Incentives, Markets and Knowledge Based Hierarchies

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Abstract

We propose an equilibrium model in which organization is choice variable for firms. Firm owners organize production in knowledge based hierarchies - designed to use and communicate knowledge efficiently. The model combines aspects of moral hazard on the side of employees and adverse selection concerning the skills of both employed and unemployed workers. Incentives are provided through termination contracts. In this context we analyze the impact of changes in labor market conditions and technology on internal organization of firms, which in turn affects wages and labor demand for different skill levels. Among other things we show that the introduction (or increase) of minimum wage may adversely affect highly skilled workers both in terms of salary levels and of employment.

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1 Introduction

Over the past decades business firms in most industrialized countries have experienced important organization transformations - changing their formal architecture, redefining the allocation of decision making authority and responsibility, redesigning their information system[1]. These evolutions in turn affected the characteristics of labor demand and wages for employees of different skill categories[2]. It is commonly admitted that some of these evolutions have been driven by the important technological progress observed in past decades[3]. However, it is doubtful that technology can explain everything as organizations differ in size (Kumar et al. (2002)), and managers' span of control ratios (Acemoglu and Newman (2001)), across countries with similar access to information technology.

How firms adapt their organization to technological change, but also to evolutions in labor market conditions or their institutional environment? In what extend such evolutions affect the structure of employment and wages? To address these questions we propose an equilibrium model in which organization structure is choice variable for firms. There is a labor market with potential employees, separated in two populations: skilled and unskilled. A firm owner hires agents on the labor market, affects them to different occupations (according to their expected skill level), chooses task repartition and wages. As far as knowledge is substantial input, organizations have the characteristics of knowledge based hierarchies - designed to use and communicate knowledge efficiently. In the model employees' productivity and wage cost endogenously depend both on technological and labor market (supply and demand, and regulatory) conditions. We analyze the extent to which changes in these conditions affect organizations' size, the allocation of decision making and span of control in the firm. In turn the latter transformations affect wages and employment

[1]Rajan and Wulf (2003) for example present evidence based on panel of american firms, that from 1986 to 1995, firms have become flatter, with less layers of managers and that the managerial span of control have increased.

[2]Caroli and van Reenen (2001) find evidence in support of the hypothesis of skill biased organizational change. They also show that technical change is complementary with human capital but the effects of organizational change are not simply due to technological change but have an independent role.

[3]Bresnahan et al (2002) among others, find that greater use of computers is associated with the employment of more educated workers, greater investment in training and s.o.

of agents from different skill level categories.

Our model is in the vein of recent literature (Radner (1992,1993), Rander and Van Zandt (1992), Bolton and Dewatripont (1994), Garicano (2000)) on hierarchies. In all this articles the optimal way for organizing production is a response to a trade-off of costly information processing (or knowledge acquisition) and costly communication. Garicano (2000) is the closest paper to ours. He shows that a knowledge based hierarchy is the way to use and communicate knowledge efficiently. Productive workers solve easy problems, and transmit the harder ones to managers. The latter specialize in solving exceptions. We preserve the assumptions driving this result thus in our framework knowledge based hierarchy is also the optimal way to produce. Unskilled agents are hired as productive workers, and skilled ones as managers. However Garicano sets incentive problems aside and focus on organization's structure. Thus the main difference of our work is in introducing opportunistic behavior of agents. We combine aspects of moral hazard on the side of employees and adverse selection concerning the skills of both employed and unemployed agents. Incentives are provided though termination contracts à la Shapiro and Stiglitz (1984). There is one wage by occupation. As there is an information asymmetry concerning agent's skill level, the wage is accompanied by a performance standard and a credible threat to fire any agent that do not meet the standard. Performance standards correspond to the degree of delegated authority on each level of the firm.

Our model enable us to connect the organization to the labor market environment. This connection passes through two channels. First evolutions of agents' supply and demand, or labor market regulation affect the cost of incentives. For example when labor demand is high employees know that being fired is not a harsh punishment, because it is relatively easy to find a new job, this increases the cost of wages. The second effect is on employees' productivity. As a principal fires any agent that do not meet the performance standard, insiders are selected over time. Their stationary distribution, and thus their productivity, depends on the size of the unemployment pool. In tight labor market workers are less selected (their distribution of talents is closer to the initial one), and thus their productivity is lower.

The rest of the paper is organized as follows. In Section 2 we present the framework and the link between production, incentives and quality of insiders' distributions. In Section 3 we analyze the program of a firm owner. Section 4 presents the characteristics of the market equilibrium, and section 5 discusses comparative static on crucial variables such as firm size, number of agents by occupation, span of control and wages. Then we conclude. All proofs are in the Appendix.

2 The Model

2.1 Framework

Economy. Our economy is composed of a continuum of measure one of homogeneous firm owners[4] and a continuum of potential employees. Each potential employee belongs to a population characterized by some education (diploma) level[5]. We assume that there are two education levels - agents can either be skilled or unskilled. N_u respectively N_s is the exogenously given size of unskilled respectively skilled agents population. An agent can not cheat on his education degree. However diploma is an imperfect signal of employees' skill level (knowledge) θ, which is one's private information. Distribution of skills in each population is common knowledge, $U(\theta)$ is the cumulative of unskilled's competence, $\theta \in [0; \overline{\theta}_u]$. Skilled are distributed on the support $[\underline{\theta}_s; \overline{\theta}_s]$, and their cumulative distribution function is $S(\theta)$. We assume that $\overline{\theta}_u < \underline{\theta}_s$ and that all firm owners have the same skill level θ^P, $\theta^P \in]\overline{\theta_s}; 1]$. Each agent is endowed with one unit of time per period, and all agents are infinitely lived and discount the future on some common rate δ ($\delta \in [0; 1]$).

Organization. A firm owner can produce alone, or create a larger organization by hiring agents on the labor market. In order to produce agents have to solve problems. Dealing with a problem requires time, solving a problem requires knowledge and effort. Complexity of a particular problem is ex ante unknown, however each agent in the economy knows problems' distribution. Thus we denote by x a problem's complexity. When $x \leq \theta$, an employee can solve the problem by spending some

[4]The size of firm owners population is exogenously given, there is no entry. We do not consider the endogenous decision to become firm owner.

[5]Diploma acquisition is exogenous.

effort. If $x > \theta$ he is able neither to solve the problem nor to identify its complexity[6]. We normalize the problems' complexity distribution so that the skill level θ, is also the proportion of problems an agent is able to solve (i.e. $x \rightsquigarrow U[0,1]$). Effort cost for solving x for an agent θ, writes[7]:

$$c^T = \begin{cases} c & if \quad x \leq \theta \\ +\infty & if \quad x > \theta \end{cases}$$

Agents can communicate their knowledge to others and thus help them solve problems. Communication is costly and as in Garicano (2000), we assume that all the communication cost is supported by the receiver. He spends his time unit dealing with a transmitted problem independently of being able or not to solve it.

To summarize, production requires skills (possibly tacit), which are embodied in individuals, and matching problems with those able to solve them is costly. Under the latter condition Garicano (2000) shows that the optimal organization is a knowledge based hierarchy. On the lowest layer there are less knowledgeable production workers (he), more knowledgeable agents occupy higher levels (managers, she). Workers specialize in production, problems arrive exclusively on their level. If a worker knows the solution of a problem he solves it and produces one unit of output, if not he can ask help to agents of the next level of the hierarchy. Thus managers specialize in dealing with problems their subordinates are not able to solve. If a manager knows the problem solution she solves it and output is realized, if not problem is transmitted to the next level and so on.

2.2 Knowledge based Hierarchy

Each firm is composed of three layers - workers, managers and firm owner (principal). The production function we presented is such that managers' job is to deal with problems that have not been solved by workers. So managers are hired in

[6]Under this assumption our setting has the principal characteristics under which a knowledge based hierarchy is the optimal organization form.

[7]The implications of this choice for effort cost are discussed below.

the population of skilled and workers are hired in the population of unskilled[8]. A principal decides the number of agents that should be employed on each level, the proportion of problems they should be able to solve (performance standard), the respective wage structure and the quality of insiders' skill distribution.

We proceed in three steps. First we characterize the production function and discuss the link between task repartition and the number of agents employed on each level. Then we analyze incentives provision, i.e. the incentive compatible wages guaranteeing that an agent of a given level performs any delegated problem he is able to solve. Finally as agents employed on each level are heterogenous, at end of a period some of them are dismissed (and replaced) for lack of competence, thus performance standards play the role of retention rules. In section 2.2.3 we characterize insiders' distribution as function of outsiders' distribution and performance standard, and discuss the selection effect of those rules.

2.2.1 Production

Production corresponds to the expected number of problems solved by an organization. All agents have limited time, and are able to deal with one problem by period. So the number of problems a firm deals with in each period corresponds to the number of productive workers[9]. A principal hires managers to help workers in solving exceptions. Their number depends on the quantity of arriving problems and on workers' solving capacity.

We first compute the proportion of problems solved on each level of the hierarchy for given performance standard. Then for given agents' productivity we obtain the number of employees a firm owner hires for each occupation.

Proportion of problems solved on each level of the organization. A performance standard $x_w \in [0, \overline{\theta}_u]$ ($x_m \in [\underline{\theta}_s, \overline{\theta}_s]$), corresponds to the set of tasks a principal delegates to workers (managers). As far as agents are heterogeneous in competence, there is a gap between formal (defined by performance standards) and

[8]For the three layer hierarchy the only alternative could be to hire only skilled agents on both layers and propose them different performance standards. We discuss this possibility below.

[9]Remember problems arrive only on the first level of the hierarchy

real (the proportion of problems really solved) productivity.

Insiders' distribution functions are $F(\theta)$ for workers and $\Phi(\theta)$ for managers. On steady state equilibrium each initial distribution of talents split into two stationary distributions, their endogenous obtention is discussed in section 2.2.3.

The expected probability of a problem to be solved at workers' level is:

$$\int_0^{x_w} \theta f(\theta) d\theta + \int_{x_w}^{\bar{\theta}_u} x_w f(\theta) d\theta \tag{1}$$

A worker whose know how is below the performance standard, solves a problem only if it is in his knowledge set i.e. only if $x \leq \theta$. A worker with $\theta > x_w$ solves any problem which difficulty is below the performance standard. Note that for given x_w, real productivity depends on the quality of insiders' distribution.

(1) is equivalent to:

$$\int_0^{\bar{\theta}_u} x_w f(\theta) d\theta - \int_0^{x_w} (x_w - \theta) f(\theta) d\theta$$

The probability for a problem to be transmitted by a worker to the next level of the hierarchy is: $t_w(x_w) = 1 - x_w + v_w$, where $v_w = \int_0^{x_w} (x_w - \theta) f(\theta) d\theta$. v_w corresponds to the gap between formal and real productivity of workers, the proportion of problems that should be solved on their level, but are not because of employees' lack of competence.

On the managers' level - the probability for a problem to be solved by a manager is the joint probability for that problem to have not been solved by a worker and to belong to the managers' knowledge and requirement set:

$$x_m - \int_{\underline{\theta}_s}^{x_m} (x_m - \theta) \varphi(\theta) d\theta - x_w + v_w \tag{2}$$

$$\Leftrightarrow x_m - v_m - x_w + v_w \text{ with } v_m = \int_{\underline{\theta}_s}^{x_m} (x_m - \theta) \varphi(\theta) d\theta$$

Thus the proportion of problems a principal have to deal with i.e. the probability for a problem not to be solved by workers nor by managers is: $t_m(x_m) = 1 - x_m + v_m$. v_m is the gap between formal and real productivity on the managers' level. We notice that the probability for a problem to be received by a principal depends only on the managers' level solving capacity, it is due to $\underline{\theta}_s \geq \overline{\theta}_u$.

Number of workers and managers. Problems arrive only on the first level of the organization. As each agent is endowed with one unit of time, which is entirely consumed in dealing with one problem, the number of problems arriving in the organization corresponds to the number of workers n_w. For a proportion $t_w(x_w)$ they ask help to the managers. Thus the number of problems arriving on the second level of the hierarchy is $n_w t_w(x_w)$. This corresponds to the number of managers n_m, a principal hires to help workers. It depends both on the number of workers (n_w) and on their solving capacity. Indeed more autonomous workers (higher performance standard x_w, and/or better skills distribution) solve larger proportion of problems and thus need fewer managers to help them.
The size of the firm is constrained by the presence on the top of one principal with one unit of time. So the number of entering problems in the organization should verify $n_w t_m(x_m) \leq 1$. n_w depends on the managers' solving capacity. more performant managers are in charge of larger teams.

2.2.2 Incentives

What is the cost for a principal of implementing higher performance standard? At the end of a period (once a cohort of arriving problems has been treated) a principal observes the difficulty of each problem, who received it, and if it has been solved or transmitted[10]. When an agent asks some help to his superior, the principal is not able to detect if the reason is shirking or lack of competence.

There are two agency problems on the employees' side: skill levels are private information, and effort is only imperfectly observed by the principal. We restrict employment contracts as follows: first all agents with the same occupation have the same reward, second, to solve moral hazard problem the owner proposes termination contracts[11].

[10] The implications of possible imperfect monitoring technology will be discussed in section **??**.

[11] Under the assumption of unique contract per level, we rule out dynamic learning. A unique contract should be optimal consistently with insiders' skill distribution, which is actually the case.

Lemma 1. *Under the previous restrictions, the optimal termination contract is a fixed wage coupled with a performance standard and a credible threat to dismiss any agent that do not meet the standard.*

A contract proposed to the agents of the layer $i = \{w; m\}$ (w for workers and m for managers), is $(x_i; s_i)$: x_i is the performance standard i.e. the set of problems an agent of level i should be able to solve in order to keep his occupation. s_i is the fixed wage paid on the beginning of each period to an employee of level i.

Each agent is in one of two states in any point of time: employed or unemployed. There is an exogenous probability $(1 - \alpha)$, for an employee at any level of the organization and independently of his type, to be separated from his job, due to relocation etc. So an employee enters the unemployment pool either for exogenous separation or because he is fired. Indeed at each period a proportion of agents whose knowledge capacities are below the corresponding performance requirement (x_i) is detected by a principal. Thus v_i, the gap between formal and real productivity on the level i, corresponds to the proportion of fired agents on that level.

$V(\theta; i)$ is the inter-temporal expected utility for an agent of type θ, working on level $i = \{w; m\}$. $V_U(\theta; j)$ is the inter-temporal expected utility of an unemployed agent of type θ, in the distribution j, where $j = \{u; s\}$. We assume that labor market is anonymous and it is hard to keep track of participants. Even if it is possible, the information about the reasons a separation occurred is not available or reliable. When an employer meets an employee she is not able to identify if he is unemployed for exogenous reasons, or because he has been fired for shirking or lack of competence.

Workers. Let us consider the case of a worker (employed unskilled agent) of type $\theta \in [0, x_w]$. In each period he could be in one of those situations:

- He receives a problem of difficulty $x \leq \theta$. If he works, he solves the problem and keeps his job with probability α. If he shirks, he is fired at the end of the period.

- He receives a problem of difficulty $x > \theta$, the problem is transmitted:

– If $x \in [\theta; x_w]$, he is fired for not meeting the performance standard.

– If $x_w < x$, he keeps his job with probability α.

When an agent receives a problem $x > \theta$, it is in his interest to transmit it[12], with probability $(1 - x_w)$ the problem is "too difficult" to be solved by a worker and he keeps his occupation, if he tries to hide the problem, he is sure to be fired.

Thus the inter-temporal expected utility respectively for an agent of type $\theta < x_w$ and $\theta \geq x_w$ writes[13]:

$$V(\theta; w) = s_w - \theta c + \delta\alpha(1 - x_w + \theta)V(\theta; w) + \delta\alpha(x_w - \theta)V_U(\theta; u) + \delta(1 - \alpha)V_U(\theta; u)$$

$$V(\theta; w) = s_w - x_w c + \delta\alpha V(\theta; w) + \delta(1 - \alpha)V_U(\theta; u)$$

When a worker receives a problem that he can (and is supposed to) solve ($x \leq \min\{\theta; x_w\}$), the effort decision depends on the comparison of the utility stream when working and the utility stream when shirking. A worker solves a problem only if:

$$(s_w - c + \delta\alpha V(\theta; w) + \delta(1 - \alpha)V_U(\theta; u)) \geq s_w + \delta V_U(\theta; u)$$

The left hand side corresponds to the expected gain when he exerts the effort and the right hand side is the expected gain when he shirks.

The incentive compatible wage on the workers' level then writes as:

$$s_w = \frac{c(1 - \delta\alpha(1 - x_w))}{\delta\alpha} + (1 - \delta)V_U(\theta; u) \tag{3}$$

Expanding the set of tasks delegated to workers, increases the probability for problem solving effort to be spent, which decreases $V(\theta; w)$ and makes the utility stream when working less attractive for the agent. Thus larger performance standard on the workers level should combined with a higher wage in order to implement effort provision.

[12]When problem's difficulty exceeds agent's competence he is not able neither to solve the problem, nor to detect its real difficulty.

[13]Individuals' instantaneous utility is separable n wages and effort $u(s, c) = s - c$.

Managers. $Z(\theta)$ corresponds to the joint probability for a problem x to be transmitted and $x \leq \theta$, $Z(\theta) = \int_0^{x_w} F(x)dx + \int_{x_w}^{\theta} dx = \theta - x_w + v_w$. For given solving capacity of workers $Z(\theta)$ is the probability for a manager of type θ to spend the effort cost.

When a principal detects a shirking manager, there are two possible punishment strategies - firing or demotion.

Lemma 2. *A shirking manager is fired by a principal.*

An unsuccessful manager would accept a demotion only if her expected utility as worker ($V(w,s)$) is larger than the expected utility as unemployed ($V_U(s)$). If it is not the case she prefers quit the organization rather than accept the new occupation. But if $V(w,s) > V_U(s)$, than the incentive cost for a principal is lower if she adopts firing as punishment strategy for shirking managers.

The inter-temporal utility of a manager with $\theta < x_m$ writes:

$$\begin{aligned} V(\theta; m) = \quad & s_m - Z(\theta)c + \delta\alpha(1 - Z(x_m) + Z(\theta))V(\theta; m) + \\ & + \delta\alpha(Z(x_m) - Z(\theta))V_U(\theta; s) + \delta(1-\alpha)V_U(\theta; s) \end{aligned}$$

The incentive compatible wage for the managers is:

$$s_m \geq \frac{c(1 - \delta\alpha(1 - x_m + x_w - v_w))}{\delta\alpha} + (1-\delta)V_U(\theta; s) \tag{4}$$

Wage is still proportional to the probability for problem solving effort to be spend. For managers this probability depends both on their performance standard, and real productivity of their subordinates. For given value of x_m, when a manager is matched with more performant workers, the probability for him to incur the effort cost is lower, and the incentive compatible wage decreases.

Lemma 3. *Incentive compatible wages do not depend on θ.*

Larger θ means larger probability to keep actual occupation (the associated gain for a worker is $\delta\alpha(V(\theta; w) - V_U(\theta; u))$), but also larger probability for spending problem solving effort c. When incentive compatibility constraint is binding the marginal gain equals the marginal cost[14], thus the inter-temporal expected utility of a worker (manager) and an unemployed unskilled (skilled) do not depend on θ. Hereafter we simplify the notations as follows: $V(w)$ (instead of $V(\theta, w)$), $V_U(u)$ (instead of $V_U(\theta, u)$), $V(m)$ (instead of $V(\theta, m)$), $V_U(s)$ (instead of $V_U(\theta, s)$).

2.2.3 Selection

Agents with given occupation have the same diploma level but are heterogenous in skills. As any detected agent of level i with $\theta < x_i$ is fired, x_w and x_m play the role of retention rules. So the choice of performance standards affects the characteristics of workers' and managers' distributions.

Initially we have the distributions of skilled and unskilled agents - $U(\theta)$ and $S(\theta)$. Each principal chooses x_w and x_m and on the steady state there are two stationary distributions for each initial population of agents - on the labor market and in the firm. In this section we discuss the link between those distributions.

For stationarity to be verified after firing and hiring at each period, the link between $F(\theta)$ (unskilled insiders' distribution) and $Q(\theta)$ (unskilled outsiders' distribution) is as follows:

$$(5) \qquad Q(\theta) = \begin{cases} \dfrac{(1-\alpha)F(\theta) + \alpha \displaystyle\int_0^\theta (x_w - u) f(u) du}{(1-\alpha+\alpha v_w)} & \text{if} \quad \theta \in [0; x_w] \\ \dfrac{(1-\alpha)F(\theta) + \alpha v_w}{(1-\alpha+\alpha v_w)} & \text{if} \quad \theta \in [x_w; \bar{\theta}_u] \end{cases}$$

Details are in the appendix (section 7.2).

For skilled agents distribution the link is very similar:

$$(6) \qquad P(\theta) = \begin{cases} \dfrac{(1-\alpha)\Phi(\theta) + \alpha \displaystyle\int_{\underline{\theta}_s}^\theta (G(x_m) - G(u)) \varphi(u) du}{(1-\alpha+\alpha v_m)} & \text{if} \quad \theta \in [\underline{\theta}_s; x_m] \\ \dfrac{(1-\alpha)\Phi(\theta) + \alpha v_m}{(1-\alpha+\alpha v_m)} & \text{if} \quad \theta \in [x_m; \bar{\theta}_s] \end{cases}$$

[14]The result of Lemma 3 hinges upon the assumption of linear solving cost function. Although simplifying this assumption does not affect our qualitative results. A more detailed discussion of its implications is provided in Section 7.1.

where $\Phi(\theta)$ is the cumulative distribution function for managers, $P(\theta)$ is the cumulative distribution function of unemployed skilled.

Lemma 4. *Insiders distributions of talent dominate in the sens of first order stochastic dominance the corresponding outsiders' distributions.*

The result of Lemma 4, corresponds to what we call the selection effect of performance standard. In each period agents with low competence are detected and fired, they are replaced by agents from the labor market. In the pool of unemployed there are agents that have been fired, but also agents that have quit their job for exogenous reasons. Thus there is an improvement of insiders' distribution.

It is immediate to notice that the exogenous turn-over affects principal's selection capacity. For given quality of outsiders' distribution, very intuitively larger α (smaller turn over) improves insiders' selection. Larger proportion of selected insiders stay in the organization.

3 Principals' problem

3.1 The program

A firm owner is "too small" to have an individual impact on market conditions. So each of them maximizes her profit for given outside options and outsiders' distributions, then reservation utility of workers and managers and insiders' and outsiders' distributions are determined consistently in general equilibrium. In this section we present the trade-offs each principal faces when choosing performance standards, then we discuss the impact of market and technological conditions on her optimal strategy.

A principal maximizes her profit[15],

$$\max_{x_w,x_m,n_w,s_w,s_m,n_m} \Pi = [n_w(\theta^P - s_w) - s_m n_m] \tag{7}$$

[15] s_i are real wages.

under the constraints:

$$
(8) \qquad \begin{cases} n_w t_w(x_w) = n_m \\ n_w t_m(x_m) = 1 \\ s_m = \dfrac{c(1-\delta\alpha(1-Z(x_m)))}{\delta\alpha} + (1-\delta)V_U(s) \\ s_w = \dfrac{c(1-\delta\alpha(1-x_w))}{\delta\alpha} + (1-\delta)V_U(u) \end{cases}
$$

and given that:

$$
(9) \qquad Q(\theta) = \begin{cases} \dfrac{(1-\alpha)F(\theta) + \alpha \displaystyle\int_0^\theta (x_w - u) f(u) du}{(1-\alpha+\alpha v_w)} & \text{if } \theta \in [0; x_w] \\ \dfrac{(1-\alpha)F(\theta) + \alpha v_w}{(1-\alpha+\alpha v_w)} & \text{if } \theta \in [x_w; \bar{\theta}_u] \end{cases}
$$

$$
(10) \qquad P(\theta) = \begin{cases} \dfrac{(1-\alpha)\Phi(\theta) + \alpha \displaystyle\int_{\underline{\theta}_s}^\theta (Z(x_m) - Z(u))\varphi(u) du}{(1-\alpha+\alpha v_m)} & \text{if } \theta \in [\underline{\theta}_s; x_m] \\ \dfrac{(1-\alpha)\Phi(\theta) + \alpha v_m}{(1-\alpha+\alpha v_m)} & \text{if } \theta \in [x_m; \bar{\theta}_s] \end{cases}
$$

For given values of outside options ($V_U(u)$ and $V_U(s)$) and outsiders' distributions ($Q(\theta)$ and $P(\theta)$) each principal chooses performance standards, wages and number of agents to be hired on each level. As it has been discussed throughout Section 2: for given characteristics of the environment, when a principal chooses performance standards she determines the expected production, the number of agents to be hired on each level, their skill characteristics and the corresponding incentive compatible wages. Thus the principals' program is equivalent to:

$$
\max_{x_w, x_m} \Pi = [n_w(x_m)(\theta^P - s_w(x_w)) - s_m(x_w, x_m) n_m(x_w, x_m)]
$$

First order condition for managers' performance standard:

$$
\frac{\partial \Pi}{\partial x_m} = \underbrace{\frac{\partial n_w}{\partial x_m}(\theta^P - s_w)}_{+} - \underbrace{\frac{\partial n_m}{\partial x_m} s_m}_{-} - \underbrace{\frac{\partial s_m}{\partial x_m} n_m}_{-} = 0 \Leftrightarrow \frac{(1-v_m^{'})}{t_m}(\theta^P - s_w - s_m t_w) - c t_w = 0
$$

The marginal cost of higher performance standard is an increase of managers' wage. The marginal gain is related to an increase of the number of problems arriving in the organization. We notice that the effective outcome of an increase of x_m on the firm's production depends on managers' distribution of skills through v'_m. Furthermore the marginal gain of an additional problem arriving in the organization is $(\theta^P - s_w - s_m t_w)$. When the principal's solving capacity (θ^P) is larger, or workers' or managers' wage is lower, then increasing the number of problems arriving in the firm is more profitable.

First order conditions for workers' performance standard:

$$\frac{\partial \Pi}{\partial x_w} = \underbrace{-n_w \frac{\partial s_w}{\partial x_w}}_{-} \underbrace{- \frac{\partial n_m}{\partial x_w} s_m}_{+} \underbrace{- \frac{\partial s_m}{\partial x_w} n_m}_{+} = 0 \Leftrightarrow -c + (1 - v_w^{'}) s_m + (1 - v_w^{'}) c t_w = 0$$

For given x_m the proportion of problems solved on the workers' level, does not directly affect the firm's output. However the gain of employing unskilled workers consists in modifying the total wage bill. Indeed there are two gains of higher performance standard on the workers' level: first, less problems arrive to the managers' level thus less skilled agents are hired, and second, the probability for a manager to spend effort is lower, which reduces the incentive compatible wage. The marginal cost of higher x_w is an increase of workers' wages.

Assumption 1. $\frac{\partial^2 v_w}{\partial x_w^2} > 1$ *and* $\frac{\partial^2 v_m}{\partial x_m^2} > 1$*, for the equilibrium values of* x_w *and* x_m*.*

Lemma 5. *Other things being equal, higher* x_m *(*x_w*) makes more valuable an increase in* x_w *(*x_m*).*

When more problems are solved by workers (higher x_w), it decreases the number of managers required to supervise them (smaller proportion of problems arriving on the managers' level), which reduces the cost of setting higher x_m. Conversely higher x_m, makes more valuable an increase in x_w.

Assumption (1) guarantees low complementarity of performance standards.

3.2 The effect of environment on principal's choice of performance standards

We focus our attention on how a principal's choice in setting x_w and x_m depends on organization's technological (solution cost, quality of firm owners' technology) or labor market environment (outside options and outsiders' distributions). In Section 4 we discuss the equilibrium effects of such evolutions.

Proposition 1. ***Comparative statics on*** x_w ***and*** x_m

1. *Both performance standards (x_w and x_m) increase if (i) the principal's solving capacity (θ^P) increases, (ii) or effort cost (c) decreases, (iii) or the unskilled agents' outside option ($V_U(u)$) decreases.*

2. *x_w decreases and x_m increases for (i) lower managers' outside option ($V_U(s)$)*

Higher θ^P makes more valuable an increase of the number of problems arriving in the organization (a larger proportion of them would be solved). Thus a more knowledgeable firm-owner is more demanding with her managers. In turn higher x_m, increases the marginal gain of increasing workers' autonomy, thus both performance standards increase.

A decrease in workers' outside option has similar effect. The cost of employing an additional worker decreases, which raises the marginal net gain of an additional problem being solved. Once again the effect on workers' performance standard passes through complementarity of x_w and x_m.

Lower outside option for skilled agents increases the marginal gain from hiring an additional worker. Simultaneously it decreases the marginal gain from higher productivity on the workers' level. Thus the direct effects on performance standards are as follows: x_m increases and x_w decreases. Under assumption 1 direct effects always dominate.

4 Market Equilibrium

Market equilibrium occurs when each firm taking as given wages and employment in other firms finds it optimal to propose the ongoing wages than other ones. Key market variables for individual firm behavior are workers' and managers' outside options, and outsiders' skill distributions. We now turn to the determination of the equilibrium values for those variables.

Outside options. While unemployed each agent receives some (exogenously given) unemployment compensation contingent on his occupation before being fired ($\underline{s_i}$ is the compensation for an agent fired from the level i with $i = \{w; m\}$). At the end of a period there is an endogenous probability to find a new job and quit the unemployment pool: p is that probability for an unskilled agent, q for a skilled one. Thus the inter-temporal utility of an unemployed unskilled (respectively skilled) writes:

$$V_U(u) = \underline{s_w} + \delta(pV(w) + (1-p)V_U(u)) \tag{11}$$

$$V_U(s) = \underline{s_m} + \delta(qV(m) + (1-q)V_U(s)) \tag{12}$$

As our attention is focused on steady state equilibria, job acquisition probability is such that the equilibrium of flows between the firms and the labor market is guaranteed.

On the market of unskilled agents: at each period the flow into the unemployment pool is $n_w(1-\alpha+\alpha v_w)$, corresponding to endogenous departures ($n_w \alpha v_w$) and exogenous quits $n_w(1-\alpha)$. The flow out is $p(N_u - n_w)$. In the population of unemployed[16] $(N_u - n_w)$ a proportion p of the agents is hired to replace departures. To guarantee the equilibrium of flows quits and entries must be equal, thus the probability for an unskilled agent to quit the unemployment pool is $p = \dfrac{n_w(1-\alpha+\alpha v_w)}{(N_u - n_w)}$. The same analysis applies for unemployed skilled. Their re-employment probability is: $q = \dfrac{n_m(1-\alpha+\alpha v_m)}{(N_s - n_m)}$.

Lower N_j, or larger n_i increase the corresponding probability to be re-employed, which increases the outside option. Higher exogenous $(1-\alpha)$ or endogenous (v_i) turn-over, also tend to increase outside options, and in fine agent's reward. Further-

[16] We made the choice to consider that an agent that quits the organization can not be immediately reemployed. We notice however that it is a convention and it doesn't affect our qualitative results.

more it is immediate to see that "no shirking" is incompatible with full employment, thus our attention will be focused on unemployment equilibria.

Skill distributions. As discussed in section 2.2.3 the initial distributions of talents are split into two distributions - inside the firm and on the labor market. However the distribution of talents for the overall population of unskilled (skilled) agents doesn't change. In stationary market equilibrium the following conditions apply:

$$U(\theta) = \frac{n_w}{N_u} F(\theta) + \frac{N_u - n_w}{N_u} Q(\theta) \tag{13}$$

$$S(\theta) = \frac{n_m}{N_s} \Phi(\theta) + \frac{N_s - n_m}{N_s} P(\theta) \tag{14}$$

Note that principal's selection capacity is constrained by labor market conditions. Lower unemployment (larger $\frac{N_j - n_i}{N_j}$), makes harder insiders' selection. As more agents are employed their distribution is closer to the initial one. Worse insiders' distribution (in the sens of first order stochastic dominance), means lower real productivity for insiders (v_i increases[17]).

When a principal decides to increase the number of employees hired on a given level of the hierarchy, she exercises double negative externality on other firm owners - by increasing outside options and deteriorating outsiders' distributions. The effect is both on wage cost and agent's productivity level.

Definition 1. *A steady state market equilibrium is a vector* $(x_w, x_m,\ n_w n_m,\ s_w, s_m,$ $F(\theta), \Phi(\theta),\ P(\theta), Q(\theta),\ V_U(u), V_U(s))$*, in which the sub-vector* $(x_w, x_m,\ n_w, n_m,\ s_w, s_m,$ $F(\theta), \Phi(\theta))$ *maximizes 7 subject to 8, 9 and 10,* $(V_U(u), V_U(s))$ *solve 11 and 12, and* $(Q(\theta), P(\theta))$ *solve 13 and 14.*

Lemma 6. *If the equilibrium exist and if* $v_w^{''} > 0$ *and* $v_m^{''} > 0$*, for any* x_w *and* x_m*, that equilibrium is unique.*

[17] The proof is in Appendix 7.4

5 Comparative statics

In this section we discuss equilibrium effects of environmental change. We consecutively consider the consequences of evolution in institutional, labor supply and technological variables on corporate structure, on demand for skilled and unskilled labor force and on wage evolutions.

5.1 Institutions

By institutions we mean labor market regulation, such as unemployment benefits and minimum wage.

5.1.1 Unemployment benefits

Unemployment compensation of unskilled agents.
A variation of the unemployment compensation affects the unskilled agents' outside option, thus affecting the wage cost of this population.

Proposition 2. ***For lower $\underline{s}_w$:***

1. *The number of workers and managers is higher, organizations are larger*[18].
2. *Workers are more autonomous i.e. the managers' span of control*[19] *increases.*
3. *Workers' wage decreases, managers' wage and firm profit increase.*

Lower $\underline{s}_w$ decreases $V_U(u)$. As shown in proposition 1 the consequence is an increase of both performance standards. Cheaper unskilled labor force makes more valuable an increase of the number of problems the firm deals with, thus more workers are hired. By complementarity, the principal also increases workers' performance standard. Workers are more autonomous and each manager can help a larger number of them (span of control increases).

As unskilled labor force becomes less expensive the demand for it increases. There are more workers in the organization, but also they are more autonomous.

[18] "firm size" corresponds to the total number of employees in the organization ($N = n_w + n_m$).
[19] "Span of control" is defined as the number of employees per manager: $span = \frac{n_w}{n_m}$.

As far as the latter is an indirect effect, it dominates the former and the demand for managers increases.

Unemployment compensation of skilled agents.
As claimed by Proposition (1) a decrease in the skilled agents' outside option leads a principal to increase x_m and decrease x_w. The following proposition summarizes the equilibrium effects on the organization's structure and equilibrium wages.

Proposition 3. ***For lower*** $\underline{s}_m$***:***

1. *The number of managers increases. The effect on the number of workers is ambiguous.*

2. *Span of control decreases.*

3. *Wages on both levels decrease, principal's profit is higher.*

As the cost of skilled labor force is lower, the demand for managers increases. Formal decision making is more decentralized in the sens that lower proportion of problems should arrive on the principal's level. In terms of real decision making - things are less clear. As the demand for managers increases significantly (more workers are hired and they solve less problems), it deteriorates the distribution of skilled insides, which increases v_m. When the latter effect is sufficiently strong managers' real productivity decreases. In these situations less workers are employed in the firms (see Figure **??** for examples). GRAPHS

5.1.2 Minimum wage

Introduction of a minimum wage is an important and contested feature of labor market regulation. The effect of such measure on (un)employment remains very controversial question. The traditional analysis claims that if the price of labor is artificially risen to a level higher than the equilibrium one there will be an increase of unemployment. However there are also theoretical models in which the effect of such measure is less clear. In matching models for example, the introduction of minimum wage has two effects: the cost of labor is higher thus less vacations are created, the expected wage increases, agents increase their search effort, which increases the matching probability. The final consequence for unemployment is ambiguous.

The employment effect of wage floor is also an important and puzzling question for a very large range of empirical studies. Some of them support the existence of zero or positive employment effect[20], others find out a significant negative effect[21].

In this section we discuss the consequences of minimum wage introduction on the organization characteristics in terms of task repartition and span of control, but also its consequences on wages and unemployment. Our model enables us to analyze the impact of minimum wage on the population directly concerned by the measure (here unskilled agents) but also on the other actors of the economy (here skilled agents and firm owners).

We introduce a constraining for the firm owners minimum wage (i.e. $s_w^* < s_{min}$). Thus each principal chooses x_w such that $s_w(x_w) = s_{min}$, which implies an increase (when possible) of the performance standard on the workers' level.

Proposition 4. ***Unskilled (un)employment:***

1. *If the principal* ***can not*** *increase x_w, the number of workers in the organization always decreases.*

2. *If the principal* ***can*** *increase x_w, the effect on unskilled unemployment is ambiguous.*

The introduction or increase of a wage floor reduces the marginal gain of employing more workers. In the case of binding workers' capacity, it is the only effect of minimum wage introduction. The demand for unskilled decreases.
However when a principal can respond to a wage floor introduction by increasing workers' performance standard, there is an additional effect playing in the opposite sens. Actually higher x_w makes less costly an increase of x_m. If the latter effect is sufficiently strong then demand for unskilled agents increases. On figure **??** we present few examples for which the positive effect dominates the negative one, and workers employment increases. GRAPH

[20]Card and Krueger (1994,1995,2000) for the United States, Machin and Manning (1994) and Dickens and al. (1999) for the United Kingdom

[21]Kim and Taylor (1995), Neumark et al. (2000) for the United States

Proposition 5. ***The effect on skilled agents employment and wages***
When a minimum wage is introduced the employment of skilled agents decreases, and the managers' remuneration is lower.

In the case a principal can not increase x_w when a minimum wage is introduced, it reduces the marginal gain of larger organization - thus decreasing both the number of workers and managers. In the case a principal can adjust the performance standard in order to reach the minimum wage level, the reduction of the number of managers is due to a transfer of tasks form managers to workers.

Our result is consistent with the facts related by Acemoglu and Newman (2001). They notice that in countries with higher minimum wage the proportion of managers in the organizations is lower.

DiNardo and al. (1996) show in empirical study based on U.S. data that a decrease of real value of the minimum wage explains a substantial proportion of the increase in wage inequality in US. In our model a reduction or withdrawal of a minimum wage increases inequality for two reasons: first workers' wages decrease, and second managers' wages increase. There is a positive spillover effect of minimum wage withdrawal on wages above the minimum.

5.2 Labor market conditions

5.2.1 When unskilled supply changes.

For higher N_u

1. Organizations are with larger number of workers and smaller number of managers. The span of control increases (i.e. more autonomous workers).

2. Wages decrease, firm profits increase.

When the number of unskilled increases it affects simultaneously the cost and quality of the unskilled distribution. The consequence is an increase of both performance standards. Larger proportion of problems is solved before the principal's level, thus more workers are hired. On the managers' level we find two effects - larger number of employees, but more autonomous. The latter seems to be stronger for very large set of parameters. Indeed higher N_u improves insiders distribution of skills. Thus the proportion of problems solved on the workers level increases both because the performance standard is higher, and because agents are better selected.

5.2.2 When skilled's supply changes

For higher (N_s):

1. Organizations are larger - more workers and managers are hired. Span of control is smaller.

2. The proportion of problems solved on the workers' level decreases, larger proportion of problems is solved by the managers.

3. Wages decrease, so is the wage-ratio. Firm profits are higher.

When the mass of skilled agents increases it decreases their outside option, but also allows better selection. Each principal increases the managers' performance standard and becomes less demanding with workers. More managers are hired, so it is on the workers' level. The wage-ratio is lower. Once again we note that this results are related to the effect that we limit our interest to evolutions of this organizational structure, and rule out considerations of some shift in the structure. If the increase supply of skilled agents is sufficiently high we could observe a shift in the organizational structure, from three layer to two layer hierarchy, with only skilled agents in the organization. Then the increase of the number of skilled is accompanied by an increase of qualified agents' wages and increase of skilled/unskilled inequalities.

5.3 Technical change

Principal's solving capacity.
θ^P can be interpreted as principal's problem solving capacity, it could also be related to the quality of a technology acquired by a principal.

Proposition 6. ***For better firm owners' technology (i.e. higher θ^P):***

1. *Organizations are larger - more workers and managers are hired. The span of control increases.*

2. Managers' and workers' wages increase.

3. The effect on firm's profit is ambiguous.

Better problem solving capacity for a principal makes more valuable an increase of the problems arriving in the organization, thus increasing demand for unskilled agents. The demand for skilled one also increases because more problems arrive in the organization and even that workers are more autonomous, the number of managers necessary for helping them increases. Wages are higher both for workers and managers, however the wage inequality measured by $\frac{s_m}{s_w}$ increases.
The ambiguity of profits evolution hinges upon the externalities a principal exercices when she increases the number of agents hired on both levels of the hierarchy. Higher employment increases outside options ($V_U(j) \nearrow$ for $j = \{u; s\}$), thus increasing cost of incentives. Furthermore insiders are less selected (i.e. lower unemployment worsens insiders' distribution), and their real productivity is lower. Thus we can exhibit situations in which the increase of organizations' size is accompanied by profits decrease.

6 Conclusion

7 Appendix

7.1 Discussion of: the type independence of utility functions

As it is mentioned in the text, the inter-temporal utility of an agent (insider or outsider) does not depend on the agent's skill level. In our setting it is due on the particular cost function we adopt. Here we discuss this assumption.

Let us consider the case of slightly different effort cost function:

$$c^T = \begin{cases} c(\theta) & if \quad x \leq \theta \\ +\infty & if \quad x > \theta \end{cases}$$

with $c'(\theta) < 0$.

In this case the incentive compatible wage for a worker of type θ, writes:

$$s_w \geq \frac{c(\theta)(1-\alpha\delta(1-x_w))}{\alpha\delta} + (1-\delta)V_U(\theta;u)$$

As $c'(\theta) < 0$, it is easy to show that the incentive compatible wage is increasing with the performance standard and decreasing with θ. It is less costly to provide incentives to more knowledgeable agents[22]. Thus in this case a principal should choose not only the performance standard, but also the smaller θ for which it's worth providing incentives.

However if we consider that problem solving cost is decreasing with agent's knowledge, then it decreases the gains from employing unskilled agents, the incentive cost on their level being potentially higher than the one of skilled agents. So in this case a three layer hierarchy becomes less profitable.

7.2 Stationary distributions

Stationarity of the workers' distribution. On a steady state equilibrium, at the beginning of each period workers' repartition function is $F(\theta)$. At the end of the period there are departures for exogenous reasons $(1-\alpha)$, and quits of fired agents. Thus at the end of the period there will be a modified repartition function $\tilde{F}(\theta)$. Before the beginning of the next period, the principal hires new agents to replace the quits. Workers are hired from the distribution of unemployed unskilled agents, which repartition function is $Q(\theta)$. The equilibrium is stationary if the distribution of insiders at the end of the period exactly corresponds to the initial one.

Thus stationarity condition writes:

$$\underbrace{(1-\alpha+\alpha v_w)}_{\text{Proportion of hired agents}} Q(u) + \underbrace{\alpha(1-v_w)}_{\text{Proportion of insiders that doesn't quit.}} \tilde{F}(u) = F(u)$$

Here is the repartition function after agents are dismissed and relocated:

[22] If $c'(\theta) > 0$, than it is more costly to provide incentives to more knowledgeable agents. The incentive compatible wage in this case would be $s_w \geq \frac{c(x_w)(1-\alpha\delta(1-x_w))}{\alpha\delta} + (1-\delta)V_U(u)$. It is immediate to notice that it doesn't depend on θ.

$$\tilde{F}(u) = \begin{cases} \dfrac{\int_0^u (1-(x_w-\theta))f(\theta)d\theta}{1-v_w} & if \quad u \in [0; x_w] \\ \dfrac{F(u)-v_w}{1-v_w} & if \quad u \in [x_w; \bar{\theta}_u] \end{cases}$$

Thus we have the following link between the insiders' and outsiders' distribution:

$$Q(u) = \begin{cases} \dfrac{F(u)(1-\alpha)+\alpha \int_0^u (x_w-\theta)f(\theta)d\theta}{1-\alpha+\alpha v_w} & if \quad u \in [0; x_w] \\ \dfrac{F(u)(1-\alpha)+\alpha v_w}{1-\alpha+\alpha v_w} & if \quad u \in [x_w; \bar{\theta}_u] \end{cases}$$

Stationarity of the managers' distribution. We apply the previous analysis.

Flows are:

- Departures :
  - Fired managers - $\alpha v_m n_m$
  - Exogenous departures - $(1-\alpha)n_m$
- Arrivals :
  - Agents hired in the pool of unemployed skilled in order to replace exogenous and endogenous departures.s

Thus we have the following link between the distributions of managers and unemployed skilled:

$$P(\theta) = \begin{cases} \dfrac{\Phi(u)(1-\alpha)+\alpha \int_{\underline{\theta}_s}^{\theta} (x_m-u)\varphi(u)du}{1-\alpha+\alpha v_m} & if \quad \theta \in [\underline{\theta}_s; x_m] \\ \dfrac{(1-\alpha)\Phi(\theta)+\alpha v_m}{1-\alpha+\alpha v_m} & if \quad \theta \in [x_m; \bar{\theta}_s] \end{cases}$$

Stationary distributions of outsiders Here we check that the conditions for stationarity of outsiders distributions are the same as the condition of insiders' one.

The pool of unemployed unskilled:

- Quits:

– Quits are corresponding to the number of persons that are hired on each period: $n_w(1-\alpha+\alpha v_w)$, with repartition function $Q(\theta)$.

- Entries:

  – Exogenous departures $n_w(1-\alpha)$, which repartition function is $F(\theta)$.

  – Fired agents $n_w \alpha v_w$, with distribution function:

$$\begin{cases} \dfrac{\int_0^\theta (x_w-\theta) f(\theta) d\theta}{v_w} & \text{if} \quad \theta \leq x_w \\ 1 & \text{if} \quad \theta > x_w \end{cases}$$

Thus stationarity condition of unemployed unskilled writes:

$$\tilde{N}_u Q(\theta) = (\tilde{N}_u - n_w(1-\alpha+\alpha v_w)Q(\theta) + n_w(1-\alpha)F(\theta) + n_w \alpha v_w \begin{cases} \dfrac{\int_0^\theta (x_w-\theta) f(\theta) d\theta}{v_w} & \text{if} \quad \theta \leq x_w \\ 1 & \text{if} \quad \theta > x_w \end{cases}$$

Which is equivalent to:

$$Q(\theta) = \begin{cases} \dfrac{(1-\alpha)F(\theta) + \alpha \int_0^\theta (x_w - u) f(u) du}{(1-\alpha+\alpha v_w)} & \text{if} \quad \theta \in [0; x_w] \\ \dfrac{(1-\alpha)F(\theta) + \alpha v_w}{(1-\alpha+\alpha v_w)} & \text{if} \quad \theta \in [x_w; \bar{\theta}_u] \end{cases}$$

The same analysis is applied to the distribution of unemployed skilled agents.

7.3 Insiders are better

We show that $Q(\theta) \geq F(\theta)$ for any θ and $Q(\theta) > F(\theta)$ for at least some θ (the proof is very similar for $P(\theta) > \Phi(\theta)$).

In the case of $\theta \geq x_w$:

$$Q(\theta) - F(\theta) = \frac{(1-\alpha)F(\theta) + \alpha v_w}{1-\alpha+\alpha v_w} - F(\theta) = \frac{\alpha v_w (1 - F(\theta))}{1-\alpha+\alpha v_w}$$

It is immediate to see that $\frac{\alpha v_w(1-F(\theta))}{1-\alpha+\alpha v_w} > 0$, for any $\theta < \overline{\theta}$, and $\frac{\alpha v_w(1-F(\theta))}{1-\alpha+\alpha v_w} = 0$ for $\theta = \overline{\theta}$.

If $\theta < x_w$:

$$Q(\theta) - F(\theta) = \frac{(1-\alpha)F(\theta) + \alpha\int_0^\theta (x_w - u)f(u)du}{1-\alpha+\alpha v_w} - F(\theta) = \frac{\alpha\int_0^\theta (x_w - u - v_w)f(u)du}{1-\alpha+\alpha v_w}$$

$\int_0^\theta (x_w - u - v_w)f(u)du$ increases with θ when $\theta < x_w - v_w$ and decreases with θs beyond this value. Thus if $\int_0^\theta (x_w - u - v_w)f(u)du \geq 0$ for $\theta = 0$ and $\theta = x_w$, then it will be positive for any θ of the interval we are interested in.

For $\theta = x_w$, we have $\int_0^{x_w} (x_w - u - v_w)f(u)du = (1 - F(x_w))v_w > 0$. For $\theta = x_w$, we have $\int_0^0 (x_w - u - v_w)f(u)du = 0$.

QED

7.4 Selection effects of higher unemployment

All proofs are for the case of unskilled agents, for skilled the same arguments apply.

The impact on outsiders' distribution

- For $\theta \geq x_w$.

We have:

$$\begin{cases} F(\theta) = \frac{Q(\theta)(1-\alpha+\alpha v_w) - \alpha v_w}{(1-\alpha)} \\ \frac{n_w}{N_u}F(\theta) + \frac{N_u - n_w}{N_u}Q(\theta) = U(\theta) \end{cases}$$

We obtain $Q(\theta) = \frac{U(\theta)N_u(1-\alpha) + \alpha v_w n_w}{N_u(1-\alpha) + \alpha v_w n_w}$, and $-\frac{\partial Q(\theta)}{\partial N_u} = \frac{(1-\alpha)\alpha v_w n_w(1-U(\theta))}{(N_u(1-\alpha) + \alpha v_w n_w^2)} < 0$ for $\theta < \overline{\theta}$, and $-\frac{\partial Q(\theta)}{\partial N_u} = 0$ for $\theta = \overline{\theta}$.

- For $\theta \leq x_w$.

$$\begin{cases} f(\theta) = \frac{q(\theta)(1-\alpha+\alpha v_w)}{(1-\alpha+\alpha(x_w-\theta))} \\ \frac{n_w}{N_u}f(\theta) + \frac{N_u - n_w}{N_u}q(\theta) = u(\theta) \end{cases}$$

We obtain $q(\theta) = \dfrac{u(\theta)N_u(1-\alpha+\alpha(x_w-\theta))}{N_u(1-\alpha+\alpha(x_w-\theta)) - \alpha n_w(x_w-\theta-v_w)}$, thus $Q(\theta)$ writes:

$$Q(\theta) = \int_0^\theta \frac{u(t)N_u(1-\alpha+\alpha(x_w-t))}{N_u(1-\alpha+\alpha(x_w-t)) - \alpha n_w(x_w-t-v_w)}dt$$

$$\Rightarrow \frac{\partial Q(\theta)}{\partial N_u} = \int_0^\theta \frac{u(t)(1-\alpha+\alpha(x_w-t))(-n_w\alpha(x_w-t-v_w))}{(N_u(1-\alpha+\alpha(x_w-t)) - \alpha n_w(x_w-t-v_w))^2}dt$$

The latter expression decreases with θ for $\theta < x_w - v_w$, and increases with θ for $\theta > x_w - v_w$. The maximal values attained by this function are for $\theta = 0$ and $\theta = x_w$. For $\theta = 0$ $\dfrac{\partial Q(0)}{\partial N_u} = 0$, and for $\theta = x_w$ $\dfrac{\partial Q(x_w)}{\partial N_u} = \dfrac{(1-\alpha)\alpha v_w n_w(1-U(x_w))}{(N_u(1-\alpha)+\alpha v_w n_w^2)} < 0$. So $\dfrac{\partial Q(\theta)}{\partial N_u} < 0$, for any $\theta \in [0, x_w]$.

So when the number of unskilled agents increases it improves the outsiders' distribution in the sens of first order stochastic dominance.

For the next two proofs we apply:

Theorem: If $\widetilde{Q}(\theta) \succ_1 Q(\theta)$, then for any strictly increasing function u, $\int u d\widetilde{Q}(\theta) > \int u dQ(\theta)$.

Remark: $\widetilde{Q}(\theta) \succ_1 Q(\theta)$ means $\widetilde{Q}(\theta) \leq Q(\theta)$ for any θ, and $\widetilde{Q}(\theta) < Q(\theta)$ for at least some θ.

$\dfrac{\partial v_w}{\partial N_u} < 0$

$v_w = \displaystyle\int_0^{x_w} f(\theta)(x_w-\theta)d\theta = (1-\alpha+\alpha v_w)\int_0^{x_w} q(\theta)\frac{(x_w-\theta)}{1-\alpha+\alpha(x_w-\theta)}d\theta$. We first notice that $\dfrac{(x_w-\theta)}{1-\alpha+\alpha(x_w-\theta)}$ is decreasing with θ. As shown above $\dfrac{\partial Q(\theta)}{\partial N_u} < 0$. We notice $\widetilde{Q}(\theta)$ the distribution of outsiders' talents after an increase of N_u, $\widetilde{Q}(\theta) \succ_1 Q(\theta)$. As $\dfrac{(x_w-\theta)}{1-\alpha+\alpha(x_w-\theta)}$ is decreasing function: $(1-\alpha+\alpha v_w)\displaystyle\int_0^{x_w} q(\theta)\frac{(x_w-\theta)}{1-\alpha+\alpha(x_w-\theta)}d\theta > (1-\alpha+\alpha v_w)\int_0^{x_w} \widetilde{q}(\theta)\frac{(x_w-\theta)}{1-\alpha+\alpha(x_w-\theta)}d\theta$. QED

The impact on v_w' The direct effect on variation of the proportion of fired workers increases with N_u.

7.5 Complementarity

$$\frac{\partial^2\Pi}{\partial x_w^2} = -\frac{1}{t_m}(v_w^{''}(s_m + ct_w) + 2(1 - v_w^{'})^2 c)$$
$$\frac{\partial^2\Pi}{\partial x_m^2} = -\frac{1}{t_m^2}v_m^{''}(1 - s_w - t_w s_m)$$
$$\frac{\partial^2\Pi}{\partial x_w \partial x_m} = \frac{1}{t_m}(1 - v_w^{'})c$$

The variation of performance standards, is given by:

$$\frac{\partial x_w}{\partial x_m} = -\frac{\frac{\partial^2\Pi}{\partial x_w \partial x_m}}{\frac{\partial^2\Pi}{\partial x_w^2}} = \frac{(1 - v_w^{'})c}{(v_w^{''}(s_m + ct_w) + 2(1 - v_w^{'})^2 c)}$$
$$\frac{\partial x_m}{\partial x_w} = -\frac{\frac{\partial^2\Pi}{\partial x_w \partial x_m}}{\frac{\partial^2\Pi}{\partial x_m^2}} = \frac{(1 - v_w^{'})ct_m}{v_m^{''}(1 - s_w - t_w s_m)}$$

For $v_w^{''} > 0$ and $v_m^{''} > 0$, both are positive.

7.6 Proof of Proposition 1

Second order conditions.

$$J = \begin{pmatrix} \Pi_{x_w,x_w} & \Pi_{x_w,x_m} \\ \Pi_{x_m,x_w} & \Pi_{x_m,x_m} \end{pmatrix}$$

The second order conditions are verified iff:

$$\begin{cases} \Pi_{x_w,x_w} + \Pi_{x_m,x_m} < 0 \\ |J| > 0 \end{cases}$$

$$\Pi_{x_w,x_w} + \Pi_{x_m,x_m} = -\frac{1}{t_m}(v_w^{''}(s_m + ct_w) + 2(1 - v_w^{'})^2 c) - \frac{1}{t_m^2}v_m^{''}(1 - s_w - t_w s_m)$$

$$|J| = \Pi_{x_w,x_w}\Pi_{x_m,x_m} - (\Pi_{x_m,x_w})^2 =$$
$$\frac{1}{t_m^3}(v_w^{''}(s_m + ct_w) + 2(1 - v_w^{'})^2 c)(v_m^{''}(1 - s_w - s_m t_w)) - \frac{1}{t_m^2}(1 - v_w^{'})^2 c^2 =$$
$$\frac{1}{t_m^2}(\frac{1}{t_m}(v_w^{''}(s_m + ct_w) + 2(1 - v_w^{'})^2 c)(v_m^{''}(1 - s_w - s_m t_w))) \geq$$
$$\frac{1}{t_m^2}((v_w^{''}(s_m + ct_w) + 2(1 - v_w^{'})^2 c)(v_m^{''}))$$

Under the assumption $v_w^{''} > 1$ and $v_m^{''} > 1$. It is easy to check that both conditions are verified.

To proof Proposition 1 we apply the following implicit function analysis:

$$\begin{cases} \Pi_{x_w}(x_w, x_m; a_1...a_n) = 0 \\ \Pi_{x_m}(x_w, x_m; a_1...a_n) = 0 \end{cases}$$

Where $a_1...a_n$ are parameters that affect the solution of the system of two equations.

To determine the sens of variation of x_i, with the parameter a_k. We first write:

$$\begin{pmatrix} \Pi_{x_w,x_w} & \Pi_{x_w,x_m} \\ \Pi_{x_m,x_w} & \Pi_{x_m,x_m} \end{pmatrix} \begin{pmatrix} \frac{\partial x_w}{\partial a_k} \\ \frac{\partial x_m}{\partial a_k} \end{pmatrix} = - \begin{pmatrix} \Pi_{x_w,a_k} \\ \Pi_{x_m,a_k} \end{pmatrix}$$

Using the Cramer rule we have:

$$\frac{\partial x_i}{\partial a_k} = -\frac{|J_i|}{|J|}$$

where J_i is the matrix obtained by replacing the ith column of the Jacobian matrix J with the vector $(\Pi_{x_w,a_k}, \Pi_{x_m,a_k})^T$.

Variation of performance standards with θ^P

$$\frac{\partial x_w}{\partial \theta^P} = -\frac{\begin{vmatrix} \Pi_{x_w,\theta^P} & \Pi_{x_w,x_m} \\ \Pi_{x_m,\theta^P} & \Pi_{x_m,x_m} \end{vmatrix}}{\begin{vmatrix} \Pi_{x_w,x_w} & \Pi_{x_w,x_m} \\ \Pi_{x_m,x_w} & \Pi_{x_m,x_m} \end{vmatrix}}$$

$$\frac{\partial x_w}{\partial \theta^P} = -\frac{\Pi_{x_w,\theta^P}\Pi_{x_m,x_m} - \Pi_{x_w,x_m}\Pi_{x_m,\theta^P}}{|J|} = -\frac{-\Pi_{x_w,x_m}\Pi_{x_m,\theta^P}}{|J|}$$

As shown above $|J| > 0$, $\Pi_{x_w,x_m} > 0$ and $\Pi_{x_m,\theta^P} = \frac{(1 - v_m^{'})}{t_m} > 0 \Rightarrow \frac{\partial x_w}{\theta^P} > 0$.

$$\frac{\partial x_w}{\partial \theta^P} = -\frac{\Pi_{x_w,x_w}\Pi_{x_m,\theta^P}}{|J|}$$

As $\Pi_{x_w,x_w} < 0$ and $\Pi_{x_m,\theta^P} > 0$, $\dfrac{\partial x_w}{\partial \theta^P} > 0$.

Variation of performance standards with $V_U(u)$

$$\frac{\partial x_w}{\partial V_U(u)} = -\frac{\Pi_{x_w,V_U(u)}\Pi_{x_m,x_m} - \Pi_{x_w,x_m}\Pi_{x_m,V_U(u)}}{|J|} = -\frac{-\Pi_{x_w,x_m}\Pi_{x_m,\theta^P}}{|J|}$$

Which is equivalent to: $\dfrac{\partial x_w}{\partial \theta^P} = \dfrac{\Pi_{x_w,x_m}\Pi_{x_m,V_U(u)}}{|J|}$ As shown in 7.5 $\Pi_{x_w,x_m} > 0$ and $\Pi_{x_m,V_U(u)} = -\dfrac{(1-v_m^{'})(1-\delta)}{t_m} > 0 \Rightarrow \dfrac{\partial x_w}{V_U(u)} < 0$.

$$\frac{\partial x_w}{\partial V_U(u)} = -\frac{\Pi_{x_w,x_w}\Pi_{x_m,V_U(u)}}{|J|}$$

As $\Pi_{x_w,x_w} < 0$ and $\Pi_{x_m,V_U(u)} < 0$, $\dfrac{\partial x_w}{\partial V_U(u)} < 0$.

Variation of performance standards with c

$$\frac{\partial x_w}{\partial c} = -\frac{\Pi_{x_w,c}\Pi_{x_m,x_m} - \Pi_{x_w,x_m}\Pi_{x_m,c}}{|J|}$$

We have

$$\Pi_{x_w,c} = -1+(1-v_w^{'})(\frac{s_m}{c} - \frac{(1-\delta)V_U(s)}{c} + t_w) = -1 + \frac{(s_m - (1-\delta)V_U(s) + t_w c)}{s_m + t_w c} < 0$$

$$\Pi_{x_m,c} = -t_w - (1-v_m^{'})(\frac{\partial s_w}{\partial c} + \frac{\partial s_m}{\partial c}t_w) < 0$$

Thus it is immediate to see that $\dfrac{\partial x_w}{\partial c} < 0$.

The same analysis applies to obtain $\dfrac{\partial x_m}{\partial c} < 0$.

Variation of performance standards with $V_U(s)$

$$\frac{\partial x_w}{\partial V_U(s)} = -\frac{\Pi_{x_w,V_U(s)}\Pi_{x_m,x_m} - \Pi_{x_w,x_m}\Pi_{x_m,V_U(s)}}{|J|}$$

$$-(\Pi_{x_w,V_U(s)}\Pi_{x_m,x_m} - \Pi_{x_w,x_m}\Pi_{x_m,V_U(s)}) = ((1-v_w^{'})\frac{1}{t_m^2}v_m^{''}(1-s_w-t_w s_m) - (1-v_w^{'})\frac{c}{t_m}(1-v_m^{'})\frac{t_w}{t_m}) > 0$$

7.7 The equilibrium program

The system of equations,

$$\begin{cases} \dfrac{\partial \Pi}{\partial x_w} = -\dfrac{\partial s_w}{\partial x_w} n_w - \dfrac{\partial n_m}{\partial x_w} s_m - \dfrac{\partial s_m}{\partial x_w} n_m = 0 \\ \dfrac{\partial \Pi}{\partial x_w} = \dfrac{\partial n_w}{\partial x_m}(\theta^P - s_w) - \dfrac{\partial s_w}{\partial x_m} n_w - \dfrac{\partial n_m}{\partial x_m} s_m - \dfrac{\partial s_m}{\partial x_m} n_m = 0 \\ V_U(\theta; w; u) = \dfrac{\underline{s_w} + (pc)/\alpha}{1-\delta} \\ V_U(\theta; m; s) = \dfrac{\underline{s_m} + (qc)/\alpha}{1-\delta} \\ U(\theta) = \dfrac{n_w}{N_u} F(\theta) + \dfrac{N_u - n_w}{N_u} Q(\theta) \\ S(\theta) = \dfrac{n_m}{N_s} \Phi(\theta) + \dfrac{N_s - n_m}{N_s} P(\theta) \end{cases}$$

can be transformed as follows:

$$\begin{cases} \dfrac{\partial \Pi}{\partial x_w} = -c + (1 - v'_w) s_m(x_w, v_w, x_m) + c(1 - v'_w) t_w(x_w, v_w) = 0 \\ \dfrac{\partial \Pi}{\partial x_m} = (1 - v'_m)(1 - s_w(x_w, v_w) - s_m(x_w, v_w, x_m) t_w(x_w, v_w)) - c t_e(x_w, v_w) t_m(x_m, v_m) = 0 \\ v_w = \displaystyle\int_0^{x_w} \dfrac{u(\theta) N_u (x_w - \theta)(1 - \alpha + \alpha v_w)}{N_u(1 - \alpha + \alpha(x_w - \theta)) - n_w \alpha (x_w - \theta - v_w)} d\theta \\ v_m = \displaystyle\int_{\underline{\theta}_s}^{x_m} \dfrac{s(\theta) N_s (x_m - \theta)(1 - \alpha + \alpha v_m)}{N_s(1 - \alpha + \alpha(x_m - \theta)) - n_m \alpha (x_m - \theta - v_m)} d\theta \end{cases}$$

We know that $v_w = \int_0^{x_w} (x_w - \theta) f(\theta) d\theta$ and $f(\theta) = \frac{u(\theta) N_u (1-\alpha+\alpha v_w)}{N_u(1-\alpha+\alpha(x_w-\theta)) - \alpha n_w (x_w - \theta - v_w)}$. So $v_w = \displaystyle\int_0^{x_w} \frac{u(\theta) N_u (x_w - \theta)(1 - \alpha + \alpha v_w)}{N_u(1 - \alpha + \alpha(x_w - \theta)) - n_w \alpha (x_w - \theta - v_w)} d\theta$. The same applies for v_m.

We now give the expressions corresponding to v'_w and v'_m, and show that they only depend on (x_w, x_m, v_w, v_m). When a principal maximizes her program she considers $q(\theta)$ and $p(\theta)$ as given. Thus $\dfrac{\partial v_w}{\partial x_w} = \dfrac{\partial\Big(\displaystyle\int_0^{x_w} \frac{q(\theta)(x_w - \theta)(1 - \alpha + \alpha v_w)}{1 - \alpha + \alpha(x_w - \theta)} d\theta\Big)}{\partial x_w}$, the latter is equivalent to:

$$\frac{\partial v_w}{\partial x_w} = \frac{1 - \alpha + \alpha v_w}{1 - \alpha} \Big(F(x_w) - \alpha \int_0^{x_w} \frac{q(\theta)(x_w - \theta)(1 - \alpha + \alpha v_w)}{(1 - \alpha + \alpha(x_w - \theta))^2} d\theta \Big)$$

$$\Leftrightarrow \frac{\partial v_w}{\partial x_w} = \int_0^{x_w} \frac{f(\theta)(1 - \alpha + \alpha v_w)}{(1 - \alpha + \alpha(x_w - \theta))} d\theta$$

and

$$\frac{\partial v_m}{\partial x_m} = \frac{1-\alpha+\alpha v_m}{1-\alpha}\Big(\Phi(x_w) - \alpha\int_{\underline{\theta}_s}^{x_m}\frac{p(\theta)(x_m-\theta)(1-\alpha+\alpha v_m)}{(1-\alpha+\alpha(x_m-\theta))^2}d\theta\Big)$$

$$\Leftrightarrow \frac{\partial v_m}{\partial x_m} = \int_{\underline{\theta}_s}^{x_m}\frac{\varphi(\theta)(1-\alpha+\alpha v_m)}{(1-\alpha+\alpha(x_m-\theta))}d\theta$$

7.8 Existence and uniqueness of the equilibrium.

$$\begin{cases} \dfrac{\partial \Pi}{\partial x_w} = -c + (1-v_e')s_m + c(1-v_e')t_e = 0 & (1)\\ \dfrac{\partial \Pi}{\partial x_m} = (1-v_c')(1-s_w-s_m t_e) - ct_e t_c = 0 & (2)\\ v_w = \int_0^{x_w}\dfrac{u(\theta)N_u(x_w-\theta)(1-\alpha+\alpha v_w)}{N_u(1-\alpha+\alpha(x_w-\theta)) - n_w\alpha(x_w-\theta-v_w)}d\theta & (3)\\ v_m = \int_{\underline{\theta}_s}^{x_m}\dfrac{s(\theta)N_s(x_m-\theta)(1-\alpha+\alpha v_m)}{N_s(1-\alpha+\alpha(x_m-\theta)) - n_m\alpha(x_m-\theta-v_m)} & (4) \end{cases}$$

Equation (3) writes:

$$\frac{v_w}{1-\alpha+\alpha v_w} = \int_0^{x_w}\frac{u(\theta)N_u(x_w-\theta)}{N_u(1-\alpha+\alpha(x_w-\theta)) - n_w\alpha(x_w-\theta-v_w)}d\theta$$

It is easy to show that the left hand side is increasing with v_w and that the right hand side is decreasing. Furthermore for $v_w = 0$ the left hand side equals to 0 and the right hand side is positive. Thus the solution of these expression is unique, thus if an equilibrium value exists (i.e. if $v_w \in [0, x_w]$), it will be unique.

Equation (4) writes as follows:

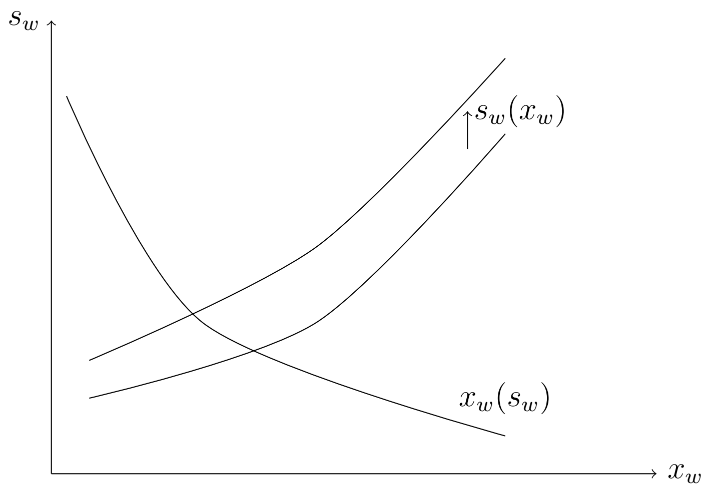
$$\frac{v_m}{1-\alpha+\alpha v_m} = \int_{\underline{\theta}_s}^{x_m}\frac{s(\theta)N_s(x_m-\theta)(1-\alpha+\alpha v_m)}{N_s(1-\alpha+\alpha(x_m-\theta)) - n_m\alpha(x_m-\theta-v_m)}$$

We apply the precedent analysis on it.

Conditions (1) and (2) are the first order condition for the principals' optimization program. To have unique solution for each of this equations we should have $\frac{\partial^2 v_w}{\partial x_w^2} > 0$ and $\frac{\partial^2 v_m}{\partial x_m^2} > 0$ for any x_w and x_m.

7.9 Proof of proposition 2

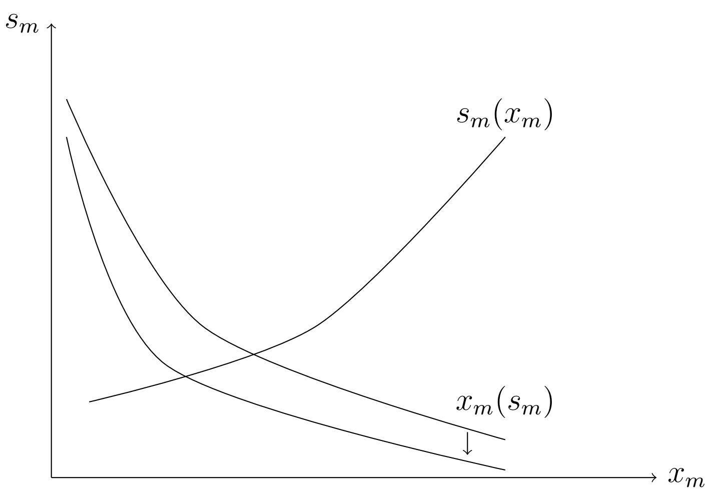
The effect of $\underline{s}_w$ on workers' wage.



Workers' wage s_w, increases with workers' performance standard. At the other side higher s_w, decreases the performance standard x_w (the proof is the same as for $\frac{\partial x_w}{\partial V_U(u)} < 0$).

For given level of x_w, an increase of the unemployment benefit increases the workers' wage. Thus on equilibrium we have $\frac{\partial x_w}{\partial \underline{s}_w} > 0$.

The effect of $\underline{s}_w$ on managers' wage.



Managers' wage increases with their performance standard x_m. At the other side when s_m increases each principal chooses lower x_m (the proof is the same as for

$\dfrac{\partial x_m}{\partial V_u(s)} < 0)$.

For given x_m, the managers' wage doesn't change. Indeed the impact of an increase of the unemployment benefit on the workers' performance standard passes through the variation of the managers' performance standard.

For given level of managers' wage when workers' unemployment benefit increases, managers' performance standard decreases. For given s_m,

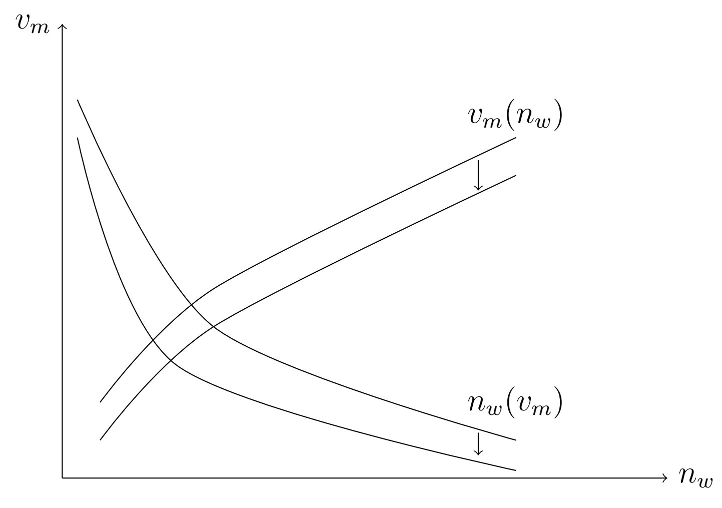
$$\frac{\partial x_m}{\partial \underline{s}_w} = \frac{(1-v'_m)\dfrac{\partial s_w}{\partial \underline{s}_w}}{-v''(\theta^P - s_w - t_w s_m) + ct_w(1-v'_m)}$$

Under assumption (1), the denominator is negative. Thus $\dfrac{\partial x_m}{\partial \underline{s}_w} < 0$.
On equilibrium we have $\dfrac{\partial s_m}{\partial \underline{s}_w} < 0$.

A similar analysis applies to show that the number of managers is a decreasing function of unskilled's unemployment benefit.

7.10 Proof of Proposition 3

Ambiguous effect on n_w



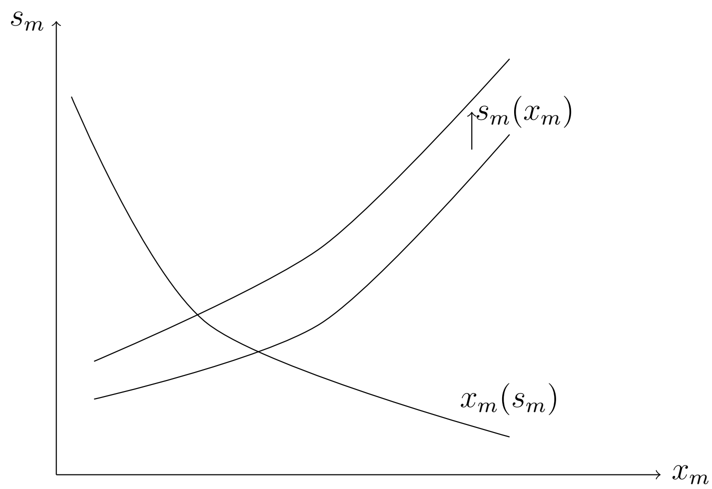
As $v_m = \int_{\underline{\theta}_s}^{x_m} \dfrac{s(\theta)N_s(x_m-\theta)(1-\alpha+\alpha v_m)}{N_s(1-\alpha+\alpha(x_m-\theta)) - n_m\alpha(x_m-\theta-v_m)}$ we notice that it increases with n_w. At the other side higher v_m means lower real productivity of managers and decreases n_w. Higher unemployment benefit for managers increases x_w and decreases x_m.

Thus for given n_w, higher $\underline{s}_m$ decreases the proportion of fired managers, as far as x_m decreases[23].

At the other side for given v_m, n_w decreases.

Thus the final effect on the number of agents that will be employed is not clear.

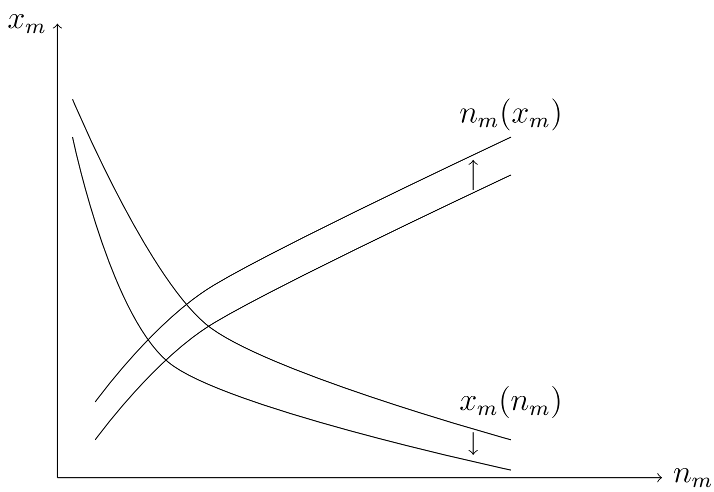
The impact of higher $\underline{s}_m$ on managers' wage.



For given x_m, s_m increases with $\underline{s}_m$. Thus when the unemployment benefit for skilled agents increases the managers' wage also increases.

7.11 Proof of Proposition 5

$\frac{\partial n_m}{\partial s_{min}} < 0$



[23]Under the Assumption 1, it is the case.

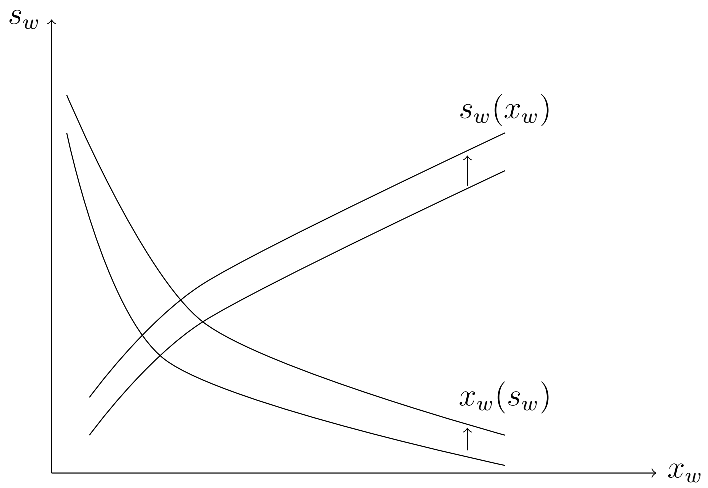
For given x_m, n_m decreases (if the principal can increase workers' performance standard), or doesn't change (in the case of binding knowledge capacity on workers' level). For given n_m, x_m decreases.

Thus the final effect of minimum wage on the number of managers is $\frac{\partial n_m}{\partial s_{min}} < 0$.

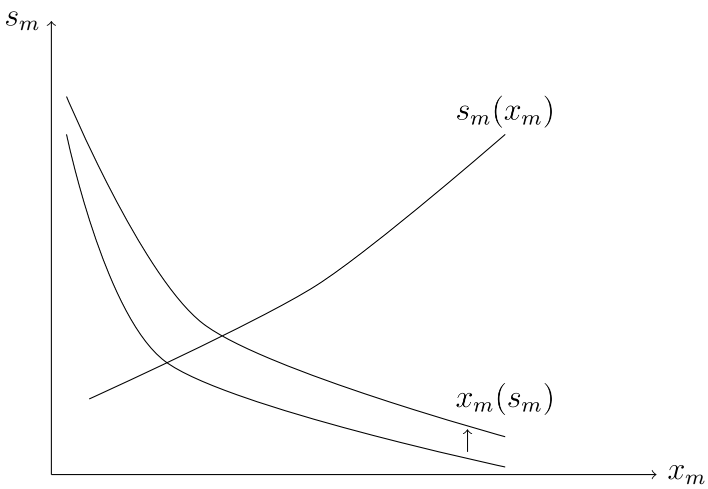
The analysis is similar for managers' wages.

7.12 Proof of Proposition 6

$\frac{\partial s_w}{\partial \theta^P} > 0$



The effect of variation in θ^P on managers' wage.



For given s_m, we have:

$$\frac{\partial^2 \Pi}{\partial x_m \partial \theta^P} = \frac{(1 - v'_m)}{t_m}$$

$$\frac{\partial^2\Pi}{\partial x_m^2} = (\theta^P - s_w - s_m t_w)(\frac{(1-v_m')}{t_m^2} - \frac{v_m''}{t_m})$$

$$\frac{\partial x_m}{\partial \theta^P} = -\frac{\frac{(1-v_m')}{t_m}}{(\theta^P - s_w - s_m t_w)(\frac{(1-v_m')}{t_m^2} - \frac{v_m''}{t_m})}$$

The denominator is negative, so $\frac{\partial x_m}{\partial \theta^P} > 0$.

The effect of variation in θ^P on the number of managers.

$$\frac{\partial n_m}{\partial \theta^P} = \frac{-(1-v_m^{'})t_m \frac{\partial x_w}{\partial x_m}\frac{\partial x_m}{\partial \theta^P} + (1-v_w^{'})t_w \frac{\partial x_m}{\partial \theta^P}}{t_m^2} =$$

$$= \frac{\frac{\partial x_m}{\partial \theta^P}((1-v_w^{'})t_w) - (1-v_m^{'})t_m \frac{\partial x_w}{\partial x_m}}{t_m^2}$$

Under assumption 1 the latter is positive. Thus the number of managers increases with θ^P.