Strongly Rational Expectations Equilibria, Endogenous Acquisition of Information and the Grossman–Stiglitz Paradox

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ABSTRACT

This paper analyzes conditions for existence of a strongly rational expectations equilibrium (SREE) in models with private information where the amount of private information is endogenously determined and where the price transmits relevant information to market participants. It is shown that the conditions for existence of a SREE known from models with exogenously given private information have to be qualified if private information is endogenously determined. A SREE exists only if informativeness of the market price falls short of a specific lower bound, which depends on the properties of the cost function associated with the individual acquisition of information. This upper bound is generally lower than that valid in case of exogenously given information. An interpretation that links our results to the famous Grossman–Stiglitz–Paradox is also given.

KEYWORDS: Eductive Learning, Private Information, Rational Expectations, Strongly Rational Expectations Equilibrium

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1. INTRODUCTION

Although playing a central role in modern economic theory, the hypothesis of rational expectations is often viewed with scepticism. Indeed, the concept of a rational expectations equilibrium (REE) is quite ambitious if we take into account the underlying severe requirements on agent's information gathering and processing capabilities. It is therefore not surprising that many attempts have been made in order to justify this concept and to state a clear set of assumptions that implicate rational expectations on the side of the agents. One such attempt is the concept of a strongly rational expectations equilibrium (SREE) proposed by Guesnerie (1992, 2002). This concept starts from the two basic hypotheses of individual Bayesian rationality and common knowledge and asks, whether an REE can be educed by rational agents, meaning that the REE is the solution of some kind of mental process of reasoning of the agents. A SREE is then a REE that is learned by agents using this 'eductive' mental process (equivalently, the REE is said to be eductively stable). As shown by Guesnerie (1992, 2002), eductive learning of rational expectations is possible, if based on a suitably specified game–form of the model, agents use an iterative process to eliminate non–best responses from their strategy sets and if this process converges to the REE. Whenever this is the case, a general belief that agents choose strategies that are in a small neighborhood of the REE will be self-enforcing. Therefore, an eductively stable REE or a SREE is not subject to expectational coordination difficulties, which would arise in case of an unstable REE where any such general belief turns out to be self-defeating.

Guesnerie (2002) provides an overview over the conditions for existence of SREE that have been derived in various economic contexts. A unanimous conclusion from this is that a REE is not necessarily a SREE, but that in general additional restrictions have to met for a SREE to exist. The class of models that have been analyzed up to now also contains models with private information, which usually exhibit quite complex rational expectations equilibria. Therefore the justification of such complex equilibria is of particular interest. Within the class of models with private information, conditions for existence of a SREE have been derived for models where agents are unable to use the information transmitted through current market prices (cf. Heinemann (2004)), as well as for models where this information can be used (cf. Desgranges et al. (2003), Desgranges (1999), Heinemann (2002)). However, a common feature of all these studies is that they assume an exogenously given amount of private information. This means that so far not only the question how this private information comes into the market has been ignored. It also means that by now it has not been analyzed, whether the endogenization of private information acquisition causes additional restrictions an REE must fulfill in order to be eductive stable.

The present paper tries to fill this gap. We will introduce endogenous information acquisition into a simple market model and derive conditions for existence of a SREE given this endogenously acquired information. The model is one where agents are able to use information contained in the current market price for their current decisions and is therefore strongly related to the theoretical analysis of the transmission of information through prices which originated from the early contributions of Grossman (1976), Green (1977) and Radner (1979). Regarding the introduction of endogenous information acquisition, we follow the seminal work of Grossman and Stiglitz (1980) and more precisely Verrecchia (1982) who has analyzed rational expectations equilibria with endogenous acquisition of information in a quite similar economic environment.

Analyses of models where prices transmit information, but with exogenously given information (cf. Desgranges et al. (2003), Desgranges (1999), Heinemann (2002)), have stressed the role of the informativeness of the market price relative to that of private signals for existence of a SREE: If prices in a REE reveal too much information, it is likely that no SREE exists. Thus, high informational efficiency of a market is likely to generate expectational coordination difficulties. The central result of this paper not only confirms these results, but states that endogenous acquisition of information can lead to even stronger conditions for existence of a SREE. Whether or not this is the case, however, particularly depends on the nature of the underlying cost function that is associated with the acquisition of private information. In simple terms, the conditions we derive say that endogenous acquisition of information will not lead not to stronger conditions for existence of a SREE whenever the marginal cost function is steep enough. If, however, the marginal cost function is flat, a SREE exists if and only if the informativeness of the market price is less than one half the informativeness of the private signals. This restriction on the informativeness of the market price is stronger than the respective condition for the case with exogenous information. Thus, we conclude that endogenous acquisition of private information in general leads to stronger conditions for existence of a SREE than that known for the case with exogenously given private information. The underlying intuition for our result is that the informativeness of the price is determined by the correlation between private information and agents' decisions. As long as the price is not very informative, this correlation is easy to predict. But, when the price is very informative, agents have an incentive to learn excessively from the price which makes their decisions very sensitive to the information that is supposed to be contained in the price. Agents' decisions then mainly depend on the beliefs about the informational content of the price and these decisions are therefore not easy to predict implicating nonexistence of a SREE. The fact that endogenous information acquisition makes it more difficult for a SREE to exist can be explained as follows: As before, when the price is very informative, no SREE exists because every agent reacts less to his private information than to his beliefs about the information revealed by the price. In this case, given that private information is not very useful to agents, but costly to acquire, the precision of the privately acquired information decreases. This last fact reinforces the stability problem of the eductive learning process. Namely, agents become much less reactive to their private information. Hence, agents' decisions depend more on their beliefs, which corresponds to a greater expectational coordination difficulties.

A considerable part of our analysis is devoted to the special case of constant marginal costs of information acquisition. This case not only allows us to derive clear cut conditions for existence of a SREE and to illustrate the eductive process in a simple fashion. Furthermore, this special case allows us to link our results with the well known Grossman–Stiglitz paradox (cf. Grossman and Stiglitz (1980)) of impossibility of informationally efficient markets. The Grossman–Stiglitz paradox says that REE where the price transmits all relevant information to the agents cannot exist if acquisition of information is a costly activity because in such a case no agent would have an incentive to acquire information costly, price will reveal anyway. If, however, no one acquires private information, prices cannot be informative. Hence, informativeness of prices in a REE is necessarily bounded from above. Our results show that even when a REE exists (implicating that the price is not fully informative), this REE can be subject expectational coordination difficulties. These coordination difficulties are absent only if informativeness of the price is far below the upper bound on informativeness of the price which, according to the Grossman–Stiglitz paradox, is compatible with existence of a REE.

The paper also provides some comparative–static exercises that can be conceived as permitting some tentative conclusions on how public policy might help to overcome possible problems of expectational coordination. Furthermore, the comparative–static results with respect to the precision of public information give occasion to confront our results with respective results derived in the literature on global games (cf. Morris and Shin (2003)). While this literature concludes that more precise public information might generate equilibrium multiplicity and, thus, coordination problems, our results are more ambiguous: It is only public information in form of the information contained in the market price, which causes expectational coordination difficulties, whereas public information in form of a priori knowledge is always tends to attenuate such difficulties

The paper is organized as follows: Section 2. presents the model and characterizes its unique REE. In Section 3. we describe the concept of a SREE, state conditions for existence of a SREE and discuss these conditions. Section 4. links the results with the Grossman–Stiglitz paradox. In Section 5. we briefly discuss two comparative–static exercises and Section 6. concludes. The proofs are gathered together in the Appendix.

2. A COMPETITIVE MARKET MODEL WITH LEARNING FROM CURRENT PRICES

The model

The model that builds the framework of our analysis is a simple model of a competitive market equivalent to the one used by Vives (1993). He examines the factors that influence the speed of learning (i.e. the rate of convergence to a REE with shared information) and, thus, looks at a dynamic version of this model where firms learn from past prices. Contrary to this, we take a static version of the model and analyze eductive stability of static equilibria where firms are able to use the information revealed by current market prices for their current decisions. This is the kind of learning from current prices analyzed especially in the static rational expectations literature in financial markets (see, for example, Grossman (1976), Grossman and Stiglitz (1980)). In fact, as demonstrated by Vives (1993), it is possible to restate the present model such that it can be interpreted as a financial market model where agents are buyers of an asset with unknown ex–post return.

In this paper, we nevertheless stick to the competitive market version of the model. This model of a competitive market consists of a continuum of risk neutral firms in $I = [0,1]$. Market demand X is random, but the inverse demand function is known to the firms:

$$p = \beta - \frac{1}{\phi} X + \varepsilon$$

Here, p is the market price, ε is a normally distributed demand shock with zero mean and precision τ_ε. $\beta > 0$ and $\phi > 0$ are known constants. Every firm faces increasing marginal costs that are affected by the parameter θ (this is a productivity shock unknown at the time where the production decision is made). With $x(i)$ denoting the output of firm i, its costs are $c(i) = \theta x(i) + \frac{1}{2}\frac{1}{\psi} x(i)^2$, where $\psi > 0$. The cost parameter θ is unknown to the firms. The firms, however, know that this parameter is drawn from a normal distribution with mean $\bar{\theta}$ and precision τ.

Private information on the side of the firms regarding the unknown parameter is introduced into the model by allowing for endogenous acquisition of information as in Verrecchia (1982) (generalizing the seminal framework of Grossman and Stiglitz (1980)). It is assumed that each firm is able to perform an experiment (independent from experiments of other firms) that reveals additional but costly information regarding the unknown parameter θ. In particular, it is assumed that each firm $i \in I$ can acquire a costly private signal $s(i)$ that reveals additional private information. The private signal is given by $s(i) = \theta + u(i)$ where the signal's noise $u(i)$ is normally distributed with mean zero and precision $\tau(i)_u$. The costs of acquiring a signal with precision $\tau(i)_u$ are given by $K(\tau(i)_u)$. The objective of a firm is to maximize the expected profit where profit $\pi(i)$ of firm i is given by:

$$\pi(i) = [p - \theta]\, x(i) - \frac{1}{2}\frac{1}{\psi}\,[x(i)]^2 - K(\tau(i)_u), \tag{1}$$

Costs of information acquisition are assumed to be increasing and convex: $K'(\tau(i)_u) > 0$ and $K''(\tau(i)_u) \geq 0$ for all $\tau(i)_u \geq 0$. Throughout the following analysis it will always be assumed that the average of the firm's private signals reveals the unknown value of the unknown parameter by the law of large numbers, such that $\int_0^1 s(i)\, di = \theta$ because $\int_0^1 u(i)\, di = 0$.

Rational expectations equilibrium

Our static equilibrium concept requires that firms are able to condition their supply decisions on the — not yet determined — equilibrium price p. One way to accomplish this is to specify an explicit game among the firms where each firm submits a supply schedule to an auctioneer. The auctioneer collects the individual supply schedules and sets the market clearing price once demand has realized. In such a game, a supply schedule specifies for every price p the optimal supply of a single firm $i \in I$ with a specific signal s_i and we look for a rational expectations equilibrium or Nash–equilibrium of this game.

Our first result establishes that there exists a unique linear REE in this model with learning from current prices.[1]

Proposition 1 *Let $\alpha = \psi/\phi > 0$. There exists a unique linear REE where every firm uses a linear supply function $x(i) = \psi\,[(1-\gamma_2^*)\, p - \gamma_0^* - \gamma_1^*\, s(i)]$ with the following properties:*

(i) If $K'(0) \geq \frac{\psi}{2\tau^2}$, then each firm $i \in I$ acquires the same level of precision $\tau_u^* = 0$.

(ii) If $K'(0) < \frac{\psi}{2\tau^2}$, then each firm $i \in I$ acquires the same level of precision $\tau_u^* > 0$. τ_u^* is the

[1] A usual question is whether there exist nonlinear equilibria besides this unique linear equilibrium. At least when supply schedules are restricted in an appropriate way such that they have bounded means and bounded variances, this is not the case. Vives (1993) provides a proof of this for a generic stage of his dynamic model that can easily be adapted to our model.

solution of the equation:

$$\sqrt{\frac{2K'(\tau_u^*)}{\psi}}\left[\frac{2K'(\tau_u^*)}{\psi}\tau_u^{*2}\,\alpha^2\tau_\varepsilon+\tau+\tau_u^*\right]=1,$$

(iii) The coefficients γ_0^*, γ_1^* and γ_2^* are given by:

$$\gamma_0^*=\frac{\beta\alpha\gamma_1^*\tau_\varepsilon+\tau\bar{\theta}-\alpha^2\gamma_1^*\gamma_0^*\tau_\varepsilon}{\tau+\tau_u+\alpha^2\gamma_1^{*2}\tau_\varepsilon},\quad \gamma_1^*=\tau_u^*\sqrt{\frac{2K'(\tau_u^*)}{\psi}},\quad \gamma_2^*=-\frac{\gamma_1^*\alpha(1-\alpha(1-\gamma_2^*))\tau_\varepsilon}{\tau+\tau_u+\alpha^2\gamma_1^{*2}\tau_\varepsilon}.$$

Proof. See Appendix. □

A rational expectations equilibrium where the firms acquire a positive amount of private information exists, as conditions (i) and (ii) make clear, only if marginal costs of information acquisition at zero (i.e. $K'(0)$), fall short of respective marginal returns of information acquisition, which are at zero equal to $\frac{\psi}{2\tau^2}$. Since we are interested in equilibria where the current market price aggregates and reveals dispersed private information, we confine the following analysis to the case where the condition $K'(0)<\frac{\psi}{2\tau^2}$ is satisfied such that a rational expectations equilibrium with $\tau_u^*>0$ exists.

Informativeness of prices

In the above described rational expectations equilibrium, firms are able to learn from current market prices the information regarding the unknown parameter θ because the equilibrium price aggregates the dispersed private information acquired by individual firms. A natural question therefore is how the informational content of market prices can be assessed.

The aggregation of information through the market price can be illustrated by substitution of the aggregated equilibrium supply schedules into the inverse demand function. Because $\int_0^1 s(i)\,di=\theta$, this yields:

$$p=\frac{\beta-\alpha\gamma_0^*-\alpha\gamma_1^*\theta+\varepsilon}{1-\alpha(1-\gamma_2^*)}$$

Thus, p is a noisy observation of the unknown θ. In fact, the observation of the market price p is equivalent to the observation of a random variable ω defined as

$$\omega\equiv\frac{p[1-\alpha(1-\gamma_2^*)]-\beta+\alpha\gamma_0^*}{\alpha\gamma_1^*}=\theta+\varepsilon\frac{1}{\alpha\gamma_1^*}$$

The variance $(\alpha\gamma_1^*)^{-2}\tau_\varepsilon^{-1}$ of the noise term in ω can be regarded as a measure of the precision of the market price. In what follows, we use this variance to measure the informational content of the market price and we accordingly define the informativeness or precision of the equilibrium market price τ_p^* as follows: [2]

$$\tau_p^* = \alpha^2 \gamma_1^{*2} \tau_\varepsilon \qquad (2)$$

According to (2) informativeness of the price depends on the endogenously determined weight γ_1^* which is given to private information in the firms' decisions and increases as firms put more weight on their private information. From Proposition 1 we get that this weight is an increasing function of the — also endogenously determined — precision of the private signals and we will discuss here two results regarding the dependence of the informativeness of the equilibrium price on exogenous parameters that might become useful later. First, an increase in the precision of public (a priori) information τ about θ decreases the equilibrium amount of information precision τ_u^*, firms will acquire. As a consequence, informativeness of prices decreases too, i.e. $\frac{\partial \tau_p^*}{\partial \tau} < 0$.[3] Second, an increase in the precision τ_ε of the noise in market demand, also decreases the equilibrium amount of information precision τ_u^*.[4] This result forms the basis of the famous Grossman–Stiglitz–Paradox (cf. Grossman and Stiglitz (1980)) which we will discuss later in more detail: As τ_ε increases, prices become more informative *ceteris paribus* such that incentives for private accumulation of information vanish. While it is true in the present model, that τ_u^* decreases as τ_ε decreases, the informativeness of the equilibrium price is still increasing in τ_ε, i.e. the overall effect is $\frac{\partial \tau_p^*}{\partial \tau_\varepsilon} > 0$.[5] Thus, as in Verrecchia (1982) in our model the positive direct effect of τ_ε on the informativeness of the price offsets the indirect negative effect via τ_u^* and γ_1^* on τ_p^*.

3. STRONGLY RATIONAL EXPECTATIONS EQUILIBRIA

A short description of the concept

Since detailed descriptions of of the concept of a SREE are already available in the literature (cf. Guesnerie (2002)), it is adequate to limit the present analysis to an informal and pragmatic

[2] Thus, the inverse of precision of the market price $(\tau_p^*)^{-1}$ is equivalent to the difference of the variances $\mathrm{Var}(\theta \,|\, p) - \mathrm{Var}(\theta)$.

[3] This follows immediately from the respective derivative of condition (ii) in Proposition 1 with respect to τ and the fact that γ_1^* is strictly increasing in τ_u^*.

[4] This result too follows immediately from the respective derivative of condition (ii) in Proposition 1 with respect to τ_ε.

[5] Without specifying the cost function $K(\tau_u)$ in more detail, this can be shown using Proposition 1 and some algebra. In section 4. this result is derived explicitly for the special case of constant marginal costs, i.e. $K''(\tau_u) = 0$.

treatment of this concept and the game–theoretical issues that are involved here. The fundamental question associated with the concept of a SREE is, how agents in a model end up in a rational expectations equilibrium, assuming nothing more than common knowledge of the model's structure and individual rationality. Usually, these two hypotheses are not sufficient to predict a unique outcome. While the set of outcomes predicted by the two hypotheses of common knowledge and individual rationality, i.e. the set of 'rationalizable' solutions, includes the rational expectations equilibrium, but it typically includes other outcomes as well.[6]

Still, under some conditions, the set of rationalizable solutions reduces to one element, and the unique outcome compatible with common knowledge and individual rationality is then the (unique) rational expectations equilibrium. Following Guesnerie (2002), we call the rational expectations equilibrium eductively stable, or a *strongly rational expectations equilibrium (SREE)* if it is the unique element of the set of rationalizable solutions. In this case the rational expectations equilibrium can be justified as result of an eductive learning process or mental process of reasoning (that is, introspection) on the side of the agents in a model. In the end, eductive stability, or existence of a SREE, means that the particular REE is not subject to problems of expectational coordination, because a general belief that firms will choose supply schedules that are in a small neighborhood of the REE is self-enforcing. It is common knowledge then that rational firms will choose supply schedules in a neighborhood of the REE.[7]

The question addressed here is to find the conditions under which the rational expectations equilibrium described above is the unique rationalizable solution. In such a case, the rational expectations equilibrium is the only outcome surviving to a process of infinitely repeated elimination of strategies that are non best responses: At the first step, we eliminate decisions that are not rational (i.e. best response to none of others' possible behavior); at the second step, we eliminate decisions that are not best response to some rational decisions of others (i.e. we eliminate decisions not compatible with the fact that 'everyone knows that everyone is rational'); at the third step, we eliminate decisions that are not best response to some decisions of others that have gone through the second step (i.e. we eliminate decisions not compatible with the fact that 'everyone knows that everyone knows that everyone is rational'). Any further step is then analogously defined.

[6]Cf. Bernheim (1984) and Pearce (1984) for the game theoretical concept of rationalizability and especially Tan and Werlang (1988) for a more formal treatment of this topic.

[7]The expectational coordination problem that is 'solved' via eductive stability is, thus, different from the coordination problem arising in case of multiplicity of REE.

Conditions for existence of a SREE with endogenous acquisition of information

As the above discussion made clear, a more formal description of the process of elimination of non best responses requires to be more specific regarding the strategies or supply schedules used by individual firms. We therefore first restrict the set of possible supply schedules used by the firms in order to be able to derive the best response dynamics.

Assumption 1 *It is common knowledge among the firms that each firm's supply schedule is an affine function of her private signal s_i and the market price p.*

More precisely, we will assume that an individual firm's supply is given by the linear function $x(i) = \psi[(1-\gamma(i)_2)\,p - \gamma(i)_0 - \gamma(i)_1\,s(i)]$ where the weights $\gamma(i)_0$, $\gamma(i)_1$ and $\gamma(i)_2$ are real numbers for all $i \in I$. This linearity assumption simplifies the following analysis considerably, because it implies that any price a firm observes is — like the unknown θ and the private signal s_i — a normally distributed random variable. Since the rational expectations equilibrium itself is characterized by linear supply schedules, the restriction imposed by Assumption 1 may seem not overly restrictive.

The respective best response mapping is then as summarized in the following Lemma:

Lemma 1 *Let $\gamma_0 = \int_0^1 \gamma(j)_0\,dj$, $\gamma_1 = \int_0^1 \gamma(j)_1\,dj$ and $\gamma_2 = \int_0^1 \gamma(j)_2\,dj$.*[8] *Aggregate demand is then*

$$\int_0^1 x(j)dj = \psi[(1-\gamma_2)p - \gamma_0 - \gamma_1\theta].$$

(so that aggregate behavior is summarized by the coefficients γ_0, γ_1 and γ_2). Then, the best response of a firm $i \in I$ to $(\gamma_0, \gamma_1, \gamma_2)$ is characterized by the coefficients $(\gamma_0'(i), \gamma_1'(i), \gamma_2'(i), \tau_u'(i))$ defined by:

$$\gamma'(i)_0 = -\frac{\alpha\gamma_1\,\tau_\varepsilon(\beta+\alpha\gamma_0)}{\tau+\tau_u'(i)+\alpha^2\gamma_1^2\,\tau_\varepsilon} \tag{3}$$

$$\gamma'(i)_1 = \frac{\tau_u'(i)}{\tau+\tau_u'(i)+\alpha^2\gamma_1^2\,\tau_\varepsilon} \tag{4}$$

$$\gamma'(i)_2 = \frac{\gamma_1\,\alpha\,(1+\alpha(1-\gamma_2))\tau_\varepsilon}{\tau+\tau_u'(i)+\alpha^2\gamma_1^2\,\tau_\varepsilon} \tag{5}$$

where $\tau_u'(i) = 0$ if $K'(0) > \frac{\psi}{2(\tau+\alpha^2\gamma_1^2\tau_\varepsilon)^2}$ and $\tau_u'(i)$ is the unique solution of

$$\frac{\psi}{2}\frac{1}{[\tau+\tau(i)_u+\alpha^2\gamma_1^2\tau_\varepsilon]^2} = K'(\tau(i)_u) \tag{6}$$

otherwise.

[8] All the measurability assumptions required are made. In particular, we assume that $\int_0^1 \gamma(j)_0\,dj$, $\int_0^1 \gamma(j)_1\,dj$ and $\int_0^1 \gamma(j)_2\,dj$ exist.

Proof. See Appendix. □

We are now able to explain in greater details the eductive process. This process is defined by means of the best response mapping defined in Lemma 1. For this, let $z(i) = \mathcal{T}(z)$ denote the best response mapping. That is, the best response of a firm $i \in I$ to an aggregate behavior $z = (\gamma_0, \gamma_1, \gamma_2, \tau_u)$ is $z(i) = \mathcal{T}(z)$ where $z(i) \equiv (\gamma(i)_0, \gamma(i)_1, \gamma(i)_2, \tau(i)_u)$ denotes the strategy of a single firm i.[9] Clearly, the REE $z^* = (\gamma_0^*, \gamma_1^*, \gamma_2^*, \tau_u^*)$ is the fixed point of this best response mapping, i.e. $z^* = \mathcal{T}(z^*)$.

Assume that the firm's strategies $z(i)$ are elements of a set $W_n \subset \mathbb{R}^3 \times \mathbb{R}_+$, i.e. assume that it is common knowledge that $z(i)_n \in W_n$ for all $i \in I$. Starting from this assumption, the eductive process then proceeds as follows: Since it is common knowledge that every firm plays a strategy from W_n, every firm knows that the resulting aggregate supply is in the convex hull $\mathcal{W}_n = \text{conv}(W_n)$ of W_n. Now, the best response to this aggregate behavior is $\mathcal{T}(\mathcal{W}_n)$ and, therefore, every firm knows that aggregate behavior in the next round of this process is in the set $\mathcal{W}_{n+1} = \mathcal{W}_n \cap \text{conv}\,(\mathcal{T}(\mathcal{W}_n))$. If the iteration of this kind of argument results in z^* as the only possible aggregate behavior, the REE is a SREE: Every firm knows that every firm knows [...] that every firm expects aggregate behavior to be z^* and, therefore, reacts playing the equilibrium strategy z^*.

From the above discussion, it is obvious that the REE is locally strongly rational if the map defined by equations (3)–(6) is contracting at the REE values $z^* = \left(\gamma_0^*, \gamma_1^*, \gamma_2^*, \tau_u^*\right)$. The next technical Lemma states the respective conditions for existence of a locally SREE:

Lemma 2 *Let η denote the elasticity of marginal costs of information acquisition with respect to τ_u (i.e. $\eta = K''(\tau_u)\tau_u / K'(\tau_u)$). The REE is a locally SREE if and only if*

$$\tau_p^* < \tau_u^* \tag{C.I}$$

and

$$\tau_p^* < \tau_u^* \left(\frac{\eta^* + 2}{\eta^* + 4}\right) + \tau \left(\frac{\eta^*}{\eta^* + 4}\right) \tag{C.II}$$

where τ_p^* is given by (2) *and η^* denotes elasticity of marginal costs at the REE.*

Proof. See Appendix. □

In Lemma 2, the respective stability condition is stated in a form which makes explicit the importance of the informativeness of the equilibrium market price for stability: Both conditions

[9]Notice from Lemma 1 that $\mathcal{T}$ is constant with respect to τ_u. Consideration of τ_u as an element of z serves only notational purposes.

imply that informativeness of the price in a REE must be bounded from above in a certain way in order for a SREE to exist. Condition (C.I) is exactly the condition derived by Heinemann (2004) for existence of SREE in the same model with exogenously given private information. In fact, (C.I) is the condition for local stability of the best response dynamics associated with equations (3) – (5) only, that is when the precision of private information is exogenously fixed. This condition says that a strongly rational expectations equilibrium in the model with learning form prices and given private information exists if and only if prices are less informative regarding the unknown θ than private signals. Condition (C.II) in Lemma 2 now shows that endogenous acquisition of information might lead to stronger conditions for existence of a SREE. However, before we go on to analyze under what circumstances endogenous acquisition of information in fact leads to stronger conditions for existence of a SREE, it is worthwhile to discuss condition (C.I) for the case of exogenous information.

SREE with exogenously given private information

Why is a high precision of the equilibrium market price harmful for stability of the best response dynamics in case exogenously given private information? The reason is that the individual firm's reaction (in terms of the weights of her supply schedule) to actions taken by other firms turns out to be too strong if condition (C.I) is not satisfied. As a consequence, the best response mapping becomes unstable and the above described eductive process will not converge towards the REE. such that no SREE exists. An intuitive explanation for this result is that a relatively high informativeness of the market price means that it is quite important for the firms to extract information regarding the unknown θ from the market price. However, extraction of information from the market price requires a precise idea of the other firms' behavior, that is an idea how the other firms use their private information as well as the information contained in the market price p. The underlying problem is identical to the well known problem of 'forecasting the forecasts of others' that is described by Keynes (1936) in his famous 'beauty contest' example. If the informativeness of the price is too high, firms have an incentive to learn excessively from the price which makes supply very sensitive to the information that is supposed to be contained in the price. This, however, implies that the actual correlation between the market price and the unknown parameter is very sensitive to the firms' beliefs which in turn makes it hard to asses the information contained in the price. Thus, without further assumptions that go beyond that of individual rationality and common knowledge, it can hardly be expected that an individual firm is able to solve such a problem in any definite way. Therefore, expectational coordination, that is coordination of the individual supply schedules on the REE, is difficult if the market price reveals too much information regarding the unknown cost parameter θ. If, on the other hand, the market price is not very informative, that is, if τ_p^* is low, it is not quite important for the individual firm to anticipate correctly what the other firms believe and do and therefore the rational expectations equilibrium is likely to be strongly rational. In this case, every firm acts nearly autonomous, with decisions based almost exclusively on private signals.

Interpreted in this way, the condition (C.I) is quite similar to that derived by Guesnerie (1992) for the the classical Muth (1961) model and also confirms a result by Desgranges et al. (2003) also obtained within the context of a model with private information but only a finite number of states and signals. They also conclude that the coordination of expectations becomes difficult, if public information becomes too informative.

One problem associated with the above stated condition (C.I) is that the informativeness of the market price τ_p^* is endogenous to the model. This implicates that it is not easy to verify this condition even if the precision of private information is assumed to be exogenously given. The Corollary below states this condition for existence of a SREE in terms of exogenous parameters only and thereby makes explicit the fact that the effect of the precision of private information τ_u on τ_p^* turns out to be non-monotonic:

Corollary 1 *With exogenously given private information τ_u, a REE always implies $\tau_p^* < \tau_u$, if $\alpha^2\tau_\varepsilon < 8\tau$. Otherwise, a REE implies $\tau_p^* < \tau_u$ if and only if*

$$\tau_u < \frac{\alpha^2\tau_\varepsilon - 4\tau - \sqrt{\alpha^2\tau_\varepsilon\,(\alpha^2\tau_\varepsilon - 8\tau)}}{8} \quad \text{or} \quad \tau_u > \frac{\alpha^2\tau_\varepsilon - 4\tau + \sqrt{\alpha^2\tau_\varepsilon\,(\alpha^2\tau_\varepsilon - 8\tau)}}{8}$$

Proof. See Appendix. □

The non-monotonic effect of τ_u is analogous to the one exhibited in Desgranges (1999) in financial market model à la Grossman (1976). It is due to the fact that the informativeness of the price τ_p^* is increasing but not linear in τ_u. According to Corollary 1 it depends on the other parameters of the model (i.e. α, τ_ε as well as τ) whether the exogenously given amount of private information τ_u is in fact relevant for existence of a SREE. If this is the case, however, it turns out that a SREE exists only for intermediate values of the precision of private information. The fact that a SREE exists for low values of τ_u is easily understood since a low τ_u means that there is not much to be learned from prices such that the above described coordination problem doesn't show up. The same is true in case of a high τ_u because τ_p^* is bounded from above by $\alpha^2\tau_\varepsilon$.[10] Hence, even though a high τ_u implies that prices are highly informative too, prices might be less informative than private signals such that, given the comparatively high precision of the private signals, it is not important for the firms to learn from prices.

[10]From $H(\gamma_1^*) = \tau_u$ it follows that $\gamma_1^* \to 1$ as $\tau_u \to \infty$.

Stronger conditions with endogenous private information

Let us now turn to the question whether endogenous acquisition of information in fact leads to stronger conditions for existence of a SREE. Endogeneity of information makes existence of a SREE more requiring whenever condition (C.I) is already implied by the second condition (C.II). Some algebra shows that this is the case whenever the following inequality holds

$$\eta^* < \frac{2\tau_u^*}{\tau} \tag{7}$$

Both sides of condition (7) contain endogenous variables and, thus, interpretation of this condition is delicate. According to this inequality, condition (C.II) is stronger than (C.I), if the elasticity of marginal costs of information acquisition η^* at the REE falls short of a certain, also endogenously determined upper bound. Still, this inequality mainly confirms the above given intuitive reason for possible coordination problems associated with a REE. If prices are very informative, it is important for the individual firm to figure out what other firms believe and do in order to extract valuable information from prices. Since this problem cannot be solved, no SREE exists if prices are too informative. In case of endogenous information, however, the individual firm's reaction to a highly informative price is not only to learn excessively from the price but also to acquire less private information. This then reinforces the underlying stability problem and in some cases results in a stronger condition for stability of the eductive process. The above inequality mainly says that condition (C.II) is stronger than (C.I), if η^* is low, which means that it is not very costly to adjust $\tau_u(i)$ for firm i. For this reason, the properties of the cost function associated with the acquisition of private information are relevant for stability.

One special case (a case we will discuss in some detail below) is the case of constant marginal costs of information acquisition $\eta^* = 0$.[11] Because the right hand side of (7) is always positive, this condition always holds if marginal costs are constant. In such a case (C.II) reduces to the condition $\tau_p^* < \frac{1}{2}\tau_u^*$, i.e. the precision of prices must be lower than half the precision of the private signals in order for a SREE to exist. Thus, in case of constant marginal costs, endogeneity of information acquisition definitely results in stronger conditions for existence of a SREE.

The next Proposition summarizes our results regarding existence of SREE with endogenous acquisition of information and again highlights the role of the informativeness of the market

[11] For instance, marginal costs are constant, if we assume that the private signals s_i are outcomes of individual sampling processes where firms make observations of the unknown θ plus some noise term with zero mean and constant variance. If, in addition, we assume that costs per observation are constant, marginal costs of information acquisition will be constant too.

price in this respect:

Proposition 2

(i) If $\eta^* \geq \frac{2\tau_u^*}{\tau}$, a necessary and sufficient condition for existence of a SREE is $\tau_p^* < \tau_u^*$.

(ii) If $\eta^* < \frac{2\tau_u^*}{\tau}$, a SREE exists if and only if condition (C.II) *is satisfied. $\tau_p^* < \tau_u^*$ is still a necessary condition, while a sufficient condition for existence of a SREE is $\tau_p^* < \frac{1}{2}\tau_u$.*

(iii) If $\eta^* = 0$, $\tau_p^* < \frac{1}{2}\tau_u$ is a necessary and sufficient condition for existence of a SREE.

Proof. See Appendix. □

Thus, in case of constant marginal costs of information acquisition the condition for existence of a SREE is definitely stronger than the respective condition for existence of a SREE with exogenously given information. Informativeness of the equilibrium price must be below a definite upper bound given by one half of the precision of the private signals. In the general case with increasing marginal costs, endogenous acquisition of information not necessarily leads to stronger conditions for existence of a SREE, but still the precision of the price must be bounded from above. The elasticity of marginal costs of information acquisition then determines the exact magnitude this upper bound. In any case, informativeness of the equilibrium price must be lower than the informativeness of private signals in order for a SREE to exist.

The case of constant marginal costs

The conditions for existence of a SREE summarized in Proposition 2 are based on local stability of the best response mapping. While these conditions guarantee that an eductive process that starts in a neighborhood of the REE actually converges towards this equilibrium, the question of global stability of the eductive process is left unanswered. In the remainder of the paper we will focus our analysis on the special case of constant marginal costs of information acquisition. This special case not only allows for some illustrations of the implications of the above derived condition for existence of a SREE. In addition, we are able to describe the whole set of rationalizable outcomes (that is the set of outcomes compatible with common knowledge of rationality and model) and show that the above derived condition implies global stability of the eductive process in this case.

The assumption of constant marginal costs enables us to analyze the essentials of the eductive process with the help of a single equation which describes the best response dynamics of τ_u only. This is possible for two reasons: First, equations (4) and (6) of the best response mapping stated in Lemma 1 are independent from γ_0 and γ_2. Second, condition (C.II), which is necessary and sufficient for existence of a SREE in case of constant marginal costs is exactly the condition for stability of this subsystem. The two equations (4) and (6) now simplify to a single equation as stated in the following Lemma.

Lemma 3 *Consider the case with constant marginal costs and denote* $Q \equiv \sqrt{\frac{\psi}{2K'}}$.

(i) If the 'amount of information in the market' *is* $\tau_u = \int_0^1 \tau_u(j)dj$ *(that is the average precision of information), then the optimal information precision of firm* $i \in I$ *is characterized by:*

$$\tau'(i)_u = T(\tau_u) = \max\left\{0, Q - \tau - \frac{\alpha^2 \tau_\varepsilon}{Q^2}\tau_u^2\right\} \tag{8}$$

(ii) The set of rationalizable information precisions is the limit set $T^\infty(\mathbb{R}_+)$*, i.e. the limit of the sequence* $\tau_{u,n}$ *defined by* $\tau_{u,n+1} = T(\tau_{u,n})$ *for all starting values* $\tau_{u,0} \in \mathbb{R}_+$.

Proof. See Appendix. □

A precise proof is given in the Appendix. The main idea is to substitute equation (4) into (6) and to use the fact these two equations also imply $\gamma_1^2 = \frac{2K'}{\psi}\tau_u^2$, which can then be used to eliminate γ_1. Equation (8) describes the best response dynamics for the endogenously acquired amount of private information (the non-negativity constraint $\tau(i)_u \geq 0$ is taken into account). According to Proposition 2, a SREE exists if and only if $\tau_p^* < \frac{1}{2}\tau_u^*$. Some computations show that $\tau_p^* < \frac{1}{2}\tau_u^*$ rewrites $\frac{3}{4}\frac{Q^2}{\alpha^2\tau_\varepsilon} < Q - \tau$. That is, in case of constant marginal costs we are able to state the condition for existence of a SREE in terms of exogenous variables only.

Lemma 3 states that the set of rationalizable information precisions coincides with the limit set of the best response dynamics associated with (8). The following Proposition 3 describes this set in more detail.

Proposition 3 *Consider the case with constant marginal costs of information acquisition (i.e.* $\eta = 0$*). Let* S^* *denote the set of rationalizable precisions which therefore represent outcomes of an eductive learning process on the side of the firms (formally,* $S^* = T^\infty(\mathbb{R}_+)$*):*

(a) If a SREE exists (that is $\frac{3}{4}\frac{Q^2}{\alpha^2\tau_\varepsilon} < Q - \tau$*),* $S^* = \{\tau_u^*\}$*, i.e.,* τ_u^* *is the unique and globally stable fixed point of the mapping* $\tau_u' = T(\tau_u)$.

(b) If no SREE exits, one of the following two cases applies:

(b.1) $\frac{3}{4}\frac{Q^2}{\alpha^2\tau_\varepsilon} \leq Q - \tau < \frac{Q^2}{\alpha^2\tau_\varepsilon}$ *such that* $S^* = [\underline{\tau}_u, \overline{\tau}_u]$*, where* $\underline{\tau}_u$ *and* $\overline{\tau}_u$ *satisfy* $0 < \underline{\tau}_u < \tau_u^* < \overline{\tau}_u < Q - \tau$.

(b.2) $Q-\tau \geq \frac{Q^2}{\alpha^2 \tau_\varepsilon}$ *such that* $S^* = T(I\!R_+) = [0, Q-\tau]$.

Proof. See Appendix. $\square$

Point (a) in Proposition 3 states that, whenever a REE is a locally SREE, the REE precision τ_u^* is the globally unique rationalizable precision of private information. Without going into the very details here, we note that this result implies global stability of the whole eductive process in case of constant marginal costs: Since equations (3) and (5) of the best response dynamics are linear in γ_0 and γ_2, respectively, global stability of the subsystem (4) and (6) implies global stability of the whole system. Thus, if the informativeness of the price is less than one half of the informativeness of the private signals at the REE, this equilibrium is a globally SREE. The remaining two cases (b.1) and (b.2) characterize the set of rationalizable precisions that result if no SREE exists, i.e. if $\tau_p^* \geq \frac{1}{2}\tau_u^*$. While the two hypotheses of common knowledge and individual rationality are not sufficient to predict the REE as the unique rationalizable outcome of the model in this case, these two assumptions still lead to some restrictions on the set of rationalizable precisions of private information (and corresponding restrictions on the remaining parameters of the firms' supply schedules).

We now illustrate the three cases in Proposition 3 and the properties of the best response mapping (8) with three examples, bearing in mind that the 'amount of information in market' τ_u is necessarily non–negative and that $Q-\tau$ represents the maximum precision ever acquired (i.e. $Q-\tau = \sup_{\tau_u \geq 0} T(\tau_u)$). Thus, we can restrict the formal analysis of the best response dynamics described by the mapping $T(\tau_u)$ to the set $S = T(I\!R_+) = [0, Q-\tau]$ without loss of generality.

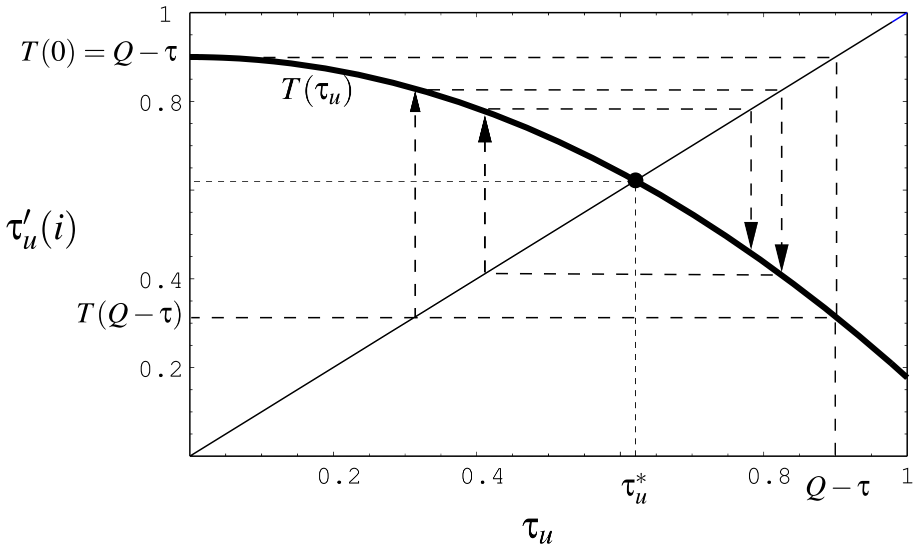
Example 1 (illustrating case (a)): Consider a numerically specified version of the model where $\alpha = -0.85$, $\psi = 1$, $\tau = 0.1$, $\tau_\varepsilon = 1$ and $\bar{\kappa} = 0.5$. From equation (4) and (6), equilibrium values can be computed as: γ_1^* =0.621, $\tau_u^* = 0.621$ and $\tau_p^* = 0.279$. A SREE exists both when the amount of private information is exogenously given and equal to τ_u^*, and when it is endogenous (the two conditions $\tau_p^* < \tau_u^*$ and $\tau_p^* < \frac{1}{2}\tau_u^*$ in Proposition 2 are both satisfied).[12] Thus, in this example, the fact that information acquisition is endogenously determined is not relevant for existence of a SREE.

Figure 1 shows how the function $T(\tau_u)$ looks like. The eductive process proceeds similarly to the well known cobweb–dynamics.[13] The first step of the process is to consider that $\tau_u \geq 0$ is common knowledge. Given that T is decreasing, this fact implies that the maximum amount

[12]In fact, in this example we have $\alpha^2 \tau_\varepsilon < 8\tau$ and according to Corollary 1 in case of exogenous information a SREE exists irrespectively of the precision of private information.

[13]This description of the process originates in Guesnerie (1992).

Figure 1: *Best response mapping $T(\tau_u)$ for example 1*



of private information a firm will ever acquire is given by $T(0) = Q - \tau > 0$. Since T and rationality are common knowledge, it is therefore also common knowledge that $\tau_u \leq T(0)$. A further step of the process shows then that no firm will ever choose $\tau(i)_u < T(T(0)) = T(Q-\tau)$. Thus, this second step restricts the set of possible precision to $[T(T(0)), T(0)]$. As indicated in the figure, the dynamics that result if this kind of reasoning is iterated converges to the REE precision τ^*_u (because the condition stated in Proposition 2 is satisfied): each firm can educe that only the precision REE $\tau^*_u = 0.621$ constitutes a possible solution under the assumptions of common knowledge of individual rationality and model.

Example 2 (illustrating case (b.1)): The precision of the noise is now $\tau_\varepsilon = 1.3$, which is larger than in example 1. From equations (4) and (6), equilibrium values can be computed as $\gamma^*_1 = 0.582$, $\tau^*_u = 0.582$ and $\tau^*_p = 0.318$. We have then $\frac{1}{2}\tau^*_u < \tau^*_p < \tau^*_u$: A SREE exists if the amount τ^*_u of private information is exogenously given, but does not exist if information is endogenously acquired.

On figure 2, we have now also plotted the function $T(\tau_u)$ and the second iterate of this function $T^2(\tau_u) \equiv T(T(\tau_u))$. As can be seen, this function possesses two additional fixed points, denoted $\underline{\tau}_u$ and $\overline{\tau}_u$. Notice too that the associated 2–cycle is stable. If we repeat the argumentation used in the discussion of the first example, we therefore get a process which converges to this 2–cycle: the first step of the process shows that $\tau_u \leq T(0) = Q - \tau$, a second step shows that $\tau_u \geq T(Q-\tau) = T^2(0)$. Clearly, iterating this argument eliminate the precisions outside the interval $[\underline{\tau}_u, \overline{\tau}_u]$, but not precisions in $[\underline{\tau}_u, \overline{\tau}_u]$. It follows that all precisions in the set $[\underline{\tau}_u, \overline{\tau}_u]$

Figure 2: *Best response mapping $T(\tau_u)$ for example 2*

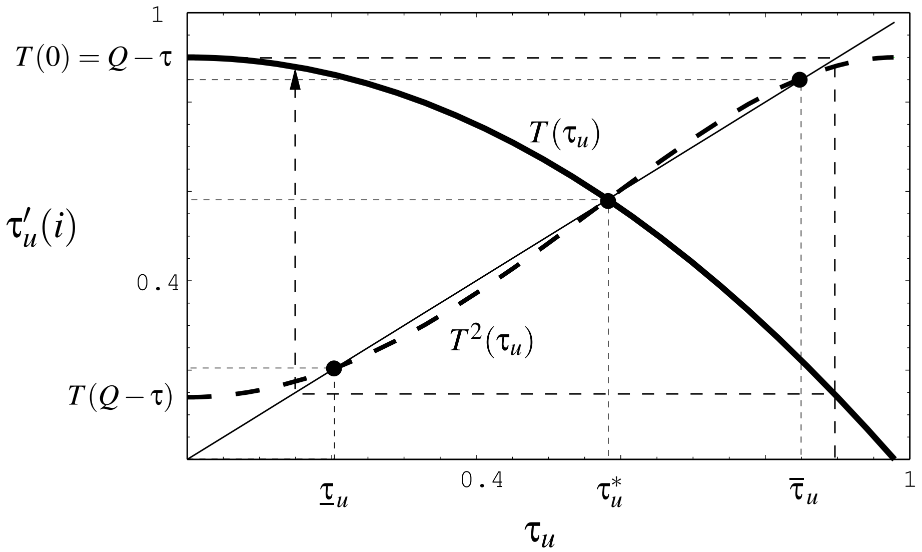
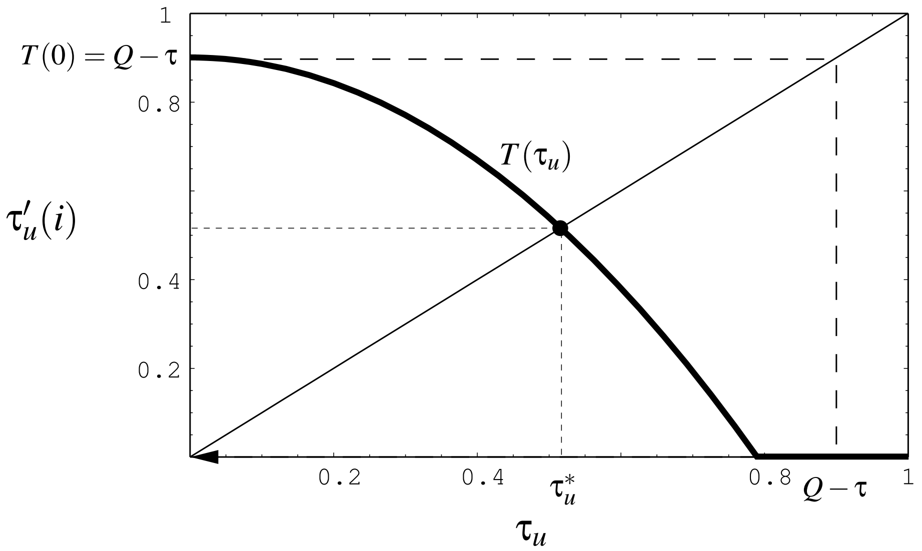


Figure 3: *Best response mapping $T(\tau_u)$ for example 3*



constitute possible solutions under individual rationality and common knowledge.

Example 3 (illustrating case (b.2)): The precision of noise is $\tau_\varepsilon = 2.0$ and, hence, larger than in examples 1 and 2. At the REE, $\tau_u^* = 0.512$ and $\tau_p^* = 0.384$. The REE is still strongly rational, if information precision τ_u^* is assumed to be exogenously given, but not (since $\tau_u^*/2 = 0.258$), when information acquisition is endogenous. The best response function $T(\tau_u)$ depicted in figure 3 reveals that in this example we have $T(Q - \tau) = 0$, i.e., now the non-negativity constraint on $\tau'(i)_u$ becomes relevant.

Again, we repeat the argumentation used in the discussion of the above examples. However, the process here immediately converges to the whole interval $[0, Q-\tau]$. Indeed, the first step of the process still shows that $\tau_u \leq T(0) = Q-\tau$. If, however, each firm acquires this maximum amount $T(0)$ of private information such that $\tau_u = T(0)$, there is so much information in the market, that it is individually optimal to stop the acquisition of information, i.e., $T(Q-\tau) = 0$. In other words, the second step of the process shows that $\tau_u \geq T(Q-\tau) = 0$. Thus, no additional restriction is created by this second step. Clearly, iterating this argument does not eliminate any precision: all the precisions in $[0, Q-\tau]$ constitute possible solutions under individual rationality and common knowledge.

4. Existence of a SREE and the Grossman–Stiglitz paradox

Existence of a SREE means that the two hypotheses of individual rationality and common knowledge are indeed sufficient to predict the REE as a reasonable outcome of the model. In such a case the fact that an eductive learning process converges to this equilibrium clearly provides a justification for the equilibrium concept of a REE. Nonexistence of a SREE, however, means that even though a REE exists, we should be sceptical regarding the justification of such an equilibrium because it is prone to expectational coordination difficulties. As the above derived results show, informativeness of the equilibrium price must be bounded from above in certain way, in order to prevent such coordination difficulties. This is especially true for the case with constant marginal costs, a case we again focus on in the following discussion. The aim is to link our above derived condition for existence of a SREE with the well known Grossman–Stiglitz paradox on the impossibility of informationally efficient markets. This paradox states that a REE with endogenous acquisition of information and a fully informative market price cannot exist because in such a case, no firm has an incentive to acquire costly the information, prices will reveal anyway. If, however, no firm acquires any information, prices cannot be informative. As a consequence, informativeness of prices in a REE must be bounded from above.

In our model, like in the model of Grossman and Stiglitz (1980), the presence of exogenous noise ε prevents the price from being fully informative regarding the unknown θ.[14] As the precision τ_ε of this noise increases, informativeness of the REE price increases too. However,

[14]Actually, our model differs in some respects from the model Grossman and Stiglitz (1980) use and is quite closer that used in Verrecchia (1982). The relevant conclusions, however, are the same.

since this in turn destroys individual incentives to acquire private information, informativeness of the REE price will be bounded from above. In case of constant marginal costs of information acquisition, this can formally be described with help of the following two equations, which determine the equilibrium precision of private information and equilibrium informativeness of the price:

$$\tau_u^* = Q - \tau - \frac{\alpha^2 \tau_\varepsilon}{Q^2} \tau_u^{*2} \tag{9}$$

$$\tau_p^* = Q - \tau - \tau_u^* \tag{10}$$

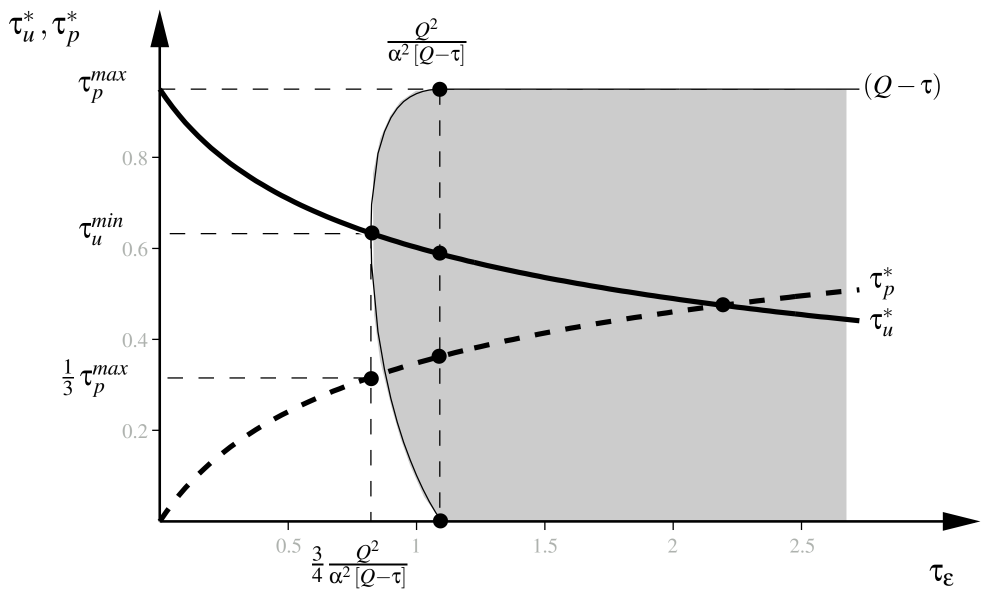
Here, equation (9) is simply the equilibrium version of (8) and (10) follows from (9) since Proposition 1 implies that in a REE $\gamma_1^* = \tau_u^*/Q$, such that τ_p^* is given by $\tau_p^* = \alpha^2 (\tau_u^*/Q)^2 \tau_\varepsilon$. From equation (9) we get that $\tau_u^* \to 0$ as $\tau_\varepsilon \to \infty$, that is as the precision of the noise approaches infinity, the precision of the acquired private information becomes smaller and smaller. This then results in an upper bound for the equilibrium informativeness τ_p^* of the market price: Even though an increase of the precision of the noise *ceteris paribus* leads to an increase of informativeness of the price, the decreasing precision of acquired information prevents informativeness of prices from increasing without any bound. From equation (10) and (9) it immediately follows that $\tau_p^* \to Q - \tau$ as $\tau_\varepsilon \to \infty$. Thus, informativeness of the REE price τ_p^* is bounded from above by $Q - \tau$, which is exogenous. This is the essence of the Grossman–Stiglitz paradox: REE with fully informative prices do not exist, because informativeness of prices in a REE must be bounded from above. Accordingly, we refer to the upper bound on the precision of prices $\tau_p^{max} \equiv Q - \tau$ as the Grossman–Stiglitz upper bound on informativeness of the market price in a REE.

Thus, within the framework of our model the Grossman–Stiglitz paradox states that no REE exists with informativeness of the price beyond the Grossman–Stiglitz upper bound τ_p^{max}. However, even if mere existence of a REE with informativeness of prices below this upper bound is no problem, these equilibria might be prone to expectational coordination difficulties because the REE is not eductively stable. The above derived condition for existence of a SREE is $\tau_p^* < \frac{1}{2}\tau_u^*$. Using (9), this condition rewrites

$$\tau_p^* < \frac{1}{3}[Q - \tau] = \frac{1}{3}\tau_p^{max}$$

Thus, in order for a SREE to exist, informativeness of prices has to be lower than one third of the Grossman-Stiglitz upper bound. All in all this means that while existence of a REE implies restrictions on the informativeness of the equilibrium price (i.e. the Grossman–Stiglitz upper

Figure 4: *The set of rationalizable strategies* ($\alpha = 0.98$, $K' = 0.5$, $\psi = 1.0$ *and* $\tau = 0.1$).



bound), the justification of such an equilibrium by means of an eductive learning process leads to even stronger restrictions. If we really want to justify such a REE only with the help of the two hypotheses of common knowledge and individual rationality, informational inefficiency of markets has to be far lower than indicated by the Grossman–Stiglitz upper bound on the informativeness of the market price.

Figure 4 serves to summarize the above discussion and, by the way, illustrates the implications of Proposition 3 from the preceeding section. Based on a numerical specification of the model, the figure shows the equilibrium precision of private information (the solid line) and the equilibrium informativeness of the market price (the dashed line) both dependent on the precision of the noise τ_ε. According to (9), τ_u^* decreases from its maximal value $Q-\tau$ towards zero as τ_ε approaches infinity, while τ_p^* increases from zero towards the Grossman–Stiglitz upper bound $\tau_p^{max} = Q-\tau$. In addition, the figure shows the set S^* of rationalizable precisions of private information τ_u as stated in Proposition 3 dependent on the precision of the noise τ_ε. As long as $\tau_\varepsilon < \frac{3}{4}\frac{Q^2}{\alpha^2[Q-\tau]}$, which is equivalent to $\tau_p^* < \tau_u^*/2$, a SREE exists. Thus, the corresponding part of the solid line also represents the set of rationalizable precisions, which coincides with the REE precision. If $\tau_\varepsilon \geq \frac{3}{4}\frac{Q^2}{\alpha^2[Q-\tau]}$, no SREE exists. The shaded area in the figure then represents all precisions that are rationalizable in this case. As can be seen, whenever the precision of prices is not too large, i.e. if $\tau_\varepsilon < \frac{Q^2}{\alpha^2[Q-\tau]}$ and case (b.1) of Proposition 3 arises, the best response dynamics exhibit a two–cycle and the assumptions of individual rationality and common knowledge lead at least to some restrictions on the set of rationalizable precisions. Even this is impossible when the price becomes too informative, i.e. if $\tau_\varepsilon \geq \frac{Q^2}{\alpha^2 [Q-\tau]}$ in which case case (b.2) of Proposition 3 arises. All in all, the figures illustrates that our results regarding existence of SREE mean that the fundamental problem described by the Grossman–Stiglitz paradox is not exclusively related to the question of existence of fully informative REE. Even REE where prices are only partially informative are subject to coordination difficulties and it might therefore be impossible to justify these equilibria with the assumptions of individual rationality and common knowledge. It is not enough to have prices which are not fully informative in order to get rid of these problems. Instead, the informativeness of prices must be below a well defined upper bound in order to achieve this.

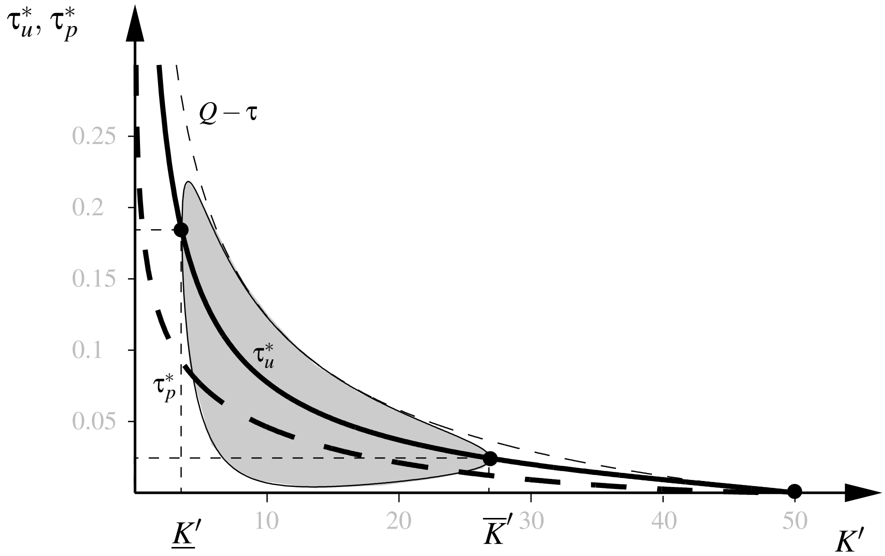
5. COMPARATIVE–STATICS: PUBLIC (A PRIORI) INFORMATION AND COSTS OF INFORMATION ACQUISITION

In this final section we want two discuss how the precision of public (a priori) information τ and the level of marginal costs of information acquisition K' impinge on the existence of a SREE. We look at these two factors because the respective comparative static results allows to derive tentative conclusions how public policy might overcome the potential expectational coordination difficulties associated with a REE. For instance, dissemination of information regarding θ through the government would lead to an increase of the precision of public (a priori) information τ. Thus, we might ask whether or not such a disclosure can serve to eliminate existing expectational coordination difficulties. Similarly, governmental policy can affect the costs of private information acquisition via taxes and subsidies, which might influence the marginal costs of information acquisition K'. We then might again ask whether public policy that aims at modifying the private acquisition of information can serve to eliminate existing expectational coordination difficulties.

Again, our analysis is restricted to the case where marginal costs of information acquisition are constant such that $K''(\tau_u) = 0$. We start with a comparative static result concerning the marginal costs of information acquisition K', which in a certain way is an analog to the result presented in Corollary 1 for the case with exogenously given private information.

Proposition 4 *If $\alpha^2 \tau_\varepsilon < 3\tau$, a SREE exists for all levels of marginal costs $K' < \frac{\psi}{2\tau^2}$ which are compatible with $\tau_u^* > 0$. Otherwise, there exist upper and lower bounds ($\overline{K}'$ and $\underline{K}'$, respectively)*

Figure 5: *Eductive stability and marginal costs of information acquisition (*$\alpha = 0.98$*,* $\tau_\varepsilon = 0.4$*,* $\psi = 1.0$ *and* $\tau = 0.1$*)*



on marginal costs given by

$$\overline{K}' = \frac{9\,\psi}{8\left(\alpha^2\,\tau_\varepsilon + \sqrt{\alpha^2\,\tau_\varepsilon\,[\alpha^2\,\tau_\varepsilon - 3\,\tau]}\right)}$$

and

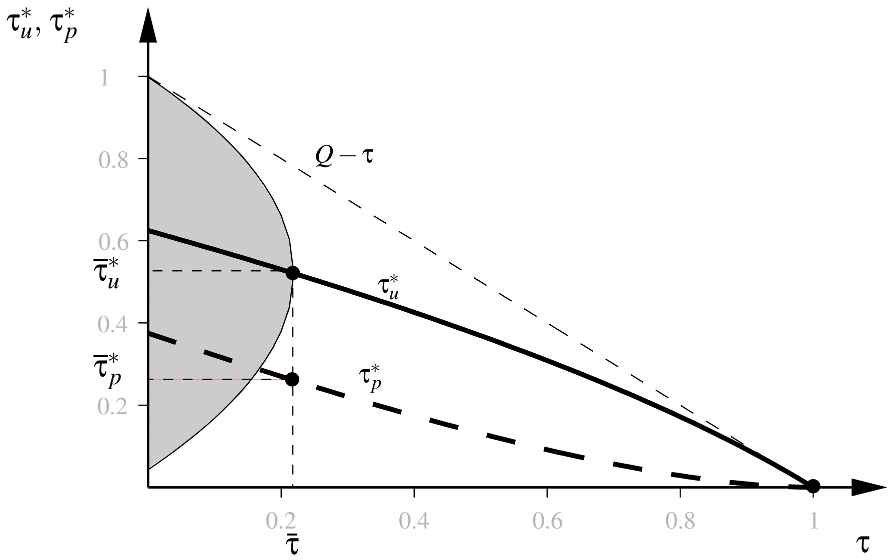
$$\underline{K}' = \frac{9\,\psi}{8\left(\alpha^2\,\tau_\varepsilon - \sqrt{\alpha^2\,\tau_\varepsilon\,[\alpha^2\,\tau_\varepsilon - 3\,\tau]}\right)}$$

such that a SREE exists whenever $K' \leq \underline{K}'$ *or* $K' \geq \overline{K}'$*.*

Proof. See Appendix. □

Thus, dependent on the other parameters of the model it might be that any change in the acquired precision of private information that is induced by changes in the level of marginal costs of information acquisition doesn't matter for existence of a SREE. If this is not the case, we end up with some kind of non monotonic effect of K' on existence of a SREE: A SREE exists for levels of marginal costs of information acquisition that are either low enough or high enough. The reason for this is the same as that given above in the explanation of Corollary 1. Figure 5 illustrates this result. Based on a numerical specification of the model, the figure shows the equilibrium precision of private information (the solid line) and the equilibrium informativeness of the market price τ_p^* both dependent on the level of marginal costs of information acquisition

Figure 6: *Eductive stability and precision of public information* ($\alpha = 0.98$, $\tau_\varepsilon = 1$, $\psi = 1$ *and* $K' = 0.5$)



K'. Note, that there is an upper bound on this level of marginal costs (in this case $\psi/2\tau = 50$) because no REE with endogenous acquisition of information exists if marginal costs are above this upper bound. The shaded area in the figure again depicts the set of rationalizable precisions of private information according to Proposition 3. As can be seen, existence of a SREE is favored by low or high marginal costs of information acquisition K'.

Let us now turn to the comparative–statics with respect to the precision of public (a priori) information τ. The effect this parameter on existence of a SREE turns out to be monotonic.

Proposition 5 *There exists a level of the precision of a priori information* $\bar{\tau} = Q - \frac{3}{4}\frac{Q^2}{\alpha^* \tau_\varepsilon}$ *such that a SREE exists whenever* $\tau > \bar{\tau}$.

Proof. See Appendix. $\square$

Depending on the other parameters of the model, the critical level of the precision of public information $\bar{\tau}$ may well be negative such that a SREE exists for all $\tau \geq 0$. In any case, there always exists a specific precision of public (a priori) information such that a SREE exists for whenever τ is above this level. Figure 6, which is again based on a numerical specification of the model, illustrates this result for the case of a positive critical level $\bar{\tau}$. The figure shows the equilibrium precision of private information (the solid line) and the equilibrium informativeness of the market price τ_p^* both dependent on the level of the precision of public (a priori) information τ. Note that τ is bounded from above by $Q = \psi/(2K') = 1$ because otherwise no REE with τ_u^* exists. The shaded area in the figure again depicts the set of rationalizable precisions of private information according to Proposition 3. As can be seen, existence of a SREE is favored by a high precision of public information τ. Under the presumption that the aim of public policy is to prevent expectational coordination difficulties, the conclusion therefore would be that acquisition and dissemination of information by the government is always helpful in this respect.

It might be interesting to confront this last result regarding the importance of public (a priori) information for existence of a SREE with results from the literature on global games (cf. Morris and Shin (2003)). In this literature, the impact of public information has been examined within the context of coordination games exhibiting strategic complementarities. It has been shown there, that in a setting where multiple equilibria would exist given common knowledge of fundamentals, there exists a unique equilibrium when there is private information which is sufficiently accurate (relative to public information). Furthermore, uniqueness of the equilibrium implies eductive stability. Thus, within this class of models, an increase in public information is harmful to eductive stability, which is in contrast with Proposition 5. However, it is important to take note of the fact that the present model contains two sources of public information: One the one hand there is public information which takes the form of a priori knowledge regarding the unknown θ, on the other hand there is public information in form of the market price p. This latter type of public information (information revealed through actions of agents) is absent from the models used in the global games literature. Our results unequivocally say that too much public information in form of the market price generates expectational coordination difficulties such that not SREE exist, while public information in form of a priori knowledge tends to attenuate such difficulties. Thus, regarding the importance of public information for coordination, our results are at best partially in line with results from the literature on global games.

6. Conclusions

In the present paper, we have shown how known results for existence of SREE must be modified, if models with endogenously acquired private information are considered. Generally, endogenous acquisition of information leads to stronger conditions for existence of a SREE than the respective conditions known for the case with exogenously given information. In particular, it was shown that prices in a REE always need to be less informative than private signals for a SREE to exist. In case of a relatively low elasticity of the marginal costs function associated with the acquisition of information with respect to the informativeness of that information, even this is not sufficient for existence of a SREE. In the limiting (and special) case of constant marginal costs and zero elasticity, informativeness of the price must be lower than one half the informativeness of the private signals. The reason is that endogenous acquisition of private information reinforces expectational coordination difficulties associated with a REE which disappear only if informativeness of prices is low enough.

Given that the conditions for existence of a SREE take the form of restrictions on informativeness of the market price, it is quite natural to look for a link between our results and the well known Grossman–Stiglitz paradox of the impossibility of informationally efficient markets. While the Grossman–Stiglitz paradox is concerned with the question of existence of a REE, our paper is concerned with the justification of an existing REE via eductive learning, that is based on the assumptions of individual rationality and common knowledge. If we regard the absence of possible expectational coordination difficulties as an important constraint to be respected when using the hypothesis of rational expectations, our results supplement the Grossman–Stiglitz paradox as follows: It is not only that mere existence of a REE necessitate a certain amount of informational inefficiency, but also the justification of such an REE based on individual rationality and common knowledge necessitates a specific amount of informational inefficiency. Furthermore, the amount of informational inefficiency required in order to avoid expectational coordination difficulties is generally greater than that required for existence of a REE.

Future work on this subject will analyze the case of increasing marginal costs of information acquisition in more detail in order to check the robustness of the results obtained for the case of constant marginal costs. Moreover, it should be analyzed whether the results carry over to financial market models with learning from current prices where risk aversion of traders is allowed for.

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APPENDIX

Proof of Proposition 1. We prove this Proposition using the best response mapping specified in Lemma 1 (the proof of this Lemma is given below):

$$\gamma'(i)_0 = -\frac{\alpha\gamma_1\tau_\varepsilon(\beta+\alpha\gamma_0)}{\tau+\tau_u'(i)+\alpha^2\gamma_1^2\tau_\varepsilon} \tag{3}$$

$$\gamma'(i)_1 = \frac{\tau_u'(i)}{\tau+\tau_u'(i)+\alpha^2\gamma_1^2\tau_\varepsilon} \tag{4}$$

$$\gamma'(i)_2 = \frac{\gamma_1\alpha(1+\alpha(1-\gamma_2))\tau_\varepsilon}{\tau+\tau_u'(i)+\alpha^2\gamma_1^2\tau_\varepsilon} \tag{5}$$

$$\frac{\psi}{2}\left(\frac{\gamma'(i)_1}{\tau_u'(i)}\right)^2 = K'(\tau_u'(i)) \tag{6}$$

A REE γ_0^*, γ_1^*, γ_2^*, τ_u^* is a fixed point of this best response mapping and we no show that there exists a unique fixed point. Combining equations (4) and (6) shows that τ_u^* and γ_1^* are the solutions to:

$$\gamma_1^* = \sqrt{\frac{2K'(\tau_u^*)}{\psi}}\tau_u^* \tag{A.1}$$

and

$$\sqrt{\frac{2K'(\tau_u^*)}{\psi}}\left[\frac{2K'(\tau_u^*)}{\psi}\tau_u^{*2}\alpha^2\tau_\varepsilon+\tau+\tau_u^*\right] = 1, \tag{A.2}$$

The LHS of the latter equation (A.2) is an increasing function of τ_u^* (increasing from $\sqrt{\frac{2K'(0)}{\psi}}\tau$ to $+\infty$ when τ_u increases from 0 to $+\infty$). This implies that there is a unique τ_u^* solving this equation. Given τ_u^*, there exists a unique positive solution γ_1^* to equation (A.1) and therefore unique solutions γ_0^* and γ_2^* of the two equations (3) and (5). □

Proof of Lemma 1. We first make precise the assumptions needed to define aggregate demand when information precisions are heterogenous among agents. Consider that the demand of every firm j is

$$x(j) = \psi[(1-\gamma(j)_2)p-\gamma(j)_0-\gamma(j)_1(\theta+u(j))],$$

with $\tau_u(j)$ the precision of $u(j)$ ($\theta+u(j)=s(j)$ is the private signal). Assume that $\int_0^1\gamma(j)_0dj$, $\int_0^1\gamma(j)_1dj$ and $\int_0^1\gamma(j)_2dj$ exist. Aggregate demand is then defined as:

$$\int_0^1 x(j)dj = \psi[(1-\gamma_2)p-\gamma_0-\gamma_1\theta-\int_0^1\gamma(j)_1u(j)dj].$$

Assume that $\int_0^1\gamma(j)_1u(j)dj=0$ almost surely (this is a law of large numbers: every $\gamma(j)_1u(j)$ is a centered Gaussian variable). It follows that aggregate demand is

$$\int_0^1 x(j)dj = \psi[(1-\gamma_2)p-\gamma_0-\gamma_1\theta],$$

and aggregate behavior is well characterized by $(\gamma_0,\gamma_1,\gamma_2)$.

Now, deriving the best response of a firm i to $(\gamma_0,\gamma_1,\gamma_2)$ is purely routine. Profit $\pi(i)$ of firm i is given by:

$$\pi(i) = [p-\theta]\,x(i) - \frac{1}{2}\frac{1}{\psi}\,[x(i)]^2 - K(\tau(i)_u)$$

Clearly, the profit maximizing output is $x(i) = \psi\,(p - E\,[\theta|p,s(i)])$. Given that $p = \beta - \frac{1}{\phi}\int_0^1 x(j)dj + \varepsilon$, we have

$$p = \frac{\beta + \alpha\,[\gamma_0+\gamma_1\theta] + \varepsilon}{1+\alpha(1-\gamma_2)}$$

and we have

$$E\,[\theta|p,s(i)] = \frac{\alpha^2\gamma_1^2\tau_\varepsilon\omega + \tau(i)_u s(i)}{\tau + \tau(i)_u + \alpha^2\gamma_1^2\tau_\varepsilon},$$

where $\omega = \theta + \frac{\varepsilon}{\alpha\gamma_1}$(the use of $\hat{p}$ makes computations simpler: consider $E\,[\theta|p,s(i)] = E\,[\theta|\hat{p},s(i)]$). We have then:

$$(1-\gamma(i)_2)p - \gamma(i)_0 - \gamma(i)_1 s(i) = p - \frac{\alpha^2\gamma_1^2\tau_\varepsilon\frac{(1+\alpha(1-\gamma_2))p-(\beta+\alpha\gamma_0)}{\alpha\gamma_1} + \tau(i)_u s(i)}{\tau+\tau(i)_u+\alpha^2\gamma_1^2\tau_\varepsilon}$$

implying:

$$\gamma(i)_0 = -\frac{\alpha\gamma_1\tau_\varepsilon\,(\beta+\alpha\gamma_0)}{\tau+\tau(i)_u+\alpha^2\gamma_1^2\tau_\varepsilon} \tag{A.3}$$

$$\gamma(i)_1 = \frac{\tau(i)_u}{\tau+\tau(i)_u+\alpha^2\gamma_1^2\tau_\varepsilon} \tag{A.4}$$

$$\gamma(i)_2 = \frac{\alpha\gamma_1\tau_\varepsilon\,(1+\alpha\,(1-\gamma_2))}{\tau+\tau(i)_u+\alpha^2\gamma_1^2\tau_\varepsilon} \tag{A.5}$$

To compute the optimal precision, consider the expected profit:

$$E[\pi(i)] = E\left([p-\theta]\,x(i) - \frac{1}{2}\frac{1}{\psi}\,[x(i)]^2\right) - K(\tau(i)_u)$$

The partial derivative with respect to $\tau_u(i)$ is then:

$$\frac{\partial E[\pi(i)]}{\partial\tau(i)_u} = \frac{\partial}{\partial\tau(i)_u}E\left([p-\theta]\,x(i) - \frac{1}{2}\frac{1}{\psi}\,[x(i)]^2\right) - K'(\tau(i)_u)$$

Straightforwardly, $E\,((p-\theta)x(i))$ does not depend on $\tau(i)_u$. Thus, some computations shows that

$$\begin{aligned}\frac{\partial E[\pi(i)]}{\partial\tau(i)_u} &= -\frac{\psi}{2}\gamma(i)_1^2\frac{\partial E\left(s(i)^2\right)}{\partial\tau(i)_u} - K'(\tau(i)_u)\\ &= \frac{\psi}{2}\left(\frac{\gamma(i)_1}{\tau(i)_u}\right)^2 - K'(\tau(i)_u)\end{aligned}$$

The first order condition is then:

$$\frac{\partial E[\pi(i)]}{\partial\tau(i)_u} \leq 0 \text{ and } \tau(i)_u\frac{\partial E[\pi(i)]}{\partial\tau(i)_u} = 0 \text{ (given the constraint } \tau(i)_u \geq 0)$$

$\frac{\partial E[\pi(i)]}{\partial\tau(i)_u} \leq 0$ rewrites

$$\frac{\psi}{2}\frac{1}{[\tau+\tau(i)_u+\alpha^2\gamma_1^2\tau_\varepsilon]^2} \leq K'(\tau(i)_u).$$

The LHS is decreasing and the RHS is increasing. This implies that $\tau(i)_u = 0$ if $\frac{\psi}{2}\frac{1}{[\tau+\alpha^2\gamma_1^2\tau_\varepsilon]^2} < K'(0)$ and $\tau(i)_u$ is the unique solution of

$$\frac{\psi}{2}\frac{1}{[\tau+\tau(i)_u+\alpha^2\gamma_1^2\tau_\varepsilon]^2} = K'(\tau(i)_u) \tag{A.6}$$

otherwise. The best response mapping is then defined by equations (A.3) – (A.5) and equation (A.6). □

Proof of Lemma 2. The relevant dynamical system is given by Eqs. (3) – (6). The total differentials of these equations evaluated at the REE are given by:

$$\begin{bmatrix} 1 & 0 & 0 & \frac{\gamma_0^*\gamma_1^*}{\tau_u^*} \\ 0 & 1 & 0 & -\frac{(1-\gamma_1^*)\gamma_1^*}{\tau_u^*} \\ 0 & 0 & 1 & \frac{\gamma_2^*\gamma_1^*}{\tau_u^*} \\ 0 & 0 & 0 & -\left(K'' + \frac{\psi\gamma_1^{*3}}{\tau_u^{*3}}\right) \end{bmatrix} \begin{pmatrix} d\gamma'(i)_0 \\ d\gamma'(i)_1 \\ d\gamma'(i)_2 \\ d\tau'(i)_u \end{pmatrix} =$$

$$\begin{bmatrix} -\frac{\alpha^2\tau_\varepsilon\gamma_1^{*2}}{\tau_u^*} & \frac{\gamma_0^*}{\gamma_1^*} - \frac{2\alpha^2\tau_\varepsilon\gamma_0^*\gamma_1^{*2}}{\tau_u^*} & 0 & 0 \\ 0 & -\frac{2\alpha^2\tau_\varepsilon\gamma_1^{*3}}{\tau_u^*} & 0 & 0 \\ 0 & \frac{\gamma_2^*}{\gamma_1^*} - \frac{2\alpha^2\tau_\varepsilon\gamma_2^*\gamma_1^{*2}}{\tau_u^*} & -\frac{\alpha^2\tau_\varepsilon\gamma_1^{*2}}{\tau_u^*} & 0 \\ 0 & \frac{2\psi\alpha^2\tau_\varepsilon\gamma_1^{*4}}{\tau_u^{*3}} & 0 & 0 \end{bmatrix} \begin{pmatrix} d\gamma_0 \\ d\gamma_1 \\ d\gamma_2 \\ d\tau_u \end{pmatrix}$$

We write this system as $Ax' = Bx$. It follows that the Jacobian of the best response dynamics at the REE is the matrix $P = A^{-1}B$. Since it turns out that P is a triangular matrix, its eigenvalues are equal the elements on its main diagonal. The respective eigenvalues $\lambda_1 \ldots \lambda_4$ are:

$$\lambda_1 = 0, \quad \lambda_2 = \lambda_3 = -\frac{\alpha^2\gamma_1^{*2}\tau_\varepsilon}{\tau_u^*}, \quad \lambda_4 = -\frac{2\,\alpha^2\gamma_1^{*2}\tau_\varepsilon}{\tau + \tau_u^* + \alpha^2\tau_\varepsilon\gamma_1^{*2} - (1-\gamma_1^*)\frac{\psi\frac{\gamma_1^*}{\tau_u^{*2}}}{K''(\tau_u^*)+\psi\frac{\gamma_1^{*2}}{\tau_u^{*3}}}}$$

The condition for stability of this dynamical system and, thus, the condition for existence of a locally SREE is that all eigenvalues are less than one in absolute value.

Using the definition (2) for τ_p^*, the stability conditions therefore are:

$$\tau_p^* < \tau_u^* \tag{A.7}$$

and

$$\tau_p^* < \tau_u^* + \tau - (1-\gamma_1^*)\frac{\psi\frac{\gamma_1^*}{\tau_u^{*2}}}{K''(\tau_u^*) + \psi\frac{\gamma_1^{*2}}{\tau_u^{*3}}} \tag{A.8}$$

Using the definition $\eta^* = \frac{K''(\tau_u^*)\tau_u^*}{K'(\tau_u^*)}$, some algebra shows that the right hand side of (A.8) is equal to $\tau_u^* - \tau_p^*\left(\frac{2}{\eta^*+2}\right) + \tau\left(\frac{\eta^*}{\eta^*+2}\right)$. Condition (A.8) then becomes

$$\tau_p^* < \tau_u^*\left(\frac{\eta^*+2}{\eta^*+4}\right) + \tau\left(\frac{\eta^*}{\eta^*+4}\right)$$

□

Proof of Corollary 1. From parts (i) and (ii) of Proposition 1 it follows that in a REE

$$\gamma_1^* \left[\alpha^2 \gamma_1^{*2} \tau_\varepsilon + \tau + \tau_u^*\right] = \tau_u^*$$

Thus, with the precision of private information τ_u exogenously given, γ_1^* is the solution of the polynomial $H(\gamma_1) \equiv \gamma_1 \left[\alpha^2 \gamma_1{}^2 \tau_\varepsilon + \tau + \tau_u\right] = \tau_u$. To prove the corollary, define $x = \sqrt{\frac{\tau_u}{\alpha^2 \tau_\varepsilon}}$. Existence of a SREE is then characterized by $\gamma_1^* < x$. $\gamma_1^* < x$ is equivalent to $H(x) > \tau_u$ (because $H' > 0$). The condition $H(x) > \tau_u$ rewrites:

$$\begin{aligned} \tau + 2\tau_u &> \sqrt{\alpha^2 \tau_\varepsilon \tau_u}, \\ \Leftrightarrow \quad \tau^2 + \left(4\tau - \alpha^2 \tau_\varepsilon\right) \tau_u + 4\tau_u^2 &> 0 \end{aligned}$$

If $\alpha^2 \tau_\varepsilon < 8\tau$, then the condition holds true. If $\alpha^2 \tau_\varepsilon > 8\tau$, then the condition becomes:

$$\tau_u < \frac{\alpha^2 \tau_\varepsilon - 4\tau - \sqrt{\alpha^2 \tau_\varepsilon (\alpha^2 \tau_\varepsilon - 8\tau)}}{8} \text{ or } \tau_u > \frac{\alpha^2 \tau_\varepsilon - 4\tau + \sqrt{\alpha^2 \tau_\varepsilon (\alpha^2 \tau_\varepsilon - 8\tau)}}{8}.$$

□

Proof of Proposition 2. Part (i) follows from the fact that the two conditions (C.I) and (C.II) are equivalent if $\eta^* = \frac{2\tau_u^*}{\tau}$ and that (C.I) is stronger than (C.II) if $\eta^* > \frac{2\tau_u^*}{\tau}$. To prove part (ii) notice that the right hand side of (C.II) is always greater than $\frac{1}{2}\tau_u^*$, while part (iii) follows immediately from (C.II). □

Proof of Lemma 3. Substitution of (4) into (6) yields:

$$\frac{\psi}{2} \left(\frac{\gamma(i)_1}{\tau(i)_u}\right)^2 = K'(\tau(i)_u)$$

With $Q = \sqrt{\frac{\psi}{2K'}}$, we therefore get $\gamma(i)_1 = \tau(i)_u / Q$ as well as $\gamma_1 = \tau_u / Q$. Using this, (4) becomes

$$\tau(i)_u = Q - \tau - \frac{\alpha^2 \tau_\varepsilon}{Q^2} \tau_u^2$$

Noting the non-negativity constraint $\tau(i)_u \geq 0$ on the best response $\tau(i)_u$, we then get (8) referred to in part (i) of the Lemma. Part (ii) follows from the fact that the best response dynamics of the precision of private information described by (8) turn out to be independent from all other parameters of the firms' supply schedules. □

Proof of Proposition 3. The proof proceeds in two steps. The first step is to derive some properties of the mapping $T^2(\tau_u)$ in order to find conditions for the existence of a 2–cycle in the best response mapping. The second step then draws the relevant conclusions.

(i) Consider the function $f(\tau_u) = Q - \tau - \frac{\alpha^2 \tau_\varepsilon}{Q^2} \tau_u{}^2$, which appears in Eq. (8) and let $f^2(\tau_u)$ denote its second iterate, i.e., $f^2(\tau_u) \equiv f(f(\tau_u))$. It is straightforward to show that (a) $f^2(\tau_u)$ is monotone and increasing and that (b) $f^2(\tau_u)$ has exactly one inflection point for $\tau_u > 0$. With respect to the derivative of this second iterate with respect to τ_u, we get:

$$f^{2\prime}(\tau_u) = f'(\tau_u) f'(f(\tau_u)) \geq 0$$

because $f'(\tau_u) \leq 0$.

The second derivative with respect to τ_u, $f^{2''}(\tau_u)$, is given by:

$$f^{2''}(\tau_u) = 4\left(\frac{\alpha^2\,\tau_\varepsilon}{Q^2}\right)^2 \left[\tau_u\, f'(\tau_u) + f(\tau_u)\right]$$

From this it follows that $f^{2''}(\tau_u) = 0$, if $f(\tau_u) = \tau_u\, f'(\tau_u)$ which is equivalent to:

$$(Q-\tau) = \tau_u^2\,\frac{\alpha^2\,\tau_\varepsilon}{Q^2}$$

With $Q-\tau > 0$, this equation possesses two real roots, such that there are two points of inflection where $f^{2''}(\tau_u) = 0$ and at most one such point in $S = [0, Q-\tau]$. Since $T(\tau_u) = \max\{0,\, f(\tau_u)\}$, it then follows, that $T^2(\tau_u)$ is monotone increasing on S with at most one point of inflection.

(ii) Consider now the case where no SREE exists. We then have $|T'(\tau_u^*)| > 1$ and therefore $T^{2'}(\tau_u^*) > 1$. From the above derived properties of $T^2(\tau_u)$ it then follows that a 2–cycle with $0 < \underline{\tau}_u < \bar{\tau}_u < Q-\tau$ and $T^2(\underline{\tau}_u) = T(\bar{\tau}_u) = \underline{\tau}_u$ exists if and only if $T^2(0) = T(Q-\tau) > 0$. Now, $T(Q-\tau)$ is given by:

$$T(Q-\tau) = \max\left\{0,\, [Q-\tau]\left(1 - \frac{\alpha^2\,\tau_\varepsilon}{Q^2}\,[Q-\tau]\right)\right\}$$

As an REE with $\tau_u^* > 0$ requires $Q-\tau > 0$, $T^2(0) > 0$ if and only if:

$$Q-\tau < \frac{\alpha^2\,\tau_\varepsilon}{Q^2} \tag{A.9}$$

If this condition is satisfied, a stable 2–cycle is a solution of the mapping $\tau_u' = T(\tau_u)$, and the best response dynamics converge to this 2–cycle. Thus, $S^* = [\underline{\tau}_u,\, \bar{\tau}_u]$ in this case. Otherwise, no such cycle exists and because τ_u^* is unstable, we have $S^* = S$.

(iii) Consider finally the case where $|T'(\tau_u^*)| < 1$ such that a SREE exists. In this case we have $T^{2'}(\tau_u^*) < 1$. Moreover, from $\tau_u^* = T(\tau_u^*)$ we get that:

$$\tau_u^* = \frac{1}{2}\,\frac{Q^2}{\alpha^2\,\tau_\varepsilon}\left(1 + \sqrt{4\,[Q-\tau]\,\frac{\alpha^2\,\tau_\varepsilon}{Q^2}} - 1\right)$$

With this, our condition for existence of a SREE becomes:

$$\begin{aligned}
\alpha^2\,\gamma_1^{*2}\,\tau_\varepsilon = \frac{\alpha^2\,\tau_\varepsilon}{Q^2}\,\tau_u^{*2} < \frac{1}{2}\,\tau_u^* \quad &\Leftrightarrow \quad \frac{\alpha^2\,\tau_\varepsilon}{Q^2}\,\tau_u^* < \frac{1}{2} \\
&\Leftrightarrow \quad (Q-\tau)\,\frac{\alpha^2\,\tau_\varepsilon}{Q^2} < \frac{3}{4}
\end{aligned}$$

As this implies $T(0) > 0$ (cf. eq. (A.9)), a 2–cycle cannot exist in this case. Hence τ_u^* is the unique stable fixed point of the mapping $\tau_u' = T(\tau_u)$ and $S^* = \tau_u^*$.

□

Proof of Proposition 4. According to Proposition 2, a SREE exists if and only if $\frac{2\tau_p^*}{\tau_u^*} < 1$. If $K' < \frac{\psi}{2\tau^2}$, which implies $Q > \tau$, τ_u^* is the unique positive solution of equation (9):

$$\tau_u^* = \frac{1}{2}\,\frac{Q^2}{\alpha^2\,\tau_\varepsilon}\left(\sqrt{1 + 4\,[Q-\tau]\,\frac{\alpha^2\,\tau_\varepsilon}{Q^2}} - 1\right)$$

Using this solution, the fraction $\frac{2\tau_p^*}{\tau_u^*}$ becomes:

$$\frac{2\,\tau_p^*}{\tau_u^*} = -1 + \sqrt{1 + 4\,[Q - \tau]\,\frac{\alpha^2\,\tau_\varepsilon}{Q^2}} \tag{A.10}$$

The expression on the right hand side of (A.10) approaches zero for $Q \to 0$ as well as $Q \to \infty$ and reaches a unique maximum at $Q = 2\tau$. If $Q = 2\,\tau$ we get

$$\frac{2\,\tau_p^*}{\tau_u^*} = -1 + \sqrt{1 + \frac{\alpha^2\,\tau_\varepsilon}{\tau}}$$

From this it follows that $\frac{2\tau_p^*}{\tau_u^*} < 1$ for all $Q < \tau$ whenever $\alpha^2\,\tau_\varepsilon < 3\,\tau$. Otherwise, the equation $\frac{2\tau_p^*}{\tau_u^*} = 1$ possesses two positive solutions with respect to Q

$$\bar{Q} = \frac{2}{3}\left(\alpha^2\,\tau_\varepsilon \pm \sqrt{\alpha^2\,\tau_\varepsilon\,[\alpha^2\,\tau_\varepsilon - 3\,\tau]}\right)$$

such that $\frac{2\tau_p^*}{\tau_u^*} < 1$ whenever Q is outside the bounds specified by these two solutions. The two bounds $\underline{K}'$ and $\overline{K}'$ referred to in the Proposition then arise from the definition of $Q = \sqrt{\frac{\psi}{2K'}}$. $\square$

Proof of Proposition 5. The proof is based on equation (A.10) used in the proof of Proposition 4. The expression on the right hand side of this equation is monotone decreasing in τ with $\frac{2\tau_p^*}{\tau_u^*} \to 0$ for $\tau \to Q$. Furthermore, from equation (A.10) it follows that $\frac{2\tau_p^*}{\tau_u^*}$ equals one if $\tau = \bar{\tau} = Q - \frac{3}{4}\,\frac{Q^2}{\alpha^*\,\tau_\varepsilon}$. $\square$