

Matching frictions, unemployment dynamics and optimal monetary policy

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Abstract

Using a New Keynesian model with search and matching frictions calibrated to match key features of the U.S. economy, I show that attempts by the monetary authority to stabilize inflation can lead to a substantial increase in mean unemployment. This result stems from several nonlinearities embedded in the search and matching framework that were first identified by Hairault et al. (2010) and Jung and Kuester (2011). By deviating from price stability, the monetary authority can reduce labor market volatility and boost mean employment. I argue that this effect of volatility on mean unemployment, which has mainly been ignored in the literature, can to some extent change the terms of the tradeoff between inflation and unemployment stabilization. I carry out a quantitative analysis of the welfare costs of following a policy of price stability and show that they are non negligible. I also find that a simple rule featuring a strong response to both inflation and output performs reasonably well.

1 Introduction

How much weight should policymakers place on employment when conducting monetary policy? This question has received renewed interest recently as many economists have started worrying about the potentially inflationary effects of the unconventional monetary policies carried out with the objective of getting unemployment and output growth out of the slump. Theories that have been developed to study the tradeoff between stabilizing inflation and real activity often come with a straightforward answer to this question; a monetary authority should focus on maintaining price stability. For instance, in the basic New Keynesian model, there is no trade-off between stabilizing inflation as closing the output gap as the two objectives are mutually compatible. The result that price stability is optimal, or nearly optimal, carries out to models augmented with search and matching frictions in the labour market. In those models, a meaningful tradeoff between inflation and unemployment stabilization can be obtained as a policymaker can use inflation to correct for an inefficient level of labor market activity. However, as shown in Faia (2009) or Ravenna & Walsh (2012), quantitatively, the terms of the tradeoff are in favor of inflation stabilization and policies of price stability are close to optimal.

Using a Mortensen Pissarides model of the labour market, Hairault et al. (2010) and Jung and Kuester (2011) show that business cycles tend to increase mean unemployment. In expansions, the positive impact of an increase in the job finding probability on employment is *dampened* by the decrease in the size of the pool of job seekers. In recessions, the negative impact of the decrease in the job finding probability on employment is *amplified* by the increase in the size of the pool of job seekers. Thus employment losses in recessions outweigh employment gains in expansions. Moreover, depending on the underlying structure of the labour market, business cycles may tend to reduce the average job finding probability. These two elements, the size of the negative covariance between unemployment and the job-finding rate and the effect on the mean job finding probability, are thus key determinants of the unemployment losses induced by business cycles and will be crucial in the analysis carried out here

In a first part of the paper, I show that different monetary policies can lead to different outcomes in terms of mean unemployment. I build a New Keynesian model with labor market frictions and real wage rigidity. I calibrate the model so that it provides a sensible description of the reality of the U.S. economy. In particular I match the model's unconditional second moments to the second moments of hp-filtered U.S data from 1951Q1 to 2007Q1, conditional on monetary policy being conducted according to a Taylor rule. In this fictitious economy, the unemployment losses arising from business cycles are of 0.25 percentage points. Under a policy of price stability, which is often advocated as nearly optimal in the literature, the unemployment losses increase to attain 0.56 percentage points. I show that mean unemployment closely depends on the volatility of

labour market variables and that a strong response to inflation in response to shocks tends to exacerbate labor market volatility. This analysis thus suggests that the monetary authority could boost mean employment by deviating from price stability in response to shocks. In a second part of the paper, I investigate the extent to which the monetary authority has an incentive to do so, that is how the effect of business cycles on mean unemployment changes the terms of the tradeoff between inflation and unemployment stabilization. Unfortunately, as I am unable to obtain analytical solutions, I have to rely on a quantitative evaluation of welfare under alternative policies. Under my baseline calibration, the costs of following a policy of price stability amounts to 0,1% of the Ramsey consumption process. This figure, however small, is non-negligible and higher than what had previously been found in the literature (Faia 2008, 2009). I show that these welfare gains depend on several characteristics of the economy, such as the value of home production (or leisure). Finally, I investigate the performance of simple interest rate rules and find that rules featuring a strong response to both inflation and output performs reasonably well.

The analysis in this paper is related to some other recent studies. Several papers have focused on the design of optimal monetary policy in the presence of matching frictions. For several reasons, most of these papers are not able to account for the effect of business cycle fluctuations on mean unemployment . Faia (2008) finds that a small response to unemployment alongside a strong response to inflation can implement the optimal policy. However, in her model, the welfare gains from deviating from price stability are small regardless of whether the steady state is efficient. By using a “tax interpretation”, Ravenna and Walsh (2012) show that monetary policy is poorly equipped to address the inefficiencies generated by labor market frictions. By adding new elements to the baseline model, several papers find that quantitatively significant deviations from price stability are warranted. Thomas (2008) introduces nominal wage staggering in an otherwise standard New Keynesian model with labor market frictions and finds that the central bank should use price inflation to avoid unemployment volatility and dispersion in hiring rates. In Blanchard and Gali’s (2010) model, fluctuations in employment have a direct utility cost since utility is decreasing and concave in the level of employment. In that case, the monetary authority has a direct incentive to deviate from price stability to stabilize unemployment fluctuations.

This paper is organized as follows. Section 2 provides an analysis of the determinants of unemployment losses along the cycle. Section 3 develops the basic model. Section 4 shows that the type of policy carried out by the monetary authority has a substantial impact on mean unemployment. Section 5 sheds light on the tradeoffs faced by the monetary authority and provide an evaluation of the welfare gains of following one policy rather than another. Section 6 concludes.

2 The non-linearity in the employment-flow equation

Labour market frictions generate asymmetries in unemployment dynamics. Along the business cycle, these asymmetries can lead to higher mean unemployment. In this section, which draws extensively from Hairault et al. (2010) and Jung and Kuester (2011), I show this formally using the employment-flow equation of the matching model. The size of the labour force is normalized to unity. In each period, workers can either be employed or unemployed. A mass u_t of unemployed workers finds a job with probability p_t and becomes productive in the following period. A mass $N_t = 1 - u_t$ of workers is employed. In each period, employment relationships are destroyed at the exogenous rate ρ . Employment evolves according to the following law of motion

$$N_t = (1 - \rho)N_{t-1} + p_{t-1}u_{t-1} \quad (1)$$

It is already possible to see that unemployment and the job finding rate enter non-linearly in equation (1). Define u as steady state (or “stabilized”, the two terms will be used interchangeably in what follows) unemployment, that is unemployment in an economy where aggregate fluctuations are eliminated and $E(u_t)$ as the unconditional average of unemployment in an economy with aggregate fluctuations. We are interested in computing $E(u_t) - u$. Assuming that all variable in the employment-flow equation (1) are covariance stationary, $E(u_t) - u$ is given by

$$E(u_t) - u = -\frac{1}{\rho + p} [Cov(p_t, u_t) + (E(p_t) - p)E(u_t)] \quad (2)$$

This proposition is directly taken from Jung and Kuester (2011) and its proof can be found in the appendix. p is the steady state value of the job finding rate. The gap between mean unemployment $E(u_t)$ and stabilized unemployment u depends on the covariance between unemployment and the job finding rate, which is strictly negative empirically, and on the gap between the mean job finding rate and the stabilized job finding rate. In an expansion, the impact on employment of an increase in the job finding rate is *dampened* by the decrease in the size of the pool of job seekers whereas, in a recession, the impact on employment of a decrease in the job finding rate is *amplified* by the increase in the size of the pool of job seekers. Depending on the underlying structure of the economy, these unemployment losses will translate in more or less substantial consumption losses. Stabilization policy, by dampening business cycle fluctuations, can then have an influence on mean unemployment and consumption. As this paper is concerned with the design of optimal monetary policy in the presence of these asymmetries, section 2 builds a model in which monetary policy has a role to play. I will show how the covariance and the gap between the mean and the stabilized job finding rates depend on the structural characteristics of the model and on how monetary policy is conducted.

3 A New Keynesian model with search and matching frictions

This section develops a model with sticky prices in which monetary policy has a meaningful role to play. It departs from the standard New Keynesian model in several ways. The labour market is not perfectly competitive but is characterized by search and matching frictions. The surplus of a match is divided between the worker and the firm according to an exogenous rule that determines the real wage. Firms have to post vacancies in order to match with workers and pay a quadratic adjustment cost à la Rotemberg (1982).

3.1 Model

3.1.1 Labour market

This subsection completes the description of the labour market initiated in section 1. Workers and firms need to match in the labour market in order to become productive. The number of matches in period t is given by a Cobb-Douglas matching function $m_t = \mu u_t^\alpha v_t^{1-\alpha}$, $u_t = 1 - N_t$ being the number of job-seekers and v_t the number of vacancies posted by firms. The parameter μ reflects the efficiency of the matching process and $\alpha \in (0,1)$ is the elasticity of the matching function with respect to unemployment. Define $\theta_t = \frac{v_t}{u_t}$ as labor market tightness. The probability q_t for a firm to fill a vacancy and the probability p_t for a worker to find a job are respectively $q_t = \frac{m_t}{v_t} = \mu \theta_t^{-\alpha}$ and $p_t = \frac{m_t}{u_t} = \mu \theta_t^{1-\alpha}$.

3.1.2 Households

Each household is thought of as large extended family which insures its members against consumption risk. The household makes the consumption and savings decisions. It has expected utility

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\sigma}}{1-\sigma} \tag{3}$$

where C_t denotes aggregate consumption in final goods. $C_t \equiv \int_0^1 \left[C_{it}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}$ is a Dixit-Stiglitz aggregator of different varieties of goods. The optimal allocation of income on each variety is given by $C_t(j) = \left[\frac{P_{jt}}{P_t} \right]^{-\varepsilon} C_t$ where $P_t = \left[\int_0^1 P_{it}^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\varepsilon/(1-\varepsilon)}$ is the price index. Households supply labour h inelastically (h is normalized to 1). Total labour income is given by $w_t N_t$ and unemployed household members receive an unemployment benefit b . Households receive profits Π_t^r from the monopolistic sector and invest in risk-free bonds that promise

a unit of currency tomorrow and cost $(1 + I_t)^{-1}$ today. They face the following per period budget constraint

$$P_t C_t + (1 + I_t)^{-1} B_{t+1} = P_t [w_t N_t + b(1 - N_t)] + B_t + P_t \Pi_t' \quad (4)$$

Households choose consumption and bonds holding so as to maximize (3) subject to (4). The household's optimal consumption path is governed by the Euler equation

$$\beta \frac{I_t}{E_t \Pi_{t+1}} \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} = 1 \quad (5)$$

3.1.3 Firms

A measure one of monopolistic firms produce differentiated goods using an identical production function

$$Y_{it} = Z_t N_{it} \quad (6)$$

where Z_t is the state of technology. Firm i chooses its level of employment N_{it} , its number of vacancies v_{it} and its prices p_{it} in order to maximize the expected sum of its discounted profits

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{\lambda_t}{\lambda_0} \left[\frac{p_{it}}{p_t} Y_{it} - \kappa v_{it} - w_{it} N_{it} - \frac{\phi^p}{2} \left(\frac{p_{it}}{p_{it-1}} - \Pi \right)^2 \right] \quad (7)$$

subject to its production technology (7), its perceived law of evolution of employment $N_{it} = (1 - \rho)N_{it-1} + v_{it-1}q(\theta_{t-1})$, the demand for its variety $Y_{it} = \left(\frac{p_{it}}{P_t} \right)^{-\epsilon} Y_t$ and taking as given the wage schedule. κ is the deadweight cost of posting a vacancy and ϕ^p captures resources devoted to adjusting prices. Define also mc_t , the lagrange multiplier associated with the demand constraint, as the marginal cost of firms. In equilibrium, all firms will choose the same price and the same number of vacancies, we can thus drop individual firm subscripts i . After rearranging the first order conditions, we obtain the following job creation and pricing equations (see the appendix for more details)

$$\frac{\kappa}{q(\theta_t)} = E_t \beta_{t+1} \left[mc_{t+1} Z_{t+1} - w_{t+1} + (1 - \rho) \frac{\kappa}{q(\theta_{t+1})} \right] \quad (8)$$

$$(1 - \epsilon + \epsilon mc_t) Z_t N_t - \phi^p \Pi_t (\Pi_t - \Pi) + E_t \beta_{t+1} \phi^p \Pi_{t+1} (\Pi_{t+1} - \Pi) = 0 \quad (9)$$

where $\beta_{t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma}$ is the stochastic discount factor of households. The first equation is an arbitrage condition for the posting of vacancies. It states that the cost of posting a vacancy, the deadweight cost κ divided by the time it takes to fill the vacancy, must be equal to the expected discounted benefit of a filled vacancy. These benefits consist of the revenues from output net of wages and the future savings on vacancy posting costs. The second equation is a non-linear expectational Phillips Curve linking marginal cost and inflation. Because of the presence of sticky prices, inflation has an influence on marginal cost. This can be seen more clearly by rewriting equation (14), the markup (which is the inverse of marginal cost) is given by

$$\mu_t = \frac{1}{mc_t} = \frac{\epsilon}{\epsilon - 1 + \phi^p \frac{\Pi_t}{Y_t} (\Pi_t - \Pi) - E_t \beta_{t+1} \phi^p \Pi_{t+1} (\Pi_{t+1} - \Pi)} \frac{1}{Y_t} \quad (10)$$

The higher is the difference between today's and tomorrow's inflation, the lower is the markup and thus the lower is the inefficiency arising from monopolistic competition. Importantly, through equation (9), lower markups (and higher marginal costs) imply greater benefits from a filled vacancy and therefore more vacancy posting. It is through this channel and through the channel of the stochastic discount factor, which is inversely related to the real interest rate, that monetary policy can have an influence on employment.

3.1.4 Wage setting

As first emphasized by Shimer (2005), the Mortensen-Pissarides model is unable to account for the volatility of labor market variables observed in U.S. data. In the case of Nash-Bargained flexible wages, the wage is too sensitive to aggregate conditions and “eats” all the incentives of firms to adjust through the employment margin. The introduction of **real wage rigidity** helps solve this problem. It is common practice in the literature (see Faia 2008 or Ravenna and Walsh 2012) to model rigid wages by assuming that wages are a weighted average between Nash-bargained wages and a wage norm

$$w_t = \gamma w_t^{Nash} + (1 - \gamma) w_t^{norm} \quad (11)$$

I will assume that the wage norm is equal to the steady state level of Nash-bargained wages $w_t^{norm} = w_{ss}$. It is central to the argument of this paper that the model generates volatility in labour market variables comparable to the one observed in U.S. data in order to quantify the mean unemployment losses due to matching frictions. I will therefore assume that γ is close to 0, i.e. substantial wage rigidity.

3.1.5 Equilibrium

Final output and home production can be used for consumption or to cover the deadweight costs of changing prices and posting vacancies

$$C_t = Z_t N_t + b(1 - N_t) - \frac{\phi^p}{2} (\Pi_t - \Pi)^2 - \kappa v_t \quad (12)$$

We can now define an equilibrium.

DEFINITION: A competitive equilibrium is a set of plans $\{C_t, N_t, mc_t, \theta_t, \pi_t, w_t\}$ satisfying equations (1), (5), (8), (9), (11), and (12) given a monetary policy $\{i_t\}$, a specification for the exogenous process $\{Z_t\}$ and initial conditions θ_{-1}, u_{-1} and N_{-1} .

Technology will be modeled as a first-order autoregressive process $e^{Z_t} = e^{\delta z Z_{t-1}} \varepsilon_t^Z$. Different specifications for monetary policy will be considered in the analysis that follows.

4 Behavior of the economy under alternative monetary policy rules

In this section, I calibrate the model to match key features of U.S data (notably the volatility of labour market variables), conditional on monetary policy being conducted according to a Taylor rule. I find that the level of unemployment is about 0.25 percentage points higher in the fluctuating economy than in steady-state. I then examine the behavior of the economy under an alternative policy rule featuring a strong response to inflation. In that case, mean unemployment is 0.3 percentage points higher than under a Taylor rule. I show that this is because policies of price stability tend to exacerbate labor market volatility.

4.1 Calibration

I calibrate the model to U.S. data. I take one period to be a month. Table 1 gives a summary of the values of the parameters

I calibrate a few parameters using conventional values. The discount factor is set to $\beta = 0.9967$, which yields an annual interest rate of 4%. The elasticity of substitution between goods is $\varepsilon = 6$, which corresponds to a steady-state markup of 20%. I choose a coefficient of relative risk aversion $\sigma = 1.5$. The price adjustment cost parameter ϕ^p is chosen according to the following logic. The linearized Phillips Curve of the model is observationally equivalent to the one derived under Calvo pricing and structural estimates of New Keynesian models find an elasticity of inflation with respect to marginal cost ω of 0.5 (Lubik and Schorfheide 2005). Since this coefficient is equal to $\frac{\varepsilon-1}{\phi^p}$, this would imply $\phi^p = 10$. Alternatively, Klenow and Kryvtov (2005) find that 26% of prices are changed every month in the U.S. Thus assuming an average contract duration of 4 months, we could choose ϕ^p so that the coefficient ω is equal to the one under Calvo pricing. This would imply $\phi^p = 60$. I choose an intermediate value $\phi^p = 50$. This is also the value chosen by Faia (2008).

Next, I calibrate the labour market parameters. I set the elasticity of matches with respect to unemployment at $\alpha = 0.4$, in line with the estimates in Blanchard and Diamond (1991). I set the steady state values of unemployment, labor market tightness, and the job finding rate to their empirical counterparts. I use the measure of the job finding probability constructed by Shimer (2007) and the seasonally-adjusted monthly unemployment rate constructed by the Bureau of Labor Statistics (BLS) from the Current Population Survey (CPS). I compute labor market tightness as the ratio of a measure of the vacancy level to the seasonally-adjusted monthly unemployment level constructed by the BLS from the CPS. The measure of the vacancy level is obtained by merging the vacancy data of the Conference Board help-wanted advertisement index for 1960-2001 and the seasonally-adjusted monthly vacancy level constructed by the BLS from JOLTS for 2001-2012. Over these periods, the mean of the job finding probability is 0.45, the mean of the unemployment rate is 5.8% and the mean of labor market tightness is 0.7. These targets imply, through the Beveridge Curve, a job destruction rate of 0.0277 and, through the definition of the job finding probability, a matching efficiency of 0.5574. Following Michaillat (2012), I set the recruiting cost to 0.32 of the real wage. I can then back out the level of the real wage from the job creation equation. I obtain $w = \frac{mc}{1 + \frac{0.32\theta v}{\mu}(\frac{1}{\beta} - 1 + \rho)} = 0.8207$ and $\kappa = 0.2626$. Finally I set the value of home production to $b = 0.4$ following Shimer (2005).

I calibrate the productivity process Z_t so as to match U.S. labour productivity standard deviation and persistence. I follow Hairault et al. and set the persistence parameter to 0.9 and the standard deviation to 0.009.

	Parameter/SS value	Justification
β	0.9967	Corresponds to an interest rate of 4% annually
ϕ^p	50	Intermediate value
σ	1.5	Conventional value
ε	6	Steady state markup of 20%
α	0.4	Blanchard & Diamond (1991)
p	0.45	Mean over the period 1951-2007
u	5.8%	Mean over the period 1951-2012
θ	0.7	Mean over the period 1951-2012
μ	0.5574	Steady state relation
κ	0.2626	32% of steady state wage
b	0.4	Shimer (2005)
ζ	0.9	Matches U.S. standard deviation and persistence
$\sigma_{\varepsilon z}$	0.009	of labor productivity

4.2 Solution method

The model is solved by taking a second-order approximation of the equilibrium conditions around the deterministic steady state. The solution method is ex-

plained in Schmitt-Grohé Uribe (2004). Using a second-order approximation to the equilibrium conditions rather than a first-order approximation has several advantages. As the main purpose of this paper is to study the implications for monetary policy of non linearities induced by matching frictions, it is crucial that I am able to capture these non linearities. First-order approximations cannot by construction account for non linearities. Second, as I will make welfare comparisons under alternative policies in section 5, it is important that I am able to compute welfare accurately. In an economy with a distorted steady state, when welfare is evaluated using a first-order approximation to the equilibrium law of motion of endogenous variables, some second order terms of the welfare function are omitted while others are included. The resulting welfare criterion will be inaccurate to order two or higher. However, when a second-order approximation to the equilibrium law of motion of endogenous variables is taken, the welfare criterion becomes accurate to order two.

4.3 First and second order moments under a Taylor rule

It is important that the model provides a sensible description of the reality of the U.S. economy. As noted in the previous section, I make sure that the model matches the mean levels of the job-finding rate, the unemployment rate and labor market tightness. I also set γ so as to match the model's unconditional second moments to the second moments of hp-filtered U.S data from 1951Q1 to 2007Q1, conditional on monetary policy being conducted according to a Taylor rule. In order to get those empirical moments, I use the data for unemployment, vacancies, labor market tightness, output, and the job finding probability described previously, and as in Shimer (2005), I use a hp-filter with a weight of 10^5 to separate fluctuations and trends. I haven't had time to do this work thus far so I present the empirical moments Michailat (2012) obtains in Table 2 below. I take special care in matching the covariance between the job finding rate and the unemployment rate as it is a crucial determinant of the mean unemployment effects of business cycles. In the data, over the period 1951Q1 to 2007Q1, this average of this covariance is equal to -8.5 (measuring both rates in percentage points). The following table reports compares the log deviations from their steady state values of some selected variables in the data and in the model. The mean of these variables in the model is also reported.

Variable	Data - Standard deviation	Model - Standard deviation	Model - Mean
u	0.170	0.178	0.0605
v	0.184	0.229	0.0403
θ	0.342	0.349	0.6996
y	0.03	0.03	0.9399
p	/	0.2093	0.4433
Z	0.02	0.02	1.002

The model does a fairly good job at reproducing the empirical second moments of the selected variables. It also reproduces the negative covariance of -0.0008

between the unemployment rate and the job finding rate. The unemployment rate is about 0.25 points higher in the fluctuating economy than in steady-state. As expected from the analysis carried out in section 2, this is due to two elements, 1) the negative covariance between the unemployment rate and the job finding rate, and 2) the fact that the mean job finding rate $E(p_t)$ is equal to 0.4430 in the fluctuating economy whereas it is higher and equal to 0.45 in steady state. In this framework, $E(p_t)$ is different from p since the stochastic discount factor and marginal cost are time-varying, which makes the job finding rate non-linear in productivity.

4.4 First and second order moments under a strong response to inflation

How would have the performance of the labour market been affected if the Fed had followed another monetary policy? In this section, I look at the behavior of my fictitious economy under a policy of price stability and explain how this policy, by affecting the volatility of labour market variables, leads to different outcomes in terms of mean employment than a Taylor rule. I focus on this specific policy as it is often considered as being nearly optimal in the literature. As emphasized in the introduction, this is the case in the baseline New Keynesian model but also in models augmented with search and matching frictions (see Faia 2008,2009 or Ravenna & Walsh 2012). The following table reports the mean and the standard deviation of selected variables of the model under price stability.

Variable	Mean	Standard deviation of the log deviations
u	0.0636	0.28632
v	0.0399	0.3129
θ	0.7131	0.5331
y	0.9369	0.0368
p	0.4397	0.3199
Z	1.002	0.02

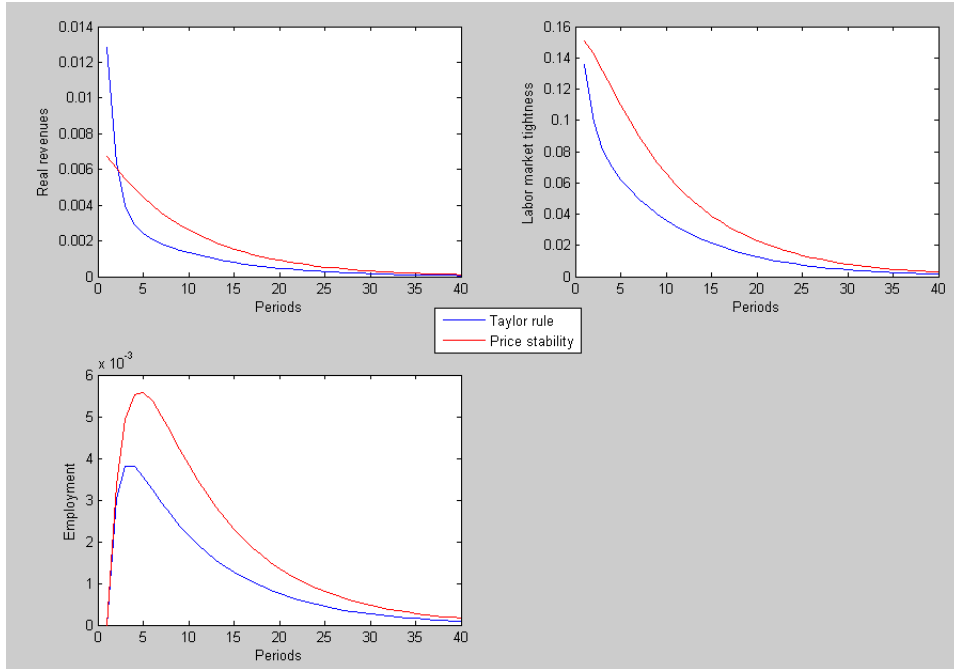
It can be seen that the labour market becomes much more volatile under price stability as the standard deviations of the log deviations of u, v, θ and jf are multiplied by about 1.5. The mean of the variables are affected, employment and accordingly output are lower by about 0.3 points. The mean of the job finding rate is also reduced and the covariance between the job finding rate and unemployment decreases at -0.0021 . Let's turn to the explanation of these results. Notice that the job creation equation can equivalently be written in the following way

$$J_t = E_t \beta_{t+1} [mc_{t+1} Z_{t+1} - w_{t+1} + (1 - \rho) J_{t+1}]$$

Solving forward, we obtain

$$\frac{\kappa}{q(\theta_t)} = \sum_{j=1}^{\infty} E_t \beta^j \left(\frac{C_{t+j}}{C_t} \right)^{-\sigma} (1-\rho)^{j-1} (mc_{t+j} Z_{t+j} - w_{t+j})$$

This equation states that vacancy posting today is driven by the sum of future expected discounted real revenues minus wage payments. Since the paths of labour productivity and real wages are sensibly identical under the policies considered, this shows that differences in vacancy posting activity must come from differences in the path of marginal cost. Under price stability, marginal cost is essentially flat whereas under a Taylor rule, marginal cost jumps up following a positive productivity shock and then becomes inferior to its steady state value. The following figure illustrates this graphically by plotting the response of expected real revenues, labor market tightness and employment under price stability and under a Taylor rule following a positive productivity shock of one standard deviation.



The sum of real revenues (the area below the curve) is unambiguously greater under price stability, which leads to greater vacancy posting following the shock. The same logic operates after a negative shock, the sum of real revenues decreases more under price stability and this leads to a greater decrease in labor market tightness. This increased volatility of labour market tightness under price stability induces greater volatility of employment, output, and the job

finding rate. The absolute value of the covariance between unemployment and the job finding rate is accordingly greater. Moreover, as the job finding rate is a concave function of labour market tightness, this implies that the average job finding rate is lower under price stability. Those two elements tend to decrease mean unemployment, which is 0.56 points higher in the fluctuating economy than in steady-state under price stability. The analysis carried out in this section suggests that monetary policy can increase mean unemployment by dampening labour market volatility. It could do so by deviating from price stability to influence the path of real revenues. The following section investigates to which extent the monetary authority has an incentive to do so.

5 The inflation-unemployment stabilization trade-off: a quantitative analysis

In this section, I compare the decentralized equilibrium and the planner equilibrium of the model to show that the monetary authority faces a tradeoff between stabilizing inflation and stabilizing unemployment, even in the absence of effect of business cycle fluctuations on mean unemployment. I emphasize that taking it into account should strengthen the case for stabilizing unemployment. I then describe the optimal monetary policy, that is the process $\{i_t\}$ associated with the competitive equilibrium that yields the highest level of welfare and carry out a quantitative evaluation of the welfare costs of following a policy of price stability rather than the optimal policy.

5.1 Distortions in the decentralized equilibrium

This economy is characterized by four main frictions: price stickiness, monopolistic competition, rigid real wages, and matching frictions in the labour market. In order to understand how these frictions interact with each other and create tradeoffs for the monetary policymaker, it is useful to compare the decentralized equilibrium with the allocation that a planner would achieve in the absence of price stickiness. The planner maximizes the discounted sum of utilities (3) with respect to C_t, v_t, N_t under the technological constraints of the economy, that is the resource constraint and the law of motion of employment. Let ω_{1t} and ω_{2t} be the lagrange multipliers associated with the two constraints. First order conditions are as follows

- $C_t^{-\sigma} = \omega_{1t}$
- $\beta^t [\omega_{1t}(Z_t - b) - \omega_{2t}] + \beta^{t+1}\omega_{2t+1} [1 - \rho - \nu\bar{m}(1 - N_t)^{\nu-1}v_t^{1-\nu}] = 0$
- $-\beta^t\kappa\omega_{1t} + \beta^{t+1}\omega_{2t+1}(1 - \nu)\bar{m}(1 - N_t)^\nu v_t^{-\nu} = 0$

Using the three equations, we obtain the job creation equation in the planner equilibrium

$$\frac{\kappa}{q(\theta_t)} = E_t \beta_{t+1} \left[(1 - \alpha)(Z_{t+1} - b) + (1 - \rho) \frac{\kappa}{q(\theta_{t+1})} - \alpha \kappa \theta_{t+1} \right]$$

This equation states that, from the point of view of society, job creation is efficient when the marginal benefit of posting a vacancy is equal to the marginal cost of posting a vacancy. The marginal benefit is given by the number of matches created by posting an extra vacancy $(1 - \alpha)q(\theta_t)$ times the discounted (because a match becomes operational in the period following its creation) difference between what workers produce on the job and what they produce at home $E_t \beta_{t+1}(Z_{t+1} - b)$. The marginal cost of posting a vacancy is equal to the fixed cost κ paid today minus the discounted savings on future vacancy posting costs in the case the match is formed. This equation should be compared with the job creation equation in the decentralized equilibrium, equation (8), which is rewritten here for the sake of clarity

$$\frac{\kappa}{q(\theta_t)} = E_t \beta_{t+1} \left[mc_{t+1} Z_{t+1} - w_{t+1} + (1 - \rho) \frac{\kappa}{q(\theta_{t+1})} \right]$$

In order for job creation to be efficient in the steady state and in response to shocks in the decentralized equilibrium, the following conditions need to hold: wages are Nash-bargained, $\alpha = \eta$, and $mc_{t+1} = 1$. Then the equation of the “optimal wage” is given by $w_{t+1}^{opt} = \alpha (mc_{t+1} Z_{t+1} + \kappa \theta_{t+1}) + (1 - \alpha)b$. This shows that the efficiency of job creation crucially depends on two elements which can introduce a wedge between the social marginal benefit and the social marginal cost of posting a vacancy; the presence of monopolistic competition and the way real wages are formed. When firms are monopolistically competitive, they set their prices as a markup over marginal cost which compresses their real revenues and leads them to post an inefficiently low level of vacancies. When the wage is different than the optimal wage, firms do not have the right incentive to post the efficient number of vacancies. When $w_{t+1} > w_{t+1}^{opt}$, the expected benefit from posting a vacancy is low and too few vacancies are posted. When $w_{t+1} < w_{t+1}^{opt}$, the expected benefit from posting a vacancy is high and too many vacancies are posted. Under the assumption of wage rigidity, even if the wage is equal to w^{opt} in steady state, job creation and unemployment fluctuations will be inefficient in response to shocks.

Since adjusting prices is costly, the monetary authority would like to maintain price stability. From equation (9), it is straightforward to see that in order to do so, it will have to keep marginal cost constant. In that case, the monetary authority will have to allow for inefficient unemployment fluctuations. On the contrary, if the monetary authority wishes to correct for inefficient unemployment fluctuations, it can only do so through the marginal cost channel. This will come at the cost of non-zero inflation. It is in this sense that we can talk

about a tradeoff between inflation and unemployment stabilization. In response to shocks the two objectives of price stability and efficient job creation are mutually exclusive.

The mean unemployment losses due to aggregate fluctuations should also influence the terms of the tradeoff between inflation and unemployment stabilization. They would not influence that tradeoff in a flexible wage world, even if the wage was set in an inefficient manner. Indeed, in such a world the rapid adjustment of the wage gives very little incentive for firms to make job creation vary in response to shocks. As a result, employment and labor market tightness will not deviate very much from their steady state value so that the amount of inefficient job creation is small. Additionally, since volatility of labour market variables is low, the mean unemployment losses of fluctuations are negligible. However, in a rigid wage world, since job creation reacts more strongly in response to shocks, the amount of inefficient job creation becomes much larger. Moreover, as we have seen in section 4, since volatility of labour market variables is higher, mean unemployment losses along the business cycle are substantial. In that case, by deviating from price stability in order to dampen labor market volatility, the monetary authority can have an effect on both the amount of inefficient job creation and on mean unemployment. This could provide an explanation for an observation made by Ravenna & Walsh (2012), namely that the costs of price stability become greater as labour market volatility increases.

5.2 Ramsey monetary policy

The optimal monetary policy is the process $\{i_t\}$ associated with the competitive equilibrium that yields the highest level of welfare. As is common practice in the literature, I first determine choices for real variables that are optimal subject to the constraints of the competitive economy. I then determine the behavior of the nominal interest rate that is consistent with these real quantities. More precisely, optimal monetary policy under commitment can be found by writing a restricted social planner's problem that involves maximizing the expected discounted sum of utility with a choices of sequences $\{C_t, N_t, \Pi_t, m_{C_t}, \theta_t\}$ subject to the resource constraint, the employment-flow equation, the job creation equation and the Phillips Curve. I have simplified the problem by substituting the definition of $u_t = 1 - N_t$ and $p_t = \mu\theta_t^{1-\alpha}$ in the employment flow equation. The Lagrangian is as follows

$$\begin{aligned}
L = E_0 \sum_{t=0}^{\infty} \beta^t & \left(\frac{C_t^{1-\sigma}}{1-\sigma} \right. \\
& + \lambda_{1t} \left[Z_t N_t - \frac{\phi^p}{2} (\Pi_t - 1)^2 - \kappa \theta_t (1 - N_t) + b(1 - N_t) - C_t \right] \\
& \left. + \lambda_{2t} [(1 - \rho)N_t + \bar{m}(1 - N_t)\theta_t^{1-v} - N_{t+1}] \right)
\end{aligned}$$

$$+ \lambda_{3t} \left[E_t \beta_{t+1} \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left[mc_{t+1} Z_{t+1} - w_{ss} + (1 - \rho) \frac{\kappa}{\bar{m}} \theta_{t+1}^v \right] - \frac{\kappa}{\bar{m}} \theta_t^v \right]$$

$$+ \lambda_{4t} [(1 - \epsilon + \epsilon mc_t) Z_t N_t - \phi^p \Pi_t (\Pi_t - \Pi) + E_t \beta_{t+1} \phi^p \Pi_{t+1} (\Pi_{t+1} - \Pi) = 0]$$

$\{\lambda_{1t}, \lambda_{2t}, \lambda_{3t}, \lambda_{4t}\}$ represent sequences of lagrange multipliers associated with the four constraints. I want to look at optimal allocations that arise when the monetary authority can commit to follow on a plan that is optimal and when the monetary authority has long been following the optimal monetary policy. I thus include artificial multipliers $\lambda_{2,-1}, \lambda_{3,-1}, \lambda_{4,-1}$ in the forward looking constraints, which values are set equal to their solution in steady-state. With these multipliers in place, the efficiency conditions take exactly the same form at any date t . The system is time-invariant. The first order conditions of the problem are presented in the appendix. I obtain a system of nine equations (the four equations just mentioned and five first-order conditions) and nine unknowns (the five endogenous variables and the four lagrange multipliers).

Importantly, as has already been noticed in Faia (2009), the optimal long-run inflation rate in that type of model is equal to zero. In steady-state, the first-order condition with respect to inflation is given by

$$-\phi^p \lambda_1 (\Pi - 1) = 0$$

Since the resource constraint is binding in equilibrium, we have that $\lambda_1 \neq 0$. Since $\phi^p > 0$, it follows that $\Pi = 1$. An intuitive explanation of this result can be found in Faia (2009) and draws from the analysis in King and Wolman (1999). I now turn to the behavior of the economy under this optimal policy.

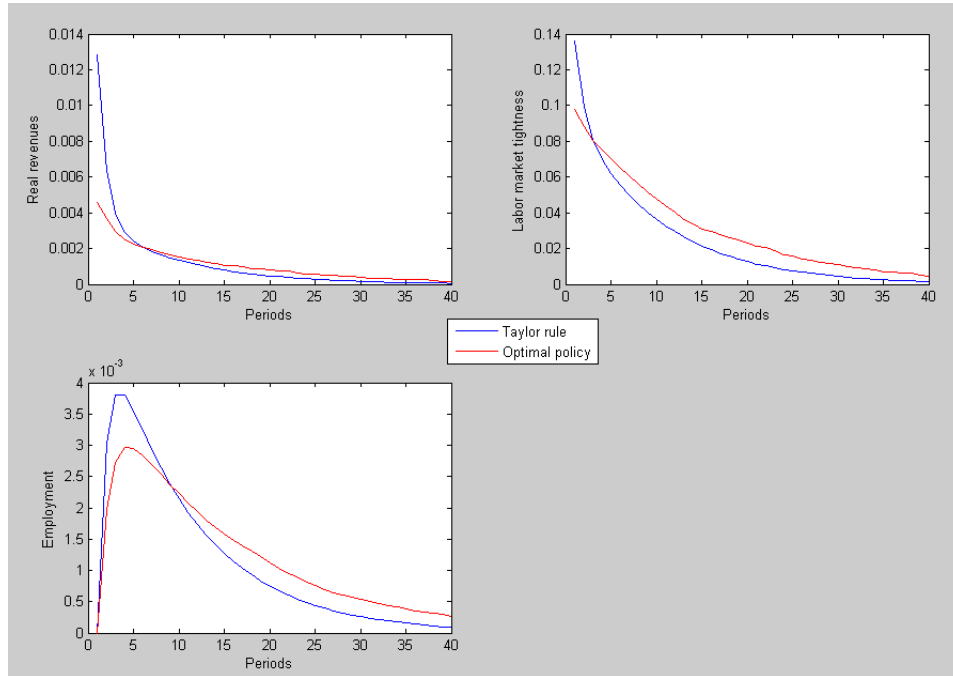
5.3 Behavior of the economy under the optimal policy

The following table reports the mean and the standard deviation of selected variables of the model under the optimal policy.

Variable	Mean	Standard deviation of the log deviations from steady state
u	0.0606	0.1854
v	0.0399	0.1941
θ	0.7006	0.3451
y	0.9369	0.0308
p	0.4438	0.2077
Z	1.002	0.02

The results are strikingly similar to what is obtained under a Taylor rule. Noticeably, the standard deviation of the log deviation of u is slightly higher, the

standard deviation of the log deviation of v is lower and the means of unemployment and the job finding rate are virtually unchanged. The following figure compare the response of expected real revenues, labor market tightness and employment under the optimal policy and under a Taylor rule following a positive productivity shock of one standard deviation



The sum of expected real revenues is lower under the optimal policy, which explains why the initial reaction of labour market tightness and employment is lower than under a Taylor rule. Expected real revenues and accordingly labor market tightness and employment decline less rapidly and more smoothly under the Ramsey policy. As expected, the Ramsey policymaker uses inflation to influence the path of marginal cost and dampen labor market volatility

5.4 Welfare costs of price stability

How do the mean unemployment gains induced by the optimal policy translate in welfare gains? This question will be hard to answer as I cannot rely on an analytical expression of the welfare function. Welfare will be influenced by many factors that I cannot isolate. Thus my goal is not to come up with a precise evaluation of the costs of following a policy of price stability, but rather to show that the costs I obtain are substantially higher than those obtained in other papers using the same framework but that do not put the emphasis on the mean unemployment gains that monetary policy can achieve. I compare the level

of utility associated a policy of price stability to the level of utility associated with the optimal policy. As emphasized previously, this welfare comparison will be accurate to order two. Welfare will be characterized conditional upon the initial steady state being the deterministic steady state. Since the deterministic steady state is the same in both regimes, this ensures that the economy begins from the same initial point under both policies. Let the equilibrium process for consumption associated with a particular policy regime be denoted by $\{c_t\}$. Welfare, V_t is measured as the conditional expectation of lifetime utility as of time t evaluated at $\{c_t\}$. Formally

$$V_0 = E_t \sum_{t=0}^{\infty} \beta^t U(c_t)$$

I assume that at time zero all state variables of the economy equal their respective Ramsey steady-state values. Consider two policy regimes, the Ramsey optimal policy regime denoted by r and the regime of price stability denoted by PS . The welfare levels associated with two regimes are

$$V_0^r = E_t \sum_{t=0}^{\infty} \beta^t U(c_t^r)$$

$$V_0^{PS} = E_t \sum_{t=0}^{\infty} \beta^t U(c_t^{PS})$$

where $\{c_t^r\}$ and $\{c_t^{PS}\}$ are the consumption processes under the Ramsey regime and the regime of price stability, respectively. The welfare cost of adopting a policy regime of price stability instead of the Ramsey policy regime, λ is measured as the fraction of the Ramsey consumption process that a household would be willing to give up to be as well off under regime PS as under the Ramsey regime. λ is implicitly defined as

$$V_0^{PS} = E_t \sum_{t=0}^{\infty} \beta^t U(c_t^r(1 - \lambda))$$

Given the form of the utility function $U(c) = \frac{c^{1-\gamma}}{1-\gamma}$, this yields

$$\lambda \approx \frac{1}{1-\gamma} (\ln V_0^r - \ln V_0^{PS})$$

The following table report the value of λ^1 in percentage points under different calibrations

¹The fraction λ is computed from the solution of the second order approximation to the model equilibrium around the deterministic steady state. See the appendix for more details

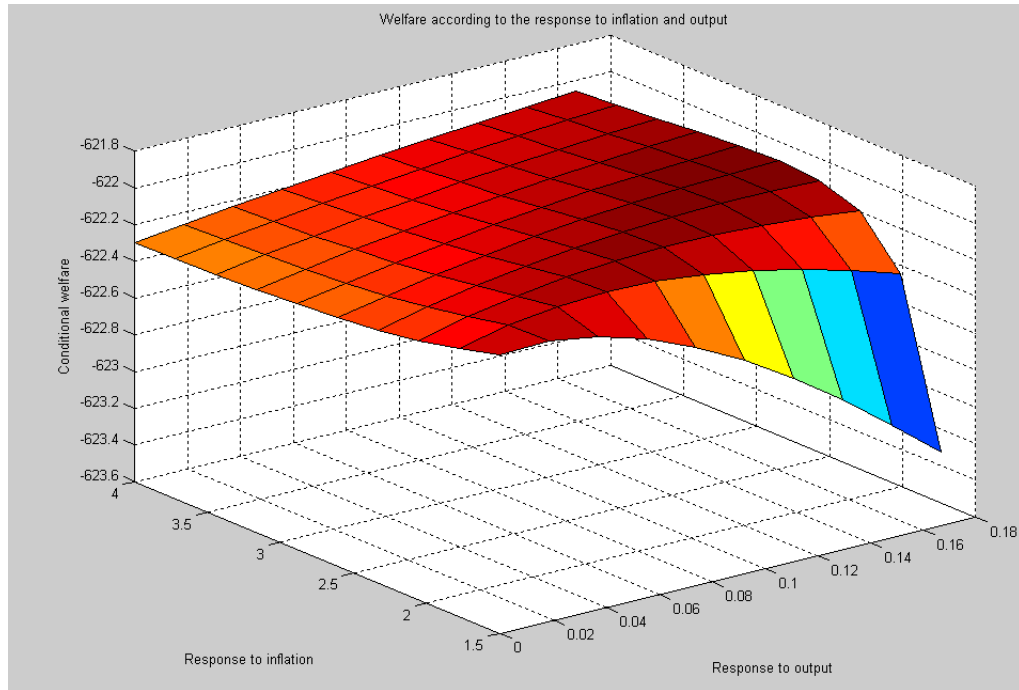
Calibration	$\lambda * 100$
Baseline	0.105%
$b = 0$	0.24%

Under the baseline calibration, the welfare costs of following a policy of price stability rather than the optimal policy amount to 0.105% of the Ramsey consumption process. This welfare cost is much higher than the one found by Faia (2008) and equivalent to the highest welfare cost reported in Ravenna & Walsh (2012). I also report the value of the welfare cost when b , the value of home production, is equal to 0. It should be noted that since wages are exogenous, making b vary has no impact on the cyclical properties of the model. Intuitively given employment gains should translate in different welfare gains according to the value of b through two main channels. First, from the point of view of society, creating an extra job comes a cost, equal to the vacancy post, and yields a benefit equal to the difference between what workers produce on the job and what they produce at home. Thus, a decrease in employment should come at a higher welfare cost when b is low. Second, households dislike consumption volatility and a given volatility of employment should translate in a higher volatility of consumption when b is low as a low value of home production makes households income more sensitive to aggregate conditions. This explains why the welfare cost of following a policy of price stability rather than the optimal policy is more than doubled when b goes from 0.4 to 0, even though the cyclical properties of the model are unchanged. I see this result as indirect proof that the employment gains account for a significant portion of the welfare gains.

5.5 Performance of simple rules

In this section, I review the performance of a simple Taylor rule compared to the optimal rule and compute the optimal simple rule, that is the rule that yields the highest level of welfare. I search the grid of parameters $\{\phi_\pi, \phi_N, \phi_y\}$ over the following intervals $[1.5, 4]$ for ϕ_π , $[0, 2]$ for ϕ_N and $[0, 2]$ for ϕ_y . Since inflation in Taylor type rules is expressed at annual rates, the parameters ϕ_π and ϕ_N must be divided by 12 given that a period in the model is a month.

Results are as follows. The level of welfare under the Taylor rule is significantly lower than under the optimal policy ($\lambda = 0.067\%$) but higher than under the policy of price stability. This is because, under a Taylor rule, the policymaker strikes a balance between stabilizing inefficient employment fluctuations, which comove with output, and keeping the price level stable. This result stands in stark contrast with findings by Faia (2008) who found that responding to output alongside inflation is welfare detrimental. As a matter of fact, the optimal simple rule features an even stronger response to output $\phi_y = 2/12$ alongside a strong response to inflation $\phi_\pi = 2.5$ and yields a level of welfare slightly superior to the one under the Taylor rule ($\lambda = 0.045\%$). The following graphs plots the conditional level of welfare according to the response to inflation and output



6 Conclusion

This paper explores the effect different monetary policies have on the level of employment in a New Keynesian model with search and matching frictions in the labour market calibrated to replicate key features of U.S. data. In that type of framework, business cycles fluctuations tend to increase mean unemployment for two reasons. First unemployment fluctuations are intrinsically asymmetric so that increased unemployment volatility leads to higher mean unemployment. Second, the job finding rate is lower in the fluctuating economy than in steady-state. I show that policies aiming at reducing the volatility of labour market variables in response to shocks can increase mean employment. A monetary authority with the sole objective of stabilizing prices will tend to exacerbate labour market volatility and lead to a mean unemployment rate about 0.3 percentage points higher than under a Taylor rule. Since prices are sticky, by deviating from price stability, the monetary authority will be able to influence the real revenues of the firms, which govern the incentives for job creation, and influence the behavior of labour market variables. I study how taking into account this effect of business cycles fluctuations on mean unemployment changes the terms of the tradeoff between inflation and unemployment stabilization and find that the welfare costs of following a policy of price stability are non negligible.

As emphasized in Ravenna & Walsh (2012), the welfare costs of fluctuations in the type of framework studied in this paper are substantial. An important part of the costs must come from the fact that fluctuations tend to increase mean unemployment and it is therefore important to build model capable of capturing this effect (i.e not relying on first order approximations to the equilibrium conditions). The real interrogation is about how potent monetary policy is in correcting for these inefficient fluctuations. I believe that the welfare costs reported here are only a lower bound for the actual costs since monetary policy only has an impact on activity through the marginal cost channel. The extent to which this channel will be operative will depend on certain assumptions such as the degree of monopolistic competition and the degree of price stickiness. Notably, a given variation in inflation leads to a lower variation in marginal cost when the degree of monopolistic competition is low than when it is high. Moreover, the assumption of rigid real wages is not a very appealing one. In a more realistic framework with sticky nominal wages, monetary policy would be able to influence real wages and would therefore have a much greater leverage of activity.

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7 Appendix

7.1 Pricing

Rewrite equation (13)

$$\text{Max}_{p_{it+j}} E_t \sum_{j=0}^{\infty} \beta^{t+j} \frac{\lambda_{t+j}}{\lambda_t} \left[\frac{p_{it+j}}{p_{t+j}} Y_{it+j} - mc_{it+j} Y_{it+j} - \frac{\phi^p}{2} \left(\frac{p_{it+j}}{p_{it+j-1}} - \Pi \right)^2 \right]$$

The demand for good z is $Y_t(z) = \left(\frac{p_t(z)}{P_t} \right)^{-\epsilon} Y_t$, we can rewrite

$$\text{Max}_{p_{it+j}} E_t \sum_{j=0}^{\infty} \beta^{t+j} \frac{\lambda_{t+j}}{\lambda_t} \left[\left(\frac{p_{it+j}}{p_{t+j}} \right)^{1-\epsilon} Y_{t+j} - mc_{it+j} \left(\frac{p_{it+j}}{p_{t+j}} \right)^{-\epsilon} Y_{t+j} - \frac{\phi^p}{2} \left(\frac{p_{it+j}}{p_{it+j-1}} - \Pi \right)^2 \right]$$

Maximization with respect to p_{it} yields

$$\begin{aligned} \beta^t \frac{\lambda_t}{\lambda_0} \left[\frac{(1-\epsilon)}{p_t} \left(\frac{p_{it}}{p_t} \right)^{-\epsilon} Y_t + \frac{\epsilon}{p_t} mc_{it} \left(\frac{p_{it}}{p_t} \right)^{-\epsilon-1} Y_t - \frac{\phi^p}{p_{it-1}} \left(\frac{p_{it+j}}{p_{it+j-1}} - \Pi \right) \right] \\ + \beta^{t+1} \frac{\lambda_{t+1}}{\lambda_0} \frac{\phi^p p_{it+1}}{p_{it}^2} \left(\frac{p_{it+j+1}}{p_{it+j}} - \Pi \right) = 0 \end{aligned}$$

Noting that in the symmetric equilibrium $p_{it} = P_t$, dividing by Y_t/P_t and defining $\Pi_t = \frac{P_t}{P_{t-1}}$, we obtain the following Phillips Curve

$$(1 - \epsilon + \epsilon mc_t) Y_t - \phi^p \Pi_t (\Pi_t - \Pi) + E_t \beta_{t+1} \phi^p \Pi_{t+1} (\Pi_{t+1} - \Pi) = 0$$

7.2 Nash Bargained wage

We first derive the marginal value of a match for both workers and firms. The value of employment for the family $W_t - U_t$ is

$$W_t - U_t = w_t - b + E_t \beta_{t+1} (1 - \rho - \theta_t q(\theta_t)) (W_{t+1} - U_{t+1}) \quad (13)$$

where $\beta_{t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma}$ is the stochastic discount factor of households. The net value of an additional employed worker in the family is the wage net of the unemployment benefits that would be received otherwise, plus the expected continuation value from the employment relationship. The value of having an employed worker rather than an unemployed one for the family is given by equation (10). Let J_t be the value of a filled job for the firm and V_t the value of an open vacancy

$$J_t = mc_t Z_t - w_t + E_t \beta_{t+1} [(1 - \rho) J_{t+1} + \rho V_{t+1}] \quad (14)$$

$$V_t = -\kappa + E_t \beta_{t+1} [q(\theta_t) J_{t+1} + (1 - q(\theta_t)) V_{t+1}] \quad (15)$$

The marginal value of a filled job depends on real revenues minus the wage plus the discounted continuation value. With probability $1 - \rho$, the job survives the exogenous separation and with probability ρ , it is destroyed in the following period. The marginal value of an open vacancy depends negatively on the vacancy posting cost that has to be paid today and positively on the discounted continuation value. With probability $q(\theta_t)$, the vacancy is filled today and the job becomes operational in the next period. Free entry implies that the value of vacancies is driven to zero at any point in time, i.e, $V_t = 0$. Using this result in (16) yields

$$\frac{\kappa}{q(\theta_t)} = E_t \beta_{t+1} J_{it+1} \quad (16)$$

which corresponds to equation (13). Nash-bargained wages are determined through a bargaining scheme between workers and employers who maximize the joint surplus of employment

$$\operatorname{argmax}_{\{w_t\}} \left[(J_t)^{1-\eta} (W_t - U_t)^\eta \right] \quad (17)$$

with η representing the exogenous part of the worker's bargaining power. This leads to the following sharing rule

$$(1 - \eta)(W_t - U_t) = \eta J_t \quad (18)$$

By using (10), (16) and (20), we find the flexible wage schedule

$$w_t^{Nash} = (1 - \eta)b + \eta (mc_t Z_t + \kappa \theta_t) \quad (19)$$

The Nash-bargained wage is a weighted average of the worker's outside option b and the real revenue of the firm plus savings on hiring costs.

7.3 Decentralized equilibrium - summary

- $\beta \frac{I_t}{E_t \Pi_{t+1}} \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} = 1$
- $\frac{\kappa}{q(\theta_t)} = \beta_{t+1} \left[mc_{it+1} Z_{t+1} - w_{it+1} + (1 - \rho) \frac{\kappa}{q(\theta_{t+1})} \right]$
- $w_t = (1 - \eta)b + \eta (mc_t Z_t + \kappa \theta_t)$ or $w^{norm} = (1 - \eta)b + \eta \left(\frac{\epsilon - 1}{\epsilon} + \kappa \theta \right)$
- $(1 - \epsilon + \epsilon mc_t) Z_t N_t - \phi^p \Pi_t (\Pi_t - \Pi) + E_t \beta_{t+1} \phi^p \Pi_{t+1} (\Pi_{t+1} - \Pi) = 0$
- $C_t = Z_t N_t - \frac{\phi^p}{2} (\Pi_t - \Pi)^2 - \kappa \theta_t u_t + b(1 - N_t)$
- $N_t = (1 - \rho) N_{t-1} + \mu u_{t-1} \theta_{t-1}^{1-\alpha}$
- $u_t = 1 - N_t$

7.4 Welfare analysis

I have

$$\lambda \approx \frac{1}{1-\gamma} (\ln V_0^r - \ln V_0^{PS})$$

I take a second order approximation of $\ln V_0^r$ and $\ln V_0^{PS}$ around the Ramsey steady state as I wish to characterize welfare conditional upon the initial state being the Ramsey deterministic steady state. I follow here the method proposed by Schmitt-Grohe & Uribe (2004). The solution to the model is a function of the state vector x_t and of the parameter σ scaling the amount of uncertainty in the economy, that is $\ln V_0^i = g^{V_0^i}(x_t, \sigma) \forall i = r, PS$. A second order approximation of the function g around the point $(x, \sigma) = (\bar{x}, 0)$ yields

$$g^{V_0^i}(x_t, \sigma) = g^{V_0^i}(x, 0) + \left[g_x^{V_0^i}(x, 0) \right]_a [(x_t - \bar{x})]_a + \frac{1}{2} \left[g_{xx}^{V_0^i}(x, 0) \right]_{ab} [(x_t - \bar{x})]_a [(x_t - \bar{x})]_b + \frac{1}{2} g_{\sigma\sigma}^{V_0^i}(x, 0) \sigma^2$$

The term $g^{V_0^i}(x, 0)$ is the same under both policies as they have the same steady state. Moreover, as we take the second order expansion around the Ramsey deterministic steady state, the terms $g_x^{V_0^i}(x, 0)$ and $g_{xx}^{V_0^i}(x, 0)$ collapse. Thus, λ can be written in the following way

$$\lambda \approx \frac{1}{1-\gamma} \left(g_{\sigma\sigma}^{V_0^r}(x, 0) - g_{\sigma\sigma}^{V_0^{PS}}(x, 0) \right) \frac{\sigma^2}{2}$$