Fire sales externalities: how liquidity ratios can help restore the liquidity insurance provided by banks

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June 21, 2013

[To be completed]

Abstract

We use a model *à la* Diamond and Dybvig where banks provide an insurance against liquidity shock to their depositors. They can liquidate their assets before maturity on a market where the supply of liquidity is fixed. Due to private and unobservable information regarding which consumer has been hit by the liquidity shock, banks need to satisfy an incentive compatibility constraint (ICC) to avoid fundamental bank runs. This model exhibits then incomplete markets features: a pecuniary externality due to a fire sales effect arises that implies a cost (and not only innocuous distributional effects). The bank is not able to provide the constrained efficient risk sharing. A policy that relaxes the ICC, such as liquidity ratios, can help get closer to the uconstrained efficiency by increasing the liquidation price. We provide a new rationale for imposing liquidity ratios on banks.

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1 Introduction

The goal of this paper is to provide a new explanation for the fact that fire sales can imply a cost for the economy, a pecuniary externality whose effects go beyond mere innocuous distributional effects. Our explanation relies on the degree of liquidity insurance provided by banks.

We build on the work by Allen and Gale (1998, 2005) on constrained risk sharing in presence of incomplete markets. By incomplete markets in this setting, they refer to an economy where trade contingent securities to provide liquidity in each state are not available. Furthermore, the supply for liquidity is inelastic in the short run: when banks are forced to sell assets to obtain liquidity in some states, this effort might turn out to be self defeating. Allen and Gale themselves build on the seminal work on externalities by Greenwald and Stiglitz (1986) and Geanakoplos and Polemarchakis (1986).

Our model does not rely on a pure Allen and Gale cash-in-the-market-pricing, since investors buying back liquidated asset also have the possibility to invest in a new productive assets and will equate marginal returns of both investment opportunities.

Several papers provide an explanation for the innefficient outcome in presence of fire sales in an incomplete market setting. In Stein (2012), banks creates too much private money which results in innefficiency.

Our contribution is to explain why fire sales effects create a negative pecuniary externality in a Diamond and Dybvig setting where banks are modeled as providers of liquidity insurance, rather than money creators. The risk sharing provided by banks is not efficient in our setting and that is the source of the externality. Banks need to satisfy the incentive compatibility constraint (ICC) that states that early withdrawers must not be paid more than late withdrawers, otherwise triggering a fundamental bank run.

In the decentralised economy, the bank does not take into account the effect of its action on the price, which turns out to be too low with respect to the constrained optimum. This results in an innefficint risk sharing where the impatient consumers do not receive enough consumption.

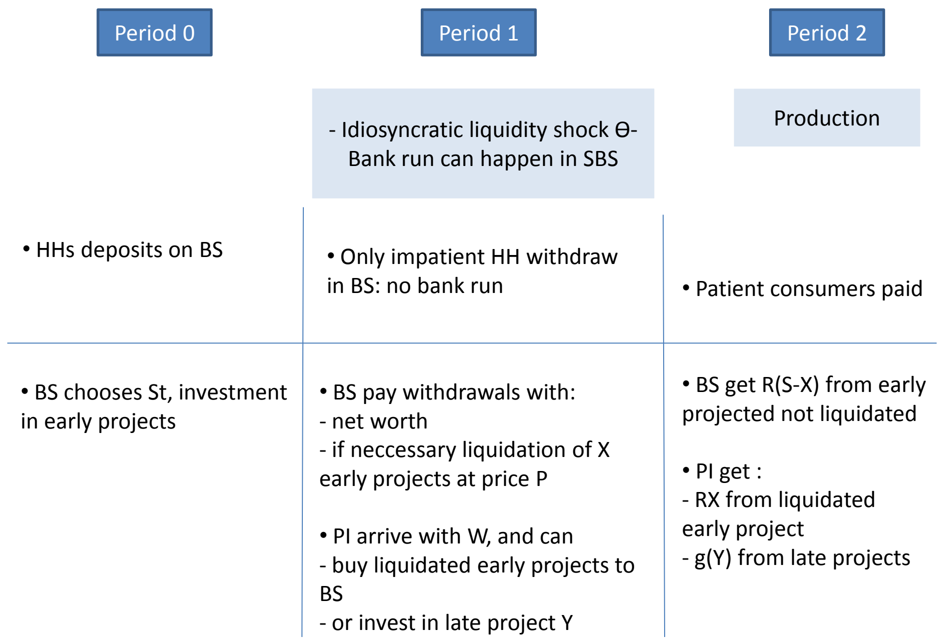
Any policy that relaxes the ICC can help restore efficiency by raising the price. Imposing liquidity ratios allows to relax the ICC and helps the decentralised ecnomy to get closer to the constrained optimum.

2 Environment

In period 0, households deposit their endowment in banks and banks invest in productive projects or in storage. In period 1, a liquidity shock hits consumers which make some depositors want to withdraw early. Consumers that are hit are called early or impatient as they care only about period 1 consumption. To pay them, banks can liquidate productive projects by selling them to patient investors. Those patient investors can either buy back liquidated early projects or invest in new productive projects, called late projects. In period 2, productive projects, both early and late projects; arrive to maturity and depositors who have not withdrawns in period 1 are paid by banks. Patient agent also receive their share of patient investors' profits.

There are 3 agents: There is a continuum of households on $[0, 1]$, one representative banks, and patient investors. There are three periods, 0, 1 and 2. There are 3 assets: a storage technology, early productive projects, and late productive projects.

2.1 Timing



In period 0, households are endowed with 1 unit of wealth which they must deposit in banks[1]. This asset is redeemable in period 1 : they have the possibility to withdraw at this moment. This is the liquid part of the households' portfolio.

As they know a liquidity shock could hit them in period 1 and are risk adverse, they need a bank pooling all resources to insure against this idiosyncratic liquidity risk, as in Diamond and Dybvig.

In period 0, banks can invest theses deposits in two different technologies: either early productive projects or storage.

[1]The assumption that households cannot access storage does not imply any loss of generality as banks can invest in storage on behalf of households and are maximising their expected utility, due to perfect competition and free entry in the banking sector.

In period 1, households are endowed with some units W of wealth, that is managed by patient investors and cannot be consumed directly in period 1. This asset is non reedemable in period 1: this is the illiquid part of households portfolio.

These patient investors are newly arrived in period 1. They can invest in late projects that start in period 1 and mature in period 2 or buy early projects liquidated by the banks. This endowment W is a totally illiquid asset that cannot be used for consumption in period 1 when the liquidity shock hits but must be invested. It can be interpreted as a non redeemable investment on a private equity fund for instance.

The reedemable asset invested in banks is normalized to 1 and the non reedemable assets placed in private equity funds of patient investors is W. Then, we assume that the respective share of liquid and illiquid assets in the households' portfolio is exogenous and not optimally chosen by households.

2.2 Liquidity shock

There is one source of uncertainty in the economy, both idiosyncratic and aggregate: in period 1, a stochastic liquidity shock hits a fraction θ of consumers, where θ has an uniform distribution in $]0,1[$. Impatient or early consumers hit by the liquidity shock only care about time 1 consumption while patient or late consumers only care about time 2 consumption.

Which consumer has been hit by the liquidity shock is a private information, unobservable by banks. They initially do not know the realisation of shock either and progressively discover the size of the shock.

As banks do not know the realization of θ in period 0 but discover it in period 1 progressively, the suspension of convertibility is not optimal to prevent bank runs (Diamond and Dybvig, 1984). The only way of preventing bank run is for the government to provide a deposit insurance. In the model, banks benefit from deposit insurance and are then not subject to sunspot banks runs.

To make clear the distinction between rational and sunspot bank runs, we define bank runs as a situation where patient agents withdraw in period 1 rather than waiting period 2. We identify two different kinds of bank run, rational bank runs, where patient agent earn more if they withdraw early even if all patient agents do not run, and sunspot bank runs, which is a pure coordination failure.

2.3 Technologies

There are 3 technologies: storage, early productive projects and late productive projects.

For 1 unit stored, storage technology yields 1 next period.

Early investments are done in period 0 by banks with a constant return to scale technology. For S invested in period 0, if they are kept until maturity, i.e. until period 2, they yield RS.

Early projects can also be liquidated in period 1, i.e. sold to patient investors. In this case, banks get the market price $P(\theta)$, that depends on the size of the liquidity shock. The patient investors that have bought back the liquidated projects get a return RS in period 1. We then rule out any technological externality whereby liquidation would imply a real cost to the economy because of a change of ownership cost assumption.

Late investments are made in period 1 by the newly arrived patient investors with a deacreasing return technology. For Y invested in period 1, late projects yield $g(y)$ in period 2, where g is a concave function.

Households do not have access to the storage technology in period 0, as explained earlier, but do have access to storage directly in period 1, to allow for the possibility of bank runs.

2.4 Paper's outline

In a first section, I derive the decentralized economy by solving for the banks' problem, the PI's problem and finally the equilibrium.

In a second section, I study the social planner's problem.

In a third section, I study the externality that arises due to a fire sale effect in the decentralized economy that creates an inefficient risk sharing. I study under which conditions liquidity ratios can enhance risk sharing.

3 Decentralized economy

3.1 Assumptions on households' preferences

First, we make some simplifying assumptions that hold all along the paper, to make sure that corner solutions are ruled out.

Assumption 1 $R > 1$

This assumption rules out solution where the bank would invest everything in the storage technology ($S = 0$).

Assumption 2 $u'(0) > RE_\theta[u'[\frac{RS}{1-\theta}]]$

This assumption rules out solution where the bank would invest everything in early projects ($L = 0$).

Assumption 3 $\frac{-cu''(c)}{u'(c)} > 1$

This assumption ensures that households are risk adverse so that they need an insurance against the unlucky outcome of being patient. Then, functional forms for the utility of the type $u(c) = \frac{c^{1-\rho}}{1-\rho}$ with $\rho > 1$ are possible.

Assumption 4 $W \geq 1$

Finally, we assume that the PI have at least as much wealth as the initial deposit in the banks (1 unit of good). This will rule out a collapse of the price of liquidated early projects below 1. Then, investing in early projects can never yield less than storage, even if the bank needs to liquidate early projects.

3.2 Patient investors

To model the market of liquidated early projects and make the return on liquidation endogeneous, we introduce a new agent, the patient investors, as in Stein (2010).

With their fixed exogenous wealth W, PI can either invest $Y(\theta)$ in new productive projects, called late projects or buy back $X(\theta)$ liquidated early projects for a price $P(\theta)$.

The amount invested in late projects $Y(\theta)$ depends on the realisation of the liquidity shock since the price of liquidated early projects depends on the amount of liquidity needed by bank to pay impatient consumers. For these late projects, PI get a technology production $g[Y(\theta)]$, where g is a concave function.

When they buy an amount $X(\theta)$ of liquidated early projects, they get the same return as original investors TBS would have get: $RX(\theta)$. We do not assume that this return is lower, in order to focus only on pecuniary externality arising from a fire sale effect. We do not want to assume any technological externality, whereby the return would be lower in case of liquidation.

As patient agents are the only ones to care about period 2 consumption, profits of patient agents necessarily incur to them. We assume that patient agents get the profits in period 2 whether they have withdrawn early or not. They receive their share of profits in either cases.

We do not need to assume that patient agents have a superior information on who has been hit by the liquidity shock and can distinguish between patient and impatient agents: impatient agents no longer care about consumption in period 2 and have no incentive to misrepresent their types. Only patient agents will ask for the profits.

They choose the amount of late projects $Y(\theta)$ and of liquidated early projects $X(\theta)$ after the liquidity shock is realised. After substituing Y with the constraint $W = P(\theta)X(\theta) + Y(\theta)$, their objective function becomes:

$$\max_{X(\theta)} g[W - P(\theta)X(\theta)] + RX(\theta)$$

The first order condition gives the price of the liquidated early projects as the ratio between the marginal return of buying back early projects and the marginal return on investing in new late productive projects.

$$P(\theta) = \frac{R}{g'(Y)}$$

With a logarithmic production function, we get:

$$P(\theta) = \frac{RW}{1 + RX}$$

So, the price of liquidated assets is decreasing on the number of early liquidated projects. Then, the attempt of banks to get liquidity can be self defeating when all banks need liquidity, which is always the case as banks are identical.

3.3 Bank's problem

3.3.1 Deposit contracts' terms

The bank is not allowed to offer contracts contingent on the realisation of the liquidity shock. Nevertheless, we stress that offering contingent contratc would achieve the efficient risk sharing as outlined in Allen and Gale work (see for instance Allen and Gale 1998).

We refer to a deposit contract as a contract by which the bank promises a fixed payment $\bar{c}$ to anyone withdrawing in period 1. The promise has to hold for anyone, and not impatient consumers only, as the bank cannot distinguish between patient and impatient agents (since it is assumed to be a private information).

If the bank is not able to pay this promised fixed amount to consumers withdrawing at period 1 and at least the same amount to remaining depositors in period 2, it must declare insolvency and liquidate all its portfolio and provides every depositor with an equal share of this liquidated portfolio.

With the above described definition of deposit contracts, we depart from the sequencial service constraint imposed in DD (1983).

Degree of insurance provided by banks
To compare the degree of insurance between autarky and banking solution, the relevant quantity is the level of consumption of a patient agent with respect to an impatient agent. If banking solution provides insurance against the unlucky outcome of being impatient, the banks will provide more consumption than what the impatient consumer would have get in autarky.

In autarky, the consumer would have gotten $c_1^A = 1$ if he were an impatient consumer and $c_2^A = R$ if he were a patient consumer.

The bank insures against the idiosyncratic risk of being impatient but cannot insure against the aggregate risk associated with the size of the shock of liquidity (variation in θ), that is non diversifiable in this setting, due to the absence of complete markets where banks could have bought contingent securities to insure against aggregate liquidity risk.

Payments to households under deposit contracts
In the case of deposit contracts, either the bank is solvent and can pay the promised rate $\overline{c}$. Or, it declares insolvency and liquidates everything, and each consumer (patient and impatient) gets an equal share of the value of its liquidated portfolio: the bank is insolvent.

In case of solvency, $c_1 = \overline{c}$ is the payment by consumer withdrawing in period 1.

In case of insolvency, the number of depositors being one, each depositor gets $c_1 = c_2 = L + SP$.

We stress that this situation of insolvency where patient agents are paid in period 1 must be distinguished from a bank run. Patient agents do not run, they only receive their share of the bank's liquidated portfolio due to a bankruptcy.

Fundamental bank run and the incentive comptability constraint
Following the literature, I define fundamental bank run as a bank run that is triggered by a comparison by patient agent between what patient agent would earn if they withdraw early and store, *conditional on every other patient agent not running*, with respect to waiting period 2 to withdraw.

This means that patient agent should not have an incentive to misrepresent their type, even in the case where they would be the only patient agent to run. The consumption of a patient agent must be at least as high as the consumption of an impatient agent, otherwise, any marginal patient agent would have an incentive to withdraw in period 1 and store until period 2, even if all patient agents wait for period 2.

As it is possible to liquidate early projects and due to deposit contratc terms, rational bank runs become possible. Indeed, patient agents may now have an incentive to run conditional on other patient agents not running, since a total liquidation of assets leaving nothing in period 2 is possible. An incentive comptability constraint is necessary to exclude this type of bank run.

Then, the banking contract must offer a rate that provides at least the same consumption to early and to late consumers to rule out fundamental bank runs. This is called in the literature the self selection constraint [2] or incentive compatibility constraint (thereafter ICC): $c_1 \leq c_2$. As the bank pays either $\bar{c}$ (no bankruptcy) or $c_1 = c_2$ (bankruptcy) to consumers withdrawing in period 1, this condition reduces to:

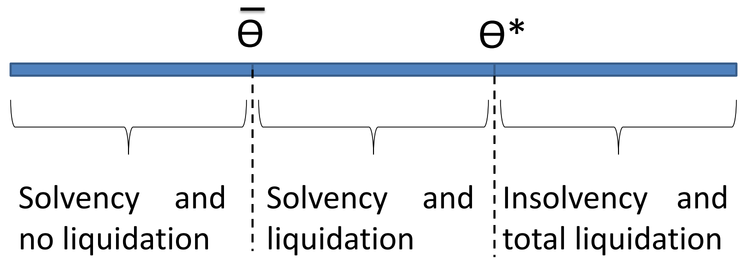
$$\bar{c} \leq c_2$$

As deposit insurance only aims at eliminating sunsport bank run, banks contracts must still include an incentive compatibility constraint.

3.3.2 Three cases

There are now three cases to consider, $i)$ the liquidity shock is so low that the bank does not need to liquidate any early projects i.e. $\theta \leq \overline{\theta}$, $ii)$ the liquidity shock is such that the bank is solvent but need to liquidate $\overline{\theta} < \theta \leq \theta^*$, and $iii)$ the liquidity shock is so high that the bank is insolvent $\theta^* < \theta$.

[2] The self selection constraints states that no agent envies the treatment by the market of other indistinguishable agents, DD 83



The liquidation treshold $\overline{\theta}$ is defined as the level of liquidity shock below which the bank does not need to liquidate to pay $\overline{c}$ to patient agents.

$$\overline{\theta} = \frac{L}{\overline{c}}$$

3.3.3 Solvency condition and ICC

Let now solve for the solvency treshold θ^*. The concept of solvency is closely linked to the ICC. It is when the ICC is no longer satisfied that the bank declares bankruptcy.

The bank is solvent if the present value of consumption promised to consumers satisfying the ICC exceeds the present value of its asset, both side of the balance sheet being evaluated at the market price:

$$\theta\overline{c} + \frac{P(\theta)}{R}(1-\theta)\overline{c} \leq L + SP(\theta)$$

The present value of the entire portfolio of the bank is evaluating considering a total liquidation of early projecs: it corresponds to the maximum liquidity the bank can pay out at period 1.

$\frac{P(\theta)}{R}$ is the price of one unit of good at date 2. It means that in order to satisfy the ICC, the bank must keep at least $\frac{P(\theta)}{R}(1-\theta)\overline{c}$ until period 2 so that the patient consumers can receive exactly $\overline{c}$ at that time. Indeed, if they were to run at time 1 and store, they would get exactly $\overline{c}$. If they were to receive less than that, a fundamental bank run would occur.

$$\theta^*\overline{c} + \frac{P^*}{R}(1-\theta*)\overline{c} = L + SP^*$$

θ^*, the insolvency treshold, is the level of liquidity shock above which the bank is insolvent. We also note $P(\theta*) = P^*$, the price of early projects when the supply is $X = S$, i.e. a total liquidation by banks.

So, the bank is always solvent when $\theta \leq \theta^*$. When the liquidity shock is so high that the bank needs to liquidate ($\theta > \theta^*$), the bank is solvent if prices are above a certain level:

$$P(\theta) \geq \frac{L - \theta\overline{c}}{\frac{(1-\theta)\overline{c}}{R} - S} \equiv P^*$$

3.4 Solution

The bank takes the price $P(\theta)$ as given. Indeed, the individual bank considers that the price is determined by aggregate quantities and that its actions have no impact on the price, due to atomicity since there is free entry and perfect competition in the banking sector.

Besides, the individual bank also takes the patient investors' profits as given.

$$\max_{\overline{c},S,L,X(\theta),c_1,c_2} E_\theta[\theta u[c_1(\theta)] + (1-\theta)u[C_2]]$$

$$\text{subject to} \begin{cases} L + S \leq 1 \\ c_1 \leq \overline{c} \text{ and } c_1 = c_2 \text{ if } c_1 < \overline{c} \\ \theta c_1 + (1-\theta)c_2 \leq R(S-X) + L + PX \\ c_1 \leq c_2 \\ X \leq S \end{cases}$$

where $C_2 = c_2(\theta) + \frac{\pi(\theta)}{1-\theta}$ is the consumption by each patient agents, including the payment by the bank only, c_2, and their share of PI profits.

The problem is solved backward. In period 1, the liquidity shock is realized, and S, L and $\overline{c}$ were chosen in the previous period so are now fixed. The bank chooses X, c_1 and c_2.

In case where $\theta \leq \overline{\theta}$, $c_1 = \overline{c}$ is the payment by impatient agents. The liquidity shock is so low that the bank does not need to liquidate: $X = 0$. Patient agents wait for period 2 and get $c_2 = \frac{L-\theta\overline{c}+RS}{1-\theta}$

In case where $\overline{\theta} < \theta \leq \theta^*$, $c_1 = \overline{c}$ is the payment by impatient. Patient agents wait for period 2 and get $c_2 = \frac{R(S-X)}{1-\theta}$. The number of liquidation is $X = \frac{\theta\overline{c}-L}{P}$.

In case of insolvency, the number of depositors being one, each depositor gets $c_1 = c_2 = L + SP$ and all early projects are liquidated : $X = S$.

In period 0, the bank chooses S, L and $\overline{c}$[3]

[3] $\overline{\theta} \leq \theta^*$ since $\overline{\theta} = L/\overline{c}$, $\theta^* = (L+SP)/\overline{c} - P/R(1-\theta)$ and$SR > (1-\theta)\overline{c}$: the maximum return the patient agents can get from the bank (no liquidation at all so all early arrive to maturity) cannot exceed the minimal return that patient can get (the rate promised to impatient) by ICC.

$$\max_{S,L,\bar{c}} \int_0^{\bar{\theta}} [\theta u(\bar{c}) + (1-\theta)u(\frac{L - \theta\bar{c} + RS\pi(\theta)}{1-\theta})]d\theta$$
$$+ \int_{\bar{\theta}}^{\theta^*} [\theta u(\bar{c}) + (1-\theta)u(\frac{R(S - \frac{\theta\bar{c}-L}{P}) + \pi(\theta)}{1-\theta}]d\theta$$
$$+ \int_{\theta^*}^{1} [\theta u(L+SP) + (1-\theta)u(L+SP+\frac{\pi(\theta)}{1-\theta})]d\theta$$

subject to:

$$S + L \leq 1$$

and where:

$$\theta^* = \frac{LR + RSP^* - \bar{c}P^*}{\bar{c}R - P^*}$$

$$\bar{\theta} = \frac{L}{\bar{c}}$$

The solvency and the ICC are integrated in the bank choice through the bound θ^* of the integral. This condition constraints the choice of $\bar{c}$ by the bank.

When choosing L, S, and $\bar{c}$ in period 1, the bank does not take into account the impact of its investment on the price of early liquidated and so on the solvency constraint of other banks and on their choice of $\bar{c}$.

The last equation makes clear that no externality is due to a change of ownership of early projects since they provide in any case the same yield RS.

The first order condition requires that:

$$\int_0^{\overline{\theta}} u'(C_2)(R-1-\theta)d\theta + \int_{\overline{\theta}}^{\theta^*} u'(C_2)(R-1/P+\theta/P) + (P^*-1)\int_{\theta^*}^1 E(C_B)d\theta$$
$$= \frac{R(P^*-1)(\overline{c}R-P^*)}{P^{*2}-R^2(L+SP^*)}\int_0^{\theta^*}\theta u'(\overline{c})d\theta$$

where C_B is the bankruptcy consumption, i.e. the consumption when the bank is forced to liquidate all the early projects.

3.5 Equilibrium

3.5.1 Analysis of the price

To analyse the price of early projects, we look at aggregate quantities that we note by adding a subscript a. In equilibrium, all banks will choose the same quantity but it is important to distinguish beteween aggregate and individual quantities to emphasize the fact that banks do not take into account that their decision will have an impact on the aggregate quantities and on the price, and so on other baks constraint when choosing $\overline{c}$.

In case where $\theta \leq \overline{\theta}$, no early projects are liquidated so as supply is zero, $P = 0$.

In case where $\overline{\theta} < \theta \leq \theta^*$, some early projects are liquidated, a quantity $X(\theta)$ that depends on θ. In this case, the price also depends on θ. Using the fact that $X(\theta) = L^A - \theta\overline{c^A}$ and the logarithmic production function:

$$P(\theta) = \frac{RW}{1 + RL^A - R\theta\overline{c^A}}$$

In case of insolvency i.e. when $\theta > \theta^*$, all early projects are liquidated : $X^A = S^A$. The quantity sold no longer depends on θ. The price no longer depends on θ either:

$$\underline{P} = \frac{RW}{1 + RS^A}$$

As $W \leq 1$, $\underline{P} \leq 1$, we get that the price cannot collaspe below 1:

$$P(\theta) \leq 1$$

3.5.2 Equilibrium first order condition

We rewrite the first order condition of the bank problem by replacing the price by the equilibirum price.

4 Social planner problem

4.1 Constrained efficient problem

We define the constrained efficient problem as an economy in which the social planner cannot distinguish between patient and impatient consumers so that he may be subject to fundamental bank runs. He thus needs to satisfy the ICC. He must also contract with households on the basis of deposit contracts and cannot offer them contingent contracts.

In period 0, he receives an endowment of 1 and chooses S, L and $\bar{c}$. In period 1, the liquidity shock hits and he receives an endowment W. He must choose Y.

There also exists a solvency treshold θ^*_{soc}. It is defined by the solvency constraint because the social planner must satisfy the ICC as he is not immune to fundamental bank run.

$$\theta\bar{c} + (1 - \theta)\bar{c}\frac{1}{g'(Y)} \leq W + L$$

The problem is solved backward. There are two cases to distinguish, either $\theta \leq \theta^*_{soc}$ or $\theta > \theta^*_{soc}$.

In period 1, if $\theta \leq \theta^*_{soc}$, then the social planner can pay the promised rate and $c_1 = \overline{c}$. In this case, $Y = W - \theta\overline{c}$. The patient consumers are paid $c_2 = \frac{g(W-\theta\overline{c})+RS}{1-\theta}$

If $\theta > \theta^*_{soc}$ the liquidity shock is too high to satisfy the ICC, and the social planner cannot pay $\overline{c}$. c_1^B and c_2^B stand for the consumption when the social planner cannot pay the promised rate. $c_2^B = \frac{RS+g(Y)}{1-\theta}$

The social planner must choose a consumption c_1^B for the early and an investment in late projects Y so as to maximize:

$$\max_{c_1,c_2} \theta u(c_1) + (1-\theta)u(c_2)$$

subject to the budget constraints $W = \theta c_1 + Y$ and $c_2 = g(Y)$. The first order condition is then:

$$u'(c_1^B) = g'(Y)u'(c_2^B)$$

The ratio $u'(c_1^B)/u'(c_2^B)$ does not depend on θ.

In period 0, the social planner must solve:

$$\max_{S,L,\overline{c}} \int_0^{\theta^*_{soc}} [\theta u(\overline{c}) + (1-\theta)u(C_2)]d\theta$$
$$+ \int_{\theta^*_{soc}}^1 [\theta u(c_1^B) + (1-\theta)u(c_2^B)]d\theta$$

5 Pecuniary externality in the decentralised economy and liquidity ratios

We can rewrite the solvency constraint:

$$L - \theta\bar{c} + P(\theta)[S - \frac{(1-\theta)\bar{c}}{R}] \leq 0$$

So, an increase in the price relaxes the constraint. Any policy that helps increasing P can restore the optimal choice of $\bar{c}$. Otherwise, the bank might choose a fixed payment $\bar{c}$ that is too low with respect to the optimum. This means that due to fire sale externality, the bank cannot optimally insure agent against the unlucky outcome of being an impatient consumers.

Crucially when choosing $\bar{c}$ under the solvency and ICC constraint combined, the bank takes the price as given, ignoring its own effect on aggregate quantities of early projects liquidated (as in equilibrium all banks take the same decision) and so on the price. The price in case of total liquidation is known from the PI problem. We can see from the last expression of P^* that:

$$\underline{P} = \frac{RW}{1 + R - RL^A}$$

Then increasing the amount of L^A, the aggregate amount of liquid asset, allows to reduce the probability of defaulting, by increasing the lower bound of prices.

The bank does not take into account the effect of its choice of S and L on the price P and so on the treshold above which the bank is bankrupt. Indeed, the individual banks analyses the treshold as being:

$$\theta^*_{dec} = \frac{LR + P^*RS - \bar{c}P^*}{\bar{c}R - P^*}$$

Instead of understanding that the choice of S and L has an impact on the price which in turn has an impact on the solvency treshold. The correct solvency treshold banks should take into account is the following, where price has been replaced by its equilibrium value from the PI's problem:

$$\theta^*_{true} = \frac{L(1+RS) + RSW - \overline{c}W}{\overline{c}(1+RS) - W}$$

It is clear from these two expressions that $\frac{\partial \theta^*_{dec}}{\partial L} \neq \frac{\partial \theta^*_{true}}{\partial L}$, $\frac{\partial \theta^*_{dec}}{\partial S} \neq \frac{\partial \theta^*_{true}}{\partial S}$ or $\frac{\partial \theta^*_{dec}}{\partial \overline{c}} \neq \frac{\partial \theta^*_{true}}{\partial \overline{c}}$ so that generally, the bank does not choose the true optimal value of its choice variable since it does not fully take into account the impact of its choice on P and so on the solvency treshold.

We can nom rewrite the incentive constraint of the planner and compare it to the ICC in the case of a decentralized economy:

$$\text{Social planner: } \theta\overline{c} + \overline{c}(1-\theta)\frac{u'(c_2^B)}{u'(c_1^B)} \leq W + L$$

The social planner can optimally choose the consumption in case of bankruptcy so that efficient risk sharing is achieved. In the decentralised economy on the contrary, the price does not ensure its role because of incomplete market and because banks do not take into account the impact of their choice on the price.

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