1}{t_{ij}} X_{ji}}{L_i P_i}, \tag{9}$$

where the second equality makes use of (5). Totally differentiating leads to

$$\hat{W}_i = \underbrace{\vartheta \widehat{\left(\frac{w_i}{P_i}\right)}}_{\text{real wage}} + \underbrace{(1-\vartheta)\frac{1}{t_{ij}}\frac{X_{ji}}{L_i P_i}}_{\text{direct effect}}\hat{t}_{ij} + \underbrace{(1-\vartheta)\frac{t_{ij}-1}{t_{ij}}}_{\text{tax-base effect}}\widehat{\left(\frac{X_{ji}}{L_i P_i}\right)}, \tag{10}$$

where $\hat{x} \equiv dx/x$ for any variable x and where ϑ denotes the share of wage income in total income. Besides the by familiar 'real wage'-term the tariff has a positive direct effect on welfare by raising at given openness and a negative indirect effect as the import reduction due to the tariff lowers the tax base. Clearly, in ACR, $\vartheta = 1$.

The objective is to express the change in welfare as a function of the observable degree of autarkiness, λ_{ii}. Thus, as the next step, with the aim of substituting out X_{ji}, we use $X_{ji} = \lambda_{ji} X_i$ and again replace X_i by (5). Solving the resulting expression for X_{ji} one obtains $X_{ji} = \frac{\lambda_{ji} t_{ij}}{t_{ij}(1-\lambda_{ji})+\lambda_{ji}} w_i L_i$. One can now rewrite the expression for welfare as

$$W_i = \frac{w_i}{P_i} \times \frac{t_{ij}}{\lambda_{ii}\left(t_{ij}-1\right)+1} \tag{11}$$

where $\lambda_{ji} = 1-\lambda_{ii}$ has been used. Clearly, in the absence of a tariff revenue, i.e., if either $t_{ij} = 1$ (free trade) or $\lambda_{ii} = 1$ (autarky), welfare is simply given by the real wage. In the presence of a tariff revenue, however, there is a second source of income for the representative consumer that is not analyzed in ACR.

3.1 Unilateral tariff reform

Next, we consider the effect of a unilateral tariff on the country imposing the tariff. Totally differentiating of (11) now leads to

$$\hat{W}_i = \vartheta \widehat{\left(\frac{w_i}{P_i}\right)} + \frac{1-\lambda_{ii}}{\lambda_{ii}\left(t_{ij}-1\right)+1}\hat{t}_{ij} + \frac{\lambda_{ii}\left(t_{ij}-1\right)}{\lambda_{ii}\left(t_{ij}-1\right)+1}\hat{\lambda}_{ii}. \tag{12}$$

To write $\hat{W}_i$ as a function of $\hat{\lambda}_{ii}$, we must reformulate the direct effect and the real wage effects. In models without export selection (either perfect or monopolistic competition), the real wage effect can again be expressed as $(1/\varepsilon)\,\hat{\lambda}_{ii}$ following the steps laid out in ACR and the detailed derivation in our Appendix.

With export selection, the analysis is more complicated than in ACR. The reason is that aggregate income affects the decision of firms to enter the country. In our setting, changes in aggregate income are not only driven by the change in the wage rate, but also by the change in tariff revenue. The effect of tariff reform on real wages is then given by

$$\frac{\widehat{w_i}}{P_i} = \frac{1}{\varepsilon_\tau}\hat{\lambda}_{ii} + \frac{\varepsilon_f}{\varepsilon_\tau}\frac{\lambda_{ii}\left(t_{ij}-1\right)}{\lambda_{ii}\left(t_{ij}-1\right)+1}\hat{\lambda}_{ii}.$$

Expressing the direct effect in terms of $\hat{\lambda}_{ii}$ involves the following steps. First, we use (3) to relate $\hat{t}_{ij}$ to endogenous variables

$$\hat{t}_{ij} = \frac{\varepsilon_w}{\varepsilon_t}\left(\hat{w}_i - \hat{w}_j\right) - \frac{1}{\varepsilon_t}\frac{1}{1-\lambda_{ii}}\hat{\lambda}_{ii}, \tag{13}$$

where we have used that the definition of λ_{ii} implies that $\hat{\lambda}_{ii} = -\left(1-\lambda_{ii}\right)\left(\hat{X}_{ji} - \hat{X}_{ii}\right)$. Relating wage adjustment to $\hat{t}_{ij}$ requires invoking the balanced trade condition $X_{ji}/t_{ij} = X_{ij}/t_{ji}$. Combining with the expression for bilateral sales, $X_{ji} = \frac{\lambda_{ji}t_{ij}}{t_{ij}(1-\lambda_{ji})+\lambda_{ji}} w_i L_i$, one obtains a link between relative wages, expenditure shares, and the tariff. Totally differentiating and using the definition of λ_{ii}, we obtain

$$\hat{w}_i - \hat{w}_j = \frac{1-\varepsilon_t}{\varepsilon_w}\frac{\tilde{\lambda}_{ii}}{1-\tilde{\lambda}_{jj}-\tilde{\lambda}_{ii}}\hat{t}_{ij}. \tag{14}$$

Employing equation (13) and reorganizing terms, an expression for the direct effect follows (see the Appendix for details).

All these considerations are summarized in the following lemma.[11]

Lemma 1 ***(Welfare effects of tariffs)***. *In ACR-class economies without export selection, the local welfare effect of a tariff can be computed using*

$$\hat{W}_i = \frac{1-\varepsilon_t}{\varepsilon_t}\left\{\frac{t_{ij}-1}{t_{ij}}(1+\Gamma)-\Gamma\right\}\tilde{\lambda}_{ii}\hat{\lambda}_{ii}, \tag{15}$$

$$\Gamma \equiv \frac{\varepsilon_w\tilde{\lambda}_{ii}}{\varepsilon_t-\varepsilon_w\varepsilon_t\tilde{\lambda}_{jj}-\varepsilon_w\tilde{\lambda}_{ii}} \geq 0, \tag{16}$$

$$\hat{\lambda}_{ii} = \left((1-\lambda_{ii})\,\varepsilon_w\left(\frac{1-\varepsilon_t}{\varepsilon_w}\frac{\tilde{\lambda}_{ii}}{1-\tilde{\lambda}_{jj}-\tilde{\lambda}_{ii}}\right)-\varepsilon_t\right)\hat{t}_{ij}. \tag{17}$$

Proof. The welfare formula follows from substituting out real wage effect and the direct effect from equation (12). The change in autarkiness is derived from combining equations (13) and (14). ■

The term Γ collects gravity elasticities weighted by domestic and foreign revenue shares $\tilde{\lambda}_{ii}$. Obviously, the welfare formula (15) is more involved than the *ex ante* analysis of liberalization reform under ACRs assumption $t_{ij} = 1$.[12] The evaluation of tariff reform requires to predict the change in autarkiness $\hat{\lambda}_{ii}$ from a change in tariffs. Moreover, we need information on the gravity elasticity ε and trade flows, which can be used to derive expenditure shares. Knowledge of the initial levels of tariffs and trade flows allow to compute revenue shares as

$$\tilde{\lambda}_{ii} = \left(1+\frac{1}{t_{ji}}\frac{X_{ij}}{X_{ii}}\right)^{-1} = \left(1+\frac{1-\lambda_{ii}}{t_{ij}\lambda_{ii}}\right)^{-1}. \tag{18}$$

Equation (15) shows that the effect of an increase in autarkiness λ_{ii} does not necessarily decrease welfare in the country imposing a tariff, as the sign of the expression in curly brackets

[11] The expression for welfare effects of tariff reform can easily be extended to cover models with export selection. It is not required, however, for the derivation of the general optimal tariff formula, which is the main focus of this paper; see section 3.2 below for details.

[12] The corresponding ACR formula is found if one combines equations (9), (10), and (11) in ACR in a two-country case.

is ambiguous. It is straightforward to see that ACR-type models imply a strictly positive optimal tariff: if $t_{ij} = 1$ initially, we have $\hat{W}_i = -\frac{1-\varepsilon_t}{\varepsilon_t}\Gamma\tilde{\lambda}_{ii}\hat{\lambda}_{ii} > 0$ so that lower openness achieved by a tariff increases welfare. In contrast, if the tariff is prohibitive to start with, i.e. $t_{ij} \to \infty$ and $\tilde{\lambda}_{ii} \to 1$, the welfare change from lowering the tariff is surely positive, since $\hat{W}_i = \frac{1-\varepsilon_t}{\varepsilon_t}\hat{\lambda}_{ii}$.

In contrast, the effect of the tariff imposed by i on the welfare of the trading partner j is always negative. This is easy to establish, in particular if one is willing to assume that $t_{ji} = 1$. Then, $W_j = w_j/P_j$ as j has no tariff revenue to be accounted for. In this case, for all ACR-class economies, the welfare formula is given by

$$\hat{W}_j = \frac{1}{\varepsilon_\tau}\hat{\lambda}_{jj} \tag{19}$$

which is negative if $\hat{\lambda}_{jj} > 0$.[13]

3.2 A general optimal tariff formula for ACR-class economies

We find the optimal tariff t^*_{ij} by setting the curly bracket in equation (15) to zero. Recognizing that, by (18), both $\tilde{\lambda}_{ii}$ and $\tilde{\lambda}_{jj}$ are functions of tariffs so that $\Gamma\left[t_{ij}\right]$, the optimal tariff solves

$$\left(\Gamma\left[t^*_{ij}\right] + 1\right)\left(t^*_{ij} - 1\right) = \Gamma\left[t^*_{ij}\right] t^*_{ij},$$

where Γ is given by equation (16). This is a quadratic equation in t^*_{ij} which can be shown to have a negative root $-\left(1 - \lambda_{ii}\right)/\lambda_{ii}$ that must be ruled out,[14] and a positive root $t^*_{ij} \geq 1$ which is the unique optimal tariff.[15]

Proposition 1 ***(Optimal tariffs.)*** *In two-country ACR-class economies, the optimal tariff formula is given by*

$$t^*_{ij} = 1 + \left(\varepsilon_t/\varepsilon_\omega - 1 - \tilde{\lambda}_{jj}\varepsilon_t\right)^{-1} \geq 1. \tag{20}$$

[13] A tariff t_{ij} reduces country $i's$ imports and, due to our two-country setup plus balanced trade, country $j's$ imports as well. So, in both countries the degree of autarkiness goes up.

[14] It would imply negative prices.

[15] The optimal tariff formula also holds in models with export selection. We prove this claim in two steps. First, we conjecture that in the welfare maximization problem the change in the real wage relative to the change in the domestic expenditure share is given by the constant $1/\varepsilon_w$ as in the models without export selection. Second, we show that evaluated at any optimal tariff, the change in the real wage relative to the change in the domestic expenditure share is indeed given by the constant $1/\varepsilon_w$.

This nests the small economy cases characterized by $\tilde{\lambda}_{jj} \to 1$ *and the following models as special cases:*

(i) Armington, Krugman (1980):

$$t^*_{ij} = 1 + \left\{ \tilde{\lambda}_{jj} \left(\sigma - 1 \right) \right\}^{-1};$$

(ii) Eaton-Kortum (2002):

$$t^*_{ij} = 1 + \left(\tilde{\lambda}_{jj} \gamma \right)^{-1};$$

(iii) Melitz (2003):

$$t^*_{ij} = \begin{cases} 1 + \left\{ \tilde{\lambda}_{jj} \left(\theta/\rho - 1 \right) \right\}^{-1} & \text{if} \quad \mu = 1 \\ 1 + \left\{ \tilde{\lambda}_{jj} \left(\theta/\rho - 1 \right) + \kappa \right\}^{-1} & \text{if} \quad \mu = 0. \end{cases}$$

where $\kappa \equiv \{ \theta - (\sigma - 1) \} / \{ \theta (\sigma - 1) \} > 0$; *all Melitz-tariffs collapse to case (i) if* $\theta \to \sigma - 1$.

Proof. In the text, details in the Appendix. ■

The fact that the optimal tariff formula appears to be the same subject to $\tilde{\lambda}_{jj}$ in widely different models is quite striking since the models exhibit very different forms of distortions. The Armington and Eaton-Kortum (2002) models feature perfect competition; yet, even if country i is a small economy in so far as its firms derive only a negligible share of revenue from domestic sales (such that $\tilde{\lambda}_{ii} \to 0, \tilde{\lambda}_{jj} \to 1$), the countries choose strictly positive tariffs $t^*_{ij} > 1$. This is because countries are specialized in specific subsets of the goods space. Since specialization is a prerequisite for the emergence of a gravity relationship, all trade models that give rise to gravity must exhibit positive import tariffs even if they are small in the sense defined above. In other words, in all models, there is a terms-of-trade rationale for optimal tariffs.[16]

A caveat is in place here: the optimal tariff depends on parameters such as σ, θ and γ and on foreign autarkiness $\tilde{\lambda}_{jj}$. Hence, in general, the optimal tariffs may very well differ across model variants. However, conditional on the gravity equation (3), and in the small economy

[16]The Krugman (1980) tariff has been derived by Gros (1987); Felbermayr, Jung and Larch (2012) have derived the Melitz (2003) tariff for the case of two large economies; Demidova and Rodriguez-Clare (2009) have presented the Melitz (2003) tariff for the small economy; Alvaraez and Lucas (2007) provide the optimal tariff formula for the small-economy Eaton and Kortum (2002) model. All these results appear as special cases in Proposition 1.

case $\left(\tilde{\lambda}_{jj} \to 1\right)$, the tariff is really the same across all those models. Without the discipline of the gravity equation, ε_t and ε_ω would not be identical and $\tilde{\lambda}_{jj}$ would differ so that different tariffs obtain. However, as we show in the next section, the optimal tariff described in Proposition 1 is sufficient for the first-best.

3.3 Alternative policies and the first-best

For the small open Melitz (2003) economy with Pareto-distributed productivities, Demidova and Rodriguez-Clare (2009) show that the outcome produced by an optimal tariff can be equivalently reproduced by the appropriate choice of a subsidy on the consumption of domestic varieties or by an export tax. A similar result holds in our more general setup encompassing a wider array of models and the case of the large economy.

Proposition 2 ***(Alternative instruments).*** *The same allocation and the same welfare levels as resulting from the choice of the optimal tariff are alternatively achieved either by the optimal choice of an subsidy on consumption of domestic varieties* $t^*_{ii} = 1/t^*_{ij} < 1$ *or of an export tax* $s^*_{ij} = 1/t^*_{ij} < 1$ *or by a combination of all three instruments such that*

$$\frac{t^*_{ij}}{t^*_{ii} s^*_{ij}} = 1 + \left(\varepsilon_t/\varepsilon_\omega - 1 - \tilde{\lambda}_{jj}\varepsilon_t\right)^{-1}.$$

Proof. In the Appendix. ■

Thus, export taxes, import tariffs and subsidies on domestically produced goods are perfect substitutes. This result is interesting because the general model features several potential non-internalized margins. In fact, the literature on commercial policies in new trade models (Demidova and Rodriguez-Clare, 2009; Felbermayr, Jung and Larch, 2012) discusses several distortions that might mandate commercial policy: (i) an entry-distortion resulting from the fact that consumers do not internalize that their choice of expenditure on imports leads to additional entry of foreign firms and hence more product variety, (ii) a markup-distortion resulting from the fact that, from a social perspective, domestic varieties are priced at a markup over opportunity costs while imports are priced at opportunity costs, and (iii) the more conventional terms-of-trade externality. The first distortion mandates an import subsidy, the second and the third an import tax. The first distortion derives from the existence of imperfect competition at the firm-level, the second from the existence of an extensive margin and an love-for-variety externality, and the third distortion from market power at the country level. The first two distortions are present in small-economy models, the latter only in a large economy setup. This rather complex picture begs the question how far the optimal tariff identified in Proposition 1 goes in remedying these distortions. In turns out that the optimal tariff fully suffices to achieve the first best outcome.

Proposition 3 ***(First-best instrument).*** *The optimal import tariff described in Proposition 1 is sufficient to implement the first-best allocation.*

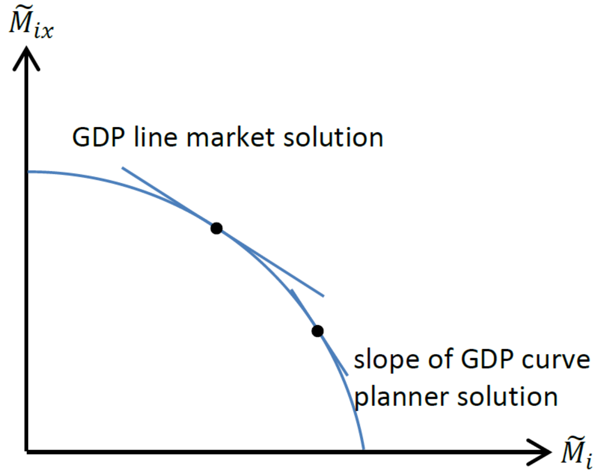
Proof. In the Appendix. ■

Proposition 3 shows that a planner, who can directly set welfare-maximizing quantities and domestic as well as export/import cutoffs, choses the same allocation than the one that results in a decentralized equilibrium in the presence of an optimal tariff. One could conjecture that the different distortions present in the class of models considered here require, in general, several independent instruments, one single instrument is enough. This implies that there is a single margin through which these distortions manifest themselves. This margin is the price of the ideal export quantity index relative to the ideal domestic quantity idex.

In Melitz, the market behaves like a social planner who maximizes GDP taking demand shifters *as given* subject to the resource constraint; see Feenstra and Kee (2008). Following Feenstra (2010), this situation can be visualized graphically in a space with ideal domestic $\tilde{M}_i$ and export quantities $\tilde{M}_{ix}$ on the axis and a concave transformation curve. GDP maximization requires that the GDP line (whose slope is the negative value of the relative demand shifter) is tangent to the transformation curve.

The social planner maximizes GDP *taking the effect on demand shifters into account* and assuming that the foreign country does not internalize market power subject to the same transformation curve and balanced trade. GDP maximization requires that the slope of the GDP curve is tangent to the transformation curve. As the social planner takes market power into account, the slope of the GDP curve differs, and the difference is driven by the elasticity of the

Figure 1: Planner vs market solution



Source: Based on Feenstra (2010)

ideal export quantity in foreign's demand shifter.

4 Comparing liberalization scenarios

In this section, we consider *ex post* welfare evaluation of trade reforms based on observed changes in openness. We will show that, for the same level of achieved openness, the welfare gains from limited trade liberalization (relative to autarky) are larger when the liberalization has involved some tariff cuts as compared to the case where the entire increase in trade is attributed to lower iceberg costs. We will also show that, for the same level of achieved openness, the welfare gains from limited trade liberalization (relative to autarky) of iceberg costs are strictly larger in the presence of tariffs compared to a world without tariffs. A simple numerical exercise suggests that the error made by wrongly attributing *all* variation in openness to iceberg trade costs can be quantitatively sizable.

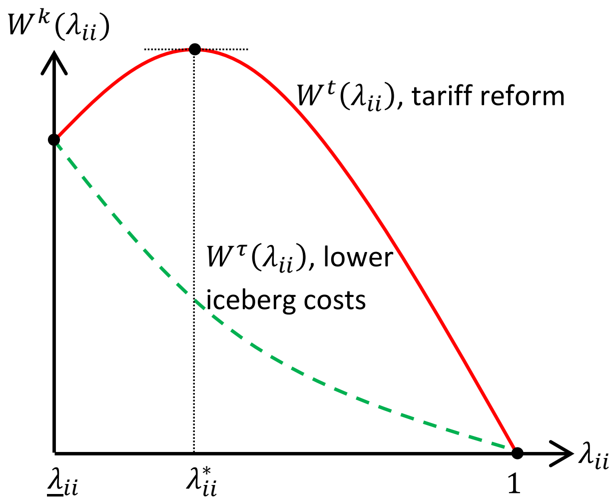
4.1 Lower iceberg costs versus tariff cuts

With the results already at hand it is rather straight-forward to compare the welfare effects of trade liberalization scenarios as functions of implied variation in observed openness λ_{ii}. For the ease of exposition, we consider liberalization of iceberg costs in the absence of tariffs. Figure 2 conveys the basic message. It plots welfare functions for two extreme scenarios: one, marked

$W^{\tau}(\lambda_{ii})$, where the entire variation in λ_{ii} is comes from changes in iceberg costs τ_{ij}, and a second, marked $W^{t}(\lambda_{ii})$, where the entire variation is induced by exogenous changes in trade policy t_{ij}. We focus on the case of unilateral trade liberalization.

On the x-axis, Figure 2 varies country i's degree of autarkiness $\lambda_{ii} \in [\underline{\lambda}_{ii}, 1)$ where $\underline{\lambda}_{ii}$ corresponds to the case of free trade ($t_{ij} = \tau_{ij} = 1$) and thus maximum openness, and $\lambda_{ii} = 1$ characterizes a situation of complete autarky where either t_{ij} or τ_{ij} are set at the prohibitive level. The y-axis reports the level of welfare $W^{k}(\lambda_{ii})$ where $k \in \{t, \tau\}$. For the sake of simplicity, assume that, for all λ_{ii}, $W^{\tau}(\lambda_{ii})$ reports the welfare level achieved from $t_{ij} = 1$ and $\tau_{ij} \in [1, +\infty)$ and $W^{t}(\lambda_{ii})$ records the welfare level obtained from $\tau_{ij} = 1$ where $t_{ij} \in [1, +\infty)$. Clearly, when trade is completely free, i.e., $t_{ij} = \tau_{ij} = 1$, we must have $W^{t}(\underline{\lambda}_{ii}) = W^{\tau}(\underline{\lambda}_{ii})$. Conversely, when trade barriers are prohibitive, i.e., $t_{ij} = 1, \tau_{ij} \to \infty$ or $t_{ij} \to \infty, \tau_{ij} = 1$, we must have $W^{t}(1) = W^{\tau}(1)$. Figure 2 identifies these common points. We also know that $W^{t}(\underline{\lambda}_{ii}) > W^{\tau}(1)$ due to the presence of gains from trade.

Figure 2: Gains from trade: unilateral trade liberalization



The key difference between $W^{t}(\lambda_{ii})$ and $W^{\tau}(\lambda_{ii})$ lies in the fact that $W^{t}(\lambda_{ii})$ must be a concave function while $W^{\tau}(\lambda_{ii})$ is strictly convex. The latter has been shown by ACR.[17] Concavity of $W^{t}(\lambda_{ii})$ follows from the fact that the existence of a unique optimal tariff t^{*}_{ij} implies a unique welfare maximizing level of autarkiness $\lambda^{*}_{ii} > \underline{\lambda}_{ii}$ such that $W(\lambda^{*}_{ii}) > W(\underline{\lambda}_{ii})$.[18] Hence,

[17] Convexity of W^{τ}_{i} in λ_{ii} follows immediately from $\hat{W}^{\tau}_{i} = \hat{\lambda}_{ii}/\varepsilon_{\tau}$.

[18] Strictly speaking, this argument establishes merely that $W^{t}(\lambda_{ii})$ is locally concave around λ^{*}_{ii}. For the special

we have that $W^{\tau}(\lambda_{ii}) > W^{t}(\lambda_{ii})$ for all $\lambda_{ii} \in [\underline{\lambda}_{ii}, 1]$.

These considerations are summarized in a corollary to Proposition 1.

Corollary 1 ***(Unilateral trade liberalization.)*** *In two-country ACR-class economies, the welfare gains relative to autarky from incomplete unilateral trade liberalization are strictly larger when liberalization consists in lower tariffs as compared to lower iceberg costs conditional on openness.*

Proof. In the text. ■

4.2 Lower iceberg costs in the presence of tariffs

We now consider welfare effects of lower iceberg trade costs in the presence of tariffs but where the variation in λ_{ii} only stems from variation in τ_{ij} and not in t_{ij}. Then, totally differentiating (11) yields

$$\hat{W}_i = \underbrace{\widehat{\left(\frac{w_i}{P_i}\right)}}_{\text{real wage}} - \underbrace{\frac{\lambda_{ii}(t_{ij}-1)}{\lambda_{ii}(t_{ij}-1)+1}}_{\text{tax-base effect}} \hat{\lambda}_{ii}. \tag{21}$$

The first term in (21) reflects the effect of a change in λ_{ii} on the real wage, the second term the effect on tariff revenue. Interestingly, the change of the real wage as triggered by an underlying variation of τ_{ji} can be shown to be $\widehat{(w_i/P_i)} = (1/\varepsilon_{\tau})\hat{\lambda}_{ii}$ for models without export selection (see the Appendix for the proof and for details on the expression for models with export selection).

In the absence of tariffs $(t_{ij} = 1)$, the welfare formula collapses to the basic equation in ACR. Since ε_{τ} is a negative number, a decrease in the degree of autarkiness $\left(\hat{\lambda}_{ii} < 0\right)$ triggered by lower iceberg costs leads to an increase in welfare of $1/\varepsilon_{\tau}$ percent. Moreover, as ε_{τ} is constant, one can integrate (21) and write $W_i = \lambda_{ii}^{1/\varepsilon_{\tau}} W_i^a$, where the constant of integration W_i^a denotes the autarky level of welfare. Also, note that the relationship between W_i and λ_{ii} triggered by changes in τ_{ij} is strictly globally convex.

In the presence of a tariff $(t_{ij} > 1)$, the negative term $1/\varepsilon_{\tau}$ is augmented by an additional negative element so that the absolute magnitude of the expression in the bracket goes up,

case of the Melitz (2003) model, global concavity can, however, be derived analytically.

magnifying the total effect of increased autarkiness on welfare. So, disregarding the existence of tariffs when studying the comparative statics of iceberg costs can be misleading.[19] The absolute size of the error increases in $\lambda_{ii}(t_{ij}-1)$. Moreover, as λ_{ii} appears in the round brackets of (21), the elasticity of welfare with respect to autarkiness λ_{ii} is no longer constant. However, it is still true that the level of W_i is a convex, decreasing function of λ_{ii}.

Importantly, the welfare formula (21) is the same regardless of microeconomic structure of the underlying trade model as long as it belongs to the ACR-class of frameworks. Thus, the isomorphism described by ACR in the absence of tariffs holds true, at least *locally.* Now, however, an *ex post* analysis of a liberalization episode requires more information than just the change in openness and the trade elasticity ε_τ. One also must know the levels of λ_{ii} and of the tariff.

The following proposition summarizes the first key result.

Corollary 2 ***(Welfare effects of lower iceberg costs in the presence of tariffs.)*** *In a two-country ACR-class model with ad valorem tariffs, conditional on the observed change in openness, lower iceberg costs increase welfare by more in the presence of tariffs than in their absence.*

Proof. Directly follows from equation (21) with $\hat{w}_i - \hat{P}_i = (1/\varepsilon_\tau)\hat{\lambda}_{ii}$ for models without export selection. The proof for models with export selection is in the Appendix. ■

4.3 Extension: reform in both countries

A simple case is obtained by imposing symmetry. This requires that trade policy is constrained such that $t_{ij} = t_{ji} = t$ (and, hence, also $\hat{t}_{ij} = \hat{t}_{ji} = \hat{t}$). However, countries are allowed to differ with respect to endowments and technology. Running through the same analysis as in section 3.1, the local welfare effect of such a reform is given by (see the Appendix for a proof)

$$\hat{W}_i = \frac{1-\varepsilon_t}{\varepsilon_t}\left\{\frac{t-1}{t} - \frac{\tilde{\lambda}_{ii}-\tilde{\lambda}_{jj}}{\frac{\varepsilon_t}{\varepsilon_w}+(1-2\varepsilon_t)\tilde{\lambda}_{jj}-\tilde{\lambda}_{ii}}\frac{1}{\lambda_{ii}t}\right\}\tilde{\lambda}_{ii}\hat{\lambda}_{ii}. \quad (22)$$

[19]The underestimation can be sizable. For reasonable numbers such as $\varepsilon_\tau = -3, \lambda_{ii} = 0.8$ and $t_{ij} = 1.1$, the elasticity $\hat{W}_i/\hat{\lambda}_{ii}$ is overestimated by about 20% when wrongly setting $t_{ij} = 1$. We offer a more elaborate quantitative perspective in section 4.4.

As before, the sign of the curly bracket is in general ambiguous. However, if one assumes perfect symmetry across the two countries, then $\tilde{\lambda}_{ii} = \tilde{\lambda}_{jj} = \tilde{\lambda}$ and $\lambda_{ii} = \lambda_{jj} = \lambda$. The welfare expression collapses to

$$\hat{W}_i = \frac{1-\varepsilon_t}{\varepsilon_t} \frac{t-1}{t} \tilde{\lambda}\hat{\lambda}. \tag{23}$$

Clearly, under symmetry, and when both countries have the same tariff, manipulating $\hat{\lambda}$ can never increase welfare and the optimal tariff is $t^* = 1$. In the absence of iceberg trade frictions, this corresponds to $\lambda^* = 1/2$. Since λ^* characterizes a welfare maximum, it must be true that $\partial W^t(\lambda)/\partial\lambda\big|_{\lambda=1/2} = 0$ and $\partial^2 W^t(\lambda)/\partial\lambda^2\big|_{\lambda=1/2} < 0$. This establishes local concavity of $W^t(\lambda)$ at λ^*; global concavity is proven in the Appendix. As in Figure 2, $W^t(\lambda)$ and $W^\tau(\lambda)$ coincide at the endpoints of the interval $[1/2, 1]$ and $W^\tau(\lambda)$ is strictly convex in λ (see ACR). It follows that $W^t(\lambda) > W^\tau(\lambda)$ for all $\lambda \in (1/2, 1)$. Hence, Proposition 1 is true even if the optimal trade policy is unrestricted trade (i.e., $t^* = 1$). Our results pertaining to the welfare comparison across our two liberalization scenarios are, therefore, not due to the existence of government revenue under $\tau-$reform.

Proposition 1 probably holds in more general environments than the ACR framework. The reason is that, starting at free trade and symmetry, a marginal tariff $dt_{ij} > 0$ has no effect on welfare as gains due to the own import tariff are exactly offset by losses due to the foreign tariff. However, $d\tau_{ij} > 0$ causes first-order losses as shown. In other words, iceberg costs come with rectangular losses while tariffs generate triangular losses.

Finally, it is interesting to solve for the tariff that country i would find optimal under the maintained assumption $t_{ij} = t_{ji} = t$. This scenario therefore supposes that i can set its own *and* the foreign tariff. We are not claiming that such a scenario mimics real world negotiation protocols such as GATT/WTO, but it does reveal countries' relevant trade policy preferences if they have to agree (e.g., vote) on a single common instrument. Setting the curly bracket in (22) to zero and solving for $i's$ optimal tariff under the simplifying restriction that $\mu = 1$ (all export fixed costs in terms of domestic labor), i's optimal tariff is

$$t_i = \frac{1 - 2\varepsilon_t \tilde{\lambda}_{jj}}{1 - 2\varepsilon_t \tilde{\lambda}_{ii}} \frac{\tilde{\lambda}_{ii}}{\tilde{\lambda}_{jj}}. \tag{24}$$

Country i desires an import tariff $t_i > 1$ if $\lambda_{ii} > \lambda_{jj}$, that is, if i is the large country, and an import subsidy if it is the small one.

4.4 Numerical quantification

Calibration. As a final step in this section, we calibrate the model and simulate trade policy reform scenarios. The aim is to gain a sense on the possible bias size when welfare calculations are entirely based on viewing trade barriers as non-revenue generating but resource-consuming iceberg trade costs. Since our theoretical frameworks are fairly stylized, we do not aim at a realistic calibration of the world economy; the rich CGE literature is better equipped for this purpose (see Balistreri and Rutherford (2012) for a survey). Rather, we model a world of only two countries. In our baseline exercise, where we study multilateral trade cost and tariff reductions, we even assume symmetry, but assume asymmetry whenever necessary for our argument. The objective of this section is not to analyze a realistic world trade reform scenario, but merely to quantify the importance of our theoretical results.

Without loss in generality, we use the Melitz (2003) model for the numerical exercise. We calibrate the model toward the US economy as around the year 2000; Table 1 summarizes our strategy. We start by assigning values to the elasticity of substitution σ and the Pareto shape parameter θ. Drawing on the estimates reported in Bernard, Eaton, Kortum, and Jensen (2003), we set $\theta = 3.3.$ and $\sigma = 3.8$. When we are interested in nesting the Melitz, Krugman, and Armington models, we choose $\sigma = \beta + 1 = 4.3$. Under that restriction, the Melitz model collapses to the Krugman model. And all models display the same optimal tariff conditional on $\tilde{\lambda}_{FF}$.

Moreover, for the year of 2000, we observe an average most favored nation tariff factor of 1.016 as evidenced in the World Bank's WITS data base, and a startup failure rate as reported by Bartelsman et al. (2004). Next, we want the model to replicate two key statistics of he US economy, namely the export participation rate and the import penetration rate, as observed in 2000. Following Bernard, Jensen, Redding, and Schott (2007), the former is 17.2% while the latter is 23.4%. These choices imply an iceberg trade cost factor of 1.37, relative market access costs $\left(f^x/f^d\right)$ of 1.75 and relative innovation costs $\left(f^e/f^d\right)$ of 5.49. These implied values compare well to the literature, where Demidova (2008) finds $f^x/f^d = 1.8$ and Obstfeld and Rogoff (2001) report $\tau = 1.3$.[20]

Table 1: Calibration

Parameter	**Value**	**Source**
Constants and parameters from the empirical literature		
Elasticity of substitution (σ)	3.8	Bernard et al.(2003)
Pareto shape parameter (θ)	3.3	Bernard et al.(2003)
Failure rate ($G(\varphi^*)$)	0.170	Bartelsmann et al. (2004)
Observed/targeted data, around 1970		
Average tariff factor (t)	1.060	World Bank WITS data base
Export participation rate (m^x)	0.104	Bernard et al.(1995)
Import penetration rate ($1-\lambda$)	0.060	Lu and Ng(2012)
Observed/targeted data, at 2000		
Average tariff factor (t)	1.016	World Bank WITS data base
Export participation rate (m^x)	0.172	Bernard et al.(2007)
Import penetration rate ($1-\lambda$)	0.234	OECD (2005)
Implied parameters, 1970		
Iceberg trade cost factor (τ)	2.23	
Relative market access costs (f^x/f^d)	0.58	
Relative innovation costs $\left(f^e/f^d\right)$	5.49	
Implied parameters, 2000		
Iceberg trade cost factor (τ)	1.37	
Relative market access costs (f^x/f^d)	1.75	
Relative innovation costs $\left(f^e/f^d\right)$	5.49	

We also calibrate the model to observed data from the 1970s. Then, the average US most favored nation import tariff was standing at 6.0%, the export participation rate was 10.4% (in 1976; Bernard et al., 1995). The import penetration rate was 6% in the year of 1970. While the tariff was about four times higher in the 70s than in the year 2000, and the import penetration rate about four times lower, the export penetration rate was only about 7 percentage points lower. This has important implication for the model parameters implied by these moments. Iceberg trade costs are 123% in 1970 relative to 37% in 2000 (replicating the fairly low 1970 import penetration rate), but *relative* market entry costs are below unity (so that the model

[20]The implied parametrization of market access costs is found by solving $\lambda = \left[1 + tm^x \left(f^x/f^d\right)\right]^{-1}$ for f^x/f^d. The implied value for τ is found by solving $m^x = t^{-\frac{\theta}{\rho}}\tau^{-\theta}\left(f^x/f^d\right)^{-\frac{\theta}{\sigma-1}}$ for τ. The implied value for f^e/f^d follows from the free entry condition

$$\frac{\sigma-1}{\theta-(\sigma-1)}p^{in}\left[1+\frac{f^x}{f^d}\left(\tau\left(\frac{f^x}{f^d}\right)^{\frac{1}{\sigma-1}}\right)^{-\theta}\right] = \frac{f^e}{f^d}.$$

replicates the observed export participation rate). Note, however, that this is perfectly compatible with falling *absolute* fixed costs of market access costs. A rising ratio f^x/f^d implies increased protection of domestic firms. Relative innovation costs f^e/f^d have been held fixed at the 2000 level, but the implied failure rate $G(\varphi^*)$ has been recalibrated.

4.5 Simultaneous liberalization in a symmetric world

Our first scenario is a simultaneous liberalization of tariffs or iceberg trade costs in a symmetric world. We compare three cases. In each, we compare equilibria anchored in observed historical openness levels with hypothetical 'free' trade or autarky equilibria. Crucially, in each comparison, we replicate observed openness levels either by choosing an appropriate value for the ad valorem tariff rate t or for the iceberg trade cost τ. Table 2 provides results.

Table 2: Simultaneous liberalization in a symmetric world

	τ	t	λ	W	ΔW
(A) 'Free' trade versus 2000					
(A0)	1.00	1.00	0.53	0.218	
(A1)	1.37	1.00	0.76	0.195	11.79%
(A2)	1.00	1.35	0.76	0.209	4.31%
(B) 1970 versus Autarky					
(B0)			1.00	0.180	
(B1)	2.23	1.00	0.94	0.183	1.89%
(B2)	1.00	2.14	0.94	0.189	5.45%
(C) 2000 versus Autarky					
(C0)			1.00	0.180	
(C1)	1.41	1.00	0.76	0.194	7.99%
(C2)	1.00	1.39	0.76	0.208	15.62%
(D) 2000 versus 1970					
(D0)	1.37	1.02	0.76	0.195	
(D1)	2.20	1.02	0.94	0.183	6.56%
(D2)	1.00	1.59	0.94	0.187	4.28%

Notes: Welfare gains relative to year (A) 2000, (B) Autarky, (C) Autarky, (D) 1970).

Scenario (A) compares 'free' trade with the status observed as of year 2000. 'Free' trade refers to a situation where all variable trade costs are zero; λ, the share of expenditure allocated to domestic goods, is still different from 0.50 (but very close to it, 0.53) due to fixed market access costs.[21] The level of welfare in this situation is 0.218, see (A0).[22] In line (A1) we reproduce the observed level of openness (more correctly: autarkiness) as of 2000, $\lambda = 0.76$, by adjusting iceberg trade costs to $\tau = 1.37$ but keeping tariffs to zero (i.e., $t = 1$). Relative to the year 2000 status, 'free' trade would feature a level of welfare higher by 11.79%. In contrast, line (A2) adjusts tariffs to $t = 1.35$ to achieve the same level of factual openness. The associated welfare gain from moving to 'free' trade is much smaller now, $\Delta W = 4.31\%$. Hence, when taking 'free' trade as the (unobserved) counterfactual, the welfare loss gap from less than free trade depends very strongly on the nature of trade frictions. Linking variation in openness to variation in iceberg costs alone can lead to substantial biases – in the case of the 'free trade versus restricted trade' scenario, welfare losses from iceberg costs are substantially bigger than those from tariffs.

Scenarios (B) and (C) take the autarky equilibrium as the starting point and contrast it with observed equilibria calibrated towards the 1970 or 2000 levels of openness. Again, the exercises differentiate between two polar cases: one where the factual levels of openness are generated by adjustment of iceberg trade costs, and one where they are generated by adjustment of tariffs. Lines (B1) and (B2) shows that the observed openness as of 1970 (6%) can be replicated by either setting $\tau = 2.23, t = 1.00$, or by setting $\tau = 1.00, t = 2.14$ (i.e., an ad valorem tariff of 114%). However, the welfare gains relative to autarky are very different: Adjustment of trade costs leads to gains from trade of 1.89% while adjustment of tariffs generates almost three times higher gains equal to 5.45%. Targeting the openness level of 2000 (24%) delivers a very similar picture. Then, adjustment of trade costs leads to a 7.99% improvement in welfare while adjustment of tariffs generates gains about twice as high (15.62%). Note that Arkolakis et al. (2012) undertake a similar "autarky versus status quo" comparison but focus on τ only. Our simple numerical results suggest that this focus can significantly understate the gains from trade.

Finally, scenario (D) compares the two factual historical situations of 1970 and 2000. Unlike in scenarios (A)-(C) before, *both* the 1970 as well as the 2000 equilibrium replicate the observed openness measures. Line (D0) refers to the equilibrium as of 2000. Line (D1) increases tariffs from the observed 2000 level (1.6%) to the observed 1970 level (6.0%), and adjusts the unobserved

[21]For this reason we use quotation marks when referring to 'free' trade.

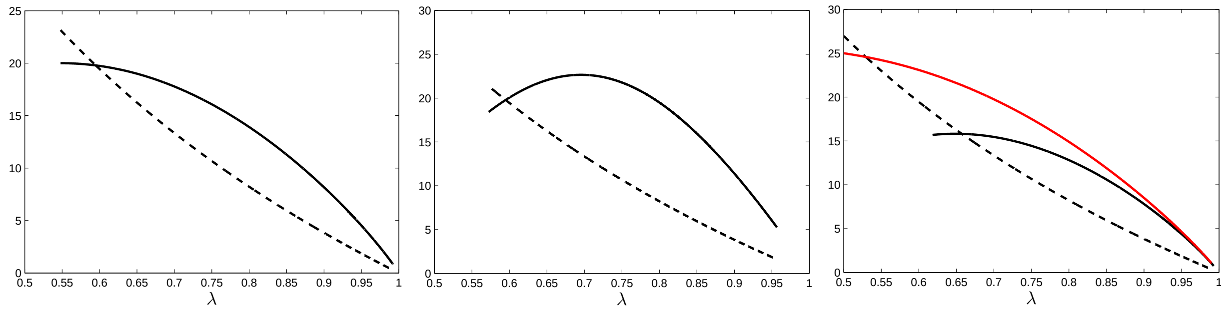
[22]Note that the absolute level of W is meaningless.

iceberg trade costs such that the 1970 openness level results. Relative to 1970, this results in year 2000 welfare lying 6.56% higher. If, instead, iceberg costs are driven to their minimum and tariffs are (counterfactually) adjusted such that the observed 1970 openness is again replicated, the welfare differential is only 4.28%. As before, the welfare calculations depend substantially on the type of trade cost adjustment assumed when calibrating the model towards some observed change in openness.

Figure 3 generalizes the insights obtained from Table 2 by looking at gains from trade (relative to the autarky case) over a more extended range of 'autarkiness' measures λ. The diagrams vary one policy parameter (τ or t at a time, keeping the other fixed at 1.06.). Diagram (a) confirms our theoretical insight derived earlier that the gains from trade are a concave function of λ when taking the underlying variation from t, but a convex curve when the underlying variation comes from τ. Over the considered range of λ, the difference between the two scenarios can be very sizable.

Figure 3: Underestimation of gains from trade

(a) Simultaneous liberalization, symmetric (b) Unilateral liberalization, symmetric (c) Simultaneous liberalization, asymmetric



Notes: Variation driving changes in openness stems from tariffs (solid curve) or iceberg trade costs (dashed curve); Home (black), Foreign (red). Simultaneous means that tariff rates are equal in both countries. Symmetric refers to a uniform distribution of the world labor endowment; asymmetric has Home hold 60% of the endowment.

4.6 Unilateral liberalization in a symmetric world

Diagram (b) of Figure 3 keeps the symmetric distribution of labor endowments across countries, but assumes that one country sets its tariff unilaterally, while the other country has the benchmark tariff of 1.06%. Because of symmetry in fundamentals, when the adjustment takes place in iceberg trade costs, curves for both countries coincide. The situation is different, when one country adjusts its tariff. This country's welfare function exhibits a hump at about $\lambda = 0.66$, implying the existence of an optimal tariff t^*. The welfare function for the other (the passive) country is as under the iceberg scenario; specifically, there is no hump. Note that we have shown this analytically for the case $t = 1$. The intuition is that, in the absence of tariffs, both definitions of λ coincide, and so the simple welfare equation (**??**) applies. This is true regardless of the fact that 'autarkiness' is shifted by a foreign shock. The message of the picture is again that looking at tariffs as compared to iceberg costs makes an important quantitative difference, that can rise to up to 4 percentage points. In all those scenarios, since we start from autarky as the reference point, focusing on iceberg costs underestimates the gains from trade.

4.7 Simultaneous liberalization in an asymmetric world

Finally, diagram (c) in Figure 3 maintains the calibration for the symmetric two country world with the only difference that labor endowments are now distributed unequally. Home (graphed in black) commands 60% of the world labor supply while Foreign (in red) commands the remainder. The scenario here is that trade liberalization is simultaneous, i.e., tariffs or iceberg costs are identical in both countries and so are their rates of change. We know from our theoretical analysis that country size does not matter for the welfare effects of lower iceberg trade costs conditional on openness. So, the loci for Home and Foreign coincide.[23]

Looking at tariffs, the picture is different. Here, market size (expressed by population shares) does matter. Again, both countries are assumed to set the same import tariffs. However, the small country (Foreign) now benefits more from an increase in openness than the large country (Home). The reason, already alluded to in our theory section, is that the large country would, if it could, set a higher tariff than the small one. This is quite visible in the diagram, where the welfare maximum for the large country is reached at a λ of about 0.66, while the welfare maximum for the small country is not reached as we restrict ourselves to $t \geq 1$. For the same value of λ and a 3:2 distribution of endowments, the gains from trade in the small country are

[23] To be more precise, the perfect coincidence of the curves has been analytically shown in the absence of tariffs ($t = 1$); in diagram (b) we have $t = 1.016$.

up to 3 percentage points larger than in the large one.

5 Conclusion

We extend the analysis of Arkolakis et al. (ACR, 2012) to *ad valorem* tariffs and endogenous trade policies. We find two main results. First, models with and without extensive margins, and with perfect or imperfect competition can be characterized by a mathematically isomorphic optimal policy, either an import tariff, an export tax or a subsidy on domestically produced goods, or a combination thereof. Conditional on gravity, these different models give rise to the same optimal tariff. Second, the optimal tariff suffices to implement the first-best. That is, the optimal tariff remedies all distortions in the model: the markup-distortion due to monopolistic competition, the entry-distortion due to the presence of an extensive margin, and the more conventional terms-of-trade externality. These distortions have a common feature: they dissociate the socially optimal relative price of the ideal quanitity indices of exported versus domestic varieties with the privately perceived one.

As a corollary, we show that modeling trade liberalization scenarios by a reduction of iceberg trade costs while in reality they may involve a reduction of import tariffs (or export taxes, or subsidies on domestic goods) leads to an underestimation of the welfare gains from trade. That underestimation can be quantitatively significant.

References

[1] Alvarez, F. and R. Lucas (2007), 'General Equilibrium Analysis of the Eaton-Kortum model of international trade', *Journal of Monetary Economics* **54**(6): 1726-1768.

[2] Anderson, J., and E. van Wincoop (2003), 'Gravity with Gravitas: A Solution to the Border Puzzle', *American Economic Review* **93**(1): 170-192.

[3] Arkolakis, C., A. Costinot, and A. Rodríguez-Clare (2012), 'New Trade Models, Same Old Gains?', *American Economic Review* **102**(1): 94-130.

[4] Arkolakis, C., S. Demidova, P. Klenow, and A. Rodríguez-Clare (2008), 'Endogenous Variety and the Gains from Trade', *American Economic Review* **98**(2): 444-450.

[5] Balistreri, E.J., R.H. Hillberry, and T.F. Rutherford (2010), 'Trade and welfare: Does industrial organization matter?' *Economics Letters* **109**: 85–87.

[6] Balistreri, E.J., R.H. Hillberry, and T.F. Rutherford (2011), 'Structural estimation and solution of international trade models with heterogeneous firms', *Journal of International Economics* **83**: 98-108.

[7] Balistreri, E.J. and J.R. Markusen (2009), 'Sub-national differentiation and the role of the firm in optimal international pricing', *Economic Modelling* **26**: 47-62.

[8] Balistreri, E.J and T.F. Rutherford (2012), 'Computing General Equilibrium Theories of Monopolistic Competition and Heterogeneous Firms', In P.B. Dicon and D.W. Jorgenson (eds.). *Handbook of Computable General Equilibrium Modeling.* North Holland.

[9] Bartelsman, E.J., J. Haltiwanger, and S. Scarpetta (2004), 'Microeconomic Evidence of Creative Destruction in Industrial and Developing Countries', *IZA Discussion Paper* No. 1374.

[10] Bernard, A., J. Eaton, B. Jensen, and S. Kortum (2003), 'Plants and Productivity in International Trade', *American Economic Review* **93**(4):1268-1290.

[11] Bernard, A., B. Jensen, and R. Lawrence (1995), 'Exporters, Jobs, and Wages in U.S. Manufacturing: 1976-1987', *Brookings Papers on Economic Activity. Microeconomics* 1995: 67–119.

[12] Bernard, A., B. Jensen, S. Redding, and P. Schott (2007), 'Firms in International Trade', *Journal of Economic Perspectives*, Summer 2007.

[13] Broda, C., N. Limao and D. Weinstein (2008), 'Optimal Tariffs and Market Power: The Evidence', *American Economic Review* **98**(5): 2032-2065.

[14] Burstein, A.T. and J. Vogel (2011), 'Factor Prices and International Trade: A Unifying Perspective', *NBER Working Paper* 16904.

[15] Chaney, T. (2008), 'Distorted Gravity: the Intensive and Extensive Margins of International Trade', *American Economic Review* **98**(4): 1707—1721.

[16] Cole, M.T. (2011a), 'Not All Trade Restrictions are Created Equally', *Review of World Economics* **147**(3): 411–427.

[17] Cole, M.T. (2011b), 'Distorted Trade Barriers', *University College Dublin, Centre for Economic Research*, WP11/05.

[18] Demidova, S. (2008), 'Productivity Improvements and Falling Trade Costs: Boon or Bane?', *International Economic Review* **49**(4): 1437-1462.

[19] Demidova, S. and A. Rodríguez-Clare (2009), 'Trade Policy under Firm-Level Heterogeneity in a Small Economy', *Journal of International Economics* **78**(1): 100-112.

[20] Dinghra, S. and J. Morrow (2011), 'Monopolistic Competition and Optimum Product Diversity under Firm Heterogeneity', mimeo: London School of Economics.

[21] Dornbusch, R., and P.A. Samuelson (1977), 'Comparative Advantage, Trade and Payments in a Ricardian Model with a Continuum of Goods', *American Economic Review* **27**(5): 823-839.

[22] Eaton, J., and S. Kortum (2002), 'Technology, Geography, and Trade', *Econometrica* 70(5): 1741-1779.

[23] Feenstra, R. (2010), 'Measuring the Gains from Trade under Monopolistic Competition', *Canadian Journal of Economics* 1-28.

[24] Feenstra, R. and H.L., Kee (2008), 'Export Variety and Country Productivity: Estimating the Monopolistic Competition Model with Endogenous Productivity', *Journal of International Economics* 500-518.

[25] Felbermayr, G., B. Jung and M. Larch (2012), 'Optimal Tariffs, Retaliation and the Welfare Loss from Tariff Wars in the Melitz Model', *Journal of International Economics*, doi: 10.1016/j.jinteco.2012.06.001.

[26] Gros, D. (1987), 'A Note on the Optimal Tariff, Retaliation and the Welfare Loss from Tariff Wars in a Framework with Intra-Industry Trade', *Journal of International Economics* 23: 357-367.

[27] Krugman, P. (1980), 'Scale Economies, Product Differentiation, and the Pattern of Trade', *American Economic Review* 70, 950–959.

[28] Lu Y, Ng T (2012), 'Import competition and skill content in U. S. manufacturing industries', *Review of Economics and Statistics*, forthcoming.

[29] Melitz, M.J. (2003), 'The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity', *Econometrica* **71**: 1695-1725.

[30] Opp, D. (2010), 'Trade Wars in the Ricardian Model with a Continuum of Goods', *Journal of International Economics* 80: 212-225.

A Gravity

Preferences given by symmetric CES aggregator function, with elasticity of substitution $\sigma > 1$, expenditure for a given variety ω from country i in country j

$$x_{ij}(\omega) = \left[\frac{p_{ij}(\omega)}{P_j}\right]^{1-\sigma} X_j$$

where $p_{ij}(\omega)$ is the c.i.f. price such that $p_{ij}(\omega) = p_i \tau_{ij} t_{ji}$. We assume linear technologies such that variable production cost per unit of output in country i is given by $w_i / \varphi(\omega)$.

A.1 Armington

Perfect competition and identical linear technology $\varphi(\omega) = 1$ across varieties such that $p_i(\omega) = w_i$. If country i produces N_i varieties, then

$$\frac{X_{ij}}{X_{jj}} = \frac{N_i}{N_j}\left(\frac{w_i}{w_j}\right)^{1-\sigma}(\tau_{ij}t_{ji})^{1-\sigma}. \tag{25}$$

Note that, different to the standard treatment, τ_{ij} and t_{ji} have the same elasticities $\varepsilon_\tau = \varepsilon_t = 1 - \sigma$ because trade flows are defined inclusive of tariff payments. Also, $\varepsilon_\omega = 1 - \sigma$.

A.2 Krugman

We continue to assume $\varphi(\omega) = 1$. However, in the Krugman (1980) model, monopolistic competition implies the f.o.b. price $p_i(\omega) = w_i / \rho$ where $\rho = (\sigma - 1)/\sigma$ is the inverse of the (constant) markup. Unlike in the Armington model, N_i is now endogenous determined by a free entry condition. However, by R1 and R2 N_i is fixed by parameters other that trade costs (real and tariffs) or wages and can, thus, be treated as constants in the present context. The gravity equation is, however, structurally identical to the one obtained in the Armington model, (25), and so the elasticities $\varepsilon_t, \varepsilon_\tau$ and ε_ω are all equal to $1 - \sigma$.

A.3 Melitz

Now, firms differ with respect to productivity φ which, in line with the literature, is assumed to follow a Pareto distribution with c.d.f. $G_i(\varphi) = T_i - \varphi^{-\theta}$, where $T_i \geq 1$ measures the location (country $i's$ lowest possibly productivity draw) and θ the shape of the distribution. Presence of a firm from i on a market j

requires payment of fixed costs $w_i^{\mu} w_j^{1-\mu} f_{ij}$, where $\mu \in [0,1]$ governs the extent to which labor from the source country i has to be employed as compared to labor from the destination country j. Only firms with $\varphi \geq \varphi_{ij}^*$ will be earning sufficiently much revenue on market j to support market presence in the presence of fixed access costs. Under these conditions the gravity equation is given by

$$\frac{X_{ij}}{X_{jj}} = \frac{N_i}{N_j}\frac{T_i}{T_j}\left(\frac{f_{ij}}{f_{jj}}\right)^{1-\frac{\theta}{\sigma-1}}\left(\frac{w_i}{w_j}\right)^{-\theta-\mu\left(\frac{\theta-\sigma-1}{\sigma-1}\right)}\tau_{ij}^{-\theta} t_{ji}^{1-\theta/\rho}, \tag{26}$$

and which collapses to the well-known form (CITE) if fixed costs are in domestic labor $(\mu = 1)$. As in the Krugman (1980) model, N_i and N_j are solved via a free-entry condition, but turn out independent from trade costs (t_{ji}, τ_{ji}) and wages. Hence, in the Melitz model with Pareto-distributed productivity, $\varepsilon_t = 1-\theta/\rho$ and $\varepsilon_\tau = -\theta$. Letting $\theta \to \sigma - 1$ to close down the selection effect, the Melitz gravity equation (26) collapses to the Krugman form as $\varepsilon_t = \varepsilon_\tau \to 1-\sigma$. In general, the elasticity ε_ω depends on μ. In the polar cases $\mu = 1$ and $\mu = 0$ we have $\varepsilon_\omega = \varepsilon_t$ and $\varepsilon_\omega = \varepsilon_\tau$, respectively.

A.4 Eaton-Kortum

The final example of a model nested in the framework is the Eaton-Kortum (2002) model. It is a perfect competition Ricardian trade model with a continuum of variety where each countries productivity φ in producing a variety ω is Fréchet distributed with $F(\varphi) = \exp\left(-T_i\varphi^{-\theta}\right)$, where T_i again governs the location and θ the shape of the distribution. That model admits a gravity equation of the form

$$\frac{X_{ij}}{X_{jj}} = \frac{T_i}{T_j}\left(\frac{w_i}{w_j}\right)^{-\theta}\left(\tau_{ij}t_{ji}\right)^{-\theta}. \tag{27}$$

B Proof of proposition 1 (Optimal tariff)

B.1 Perfect competition

The change in the real wage is given by

$$\hat{w}_i - \hat{P}_i = \frac{1}{\varepsilon}\hat{\lambda}_{ii}.$$

This claim directly follows from noting that ACR's proof of proposition 1 (pp. 119-122) only involves changes in marginal costs $\hat{c}_{ij}$ irrespectively of the components of c_{ij} and does not depend on changes in income. In our setting, the expression for c_{ij} extends to

$$c_{ij} = w_i \tau_{ij} t_{ji}.$$

The optimal tariff then immediately follows from equation ().

B.2 Monopolistic competition

In the absence of export selection, the proof resembles the proof for perfect competition models. The reason is the mass of firms is determined independently of tariffs, and therefore does not enter the equations for changes in the price index and changes in expenditure shares.

For models with export selection, we proceed in 4 steps.

Step 1: The change in the real wage is given by

$$\begin{aligned} \widehat{\frac{w_i}{P_i}} &= \frac{\theta - (\sigma - 1)}{\theta (\sigma - 1)} \left(\hat{w}_i - \hat{E}_i \right) + (1 - \lambda_{ii}) \hat{w}_i + (1 - \lambda_{ii}) \hat{\tau}_{ji} \\ &+ (1 - \lambda_{ii}) \left(\frac{\sigma (\theta - (\sigma - 1))}{\beta (\sigma - 1)} + \frac{\sigma - 1}{\beta} \right) \hat{t}_{ij} + (1 - \lambda_{ii}) \hat{w}_j - (1 - \lambda_{ii}) \frac{\mu (\theta - (\sigma - 1))}{\beta (\sigma - 1)} \widehat{\frac{w_i}{w_j}}. \end{aligned} \tag{28}$$

The price index in levels is given by equation (). Totally differentiating this expression, we obtain

$$\begin{aligned} \widehat{\frac{w_i}{P_i}} &= (1 - \lambda_{ii}) \hat{w}_i - \lambda_{ii} \frac{\sigma - 1 - \theta}{1 - \sigma} \hat{\varphi}^*_{ii} \\ &- (1 - \lambda_{ii}) \frac{\sigma - 1 - \theta}{1 - \sigma} \hat{\varphi}^*_{ji} - (1 - \lambda_{ii}) \left(\hat{t}_{ij} + \hat{\tau}_{ji} + \hat{w}_j \right). \end{aligned} \tag{29}$$

Totally differentiating the zero cutoff profit condition, we obtain

$$\hat{\varphi}^*_{ji} = \frac{1 - \mu}{\sigma - 1} \hat{w}_i + \frac{\mu}{\sigma - 1} \hat{w}_j - \frac{1}{\sigma - 1} \hat{E}_i + \hat{\tau}_{ji} + \hat{w}_j + \frac{\sigma}{\sigma - 1} \hat{t}_{ij} - \hat{P}_i. \tag{30}$$

Substituting out $\hat{\varphi}^*_{ii}$ and $\hat{\varphi}^*_{ji}$ from equation (29) and collecting terms, we obtain equation (28).

Step 2: The change in the real wage is given by

$$\frac{\widehat{w_i}}{P_i} = -\frac{1}{\theta}\hat{\lambda}_{ii} - \frac{\theta - (\sigma - 1)}{\theta(\sigma - 1)}\left(\hat{w}_i - \hat{E}_i\right). \tag{31}$$

The share of income country i spends on goods from country j is given by

$$\lambda_{ji} = \frac{N_j w_j^{1-\sigma}\tau_{ji}^{1-\sigma}t_{ij}^{1-\sigma}\left(\varphi_{ji}^*\right)^{\sigma-1-\beta}}{\sum_k N_k w_k^{1-\sigma}\tau_{ik}^{1-\sigma}t_{ik}^{1-\sigma}\left(\varphi_{ki}^*\right)^{\sigma-1-\beta}}.$$

Totally differentiating the import spending share relative to the domestic spending share, we obtain

$$-\frac{1}{1-\lambda_{ii}}\hat{\lambda}_{ii} = (1-\sigma)\left(\hat{w}_j - \hat{w}_i + \hat{\tau}_{ji} + \hat{t}_{ij}\right) + (\sigma - 1 - \beta)\left(\hat{\varphi}_{ji}^* - \hat{\varphi}_{ii}^*\right), \tag{32}$$

where we have used that spending shares add to unity.

Totally differentiating the domestic zero cutoff profit condition relative to the import zero cutoff profit condition, we obtain

$$\hat{\varphi}_{ji}^* = \hat{\varphi}_{ii}^* + \frac{1}{\rho}\hat{t}_{ij} + \hat{\tau}_{ji} + \frac{-\mu - \sigma + 1}{\sigma - 1}\frac{\widehat{w_i}}{w_j}. \tag{33}$$

Substituting out $\hat{\varphi}_{ji}^*$ from equation (32) and using the resulting expression in equation (28), we obtain equation (31).

Step 3: The change in the real wage is given by

$$\frac{\widehat{w_i}}{P_i} = -\frac{1}{\theta}\hat{\lambda}_{ii} - \frac{\theta - (\sigma - 1)}{\theta(\sigma - 1)}\left(\frac{\lambda_{ii}(t_{ij} - 1)}{\lambda_{ii}(t_{ij} - 1) + 1}\hat{\lambda}_{ii} - \frac{1 - \lambda_{ii}}{\lambda_{ii}(t_{ij} - 1) + 1}\hat{t}_{ij}\right). \tag{34}$$

Aggregate expenditure is given by

$$E_i = \frac{t_{ij}}{\lambda_{ii}(t_{ij} - 1) + 1} w_i L_i.$$

Totally differentiating this expression, we obtain

$$\hat{w}_i - \hat{E}_i = \frac{\lambda_{ii}(t_{ij} - 1)}{\lambda_{ii}(t_{ij} - 1) + 1}\hat{\lambda}_{ii} - \frac{1 - \lambda_{ii}}{\lambda_{ii}(t_{ij} - 1) + 1}\hat{t}_{ij}.$$

Substituting out $\hat{w}_i - \hat{E}_i$ from equation (31), we obtain equation (34).

Step 4: The optimal tariff is derived under the conjecture that $\hat{w}_i - \hat{P}_i = \hat{\lambda}_{ii}/\varepsilon_t$. We show that evaluated at the optimal tariff, this conjecture holds. Substituting out

$$\hat{t}_{ij} = \frac{\varepsilon_w}{\varepsilon_t}\frac{\widehat{w_i}}{w_j} - \frac{1}{\varepsilon_t}\frac{1}{1-\lambda_{ii}}\hat{\lambda}_{ii},$$

where

$$\frac{\widehat{w_i}}{w_j} = -\frac{1-\varepsilon_t}{\varepsilon_t}\frac{1}{1-\tilde{\lambda}_{jj}\varepsilon_w - \tilde{\lambda}_{ii}\frac{\varepsilon_w}{\varepsilon_t}}\frac{\tilde{\lambda}_{ii}}{1-\lambda_{ii}}\hat{\lambda}_{ii}$$

and noting that the optimal tariff solves

$$-\frac{\varepsilon_w}{\varepsilon_t}\frac{\tilde{\lambda}_{ii}}{1-\tilde{\lambda}_{jj}\varepsilon_w - \tilde{\lambda}_{ii}\frac{\varepsilon_w}{\varepsilon_t}} + \lambda_{ii}\left(t_{ij}-1\right) = 0,$$

it follows that

$$\begin{aligned}\frac{\widehat{w_i}}{P_i} &= -\frac{1}{\theta}\hat{\lambda}_{ii} - \frac{\theta-(\sigma-1)}{\theta\left(\sigma-1\right)}\left(\frac{\lambda_{ii}\left(t_{ij}-1\right)}{\lambda_{ii}\left(t_{ij}-1\right)+1}\hat{\lambda}_{ii} - \frac{1-\lambda_{ii}}{\lambda_{ii}\left(t_{ij}-1\right)+1}\hat{t}_{ij}\right)\\ &= \frac{1}{\varepsilon_t}\hat{\lambda}_{ii}.\end{aligned}$$

Intermediate steps of calculus:

$$
\begin{aligned}
\frac{\widehat{w_i}}{P_i} &= -\frac{1}{\theta}\hat{\lambda}_{ii} - \frac{\theta-(\sigma-1)}{\theta(\sigma-1)}\left(\frac{\lambda_{ii}(t_{ij}-1)}{\lambda_{ii}(t_{ij}-1)+1}\hat{\lambda}_{ii} - \frac{1-\lambda_{ii}}{\lambda_{ii}(t_{ij}-1)+1}\hat{t}_{ij}\right). \\
&= -\frac{1}{\theta}\hat{\lambda}_{ii} - \frac{\theta-(\sigma-1)}{\theta(\sigma-1)}\left(\frac{\lambda_{ii}(t_{ij}-1)}{\lambda_{ii}(t_{ij}-1)+1}\hat{\lambda}_{ii} - \frac{1-\lambda_{ii}}{\lambda_{ii}(t_{ij}-1)+1}\left(\frac{\varepsilon_w}{\varepsilon_t}\frac{\widehat{w_i}}{w_j} - \frac{1}{\varepsilon_t}\frac{1}{1-\lambda_{ii}}\hat{\lambda}_{ii}\right)\right) \\
&= -\frac{1}{\theta}\hat{\lambda}_{ii} - \frac{\theta-(\sigma-1)}{\theta(\sigma-1)}\left(\frac{\lambda_{ii}(t_{ij}-1)}{\lambda_{ii}(t_{ij}-1)+1}\hat{\lambda}_{ii} - \frac{1-\lambda_{ii}}{\lambda_{ii}(t_{ij}-1)+1}\frac{\varepsilon_w}{\varepsilon_t}\frac{\widehat{w_i}}{w_j} + \frac{1}{\varepsilon_t}\frac{1}{\lambda_{ii}(t_{ij}-1)+1}\hat{\lambda}_{ii}\right) \\
&= \left[-\frac{1}{\theta} - \frac{\theta-(\sigma-1)}{\theta(\sigma-1)}\left(\frac{\lambda_{ii}(t_{ij}-1)}{\lambda_{ii}(t_{ij}-1)+1} + \frac{\frac{1-\varepsilon_t}{\varepsilon_t}\lambda_{ii}(t_{ij}-1)}{\lambda_{ii}(t_{ij}-1)+1} + \frac{1}{\varepsilon_t}\frac{1}{\lambda_{ii}(t_{ij}-1)+1}\right)\right]\hat{\lambda}_{ii} \\
&= \left[-\frac{1}{\theta} - \frac{\theta-(\sigma-1)}{\theta(\sigma-1)}\frac{1}{\varepsilon_t}\left(\frac{\lambda_{ii}(t_{ij}-1)}{\lambda_{ii}(t_{ij}-1)+1} + \frac{1}{\lambda_{ii}(t_{ij}-1)+1}\right)\right]\hat{\lambda}_{ii} \\
&= -\frac{1}{\theta}\frac{1}{\varepsilon_t}\left[\varepsilon_t + \frac{\theta-(\sigma-1)}{\sigma-1}\right]\hat{\lambda}_{ii} \\
&= -\frac{1}{\theta}\frac{1}{\varepsilon_t}\left[1-\frac{\theta}{\rho} + \frac{\theta-(\sigma-1)}{\sigma-1}\right]\hat{\lambda}_{ii} \\
&= -\frac{1}{\theta}\frac{1}{\varepsilon_t}\left[-\frac{\theta}{\rho} + \frac{\theta}{\sigma-1}\right]\hat{\lambda}_{ii} \\
&= -\frac{1}{\varepsilon_t}\left[-\frac{\sigma}{\sigma-1} + \frac{1}{\sigma-1}\right]\hat{\lambda}_{ii} \\
&= \frac{1}{\varepsilon_t}\hat{\lambda}_{ii}.
\end{aligned}
$$

C Proof of proposition 2 (First-best instrument)

C.1 Perfect competition models

In a closed economy, the laissez-faire equilibrium must be the first-best outcome because their is no distortion. In an open economy, however, the economy could do better than the laissez-faire equilibrium. The reason is that any country has market power (even if it is "small") as it is the only producer of a certain variety. This is the terms-of-trade externality, which can be exploited by imposing the optimal tariff.

C.2 Monopolistic competition models

We prove that the tariff is the first-best instrument for the case in which all export fixed are paid in terms of domestic labor ($\mu = 1$). Note that the Krugman (1980) case is nestet for $\theta \to \sigma - 1$.

- Let the social planner choose the country's domestic entry cutoff, its export cutoff, quantities produced for the domestic and the export market, the mass of domestic firms and the spending on imports K_m
- Utility is maximized subject to labor market clearing and balanced trade
- The social planner takes labor market clearing in Foreign as given
- We follow Demidova and Rodriguez-Clare (2009) in the use of notation
  - $q[\varphi]$: quantity consumed
  - $Q[\varphi]$: quantity produced
  - N_i is the mass of firms active in country i (which is different from the notation in the body of the paper)
- Utility maximization yields relative demand for domestic varieties

$$\frac{q[\varphi_1]}{q[\varphi_2]} = \left(\frac{p[\varphi_1]}{p[\varphi_2]}\right)^{-\sigma} = \left(\frac{\varphi_1}{\varphi_2}\right)^{\sigma}$$

- Export revenue maximization

$$\frac{Q[\varphi_1] - q[\varphi_1]}{Q[\varphi_2] - q[\varphi_2]} = \left(\frac{\varphi_1}{\varphi_2}\right)^{\sigma}$$

- An optimal allocation would have

$$\begin{aligned} q[\varphi] &= \phi\varphi^{\sigma}, \phi > 0 \\ Q[\varphi] - q[\varphi] &= \alpha\varphi^{\sigma}, \alpha > 0 \end{aligned}$$

- Moreover, if a variety with φ_1 is consumed, then all varietiey with $\varphi > \varphi_1$ must be consumed. The same argument holds for exported varieties
- Ultility maximization also yields relative demand for imported varieties

$$\frac{q[\varphi_1]}{q[\varphi_2]} = \left(\frac{p[\varphi_1]}{p[\varphi_2]}\right)^{-\sigma} = \left(\frac{\varphi_1}{\varphi_2}\right)^{\sigma},$$

where we have assumed that firms in Foreign impose a constant mark-up over marginal costs

- We then have

$$q_m[\varphi] = i\varphi^{\sigma}, i > 0$$

The least productive firm entering Home is indexed by φ^*_{FH}. Note that $w_F = 1$. Hence,

$$i = \frac{(\sigma - 1) f^x}{(\varphi^*_{FH})^{\sigma-1}}$$

The value of imports is given by

$$\begin{aligned} & N_F \int_{\varphi^*_{FH}} \frac{1}{\rho\varphi} i\varphi^{\sigma} \frac{dG[\varphi]}{1 - G[\varphi^*_{FF}]} \\ = & \frac{\theta}{\theta - (\sigma - 1)} \sigma N_F m_{FH} f^x \end{aligned}$$

- Labor market clearing in Foreign implies that the value of imports is given by

$$\begin{aligned} & \frac{\theta}{\theta - (\sigma - 1)} \sigma \frac{\rho}{\theta f^e} L_F (\varphi^*_{FF})^{-\theta} \left(\frac{\varphi^*_{FF}}{\varphi^*_{FH}}\right)^{\theta} f^x \\ = & \frac{\theta}{\theta - (\sigma - 1)} \sigma \frac{\rho}{\theta f^e} L_F (\varphi^*_{FH})^{-\theta} f^x \end{aligned}$$

Hence, choosing the optimal level of spending on imports is equivalent to choosing the optimal import cutoff φ^*_{FH}

- Export revenues per firm

$$(Q[\varphi] - q[\varphi]) p_x[\varphi]$$

Recall from above that

$$Q[\varphi] - q[\varphi] = A(p_x[\varphi])^{-\sigma} \Leftrightarrow A^{\frac{1}{\sigma}} (Q[\varphi] - q[\varphi])^{-\frac{1}{\sigma}} = p_x[\varphi]$$

Hence, export revenue per firm is

$$A^{\frac{1}{\sigma}} (Q[\varphi] - q[\varphi])^{\rho}$$

The value of total exports is given by

$$
\begin{aligned}
& N_H \int_{\varphi^*_{HF}} \varphi^{\sigma-1} \frac{dG[\varphi]}{1-G[\varphi^*_{HH}]} \\
= \; & N_H \alpha^\rho A^{\frac{1}{\sigma}} \frac{\theta}{\theta-(\sigma-1)} \left(\frac{\varphi^*_{HH}}{\varphi^*_{HF}}\right)^\theta \left(\varphi^*_{HF}\right)^{\sigma-1}
\end{aligned}
$$

- Note that A is assumed to be a constant in the small open economy case. The social planner of a large open economy takes into account changes in $A = Y_F P_F^{\sigma-1}$. Foreign's domestic entry cutoff condition reads

$$Y_F P_F^{\sigma-1} \left(\rho \varphi^*_{FF}\right)^{\sigma-1} = \sigma f^d \Leftrightarrow Y_F P_F^{\sigma-1} = \sigma f^d \left(\rho \varphi^*_{FF}\right)^{1-\sigma}$$

Note that

$$\tilde{\lambda}_{FF} = \frac{1}{1+\left(\frac{\varphi^*_{FF}}{\varphi^*_{FH}}\right)^\theta \frac{f^x}{f^d}}.$$

Free entry implies

$$\frac{f^x}{f^d}\left(\varphi^*_{FH}\right)^{-\theta} = \frac{f^e}{f^d}\frac{\theta-(\sigma-1)}{\sigma-1} - \left(\varphi^*_{FF}\right)^{-\theta}$$

Hence,

$$
\begin{aligned}
\tilde{\lambda}_{FF} &= \frac{1}{1+\left(\varphi^*_{FF}\right)^\theta \left(\frac{f^e}{f^d}\frac{\theta-(\sigma-1)}{\sigma-1} - \left(\varphi^*_{FF}\right)^{-\theta}\right)} \\
&= \frac{1}{1+\frac{f^e}{f^d}\frac{\theta-(\sigma-1)}{\sigma-1}\left(\varphi^*_{FF}\right)^\theta - 1} \\
&= \frac{f^d}{f^e}\frac{\sigma-1}{\theta-(\sigma-1)}\left(\varphi^*_{FF}\right)^{-\theta}
\end{aligned}
$$

or

$$\varphi^*_{FF} = \left(\frac{\theta-(\sigma-1)}{\sigma-1}\frac{f^e}{f^d}\tilde{\lambda}_{FF}\right)^{-\frac{1}{\theta}}$$

Then,

$$
\begin{aligned}
A &= Y_F P_F^{\sigma-1} \\
&= \sigma f^d \left(\rho \varphi^*_{FF}\right)^{1-\sigma} \\
&= \sigma f^d \rho^{1-\sigma} \left(\frac{\theta-(\sigma-1)}{\sigma-1}\frac{f^e}{f^d}\tilde{\lambda}_{FF}\right)^{\frac{\sigma-1}{\theta}}
\end{aligned}
$$

Hence, in the limiting case $\tilde{\lambda}_{FF} \to 1$, A is a constant

$$A = \sigma f^d \rho^{1-\sigma} \left(\frac{\theta-(\sigma-1)}{\sigma-1}\frac{f^e}{f^d}\right)^{\frac{\sigma-1}{\theta}}$$

- The value of Home's exports can be rewritten as

$$
\begin{aligned}
& N_H \alpha^{\rho} \left(\varphi^*_{FF}\right)^{-\rho} \frac{\theta \left(\sigma f^d\right)^{\frac{1}{\sigma}} \rho^{-\rho}}{\theta-(\sigma-1)} \left(\frac{\varphi^*_{HH}}{\varphi^*_{HF}}\right)^{\theta} \left(\varphi^*_{HF}\right)^{\sigma-1} \\
= & \frac{\theta}{\theta-(\sigma-1)} N_H \alpha^{\rho} \left(\sigma f^d\right)^{\frac{1}{\sigma}} \rho^{-\rho} \left(\frac{f^e}{f^d}\frac{\theta-(\sigma-1)}{\sigma-1} - \frac{f^x}{f^d}\left(\varphi^*_{FH}\right)^{-\theta}\right)^{\frac{\rho}{\theta}} \left(\frac{\varphi^*_{HH}}{\varphi^*_{HF}}\right)^{\theta} \left(\varphi^*_{HF}\right)^{\sigma-1}
\end{aligned}
$$

- The welfare maximization problem reads

$$
\max_{\varphi^*_{HH},\varphi^*_{HF},\varphi^*_{FH},N_H,\alpha,\phi} \frac{\theta}{\theta-(\sigma-1)} \left\{ N_H \phi^{\rho} \left(\varphi^*_{HH}\right)^{\sigma-1} + (\sigma-1)^{\rho} \frac{\rho L_F}{\theta f^e} \left(\frac{f^x}{\tau_{FH}}\right)^{\rho} \left(\varphi^*_{FH}\right)^{-(\theta-\rho)} \right\} \text{ s.t.}
$$

$$
\begin{aligned}
N_H \left[f^d + \frac{\theta}{\theta-(\sigma-1)} \phi \left(\varphi^*_{HH}\right)^{\sigma-1} + \left(\frac{\varphi^*_{HH}}{\varphi^*_{HF}}\right)^{\theta} \left(f^x + \frac{\theta}{\theta-(\sigma-1)} \alpha \left(\varphi^*_{HF}\right)^{\sigma-1} \right) + f^e \left(\varphi^*_{HH}\right)^{\theta} \right] = \\
A^{\frac{1}{\sigma}} \left(\frac{\varphi^*_{HH}}{\varphi^*_{HF}}\right)^{\theta} \left(\varphi^*_{HF}\right)^{\sigma-1} \frac{\theta}{\theta-(\sigma-1)} - \sigma f^x \frac{\rho}{\theta f^e} L_F \left(\varphi^*_{FH}\right)^{-\theta} \frac{\theta}{\theta-(\sigma-1)} = L_H
\end{aligned}
$$

where $A^{\frac{1}{\sigma}} \equiv \left(\sigma f^d\right)^{\frac{1}{\sigma}} \rho^{-\rho} N_H \alpha^{\rho} \left(\frac{f^e}{f^d}\frac{\theta-(\sigma-1)}{\sigma-1} - \frac{f^x}{f^d}\left(\varphi^*_{FH}\right)^{-\theta}\right)^{\frac{\rho}{\theta}}$.

- Compared to the small open economy case in which A is constant, the derivatives of the Lagrangian wrt N_H, φ^*_{HH}, φ^*_{HF}, φ^*_{FH}, α, and one of the Lagrange multipliers (balanced trade condition) are different.

- Let ψ^{LMC} and ψ^{BT} denote the Lagrange multipliers wrt labor market clearing and balanced trade, respectively

$$
\begin{aligned}
\mathcal{L} = & \frac{\theta}{\theta-(\sigma-1)} \left\{ N_H \phi^{\rho} \left(\varphi^*_{HH}\right)^{\sigma-1} + (\sigma-1)^{\rho} \frac{\rho L_F}{\theta f^e} \left(\frac{f^x}{\tau_{FH}}\right)^{\rho} \left(\varphi^*_{FH}\right)^{-(\theta-\rho)} \right\} \\
& -\psi^{LMC} \left[N_H \left(f^d + \frac{\theta}{\theta-(\sigma-1)} \phi \left(\varphi^*_{HH}\right)^{\sigma-1} + \left(\frac{\varphi^*_{HH}}{\varphi^*_{HF}}\right)^{\theta} f^x + \frac{\theta}{\theta-(\sigma-1)} \left(\frac{\varphi^*_{HH}}{\varphi^*_{HF}}\right)^{\theta} \alpha \left(\varphi^*_{HF}\right)^{\sigma-1} + f^e \left(\varphi^*_{HH}\right)^{\theta} \right) \right. \\
& +\psi^{BT} \left[\left(\sigma f^d\right)^{\frac{1}{\sigma}} \rho^{-\rho} N_H \alpha^{\rho} \left(\frac{f^e}{f^d}\frac{\theta-(\sigma-1)}{\sigma-1} - \frac{f^x}{f^d}\left(\varphi^*_{FH}\right)^{-\theta}\right)^{\frac{\rho}{\theta}} \left(\frac{\varphi^*_{HH}}{\varphi^*_{HF}}\right)^{\theta} \left(\varphi^*_{HF}\right)^{\sigma-1} \frac{\theta}{\theta-(\sigma-1)} \right. \\
& \left. -\sigma f^x \frac{\rho}{\theta f^e} L_F \left(\varphi^*_{FH}\right)^{-\theta} \frac{\theta}{\theta-(\sigma-1)} \right]
\end{aligned}
$$

The set of first order conditions is given by

$$\frac{\partial\mathcal{L}}{\partial\varphi^*_{HH}}=\frac{\theta\left(\sigma-1\right)}{\theta-\left(\sigma-1\right)}N_H\phi^\rho\left(\varphi^*_{HH}\right)^{\sigma-1}\frac{1}{\varphi^*_{HH}}-\psi^{LMC}N_H\frac{\theta\left(\sigma-1\right)}{\theta-\left(\sigma-1\right)}\phi\left(\varphi^*_{HH}\right)^{\sigma-1}\frac{1}{\varphi^*_{HH}}$$
$$-\psi^{LMC}\frac{N_H\theta}{\varphi^*_{HH}}\left(\frac{\varphi^*_{HH}}{\varphi^*_{HF}}\right)^\theta f^x-\psi^{LMC}N_H\frac{\theta^2}{\theta-\left(\sigma-1\right)}\left(\frac{\varphi^*_{HH}}{\varphi^*_{HF}}\right)^\theta\frac{\alpha\left(\varphi^*_{HF}\right)^{\sigma-1}}{\varphi^*_{HH}}-\psi^{LMC}\theta N_Hf^e\frac{\left(\varphi^*_{HH}\right)^\theta}{\varphi^*_{HH}}$$
$$+\psi^{BT}\theta N_H\alpha^\rho A^{\frac{1}{\sigma}}\left(\frac{\varphi^*_{HH}}{\varphi^*_{HF}}\right)^\theta\left(\varphi^*_{HF}\right)^{\sigma-1}\frac{\theta}{\theta-\left(\sigma-1\right)}\frac{1}{\varphi^*_{HH}}=0$$

$$\begin{aligned}\frac{\partial\mathcal{L}}{\partial\varphi^*_{HF}} &= \theta\psi^{LMC}N_H\left(\frac{\varphi^*_{HH}}{\varphi^*_{HF}}\right)^\theta f^x\frac{1}{\varphi^*_{HF}}-\psi^{LMC}N_H\frac{\theta\left(\sigma-1-\theta\right)}{\theta-\left(\sigma-1\right)}\left(\frac{\varphi^*_{HH}}{\varphi^*_{HF}}\right)^\theta\alpha\left(\varphi^*_{HF}\right)^{\sigma-1}\frac{1}{\varphi^*_{HF}}\\ &+ \psi^{BT}\left(\sigma-1-\theta\right)N_H\alpha^\rho A^{\frac{1}{\sigma}}\left(\frac{\varphi^*_{HH}}{\varphi^*_{HF}}\right)^\theta\left(\varphi^*_{HF}\right)^{\sigma-1}\frac{\theta}{\theta-\left(\sigma-1\right)}\frac{1}{\varphi^*_{HF}}=0\end{aligned}$$

$$\frac{\partial\mathcal{L}}{\partial\varphi^*_{FH}}\varphi^*_{FH}=-\frac{\left(\theta-\rho\right)\theta}{\theta-\left(\sigma-1\right)}\left(\sigma-1\right)^\rho\frac{\rho L_F}{\theta f^e}\left(\frac{f^x}{\tau_{FH}}\right)^\rho\left(\varphi^*_{FH}\right)^{-\left(\theta-\rho\right)}$$
$$+\frac{\rho\theta}{\theta}\psi^{BT}N_H\alpha^\rho A^{\frac{1}{\sigma}}\left(\frac{\varphi^*_{HH}}{\varphi^*_{HF}}\right)^\theta\left(\varphi^*_{HF}\right)^{\sigma-1}\frac{\theta}{\theta-\left(\sigma-1\right)}\frac{\frac{f^x}{f^d}\left(\varphi^*_{FH}\right)^{-\theta}}{\frac{f^e}{f^d}\frac{\theta-(\sigma-1)}{\sigma-1}-\frac{f^x}{f^d}\left(\varphi^*_{FH}\right)^{-\theta}}$$
$$+\psi^{BT}\sigma f^x\frac{\rho}{\theta f^e}L_F\left(\varphi^*_{FH}\right)^{-\theta}\frac{\theta^2}{\theta-\left(\sigma-1\right)}=0$$

Using $\frac{f^x}{f^d}\left(\frac{\varphi^*_{FF}}{\varphi^*_{FH}}\right)^\theta=\frac{1-\tilde{\lambda}_{FF}}{\tilde{\lambda}_{FF}}$ We can rewrite $\frac{\partial\mathcal{L}}{\partial\varphi^*_{FH}}\varphi^*_{FH}$ as

$$\frac{\partial\mathcal{L}}{\partial\varphi^*_{FH}}\varphi^*_{FH}=-\frac{\left(\theta-\rho\right)\theta}{\theta-\left(\sigma-1\right)}\left(\sigma-1\right)^\rho\frac{\rho L_F}{\theta f^e}\left(\frac{f^x}{\tau_{FH}}\right)^\rho\left(\varphi^*_{FH}\right)^{-\left(\theta-\rho\right)}$$
$$+\rho\psi^{BT}N_H\alpha^\rho A^{\frac{1}{\sigma}}\left(\frac{\varphi^*_{HH}}{\varphi^*_{HF}}\right)^\theta\left(\varphi^*_{HF}\right)^{\sigma-1}\frac{\theta}{\theta-\left(\sigma-1\right)}\frac{1-\tilde{\lambda}_{FF}}{\tilde{\lambda}_{FF}}$$
$$+\psi^{BT}\sigma f^x\frac{\rho}{\theta f^e}L_F\left(\varphi^*_{FH}\right)^{-\theta}\frac{\theta^2}{\theta-\left(\sigma-1\right)}=0$$

$$\frac{\partial\mathcal{L}}{\partial N_H}=\frac{\theta}{\theta-\left(\sigma-1\right)}\phi^\rho\left(\varphi^*_{HH}\right)^{\sigma-1}$$
$$-\psi^{LMC}\left(f^d+\frac{\theta}{\theta-\left(\sigma-1\right)}\phi\left(\varphi^*_{HH}\right)^{\sigma-1}+\left(\frac{\varphi^*_{HH}}{\varphi^*_{HF}}\right)^\theta f^x+\frac{\theta}{\theta-\left(\sigma-1\right)}\left(\frac{\varphi^*_{HH}}{\varphi^*_{HF}}\right)^\theta\alpha\left(\varphi^*_{HF}\right)^{\sigma-1}+f^e\left(\varphi^*_{HH}\right)^\theta\right)$$
$$+\psi^{BT}\alpha^\rho A^{\frac{1}{\sigma}}\left(\frac{\varphi^*_{HH}}{\varphi^*_{HF}}\right)^\theta\left(\varphi^*_{HF}\right)^{\sigma-1}\frac{\theta}{\theta-\left(\sigma-1\right)}$$

$$\frac{\partial\mathcal{L}}{\partial\alpha}=-\psi^{LMC}N_H\frac{\left(\varphi^*_{HF}\right)^{\sigma-1}\theta}{\theta-\left(\sigma-1\right)}\left(\frac{\varphi^*_{HH}}{\varphi^*_{HF}}\right)^\theta+\psi^{BT}\rho N_H\alpha^\rho A^{\frac{1}{\sigma}}\left(\frac{\varphi^*_{HH}}{\varphi^*_{HF}}\right)^\theta\frac{\left(\varphi^*_{HF}\right)^{\sigma-1}\theta}{\theta-\left(\sigma-1\right)}\frac{1}{\alpha}=0$$

$$\frac{\partial \mathcal{L}}{\partial \phi} = \frac{\theta\rho}{\theta-(\sigma-1)} N_H \phi^\rho \left(\varphi^*_{HH}\right)^{\sigma-1} \frac{1}{\phi} - \psi^{LMC} N_H \frac{\theta}{\theta-(\sigma-1)} \left(\varphi^*_{HH}\right)^{\sigma-1} = 0$$

$$\frac{\partial \mathcal{L}}{\partial \psi^{LMC}} = N_H \left(f^d + \frac{\left(\varphi^*_{HH}\right)^{\sigma-1}\theta}{\theta-(\sigma-1)}\phi + \left(\frac{\varphi^*_{HH}}{\varphi^*_{HF}}\right)^\theta f^x + \frac{\left(\varphi^*_{HF}\right)^{\sigma-1}\theta}{\theta-(\sigma-1)} \left(\frac{\varphi^*_{HH}}{\varphi^*_{HF}}\right)^\theta \alpha + f^e \left(\varphi^*_{HH}\right)^\theta \right) - L_H = 0$$

$$\frac{\partial \mathcal{L}}{\partial \psi^{BT}} = N_H \alpha^\rho A^{\frac{1}{\sigma}} \left(\frac{\varphi^*_{HH}}{\varphi^*_{HF}}\right)^\theta \left(\varphi^*_{HF}\right)^{\sigma-1} \frac{\theta}{\theta-(\sigma-1)} - \sigma f^x \frac{\rho}{\theta f^e} L_F \left(\varphi^*_{FH}\right)^{-\theta} \frac{\theta}{\theta-(\sigma-1)} = 0$$

- Rearranging terms, we obtain (in reverse order)

$$N_H \alpha^\rho A^{\frac{1}{\sigma}} \left(\frac{\varphi^*_{HH}}{\varphi^*_{HF}}\right)^\theta \left(\varphi^*_{HF}\right)^{\sigma-1} = \sigma f^x \frac{\rho}{\theta f^e} L_F \left(\varphi^*_{FH}\right)^{-\theta}$$

$$N_H \left(f^d + \frac{\theta}{\theta-(\sigma-1)}\phi \left(\varphi^*_{HH}\right)^{\sigma-1} + \left(\frac{\varphi^*_{HH}}{\varphi^*_{HF}}\right)^\theta f^x + \frac{\theta}{\theta-(\sigma-1)} \left(\frac{\varphi^*_{HH}}{\varphi^*_{HF}}\right)^\theta \alpha \left(\varphi^*_{HF}\right)^{\sigma-1} + f^e \left(\varphi^*_{HH}\right)^\theta \right) = L_H$$

$$\rho\phi^{\rho-1} = \psi^{LMC}$$

$$\psi^{LMC} = \psi^{BT} \rho \alpha^{\rho-1} A^{\frac{1}{\sigma}}$$

$$\frac{\theta}{\theta-(\sigma-1)} \phi^\rho \left(\varphi^*_{HH}\right)^{\sigma-1} = \psi^{LMC} \frac{L_H}{N_H} - \psi^{BT} \alpha^\rho A^{\frac{1}{\sigma}} \left(\frac{\varphi^*_{HH}}{\varphi^*_{HF}}\right)^\theta \left(\varphi^*_{HF}\right)^{\sigma-1} \frac{\theta}{\theta-(\sigma-1)}$$

$$\begin{aligned} & (\theta-\rho)(\sigma-1)^\rho \frac{\rho L_F}{\theta f^e} \left(\frac{f^x}{\tau_{FH}}\right)^\rho \left(\varphi^*_{FH}\right)^{-(\theta-\rho)} \\ = \; & \rho \psi^{BT} N_H \alpha^\rho A^{\frac{1}{\sigma}} \left(\frac{\varphi^*_{HH}}{\varphi^*_{HF}}\right)^\theta \left(\varphi^*_{HF}\right)^{\sigma-1} \frac{1-\tilde{\lambda}_{FF}}{\tilde{\lambda}_{FF}} + \theta \psi^{BT} \sigma f^x \frac{\rho}{\theta f^e} L_F \left(\varphi^*_{FH}\right)^{-\theta} \end{aligned}$$

$$\theta \psi^{LMC} N_H \left(\frac{\varphi^*_{HH}}{\varphi^*_{HF}}\right)^\theta f^x - \psi^{LMC} N_H \frac{\theta(\sigma-1-\theta)}{\theta-(\sigma-1)} \left(\frac{\varphi^*_{HH}}{\varphi^*_{HF}}\right)^\theta \alpha \left(\varphi^*_{HF}\right)^{\sigma-1}$$

$$= \psi^{BT} (\theta-(\sigma-1)) N_H \alpha^\rho A^{\frac{1}{\sigma}} \left(\frac{\varphi^*_{HH}}{\varphi^*_{HF}}\right)^\theta \left(\varphi^*_{HF}\right)^{\sigma-1} \frac{\theta}{\theta-(\sigma-1)}$$

$$\theta \psi^{LMC} f^x - \psi^{LMC} \frac{\theta(\sigma-1-\theta)}{\theta-(\sigma-1)} \alpha \left(\varphi^*_{HF}\right)^{\sigma-1} = \frac{\psi^{LMC}}{\rho \alpha^{\rho-1} A^{\frac{1}{\sigma}}} (\theta-(\sigma-1)) \alpha^\rho A^{\frac{1}{\sigma}} \left(\varphi^*_{HF}\right)^{\sigma-1} \frac{\theta}{\theta-(\sigma-1)}$$

$$f^x + \alpha \left(\varphi^*_{HF}\right)^{\sigma-1} = \frac{1}{\rho} \alpha \left(\varphi^*_{HF}\right)^{\sigma-1}$$

$$\alpha \left(\varphi^*_{HF}\right)^{\sigma-1} = (\sigma-1) f^x$$

entry cutoff

$$\frac{\theta\left(\sigma-1\right)}{\theta-\left(\sigma-1\right)}\phi^{\rho}\left(\varphi_{HH}^{*}\right)^{\sigma-1} \tag{35}$$

$$=\psi^{LMC}\frac{\theta\left(\sigma-1\right)}{\theta-\left(\sigma-1\right)}\phi\left(\varphi_{HH}^{*}\right)^{\sigma-1}+\psi^{LMC}\theta\left(\left(\frac{\varphi_{HH}^{*}}{\varphi_{HF}^{*}}\right)^{\theta}f^{x}+\frac{\theta}{\theta-\left(\sigma-1\right)}\left(\frac{\varphi_{HH}^{*}}{\varphi_{HF}^{*}}\right)^{\theta}\alpha\left(\varphi_{HF}^{*}\right)^{\sigma-1}+f^{e}\left(\varphi_{HH}^{*}\right)^{\theta}\right) \tag{36}$$

$$-\psi^{BT}\theta\alpha^{\rho}A^{\frac{1}{\sigma}}\left(\frac{\varphi_{HH}^{*}}{\varphi_{HF}^{*}}\right)^{\theta}\left(\varphi_{HF}^{*}\right)^{\sigma-1}\frac{\theta}{\theta-\left(\sigma-1\right)}$$

$$\Leftrightarrow\frac{\theta\left(\sigma-1\right)}{\theta-\left(\sigma-1\right)}\phi^{\rho}\left(\varphi_{HH}^{*}\right)^{\sigma-1}=$$

$$\psi^{LMC}\frac{\theta\left(\sigma-1\right)}{\theta-\left(\sigma-1\right)}\phi\left(\varphi_{HH}^{*}\right)^{\sigma-1}+\psi^{LMC}\theta\left(\left(\frac{\varphi_{HH}^{*}}{\varphi_{HF}^{*}}\right)^{\theta}f^{x}+\frac{\theta}{\theta-\left(\sigma-1\right)}\left(\frac{\varphi_{HH}^{*}}{\varphi_{HF}^{*}}\right)^{\theta}\alpha\left(\varphi_{HF}^{*}\right)^{\sigma-1}+f^{e}\left(\varphi_{HH}^{*}\right)^{\theta}\right) \tag{37}$$

$$+\theta\left(\frac{\theta}{\theta-\left(\sigma-1\right)}\phi^{\rho}\left(\varphi_{HH}^{*}\right)^{\sigma-1}-\psi^{LMC}\frac{L_{H}}{N_{H}}\right)$$

$$\Leftrightarrow\frac{\theta\left(\sigma-1-\theta\right)}{\theta-\left(\sigma-1\right)}\phi^{\rho}\left(\varphi_{HH}^{*}\right)^{\sigma-1}$$

$$=\psi^{LMC}\frac{\theta\left(\sigma-1\right)}{\theta-\left(\sigma-1\right)}\phi\left(\varphi_{HH}^{*}\right)^{\sigma-1}+\psi^{LMC}\theta\left(\left(\frac{\varphi_{HH}^{*}}{\varphi_{HF}^{*}}\right)^{\theta}f^{x}+\frac{\alpha\left(\varphi_{HF}^{*}\right)^{\sigma-1}\theta}{\theta-\left(\sigma-1\right)}\left(\frac{\varphi_{HH}^{*}}{\varphi_{HF}^{*}}\right)^{\theta}+f^{e}\left(\varphi_{HH}^{*}\right)^{\theta}\right)-\theta\psi^{LMC}\frac{L_{H}}{N_{H}} \tag{38}$$

$$-\phi^{\rho}\left(\varphi_{HH}^{*}\right)^{\sigma-1}=\psi^{LMC}\frac{\left(\sigma-1\right)\phi\left(\varphi_{HH}^{*}\right)^{\sigma-1}}{\theta-\left(\sigma-1\right)}+\psi^{LMC}\left(\left(\frac{\varphi_{HH}^{*}}{\varphi_{HF}^{*}}\right)^{\theta}f^{x}+\frac{\alpha\left(\varphi_{HF}^{*}\right)^{\sigma-1}\theta}{\theta-\left(\sigma-1\right)}\left(\frac{\varphi_{HH}^{*}}{\varphi_{HF}^{*}}\right)^{\theta}+f^{e}\left(\varphi_{HH}^{*}\right)^{\theta}\right)-$$

$$-\frac{\phi^{\rho}\left(\varphi_{HH}^{*}\right)^{\sigma-1}}{\psi^{LMC}}=\frac{\left(\sigma-1\right)\phi\left(\varphi_{HH}^{*}\right)^{\sigma-1}}{\theta-\left(\sigma-1\right)}+\left(\frac{\varphi_{HH}^{*}}{\varphi_{HF}^{*}}\right)^{\theta}f^{x}+\frac{\theta\alpha\left(\varphi_{HF}^{*}\right)^{\sigma-1}}{\theta-\left(\sigma-1\right)}\left(\frac{\varphi_{HH}^{*}}{\varphi_{HF}^{*}}\right)^{\theta}+f^{e}\left(\varphi_{HH}^{*}\right)^{\theta}$$

$$-\left(f^{d}+\frac{\theta\phi\left(\varphi_{HH}^{*}\right)^{\sigma-1}}{\theta-\left(\sigma-1\right)}+\left(\frac{\varphi_{HH}^{*}}{\varphi_{HF}^{*}}\right)^{\theta}f^{x}+\frac{\theta\alpha\left(\varphi_{HF}^{*}\right)^{\sigma-1}}{\theta-\left(\sigma-1\right)}\left(\frac{\varphi_{HH}^{*}}{\varphi_{HF}^{*}}\right)^{\theta}+f^{e}\left(\varphi_{HH}^{*}\right)^{\theta}\right)$$

$$-\frac{\phi^{\rho}\left(\varphi_{HH}^{*}\right)^{\sigma-1}}{\psi^{LMC}}=\frac{\sigma-1}{\theta-\left(\sigma-1\right)}\phi\left(\varphi_{HH}^{*}\right)^{\sigma-1}-f^{d}-\frac{\theta}{\theta-\left(\sigma-1\right)}\phi\left(\varphi_{HH}^{*}\right)^{\sigma-1}$$

$$\frac{\phi\left(\varphi_{HH}^{*}\right)^{\sigma-1}}{\rho}=\phi\left(\varphi_{HH}^{*}\right)^{\sigma-1}+f^{d}$$

$$\phi\left(\varphi_{HH}^{*}\right)^{\sigma-1}=\left(\sigma-1\right)f^{d}$$

- Hence, the system of FOCs is given by (again in reverse order)

$$\phi \left(\varphi_{HH}^{*}\right)^{\sigma-1} = (\sigma-1) f^{d} \tag{39}$$

$$\alpha \left(\varphi_{HF}^{*}\right)^{\sigma-1} = (\sigma-1) f^{x} \tag{40}$$

$$(\theta-\rho)(\sigma-1)^{\rho} \frac{\rho L_F}{\theta f^{e}} \left(\frac{f^{x}}{\tau_{FH}}\right)^{\rho} \left(\varphi_{FH}^{*}\right)^{-(\theta-\rho)} = \rho \psi^{BT} N_H \alpha^{\rho} A^{\frac{1}{\sigma}} \left(\frac{\varphi_{HH}^{*}}{\varphi_{HF}^{*}}\right)^{\theta} \left(\varphi_{HF}^{*}\right)^{\sigma-1} \frac{1-\tilde{\lambda}_{FF}}{\tilde{\lambda}_{FF}} + \theta \psi^{BT} \sigma f^{x} \frac{\rho}{\theta f^{e}} L_F (\tag{41}$$

$$f^{d} \left(\varphi_{HH}^{*}\right)^{-\theta} + f^{x} \left(\varphi_{HF}^{*}\right)^{-\theta} = f^{e} \frac{\theta-(\sigma-1)}{\sigma-1} \tag{42}$$

$$\psi^{LMC} = \psi^{BT} \rho \alpha^{\rho-1} A^{\frac{1}{\sigma}} \tag{43}$$

$$\rho \phi^{\rho-1} = \psi^{LMC} \tag{44}$$

$$N_H = \frac{\rho}{\theta f^{e}} L_H \left(\varphi_{HH}^{*}\right)^{-\theta} \tag{45}$$

$$N_H \left(\frac{\varphi_{HH}^{*}}{\varphi_{HF}^{*}}\right)^{\theta} \alpha^{\rho} A^{\frac{1}{\sigma}} \left(\varphi_{HF}^{*}\right)^{\sigma-1} = \sigma f^{x} \frac{\rho}{\theta f^{e}} L_F \left(\varphi_{FH}^{*}\right)^{-\theta} \tag{46}$$

where the forth line (FE) follows from

$$N_H \frac{\theta}{\theta-(\sigma-1)} \phi^{\rho} \left(\varphi_{HH}^{*}\right)^{\sigma-1} = \psi^{LMC} L_H + N_H \psi^{BT} \alpha^{\rho} A^{\frac{1}{\sigma}} \left(\varphi_{HF}^{*}\right)^{\sigma-1} \left(\frac{\varphi_{HH}^{*}}{\varphi_{HF}^{*}}\right)^{\theta} \left(\varphi_{FH}^{*}\right)^{\theta}$$

$$N_H \left(\frac{\theta}{\theta-(\sigma-1)} \phi^{\rho} \left(\varphi_{HH}^{*}\right)^{\sigma-1} - \psi^{BT} \alpha^{\rho} A^{\frac{1}{\sigma}} \left(\varphi_{HF}^{*}\right)^{\sigma-1} \left(\frac{\varphi_{HH}^{*}}{\varphi_{HF}^{*}}\right)^{\theta} \left(\varphi_{FH}^{*}\right)^{\theta}\right) = \psi^{LMC} L_H$$

$$\frac{\rho}{\theta f^{e}} L_H \left(\varphi_{HH}^{*}\right)^{-\theta} \left(\frac{\theta}{\theta-(\sigma-1)} \phi^{\rho} \left(\varphi_{HH}^{*}\right)^{\sigma-1} - \psi^{BT} \alpha^{\rho} A^{\frac{1}{\sigma}} \left(\varphi_{HF}^{*}\right)^{\sigma-1} \left(\frac{\varphi_{HH}^{*}}{\varphi_{HF}^{*}}\right)^{\theta} \left(\varphi_{FH}^{*}\right)^{\theta}\right) = \psi^{LMC} L_H$$

$$\frac{\rho}{\theta f^{e}} \left(\varphi_{HH}^{*}\right)^{-\theta} \left(\frac{\theta}{\theta-(\sigma-1)} \phi^{\rho} \left(\varphi_{HH}^{*}\right)^{\sigma-1} - \psi^{BT} \alpha^{\rho} A^{\frac{1}{\sigma}} \left(\varphi_{HF}^{*}\right)^{\sigma-1} \left(\frac{\varphi_{HH}^{*}}{\varphi_{HF}^{*}}\right)^{\theta} + \frac{\psi^{LMC}}{\psi^{BT} \rho \alpha^{\rho-1} A^{\frac{1}{\sigma}}} \frac{\theta}{\theta-(\sigma-1)}\right) = \psi^{LM}$$

$$\frac{\theta}{\theta-(\sigma-1)} \frac{\rho}{\theta f^{e}} \left(\varphi_{HH}^{*}\right)^{-\theta} \left(\phi^{\rho} \left(\varphi_{HH}^{*}\right)^{\sigma-1} + \alpha \left(\varphi_{HF}^{*}\right)^{\sigma-1} \frac{\psi^{LMC}}{\rho} \left(\frac{\varphi_{HH}^{*}}{\varphi_{HF}^{*}}\right)^{\theta}\right) = \psi^{LMC}$$

$$\frac{\theta}{\theta-(\sigma-1)} \frac{\rho}{\theta f^{e}} \left(\varphi_{HH}^{*}\right)^{-\theta} \left(\phi^{\rho-1} (\sigma-1) f^{d} + (\sigma-1) f^{x} \frac{\psi^{LMC}}{\rho} \left(\frac{\varphi_{HH}^{*}}{\varphi_{HF}^{*}}\right)^{\theta}\right) = \psi^{LMC}$$

$$\frac{\theta}{\theta-(\sigma-1)} \frac{\rho}{\theta f^{e}} \left(\varphi_{HH}^{*}\right)^{-\theta} \left(\frac{\psi^{LMC}}{\rho} (\sigma-1) f^{d} + (\sigma-1) f^{x} \frac{\psi^{LMC}}{\rho} \left(\frac{\varphi_{HH}^{*}}{\varphi_{HF}^{*}}\right)^{\theta}\right) = \psi^{LMC}$$

$$\frac{\sigma-1}{\theta-(\sigma-1)} \left(\varphi_{HH}^{*}\right)^{-\theta} \left(f^{d} + f^{x} \left(\frac{\varphi_{HH}^{*}}{\varphi_{HF}^{*}}\right)^{\theta}\right) = f^{e}$$

- Check whether market equilibrium coincides with market equilibrium for

$$t_{HF} = 1 + \frac{1}{\left(\frac{\theta}{\rho}-1\right) \tilde{\lambda}_{FF}}$$

Clearly, FE and LMC coindice. Let

$$\begin{aligned}\phi &= Y_H P_H^{\sigma-1}\left(\frac{\rho}{w_H}\right)^{\sigma}\\ \alpha &= Y_F P_F^{\sigma-1}\left(\frac{\rho}{w_H}\right)^{\sigma}\end{aligned}$$

Then, the domestic entry and the export cutoff conincide. Moreover, balanced trade coincides as

$$\begin{aligned}N_H\left(\frac{\varphi_{HH}^*}{\varphi_{HF}^*}\right)^{\theta}\alpha^{\rho}A^{\frac{1}{\sigma}}\frac{(\sigma-1)f^x}{\alpha} &= \sigma f^x\frac{\rho}{\theta f^e}L_F\left(\varphi_{FH}^*\right)^{-\theta}\\ \frac{\rho}{\theta f^e}L_H\left(\varphi_{HH}^*\right)^{-\theta}\left(\frac{\varphi_{HH}^*}{\varphi_{HF}^*}\right)^{\theta}\alpha^{-\frac{1}{\sigma}}A^{\frac{1}{\sigma}}(\sigma-1)f^x &= \sigma f^x\frac{\rho}{\theta f^e}L_F\left(\varphi_{FH}^*\right)^{-\theta}\\ \frac{\rho}{\theta f^e}L_H\left(\varphi_{HH}^*\right)^{-\theta}\left(\frac{\varphi_{HH}^*}{\varphi_{HF}^*}\right)^{\theta}\left(Y_F P_F^{\sigma-1}\left(\frac{\rho}{w_H}\right)^{\sigma}\right)^{-\frac{1}{\sigma}}\left(Y_F P_F^{\sigma-1}\right)^{\frac{1}{\sigma}}(\sigma-1)f^x &= \sigma f^x\frac{\rho}{\theta f^e}L_F\left(\varphi_{FH}^*\right)^{-\theta}\\ L_H\left(\varphi_{HF}^*\right)^{-\theta}\left(\frac{\sigma-1}{\rho}\right)w_H &= \sigma L_F\left(\varphi_{FH}^*\right)^{-\theta}\\ L_H\left(\varphi_{HF}^*\right)^{-\theta}w_H &= L_F\left(\varphi_{FH}^*\right)^{-\theta}\end{aligned}$$

Note that

$$\begin{aligned}\psi^{LMC} &= \rho\phi^{\rho-1}\\ &= \rho\left(Y_H P_H^{\sigma-1}\left(\frac{\rho}{w_H}\right)^{\sigma}\right)^{\rho-1}\\ &= Y_H^{-\frac{1}{\sigma}}P_H^{-\rho}w_H\end{aligned}$$

and

$$\begin{aligned}\psi^{BT} &= \frac{\psi^{LMC}}{\rho\alpha^{\rho-1}A^{\frac{1}{\sigma}}}\\ &= \frac{Y_H^{-\frac{1}{\sigma}}P_H^{-\rho}w_H}{\rho\left(Y_F P_F^{\sigma-1}\left(\frac{\rho}{w_H}\right)^{\sigma}\right)^{\rho-1}\left(Y_F P_F^{\sigma-1}\right)^{\frac{1}{\sigma}}}\\ &= \frac{Y_H^{-\frac{1}{\sigma}}P_H^{-\rho}}{\left(Y_F P_F^{\sigma-1}\right)^{-\frac{1}{\sigma}}\left(Y_F P_F^{\sigma-1}\right)^{\frac{1}{\sigma}}}\\ &= Y_H^{-\frac{1}{\sigma}}P_H^{-\rho}\end{aligned}$$

Finally, compare import cutoff under social optimum equilibrium

$$
\begin{aligned}
(\theta-\rho)(\sigma-1)^{\rho}\frac{\rho L_F}{\theta f^e}\left(\frac{f^x}{\tau_{FH}}\right)^{\rho}\left(\varphi^*_{FH}\right)^{-(\theta-\rho)} &= \rho\psi^{BT}\sigma f^x\frac{\rho}{\theta f^e}L_F\left(\varphi^*_{FH}\right)^{-\theta}\frac{1-\tilde{\lambda}_{FF}}{\tilde{\lambda}_{FF}}+\theta\psi^{BT}\sigma f^x\frac{\rho}{\theta f^e}L_F\left(\varphi^*_{FH}\right)^{-} \\
(\theta-\rho)(\sigma-1)^{\rho}\frac{\rho L_F}{\theta f^e}\left(\frac{f^x}{\tau_{FH}}\right)^{\rho}\left(\varphi^*_{FH}\right)^{-(\theta-\rho)} &= \left(\frac{\rho}{\theta}\frac{1-\tilde{\lambda}_{FF}}{\tilde{\lambda}_{FF}}+1\right)\theta\frac{\rho}{\theta f^e}L_F\left(\varphi^*_{FH}\right)^{-\theta}\psi^{BT}\sigma f^x \\
(\theta-\rho)(\sigma-1)^{\rho}\left(\frac{f^x}{\tau_{FH}}\right)^{\rho}\left(\varphi^*_{FH}\right)^{\rho} &= \left(\frac{\rho}{\theta}\frac{1-\tilde{\lambda}_{FF}}{\tilde{\lambda}_{FF}}+1\right)\theta\psi^{BT}\sigma f^x \\
\left(\varphi^*_{FH}\right)^{\rho} &= \frac{\theta\left(\frac{\rho}{\theta}\frac{1-\tilde{\lambda}_{FF}}{\tilde{\lambda}_{FF}}+1\right)Y_H^{-\frac{1}{\sigma}}P_H^{-\rho}\sigma f^x}{(\theta-\rho)(\sigma-1)^{\rho}\left(\frac{f^x}{\tau_{FH}}\right)^{\rho}}
\end{aligned}
$$

to import cutoff under market equilibrium with optimal tariff

$$
\begin{aligned}
Y_H P_H^{\sigma-1}t_{FH}^{-\sigma}\left(\frac{\rho\varphi^*_{FH}}{\tau_{HF}}\right)^{\sigma-1} &= \sigma f^x \Leftrightarrow \\
\left(\varphi^*_{FH}\right)^{\sigma-1} &= Y_H^{-1}P_H^{1-\sigma}t_{FH}^{\sigma}\sigma f^x\tau_{HF}^{\sigma-1}\rho^{1-\sigma} \Leftrightarrow \\
\left(\varphi^*_{FH}\right)^{\rho} &= Y_H^{-\frac{1}{\sigma}}P_H^{-\rho}t_{FH}(\sigma)^{\frac{1}{\sigma}}\left(f^x\right)^{\frac{\rho}{\sigma-1}}\tau_{HF}^{\rho}\rho^{-\rho}
\end{aligned}
$$

Check

$$
\begin{aligned}
Y_H^{-\frac{1}{\sigma}}P_H^{-\rho}\sigma\frac{\theta}{\theta-\rho}(\sigma-1)^{-\rho}\left(\frac{f^x}{\tau_{FH}}\right)^{1-\rho}\left(1+\frac{\rho}{\theta}\frac{1-\tilde{\lambda}_{FF}}{\tilde{\lambda}_{FF}}\right) &= Y_H^{-\frac{1}{\sigma}}P_H^{-\rho}\left(1+\frac{1}{\left(\frac{\theta}{\rho}-1\right)\tilde{\lambda}_{FF}}\right)(\sigma)^{\frac{1}{\sigma}}\left(f^x\right)^{\frac{\rho}{\sigma-1}}\tau_{HF}^{\rho} \\
\sigma\frac{\theta}{\theta-\rho}(\sigma-1)^{-\rho}\left(1+\frac{\rho}{\theta}\frac{1-\tilde{\lambda}_{FF}}{\tilde{\lambda}_{FF}}\right) &= \left(1+\frac{1}{\left(\frac{\theta}{\rho}-1\right)\tilde{\lambda}_{FF}}\right)(\sigma)^{\frac{1}{\sigma}}\left(\frac{\sigma-1}{\sigma}\right)^{-\rho} \\
\sigma\frac{\theta}{\theta-\rho}\left(1+\frac{\rho}{\theta}\frac{1-\tilde{\lambda}_{FF}}{\tilde{\lambda}_{FF}}\right) &= \left(1+\frac{1}{\left(\frac{\theta}{\rho}-1\right)\tilde{\lambda}_{FF}}\right)(\sigma)^{\frac{\sigma-1}{\sigma}+\frac{1}{\sigma}} \\
\frac{\theta}{\theta-\rho}+\frac{\rho}{\theta-\rho}\frac{1-\tilde{\lambda}_{FF}}{\tilde{\lambda}_{FF}} &= 1+\frac{\rho}{\theta-\rho}\frac{1}{\tilde{\lambda}_{FF}} \\
\frac{\rho}{\theta-\rho}\frac{1}{\tilde{\lambda}_{FF}}\left(1-\tilde{\lambda}_{FF}-1\right) &= 1-\frac{\theta}{\theta-\rho} \\
-\frac{\rho}{\theta-\rho} &= \frac{\theta-\rho-\theta}{\theta-\rho} \\
0 &= 0
\end{aligned}
$$

- Uniqueness of the planner solution. Following DRC, we reduce the system of equations to a single equation. First, we exclude ψ^{LMC} and ψ^{BT}

$$
\begin{aligned}
(\theta-\rho)(\sigma-1)^{\rho}\frac{\rho L_F}{\theta f^e}\left(\frac{f^x}{\tau_{FH}}\right)^{\rho}\left(\varphi_{FH}^{*}\right)^{-(\theta-\rho)} &= \rho\psi^{BT}\sigma f^x\frac{\rho}{\theta f^e}L_F\left(\varphi_{FH}^{*}\right)^{-\theta}\frac{1-\tilde{\lambda}_{FF}}{\tilde{\lambda}_{FF}}+\theta\psi^{BT}\sigma f^x\frac{\rho}{\theta f^e}L_F\left(\varphi_{FH}^{*}\right)^{-}\\
(\theta-\rho)(\sigma-1)^{\rho}\left(\frac{f^x}{\tau_{FH}}\right)^{\rho}\left(\varphi_{FH}^{*}\right)^{\rho}\left(\frac{\phi}{\alpha}\right)^{1-\rho} &= \frac{\rho}{A^{\frac{1}{\sigma}}}\sigma f^x\frac{1-\tilde{\lambda}_{FF}}{\tilde{\lambda}_{FF}}+\frac{\theta}{A^{\frac{1}{\sigma}}}\sigma f^x
\end{aligned}
$$

We use the domestic entry condition and the export cutoff condition to eliminate ϕ/α in import cutoff

$$
\left(\frac{\phi}{\alpha}\right)^{1-\rho}=\left(\frac{\varphi_{HF}^{*}}{\varphi_{HH}^{*}}\right)^{\rho}\left(\frac{f^x}{f^d}\right)^{1-\rho}
$$

$$
(\theta-\rho)(\sigma-1)^{\rho}\left(\frac{f^x}{\tau_{FH}}\right)^{\rho}\left(\varphi_{FH}^{*}\right)^{\rho}\left(\frac{\varphi_{HF}^{*}}{\varphi_{HH}^{*}}\right)^{\rho}\left(\frac{f^x}{f^d}\right)^{1-\rho}=\theta\frac{\sigma f^x}{A^{\frac{1}{\sigma}}}\left(\frac{\rho}{\theta}\frac{1-\tilde{\lambda}_{FF}}{\tilde{\lambda}_{FF}}+1\right),
$$

Hence,

$$
(\theta-\rho)(\sigma-1)^{\rho}\left(\frac{f^x}{\tau_{FH}}\right)^{\rho}\left(\varphi_{FH}^{*}\right)^{\rho}\left(\frac{f^x}{f^d}\right)^{1-\rho}=\theta\frac{\sigma f^x}{A^{\frac{1}{\sigma}}}\left(\frac{\varphi_{HH}^{*}}{\varphi_{HF}^{*}}\right)^{\rho}\left(\frac{\rho}{\theta}\frac{1-\tilde{\lambda}_{FF}}{\tilde{\lambda}_{FF}}+1\right)
$$

Balanced trade and labor market clearing and export cutoff

$$
\begin{aligned}
\left(\varphi_{HF}^{*}\right)^{\sigma-1-\theta} &= \frac{\sigma f^x L_F\left(\varphi_{FH}^{*}\right)^{-\theta}}{A^{\frac{1}{\sigma}}L_H\left(\frac{(\sigma-1)f^x}{\left(\varphi_{HF}^{*}\right)^{\sigma-1}}\right)^{\rho}}\\
\varphi_{HF}^{*} &= \left(\frac{\sigma f^x L_F\left(\varphi_{FH}^{*}\right)^{-\theta}}{A^{\frac{1}{\sigma}}L_H\left(\frac{(\sigma-1)f^x}{\left(\varphi_{HF}^{*}\right)^{\sigma-1}}\right)^{\rho}}\right)^{-\frac{1}{\theta-(\sigma-1)}}
\end{aligned}
$$

By free entry

$$
\varphi_{HH}^{*}=\left(\frac{f^e}{f^d}\frac{\theta-(\sigma-1)}{\sigma-1}-\frac{f^x}{f^d}\left(\varphi_{HF}^{*}\right)^{-\theta}\right)^{-\frac{1}{\theta}}
$$

Hence,

$$
\begin{aligned}
(\theta-\rho)(\sigma-1)^{\rho}\left(\frac{f^x}{\tau_{FH}}\right)^{\rho}\left(\varphi_{FH}^{*}\right)^{\rho}\left(\frac{f^x}{f^d}\right)^{1-\rho} &= \theta\frac{\sigma f^x}{A^{\frac{1}{\sigma}}}\left(\frac{f^e}{f^d}\frac{\theta-(\sigma-1)}{\sigma-1}-\frac{f^x}{f^d}\left(\varphi_{HF}^{*}\right)^{-\theta}\right)^{-\frac{\rho}{\theta}}\left(\varphi_{HF}^{*}\right)^{-\rho}\left(\frac{\rho}{\theta}\frac{1-\tilde{\lambda}_{FF}}{\tilde{\lambda}_{FF}}+\right.\\
&= \theta\frac{\sigma f^x}{A^{\frac{1}{\sigma}}}\left(\frac{f^e}{f^d}\frac{\theta-(\sigma-1)}{\sigma-1}\left(\varphi_{HF}^{*}\right)^{\theta}-\frac{f^x}{f^d}\right)^{-\frac{\rho}{\theta}}\left(\frac{\rho}{\theta}\frac{1-\tilde{\lambda}_{FF}}{\tilde{\lambda}_{FF}}+1\right)\\
&= \theta\frac{\sigma f^x}{A^{\frac{1}{\sigma}}}\left(\frac{f^e}{f^d}\frac{\theta-(\sigma-1)}{\sigma-1}\left(\frac{\sigma f^x L_F\left(\varphi_{FH}^{*}\right)^{-\theta}}{A^{\frac{1}{\sigma}}L_H\left(\frac{(\sigma-1)f^x}{\left(\varphi_{HF}^{*}\right)^{\sigma-1}}\right)^{\rho}}\right)^{-\frac{\theta}{\theta-(\sigma-1)}}-\frac{f^x}{f^d}\right)^{-\frac{\rho}{\theta}}\left(\frac{\rho}{\theta}\frac{1-\tilde{\lambda}_{FF}}{\tilde{\lambda}_{FF}}+1\right)
\end{aligned}
$$

Recall that

$$
A^{\frac{1}{\sigma}}=\left(\sigma f^d\right)^{\frac{1}{\sigma}}\rho^{-\rho}\left(\frac{f^e}{f^d}\frac{\theta-(\sigma-1)}{\sigma-1}-\frac{f^x}{f^d}\left(\varphi_{FH}^{*}\right)^{-\theta}\right)^{\frac{\rho}{\theta}}
$$

is increasing in φ^*_{FH}. Moreover, $\tilde{\lambda}_{FF}$ is increasing in φ^*_{FH} due to free entry and

$$\tilde{\lambda}_{FF} = \frac{f^d}{f^e}\frac{\sigma-1}{\theta-(\sigma-1)}\left(\varphi^*_{FF}\right)^{-\theta}$$

Moreover, $\frac{\partial \frac{1-\tilde{\lambda}_{FF}}{\tilde{\lambda}_{FF}}}{\partial \tilde{\lambda}_{FF}} = \frac{-\tilde{\lambda}_{FF}-\left(1-\tilde{\lambda}_{FF}\right)}{\tilde{\lambda}^2_{FF}} = -\frac{1}{\tilde{\lambda}^2_{FF}} < 0$. Then, the RHS is decreasing in φ^*_{FH}, while the LHS is increasing in φ^*_{FH}, and there is a unique solution.

C.3 Representation of social planner problem in "ideal quantity" space

The system of first-order conditions of the social planner problem (39)-(46) shows that labor market clearing and trade balance hold. We can therefore restate the social planner as maximizing GDP taking the effect on demand shifters into account subject to the resouce constraint. Following Feenstra (2010), the resouce constraint can be rewritten in terms of a transformation curve between the domestic ideal quantity $\tilde{M}_i$ and the export ideal quantity $\tilde{M}_{ix}$. Hence, the social planner problem becomes

$$\max_{\tilde{M}_i,\tilde{M}_{ix}} \tilde{M}_i + \frac{A_j}{A_i}\tilde{M}_{ix}.$$

Let $A \equiv \frac{A_j}{A_i}$. Then, the optimality condition is given by

$$-MRT = -A\frac{1+\frac{\partial A}{\tilde{M}_{ix}}\frac{\tilde{M}_{ix}}{A}}{1+\frac{\partial A}{\tilde{M}_i}\tilde{M}_{ix}},$$

where MRT is the marginal rate of transformation between

One can write A as a function of $\tilde{M}_{ix}$ only; then, $\frac{\partial A}{\tilde{M}_i} = 0$. One can also show that $\frac{\partial A}{\tilde{M}_{ix}}\frac{\tilde{M}_{ix}}{A} < 0$. Hence, in a diagram with $\tilde{M}_{ix}$ on the y-axis and $\tilde{M}_i$ on the x-axis, the social planner soluation implies a tangent that is steeper than under the market solution. The social planner restricts export quantity in order to improve the terms of trade.

A tariff can be used to manipulate A in the market solution. An increase in t_{ij} leads to a higher $A = A_j/A_i$. The optimal tariff is

$$t = \frac{1}{1+\frac{\partial A}{\tilde{M}_{ix}}\frac{\tilde{M}_{ix}}{A}}.$$

D Proof of corollary 2 (iceberg costs in the presence of tariffs)

D.1 Perfect competition

We prove in 3 steps that welfare is decreasing and convex in the domestic expenditure share

$$\frac{\partial^2 W_i}{\partial \lambda_{ii}^2} < 0.$$

Step 1: The change in the real wage is given by

$$\hat{w}_i - \hat{P}_i = \frac{1}{\varepsilon}\hat{\lambda}_{ii}.$$

This claim directly follows from noting that ACR's proof of proposition 1 (pp. 119-122) does not depend on changes in income. The change in welfare can therefore be written as

$$\hat{W}_i = \left(\frac{1}{\varepsilon} - \frac{\lambda_{ii}\left(t_{ij}-1\right)}{\lambda_{ii}\left(t_{ij}-1\right)+1}\right)\hat{\lambda}_{ii}$$

Step 2: Welfare is decreasing in the domestic expenditure share. The claim follows from rearranging terms and noting that $\varepsilon < 0$ and $t_{ij} \geq 1$

$$\frac{\partial W_i}{\partial \lambda_{ii}} = \frac{W_i}{\lambda_{ii}}\left(\frac{1}{\varepsilon} - \frac{\lambda_{ii}\left(t_{ij}-1\right)}{\lambda_{ii}\left(t_{ij}-1\right)+1}\right) < 0. \tag{47}$$

Step 3: The sign of $\partial^2 W_i/\partial\lambda_{ii}^2$ is given by

$$2\left[\lambda_{ii}\left(t_{ij}-1\right)\left(1-\varepsilon\right)+1\right]^2 - \varepsilon\left[\lambda_{ii}\left(t_{ij}-1\right)+1\right]\left(1+\lambda_{ii}\left(t_{ij}-1\right)\right) + \varepsilon^2\lambda_{ii}^2\left(t_{ij}-1\right)^2 > 0,$$

where the inequality follows from $\varepsilon < 0$.

Differentiating equation (47) with respect to λ_{ii}, we obtain

$$\frac{\partial^2 W_i}{\partial \lambda_{ii}^2} = \frac{W_i}{\lambda_{ii}^2}\left(\frac{\partial W_i}{\partial \lambda_{ii}}\frac{\lambda_{ii}}{W_i}\left(\frac{\partial W_i}{\partial \lambda_{ii}}\frac{\lambda_{ii}}{W_i} - 1\right) - \frac{\lambda_{ii}\left(t_{ij}-1\right)}{\left[\lambda_{ii}\left(t_{ij}-1\right)+1\right]^2}\right).$$

Substituting out $\frac{\partial W_i}{\partial \lambda_{ii}} \frac{\lambda_{ii}}{W_i} = \frac{\lambda_{ii}(t_{ij}-1)(1-\varepsilon_\tau)+1}{\varepsilon(\lambda_{ii}(t_{ij}-1)+1)} < 0$, the sign of $\partial^2 W_i / \partial \lambda_{ii}^2$ is given by

$$
\begin{aligned}
& \left[\lambda_{ii}\left(t_{ij}-1\right)\left(1-\varepsilon\right)+1\right]^2 - \varepsilon\left[\lambda_{ii}\left(t_{ij}-1\right)\left(1-\varepsilon\right)+1\right]\left(1+\lambda_{ii}\left(t_{ij}-1\right)\right) - \varepsilon^2 \lambda_{ii}\left(t_{ij}-1\right) \\
= & \left[\lambda_{ii}\left(t_{ij}-1\right)\left(1-\varepsilon\right)+1\right]^2 - \varepsilon\left[\lambda_{ii}\left(t_{ij}-1\right) - \varepsilon\lambda_{ii}\left(t_{ij}-1\right)+1\right]\left(1+\lambda_{ii}\left(t_{ij}-1\right)\right) - \varepsilon^2 \lambda_{ii}\left(t_{ij}-1\right) \\
= & \left[\lambda_{ii}\left(t_{ij}-1\right)\left(1-\varepsilon\right)+1\right]^2 - \varepsilon\left[\lambda_{ii}\left(t_{ij}-1\right)+1\right]\left(1+\lambda_{ii}\left(t_{ij}-1\right)\right) + \varepsilon^2 \lambda_{ii}^2\left(t_{ij}-1\right)^2 > 0.
\end{aligned}
$$

D.2 Monopolistic competition

The proof for models without export selection resembles the proof for models with perfect competition. For models with export selection, we proceed in 4 steps.

Step 1: The change in real income is given by

$$
\frac{\widehat{w_i}}{P_i} = \frac{1}{\varepsilon_\tau}\hat{\lambda}_{ii} + \frac{\varepsilon_\tau - (\sigma-1)}{\varepsilon_\tau(\sigma-1)}\left(\frac{\lambda_{ii}\left(t_{ij}-1\right)}{\lambda_{ii}\left(t_{ij}-1\right)+1}\hat{\lambda}_{ii}\right).
$$

This claim immediately follows from equation (34) with $\hat{t}_{ij} = 0$.

Step 2: The change in welfare can be written as

$$
\hat{W}_i = \left(\frac{1}{\varepsilon_\tau} - \frac{\varepsilon_t}{\varepsilon_\tau}\frac{\lambda_{ii}\left(t_{ij}-1\right)}{\lambda_{ii}\left(t_{ij}-1\right)+1}\right)\hat{\lambda}_{ii}.
$$

The claim follows from substituting out the change in the real wage from equation (21) with $\hat{t}_{ij} = 0$. As $\varepsilon_t/\varepsilon_\tau > 0$, this equation implies that, conditional on the change in the domestic expenditure share, welfare gains are underestimated if one abstracts from tariffs.

Step 3: Welfare is decreasing in the domestic expenditure share. The claim follows from rearranging terms and noting that $\varepsilon_\tau, \varepsilon_t < 0$ and $\mathrm{t}_{ij} \geq 1$

$$
\frac{\partial W_i}{\partial \lambda_{ii}} = \frac{W_i}{\lambda_{ii}}\left(\frac{1}{\varepsilon_\tau} - \frac{\varepsilon_t}{\varepsilon_\tau}\frac{\lambda_{ii}\left(t_{ij}-1\right)}{\lambda_{ii}\left(t_{ij}-1\right)+1}\right) < 0. \tag{48}
$$

Step 4: The sign of $\partial^2 W_i/\partial\lambda_{ii}^2$ is given by

$$\left[\lambda_{ii}\left(t_{ij}-1\right)\left(1-\varepsilon_t\right)+1\right]^2-\varepsilon_\tau\left[\lambda_{ii}\left(t_{ij}-1\right)+1\right]\left(1+\lambda_{ii}\left(t_{ij}-1\right)\right)+\varepsilon_\tau\varepsilon_t\lambda_{ii}^2\left(t_{ij}-1\right)^2>0,$$

where the inequality follows from $\varepsilon<0$.

Differentiating equation (47) with respect to λ_{ii}, we obtain

$$\frac{\partial^2 W_i}{\partial\lambda_{ii}^2}=\frac{W_i}{\lambda_{ii}^2}\left(\frac{\partial W_i}{\partial\lambda_{ii}}\frac{\lambda_{ii}}{W_i}\left(\frac{\partial W_i}{\partial\lambda_{ii}}\frac{\lambda_{ii}}{W_i}-1\right)-\frac{\varepsilon_t}{\varepsilon_\tau}\frac{\lambda_{ii}\left(t_{ij}-1\right)}{\left[\lambda_{ii}\left(t_{ij}-1\right)+1\right]^2}\right)$$

Substituting out $\frac{\partial W_i}{\partial\lambda_{ii}}\frac{\lambda_{ii}}{W_i}=\frac{\lambda_{ii}(t_{ij}-1)(1-\varepsilon_t)+1}{\varepsilon_\tau(\lambda_{ii}(t_{ij}-1)+1)}<0$, the sign of $\partial^2 W_i/\partial\lambda_{ii}^2$ is given by

$$\begin{aligned}
&\left[\lambda_{ii}\left(t_{ij}-1\right)\left(1-\varepsilon_t\right)+1\right]^2-\varepsilon_\tau\left[\lambda_{ii}\left(t_{ij}-1\right)\left(1-\varepsilon_t\right)+1\right]\left(1+\lambda_{ii}\left(t_{ij}-1\right)\right)-\varepsilon_\tau\varepsilon_t\lambda_{ii}\left(t_{ij}-1\right)\\
=\;&\left[\lambda_{ii}\left(t_{ij}-1\right)\left(1-\varepsilon_t\right)+1\right]^2-\varepsilon_\tau\left[\lambda_{ii}\left(t_{ij}-1\right)-\varepsilon_t\lambda_{ii}\left(t_{ij}-1\right)+1\right]\left(1+\lambda_{ii}\left(t_{ij}-1\right)\right)-\varepsilon_\tau\varepsilon_t\lambda_{ii}\left(t_{ij}-1\right)\\
=\;&\left[\lambda_{ii}\left(t_{ij}-1\right)\left(1-\varepsilon_t\right)+1\right]^2-\varepsilon_\tau\left[\lambda_{ii}\left(t_{ij}-1\right)+1\right]\left(1+\lambda_{ii}\left(t_{ij}-1\right)\right)+\varepsilon_\tau\varepsilon_t\lambda_{ii}^2\left(t_{ij}-1\right)^2>0.
\end{aligned}$$