Risk, Return and Portfolio Allocation under Alternative Pension Systems with Incomplete and Imperfect Financial Markets

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Abstract:

This paper uses stochastic simulations on a calibrated model to assess the impact of different pension reform strategies in an environment where financial markets are less than perfect. Surprisingly little is known about the optimal split between funded and unfunded systems when there are sources of uninsurable risk that are allocated in different ways by different types of pension system and where there are imperfections in financial markets (e.g. transactions costs or adverse selection). This paper calculates the expected welfare of agents of different cohorts under various policy scenarios. We estimate how the optimal level of unfunded, state pensions depends on rate of return and income risks and also upon the actuarial fairness of annuity contracts.

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Résumé :

Ce papier fait des simulations stochastiques sur un modèle calibré pour évaluer l'effet de différents régimes de retraite dans un environnement où les marchés financiers sont moins que parfaits. Il est étonnant qu'aussi peu soit connu du partage optimal entre systèmes de capitalisation et de répartition, sachant que les sources non assurables de risque ne sont pas allouées de la même manière par les différents systèmes de retraite et que les marchés financiers sont imparfaits (coûts de transaction, anti-sélection, …). Ce papier calcule l'espérance du bien-être d'agents de différentes générations sous divers choix de politique. Nous estimons en quoi le niveau optimal des retraites publiques par répartition dépend des taux de rendement, des risques de revenu et également de l'équité actuarielle des contrats de rentes.

I. Introduction:

The old age dependency ratio in nearly all developed economies (the ratio of those of pensionable age to those of working age) will be substantially higher in the future; in many cases (Germany, Italy, France, Japan) the ratio is likely to double. If unfunded (pay-as-you-go), state pensions are to continue to provide a large part of retirement incomes, then contribution rates in most countries will have to be substantially higher to balance the system. The desirability of providing a significant proportion of retirement income from unfunded pensions is therefore a key policy issue. It has generated a large literature on the reform of pension systems (see, for example, Feldstein (1996); Feldstein and Samwick (1998); OECD (1996); Mitchell and Zeldes (1996); Disney (1996); Kotlikoff (1996); Huang, Imrohoroglu, and Sargent (1997); Miles and Timmerman (1999); Sinn (1999) and Campbell and Feldstein (2001)). If unfunded pensions have substantial advantages then it might be worth paying the costs of higher contribution rates to preserve them. But if greater reliance on other sources of pension income, most obviously income from funded pensions (or more generally from private saving), can replace unfunded pensions without adverse effects (for example on the allocation of risk), then there would be associated long-run benefits of a higher capital stock and lower, potentially less distortionary, labor taxes. But funded and unfunded pension systems allocate risk in different ways, so any analysis of the implications of different degrees of reliance on funded and unfunded pensions has to consider the welfare implications of different risk allocation mechanisms. It also has to address transitional issues – how does one engineer the move towards a system with a different degree of reliance upon funded pensions given existing pension obligations?

The central policy issue we address in this paper are ones which are relevant in all economies: what is the desirable split between funded and unfunded systems when there are sources of uninsurable risk that affect risk averse agents and where those risks are allocated in different ways by different types of pension system? How does reform re-allocate resources between generations on a transition where pension arrangements and demographics are changing? How does the distribution of welfare evolve both within and between different generations?

In this paper we use a calibrated model to assess the issues. In our model we allow for the impact of changing demographics and focus not just on steady states[2]. We also allow for less than perfect risk sharing opportunities: idiosyncratic risk is significant and cannot be fully insured against; longevity risk exists in an environment where although annuities markets exist, they are less than perfect. Returns on some financial assets are risky.

There already exists a substantial literature that uses calibrated models to address issues of risk allocation in a world with less than perfect risk sharing opportunities. Much of that literature focuses on the US economy – Hubbard and Judd (1987); Imrohoroglu and Imrohoroglu and Joines (1995); Storesletten, Telmer and Yaron (1999); and Campbell, Cocco, Gomes and Maenhout (2001) present results from models calibrated to the US economy. One of the aims of this paper is to see whether results that hold for the US carry over to the case where population growth is likely to be much slower (and populations may even decline) and where the process of ageing is likely to be much more rapid. More rapid ageing poses more problems for the design of pension systems so results on models calibrated to the US may be very misleading fro Germany, Italy, France and Japan. A second aim is to model transitions, where demographics and pension arrangements are changing, rather than focus just on steady states. In doing so we allow for the endogeneity of rates of return and of wages. A third aim is to allow for less than perfect annuities markets. In much of the literature the assumption is made either that annuity markets are entirely absent

[2] In the simulations we will illustrate with demographic data for Japan. Japan is interesting because it has unusually high life expectancy and low fertility. But Japan is not atypical of many Continental

or that they are perfect. In practice annuities markets in some form exist in most economies but they are less than perfect. We explore the impact of variations in the degree of imperfection.

Any model that wants to say something useful about risk and uncertainty must take account of several factors:

1. that individuals face substantial, largely idiosyncratic, risks that affect their labor income and are unlikely to be able to insure against such risks;
2. that borrowing against human capital is difficult, if not impossible;
3. that returns on most financial assets are volatile and uncertain;
4. that to the extent that individuals depend upon their own accumulated funds for retirement resources the way in which annuities markets work is important (state run, unfunded systems will be unaffected by the efficiency of annuities markets because the government is effectively providing insurance itself);
5. if it is to be useful for policy purposes the model should, ideally, consider the transition from one policy regime to another and allow for the endogeneity of asset prices and wages.

It is the existence of multiple sources of uncertainty that makes these models hard to work with. But uncertainty in asset returns and in labour income are so central to the issues about the nature of optimal pension regimes that to omit them is to risk generating seriously misleading results. One of the issues we are keen to explore is the link between optimal portfolio allocation and the structure of pension arrangements. With no uncertainty about asset returns there is no portfolio allocation decision since arbitrage will ensure that returns on al assets are equal. So it is essential to allow for stochastic returns on at least some financial assets.

Solving models with all these features is difficult. Imrohoroglu, Imrohoroglu and Joines (1995 and 1999a) investigate the role of social security in a general equilibrium setting with labor income uncertainty but non-stochastic rates of return and no annuities. Huang, Imrohoroglu and Sargent (1997) focus on the intergenerational

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European countries where life expectancy is projected to rise steadily and where population may impact of various social security systems on transition paths. They allow for stochastic labor income but there is no uncertainty on rates of return. Hubbard and Judd (1987) focus on the impact of credit restrictions, but in a model with no uncertainty about earnings or rates of return. Storesletten, Telmer and Yaron (1999) focus on the risk sharing implications of alternative social security systems; they concentrate on stationary states in a model with no uncertainty about rates of return. (See Imrohoroglu, Imrohoroglu and Joines (1999b) for an excellent survey on computational models of social security)).

In some ways the nearest paper to our own is Campbell, Cocco, Gomes and Maenhout (2001) because they consider the portfolio allocation implications of various pension arrangements. They consider the long-run pattern of lifetime savings and portfolio allocation in the presence of income and rate of return uncertainty and with various pension arrangements. They do not consider the impact of varying degrees of imperfection in annuity markets but do consider fixed costs of entering the equity market. Since the efficiency of annuity contracts is central to the desirability of private, funded pension systems - particularly of individual retirement accounts - we consider modeling the impact of different degrees of imperfection in this market to be important. We also endogenise asset prices and allow for transitions generated by policy changes and shifting demographics.

The central policy issue we address is what is the optimal degree of reliance upon personal, funded pensions. Personal funded pensions may allow people to insure perfectly against some risks – if annuities are available at actuarially fair rates then length of life risk can be avoided. But personal pensions mean that labor income risk from working years, which will have an impact on the contributions to a personal pension fund, have lasting effects upon pension income; such pensions obviously also generate rate of return risk. Given this we consider what role might be played by unfunded, state pensions that give varying degrees of insurance against labor income risk and are not dependent on rate of return risk. We take into account shifting demographics that alter the contribution rate needed to balance an unfunded, state run system. Shifting demographics also alter asset prices and changes in returns on

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actually fall.

financial assets affect the relative advantages of unfunded and funded pension systems.

In calculating the optimal behavior of individuals we take account of all the sources of risk we noted above were essential. In particular, we assume that individuals face random shocks to labor income throughout their working life; some of these shocks are transitory but some are highly persistent. We also assume that they face uncertainty about the returns they will earn on at least some sorts of financial assets. We assume that there is a safe asset but there are also risky assets which, on average, earn higher returns. We assume individuals are risk averse and that they understand the risks of investing in different sorts of assets and are also aware of the uncertainty over how long they will live. We then use numerical techniques to calculate optimal profiles of consumption, saving and portfolio allocation for individuals over their lives. We aggregate these decisions to construct the macroeconomic aggregates and also construct measures of welfare. Solving this sort of model is difficult and we use numerical techniques to work out optimal paths. We set the critical parameters in the model (parameters of the utility functions such as degrees of risk aversion and rates of time preference, and characteristics of the labor income profile over life) by reference to recent data from the Japanese economy. We simulate the model using different settings for the key policy variables. We also consider how differences in the investment environment, particularly the efficiency of annuities contracts and the risk return trade-off on risky assets, affect the economy. We are able to show how different degrees of generosity of unfunded pensions affects the evolution of overall saving rates, levels of national income, and the allocation of savings across different assets. We are also able to make welfare comparisons. We construct a measure of welfare by estimating the expected utility of individuals born at various times. We can calculate the average gains and losses for people of different ages of various reform strategies. We are also able to calculate the distribution of gains and losses to agents of a particular cohort.

Our results indicate several things.

I. The capital stock, the level of national income and portfolio allocation are extremely sensitive to differences in the generosity of unfunded pensions. In the long run the aggregate stock of wealth might be twice as high if unfunded pensions were, on average, worth only 25 percent of average earnings, as opposed to 50 percent.

II. Long-run benefits to future generations of a move towards greater reliance upon funded pensions are likely to be substantial; losses to the current generation of workers are smaller but not insignificant.

III. How much of financial wealth is invested in risky assets is very sensitive to both the level of state pensions and the efficiency of financial markets. Even with quite low risk aversion (a coefficient of relative aversion of 3) we can explain substantial holdings of safe assets (often over 50% of portfolios) if state pensions are low. We do not need to assume extreme risk aversion or fixed costs of investing in risky assets to generate substantial investment in safe assets. This is so even though we use common assumptions about risk premia and the volatility of risky assets.

IV. The effects of reducing the generosity of unfunded pensions upon welfare, savings, portfolio allocation and national income depends on the efficiency of annuities contracts. They are also sensitive to the size of the equity risk premium.

V. Credit restrictions affect the answers substantially. Individuals find it difficult to borrow against future labor income (that is their human capital) and therefore any model with uncertainty over income and over length of life is one in which individuals naturally face borrowing constraints. We find that these constraints are likely to matter significantly. We also find that how serious borrowing constraints are, particularly amongst the elderly, depends very much on the pension environment.

VI. A key finding is that longer run gains from a switch towards greater reliance upon funding, and away from an unfunded system where pensions are linked to salaries, do not go disproportionately to the better off.

II. The model:

Given stochastic processes for labor income and for rates of return (and conditional on pensions arrangements and mortality rates) agents choose consumption (and therefore saving) and portfolio allocation in each period to maximise expected lifetime utility. We assume an additively separable form of the agent's lifetime utility function. We also assume a constant coefficient of risk aversion, the inverse of the intertemporal substitution elasticity. Agents are assumed to know the probabilities of surviving to given ages. Agent k who is aged j at time t maximises:

$$U_k = E_t \left[\sum_{i=0}^{i=T-j} s_{ij} \{ [c_{kt+i}]^{1-\zeta} / (1-\zeta) \} / (1+\rho)^i \right] \quad (1)$$

where T is the maximum length of life possible and the probability of surviving i more periods conditional on reaching age j is $s_{ij.}$ (s_{0j} =1). ρ is the rate of pure time preference; c_{kt+i} is consumption of the agent in period t+i.

ζ is the coefficient of relative risk aversion.

Agents face two constraints:
First there is a budget constraint governing the evolution of financial assets taken from one period to the next.

$$W_{k\,t+1} = [W_{k\,t} + \exp(y_{kt}).(1-\tau) - c_{kt} + b_{kt}].(\lambda \exp(r_{st}) + (1-\lambda)\exp(r_{ft})) \quad (2)$$

$W_{k\,t}$ is the stock of wealth of agent k in period t
y_{kt} is the log of gross labor income
τ is the tax rate on labor income. Tax paid is simply a proportion of gross income
b_{kt} is the level of the unfunded, state pension received by an agent
λ is the proportion of financial assets invested in risky assets
r_{st} is the one period (log) rate of return on risky financial assets held between period t and period t+1
r_{ft} is the one period (log) rate of return on safe financial assets held between period t and period t+1

We assume independent, normally distributed shocks affect the log returns on risky assets and a different set of idiosyncratic shocks affect log incomes; it is therefore natural to use log returns and log incomes in (2).

For ease of notation we have not given agent-specific subscripts to asset returns but we will allow for returns to depend on characteristics of the investor because probabilities of death are specific to agents of a given age. Rates of return on assets that might have annuities features will therefore be agent specific. We will describe shortly how rates of return on financial investments are determined and what role annuities play.

Agents also face a borrowing constraint; wealth cannot be negative:

$W_{kt} \geq 0$ for all k and t.

This constraint may bind in various periods. Whether it does so depends in a complex way upon the profile of the deterministic component of labor income, the realisations of income and rate of return shocks, portfolio choices, the degree of risk aversion and the volatility of shocks. It also depends on the tax rate and the generosity of state pensions.

We assume agents cannot take short positions in either safe or risky assets:

$0 \leq \lambda \leq 1$

In the model agents born before 1961 work from age 20 to the end of their 63rd year (if they survive that long) and are retired thereafter. We allow for the retirement age to move up so that by 2030 retirement comes at age 65[3]. We assume that the profile of gross of tax labor income reflects three factors. First, there is a time-related rise in general labor productivity. Second, there is an age-related element to the growth of labor income over an agent's life. This is modelled as a quadratic in age.
The age-specific part of the log of labor income is:

$$\alpha + \gamma age - \theta age^2 \quad (3)$$

We set γ and θ so that the age-income profile matches patterns that are typical (we discuss calibration issues in detail in the next section).

There are also idiosyncratic (agent specific) stochastic elements of labor income. The log of labor income for an agent is the sum of the age-related element, the time related element and the additive income shock. The income shock has a transitory component

[3] In most developed economies there are now plans for mild increases in the age at which people will become eligible for full receipt of state pensions.

(ω) and a persistent component (u). Denoting the log of gross labor income of agent k who is aged j in period t as y_{kt} we have:

$$y_{kt} = \psi + gt + \gamma.j - \theta.j^2 + u_{kt} + \omega_{kt} \qquad (4)$$

$$u_{kt} = \phi\, u_{kt-1} + e_{kt}$$

where $e \sim N(0, \sigma_e)$; $\omega \sim N(0, \sigma_\omega)$ and e and ω are iid and uncorrelated.

ψ is a constant.

g is the rate of growth of labor productivity over time.

φ reflects the degree of persistence in the non-transitory idiosyncratic shocks to labor income; empirical evidence from a range of countries suggests φ is high and that idiosyncratic shocks to income typically have a high degree of persistence (We discuss the exact calibration of the model in Section IV).

We assume that rates of return on risky financial wealth vary across periods due to random shocks that hit stock and bond markets. There may also be a slow-moving evolution of the mean rates of return on safe and risky assets due to changes in the stock of capital relative to the stock of labor. In the simulations where desired holdings of wealth by residents match the stock of capital used in production we will allow both the safe rate of return and the mean of the risky rate of return to depend upon the stock of aggregate wealth (relative to labor) held in the economy. There is an underlying production function (of the Cobb Douglas sort) and an assumption that the capital stock used in production moves in line with the desired holding of wealth by the private sector. Movements in the aggregate capital to labor ratio are assumed to drive the mean returns on assets; they do not affect the stochastic part of the return on risky assets. Movements in the capital–labor ratio also affect the path of average real wages.

We assume that rates of return on savings at a particular time - both on safe and risky assets - differ between individuals because financial institutions take into account the probabilities of death of agents and offer age-related investment products. More specifically, financial institutions offer the following contracts. For every $ invested in period t, with a given risky/safe split, the investor receives the market return adjusted for a probability of survival to the next period. If markets are perfect the probability used in making this adjustment is the true survival probability. But we allow for imperfections (stemming from adverse selection or some other types of cost) which mean that the two are not equal. If the agent dies the institution keeps the funds. With no bequest motives agents will always chose these contracts over ones which just pay the market rate of return.[4]

If the insurance element of this contract is offered on actuarially fair terms the ex-post rate of return on a $ invested in the risky asset during period t by an agent k who is aged j and who survives to the next period is given by:

$$\exp(r_{st}) / s_{1j} \qquad (5)$$

s_{1j} is the probability of surviving one more year conditional on reaching age j

We assume r_s is the sum of the mean log return and an unpredictable shock.

$r_{st} = r_t + v_t$

r_t is the mean rate of return on risky assets at time t

v_t is the random element of the rate of return on assets in period t.

We assume v is iid and normal: $v \sim N(0, \sigma_r)$

For a $ invested in the safe asset the return to an agent aged j is:

$$\exp(r_{ft}) / s_{1j} \qquad (6)$$

We can write the log returns on actuarially fair investments in risky and safe assets respectively to an agent aged j at time t as:

$$r_t + v_t - \ln(s_{1j}) \qquad (7)$$

[4] Demand for annuities currently across OECD countries is limited. This reflects the generosity of state pensions (which are effectively annuities), imperfections in financial markets and, perhaps, bequest motives. We do consider the first two factors in our model but, as noted already, we follow Campbell et al (2001) in abstracting from bequest issues. Horioka (2001) presents evidence that even in Japan, where people have considered the bequest to be strong, the desire to leave bequests is not a powerful factor.

$r_{ft} - \ln(s_{1j})$ (8)

If markets are perfect this financial contract can be offered at no risk by financial institutions because they pass on all the rate of return risk to investors and are assumed to be able to take advantage of the law of large numbers and face no uncertainty about the proportion of agents who will survive. It seems natural to assume that financial firms will offer insurance against risks that are idiosyncratic (individual length of life risk) but do not offer insurance against market risk (rate of return risk). The contracts offered by financial institutions can be thought of as highly flexible individual retirement accounts (or personal pension schemes). Effectively agents have their own pot of assets into which they pay contributions and make deductions. Contribution rates and drawdowns from the fund are subject only to the constraint that the pot of assets can never fall below zero. The average rates of return on the fund increase with age since survival probabilities decline with age. Just as standard flat annuities available for a given sum rise with age, so the average rate of return offered by financial institutions increases with age.

In effect we are assuming that financial institutions offer one period annuities. These are the vehicles through which agents save for retirement. Agents are able to draw down such accounts in a flexible way in retirement. Individuals may decide to mimic the payments from standard flat annuities by having the "pot" size (i.e. W) decline with age at a rate that is offset by rising average rates of return[5]. (Appendix 1 shows this result formally)

By assuming the availability of one period annuity contracts we are giving agents more options on lifetime accumulation and decumulation of assets than with standard

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[5] A simple example shows how such assets can be used to mimic annuities. Suppose the probability of death is invariant with respect to age and is at a constant rate p. Assume a non-stochastic, constant rate of return rf. With an initial stock of wealth at retirement of W an agent could buy a standard annuity from a firm offering an actuarially fair deal which pays W(rf+p)/(1+rf) each period until death. We assume here that the first of these level payments is made immediately and then come at the start of each subsequent period so long as the agent is alive. One period savings contracts of the sort we envisage pay a return per dollar invested of (1+rf)/(1-p) if the agent survives and nothing otherwise. An agent starting with wealth of W could immediately take W(rf+p)/(1+rf) out and reinvest the rest for one period. If they survive their wealth at the start of the next period is: W [1 – (rf+p)/(1+rf)].{(1+rf)/(1-

annuities or with standard retirement accounts. Agents will value flexibility in annuitising and are unlikely to want a flat drawdown of their accumulated fund. The optimal rate of accumulation and decumulation of funds over time is a complicated function of all the parameters in the model and depends on the realization of shocks; it can only be ascertained by simulations.

But in assuming that agents are offered these savings vehicles on actuarially fair terms we would be making a strong assumption that factors that seem to be important in the real world, and that make rates of return implicit in annuities contracts tend to be less than actuarially fair, are absent. (See, for example, Friedman and Warshawsky (1988); Mitchell, Poterba and Warshawsky (1997) and Brown, Mitchell and Poterba (1999)). It is important to allow for problems that make annuities less than fair. We introduce a measure of the efficiency of annuities markets. When this measure, β , is 1 the annuities market work perfectly. When $\beta = 0$ annuities are, effectively, not offered. The survival probability implicit in the contract offered by a financial institution is a weighted average of the true survival probability, s_{1j} , and the rate when no annuity is offered, an effective survival probability of unity. β is the weight placed on the actuarially fair survival probability

The rate of return paid on one period savings invested with λ in the risky asset and $(1-\lambda)$ in the safe asset for an agent aged j at time t becomes:

$$\{\lambda \exp[r_t + v_t] + (1-\lambda)\exp[r_{ft}]\} / [\beta s_{1j} + (1-\beta)] \qquad (9)$$

This way of modeling the efficiency of annuity contracts allows the departure from actuarially fair contracts to vary with age. The greater is age, the lower the probability of surviving and for all $\beta < 1$ the greater is the departure from actuarially fair contracts. Recent empirical evidence from the US suggests that annuity rates do become increasingly less favorable with age. Mitchell, Poterba and Warshawsky (1999) estimate that the average US annuity in 1995 delivered payouts with expected present value of between 80% and 85% of each $ annuity premium for 65 year olds;

p)} = W. Obviously this policy can be sustained indefinitely. Thus the standard annuity contract can be replicated exactly by rolling forward one period contracts.

but the payout ratio was less for older people. A payout ratio of 80% of the actuarially fair value for a 65 year old corresponds in our simulations[6] to a value of β of about 0.3 if the rate of return on assets is a flat 6%. Friedman and Warshawsky (1988) report US payout ratios from the 1970's and 1980's of around 75% which corresponds to a β at current life expectancies of about 0.2. Brown, Mitchell and Poterba (1999) provide some evidence that in the UK annuities average about 90% of the actuarially fair rates. This corresponds to a β of around 0.55. In Continental Europe and in Japan there is less evidence on annuity market efficiency and a less deep market. The depth of the market is linked to the generosity of state pensions. The issue of what assumption to make about what annuity market efficiency will be with different pension arrangements is important. Our strategy is to consider two values for efficiency – a low value of 0.25 and a value that broadly corresponds to the latest estimates of the efficiency in the UK and US (β=0.5).

This modeling of the structure of financial market efficiency is highly stylized. In order to ensure that funds are fully accounted for we are implicitly assuming that any departure from full actuarial efficiency reflects resources used up in the process of intermediation. So when we take a value of our efficiency parameter β of 0.25, which implies a pay out ratio of around 85%, we are assuming that the cost of offering savings products with annuities features is around 15% of the net present value of the payouts. Given adverse selection issues in the market for insurance products, and the costs of intermediaries in trying to offset those costs with screening, substantial costs are not implausible. But our characterisation of contracts remains ad hoc and we have not attempted to match the value of β closely to carefully estimated values of the costs of financial intermediation. Instead we take two values of β that – a high efficiency value and a lower efficiency value – that allow us to gauge the importance of plausible degrees of variation in the scale of efficiency in financial intermediation.

State pensions

We model the PAYGO pension system with a flat rate element and an earnings related element. We assume that for the typical worker the flat rate pension initially

[6] This is based on current Japanese mortality rates. These are likely to be relevant in European countries as their life expectancies move up to current Japanese levels.

generates around 1/3 of pension benefits and 2/3 come from a salary related pension. We use these ratios (1/3 flat rate; 2/3 salary related) and model the earnings related element as depending on final salary. The system is financed by the proportional tax on labor income levied on all those working. The tax rate is set to balance the unfunded state pension system in every period. We define the average replacement rate of the state pension as the ratio between the average pension paid in period t to someone just retired and the average gross income of those in the last year of their working life at period t-1. Pensions paid at retirement are therefore linked to movements in wages, but pensions subsequently are fixed in real terms.

The tax rate to finance state pensions of a given generosity is proportional to the replacement rate of the unfunded system. The factor of proportionality reflects the support ratio which is likely to change sharply over the next few decades.

When we analyse various reform strategies we make one very strong assumption about private sector behavior. We assume a form of super-rationality. Individuals work out the implications of any reform strategy and calculate what tax rates (or social security contribution rates) need to be to balance the system. In fact for technical reasons we make an even stronger assumption which is that agents know in advance which reform strategy will be chosen. This is clearly counter-factual. But all the reform strategies for state pensions we consider involve gradual changes in the levels of benefits and taxes – there are no sudden jumps – and for the most drastic change there is a 10 year gap between when reform is announced and when it come it gradually begins to affect taxes and pension levels (relative to base). What this means is that the impact on those close to retirement when reforms are unveiled is small and agents who will ultimately be most affected by reform have many years to respond. So the assumption that people always knew what was coming is less important that at first it might appear.

Nonetheless the super-rationality assumption is extreme. Furthermore our ruling out a bequest motive and making labour supply exogenous are strong assumptions.

III. Solving the Model:

The set of first order conditions from individual k's optimisation problem are:

if $\quad c_{kt} < [W_{kt} + \exp(y_{kt}).(1-\tau) + b_{kt}]$

then (10)

$$U`(c_{kt}) = E_t [s_{1j} \{ U`[c_{kt+1}].\{ \lambda \exp(r_t + v_t) + (1-\lambda)\exp(r_{ft}) \}/ [\beta s_{1j} + (1-\beta)] \} / (1+\rho)]$$

else:

$c_{kt} = [W_{kt} + \exp(y_{kt}).(1-\tau) + b_{kt}]$

and (11)

$$U`(c_{kt}) \geq E_t [s_{1j} \{ U`[c_{kt+1}]. \{ \lambda \exp(r_t + v_t) + (1-\lambda)\exp(r_{ft}) \} / [\beta s_{1j} + (1-\beta)] \} / (1+\rho)]$$

where $U`(c_{kt})$ is $\partial U_k / \partial c_{kt}$

We also require a condition for optimal portfolio allocation:

Either :

$0 = E_t [U`[c_{kt+1}].\{ \exp(r_t + v_t) - \exp(r_{ft}) \}]$

and $0 \leq \lambda \leq 1$

or

$0 < E_t [U`[c_{kt+1}].\{ \exp(r_t + v_t) - \exp(r_{ft}) \}]$

and $\lambda = 1$ (12)

else

$0 > E_t [U`[c_{kt+1}].\{ \exp(r_t + v_t) - \exp(r_{ft}) \}]$

and $\lambda = 0$.

(10) holds when the borrowing constraint is not binding. When the constraint binds complementary slackness implies that (11) holds. (12) is a standard condition for optimal portfolio allocation. Corner solutions may arise where agents wish to only invest in the safe asset or in the risky asset; for an internal solution the first equality at (12) must hold.

Although characterising optimal plans is easy enough solving explicitly for optimal consumption and for the optimal accumulation path for funds is not possible. Instead we have to turn to numerical methods. We solve the problem backwards in a now standard way (see Deaton 1991, Zeldes 1989, and Hubbard et al. 1995.) This involves constructing large grids in the state space and solving for optimal saving and portfolio allocation at those grid points using numerical integration and interpolation. We first solve the saving/consumption/portfolio allocation problem for a given path of interest rates and wages. We then generate life histories for cohorts each of size 7000 households born at different times[7]. To do that we create a set of paths for the idiosyncratic shocks to income for each member of every cohort. We then work out the optimal consumption-saving-portfolio allocation profile for every agent. Then we aggregate the decisions made by all cohorts alive at each date, taking account of the relative numbers of agents of each age at each point. We aggregate the saving and labor supply of all agents alive at each time and construct a time series for the aggregate stock of effective units of labor relative to capital. In the initial set of simulations we assume that investment in capital used within the economy moves in line with domestic saving. Put another way, the capital stock used in production is equal to the stock of wealth that individuals wish to hold. This assumption is clearly at odds with the fact that most developed economies do not, and have not, had a zero capital account on the balance of payments. So we consider an alternative assumption later in the paper where we take the rate of return and the evolution of average real wages as exogenous to domestic saving. We are able to assess what difference the various assumptions about the degree of exogeneity of factor prices make. This turns out to be important and suggests that how successful various pension reform strategies are depends upon the portfolio allocation decisions of the private sector in an environment of increasing reliance upon private saving for retirement resources.

In the case where factor prices are endogenous the future path of the mean rates of return on assets, and the evolution of mean real wages, depends upon how the aggregate capital to labor ratio (denoted k_t) evolves relative to its value in the 1990's. We calibrate the model to give a mean expected rate of return on risky assets over the 1990's of 6.5 and a safe rate of 2%. Denoting the average 1990's value for the

[7] We experimented with larger cohort sizes than 7000 and found this did not significantly affect the

aggregate capital to (effective) labor units ratio as k_0 we have that for some future date t the mean rate of return on risky assets is:

$$r_t = 0.065 \text{ x } (k_t / k_0)^{\alpha-1} \quad (13)$$

where we set the coefficient on capital in the implicit underlying Cobb Douglas production function equal to α (which is the share of profits in GDP which we set at 0.3). We consider two alternative assumptions about how the safe rate evolves. The first assumption is that the safe rate evolves according to:

$$r_{ft} = 0.02 \text{ x } (k_t / k_0)^{\alpha-1} \quad (14)$$

This implies that the risk premium on risky assets evolves according to:

$r_t - r_{ft} = (0.065- 0.02) \text{ x } (k_t / k_0)^{\alpha-1}$

So that the risk premium becomes proportional to the evolution of the capital labor ratio. An alternative assumption is that the risk premium remains at 4.5%, that the expected return on risky assets is still given by equation (13) and that the safe rate is always 4.5% less than this rate. These alternative assumptions, not surprisingly, turn out to have rather different implications for the evolution of the share of wealth held in risky assets.

The real wage per effective unit of labor (before we allow for the influence of exogenous, time related productivity growth) is different from the average level for 1990 by the factor:

$$(k_t / k_0)^{\alpha} \quad (15)$$

Equations (14) and (15) ensure that in the simulations with endogenous factor prices the average return on assets declines (rises) if the capital to labor ratio increases (falls). We also allow for real wages to be higher the greater is the capital to labor ratio (by (15)).

results.

Once we have solved the optimising problem at a given set of average rates of return and for a given path of wages, and calculated aggregate saving and labor supply, we update the path of rates and return and wages using equations (13) – (15). We then revise the optimising problem for each cohort, simulate the life paths for every cohort, aggregate once more and re-calculate the new paths for factor prices. This process is continued until convergence - that is until the evolution of average factor prices is consistent with the aggregate decisions made by agents who base their decisions upon that path of average prices.

What we end up with is 7000 profiles of consumption and wealth for each cohort. From this set of profiles we calculate for each cohort the expected utility of someone about to start their life. We take the actual life histories of the 7000 in the cohort and calculate the average lifetime utility of this group, a calculation that takes into account both time discounting and survival probabilities. This figure we take to be the ex-ante expected utility of the cohort

We can also use the individual profiles to calculate national income by aggregating labor income and capital income (the product of rates of return and the stock of wealth) across all agents at each point in time.

In this solution procedure we do, effectively, distinguish between the market value of the aggregate financial wealth of individuals and the value of the physical capital used for production. To capture the fact that variation in the value of physical capital is likely to be lower than variation in financial wealth (ie. there is volatility in Tobin's q), we model the value of physical capital as a moving average of the value of financial wealth. It is as if the value of physical capital is equal to financial wealth generated by zero shocks to the rate of return. We return to this point below having first discussed the calibration of the model.

Where factor prices are exogenous the simulations are much easier since we do not iterate towards a fixed point for mean returns and wages.

Initially we will stick to the assumption that national saving is closely linked to the evolution of capital used in production – ie that factor prices are endogenous. We consider this our base case and note that while the assumption that domestic saving and investment in domestic capital are equal is obviously unrealistic, an alternative assumption of a completely global capital market is also at odds with many facts. At the national level there remains a significantly higher correlation between savings and investment than one might expect with a global capital market - the Feldstein and Horioka puzzle remains puzzling. Portfolios of assets held by the private sector in developed economies remain substantially invested at home. Even to the extent that households in the developed economies invest overseas, they do so largely within other economies that also face an ageing problem. Arguably it is more reliable to assume that we are effectively in a closed economy (in terms of net capital flows) rather than assume that there is some exogenous world rate of return on assets that citizens in developed economies will be able to earn even as the population structure and pension arrangements change substantially.

We will show three types of simulation. In the first, state pension generosity is preserved at roughly its current level. We assume that the unfunded pension for someone earning average wages at retirement is worth about 60% of their gross final earnings. This generates an average ratio of pensions at retirement to net of contributions final period wages of about 70% - a figure that is typical in countries where state pensions have provided the major part of retirement resources. In the second set of simulations we assume that the contribution rate (the tax rate) is preserved at roughly the level needed to balance the system now – this rate turns out be around 17%. This will require that the typical replacement rate of the state pension decline gradually as the population ages. In the third set of simulations we assume that the government takes a radical course and announces now that from 2010 they will gradually reduce the typical replacement rate of unfunded state pensions in a way that leads to a fall in the replacement rate to only around 14% (one fifth its assumed current level) by 2050.

IV. Calibration

The key parameters in the model reflect degrees of risk aversion and the substitutability of consumption over time (that is the inter-temporal substitution of consumption), the rates of pure time preference, the degree of efficiency of financial markets and also the share parameter of the production function. There is considerable uncertainty about the magnitude of many of these parameters. Least controversial is probably the parameter from a simple Cobb Douglas production function that reflects a share of capital income in GDP (α) which we set equal to 0.3. In the base case we assume a relatively low inter-temporal substitutability of consumption (equal to one third, which implies a coefficient relative risk aversion of 3), we also assume a slightly negative rate of pure time preference, which may seem strange but is consistent with the limited empirical evidence available. In the absence of bequests, we find that a negative discount rate is needed to generate the level of savings observed in Germany, France, Italy and Japan (where savings rates have been significantly higher than in the US and the UK). A negative discount rate is not inconsistent with positive equilibrium real rates of return; see Benninga (1990), and Kocherlakota (1990). Other researchers have used a negative rate of pure time preference to model household decisions. Kato (1998) calibrates an OLG model with a -7.5% discount rate and Kato (2000) uses the rate of -3.5%. Imrohoroglu, Imrohoroglu and Joines (1999) use a negative discount rate of just under -1% in their numerical simulations. The empirical work of Hurd (1989), based on US data, is also consistent with negative rates of pure time preference. We also show results with a much higher rate of pure preference where agents discount the future at +1.5% a year. This is a very different assumption. With a rate of pure preference of –1.5% an agent at age 30 attaches four and a half times as much weight to outcomes at age 80 as when the rate of pure preference is +1.5%.

The coefficient of risk aversion and the intertemporal elasticity of substitution (which in our specification are the inverse of each other) is also controversial; Cooley and Prescott (1995) use unity for their simulations whereas Auerbach and Kotlikoff (1987) use a coefficient of relative risk aversion of 4, implying the elasticity of substitution is only 0.25. Empirical work by Hansen and Singleton (1983) and Mankiw, Rotemberg and Summers (1985) suggest values a little over unity for intertemporal substitutability implying, in our framework, a coefficient of relative risk aversion a little under unity. Grossman and Shiller (1981), Mankiw (1985) and Hall (1980) found, using US data, values between 0 and 0.4. for the intertemporal elasticity suggesting coefficients of risk aversion well in excess of 2. Hubbard, Skinner and Zeldes (1995) use a relative risk aversion of 3 in their simulations. Zeldes (1989) estimated the risk aversion coefficient as 2.3. We consider a value of 3 for the risk aversion coefficient is a central estimate but clearly the evidence makes it hard to be confident about what a plausible figure is. We will also show results where the coefficient of risk aversion is set equal to 6. We assume in this simulation that aggregate labor productivity increases at an exogenous rate of 2% a year.

This set of parameters yields an equilibrium path in the stochastic OLG model where the savings rate over the 1990s averages about 20% and the capital output ratio is around 3.3. These are plausible magnitudes.

Income profile

Cross section profiles of incomes in developed economies suggest that it is typical for earnings to peak at around age 50. We set the parameters of the earnings process so that on average the income of workers peaks at the age of 50 when it is roughly double earnings at age 20. With productivity growth at 2% this leads to the following average income profile

$$y_t = \ln 2 - (\ln 2)/900 * (age-50)^2 + 0.02t.$$

Labor income for a particular cohort is the product of the number of units of effective labor supplied and the post-tax real wage per unit. The latter is endogenous and depends on the tax rate needed to balance the state pension scheme and on the aggregate capital to labor ratio that, via the Cobb Douglas production function, determines the marginal productivity of labor.

Income volatility

Setting the volatility of the shock to labor income is particularly important for the simulations. As noted above, a significant part of the shocks to individual incomes is likely to be persistent. Hubbard, Skinner and Zeldes (1995) use a model of income dynamics to simulate the impact of social security which is based on characteristics of US household income data. Their model for the log income of household k at time t is the same as specified at (4) above and can be written:

$$y_{kt} = f(age_{kt}) + u_{kt} + \omega_{kt}$$

$$u_{kt} = \phi u_{kt-1} + e_{kt}$$

where ω and e are iid shocks that are not correlated and $f(age_{kt})$ is a deterministic function.

A measure of the unconditional volatility of log income is:

$\sigma^2_{\omega} + \sigma^2_e / (1 - \phi^2)$

Typical values for ϕ, σ_{ω} and σ_e used by Hubbard et al are 0.955, 0.158 and 0.158. These imply that some income shocks are highly persistent. With these values their measure of the unconditional standard deviation of the shock to log income is 0.56[8].

The dispersion of wages in Continental European countries and in Japan is lower than in the US, but data on how individual incomes are correlated over time suggest that the persistence in shocks is also very high. We used information on the distribution of labor incomes of Japanese households over time to find the key parameters of the stochastic process. Data on how the cross-section distribution of incomes for people aged 20 and aged 40 in 1981 evolved over the period to 1996 was used to pick the parameters of equation (4) so as to best match the empirical moments of the distribution. We found that the best fit was achieved when we set the persistence parameter at 0.968 and the standard deviation of the persistent income shocks (e) at 0.1191. The standard deviation of the temporary shock to log incomes was set at 0.076. This generates a cross section standard deviation of log incomes amongst those close to retirement of around 0.46.

Returns

The historical real returns on global stocks in developed economies over the past few decades have a mean of around 7-8% with standard deviation of 22-25% depending on the precise period one considers (we have looked at 1960-2000, 1970-2000, 1980-2000). These figures are for gross returns; net of charges annual returns to individuals are likely to be lower by at least 50 basis points, and perhaps by much less.

Returns on bond portfolios are less volatile than on equity portfolios. Miles and Timermann (1999) suggest that a mixed bond and stock portfolio in developed

[8] In fact Hubbard et al set different values of ϕ, σ_{ω} and σ_e for those with no high school, high school and College education. The implied unconditional standard deviation of the shocks to log income for these three groups are 0.64, 0.51 and 0.44 respectively.

countries would have generated a lower average real return than an equity portfolio and have a significantly lower annual volatility. Such a mixed portfolio might generate an average real return of about 6-7% a year with annual standard deviations of around 17.5%. Stock returns with these characteristics have been typical in many developed countries in the past. The above figures are before any deductions for charges, and for this reason we think of net returns on risky assets with a mean of 6.5% and volatility of 17.5% as relatively optimistic. We calibrate the model so that in the 1990's the mean of the risky return distribution is 6.5% and the safe rate is 2%. When we make factor prices endogenous we adjust subsequent mean rates of return to reflect movements in the capital labor ratio from their average levels in the 1990's.

V. Results

There are three different reform scenarios. Figure 1 shows the average gross replacement rate at retirement of the unfunded, state pension in each. Figure 2 shows the path of the contribution rate that balances the PAYGO system in each case. If the pension relative to *net* earning remains constant at about 70% then relative to gross earnings it drifts down slightly as the contribution rate increases. So the line marked "flat rate" in figure 1 , which shows the average pension relative to average gross earnings, drifts down from about 60% in 2001 to about 53% by 2050. The contribution rate needed to balance the system with a flat net replacement rate needs to rise from about 17.5% to about 27%. The rise is not steady since we allow for an assumed increase in the retirement age that will take place around 20 years from now which temporarily allows the contribution rate to fall slightly. A decision to keep the contribution rate at its current level requires that ultimately the replacement rate would need to fall from around 59% of gross incomes to about 36% by 2050. A decision to phase down dramatically the gross replacement rate from 59% to 13% would allow the contribution rate needed to balance the system to fall to about 7% - well under half its current level.

Using these different paths for the contribution rates and for the value of unfunded, state pensions we then undertake simulations with 7000 individuals *of each and every* cohort followed through their lives. Both the aggregate outcomes and, of course, the individual outcomes, depend on the realisation of shocks. The income shocks are idiosyncratic and so tend to get averaged out for the aggregate outcomes. But the rate of return shocks are common financial market shocks. So when we undertake dynamic stochastic simulations we need to make some common assumptions on the realisation of rate of return shocks to be able to compare different simulations with different pension arrangements.

We consider 5 different paths for the stochastic element of the return on risky assets.

1. The realisation for the random shock to the rate of return on risky assets is zero in all periods (although people make decisions based on a standard deviation of 17.5%).
2. Returns on risky assets are predominantly better than average. More specifically we generate a set of outcomes where returns regularly move from being 1 standard deviation above the mean to being 1 standard deviation below it, but in such a way that there are 3 above average returns for every 2 below average returns.
3. Returns on risky assets are predominantly below average. Here we generate a set of outcomes where returns regularly move from being 1 standard deviation above the mean to being 1 standard deviation below it, but in such a way that there are 3 below average returns for every 2 above average returns.
4. Returns alternate each year between being one standard deviation above the mean, and one standard deviation between the mean.
5. Returns come from one draw from the relevant distribution for the time series from 1900 to 2150.

Tables 1a and 1b, and figures 3-6, summarise the main macroeconomic implications of the three simulations. Here the return shock alternates between +1 and –1 standard deviation. Table 1a shows results where the risk premium isn proportional to the capital labor ration and Table 1b shows a fixed risk premium of 4.5%. Table 1a shows that the savings rate is projected to move sharply as a result of ageing. How great this effect is depends very much on pension reform. If unfunded pensions remain, on average, worth 59% of gross wages (70% of net wages) the savings rate is predicted to fall sharply – from around 19% in 2000 to about 11% by 2060. The path is, however, not steady since the rise in the retirement age that comes through between 2025 and 2030 significantly boosts total private savings for a while. If the contribution rate to balance an unfunded system is kept at 17.5% the decline in the saving rate is much more gentle. The saving rate in 2060 is around 13%. If the state pension is scaled back dramatically so it becomes a very small fraction of wealth at retirement (worth on average only 14% of salary at retirement) the aggregate saving rate actually rises slightly over the next 50 years.

Table 1b shows that the general trend in savings rates is similar if we assume a fixed risk premium.

Figure 4 shows how influenced by the realization of shocks to rates of return on risky assets the aggregate savings rates can be. The five panels in figure 4 show the evolution of savings under 5 different assumptions about the rate of return; here (and in all the figures) we just show results for the case where the future evolution of the risk premium is proportional to the capital labor ratio. If there are predominantly above average rates of return (the second panel of figure 4) savings rates will be much lower in the future; if returns are predominantly below average savings rates are very much higher. Unusually high returns increase financial wealth substantially and encourage lower saving; bad returns erode financial wealth and households save faster to rebuild assets for retirement.

Not surprisingly when there is either a partial or almost complete switch to funding both savings and the stock of wealth (Table 1) are higher. The impact on the stock of assets is particularly marked. With a constant contribution rate to the state pension system the stock of assets per capita by 2060 is almost 40% higher than it would be with a constant replacement rate. The stock of per capita assets would be over 80% higher if the net replacement rate is reduced to 14% (from about 70%).

All this has implications for rates of return. Figure 3 shows how the mean return on risky assets evolves as a result of changes in the capital/labor ratio. How great the fall in the rate of return over the next few decades will be, and whether that fall is reversed, depends very much on the nature of pension reform. With no reform (by which we mean a constant net replacement rate) the decline is limited; mean rates of return on risky assets fall from an average of 6.5%in the 1990s to about 5.6% by 2040. But in 2050 the rate is close to 6.0%. With a fixed contribution rate the decline in the rate of return is larger and more sustained. The mean rate of return falls to 5%

by 2035 and to 4.6% by 2060. With an even more substantial phasing back of the role of PAYGO pensions the rate of return ultimately falls to about 3.5% - not much more than half its 2001 level.

It might appear strange that the rate of return on capital (which in this model is the rate of return that people earn on savings) declines even in the scenarios when the savings rate falls significantly. One might suppose that with a lower savings rate capital would be scarce and this would drive the mean rates of return up. But with a Cobb Douglas production function what really matters is the capital labor ratio, or the capital output ratio. What is likely to happen in developed economies is that the supply of labor is likely to be significantly lower as a result of population ageing than it would be if population structures were unchanging. So while the savings rate might be lower, and this reduces the amount of capital relative to what it would have been with a constant savings rate, the impact of ageing upon aggregate labor supply is larger and the capital labor ratio is likely to be *higher* then it otherwise would be. This means that capital becomes *relatively* abundant and as a result the rate of return on that capital falls.

Figure 5 shows the evolution of the share of aggregate wealth held in risky assets. Again the figure takes the case where the risk premium is affected by the capital labor ratio. The figure shows that the portfolio share in risky assets is highly sensitive to pension arrangements but not very sensitive to changes in overall mean rates of return. The reason why portfolio shares do not react strongly to downward movements in mean rates of return on risky assets is that we assume that a rising capital to labor ratio simultaneously reduces returns on safe assets so that the risk premium is less affected, though it still shrinks. But if households have to rely for a much greater share of their retirement consumption on their own funds, and less on state pensions that are independent of financial market risk, they react by switching a large part of their portfolios away from risky assets and into safe assets. The greater is reliance upon funding, and the lower are PAYGO pensions, the larger is the share of wealth that is invested in safe assets. The scale of the effect is very large. With pensions worth, on average, 70% of net final salary usually more than 90% of wealth is invested in risky assets. (Though notice how different this is if returns turn out to be predominantly well above average – see the second panel of figure 5). If the net replacement rate falls to 14% that share eventually falls to well under 50%.

Results on portfolio allocation are different if we assume the risk premium is steady at 4.5%. Comparing tables 1a and 1b shows that the share of wealth put in risky assets is very much higher when the risk premium is constant. The difference is particularly marked when state pensions are scaled back dramatically. By 2060 the share of savings in risky assets falls to 50% if both the expected risky rate and the safe rate fall together as the capital to labor ratio rises. But if the risk premium is steady at 4.5%, so that the real safe rate falls to a slightly negative value, around 85% of savings are directed towards risky assets.

The evolution of aggregate consumption (relative to a base case of a constant average net replacement rate) is illustrated in Figure 6. Transition towards greater reliance upon funding requires a period when aggregate consumption falls below its path under a policy of preserving the generosity of unfunded pensions. But ultimately aggregate consumption rises significantly above that level when funding becomes more important. How great the decline in consumption is, how long it takes before consumption then overtakes the path under a policy of no reform, and by how much consumption in the long term can be higher depend both upon how great the move to funding is and what happens to rates of return. Under a policy of holding the contribution rate constant, and in the absence of either unusually good or bad return outcomes, aggregate consumption is relatively low until about 2020. But in the long run it is about 20% higher. Under a policy of reducing the net replacement rate for the state pension to 14% (staring from 2010) aggregate consumption is slightly lower until about 2030 but ultimately very much higher.

The paths for aggregate consumption suggest that some generations might be worse off as a result of a move to funding. In particular, cohorts who are relatively early in the working lives at the initiation of reform find that they need to continue paying substantial contributions to finance PAYGO pensions to their parents generation but will receive a relatively small pension by the time they retire 30 or 40 years hence. Consider the generation aged 20 in 2000 who have just started work at the date when reforms are undertaken. Figure 7 shows the evolution of average consumption for this cohort under each of the three pension scenarios and for the 5 different outcomes for random returns. The cross-section average of consumption for this cohort throughout its life remains lower in the case where there is pension reform and a movement towards greater reliance upon funding. Figure 8 reveals why: on average this cohort needs to build up a much greater stock of financial assets knowing that receipts of state, unfunded pensions will be much lower by the time it retires.

It is what happens to the *welfare* of agents of different generations that really matters. Table 2 shows in some detail how agents of different ages are affected by various reform strategies. We take as the base case a situation that we have called "no reform" - PAYGO pensions continue to be paid at a generous level (assumed to average around 70% of net wages). Table 2 gives an indication of the effect upon the lifetime utility of people of different ages as a result of either pegging the contribution rate at its current level or scaling back pensions from 2010 towards a 14% average net replacement rate by 2050. We measure the welfare implications of phasing out pensions by calculating a measure of the expected utility of agents of various cohorts (from those who are aged 60 today to those who will not be born for another 40 years). We compare how each cohort does in the base case where the state pension stays constant (relative to average earnings) with how they do when the ratio of the state pension to average earnings starts to fall as a move is made towards funding. In all cases we look at a scenario where the random component of the return on risky assets alternates between +1 and -1 standards deviation.

The first column for each reform strategy shows whether those in the relevant age cohort gain (G) or lose (L). The second column is an estimate of the scale of the gain or loss as in figure 9; it is the percentage by which consumption in the base case simulation would need to have been higher or lower to generate the same level of welfare as is given by the transition path. This is the compensating variation in consumption. The main figure is for simulations assuming a variable risk premium; the figures in parenthesis are for a fixed risk premium. So for people aged 50 in 2002 (born in 1952) the average decline in utility generated by pegging the contribution rate is the equivalent of a 3% cut in lifetime consumption. Their average loss if pensions are reduced gradually from 2010 is the equivalent of a cut in lifetime consumption of about 1%

Under a strategy of dramatically scaling back the level of PAYGO pension benefits more of those currently alive lose out than if the contribution rate is pegged. But the long-run gains are very much higher. With a fixed risk premium the safe rate is driven down more than when the risk premium is variable and that is bad for the welfare of future generations of savers so that the future gains from pension reform are somewhat lower with a fixed risk premium.

The result that a large proportion of those alive now would be worse off if the unfunded state scheme is phased out - even though *every* future generation is better off - illustrates the nature of the transition problem rather clearly. Democratically elected governments facing voters who focus on the direct implications to them (and not to all future generations) of changes to state pension systems would find it hard to get support for this kind of transition plan. Table 2 suggests that once a transition from an unfunded to a funded scheme is complete welfare for *all* subsequent generations will be higher, but without relying on deficit financing the transition will cause certain generations to be worse off, and those generations could form a majority of voters thus permanently blocking any change.

The losses of the transition generations are relatively small and the gains of the future generations, and of the current young, are very large. So optimal policy might well be to scale back significantly the generosity of PAYGO pensions. But the table suggests that there is unlikely to be a painless (i.e. Pareto improving) way of achieving this. But as we will see these welfare calculations are sensitive to assumptions about discount rates and rates of risk aversion.

It is important to stress that these welfare calculations are all about *expected utility at birth.* The fact that expected welfare for an agent born in, say, 2012 is higher when there has been a substantial movement towards funding and a cut in the generosity of unfunded pensions does not tell us that all individuals born then will turn out to be better off under a reform strategy that scales back PAYGO pensions. What it does tell us is that someone born then who understand the risks they faced *and did not know the realizations of income and rate of return shocks* would judge, at the start of their life, that they are better off in a world with more funding and less reliance upon

PAYGO pensions. There will be individuals who turn out subsequently to be worse off with more funded pensions. One way in which we can assess what the distribution of gainers and losers (ex-post) looks like is to compare the distribution of retirement consumption and of ex-post utility for different cohorts under different pension regimes. This is what we consider in the next section.

Before that we need to repeat some warnings about the interpretation of the results. Table 2 shows huge gains for future generations fro pension reform and much smaller losses for current generations. A large part of the gains are from the capital stock being much higher when reforms scale back the generosity of PAYGO pensions. Those gains will turn out to be much lower if we make a small open economy assumption which means that real wages are not affected by domestic savings and that there is no change to the capital labor ratio. A second warning is that we have assumed that agents are super-rational; they know what is coming and respond accordingly. Indeed they always knew what is coming. This is likely to bias downward the scale of the cost to those generations that do lose out. The assumption that people know in advance which reform path is chosen is clearly unsatisfactory. But there is one indirect indicator that it may not make a huge difference to the results. Tables 1a and 1b show that the macroeconomic aggregates that the model generates for the year 2000 are very close whichever future reform path is chosen. This implies that the behavior of agents in 2000, although it reflects a different forward looking environment in each scenario, does not substantially influence aggregate behavior at that point. Because reforms are only introduced gradually – and with the third scenario only with a 10 year delay – they have a muted impact on behavior at the start .

VI: The distribution of retirement consumption and of lifetime utility under different pension arrangements.

Different pension systems generate different distributions of retirement consumption and of ex-post lifetime utility. Here we focus on the spread of retirement consumption and of lifetime utility for different cohorts under different pension scenarios. We consider 4 cohorts: those aged 60 in 2002; those aged 35 in 2002; those born in 2002 (who will not start work until 2022); and those born in 2030 who will not start work until 2050. We consider the distribution of consumption at age 65 and of lifetime utility for members of each of these cohorts under the 3 different pension regimes. For each pension regime we report results on the distribution of outcomes where return shocks alternate between +1 and –1 standard deviation. Table 3 looks at the distribution of ex-post utility of various cohorts. There are three pension regimes, as before: a regime in which unfunded, PAYGO pensions on average remain worth 70% of net final salary; a regime with a contribution rate of 17.5%; a regime in which PAYGO pensions by 2050 are worth on average only 14% of net final salary.

For those born in 2030 (and whose age 65 consumption is made in 2095) virtually everyone has a higher level of consumption *at retirement* and higher lifetime utility if there has been a move towards greater reliance upon funding. The consumption of the household in this cohort at the lowest percentile is consistently higher the greater is reliance upon funded pensions. This is so even when we consider scenarios in which the history of rates of return is one with disproportionately below average returns.

In a world in which PAYGO pensions have been scaled right down (to generate only a 14% net replacement rate) the least well off percentile at retirement have a level of consumption around 27% higher than the corresponding household in a world where PAYGO pensions continue to generate pensions that on average are worth 70% of net final salary. The median household has consumption that is 40% higher at age 65.

For the cohorts who do not live through a transition towards a more funded system (and therefore do not face the double payment burden) almost everyone is better off at retirement. But as we consider cohorts born earlier, and who to different extents do live through the transition, the position is different. Consider the cohort born in 1967, who enter work in 1987 and who are aged 35 in 2002. Right across the distribution of retirement resources we find that this cohort are worse off the greater is the scale of the move towards funding. They enter retirement in year 2032. At that point consumption is lower right across the distribution than it would be if PAYGO pension generosity had been left unchanged. The scale of the loss in consumption at retirement is substantial and fairly equal across the distribution: generally between 11% and 15% with greater losses for those with more resources. Lifetime welfare for this cohort falls by the equivalent of a cut in lifetime consumption of about 5% across the distribution. Those aged 60 in 2002 (born in 1942) are generally small losers from pension reform.

The important point to emerge from this distributional analysis is that in the longer run pretty much everyone gains from a switch to funding - even those with very low lifetime resources. But for those who face the double payment burden (at least for the fairly simple transitional arrangement simulated here) the losses are also very hard to avoid – right across the income distribution there is lower consumption at retirement. This is especially marked for those in their mid thirties today.

VII: Alternative Parameterisations:

In this section we consider how the simulation results are affected by changes in key parameters. We consider four alternatives: first we take a much higher rate of risk aversion, increasing the coefficient of relative aversion from 3 to 6. Second we take a much higher rate of time preference, raising the discount rate from –1.5% to +1.5%. Third we consider the implications of a lower degree of efficiency of annuity contracts; we reduce the efficiency parameter β from 0.5 to 0.25, which corresponds to a reduction in the money's worth ratio at retirement from an average of approximately 88% to around 81%.

Tables 5-7 summarise the results. In constructing these figures we once again have calculated optimal decisions for individuals based on draws for income and rate of return shocks. For each cohort we then aggregate over 7000 agents and show the average results for each regime, at each point and for a given evolution of the random rate of return. In these tables we show results where the random component of the rate of return alternates between +1 and –1 standard deviation..

Table 5 summarises the aggregate results when the coefficient of risk aversion is much higher. Comparing those results with those in table 1 reveals that with higher risk aversion a very substantially smaller share of wealth is invested in risky assets. When state pensions are reduced very sharply the share of risky assets in portfolios by 2060 falls to only around 20% with high risk aversion as against a figure of around 50% with a coefficient of risk aversion of 3. The saving rate is also consistently lower with high risk aversion – the incentive to accumulate wealth is pulled in two directions by greater risk aversion. On the one hand there is a tendency for savings to rise because of greater precautionary demands. But with risky assets so much less attractive, the overall desirability of saving is actually lower. The capital output ratio is significantly lower when there is more risk aversion.

But the key welfare conclusion from the analysis is little changed with higher risk aversion. Agents of working age now tend to lose out slightly from a decision to move towards funding. Those born in the future stand to gain a great deal, and their gain is far greater than the loss of the working generations that live through the transition. GDP is very much higher in the long run when pensions are scaled back – indeed much more so with higher risk aversion than with a coefficient of risk aversion of 3.

Table 6 shows aggregate outcomes when the rate of time preference is +1.5%. Not surprisingly the saving rate is consistently very much lower than in Table 1 where the rate of time preference was *minus* 1.5%. (The reason for using a negative rate of preference was largely to try to match the relatively high saving rate in Europe and in Japan). As a result the capital/output ratio is much lower with positive time preference. But once again savings rates and capital output ratios are ultimately boosted very significantly by a move towards funding of pensions and a decline in the generosity of unfunded pensions. The overall pattern of the welfare implications of reforms is little affected by assumptions about the discount rate. But the long run gains are much greater with a much higher rate of time preference. The reason is straightforward. A major effect of phasing out unfunded, state pensions is that individuals do not need to make significant compulsory contributions from labor income. This is very valuable early in life to agents facing credit restrictions. The scale of those credit restrictions is greater the more impatient consumers are. *It is important to note that a significant part of the gain from a switch towards giving people discretion about the scale and timing of contributions towards their own personal pension pot is the value of the flexibility this gives – something which is absent in most unfunded social security systems where contributions are typically a given proportion of earnings.*

Table 7 shows the impact of less efficient annuities markets. Here we cut the efficiency parameter to 0.25. This has the result of increasing saving rates slightly (as people save more to guard against unusually long lives in an environment where annuities contracts are less effective). Portfolio decisions are little changed and once again the broad shape of the welfare implications of alternative reform strategies is little affected.

Small Open Economy results:

Table 8 summarises the results when the rate of return on assets is independent of the stock of wealth of domestic citizens; here sharp movements in the saving rate are assumed not to influence the capital labor ratio as flows of funds across the capital account react to swings in the domestic savings/investment balance. When we allow for movements in domestic savings and in the labor force to influence rates of return the mean return on risky assets and the return on safe assets falls and the scale of this decline is large. When there is a wholesale move towards funding the mean return on risky assets falls from 6.5% to around half that level. When the mean rate of return is held constant savings rates are projected to be slightly higher. The impact upon the aggregate wealth to labor ratio is more significant. Wealth to labor ratios are consistently higher when rates of return are assumed to be exogenous.

There are also significant affects upon aggregate portfolio allocation. With exogenous rates of return there is no shrinkage in the equity risk premium as funded pensions become more important and as the labor force shrinks. As a result the share of wealth invested in risky assets is consistently higher when rates of return are exogenous. With endogenous rates of return the share of wealth in risky assets is 50% when state pensions are run down to a low level by 2060 (see Table 1). With exogenous rates of return that share is 69%.

But once again the same broad set of welfare implications of different pension reform strategies emerges. A majority of those of working age at the initiation of pension reform are worse off as a result of a switch to funding, though the average loss is relatively small and much smaller than the welfare gain to future generations. But the scale of the long run gains is much lower when we do not allow for factor prices to react to changes in savings and in the labor force. When rates of return and real wages react to swings in domestic saving the gains in welfare of cutting the net replacement rate of PAYGO pensions to an average of 14% for people born in 2002 are the equivalent of rise of between 15% and 25% in consumption. With exogenous factor prices the scale of the gain is only around one third as great. The key factor here is that labor incomes are significantly higher for future generations when the higher stock of wealth boosts the capital labor ratio which drives up real wages. This more than offsets the disadvantage of lower mean rates of return when factor prices are endogenous. With endogenous factor prices the gain in long run output from a substantial switch to funding is three times as great as the percentage gain in national income when extra saving is channelled abroad.

VIII Conclusions:

Our key findings are these.

- The overall saving rate, and particularly the aggregate stock of assets, is likely to be *highly* sensitive to the generosity of unfunded state pensions.
- In the absence of prolonged bear markets it is likely that a long-run implication of a switch to much greater reliance upon funded pensions is that consumption and welfare for future generations will be *very* significantly higher.
- If most of the extra savings that would be generated by a switch to funding are invested domestically (or at least within the developed world where ageing is similar across countries) the capital labor ratio is likely to rise significantly and drive down mean rates of return. In itself this is not helpful for the whole pension reform process. But there is an upside. With a much higher capital labor ratio wages are also higher. Indeed our simulation results suggest that the gain to future generations from higher wages more than offsets the loss from lower returns. We find that if factor prices are independent of pension reform – as they would be if Japan were a small open economy and if extra savings were largely channeled overseas – the gains to future generations of a switch to funding are significantly lower.
- Yet it is likely to be hard to engineer a transition to much greater reliance upon funding without leaving a substantial proportion of today's adults at least slightly worse off.

- Annuity market efficiency is important - in a model with no bequests the degree of efficiency of annuity markets has a significant impact on the scale of the potential gains from relying more on private saving to finance retirement consumption. The more significant are bequests the less important is this factor likely to be.
- There is a powerful link between overall portfolio allocation and the pension system. The more generous are state, unfunded pensions –the value of which is only indirectly (and perhaps weakly) linked to rates of return on financial assets - the greater is the proportion of wealth invested in risky assets. If people come to rely heavily upon funded pensions that expose their retirement resources to financial market risk then they are likely to respond by holding a larger share of their financial wealth in safe assets. But how great that portfolio switch will be depends very much on whether the risk premium on risky assets stays constant or is squeezed as rising capital labor ratios drive down expected returns on risky assets *and* on safe assets by the same proportion.
- It is important to note that a significant part of the gain of a switch towards funding stems from giving people discretion about the scale and timing of contributions towards their own personal pension pot. There is a significant value to the flexibility this brings – something which is absent in most unfunded social security systems where contributions are typically a given proportion of earnings. One important implication of this is that there may be substantial gains to be had by increasing the flexibility in the timing of contributions within an existing unfunded, PAYGO pension system.
- A key finding is that longer run gains from a switch towards greater reliance upon funding, and away from an unfunded system where pensions are linked to salaries, do NOT go disproportionately to the better off. We find no tendency for reform to have adverse distributional consequences

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Appendix 1:

To show how the one period contracts we assume are available allow agents to create standard annuities – should they so wish – consider someone who just invests in the safe asset (assumed to generate a constant return at rate r_f). As before we focus on an agent aged j at time t who has wealth W.

A standard, actuarially-fair annuity contract would promise to pay to a j year old an annual amount of A (until death) in exchange for a lump sum of W, where A satisfies:

$$W = A\,[\,1 + s_{1j.}\,e^{-rf} + s_{2j.}\,e^{-2rf} + \ldots\ldots\ldots\ldots. + s_{nj.}\,e^{-nrf}\,] \qquad n \to \infty \qquad \text{(A1)}$$

Here we are assuming payments are made at the start of each period and the first payment is made immediately. Let:

$$\Delta \equiv [\,1 + s_{1j.}\,e^{-rf} + s_{2j.}\,e^{-2rf} + \ldots\ldots\ldots\ldots. + s_{nj.}\,e^{-nrf}\,] \qquad n \to \infty \qquad \text{(A2)}$$

So $\quad A = W / \Delta$

Note the link between one period survival probabilities at different ages:

$$s_{2j} = s_{1j}\, s_{1j+1}$$

and more generally:

$$s_{nj} = s_{1j}\, s_{1j+1}\, s_{1j+2} \ldots\ldots\ldots.\, s_{1j+n-1} \qquad \text{(A3)}$$

If a j year old agent has wealth W in a fund and withdraws an amount $A= W/\phi$ and reinvests the remainder with our one-period contracts, their wealth at the start of the next period is:

$$W[\,1 - 1/\Delta\,]\, e^{rf} / s_{1j}$$

Withdrawing the same amount in the second period, when the agent is aged j+1, generates a fund at the start of the third period of:

$$W[\{(1 - 1/\Delta).\, e^{rf} / s_{1j} - 1/\Delta\}.\, e^{rf} / s_{1j+1}\,] = W[(1 - 1/\Delta)\, e^{2rf} / s_{2j} - (1/\Delta)\, e^{rf} / s_{1j+1}]$$

Assuming a constant per period withdrawal rate of W/ϕ the level of funds at the start of period n+1 is:

$$W[\ (1 - 1/\Delta)\ e^{nrf} / s_{nj}\ -\ (1/\Delta)\{\ e^{(n-1)rf} / s_{n-1\ j+1}\ +\ e^{(n-2)rf} / s_{n-2\ j+2}\ +\ e^{(n-3)rf} / s_{n-3\ j+3}\ +$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ +\ e^{rf} / s_{1\ j+n-1}\ \}\ ] \qquad \text{(A4)}$$

Which we can write:

$$W(e^{nrf} / s_{nj})\ [1 - (1/\Delta).\{1 + s_{1j.}\ e^{-rf}\ +\ s_{2j.}\ e^{-2rf}\ +\ \ldots\ldots\ldots\ +\ s_{n-1\ j.}\ e^{-(n-1)rf}\ \}] \qquad \text{(A5)}$$

From (A2) we have that for finite n

$$0\ <\ (1/\Delta).\{1 + s_{1j.}\ e^{-rf}\ +\ s_{2j.}\ e^{-2rf}\ +\ \ldots\ldots\ldots + s_{n-1\ j.}\ e^{-(n-1)rf}\ \}\ <\ 1$$

So (A5) is always positive. This proves that the one period annuity contracts allow agents to mimic the returns from standard (open-ended) annuity contracts and satisfy budget constraints.

Table 1a:
Stochastic Simulations: Aggregate Outcomes: variable risk premium

	Keep average net replacement rate of state pension at 70%	Keep contribution rate constant at 17.5%	Phase down net replacement rate to 14% by 2040
Savings Rate			
2000	19%	19%	20%
2020	14%	17%	16%
2040	13%	16%	21%
2060	11%	13%	20%
Capital-Output Ratio			
2000	3.4	3.4	3.4
2020	3.9	3.8	3.5
2040	3.9	4.3	3.7
2060	4.0	4.8	5.3
Share of Risky Assets			
2000	94.1%	95.7%	93.8%
2020	92.7%	88.7%	95.8%
2040	90.7%	80.9%	77.5%
2060	91.2%	74.2%	50.6%
Mean Risky Rate			
2000	6.2%	6.3%	6.2%
2020	6.0%	5.5%	6.3%
2040	5.6%	4.9%	4.6%
2060	5.8%	4.6%	3.5%
GDP relative to base			
2000		0%	-1%
2010		2%	-3%
2020		7%	-1%
2030		7%	4%
2040		10%	14%
2050		14%	29%
2060		15%	37%

Notes:

Figures show the simulation results when shocks alternate between one standard deviation above the mean and one standard deviation below the mean.
In these simulations annuity market efficiency is set to 0.5. The coefficient of risk aversion is set at 3.0 and the rate of pure time preference is -1.5% p.a.. State, unfunded pensions are related to final salary.

Table 1 b:

Stochastic Simulations: Aggregate Outcomes: risk premium fixed at 4.5%

Savings Rate			
2000	19%	19%	20%
2020	14%	18%	17%
2040	14%	17%	23%
2060	12%	15%	24%
Capital-Output Ratio			
2000	3.3	3.3	3.3
2020	3.8	3.7	3.6
2040	3.8	4.0	3.6
2060	3.9	4.5	5.0
Share of Risky Assets			
2000	96.2%	96.5%	95.6%
2020	95.9%	95.9%	96.6%
2040	96.3%	95.8%	96.6%
2060	95.8%	93.4%	84.8%
Mean Risky Rate			
2000	6.2%	6.3%	6.2%
2020	5.9%	5.5%	6.3%
2040	5.5%	4.8%	4.7%
2060	5.7%	4.6%	3.5%
GDP relative to base			
2000		0%	0%
2010		2%	-2%
2020		7%	-1%
2030		8%	3%
2040		11%	13%
2050		16%	30%
2060		18%	40%

Notes:

Figures show the simulation results when shocks alternate between one standard deviation above the mean and one standard deviation below the mean.
In these simulations annuity market efficiency is set to 0.5. The coefficient of risk aversion is set at 3.0 and the rate of pure time preference is -1.5% p.a.. State, unfunded pensions are related to final salary.

Table 2: Gainers and Losers from Pension reform

Compensating Variation in Consumption - equivalent variation in lifetime consumption to keep utility at base level: gains (+) or losses (-)

Figures are for variable risk premium, in parenthesis figures are for fixed risk premium

Cohort age in 2002 (and year when born)	Keep contribution rate constant at 17.5%		Phase down replacement rate to 10% by 2040	
60 (1942)	L	-2% (-1%)	L	-1% (0%)
50 (1952)	L	-3% (-3%)	L	-1% (-2%)
40 (1962)	G	-1% (-2%)	L	-5% (-4%)
30 (1972)	G	2% (0%)	L	-6% (-7%)
20 (1982)	G	3% (+2%)	L	-6% (-7%)
10 (1992)	G	12% (+9%)	G	2% (-2%)
0 (2002)	G	17% (+14%)	G	25% (+10%)
-10 (2012)	G	19% (+19%)	G	29% (+29%)
-20 (2022)	G	26% (+25%)	G	59% (+52%)
-30 (2032)	G	30% (+29%)	G	79% (+69%)
-40 (2042)	G	31% (+31%)	G	82% (+80%)

G = gain; L = lose;

Notes:

Figures show the simulation results when shocks alternate between one standard deviation above the mean and one standard deviation below the mean.
In these simulations annuity market efficiency is set to 0.5. The coefficient of risk aversion is set at 3.0 and the rate of pure time preference is -1.5% p.a.. State, unfunded pensions are related to final salary.

Table 3: Ex-post Utility of agents born in 1942, 1967, 2002 and 2030

Percentile of Distribution	Keep net replacement rate of state pension at 70%	Keep contribution rate constant at 17.5%		Phase down replacement rate to 14% by 2040	
		1942 cohort			
			% gain		% gain
1%	0.043	0.043	-0.5%	0.043	0.3%
5%	0.052	0.051	-0.7%	0.052	0.1%
20%	0.065	0.064	-0.8%	0.065	0.0%
50%	0.080	0.079	-1.1%	0.080	-0.3%
75%	0.094	0.093	-1.2%	0.093	-0.4%
95%	0.116	0.115	-1.3%	0.115	-0.6%
		1967 cohort			
1%	0.041	0.041	-0.7%	0.039	-4.6%
5%	0.050	0.050	-0.5%	0.047	-4.8%
20%	0.062	0.062	-0.3%	0.059	-5.0%
50%	0.077	0.076	-0.5%	0.073	-5.3%
75%	0.090	0.090	-0.6%	0.085	-5.2%
95%	0.112	0.111	-1.1%	0.105	-5.6%
		2002 cohort			
1%	0.040	0.046	14.5%	0.045	14.3%
5%	0.048	0.055	14.8%	0.054	13.2%
20%	0.060	0.069	16.0%	0.068	13.1%
50%	0.074	0.085	15.7%	0.083	12.2%
75%	0.087	0.100	15.5%	0.097	11.9%
95%	0.108	0.124	15.2%	0.120	11.6%
		2030 cohort			
1%	0.039	0.049	26.0%	0.065	66.4%
5%	0.047	0.060	28.1%	0.080	71.2%
20%	0.058	0.075	29.4%	0.100	72.4%
50%	0.072	0.093	28.9%	0.125	73.7%
75%	0.085	0.109	28.8%	0.148	74.5%
95%	0.105	0.136	29.0%	0.185	76.1%

Notes:

Figures show level of utility for member of cohort at given percentiles of utility distribution. “% gain” is the equivalent change in lifetime consumption to generate the change in utility relative to the base case of a constant average net replacement rate for the state pension. This is the compensating variation in consumption.

Rates of return alternate between being one standard deviation above and one standard deviation below the average.

Table 4: Ex-post Consumption at age 65 of agents born in 1942, 1967, 2002 and 2030

Percentile of Distribution	Keep net replacement rate of state pension at 70%	Keep contribution rate constant at 17.5%		Phase down net replacement rate to 14% by 2040	
		1942 cohort			
			% gain		% gain
1%	0.653	0.643	-1.6%	0.651	-0.3%
5%	0.819	0.801	-2.2%	0.816	-0.3%
20%	1.076	1.046	-2.8%	1.076	0.0%
50%	1.461	1.406	-3.8%	1.457	-0.3%
75%	1.888	1.810	-4.2%	1.888	0.0%
95%	2.749	2.617	-4.8%	2.745	-0.1%
		1967 cohort			
1%	0.630	0.615	-2.4%	0.562	-10.9%
5%	0.784	0.763	-2.6%	0.697	-11.1%
20%	1.027	1.004	-2.3%	0.904	-12.0%
50%	1.388	1.359	-2.1%	1.205	-13.2%
75%	1.791	1.750	-2.3%	1.543	-13.9%
95%	2.598	2.531	-2.6%	2.211	-14.9%
		2002 cohort			
1%	0.611	0.647	6.0%	0.609	-0.2%
5%	0.763	0.818	7.2%	0.770	0.9%
20%	1.005	1.085	8.0%	1.042	3.7%
50%	1.368	1.467	7.2%	1.445	5.6%
75%	1.774	1.892	6.7%	1.882	6.1%
95%	2.583	2.745	6.3%	2.783	7.7%
		2030 cohort			
1%	0.605	0.685	13.2%	0.771	27.3%
5%	0.753	0.868	15.3%	1.001	33.0%
20%	0.989	1.150	16.2%	1.361	37.6%
50%	1.352	1.559	15.4%	1.890	39.9%
75%	1.752	2.010	14.8%	2.476	41.3%
95%	2.572	2.926	13.8%	3.698	43.8%

Notes:

Figures show level of consumption at age 65 for member of cohort at given percentiles of age 65 consumption distribution.

Rates of return alternate between being one standard deviation above and one standard deviation below the average.

Table 5:
Stochastic Simulations: Coefficient of Risk version = 6

	Keep net replacement rate of state pension at 70%	Keep contribution rate constant at 17.5%	Phase down net replacement rate to 14% by 2040
Savings Rate			
2000	13%	14%	13%
2020	9%	13%	10%
2040	9%	12%	18%
2060	7%	10%	19%
Capital-Output Ratio			
2000	2.7	2.7	2.6
2020	3.3	3.1	2.7
2040	3.2	3.5	2.9
2060	3.3	4.0	4.6
Share of Risky Assets			
2000	69.5%	72.9%	70.7%
2020	67.0%	60.2%	72.5%
2040	62.0%	49.4%	43.0%
2060	62.6%	42.5%	19.6%
Mean Risky Rate			
2000	6.1%	6.3%	6.2%
2020	5.8%	5.3%	6.0%
2040	5.3%	4.5%	3.9%
2060	5%	4%	3%
GDP relative to base			
2000		0%	-1%
2010		0%	-4%
2020		9%	0%
2030		10%	10%
2040		14%	27%
2050		20%	55%
2060		24%	76%

Notes:

Figures show the simulation results when shocks alternate between one standard deviation above the mean and one standard deviation below the mean.
In these simulations annuity market efficiency is set to 0.5. The coefficient of risk aversion is set at 6.0 and the rate of pure time preference is –1.5% p.a.. State, unfunded pensions are related to final salary.

Table 6:

Stochastic Simulations: Rate of pre time preference +1.5%

	Keep net replacement rate of state pension at 70%	Keep contribution rate constant at 17.5%	Phase down net replacement rate to 14% by 2040
Savings Rate			
2000	10%	11%	11%
2020	8%	11%	10%
2040	8%	10%	14%
2060	6%	8%	15%
Capital-Output Ratio			
2000	1.8	1.9	1.8
2020	2.2	2.0	2.6
2040	2.1	2.5	1.9
2060	2.2	2.9	3.3
Share of Risky Assets			
2000	98.3%	99.0%	97.6%
2020	97.1%	94.5%	98.7%
2040	96.0%	87.5%	85.3%
2060	96.2%	79.4%	43.9%
Mean Risky Rate			
2000	6.1%	6.4%	5.9%
2020	5.5%	5.1%	6.4%
2040	5.2%	4.5%	4.3%
2060	5%	4%	3%
GDP relative to base			
2000		0%	1%
2010		0%	1%
2020		10%	-5%
2030		13%	-5%
2040		14%	15%
2050		19%	47%
2060		22%	65%

Notes:

Figures show the simulation results when shocks alternate between one standard deviation above the mean and one standard deviation below the mean.
In these simulations annuity market efficiency is set to 0.5. The coefficient of risk aversion is set at 3.0 and the rate of pure time preference is +1.5% p.a.. State, unfunded pensions are related to final salary.

Table 7:

Stochastic Simulations: Efficiency of Annuity Contracts parameter (b) = 0.25

	Keep net replacement rate of state pension at 70%	Keep contribution rate constant at 17.5%	Phase down net replacement rate to 14% by 2040
Savings Rate			
2000	20%	20%	20%
2020	15%	19%	18%
2040	15%	17%	23%
2060	13%	15%	22%
Capital-Output Ratio			
2000	3.4	3.3	3.3
2020	4.0	3.7	3.6
2040	4.0	4.2	3.8
2060	4.0	4.7	5.3
Share of Risky Assets			
2000	95.3%	97.1%	95.3%
2020	94.6%	91.3%	97.2%
2040	93.0%	84.0%	81.7%
2060	93.8%	77.2%	53.6%
Mean Risky Rate			
2000	6.1%	6.3%	6.2%
2020	6.1%	5.5%	6.3%
2040	5.8%	4.9%	4.8%
2060	6%	5%	4%
GDP relative to base			
2000		1%	-1%
2010		2%	-2%
2020		8%	-1%
2030		8%	4%
2040		12%	13%
2050		16%	28%
2060		18%	37%

Notes:

Figures show the simulation results when shocks alternate between one standard deviation above the mean and one standard deviation below the mean.
In these simulations annuity market efficiency is set to 0.25. The coefficient of risk aversion is set at 3.0 and the rate of pure time preference is -1.5% p.a.. State, unfunded pensions are related to final salary.

Table 8:

Stochastic Simulations: Small Open Economy assumption

– rates of return exogenous.

	constant replacement rate at	**fixed contribution rate**	**phase down replacement rate**
Savings rate			
2000	21%	22%	21%
2020	15%	20%	20%
2040	14%	18%	23%
2060	12%	15%	20%
Capital-Output Ratio			
2000	3.9	3.9	3.9
2020	4.3	4.8	4.4
2040	4.3	5.3	5.5
2060	4.1	5.5	6.8
Share of Risky Asset			
2000	93.8%	94.6%	94.2%
2020	92.3%	88.1%	93.0%
2040	92.6%	84.7%	81.0%
2060	93.0%	81.6%	68.9%
Mean Risky Rate			
2000	6.5%	6.5%	6.5%
2020	6.5%	6.5%	6.5%
2040	6.5%	6.5%	6.5%
2060	6.5%	6.5%	6.5%
national income relative to base			
2000		0.9%	0%
2010		4.0%	1%
2020		6.2%	5%
2030		4.5%	9%
2040		5.7%	11%
2050		6.9%	14%
2060		6.8%	13%

Notes:
Figures show the simulation results when shocks alternate between one standard deviation above the mean and one standard deviation below the mean.
In these simulations annuity market efficiency is set to 0.5. The coefficient of risk aversion is set at 3.0 and the rate of pure time preference is -1.5% p.a.. State, unfunded pensions are related to final salary. The mean of the risky rate of return is set equal to 6.5% and the safe rate is set at 2%.

Figure 1:

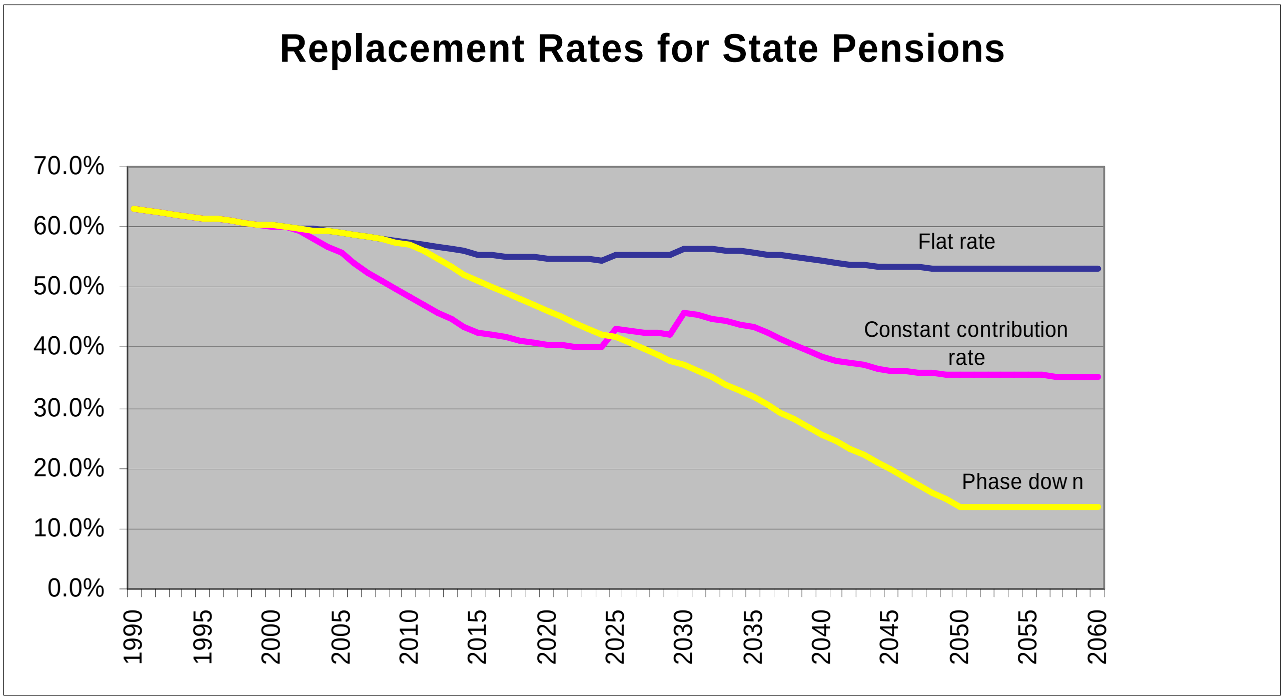


Figure 2:

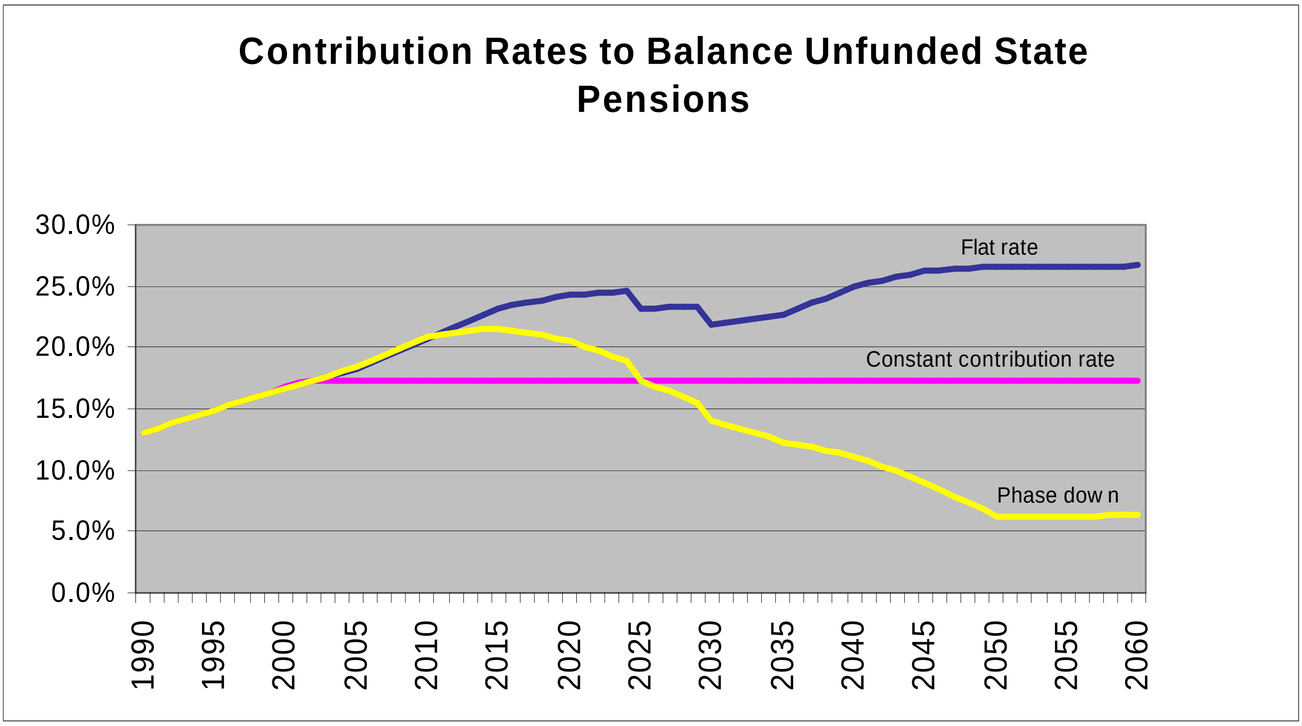
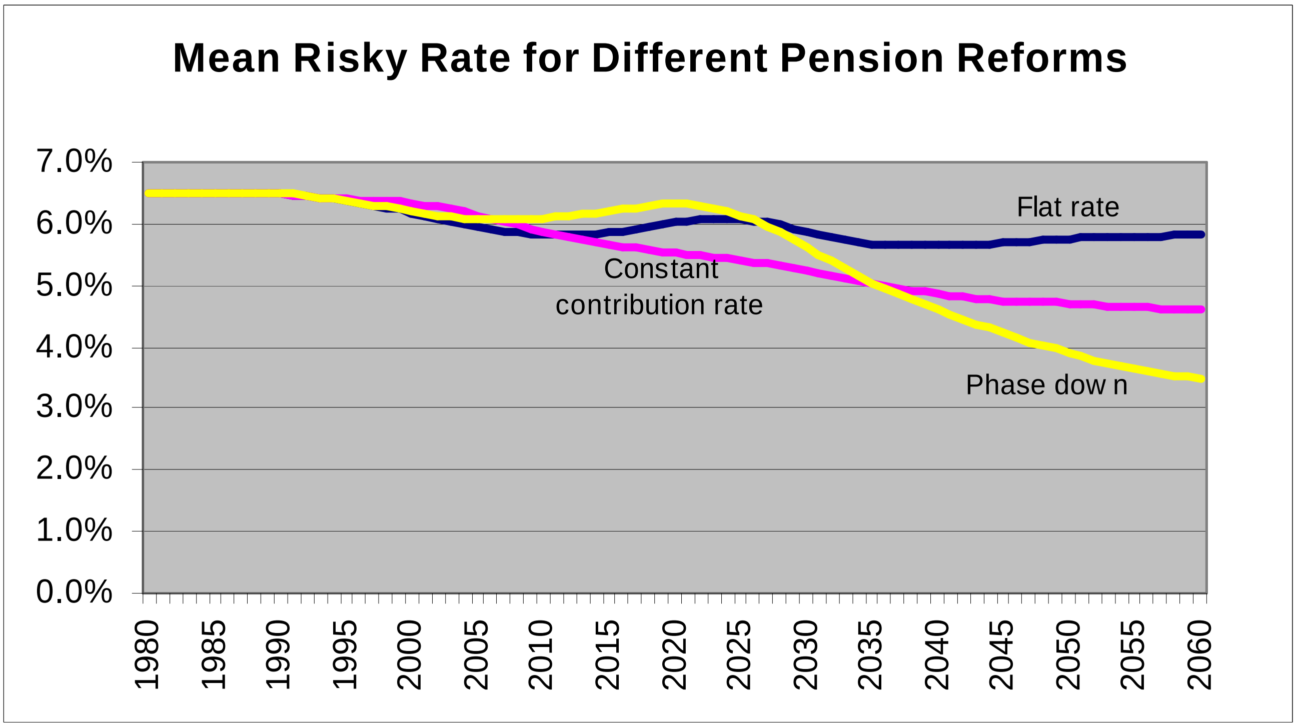


Figure 3:

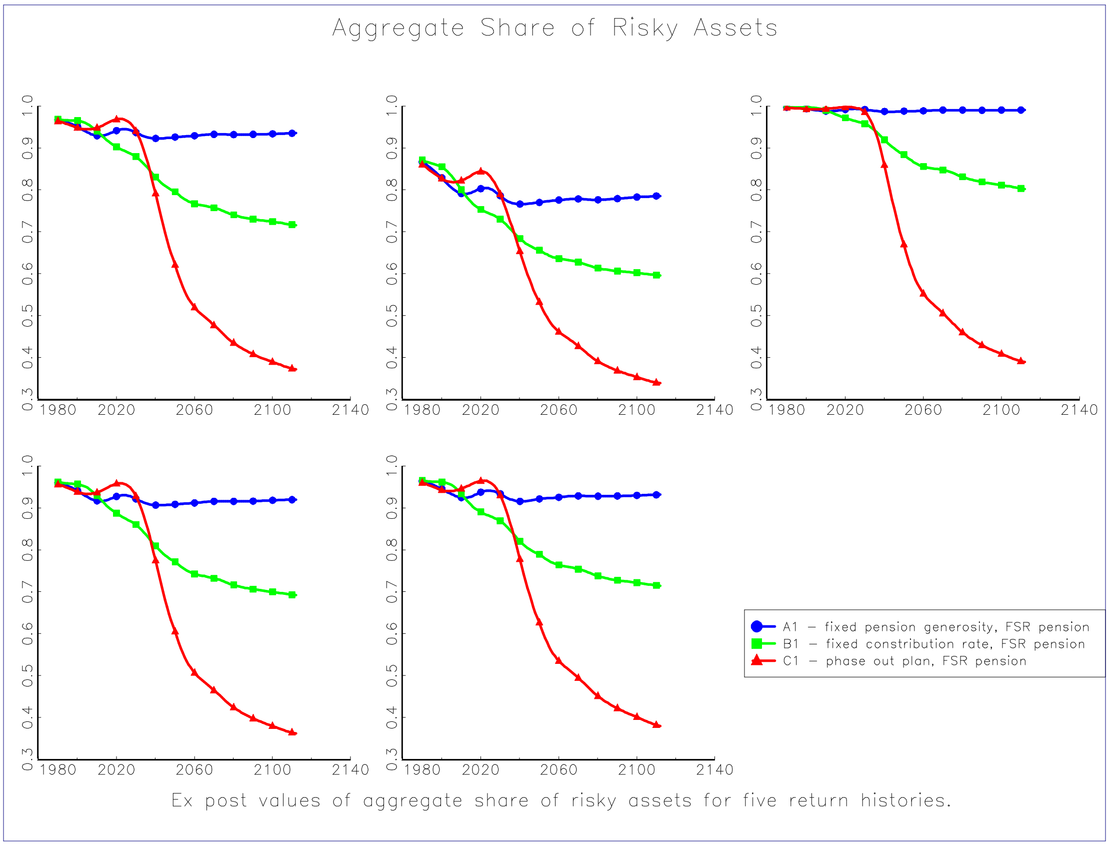


Mean Risky Rate for Different Pension Reforms
7.0%
6.0%
5.0%
4.0%
3.0%
2.0%
1.0%
0.0%
Flat rate
Constant
contribution rate
Phase dow n
1980
1985
1990
1995
2000
2005
2010
2015
2020
2025
2030
2035
2040
2045
2050
2055
2060

Figure 4: [9]

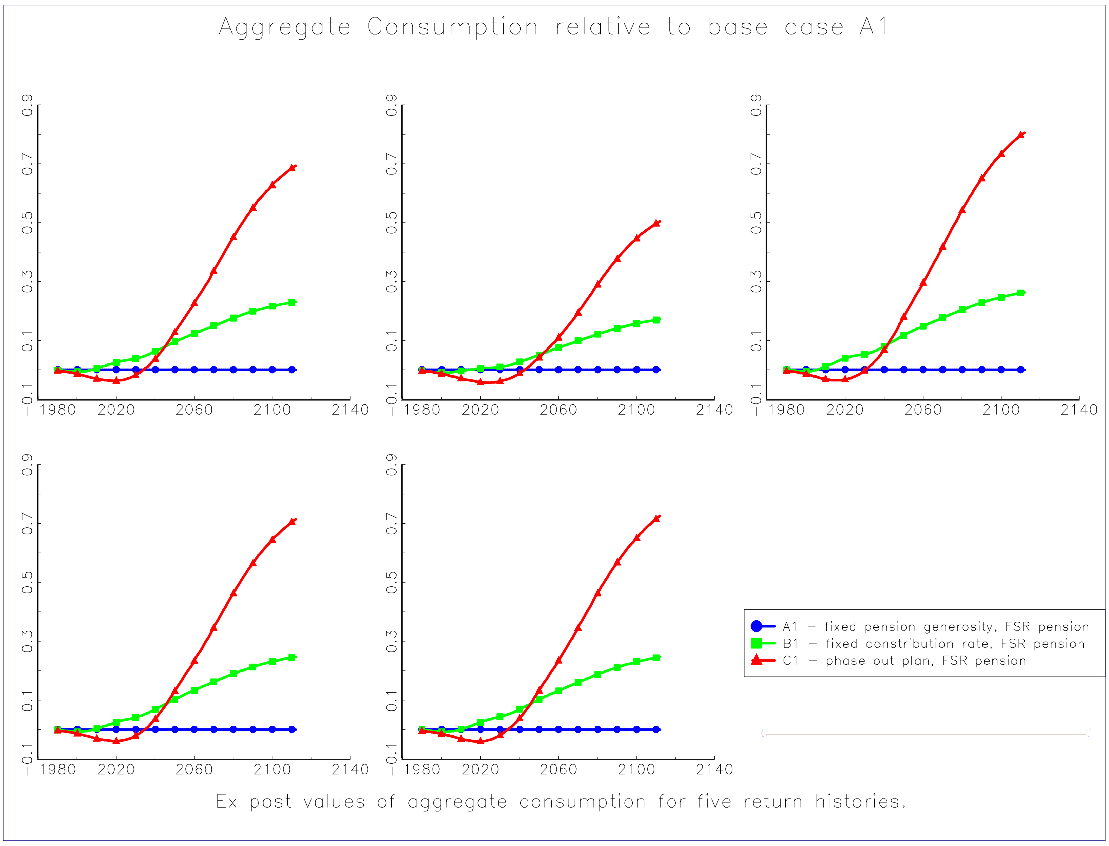
[9] The five panels correspond to five different return scenarios. Left to right, top to bottom they are i) zero return shocks, ii) 60% positive shocks, 40% negative shocks, size one standard deviation from the mean; iii) 40% positive shocks, 60% negative shocks; iv) 50% positive shocks, 50% negative shocks; v) randomly generated IID shocks.

Figure 5: [10]



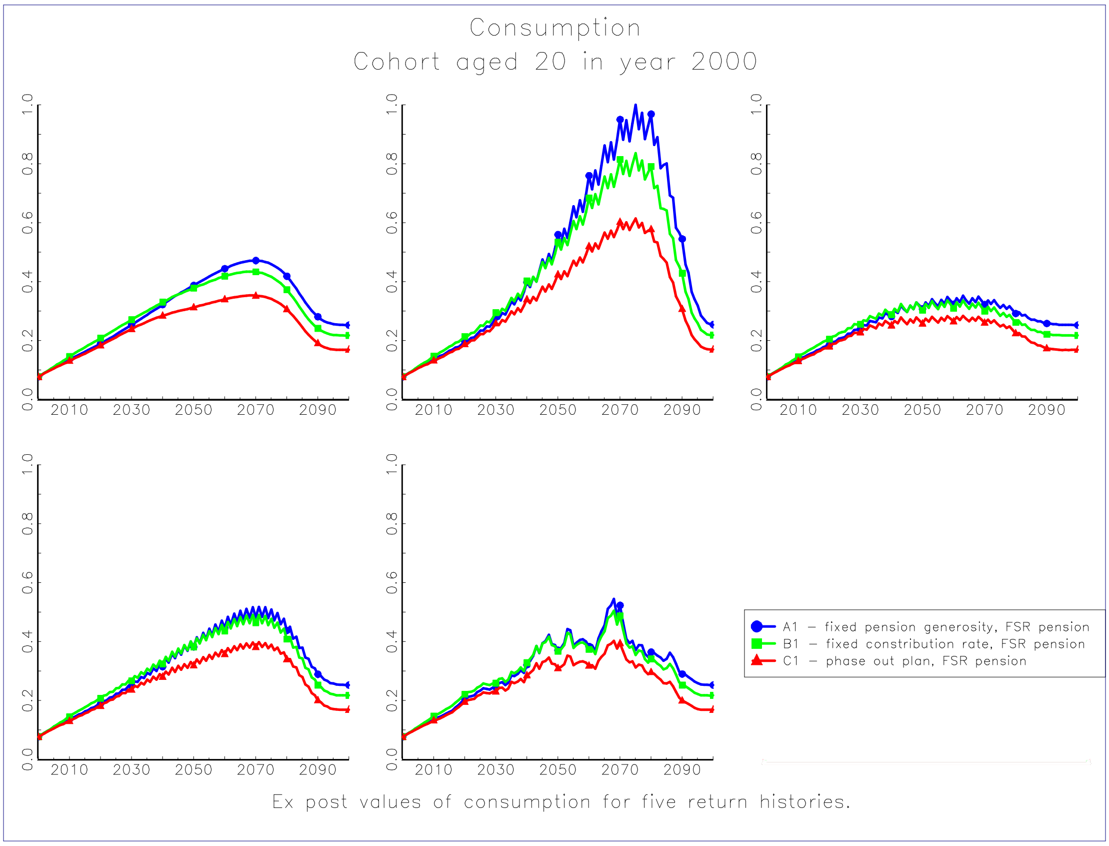
[10] The five panels correspond to five different return scenarios. Left to right, top to bottom they are i) zero return shocks, ii) 60% positive shocks, 40% negative shocks, size one standard deviation from the mean; iii) 40% positive shocks, 60% negative shocks; iv) 50% positive shocks, 50% negative shocks; v) randomly generated IID shocks.

Figure 6: [11]



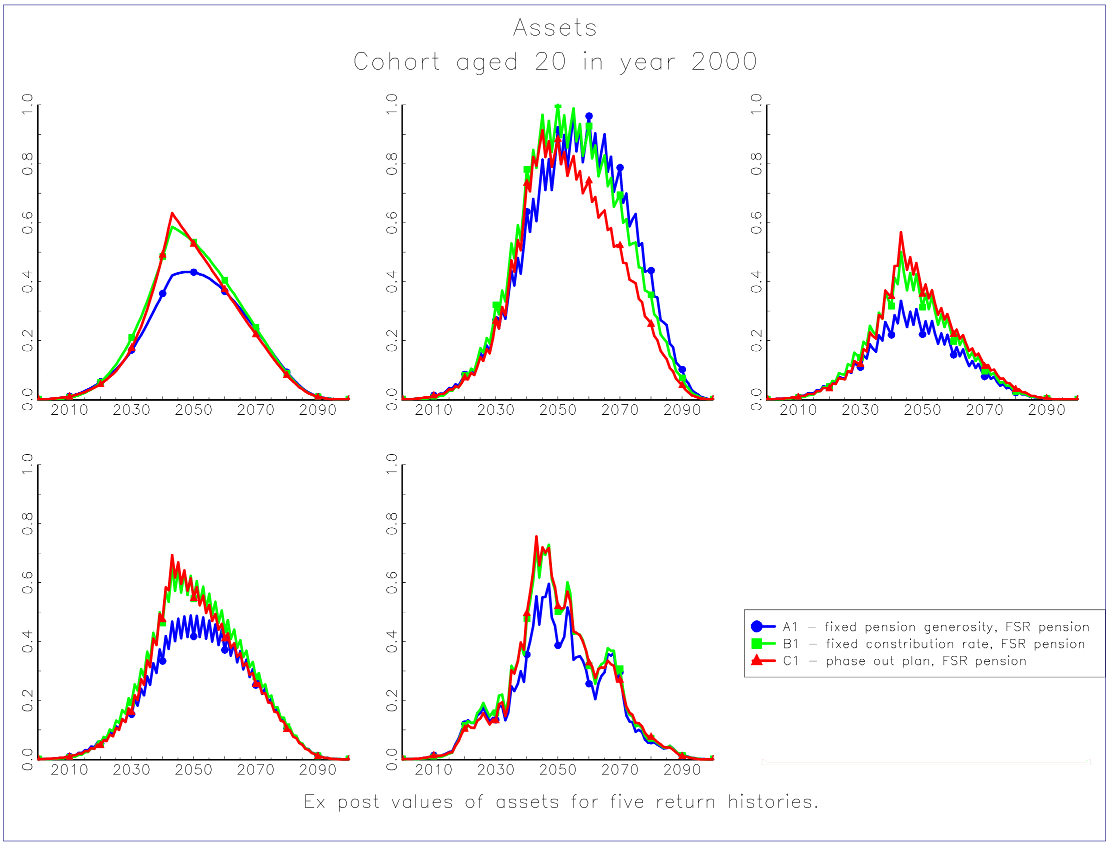
[11] The five panels correspond to five different return scenarios. Left to right, top to bottom they are i) zero return shocks; ii) 60% positive shocks, 40% negative shocks, size one standard deviation from the mean; iii) 40% positive shocks, 60% negative shocks; iv) 50% positive shocks, 50% negative shocks; v) randomly generated IID shocks.

Figure 7: [12]



[12] The five panels correspond to five different return scenarios. Left to right, top to bottom they are i) zero return shocks; ii) 60% positive shocks, 40% negative shocks, size one standard deviation from the mean; iii) 40% positive shocks, 60% negative shocks; iv) 50% positive shocks, 50% negative shocks; v) randomly generated IID shocks.

Figure 8: [13]



[13] The five panels correspond to five different return scenarios. Left to right, top to bottom they are i) zero return shocks; ii) 60% positive shocks, 40% negative shocks, size one standard deviation from the mean; iii) 40% positive shocks, 60% negative shocks; iv) 50% positive shocks, 50% negative shocks; v) randomly generated IID shocks.