March 2, 2007

"The political economy of income taxation in a two-party democracy"

by

John E. Roemer
Yale University
john.roemer@yale.edu

Abstract. Parties compete on a large policy space. Only a fraction of each voter type will vote for each party, perhaps because of party reputations, or issues not modeled here. Each party's policy makers comprise two factions, one concerned with maximizing the welfare of its constituency, the other with maximizing vote share. These factions are particularly concerned with the party's base or core voters and the swing voters, respectively. All these concepts (constituency, core, swing) are endogenous to the policy choice. An application to competition over redistribution produces equilibria in which each party proposes a piece-wise linear tax schedule, and these schedules coincide in their treatment of a possibly large interval of middle-income voters. This appears to conform with recent tax history in the US.

March 2, 2007

> The spirit of a people, its cultural level, its social structure, the deeds its policy may prepare—all this and more is written in its fiscal history, stripped of all phrases. He who knows how to listen to its message here discerns the thunder of world history more clearly than anywhere else.[1]

1. Introduction

In the United States, the most important tax, since at least the second world war, has been the income tax, and to apply Schumpeter's dictum, it behooves us to study it. There are, indeed, a good number of historical studies of the American income tax. A useful 'short history' is provided by Brownlee (2004), which also contains a full guide to the literature.

Formal political-economic analysis of taxation has been in the main of a schematic nature: I mean by this, that existing models of income taxation usually assume that taxation is an affine function of income[2]. In reality, income-tax policy is extremely complex, and this reflects the fact that many competing interests must be satisfied, or attended to. In this paper, I attempt to capture some of this complexity by modeling political competition over the income tax as taking place on an infinite dimensional space of functions. Each political party will propose a function which will define the net taxes paid, for every possible realization of pre-tax income, and these functions will be chosen from a large space, constrained only by upper and lower bounds on what the marginal tax rates can be[3].

We will suppose that two parties are competing in a general election, and that the platform of each party consists in a proposal of such an income-tax function. We attempt to capture what appear to be an important aspect of party competition: that parties

---

[1] Schumpeter (1954 [1918]), as quoted in Brownlee (2004).

[2] A number of papers, for example Dixit and Londregan (1998), study 'pork barrel politics,' in which parties propose payments to each of a finite number of voter types. Here, the policy space is finite dimensional, but could be of high dimension.

[3] I first studied taxation on this policy space in Roemer (2006). The present paper is a substantial variation on some themes developed there.

are concerned with the base (or core) voters, and with winning the undecided (or swing) voters[4]. A simple way of formalizing these aims of a party is to assume that there are intra-party factions concerned, respectively, with these two problems – of satisfying the core constituency, and of appealing to the swing voters. Analytically, all that is important is that we model the party's decision procedure as a problem of satisfying two payoff functions, rather than one. Doing so, we show, solves the problem of the existence of a (Nash-type) equilibrium in pure strategies in the game of party competition, even when the parties are choosing strategies from an infinite dimensional space.

Besides modeling parties as complex organizations (in the sense that a party does not maximize a single payoff function), we depart from traditional formal approaches in the study of political competition in another way. The polity consists of a continuum of voter types, where 'type' is defined by the pre-tax income of the agent or household. In many – perhaps most -- formal papers about political competition, parties represent no constituencies. This is the case with the Downs model, where each party is only a vehicle of a candidate, who seeks election. It is as well the case with the citizen-candidate models of Osborne and Slivinski (1996) and Besley and Coate (1997), where candidates run on their own ideal policies. In models where parties represent constituencies – see for instance Dixit and Londregan (1998), Austen-Smith(2000), Levy (2004), and Roemer (1999, 2001,2006) -- it is supposed that each party represents an element of some partition of the polity. Here, we depart from these practices, by recognizing that, in reality, it is never the case that the sets of voters who support the various parties form an easily defined partition. For instance, in American elections, a quite substantial fraction of voters *at every income level* supports each of the two parties[5]. Of course, one can say that, if the *space of types* had sufficiently large dimensionality, there would always be characteristics of voters that would enable us to define the set voting for a particular party as an element of a partition of that space. We prefer,

---

[4] See Cox (2006) for a recent review of the formal literature which attempts to model parties' concerns with swing and core voters.

[5] See McCarty, Poole and Rosenthal (2006, chapter 3).

however, to take a more statistical approach -- to say that, from the viewpoint of parties, there is a random element in voting, and therefore, if a party is concerned to represent its constituents, it has to attempt to represent every household type, at least to some extent.

We will define an equilibrium concept reflecting these concerns, and then characterize a family of equilibria in the income-tax competition game. The main characteristics of the equilibria are these:

1. In every equilibrium, there is a 'Left' and a 'Right' party. The Left party puts more weight on the interests of voters the poorer they are, and the Right puts more weight on the interests of voters, the richer they are;
2. Each party proposes a piece-wise linear tax policy, which gives post-fisc income of as a function of pre-fisc income;
3. In every equilibrium, the policy proposed by Left entails an increasing average rate of taxation on the whole domain of incomes; the policy proposed by the Right entails an average rate of taxation that increases up to a point, and then decreases;
4. There is a two dimensional manifold of equilibria, where a particular equilibrium can be viewed as being characterized by the relative strength of the 'swing' versus 'core' factions within the parties (thus, two numbers);
5. In every equilibrium, the two parties propose exactly the same tax treatment for what may be a substantial interval of middle-income voters. The greater the focus upon swing voters the parties place, the larger will be the size of this interval;
6. All equilibrium policies are progressive.

Recent empirical work on the history of income taxation in the United States enables us to check some of these predictions. In particular, there appears to be quite striking confirmation of the fifth prediction.

In section 2, we propose several concepts of political equilibrium. In section 3, we characterize one set of equilibria. In section 4, we examine US income-tax data to see how well reality conforms to the model's predictions, and section 5 concludes.

2. A concept of political equilibrium in two-party politics

Let the policy space be a convex set $\Im$; the generic policy is denoted *X.* The space of voter types is $H \subset \mathbb{R}$; voters are distributed according to cdf *F* on *H.* There is a continuum of voters of each type. Voters behave with preferences represented by a non-negative utility function $v : \Im \times H \to \mathbb{R}_+$, where *v* is concave on $\Im$. Voters behave stochastically. When facing two policies X^a, X^b from parties *a* , *b,* a voter of type *h* votes for policy X^a when

$$\log \frac{v(X^a;h)}{v(X^b;h)} > \varepsilon, \qquad (2.1)$$

where ε is a normal variate, with mean zero, whose cdf is *N.* Thus, if $v(X^a;h) = v(X^b;h)$ then one-half the voters of type *h* will vote for each policy, by the continuum assumption[6].

We propose a concept of political equilibrium in which parties are endogenous, and each party contains political entrepreneurs who adopt different strategies. One strategy is to attempt to represent the *constituency* of the party with special attention to its *core voters;* the other strategy is to attempt to maximize the party's vote share and to target, in particular, *swing voters.* The sets of core and swing voters are themselves endogenous.

Note, from (2.1), that given two policies X^a, X^b, the fraction of voters of type *h* who vote for policy X^a is:

$$\theta(h; X^a, X^b) = N\left(\log \frac{v(X^a;h)}{v(X^b;h)} \right). \qquad (2.2)$$

We will usually write this simply as $\theta(h)$ when the policies are clear. Given a pair of policies (X^a, X^b) the fraction of the vote share going to party *a* is:

$$\sigma^a(X^a, X^b) = \int \theta(h; X^a, X^b)\, dF(h). \qquad (2.3)$$

Given a pair of policies (X^a, X^b) proposed by parties *a* and *b*, we say that the *core* of party *a* with respect to this policy pair is the set of types:

---

[6] I indicate below a generalization of the model to the case in which different types have biases towards different parties.

$$C^a(X^a, X^b) = \{h \mid \theta(h) > \frac{1}{2}\},$$

with the analogous definition for the core of party *b.* We say the *swing voters* with respect to a pair of policies constitute the set of types:

$$S(X^a, X^b) = \{h \mid \theta(h) = \frac{1}{2}\}.$$

Thus, under this construal, the cores of the two parties and the swing voters together comprise a three-element partition of the polity.

We now discuss the behavior of political entrepreneurs, who set policy for the parties. We assume there are two parties. Parties exist for a long time; they build a reputation by representing certain constituencies. With stochastic voting, the constituency of a party is hard to define, because one can never be sure exactly who will vote for the party. Nevertheless, from a statistical viewpoint, the constituency of a party may be quite clear. Suppose at the last election, the vote shares for party *a* are described by the function $\theta(\cdot)$; thus, for every *h*, fraction $\theta(h)$ of type *h* voted for party *a.* I propose that those politicians who attempt to represent their constituency, in the present election (for which policies are now being formulated) will be concerned with maximizing the function:

$$\int \theta(h) v(X;h) dF(h);$$

that is, they will attempt to maximize the average welfare of their statistical constituency, by weighting the welfare of every type by the fraction of that type that comprise the constituency of the party in that type[7]. These politicians, are eclectic; they attempt to represent the *entire polity*, but skewed in a way that reflects the degree of support the party received from the different types. We depart from the more familiar formulation that each party represents a distinct set of voter types.

---

[7] Some aggregation principle (i.e., social welfare function) other than summing up could be used. The key point is that types are weighted by their historical loyalty to the party.

The second faction of politicians in a party have the strategy of maximizing the party's vote share: they are concerned, in particular, with not ceding swing voters to the other party.

We now propose a concept of political equilibrium. Our first concept is of a *sequence* of political equilibria over time. Note, from what I said above, that the party's constituency is defined for it by the last election.

We suppose that the distribution of types, *F*, is unchanging over time. This is a way of postulating that the distribution of types changes slowly compared to the period of the election cycle.

Definition 1 A *history of political equilibria given a function* $\theta_0 : H \to [0,1]$ is a sequence of policies $\{(X_t^L, X_t^R) \in \Im \times \Im \mid t = 1,2,...\}$, and a sequence of functions $\{\theta_t : H \to [0,1] \mid t = 1,2,...\}$ such that:

(1) for every *t*=1,2,…, and for all $h \in H$:

$$\theta_t(h) = N\left(\log \frac{v(X_t^L;h)}{v(X_t^R;h)}\right);$$

(2a) for every *t*=1,2,… policy X_t^L solves the following program:

$$\max_{X \in \Im} \int \theta_{t-1}(h) v(X;h) dF(h)$$

$$\text{subj. to } \sigma^L(X, X_t^R) \geq \sigma^L(X_t^L, X_t^R) \quad (L1)$$

$$(\forall h)(\theta_{t-1}(h) \geq \frac{1}{2} \Rightarrow v(X;h) \geq v(X_t^R;h)) \quad (L2)$$

(2b) for every *t* =1,2,…, policy X_t^R solves the program:

$$\max_{X \in \Im} \int (1 - \theta_{t-1}(h)) v(X;h) dF(h)$$

$$\text{subj. to } 1 - \sigma^L(X, X_t^R) \geq 1 - \sigma^L(X_t^L, X_t^R) \quad (R1) \quad .$$

$$(\forall h)(\theta_{t-1}(h) \leq \frac{1}{2} \Rightarrow v(X;h) \geq v(X_t^L;h)) \; (R2)$$

From (1), the function θ_t gives the fraction of each type that votes for party *L* in the election at date *t.* In words, a history of political equilibria comprises a sequence of pairs of policies such that, given the party's conception of its statistical constituency from the election held at date $t-1$, the policy of the party at date *t* cannot be dominated by any other policy with respect to *both* weighted average welfare of the party's historical constituency *and* party vote share, subject to providing at least as well as the opposition proposes to provide for the party's core and swing voters. The last clause is guaranteed by the constraints (L2) and (R2).

Condition (2a) represents *bargaining* between the two factions of politicians in party *L:* one faction is attempting to maximize the welfare of the constituency, and the other focuses upon vote share. The condition states that there is no further room for improving the payoff of both of these factions of political entrepreneurs, subject to an insistence that no core or swing voters be abandoned to the opposition. Condition (2b) makes a similar statement for the R party.

The datum of the equilibrium concept is the pair of functions (F,θ_0). We may view θ_0 as the initial conjecture of the two parties concerning their statistical constituencies.

In general, it will be difficult to characterize the set of stationary equilibria. If $\Im$ is a large space – in our application below it will be an infinite dimensional space-- then the interaction between the policies of the two parties which is formulated in conditions (2a) and (2b) will make the characterization of equilibrium hard. In particular, conditions (L1) and (R1) will generally render the two programs in those conditions non-convex, even when $\Im$ is a convex set and *v* is concave in *X.* (The opportunity sets of the two programs in (2a) and (2b) will generally not be convex.) So there is little hope for a general solution.

We can expect that there will be many possible histories of political equilibria. If one is interested in modeling general elections to understand the *underlying long-range* political conflicts in a society, then one should be interested in stationary points of these histories. An interest in stationary points must, of course, be justified by a view that the underlying distribution of preferences, represented by F, is changing slowly relative to the frequency of political events, that is, of elections – as we have assumed.

I propose a concept of stationarity which entails that there is a function θ_* such that $\theta_t \to \theta_*$: thus, the constituency of each party becomes stationary. We define a *stationary equilibrium* as a function θ_* and a pair of policies (X_*^L, X_*^R) such that:

(α1) For all $h \in H$, $\theta_*(h) = N\left(\dfrac{\log v(X_*^L;h)}{\log v(X_*^R;h)}\right)$

(α2a) policy X_*^L solves the program:

$$\max_{X \in \Im} \int \theta_*(h) v(X;h) dF(h)$$

$$\text{subj. to } \sigma^L(X, X_*^R) \geq \sigma^L(X_*^L, X_*^R) \quad (L1)$$

$$(\forall h)(\theta_*(h) \geq \frac{1}{2} \Rightarrow v(X;h) \geq v(X_*^R;h)) \quad (L2)$$

(α2b) policy X_*^R solves the program:

$$\max_{X \in \Im} \int (1 - \theta_*(h)) v(X;h) dF(h)$$

$$\text{subj. to } 1 - \sigma^L(X, X_*^R) \geq 1 - \sigma^L(X_*^L, X_*^R) \quad (R1)$$

$$(\forall h)(\theta_*(h) \leq \frac{1}{2} \Rightarrow v(X;h) \geq v(X_*^L;h)) \; (R2)$$

The above remark about the non-convexity of the optimization programs is true here as well. Nevertheless, there is an equilibrium concept which is a refinement of stationary equilibrium and that is relatively easy to solve, which I present next.

<u>Definition</u> <u>2</u> A β-*stationary equilibrium* is a function θ_*, a pair of policies (X_*^L, X_*^R), and an ordered pair $(h_*, y) \in \mathbb{R}_+^2$ such that:

(β1) For all $h \in H$, $\theta_*(h) = N\left(\dfrac{\log v(X_*^L;h)}{\log v(X_*^R;h)}\right)$

(β2) X_*^L solves the program:

$$\max_{X \in \Im} \int \theta_*(h) v(X;h) dF(h)$$

$$\text{subj. to } v(X;h^*) \geq y \quad (L3)$$

(β3) X_*^R solves the program

$$\max_{X \in \Im} \int (1-\theta_*(h)) v(X;h) dF(h)$$
$$\text{subj. to } v(X;h^*) \geq y \quad (R3)$$

(β4) $v(X_*^L;h_*) = y = v(X_*^R;h_*)$.

In this concept, it is as if the vote-share-seeking faction is concentrating on not losing the loyalty of one swing voter type, namely *h**. But this interpretation is not essential. What is important is the relationship of β-stationary equilibrium to stationary equilibrium.

<u>Proposition</u> <u>1</u> Every β-stationary equilibrium is a stationary equilibrium.

<u>Proof</u>:

1. Let $(\theta_*, X_*^L, X_*^R, h_*, y)$ be a β-stationary equilibrium. By definition of θ_*, X_*^L satisfies condition (L2) of (α2a). For suppose, for some *h*, $v(X_*^L;h) < v(X_*^R;h)$. Then, from the definition of θ_* in (β1), it follows that $\theta_*(h) < \frac{1}{2}$. Now by (β4), the constraint (L3) is equivalent to a constraint $v(X;h_*) \geq v(X_*^R;h_*)$. But this constraint is a weaker constraint than (L2). The upshot of this paragraph is that (by the Le Chatelier principle) X_L^* is also the solution of the program:

$$\max_{X \in \Im} \int \theta_*(h) v(X;h) dF(h)$$
$$\text{subj. to } (\forall h)(\theta_*(h) \geq \frac{1}{2} \Rightarrow v(X;h) \geq v(X_*^R;h)) \quad (L2)$$

2. But, trivially, X_*^L satisfies condition (L1). Therefore, again invoking the Le Chatelier principle, X_*^L is a solution to the program stated in (α2a).

3. In like manner, X_*^R solves the program stated in (α2b). Since (β1) is simply a restatement of (α1), the argument is complete. ■

Now examine the program in condition (β2). It is a convex programming problem. There is no interaction between the L and R policies – so in principle it can be solved. The same goes for the program in (β3). In other words, the 'refined' equilibrium concept of β-stationary equilibrium is quite tractable.

I will proceed by applying this concept to the study of income taxation, and will then argue, by looking at the history of income tax reform, that the equilibrium concept appears to explain quite well some of its main features.

3. Income taxation

We assume that the (pre-tax) income distribution is given by a cdf F on $H = \mathbb{R}_+$; the generic pre-tax income is denoted *h,* and the mean pre-tax income is denoted μ. The policy space $\Im$ consists of functions $X : \mathbb{R}_+ \to \mathbb{R}_+$ such that:

$(P0) X$ is continuous,

$(P1) \alpha \le X'(h) \le 1,$ where X is differentiable,,

$(P2) \int X(h) dF(h) = \mu$

where α is a number, $0 \le \alpha < 1$. *X* is interpreted as a post-fisc distribution of income; the two conditions (P1) and (P2) state that the derivative of *X,* where it exists, lies between α and 1, and that *X* redistributes pre-tax income fully.

The preferences of voters are given by:

$$v(X;h) = X(h);$$

that is, each voter wishes to maximize his/her post-fisc income. If the policy is *X,* then the net taxes paid by an individual *h* are $t(h;X) = h - X(h)$. Hence the marginal tax rate for *h* at policy *X* is $1 - X'(h)$, which is bounded below and above by zero and $1 - \alpha$.

I do not to put leisure in the utility function for reasons of tractability: the equilibrium analysis would otherwise become unmanageable. I attempt to recognize the elasticity of labor supply by requiring that the marginal tax rate be at most $1-\alpha$. In other words, political parties agree not to consider policies that have very high marginal tax rates, because of the deleterious labor-supply effects; α is a parameter of the model. Alternatively put, we are assuming that when marginal tax rates lie in the interval $[0,1-\alpha]$, labor-supply elasticity is very small and can be ignored.

Notice that the space $\Im$ is infinite dimensional: we allow parties to choose any continuous function as the post-fisc income distribution, subject to the proviso on maximum and minimal marginal rates. The choice to work on this large space is dictated by reality, where income-tax policies are generally very complex.

The main theorem of this section will characterize a two dimensional family of β-stationary equilibria.

First, we define two families of piece-wise linear functions. Fix a number $h_* > 0$. The first family is

$$M_a(h_*) = \left\{ \begin{array}{l} X \in \Im \mid \exists (x_a, h_1) \in \mathbb{R}_+^2 \text{ such that } h_1 \le h_* \text{ and} \\ X(h) = \left\{ \begin{array}{c} x_a + \alpha h, \text{ if } h \le h_1 \\ x_a + \alpha h_1 + (h - h_1), \text{ if } h_1 < h \le h^* \\ x_a + \alpha h_1 + (h^* - h_1) + \alpha(h - h^*), \text{ if } h > h^* \end{array} \right. \end{array} \right\}.$$

A typical function in the family is graphed in figure 1. $M_a(h_*)$ is a unidimensional family of functions which we may view as being parameterized by h_1. By construction, the policies $X \in M_a(h_*)$ satisfy (P0) and (P1). The budget- balance condition (P2) gives one equation in the two unknowns (x_a, h_1): hence, the unidimensionality of this family.

The second family is:

$$M_b(h_*) = \left\{ \begin{array}{l} X \in \Im \mid \exists (x_b, h_2) \in \mathbb{R}_+^2 \text{ such that } h_2 \ge h_* \text{ and} \\ X(h) = \left\{ \begin{array}{c} x_b + h, \text{ if } h \le h_* \\ x_b + h_* + \alpha(h - h_*), \text{ if } h_* < h \le h_2 \\ x_b + h_* + \alpha(h_2 - h_*) + (h - h_2), \text{ if } h > h_2 \end{array} \right. \end{array} \right\}.$$

Likewise, $M_b(h_*)$ is a unidimensional family of piece-wise linear policies, which is parameterized by h_2; a typical policy is also graphed in figure 1.

I have defined these two families in the natural way. It will, however, be useful to re-write their characterization using the values of the policies at the point h_*. To this end, for any policy $X \in M_a(h_*)$ define:

$$y = X(h_*) = x_a + \alpha h_1 + h_* - h_1 . \qquad (3.1)$$

Then, we may view $M_a(h_*)$ as parameterized by *y*. In like manner, for the family $M_b(h_*)$, we may write:

$$y = X(h_*) = x_b + h_* , \qquad (3.2)$$

and then we can view $M_b(h_*)$ as parameterized by *y*.

We will be interested in policy pairs in $M_a(h_*) \times M_b(h_*)$ which share a common value of $X(h_*) = y$. The next proposition tells us exactly what the admissible range is for *y*.

<u>Proposition</u> <u>2</u> Let $h_* > 0$, and let *y* lie in the interval

$$\max[(1-\alpha)\mu + \alpha h_*, h_*] \le y \le h_* + (1-\alpha)\int (h - h^*) dF(h) . \qquad (3.3)$$

Then:

A. There exist unique policies $X_a \in M_a(h_*)$, $X_b \in M_b(h_*)$ such that

$$X_a(h_*) = y = X_b(h_*) . \qquad (3.4)$$

B. Conversely, if *y* does not lie in the interval defined by (3.1), then there is no pair of policies in the two families for which (3.4) holds.

C. The number x_a is positive, and the number x_b is non-negative, and positive except in a singular case.

<u>Proof:</u>

1. Write the budget constraint for a policy $X \in M_a(h_*)$:

$$x_a + \alpha\int_0^{h_1} h dF(h) + \alpha h_1(1-F(h_1)) + \int_{h_1}^{h_*}(h-h_1)dF(h) +$$
$$(h_* - h_1)(1-F(h_*)) + \alpha\int_{h_*}^{\infty}(h-h_*)dF(h) = \mu .$$

We can rewrite this equation as:

$$x_a = (1-\alpha)\left(\int_0^{h_1} h dF(h) + \int_{h_*}^{\infty}(h-h_*)dF(h) + h_1(1-F(h_1))\right).$$

2. Viewing $M_a(h_*)$ as parameterized by h_1, and differentiating eqn. (3.1) w.r.t. h_1, we have:

$$\frac{dy}{dh_1} = \frac{dx_a}{dh_1} - (1-\alpha).$$

Now differentiating the expression derived in step 1 for x_a w.r.t. h_1 gives:

$$\frac{dx_a}{dh_1} = (1-\alpha)(1-F(h_1)).$$

These two equations together tell us that:

$$\frac{dy}{dh_1} = (\alpha - 1)F(h_1) < 0.$$

Therefore the smallest (largest) value of y compatible with a policy's being in $M_a(h_*)$ is associated with $h_1 = h_*$ (respectively, $h_1 = 0$). Using the equation for x_a in step 1, we have:

$$x_a(h_*) = (1-\alpha)\mu, \quad x_a(0) = (1-\alpha)\int_{h_*}^{\infty}(h-h_*)dF(h),$$

and so, using (3.1), these two values of y are given by:

$$y_a(h_*) = (1-\alpha)\mu + \alpha h_*, \quad y_a(0) = h_* + (1-\alpha)\int_{h_*}^{\infty}(h-h_*)dF(h).$$

3. We perform a similar analysis of policies in $M_b(h_*)$. For any such policy, we may rewrite the budget constraint as:

$$x_b = (1-\alpha)\int_{h_*}^{h_2} h\, dF(h) + (1-\alpha)h_2(1-F(h_2)) - (1-\alpha)h_*(1-F(h_*)).$$

Differentiating this equation w.r.t. the parameter h_2 gives:

$$\frac{dx_b}{dh_2} = (1-\alpha)(1-F(h_2)) > 0\,;$$

now using (3.2), we have:

$$\frac{dy}{dh_2} = \frac{dx_b}{dh_2} > 0\,.$$

Therefore, the smallest (largest) value of y compatible with a policy's being in $M_b(h_*)$ is associated with $h_2 = h_*$ (respectively, $h_2 = \infty$). These two values of y are:

$$y_b(h_*) = h_*, \quad y_b(\infty) = (1-\alpha)\int_{h_*}^{\infty}(h-h_*)dF(h) + h_*\,.$$

4. To summarize, the number y is associated with a policy in $M_a(h_*)$ if and only if

$$y_a(h_*) \le y \le y_a(0),$$

and y is associated with a policy in $M_b(h_*)$ if and only if

$$y_b(h_*) \le y \le y_b(\infty)\,.$$

Notice that $y_a(0) = y_b(\infty)$; parts A and B of the proposition follow immediately.

5. We prove part C. We have shown that the smallest value of x_a is $(1-\alpha)\int_{h_*}^{\infty}(h-h_*)dF(h)$ which is positive, as long as F has some support on (h_*,∞). The argument in step 3 above shows that $x_b > 0$ except in the singular case that $h_* = h_2$. In that case, the policy X_b is the laissez-faire policy $X_b(h) = h$. ■

Define:

$$\Gamma = \{(h_*, y) \in \mathbb{R}_+^2 \mid \max[(1-\alpha)\mu + \alpha h_*, h_*] \le y \le h_* + (1-\alpha)\int(h-h^*)dF(h)\}\,.$$

For later use, we define:

$$y_{\min}(h_*) = \max[(1-\alpha)\mu + \alpha h_*, h_*], \quad y_{\max}(h_*) = h_* + (1-\alpha)\int(h-h^*)dF(h)\,.$$

Proposition 2 tells us that for any $(h_*, y) \in \Gamma$, there is a unique pair of policies $X^a \in M_a(h_*)$ and $X^b \in M_b(h_*)$ such that

$$X^a(h_*) = y = X^b(h_*)\,.$$

To avoid notational complexity, let us fix $(h_*, y) \in \Gamma$ and denote these two functions simply by X^a and X^b. Figure 1 displays the graphs of a typical pair of such functions. Note, in particular, that these two policies coincide on the interval $[h_1, h_2]$. Suppose, now, that these two policies are being proposed by the parties, *a* and *b,* and define the function $\theta(\cdot; X^a, X^b)$ by equation (2.2). We have:

Proposition 3. The function $\theta(\cdot; X^a, X^b)$ is decreasing on the interval $[0, h_1]$, constant and equal to one-half on the interval $[h_1, h_2]$, and decreasing on the interval (h_2, ∞).

Proof:

Easily verified from the definition of the functions X^a and X^b. ■

We may now state our main result:

Theorem 1 Let $(h_*, y) \in \Gamma$, and let $X^a \in M_a(h_*)$, $X^b \in M_b(h_*)$ such that $X^a(h_*) = y = X^b(h_*)$. Let $\theta(\cdot; X^a, X^b)$ be as above. Then (θ, X^a, X^b) is a β- stationary equilibrium.

Proof:

1. The theorem will be proved if we can show that X^a and X^b solve the programs in conditions $(\beta 2)$ and (β3) of definition 2, respectively. We first address $(\beta 2)$. Let the density function of *F* be denoted *f.* The numbers h_1 and h_2 come with the functions X^a and X^b.

2. Define the number ρ, the functions $r(h)$ on $[0,h_1]$, $s(h)$ on $[h_1, h_*]$ and $t(h)$ on $[h_*, \infty)$, and the number λ as follows.

(i) $\rho = \dfrac{\int_0^{h_1} \theta(h) dF(h)}{F(h_1)}$,

(ii) $r(0) = 0$ and $r'(h) = (\theta(h) - \rho) f(h)$ on $[0, h_1]$,

(iii) $s(h_1) = 0$ and $s'(h) = (\rho - \theta(h)) f(h)$ on $[h_1, h_*]$,

(iv) $t(h_*) = \rho(1 - F(h_*)) - \int_{h_*}^{\infty} \theta(h) dF(h)$ and

$t'(h) = (\theta(h) - \rho) f(h)$ on $[h_*, \infty)$

(v) $\lambda = s(h_*) + t(h_*)$.

Note that $\rho > 0$. Note, from Proposition 3, that the function r is first increasing and then decreasing. Compute that $r(h_1) = r(0) + \int_0^{h_1} r'(h)dh = 0$. Therefore *r* is a non-negative function on its domain. Note from Proposition 3 that *s* is an increasing function on its domain: since θ is constant on $[h_1, h_*]$, by Proposition 3, we know that $\rho > \theta(h)$ on this interval. Therefore *s* is a non-negative function on its domain, and $s(h_*) = \rho(F(h_*) - F(h_1)) - \int_{h_1}^{h_*} \theta(h)dF(h) > 0$. Note that *t* is decreasing on its domain, and $t(\infty) = t(h_*) + \int_{h_*}^{\infty} t'(h)dh = 0$. Therefore *t* is non-negative on its domain. Finally, note that $\lambda > 0$.

4. Suppose that X^a were not the solution to the program (β2) of definition 2, and that the true solution is some other policy X. Define the function *g* by the equation $X(h) = X^a(h) + g(h)$. Now define the function $\Delta : \mathbb{R} \to \mathbb{R}$ as follows.

$$\begin{aligned}\Delta(\varepsilon) = &\int_0^{\infty} (X^a(h) + \varepsilon g(h))\theta(h)dF(h) + \int_0^{h_1} \left(X^{a\prime}(h) + \varepsilon g'(h) - \alpha\right) r(h)dh + \\ &\int_{h_1}^{h_*} \left(1 - (X^{a\prime}(h) + \varepsilon g'(h))\right) s(h)dh + \int_{h_*}^{\infty} \left(X^{a\prime}(h) + \varepsilon g'(h) - \alpha\right) t(h)dh + \\ &\lambda\left(X^a(h_*) + \varepsilon g(h_*) - y\right) + \rho\left(\mu - \int_0^{\infty} (X^a(h) + \varepsilon g(h))dF(h)\right)\end{aligned}.$$

Note that Δ is a linear function, and that $\Delta(0) = \int_0^{\infty} X^a(h)\theta(h)dF(h)$: this is the objective of program (β2) evaluated at the policy X^a. Note as well that when $\varepsilon = 1$, all the terms in the expression defining Δ are non-negative: this follows from the fact that *r,s,t,* λ and ρ are all non-negative functions or numbers, and that $X \in \Im$. Suppose we can show that $\Delta'(0) = 0$: then Δ will be equal to a constant, and consequently $\Delta(0) = \Delta(1)$. But this implies that the value of the objective function of (β2) at X^a is at least as large as its value at *X*: a contradiction. Thus we will have proved that X^a solves program (β2) if we can show that $\Delta'(0)=0$.

5. Compute that

$$\Delta'(0)=\int_0^\infty \theta(h)g(h)dF(h)+\int_0^{h_1} g'(h)r(h)dh-\int_{h_1}^{h_*} g'(h)s(h)dh$$
$$+\int_{h_*}^\infty g'(h)t(h)dh+\lambda g(h_*)-\rho\int_0^\infty g(h)dF(h)$$

.

Hence, integrating three times by parts, we have:

$$\Delta'(0)=\int_0^\infty \theta(h)g(h)dF(h)+g(h)r(h)\Big|_0^{h_1}-\int_0^{h_1} r'(h)g(h)dh-g(h)s(h)\Big|_{h_1}^{h_*}$$
$$+\int_{h_1}^{h_*} s'(h)g(h)dh+g(h)t(h)\Big|_{h_*}^{\infty}-\int_{h_*}^\infty t'(h)g(h)dh+\lambda g(h_*)-\rho\int_0^\infty g(h)dF(h).$$

We next re-group terms and write:

$$\Delta'(0)=\int_0^{h_1}\big((\theta(h)-\rho)f(h)-r'(h)\big)g(h)dh+\int_{h_1}^{h_*}\big(s'(h)-(\rho-\theta(h))f(h)\big)g(h)dh+$$
$$\int_{h_*}^\infty\big((\theta(h)-\rho)f(h)-t'(h)\big)g(h)dh+g(h_*)(\lambda-s(h_*)-t(h_*))-g(0)r(0)+g(h_1)(r(h_1)+s(h_1))$$
$$+t(\infty)g(\infty).$$

Now check, by the definitions of *r,s,t* and λ that every term on the r.h.s. of this equation vanishes, which proves that $\Delta'(0)=0$.

6. We proceed to prove that X^b is the solution to program $(\beta 3)$ of definition 2.

Suppose that the true solution is *X* and now define the function *g* by $X=X^b+g$. We now define functions *R,S,* and *T,* and numbers γ and δ as follows:

(i) $\delta=\int_{h_2}^\infty (1-\theta(h))dF(h)/(1-F(h_2)),$

(ii) $R(0)=0$ and $R'(h)=(\delta-(1-\theta(h))f(h)$ on $[0,h_*]$,

(iii) $S(h_*) = \int_{h_2}^{h_*} (\delta - (1-\theta(h))dF(h)$ and $S'(h) = (1-\theta(h) - \delta) f(h)$ on (h_*, h_2),

(iv) $T(h_2) = 0$ and $T'(h) = (\delta - (1-\theta(h)) f(h)$ on (h_2, ∞),

(v) $\gamma = R(h_*) + S(h_*)$.

Since the function $1-\theta(h)$ is (weakly) increasing (see Proposition 3), it follows from the definition of δ that $R' \geq 0, S' \leq 0$, and that $S'(h_2) = 0$. The functions *R,S,* and *T* are non-negative on their domains. As well, $R(h_*), S(h_*)$ and γ are positive.

7. We now define the function Φ by:

$$\Phi(\varepsilon) = \int_0^\infty (1-\theta(h))(X^b(h) + \varepsilon g(h))dF(h) + \int_0^{h_*} \left(1 - (X^{b\prime}(h) + \varepsilon g'(h)\right) R(h)dh +$$
$$\int_{h_*}^{h_2} \left(X^{b\prime}(h) + \varepsilon g'(h) - \alpha\right) S(h)dh + \int_{h_2}^{\infty} \left(1 - (X^{b\prime}(h) + \varepsilon g(h))\right) T(h)dh + \gamma \left(X^b(h_*) + \varepsilon g(h_*)\right)$$
$$+\delta \left(\mu - \int_0^\infty (X^b(h) + \varepsilon g(h))dF(h) \right).$$

All the terms on the r.h.s. of this equation are non-negative, and so, as we argued above, if we can demonstrate that $\Phi'(0) = 0$, then we will have proved that X^b solves the program in condition $(\beta 3)$ of definition 2.

8. Compute that

$$\Phi'(0) = \int_0^\infty (1-\theta(h))g(h)dF(h) - g(h)R(h)\Big|_0^{h_*} + \int_0^{h_*} R'(h)g(h)dh + g(h)S(h)\Big|_{h_*}^{h_2}$$
$$-\int_{h_*}^{h_2} S'(h)g(h) - g(h)T(h)\Big|_{h_2}^{\infty} + \int_{h_2}^{\infty} T'(h)g(h)dh + \gamma g(h_*) - \delta \int_0^\infty g(h)dF(h).$$

Re-grouping terms, we have:

$$\Phi'(0)=\int_0^{h_*}((1-\theta(h)-\delta)f(h)+R'(h))g(h)dh+\int_{h_*}^{h_2}((1-\theta(h)-\delta)f(h)-S'(h))g(h)dh+$$

$$\int_{h_2}^{\infty}((1-\theta(h)-\delta)f(h)+T'(h))g(h)dh+(\gamma-R(h_*)-S(h_*))-g(0)R(0)+g(h_2)(S(h_2)+T(h_2))$$

$$-g(\infty)T(\infty).$$

From the definitions of the functions R,S,T and the numbers δ and γ, we observe that all terms on the r.h.s. of this equation vanish, which proves the theorem[8]. ■

We next remark that we can generalize the model to conform with the literature on 'pork barrel politics.' In the model of Dixit and Londregan (1998), there are two parties, which have ideological characters: each party has certain political positions with which it is identified, and will not change during the election. What the parties compete over is the distribution of post-fisc income. Suppose we take this to be the case here. A simple way of modeling the predisposition of the parties to fixed positions on non-economic issues is to suppose that there is a distribution of 'Left' and 'Right' political views – on these other issues – in each type, and so a voter with income *h* will vote for the Left policy if (2.1) holds, but we now specify that ε is distributed according to a normal variate N^h and that $h'<h\Rightarrow N^{h'}$ fosd N^h. (fosd = first order stochastic dominates) Thus, the larger the voter's pre-tax income, the *less* likely she is to vote Left, ceteris paribus. (Dixit and Londregan (1998) propose this association.) Then the fraction of a type who will vote Left when facing a policy pair (X^L,X^R) is:

$$\hat{\theta}(h;X^L,X^R)=N^h(\log\frac{X^L(h)}{X^R(h)}).$$

We now observe that if (X^L,X^R) satisfies the premise of Theorem 1, then *a fortiori* the function $\hat{\theta}$ is decreasing. Since the only property of the function θ used in the proof of

[8] The convention that "$t(\infty)=0=T(\infty)$" is a stand-in for a transversality condition. The proof can be rigorously completed by checking that $\lim_{h\to\infty}t(h)g(h)=0$ and $\lim_{h\to\infty}T(h)g(h)=0$.

theorem 1 was that it is monotone decreasing, it follows that Theorem 1 continues to hold for this modified model.

Theorem 1 gives us a stationary equilibrium for each $(h_*, y) \in \Gamma$: thus, a two-parameter family of equilibria. We propose an interpretation of the political nature of these various equilibria, which follows from the next result.

Theorem 2

A. Consider a point $(h_*, y_{\max}(h_*))$ on the upper envelope of the manifold Γ. Let the two policies of the β-stationary equilibrium at this point be denoted X^L and X^R. Then $h_1 = 0, \quad h_2 = \infty$ and $X^L = X^R = X^*$, where X^* is the ideal policy in $\Im$ of voter h_*.

B. Consider a point $(h_*, y_{\min}(h_*))$ on the lower envelope of Γ, with its associated β-stationary equilibrium (X^L, X^R). If $h_* \le \mu$ then X^L is the ideal policy in $\Im$ of Left's constituency[9], and if $h_* \ge \mu$ then X^R is the ideal policy in $\Im$ of Right's constituency.

Proof:

1. It is clear that the ideal policy for a type h_* -- the policy in $\Im$ that maximizes its (post-fisc) income -- has some value y at h_*, increases as slowly as possible for $h > h_*$, and decreases from the (h_*, y) as rapidly as possible for $h < h_*$. This is the way to spend as few resources as possible on everyone other than h_*. Thus the ideal policy for h_* is defined by:

$$X^*(h) = \begin{cases} x_0 + h, & h \le h_* \\ x_0 + h_* + \alpha(h - h_*), & h > h_* \end{cases}$$

where x_0 is such that this policy integrates to μ. But this is precisely the policy in $M_a(h_*) \cap M_b(h_*)$ when $y = y_{\max}(h_*)$.

2. If $h_* \le \mu$ and $y = (1-\alpha)\mu + \alpha h_*$ then the policy $X^a \in M_a(h_*)$ is a line of slope α such that $x_a = (1-\alpha)\mu$. We can prove, using the variational technique of the proof of theorem 1, that this is the policy that maximizes $\int \theta(h) X(h) dF(h)$ on $\Im$. But the fact is

[9] That is, X^L maximizes $\int \theta(h) X(h) dF(h)$, for $X \in \Im$, where $\theta(h) \equiv \theta(h; X^L, X^R)$.

intuitively clear. Because θ is a decreasing function, the objective wishes to push resources as much as possible to the poorest. The solution is to maximize what is given to *h*=0, which means to increase as slowly as possible (that is, at rate α) on the whole positive line, subject to having given just enough to *h*=0 so that the policy integrates to μ.

3. If $h_* \geq \mu$ and $y = y_{\min}(h_*) = h_*$ then the policy $X^b \in M_b(h_*)$ is the laissez-faire policy $X(h) = h$. It is also intuitively clear that this is the policy that maximizes $\int (1-\theta(h))X(h)dF(h)$: for now, the objective wishes to push resources to the very rich. Once it is decided how much the very rich get, the strategy must be to decrease as fast as possible (i.e., at rate one) for *h* smaller. This yields in the limit the laissez-faire policy. Of course, this can also be proved using the variational technique of theorem 1. ■

Fix 'pivot type' h_* and begin at the upper envelope of Γ at h_*. In the stationary equilibrium at this point, both parties propose the ideal policy of the pivot type, h_*. In the standard case where all voters' behavior is described by the same normal variate *N,* each party receives half the vote. Here we have politics where the concern for swing voters is very strong in both parties: the factions representing constituent interests have no pull. As we start to move down the manifold, decreasing *y* and holding h_* fixed, the two policies diverge. The factions concerned with core voters (who weight the interests of a type more highly, the greater the loyalty of the type to the party) become more powerful in intra-party bargaining. When we reach the lower envelope of Γ, if $h_* < \mu$, then this faction is entirely dominant in the Left party in the sense that the L party is playing as if it is only concerned with constituent interests; if $h_* > \mu$, then constituent interests are dictating policy in the Right party. In the singular case that $h_* = \mu$, both parties are maximizing over $\Im$ the average utility of their statistical constituencies.

For a policy *X,* define the average tax rate at *h* as:

$$t(h;X) = \frac{h - X(h)}{h}.$$

Define a policy as progressive if it unambiguously redistributes from the rich to the poor, in the following sense:

Definition 3. A policy X is *progressive* if there exists $\hat{h}$ such that:

$$h \leq \hat{h} \Rightarrow X(h) \geq h$$
$$h > \hat{h} \Rightarrow X(h) \leq h$$

and at least one (some) of these inequalities hold(s) strictly for some h.

We have:

Proposition 4 Consider any β-stationary equilibrium (X^L, X^R) of theorem 1. Then:

A. $t(\cdot; X^L)$ is increasing on $\mathbb{R}_+$, and $t(\cdot; X^R)$ is increasing on $[0, h_2)$ and decreasing on (h_2, ∞), except in the singular case that X^R is the laissez-faire policy.

B. Both policies are progressive, except in the singular case that X^R is the laissez-faire policy.

Proof:

A. The condition $\frac{d}{dh} t(h; X) > 0$ is equivalent to $hX'(h) < X(h)$. It is easy to check (for instance, examine Figure 1) that this condition is true for X^L: formally, this follows from the fact that $x_a > 0$ and $x_b \geq 0$ (see Prop. 2(c)). For X^R, it is also easy to check that the condition holds if and only if $h < h_2$. Now note that if the segment of the graph of X^R on the domain $[h_2, \infty)$ is extended into a line, it passes below the origin. (For if it passed through or above the origin , the policy X^R would dominate the policy $X(h) \equiv h$, and would not be feasible, as it would integrate to more than μ.) This means that $hX' > X$ on $[h_2, \infty)$.

B. This follows immediately from the fact that $x_a > 0$ and $x_b > 0$ (except in the laissez-faire policy). ■

We end this section with some simulations showing these stationary equilibria. We choose F to be the lognormal distribution with mean 50 and median 40, which is approximately the household distribution of income in the United States in thousands of dollars. We choose N to be the normal variate with mean zero and standard deviation

0.720: this means that $N(\log[1.2]) = 0.6$, saying that a voter of any type has a 60% probability of voting for a party that promises her 20% higher post-fisc income than the other party (see (2.1)). We chose $\alpha = 0.5$. Figure 2 shows the manifold Γ for $h_* \in [20,90]$. Note that for $h_* \geq \mu$, the lower envelope of Γ coincides with the 45^0 ray.

In figure 3, we graph the β-stationary equilibrium of theorem 1 for five values of *y,* holding h_* fixed at 40 (the median of the income distribution). When $y = y_{\max}(h_*)$ (figure 3a), the two policies coincide at the ideal policy of h_*; when $y = y_{\min}(h_*)$ (figure 3c), Left is playing the ideal policy of its average constituency, since $h_* < \mu$. Table 1 presents the values of h_1 and h_2 at these five values of *y,* and gives in its last column the fraction of the polity for which the Left and Right policies coincide. We see that, even on the lower envelope of Γ, the policies coincide for 30% of the polity.

Out[35]//TableForm=

y	h1	h2	F[h2]-F[h1]
48.6987	0	∞	1
47.6987	25.9348	124.589	0.697199
46.6987	32.253	94.8931	0.52838
45.6987	37.0743	78.7839	0.390123
45.	40.	70.7935	0.303603

Table 1 Data for the policies in Figure 3a-3c

In Figure 4, we plot the average tax rate functions for the Left and Right policies, at a point in Γ. The Right imposes a higher average tax rate up to h_1; of course average tax rates of the two policies coincide on the interval $[h_1, h_2]$; the Left policy imposes a higher average tax rate on (h_2, ∞). Moreover, the Left average tax rate is monotone increasing on the whole domain, while the Right average tax rate is increasing until h_2, and then monotone decreasing asymptotically to zero thereafter.

We next ask what is the effect of a change in h_* on equilibrium policies. Rather than prove a theorem, we graph some examples to show the contrast. In figure 5, we present the average tax rate functions associated with Left and Right policies for two values, $h_* \in \{20, 80\}$. In each case we plot policies for $(h_*, \frac{y_{\min}(h_*) + y_{\max}(h_*)}{2}) \in \Gamma$. We see that the effect of increasing the 'pivot' h_* is to flatten out the average tax rate functions. For large values of the pivot, both parties propose net tax rates of close to zero for middle-income voters.

Finally, we address the question of vote share at the various equilibria in Γ. We assume the standard case, when the same normal variate with a mean of zero governs voter behavior at all income levels (so vote share is given by eqn. (2.2)). Obviously, at any point on the upper envelope of Γ, each party wins half the votes, because the parties propose the same policy. It is difficult to characterize the behavior of vote share analytically in a clean way. One can ask: What happens to vote share when we begin at a point (h_*, y) on the upper envelope of Γ and then decrease *y*? If the standard deviation of *N* is sufficiently large, then the vote share decreases, which means that the *L* party wins at least 50% of the vote every in Γ except on its upper envelope. This presumably is a consequence of the fact that the median of *F* is less than its mean. However, if the standard deviation is sufficiently small, we can find points in the interior of Γ where the vote share is increasing as we decrease *y.* Whether the vote share can ever be less than 50% for the L party, when the median of *F* is less than its mean, is an open question.

In American elections, the vote share of each party has been, in recent decades, quite close to one-half. This suggests that we find the policies in Γ where the vote shares are close to 0.5. Figure 6 presents the points in Γ where the Left policy defeats the Right policy by a margin of 0.025 or less. (We randomly generated 600 points in Γ, choosing $h_* \in (20, 80)$ and $y \in (y_{\min}(h_*), y_{\max}(h_*))$, and recorded the ones with this property.) We see that the equilibria are clustered quite close to the upper envelope of Γ. This suggests that, if the model is an accurate description of American politics, we should see the two parties offering tax policies which are quite similar, in the precise sense that they will coincide for a large fraction of the polity – corresponding in the model to the income interval $[h_1, h_2]$.

4. <u>Income tax rates in the United States</u>

In this section, we ask how well the model performs in light of the recent historical record in the United States. We use the data on income taxation assembled by

Piketty and Saez (2006). In their research, Piketty and Saez have used the public micro-file tax data of the IRS, and have computed the sum of four federal taxes for US taxpayers: the income tax, the social security and medicare payroll tax, the estate tax, and the corporate tax. The corporate tax is allocated to households in proportion to their holdings of corporate equity. The authors then produce the distribution of taxes paid, annually for the years 1960 to 2004, and consequently the distribution of post-tax income.

Post-tax income, so computed, does not correspond exactly to the theoretical concept that we used in sections 2 and 3 of post-fisc income. Adding in transfer payments and benefits, however, would affect the results only for the bottom quintile or perhaps two quintiles of the income distribution. (Although higher quintiles do receive some transfer payments, they will constitute a negligible fraction of their net income.)

In recent US fiscal history, the main tax reforms were the following[10]:

- In 1981, the Economic Reform Tax Act was passed under R. Reagan, which reduced to top marginal income tax rate from 70 to 50%, and continued to cut rates over three years;
- In 1986, the Tax Reform Act was signed by Reagan;
- In 1993, the top personal income tax rate was raised under B. Clinton to 39.6%;
- In 1997, the Tax Payer Relief Act cut the top rate on capital gains from 28 to 20%;
- In 2001, under G.W. Bush, the Economic Growth and Tax Relief Reconciliation Act reduced the tax rate in the lowest bracket to 10%, reduced the highest marginal rate to 35%, and reduced the marriage penalty. In addition, the estate tax was to be reduced over a ten year period to the vanishing point.

Our model supposes that each of the two political parties proposes a tax policy as part of its electoral strategy. This is, of course, a stylization of reality. In order to confront the data, we will assume that when a major tax reform occurs, the policy that is enacted is the equilibrium policy of the president's party. That policy continues to hold until the next major tax reform. Thus, for example, we will assume that the policy in

[10] See Brownlee (2004).

force prior to 1981 was the Democratic Party's equilibrium policy; that the policy after 1986 was the Republican Party's equilibrium policy; that the policy after 1993 was the Democratic Party's equilibrium policy; and that the policy after 2001 was the Republican Party's equilibrium policy. Our strategy will be to examine the *de facto* changes in the distribution of post-tax income, as given by the Piketty-Saez data, before and after these major tax reforms, identifying the results that we observe with equilibrium party policies as described.

Figure 7 (various panels) presents the average federal tax rates, from Piketty and Saez (2006), but normalized to control for the total taxes collected as a fraction of national income, a number which changes slowly over time. Thus, a height of '1' of a bar in the figure represents a tax rate which is exactly equal to the average fraction of national income collected in federal taxes. Groups whose average tax rate is less (greater) than one pay a smaller (larger) fraction of their income in taxes than the average. In figure 7a, we present the average tax rates for the various quantile groups reported by Piketty and Saez, before and after the Reagan tax reforms. We maintain the conventional US color coding [in the electronic version of this paper]: Democratic policies are blue, Republican are red. Piketty and Saez are particularly interested in the tax treatment of the very rich; we see that they disaggregate the top decile of the income distribution into six quantile groups, where the top one refers to the top 0.01% of the income distribution.

The main observations from figure 7a are that the Reagan tax reforms substantially reduced the tax rates on the top five quantile groups, from the 95^{th} centile up, that the tax rate on the bottom quintile was substantially increased, and that tax rates on the groups from the 20^{th} centile to the 95^{th} centile stayed about the same. If we interpret the pre- and post- Reagan reform tax rates as associated with Democratic and Republican equilibrium policies, respectively, these characteristics conform to the model's predictions.

While *de jure* income-tax schedules are indeed piece-wise linear, we cannot assert that the tax data from the Piketty-Saez aggregation of several taxes conform to a piece-wise linear rule.

One characteristic of our equilibrium policies that does not conform to the data is the predicted decrease in the average tax rate proposed by the Right party for the upper end of the distribution (incomes greater than h_2). In addition, the equilibrium policies in our model either tax at the minimal marginal tax rate (zero) or the maximal marginal tax rate($(1-\alpha)$); this is not a feature of observed tax rates. We do not observe negative net tax rates at the bottom part of the distribution (as the model predicts): of course, were we to add in transfer payments, as we should to conform to the model, we might observe negative average tax rates for the bottom quintile.

Figure 7b presents the normalized tax rates by quantile groups in 1988 and 1996, to attempt to capture the effect of the Clinton tax reform of 1993. The most obvious features of this figure are that the Democrats raised the tax rates on the top groups, lowered them on the bottom groups, while the treatment of the middle groupings, from the 60th to the 99th centiles, did not change much. This conforms to our predictions.

Figure 7c presents the tax rates before and after the Bush tax bill of 2001. The Bush tax bill decreased the rates on the rich, as expected, but it also decreased the rates on the bottom two quintiles. This was probably a consequence of the reduction of the bottom marginal rate to 10%. Tax rates from the 60th to the 99.5th centile stayed about the same. We do not know what the treatment of the bottom two quintiles would be if transfer payments were added in to income.

In Figure 7d, we present the tax rates before and after a minor tax bill enacted in 1969 under the Nixon administration. This is the one example that does not fit the theory at all. We interpret the 1968 taxes as that of the Kennedy – Johnson administrations. Nixon evidently raised tax rates on the very rich, but taxes on the rest of the distribution remained about the same[11].

---

[11] We quote *Time Magazine* wrote about this bill, to indicate that it was not a very partisan piece of legislation: “The tax bill worked out by a Senate-House conference in a series of exhausting 16-hour sessions last week provides plenty of tax relief but relatively little in the way of long-term reforms. What started out as an effort to close tax loopholes turned into a tax-cutting binge designed to win friends for Congress in an election year.

As a final contrast, we present in figure 7e tax rates in 1960 and 2004. These years are too far apart for us to interpret the tax treatments as a pair of equilibrium policies. The figure shows that there has been a massive decrease in the taxation of the top centile of the income distribution, and also a significant decrease in the taxation of the bottom quintile, much of which is probably due to the earned income tax credit, enacted initially in 1986.

5. Conclusion

I have attempted to model political competition in a general election between two parties, incorporating two features of what appears to be American political reality: that parties compete on a very large policy space, and that they appear to conflict internally over whether to represent their core voters, or appeal to swing voters.

We modeled the internal conflict as one between two preference orders over policies within each party (each preference order belonging to a 'faction'). Thus a *party* possesses only a preference quasi-order over the cross-product $\Im \times \Im$, defined as the intersection of the binary relations representing the preference orders on $\Im \times \Im$ of its two factions. A stationary political equilibrium is a Nash equilibrium in the game played between these quasi-orders. Characterizing the entire set of stationary equilibria is an intractable problem; we elected to characterize the equilibria of a refinement, called β-stationary equilibrium. The model produces a two dimensional manifold of such equilibria. Roughly speaking, a point in this manifold is determined by the relative bargaining power of the factions in each of the parties (thus, two numbers)[12]. We

---

In the short run, the bill will increase federal revenues. Eventually, however, as tax reductions take effect, federal intake will decline sharply, creating what one Treasury man calls "the revenue crunch of the '70s." The bill represents a Democratic attempt to win the affections of Nixon's middle-class constituency by offering ample benefits to middle-income taxpayers. (*Time,* December 26, 1969)

[12] Roughly speaking, because I did not specify the bargaining game. For party unanimity Nash equilibria when the policy space is finite dimensional, I have specified a precise bargaining game where the relative bargaining strengths of factions parameterize the

presented a different parameterization of the manifold of equilibria, Γ, where each equilibrium is parameterized by a 'pivot' voter for whose loyalty the two parties compete, and the post-fisc income they promise to the pivot income type. Equilibria where the pivot is promised a high income correspond to situations where the swing-voter factions are strong in the two parties. In equilibria where the pivot voter is proposed the maximum feasible amount, both parties play the same policy, the ideal policy of the pivot type – thus, we have a kind of generalization of the median-voter theorem to an infinite dimensional policy space[13]. Equilibria where the pivot voter is promised a relatively small income correspond to situations where the core-voter factions are strong in the two parties. Indeed, on the lower envelope of the manifold Γ, at least one of the parties is playing the ideal policy of its core-voter faction.

Voters in our model behave stochastically; within an income type, there is a distribution of voter behavior, perhaps reflecting the fact that the Left and Right parties have reputations (e.g., positions on other issues not modeled here) which attract or repel some voters. Thus, we said that only a fraction of the voters of an income type will vote for the party that proposes to give it a larger post-fisc income, and that fraction is an increasing function of the income advantage that that party proposes.

In the application of the model to competition over tax policy, we chose the policy space to consist of all continuous functions restricted only by a budget constraint, and by a requirement that marginal tax rates lie everywhere in an interval $[0,1-\alpha]$. Choosing $\alpha < 1$ was our simple strategy for capturing concerns with labor supply elasticity. The stationary equilibria of our refinement have the following properties: both parties propose increasing, piece-wise linear post-fisc income policies; average tax rates proposed by the Left party increase on the whole domain, while average taxes proposed by the Right party increase to a point and then decrease, and both policies are progressive. The parties propose the same treatment for what may be a quite large interval of middle-income voters. The more 'swing-voter' concerns dominate in the

---

equilibrium manifold (see Roemer (2001, section 8.2). The problem is complicated here by the intervention of the β refinement.

[13] Of course, in the Downs model, parties are *only* concerned with the swing voter – the median type. They do not represent core voters at all.

parties, the larger will this interval be. But even on the lower envelope of the equilibrium manifold, the policies will coincide for a non-negligible fraction of the income distribution.

Finally, we tested the model's predictions by examining the income-tax data assembled by Piketty and Saez (2006). Some of the features of our equilibria appear to hold, and some do not. Certainly legislated tax policies in the US are piece-wise linear; however, our model produces equilibrium policies with only two marginal tax rates, zero and $1-\alpha$, and three pieces for each party (although different sets of pieces for the two parties). We do find that Left policies tax the poor more lightly, and the rich more heavily, than Right policies: in this, reality and the model agree. We also appear to find in the data that there is a significant interval of middle-income voters who receive the same tax treatment from both parties. We take this to be the strongest evidence for the model. Finally, both Democratic and Republican tax policies appear to increase the average tax rates on the whole domain: this does not conform with the model, which predicts that the average tax rate should decrease for the Right policy, for very rich voters.

It is perhaps appealing to view certain aspects of tax policy as being due to *simplicity* or *inertia*: thus, one might conjecture that piece-wise linear policies with a small number of pieces are adopted for reasons of simplicity, and that policies do not change much between Left and Right administrations for middle-income voters for reasons of both simplicity (costly to change the entire tax code) and inertia. We have shown, however, that these characteristics of policies derive from political competition—they are equilibrium characteristics. We need not appeal to simplicity and inertia.

One could point to many ways in which the model simplifies real politics. One of the most important of such ways is that actual tax policy is not proposed by parties in general elections: it is the consequence of legislation, and in particular, of legislative bargaining between the parties. Modeling the problem of tax policy as a legislative bargaining problem could improve the fit of reality to theory. Nevertheless, if we take Schumpeter's dictum seriously, as stated in the paper's epigram, and also Riker's dictum, that the heart of democracy is the popular election, then the investigation reported upon here is a recommended undertaking[14].

[14] Riker (1982).

References

Austen-Smith, D., 2000. “Redistributing income under proportional representation,” *J. Political Economy* 108, 1235-1269

Besley, T. and S. Coate, 1997. . “An economic model of representative democracy,” *Quarterly J. Economics* 112, 85-114

Brownlee, W. E., 2004. *Federal taxation in America: A short history,* New York: Cambridge University Press

Cox, G. 2006. “Swing voters, core voters and distributive politics,” Working Paper, http://www.yale.edu/polisci/info/conferences/Representation/papers/Cox.pdf

Dixit, A. and J. Londregan, 1998. “Ideoglogy, tactics, and efficiency in redistributive politics,” *Quarterly J. Econ.* 113, 497-529

Levy, G. , 2004. “A model of political parties,” *J.Econ.Theory* 115, 250-277

McCarty, N.., K. Poole and H. Rosenthal, 2006. *Polarized America,* Cambridge, Ma. : MIT Press

Osborne, M. and A. Slivinski. 1996. “A model of political competition with citizen-candidates,” *Quarterly J. Economics* 111, 65-96

Piketty, T. and E. Saez, 2006. “How progressive is the US federal tax system? An historical and international perspective,” http://elsa.berkeley.edu/~saez/piketty-saezNBER06taxprog.pdf (In press at *J.Econ.Perspectives)*

Riker, W. 1982. *Liberalism against populism: A confrontation between the theory of democracy and the theory of social choice*, San Francisco: WH Freeman

Roemer, J.E., 1999. “The democratic political economy of progressive income taxation,” *Econometrica* 67, 1-19

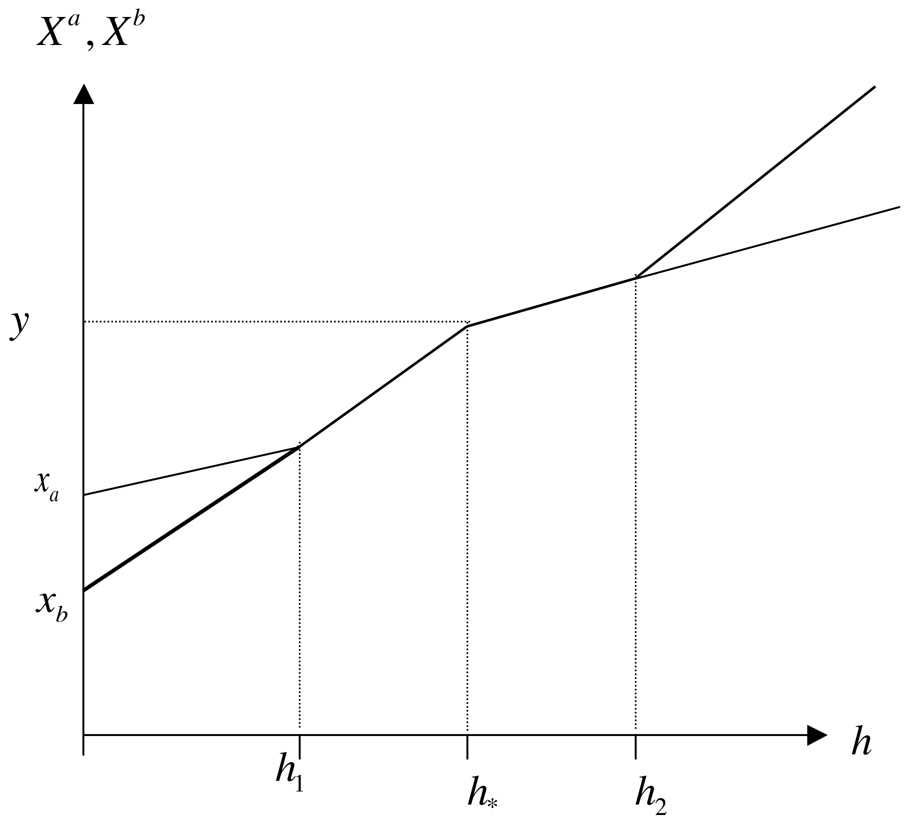
-- 2001. *Political competition,* Cambridge, Ma.: Harvard University Press

-- 2006. *Democracy, education and equality,* New York: Cambridge University Press

Schumpeter, J. [1918] . "The crisis of the tax state," in A. Peacock (ed.), 1954, *International economic papers: Translations prepared for the International Economic Association,* no. 4, 5 – 38, London: Macmillan

March 2, 2007

Figure 1 Policies $X^a \in M_a(h_*)$ [thin line] and $X^b \in M^b(h_*)$ [bold line] which share a common value y at h_*



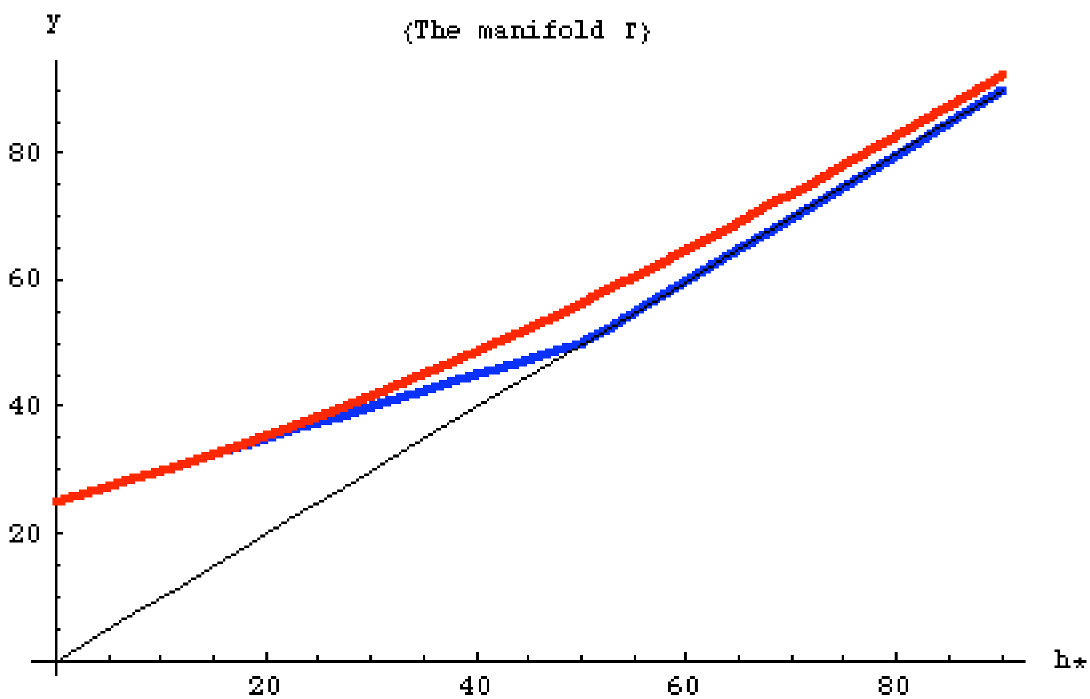


Figure 2 A graphs of the functions $y^{\max}(\cdot), y^{\min}(\cdot)$ and the ray $y = h_*$. The manifold Γ is the set between and including the two bold curves.

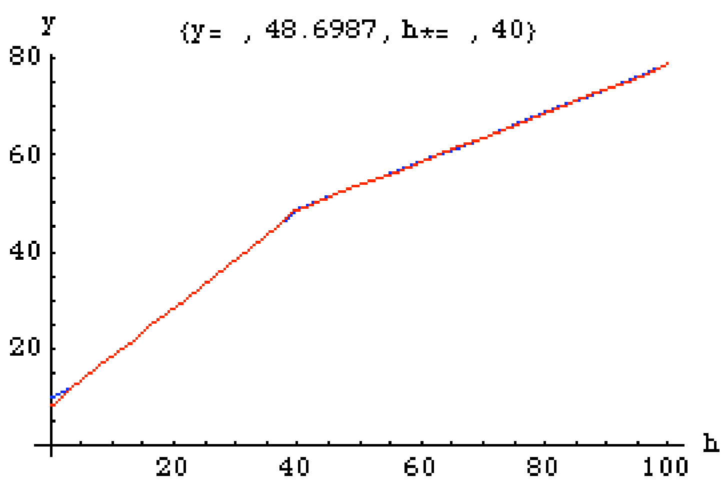


Figure 3a. The two policies coincide on the upper envelope of Γ

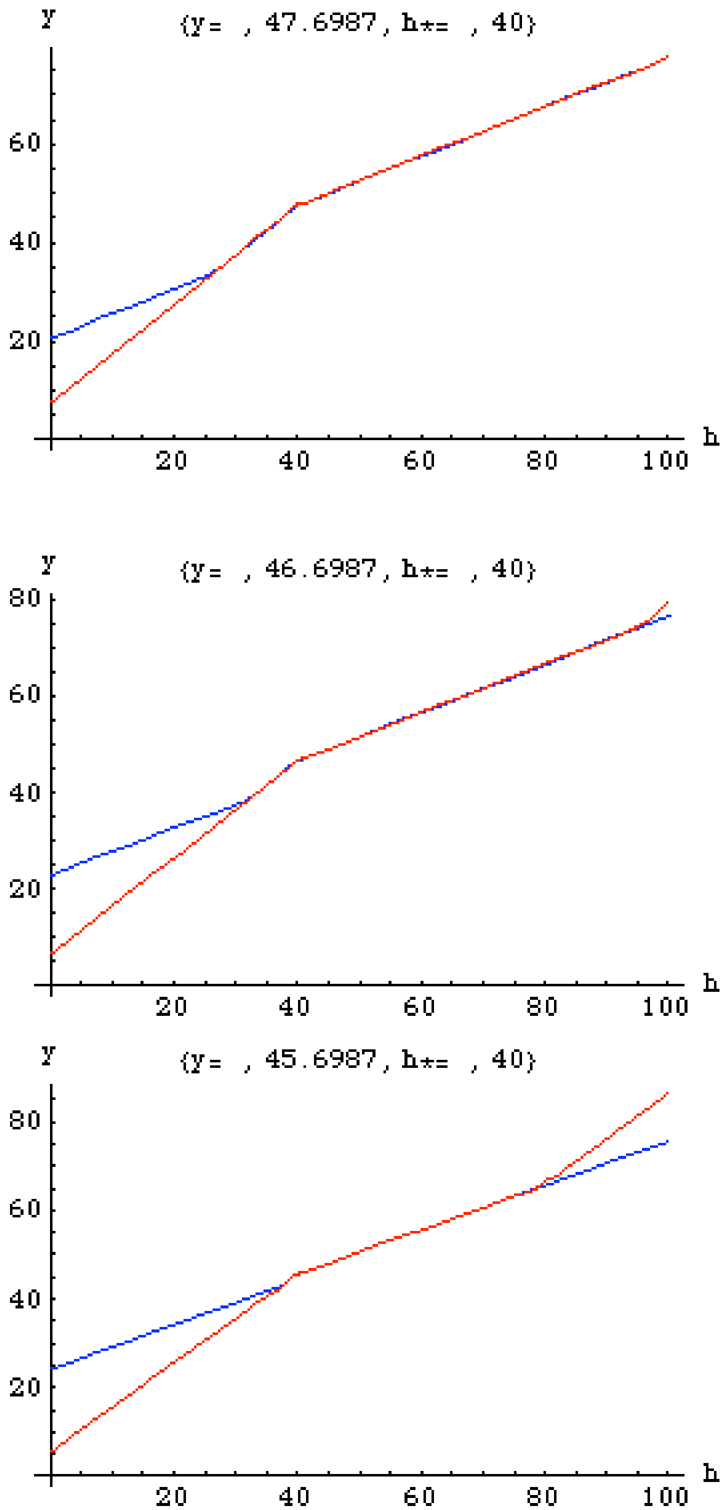


Figure 3b. Graphs of stationary equilibria as *y* decreases in Γ at constant h_*

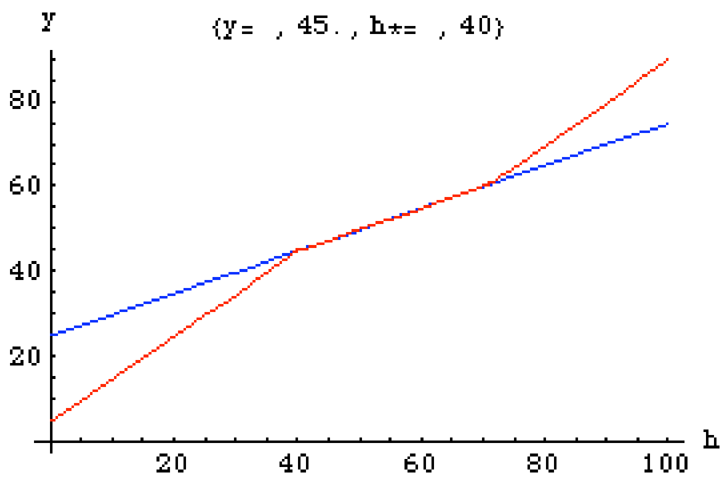


Figure 3e. Stationary equilibrium on the lower envelope of the Γ, where Left plays the ideal policy of its average constituency

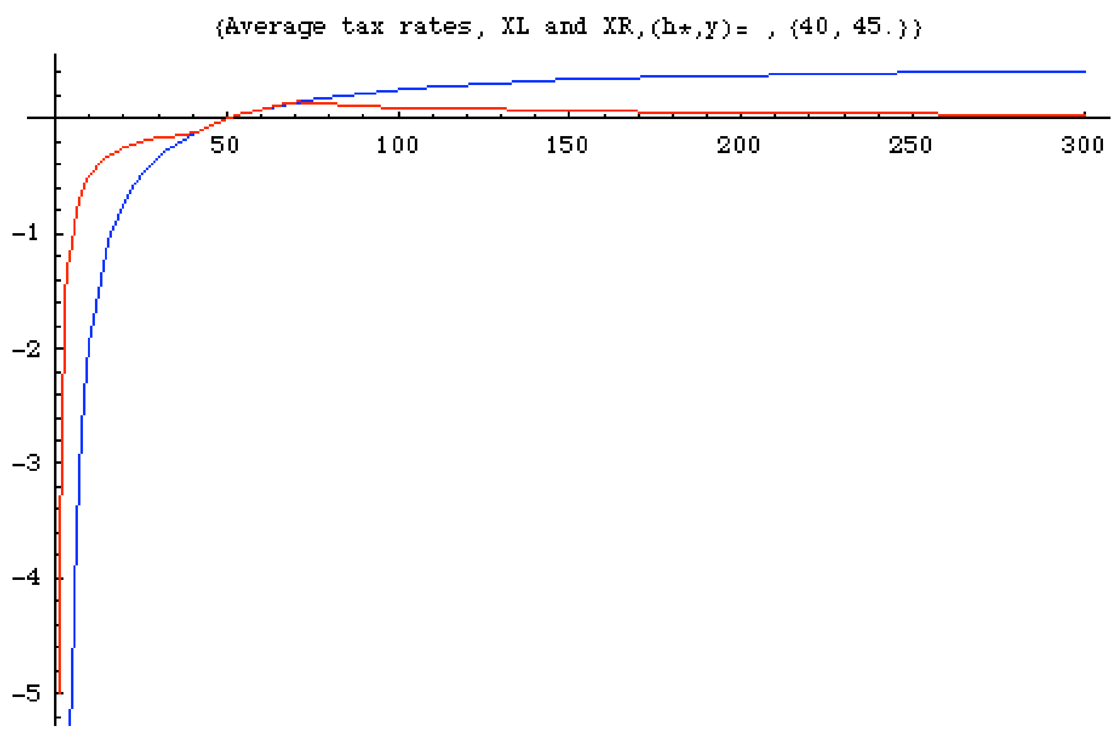


Figure 4 The average tax rate functions for a pair of equilibrium policies

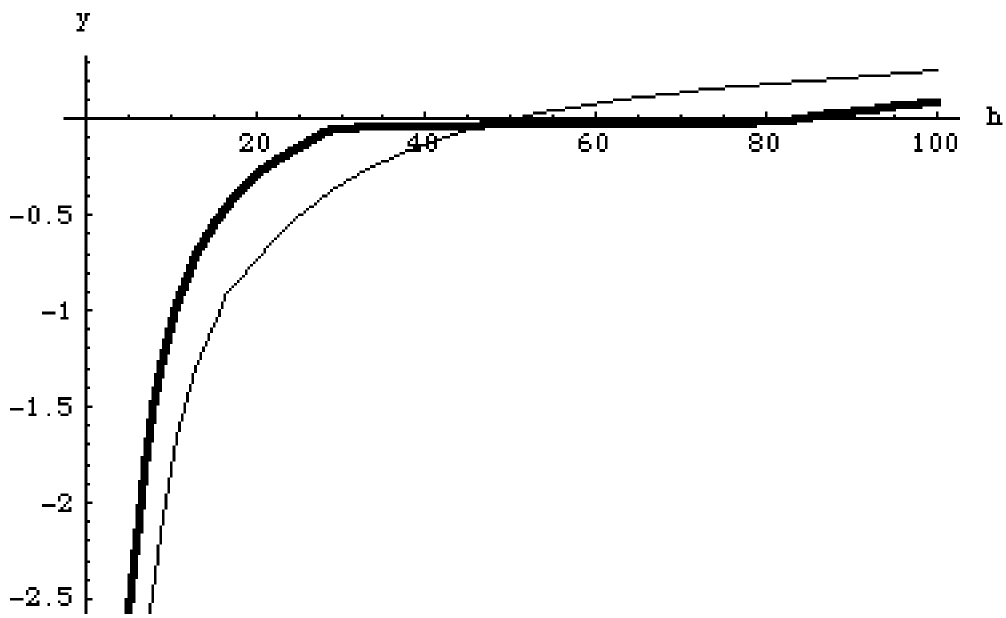


Figure 5a. Average tax rates, Left policy, at $h_* = 20$ (plain) and $h_* = 80$ (bold)

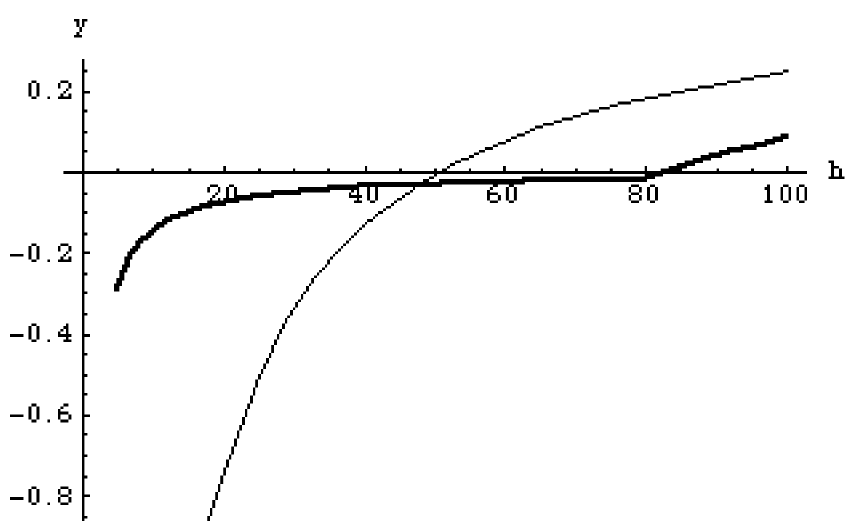


Figure 5b. Average tax rates, Right policy, at $h_* = 20$ (plain) and $h_* = 80$ (bold)

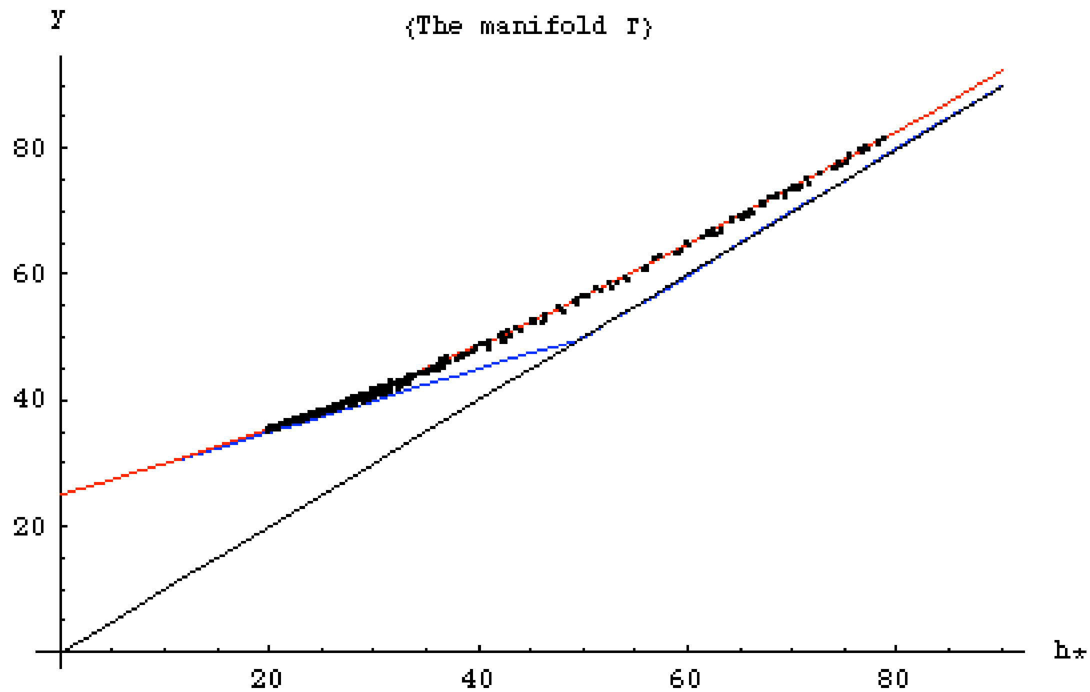


Figure 6 Equilibria where the Left vote share exceeds the Right vote share by less than 0.025

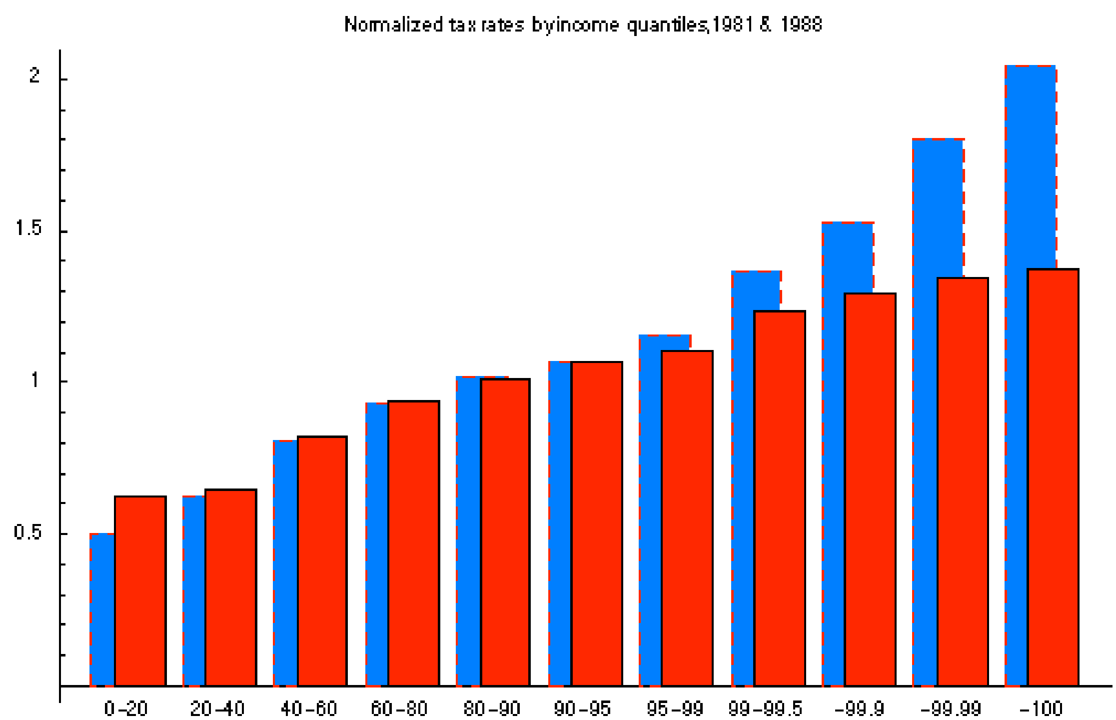


Figure 7a Tax rates, various quantile groups, before and after the Reagan tax reforms: Democrats in blue, Republicans in red

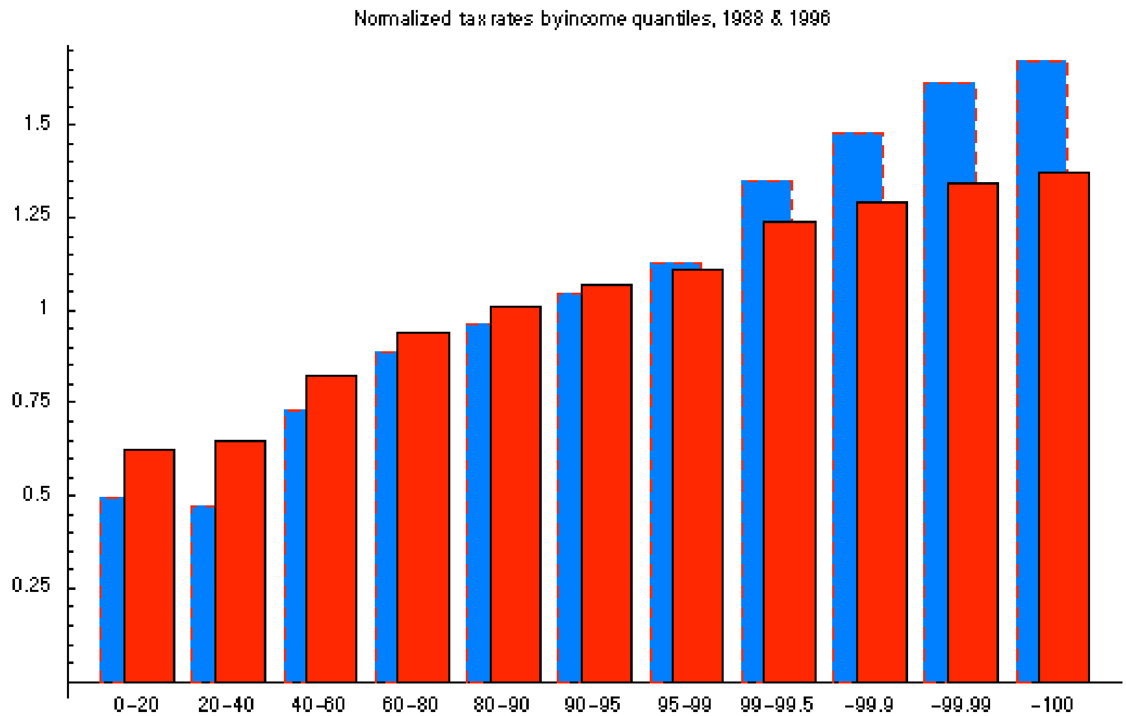


Figure 7b. Tax rates before and after the Clinton tax reform of 1993 (Democrats in blue)

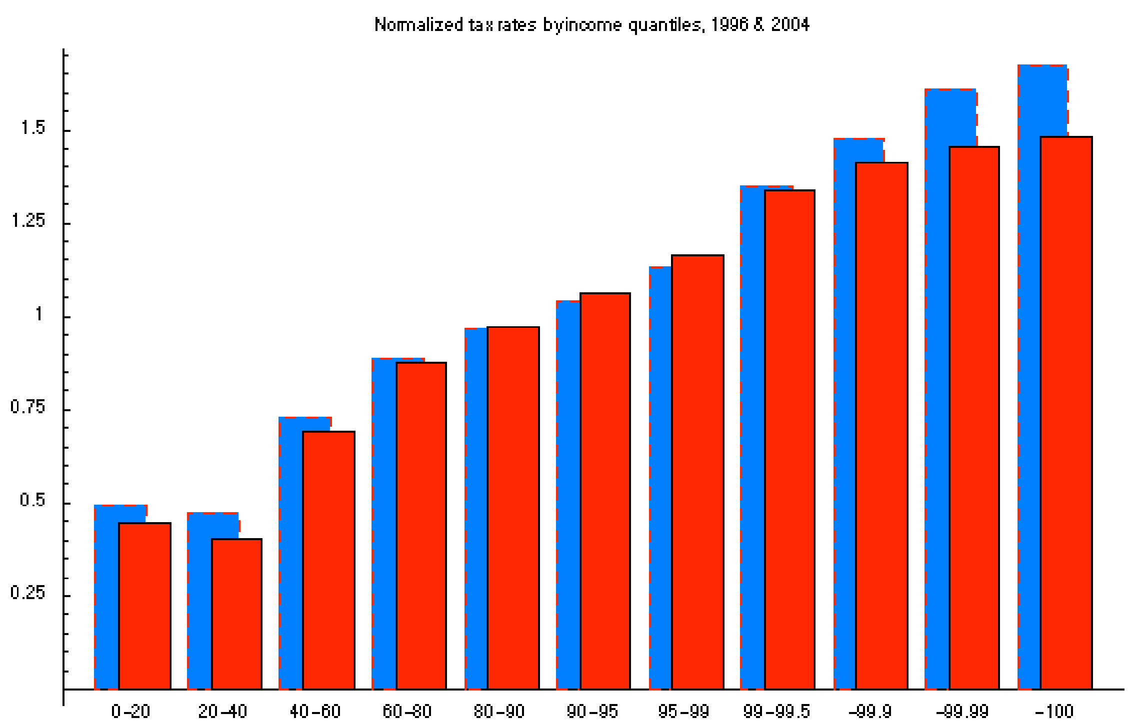


Figure 7c. Tax rates before and after the Bush tax reform of 2001

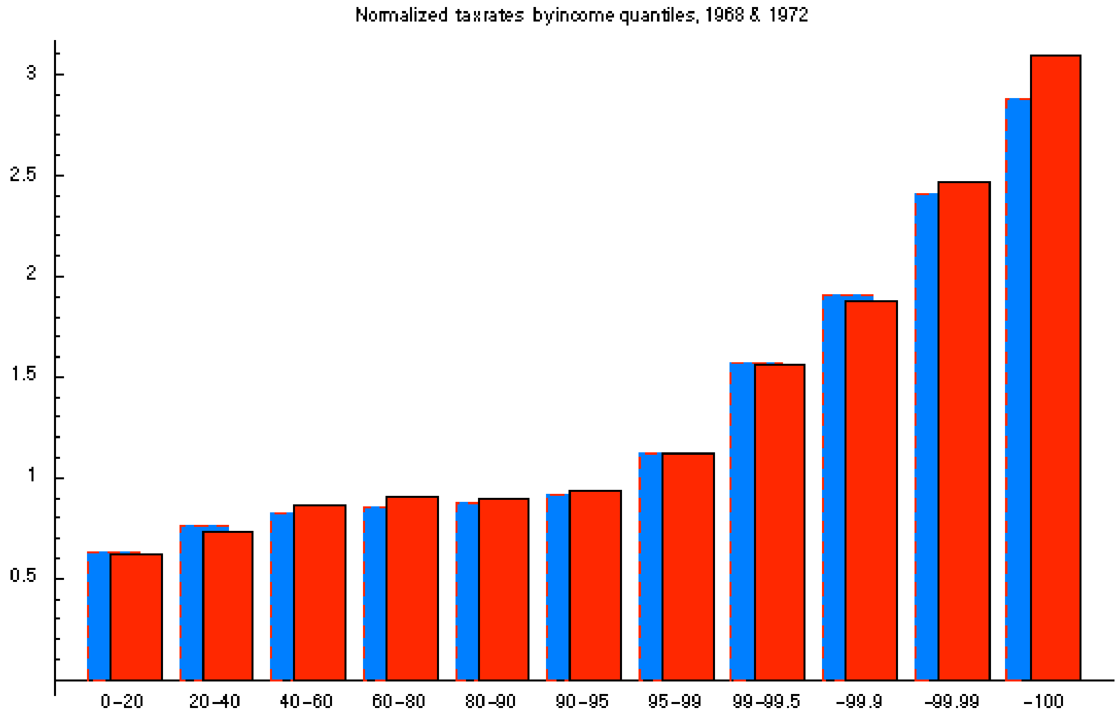


Figure 7d. Tax rates before and after Nixon's 1969 tax bill

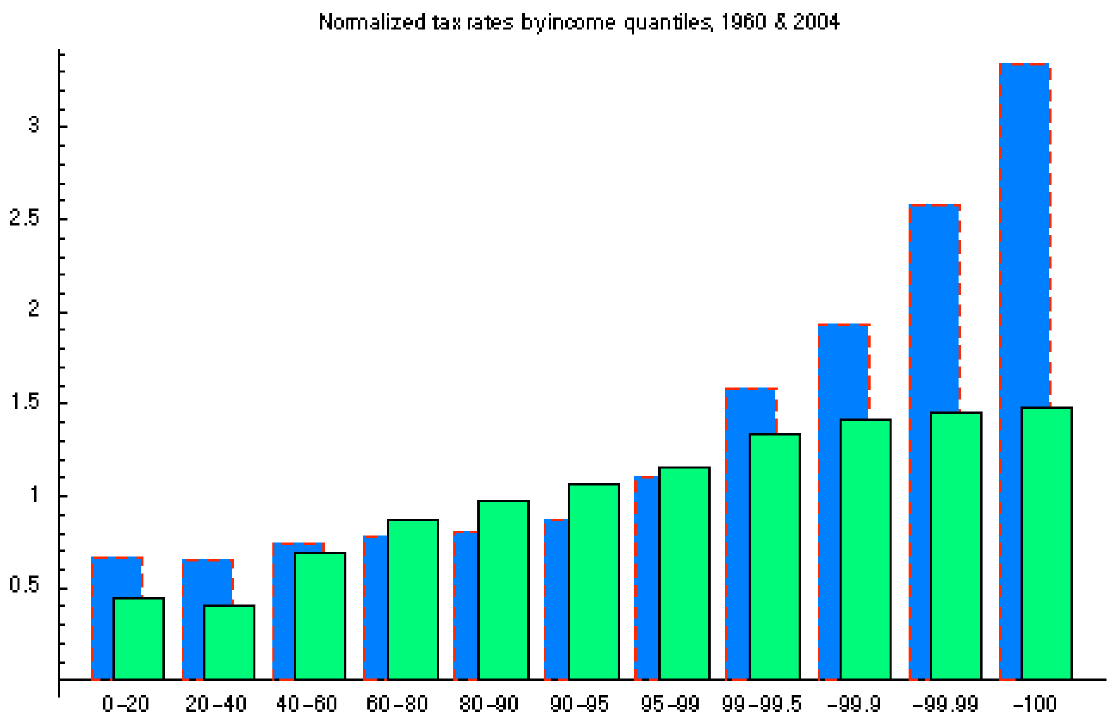


Figure 7e. Tax rates in 1960 and 2004