Unemployment Dispersion and City Configuration*

Vincent Boitier†

Abstract

The aim of this theoretical article is to answer the following question: what drives unemployment dispersion in big cities? Endogenizing land consumption and workers' search intensity, I highlight the role of residential externalities in a urban search model *à la* Wasmer and Zenou (2002). In presence of these externalities, I demonstrate the existence of endogenous spatial distributions of workers generating endogenous unemployment dispersion. I also show that the obtained dispersion does not affect the unemployment rate. Indeed, in line with Wasmer and Zenou (2002), the latter only relies on unemployed workers' allocation in space.

JEL Codes : E24, J41, R14.
Keywords: matching model, mixed pattern, unemployment dispersion.

*Work in Progress, Version June 2013.

†Paris School of Economics and University Paris 1 Panthéon-Sorbonne, vincent.boitier@univ-paris1.fr.

1 Introduction

Spatial variations in the local unemployment rate within big cities are well documented (see Marston (1985) and Blanchflower and Oswald (1994)). For instance, the latter is dispersed between 1.9 % and 33.7 % in Brussels (see Dujardin, Selod and Thomas (2008, Figure 2)), 4 % and 37.3 % in Lyon (see Dujardin and Goffette-Nagot (2010, Figure 1)) and 5 % and 20 % in Paris (see Gobillon and Selod (2007, Carte 1)). Surprisingly, we do not have a clear theoretical understanding of this empirical phenomenon. Indeed, in urban models dealing with mixed land use (i.e. Fujita and Ogawa (1980, 1982), Lucas (2001), Lucas and Rossi-Hansberg (2002), Wheaton (2003)), unemployment is nowhere to be found. In urban search models (hereafter USMs) (see Zenou (2009 a,b) for a review of the literature), only completely segregated cities arise since land is allocated according to the bid rent theory which prevents the existence of situations where unemployed and employed live in the same places of residence.[1]

The purpose of this paper is to account for unemployment dispersion in big cities. To this end, I develop and study the properties of a USM without bid rent theory. Nevertheless, in absence of this principle, how land prices are determined and what drives allocation of workers in space? My approach is as follows. Motivated by a recent empirical literature (Topa (2001), Wahba and Zenou (2005), Bayer et al. (2008), Brueckner and Largey (2008)), I emphasize the role of residential externalities in a model *à la* Wasmer and Zenou (2002).[2] The first residential externality appears in land prices. Endogenizing land consumption and considering land prices as the outcome of a simple Walras equilibrium, I point out that residential density in a given area increases local demand for land as well as land prices. It is a fairly robust and standard economic mechanism (see Combes and al. (2011, 2013), Mangum (2012)). The second externality occurs with the probability of finding a job. Namely, endogenizing workers' search intensity, I underline that the concentration of unemployed workers in a given area lowers their probabilities to receive a job offer (see Topa (2001) and Wahba and Zenou (2005) for an empirical proof of this mechanism). Thus, I demonstrate that players' utilities depend both on the residential density of unemployed

[1] Wheaton (2003, p. 421): "In the traditional theory of competitive spatial markets, land use at each such location is exclusively of one type-deterministically based on which use offers the highest rent (Alonso [1]). By definition this precludes land use mixing, except possibly in the case where the rent from two uses is identical. Even in the case where rents are "tied," the exact fraction of land that is assigned to each use is undetermined. As a result, traditional spatial theory tends to create land use patterns in which there are exclusive zones or rings for each use".

[2] The model is *à la* Wasmer and Zenou (2002) for two reasons. First, labor market is governed by a Mortensen-Pissarides search and matching model: wages are bargained according to a Nash game, new jobs and equilibrium unemployment are determined by a job creation equation and a Beveridge curve. Second, distance to jobs has a negative effect on workers' search intensity.

workers and on the residential density of employed workers. The existence of these two residential effects changes workers' location decisions. Indeed, workers become strategic in the sense that they endogenously choose a place of residence according to their spatial preferences and the locations of other agents summarized by these residential densities. As a consequence, the allocation of workers in space is not determined by a bid rent principle but by a Nash equilibrium.

Two results emerge. First, I prove the existence of a unique market equilibrium in which a closed form solution for the local unemployment rate is obtained. Within this framework, I show that four urban patterns can emerge as outcomes: a completely mixed city where the local unemployment rate is uniformly distributed, a mixed city where the latter is continuously distributed, a completely segregated cities where unemployment distribution is degenerated (*i.e.* 0 % or 100 %) and an incompletely mixed (or incompletely segregated) cities if the local unemployment rate is continously diffused in a zone of the city while in other areas it is degenerated. Then, I demonstrate that the predominance one of these patterns depends on the balancing between two standard forces: centripetal (*i.e.* agglomeration) forces summarized by workers transport costs and commuting time versus centrifugal (*i.e.* dispersion) forces captured by workers' preferences for land and information about jobs. Second, I find that unemployment dispersion does not affect unemployment rate. In line with Wasmer and Zenou (2002), the latter only depends on the unemployed allocation in space, that is to say, on the centralization of the city.[3] However, the effect of centralization on global unemployment rate is ambiguous as it generates two opposite effects on the probability of finding a job. It increases workers' search efficiency by improving information on jobs since they live closer to jobs (i.e. positive distance effect) but it also decreases the latter by lowering transfer of networks as unemployed workers are more spatially concentrated (i.e. negative residential effect).

The article is organized as follows. Section 2 presents the new USM. Section 3 concludes.

2 USM with residential externalities

2.1 Environment

Let $\mathcal{X} = [0, 1]$ be a linear city composed of a continuum of locations denoted by $x \in \mathcal{X}$. The city is assumed to be monocentric in the sense that $x = 0$ is considered as the Business

[3]Using Massey and Danton (1988), centralization of a city in this article refers to the degree to which unemployed workers are spatially concentrated near jobs.

District (hereafter BD) where all firms are exogenously located.[4] Consequently, x also represents distance to city center and access to jobs. I suppose a land market and a labor market in a steady state which are populated with three types of agents: firms, workers and absent landlords. Time is continuous.

2.1.1 Job matching

The labor market under study gathers a continuum of homogenous and infinitely lived unemployed with mass $u \in [0,1]$ and a continuum of identical and infinitely lived employed represented by a mass $e = 1 - u \in [0,1]$.[5] As a consequence, u (*respectively* e) stands for the unemployment (*respectively* employment) rate. These workers are spatially dispersed following three endogenous distributions $\mu_U, \mu_W, \mu \in \mathcal{M}(\mathcal{X})$ defined as

$$\begin{cases} \mu_U : \mathcal{X} \to \mathbb{R}_+ \\ \int_{\mathcal{X}} \mu_U(x)dx = u \\ \mu_W : \mathcal{X} \to \mathbb{R}_+ \\ \int_{\mathcal{X}} \mu_W(x)dx = 1 - u \\ \mu : \mathcal{X} \to \mathbb{R}_+ \\ \int_{\mathcal{X}} \mu(x)dx = 1 \end{cases} \tag{1}$$

where $\mathcal{M}(\mathcal{X})$ is the set of absolutely continuous spatial distributions over $\mathcal{X}$ with respect to the Lebesgue measure, $\mu_U(x)$ is the density of unemployed workers located in x, $\mu_W(x)$ is the density of employees residing in x and $\mu(x) = \mu_U(x) + \mu_W(x)$ the residential density. There also exists a continuum of vacant jobs with mass $v \in [0,1]$ where v is referred as to the vacancy rate. Job seekers find a job and vacancies are filled according to two random processes. These processes are governed by a matching function with constant return to scales leading to the following aggregate number of contacts per unit of time

$$m(\bar{s}u, v) \tag{2}$$

with $\bar{s}$ the endogenous average search intensity of workers.[6] Thus, in this city, the filling rate is

$$q(\theta) = \frac{m(\bar{s}u, v)}{v} \tag{3}$$

[4]By doing so, I follow the literature (see Zenou (2009a, 2009b)). This assumption is useful since: it "*greatly simplifies the analysis: for example, commuting trips can be exactly specified if the residential locations are known, and with the assumption of a linear or circular city, the spatial characteristics of each location in the city can be described simply by the distance from the CBD*" in Fujita and Owaga (1980, p. 455).

[5]Unemployed workers are considered as job seekers (*i.e.* no on-the-job search).

[6]A complete defintion of $\bar{s}$ is given in Section 2.2.2.4.

with $\theta = \frac{v}{\bar{s}u}$ the labor market tightness in intensity units and such that $q'(\theta) < 0$.[7] Likewise, the local finding rate is

$$f(\theta, x) = \frac{s(x)}{\bar{s}} \frac{m(\bar{s}u, v)}{u} = f(\theta)s(x) \tag{4}$$

such that $f(\theta) = \theta q(\theta)$ and $f'(\theta) > 0$. $s(x)$ is the endogenous search intensity of unemployed residing at a distance x to jobs. It is defined as the amount of information about jobs. Especially, job seekers have information about job openings in different ways. First, they commute to the CB to look for a job using resources from employment agency. When they are in the city center, they spend $\mathcal{T}_0(x)$ unit of time collecting $\mathcal{T}_0(x)$ units of information. Second, when they are at home, they receive a fixed level of information denoted by $0 < \eta < 1$ according a Poisson process with rate $\frac{1}{\mu_U(x)}$. Therefore, they spend $\mathcal{T}(x) = \mu_U(x)$ unit of time to obtain it. Observe that information is exclusive (*i.e.* the information available in the city center is not accessible at home and *vice versa*) and search in the BD is more effective than at home (*i.e.* $\eta < 1$). Thus, workers' search intensity is

$$s(x) = \mathcal{T}_0(x) + \eta\mathcal{T}(x) = \mathcal{T}_0(x) + \eta\mu_U(x) \tag{5}$$

As the total amount of time $\mathcal{H}$ is given by

$$\mathcal{H} = l + \tau x + \mathcal{T}_0(x) + \mathcal{T}(x) \tag{6}$$

with l the time dedicated to leisure and τx the commuting time, using equation (6) and assuming (for the sake of simplicity) that $\mathcal{H} - l = 1$, local search intensity becomes

$$s(x) = 1 - \tau x - \chi\mu_U(x) \tag{7}$$

with $\chi = 1 - \eta$. Hence, when commute time and unemployed density are larger, this lowers time dedicated to look for a job in the BD and, therefore, search intensity decreases. Conversely, if the amount of information increases (*i.e.* χ decreases), by definition, search intensity is higher.

2.1.2 Worker

A worker can remain in two different states: either employed or unemployed.

Employed worker If the worker is employed, he is endowed with the following hyperbolic utility function à la Mossay and Picard (2011)[8]

$$\mathcal{Z}(\sigma_W(x), \zeta_W(x)) = \sigma_W(x) - \frac{\phi}{2\zeta_W(x)} \tag{8}$$

[7] Throughout this article, $h'(\bullet)$ denotes the first derivative of h with respect to $\bullet$.

[8] Hyperbolic preferences are a special case of quasi-linear preferences. This means that the income effect in the land consumption is eliminated (i.e. indifference curves are parallel). I use these particular

with $\sigma_W \in \mathbb{R}_+$ (*respectively* $\zeta_W(x) \in \mathbb{R}_+$) the amount of composite good (*respectively* land) consumed by an employed person residing in x and $\phi \in \mathbb{R}_+$ a parameter capturing the preference for land. He is also endowed with one unit of labor, a level of productivity $y \in \mathbb{R}_+$, earns a wage $\omega \in \mathbb{R}_+$, faces a Poisson rate $\delta \in \mathbb{R}_+$ of losing his job, commutes to the BD to work incurring a linear transport cost $t_W \in \mathbb{R}_+$ per unit of distance and pays $R(x)$ per unit of land to absent landlords.[9] Therefore, his budget constraint is

$$\sigma_W(x) + R(x)\zeta_W(x) + t_W x = \omega \tag{9}$$

and he has the following maximization program

$$\max_{\sigma_W(x),\zeta_W(x)} \mathcal{Z}(\sigma(x), \zeta(x)) \tag{10}$$

subject to constraint (9). Maximizing equation (10) with respect to $\zeta_W(x)$ subject to constraint (9) gives the Marshallian (uncompensated) demand function of an employed

$$\zeta_W^*(x) = \sqrt{\frac{\phi}{2R(x)}} \tag{11}$$

Moreover, in the land market, demand and supply are equal such that

$$\mu_U(x)\zeta_U^*(x) + \mu_W(x)\zeta_W^*(x) = \iota = 1 \tag{12}$$

with $\mu_U(x)\zeta_U^*(x)$ (*respectively* $\mu_W(x)\zeta_W^*(x)$) the demand for land made by unemployed (*respectively* employed) workers and $\iota = 1$ the land intensity.[10] Using equation (11) and equation (20), I obtain

$$R(x) = \frac{\phi}{2}\mu(x)^2 \tag{13}$$

Integrating this result in equation (11), the instantaneous indirect utility of an employee is

$$W(x) = \omega - t_W x - \phi\mu(x) \tag{14}$$

preferences for the sake of simplicity. Indeed, they have the convenient property of the instantaneous indirect utility function linearly depends on residential density of workers (see equation (14) and equation (21)). This simplification allows me to derive the important analytical results of this article. I could perform my analysis with other quasi-linear preferences: log-linear preferences *à la* Beckmann (1976) or square root functions à la Zenou (2003). These alternatives do not affect my results. I also use this utility form because results do not depend on the existence of wealth effects (*i.e.* Cobb Douglas, log-linear or CES functions lead to the exactly same findings).

[9]Transport costs are independent of residential density meaning that the city is free of congestion. Moreover, for analytical convenience, they are supposed to be linear. However, the model remains true and tractable for continuous general functions $t_W(x)$ and $t_U(x)$.

[10]This assumption (i.e. $\iota = 1$) is standard in urban search economics (see Zenou (2009a, 2009b)).

Thence, the expected utility function of an employed in x denoted by $\mathcal{W}(x)$ satisfies the following Bellman equation

$$r\mathcal{W}(x) = W(x) - \delta\left[\mathcal{W}(x) - \mathcal{U}(x)\right] \tag{15}$$

with r the risk-free interest rate and $\mathcal{W}(x) - \mathcal{U}(x)$ the local worker's surplus.[11] Given this setup, an employed person chooses his residential location according the following program

$$\max_{x\in\mathcal{X}} \mathcal{W}(x) \tag{16}$$

Observe that equation (16) states that worker's choice is strategic because he selects a residential location according to his preferences and the strategies of other workers summarized by the endogenous densities $\mu_U(x)$ and $\mu_W(x)$.

Unemployed worker If the worker is unemployed, he is associated with the same hyperbolic utility function than the employed person

$$\mathcal{Z}(\sigma_U(x), \zeta_W(x)) = \sigma_U(x) - \frac{\phi}{2\zeta_U(x)} \tag{17}$$

with $\sigma_U \in \mathbb{R}_+$ (*respectively* $\zeta_U(x) \in \mathbb{R}_+$) the amount of composite good (*respectively* land) consumed by an unemployed person locating in x.[12] He also earns a level of benefits $z \in \mathbb{R}_+$, goes to the BD to look for a job incurring a linear transport cost t_U such that $t_W \geq t_U$, faces a Poisson rate $f(\theta, x)$ to have a job and pays $R(x)$ per unit of land. In this case, his budget constraint is

$$\sigma_U + \zeta_U(x)R(x) + t_U x = z \tag{18}$$

and he has to solve the following problem

$$\max_{\sigma_U(x),\zeta_U(x)} \mathcal{Z}(\sigma(x), \zeta(x)) \tag{19}$$

subject to constraint (18). Following the same mathematical reasoning developed above, the Marshallian (uncompensated) demand function of an unemployed residing in x is

$$\zeta_U^*(x) = \sqrt{\frac{\phi}{2R(x)}} \tag{20}$$

[11]It is possible to assume that when an employed is hit by the exogenous shock δ, changing his employment status to unemployed, he moves optimally to a new location $x' = arg \max_{x\in\mathcal{X}} \mathcal{W}(x)$ without any reallocation cost. In this case, the Bellman equation becomes

$$r\mathcal{W}(x) = W(x) - \delta\left[\mathcal{W}(x) - \mathcal{U}(x')\right]$$

This assumption leads to results identical to those in the baseline model.

[12]This simplifying assumption (i.e. workers have identical preferences for good and land) is standard in urban search economics (see Smith and Zenou (2003), Zenou (2009a, 2009b)) and does not determine the nature of the results.

the instantaneous indirect utility of an unemployed person locating in x is

$$U(x) = z - t_U x - \phi \left[\mu_U(x) + \mu_W(x) \right] \tag{21}$$

and the expected utility function of an unemployed in x denoted by $\mathcal{U}(x)$ is determined by the following Bellman equation

$$r\mathcal{U}(x) = U(x) + f(\theta, x) \left[\mathcal{W}(x) - \mathcal{U}(x) \right] \tag{22}$$

Given this environment, an unemployed person chooses strategically his residential location so that

$$\max_{x \in \mathcal{X}} \mathcal{U}(x) \tag{23}$$

2.1.3 Centripetal forces versus centrifugal forces

Workers do not have the same incentives to agglomerate close to the CBD (*i.e.* different centripetal forces): unemployed workers minimize their transport costs $t_U x$ and their losses of information $f(\theta)\tau x$ whereas employed workers only minimize their transport costs $t_W x$. Likewise, workers do not have the same motives to disperse throughout space (*i.e.* different centrifugal forces). The unemployed disperse because of rent prices $R(x)$ and of the probability of finding a job $f(\theta)\chi\mu_U(x)$, while employed disperse only because of rents. Thus, residential externalities on the land market are symmetric, although residential externalities on the labor market are asymmetric.

2.2 Market equilibrium $(\mu_U^*, \mu_W^*, \omega^*, \theta^*, u^*)$

A market equilibrium is composed of two partial equilibria: a spatial equilibrium and a labor market equilibrium. A spatial equilibrium determines the allocation of unemployed and employed in space according to two endogenous distributions μ_U^* and μ_W^*. A labor market equilibrium leads to a wage equation ω^*, a labor market tightness index θ^* and an unemployment rate u^*.[13]

2.2.1 Spatial equilibrium (μ_U^*, μ_W^*)

Assume that the labor market is in equilibrium.

2.2.1.1 Definition, existence and uniqueness

An unemployed (*respectively* employed) person chooses his residential location according to program (23) (*respectively* program (16)). As workers' decisions are strategic, the suited

[13] A complete definition of a market equilibrium is given in Section 2.2.3.

equilibrium is a Nash equilibrium. Nevertheless, with infinite number of agents and interactions summarized by densities, it is shown that the Nash equilibrium takes the following form.[14]

Definition 1 *A spatial equilibrium is a spatial distribution of unemployed workers $\mu_U^* \in \mathcal{M}(\mathcal{X})$ and a spatial distribution of employed workers $\mu_W^* \in \mathcal{M}(\mathcal{X})$ such that*

$$\begin{cases} Supp(\mu_U^*) \subset arg\max_{x\in\mathcal{X}} \mathcal{U}(x) \\ Supp(\mu_W^*) \subset arg\max_{x\in\mathcal{X}} \mathcal{W}(x) \end{cases} \tag{24}$$

Using Definition 1, I find that[15]

Proposition 1 *For a given labor market equilibrium $(\omega^*, \theta^*, u^*)$, there exists a unique spatial equilibrium (μ_U^*, μ_W^*).*

2.2.1.2 Unemployment dispersion

Solving system (24), I find the following closed form solution for the endogenous local unemployment rate.

Proposition 2 *The endogenous local unemployment rate is*

$$u(x) = \frac{\mu_U^*(x)}{\mu_U^*(x) + \mu_W^*(x)}$$

such that

C1U: *if $t_W - t_U - f(\theta^*)\tau\epsilon^* < -2u^* f(\theta^*)\chi\epsilon^* < 0$ (i.e. if μ_U^* decreases with distance and this decrease is high), then*

$$\mu_U^*(x) = \sqrt{\frac{2u^*\left[t_U - t_W + f(\theta^*)\tau\epsilon^*\right]}{f(\theta^*)\chi\epsilon^*}} + \frac{t_W - t_U - f(\theta^*)\tau\epsilon^*}{f(\theta^*)\chi\epsilon^*} x$$

for all x in $\left[0, \sqrt{\frac{2u^* f(\theta^*)\chi\epsilon^*}{t_U - t_W + f(\theta^*)\tau\epsilon^*}}\right]$.

C2U: *if $t_W - t_U - f(\theta^*)\tau\epsilon^* > 2u^* f(\theta^*)\chi\epsilon^* > 0$ (i.e. if μ_U^* increases with distance and this increase is high), then*

$$\mu_U^*(x) = \frac{1}{f(\theta^*)\chi\epsilon^*} - \sqrt{\frac{2u^*\left[t_U - t_W + f(\theta^*)\tau\epsilon^*\right]}{f(\theta^*)\chi\epsilon^*}} + \frac{t_W - t_U - f(\theta^*)\tau\epsilon^*}{f(\theta^*)\chi\epsilon^*} x$$

[14]See Lions (2008), Sandholm (2001, 2009), Lachapelle (2012), Mas Colell (1983).

[15]Proofs of this article are gathered in Appendix A.

for all x in $\left[1-\sqrt{\frac{2u^*f(\theta^*)\chi\epsilon^*}{t_W-t_U-f(\theta^*)\tau\epsilon^*}},1\right]$.

C3U: *if* $-2u^*f(\theta^*)\chi\epsilon^* < t_W - t_U - f(\theta^*)\tau\epsilon^* < 0$ *(i.e.* μ_U^* *decreases with distance and this decrease is low), then*

$$\mu_U^*(x) = u^* - \frac{t_W - t_U - f(\theta^*)\tau\epsilon^*}{2f(\theta^*)\chi\epsilon} + \frac{t_W - t_U - f(\theta^*)\tau\epsilon^*}{f(\theta^*)\chi\epsilon^*}x$$

for all x in $[0,1]$.

C4U: *if* $2u^*f(\theta^*)\chi\epsilon^* > t_W - t_U - f(\theta^*)\tau\epsilon^* > 0$ *(i.e.* μ_U^* *increases with distance and this increase is low), then*

$$\mu_U^*(x) = u^* - \frac{t_W - t_U - f(\theta^*)\tau\epsilon^*}{2f(\theta^*)\chi\epsilon} + \frac{t_W - t_U - f(\theta^*)\tau\epsilon^*}{f(\theta^*)\chi\epsilon^*}x$$

for all x in $[0,1]$.

C1W: *if* $\phi(t_W - t_U) - (t_W\chi - \tau)f(\theta^*)\epsilon^* < -2e^*f(\theta^*)\chi\epsilon^* < 0$ *(i.e.* μ_W^* *decreases with distance and this decrease is high), then*

$$\mu_W^*(x) = \sqrt{\frac{2e^*\left[\phi(t_W - t_U) - (t_W\chi - \tau)\, f(\theta^*)\epsilon^*\right]}{f(\theta^*)\chi\epsilon^*}} - \frac{\phi(t_W - t_U) - (t_W\chi - \tau)\, f(\theta^*)\epsilon^*}{f(\theta^*)\chi\epsilon^*}x$$

for all x in $\left[0,\sqrt{\frac{2e^*f(\theta^*)\chi\epsilon^*}{\phi(t_W-t_U)-(t_W\chi-\tau)f(\theta^*)\epsilon^*}}\right]$.

C2W: *if* $\phi(t_W - t_U) - (t_W\chi - \tau)f(\theta^*)\epsilon^* > 2e^*f(\theta^*)\chi\epsilon^* > 0$ *(i.e.* μ_W^* *increases with distance and this increase is high), then*

$$\mu_W^*(x) = \frac{1}{f(\theta^*)\chi\epsilon^*} - \sqrt{\frac{2e^*\left[\phi(t_W - t_U) - (t_W\chi - \tau)\, f(\theta^*)\epsilon^*\right]}{f(\theta^*)\chi\epsilon^*}} - \frac{\phi(t_W - t_U) - (t_W\chi - \tau)\, f(\theta^*)\epsilon^*}{f(\theta^*)\chi\epsilon^*}x$$

for all x in $\left[1-\sqrt{\frac{2e^*f(\theta^*)\chi\epsilon^*}{\phi(t_W-t_U)-(t_W\chi-\tau)f(\theta^*)\epsilon^*}},0\right]$.

C3W: *if* $-2e^*f(\theta^*)\chi\epsilon^* < \phi(t_W - t_U) - (t_W\chi - \tau)f(\theta^*)\epsilon^* < 0$ *(i.e.* μ_W^* *decreases with distance and this decrease is low), then*

$$\mu_W^*(x) = e^* + \frac{\phi(t_W - t_U) - (t_W\chi - \tau)\, f(\theta^*)\epsilon^*}{2f(\theta^*)\chi\epsilon} - \frac{\phi(t_W - t_U) - (t_W\chi - \tau)\, f(\theta^*)\epsilon^*}{f(\theta^*)\chi\epsilon^*}x$$

for all $x \in [0,1]$.

C4W: *if* $2e^* f(\theta^*)\chi\epsilon^* > \phi(t_W - t_U) - (t_W\chi - \tau) f(\theta^*)\epsilon^* > 0$ *(i.e.* μ^*_W *increases with distance and this increase is low), then*

$$\mu^*_W(x) = e^* + \frac{\phi(t_W - t_U) - (t_W\chi - \tau)\, f(\theta^*)\epsilon^*}{2f(\theta^*)\chi\epsilon} - \frac{\phi(t_W - t_U) - (t_W\chi - \tau)\, f(\theta^*)\epsilon^*}{f(\theta^*)\chi\epsilon^*}x$$

for all $x \in [0, 1]$.

Several comments are in order.

First, the geographical distribution of unemployed workers decreases with distance if and only if

$$t_W - t_U < f(\theta^*)\tau\epsilon^* \tag{25}$$

This condition is very intuitive. In line with Wasmer and Zenou (2002, Condition 1, p. 522), the unemployed live (in proportion) near the city center if the expected return of being more effective in terms of search by being marginally nearer to the CBD is larger than the differential in transport costs. Another interpration of this equation is to say that unemployed location is driven only by a relative centripetal motive. Indeed, they live (in proportion) close to the CBD because they bear more important centripetal costs than the one bear by employed (see Section 2.1.3). Obviously, the distribution is uniform if $t_W - t_U = f(\theta^*)\tau\epsilon^*$ and increases with distance if $t_W - t_U > f(\theta^*)\tau\epsilon^*$. Likewise, the spatial distribution of employed workers is decreasing if and only if

$$\phi(t_U - t_W) < (\tau - t_W\chi) f(\theta^*)\epsilon^* \tag{26}$$

Allocation of employed in space is determined by both centripetal and centrifugal forces. Notice that, since $t_U - t_W < 0$, a sufficient condition such that equation (26) holds is $\tau > t_W\chi$. Obviously, the distribution is uniform if $\phi(t_U - t_W) = (\tau - t_W\chi) f(\theta^*)\epsilon^*$ and increases with distance if $\phi(t_U - t_W) > (\tau - t_W\chi) f(\theta^*)\epsilon^*$. Using equation (25) and equation (26), observe that labor elements as worker's productivity, unemployment benefits, worker's bargaining power and the cost of a job do not affect directly spatial equilibrium but indirectly via labor market tightness and local surplus.

Second, many urban patterns can emerge as outcomes. For analytical simplicity and without any loss of generality, I underline urban equilibria where land is not vacant (*i.e.* $\forall x \in \mathcal{X}$, $x \in Supp(\mu^*_W)$ and/or $x \in Supp(\mu^*_U)$). The predominance of one of these patterns is obtained comparing the slopes of spatial distributions of employed and unemployed workers. Within this framework, the following urban patterns can be underlined.

• Cities are said to be *completely mixed* if the local unemployment rate is uniformly distributed throughout space. These patterns arise if $\mu^*_U(x) > 0$, $\mu^*_W(x) > 0$ for all x in $\mathcal{X}$

and

$$u'(x) = 0 \Leftrightarrow -u^* \left[\phi(t_W - t_U) - (t_W\chi - \tau)f(\theta^*)\epsilon^*\right] = (1-u^*)\left[t_W - t_U - f(\theta^*)\tau\epsilon^*\right] \quad (27)$$

that is to say, when centripetal forces are eliminated ($i.e.$ $t_W = t_U = \tau = 0$).

• Cities are said to be *mixed* if the local unemployment rate is continuously but non uniformly dispersed over space. These situations occur if $\mu_U^*(x) > 0$ and $\mu_W^*(x) > 0$ for all x in $\mathcal{X}$. The latter emerge when conditions C3U&C3W or C3U&C4W or C4U&C3W or C4U&C4W are verified, that is to say, if centripetal and centrifugal forces are low (i.e. when difference in transport costs, commuting time, preference for land and information about jobs are small). More precisely, the local unemployment rate continuously decreases with distance if C3U&C4W or if C3U&C3W and the following equation holds

$$u^*(x) < 0 \Leftrightarrow -u^* \left[\phi(t_W - t_U) - (t_W\chi - \tau)f(\theta^*)\epsilon^*\right] > (1-u^*)\left[t_W - t_U - f(\theta^*)\tau\epsilon^*\right] \quad (28)$$

Conversely, the local unemployment rate continuously increases with distance if C4U&C3W or if C4U&C4W and the equation (28) is not verified.

• Cities are said to be *segregated* if the distribution of the local unemployment rate is degenerated in the sense that there are only two unemployment rates in the city: 0% and 100%. These patterns prevail if $\forall x \in Supp(\mu_U^*)$, $x \notin Supp(\mu_W^*)$ and $\forall x \in Supp(\mu_W^*)$, $x \notin Supp(\mu_U^*)$, that is to say, when centripetal and centrifugal forces are very large ($i.e.$ difference in transport costs, commuting time, preference for land and information about jobs are very high). Two types of completely segregated cities can be found[16]: a situation where the unemployed live close to jobs and the employed reside at the fringes of the city if C1U, C2W and the following equation holds

$$\sqrt{\frac{2u^* f(\theta^*)\chi\epsilon^*}{t_U - t_W + f(\theta^*)\tau\epsilon^*}} = 1 - \sqrt{\frac{2e^* f(\theta^*)\chi\epsilon^*}{\phi(t_W - t_U) - (t_W\chi - \tau)\, f(\theta^*)\epsilon^*}} \quad (29)$$

or a state where employed reside near the city center and unemployed live in the outskirts of the city arises when C1W, C2W and equation (29) are satisfied.

• Cities are said to be *incompletely mixed* or *incompletely segregated* if the local unemployment rate is continously diffused in a zone of the city while in other areas it is

[16]This two sub-patterns are in line with Wasmer and Zenou (2002, p. 521-522): "*In the first one (Equilibrium 1), the unemployed reside in the vicinity of the CBD and the employed at the outskirts of the city. We call such a city the "integrated city" because the unemployed have a good access to jobs. By contrast, in the other one (Equilibrium 2), the unemployed locate at the outskirts of the city and the employed close to the city-center. This city is called the "segregated city" because the unemployed have a bad access to jobs*".

degenerated (*i.e.* either 0% or 100%). These cases exist if for some x in $\mathcal{X}$, $\mu_U^*(x) = 0$ and/or $\mu_W^*(x) = 0$. Three sub-patterns can be underlined: a central core of unemployed (*respectively* employed) surrounded by a peripheral mixed ring of workers if C3U&C2W or C4U&C2W (*respectively* C2U&C3W or C2U&C4W) are obtained, a central mixed ring of workers with a peripheral core of unemployed (*respectively* employed) if C3U&C1W or C4U&C1W (*respectively* C2U&C1W or C1U&C4W) holds and both a central core and an peripheral ring of unemployed (*respectively* employed) separated by an intermediate ring of mixed workers if

$$\sqrt{\frac{2u^* f(\theta^*)\chi\epsilon^*}{t_U - t_W + f(\theta^*)\tau\epsilon^*}} \neq 1 - \sqrt{\frac{2e^* f(\theta^*)\chi\epsilon^*}{\phi(t_W - t_U) - (t_W\chi - \tau)\, f(\theta^*)\epsilon^*}} \tag{30}$$

and C1U and C2W (*respectively* C2U and C1W) are filled.

2.2.1.3 City centralization

Following Massey and Danton (1988), I introduce the notion of centralization as follows.

Definition 2 *Centralization of a city is the degree to which unemployed workers are spatially concentrated near jobs.*

To measure it, I create the following centralization index denoted by IC

$$IC = \frac{1}{\chi}\left[\tau + \frac{t_U - t_W}{f(\theta^*)\epsilon^*}\right] \tag{31}$$

The higher IC is, the more centralized the city is.

2.2.2 Labor market equilibrium $(\omega^*, \theta^*, u^*)$

Assume that the spatial market is in equilibrium.

2.2.2.1 Firm

A firm consumes no space and can remain in two different situations: either productive or unproductive.

Productive firm If the firm is productive, she is associated with a worker and makes the following instantaneous profit function

$$J = y - \omega \tag{32}$$

As jobs are destroyed according to a Poisson rate δ, the expected profit of a productive firm (i.e. a filled job) denoted by $\mathcal{J}$ satisfies the following Bellman equation

$$r\mathcal{J} = J - \delta\left(\mathcal{J} - \mathcal{V}\right) \tag{33}$$

where $\mathcal{V}$ is the expected profit of an unproductive firm and $\mathcal{J} - \mathcal{V}$ firm's surplus.

Unproductive firm If the firm is unproductive, she is unfilled by a worker. As a result, she posts a unique vacancy at cost $\kappa \in \mathbb{R}_+$. Notice that the posted job vacant is filled at Poisson rate $q(\theta)$. Hence, the instantaneous profit function for an unproductive firm is

$$V = -c \tag{34}$$

and the expected profit of an unproductive firm (i.e. a vacancy) is driven by the following Bellman equation

$$r\mathcal{V} = V + q(\theta)\left(\mathcal{J} - \mathcal{V}\right) \tag{35}$$

Using equation (33), equation (35) and free entry (i.e. $\mathcal{V} = 0$), I obtain the condition determining the labor market tightness

$$\frac{y - \omega^*}{r + \delta} = \frac{c}{q(\theta^*)} \tag{36}$$

This standard equation, showing an inverse relation between the labor market tightness and the wage, has the straightforward following explanation. In equilibrium, the average benefit of a filled job (i.e. the benefit of a filled job multiplied by the expected average duration of a filled job) is equal to the average search cost of a vacancy (i.e. the cost of a vacancy multiplied by the average duration of a job vacant).

2.2.2.2 Wage setting

Following Gautier (2002), Wasmer and Zenou (2002, 2006), Gautier and Zenou (2008) and Zenou (2006), firms cannot observe the locations of workers (i.e. workers' locations are impossible to verify) but think as if they bargain with a worker located in the outskirts of the city (*i.e.* $x = 1$) and in a pure segregated area (*i.e.* $\mu_U^*(1) = u$) such that[17]

$$r\mathsf{W} = \omega - t_W - \phi - \delta(\mathsf{W} - \mathsf{U}) \tag{37}$$

and

$$r\mathsf{U} = z - t_U - \phi + f(\theta)(1 - \tau - u)(\mathsf{W} - \mathsf{U}) \tag{38}$$

The total intertemporal surplus $S = \mathsf{W} - \mathsf{U} + \mathcal{J} - \mathcal{V}$ is shared according to a generalized Nash bargaining game. Therefore, the wage is determined by

$$\omega = argmax\,(\mathsf{W} - \mathsf{U})^{\gamma}\,(\mathcal{J} - \mathcal{V})^{1-\gamma} \tag{39}$$

with γ the worker's bargaining power. Maximizing (39) leads to the following sharing rule

$$(1 - \gamma)\,(\mathsf{W} - \mathsf{U}) = \gamma(\mathcal{J} - \mathcal{V}) \tag{40}$$

Following Pissarides (2000, Chapter 1), I find the following modified wage equation (hereafter MWE)

$$\omega = (1 - \gamma)(z + t_W - t_U) + \gamma\,[y + c(1 - \tau - \chi)\theta] \tag{41}$$

where $y + c(1 - \tau - \chi)\theta$ is the outside option and where $z + t_W - t_U$ is the reservation wage. The latter represents the compensation paid by the firm to prompt the unemployed to accept the offer. The firm has to compensate at least the loss of unemployment benefits and the loss of transport costs. This result is consistent with empirical studies (see among others Barber (1998) and Potter et al. (2006)). The effects of non spatial parameters are classical according to Pissarides (2000). The impact of commuting time is in line with Wasmer and Zenou (2002). The role of other parameters related to neighborhood are well established. Preference in housing consumption does not affect the MWE since firms do not compensate workers for their housing costs. Moreover, less information available in the informal market (i.e. a larger χ) pulls down the outside option effect as well as the wage charged by firms. The underlying intuition is as follows. Workers are less effective in their searches due to the lack of information. Thereby, during negotiation, firms are well conscious that the chosen worker has a lower outside option effect. Consequently, they

[17] Several alternatives can be considered. Following Zenou (2009, Appendix B), I can suppose that firms evolve in a full imperfect information framework in the sense that they only bargain over observable factors. In this case, the wage equation is

$$\omega = (1 - \gamma)z + \gamma(y + \kappa\theta)$$

Finally, following Zenou (2006, 2009), I can assume that firms observe worker's locations. Given this set-up, the wage equation becomes

$$\omega = (1 - \gamma)\,[z + (t_W - t_U)x] + \gamma\,\{y + c\,[1 - \tau x - \chi\mu_U^*(x))]\,\theta\}$$

All these alternatives deliver results in line with the baseline model.

can lower the wage. To conclude, notice that the more centralized the city is, the lower the wage is. To get a sense of this result, observe that, as stated earlier in equation (31), centralization increases with commuting time and transport costs bear by the unemployed and decreases with employed transport costs. According to the MWE, this lowers the reservation wage and the outside option of workers, as well as the wage paid by firms.

2.2.2.3 Job creation equation

Plugging the MWE in equation (36) gives

$$\frac{y-(1-\gamma)z-\gamma\left[y+c(1-\tau-\chi)\theta^{*}\right]}{r+\delta}=\frac{c}{q(\theta^{*})} \tag{42}$$

Equation (42) is referred as the modified job creation equation (hereafter MJCE). The impacts of non spatial parameters are standard to Pissarides (2000) and the effect of commuting time is similar to Wasmer and Zenou (2002). A larger χ ends up to higher job creation since it lowers the outside options of workers and so the wage. Preference for housing consumption is nowhere to be found as firms do not compensate workers for their housing costs. Finally, as wage decreases with the city centralization, the latter naturally leads to an increase in labor market tightness.

2.2.2.4 Modified Beveridge curve

Following Wasmer and Zenou (2002, 2006), the average search efficiency is defined as[18]

$$s(\overline{x})=1-\tau\overline{x}-\chi\mu_U^{*}(\overline{x}) \tag{44}$$

where $\overline{x}=\int_{Supp(\mu_U^*)}x\mu_U^*(x)dx$ is the average location of unemployed workers. This assumption is standard in urban search economics (see Zenou (2009a)). Observe that the impact of city centralization on the average search intensity is ambiguous since two opposite effects come into play: positive distance effect versus negative neighborhood effect. On the one hand, when city is more centralized, the unemployed live close to jobs. This lowers commuting time and increases time spent in the BD to look for a job. On the other hand, city centralization requires a stronger concentration of unemployed workers around the BD. This decreases average search intensity since they spend less time in the BD. Using

[18] Another modeling would be considering the average search intensity as the mean of all local search intensities such that

$$s(\overline{x})=\int_{Supp(\mu_U^*}s(x)\mu_U^*(x)dx \tag{43}$$

This alternative does not alter any of my results.

equation (44), the dynamics of the global unemployment rate is[19]

$$\dot{u} = \delta(1-u) - f(\theta^*)s(\overline{x})u \tag{45}$$

with $\dot{u}$ the variation of unemployment with respect to time, $\delta(1-u)$ is the number of employed workers entering unemployment and $f(\theta^*)s(\overline{x})u$ is the number of unemployed workers finding a job. In steady state, the flows are equal such that

$$u^* = \frac{\delta}{\delta + f(\theta^*)s(\overline{x})} \tag{46}$$

It is the modified Beveridge curve (hereafter MBC) showing an inverse relationship between the unemployment rate and the vacancy rate. For a given $\overline{s}$, the effects of non spatial parameters are identical to Pissarides (2000). Furthermore, the impact of the average search efficiency is in line with Wasmer and Zenou (2002). Thus, the local unemployment dispersion does not determine the level of the global unemployment rate. This latter is driven by the centralization of the city. However, the effect of centralization is ambiguous as I demonstrated that the impact of centralization on average search intensity is undetermined.

2.2.2.5 Definition, existence and uniqueness

To complete the model, we define and prove the existence and the uniqueness of a labor market equilibrium.

Definition 3 *A labor market equilibrium consists in finding a labor market tightness index θ^* solving the job creation equation (42) and an unemployment rate u^* solving the Beveridge curve (46).*

Proposition 3 *For a given land market equilibrium (μ_U^*, μ_W^*), there exists a unique labor market equilibrium $(\omega^*, \theta^*, u^*)$.*

[19] Another modeling would be considering the dynamics of the global unemployment rate as the average unemployment rates such that

$$\dot{u} = \int_{Supp(\mu_W^*)} \delta\left[1 - \mu_W^*(x)\right] dx - \int_{Supp(\mu_U^*)} f(\theta^*, x, \mu_U^*(x))\mu_U^*(x)dx$$

that is

$$\dot{u} = \delta(1-u^*) - f(\theta^*)\overline{s}$$

with

$$\overline{s} = u - \tau\overline{x} - \chi \int_{Supp(\mu_U^*)} \mu_U^*(x)^2 dx$$

Once again, this reasoning would have complicated the model without altering any of the results.

2.2.3 Definition, existence and uniqueness

Definition 4 *A market equilibrium* $(\mu_U^*, \mu_W^*, \omega^*, \theta^*, u^*)$ *is such that a land market equilibrium* (μ_U^*, μ_W^*) *and a labor market equilibrium* $(\omega^*, \theta^*, u^*)$ *are solved for simultaneously.*

Proposition 1 and Proposition 3 insure the existence and the uniqueness of a market equilibrium.

3 Conclusion

Urban search models represent an important literature to understand the relationship between the land market and the labor market in big cities. Although spatial variation in local unemployment rate is well established in empirical studies, this literature focuses on the analysis of pure segregated cities because land is determined by a bid rent theory. This paper aims at building a simple model able to breed and explain the effect of unemployment dispersion. For this purpose, I show, in a USM *à la* Wasmer and Zenou (2002), the existence of two residential externalities. I highlight an externality in the land market and another in the finding rate. In this new context, I endogenize the spatial distribution of workers. The obtained model easily generates dispersion in the local unemployment rate. The latter is explained by transport costs, commuting time, information about jobs, finding rate and preference for land consumption. I then show that this endogenous dispersion has no effect on the global unemployment rate. The impact of the land market transits trough the centralization of the city.

References

[1] Alonso W., 1964, Location and Land Use, Harvard University Press (Massachusetts).

[2] Barber A., 1998, Recruiting Employees: Individual and Organizational Perspectives, Sage Publications, London.

[3] Benabou R., 1993, Workings of a City: Location, Education, and Production, Quarterly Journal of Economics, CVII, 619-652.

[4] Blanchet A. and Carlier G., 2012, Optimal transport and Cournot-Nash equilibria, Working Papers hal-00712488, HAL.

[5] Blanchet A., Mossay P., Santambrogio F., 2011, Working Paper.

[6] Blanchflower D.G. and Oswald A.J., 1994, The Wage Curve, MIT Press, Cambridge, MA.

[7] Brueckner J.K. and Largey A.G., 2008, Social interaction and urban sprawl, Journal of Urban Economics, Elsevier, vol. 64(1), pages 18-34.

[8] Burdett K. and Judd K., 1983, Equilibrium Price Distributions, Econometrica, 51:955-970.

[9] Coulson E., Laing, D. and Wang P., 2001, Spatial mismatch in search equilibrium, Journal of Labor Economics, 19:949-972.

[10] Debreu G., 1952, A social equilibrium existence theorem, Proc. Nat. Acad. Sci. U.S.A.

[11] Barron J.M. and Gilley O.W., 1981, Job serach and vacancy contacts: note, American Economic Review, 71, 747-752.

[12] Bleakley H. and Fuhrer J.C., 1997, Shifts in the beveridge curve, job matching and labor market dynamics, New England Econ. Rev., 9:65-89.

[13] Crane J., 1991, The epidemic theory of ghettos and neighborhood effects on dripping out and teenage childbearing, American Journal of Sociology, 96:1226-1256.

[14] Dujardin C., Selod H. and Thomas I., 2004, Le chômage dans l'agglomération bruxelloise: une explication par la structure urbaine, Revue d'Economie Régionale et Urbaine, 3-28.

[15] Glaeser E.L., Kahn M.E. and Rappaport J., 2008, Why do the poor live in cities? The role of public transportation, Journal of Urban Economics, Elsevier, vol. 63(1), pages 1-24, January.

[16] Holzer H., 1987, Informal job search and black youth unemployment, American Economic Review 77:446-452.

[17] Holzer H. and Reaser J., 2000, Black applicants, black employees, and urban labor market policy, Journal of Urban Economics, 48:365-387.

[18] Ihlanfeldt R.K., 1997, Information on the spatial distribution of job opportunities within Metropolitan Areas, Journal of Urban Economics, 41:218-242.

[19] Jolivet G., Postel-Vinay F. and Robin J.M., 2006, The Empirical Content of the Search Model: Labor Mobility and Wage Distributions in Europe and the US, European Econonomic Review, Vol. 50:877-907.

[20] Fujita M., 1989, Urban Economic Theory. Cambridge University Press, Cambridge.

[21] Fujita M, Ogawa H, 1982, Multiple equilibria and structural transition of nonmonocentric urban configurations, Regional Science and Urban Economics 12:161-196.

[22] emoy R., Bertin E. and Jensen J., 2011, Socio-economic utility and chemical potential, EPL 93, 38002.

[23] Lions P.L., 2008, Cours au College de France.

[24] Lucas R. E. and Rossi-Hansberg E., 2002, On the Internal Structure of Cities. Econometrica, 70 (4).

[25] Marston S.T., 1985, Two views of the geographic distribution of unemployment, Quarterly Journal of Economics, 100, 57-79.

[26] Mortensen D.T, 2003, Wage Dispersion: Why are Similar Workers Paid Differently?, MIT. Press

[27] Pissarides C.A., 2000, Equilibrium Unemployment Theory, second edition, MIT Press, Cambridge, MA.

[28] Potter S. et al, 2006, The tax treatment of employer commuting support: an international review, Transport reviews, 26 (2), pp. 221-237.

[29] Sandholm W.H., 2001, Potential games with continuous player sets, Journal of Economic Theory, 97:81-108.

[30] Sato Y. 2001, Labor heterogeneity in an urban labor market, Journal of Urban Economics 50,313?337.

[31] Sato Y., 2004, City structure, search, and workers? job acceptance behavior, Journal of Urban Economics 55:350-370.

[32] Smith T.E. and Zenou Y., 1997, Dual labor markets, urban unemployment, and multicentric cities. Journal of Economic Theory 76, 185?214.

[33] Timothy D. and Wheaton W.C., 2001, Journal of Urban Economics, 50, 338-366.

[34] Topa G., 2001, Social interactions, local spillovers and unemployment, Review of Economic Studies 68, 261-295.

[35] S. Turner, 1997, Barriers to a better break: Employers discrimination and spatial mismatch in Metropolitan Detroit, Journal of Urban Affairs, 19, 123-141.

[36] Von Thunen J.H., 1826, Der isolierte Staat in Beziehung auf Landwirtschaft und Nationalokonomie (Hamburg). English translation by C.M. Wartenberg (1966), edited by P.Hall. Von Thunen;s Isolated State. Pergamon (London)

[37] Wahba J. and Zenou Y., 2004, Density, Social Networks and Job Search Methods: Theory and Application to Egypt, Working Paper Series 629, Research Institute of Industrial Economics

[38] Wasmer E. and Zenou Y., 2002, Does City Structure Affect Job Search and Welfare ?, Journal of Urban Economics, 51, 515-541.

[39] Wasmer E. and Zenou Y., 2006, Equilibrium Search Unemployment with Explicit Spatial Frictions, Labour Economics, 13, 143-165.

[40] Zax J. and Kain, J.F., 1996, Moving to the suburbs: do relocating companies leave their black employees behind? Journal of Labor Economics, 14, 472?493.

[41] Zenou, Y, 2009a, Urban Labor Economics, Cambridge University Press,Cambridge.

[42] Zenou, Y, 2009b, Endogenous job destruction and job matching in cities, Journal of Urban Economics 65,323?336.

[43] Zenou Y., 2009c, Search in cities, European Economic Review, 53, 607-624.

[44] Zenou Y., 2011, Search, wage posting and urban spatial structure, Journal of Economic Geography, 11, 387-416.

4 Proofs

Proof 1 *A land market equilibrium is a Nash equilibrium* (μ_U^*, μ_W^*) *such that*

$$\begin{cases} Supp(\mu_U^*) \subset arg\max_{x\in\mathcal{X}} \mathcal{U}(x, \mu_U^*(x), \mu_W^*(x)) \\ Supp(\mu_W^*) \subset arg\max_{x\in\mathcal{X}} \mathcal{W}(x, \mu_U^*(x), \mu_W^*(x)) \end{cases}$$

In other words, a land market equilibrium is a situation where unemployed spatial denisty and employed spatial density are concentrated where functions $\mathcal{U}$ *and* $\mathcal{W}$ *realize there maximum values* $\mathcal{U}^*$ *and* $\mathcal{W}^*$*. Thus, using Blanchet et al. (2012), the equilibrium can be rewritten as*

$$\begin{cases} \mathcal{U}(x, \mu_U^*(x), \mu_W^*(x)) \leq \mathcal{U}^* & \text{for almost every } x \in \mathcal{X} \\ \mathcal{W}(x, \mu_U^*(x), \mu_W^*(x)) \leq \mathcal{W}^* & \text{for almost every } x \in \mathcal{X} \\ \mathcal{U}(x, \mu_U^*(x), \mu_W^*(x)) = \mathcal{U}^* & \text{for almost every } x \in \mathcal{X} \text{ such that } \mu_U^*(x) > 0 \\ \mathcal{W}(x, \mu_U^*(x), \mu_W^*(x)) = \mathcal{W}^* & \text{for almost every } x \in \mathcal{X} \text{ such that } \mu_W^*(x) > 0 \end{cases}$$

that is

$$\begin{cases} \mathcal{U}(x, \mu_U^*(x), \mu_W^*(x)) = \mathcal{U}^*, \forall x \in \mathcal{X} \\ \mathcal{W}(x, \mu_U^*(x), \mu_W^*(x)) = \mathcal{W}^*, \forall x \in \mathcal{X} \\ \mu_U^*(x) \geq 0, \forall x \in \mathcal{X} \\ \mu_W^*(x) \geq 0, \forall x \in \mathcal{X} \\ \int_{\mathcal{X}} \mu_U^*(x)dx = u^* \\ \int_{\mathcal{X}} \mu_W^*(x)dx = 1 - u^* \end{cases}$$

Assume that, for every $x \in \mathcal{X}$, $\mu_U^*(x) \neq 0$ *and* $\mu_W^*(x) \neq 0$*. Integrating equation (19) and equation (23) on* $\mathcal{X}$ *and using system (25) yields*

$$\begin{cases} r\mathcal{U}^* = z - \frac{t_U}{2} - \phi + f(\theta^*)(1 - \frac{\tau}{2} - \chi u^*)(\mathcal{W}^* - \mathcal{U}^*) \\ r\mathcal{W}^* = \omega - \frac{t_W}{2} - \phi + \delta(\mathcal{U}^* - \mathcal{W}^*) \end{cases}$$

This system admits a unique solution for $\mathcal{U}^*$ *and* $\mathcal{W}^*$*. Rewritting equation (19) and equation (23) using system (25), we get*

$$\begin{cases} r\mathcal{U}^* = z + f(\theta^*)\epsilon^* - [t_U + f(\theta^*)\tau]\, x - \phi\mu_W^*(x) - [\phi + \chi f(\theta^*)\epsilon^*]\, \mu_U^*(x) \\ r\mathcal{W}^* = \omega - \delta\epsilon^* - t_W x - \phi\mu_U^*(x) - \phi\mu_W^*(x) \end{cases}$$

with $\epsilon^* = \mathcal{W}^* - \mathcal{U}^*$*. As there exists a unique* $\mathcal{U}^*$ *and* $\mathcal{W}^*$*, the above system admits a unique solution for* $\mu_W^*(x)$ *and* $\mu_U^*(x)$*. Consider, for some* $x \in \mathcal{X}$, $\mu_U^*(x) = 0$ *and/or*

$\mu_W^*(x) = 0$. *Integrating equation (19) and equation (23) on $\mathcal{X}$ and using system (25), we find*

$$\begin{cases} r\mathcal{U}^*\check{x}_U = z\check{x}_U - t_U x_U^{**} - \phi + f(\theta^*)(\check{x}_U - \tau x_U^{**} - \chi u)(\mathcal{W}^* - \mathcal{U}^*) \\ r\mathcal{W}^*\check{x}_W = \omega\check{x}_W - t_W x_W^{**} - \phi + \delta(\mathcal{U}^* - \mathcal{W}^*)\check{x}_W \end{cases}$$

with $x_U^{**} = \int_{Supp(\mu_U)} xdx \leq \frac{1}{2}$, $x_W^{**} = \int_{Supp(\mu_W)} xdx \leq \frac{1}{2}$, $\check{x}_U = max\left\{\widehat{x}_U, 1\right\} - min\left\{\widehat{x}_U, 0\right\}$ and $\check{x}_W = max\left\{\widehat{x}_W, 1\right\} - min\left\{\widehat{x}_W, 0\right\}$ where $\widehat{x}_i$ is found such that $\mu_i^*(\widehat{x}_i) = 0 \ \forall i \in \{U, W\}$. This system admits a unique solution for $\mathcal{W}^*$ and $\mathcal{U}^*$, that is to say, a unique solution for $\mu_W^*(x)$ and $\mu_U^*(x)$.

Proof 2 *Using Proof 1, we have*

$$\begin{cases} \left[\phi + f(\theta^*)\chi\epsilon\right]\mu_U^*(x) = z - \left[t_U + f(\theta^*)\tau\epsilon\right]x - \phi\mu_W^*(x) + f(\theta^*)\epsilon - r\mathcal{U}^* \\ \phi\mu_W^*(x) = \omega - t_W x - \phi\mu_U^*(x) - \delta\epsilon - r\mathcal{W}^* \end{cases}$$

that is

$$\begin{cases} \left[\phi + f(\theta^*)\chi\epsilon\right]\mu_U^*(x) = z - \left[t_U + f(\theta^*)\tau\epsilon\right]x + f(\theta^*)\epsilon - rU - \left[\omega - t_W x - \phi\mu_U^*(x) - \delta\epsilon - rW\right] \\ \phi\mu_W^*(x) = \omega - t_W x - \delta\epsilon - rW - \frac{\phi}{\phi + f(\theta^*)\chi\epsilon}\left[z - \left[t_U + f(\theta^*)\tau\epsilon\right]x - \phi\mu_W^*(x) + f(\theta^*)\epsilon - rU\right] \end{cases}$$

$$\mu_U^*(x) = \frac{1}{f(\theta^*)\chi\epsilon^*}\left\{z - \omega + \left[r + \delta + f(\theta^*)\right]\epsilon^* + \left[t_W - t_U - f(\theta^*)\tau\epsilon^*\right]x\right\}$$

$$\mu_U^*(x) = \frac{1}{f(\theta^*)\chi\epsilon^*}\left\{\phi\right\}$$

Assume that

$$\mu_U^{*\prime}(x) < 0 \Leftrightarrow t_W - t_U < f(\theta^*)\tau\epsilon^*$$

$$\mu_U^*(\widehat{x}_U) = 0 \Leftrightarrow \widehat{x}_U = \frac{z - \omega + \left[r + \delta + f(\theta^*)\right]\epsilon^*}{t_U - t_W + f(\theta^*)\tau\epsilon^*}$$

$$\widehat{x}_U > 0 \Leftrightarrow \left[r + \delta + f(\theta^*)\right]\epsilon^* > \omega - z \Leftrightarrow \frac{\gamma}{1 - \gamma}\frac{\kappa\epsilon^*}{q(\theta^*)} > 1$$

Assume that $\widehat{x}_U < 1$.

$$\int_0^{\widehat{x}_U} \mu_U^*(x) = u^*$$

that is

$$\frac{\{z-\omega+[r+\delta+f(\theta^*)]\,\epsilon^*\}^2}{2f(\theta^*)\chi\epsilon^*\,[t_U-t_W+f(\theta^*)\tau\epsilon^*]}=u^*$$

$$\widehat{x}_U=\sqrt{\frac{2u^*f(\theta^*)\chi\epsilon^*}{t_U-t_W+f(\theta^*)\tau\epsilon^*}}<1$$

Integrating this result in unemployed distribution

$$\mu_U^*(x)=\sqrt{\frac{2u^*\,[t_U-t_W+f(\theta^*)\tau\epsilon^*]}{f(\theta^*)\chi\epsilon^*}}+\frac{t_W-t_U-f(\theta^*)\tau\epsilon^*}{f(\theta^*)\chi\epsilon^*}x$$

Assume that $\widehat{x}_U\geq 1$.

$$\int_0^1\mu_U^*(x)dx=u^*$$

that is

$$\frac{z-\omega+[r+\delta+f(\theta^*)]\,\epsilon^*}{f(\theta^*)\chi\epsilon^*}+\frac{t_W-t_U-f(\theta^*)\tau\epsilon^*}{2f(\theta^*)\chi\epsilon^*}=u^*$$

Introducing this relation in the spatial distribution of unemployed workers μ_U^**, we get*

$$\mu_U^*(x)=u^*-\frac{t_W-t_U-f(\theta^*)\tau\epsilon^*}{2f(\theta^*)\chi\epsilon}+\frac{t_W-t_U-f(\theta^*)\tau\epsilon^*}{f(\theta^*)\chi\epsilon^*}x$$

$$\widehat{x}_U=1+\frac{u^*f(\theta^*)\chi\epsilon^*}{t_U-t_W+f(\theta^*)\tau\epsilon^*}>1$$

Assume that

$$\mu_U^*{}'(x)>0\Leftrightarrow t_W-t_U>f(\theta^*)\tau\epsilon^*$$

$$\widehat{x}_U<1$$

Assume

$$\widehat{x}_U>0\Leftrightarrow[r+\delta+f(\theta^*)]\,\epsilon^*<\omega-z\Leftrightarrow\frac{\gamma}{1-\gamma}\frac{\kappa\epsilon^*}{q(\theta^*)}<1$$

$$\int_{\widehat{x}_U}^1\mu_U^*(x)=u$$

that is

$$\frac{z-\omega+[r+\delta+f(\theta^*)]\,\epsilon^*}{f(\theta^*)\chi\epsilon^*}+\frac{t_W-t_U-f(\theta^*)\tau\epsilon^*}{2f(\theta^*)\chi\epsilon^*}-\frac{\{z-\omega+[r+\delta+f(\theta^*)]\,\epsilon^*\}^2}{2f(\theta^*)\chi\epsilon^*\,[t_W-t_U-f(\theta^*)\tau\epsilon^*]}=u^*$$

.....

$$\mu_U^*(x) = \frac{1}{f(\theta^*)\chi\epsilon^*} - \sqrt{\frac{2u^*\left[t_U - t_W + f(\theta^*)\tau\epsilon^*\right]}{f(\theta^*)\chi\epsilon^*}} + \frac{t_W - t_U - f(\theta^*)\tau\epsilon^*}{f(\theta^*)\chi\epsilon^*}x$$

$$\widehat{x}_U = 1 - \sqrt{\frac{2u^* f(\theta^*)\chi\epsilon^*}{t_W - t_U - f(\theta^*)\tau\epsilon^*}}$$

$$\mu_W^*(x) = \frac{1}{\phi f(\theta^*)\chi\epsilon^*}\left[(\omega - \delta\epsilon^* - r\mathcal{W}^*)f(\theta^*)\chi\epsilon^* - \phi(z - \omega + \left[r + \delta + f(\theta^*)\right]\epsilon^*)\right]...$$

$$... - \frac{1}{\phi f(\theta^*)\chi\epsilon^*}\left[\phi(t_W - t_U) - \left(\frac{t_W\chi - \tau}{\tau}\right) f(\theta^*)\tau\epsilon^*\right] x$$

$${\mu_W^*}'(x) < 0 \Leftrightarrow \frac{\phi\tau}{t_W\chi - \tau}(t_W - t_U) < f(\theta^*)\tau\epsilon^*$$

$$\widehat{x}_W = \frac{(\omega - \delta\epsilon^* - r\mathcal{W}^*)f(\theta^*)\chi\epsilon^* - \phi(z - \omega + \left[r + \delta + f(\theta^*)\right]\epsilon^*)}{\phi(t_W - t_U) - \left(\frac{t_W\chi - \tau}{\tau}\right) f(\theta^*)\tau\epsilon^*} > 0$$

Assume $\widehat{x}_W < 1$

$$\mu_W^*(x) = \sqrt{\frac{2e^*\left[\phi(t_W - t_U) - \left(\frac{t_W\chi - \tau}{\tau}\right) f(\theta^*)\tau\epsilon^*\right]}{f(\theta^*)\chi\epsilon^*}} - \frac{\phi(t_W - t_U) - \left(\frac{t_W\chi - \tau}{\tau}\right) f(\theta^*)\tau\epsilon^*}{f(\theta^*)\chi\epsilon^*}x$$

$$\widehat{x}_W = \sqrt{\frac{2e^* f(\theta^*)\chi\epsilon^*}{\phi(t_W - t_U) - \left(\frac{t_W\chi - \tau}{\tau}\right) f(\theta^*)\tau\epsilon^*}}$$

Assume that $\widehat{x}_W \geq 1$.

$$\mu_W^*(x) = e^* + \frac{\phi(t_W - t_U) - \left(\frac{t_W\chi - \tau}{\tau}\right) f(\theta^*)\tau\epsilon^*}{2f(\theta^*)\chi\epsilon} - \frac{\phi(t_W - t_U) - \left(\frac{t_W\chi - \tau}{\tau}\right) f(\theta^*)\tau\epsilon^*}{f(\theta^*)\chi\epsilon^*}x$$

$$\widehat{x}_W = 1 + \sqrt{\frac{2e^* f(\theta^*)\chi\epsilon^*}{\phi(t_W - t_U) - \left(\frac{t_W\chi - \tau}{\tau}\right) f(\theta^*)\tau\epsilon^*}}$$

$${\mu_W^*}'(x) > 0 \Leftrightarrow \frac{\phi\tau}{t_W\chi - \tau}(t_W - t_U) > f(\theta^*)\tau\epsilon^*$$

$$\widehat{x}_W < 1$$

Assume $\widehat{x}_W \leq 0$ *same case.*

Assume $\widehat{x}_W > 0$.

$$\phi(z - \omega + [r + \delta + f(\theta^*)]\, \epsilon^*) > (\omega - \delta\epsilon^* - r\mathcal{W}^*) f(\theta^*)\chi\epsilon^*$$

Proof 3 *First, we prove the existence and the uniqueness of a labor market tightness. For this purpose, we rewrite the MJCE as*

$$y - (1-\gamma)(z + t_W - t_U) - \gamma\,[y + c(1-\tau-\chi)\theta] = c(r+\delta)q(\theta)^{-1}$$

Let us defined two functions a *and* b *such that* $a(\theta) = c(r+\delta)q(\theta)^{-1}$ *and* $b(\theta) = y - (1-\gamma)(z + t_W - t_U) - \gamma\,[y + c(1-\tau-\chi)\theta]$*. The function* a *is increasing and concave with* $a(0) = 0$ *and* $\lim_{\theta \to +\infty} a(\theta) = +\infty$ *while the function* b *is decreasing and linear with* $b(\theta) = y - (1-\gamma)(z + t_W - t_U)\gamma y > 0$ *and* $\lim_{\theta \to +\infty} b(\theta) = -\infty$*. This implies the existence of a unique labor market tightness index* θ*. Second, we prove the existence of a unique unemployment rate* u*. We follow the same method as before. Using Proof 1 and Proof 2, we can state that there exists a constant* ϵ^* *such that*

$$\int_{\mathcal{X}} max\,\{\mu_U^*(x,\epsilon^*), 0\}\,dx = u^*$$

where $\mu_U^*(x,\epsilon^*)$ *is defiend in equation (27). Hence, there exists for all* ϵ *in the set* $[\underline{\epsilon}, \overline{\epsilon}]$ *a function* g *defined as*

$$g(\epsilon) = \int_{\mathcal{X}} max\,\{\mu_U^*(x,\epsilon), 0\}\,dx \in [0,1]$$

and such that $g(\underline{\epsilon}) = 0 \Leftrightarrow g(\overline{\epsilon}) = 1$ *or* $g(\underline{\epsilon}) = 1 \Leftrightarrow g(\overline{\epsilon}) = 0$*. Since* μ_U^* *is continuous and monotone,* g *is continuous and monotone. Moreover, using Section 2.4.4, we also can state that there exists a function* h *defined as*

$$h(\epsilon) = \frac{\delta}{\delta + f(\theta)s(x,\epsilon)} \in [0,1]$$

for all ϵ *in the set* $[\underline{\epsilon}, \overline{\epsilon}]$ *and with*

$$s(x,\epsilon) = 1 - s\overline{x}(\epsilon) - \chi\mu_U^*(\overline{x}(\epsilon)) \in [0,1]$$

and

$$\overline{x}(\epsilon) = \int_{\mathcal{X}} max\,\{\mu_U^*(x,\epsilon), 0\} \in [0,1]$$

Due to continuity and monotony of μ_U^*, $\overline{x}$, s *and* h *are continuous and monotone. This implies the existence of a unique unemployment rate* u*.*