

Sector Specialisations, Nonhomothetic Demand and Welfare*

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Abstract

We propose a model of classical inspiration (of the Dornbush, Fisher and Samuelson (1977) type) which incorporates a nonhomothetic demand function, the microeconomic foundations of which express hierarchic needs. The model proposed is applied to the case of trade between two economies with different levels of development and it is used to study the impact on welfare of technological shocks. In particular, we show that the developing country can experience a fall in utility as a result of technical progress in the developed country. This configuration depends partly on the type of technological shock assumed (biased progress) and partly on the development gap.

Keywords : Dornbush-Fisher-Samuelson ricardian model ; Technology and Trade ; North-South Trade ; Nonhomothetic preferences ; Hierarchic needs

JEL Classification : F11, O11

1 Introduction

What are the determinants of the real income gap between two open economies with different levels of development? What effect does trade in general, and the nature of sector specialisations in particular, have on this gap? In the literature, under the hypothesis of full employment of factors of production, the difference between relative real incomes depends, roughly speaking, on the combination of two effects:

- *productivity growth rate differentials (or factor supply differentials)* between the two countries, which determine the differences in volume growth.

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- *the nature of preferences* defines the evolution of the sectorial demand which accompanies volume growth. It determines the variation in the real exchange rate (terms of trade) required for the production to be sold.

When the "volume" effect is more powerful than the "terms of trade" effect, the productivity growth differential is expressed by a difference in real income growth, to the benefit of the country enjoying higher productivity growth. When the "terms of trade" effect is stronger, on the other hand, immiserising growth appears (Bhagwati (1958)). Under certain conditions, the two effects can cancel each other out, in which case the difference in productivity growth has no effect on the growth in real income (Dornbush, Fisher and Samuelson (1977) and Grossman-Helpman (1991, chapter 7)).

The application of endogenous growth models in the context of international trade has helped to refine the analysis of the "productivity effect", especially by taking into account multiple determinants of the difference in technical progress between countries (research and development activity, imitation, technological spillovers, the degree of knowledge appropriation, etc.). In particular, under the hypothesis of increasing returns, some of these models confirm the importance of the nature of specialisations, by introducing a relation between the rate of technical progress and sector specialisation¹.

For agents' preferences, the most widely-adopted specification is that of a homothetic utility function. In this case, the spending distribution is fixed, and any eventual divergence in real income is determined by the agents' reactions to price variations (Lucas, 1988). When income elasticities are assumed to be non-unitary, on the other hand, the rise in income generated by productivity growth is largely used to consume goods produced in the country which has the more favourable income elasticities. The result is a worsening in the terms of trade for the other country (Matsuyama, 2000). Just as the introduction of increasing returns in the models of international trade leads to the definition of "good" and "bad" specialisations in terms of productivity growth, so the non-uniform evolution of sectorial demand can lead to the same distinction, through effects linked to the variation in terms of trade.

Our aim in this paper is to present a static model of international trade that takes into account this relation between specialisations and the dynamism of external demand, under the hypothesis of nonhomothetic preferences. The purpose of our model is to analyse the impact of sector specialisations on the real income (welfare) of two economies with differing levels of development when we assume productivity shocks.

In the following section, we shall run briefly through the stylised facts that provide an empirical justification for the hypothesis of nonhomothetic demand. In the third section, we shall present a closed-economy model where the specification of the utility function leads to demand behaviour that fits in with these stylised facts. In the fourth section, we shall incorporate this into a Ricardian model of the type developed by Dornbush, Fisher and Samuelson (DFS, 1977)

¹This is the case for what are called Ricardian models of endogenous growth, which distinguish between sectors according to their different rates of productivity growth. The cumulative processes associated with dynamic increasing returns can lead to divergent growth patterns.

and so obtain a model of North-South trade where the income elasticities of external demand are non-unitary and micro-founded. In the final section, we shall study the impact of technological shocks. We do this through simulations, interpreting our results by comparing them with two other Ricardian models of international trade which constitute the borderline cases of our approach (Dornbush, Fisher and Samuelson (1977) and Matsuyama (2000)).

2 Empirical foundations

Since the first works of Magee and Houthakker (1969), the calculation of import and export demand functions has systematically produced non-unitary income elasticities of trade². However, this empirical result obtained at the aggregate level is not conclusive proof that preferences are nonhomothetic. From a theoretical point of view, two types of explanation are possible, depending on the roles attributed to nonhomothetic preferences and sector specialisations.

According to Mc Gregor and Swales (1986, 1991) and Krugman (1989), this result is a statistical artefact resulting from a composition effect: countries benefiting from the reorientation of world demand in their favour are those which enjoy stronger *growth* in the differentiation of intra-sector supply. In this case, by hypothesis, the nature of sector specialisations has no impact on external demand: the demand function is homothetic, but there is uniform reorientation of consumption as and when new varieties are created³.

On the other hand, nonhomothetic preferences occupy a central position in the post-Keynesian analysis of income elasticities of trade (Thirlwall (1979), Kaldor (1970)). In accordance with Engel's law, luxury goods are distinguished from necessities, and specialisation in one or the other of these categories determines a country's income elasticities of external demand.

Two arguments can be put forward in favour of the nonhomothetic approach.

Firstly, econometric works introducing variables for intra-sector non-price competitiveness (growth in the supply of varieties or in the quality or technological content of the goods) can be considered as tests of the "statistical artefact" hypothesis. If the incorporation of these variables into the calculation of import and export functions were to produce unitary income elasticities of external demand, then the hypothesis would be confirmed. However, these elasticities remain significantly non-unitary, which argues in favour of the role attributed to sector specialisations (Amable (1992), Fagerberg (1988), Feenstra (1994)).

²In particular, we can refer to works dealing with the relation between the value of a country's income elasticities of trade and its relative growth rate (Thirlwall's simplified equation (1978)); Atesoglu (1993, 1994) devoted to Germany and Canada; Bairam-Dempster (1991) and Perraton (2003) devoted to developing countries; Muscatelli et al. (1994, 1995) devoted to new industrialised countries.

³In particular, Krugman (1989) proposed a model where this type of effect can be generated by using a Dixit-Stiglitz (1977) utility function of preference for variety.

Secondly, a series of macro- and microeconomic studies on demand behaviour provide direct validation for the hypothesis of nonhomothetic preferences. In particular, we can draw four stylised facts from these works:

SF1 - There is a relation between the characteristics of the goods supplied by a country (their technological content) and the value of that country's income elasticities of external demand⁴ (Meliciani (2002), Verspagen (1993), chapter 4).

SF2 - There is a relation between the level of agents' incomes and the concentration of their consumption. In other words, when their income rises, agents do not distribute this increase uniformly over all the goods they buy (Hunter (1991), Hunter and Markusen (1988), Jackson (1984), Fillat, Francois(2004)).

SF3 - There is a relation between the level of income and the number of varieties consumed. In other words, a rise in income is accompanied by an extension in the range of goods consumed by agents (Falkinger and Zweimuller (1996), Jackson (1984)).

SF4 - There is a relation between the composition of agents' baskets of goods (the characteristics of the goods consumed) and their levels of income. Agents with high incomes consume higher qualities (Bills and Klenow (2000)) and more varieties in certain sectors (Jackson (1986)). The empirical validation of Linder's hypothesis⁵ also argues in favour of this (Thursby and Thursby (1987), Francois and Kaplan (1996), Bergstrand (1990), Hallak (2003)).

From these stylised facts we can conclude that the heterogeneity of agents' baskets of consumption depends partly on their levels of income and partly on the characteristics of the goods supplied. Changes in consumption behaviour resulting from an increase in income then take the form of access to what had initially been non-priority (SF3) and less sophisticated goods (SF4 and SF1), together with a variation in the relative spending on each good (SF2).

In the following section, we present a static (closed-economy) model where the utility function aims to take these stylised facts into account.

3 A nonhomothetic closed-economy model

3.1 The properties of the prioritised consumption function

We use the quasi-homothetic utility function developed by Jackson (1984), which differs from a standard Cobb-Douglas function by the introduction of a constant term (γ_i).

⁴This stylised fact is not the same as Krugman's composition effect (1989). Here, a country's trade income elasticities depend on the technological level of its supply (and not on its growth). In other words, it is the characteristics of the goods as such, and not the growth in goods supplied, which determine the dynamism of external demand.

⁵In this case, on an aggregate level, a relation is established between the share of intra-industrial transactions and the level of per capita income in a country.

$$U = \sum \beta_i \log(q_i + \gamma_i)$$

where q_i corresponds to the quantity of i consumed.

We modify this function in three ways: firstly by a linear transformation so that only the goods consumed enter into the agent's final utility⁶; secondly, it is expressed in continuum so that we can obtain an expression for the marginal good; thirdly we assume that $\beta_i = 1$. This transformation brings our utility function closer to that used by Young (1991) and Markusen (1986)⁷. It can be written:

$$U = \int_0^{\infty} \log\left(\frac{q_i}{\gamma_i} + 1\right) di \quad (1)$$

The maximisation programme can be written:

$$\text{Max } U$$

$$\text{sc } \int_0^{\infty} p_i q_i di = y$$

and $q_i \geq 0$

where p_i and y correspond to the price of good i and the agent's income respectively.

We can write the Khun-Tucker conditions:

$$\text{for } i \in K, \quad \frac{1}{p_i \gamma_i} > \lambda \quad \Rightarrow \quad q_i = \frac{1}{p_i \lambda} - \gamma_i \quad (2)$$

$$\text{for } i \notin K, \quad \frac{1}{p_i \gamma_i} < \lambda \quad \Rightarrow \quad q_i = 0 \quad (3)$$

$$\text{where } K = \left\{ i \mid \frac{1}{p_i \gamma_i} > \lambda \right\}$$

The purpose of transforming Jackson's equation (1984) into a continuum is to be able to determine a marginal good. For such a good to exist, it must be

⁶Jackson (1984), considers I goods in the economy. For all goods consumed $i \in K$, utility is determined by $(q_i + \gamma_i)$. Non-consumed goods (i.e. $i \notin K$, $q_i = 0$) enter also into the agent's utility through the term γ_i . Our transformation removes this effect (equation 1).

⁷The utility function in Young's model is similar to our own, except that $\gamma_i = 1$. However, he did not explore the implications of this choice of form, although it enabled him to link the endogenous evolution of specialisations, consumption and well-being. Markusen (1986) considers a Stone-Geary utility function. The main difference is that his approach is based on minimum consumption for an exogenous number of sectors, whereas our approach relies more on an endowment effect for an endogenous number of consumed products.

assumed that the criterion $\frac{1}{p_i \gamma_i}$ (which determines hierarchic consumption) follows a decreasing monotonic function. We shall see that at general equilibrium, the model presents this property, which enables us to order the consumption process explicitly, and to divide the continuum into two segments: that of consumed goods $i \in K = [0, J]$ and that of non-consumed goods $i \notin K = [J, \infty[$. (cf. figure 1)

Under this hypothesis, the third Kuhn-Tucker condition leads to the following expression for the marginal good J :

$$\text{for } i = J, \quad \frac{1}{p_J \gamma_J} = \lambda \quad \Rightarrow \quad q_i = 0 \quad (4)$$

We can then write the Lagrange multiplier:

$$\lambda = \frac{J}{y + \int_0^J p_i \gamma_i di} \quad (5)$$

où $J = \int_{i \in K} di$

We shall now present the main properties of our demand function under the hypothesis $p_i \gamma_i$ ranked in increasing order. We shall demonstrate that the composition and evolution of the agent's basket of consumption depend on his level of income (y) and on the rating of the goods in the continuum (i).

Theorem 1: *The number of varieties consumed J is an increasing function of y*

Using equations 4 and 5, we can determine an implicit equation in J

$$J = \frac{y + \int_0^J p_i \gamma_i di}{p_J \gamma_J} \quad (6)$$

Using this equation, we can demonstrate graphically that J increases with y (cf. figure 2). When y increases, the area of the triangle above the curve $p_i \gamma_i$ increases, which is expressed by an increase in J and $p_J \gamma_J$. For given prices of goods, the values of these two variables therefore represent a wealth effect.

Theorem 2: *The quantity consumed q_i of each variety is an increasing function of y*

From equations 2 and 5, we can define the following demand function:

$$q_i = \frac{y + \int_0^J p_i \gamma_i di}{p_i J} - \gamma_i \quad (7)$$

After transformation (using equation 6), this can be written:

$$\begin{aligned}
q_i &= \frac{p_J \gamma_J}{p_i} - \gamma_i & \text{if } & p_i \gamma_i < p_J \gamma_J \\
q_i &= 0 & \text{if } & p_i \gamma_i > p_J \gamma_J
\end{aligned} \tag{8}$$

From theorem 1, according to which J and therefore $p_J \gamma_J$ are an increasing function of y , it follows that a rise in income is translated into a rise in the quantities consumed of all goods.

Theorem 3: *The amount spent on each good $p_i q_i$ is a decreasing function of i*

According to equation 8, the amount spent on a good $p_i q_i$ is given by the gap between the threshold effect value of the marginal good and the threshold effect value of the good i . The amount spent on each good is therefore a decreasing function of that good's position in the continuum (when $i < J$; otherwise $p_i q_i = 0$)

Theorem 4: *Income elasticity η_y^i is an increasing function of i*

From equation 7, we can deduce the expression of income elasticities:

$$\eta_y^i = \frac{1}{J} \cdot \frac{y}{p_i q_i} \tag{9}$$

Theorem 3 ensures that income elasticities increase along the continuum. The last good to enter the consumption basket is the one which has the lowest share of spending and therefore the highest income elasticity value. Thus, every good, except the first one, which is always consumed, can be considered as a "luxury good" the first time it enters the consumption basket. Subsequently, a process of progressive standardisation takes place with the entry of new goods. In other words, each good tends to behave more and more like a necessity as and when its weight in the consumption increases.

Theorem 5: *Income elasticity η_y^i is a decreasing function of y*

Given that income elasticities increase along the continuum (theorem 4), we denote k ($\eta_y^k = 1$) the good which marks the limit between the "luxury goods" segment ($\eta_y^i > 1$) and the "necessity goods" segment ($\eta_y^j < 1$). Using equations 8 and 9, we can write for this good:

$$p_k \gamma_k = p_J \gamma_J - \frac{y}{J}$$

After transformation using equation 6, this gives:

$$p_k \gamma_k = \frac{\int_0^J p_i \gamma_i di}{J} \tag{10}$$

To prove theorem 5, we must verify that when income increases, the good k (which satisfies the condition $\eta_y^k = 1$) corresponds to a higher index value in the continuum of goods. In other words:

$$\frac{\partial(p_k \gamma_k)}{\partial y} = \frac{\partial(p_k \gamma_k)}{\partial J} \cdot \frac{\partial J}{\partial y} > 0$$

Following theorem 1:

$$\frac{\partial J}{\partial y} > 0$$

We then calculate the first partial derivative and simplify the result (using equation 6). We obtain:

$$\begin{aligned} \frac{\partial(p_k \gamma_k)}{\partial J} &= \frac{p_J \gamma_J}{J} - \frac{\int_0^J p_i \gamma_i di}{J^2} \\ &= \frac{y}{J^2} > 0 \end{aligned}$$

In other words, the lower the agent's relative level of income, the more sensitive demand for good i will be to variations in income.

Theorem 6: *The absolute value of price elasticity $|\eta_p^i|$ is an increasing function of i*

As with income elasticities, the function of hierarchic consumption displays non-unitary price elasticities. From equation 8, we can deduce:

$$\eta_p^i = -\frac{p_J \gamma_J}{p_i q_i} \quad (11)$$

$$= -1 - \frac{p_i \gamma_i}{p_i q_i} \quad (12)$$

This theorem can be proved by simple application of theorem 3 (in equation 11), according to which, for a given level of income, $p_i q_i$ is a decreasing function of i . The last goods to enter the consumption basket therefore have higher price elasticities (in absolute value). As with income elasticities, a good's level of sensitivity to price variations depends on its novelty. Here, unlike the standard demand function, consumers do not re-evaluate their baskets uniformly when prices change. They tend to adapt their consumption in favour of the most recent goods to enter their baskets when there is a uniform fall in prices, and to the detriment of these same goods when there is a rise in prices.

Theorem 7: *The absolute value of price elasticity $|\eta_p^i|$ is a decreasing function of y*

According to theorem 2, $p_i q_i$ is an increasing function of y . On the basis of equation 12, this is sufficient to prove this theorem. For any given good, the higher an agent's income, the lower their sensitivity to price variations.

The continual process of consumption and the seven theorems we have just set out are founded on the hypothesis of a monotonic increasing function $p_i\gamma_i$. We shall now present a case of general equilibrium which displays this property.

3.2 General equilibrium in a closed economy

We assume a single-input (labour) production function with constant returns. We denote a_i , the quantity of labour required to produce one unit of good i . We define q_i as the production of good i and l_i the labour required for the production of these q_i quantities.

$$l_i = q_i \cdot a_i \quad (13)$$

We also assume perfect competition, so that the prices of goods are given by their production costs:

$$p_i = a_i w \quad (14)$$

where w denotes wages.

There are no profits in the economy. National income is given by total wages:

$$Y = wL$$

Without losing generality, we can assume that goods are ranked in increasing order so that the parameters of the model satisfy the following relation:

$$a_i\gamma_i \text{ increasing function of } i \quad (15)$$

At equilibrium, using equation 14, $p_i\gamma_i$ is also an increasing function of i . All the properties of the consumption function described above are therefore maintained.

It is this proposition 15 which gives our approach its specificity. We describe this ranking (which determines the order of entry of the different goods) as a *techno-utility ranking*, as it combines a technological criterion⁸ (a_i) and a criterion based on the utility function (γ_i). It expresses the fact that a technological shock (via a_i) or a preference shock (via γ_i) have the same impact on the composition and evolution of consumption⁹. This equivalence is founded on the intuition that when a technical advance results in the creation of new goods¹⁰,

⁸Fixed capital is not explicitly introduced into the model. However, we consider that the input of labour represents effective labour, i.e. a composite (comprising labour, capital, and human capital).

⁹Given the stylised facts SF1 and SF4 presented in the second section, we consider that this *techno-utility* ranking expresses the *relative sophistication* of goods. These two terms should therefore be considered interchangeable in the rest of the paper.

¹⁰Like Young (1991), we have not modelised the innovation of goods in our model: an infinite number of goods are theoretically available for consumption, but some of them are too expensive to be produced. Technical progress makes it possible to produce these goods by causing a reduction in production costs.

it also entails the creation of new needs and modifies agents' perceptions of pre-existing goods (in terms of need). This modification is expressed formally by a change in the value of the income and price elasticities associated with each good (theorems 4-7)¹¹. The effects of a technological shock are therefore similar in spirit to a shock having a direct influence on agents' preferences through the parameter (γ_i).

4 Specialisations, Structure of Consumption and Transactions

4.1 The hypotheses of the nonhomothetic Ricardian model

We consider two economies with different levels of development. The more developed economy is the foreign one, and it is denoted by an asterisk. The general structure of the open-economy model is established in three stages.

* Firstly, we consider the developed economy, for which the ranking of goods in the continuum is defined by the techno-utility criterion described in the closed economy (relation 15).

* Then we model the supply sphere following the standard hypotheses of DFS (1977):

Hypothesis 1: *Competition in the two countries is perfect. Prices are determined by production costs.*

There is therefore an international price p_i^m which corresponds to the trade equilibrium at:

$$p_i^m = \text{Min} \{p_i, p_i^*\} \quad (\text{H1})$$

Hypothesis 2: *The foreign wage has the value*

$$w^* = 1 \quad (\text{H2})$$

Hypothesis 3: *The foreign economy is more developed in the sense that it is more productive, in absolute terms, in every sector.*

$$a_i^* < a_i \quad (\text{H3})$$

Hypothesis 4: *The productivity advantage of the developed country increases along its goods continuum*

$$\frac{a_i}{a_i^*} \text{ increasing function of } i \quad (\text{H4})$$

* Thirdly, we demonstrate that under the hypotheses of a technological gap that increases along the whole goods continuum of the developed country (H4)

¹¹Borrowing from Pasinetti (1981), we could say that the boundary line between essential needs and psychological needs is modified.

and an identical utility function in the two countries, the ranking of goods in the developed country entails an identical ranking in the developing country.

Hypothesis 5: *The two economies have identical utility functions and the utility function is individual (equation 1)*

$$U = U^* \tag{H5}$$

Theorem 8: *The ranking of goods in order of priority is identical in both countries*

$$\begin{array}{ll} \text{if} & \frac{\partial a_i^* \gamma_i}{\partial i} > 0 \\ \text{and} & \frac{\partial \frac{a_i}{a_i^*}}{\partial i} > 0 \\ \text{then} & \frac{\partial a_i \gamma_i}{\partial i} > 0 \end{array}$$

In other words, the ranking of goods initially established for the developed economy is an "international" ranking. We can therefore generalise hypothesis 4, by considering that the developed country has a comparative advantage in the goods regarded as the most sophisticated in both countries (in other words the goods with the highest techno-utility content). This enables us to interpret the DFS model in techno-utility terms. We thus establish a relation between the characteristics of a country (its level of development) and the characteristics of the goods in which it specialises (relative techno-utility content/sophistication of the goods).

4.2 The trade equilibrium

*Under hypothesis H4, we can determine the expression of the good \bar{i} which marks the limit between the specialisations of the two countries and which also corresponds to the number of varieties in which the developing economy specialises:

$$\begin{aligned} a_{\bar{i}}^* &= w a_{\bar{i}} \\ w &= \frac{a_{\bar{i}}^*}{a_{\bar{i}}} \end{aligned} \tag{16}$$

Given the hypothesis of an absolute advantage in productivity for the developed economy (H3), trade can only take place if, for all $i \in [0, \bar{i}]$, $p_i^ > p_i$ (H1). In other words, for the developing country to be competitive in this goods segment, the minimum condition is:

$$w < 1$$

The marginal goods consumed in the two countries can be determined using equation 6:

$$J = \frac{w + \int_0^J p_i^m \gamma_i di}{\gamma_J P_J^m} \quad (17)$$

$$J^* = \frac{1 + \int_0^{J^*} p_i^m \gamma_i di}{\gamma_{J^*} P_{J^*}^m} \quad (18)$$

J = number of varieties consumed by the developing country.

J^* = number of varieties consumed by the developed country.

*We make the standard assumption of a balance of trade equilibrium constraint:

$$X = M$$

$$\int_0^{\bar{i}} p_i^m q_i^* L^* di = \int_{\bar{i}}^J p_i^m q_i L di \quad (19)$$

X = exports of the developing country (imports of the developed country)

M = imports of the developing country (exports of the developed country)

q_i = individual consumption, and therefore $q_i L$ = total consumption

On the basis of equations 8, 14, H1 and H2, we can write:

$$\int_0^{\bar{i}} (a_{J^*}^* \gamma_{J^*} - w a_i \gamma_i) L^* di = \int_{\bar{i}}^J (a_J^* \gamma_J - a_i^* \gamma_i) L di$$

$$\bar{i} L^* a_{J^*}^* \gamma_{J^*} - w L^* \int_0^{\bar{i}} a_i \gamma_i di = (J - \bar{i}) L a_J^* \gamma_J - L \int_{\bar{i}}^J a_i^* \gamma_i di \quad (20)$$

Now, equation 17 can be re-written:

$$\int_{\bar{i}}^J a_i^* \gamma_i di = J a_J^* \gamma_J - w - w \int_0^{\bar{i}} a_i \gamma_i di$$

When we introduce this expression into equation 20, we obtain, after simplification:

$$w = \frac{\bar{i} [L^* a_{J^*}^* \gamma_{J^*} + L a_J^* \gamma_J]}{L + (L + L^*) \int_0^{\bar{i}} a_i \gamma_i di} \quad (21)$$

We obtain a system of four equations (16-18, 21) with four unknowns, which must satisfy the following conditions: $0 < \bar{i} < J < J^*$. The general model obtained in this way is too complex to be resolved analytically. We can, however, present its main properties in an open economy. In particular, in accordance with our initial objective, we can present an endogenous and micro-founded demonstration of a relation between trade specialisation, structure of consumption and dynamism of external demand. Notably, it appears that the differences in income and price elasticities between a developing and a developed country favour the latter.

4.3 Characteristics of the two countries' demand behaviour at trade equilibrium

Because agents in the two countries face identical prices for goods, the difference in relative wage between the two countries expresses differences in real income. We can therefore apply the main theorems defined in the closed economy. Thus, we set down:

$$\begin{aligned} y &= w \\ y^* &= 1 > w \\ L &= L^* \end{aligned}$$

Corollary 1: *The developed country consumes more varieties and these varieties are, by hypothesis, more sophisticated*

According to theorem 1 and relation 15, and given that $w < 1$, we can write:

$$\begin{aligned} J^* &> J \\ \text{and therefore } p_{J^*}^m \gamma_{J^*} &> p_J^m \gamma_J \end{aligned}$$

Corollary 2: *The developing country specialises in a narrower range of varieties*

According to theorem 3, for all $j > i$, we verify:

$$p_i^m q_i > p_j^m q_j$$

Using theorem 2, we can write, for all $i \in [0, \bar{i}]$,

$$p_i^m q_i^* > p_i^m q_i$$

We can deduce:

$$p_i^m q_i^* > p_j^m q_j$$

Because of the condition of trade equilibrium (equation 19), we can therefore write:

$$[0, \bar{i}] < [\bar{i}, J]$$

According to these two corollaries, the structure of consumption and specialisations induced by the model can be summarised in table 1.

We can complete this table with two other corollaries concerning the dynamic of demand behaviour.

Corollary 3: *The income elasticities of external demand are more favourable to the developed country*

We denote: $\eta_{y^*}^i$ the income elasticity of demand for imports in the developed country for a good $i \in [0, \bar{i}]$ produced by the developing country.

η_y^i , the income elasticity of demand in the developing country for each of its home-produced goods.

η_y^j , the income elasticity of demand for imports in the developing country for a good $j \in [\bar{i}, J]$ produced by the developed country.

On the basis of theorems 4 and 5, we can write for all $i \in [0, \bar{i}]$ and all $j \in [\bar{i}, J]$:

$$\eta_{y^*}^i < \eta_y^i < \eta_y^j$$

In other words, for each specialisation i (of the developing country) and j (of the developed country), the income elasticity of demand for imports of the developing country is greater than that of the developed country.

As regards the overall income elasticities of external demand for each country, because of corollary 2, the developing country can never compensate for its "poor" sector specialisations by specialising in a relatively wider range of varieties ($[0, \bar{i}] > [\bar{i}, J]$ is impossible).

So on a global level:

$$\frac{\partial X}{\partial y^*} \cdot \frac{y^*}{X} < \frac{\partial M}{\partial y} \cdot \frac{y}{M}$$

is always verified when $\partial y = \partial y^*$

Corollary 4: *The price elasticities of external demand (in absolute value) are higher for the developed country*

Using theorems 6 and 7 and applying the same reasoning as above, we can establish the following relation:

$$|\eta_p^{*i}| < |\eta_p^i| < |\eta_p^j|$$

Here again, by application of corollary 2, this relation is verified on an aggregate level. So, in our model, the specialisations of the developing country lead it to have weaker price elasticities of external demand (in absolute value) than

the developed country. This result is not inconsistent with the idea that developed countries specialise in relatively more sophisticated goods which are more likely to be subject to non-price competitiveness. But here, non-price competitiveness is not modeled as price insensitivity. It is mainly connected with the relative priority of consumptions. Thus, the previous relation implies that given an identical fall in price for all goods, agents will favour the goods produced by the developed country in their additional consumption. Likewise, when prices increase, agents will reorientate their consumption to the detriment of the goods produced by the developed country (considered to have lower priority).

The implications of our formulation of the demand function for the analysis of welfare will be presented using simulations. To do so, we shall specify functional forms and thus propose a version of the general model which, though simplified, keeps the same properties. We shall then carry out comparative statics analysis (by assuming technological shocks). We interpret our results by comparison with two other Ricardian models: the standard model of Dornbush, Fisher and Samuelson (DFS, 1977) and that of Matsuyama (2000). These two models can be considered as borderline cases of our approach.

5 The effect of technological shocks on welfare

5.1 Simplification of the model, simulation procedure and interpretation of the results

5.1.1 The choice of functional form

In the standard interpretation of Dornbush, Fisher and Samuelson (1977), the continuum of goods involves different sectors, the characteristics of which are not defined. The only precision made concerning sector characteristics is a ranking of goods along the continuum as a function of the relative productivity gap between the two countries (H4). Given the lack of interpretation of the sector differences in absolute terms, the choice of cost function for the simulation is rather arbitrary. Symmetrical cost functions are often adopted for the two countries, for example¹². We prefer not to use functional forms of this type, because they are incompatible with several of our hypotheses on the supply structure (H4 and H3).

Krugman (1990) proposed an extension of the theoretical interpretation that can be made of the DFS model: he argued that the ranking of goods according to the productivity gap (H4) corresponds to the ranking of goods according to their technological content. However, choosing a functional form which can take into account the link between technological content and sector productivity is no easy task, insofar as that if the continuum involves different sectors, the

¹²This is the case in the simulation of the Ricardian model presented by Hummels, Ishi and Yi (1999).

respective units of each good are likely to be different. In other words, it is impossible directly to establish the relation between a_i and i .

In our model, we get around this problem by not directly fixing the functional form of a_i , but only that of $a_i\gamma_i$, which we have defined as an increasing function of i . We propose to use the following functional forms:

$$a_i\gamma_i = \alpha + \beta.i \quad (22)$$

$$a_i^*\gamma_i = \alpha^* + \beta^*.i \quad (23)$$

In addition, our hypotheses on the technological gap will enable us to constrain parameter values. For H3 and H4 always to be verified, it is sufficient that:

$$\frac{\beta}{\beta^*} > \frac{\alpha}{\alpha^*} > 1$$

In this case, the ratio of production costs ($\frac{\alpha^*}{\alpha_i}$) decreases along the whole continuum with an asymptote tending to $\frac{\beta^*}{\beta}$.

5.1.2 The new equilibrium conditions

We rewrite the general equilibrium model by applying the new equations (22 and 23)¹³.

$$w = \frac{\alpha^* + \beta^*.\bar{i}}{\alpha + \beta.\bar{i}} \quad (24)$$

$$J^2 = \bar{i}^2 + \frac{w}{\beta^*}(2 - \beta\bar{i}^2) \quad (25)$$

$$J^{*2} = \bar{i}^2 + \frac{(2 - w\beta\bar{i}^2)}{\beta^*} \quad (26)$$

Consequently, it is indeed verified that $J^* > J$ when $w < 1$.

On the basis of these three equations, we can determine a polynomial for \bar{i} :

$$\begin{aligned} P(\bar{i}) &= \\ & \alpha^* \left(l - (1+l)\frac{\beta\bar{i}^2}{2} \right) \\ & + \beta^*\bar{i} \left(l + (1+l)\bar{i} \left(\alpha + \frac{\beta\bar{i}}{2} \right) - (\alpha + \beta\bar{i})(J^* + lJ) \right) \\ & = 0 \end{aligned} \quad (27)$$

With

$$l = \frac{L}{L^*}$$

¹³Details of the calculations can be obtained from the author upon request.

5.1.3 Verification of the existence of an analytical solution

We shall now demonstrate that the polynomial which determines the expression of \bar{i} presents at least one economically possible solution ($\bar{i} > 0$). To do so, we simply need to verify that the solution can be flanked by two values (one positive and one negative). We therefore flank $P(\bar{i})$ by two extreme values of \bar{i} :

- The developing country does not specialise in any good ($\bar{i} = 0$). If we define $\bar{i} = \bar{i}_0 = 0$, then equation 27 can be written:

$$P(\bar{i}_0) = \alpha^* l > 0 \quad (28)$$

- The developed country satisfies all its needs through its domestic production ($\bar{i} = J$). According to the expression of the marginal good consumed by the domestic economy (equation 25), the value of \bar{i} corresponding to this configuration is $\bar{i} = \bar{i}_1 = \sqrt{\frac{2}{\beta}}$. With this value, we obtain:

$$P(\bar{i}_1) = -\alpha^* + \beta^* \bar{i}_1 l + \beta^* \frac{2}{\beta} (1+l) \left(\alpha + \frac{\beta \bar{i}_1}{2} \right) - \beta^* \bar{i}_1 (\alpha + \beta \bar{i}_1) (J^* + lJ) \quad (29)$$

We can then verify that¹⁴:

$$P(\bar{i}_1) < -\alpha^* - \beta^* \bar{i}_1 < 0$$

So, by flanking \bar{i} in this way, we have demonstrated that our polynomial has at least one root and that this root, with a value between \bar{i}_0 and \bar{i}_1 , is economically possible.

5.1.4 Simulation procedure

We use simulation to calculate the values of the equilibriums associated with equations (24-27). We consider two different configurations, in order to measure the extent to which the qualitative results produced by the simulations are sensitive to initial conditions. These configurations differ in the value given to the parameter β^* .

- simulation 1: low technological gap ($\beta^* = 0.003$)
- simulation 2: large technological gap ($\beta^* = 0.0003$)
- simulation 3 : very large technological gap¹⁵ ($\beta^* = 0,0000198$)

With, for these three cases: $\alpha = 0.045$; $\alpha^* = 0.0225$; $\beta = 0.020$; $L = L^* = 1$

Table 3 presents the values taken by the different variables for these three initial equilibriums.

We adopt a procedure of comparative statics. We assume a technical shock which results in a reduction in production costs. We then calculate the new

¹⁴See annex 1 for the demonstration of this.

¹⁵The technological gap in this case is particularly improbable. Nevertheless, it allows us to see the extent to which our results depend on the width of the gap.

equilibrium after the shock. The new equilibrium values are systematically compared with the corresponding initial equilibrium values (i.e. results on table 3).

We study the effects of three types of technical shock: a world uniform technical shock (WUTS, identical variation in $\alpha, \alpha^*, \beta, \beta^*$) ; a foreign uniform technical shock (FUTS, identical variation in α^*, β^*) and a foreign non uniform technical shock (FNUTS, variation in β^* which biased the technical progress towards the most sophisticated goods in developed country). In each case, we assume that this shock reduces the values of the parameters concerned by one half¹⁶.

For each simulation, we analyse especially the evolution and the determinants of consumer utility in the developing country.

5.1.5 Interpretation of the results

First, to analyse the determinants of changes in utility, we propose to consider the evolution of the "real price" of goods consumed. We report for each good (along the continuum) its world price (p_i^m) divided by the relative wage (w). For a given good, a technical shock hurts the purchasing power of the consumer if the deterioration of relative wage is greater than the productivity gain. A negative (positive) evolution of this ratio can thus inform us about the way the consumer in developing country gains (or loses) from technical progress.

Second, the results obtained from these simulations are systematically (but briefly) compared with those obtained by DFS (1977) and Matsuyama (2000) for the same shocks (their main results are presented annex 2b). This comparison will help us to interpret the mechanisms underlying our model, to the extent that these two approaches can be considered as two borderline cases to our approach¹⁷. Table 2 summarises the main properties of the demand function of the three approaches. We can see that our model thus incorporates both the price and non-price components of competitiveness, whereas Matsuyama's model (2000) fails to take price competitiveness into account (quantity demand is insensitive to price variations), and the DFS model fails to take non-price competitiveness into account (the sensitivity of demand to income variations is uniform for all goods).

¹⁶As we are working in comparative statics, we could also assume a succession of "small" shocks. We have, in fact, systematically verified that the size of the shock has no effect on the qualitative results.

¹⁷To obtain DFS model from our model it is just necessary to assume $\gamma_i = 0$. For Matsuyama model, one need to assume $\gamma_i = -1$ and $q_i = 1$. Details of this demonstration can be obtained from the author upon request.

5.2 The effects of technological shocks on welfare

5.2.1 Uniform international technical progress: non-price competitiveness is the key to the North's appropriation of some of the South's productivity gains

With the introduction of our nonhomothetic demand function, the developing country benefits relatively less from its own technical progress because of its sector specialisations. Effectively, given the relative value of its price and income elasticities (corollaries 3 and 4), the additional real income derived from uniform technical progress in the two countries is more likely to be used to demand goods produced by the developed country rather than those produced by the developing country. As a consequence, the balance of trade equilibrium condition entails a fall in relative wage (tables 3 and 4). This worsening in the terms of trade of the developing country explains its relative difficulty in appropriating its own productivity gains. However, immiserising growth can never appear, whatever the size of the productivity gap¹⁸. In other words, the fall in relative wage is always smaller than the fall in production costs (figures 3). The utility of both countries therefore rises.

It should be noted that this result provides a sharp contrast with the DFS model, where, under the hypothesis of homothetic demand, this type of shock has no effect on relative wage. The difference between the two models arises out of the fact that in the DFS model, the rise in real income resulting from technical progress is distributed uniformly over the goods produced by both countries (because of the unitary income and price elasticities). Therefore, the balance of trade equilibrium is not modified: the two countries draw equivalent benefit from their own technical progress. Matsuyama (2000), on the other hand, presented similar results to our own. The function of hierarchic desires he uses also takes nonhomothetic preferences into account. Thus, given an identical rise in real income in both countries, demand behaviour causes a disequilibrium in the balance of trade, which can only be resolved by a fall in relative wage. In Matsuyama's model, however, immiserising growth can appear in the developing country, because of the hypothesis of the price insensitivity of (quantity) demand.

5.2.2 Uniform technical progress in the North: price competitiveness is the key to the diffusion of productivity gains from the North to the South

Given uniform technical progress in the developed country, the productivity gap between the two countries increases uniformly. According to the specialisation equation (16), this entails a fall in relative wage. But this fall in relative wage induces an increase in specialisation and stimulates demand for goods produced by the developing country, which limits the worsening of its terms of trade (tables

¹⁸ "Simul 3" is illustrative of this proposition. Effectively, for this simulation we can observe an utility increase despite the unrealistic technological gap assumed.

3 and 5). At the new equilibrium, the fall in relative wage does not completely offset the initial productivity gains. welfare increases in both countries (figures 4).

This result tallies with that of DFS, but differs from that of Matsuyama, where the developing country is totally impervious to the developed country's technical progress. Under the hypothesis of saturation of the quantity demanded (of goods produced by the developing country), the fall in production costs in the developed country has no impact on the demand for goods produced by the developing country. The balance of trade equilibrium condition therefore presupposes a fall in relative wage equivalent to the initial fall in production costs. The developing country consequently remains at its initial level of utility.

5.2.3 Biased technical progress in the North: the differences between the countries is the key to the diffusion of technical progress gains from the North to the South?

As the result of biased technical progress in the developed country (growing over $[\bar{z}, J^*]$), the utility of the developing country may fall (cf. simul 2 and 3 in tables 3 and 6). The appearance of this configuration depends on the technological gap between the two countries (cf.figure 5).

We propose the following explanation. Firstly, the fall in relative wage needed to return to equilibrium is, according to the condition of specialisation (equation 16), all the greater when the productivity gap is wider. Secondly, our modeling of preferences entails that the development gap (the difference in per capita income) determines the differences in income and price elasticities between the two countries (theorems 5 and 7). We can make two observations about this effect. Firstly, in a similar fashion to the uniform shock described above, the developing country is relatively penalised by its income elasticities of trade. Secondly, according to theorem 6, we have seen that in our model, price elasticities increase along the continuum. Now, under the hypothesis of a shock biased towards the most technological goods, the goods which benefit from the highest price elasticities are also those which benefit from the largest falls in price. The price elasticity and income elasticity effects therefore contribute jointly to a pronounced change in spending distribution, in favour of the goods produced by the developed country. The balance of trade equilibrium condition therefore requires a fall in relative wage such that the developing country may see a fall in its aggregate real income. We thus demonstrate in figures 6 that for the developing country, the real costs of goods located at the beginning of the specialisation segment of the developed country have risen. For these goods, in other words, the fall in relative wage has more than offset the initial productivity gains.

It should be noted that the appearance of this type of configuration is totally specific to our model. With Matsuyama (2000) and DFS (1977), the following

Ricardian property is always verified: *Trade gains are derived from the existence of differences between the countries. When these differences grow larger (smaller), i.e. when technological shocks augment (reduce) the existing terms of comparative advantage, the transaction gains increase (decrease).*

If this "law" is not verified here, this is because the return to equilibrium (after the shock) imposes an extension of the specialisations of the developing country, despite the reduction in its comparative advantages. The wider the development gap between the two countries, the more this extension will penalise the developing country.

6 CONCLUSION

We have proposed a static Ricardian model in which goods are ranked along a continuum using a techno-utility criterion. The model thus establishes a correspondence between the country's characteristics (level of development) and the nature of its specialisation (namely, the techno-utility content of the goods, which we have also described as an expression of the relative sophistication of the goods). This ranking originates in the introduction into the model of a nonhomothetic demand function, for which the micro-economic foundations express hierarchic needs. Goods are thus consumed according to an order of priority, and we have assumed that the goods produced by the developed country have relatively less priority. Our model thus incorporates a relation between the nature of specialisations and the evolution of demand, in terms of both the composition of demand (the number and type of goods consumed) and the distribution of spending.

We have assumed shocks, expressing technical progress, which have led to three results concerning the link between specialisations and welfare. Firstly, we have shown that specialisations are a determinant of the relative capacity of developing countries to transform their own technical progress into gains in welfare. Secondly, we have shown that price competitiveness ensures (through trade) a diffusion of gains in welfare when there is uniform technical progress in the developed country. Finally, we have shown that our model can present results that are *a priori* paradoxical in a Ricardian context : when there is technical progress in the North biased towards the most technological goods, the greater the difference between the two countries, the less the developing country gains from trade.

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TABLES

Table 1 : Structure of consumption and specialisations in the two countries

Specialisations	Price	Country of production	Country of consumption
$[0, \bar{i}]$	wa_i	Developing country	Both countries
\bar{i}, J	a_i^*	Developed country	Both countries
$[J, J^*]$	a_i^*	Developed country	Developed country

Table 2 : Main properties of the demand function of the three models

The equations of the DFS and Matsuyama models underlying the preparation of this table are presented in annex 2a :

	DFS (1977) Cobb Douglas	Mats (2000) Hierarchics desires	Current model Hierarchics purchases
$\eta_y^i =$	1	0	$\eta_y^i(y, i)$
$\eta_p^i =$	1	0	$\eta_p^i(y, i)$
$J =$	$J^* = cst$	$J(y, p)$	$J(y, p)$
$q_i =$	$q_i(y, p)$	1	$q_i(y, p)$
$\frac{p_i q_i}{y} =$	$\beta_i = cst$ ($=\frac{1}{J}$ when $\beta_i=1$)	$\frac{p_i q_i}{y}(J, p)$	$\frac{p_i q_i}{y}(J, q_i, p)$

Table 3 : Initial equilibriums according the technological gap

Initial equilibriums (for each simul : $\alpha/\alpha^* = 2$)	U	U^*	w	\bar{i}	J	J^*
Simul 1 ($\beta/\beta^* = 6, 7$)	6,7	15,0	0,31	2,6	14,2	25,7
Simul 2 ($\beta/\beta^* = 66, 7$)	7,2	27,4	0,19	4,0	32,7	80,5
Simul 3 ($\beta/\beta^* = 1010, 1$)	6.8	38.9	0.14	5.8	97.0	310.2

Table 4 : Equilibriums after world uniform technical shock (WUTS)

Equilibriums after WUTS (for each simul : $\alpha/\alpha^* = 2$)	U	U^*	w	\bar{i}	J	J^*
Simul 1 ($\beta/\beta^* = 6, 7$)	10.5	23.9	0.29	3.5	19.3	36.4
Simul 2 ($\beta/\beta^* = 66, 7$)	11.5	47.2	0.16	5.2	43.2	114.3
Simul 3 ($\beta/\beta^* = 1010, 1$)	10,9	73,0	0,11	7,9	125,4	441,7

Table 5 : Equilibriums after foreign uniform technical shock (FUTS)

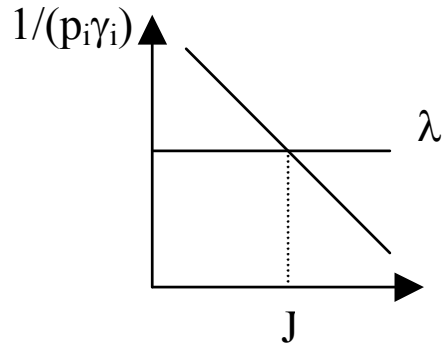
Equilibriums after FUTS (for each simul : $\alpha/\alpha^* = 4$)	U	U^*	w	\bar{i}	J	J^*
Simul 1 ($\beta/\beta^* = 13, 3$)	6.9	23.6	0.16	2.1	14.6	36.4
Simul 2 ($\beta/\beta^* = 133, 3$)	7.5	46.5	0.10	3.5	34.6	114.8
Simul 3 ($\beta/\beta^* = 2020, 2$)	7.0	72,0	0,07	5,5	101,7	444,5

Table 6 : Equilibriums after foreign non uniform technical shock (FNUTS)

Equilibriums after FNUTS (for each simul : $\alpha/\alpha^* = 2$)	U	U^*	w	\bar{i}	J	J^*
Simul 1 ($\beta/\beta^* = 13, 3$)	7.0	18.6	0.26	2.9	18.0	36.2
Simul 2 ($\beta/\beta^* = 133, 3$)	7.1	31.0	0.17	4.5	42.7	113.5
Simul 3 ($\beta/\beta^* = 2020, 2$)	6,77	40,6	0,13	6,1	130,6	438,0

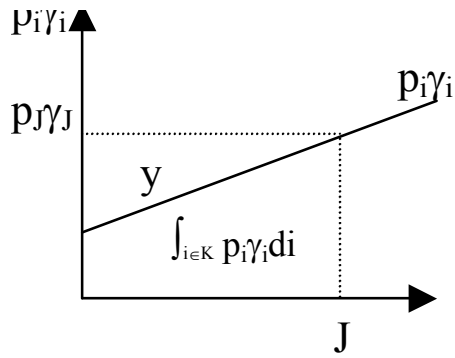
FIGURES

Figure 1 : Consumption of varieties according to the entry criterion $(\frac{1}{p_i \gamma_i})$



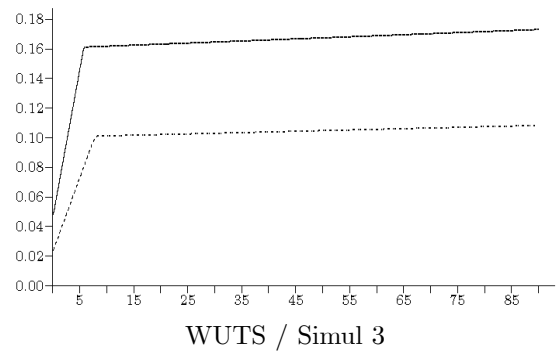
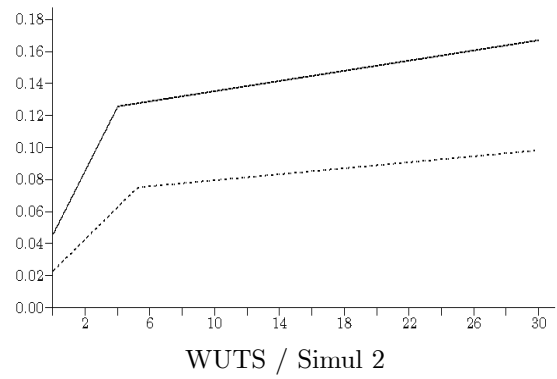
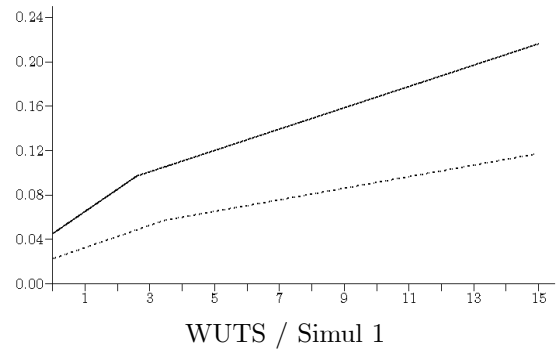
$(0, J)$ represents the segment of consumed goods under the assumption of a decreasing monotonic function for $\frac{1}{p_i \gamma_i}$

Figure 2 : Proof of theorem 1 : *The number of varieties consumed J is an increasing function of y*



As y increases, the area of the triangle above the curve $p_i \gamma_i$ increases, which is expressed by an increase of varieties consumed.

Figures 3 : Effect of WUTS on real prices of goods ($\frac{p_i^m}{w}$) consumed by developing country¹⁹



¹⁹ For figures 3, 4 and 6, the vertical axis represents $\frac{p_i^m}{w}$ and the horizontal axis the segment of goods i consumed by the developing country at initial equilibriums (simul 1, 2 and 3). Each ordinary line represents the real price of goods at initial equilibriums and each dotted line reports this value for equilibriums after the shock considered.

Figures 4 : Effect of FUTS on real prices of goods ($\frac{P_i^m}{w}$) consumed by developing country

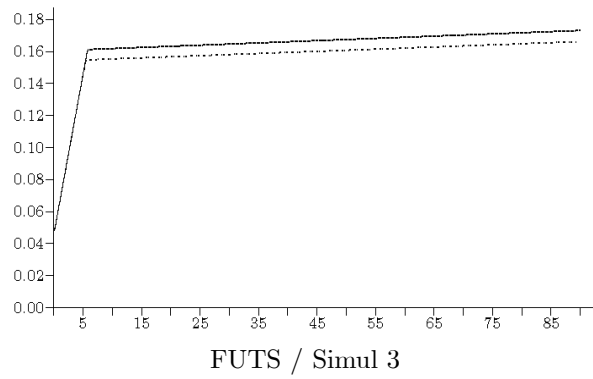
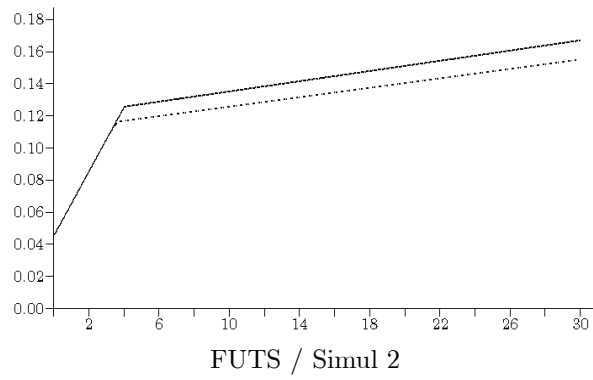
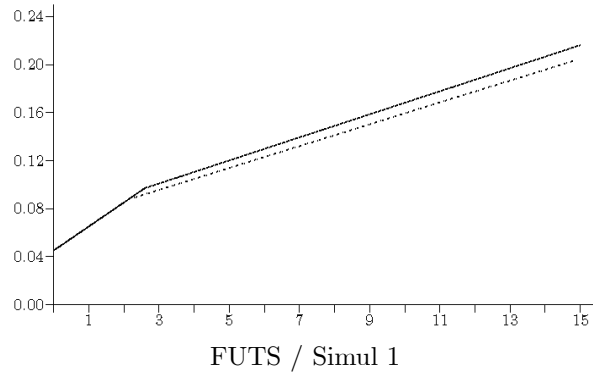
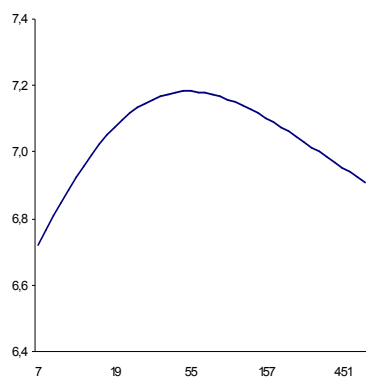
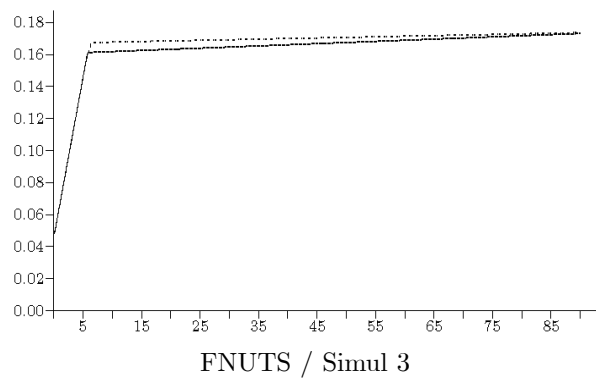
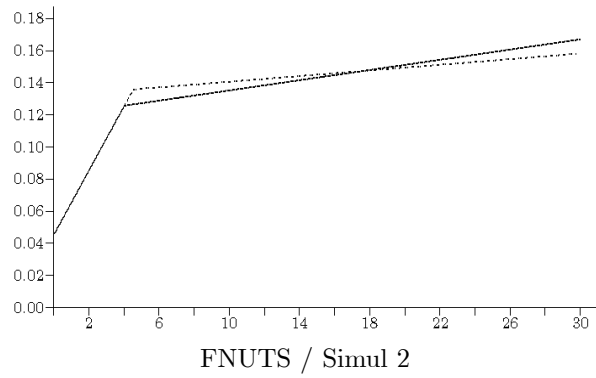
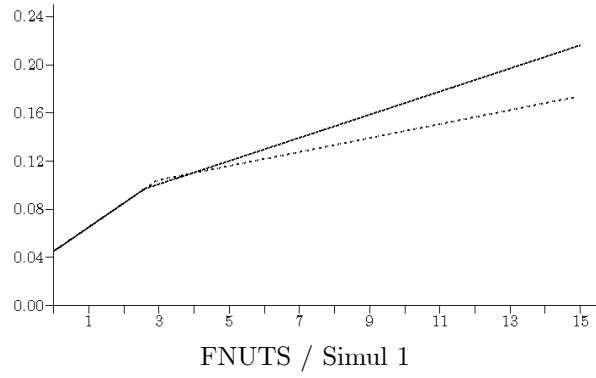


Figure 5 : Utility in developing country and technological gap



The vertical axis represents the utility of developing country and the horizontal axis reports the value taken by β/β^* (under the assumptions $\alpha/\alpha^* = 2$ and constant)

Figures 6 : Effect of FNUTS on real prices of goods ($\frac{P_i^m}{w}$) consumed by developing country



ANNEXES

ANNEX 1 : Verification of the existence of an analytical solution (section 5.1.3)

To determine the sign of the polynomial (equation 29), we proceed as follows:

With $\bar{i}_1 = \sqrt{\frac{2}{\beta}}$, we know that $\bar{i} = J$ and $J^* > J = \bar{i}_1$.

Consequently, we can write the following relation for the last term of the polynomial:

$$\begin{aligned} -\beta^* \bar{i}_1 (\alpha + \beta \bar{i}_1) (J^* + lJ) &< -\beta^* \bar{i}_1 (\alpha + \beta \bar{i}_1) (\bar{i}_1 + \bar{i}_1 l) \\ -\beta^* \bar{i}_1 (\alpha + \beta \bar{i}_1) (J^* + lJ) &< -\beta^* (\bar{i}_1)^2 (\alpha + \beta \bar{i}_1) (1 + l) \\ -\beta^* \bar{i}_1 (\alpha + \beta \bar{i}_1) (J^* + lJ) &< -\beta^* \frac{2}{\beta} (1 + l) \alpha - 2\beta^* \bar{i}_1 (1 + l) \end{aligned}$$

By substituting the right-hand expression for the last term of equation 29, we verify:

$$\begin{aligned} P(\bar{i}_1) &< -\alpha^* + \beta^* \bar{i}_1 (1 + 2l) + \beta^* \frac{2}{\beta} (1 + l) \alpha - \beta^* \frac{2}{\beta} (1 + l) \alpha - 2\beta^* \bar{i}_1 (1 + l) \\ P(\bar{i}_1) &< -\alpha^* - \beta^* \bar{i}_1 < 0 \end{aligned}$$

ANNEX 2 : Comparaison between the DFS and Matsuyama models (section 5.2)

a/ The equations of the DFS and Matsuyama models

The hypotheses (H1-H4) are maintained. Consequently, the condition of specialisation, i.e. equation 16, is identical in all three models. The differences in the choice of utility function, on the other hand, induce different equations for the balance of trade equilibrium. To make it easier to compare the three models, the equilibrium equations are presented under the hypothesis $\beta_i = 1$.

DFS (1977)	Matsuyama (2000)	
$Max \quad U = \int_0^J \beta_i \ln q_i di$	$Max \quad U = \int_0^\infty \beta_i q_i di$	(A1)
$sc \quad w = \int_0^J p_i q_i di$	$sc \quad w = \int_0^\infty p_i q_i di$	(A2)
$X_i = p_i^m q_i^* \cdot L^* = \frac{1}{J} \cdot L^*$	$X_i = p_i^m q_i^* L^* = w a_i L^*$	(A3)
$M_i = p_i^m q_i \cdot L = \frac{w}{J} \cdot L$	$M_i = p_i^m q_i L = a_i^* L$	(A4)
$J = J^* = cste$	$wL = Lw \int_0^{\frac{J}{i}} a_i di + L \int_{\frac{J}{i}}^J a_i^* di$	(A5)
	$L^* = L^* w \int_0^{\frac{J}{i}} a_i di + L^* \int_{\frac{J}{i}}^{J^*} a_i^* di$	(A6)
$\frac{X=M}{w} \Rightarrow \frac{J}{J-i} \frac{L^*}{L}$	$\frac{X=M}{L+L^*} \Rightarrow \frac{J}{i} \int_0^{\frac{J}{i}} a_i di$	(A7)

- (A1) and (A2) represent the agent's maximisation programme.

- (A2) and (A3) represent the demand for imports per product addressed to the developing country and the demand for exports that this country addresses to the developed country.

- The balance of trade equilibrium condition (A7) is obtained through (A2) and (A3). For Matsuyama, this expression is simplified with the help of equation (A5).

- (A5) and (A6) represent the exhaustion of the budget constraint in the two countries and makes it possible to determine the number of varieties consumed in Matsuyama's model.

b/ The effect of technological shocks

The line A represents the condition of specialisation (equation 16) and the line B represents the balance of trade equilibrium condition (equations A7). B' and A' represent the shifting of these lines with the occurrence of technical progress (TP). It should be noted that the graphic representation proposed here is deliberately simplified²⁰. It should be read in the following manner:

- Along the y axis, the shift from one equilibrium to another gives the size of the variation in relative wage

- The distance between A and A', at any given point on the x axis, tells us the size of the productivity gains for a good i .

From the comparison of these two distances, we can deduce the variation in the purchasing power of an agent for every good consumed.

²⁰This presentation is intuitive. For more rigour, the reasoning in terms of variations would require modification of the scale in logarithm or the representation of shifts in a non-linear form. This would complicate the presentation without modifying the results.

	DFS (1977)	Matsuyama (2000)
Uniform international TP		
Uniform TP in the developed country		
Biased TP in the developed country		