Coordination and Eligible Collateral: Monetary Policy and Asset Prices.

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Very Preliminary. Please do not circulate.

Abstract

What is the consequence of monetary policy when assets are complements for cash instead of being substitutes? This paper analyses asset prices in a model where liquidity is endogenous: agents do not only base their portfolio decision on the payoffs of available assets in the economy, but also on their anticipation of future liquidity provision by the central bank. As a result, implicit guarantees of extension of the collateral framework in times of stress create distortions in asset prices, because agents correlate their risk in their portfolio choices. Therefore, even when the monopoly of central bank money as the settlement asset is assumed, other money-like assets emerge from the private sector which impair central banks' independency. This paper provides a theoretical framework, as well as an empirical assessment of the liquidity premium associated with eligibility at the European Central Bank.

Introduction

When an asset is eligible at the central bank, it automatically becomes liquid from the banks' viewpoint in the sense that it becomes instantly and risklessly convertible into cash. In recent years, central banks have been changing the lists of their eligible assets, arguing that they had to remove the tail risk associated with liquidity in the financial system. This, together with other extraordinary liquidity measures, has resulted in a considerable extension of central banks' balance sheets, which in turn raised considerable attention, if not concern, of the financial markets and the media[1]. Although the motives for extending the list of eligible assets were clear, there was no precise rationale for the size of those extensions. And yet, by doing so, the central bank accepted more risk on its balance sheet. It also created moral hazard because of the implicit bailout guarantee that it provides to banks in times of crisis. In this paper, we propose, as a first step, to study asset prices in response to the changes in collateral frameworks, in a rational expectations model where liquidity matters.

This paper is related to several strands of the literature on banking and monetary policy. To start with, it follows the idea of Farhi and Tirole [7], that the central bank is unable to commit and that the anticipation of this lack of commitment in times of crisis creates imbalances in agents' portfolio choices ex ante. However, it differs from Farhi and Tirole in the sense that it considers specifically the extension of the list of eligible collateral, while they proposed a general monetary policy instrument based on the interest rate. They argue that at the zero lower bound, extending the list of eligible collateral is a substitute for a lower interest rate. In this paper nevertheless, we refine the analysis by studying a setup where mulitple assets compete, and where payoffs, variances and other characteristics matter. As in their paper there are multiple equilibria, but we focus on belief formation and the endogenous emergence of one asset as a liquid asset. From the latter point of view, this work contributes to the body of literature on coordination games and currency crisis in the way the central bank intervention is designed. We rely on Angeletos and Werning (2006, [2]) for the coordination of agents in their portfolio choices: prices convey information. Similar intuitions can be found in Hellwig Mukherji and Tsyvinski (2006) or Metz (2002). However, the collateral game calls for a space of strategies that is more complex than currency crises: instead of two actions, we have a continuous portfolio decision from investors. It is also reminiscent to a question that is raised in Xavier Gabaix's Variable Rare Disasters. Indeed, the latter shows how asset prices are mainly driven by their payoffs in very bad states of the world, that Barro and Ursua (2008) define as disasters. Our argument here is that the role of central banks as lenders of last resort might be anticipated in driving up some asset prices in times of crisis, namely the eligible collateral's prices. Hence, anticipating lax monetary

[1] *The cost of global central bank balance sheet expansion*, by Izabella Kaminska, FTalphaville, Dec 16, 2011. - *Central bank existential crisis confirmed*, by Izabella Kaminska, FTalphaville, Jun 25, 2012. - *The decline of safe assets*, by Cardiff Garcia, FTalphaville, Dec 5, 2011.

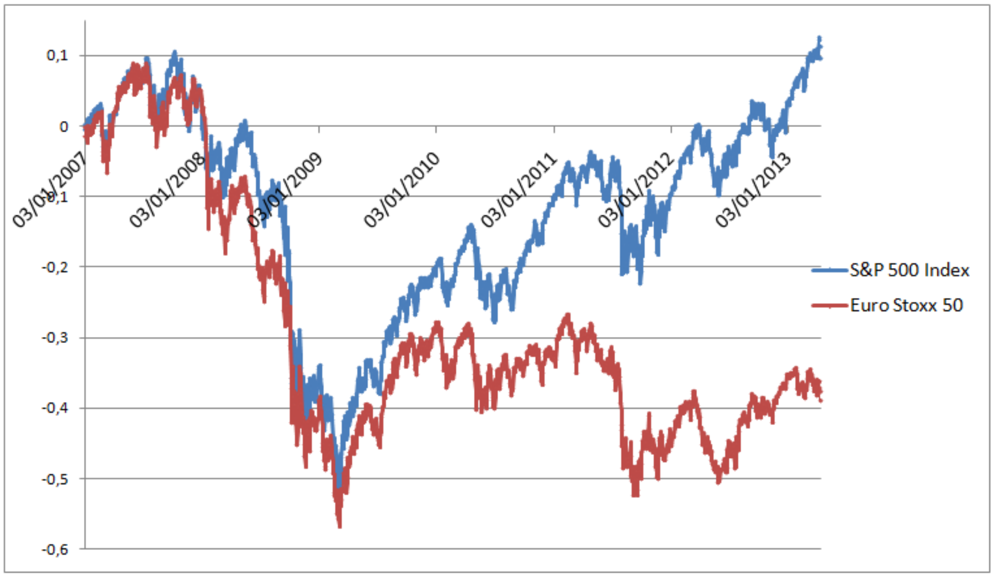
policy in a financial crisis, the variable disaster model of Gabaix might in fact become an endogenous variable rare disaster model. Finally, we claim that this work is related to the on-going interrogations on the role of central banks in the crisis. Ultimately, we want to show that accepting not-so-good collateral on central banks' balance sheets creates sources of instability that selective government intervention can avoid. Although this paper does not study the question of monetary or fiscal dominance, it was inspired by Gourinchas and Jeanne's Global Safe Assets (2012), and attempts to give a measure of the distortions in asset prices related to liquidity injections associated with private bonds as collateral: the latter distortion is the main reason why, according to Gourinchas and Jeanne, the central bank should "bailout" the government instead of buying more private bonds.

We base our work on Lester Postelwaite and Wright [14]. In their paper, they ask how to model a liquidity premium in a tractable model where assets are used as collateral to borrow from one another. They describe an infinite-horizon economy, where the decomposition of each period into two subperiods enables them (1) to describe liquidity, (2) to keep their model tractable, in the sense that they do not need to track the distribution of wealth of agents although the latters face idiosyncratic shocks. We keep those two features, but depart from their description of a decentralized market with bilateral trade. We focus on the role of central banks in the determination of the liquidity of assets: therefore, we dedicate their decentralized market to bilateral trade with the central bank: agents facing a liquidity shock have no choice but going to the central bank and using their eligible assets to get short term loans for their immediate consumption. Contrary to the usual view that a liquidity shortage creates crises, like in Farhi and Tirole (2011a), we reverse the statement and suggest that it is rather the anticipation that central banks will flood the market with new liquidity in times of crisis that creates imbalances, and volatile asset prices. Consequently, while Fahri and Tirole argue (in [8]) that "liquidity is self-fulfilling: A perception of future illiquidity creates current illiquidity", this paper on the contrary focuses on the idea that "A perception of future liquidity creates current liquidity".

Contents

1 Stylized Facts on Asset Prices

Although the American and the European economies have not yet recovered from the crisis (high unemployment levels, low credit, low growth), Wall Street experiences in 2013 record-high levels of stock prices, contrary to European market-places. Investors and policy-makers recently emphasized that those prices are far above fundamentals and that the Federal Reserve's lax monetary policy might have sown the seeds of the future crisis[2].



This controversial issue is backed by anecdotal evidence. On 23 April 2013, the impact of the release of Markit Economics PMIs (Performance Monitoring Indicators) on stock prices restarted the debate on the anticipation of non-conventional monetary policy by the market. Although it was no proof that implicit bailout guarantees are responsible for crises, this event suggests that the market has been learning from past non-conventional monetary policy instruments. Markit is an independent corporation that provides regular financial information on the state of world economies, considered as an important reference by financial markets. On April 23, between 8AM and 9AM, the release of Eurozone, Germany and France PMIs revealed a new decline in growth and further economic depression. The market's reaction at the next marketplace opening at 9AM was a large increase in stock prices (see figure 1). Several newspapers interpreted this paradoxical effect as the result of anticipation of renewed non-conventional monetary policy[3].

[2]See for example *Fed contre BCE: le match des politiques monétaires face à la crise*, published in Les Echos, 17 april 2013.

[3]See Les Echos, *Les marchés placent leurs espoirs dans un nouveau geste de la BCE*, 24/04; or on FT.com, *Stocks rise on hopes for more central bank help*, 24/04.

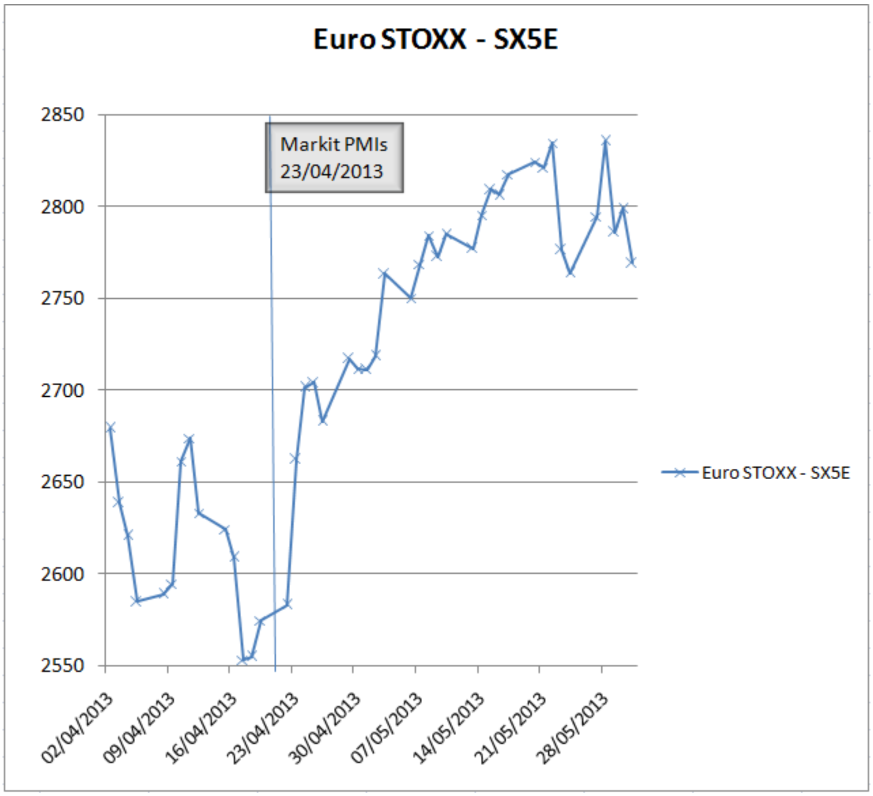


Figure 1: Euro Stoxx price after the release of Markit PMIs on April, 23 2013. Source: Stoxx.com.

In this paper, we ask how to measure a liquidity premium associated with the non-conventional arm of monetary policy. We analyse the effect of the anticipation of future bailouts from the central bank, which consists in lowering the haircut on various asset classes, on the price of assets. This analysis can be seen as a refinement of the Vissing-Jorgensen and Krishnamurthy (2012, [13]) model of liquidity demand. In their paper, liquidity is assimilated to treasury debt: the higher the volume of public debt, the lower the price of liquidity, see figure 2. In the latter figure, we added three red points, which represent the values of the Aaa-Treasury spread on long yields to the value of US government debt to GDP. Our point is that the post-crisis figures do not seem consistent with their simplifying assumption that government debt is the only safe heaven. We argue that there are other money-like assets, and that their price might depend on the anticipation of the scope of future monetary policy. Consistently with this idea, the bottom graph in figure 3 shows that the spread between the yield of BBB assets and AAA assets rose sharply at the time where liquidity became threateningly scarce in 2008, before the Fed extended the list of collateral asset to less highly graded bonds: corporate AAA-rated assets attracted investors in their flight to quality.

Finally, figure 4 shows respectively the eligible collateral of the European Central Bank (in volume) and the volume of collateral effectively posted by banks during the same period.

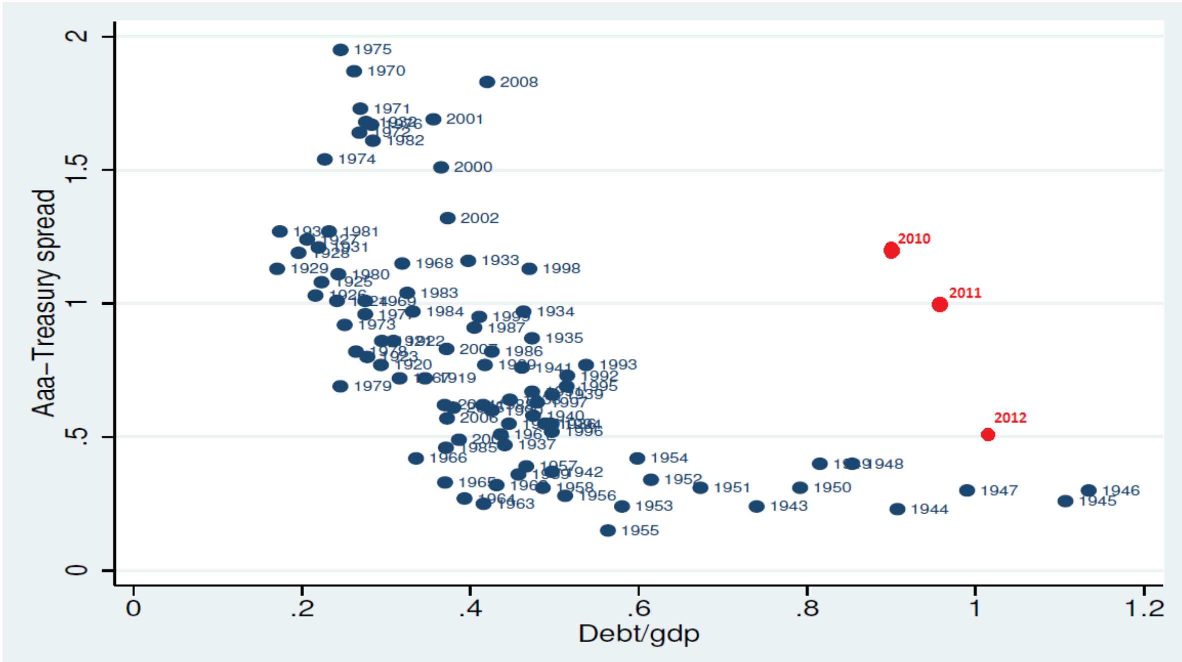


Figure 2: Spread AAA-corp. bonds vs. T-bill, as a function of endebtment - US data. Source: Krishnamurthy & Vissing-Jorgensen (2012).

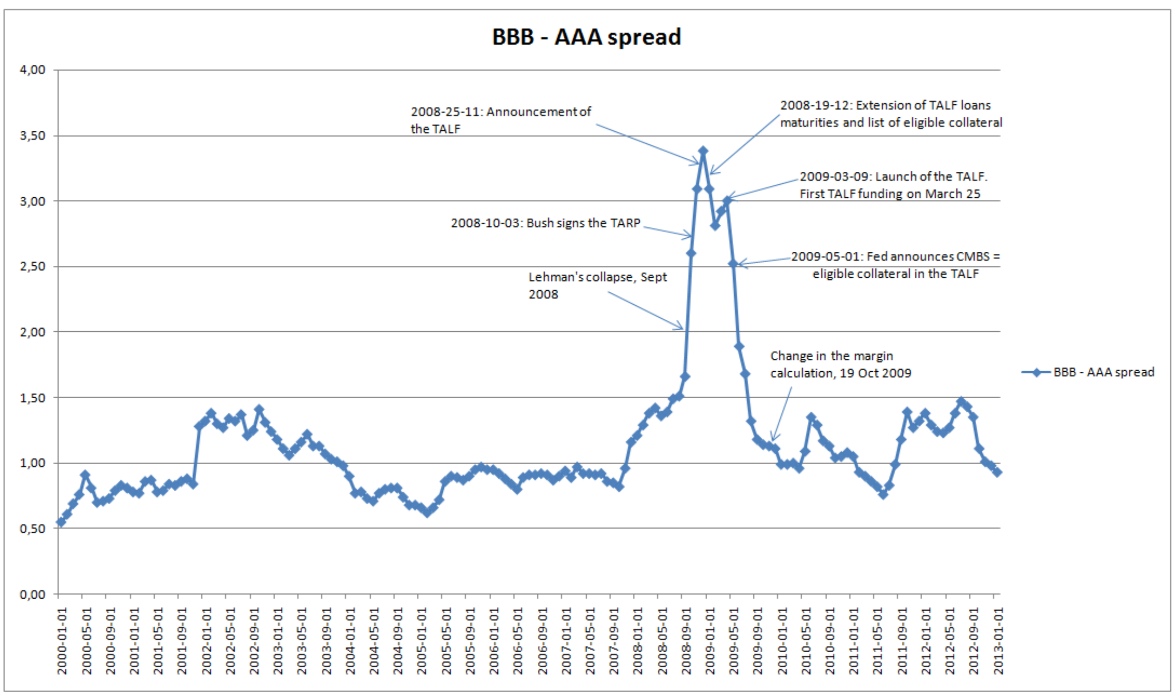


Figure 3: BBB - AAA bond spread Source: Datastream.

The fraction of private bonds being used as collateral increased dramatically, which might be a sign of banks' highly encumbered balance sheets. As a result, eligible corporate bonds have a higher value, reflecting their liquidity.

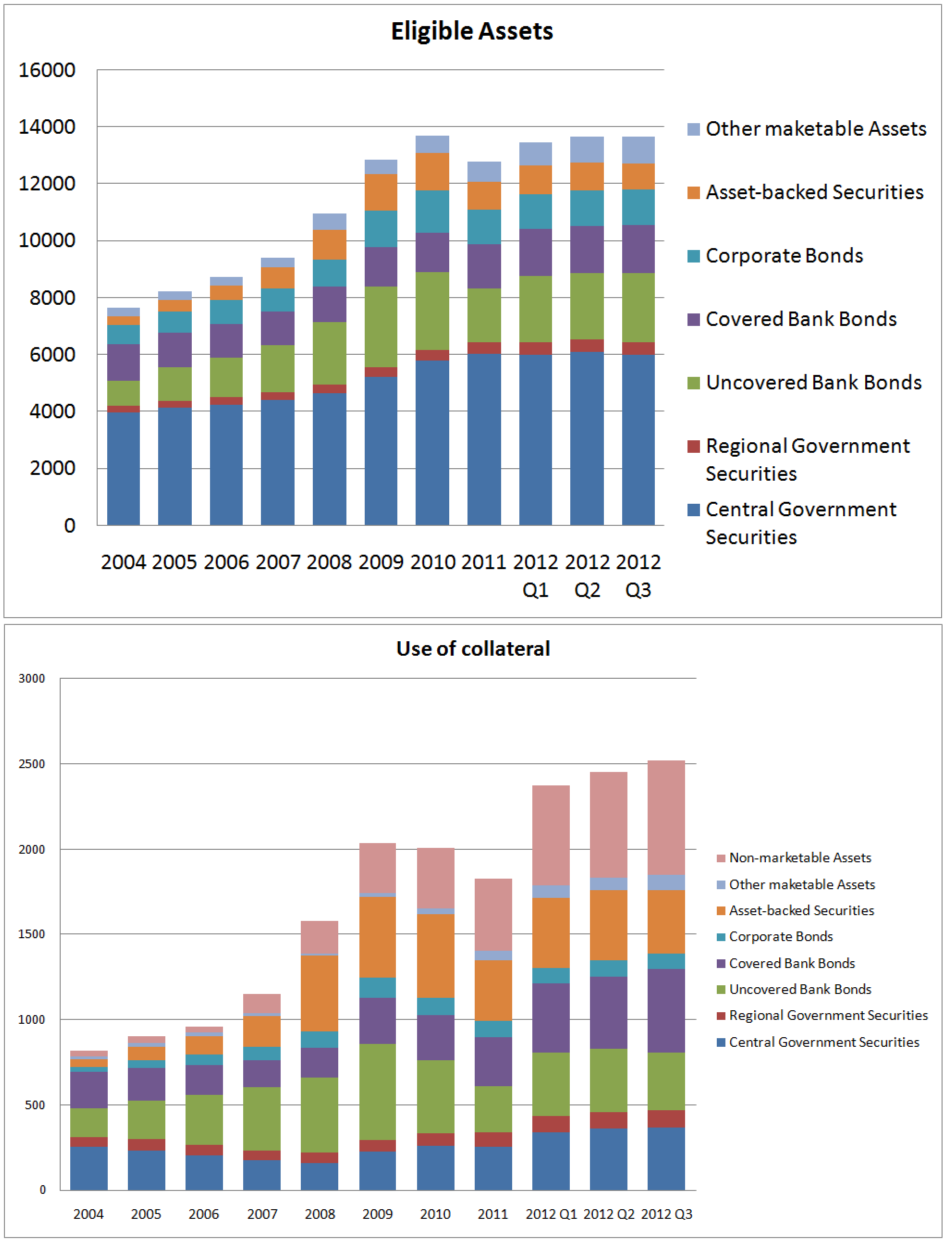


Figure 4: top: Eligible collateral at the European Central Bank, in volume, between 2004 and 2012. bottom: Collateral posted at the European Central Bank between 2004 and 2012. Source: ECB.

2 The Setup with N assets

We strongly ground this model on Lester, Postelwaite and Wright (2011) (henceforth LPW), and use some insights from Berensten and Monnet [3]. The reason for this choice lies in the connection of those contributions with macro-based asset pricing models: we give a high value to the ability to calibrate and give quantitative results. In this setup, the infinite time horizon solution remains tractable in spite of the existence of idiosyncratic shocks: we break down every period into two subperiods, one where agents are constrained, and the other where no constraint binds so that all agents keep holding the same portfolio. The present paper constructs a consumption-based asset pricing model in order to explain and quantify the macro impacts of micro frictions. Traditionally, macroeconomists have taken the view that frictions in asset trades are small enough to be neglected in the analysis: asset prices are set "as if" there were no intermediaries but instead a grand walrasian auction directly between consumers. But this approach has largely shown its limits since Mehra and Prescott's equity premium puzzle, and more generally to understand what liquidity is in the macroeconomy. Here, we intend to embed a stylized model of early and late consumers where liquidity is provided by the central bank, and where the anticipation of monetary policy creates distortions in prices.

2.1 Liquidity premium of eligible assets

There is a $[0,1]$ continuum of infinitely-lived agents. Time is discrete. Each period t, two perfectly competitive markets open sequentially. First, at say, $t+0$, agents are in a centralized market (CM), where they can buy and sell bonds between them, so as to consume the main good X and get the utility $U(X)$. Then, at, say, $t+1/2$, they face a liquidity shock with probability λ, in the sense that they need to consume another good q, and enter a decentralized market (DM) where they can get cash from the central bank. The remaining $1-\lambda$ agents, call them sellers, can produce the good that buyers want to consume but they cannot get paid immediately. They can however grant the buyers with credit, and we assume that they only accept cash as collateral[4]. They produce a quantity q of the good at cost q, and get paid at their marginal cost 1. In order to consume more than their cash holdings would enable them to, buyers might go to the central bank, post collateral, get cash in exchange, buy the good q with the money and consume it, getting the utility $u(q)$. In the next CM, the walrasian auction takes place all over again, buyers get back the money from

[4]In LPW, agents accept various assets as collateral on the decentralized market, and the equilibrium set of liquid assets depends on one of their characteristics, a cognitive cost κ that varies across assets. Here we focus on the role of the central bank in the process of liquidity provision, and therefore rule out the possibility that sellers recognize any asset but money in the decentralized market: we do not discuss what are the properties of a durable good that can make it liquid, but how the anticipation of future non-conventional monetary policy affects the prices of assets before crises.

sellers and pay them using their initial portfolio, while giving back the money to the central bank. As a result, the money created by the central bank at $t + 1/2$ for the consumption of good q goes back to the central bank in the next period, so that there is no definitive money creation on the DM: this is because default never occurs. The idea of consumers' liquidity shock in the interim period purposely recalls the Diamond-Dybvig concept of patient and impatient consumers.

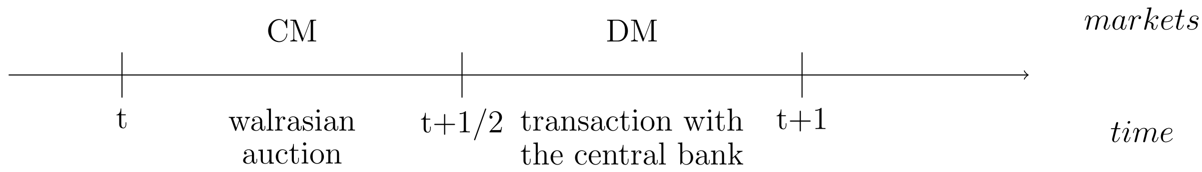


Figure 5: Timing.

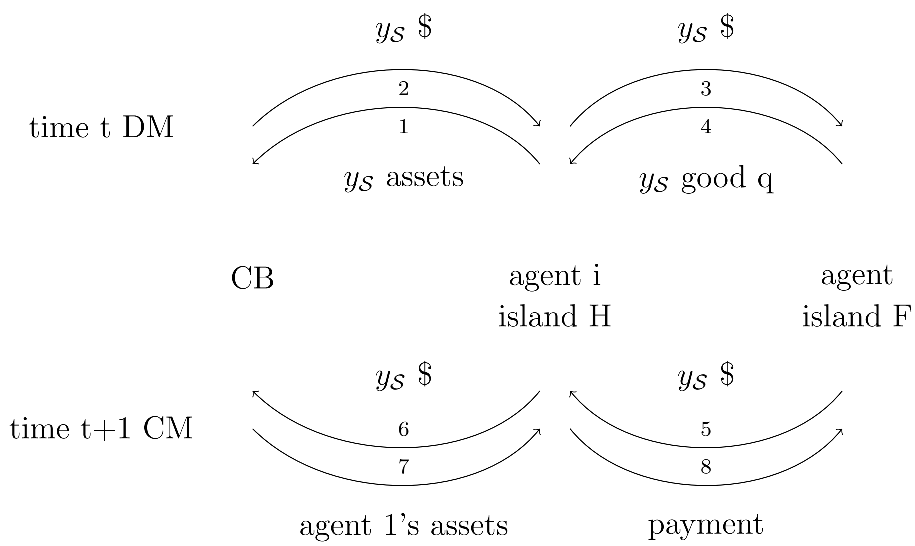


Figure 6: Timing.

In this setup, only in the DM there is a problem of distinguishing between high- and low-liquidity assets, because the central bank only considers a fraction of assets as eligible for their credit operations. Let $\{1, 2, ..., n\}$ be the index of assets in the economy[5], and call $\mathcal{P}$ its power set. Call $\mathcal{S}$ the list of liquid assets[6]. Then ρ_S is the probability that $\mathcal{S}$ will be the list of liquid assets at $t + 1/2$. For now, we take it as exogenous. Later on, we will consider the central banker's problem and endogenize ρ_S.

[5]Pour l'instant on reste dans un environnement discret comme dans LPW car passer a un continuum d'actifs complique beaucoup la resolution du probleme: il faut alors maximiser la value function de l'agent non plus sur un sous-espace de $\mathbb{R}^n$ mais sur un ensemble de fonctions $\{a : [0, 1] \to \mathbb{R}^+\}$.

[6]That is, assets that can be traded on the decentralized market.

An asset $j \in \{1, 2, ..., n\}$ yields a dividend δ_j and has a price ϕ_j at the beginning of each period. Let a_j be the quantity of asset j held by an agent. Hence, the amount of liquid wealth is $y_S(\mathbf{a}) = \sum_{j \in S}(\delta_j + \phi_j)a_j$. Total wealth is larger than liquid wealth: $y_S(\mathbf{a}) \leq y(\mathbf{a}) = \sum_{j=1}^{n}(\delta_j + \phi_j)a_j$.

Let $V(\mathbf{a})$ be the value function of an agent in the DM and $W(y(\mathbf{a}))$ the value function in the CM. Because all assets are recognized in the CM, the value function does not depend on the specific portfolio but on total wealth. In other words, the CM works like a usual Arrow Debreu world, whereas the DM is subject to constraints and consists only in transactions with the central bank. The CM problem is:

$$W(y) = \max_{X \in \mathbb{R}, H \in [0, \bar{H}], \mathbf{a}' \in \mathbb{R}_+^n} \{U(X) - H + \beta E_\delta[V(\mathbf{a}')]\}$$

s.t.

$$X + \sum_j \phi_j a'_j = H + y(\mathbf{a})$$

In this formulation of the problem, we assumed linear production technology, so that one unit of labor H produces one unit of good X, the numeraire. A perfectly competitive production sector thus implies that real wage is equal to 1. On the RHS, the expectation is taken on δ: uncertainty here is on the future dividends, and the list of eligible assets $\mathcal{S}$ might also be state-contingent. Assume that the constraint $H \in [0, \bar{H}]$ is not binding. Then we can rewrite the problem as:

$$W(y) = U(X^*) - X^* + y + \max_{\mathbf{a}' \in \mathbb{R}_+^n} \left\{ -\sum_{j=1}^{n} \phi_j a'_j + \beta E_\delta[V(\mathbf{a}')] \right\}$$

where X^* comes from the maximization of $U(X) - X$ over $\mathbb{R}^+$. Because the agent is able to choose between leisure and consumption in the CM, and because he can disentagle this trade-off from the portfolio decision (after we replace H with the expression given by the budget constraint, we still maximize U over X), there is no CM-pricing kernel modifying the price of assets, as would be the case in a usual asset pricing framework. However, a pricing kernel will arise from the continuation value V, where the composition of the portfolio matters to the agent, for his interim consumption. The FOC on the RHS writes:

$$\beta E_\delta \left[\frac{\partial V}{\partial a'_j} \right] = \phi_j \text{ if } a'_j > 0 \tag{1}$$

$$\leq \phi_j \text{ if } a'_j = 0 \tag{2}$$

We can now express V as a function of W and the utility coming from the consumption of good q, that is to say, the problem of agents on the DM. In the DM, agents trade with the central bank to fulfill their need of cash for consumption at the interim period. The transaction takes the form of a repurchase agreement: in order to get a loan, agents have to post assets as collateral at the central bank (a secured loan), until they repay their loan in the next period's CM. All assets $j \in \mathcal{S}$ are eligible as collateral. Because the function $W(y)$ is linear in y, we can write:

$$V(\mathbf{a}) \equiv \max_q \{W(y(\mathbf{a}) - q) + u(q)\} = W(y(\mathbf{a})) + \max_q \{u(q) - q\}$$

s.t.[7]

$$q \leq y_s(\mathbf{a}) \tag{3}$$

Let q^* solve $u'(q) = 1$. If $q^* > y_s(\mathbf{a})$, then the equilibrium value of q is $q_{\mathcal{S}}(\mathbf{a}) = y_{\mathcal{S}}(\mathbf{a})$. If on the contrary $q^* \leq y_s(\mathbf{a})$ then $q_{\mathcal{S}}(\mathbf{a}) = q^*$. The expectation of V is then:

$$E_\delta[V(\mathbf{a})] = W(y(\mathbf{a})) + \sum_{\mathcal{S}\in\mathcal{P}_j} \rho_{\mathcal{S}}\{u(q_{\mathcal{S}}(\mathbf{a})) - q_{\mathcal{S}}(\mathbf{a})\}$$

We then derivate this expression so as to rewrite equation (1):

$$E_\delta\left[\frac{\partial V}{\partial a'_j}\right] = W'(y) \times \frac{\partial y}{\partial a_j} + \sum_{\mathcal{S}\in\mathcal{P}_j} \rho_{\mathcal{S}}\left\{u'(q_{\mathcal{S}}(\mathbf{a})) \times \frac{\partial q_{\mathcal{S}}}{\partial a_j} - \frac{\partial q_{\mathcal{S}}}{\partial a_j}\right\} \tag{4}$$

Assume the constraint is binding. Then, for all j:

$$E_\delta\left[\frac{\partial V}{\partial a'_j}\right] = 1 \times (\delta_j + \phi_j) + \sum_{\mathcal{S}\in\mathcal{P}_j} \rho_{\mathcal{S}}(\delta_j + \phi_j)\left\{u'(q_{\mathcal{S}}(\mathbf{a})) - 1\right\}$$

We assume that assets are in fixed supply $\mathbf{A}$. Rewriting (1) with the market clearing condition $\mathbf{a} = \mathbf{A}$:

$$\phi_{j,t} = \beta E_\delta\left[\frac{\partial V}{\partial a'_j}\right] = \beta E_\delta\left[(\delta_{j,t+1} + \phi_{j,t+1})\left\{1 + \sum_{\mathcal{S}\in\mathcal{P}_j} \rho_{\mathcal{S}}\left\{u'(q_{\mathcal{S}}(\mathbf{A})) - 1\right\}\right\}\right] \tag{5}$$

$\forall j$, and where $u'(q_{\mathcal{S}}) - 1$ is the liquidity premium: liquid assets have a higher value than their fundamental given by the stream of dividends δ_j, because they are useful in the DM. In the next section, we consider an economy where there are "normal times" and "crises". Agents then make their portfolio decision so that assets that are not eligible during normal times become eligible in crises: the anticipation of future non-conventional monetary policy

[7] Note that we put no haircut for now, though it might be interesting this other degree of freedom has been extensively used by central bankers who extended the list of eligible assets. Moreover, the LPW-model is here importantly simplified, since we have no such thing as nash bargaining and search mechanism: the DM consists in a simple loan from the central bank, and for now the interest rate on this loan is set equal to zero.

results in an inefficient liquidity premium - this is what we will call a bubble. This result is in line with Farhi and Tirole's idea that bubbles arise when liquidity is scarce. The premium is stricly larger than 0 if and only if the supply of safe asset is low enough for the constraint (3) to be binding. Otherwise, $q_{\mathcal{S}}(\mathbf{A}) = q^*$ and $u'(q_{\mathcal{S}}(\mathbf{A})) = 1$.

Conclusion: Equilibrium asset prices are given by any sequence $\{\phi_{j,t}\}_{t=0}^{\infty}$ satisfying (5), a non-negativity condition, and a boundedness condition.

We now endogeneize the size of the shock on the banking system, and specify the central banker's decision rule, in order to study strategic complementarities: the central bank is forced to intervene because banks coordinate in their portfolio decision.

2.2 The central banker's decision rule

We consider a welfare-maximizing central banker. If there was no cost of accepting every asset as collateral, the central banker would implement $\mathcal{S} = \{1, ..., n\}$, reducing the liquidity constraint in the DM to its minimum. Yet, we observe that most central banks have strict collateral rules that do not include all asset classes. In this section, we microfound this feature using information costs and the existence of counterfeits, hence following an idea by LPW.

Nevertheless, we depart from the usual way in which central banks are modeled in macroeconomics. In most macroeconomic or asset pricing models, the central bankers' policy instrument is exclusively the interest rate, which is set in such a way that the inflation target is reached. Here, we focus on the other arm of monetary policy, namely haircuts and the choice of eligible collateral, and we argue that they may play a complementary yet crucial role. We consider only the case where the monetary policy interest rate is 0, so the economy is at the zero lower bound. Inflation is managed directly by choosing the supply of money, say $M = A_1$ if money is asset 1, so the central banker has two degrees of freedom: M, money supply, and $\mathcal{S}$, the list of eligible assets. This new way of understanding monetary policy fits, for instance, the very fundamental rules defined by the European Monetary Institute when establishing the European System of Central Banks, between 1995 and 1999: on December 1st 1998, the Governing Council of the EDM agreed on the reference value for monetary growth and the definition of the specific broad monetary aggregate for which the reference value would be announced. The chosen monetary aggregate was M3[8], and the target was set to 4.5% growth per year. Similarly, the present model allows for the liquidity of assets

[8]Press release December 1, 1998: "M3 will consist of currency in circulation plus certain liabilities of Monetary Financial Institutions (MFIs) resident in the euro area and, in the case of deposits, the liabilities of some institutions that are part of central government (such as Post Offices and Treasuries). These liabilities included in M3 are: overnight deposits; deposits with an agreed maturity of up to two years; deposits redeemable at notice up to three months; repos; debt securities with maturity of up to two years; unit/shares of money market funds and money market paper (net)".

to play a role, and for the central bank to have liquidity-setting powers. Here we view $y_{\mathcal{S}}$ as the monetary aggregate M3, while A_1 would be the monetary base M0. The model is yet too stylized to distinguish between M1, M2 or M3, as the press suggests one should do[9]: a description of the interbank market would allow to distinguish between those different monetary aggregates, but for the time being would complicate the model without providing important additional insights.

We consider $\mathcal{S} \subset \mathcal{S}' \subset \{1, ..., n\}$. Once in the DM, let $\Pi(\mathcal{S}, \mathcal{S}')$ denote the benefit to extend the list of eligible collateral from $\mathcal{S}$ to $\mathcal{S}'$. This benefit comes from the surplus that goes to agents facing a liquidity shock at $t + 1/2$, $u(y_{\mathcal{S}}) - y_{\mathcal{S}}$ if $\mathcal{S}$ is implemented, and $u(y_{\mathcal{S}'}) - y_{\mathcal{S}'}$ if $\mathcal{S}'$. Then:

$$\Pi(\mathcal{S}, \mathcal{S}') = \lambda \left[u(y_{\mathcal{S}'}) - y_{\mathcal{S}'} - u(y_{\mathcal{S}}) + y_{\mathcal{S}}\right]$$

We now assume that there is a technological cost $\bar{\kappa}$ of accepting an asset j as collateral, that allow the central banker to recognize the quality of j[10]. For example, central banks have important risk management divisions, controlling asset flows on their balance sheets associated with discount window lending: if more assets have to be controlled, more workforce is also needed[11]. Then the central banker will accept to extend the list of eligible assets if and only if the benefit of the extension is larger than the cost:

$$\Pi(\mathcal{S}, \mathcal{S}') > \bar{\kappa}$$

In this model, they will always set the haircut to be 100%, since there is no residual risk on the asset once on the DM and no inflation associated with the volume of liquidity $q_{\mathcal{S}}$.

[9] *The collateral crunch gets monetary*, by Izabella Kaminska, FT alphaville, January 3, 2012.

[10] There are arguments against the fixed technological cost that we propose in this paper, as well as the absence of uncertainty on assets in the DM. Since 2008, it seems that one reason why central banks do not accept every asset as collateral in their credit operations is that they want to limit the risk taken on their balance sheets, so as to reduce capital loss and subsequent inflation risks. This effect could be analysed only if there was some residual uncertainty between the period-t DM and the period-t+1 CM. Such a model should then also take into account the definition of haircuts on eligible collateral by the central bank. However, it seems that the risk related to balance sheet expansion is not the only explanation for collateral restrictions. In fact, we observe that all assets that are used as collateral in central banks are given haircuts below 50%, creating an empty haircut intervalle $[0.5, 1]$, where no eligible asset stands. Consequently, non-eligible assets are not necessarily those for which haircut is above 100%. Some assets, for which rational haircuts should stand between 0.5 and 1, remain excluded from central banks' eligibility criterium. A simple way to model this is by introducing this cognition cost $\bar{\kappa}$.

[11] This ad-hoc explanation is however just for heuristics. Philippon and Skreta (2012) provide a micro-foundation for the cost of accepting assets as collateral, based on adverse selection. In their setup, firms finance risky projects by contracting debt with the market. For any given level of investment, they exhibit a cost-minimizing intervention: they show that when the central banker accepts an asset as collateral and provides banks with cash, he always makes a loss above some lower bound. They find that debt contracts can then enable the central bank to achieve the cost-minimizing intervention.

Once they have accepted to pay the fixed cost $\bar{\kappa}$, they find it optimal to accept the entire volume of asset-holdings in $\bar{\mathcal{S}}$ in the cash provision process so as to maximize consumption on the DM.

3 Equilibrium asset prices in the three-asset case

To start with, we assume that there are only 3 assets $\{1, 2, 3\}$. There are two states of the world: $\{\bar{\delta}_1, \bar{\delta}_2, \bar{\delta}_3\}$ the state of the world 1 occurring with probability $1 - \pi$, and $\{\underline{\delta}_1, \underline{\delta}_2, \underline{\delta}_3\}$ the state of the world 2, occuring with probability π. One can say that π is the probability of a crisis occuring, if all $\underline{\delta}_j \leq \bar{\delta}_j$. We assume that $\mathcal{S} = \{1\}$, but can be extended to $\mathcal{S}' = \{1, 2\}$ or $\mathcal{S}' = \{1, 3\}$ if agents are "too much" liquidity constrained: at the zero lower bound, the central banker can increase liquidity only by extending the list of eligible assets. We first describe what determines that $\bar{\mathcal{S}} = \{1, 2\}$ rather than $\{1, 3\}$ or $\{1, 2, 3\}$, for given prices that the central banker observes at $t + 1/2$. In a second subsection, we describe how multiple equilibria emerge from the anticipation of the central bank's collateral rule. Finally, we break down the number of equilibria to one, thanks to coordination and information games.

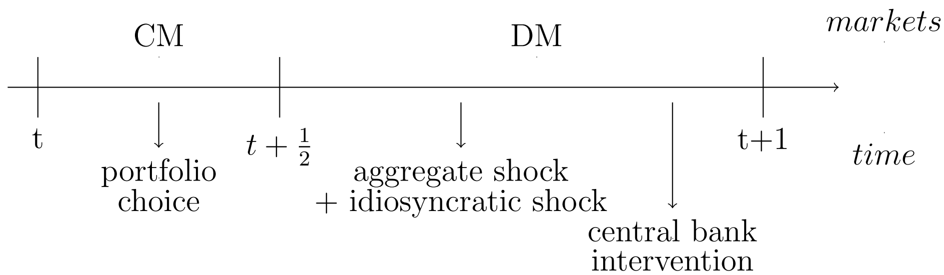


Figure 7: Central banker's decision rule and shocks

3.1 Computation of the liquidity premium and self-fulfilling equilibria

We study self-fulfilling equilibria, which are created by the anticipation of collateral list extensions by the central bank. We show that, if everybody anticipates that in the bad state of the world, the central banker will extend the list of eligible collateral to a new set $\bar{\mathcal{S}}$, then indeed, when it occurs, the central banker is forced into accepting $\bar{\mathcal{S}}$. When there is no anticipation of the list extension on the contrary, the liquidity will be sufficient for the central bank to be able to keep $\mathcal{S}$ unchanged: there are multiple self-fulfilling equilibria.

We proceed by showing that there exists $\bar{\kappa}$ such that multiple equilibria exist. Let us call $g : x \mapsto u(x) - x$. Then the benefit arising from the extension of the eligible asset list, from the central banker's viewpoint, is $\Pi(\mathcal{S}, \mathcal{S}') = \lambda \left[g(y_{\mathcal{S}'}) - g(y_{\mathcal{S}})\right]$. Mulitple equilibria exist if:

- With anticipation of the change in central bank collateral framework, there is a liquidity shortage with $\mathcal{S} = \{1\}$, compared with $\mathcal{S}' = \{1, 2\}$, only in the bad state of the world. In other words, the gain from extending the list of eligible assets is larger than the cost during a crisis.
- With no anticipation of the change in the collateral framework, there is no sufficient liquidity shortage in either state to compensate for the cost $\bar{\kappa}$: the central banker keeps $\mathcal{S}$ as the list of eligible assets in either state of the world.

In the case where agents anticipate an extension of the collateral framework from $\mathcal{S}$ to $\bar{\mathcal{S}}$ with probability 1, prices are denoted ϕ_j^{nc} (the superscript nc is for "no commitment"). The central banker decides to extend the list of eligible assets in the bad state of the world if:

$$g(\underline{y}_1^{nc} + \underline{y}_2^{nc}) - g(\underline{y}_1^{nc}) > \frac{\bar{\kappa}}{\lambda}$$

The condition for the central banker to leave the collateral list unchanged in the good state of the world writes:

$$g(\bar{y}_1^{nc} + \bar{y}_2^{nc}) - g(\bar{y}_1^{nc}) < \frac{\bar{\kappa}}{\lambda}$$

On the other hand, in the case where agents anticipate no change in the collateral list $\mathcal{S}$, prices are denoted ϕ_j^c. Then the condition for the central banker never to change the list of eligible assets in either state of the world writes:

$$g(\bar{y}_1^c + \bar{y}_2^c) - g(\bar{y}_1^c) < \frac{\bar{\kappa}}{\lambda}$$

$$g(\underline{y}_1^c + \underline{y}_2^c) - g(\underline{y}_1^c) < \frac{\bar{\kappa}}{\lambda}$$

A cost $\bar{\kappa} \in \mathbb{R}$ such that the central banker extends the list of eligible assets only when agents anticipate this change exists if and only if all the three following conditions are verified:

$$g\left(\sum_{j=1,2} (\underline{\delta}_j + \phi_j^{nc})A_j\right) - g\left((\underline{\delta}_1 + \phi_1^{nc})A_1\right) > g\left(\sum_{j=1,2} (\underline{\delta}_j + \phi_j^{c})A_j\right) - g\left((\underline{\delta}_1 + \phi_1^{c})A_1\right) \quad (6)$$

$$g\left(\sum_{j=1,2} (\underline{\delta}_j + \phi_j^{nc})A_j\right) - g\left((\underline{\delta}_1 + \phi_1^{nc})A_1\right) > g\left(\sum_{j=1,2} (\bar{\delta}_j + \phi_j^{nc})A_j\right) - g\left((\bar{\delta}_1 + \phi_1^{nc})A_1\right) \quad (7)$$

$$g\left(\sum_{j=1,2} (\underline{\delta}_j + \phi_j^{nc})A_j\right) - g\left((\underline{\delta}_1 + \phi_1^{nc})A_1\right) > g\left(\sum_{j=1,2} (\bar{\delta}_j + \phi_j^{c})A_j\right) - g\left((\bar{\delta}_1 + \phi_1^{c})A_1\right) \quad (8)$$

Consider the function $h : (x, y) \mapsto g(x + y) - g(x)$. Because the agent is assumed to be constrained on the decentralized market, $u' < 1$, and because of the concavity of the utility function, $u'' < 0$, we get that $\partial_x h < 0$ and $\partial_y h > 0$. So a set of sufficient conditions for equation (6) to hold in equilibrum is: $\underline{y}_1^{nc} < \underline{y}_1^c$ and $\underline{y}_2^{nc} > \underline{y}_2^c$. We show in appendix 6.1 that $\phi_1^c > \phi_1^{nc}$ and $\phi_2^{nc} > \phi_2^c$. This result is quite intuitive. It says that the liquidity premium on asset 1 is larger when the central banker is able to commit to keep $\mathcal{S}$ unchanged. Indeed, in the no commitment case, when agents anticipate that the central banker will extend the list of eligible assets in the bad state of the world to $\{1, 2\}$, part of the liquidity premium on asset 1 will be "transferred" to asset 2. As a result, we expect to see the price of asset 1 decrease, and that of asset 2 increase. We also show in the proof that the equilibrium price exists if and only if:

$$\frac{A_1^-}{A_1}\beta(1 - \lambda) < 1$$

This condition means that agents do not anticipate an excessive shortage of A_1 at the steady-state. If it is not verified, then the stationary equilibrium does not exist. In LPW, they even assume that:

$$\frac{A_j^-}{A_j}\beta < 1 \quad \Leftrightarrow \quad \gamma_j > \beta - 1$$

where $1 + \gamma_j = A_j/A_j^-$ is an inflation rate of asset j. They call this the Friedman rule. In our model, we need this condition only for central-bank-eligible assets. If the latter is verified in our model, then the equilibrium exists and the price ϕ^{nc} is lower than ϕ^c.

Therefore (6) holds. For the two remaining equations, first note that since $\phi_1^{nc} < \phi_1^c$ and $\phi_2^c < \phi_2^{nc}$, if (7) holds then it implies that (8) holds as well. To start with, assume that $\underline{\delta}_2 = \bar{\delta}_2$. Then (7) can be written $h((\underline{\delta}_1 + \phi_1)A_1, y_2) > h((\bar{\delta}_1 + \phi_1)A_1, y_2)$. Since $\underline{\delta}_1 < \bar{\delta}_1$ and $\partial_x h < 0$ for all $y > 0$, then the latter strict inequality holds. Then by continuity of h as a function of y, there exists η such that $\forall \bar{\delta}_2 \in [\underline{\delta}_2; \underline{\delta}_2 + \eta[$ equation (7) still strictly holds.

Those three inequalities are sufficient to ensure that there exists values of the cost $\bar{\kappa}$ such that the self-fulfilling equilibrium exists.

Note that it remains to verify our assumption that the prices ϕ_1^c, ϕ_1^{nc} and ϕ_2^{nc} are such that $u'(y_{\mathcal{S}}) > 1$ and $u'(y_{\bar{\mathcal{S}}}) > 1$. In appendix 6.2, we show that this is the case when dividends and asset supply verify certain conditions, more specifically when A_1 is low enough so that there is an interest of holding asset 1 for its liquidity property.

Proposition 1. *If everybody anticipates that in the bad state of the world, the central banker will extend the list of eligible collateral to* $\bar{\mathcal{S}} = \{1, 2\}$*, then indeed, when it occurs, the central* *banker is forced to intervene. In such an equilibrium, the price of the $\mathcal{S}$-asset is lower than in the commitment equilibrium:*

$$\phi_1^{nc} < \phi_1^c$$

and the price of the $\bar{\mathcal{S}}$-asset is higher:

$$\phi_2^{nc} > \phi_2^c$$

The externality creating the bubble $\bar{\mathcal{S}}$ does not occur if the central bank, instead of extending the list $\mathcal{S}$ in the bad state of the world, allows the government to increase the quantity of T-bills A_1 in times of liquidity shortage.

It is straightforward to see that the same reasoning holds for $\mathcal{S}' = \{1, 3\}$, or $\mathcal{S}' = \{1, 2, 3\}$. As a result, for some values of $\bar{\kappa}$, if all agents anticipate that $\mathcal{S}$ will be extended to $\mathcal{S}'$ in the bad state of the world, then assets in $\mathcal{S}'$ bear a liquidity premium which forces central bank intervention ex post.

3.2 Endogenous formation of $\bar{\mathcal{S}}$ in $\mathcal{P} \backslash \mathcal{S}$

This section answers to an important question in macroeconomics which is: why is money liquid? Why is it not replaced by other assets? And among all available assets, which ones emerge as liquid assets? Here we focus on the role of a central bank in determining the set of collateralizable assets. OLG is not needed in the sense that intergenerational transfers do not matter. Only the role of an asset as eligible collateral does. A similar issue is treated in Gorton and Ordoñez [9]: however, although they have collateral of different qualities, their identity is unique, and can only be land. LPW propose an explanation based on information costs: they argue that liquid assets are the ones that are more easily recognizable. Although this result is consistent with the use of money, it seems of little help to understand ABS's liquidity.

We propose to study how liquid assets rationally emerge from the set of all assets in $\mathcal{P}$ with dynamics. Until now, we have proved that multiple equilibria exist in a model where everybody anticipates the same extension of the list of central bank eligible assets in the bad state of the world: $\bar{\mathcal{S}} = \{1\}, \ \{1, 2\}, \ \{1, 3\}$ or $\{1, 2, 3\}$. In this section, we introduce the existence of noise traders, and show how their effect on the prices is amplified by arbitrageurs' rational anticipation of central bank intervention: in other words, we endogeneize the process through which liquid assets emerge out of a wide range of similar assets.

This section is reminiscent of the literature on central banking in a coordination game. We show that imperfect information on the choice of assets for the extension of central banks' eligible asset lists leads to agents' coordination. This is in the line of Hellwig Mukherji Tsyvinski (2006), or Angeletos and Werning (2006), but differs in its time horizon, and in the set of possible actions: while strategies are limited to "attack" and "do not attack" in those papers, here investors have to make portfolio choices similar to that of standard asset pricing models. Moreover, we use techniques that are developped in Angeletos and La'o [1]'s game of endogenous learning, in the formation of beliefs represented by a space Ω. The dynamics of our model is due to the dynamics of beliefs, as in their paper.

Assumptions, priors and prices: The centralized market now starts with belief formation about $\bar{\mathcal{S}}$. Agents anticipate that the list of eligible collateral will be extended in the bad state of the world; however, they disagree on which one of the identical assets 2 and 3 will become eligible. In what follows we assume that an extension of the list of eligible collateral can at most add one asset to the list $\mathcal{S}$. While this assumption is not necessary for the result, it is essential to ensure tractability[12]. At date 0, each agent receives a prior $\rho^i = (\rho^i_{10}, \rho^i_{20}, \rho^i_{30})$, where $\rho^i_{j,0}$ is the probability that $\mathcal{S}$ is extended to asset j in the bad state of the world. Their prior depends on the private information they get about the cost of an extension for the central bank, $\bar{\kappa}_j$. The probability is $\rho^i_{2t} = \mathbb{P}\left[2 \in \bar{\mathcal{S}}\right]$, which writes:

$$\rho^i_{2t} = \mathbb{P}\left[2 \in \bar{\mathcal{S}} \middle| g(y_{2t}) - g(y_{3t}) > \Delta\kappa^i\right] \times \mathbb{P}\left[g(y_{2t}) - g(y_{3t}) > \Delta\kappa^i\right] \tag{9}$$

We assume that there are noise traders affecting the supply of assets 2 and 3. If Φ is the cdf of A_2, the previous equality becomes:

$$\rho^i_{2t} = \Phi\left(\frac{g^{-1}\left(\kappa^i_2/\lambda + g(y_{1t})\right)}{\underline{\delta}_2 + \phi^e_{2t+1}}\right)\mathbb{P}\left[g(y_{2t}) - g(y_{3t}) > \Delta\kappa^i\right]$$

The probability ρ^i hence reflects both an agents' beliefs about the costs κ^i_j. As in Angeletos and Werning, information is conveyed through prices: agents use ϕ_{2t} and ϕ_{3t} every period to find others' beliefs and infer which one of asset 2 or 3 is more likely to become eligible next period. As we did in the previous section, we consider that u is CARA, $u(c) = c^{1-\sigma}/(1-\sigma)$ with $\sigma \leqslant 1$. For each agent i, the portfolio choice given a price ϕ_2 is given by the foc:

$$\phi_{2,t} = \beta\pi(\bar{\delta}_2 + \phi^e_{2,t+1}) + \beta(1-\pi)(\underline{\delta}_2 + \phi^e_{2,t+1})\left[1 + \lambda\rho^i_{2t}(u'(\sum_{j=1,2}(\underline{\delta}_j + \phi^e_{j,t+1})a^i_j) - 1)\right] \tag{10}$$

ϕ^e_{jt+1} is agent i's expected $t+1$ price of asset j at t. Modifying the previous equation to take out the demands, we get a closed from:

$$\sum_j(\underline{\delta}_j + \phi^e_{j,t+1})a^i_j = \left(\frac{\Lambda_1(\phi_2)}{\rho^i_{2t}} + 1\right)^{\frac{-1}{\sigma}}$$

[12]For instance, if the assets are such that $g'(\underline{\delta}_1 A_1 + \underline{\delta}_j A_j)M < \bar{\kappa}/\lambda$ for $j = 2, 3$, where M is the upper bound of y_j, then $|\bar{\mathcal{S}} \backslash \mathcal{S}| \leqslant 1$. I relax this assumption in the numerical solution of the next section.

Λ_1 does not depend on ρ_2^i or the demands a_j^i, but depends on the price and the price expectation:

$$\Lambda_1 = \frac{1}{\lambda}\frac{\phi_{2,t} - \beta(E_t[\delta_2] + \phi^e_{2,t+1})}{\beta(1-\pi)(\underline{\delta}_2 + \phi^e_{2,t+1})}$$

Suppose for instance that the distribution of priors is uniform $\rho_{20}^i \in [\underline{\rho}, \bar{\rho}]$. $\bar{\rho} - \underline{\rho}$ is known, while the mean of $\bar{\rho}$ and $\underline{\rho}$ is not. Agents are interested in learning about the mean ρ_μ for their own portfolio choice. They will use prices as a signal.

3.2.1 Simple case: myopic agents

We first consider the case where $\phi^e_{jt+1} = \phi_{jt}$: this is not compatible with rational expectations, and REE will be solved numerically in the next section. However, as the myopic agent case contains the main insights of this model, we choose to develop it more. We also assume $\sigma = 1$ so as to be able to solve the problem. Then summing over all agents i we get:

$$\sum_j (\underline{\delta}_j + \phi_j)A_j = \int_{\underline{\rho}}^{\bar{\rho}} \frac{\rho}{\Lambda_1(\phi_2) + \rho}\frac{d\rho}{\bar{\rho} - \underline{\rho}}$$

A_j for $j = 1, 2$ is the supply of asset j. The integrand is of the form of $1 + \Lambda_1 \frac{f'}{f}$ where $f(x) = x - \Lambda_1$. As a result, $\phi_{2,t}$ is the fixed point of the following equation:

$$\bar{\rho} - \underline{\rho} - \Lambda_1(\phi_2)\ln\left[\frac{\Lambda_1(\phi_2) + \bar{\rho}}{\Lambda_1(\phi_2) + \underline{\rho}}\right] = (\bar{\rho} - \underline{\rho})\sum_{j=1,2}(\underline{\delta}_j + \phi_j)A_j \qquad (11)$$

We rewrite $\bar{\rho} = \rho_\mu + \eta$ and $\underline{\rho} = \rho_\mu - \eta$. This expression becomes:

$$\frac{\Lambda_1(\phi_{2t})}{2\eta}\ln\left[\frac{\Lambda_1(\phi_{2t}) + \rho_\mu - \eta}{\Lambda_1(\phi_{2t}) + \rho_\mu + \eta}\right] = \sum_{j=1,2}(\underline{\delta}_j + \phi_{jt})A_j - 1$$

and has the form $f_1(\underset{-}{\phi_2}, \underset{+}{\rho_\mu}) = f_2(\underset{+}{\phi_2})$, where f_1 and f_2 are continuous differentiable functions on their definition sets. Therefore, the higher the observed price ϕ_2, the higher agents' beliefs about ρ_2 will be. Agents can compute ρ_μ after observing prices. Let us call $y_2 = (\phi_1 + \underline{\delta}_1)A_1 + (\phi_2 + \underline{\delta}_2)A_2$. Then:

$$\rho_{2\mu} = \eta\frac{1 + e^{2\eta(y_2-1)/\Lambda_1}}{1 - e^{2\eta(y_2-1)/\Lambda_1}} - \Lambda_1 = \eta\coth(\eta(1-y_2)/\Lambda_1) - \Lambda_1$$

The same reasoning holds for asset 3. In figure 8, we show what value of $\rho_{2\mu}$ is inferred from the observation of prices ϕ_{1t} and ϕ_{2t}, for a given value of A_2.

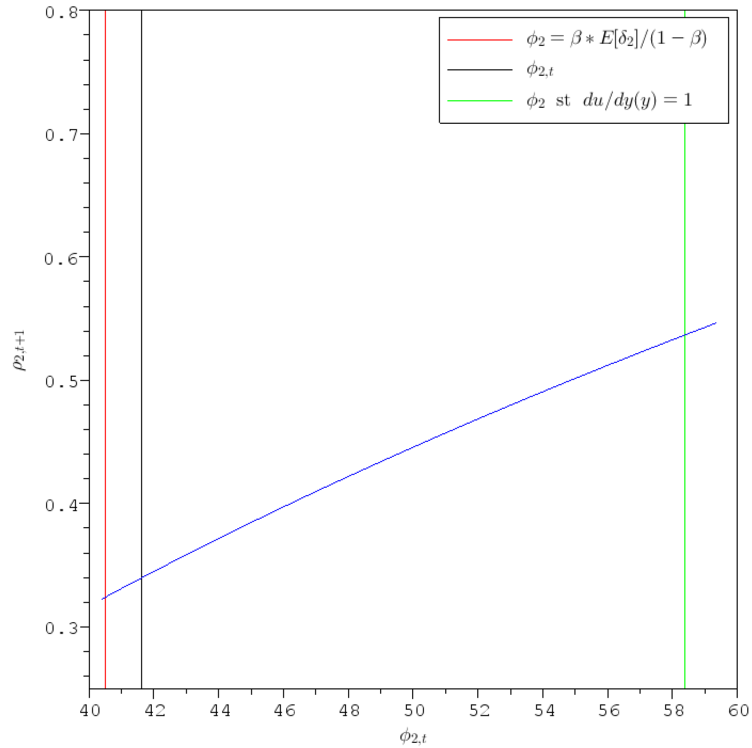


Figure 8: Posterior belief about ρ_2. The parameter values are $\beta = 0.9$, $\pi = 0.9$, $A_2 = 0.01$, $\bar{\delta}_j = 5$, $\underline{\delta}_j = 0$.

As there are noise traders, the value of A_2 and A_3 are in fact unknown. Therefore, agents have to take expectations on A_j to compute expected probabilities:

$$E[\rho_{2\mu}] = \eta \int_{A_2} \coth(\eta(1 - y_2(A_2))/\Lambda_1) f(A_2) dA_2 - \Lambda_1$$

Assume for instance that $A_2 \sim \mathcal{U}([\underline{A}_2, \bar{A}_2])$. Then we get:

$$E[\rho_{2\mu}] = \frac{\Lambda_1}{(\bar{A}_2 - \underline{A}_2)(\phi_2 + \underline{\delta}_2)} \ln\left(\frac{\sinh(\eta(1 - y_2(\underline{A}_2))/\Lambda_1)}{\sinh(\eta(1 - y_2(\bar{A}_2))/\Lambda_1)}\right) - \Lambda_1$$

Belief formation with price observation: What is crucial for the dynamics of asset prices is the update of beliefs about ρ_2 and ρ_3, and expectations about future prices. Note that it is not crucial that the agents and the central bank should have the same expectations of asset prices. The equation (11) and its equivalent for asset 3 enables the agents to compute the values of $\bar{\rho}$ and $\underline{\rho}$, given the existence of noise traders. If for instance $\phi_{2t} >> \phi_{3t}$, then

$\rho_{2\mu,t+1} > \rho_{3\mu,t+1}$, which means that the price signals that agents' beliefs on the probability that asset 2 will be eligible should be revised upwards. All agents get the same information out of date t prices, independently of their beliefs. Once they computed this public signal, they update their belief by taking the mean of all their past signals, according to:

$$\rho^i_{2,t+1} = \frac{t-1}{t}\rho^i_{2t} + \frac{1}{t}\rho_{2\mu,t} \tag{12}$$

As a result, their beliefs converge to the same probability ρ_2 when time goes to infinity. Moreover, if in period t their beliefs are $\rho^i_{2t} \sim \mathcal{U}([\rho_{\mu,t-1} - \eta; \rho_{\mu,t-1} + \eta])$, then in the next period beliefs are $\rho^i_{2,t+1} \sim \mathcal{U}([(t-1/t)\rho_{\mu,t-1} + (1/t)\rho_{\mu,t} - \eta; (t-1/t)\rho_{\mu,t-1} + (1/t)\rho_{\mu,t} + \eta])$: the variance 2η goes to 0, and at every date t, the distribution of beliefs remains uniform; only the mean is moved away from its initial value. What we show next is that this probability can be biased towards one or the other asset.

Dynamics of asset prices: We first describe an example of a path of equilibrium prices and beliefs. Assume that at date 0, there is a low realisation of A_2 and a high realisation of A_3, meaning that noise traders purchased a huge amount of asset 2 for some exogeneous reason, leaving little supply to agents in the centralized market. Given the observation of prices $(\phi_{10}, \phi_{20}, \phi_{30})$, agents in period 1 compute high values for $\bar{\rho}$ and $\underline{\rho}$ using equations (11) and its equivalent for asset 3. As a consequence, even if the supply of A_2 is (reasonably) higher than the supply of A_3 in period 1 (noise traders are not persistent), the fact that beliefs are distorted towards $2 \in \bar{\mathcal{S}}$ in period 1 because of price observation in $t = 0$ can result in $\phi_{21} > \phi_{31}$. Hence, the history of shocks matters, and even for non-persistent noise-trading, first shocks might be decisive in creating an investment bias towards one or the other asset.

The previous mechanism is highlighted with computational resolution. The calibration is given in table 1.

Parameters											
$\bar{\delta}_j$	$\underline{\delta}_j$	λ	β	π	$\rho_{\mu 2,0}$	$\rho_{\mu 2,0}$	η_ρ	A^0_1	A^0_2	A^0_3	η_A
1	0	0.5	0.9	0.9	0.3	0.3	0.1	0.01	0.005	0.005	0.003

Table 1: Calibration.

Figure 9 shows the impulse response function of prices to a negative noise trader shock to the supply A_2. In the case where agents do not update their priors $\rho^i_{j,0}$ (dashed lines), prices come back to their fundamental right after the shock (period 1). On the contrary, in the case where agents update their beliefs, the effect of a negative shock to A_2 remains in the subsequent periods: once updated upwards, the probability that asset 2 will be eligible next period stays higher than that of asset 3, which is reflected in their prices. The reason why the price of asset 3 also increases in the periods following a shock is because of noise trading shocks' effects on the price of asset 1, which is revised downwards, while agents do not internalize the effect of their probability updates. Finally, we compare the case where agents base their beliefs for the $t + 1$ period based on the probability they computed in period t (dash-dotted lines) to the case where they use all past information to compute their new ρ_{jt+1} (solid lines). Those two responses are quite similar, although in the first case the response is slightly more sensitive to present shocks, since only this shock determines all that agents believe for the future. We think that this case is worth considering since agents are myopic about the prices, and there is no reason they should not be so about eligibility probabilities.

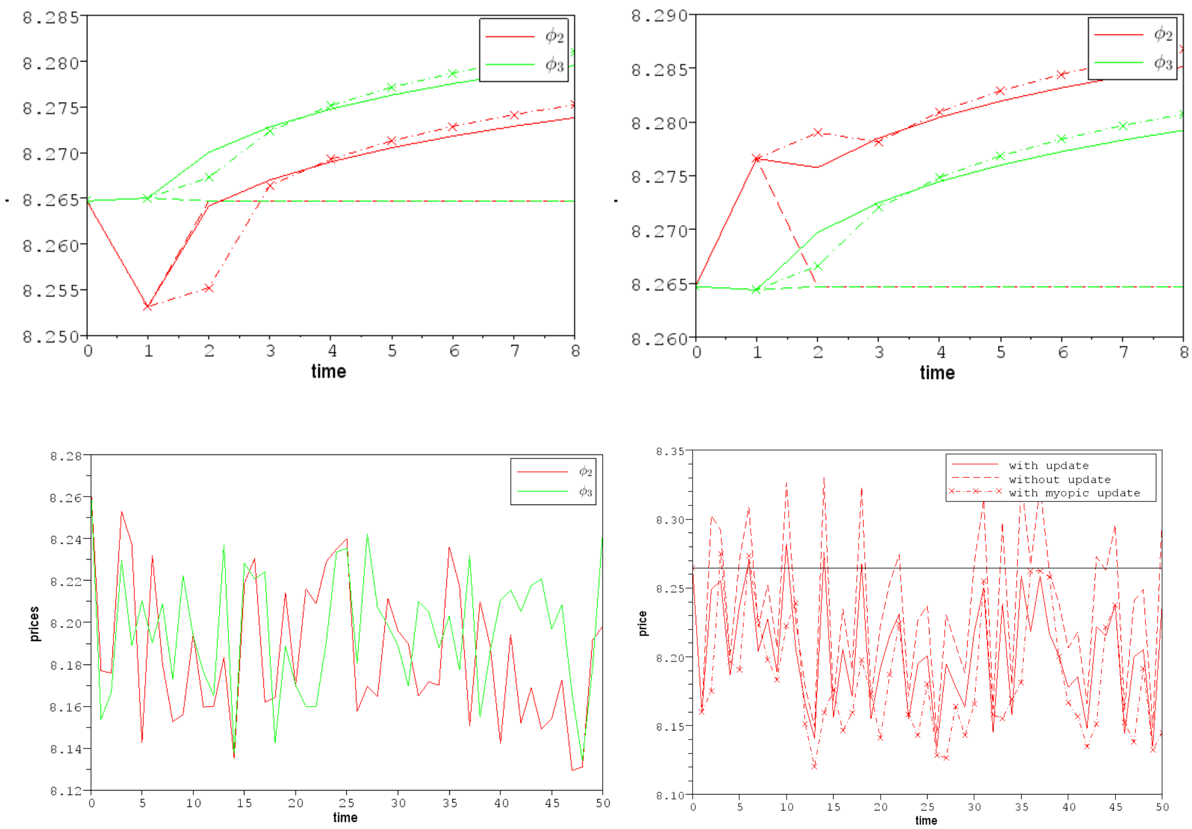


Figure 9: IRF to $A_{2,1} = 1.2 \times A_{2\mu}$ (top left) and $A_{2,1} = 0.8 \times A_{2\mu}$ (top right), and resulting fluctuations comparing prices ϕ_2 and ϕ_3 (bottom left) and different learning rules (bottom right).

Asset price volatility and the probability of disasters: How does asset prices' volatility vary with $1-\pi$, the probability of a crisis? We show that volatility increases when $1-\pi$ increases. The reason is that, as $1-\pi$ goes up, the weight on the component associated with liquidity in the fixed point equation (10) increases, making prices more sensitive to noise trading shocks. Yet, it is worth noting that the computational resolution shows the opposite effect of π on probabilities ρ_j. First, in the left-hand side of figure 10, we plot the updated beliefs about the probability ρ_2 when the probability of a crisis $1-\pi$ and the price of asset 2 vary. When the probability of a crisis is high (π is low), beliefs are less sensistive to price changes. This is due to the fact that, when the weight on the crisis-specific term of the price is low, a small variation of the price could reflect a huge mistake in beliefs ρ_2^i. As a result, *beliefs* are less volatile. However, *prices* are more volatile when a crisis is more imminent: figure 10 shows that the inflation of asset prices, comupted as $(\phi_{t+1}-\phi_t)/\phi_t$, responds more to noise trading shock when $1-\pi$ is high (π is low). As a result, although probabilities are less responsive when the probability of a crisis increases, prices are more. Finally, table 3.2.1 explicitely calculates the volatility of asset prices associated with the simulation plotted in figure 11.

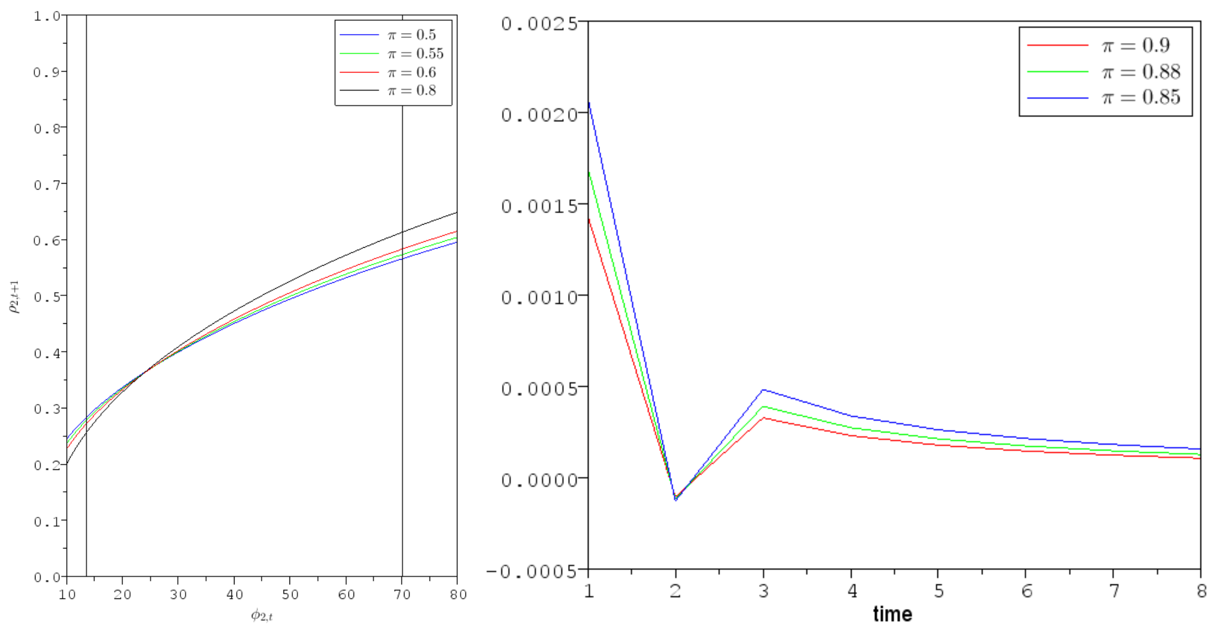


Figure 10: Price response to noise trader shocks for different values of π, the probability of a crisis - IRF.

Variance of asset prices		
$\pi = 0.9$	$\pi = 0.88$	$\pi = 0.85$
0.0019	0.0026	0.0038

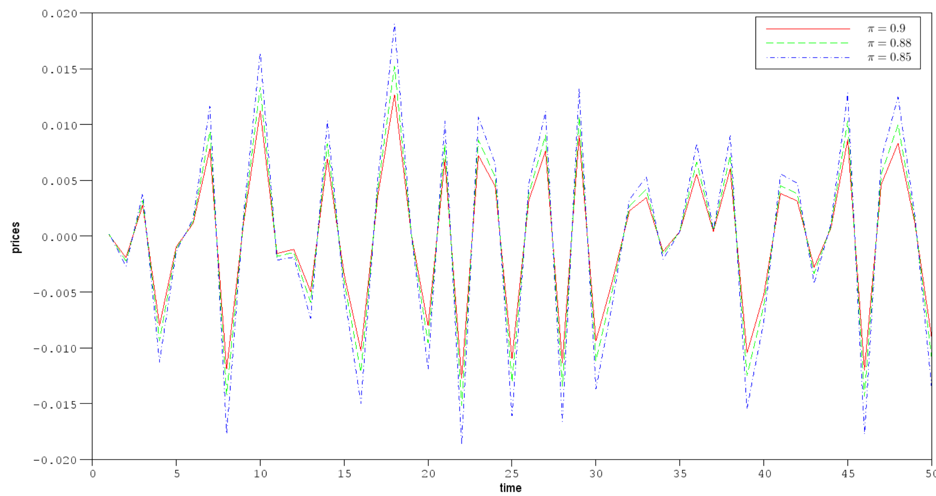


Figure 11: Price response to noise trader shocks for different values of π, the probability of a crisis - fluctuations.

Relaxing the assumption of one-degree collateral framework extension: In this section, we assumed for the sake of tractability that agents anticipate the probability of an extension of the collateral framework to $\bar{\mathcal{S}} = \{1, 2, 3\}$ to be zero. This is valid as long as dividends $\underline{\delta}_2$ and $\underline{\delta}_3$ are high enough compared to the cost $\bar{\kappa}$. It is convenient as long as we want analytical results on price and probability updates. However this assumption is irrelevant when deciding to solve the equilibrium with a computer, in the sense that letting $\{1, 2, 3\}$ be one of the possible outcomes of the central banker's choice does not make numerical resolutions of the equilibrium unfeasible. However we do not implement it, since this simple model already sheds light on the main issues we are interested in, and we think that relaxing this assumption is superfluous.

3.2.2 Complete model with rational expectations hypothesis

We solve the previous problem when agents have rational expectations of future prices. As it is not clear whether the problem can be solved for a continuum of agents, we start this paragraph with the set of agents being $\mathcal{I}$. As before, the problem is essentially static and the dynamics of portfolio choices only arises from the dynamics of beliefs. All agents are aware that they do not have the same information as others in every period. Indeed, if priors in period 0 are uniformely distributed on $[\underline{\rho}, \bar{\rho}]$, the learning rule 12 implies that those differences remain important for subsequent periods expectations of ρ_j^i, and beliefs might remain rationally dispersed. As a result, all prices and individual demands depend on the realized distributions of $\Omega = \{\rho_{jt}^i\}_{i \in \mathcal{I}}$ for all assets j. We thus define an equilibrium as follows:

Definition: An equilibrium consists of portfolio choices $a_j(\rho, \Omega)$, prices $\phi_j(\Omega)$ such that:

(i) given current prices and the expectations of future prices, the allocations are optimal for households and firms.

(ii) prices clear the market for each asset j:

$$A_j = \int_{i \in \mathcal{I}} a_j^i(\rho^i, \Omega)$$

(iii) expectations in the CM are formed based on all available information at t, ρ_j^i and $(\phi_{js})_{s=0}^t$.

Agent i has the demand functions:

$$\begin{aligned}
a_1^i &= \frac{\beta \pi \lambda}{\phi_1 - \beta(1-\pi)(\underline{\delta}_1 + \phi_1^{ei})(1 + \lambda(\Lambda_2^i + \Lambda_3^i)) - \beta\pi(\bar{\delta}_1 + \phi_1^{ei})(1-\lambda)} \\
a_2^i &= \frac{1}{\underline{\delta}_2 + \phi_2^{ei}} \left(\frac{\rho_2^i}{\Lambda_2^i + \rho_2^i} - (\underline{\delta}_1 + \phi_1^{ei}) a_1^i \right) \\
a_3^i &= \frac{1}{\underline{\delta}_3 + \phi_3^{ei}} \left(\frac{\rho_3^i}{\Lambda_3^i + \rho_3^i} - (\underline{\delta}_1 + \phi_1^{ei}) a_1^i \right)
\end{aligned}$$

As a result, price anticipations $\phi_j^e(\rho^i, \Omega)$ are the solutions to the following equations: $\forall i$,

$$\begin{aligned}
A_1 &= \int_\Omega \frac{\beta \pi \lambda}{\phi_1^e(\rho^i, \Omega) - \beta(1-\pi)(\underline{\delta}_1 + \phi_1^e(\rho, \Omega))(1 + \lambda(\Lambda_2^i + \Lambda_3^i)) - \beta\pi(\bar{\delta}_1 + \phi_1^e(\rho, \Omega))(1-\lambda)} F(\rho) d\rho \\
A_2 &= \int_\Omega \frac{1}{\underline{\delta}_2 + \phi_2^e(\rho, \Omega)} \left(\frac{\rho_2}{\Lambda_2^i + \rho_2} - (\underline{\delta}_1 + \phi_1^e(\rho, \Omega)) a_1(\rho, \Omega) \right) F(\rho) d\rho \\
A_3 &= \int_\Omega \frac{1}{\underline{\delta}_3 + \phi_3^e(\rho, \Omega)} \left(\frac{\rho_3}{\Lambda_3^i + \rho_3} - (\underline{\delta}_1 + \phi_1^e(\rho, \Omega)) a_1(\rho, \Omega) \right) F(\rho) d\rho
\end{aligned}$$

In order to find the solution $\phi_j^e(., \Omega) : \rho \mapsto \phi^e(\rho, \Omega)$ to the previous system, for $j = 1, 2, 3$, we discretize the interval $\mathcal{I} = [0, 1]$ in individuals $\{1, ...N\}$, which is equivalent to calculating the riemann series.

Results: The computation requires:

- a programme calculating $\phi_j^e(\rho, \Omega)$
- a programme calculating $a_j(\rho, \Omega)$
- a programme calculating the new prices ϕ_1, ϕ_2, ϕ_3 after a supply shock
- a programme calculating probability rational updates $\rho_j(\phi_1, \phi_2, \phi_3, \Omega)$ after a supply shock, and hence $\Omega'(\phi_1, \phi_2, \phi_3, \Omega)$.

Results have the following form:

Parameters												
$\bar{\delta}$	$\underline{\delta}$	λ	β	π	A_1	A_2	A_3	η_A	N	$\rho_{\mu,2}$	$\rho_{\mu,3}$	η_ρ
1	0	0.07	0.9	0.9	0.005	0.005	0.005	0.001	50	0.22	0.18	0.001

Shock: A_2 is hit down to 0.00499.

Results		
ϕ_1^e	ϕ_2^e	ϕ_3^e
78.819353	8.2482815	8.2209867
ϕ_1	ϕ_2	ϕ_3
78.8194	8.2482865	8.2209867
$\rho'_{2\mu}$	$\rho'_{3\mu}$	η'_ρ
0.2200738	0.18	0.001

Conjecture 1. *If Ω is an interval and ρ_j is uniformely distributed over that interval, then all agents have the same price expectations and Ω' is also an interval, over which agents are uniformely distributed. Hence, the same game can start in the following period, with beliefs distorted towards one or the other asset.*

This conjecture is formed based on the results of the computational resolution. Indeed, we find that agents compute a posterior $\rho'_{j\mu}$ that takes the form of a linear function of their priors $\rho_{j\mu}$, and the variance of the signal remains the same across assets. Therefore, we can iterate the reasoning and get the whole profile of ρ updates across time. This result is robust to changes in priors. However, the fact that we discretized the intervalle $\mathcal{I}$ so that we can solve a system of N equations, where N is the number of agents, introduces some convergence issues. Analytical progress is required to get more precise results. For an example of the programme's output, see figure 12.

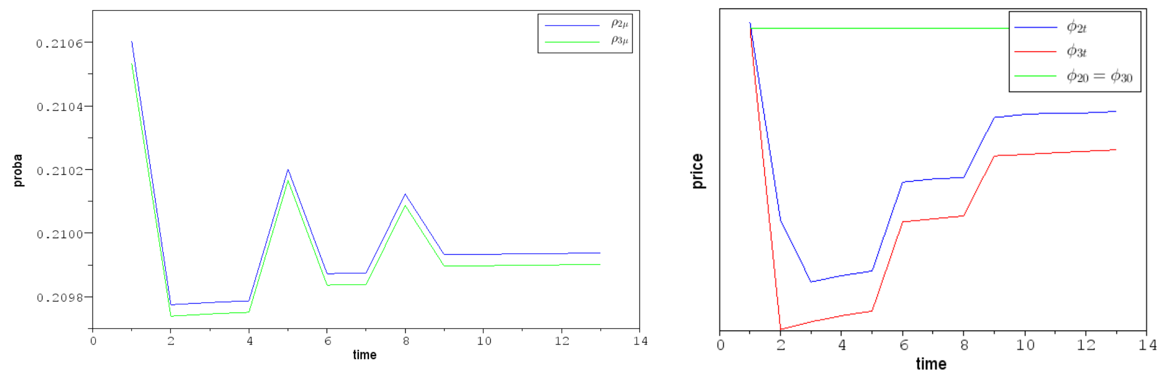


Figure 12: Prices and probabilities in a REE equilibrium, after a negative shock on A_2 in period 1 - starting with $\rho_2 = 0.21$ and $\rho_3 = 0.21$.

4 Extension: Learning from previous bailouts

This model is also helpful in understanding post central-bank-intervention developments in asset markets such as stock markets and treasury bill-markets. This analysis consists in studying the irreversible impact of an extension of the list of eligible assets in bad times on asset prices: each extension of the eligible collateral framework reveals information on the central banker's type. For this purpose, it is sufficient to assume that $\mathcal{P} = \{1, 2\}$ and that the central banker's rule is to extend the list of eligible assets to $\bar{\mathcal{S}}$ if and only if:

$$\varphi\Pi(\mathcal{S}, \mathcal{S}') = \varphi\lambda\left[u(y_{\mathcal{S}'}) - y_{\mathcal{S}'} - u(y_{\mathcal{S}}) + y_{\mathcal{S}}\right] > \bar{\kappa} \tag{13}$$

In other words, there is a new variable φ which represents the degree of "fiscal dominance" of the central bank. φ is a random variable. Let Φ be its cdf.

Before central bank extension of eligible collateral: We first compute equilibrium prices before any central bank intervention. If agents take as granted that the central bank's list of eligible asset will remain $\mathcal{S} = \{1\}$, the price of asset 2 is equal to the fundamental $\phi_2^- = \beta\mathbb{E}[\delta_2] + \beta\phi_2$. The price of asset 1, however, is ϕ_1^c as given by section 3.1. We call those prices (ϕ_1^0, ϕ_2^0). Given those prices, the probability that the list of eligible collateral will be extended to $\mathcal{P} = \{1, 2\}$ is:

$$\rho_{\mathcal{P}}^0 = \mathbb{P}(\bar{\mathcal{S}} = \{1, 2\} | \phi_1^0, \phi_2^0) = \mathbb{P}(\underline{y} > (\underline{\delta}_1 + \phi_1^0)A_1 | \phi_1^0) = 1 - \Phi((\underline{\delta}_1 + \phi_1^0)A_1) > 0 \tag{14}$$

while the probability that it remains $\mathcal{S} = \{1\}$ is:

$$\rho_{\mathcal{S}}^0 = \mathbb{P}(\bar{\mathcal{S}} = \{1\} | \phi_1^0, \phi_2^0) = \mathbb{P}(\underline{y} < (\underline{\delta}_1 + \phi_1^0)A_1 | \phi_1^0) = \Phi((\underline{\delta}_1 + \phi_1^0)A_1) > 0 \tag{15}$$

To be consistent, agents have to take into account this probability in computing their demands, and ϕ_2 now bears a liquidity premium:

$$\phi_2^- = \beta\pi(\bar{\delta}_2 + \phi_2) + \beta(1-\pi)(\underline{\delta}_2 + \phi_2)\rho_{\mathcal{S}}^0 \left[1 - \lambda + \lambda u'(\sum_{j=1,2}(\underline{\delta}_j + \phi_j)A_j)\right] \tag{16}$$

and the price of asset one decreases:

$$\phi_1^- = \beta\pi(\bar{\delta}_1 + \phi_1)(1-\lambda+\lambda u'((\bar{\delta}_1+\phi)A_1)) + \beta(1-\pi)(\underline{\delta}_1+\phi_1)\sum_{\Omega=\mathcal{S},\mathcal{P}}\rho_\Omega^0\left[1-\lambda+\lambda u'(\sum_{j\in\Omega}(\underline{\delta}_j+\phi_j)A_j)\right] \tag{17}$$

Finally, we can define a sequence of prices and probabilities $(\rho_{\mathcal{P}}^n, \rho_{\mathcal{S}}^n, \phi_1^n, \phi_2^n)_{n=0}^{+\infty}$ iteratively, following the four equations 14 to 17. $\rho_{\mathcal{P}}^n$ and ϕ_1^n are decreasing, and $\rho_{\mathcal{S}}^n$ and ϕ_2^n are increasing. Monotone series on a compact set converge. Therefore, at equilibrium, beliefs converge to the limits $(\rho_{\mathcal{P}}^\infty, \rho_{\mathcal{S}}^\infty, \phi_1^\infty, \phi_2^\infty)$, such that $\phi_1^\infty < \phi_1^0$ and $\phi_2^\infty > \phi_2^0$.

After the central bank intervenes: Let us assume that at a date t_0, the bad state of the world occurs and the central bank extends the list of eligible assets to asset 2. In order to keep the distribution of the central banker's type unchanged, we assume that Φ is the cdf of a uniform distribution on $[\underline{\Phi}^0, \bar{\Phi}^0]$. Observing the extension occuring at date t_0 and using equation (13), agents learn that:

$$\varphi \geqslant \frac{\bar{\kappa}}{u(y_{\mathcal{P}}) - y_{\mathcal{P}} - u(y_{\mathcal{S}}) + y_{\mathcal{S}}}$$

If the right-hand side is larger than $\underline{\Phi}^0$, then this event delivers information about the central bank to agents, who update their belief about central bank's type distribution set: $\varphi \in [\underline{\Phi}^{t_0}, \bar{\Phi}^{t_0}] \subset [\underline{\Phi}, \bar{\Phi}]$. The game is over if there are only two assets in the economy. But if there are more assets, and the central bank extends its collateral framework to only one asset in period t_0, then learning the central bank's type affects the expected liquidity premium of all asset classes upwards, even that of assets that are not yet eligible at the central bank.

5 Empirical Analysis

5.1 Panel analysis of the daily updates of central bank eligible-asset-list

Currently gathering daily updates of the list of eligible assets since Feb 20, 2013. This will allow a panel data analysis of the liquidity premium on cb-eligible assets. Based on Vissing-Jorgensen and Krishnamurthy (2008), we want to study the relationship between the quantity of liquid assets and the liquidity premium. The data arrives in tables, as in figure 13. We focus on the liquidity premium associated with the fact of being eligible at the central bank. We use data on the volume of eligible assets, available on datastream. There are two distinct effects. *First*, when an existing asset becomes eligible, it is possible to measure the premium associated with eligibility by considering the spread just before and just after announcement of the updated list of eligible assets. This figure will be a measure of the premium associated with the characteristic of being eligible at the central bank.

$$asset\_price_t^i = eligible_t^i + u_i + \varepsilon_t^i$$

Second, when new assets enter the list and other leave the list, the total volume of eligible assets changes. By considering the impact of those changes in volumes on the prices of assets that were already eligible before the list update, we can measure the variations of the liquidity premium with the quantity of available liquidity in the economy.

$$asset\_price_t^i = eligible_t^i + list\_volume_t * eligible_t^i + u_i + \varepsilon_t^i$$

Note that we will have to take into account the haircut, since a high haircut is equivalent to a lower volume of liquidity:

$$asset\_price_t^i = eligible_t^i + haircut_t^i * eligible_t^i + \left(\sum_{j \neq i} volume\_asset_t^j * haircut_t^j \right) * eligible_t^i + u_i + \varepsilon_t^i$$

ISIN_CODE	ISSUANCE_DATE	MATURITY_DATE	COUPON_RATE	ISSUER_NAME	VALUATION_HAIRCUT	CHANGE
AT0000325550	07/03/2003	07/03/2043	0,617	Landes-Hypothekenbank Stei	6,5	update
AT000B012646	07/03/2012	07/03/2016	3,15	Raiffeisen Bank International	8,5	update
AT000B012661	07/03/2012	07/03/2016	2,68	Raiffeisen Bank International	8,5	update
AT000B012687	07/03/2012	07/03/2018	3,29	Raiffeisen Bank International	11	update
AT000B042379	07/12/2011	09/12/2013	2,202	UniCredit Bank Austria AG	6,5	update
AT000B042668	12/07/2012	12/01/2015	0,475	UniCredit Bank Austria AG	8,5	new
AT000B042775	16/11/2012	16/11/2016	2,125	UniCredit Bank Austria AG	11	new
AT000B049317	07/12/2012	09/12/2013	0,03	UniCredit Bank Austria AG	1,5	update
AT000B064340	07/06/2011	07/06/2015	0,75	Raiffeisenlandesbank Vorarlb	6,5	update
AT000B065164	08/03/2013	08/03/2021	1,92	Raiffeisenlandesbank Vorarlb	8,5	new
AT000B076740	07/03/2011	07/03/2016	3	RAIFFEISENLANDESBANK NIEI	8,5	update
AT000B077490	07/03/2013	07/03/2018	2,25	RAIFFEISENLANDESBANK NIEI	11	update
BE6244236558	26/10/2012	26/04/2013		BNP Paribas Fortis SA	6,5	update
BE6247203159	09/01/2013	08/03/2013		Sumitomo Mitsui Banking Co	6,5	delete

Figure 13: Data - extract from a daily update from the ECB.

5.2 Event study of LTROs and other extraordinary liquidity measures

5.2.1 Liquidity premium and the central bank

Until recently, it has never been possible to distinguish between the liquidity premium associated with the fact of being AAA-rated and central bank eligibility, as central banks had defined their eligibility criterium as being AAA. This is not true any more, as the implementation of extraordinary liquidity measures of the Federal Reserve and the EDM since 2008 (such as LTROs) have led to the extension of the collateral frameworks to lower rated bonds, down to BB^{-}. I propose an event study of the liquidty premium associated with central bank eligibility, as opposed to the fact of being rated AAA, on aggregated data.

$$spread_t = 1 + eligible_t + ext_t + LTRO1_t + LTRO2_t + ext_t * eligible_t + \sum_k LTROk_t * eligible_t + \varepsilon_t$$

where *spread* is a vector of spreads with components according to asset classes AAA, AA, A, BBB, etc. 1 symbolizes the constant. *ext* is a dummy equal to 1 after the collateral list extension from AAA assets to a broader list including BB assets just after the failure of Lehman. $LTRO1$ and $LTRO2$ are dummies equal to one after each one of the two LTROs.

5.2.2 Learning from previous bailouts

Observing that the central bank extends the eligible assets list in bad times, agents might update their beliefs about the probability of further extensions, creating even more asset price volatility.

5.2.3 Real effects of changes in the collateral framework

Furthermore, I consider studying the “real” effects of central bank eligibility, by distinguishing the financial accelerator effect from collateral effect: that is, we want to distinguish whether there is a transmission of monetary policy to the whole economy, or if potential improvements in investment are only due to new bonds issued by eligible firms, which are big firms such as EDF or Veolia, or bonds from the banking sector. Compare the effect on inflation and output of: private bonds as collateral (SMP) or Greek bonds as collateral.

Conclusion

[...]

References

[1] Angeletos, George-Marios & Jennifer La'o, 2013. *Efficiency and Policy with endogenous learning.*

[2] Angeletos, George-Marios & Iván Werning, 2006. *Crises and Prices: Information Aggregation, Multiplicity, and Volatility,* American Economic Review, American Economic Association, vol. 96(5), pages 1720-1736, December.

[3] Berentsen, Aleksander & Cycil Monnet, (2008). *Monetary policy in a channel system*, Speech, Federal Reserve Bank of St. Louis, issue Mar 28.

[4] Bernanke, Ben & Gertler, Mark & Gilchrist, Simon, 1996. *The Financial Accelerator and the Flight to Quality*, The Review of Economics and Statistics, MIT Press, vol. 78(1), pages 1-15, February.

[5] Edmond, Chris & Weill, Pierre-Olivier, 2012. *Aggregate implications of micro asset market segmentation,* Journal of Monetary Economics, Elsevier, vol. 59(4), pages 319-335.

[6] Farhi, E. & J. Tirole, 2011a. *Bubbly liquidity.*

[7] Farhi, E. & J. Tirole, 2011b. *Collective Moral Hazard, Maturity Mismatch and Systemic Bailouts.*

[8] Farhi, E. & J. Tirole, 2012. *Information, Tranching and Liquidity*, IDEI Working Papers 736, Institut d'Économie Industrielle (IDEI), Toulouse.

[9] Gorton Ordoñez, 2013. Collateral Crises, forthcoming.

[10] Heider, F. & M. Hoerova, 2009. *Interbank Lending, Credit Risk Premia and Collateral*, ECB working papers no. 1107.

[11] Holmstrom, Bengt & Jean Tirole, 1998. *Private and Public Supply of Liquidity*, Journal of Political Economy, University of Chicago Press, vol. 106(1), pages 1-40, February.

[12] Kocherlakota, 1998. *Money is Memory*, Journal of Economic Theory 81, 232-251 (1998).

[13] Krishnamurthy, Arvind and Vissing-Jorgensen, Annette, (2012). *The Aggregate Demand for Treasury Debt*, Journal of Political Economy, 120, issue 2, p. 233 - 267.

[14] Lester, B., Postlewaite, A. & R. Wright, 2008. *Information, Liquidity and Asset Prices,* PIER Working Paper Archive.

6 Appendix

6.1 Proof of the existence of a self-fulfilling equlibrium

Let us show that $\phi_1^{nc} < \phi_1^c$.

If A_j^- is the supply of assets at t, and A_j at $t+1$, then stationarity requires that: $\phi_j^- A_j^- = \phi_j A_j$. Then formula (5) gives the equilibrium prices:

with commitment:

$$\begin{aligned} \phi_1^c \tfrac{A_1}{A_1^-} = \quad & \beta\pi(\bar{\delta}_1 + \phi_1^c)\Big[1 + \lambda(u'((\bar{\delta}_1 + \phi_1^c)A_1) - 1)\Big] \\ & + \beta(1-\pi)(\underline{\delta}_1 + \phi_1^c)\Big[1 + \lambda(u'((\underline{\delta}_1 + \phi_1^c)A_1) - 1)\Big] \end{aligned}$$

without commitment:

$$\begin{cases} \phi_1^{nc} \tfrac{A_1}{A_1^-} = & \beta\pi(\bar{\delta}_1 + \phi_1^{nc})\Big[1 + \lambda(u'((\bar{\delta}_1 + \phi_1^{nc})A_1) - 1)\Big] \\ & + \beta(1-\pi)(\underline{\delta}_1 + \phi_1^{nc})\Big[1 + \lambda(u'(\sum_j(\underline{\delta}_j + \phi_j^{nc})A_j) - 1)\Big] \\ \phi_2^{nc} \tfrac{A_2}{A_2^-} = & \beta\pi(\bar{\delta}_2 + \phi_2^{nc}) + \beta(1-\pi)(\underline{\delta}_2 + \phi_2^{nc})\Big[1 + \lambda(u'(\sum_j(\underline{\delta}_j + \phi_j^{nc})A_j) - 1)\Big] \end{cases}$$

Let us call:

$$f(x,y) = \frac{A_1^-}{A_1}\beta\pi(\bar{\delta}_1 + x)\left[1 + \lambda(u'((\bar{\delta}_1 + x)A_1) - 1)\right] + \frac{A_1^-}{A_1}\beta(1-\pi)(\underline{\delta}_1 + x)\left[1 + \lambda(u'((\underline{\delta}_1 + x)A_1 + y) - 1)\right]$$

Then ϕ^{nc} and ϕ^c are the following fixed points:

$$\begin{cases} \phi_1^{nc} = f(\phi_1^{nc}, y^{nc}) \\ \phi_1^c = f(\phi^c, 0) \end{cases}$$

with $y^{nc} = (\underline{\delta}_2 + \phi_2^{nc})A_2 > 0$. We now show that $\phi^{nc} < \phi^c$. The latter equation is true iff:

$$\frac{\partial f}{\partial x}(\phi^{nc}) < 1$$

Assume that u is CRRA:

$$-\frac{qu''(q)}{u'(q)} = \sigma$$

We compute the partial derivative of f:

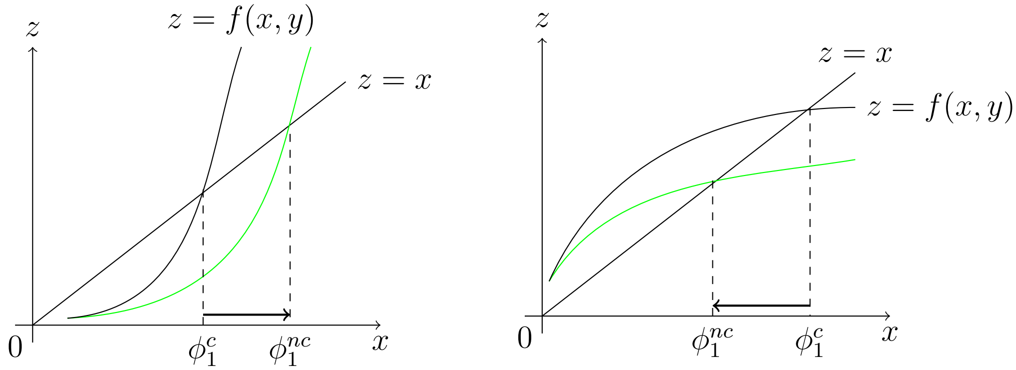


Figure 14: Variation of the fixed point x^* when y increases (green curve), depending on whether the slope of f is larger (left) or smaller (right) than 1.

$$\frac{\partial f}{\partial x}(x) = \frac{A_1^-}{A_1}\beta\Big(\pi\left[1-\lambda+\lambda u'((\bar{\delta}_1+x)A_1)\right] + (1-\pi)\left[1-\lambda+\lambda u'((\underline{\delta}_1+x)A_1+y^{nc})\right] + \pi\lambda A_1(\bar{\delta}_1+x)u''((\bar{\delta}_1+x)A_1) + (1-\pi)\lambda A_1(\underline{\delta}_1+x)u''((\bar{\delta}_1+x)A_1+y^{nc})\Big)$$

$$\frac{\partial f}{\partial x}(x) = \frac{A_1^-}{A_1}\beta\Big(1-\lambda+\pi\lambda u'((\bar{\delta}_1+x)A_1) + (1-\pi)\lambda u'((\underline{\delta}_1+x)A_1+y^{nc}) - \pi\lambda\sigma u'((\bar{\delta}_1+x)A_1) - \sigma(1-\pi)\lambda\frac{A_1(\underline{\delta}_1+x)}{A_1(\underline{\delta}_1+x)+y^{nc}}u'((\bar{\delta}_1+x)A_1+y^{nc})\Big)$$

$$\frac{\partial f}{\partial x}(x) = \frac{A_1^-}{A_1}\beta\Big(1-\lambda+\pi\lambda(1-\sigma)u'((\bar{\delta}_1+x)A_1) + (1-\pi)\lambda\left[1-\sigma\frac{A_1(\underline{\delta}_1+x)}{A_1(\underline{\delta}_1+x)+y^{nc}}\right]u'((\underline{\delta}_1+x)A_1+y^{nc})\Big)$$

$u' > 0$ implies that f is increasing if we assume that $\sigma < 1$. Moreover, we get the second derivative:

$$\frac{\partial^2 f}{\partial x^2}(x) = \frac{A_1^-}{A_1}\beta\Big(\pi\lambda(1-\sigma)A_1u''((\bar{\delta}_1+x)A_1) - (1-\pi)\lambda\sigma\frac{A_1y^{nc}}{(A_1(\underline{\delta}_1+x)+y^{nc})^2}u'((\underline{\delta}_1+x)A_1+y^{nc}) + (1-\pi)\lambda\left[1-\sigma\frac{A_1(\underline{\delta}_1+x)}{A_1(\underline{\delta}_1+x)+y^{nc}}\right]A_1u''((\bar{\delta}_1+x)A_1+y^{nc})\Big)$$

$$\frac{\partial^2 f}{\partial x^2}(x) = A_1^-\beta\lambda\Big(\pi(1-\sigma)u''((\bar{\delta}_1+x)A_1) + (1-\pi)\frac{(1-\sigma)A_1(\underline{\delta}_1+x)+2y^{nc}}{A_1(\underline{\delta}_1+x)+y^{nc}}u''((\bar{\delta}_1+x)A_1+y^{nc})\Big)$$

The latter expression is negative under the assumption $\sigma < 1$. Hence the function f is concave. Therefore it's the right-hand picture in figure 14 that applies here, since we also know that: $f(0, y) > 0\ \forall y$. The equilibrium price exists if and only if the slope of f tends to a value less than 1 when x tends to infinity, that is:

$$\frac{A_1^-}{A_1}\beta(1-\lambda) < 1$$

This condition means that agents do not anticipate an excessive shortage of A_1 at the steady-state. If it is not verified, then the stationary equilibrium does not exist. In Lester Postelwaite and Wright, they even assume that:

$$\frac{A_j^-}{A_j}\beta < 1 \quad \Leftrightarrow \quad \gamma_j > \beta - 1$$

where $1+\gamma_j = A_j/A_j^-$ is an inflation rate of asset j. They call this the Friedman rule. In our model, we need this condition only for central-bank-eligible assets. If the latter is verified in our model, then the equilibrium exists and the price ϕ^{nc} is lower than ϕ^c. QED.

6.2 Conditions for a liquidity premium in all states of the world

First it is easier to see that there has to be a sufficient supply A_1 of asset 1 for equilibrium prices to be free of liquidity premium. Suppose that $(\bar{\delta}_1+\phi_1)A_1 \geqslant (\underline{\delta}_1+\phi_1)A_1 > y^*$. Then the price is free of liquidity premium: $(1+\gamma_1)\phi_1 = \beta\pi(\bar{\delta}_1+\phi_1) + \beta(1-\pi)(\underline{\delta}_1+\phi_1)$. This implies that:

$$\phi_1 = \frac{\beta}{\gamma_1 - (\beta - 1)}\mathbb{E}(\delta_1)$$

where the Friedman rule $\gamma_1 > \beta - 1$ ensures strictly positive and finite asset prices. The condition $(\underline{\delta}_1+\phi_1)A_1 > q^*$ for having unconstrained agents on the CM then implies that:

$$\underline{\delta}_1 + \frac{\beta}{\gamma_1 - (\beta-1)}\mathbb{E}(\delta_1) > \frac{y^*}{A_1}$$

Obviously, the latter equation is not true when A_1 is sufficiently small: in such a case, agents will be constrained at least in the bad state of the world: $(\underline{\delta}_1+\phi_1)A_1 \geqslant y_{\mathcal{S}}$ binds. Consequently $u'(y_{\mathcal{S}}) > 1$ and prices verify $(1+\gamma_1)\phi_1 > \beta\pi(\bar{\delta}_1+\phi_1) + \beta(1-\pi)(\underline{\delta}_1+\phi_1)$. But it is still possible that the constraint is not binding in the good state of the world. This is equivalent to: $u'((\bar{\delta}_1+\phi_1)A_1) \leqslant 1$, that is to say: $\bar{\delta}_1 + \phi_1 \geqslant y^*/A_1$. This is always true if $\bar{\delta}_1$ is high enough, even if we might have to assume that ϕ is low enough for the low-SOW-equilibrium condition to hold. Finally, the all-constrained equilibrium exists if the fiwed point given by $\phi_1 = f(\phi_1, y)$ is such that $\phi_1 < y^*/A_1 - \bar{\delta}_1$. λ sufficiently small or γ_1 sufficiently high are sufficient conditions. Note that in the case of CRRA utilities, $y^* = 1$.

6.3 Computational details in the case of unknown noise traders

For given prices, the probability that one asset will become eligible in times of crisis is higher when supply is believed to be larger. The probability ρ^i hence reflects an agent's belief about noise traders' permanent bias towards one or the other asset: it is associated to a belief about the mean of A_2 and A_3, that we call $\bar{A}_2(\underset{+}{\rho_2^i})$ and $\bar{A}_3(\underset{+}{\rho_3^i})$. For instance, if A_2 is normally distributed $\mathcal{N}(A_\mu, \sigma)$ and ρ_2 is known, then $\bar{A}_2(\rho_2^i)$ is given by the equation $\rho_2 = 1/2 \times (1 + \mathrm{erf}((C - A_\mu)/\sigma\sqrt{2}))$, where $C = g^{-1}\left(\bar{\kappa}/\lambda + g(y_{1t})\right)/(\delta_2 + \phi_{2t})$. While agents know their own priors, they are uncertain about other agents' beliefs on the distribution of A_2 and A_3, hence about which asset will be in $\bar{\mathcal{S}}$.

Equation 11 can be rewritten as:

$$\rho_{\mu 2} = \eta\frac{1 + e^{2\eta(y_2-1)/\Lambda_1}}{1 - e^{2\eta(y_2-1)/\Lambda_1}} - \Lambda_1$$

where $y_2 = (\phi_1 + \underline{\delta}_1)A_1 + (\phi_2 + \underline{\delta}_2)A_2$. Since there are noise traders affecting A_2, when agents compute their posterior about $A_{2\mu}$ after observing the price by taking the expectation of $\rho_{2\mu}$ given prices:

$$E_A[\rho_{2\mu}] = \int_{A_2} \left(\eta \frac{1 + e^{2\eta(y_2-1)/\Lambda_1}}{1 - e^{2\eta(y_2-1)/\Lambda_1}} - \Lambda_1 \right) \frac{dA_2}{2\sigma}$$

$$= \frac{\eta\Lambda_1}{\sigma(\phi_2 + \underline{\delta}_2)} \ln \left(\frac{e^{-\eta(\bar{y}_2-1)/\Lambda_1} - e^{\eta(\bar{y}_2-1)/\Lambda_1}}{e^{-\eta(\underline{y}_2-1)/\Lambda_1} - e^{\eta(\underline{y}_2-1)/\Lambda_1}} \right) - \Lambda_1$$

$$= \frac{\eta\Lambda_1}{\sigma(\phi_2 + \underline{\delta}_2)} \ln \left(\frac{\sinh(\eta(\bar{y}_2 - 1)/\Lambda_1)}{\sinh(\eta(\underline{y}_2 - 1)/\Lambda_1)} \right) - \Lambda_1$$

In this example, A_2 is uniformely distributed over the interval $[A_{2\mu} - \sigma; A_{2\mu} + \sigma]$, and $\bar{y}_2$ and $\underline{y}_2$ are the corresponding bounds for y_2. For given prices, the probability $E[\rho_2\mu]$ given by this equation is higher for a higher value of $A_{2\mu}$. This is because as asset supply grows, agent are less and less constrained in the DM when the crisis occurs. When they are little constrained, a small variation of the price reflects large variations in the probability $\rho_{2\mu}$, since the weight on $\rho_{2\mu}$, $u'(y_2)$, is lower and tends to decrease this effect. This is why $\rho_{2\mu}$ is an increasing function of A_2 in figure 15. On the other hand, when $\rho_{2\mu}$ is held constant, a small increase in the price creates a lower expectation of $A_{2\mu}$: agents, given other people's belief, understand that there has been noise trading activity on asset 2. Finally, $\rho_{2\mu}$ is an increasing function of the price, when $A_{2\mu}$ is held constant: if asset supply is known, a higher price reflects a higher (exogenous) probability of getting eligible (see figure 15).

However, in the previous equation, asset supply is not known, and the probabilities ρ_j^i represent private information. The ρ_j^i are in fact isomorphic to A_j^i through the equation 9, which can be rewritten, for uniform distribution of noise trading activities:

$$\rho_2^i = \left(\frac{\max(A_{2\mu} - A_{3\mu}, 0)}{2\eta_A} + \frac{(2\eta_A + A_{2\mu} - A_{3\mu})(2\eta_A - |A_{2\mu} - A_{3\mu}|)}{8{\eta_A}^3} \right) \times \frac{A_2^i + \eta_A - C}{2\eta_A}$$

Taking the expected value over all agents' belief gives:

$$E[\rho_{2\mu}] = \frac{1}{4\eta_A} \left(\frac{(A_{2\mu} - A_{3\mu} + 2\eta_A)^2}{4\eta_A} + \frac{\eta_A}{3} + \frac{(A_{2\mu} - A_{3\mu} + 2\eta_A)}{2} - \frac{((A_{2\mu} - A_{3\mu} + 2\eta_A)^3}{24{\eta_A}^2} - \frac{(A_{2\mu} - A_{3\mu})^3}{24{\eta_A}^2} \right) \times \frac{A_{2\mu} - C + \eta_A}{2\eta_A}$$

Taking both expression we computed for $\rho_{2\mu}$, we can take out the value of $A_{2\mu}$ and $A_{3\mu}$ as a result of asset prices. As we do that computationally, we do not find the true values of asset supplies. Therefore there must be a mistake somewhere. The mistake must be conceptual. In particular, it does not seem correct that all variables can be uniformely distributed.

Belief formation with price observation: What is crucial for the dynamics of asset prices is the update of beliefs about ρ_2 and ρ_3, and the subsequent expectations about future prices. Note that it is not crucial that the agents and the central bank should have the same expectations of asset prices. The equation (11) and its equivalent for asset 3 enables the agents to compute the values of $\bar{\rho}$ and $\underline{\rho}$, given the existence of noise traders. If for instance $\phi_{2t} >> \phi_{3t}$, then $\rho_{2\mu,t+1} > \rho_{3\mu,t+1}$, which means that the price signals that agents' beliefs on the probability that asset 2 will be eligible should be revised upwards. An agent who has the belief $\bar{A}_2(\rho_2^i)$ about asset 2 and $\bar{A}_3(\rho_3^i)$ about asset 3 indeed finds a posterior $[\rho_\mu - \eta; \rho_\mu + \eta]$ from

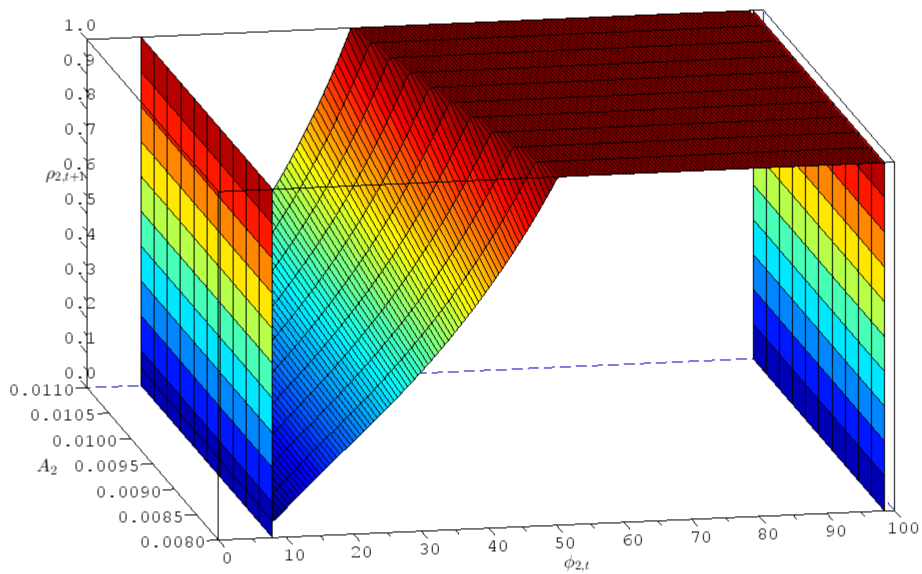


Figure 15: The expectation of $\rho_{2\mu}$ as a function of $A_{2\mu}$ and ϕ_2. The vertical surfaces represent the minimum and maximum values of ϕ_2, resp. when there is no liquidity premium and when agents are satiated.

11. The most pessimistic agent, we call him i^L, $(\bar{A}_2(\rho_2^{i^L}) = \bar{A}_2(\rho_\mu - \eta))$ gets the lower bound next period: $\underline{\rho}_{2,t+1} = \rho_{2,t+1}^{i^L} < \rho_{2,t+1}^i$, $\forall i \in [0,1]$. The time-t problem does not stop when agent i has computed his first update ρ_μ. In fact, updating ρ_μ changes his belief about the distribution of A_2, according to (9). When ρ_μ increases, A_2 is shifted upwards. Then the agent uses this new value of A_2 to compute ρ_μ: at the limit, ρ_μ^∞ is even larger than ρ_μ^0: there is a further amplification spiral in the process of coordination of beliefs through this quantity effect. The sequence of ρ_μ^n and A_μ^n computed by each agent follows the rule:

$$\rho_\mu^0 = \rho_2^i \text{ given }, A_\mu^0 = a(\rho_\mu^0), \text{ and } \forall n \geqslant 1, \begin{cases} \rho_\mu^n & = b\left(A_\mu^{n-1}\right) \\ A_\mu^n & = a\left(\rho_\mu^n\right) \end{cases}$$

The function a is given by (9), while b is given by (11). ρ_μ^∞ is thus given by the fixed point: $\rho_\mu^\infty = (b \circ a)(\rho_\mu^\infty)$, and is the same for every agent. Once they computed this public signal, they update their belief by taking the mean of all their past signals, according to:

$$\rho_{2,t+1}^i = \frac{t-1}{t}\rho_t^i + \frac{1}{t}\rho_\mu^\infty$$

As a result, their beliefs converge to the same probability ρ_2 when time goes to infinity. Moreover, if in the initial period their beliefs are $\rho_{2t}^i \sim \mathcal{U}([\rho_{\mu,t}^\infty - \eta; \rho_{\mu,t}^\infty + \eta])$, then in the next period beliefs are $\rho_{2,t+1}^i \sim \mathcal{U}([(t-1/t)\rho_{\mu,t}^\infty + (1/t)\rho_{\mu,t+1}^\infty - \eta; (t-1/t)\rho_{\mu,t}^\infty + (1/t)\rho_{\mu,t+1}^\infty + \eta])$: the variance 2η remains the same, and so does the uniformity of the distribution; only the mean is moved away from its initial value. What we show next is that this probability can be biased towards one or the other asset.