

Differentiated Duopoly with ‘Elimination By Aspects’

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October 5, 2006

Abstract

“Elimination by aspects” (EBA) is a discrete model of probabilistic choice worked out by Tversky in 1972 which supposes that decision makers follow a particular heuristic during a process of sequential choice. Goods are described by their attributes and, at each decision stage, consumers eliminate all the products not having an expected specific attribute, until only one option remains. In this paper, probabilities resulting from the EBA model are used to construct demands of a differentiated duopoly with imperfectly rational consumers. These demands are also consistent with partial heterogeneity of preferences and may be linked with a spatial framework in which consumers have convex perception of distance. In this model, a price Nash equilibrium in pure strategies exists under two conditions on attributes level and unit costs. At the outcome, the “differentiation by attributes” constitutes a general framework which embodies both horizontal and vertical differentiation. When the equilibrium does not exist, the interaction of best response functions of the firms induces an Edgeworth cycle instead of an exit of the lowest attributes level firm. This result underlines the role of cost asymmetries in the existence of such a cycle.

JEL CLASSIFICATION : D11, D43, L13.

KEYWORDS : discrete choices, product differentiation, oligopoly theory, elimination-by-aspects, Edgeworth cycles.

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1 Introduction

The growing number of products available in the marketplace, together with the multiplication of their attributes, makes consumers' choice more and more difficult. In such an environment, the bounded cognitive capacity of consumers is likely to lead them to use *decision heuristics*, which are simple rules of reasoning that reduce the complexity of a decision-making problem. This observation justifies the introduction of imperfectly rational consumers into the oligopoly theory, which rests on the assumption of perfectly rational agents. The paper explores this new line of research by applying the “elimination by aspects” (or EBA) heuristic proposed by Tversky (1972a, b)¹ to a differentiated duopoly.

As emphasized by McFadden (2001), the literature devoted to the analysis of individual choices among a *discrete* number of options is largely dominated by random utility models. Such models suppose that the utility of consumers confronted with various options comprises a random component. This one expresses changing state of minds (cognitive interpretation) or incapacity of the modeler to apprehend individual behaviors (econometric interpretation). The most widespread models are the logit (Luce and Suppes, 1965; McFadden, 1974), nested logit (Ben-Akiva, 1973), mixed logit (McFadden and Train, 2000) and, more generally, models with “generalized extreme values” (McFadden, 1978). Their success can be explained by their proximity with the standard approach of deterministic utility maximization, while integrating a particular form of imperfect rationality (Chen and al, 1997).

Nevertheless, among discrete choice models, the class of *random decision rule* models has not yet been explored by economists, whereas these models are widely diffused in psychology or marketing. The models proposed by Luce (1959) or Tversky (1972a, b) belong to this class. In a context that casts doubt on the relevance of utility maximization, these models clearly fit the cognitive interpretation. They postulate that the utility assigned to the possible options is deterministic, but that the decision rule used by consumers is intrinsically probabilistic. As observed by Tversky himself, “when faced with a choice among several alternatives, people often experience uncertainty and exhibit inconsistency. That is, people are often not sure which alternative they should select, nor do they always make the same choice under seemingly identical conditions” (1972a, p 281).

In the EBA model, choice options are represented by bundles of characteristics: for example, if an option is a product, the latter is described by the set of attributes it possesses, an approach which recalls that of Lancaster (1966). Moreover, Tversky suggests to describe individual choices as the result of a stochastic process involving a successive elimination of the products :

- (a) the attributes common to all goods are eliminated because they can not permit to discriminate between goods during the choice process ;
- (b) an attribute is randomly selected and all the products that do not have it are eliminated. The higher the utility of a characteristic is, the larger the probability of selecting this characteristic is ;
- (c) if the remaining goods still have specific attributes, one goes back to the first stage. On the contrary, if all goods have the same attributes, the procedure ends. If only one product remains, it is selected by the consumer. Otherwise, all the remaining goods have the same probability to be selected.

This heuristic fits well the paradigm of “adaptive toolbox” developed by Gigerenzer and Selten (2001). According to them, decision makers use fast and frugal rules, forming a *toolbox*, to solve their decision

¹Tversky employed the term of “aspects”: for our part, we indifferently use “attributes” or “characteristics”.

problems, often tackled in situation of uncertainty, limited resources and constrained time. This toolbox is *adaptive* because the heuristics used depend on context, importance of decision, and so on. As shown by Payne and Bettman (2001), the EBA heuristic is “efficient” in that it realizes a good balance between the quality of the decision finally taken and the cognitive effort of the decision-making. Tversky’s EBA model thus makes it possible to represent imperfectly rational consumers choosing on markets of differentiated products, while keeping a flexibility similar to that of random utility models, such as nested logit or probit.

In this paper, EBA-like probabilities are assumed to depict consumer’s behavior, thus leading to the construction of individual demands. We then show that the existence of a price Nash equilibrium in a differentiated duopoly depends on conditions relative to the levels of attributes of the goods as well as on the degree of unit cost asymmetry. Furthermore, introducing imperfectly rational consumers has some unexpected consequences in that it leads to a *general framework of product differentiation*, which embodies both vertical and horizontal differentiation. Recall that, in the former case, goods differ by their quality, thus implying that each consumer chooses the highest quality good when all prices are equal (Mussa and Rosen, 1978). In the latter case, when several varieties of the same base product are supplied at equal prices, each consumer chooses its most preferred variety (Hotelling, 1929).

In the EBA model, *goods are differentiated by their specific attributes*. Consequently, when the specific attributes of the two goods provide to the consumers the same positive level of utility, differentiation is horizontal. When a single product has all the specific attributes available on the market, and the other none, differentiation is vertical. Finally, when each good has some specific attributes but that provide different utility levels to the consumers, the two previous dimensions of differentiation are simultaneously taken into account. In the literature, such double differentiation has only been analyzed by Neven and Thisse (1990) and the EBA duopoly possesses new interesting properties : in addition to a more realistic formulation of consumers behavior, it provides a more compact formalism (only two parameters of differentiation against four in Neven and Thisse’s model) and a more “natural” representation of firms’ choice for product specifications.

Introducing imperfect rationality on the consumer side affects also competition between firms. Thus, the firm offering the highest level of attributes plays the role of a *reference* for the pricing choice of its rival. Formally, this means that the price of the latter does not depend on its unit cost but only upon that of the former. The existence of such a pricing relation was already observed on some markets (Lazer, 1957). It has also been analyzed theoretically with the focal prices à la Schelling (1960) and the *imitation equilibrium* of Ostmann and Selten (2001).

When price equilibrium in pure strategies does not exist, the interaction of firms’ best response functions during a succession of periods leads to a trend of prices which takes the form of an Edgeworth cycle. Recall that such a cycle, studied by Edgeworth (1925) and Maskin and Tirole (1988), comprises a long price war phase followed by an abrupt increase in prices when firms’ margins become too low. Noel (2004) recently showed that such a cycle can be a Markov Perfect Equilibrium when products are horizontally differentiated. Without using this particular concept of equilibrium, our analysis reveals that these cycles emerge with less sophisticated strategies of the producers and with a more general structure of product differentiation.

This paper is organized as follows. The EBA probabilities of products choice are described in section

2, whereas the properties of the corresponding demand functions are studied in section 3. The conditions for the existence of a price equilibrium are analyzed in section 4. Section 5 discusses the properties of this equilibrium and compares them with the existing literature. Section 6 is devoted to the strategic interactions between firms when such an equilibrium does not exist and highlights the price cycle properties. Our conclusions are presented in section 7.

2 Choice probabilities in Tversky's EBA model

Suppose that a consumer must choose a product among a set of two goods. Each good $i \in \{1, 2\}$ possesses a set of *attributes* and each attribute can be either *specific* to the considered good or *shared* by the two goods, as shown in the following figure :

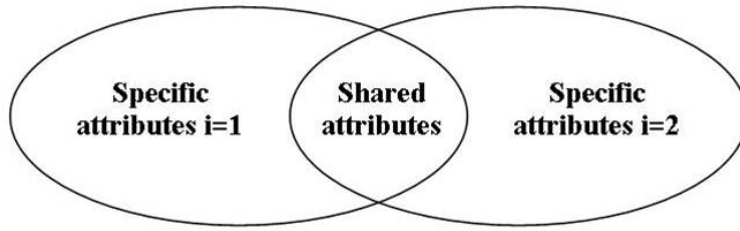


Figure 1: Specific and shared attributes

The consumers following the EBA heuristic do not care about the shared attributes, since they do not allow to discriminate between goods : they rather focus on specific attributes. Each *specific* attribute k provides to the consumer a utility u_{ik} (utilities are supposed to be additive). In order to choose one of the two products, each consumer selects a specific attribute and eliminates the good not possessing it. When K_i is the set of specific attributes of the good i , the probability of buying this good is given by Tversky's formula (1972a) of the EBA model :

$$P_i = \sum_{k \in K_i} P_k \cdot P_i^k \quad (2.1)$$

where P_k is the probability of choosing attribute k among the set $K = K_1 \cup K_2$ of the two goods' specific attributes, and P_i^k is the probability of choosing the product i when attribute k has been selected.

Since the discriminating attributes are specific, that implies necessarily that $P_i^k = 1, \forall i, k$ (only true in such two-products set). Besides, suppose for the moment that the goods are sold at the same price : then, in the EBA model, the probability of choosing a discriminating attribute k equals the ratio between the utility of this attribute and the sum of utilities of all the specific attributes :

$$P_k = \frac{u_{ik}}{\sum_{k \in K_1} u_{1k} + \sum_{k \in K_2} u_{2k}} \quad (2.2)$$

By defining $u_i = \sum_{k \in K_i} u_{ik}$ as the utility of all the attributes specific to i , the choice probability becomes :

$$P_i = \frac{u_i}{u_1 + u_2} \quad (2.3)$$

If a good does not have any specific attribute, its utility is null and it will never be chosen by the consumers. Note that, in the case of two goods, the EBA model of Tversky is formally equivalent to the Luce model (1959) if one interprets the parameters u_i ($i \in \{1, 2\}$) as the “utility of the goods” (instead of the “utility of the specific attributes” of the goods). Thus, by replacing “ u_i ” by “ $\exp(u_i/\mu)$ ”, the probabilities equals those of the binomial logit.

However, *this equivalence ceases to be true when the prices of the goods differ* : in the logit model, the price is viewed as a component of the global utility of a product, whereas the integration of prices in the EBA model follows an other logic. Indeed, Rotondo (1986) showed that the price differences between goods should be considered as attributes in the EBA model. Such attributes represent *the money saved* when the consumers buy the cheapest good rather than the other, what brings to consumers a specific utility. In a set of two goods, called i and j , let us now distinguish between “price” attributes, having the index $k = 0$, and “non-prices” attributes, indicated with $k \geq 1$. Suppose also that $p_i > p_j$. Then, according to Rotondo, good j has an additional “price” attribute compared to good i such that :

$$u_{j0} = \int_{p_j}^{p_i} w(\lambda) d\lambda \quad (2.4)$$

The shape of price difference depends on the function w retained: tests carried out by Rotondo show that a linear price difference ($w(\lambda) = 1$) constitutes a good approximation of individual behaviors. Thus, we suppose that the utility of price attribute for the product j takes the form :

$$u_{j0} = \theta(p_i - p_j) \quad (2.5)$$

The choice probabilities become $P_i = u_i/(u_i + u_j + u_{j0})$ and $P_j = (u_j + u_{j0})/(u_i + u_j + u_{j0})$ with now $u_i = \sum_{k \in K_i | k \geq 1} u_{ik}$ (and u_j is similar). Thus, consumers can eliminate a good because it does not have a particular non-price attribute or because its price is too high compared to that of the other good. For instance, a consumer who chooses between two cars can eliminate one of them because it is too expensive (if the discriminating attribute is the price) or because it does not have a specific equipment, as the airbag (if it is the discriminating attribute). In a set of homogeneous goods ($u_i = u_j = 0$), the least expensive good would be always selected.

Finally, the EBA probabilities do not have the empirically irrelevant property of “independence of irrelevant alternative” (IIA), revealed by Debreu (1960), conversely to the logit model. The EBA framework provides thus an interesting alternative to complex random utility models, as nested logit or probit models, by keeping a high degree of flexibility.²

3 Properties of the EBA demands

In this section, we study the shape of demands when price varies. Then, we show that a partial heterogeneity of preferences of the consumers can be introduced in the EBA model, which belongs to the

²A more complete presentation of the EBA model and its links with various discrete choice models is provided by Laurent (2006a)

representative consumer paradigm. Finally, we show that this model may also be linked with a spatial approach of differentiation.

3.1 A kinked demand curve

Consider now a market with N representative consumers similar to those of the previous section and buying exactly one unit of one product.³ In this duopoly, each firm⁴ produces a differentiated good i sold at the price p_i , for $i = \{1, 2\}$. The EBA model is used to construct the demands $X_i = NP_i$ where P_i corresponds to the probability of choosing i , given by equation (2.1). This demand for product i depends on the price hierarchy retained :

- if $p_i \geq p_j$,

$$X_i = \frac{Nu_i}{u_i + u_j + \theta(p_i - p_j)} \quad (3.1)$$

- if $p_j \geq p_i$,

$$X_i = \frac{N(u_i + \theta(p_j - p_i))}{u_i + u_j + \theta(p_j - p_i)} \quad (3.2)$$

The parameter θ can be interpreted as the relative importance of price attributes compared to non-price attributes. Each demand for a product is clearly decreasing with its price but the study of concavity gives the following result :

$$\left. \frac{\partial^2 X_i}{\partial p_i^2} \right|_{p_i \geq p_j} = \frac{2N\theta^2 u_i}{(u_i + u_j + \theta(p_i - p_j))^3} > 0 \quad (3.3)$$

$$\left. \frac{\partial^2 X_i}{\partial p_i^2} \right|_{p_j \geq p_i} = \frac{-2N\theta^2 u_j}{(u_i + u_j + \theta(p_j - p_i))^3} < 0 \quad (3.4)$$

The demand is strictly concave as long as $p_j > p_i$ but becomes strictly convex as soon as $p_i > p_j$. It can be represented graphically when the price varies. Here an example with $N = 1$, $p_2 = 4$, $\theta = 1$, $u_1 = 2$ and $u_2 = 1$.

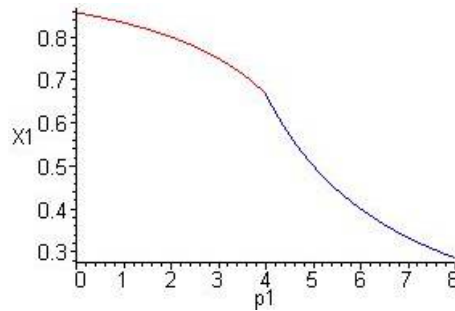


Figure 2: Evolution of demand according to the price in the EBA model

Thus, the demand has a kink implying its non-concavity. This type of kink is not very widespread in the literature but appears especially in models with “market inertia” or “switching costs” (Klemperer,

³This assumption will be discussed in section 5.4

⁴The firm selling good i will be described as “firm i ”

1984, Scotchmer, 1986, Farrel and Shapiro, 1987). These models also use price differences and have sometimes the same properties than the framework we develop here, as it will be specified later.

3.2 Representative consumer or heterogeneous preferences ?

If consumers choices are heterogenous by the random nature of the decision rule, we have previously made the strong assumption that the individuals have the same preferences for the products. Thus, the parameter u_i is interpreted as the perceived utility of specific attributes for the product i by a representative consumer. However, the EBA model is not inconsistent with a partial heterogeneity of preferences.

Consider a population of size N divided in $m = \{1, \dots, M\}$ groups of consumers with $\sum_{m=1}^M N_m = N$ where N_m denotes the number of individuals in group m . The taste for specific attributes varies between groups but not within each group: an agent of group m receives a utility u_{im} when consuming the product i . Suppose that all consumers have the same coefficient of preference θ and that the firm cannot discriminate between groups. In this case, if $p_1 \geq p_2$, demands are given by :

$$X_1 = \sum_{m=1}^M \frac{N_m u_{1m}}{u_{1m} + u_{2m} + \theta(p_1 - p_2)} \quad (3.5)$$

and

$$X_2 = \sum_{m=1}^M \frac{N_m(u_{2m} + \theta(p_1 - p_2))}{u_{1m} + u_{2m} + \theta(p_1 - p_2)} \quad (3.6)$$

Such expressions can be simplified if we make the following assumption :

Assumption 1 : *The sum of specific non-price attributes utilities for the set of goods is constant between groups of individuals* : $\sum_{i=1}^2 u_{im} = \sum_{i=1}^2 u_{il} \quad \forall l \neq m$

This assumption means that each consumer values identically the total set of attributes. For instance, if those attributes are the accessories of a car, this means that each individual associates the same utility to the set of all possible existing accessories. This assumption seems acceptable when the number of attributes is sufficiently high: in this case, the global utility is almost equivalent for everyone.

By fixing $u = u_{1m} + u_{2m}, \forall m$, demands become :

$$X_1 = \frac{(\sum_{m=1}^M N_m u_{1m})}{u + \theta(p_1 - p_2)} = \frac{N(\sum_{m=1}^M \frac{N_m}{N} u_{1m})}{u + \theta(p_1 - p_2)} \quad (3.7)$$

and

$$X_2 = \frac{(\sum_{m=1}^M N_m u_{2m} + N\theta(p_1 - p_2))}{u + \theta(p_1 - p_2)} = \frac{N(\sum_{m=1}^M \frac{N_m}{N} u_{2m} + \theta(p_1 - p_2))}{u + \theta(p_1 - p_2)} \quad (3.8)$$

Thereafter, if $u_i = (1/N) \sum_{m=1}^M N_m u_{im}$ then one finds the traditional formulation of demand. In this case, the parameter u_i can be interpreted differently: it is the *weighted average utility in the population* for

the specific attributes set of good i . We thus show that it is possible to introduce a certain degree of preference heterogeneity if Assumption 1 holds.

3.3 Demands and spatial differentiation

In this section, we show that the demands obtained with the probabilistic EBA model can also be constructed with a particular deterministic model of localized competition. These Kaldorian models suppose that the consumers differ by their tastes, the preferences being described by a space of characteristics. They are thus known as “spatial” models, like Hotelling (1929) and Salop (1979) models. The intensity of competition is stronger between the firms localized closer because their products are closer substitutes.

Anderson, De Palma and Thisse (1992) previously proved that discrete choice models may be linked both with Kaldorian models of localized competition and with Chamberlinian models of non-localized competition. In particular, they show that the logit demands can also be obtained in a spatial framework for a specific density function of consumers preferences (op. cit. p 118). The conditions of such equivalence for the EBA model are now identified.

Suppose that two firms are localized at the bounds of a segment (firm 1 on “left”) whose length is l . The N consumers are uniformly distributed with a density N/l along the segment and buy a single unit of good. Each consumer is supposed to bear a linear cost per unit of distance : thus, a consumer located at point x on the segment bears $C_1(x) = tx$ to go to firm 1 and a cost $C_2(x) = t(l - x)$ to go to firm 2.⁵ The highest priced good brings a utility equal to the utility of its specific attributes. But for the other good, the utility of specific attributes is increased by the price difference, which corresponds to the saving made by the consumers choosing this good. When $p_1 > p_2$, we have $U_1^S = u_1$ and $U_2^S = u_2 + \theta(p_1 - p_2)$.

Finally, suppose that the consumers choose the product which gets them the highest specific utility per unit of distance $U_i^S/C_i(x)$. Such a decision rule is not usual in models of spatial differentiation in which consumers maximize their mill utility, the converse of the mill prices (which comprises the price of the product and the cost of distance). In the Hotelling model, this utility decreases in a linear way with the distance. In the quadratic Euclidean distance model (D’Aspremont, Gabszewicz and Thisse, 1979), consumers have a decreasing and concave utility with the distance. However, when consumers maximize their utility per unit of distance, as in the EBA model, *their utility is decreasing and convex with the distance*.

Let us now establish the position of the indifferent consumer between buying at firm 1 and at firm 2, which verifies $(U_1^S/C_1(x)) = (U_2^S/C_2(x))$. As the parameter t plays no role here, this equality leads to :

$$(l - x)u_1 = x(u_2 + \theta(p_1 - p_2)) \Leftrightarrow x = \frac{lu_1}{u_1 + u_2 + \theta(p_1 - p_2)} \quad (3.9)$$

Demands are then given by

$$D_1 = Nx = \frac{lNu_1}{u_1 + u_2 + \theta(p_1 - p_2)} \quad \text{and} \quad D_2 = N(l - x) = \frac{lN(u_2 + \theta(p_1 - p_2))}{u_1 + u_2 + \theta(p_1 - p_2)}$$

and for $l = 1$, we find demand functions (3.1) and (3.2). Thus, the EBA demands are consistent with an “address” model in which decision depends on the utility of relative advantages per unit of distance.

⁵These assumptions are rather common in the literature: this cost can be interpreted as a physical cost of transportation or as a cost related to the distance of the preferred product variety.

Note that establishing a link between discrete choice models and address approach implies, for random utility models (like the logit model), a modification of the *density function which affects the utility* of the product compared to the basic spatial model. But for a random decision rule model (like the EBA model), establishing such a link is made possible by the modification of the *decision rule*, compared to the spatial model.

4 Existence and uniqueness of the price equilibrium

After having studied the EBA demands, we show now that a price equilibrium exists in such a duopoly. Each firm plays a non-cooperative game with its rival in which the strategy, belonging to the set $S_i \subseteq \mathbb{R}_+$, consists in price determination. Suppose that each firm bears a unit cost c_i and a fixed cost F_i , the latter being sufficiently weak to guarantee the positivity of profits, of which here expressions:

- if $p_i \geq p_j$,

$$\Pi_i = \frac{Nu_i(p_i - c_i)}{u_i + u_j + \theta(p_i - p_j)} - F_i \quad (4.1)$$

- if $p_j \geq p_i$,

$$\Pi_i = \frac{N(u_i + \theta(p_j - p_i))(p_i - c_i)}{u_i + u_j + \theta(p_j - p_i)} - F_i \quad (4.2)$$

We assume that firms are risk neutral and use the concept of Nash equilibrium in pure strategies. Thus, in a two goods market, a “price equilibrium” is a price vector $(p_1^*; p_2^*)$ such that each firm i ($i = \{1, 2\}$) maximizes its profit for the value p_i^* of p_i conditionally to the price p_j ($j \neq i$) set by the other firm j . Formally, this means that :

$$\Pi_i(p_i^*, p_j^*) \geq \Pi_i(p_i, p_j^*) \quad \forall p_i \in S_i, \quad \forall i, j \in \{1, 2\}, i \neq j$$

When the demand X_i is concave with prices, it is not necessary to prove the existence of such an equilibrium: Caplin and Nalebuf (1991) showed that if $1/X_i$ is increasing and convex with prices, then profit is concave, which guarantees the existence of price equilibrium. However, the EBA demands are kinked and their concavity is not proven : that is why the theorem of Caplin and Nalebuf cannot be mobilized here. For which values of unit costs does such an equilibrium exist ?

PROPOSITION 1 *Necessary and sufficient conditions to the existence of a Nash equilibrium in $p_i \geq p_j$, with $i, j \in \{1, 2\}$ and $i \neq j$, are*

$$u_i \geq u_j \quad (4.3)$$

and

$$c_i - c_j \geq \frac{\sqrt{u_i u_j} - u_i}{\theta} \quad (4.4)$$

If this equilibrium exists, then it is unique.

Proof : the proof of this proposition is presented in Appendix A.

The proof adopts the following method. In a first time, we establish the best response function of firm i for a given price p_j , according to whether i chooses $p_i \leq p_j$ or $p_i \geq p_j$. It follows from this analysis that the best response functions *on all the interval of definition*, established in a second time, depend on the hierarchy between u_i and u_j . Finally, symmetrical and asymmetrical price equilibrium and their conditions of existence are identified.⁶

The couple of equilibrium price when $p_i \geq p_j$ is given by⁷:

$$p_i^* = \frac{u_i + \sqrt{\Delta}}{2\theta} + c_i \quad (4.5)$$

$$p_j^* = \frac{u_i + u_j}{\theta} + c_i \quad (4.6)$$

where $\Delta = u_i^2 + 4u_i(u_i + u_j + \theta(c_i - c_j))$. At this outcome, the firm i , whose product is the most appreciated by consumers, sets a higher price than its rival. This equilibrium exists when the unit cost of i is higher than that of j , which seems intuitive. However, firm j may also have a higher unit cost than i since $\sqrt{u_i u_j} - u_i < 0$ but the gap of unit costs should necessarily be weak. Properties of this equilibrium are detailed in section 5. Finally, note that the two necessary and sufficient conditions do not guarantee the existence of equilibrium for all values of cost or utility parameters. The case in which no equilibrium exists will be analyzed in section 6.

5 Economic properties of price equilibrium

This section studies equilibrium configurations when attributes verify, for example, the hierarchy $u_1 \geq u_2$. First of all, we show that this model covers many existing forms of differentiation, depending on the values of the parameters. The analysis of equilibrium prices will also show that firm 1 is used as a reference point by its rival for its pricing choices. Moreover, profits and market shares are compared. The last section shows that the parameter θ plays a role similar to a participation constraint in this model.

5.1 What kind of differentiation ?

Various configurations exist according to the values of the parameters u_1 and u_2 .

5.1.1 Pure horizontal differentiation

When $u_1 = u_2 = u > 0$, two conclusions can be drawn: on the one hand, goods are differentiated and, on the other hand, the specific attributes of each good are appreciated in the same way within the population of consumers. Such a configuration, in which all varieties have a positive demand when they are sold at a same price, refers to a *pure horizontal differentiation*.

⁶An other method consists in determining local equilibrium existing on an interval and, if necessary, to check if this local equilibrium is global. This method leads obviously to the same result, but at the cost of more tiresome calculations...

⁷Note that the Nash equilibrium is weak in the sense that the firm i could obtain the same profit by choosing another price belonging to an interval $[p_j; +\infty[$

In this framework, there is always a Nash price equilibrium. If $c_1 = c_2 = c$, the equilibrium prices are given by : $p_1 = p_2 = (2u/\theta) + c$. Prices are higher than marginal costs, which traduces the differentiation and this one is horizontal because prices are equal when marginal costs are equal: it is, for example, the case in the Hotelling model (1929). This type of differentiation is also present in the standard version of the binomial logit model, in which prices are $p = 2\mu + c$, where μ is an index of consumers taste heterogeneity. In such configuration, the u/θ ratio in the EBA model thus plays a role similar to that of the parameter μ in the logit model.

If $c_i > c_j$, we find an equilibrium in $p_i > p_j$ with $p_i = c_i + ((u + \sqrt{9u^2 + 4u\theta(c_i - c_j)})/2\theta)$ and $p_j = (2u/\theta) + c_i$. This result, according to which the firm having the highest marginal cost fixes the highest price, is consistent with an Hotelling model in which costs are asymmetric.

5.1.2 Pure vertical differentiation

When $u_i > 0$ and $u_j = 0$, the product i has all the specific attributes of the market. Thus, at equal price, all consumers would prefer having the good i rather than the good j : the existence of such a preference hierarchy is the sign of a *pure vertical differentiation*. Such models have been developed by Mussa and Rosen (1978), Gabszewicz and Thisse (1979) and Shaked and Sutton (1982). Traditionally, these models suppose that the firms sell products of different “qualities”, this heterogeneity being objective or the result of consumers’ different valuations. But, in the EBA model, the quality is viewed as the supplement of utility get by the consumers for all the *specific* attributes of a good. The attributes shared between goods are not taken into account since they do not intervene in the choice process.

In the EBA model with vertical differentiation, the only possible equilibrium when $u_i > u_j$ verifies $p_i > p_j$ and the remaining equilibrium condition (4.4) is reformulated into $\theta(c_i - c_j) \geq -u_i$. If the latter is verified, equilibrium prices are given by $p_i = c_i + ((u_i + \sqrt{u_i^2 + 4u_i(u_i + \theta(c_i - c_j))})/2\theta)$ and $p_j = (u_i/\theta) + c_i$. The highest quality firm sets the highest price, which is standard in this type of model.

5.1.3 Vertical and horizontal differentiation

The general case with $u_i > u_j > 0$ corresponds to a *double differentiation*: differentiation is horizontal up to the level u_j , goods offering to the consumer a comparable utility, and vertical for a level $u_i - u_j$, product i proposing also additional attributes. Thus, the EBA model allows to represent in a simple way a double differentiation coupled with a costs asymmetry.

Such models are rare in the literature : Neven and Thisse (1990) were the only to propose a duopoly allowing the triple analysis of price, quality and variety equilibria. This model supposes null production costs, a horizontal differentiation à la Hotelling according to a variety y_i (with a quadratic transport cost) and a vertical differentiation à la Mussa and Rosen (1978) according to a quality q_i (with a uniform distribution of consumers taste for quality). Thus, Neven and Thisse suppose that the firms choose independently a vertical and a horizontal positioning. This assumption is questionable : it seems more plausible to suppose that firms basically choose the attributes of the good and that this choice affects thereafter simultaneously the horizontal and vertical positioning of the product. For instance, adding a sun roof to a car affects both its vertical position on the market (highest quality) and its horizontal one (new “target” consumers).

Besides, in the model of Neven and Thisse, equilibrium prices differ for a same segment of demand according to whether the gap of differentiation is more important for the vertical characteristic or for the horizontal one. Thus, for two goods noted i and j , the results of the model differ according to whether $q_i - q_j \lesseqgtr y_i - y_j$. Such a property is not highlighted in the EBA model, which can be explained as well by the nature of differentiation as by the shape of demands, and thus by the consumers' imperfect rationality of probabilistic type.

5.1.4 The special case of Bertrand

When $u_1 = u_2 = 0$ or $\theta \rightarrow +\infty$, any form of differentiation vanishes. For $c_1 = c_2 = c$, one finds the price competition of a Bertrand type, with homogeneous goods, and in which $p_1 = p_2 = c$.

5.2 Equilibrium prices and the “reference firm”

We consider now the most general structure of differentiation with $u_1 > u_2 > 0$ and study the properties of equilibrium prices.

Firstly, the price $p_1^* = c_1 + (u_1/2\theta) + (\sqrt{\Delta}/2\theta)$ is of a rather classical formulation : it equals to the unit cost of the firm plus a margin, since products are differentiated. This margin of firm 1 $m_1 = (u_1 + \sqrt{u_1^2 + 4u_1(u_1 + u_2 + \Delta c)})/2\theta$ increases with u_1 , because differentiation is valued by the consumer, but is also positively correlated (although slightly) with u_2 and $\theta(c_1 - c_2)$. The former relation means that the price is increasing with the *global* degree of differentiation on the market: an effort of differentiation from a competitor will profit to all the protagonists. The latter relation is more original: the margin of firm 1 is increasing with its cost disadvantage compared to firm 2.

These two elements also seem at the origin of the non-conventional form of $p_2^* = (u_1 + u_2/\theta) + c_1$. This price grows with u_1 and u_2 , which confirm that prices increases with the total effort of differentiation on the market. But, here again, it could seem surprising that the price does not depend on c_2 but increases with c_1 .

An analysis of the strategic interactions between firms allows to highlight these unusual relations. We have previously mentioned that the firm 1 can obtain the same profit by deviating from the equilibrium price p_1^* inside the interval $[p_2; +\infty[$. Suppose that the firm 1 decide to modify its price, then the best response of firm 2 is to modify its own price in the same sense (case J1 in Appendix A, section 9.2.2) : this means that the firm 2 *imitates* the sense of price variation of the firm 1. Indeed, for a particular gap of levels of attributes and a particular gap of costs, a fixed and strict gap of price is required for the existence of equilibrium on the market. These practices of pricing imitation recall those described by Lazer (1957 p. 130-131), and particularly the case in which the firm selling the “best quality” good sets a *reference price* on the market (or “focal price”). In this context, the other firms chooses the reference price minus a certain amount, which depends on the quality gap with the reference firm.⁸ Such an interpretation allows to explain both why a price equilibrium exists when $c_1 \gg c_2$ and why p_2 increases with c_1 and not with c_2 .

In the asymmetric case, the introduction of imperfectly rational consumers can thus involve a modification of relationships between firms, the firm selling the most appreciated good acquiring a statute of

⁸Recall however that the information is perfectly symmetrical in the concept of equilibrium used here.

“reference”. Note that a model of price determination based on imitation between firms in a Bertrand oligopoly has been developed by Ostmann and Selten (2001), in a more symmetrical framework.

5.3 Equilibrium demands and profits

At the equilibrium, the demands take the values :

$$X_1 = NP_1^* = \frac{2Nu_1}{u_1 + \sqrt{\Delta}} \quad \text{and} \quad X_2 = NP_2^* = \frac{N\sqrt{\Delta} - u_1}{(u_1 + \sqrt{\Delta})}$$

with still $\Delta = u_1^2 + 4u_1(u_1 + u_2 + \theta(c_1 - c_2))$. Let us determine under which conditions a firm can obtain the largest market share. Comparison shows that:

$$\begin{aligned} X_1^* > X_2^* &\Leftrightarrow 3u_1 > \sqrt{\Delta} \\ &\Leftrightarrow \theta(c_1 - c_2) < u_1 - u_2 \end{aligned} \tag{5.1}$$

This condition stresses the importance of cost parameters in the determination of firms’ market shares: the firm selling the “most appreciated” product (such that $u_1 > u_2$) will make the largest market share if the gap of differentiation is sufficiently high compared to the gap of costs. Indeed, a too strong difference in costs would generate a too high price for firm 1 and a transfer of demand in the direction of firm 2.

A similar analysis can be made about profits which take the outcome values :

$$\Pi_1^* = \frac{Nu_1}{\theta} - F_1 \quad \text{and} \quad \Pi_2^* = \frac{N(u_1 + u_2 + \Delta c)(\sqrt{\Delta} - u_1)}{\theta(\sqrt{\Delta} + u_1)} - F_2$$

The calculus of first order conditions allows to check that Π_i^* increases with u_i , $\forall i$. When fixed costs are identical $F_1 = F_2$, the comparison of profits leads to the following result :

$$\Pi_1^* \geq \Pi_2^* \Leftrightarrow \theta(c_1 - c_2) < u_1 - u_2$$

Thus, if the condition (5.1) is verified, the highest market share firm has also the highest profit. This omnipresence reveals a convergence of firms’ goals in this model: maximization of profit, i.e. short term objective, is consistent with maximization of market share, i.e. long term objective. Such a convergence is also present in the “switching costs” models previously evoked, as Farrel (1986) notes it. When $c_1 = c_2$, as in many models of product differentiation, one finds the standard result : the firm selling the “most appreciated” good always makes the highest profit.

5.4 Market participation for consumers and outside option

We recall initially how the outside option is generally integrated in models of differentiation. Then we show that the assumption of complete coverage of market is not essential in the EBA model.

5.4.1 Outside option in models of differentiation

Up to now, we have made the assumption of complete coverage of the market, which means that each consumer buys exactly a unit of good. However, differentiated products models introduce sometimes the

possibility that some consumers decide not to buy at all. These consumers prefer to choose an “outside option” or keep their money, which brings to them some utility.

The interest of non-consumption is generally represented by a parameter in the model and, according to the interval to which this parameter belongs, the market is covered or not: that defines the participation constraint of the model. This parameter may be a utility of reservation U_0 , as in vertical differentiation models or in the logit oligopoly. In all these structures, when U_0 varies, equilibrium prices and margins decrease and generally the profits too. Indeed, the possibility that the consumers leave the market limits the firms price level.

In vertical differentiation, the role played by U_0 on demands is complex. Let us take the example of the duopoly of Gabszewicz and Thisse (1979) in which the firm A sells the highest quality good and the firm B a lower quality good (at a lower price)⁹. In this model, there exists two modes :

- when the market is covered (mode D_2) and for some values of the parameters¹⁰, the demand of firm B grows with U_0 whereas that of firm A decreases with it. In this mode, an increase in U_0 results only in a transfer to firm B of the demand previously addressed to firm A.
- when the market is not covered (mode D_1), the demands of the two firms decrease with U_0 (but this decrease is larger for firm A). In this mode, the increase in U_0 induces a market exit of B’s consumers. However, as a large part of A’s demand is transferred from A to B, the reduction of B’s demand is lower than that of A.

5.4.2 Outside option in the EBA model

Let us see now how to model an outside option in the EBA framework. Note that it is impossible to use here a utility of reservation because choices are based on comparisons between specific attributes and prices : the “own” utilities of options are not taken into account. Nevertheless, the parameter θ plays in the EBA model a similar role to the parameter U_0 in the model of Gabszewicz and Thisse (1979).

First of all, firms’ prices and margins decrease with θ . We have previously noted that this parameter measures the consumers relative preference of prices compared to non-price attributes : it thus seems logical that p_1 decreases with θ but more surprising than p_2 also diminishes. This apparent paradox is solved by recalling that the firm 1 plays a role of “reference” for its rival in price determination : since p_1 decreases with θ , it is also true for p_2 .

Moreover, it seems plausible that the consumers do not enter in the market when they attach to prices a great importance compared to non-price attributes. That’s why we suppose now that each consumer decide not to buy any good with the probability $\mu(\theta)$. This probability verifies $\mu(\theta) = 0$ if $\theta \leq \bar{\theta}$ (where $\bar{\theta}$ is a threshold) and $\mu(\theta) > 0$ with $\mu'(\theta) > 0$ if $\theta > \bar{\theta}$.

This participation constraint is not linked directly to the prices but only to θ , a “preference” parameter of prices. This assumption is not so surprising if one distinguishes between the consumer’s decision of *entering in the market*, at a first stage, (“I need a car”) and the choice among all the goods proposed on this market (“I choose between a Chrysler and a Ford”), at a second stage. The EBA heuristic applies exclusively to the second stage, in which pricing arbitrages occur, which is not necessarily true for the first stage. Indeed, in mature markets of differentiated products, the entering motivation comes rather

⁹We will not consider here the case in which one firm is excluded from the market

¹⁰Notably when the fixed part of consumers income is not too high

from the *services rendered by the good* which includes the *common attributes* of all the available goods, not considered in the EBA heuristic up to now. If θ is high, then, for a high proportion of consumers, the non-price attributes of the product do not bring a sufficient utility compared to the lost of utility because of the money spent.

In such a framework, the demand becomes $X_i = N(1 - \mu(\theta))P_i$ and two cases must be distinguished :

- Firstly, when $\mu(\theta) = 0$, the market is completely covered. X_1 decreases with θ whereas X_2 increases with it. These results correspond to the mode D_2 in the model of Gabszewicz and Thisse.

- But when $\mu(\theta) > 0$, the market is not covered. For simplification, suppose that $c_1 - c_2 \geq 0$ and let us study how demands vary with θ :

$$\left. \frac{\partial X_1}{\partial \theta} \right|_{\mu(\theta) > 0} = \frac{2Nu_1[-\mu'(\theta)(u_1 + \sqrt{\Delta})\sqrt{\Delta} - 2u_1(c_1 - c_2)(1 - \mu(\theta))]}{\sqrt{\Delta}(u_1 + \sqrt{\Delta})^2} < 0 \quad (5.2)$$

$$\left. \frac{\partial X_2}{\partial \theta} \right|_{\mu(\theta) > 0} = \frac{4Nu_1[-\mu'(\theta)(u_1 + u_2 + \theta(c_1 - c_2))\sqrt{\Delta} + u_1(c_1 - c_2)(1 - \mu(\theta))]}{\sqrt{\Delta}(u_1 + \sqrt{\Delta})^2} \quad (5.3)$$

with $\Delta = u_1^2 + 4u_1(u_1 + u_2 + \theta(c_1 - c_2))$. The demand of product 1 is still decreasing with θ . But the sign of demand's derivative for the product 2 is unknown in the general case, since each of the two effects previously evoked, the “consumers’ exit” effect and the “inter-firms demand transfer” effect, plays against the other. Besides, when the former dominates the latter, this slope is negative and it is true for many values of the parameters, as when $c_1 = c_2$ for example. If this property holds, one finds the characteristic relation of mode 1 in the model of Gabszewicz and Thisse: the rise of θ benefits to the outside option and the demand for product 2 decreases with θ .

6 Non-existence of an equilibrium and Edgeworth cycle

Let us analyse now the properties of the EBA duopoly when a condition of existence of price equilibrium holds but not the other.¹¹ After having highlighted the existence of an Edgeworth cycle in this framework, we link this result with the existing literature.

6.1 Edgeworth cycles in the EBA model

Suppose in this section that the condition (4.3) is verified but (4.4) is violated, which implies the inequality $c_i - c_j \leq (\sqrt{u_i u_j} - u_i)/\theta$. Consider a sequential game with alternate moves of firms in which these firms make a choice in the space of pure price strategies. Each firm is unaware of the reaction function of its rival, which limits its temporal horizon of profit maximization to one period. Thus, each time t its “turn” to play occurs, the firm i observes p_j^{t-1} and chooses the price p_i^t maximizing its profit for the current period. Prices evolution is thus described by successive and “naive” interactions of the firms’ reaction functions, previously established in appendix A, section 9.2.2.

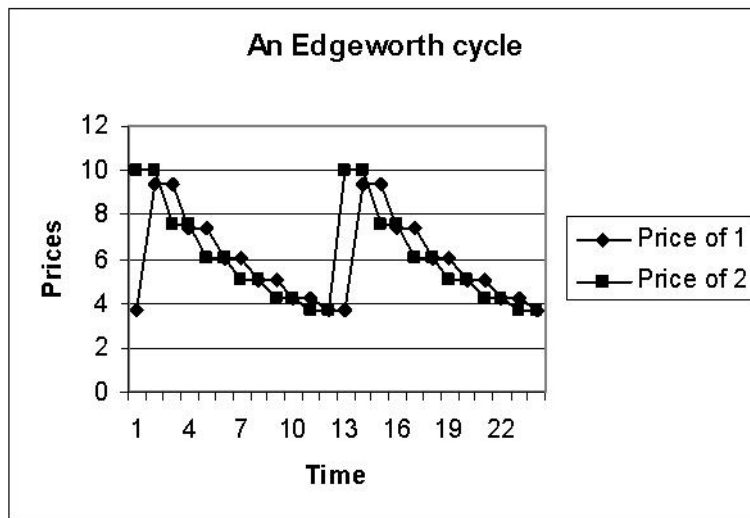
In this framework, we show that the strategic interaction of firms generates a cycle of a specific form. In the downward phase of the cycle, prices decrease during several steps, but when they fall under a particular threshold, a firm decides to increase its price brutally. This raise, which characterizes the ascending phase of the cycle, is unique. This shape of cycle has already been studied in the literature:

¹¹If the two conditions were simultaneously violated, another equilibrium configuration would be obtained.

PROPOSITION 2 *When the equilibrium conditions are not verified simultaneously, firms' pure strategies interactions in an alternate move game lead to an Edgeworth price cycle.*

Proof : The proof of this proposition is presented in Appendix B.

In the absence of reservation price or outside option, the maximum profit in the ascending phase is reached for $p \rightarrow +\infty$, which characterizes an extremely large raise of prices and implies an extremely long downward phase. Such a width is obviously not observable in reality.¹² A little modification of the EBA model reducing the width of fluctuations could however make it possible to draw a more representative cycle. An example is provided here when maximum price has a finite bound $p_{max} \rightarrow 10$ with $u_1 = 2$, $u_2 = 1$, $c_1 = 0$, and $c_2 = 1$ (no equilibrium for these values):



Thus, in the EBA model, when a firm cumulates disadvantages in term of unit cost and of specific attributes, this firm chooses to strongly increase its price and the form of differentiation on the market allows it to keep a positive profit in spite of a negligible market share (which recall the “niche” strategy). As this strategy is better than the market exit, which gets a null profit, it triggers the price fluctuations. Afterwards, each firm can obtain a larger profit by decreasing its price and price war continues until the situation becomes untenable for the high cost firm. This result seems to confirm that the market exit is less frequent in double differentiation models than in vertical ones (no “finiteness” property) : indeed, Anderson, De Palma and Thisse (1992) have already underlined, by using the framework of Neven and Thisse (1990), that an entering firm can always make a positive market share by choosing a new horizontal variety with the highest quality on the market.

6.2 A setting in prospect of the results

The existence of cycles characterized by a price war during a long period followed by a brutal increase in prices, was firstly established by Edgeworth (1925) in the case of homogeneous goods. According to

¹²But the aim of this proposition is only to show that the fundamental properties of the Edgeworth cycles are verified in the absence of equilibrium in the EBA model, not to confront the cycle obtained to a specific market

him, each firm sets initially a lower price than its rival to increase its market share but did not obtain the totality of the demand because of capacity constraints. Thus, firms decrease their price until the war becomes too expensive: then a firm increases its price, immediately followed by its rivals. Finally, price war starts again.

Such cycles were then studied by Maskin and Tirole (1988) in a framework of dynamic and symmetrical duopoly with price competition, identical costs and homogeneous goods. These authors also show that the Edgeworth cycles can be Markov Perfect Equilibria for particular mixed strategies maximizing the firms' discounted profits, if the discount factor is sufficiently high. Such equilibria are obtained in games with simultaneous moves or with alternate ones. This result was extended by Eckert (2003) to the case of asymmetric market share duopolies (but identical prices). An extension in several directions was realized by Noel (2004), which studies notably an alternate move game in a Hotelling model and shows that Edgeworth cycles can occur if differentiation is not too strong. Indeed, according to the author, the differentiation should decrease the probability of Edgeworth cycles existence, since it gets a positive market share (and then a positive profit) to the firm with the highest price, which can destroy the cycle.

Compared to the literature, note that the cycles in the EBA model are obtained by simple interaction of best response functions in a one period scale and in pure strategies : cycles require neither mixed strategy nor unit costs fluctuation (as in Noel) but, on the other hand, do not constitute equilibria. Besides, Markov Perfect Equilibria cannot be identified within this complex framework, with asymmetric reaction functions and price choice on a continuous interval. Pure strategic interactions require only short term opportunism from firms whereas Markov Perfect Equilibria require a planning by firms on a potentially high number of periods, a capacity to use complex strategies and a perfect information on past actions.

In the above mentioned papers, main decisive factors for price cycles existence seem to be the nature of reaction functions (Maskin and Tirole, 1988) and the degree of differentiation (Noel, 2004) but *cost asymmetry* is rarely mentioned as a deciding factor, whereas it is the case in the EBA model. Moreover, by supposing that the condition (4.3) holds, the probability of Edgeworth cycle existence corresponds to the probability of violation of condition (4.4). However this probability decreases with the degree ($u_i - u_j$) of vertical differentiation, and a pure horizontal differentiation supposes necessarily the absence of vertical differentiation. Thus, *horizontal differentiation seems to induce Edgeworth cycles more frequently than vertical differentiation*. This result is consistent with the conclusion of Gabszewicz and Thisse (1986) according to which a stable equilibrium exists more frequently in vertical differentiation models than in horizontal ones. On the other hand, the width of horizontal differentiation does not play here any role in the existence of cycle, which differs from the conclusion of Noel. In the EBA model, the cycle existence is related to the capability of market structure to support a cost asymmetry at the advantage of the most appreciated product firm.

The structure of Edgeworth cycles obtained in the EBA model is relatively standard. Nevertheless, note that prices remain over marginal costs, which is not always true in the homogenous products models or in the Hotelling form of differentiation used by Noel.

7 Conclusion

In this paper, the EBA probabilities have been used to construct the demands of a duopoly. Consumers' choice of products differ, as they follow a random decision rule, but these demands are consistent both with a representative consumer approach and with heterogeneity of preferences. Moreover, these demands may be obtained within a spatial framework in which consumers have convex perception of distance (whereas it is generally concave in the literature, for instance when transport costs are quadratic). All these links underline the high level of generality of the EBA demands.

By using these demands, we develop a discrete choice oligopoly of product differentiation. We showed that a price equilibrium in the EBA model exists even if the unit costs are asymmetrical. The firm proposing the most appreciated product can sell it at the highest price. The existence of equilibrium requires however that its cost is not too low compared to that of its rival.

The properties of this outcome are interesting in more than one way. First of all, by setting the attributes at the center of the decision process of firms and consumers, the model allows a more intuitive representation of product differentiation. This framework embodies both vertical and horizontal differentiation, while keeping a rather light formalism. The introduction of imperfectly rational consumers affects also the relations between the firms: the firm selling the most appreciated good plays a role of "reference" for its rival during the price fixation. Finally, we showed that the parameter θ has in the EBA model the same function than a participation constraint in other models: the assumption of complete coverage of the market cannot thus constitute a limit at the structure we propose.

When no price equilibrium exists, the strategic interactions of firms lead to an Edgeworth cycle. Economic theory generally explain the existence of these frequently observed cycles by complex strategies of the agents. This model shows, on the one hand, that this assumption is not always required and, on the other hand, that costs asymmetries can play a role of trigger for such cycles.

Several extensions of the EBA model remain to do. Firstly, a study of firms' attributes choice with endogenous unit or fixed costs allows to integrate arbitrages between innovation and imitation : we propose such an analysis in an other paper, still preliminary (Laurent, 2006b). Moreover, the properties of the EBA oligopoly with more than two firms remain to study, but the formulation of demands is less "natural" and the integration of price more complex. More generally, it seems that considering credible decision rules followed by consumers to choose among a set of complex differentiated product leads also to a more general framework to describe the product differentiation in the market. That is why it would be interesting to develop new differentiated oligopolies by considering the other potential heuristics used by consumers in such a situation.

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9 Appendix A

9.1 Best response functions conditional to price hierarchy

Initially, let us establish the firms' best response functions conditionally to the price hierarchy retained. Which price p_i constitutes the best response of firm i knowing the price p_j of its rival?

9.1.1 Best response in $p_i \geq p_j$

When $p_i \geq p_j$, the profit of i is given by : $\Pi_i = \frac{Nu_i(p_i - c_i)}{(u_i + u_j + \theta(p_i - p_j))} - F_i$

For the firm i , the first derivative is :

$$\frac{\partial \Pi_i}{\partial p_i} = \frac{Nu_i(u_i + u_j + \theta c_i - \theta p_j)}{(u_i + u_j + \theta(p_i - p_j))^2} \quad (9.1)$$

The sign of this derivative does not depend on the value of p_i and there are three cases :

- If $p_j < \frac{u_i + u_j}{\theta} + c_i$, then $\frac{\partial \Pi_i}{\partial p_i} > 0$ and the best response is $p_i^*(p_j) \rightarrow +\infty$.
- If $p_j = \frac{u_i + u_j}{\theta} + c_i$, then $\frac{\partial \Pi_i}{\partial p_i} = 0$ and the best response belongs to an interval $p_i^*(p_j) \in [p_j; +\infty[$.
- If $p_j > \frac{u_i + u_j}{\theta} + c_i$, then $\frac{\partial \Pi_i}{\partial p_i} < 0$ and the best response is $p_i^*(p_j) = p_j$.

9.1.2 Best response in $p_j \geq p_i$

When $p_j \geq p_i$, the profit of firm i is given by $\Pi_i = \frac{N(u_i + \theta(p_j - p_i))(p_i - c_i)}{u_i + u_j + \theta(p_j - p_i)} - F_i$

The first derivative is :

$$\frac{\partial \Pi_i}{\partial p_i} = \frac{N[(u_i + u_j + \theta(p_j - p_i))(u_i + \theta(p_j - p_i)) - \theta u_j (p_i - c_i)]}{(u_i + u_j + \theta(p_j - p_i))^2} \quad (9.2)$$

This function has a single extremum whose expression is :

$$\frac{\partial \Pi_i}{\partial p_i} = 0 \Leftrightarrow \theta^2 p_i^2 - 2\theta p_i (u_i + u_j + \theta p_j) + (u_i + \theta p_j)(u_i + u_j + \theta p_j) + \theta u_j c_i = 0 \quad (9.3)$$

Only one root of this polynomial can verify $p_j \geq p_i$ (the other never respects this hierarchy) :

$$p_i^c(p_j) = \frac{u_i + u_j + \theta p_j - \sqrt{u_j(u_i + u_j + \theta(p_j - c_i))}}{\theta} \quad (9.4)$$

We study if this extremum is a maximum :

$$\left. \frac{\partial^2 \Pi_i}{\partial p_i^2} \right|_{\frac{\partial \Pi_i}{\partial p_i} = 0} = \frac{2N(\theta p_i - u_i - u_j - \theta p_j)}{(u_i + u_j + \theta(p_j - p_i))^2} \quad (9.5)$$

This derivative is always negative in the interval $p_j \geq p_i$ and the profit is thus concave. Moreover, $p_i^c(p_j)$ is in the interval of definition if :

$$p_j \geq \frac{u_i (u_i + u_j)}{u_j} + c_i \quad (9.6)$$

The best response function can thus take two possible forms :

- if $p_j \geq \frac{u_i (u_i + u_j)}{u_j} + c_i$, the profit is concave in the interval and the best response is

$$p_i^*(p_j) = \frac{u_i + u_j + \theta p_j - \sqrt{u_j(u_i + u_j + \theta(p_j - c_i))}}{\theta}$$

- if $p_j < \frac{u_i (u_i + u_j)}{u_j} + c_i$, the profit is increasing in the interval and the best response is $p_i^*(p_j) = p_j$.

9.2 Non-conditional best response functions

9.2.1 Asymmetry of best response functions

From the conditional reaction functions, it is possible to establish each firm's best response function whatever the price hierarchy. However, the condition (9.6) indicates that the shape of the reaction function depends on the hierarchy of specific attributes utilities.

Thus, when $u_i \geq u_j$, if the profit is concave on $[0; p_j]$, it decreases necessarily on $[p_j; +\infty[$ because $\frac{u_i (u_i + u_j)}{u_j} + c_i \geq \frac{(u_i + u_j)}{\theta} + c_i$. However, since the demand is continuous, the profit is also continuous and the global maximum is necessarily in the interval $[0; p_j]$. On the other hand, when $u_j \geq u_i$, the profit may be simultaneously concave in $[0; p_j]$ and increasing in $[p_j; +\infty[$ in which case the determination of global maximum requires to compare the profits on the two intervals. In all the other cases, there are only one local maximum.

Best response functions are thus asymmetric according to the hierarchy between u_i and u_j . *Let us suppose now that $u_j \geq u_i$ and let us solve this indetermination on firm j's best response by comparing maximum profits on each of the two intervals $[0; p_i]$ et $[p_i; +\infty[$.*

When $p_j = (u_i + u_j + \theta p_i - \sqrt{u_i(u_i + u_j + \theta(p_i - c_j))})/\theta$, the profit equals :

$$\Pi_j^c = \frac{N(\sqrt{u_i + u_j + \theta(p_i - c_j)} - u_i)^2}{\theta u_i} \quad (9.7)$$

When $p_j \rightarrow +\infty$, the profit is

$$\Pi_j^{cc} = \frac{Nu_j}{\theta} \quad (9.8)$$

The comparison of profits leads to the following condition :

$$\Pi_j^c > \Pi_j^{cc} \Leftrightarrow p_i > \frac{2\sqrt{u_i u_j}}{\theta} + c_j \quad (9.9)$$

Moreover, it is easy to show that $\frac{u_j}{u_i}(u_i + u_j) \leq 2\sqrt{u_i u_j} \leq u_i + u_j$ when $u_j \geq u_i$. This result allows us to establish the best response functions of the firms, independently of the initial hierarchy of prices.

9.2.2 Expression of the best response functions

a) The best response function of i is :

- if $p_j > \frac{u_i(u_i + u_j)}{u_j \theta} + c_i$, or $p_j = \frac{u_i(u_i + u_j)}{u_j \theta} + c_i$ and $u_i > u_j$, then the profit is concave in $[0; p_j]$ and strictly decreasing in $[p_j; +\infty[$. In this case, the best response function of i is

$$p_i^*(p_j) = \frac{u_i + u_j + \theta p_j - \sqrt{u_j(u_i + u_j + \theta(p_j - c_i))}}{\theta} \text{ which verifies } p_i \leq p_j. \text{ (case I1)}$$

- if $\frac{u_i + u_j}{\theta} + c_i < p_j < \frac{u_i(u_i + u_j)}{u_j \theta} + c_i$ (which supposes that $u_i > u_j$), then the profit is strictly increasing in $[0; p_j]$ and strictly decreasing in $[p_j; +\infty[$. In this case, the best response of i is ‘‘on the kink’’: $p_i^*(p_j) = p_j$. (Case I2)

- if $p_j = \frac{u_i + u_j}{\theta} + c_i$ then the profit is strictly increasing in $[0; p_j]$ and constant in $[p_j; +\infty[$. The best response belongs to the interval $p_i^*(p_j) \in [p_j; +\infty[$. (Case I3)

- if $p_j < \frac{u_i + u_j}{\theta} + c_i$, then the profit of i is strictly increasing in all the interval of definition and the best response is $p_i^*(p_j) \rightarrow +\infty$. (Case I4)

b) The best response function of j is :

- if $p_i > \frac{2\sqrt{u_i u_j}}{\theta} + c_j$, or $p_i = \frac{2\sqrt{u_i u_j}}{\theta} + c_j$ and $u_i > u_j$, then the global maximum is in the interval $[0; p_i]$ in which the profit is concave and the best response of j is

$$p_j^*(p_i) = \frac{u_i + u_j + \theta p_i - \sqrt{u_i(u_i + u_j + \theta(p_i - c_j))}}{\theta} \text{ which verifies } p_j \leq p_i. \text{ (case J1)}$$

- if $p_i = \frac{2\sqrt{u_i u_j}}{\theta} + c_j$ and $u_i = u_j$, then the profit is strictly increasing in $[0; p_i]$ and constant in $[p_i; +\infty[$ and the best response belongs to the interval $p_j^*(p_i) \in [p_i; +\infty[$. (case J2)

- if $p_i < \frac{2\sqrt{u_i u_j}}{\theta} + c_j$, then the global maximum is reached in the interval $[p_i; +\infty[$ in which the profit is increasing and the best response of j is $p_j^*(p_i) \rightarrow +\infty$. (Case J3)

9.3 Identification of the equilibria

The Nash price equilibrium is given by the intersection of the best response functions. Let us prove now that the equilibrium in $p_i \geq p_j$ exists when conditions (4.3) and (4.4) hold.

A symmetric Nash equilibrium exists when $p_i^*(p_j) = p_j$ and $p_j^*(p_i) = p_i$, which restricts the analysis to the cases I2 and I3 for i and J2 for j . Moreover, the reaction function of j verifies $p_j^*(p_i) = p_i$ if and only if $u_i = u_j = u$ and $p_i = \frac{2u}{\theta} + c_j$. But when $u_i = u_j$, the firm i will choose an identical price only if $p_j = \frac{2u}{\theta} + c_i$. The condition $c_i = c_j = c$ is also required for the symmetric equilibrium existence. In this case, $p_i^*(p_j) = \frac{2u}{\theta} + c$ is one of the bests responses of the firm i . We thus have a price Nash equilibrium.

Let us now study the possibility of asymmetric price equilibria of the form $p_i \neq p_j$.

First, the cases in which $p_i \rightarrow +\infty$ and $p_j \rightarrow +\infty$ can not constitute equilibria. Indeed, if the firm i chooses $p_i \rightarrow +\infty$, then the best response of j is : $p_j^*(p_i) = \bar{p} = \frac{u_i + u_j + \theta p_i - \sqrt{u_i(u_i + u_j + \theta(p_i - c_j))}}{\theta}$.

Moreover $\lim_{p_i \rightarrow +\infty} \bar{p} \rightarrow +\infty$, which violates the condition $p_j < \frac{(u_i + u_j)}{\theta} + c_i$ under which the firm i would choose this price $p_i \rightarrow +\infty$ (the demonstration is similar for $p_j \rightarrow +\infty$). The cases I4 and J3 are thus eliminated from the analysis. Finally, the only possible equilibrium corresponding to the case J2 has already been revealed, which also exclude it of this analysis.

Consequently, if an asymmetric equilibrium exists, it verifies necessarily $p_i > p_j$ and corresponds to the case J1 coupled with the case I3. This last case implies that: $p_j^c = \frac{(u_i + u_j)}{\theta} + c_i$. For which value of p_i leads the reaction function of j to the choice of such a p_j ?

By using this reaction function, we find that p_i must verify $\theta(p_i - c_i) = \sqrt{u_i(u_i + u_j + \theta(p_i - c_j))}$. This equation may be rewritten as the following second degree polynomial : $\theta^2 p_i^2 - \theta p_i(u_i + 2\theta c_i) - u_i(u_i + u_j - \theta c_j) + \theta^2 c_i^2$. Only one root verifies $p_i > p_j$: $p_i^c = \frac{u_i + \sqrt{\Delta}}{2\theta} + c_i$ with $\Delta = u_i^2 + 4u_i(u_i + u_j + \theta(c_1 - c_2))$.

But this price should also verify the inequality $p_i \geq \frac{2\sqrt{u_i u_j}}{\theta} + c_j$ so that firm j really chooses p_j^c . And this condition is verified if and only if $c_i - c_j \geq \frac{\sqrt{u_i u_j} - u_i}{\theta}$. Prices are within the intervals of definition of the cases I3 and J1 which thus constitute a Nash equilibrium.

The proof of uniqueness may now be realized. Consider the first condition between u_1 and u_2 : there are three possible senses of hierarchy between these parameters.

- if $u_1 > u_2$ then the only possible price equilibrium verifies $p_1 > p_2$
- if $u_2 > u_1$ then the only possible price equilibrium verifies $p_2 > p_1$
- if $u_2 = u_1$ then the nature of equilibrium depends on the second necessary condition. When $c_1 > c_2$, the only possible price equilibrium verifies $p_1 > p_2$. Conversely, if $c_2 > c_1$, this equilibrium takes the form $p_2 > p_1$. Finally, if $c_1 = c_2$, an equilibrium with $p_1 = p_2$ is reached.

In all these various cases, if the equilibrium exists, then it is unique.

10 Appendix B

We show here that the interaction of reaction functions gives raise to an Edgeworth cycle. This proof is organized in two steps : on the one hand, we show that the downward phase of the cycle has several stages and, on the other hand, we prove that the ascending phase comprise a unique increase in price by the highest unit cost firm.

LEMMA 3 *When the condition (4.3) is verified and the condition (4.4) is violated, the downward phase of the cycle has several steps.*

Proof: Let us take again the reaction functions of the appendix A, section 9.2.2, when $u_i > u_j$ and suppose that the firm j chooses a very high price (because its profit is strictly increasing), $p_j \rightarrow +\infty$. In this case, the best response of i is to choose $p_i^*(p_j) = \frac{u_i + u_j + \theta p_j - \sqrt{u_j(u_i + u_j + \theta(p_j - c_i))}}{\theta} < p_j$ but verifying $\lim_{p_j \rightarrow +\infty} p_i^*(p_j) \rightarrow +\infty$. Consequently, the firm j is also incited to set a lower price $p_j^*(p_i) = \frac{u_i + u_j + \theta p_i - \sqrt{u_i(u_i + u_j + \theta(p_i - c_j))}}{\theta} < p_i$ but which still remains very high, and so on. The decreasing period thus comprises several stages. ■

LEMMA 4 *When the condition (4.3) is verified and the condition (4.4) violated, the firm j is the first to increase its price. This raise, which characterizes the ascending phase of the cycle, is unique.*

Proof: The first part of this lemma will be proven by contradiction : we show that prices cannot decrease in the area in which Π_i is increasing, which proves that Π_j is increasing “before” Π_i . Let us suppose that we are in the area in which Π_i is increasing for a particular period t . In this case, p_j^{t-1} necessarily verifies $p_j^{t-1} < \frac{(u_i + u_j)}{\theta} + c_i$. By using the proof of asymmetric equilibrium existence, realized in section 9.2.2, we deduced from it that p_i^{t-1} should verify both $p_i^{t-1} < \frac{u_i + \sqrt{u_1^2 + 4u_1(u_1 + u_2 + \theta(c_1 - c_2))}}{2\theta} + c_i$ and $p_i^{t-1} > \frac{2\sqrt{u_i u_j}}{\theta} + c_j$. However that is impossible since the condition (4.4) is violated.

Consequently, at the end of the downward phase, firm i will end up choosing a price such that Π_j increases and j will choose $p_j^* \rightarrow +\infty$ at the following period. For this price, the best response of i will be to propose a lower price which will initiate a new downward phase. The ascending phase can thus comprise only one stage. ■

Note that this lemma just guarantees that $\Pi_i(p_j^*)$ is not strictly increasing when $\Pi_j(p_i^*)$ increases. Thus, after the ascending phase, the best response of firm i can consist in fixing a price either inferior or identical to that of its rival.

By combining the two previous lemmas, we prove the proposition 2.