Is One Shock Enough? The Relative Price of Investment in the Short Run

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Abstract

A number of authors argue that investment-specific productivity shocks play an important role in shaping the business cycle, based on the fact that movements in the relative price of investment are both large and countercyclical. This paper shows that in fact there is no need to assume sector-specific shocks in order to explain these movements. Aggregate shocks, together with the assumptions of time-varying markups through imperfect competition and of rigidities in the reallocation of factors across sectors are shown to be able to generate the same amount of countercyclical variability in the relative price of investment as is observed in the data.

1 Introduction

Sector-specific productivity shocks have become a standard feature in technological shocks-driven models of the business cycle at least since Greenwood, Hercowitz, and Krusell's (2000) seminal paper on the subject. Studies rely on investment-specific shocks to explain the negative correlation between investment and its relative price (in terms of consumption goods) that is observed in U.S. data, the idea being that a positive shock on productivity in the investment sector pushes up output in that sector while pushing down its relative price. A number of authors have concluded that those shocks play a quantitatively important role in shaping the business cycle: Greenwood, Hercowitz, and Krusell (2000) argue that thirty percent of output fluctuations
are due to such shocks, while Fisher (2004) estimates that their contribution is closer to fifty percent.

One of the reasons for why one might care about whether shocks are mainly aggregate or sector-specific is that the existence of sizeable sector-specific productivity shocks tends to support technology-driven views of the cycle. Indeed, while it is difficult to find another label than "technology" for sector-specific shocks, this is not the case for aggregate shocks: for example, Fisher (2004) states that "a plausible interpretation regarding the neutral term is that it represents 'productivity' shocks, disturbances to production possibilities more generally conceived and not necessarily due to technological change, such as taxes, regulation and market structures." Also, Kamihigashi (1996) shows that Real Business Cycle (RBC) models with a single aggregate shock are observationally equivalent to models of endogenous business cycles, or animal spirits, and Francis and Ramey (2005) find that, while the data seem to be at odds with the predictions of the technology-driven real business cycles hypothesis, abandoning this framework does not require adherence to sticky price assumptions. In other terms, RBC models driven by aggregate shocks might still constitute an acceptable modeling framework even if it turns out that the main shocks which drive the cycle are not of a technological nature.

The present paper shows that in fact there is no need to assume sector-specific productivity shocks to explain the observed movements in the relative price of investment: purely aggregate shocks, associated with the assumption of imperfect competition, already do the trick. The idea is that competition among firms increases during booms, which results in a fall in output prices relative to factor prices; in other words, markups are countercyclical, which is supported by empirical evidence (see for example Murphy, Shleifer, and Vishny, 1989, and Rotemberg and Woodford, 1999). Since investment varies much more than consumption over the business cycle, this effect will be much stronger within the investment sector; it follows that during a boom, the price of investment in terms of consumption goods is bound to fall, causing the previously mentioned negative correlation between investment and its relative price.

A dynamic stochastic general equilibrium model with aggregate productivity shocks is then set up to assess whether the above idea makes sense from a quantitative point of view. Arguably, in order to correctly evaluate the impact of a given modelling framework on the evolution of the relative price of investment, the model under consideration should replicate the high amount of co-movement between the investment and consumption sectors which we observe in the data, since this feature (or its absence) is likely to affect significantly the behaviour of the relative price which any given model implies.
Boldrin, Christiano, and Fisher (2001) show that habit persistence in consumption, together with short-term rigidity in the cross-sectoral reallocation of factors, can generate an amount of sectoral co-movement which is broadly consistent with the data. As they explain, a positive shock in aggregate productivity entails a sharp rise in the demand for investment goods, along with a corresponding fall in the demand for consumption goods. However, because of the short-term rigidities, factors cannot be reallocated instantly to the investment sector to satisfy the rise in demand. The presence of habit persistence then raises the value of consumption in subsequent periods, ensuring that consumption does not fall sharply shortly afterwards.

The monopolistic competition framework proposed by Galí and Zilibotti (1995) is adopted, in which each industry is composed of an endogenous number of firms paying a fixed operating cost and operating under Cournot competition, which results in countercyclical markups. Average markups are assumed to be the same across sectors.

It is shown that, for parameters which imply an average markup which is consistent with available evidence, a calibrated version of the model described above is able to replicate both the negative correlation between investment and its relative price, and the variance of this relative price.

The remainder of the paper is organised as follows: section 2 describes the model; section 3 deals with its quantitative implications; section 4 looks at the results; and section 5 concludes.

2 The Model

The general specification of the model follows Boldrin, Christiano, and Fisher (2001) in assuming habit persistence in consumption, rigidities in the reallocation of factors across sectors, and the existence of a unit root in the law of motion of aggregate productivity. The imperfect competition framework which is used originates from Galí and Zilibotti (1995). Time is discrete; time subscripts are omitted whenever there is no risk for confusion.

2.1 Preferences

The preferences of the representative agent are given by

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \left[ \ln \left( C_t - bC_{t-1} \right) - \kappa_t L_t \right],$$

$$0 < \beta < 1, b \geq 0,$$
where $C_t$ and $L_t$ denote consumption and labour at time $t$, respectively. When $b > 0$, agents’ preferences display habit formation. This particular way of modelling habit formation implies that households link their current consumption level with their own past consumption. In order to ensure stationarity in hours worked, $\kappa$ is assumed to grow at the same rate as $C$.

2.2 Technology

The economy contains one sector producing consumption goods $C$ and one sector producing investment goods $I$. Total output in the economy is then given by

$$Y = P_c C + P_i I,$$

where $P_c$ and $P_i$ stand for the prices of consumption and investment. For convenience, $P_c$ is normalised to one, so that $P_i$ corresponds to the price of investment in terms of consumption goods.

Final Goods  Firms producing final output in each sector $X$ operate under perfect competition and use sector-specific intermediate goods $X_m, m \in [0, 1]$ as inputs. The output of the representative firm in sector $X$ is given by

$$X = \left( \int_0^1 X_m^{\frac{\sigma-1}{\sigma}} \, dm \right)^{\frac{\sigma}{\sigma-1}}, X = C, I.$$

Intermediate Goods  Each intermediary good $X_m$ is produced by a finite number of firms indexed by $n$ which operate under Cournot competition. Each of those firms has access to a production technology given by

$$C_{mn} = K_{c\!mn}^\alpha (AL_{c\!mn})^{1-\alpha} - \phi_c,$$

$$I_{mn} = K_{i\!mn}^\alpha (AQL_{i\!mn})^{1-\alpha} - \phi_i,$$

where $A$ is a measure of economy-wide aggregate productivity, $Q$ is technological progress which is specific to the investment sector, and $K_{xij}$ and $L_{xij}$ are the stocks of capital and labour used by firm $j$ to produce the intermediate good $X_i$. $\phi_c$ and $\phi_i$ are fixed costs to be paid each period by active firms and are assumed to grow at the same rate as productivity in the consumption and investment sectors respectively. Total output for each intermediate good $X_m$ is then given by

$$X_m = \sum_{n=1}^{N_{xm}} X_{mn},$$
where $N_{xm}$ is the number of firms producing the intermediate good $X_m$. While there is a fixed number of intermediate goods in each sector, the number of firms producing each intermediate good $X_m$ is determined each period under free entry.

Likewise, the amount of capital and labour used in the production of each intermediate good can be written as

$$Z_{xm} = \sum_{n=1}^{N_{xm}} Z_{xmn}, Z = K, L.$$  

(7)

**Resource Constraints**  The aggregate resource constraints for capital and labour are then

$$K = \int_0^1 K_{cm} dm + \int_0^1 K_{im} dm = K_c + K_t,$$  

(8)

$$L = \int_0^1 L_{cm} dm + \int_0^1 L_{im} dm = L_c + L_t;$$  

(9)

while the accumulation constraints for the aggregate stock of capital is given by

$$K_{t+1} = (1 - \delta) K_t + I_t,$$  

(10)

where $\delta$ stands for the physical depreciation rate of capital.

**Aggregate Productivity**  Aggregate productivity is assumed to follow a random walk with drift:

$$A_t = \exp (\varepsilon_t) A_{t-1}, \varepsilon_t \sim N \left[ \log (\gamma_a), \sigma_{\varepsilon}^2 \right],$$  

(11)

where $\gamma_a$ is the expected growth rate of $A$. As mentioned in the introduction, $\varepsilon_t$ is the only shock hitting the economy in this model.

I follow Boldrin, Christiano, and Fisher (2001) in assuming that the allocation of capital and labour across sectors at time $t$ has to be decided before the realisation of the shock $\varepsilon_t$ is known. This reflects the fact that it is often difficult for firms to immediately reallocate their production factors following the release of new information. Note that because of this feature, the present model cannot be reformulated in terms of a one-sector model in which output can be transformed into capital at rate $q_t$, as in Greenwood, Herckowitz and Krusell (1997, 2000).
2.3 Competitive Equilibrium

2.3.1 Consumers

The representative agent maximises (1) subject to his budget constraint

$$(1 + r_t) P_{it} K_t + w_t L_t - P_{i,t+1} K_{t+1} \geq 0,$$

where $r$ and $w$ stand for the rental rates of capital and labour respectively, net of depreciation. The first order conditions with respect to $K_{t+1}$ and $L_{t+1}$ are given by

$$H_t/H_{t+1} = \beta (1 + r_t) \frac{P_{i,t+1}}{P_{it}}, \quad (12)$$

$$w_t = C_t - b C_{t-1}, \quad (13)$$

where $H_t = \frac{1}{C_t - b C_{t-1}} - \frac{\beta b}{C_{t+1} - b C_t}$.

2.3.2 Firms

Final Goods  The representative firm producing final goods in sector $X$ chooses $X_m$ to maximise profits given intermediate goods prices $P_{x_i}$:

$$\pi^*_x = \max_{X_m} \left[ P_x \left( \int_0^1 X_{m-1} dm \right) \frac{X}{X_m} - \int_0^1 P_{xm} X_m dm \right], X = C, I. \quad (14)$$

The first order condition on $X_m$ yields the demand for good $X_m$:

$$P_{xm} = \left( \frac{X}{X_m} \right)^{1/\sigma} P_x. \quad (15)$$

Intermediate Goods  A given firm $n$ producing the intermediate good $X_{mn}$ chooses its factor inputs $K_{xmn}$ and $L_{xmn}$ to maximise expected profits given factor prices $r$ and $w$:

$$\pi^*_{xmn} = \max_{K_{xmn}, L_{xmn}} E \left\{ X_{mn} P_{xm} - (r - \delta) K_{xmn} - w L_{xmn} \right\}, \quad (16)$$

where $C_{mn}$ and $I_{mn}$ are given by equations (4) and (5) respectively, and $P_{xm}$ is given by the demand for good $X_m = \sum_{j=1}^{N_x} X_{mn}$ in equation (15). Considering only symmetric equilibria and integrating over all intermediate goods in each sector, the first order conditions with respect to $K_{xmn}$ and $L_{xmn}$ yield the following two factor price equations for each sector $X$:

$$P_i (r + \delta) = \left( 1 - \frac{1}{\sigma N_x} \right) \alpha \frac{X}{K_x} P_x, \quad (17)$$

$$w = \left( 1 - \frac{1}{\sigma N_x} \right) (1 - \alpha) \frac{X}{L_x} P_x, \quad (18)$$
where $\mu_x = \frac{1}{\sigma N_x}$ corresponds to the markup in sector $X$.

Under free entry, the number of firms in each sector adjusts to ensure that expected profits are zero in all sectors:

$$E(\pi_{xmn}) = 0, \forall x, m, n.$$  \hspace{1cm} (19)

Ignoring integer constraints, the number of firms $N_x$ producing each intermediate good $X_m$ is then given by

$$N_x = \frac{X}{\sigma \phi_x},$$  \hspace{1cm} (20)

so that the markup in sector $X$ is equal to $\mu_x = \frac{1}{\sigma N_x} = \frac{\phi_x}{\sigma X}$, which implies that the markup in a given sector is negatively correlated with output in that same sector. As mentioned earlier, in order to ensure that markups in each sector remain constant along a balanced growth path, the fixed costs of production $\phi_c$ and $\phi_i$ are assumed to grow at the same rates as productivity in the consumption and investment sectors respectively. Also, I assume that $\phi_c/\phi_i$ is equal to the average consumption / investment ratio, so that average markups are the same in both sectors.

2.3.3 Equilibrium

An equilibrium for this model is defined as a sequence $\{X_t\}_{t=0}^{\infty}$ for $X = [K_c, K_i, L_c, L_i, A, r, w, P_i]$ which satisfies the first order conditions for the households’ problem (12) and (13), the two pairs of factor price equations given by equations (17) and (18), the law of motion for capital (10) and the law of motion for aggregate productivity (11).

2.4 Balanced Growth

The aim of this section is to transform the model in a way which makes all variables constant along a non-stochastic balanced growth path; normalised variables are written in lower-case letters. First, note that equations (4) and (5) imply that $C$ and $I$ grow at an expected rate of $\gamma_c = \gamma_a^a \gamma_q^a$ and $\gamma_i = \gamma_a^a \gamma_q^a$ respectively. Also, the resource constraints given by equations (8), (9) and (10) imply that $K_c$, $K_i$ and $K_i$ also grow at rate $\gamma_i$. Given that aggregate productivity $A$ follows a random walk with drift, the appropriate transformation consists in dividing each time $t$ variable by an appropriate function of $A_{t-1}$, so that $c_t = C_t / (A_{t-1} \gamma_q^{a t})$, $x_t = X_t / (A_{t-1} \gamma_q^{a t})$ for $X_t = I, K_{t}, K_{c_{t}}, x_{t} = X_{t}$ for $X_t = L, L_c, L_i$, $a_t = A_t / A_{t-1}$ and $p_t = P_t / \gamma_q^{a-1}$.
The equations characterising the transformed model are then given by

\[ \frac{h_t}{h_{t+1}} g_{it} = \beta \left[ 1 + (1 - \mu_i) \alpha \frac{k_{it}}{k_{it+1}} - \delta \right] \frac{p_{i,t+1}}{p_{i,t}}, \]  
\[ \frac{k_{it}}{k_{ct}} = \frac{l_{it}}{l_{ct}}, \]  
\[ k_{it+1} g_{it} = (1 - \delta) k_t + \nu_t, \]  
\[ 1 = (1 - \mu_c) (1 - \alpha) \frac{c_{t+1} h_{t+1}}{l_{ct}}, \]  
\[ a_t = e^{\tau_t}, \]

where \( h_t = \frac{1}{c_s - \beta g_c - \gamma_s} - \frac{\beta b}{c_s - \beta g_c - \gamma_s}, \) \( p_{i,t} = \frac{\mu_i}{k_{ct} k_{it}} \) \( k_{it} = a_t \gamma_q^o \) and \( g_{it} = a_t \gamma_q. \)

Given that this problem does not have a closed-form solution, a second-order approximation is obtained by using a methodology proposed by Schmitt-Grohe and Uribe (2004).

### 3 Calibration

The parameters which need to be determined are the discount rate \( \beta \), the habit formation parameter \( b \), the parameters related to the income share of capital \( \alpha \), the depreciation rate of capital \( \delta \), the parameter determining the elasticity of substitution between intermediary goods \( \sigma \), the fixed per-period costs of production \( c_c \) and \( c_i \), the variance of the aggregate shock \( \sigma_c \), and the steady-state growth rates of \( A, \gamma_A, \) and \( Q, \gamma_q. \) Note that, since \( c_{i,c} \) and \( \sigma \) always appear together in the model, it is not possible nor indeed necessary to estimate them separately. Instead, the relevant parameter for calibration is given by the average markup, \( \mu = \sqrt{\frac{\phi_c}{\phi_c}}. \)

**Parameter Values Based on Average U.S. Data** \( \alpha, \beta, \delta, \gamma_a \) and \( \gamma_q \) are set so that the model’s balanced growth path displays a number of features which are observed over the long run in U.S. data.

The balanced growth path satisfies the following two equations:

\[ \gamma_i = \beta \left[ 1 + (1 - \mu) \alpha \frac{I}{K} - \delta \right], \]
\[ \gamma_i = (1 - \delta) + I/K, \]

\(^1\text{As mentioned above, markups are assumed to be the same in both sectors.}\)
while average quarterly values of aggregate U.S. data for the time period 1954-2000 yield the following five additional equations:

\[
\begin{align*}
\alpha &= .36, \\
I/K &= .036, \\
I/Y &= .204, \\
\gamma_c &= 1.0047, \\
\gamma_i &= 1.0109.
\end{align*}
\]

Remaining Parameters The remaining parameters are determined in order to replicate a set of cyclical features observed in logged and Hodrick-Prescott filtered quarterly U.S. data over the same time period: the habit formation parameter \( b \), the variance of the shock to aggregate productivity \( \sigma_z \) and the average markup \( \mu \) are set to match the variance in consumption \( C \), output \( Y \) and in the relative price of investment \( P_i \) that are observed in the data.

4 Results

The calibrated parameter values are \( \beta = .9833, \delta = .0251, b = .5, \gamma_a = 1.0012, \gamma_q = 1.0097, \sigma_z = .01299 \) and \( \mu = .163 \).

The main result of the paper is that, if one assumes that movements in the relative price of investment are mainly due to the fact that markups are countercyclical, then the implied average markup is 16.3 percent. While empirical estimates of average markups for the U.S. economy are inconclusive to say the least\(^2\), Galí, Gertler, and López-Salido (2005) find that “a value of 0.15 to 0.20 is plausible for the steady state price markup.”

Table 1 contains a number of statistics characterising the behaviour of the U.S. economy over the sample period, along with the corresponding statistics for the model, while figure 1 illustrates the impulse responses of a number of endogenous variables in the model to a one percent aggregate shock.

Given that the model relies on the fact that investment is more volatile than consumption to generate movements in the relative price of investment, particular attention should be given to the relative volatility of these two aggregates. The fact that investment is somewhat more volatile in the model economy than it is in the data implies that the impact of aggregate shocks on the relative price is bound to be slightly overestimated. Also, sectoral co-movement is not quite as pronounced in the model as it is in the data.

\(^2\)See for example Chevalier and Scharfstein (1996).
Table 1: Business Cycle Statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>Standard deviation (%)</th>
<th>Cross-correlation with output</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data</td>
<td>Model</td>
</tr>
<tr>
<td>Output</td>
<td>2.01</td>
<td>2.01</td>
</tr>
<tr>
<td>Consumption</td>
<td>1.23</td>
<td>1.23</td>
</tr>
<tr>
<td>Investment</td>
<td>7.56</td>
<td>8.98</td>
</tr>
<tr>
<td>Hours</td>
<td>1.47</td>
<td>2.03</td>
</tr>
<tr>
<td>Relative Price</td>
<td>0.98</td>
<td>0.98</td>
</tr>
<tr>
<td>Markup</td>
<td>–</td>
<td>.41</td>
</tr>
</tbody>
</table>

Nevertheless, the model is broadly consistent with key features of the data while at the same time replicating the observed variability in the relative price of investment. A somewhat striking feature of the model is that the (implied) variance in the markup which is needed to generate this variability is actually quite small.

The fact that the (negative) correlation between investment and its relative price is counterfactually high in the model is not really surprising, given that in the latter movements in the relative price are caused exclusively by movements in the level of investment. Obviously the relative price may be affected by other factors; this paper just argues that markups appear to be an important part of the story.

3Results for the model are based on 500 replications of sample size 184. The statistics are calculated using data that was logged and Hodrick-Prescott filtered; the dataset is described in the appendix.

4Cross-correlation with investment.
5 Conclusions

The purpose of this paper was to show that the observed countercyclical movements in the price of investment in terms of consumption goods can be easily accounted for without having to rely on sector-specific technological shocks. To this purpose, a simple two-sector, aggregate shock-driven model featuring imperfect competition with variable markups, as well as habit formation in consumption, was set up. The model was then calibrated to match the observed variance in the relative price of investment as well as a number of other features of the data; the average markup implied by the calibrated model is around 16 percent, which is broadly in line with empirical estimates. The calibration exercise suggests that a specification which includes only one (aggregate) shock is indeed enough to explain the variability in the relative price of investment. This result gives support to non-technological interpretations of the business cycle, since arguably it is easier to find non-technological explanations for aggregate shocks than it is for sector-specific ones.
Appendix A Data

The variables in the model’s resource constraint, namely $Y$, $C$, $I$ and $L$, are matched up with the corresponding real variables and their price series from quarterly U.S. NIPA data. When series are added together they are chain-weighed. The sample period is 1955:1 - 2000:4.

A.1 Aggregate Quantities

$C$ is matched up with private consumption, which includes durable and non-durable goods and services; $I$ corresponds to private investment, which includes equipment and software, residential and non-residential structures, as well as inventories. Net exports are included in output $Y$; while government expenditures are netted out. $L$ is given by total hours employed per week from the Household Survey data.

A.2 Prices

$P_i$, the relative price of investment, is measured as the price index for investment divided by the price index for consumption.

Given that there is a considerable upward bias in NIPA’s equipment and software price index, Gordon’s (1990) index is used. The index has been extended by Cummins and Violante (2002) and estimated for quarterly data by Fisher (2004).

References


