Investigating the Structural Stability of the Phillips Curve Trade-Off*

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Abstract

The reduced form correlation between inflation and measures of real activity has changed substantially for the main developed economies over the post-WWII period. In this paper we attempt to describe the observed inflation dynamics in the United Kingdom, the United States and the Euro area with a sequence of New Keynesian Phillips curve (NKPC) equations that are log-linearised around different, non-zero, steady state inflation levels. In doing this, we follow a two-step estimation strategy. First, we model the time-variation in the trade-off between inflation and a real cost-based measure of activity through a Markov switching vector autoregressive model. We then impose in each inflation regime the cross-equation restrictions of a Calvo pricing-based NKPC under non-zero steady state inflation and estimate the structural parameters by minimising for each inflation regime the distance between the restricted and unrestricted vector autoregressive parameters. The structural estimation results indicate that for all the economies there is evidence for a structurally invariant NKPC, albeit with a significant backward-looking component.

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1 Introduction

One of the main building blocs of modern day macroeconomic models, both in academia and at central banks, is the New Keynesian Phillips curve (NKPC) equation, and this relationship

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is essential for interpreting observed short-run inflation dynamics. Within the context of the NKPC equation, there is staggered price setting, either motivated using the staggered contract setting of Taylor (1980) or the probabilistic setting of Calvo (1983), and firms that are allowed to change their prices do that in a forward-looking manner based on their future expected real marginal costs. As a result short-run inflation dynamics are not determined within these models by short-term trade-off relationships between inflation and the output gap or between inflation and unemployment, but rather by a short-run trade-off between inflation and a measure of real marginal costs.

When researchers attempt to estimate the *structural* parameters that underlie the NKPC equation, they often assume that the relationship has not changed significantly over time, see eg Galí and Gertler (1999) and Sbordone (2002). There is, however, substantial empirical *reduced form* evidence that the inflation process has changed over time, as witnessed by the analysis in Cogley and Sargent (2002, 2005) for the United States, Benati (2004) for the United Kingdom and Levin and Piger (2004) for twelve main OECD economies (including France, Germany, Italy, Japan, the UK and the US). These changes in the inflation process seem at first sight to be related to shifts in the monetary policy regimes in the respective economies, and therefore, if valid, the NKPC relationship should be structurally invariant, as only equilibrium inflation rates would have shifted over time. This is a feature of the NKPC relationship, which can be used to test its empirical validity, that up to now has not been exploited often. A notable exception is Cogley and Sbordone (2005), who use a Calvo (1983) pricing-based NKPC relation log-linearised around a non-zero steady state rate of inflation and then, by mapping this NKPC relationship into a time-varying parameter vector autoregressive model, attempt to describe post-WWII US inflation dynamics by a combination of linearised NKPCs across various levels of steady state inflation. Cogley and Sbordone (2005) claim that this approach provides evidence that the US NKPC relationship has been structurally invariant across the different inflation regimes. Our paper is in the same vein as Cogley and Sbordone (2005).

We use a two-step indirect estimation method to estimate a Calvo (1983) pricing-based NKPC that is log-linearised around a non-zero equilibrium inflation rate. First, we estimate a reduced form Markov switching vector autoregressive (MS-VAR) model to proxy the shifts in the inflation process. Then in the second step the structural parameters in the NKPC are estimated by minimising the distance between the unrestricted MS-VAR model and a version of the estimated reduced form MS-VAR model on which we impose the cross-equation restrictions of the NKPC relationship under non-zero equilibrium inflation. This two-step structural estimation method is comparable to that used in Cogley and Sbordone (2005), but our auxiliary model is now a MS-VAR model instead of a time-varying parameter VAR model, and the method has its roots in the structural (micro-)econometrics literature that started with, amongst others, Chamberlain (1982). We prefer the Markov switching VAR model as the auxiliary model for our structural estimation over the time-varying parameter VAR framework, as it allows us to identify the VAR parameters that correspond with the different inflation regimes in an objective, data-driven manner. Also, it treats the shifts in the inflation process as stochastic, which potentially could also be non-monotonic (see Sims and Zha (2006)), and its probabilistic selection of the inflation regimes is in our view more compatible with the theoretical framework.

Our results can be summarised as follows. We estimate for the United Kingdom, the United States and the Euro area a MS-VAR model for GDP deflator inflation, GDP growth, the labour share and a proxy for the nominal discount rate where the VAR coefficients are regime dependent and identify the different states or regimes based on differences in the VAR-implied long-run level of inflation. We simultaneously allow for an unobserved break in the covariance matrix for the VAR that can occur independently of the regime switches in the VAR coefficients. For all three economies, we identify two regimes that correspond to high trend or equilibrium inflation (where the equilibrium level is quantitatively similar across the economies) and low equilibrium inflation. When we map our NKPC under non-zero equilibrium inflation into these estimated MS-VAR models, we do find that for all economies that we examined the character of the reduced form short-run trade-off between inflation and real marginal costs has shifted over time due to changes in equilibrium inflation: under high levels of steady state inflation firms put more emphasis on the *expected future* levels of their real marginal costs when they change their prices, whereas under low levels of steady state inflation they put more emphasis in the *current* level of real marginal costs. We find little direct evidence that the structural parameters that underlie this inflation - real costs trade-off also shift across the different inflation regimes for the United Kingdom, the United States and the Euro Area.

The outline of the remainder of this paper is as follows. In Section 2 we discuss how the most commonly used NKPC relationships change when they are derived under the assumption of a non-zero rate of steady state (or equilibrium) inflation, and we also explain how we can map such a structural NKPC relationship into a reduced form VAR model in order to be able to estimate the underlying structural parameters. Our reduced form representation of the data will be based on a Markov switching VAR model, and the estimation of such a Markov switching VAR model is discussed in Section 3, where we also report the reduced form estimation results for the UK and US economies. Structural estimation results based on the estimated Markov switching VAR models are reported in Section 4 and we conclude the paper in Section 5.

2 The Phillips curve trade-off under positive equilibrium inflation

Within the New Keynesian framework the presence of nominal rigidities in a world of forward-looking agents does not imply the existence of the traditional short-term Phillips curve trade-off between inflation and either the output gap or unemployment. Instead short-run inflation dynamics depends on the trade-off between inflation and real costs, usually proxied by the labour share, in a forward-looking manner, and we summarise this result in Section 2.1. Typically, one log-linearises the model around a zero steady state rate of inflation and take the resulting NKPC equation to the data. However, if we want to fit the NKPC relationship on an inflation process that shifts over time, one needs to log-linearise the model around non-zero levels of equilibrium inflation. The consequences for the NKPC are discussed in Section 2.2. Finally, we go through the basic two-step indirect estimation of the structural parameters that underlie the NKPC equation in Section 2.3.

2.1 The standard New Keynesian Phillips curve

The typical New Keynesian Phillips curve (NKPC) equation used in the literature, is based on the Calvo (1983) staggered pricing scheme for an individual firm. Within this framework the individual firm faces a certain fixed probability $(1-\alpha)$ of receiving a signal to re-optimise its profit-maximising pricing plan, and as the firm sets its price as a mark-up over its nominal marginal costs when it re-optimises, this optimal price increase reflects the expected future path of real marginal costs. Assuming that capital is firm-specific, and thus one allows for discrepancies between individual and aggregate marginal costs, we write the equilibrium condition for the price setting firm, see Cogley and Sbordone (2005), as

$$0 = E_t \sum_{j=0}^{\infty} \alpha^j R_{t,t+j} \times \left\{ \prod_{k=1}^{j} \gamma_{y,t+k} \prod_{k=1}^{j} \pi_{t+k}^{\theta} \prod_{k=0}^{j-1} \pi_{t+k}^{-\varrho(\theta-1)} \left(x_t^{1+\theta\omega} - \frac{\theta}{\theta-1} mc_{t+j} \prod_{k=1}^{j} \pi_{t+k}^{1+\theta\omega} \prod_{k=0}^{j-1} \pi_{t+k}^{-\varrho(1+\theta\omega)} \right) \right\} \quad (1)$$

Within (1) $R_{t,t+j}$ is the nominal discount rate between periods t and $t+j$, $\gamma_{y,t}$ is the gross rate of output growth in t, the gross rate of inflation in t is denoted by π_t, mc_t are the aggregate real marginal costs in t, the elasticity of marginal costs to the firm's own output, and θ is the Dixit-Stiglitz elasticity of substitution amongst the goods produced by the individual firms, which determines the mark-up that firms can demand over their marginal costs, as this mark-up equals $\frac{\theta}{\theta-1}$.

This leaves the α fraction of firms who do not receive a signal to set a new price. A common practice is to follow Christiano *et al.* (2005), and let these firms index their price increase to lagged inflation with an indexation parameter $0 \leq \varrho \leq 1$, and thus one can write the evolution of aggregate prices as

$$\left[(1-\alpha) x_t^{1-\theta} + \alpha \pi_{t-1}^{\varrho(1-\theta)} \pi_t^{-(1-\theta)} \right]^{\frac{1}{1-\theta}} = 1 \quad (2)$$

in which x_t is the relative price (relative to the aggregate price level) set in t by the representative optimising firm.

When one takes this set-up and log-linearises the aggregated pricing equation around a zero steady-state rate of inflation we get a typical hybrid NKPC equation:[1]

$$\hat{\pi}_t = \left(\frac{\varrho}{1+\varrho\beta} \right) \hat{\pi}_{t-1} + \left(\frac{\beta}{1+\varrho\beta} \right) E_t(\hat{\pi}_{t+1}) + \left(\frac{(1-\alpha)(1-\alpha\beta)}{\alpha(1+\theta\omega)(1+\varrho\beta)} \right) \hat{mc}_t + u_t \quad (3)$$

In (3) β is the real discount factor with which the firms discounts the future expected path of real marginal costs and u_t is a zero-mean, stationary term to capture any approximation error that might occur; the hat variables indicate that they are log-linearised around their steady state levels (eg $\hat{\pi}_t = \ln(\pi_t/\bar{\pi})$).

2.2 The NKPC under a non-zero equilibrium inflation rate

The assumption of a zero steady state rate of inflation in (3), however, can be undesirable. Ascari (2004), for example, shows how a positive steady state inflation rate results in a different optimal monetary policy than if one assumes zero steady state inflation. Bakshi *et al.* (2003) and Sahuc (2006) show that ignoring positive steady state inflation in specifying the NKPC equation, would lead the researcher to erroneously conclude that a hybrid NKPC, which mixes backward- and forward-looking behaviour as in (3), provides the best fit of the data even when there is no backward-looking behaviour present. The purpose of this paper is to assess whether

[1] Detailed derivations can be found in eg Galí and Gertler (1999) and Sbordone (2002).

'deep parameters' that underpin the Phillips curve trade-off have remained stable over the post-WWII era, despite the monetary policy regime changes we observed for major economies like the United States and the United Kingdom over this period. And as inflation in the long-run is determined by the central bank, these monetary regime changes implied different steady state rates of inflation. Hence, we have to use a NKPC equation derived under positive steady state inflation to assess the structural stability of the NKPC relationship.

Cogley and Sbordone (2005) derive a concise and empirically usable Calvo pricing-based NKPC framework under positive equilibrium inflation. In Cogley and Sbordone (2005, Appendix A) it is shown that under positive steady state inflation, or equilibrium inflation, we have the following relation between the steady state values of π_t and mc_t in order to ensure that equilibrium condition (2) holds in steady state:

$$\left(1-\alpha\bar{\pi}^{(\theta-1)(1-\varrho)}\right)^{\frac{1+\theta\omega}{1-\theta}}\left(\frac{1-\alpha\bar{R}\bar{\gamma}_y\bar{\pi}^{1+\theta(1-\varrho)(1+\omega)}}{1-\alpha\bar{R}\bar{\gamma}_y\bar{\pi}^{\theta-\varrho(\theta-1)}}\right)=(1-\alpha)^{\frac{1+\theta\omega}{1-\theta}}\left(\frac{\theta}{\theta-1}\right)\bar{mc} \tag{4}$$

where the variables with bars indicate steady state equivalents of the variables that have been defined earlier. By log-linearising (1) and (2) around a steady state rate of inflation equal to $\bar{\pi}>0$, Cogley and Sbordone (2005) show that the zero equilibrium inflation rate NKPC (3) changes to

$$\hat{\pi}_t=\tilde{\varrho}\hat{\pi}_{t-1}+\zeta\hat{mc}_t+b_1E_t(\hat{\pi}_{t+1})+b_2E_t\sum_{j=2}^{\infty}\gamma_1^{j-1}\hat{\pi}_{t+j}+\chi(\gamma_2-\gamma_1)(P_{\hat{R}_t}+P_{\hat{\gamma}_{y,t}})+u_t \tag{5}$$

with

$$P_{\hat{R}_t}=E_t\sum_{j=0}^{\infty}\gamma_1^j\hat{R}_{t+j,t+j+1}$$
$$P_{\hat{\gamma}_{y,t}}=E_t\sum_{j=0}^{\infty}\gamma_1^j\Delta\hat{y}_{t+j+1}$$

Compared to (3), NKPC (5) has two extra righthand side terms: an extra forward-looking term related to future inflation and one related to future stochastic movements in output growth and the nominal discount term, and it precisely is because (3) lacks these terms that Sahuc (2006) argues that only a hybrid version of (3) (i.e. $\varrho>0$) can fit the data well. The coefficients in (5) are non-linear functions of the structural parameters α, θ, ϱ and ω, which are identical as in (3), as well as equilibrium inflation $\bar{\pi}$ and $\tilde{\beta}=\bar{R}\bar{\pi}\bar{\gamma}_y$, which is the steady state value of the real discount factor. Hence, the coefficients in (5) can be defined as follows[2]

$$
\begin{aligned}
\tilde{\varrho} &= \frac{\varrho}{\Delta} \\
\zeta &= \frac{\frac{(1-\alpha\xi_1)(1-\gamma_2)}{\alpha\xi_1(1+\theta\omega)}}{\Delta} \\
b_1 &= \frac{\phi_1\left(\frac{1-\alpha\xi_1}{\alpha\xi_1}\right)+\gamma_2}{\Delta} \\
b_2 &= \frac{\frac{1-\alpha\xi_1}{\alpha\xi_1}\left(\frac{\theta(1-\varrho\gamma_1)+\varrho\gamma_1}{1+\theta\omega}\right)(\gamma_2-\gamma_1)}{\Delta} \\
\chi &= \frac{\frac{1-\alpha\xi_1}{\alpha\xi_1(1+\theta\omega)}}{\Delta}
\end{aligned}
\tag{6}
$$

where we use

$$
\begin{aligned}
\xi_1 &= \bar{\pi}^{(\theta-1)(1-\varrho)} \\
\xi_2 &= \bar{\pi}^{\theta(1-\varrho)(1+\omega)} \\
\gamma_1 &= \alpha\tilde{\beta}\xi_1 \\
\gamma_2 &= \alpha\tilde{\beta}\xi_2 \\
\Delta &= 1+\varrho\gamma_2-\left(\frac{1-\alpha\xi_1}{\alpha\xi_1}\right)\phi_0 \\
\phi_0 &= \frac{\varrho\theta(\gamma_1-(1+\omega)\gamma_2)-\varrho\gamma_1}{1+\theta\omega} \\
\phi_1 &= \frac{1}{1+\theta\omega}\left(\gamma_2(1+\theta\omega)+(\gamma_2-\gamma_1)(\theta(1-\varrho\gamma_1)+\varrho\gamma_1)\right)
\end{aligned}
\tag{7}
$$

In the end, our purpose is not necessarily to use (5) as a better alternative to (3), but whether there is a inflation - real activity trade-off that is deeply entrenched in micro-founded behaviour of agents. More specifically, if we can describe the changes in monetary policy regimes over time through shifts in the steady state rate of inflation $\bar{\pi}$ then if (5) is structurally invariant over the regimes, any variation in the coefficients of (5) is only due to variations in $\bar{\pi}$. The underlying 'deep' parameters of the Calvo pricing model, i.e. α, ϱ and θ, would therefore be unaffected. The observed time-variation in the inflation - real activity trade-off would then be solely due to changes in the monetary policy regime.

2.3 Two-step structural parameter estimation

Before we discuss the issue of modelling time-variation amongst macroeconomic time series, it is appropriate at this point to discuss first the general methodology of estimating the structural parameters in a NKPC equation like (5). In principle, there are two strands in the NKPC literature when it comes to the estimation of the structural parameters in a NKPC equation like (3) or (5). One strand attempts to proxy the representative firm's conditional information set for the expectations terms in (3) or (5) using instrument variables and the resulting conditional

[2] Cogley and Sbordone (2005) provide a detailed derivation.

moment restrictions are than used in a generalised method of moment (GMM) procedure to estimate the structural parameters, see eg Galí and Gertler (1999). Sbordone (2002), on the other hand, proposes to use a reduced-form times series model, typically a vector autoregressive (VAR) model, as a representation of the data, after which values for the structural parameters are chosen such that they minimise the distance between an unrestricted estimate of the VAR and a representation of this estimated VAR on which the cross-equation restrictions of a NKPC equation like (3) or (5) are imposed. We will employ this latter two-step minimum distance estimation approach, as it is more convenient to incorporate time-variation in the structure of the data within this framework. The two-step minimum distance estimation of structural parameters based on an auxiliary model that provides a reduced form representation of the data, has a long tradition in structural econometrics that goes back to Chamberlain (1982).[3] Kodde *et al.* (1990), for example, apply the approach to estimate a dynamic rational expectation factor demand model for the Dutch manufacturing sector, and they also show that in general the method yields asymptotically efficient estimates for a large range of econometric models.

Lets assume for the moment that in our case the data can be represented by a constant parameter VAR model

$$Y_t = \mu + \sum_{j=1}^{p} A_j Y_{t-j} + \Omega^{\frac{1}{2}} \nu_t \tag{8}$$

where Y_t denotes a 4×1 data vector: $Y_t = \{\pi_t, \Delta y_t, mc_t, R_t\}'$, μ is the 4×1 vector of intercepts, A_j is the 4×4 matrix of coefficients for the j^{th} lags of the endogenous variables collected in Y_{t-j}, Ω is the 4×4 covariance matrix of the VAR disturbances, and the 4×1 vector ν_t with $\nu_t \sim N(\mathbf{0},\ I_4)$. In VAR(1) form (8) reads

$$\tilde{Y}_t = \tilde{\mu} + \tilde{A}\tilde{Y}_{t-1} + \tilde{\Omega}\tilde{\nu}_t \tag{9}$$

with $\tilde{Y}_t = (Y_t' \; Y_{t-1}' \cdots Y_{t-p+1}')'$, $\tilde{\mu} = (\mu' \; 0 \cdots 0)'$, $\tilde{\nu}_t = (\nu_t' \; 0 \cdots 0)'$ and the $4p \times 4p$ companion matrix

$$\tilde{A} = \begin{pmatrix} A_1 & A_2 & \cdots & A_{p-1} & A_p \\ I & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & I & & \mathbf{0} & \mathbf{0} \\ \vdots & & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & I & \mathbf{0} \end{pmatrix}$$

where I and $\mathbf{0}$ are an identity matrix and a matrix of zeros respectively with appropriate dimensions. In a first step we can use the estimate of (9) to construct empirical equivalents of the log-linearised data on which (5) is based, i.e.

$$\hat{Y}_t = \tilde{Y}_t - \left(I - \tilde{A}\right)^{-1} \tilde{\mu} \tag{10}$$

Next, (9) and (10) can be used to proxy the conditional expectations, current levels as well as the lagged levels on the righthand side in (5), by considering the inflation rate that is determined according to (5) as an expectation for present inflation conditional on $\hat{Y}_{t-1}$. This results in a set

[3] Of course, the general idea of incorporating cross-equation restrictions of a structural model in a reduced form presentation of the data to estimate the parameters of this model goes back to Hansen and Sargent (1981), who propose a one-step maximum likelihood procedure using a VAR representation of the data.

of non-linear cross-equation restrictions which should be equal to the unrestricted coefficients in $\tilde{A}$ that determine the dynamics of the inflation rate (see also Cogley and Sbordone (2005)):

$$\begin{aligned} e'_\pi \tilde{A} = \tilde{\varrho} e'_\pi + \zeta e'_{mc} \tilde{A} + b_1 e'_\pi \tilde{A}^2 + b_2 e'_\pi \gamma_1 (I - \gamma_1 \tilde{A})^{-1} \tilde{A}^3 \\ + \chi(\gamma_2 - \gamma_1) e'_R (I - \gamma_1 \tilde{A})^{-1} \tilde{A} + \chi(\gamma_2 - \gamma_1) e'_{\Delta y} (I - \gamma_1 \tilde{A})^{-1} \tilde{A} \end{aligned} \quad (11)$$

where e_K is an identity vector that corresponds with the equation for variable K (which can be π, mc, Δy or R), and ζ, b_1, b_2, γ_1, γ_2 and χ are defined in (5). This then yield the following $4p \times 1$ vector $\digamma_1$ of moment conditions:

$$\digamma'_1(\tilde{\mu}, \bar{\pi}, \tilde{\beta}, \tilde{A}, \psi) = e'_\pi \tilde{A} - g(\tilde{A}) \quad (12)$$

with $g(\tilde{A})$ equals the righthand side in (11), $\tilde{\beta}$ is the stochastic real discount factor defined as

$$\tilde{\beta} = \bar{R} \bar{\pi} \Delta \bar{y} \quad (13)$$

the empirical equivalents of the steady state values are equal to

$$\begin{pmatrix} \bar{\pi} \\ \Delta \bar{y} \\ \bar{mc} \\ \bar{R} \end{pmatrix} = \begin{pmatrix} \exp\left(e'_\pi \left(I - \tilde{A} \right)^{-1} \tilde{\mu} \right) \\ \exp\left(e'_{\Delta y} \left(I - \tilde{A} \right)^{-1} \tilde{\mu} \right) \\ \exp\left(e'_{mc} \left(I - \tilde{A} \right)^{-1} \tilde{\mu} \right) \\ \left(e'_R \left(I - \tilde{A} \right)^{-1} \tilde{\mu} \right) \end{pmatrix} \quad (14)$$

and ψ is the vector of structural parameters

$$\psi = (\alpha \quad \theta \quad \varrho \quad \omega)' = (\psi'_1 \quad \omega)' \quad (15)$$

with ω is a function of the production function capital elasticity κ

$$\omega = \frac{\kappa}{1 - \kappa}; \quad 0 < \kappa < 1$$

which we calibrate (see Section 3.2).

Equilibrium condition (4) yields another moment condition,

$$\begin{aligned} \digamma_2(\tilde{\mu}, \bar{\pi}, \bar{R}, \Delta\bar{y}, \bar{mc}, \tilde{A}, \psi) = \left(1 - \alpha \bar{\pi}^{(\theta - 1)(1 - \varrho)} \right)^{\frac{1 + \theta\omega}{1 - \theta}} \left(\frac{1 - \alpha \bar{R} \Delta \bar{y} \bar{\pi}^{1 + \theta(1 - \varrho)(1 + \omega)}}{1 - \alpha \bar{R} \Delta \bar{y} \bar{\pi}^{\theta - \varrho(\theta - 1)}} \right) \\ - (1 - \alpha)^{\frac{1 + \theta\omega}{1 - \theta}} \left(\frac{\theta}{\theta - 1} \right) \bar{mc} \end{aligned} \quad (16)$$

We will summarise (12) and (16) as the $(4p + 1) \times 1$ vector

$$\digamma(\tilde{\mu}, \bar{\pi}, \bar{R}, \Delta\bar{y}, \bar{mc}, \tilde{A}, \psi) = (\digamma'_1(\tilde{\mu}, \bar{\pi}, \tilde{\beta}, \tilde{A}, \psi) \ \digamma'_2(\tilde{\mu}, \bar{\pi}, \bar{R}, \Delta\bar{y}, \bar{mc}, \tilde{A}, \psi)' \quad (17)$$

After estimating the reduced form VAR model, which will yield estimates of $\tilde{\mu}$ and $\tilde{A}$, one can estimate the parameters in ψ by minimising the following objective function:

$$\min_{\psi_1} \digamma'(\tilde{\mu}, \bar{\pi}, \bar{R}, \Delta\bar{y}, \bar{mc}, \tilde{A}, \psi)\digamma(\tilde{\mu}, \bar{\pi}, \bar{R}, \Delta\bar{y}, \bar{mc}, \tilde{A}, \psi) \tag{18}$$

Numerical methods have to be used to solve (18) for ψ, and often when the aforementioned two-step minimum distance estimation is used to estimate the structural parameters that underly the NKPC equation, one employs a grid search algorithm. Although slow in terms of reaching the point of convergence, the grid search approach is robust to any 'odd shapes' in the contour of objective function (18) and hence we employ this approach to get estimates of the structural parameter vector ψ_1.

3 Modelling time-variation in macroeconomic time series: Markov switching VAR approach

Cogley and Sargent (2002, 2005) as well as Sims and Zha (2006) convincingly showed for the US that the joint time series behaviour of macroeconomic series such as inflation and real economic growth has shifted several times over the post-WWII period and although the jury is still out on whether these shifts are due to changes in the monetary policy regime or shifts in structural shock processes, our prior is that these shifts in the joint time series behaviour coincided with shifts in the steady state rate of inflation. In order to be able to estimate the structural parameters that underlie (5), we have to choose a specification that is able to capture these shifts properly.

Cogley and Sbordone (2005) and Cogley and Sargent (2002, 2005) use a time-varying parameter VAR (TVP-VAR) to model the shifts in the data. Sims and Zha (2006), on the other hand, employ a Markov switching framework, which allows them to treat shifts in the joint behaviour of time series as stochastic and potentially non-monotonic (i.e. the multivariate time series process can potentially shift back to a regime that had been in place in the past) in nature. This is appealing for our framework, as it would assume that when the representative firm has to re-optimise its pricing plan, it makes a probabilistic assessment of whether the economy is operating under a high or a low steady state inflation regime. Also, the Markov switching framework allows us to identify the VAR coefficients that corresponds with different levels of steady state inflation in an objective, data-driven manner, whereas with a TVP-VAR model the researcher has to make a judgement about which VAR coefficient matrix belongs to which steady state inflation level.[4] Note, however, that the TVP-VAR offers flexibility in terms of allowing for independent variation in VAR coefficients and volatility of the reduced form shocks.

Our empirical model tries to bring together this flexibility of the TVP-VAR and the greater 'structure' offered by the Markov switching VAR (MS-VAR). In particular, we use MS-VAR models where we allow for M possible 'trend inflation regimes' while simultaneously allowing for an unobserved break in the variance covariance matrix. This latter structural break specification for independent shifts in the variance is inspired by Sensier and Van Dijk (2004), who show that for a majority of US macroeconomic time series it is more appropriate to model time-variation in volatility as instantaneous breaks than as gradual changes.

[4] This is because within a TVP-VAR model for T time series observations, one would end up with a $(4p+1)\times T$ matrix of moment conditions $\digamma(\tilde{\mu}_t, \tilde{A}_t, \psi)$ for $t = 1, \ldots, T$, which yields a too high-dimensional computational exercise. Cogley and Sbordone (2005) therefore pick the VAR coefficient matrices on four dates, which they assume, based on anecdotal evidence and judgement, to be representative for different monetary policy regimes.

In Section 3.1 we describe the MS-VAR framework, summarise the estimation issues that are related to this framework and describe the procedure we use to select the number of regimes in the MS-VAR. After a brief data description in Section 3.2, we report the reduced form estimation results for the MS-VAR models in case of the United Kingdom, the United States and the Euro area can be found in Section 3.3

3.1 Methodology

We assume that the data can be described by the following Markov Switching VAR (MS-VAR) model

$$Y_t = \mu_s + \sum_{j=1}^{p} A_{j,s} Y_{t-i} + \Omega_S^{\frac{1}{2}} \nu_t \tag{19}$$

where Y_t is the 4×1 data vector from (8) and the ν_t is also the same as in (8). The subscript 's' denotes the unobserved regime related to the VAR coefficients and it can take on M discrete values, $s = 1, 2, ...M$. Similarly, $S = 1, 2$ denotes the unobserved regime related to the disturbance covariance matrix (with $diag\,(\Omega_1) > diag\,(\Omega_2)$) and is assumed to be independent of s.[5] Therefore, in (19) μ_s, $A_{j,s}$ and Ω_S are the regime-specific versions of intercept vector μ, coefficient matrix A_j and disturbance covariance matrix Ω in (8).

Following Hamilton (1989) and Hamilton (1994, Chapter 22), the state variable s is modelled as a stationary, time homogeneous, first order Markov Chain with a transitional probability matrix

$$\tilde{P} = \begin{pmatrix} p_{11} & p_{21} & \cdots & p_{M1} \\ p_{12} & p_{22} & \cdots & p_{M2} \\ \vdots & \vdots & \cdots & \vdots \\ p_{1M} & p_{2M} & \cdots & p_{MM} \end{pmatrix} \tag{20}$$

where $p_{ij} = Pr\,(s_{t+1} = j | s_t = i)$ for $i, j = 1, \ldots, M$.

The state variable S follows a first order Markov chain with the following *restricted* transition matrix

$$\tilde{Q} = \begin{pmatrix} q_{11} & 0 \\ q_{12} & 1 \end{pmatrix} \tag{21}$$

where $q_{ij} = Pr\,(S_{t+1} = j | S_t = i)$. The restrictions in (21) make regime $S = 2$ an absorbing state and essentially implies that we model one unknown breakpoint in the evolution of Ω (see Kim and Nelson (1999b)). This simple formulation captures time-variation in volatility highlighted by Kim and Nelson (1999b) and Sims and Zha (2006) while still maintaining model parsimony.

Following Albert and Chib (1993) and Kim and Nelson (1999a, Chapter 9) we use Bayesian simulation methods to estimate the MS-VAR models[6]. In particular, we use Gibbs sampling to simulate draws from the posterior distribution. Details of the prior and the posterior distributions are confined to Appendix A. Here, we briefly describe the main steps in the algorithm.

[5]Indeed, Sensier and Van Dijk (2004) find that for 80% of 214 US macroeconomic time series over 1959-1999 most of the observed reduction in volatility is due to a reduction in conditional volatility rather than breaks in the conditional mean.

[6]The likelihood function for the model can be calculated using the non-linear filter described in Hamilton (1994, Chapter 22) and Kim and Nelson (2000). Although standard numerical techniques are readily available for maximising the likelihood function, the large number of free parameters make this a challenging task especially with independent regime switching in the VAR coefficients and the covariance matrix. In addition, model selection is greatly simplified in the Bayesian framework.

1. **Sampling s_t and $\mathbf{S}_t$:**
   Following Kim and Nelson (1999a, Chapter 9) we use Multi-Move Gibbs sampling to draw s_t from the joint conditional density $f(s_t|Y_t, \mu_s, A_{1,s}, \ldots, A_{p,s}, P, S_t)$ and S_t from the joint conditional density $f(S_t|Y_t, \mu_s, A_{1,s}, \ldots, A_{p,s}, P, s_t)$.

2. **Sampling $\mu_s,\ A_{1,s}, \ldots, A_{p,s},\ \Omega_S$:**
   Conditional on a draw for s_t and S_t the model is simply a sequence of Bayesian VAR models. The regime specific VAR coefficients are sampled from a Normal distribution and the covariances are drawn from an inverted Wishart distribution.

3. **Sampling $\tilde{P}$ and $\tilde{\mathbf{Q}}$**:
   Given the state variables s_t and S_t, the transition probabilities are independent of Y_t and the other parameters of the model and have a Dirichlet posterior.

This sampling algorithm is complicated due to the possibility of 'label switching'. That is, the likelihood function of the model is exactly the same if $\mu_m, A_{j,m}, \tilde{P}_m$ are replaced with $\mu_n, A_{j,n}, \tilde{P}_n$ for $m \neq n$. This may imply that the resulting posterior distribution is multi-modal. We identify the regimes by imposing inequality restrictions on the level of mean inflation implied by the model across regimes. For example, when $M = 2$ we require that $\bar{\pi}_1 > \bar{\pi}_2$.

We set the lag length $p = 2$. The choice of the number of (coefficient) regimes, M, is a crucial specification issue as this may have a substantial impact on our estimates for $\bar{\pi}$. Following Sims and Zha (2006) we select M by comparing marginal likelihoods across models with $M = 1, \ldots, 4$. In contrast, we fix the number of variance regimes to 2 and assume one break in the volatility of the reduced form shocks. This assumption about the number of volatility regimes is based on the following observations. First, there is some evidence for a single major break in volatility (of some of the variables in our VAR models) presented by Kim and Nelson (1999b), McConnell and Perez-Quiros (2000), Sensier and Van Dijk (2004) and Benati (2004). Next, given the computational burden involved in estimating MSVAR models (with independent shifts in the coefficients and the covariance matrix), one of our aims is to keep the model as parsimonious and simple as possible while simultaneously allowing for enough time-variation to capture important shifts in trend values of the endogenous variables.

For the model selection exercise we estimate each MS-VAR using 30000 replications of the Gibbs sampler discarding the first 27000 as burn-in. Finally, the selected model is re-estimated using 250000 replications with first 240000 discarded as burn-in.

3.2 The data

Our aim is to estimate the structural parameters in the NKPC equation (5) for the United Kingdom and the United States, and for us to be able to do that we need data on inflation, real output growth, the discount rate and real marginal costs of the representative firm. Inflation and real output growth can be constructed in a fairly straightforward manner from the available data. Also, the nominal discount rate R_t can be constructed without any major effort from nominal interest rates data, i.e.

$$R_t = \frac{1}{1 + i_t} \tag{22}$$

where i_t is a nominal interest rate with appropriate maturity. Constructing real marginal costs is a bit more involved as these are unobserved and one therefore has to come up with some approximation. Following Galí and Gertler (1999) and Sbordone (2002), we assume that the output of an economy is produced through a Cobb-Douglas production function,

$$Y_t = \Lambda_t N_t^{1-\kappa} K_t^{\kappa}; \quad 0 < \kappa < 1 \tag{23}$$

where Y_t is the level of output in real terms, N_t is the total amount of labour input, K_t is the amount of capital input and κ is the long-run share of capital in output. Abstracting from capital, the real marginal costs of the representative firm equals

$$MC_t = \left(\frac{W_t}{P_t}\right)\left(\frac{\partial Y_t}{\partial N_t}\right) \tag{24}$$

with W_t is the wage rate. Therefore, through (23) and (24), we can write the real marginal costs in logarithm as

$$mc_t = \ln\left(\frac{W_t N_t}{P_t Y_t}\right) - \ln(1-\kappa) = \ln(ULC_t) - \ln(1-\kappa) \tag{25}$$

Hence, by constructing unit labour costs or labour share data for each of our economies we can approximate the log real marginal costs through (25) where we assume that $\kappa = 1/3$[7].

For the United Kingdom, we use quarterly data over the sample 1963.I - 2005.II. The inflation rate π_t is computed as the relative annualised quarter-to-quarter change in the implicit gross domestic product (GDP) deflator, real output growth Δy_t is set equal to the relative annualised quarter-to-quarter change in real GDP, and the nominal discount rate R_t is constructed through (22) using the three-month Treasury bill interest rate (divided by 100). Our interest data are taken from the *Global Financial Data* website, whereas the GDP related data are from the *Office for National Statistics*. The labour share data that is used to construct mc_t through (25) for the United Kingdom is an update of the labour share data used in Batini *et al.* (2000, 2005).[8]

Quarterly data for a 1955.I - 2005.II sample are used in case of the United States. We construct the inflation and real output growth data in a comparable way as for the United Kingdom, i.e. relative annualised quarter-to-quarter changes in the implicit GDP deflator and real GDP respectively, and these data are extracted from the Federal Reserve Bank of St. Louis' *FRED*® database. Similarly, the nominal discount rate (22) is based on the three-month Treasury bill rate, which is taken from the *Global Financial Data* website. To construct the US labour share data, we use data on labour income from the US Bureau of Economic Analysis' *National Income and Product Accounts*. Based on a Cobb-Douglas production function decomposition of GDP we use this labour income data to construct a labour share series that strips out ambiguous components of labour income as well as the impact of the government sector; see Appendix B for more details.

Finally, for the Euro area we use in our estimations quarterly data for the period 1970.I - 2005.IV. All the data are either directly taken from or constructed using series taken from the latest update of the data set that corresponds with the ECB's Area-Wide Model (AWM)[9], which we retrieved from the the Euro Area Business Cycle Network (see http://www.eabcn.org/data/awm/index.htm). Inflation and real output growth are, again, constructed as annualised relative quarter-to-quarter changes in, respectively, the implicit GDP deflator and real GDP, whereas the nominal discount rate (22) is constructed using the short-term

[7]This is broadly in line with the long-run average of the capital share across the developed economies, see Backus *et al.* (1992)

[8]See Appendix B for more details regarding these labour share data.

[9]See Fagan *et al.* (2001) for a description of this data set.

Table 1: Log marginal likelihoods for MS-VAR model (19) estimated for the UK and US

Model	UK	US	Euro Area
BVAR	-699.3	-701.8	-701.4
M=1	-700.5	-700.9	-700.7
M=2	-674.8	-687.0	-699.4
M=3	-672.8	-694.8	-701.0
M=4	-690.2	-692.9	-700.8

Notes: The entries are log marginal likelihoods computed for MS-VAR model (19) estimated for lag orders $p = 1,\ 2$ and number of regimes $M = 1, \ldots, 4$. The modified harmonic mean method of Gelfand and Dey (1994) is used for computing the marginal likelihoods.

interest rate measure from the extended AWM database. The quality of the labour share data for the Euro area is less than for the other economies. We approximate the Euro area equivalent of this variable with the ratio of nominal income of employees, which we corrected for the presence of self-employed earners, over nominal GDP at factor costs; see Appendix B.

3.3 Reduced form estimation results

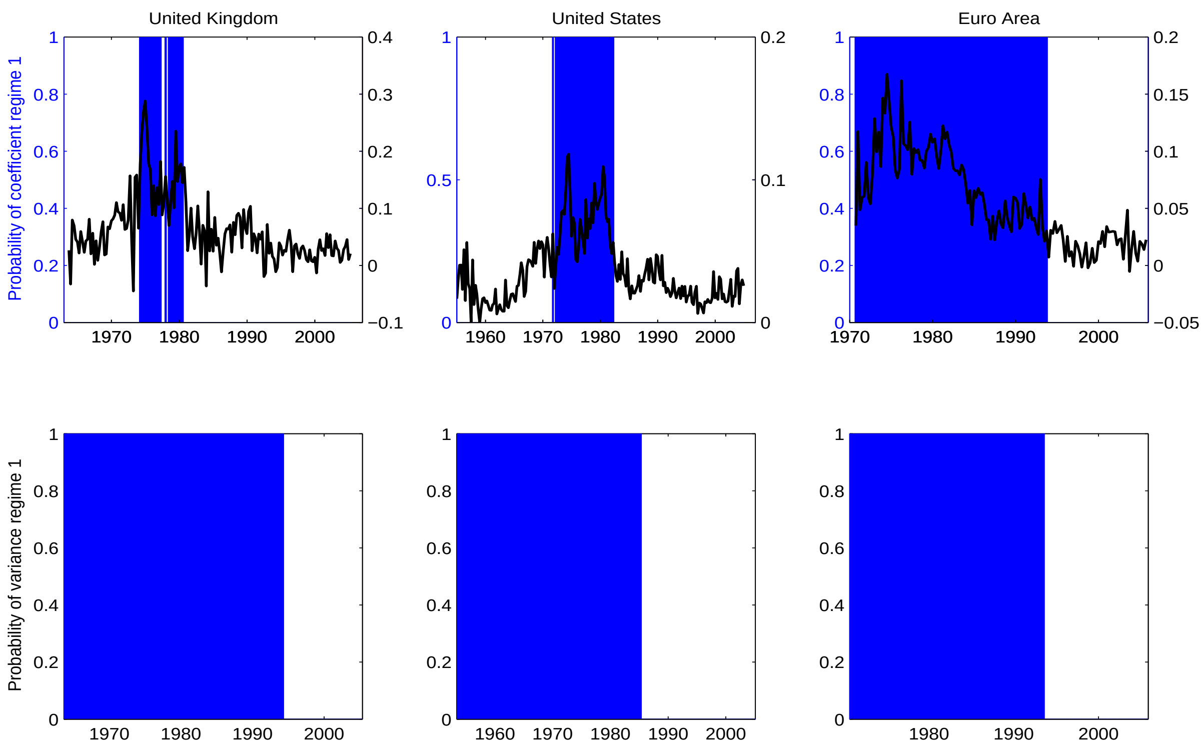
We use the described in Section 3.2 to fit a MS-VAR for π_t, Δy_t, mc_t and R_t; see also (19). In this subsection we report the estimation results of the optimal MS-VAR fitted for each economy.

A first step in the estimation of these MS-VAR models is to determine the optimal number of regimes M. Our model selection procedure involves the estimation of models with $M = 1, \ldots, 4$ and then selecting the MS-VAR with the highest marginal likelihood. Table 1 reports the estimated log marginal likelihoods for the UK, the US and the Euro area. Note that the first row of the table presents results for a time-invariant Bayesian VAR, while the remaining rows display results from models that incorporate the break in the covariance matrix. These marginal likelihoods are approximated using the modified harmonic mean method proposed by Gelfand and Dey (1994)[10].

Table 1 indicates that models with $M > 1$ are preferred to linear VAR models, and this is not a surprising result given the large literature that indicates the presence of shifts in persistence and volatility of inflation and output for these economies (e.g. Cogley and Sargent (2002, 2005), Benati (2004) and Levin and Piger (2004)). For the UK, an MS-VAR with $M = 3$ is the preferred model. However, the two regime model is a close second with a relatively small difference in the estimated log marginal likelihoods. Moreover, our estimates for these two specifications indicate that the third regime adds little in terms of capturing significant shifts in trend inflation, while significantly inflating the number of estimated coefficients in the model. Therefore, we base our

[10]See Sims and Zha (2006) for description of how this method is applied to Markov switching models.

Figure 1: Probability of coefficient and variance regimes

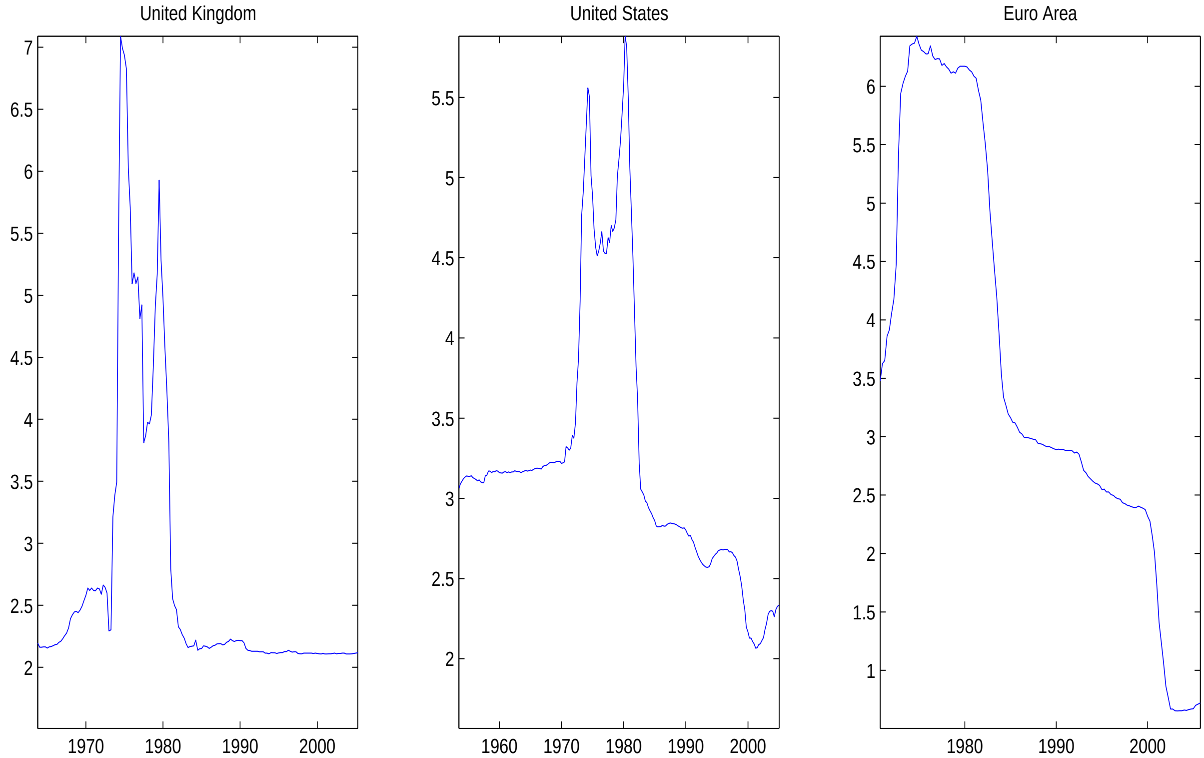


The blue shaded areas in the top two figures are the probabilities for the UK, the US and the Euro area that $p(s_t = 1)$ estimated from the draws of s_t in the Gibbs sampling estimation of MS-VAR model (19) with $p = 2$ and $M = 2$ for the UK, the US and the Euro area. The blue shaded areas in the lower two figures indicate when the disturbance covariance matrix of the aforementioned MS-VAR models is in variance regime 1 (i.e. the era before the structural break in this covariance matrix).

main results on the MS-VAR model with $M = 2$ but report results for the three regime model in Appendix C. For the US, the evidence clearly points to the presence of two coefficient regimes and similarly, but less clearly, this is the case for the Euro area.

The top panel of Figure 1 plots the probability of each (coefficient) regime for the UK and the US $p(s_t = i)$ estimated from the draws of s_t. For the UK and the US economies the timing of the trend inflation regimes is quite similar. The high trend inflation regimes occur during the 1970's and the early 1980's for both countries. Similarly the pre-1970's period, the 1990's and beyond are largely characterised by the low trend inflation regime for both countries. In case of the Euro area, the high trend inflation regime lasts throughout the 1970s and 1980s until the early 1990s - the shift to a low trend inflation regime takes place at the end of 1993. This is quite similar to the break dates that Levin and Piger (2004) identified for GDP deflator inflation in the Euro area member states in their sample (France, Germany, Italy and the Netherlands): using a Bayesian approach they identify break dates for this inflation measure in these countries

Figure 2: MS-VAR (19) implied trend inflation levels



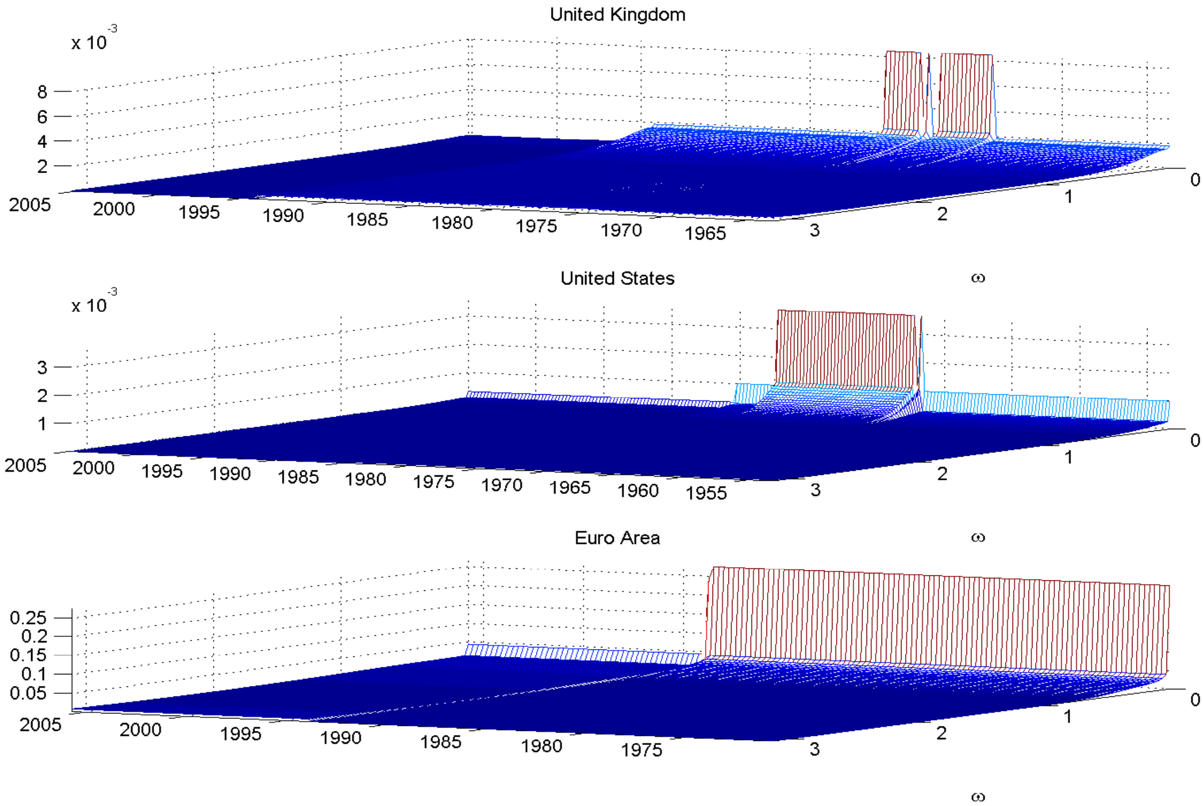
The 'trend' levels of the respective GDP deflator inflation series equal the medians across the Gibbs sampling replications where in each replication we construct a weighted average of the regime-specific long-run values $\bar{\pi}$ (see (14)) with the weights derived from the draw of the state variable s_t

in the first half of the 1990s.

The bottom panel of Figure 1 shows the probability of the first variance regime for the economies. For both the UK and the Euro area, a break in the variance occurred in the early 1990s, coinciding with the introduction of inflation targeting in 1992 and independence of the Bank of England in 1997 in the UK as well as the aftermath of the break-down of the European exchange rate mechanism (ERM I). Interestingly for the Euro area the variance break practically coincides with the shift in trend inflation regime, where the variance break occurs one to two quarters earlier than the trend inflation regime shift, despite the fact that we do not impose this in the estimation. For the US, this break occurred in the mid 1980s after the Volcker experiment of non-borrowed reserves targeting. This US finding is close to that reported in Kim and Nelson (1999b) and McConnell and Perez-Quiros (2000).

How can we typify the different trend inflation regimes for the economies? Figure 2 plots our median estimates of the evolution of annual trend inflation in the economies as estimated by the MS-VAR models. The estimates for these trends are constructed as a weighted average of $\bar{\pi}$ where the weights are derived from the draw of s_t. For the UK, the 1970's and the early 1980's were clearly a time of high trend inflation with the estimates of $\bar{\pi}$ reaching a maximum of around 7% in the mid-1970s. Since the early 1980s, trend GDP deflator inflation has remained

Figure 3: Normalised spectral densities for UK and US inflation



The normalised spectral densities in the figure equal the medians across the Gibbs sampling replications of the UK, US and Euro area MS-VAR models (19) where in each replication we construct a weighted average of the regime-specific normalised spectral densities based on (27) with the weights derived from the draws of the state variable s_t.

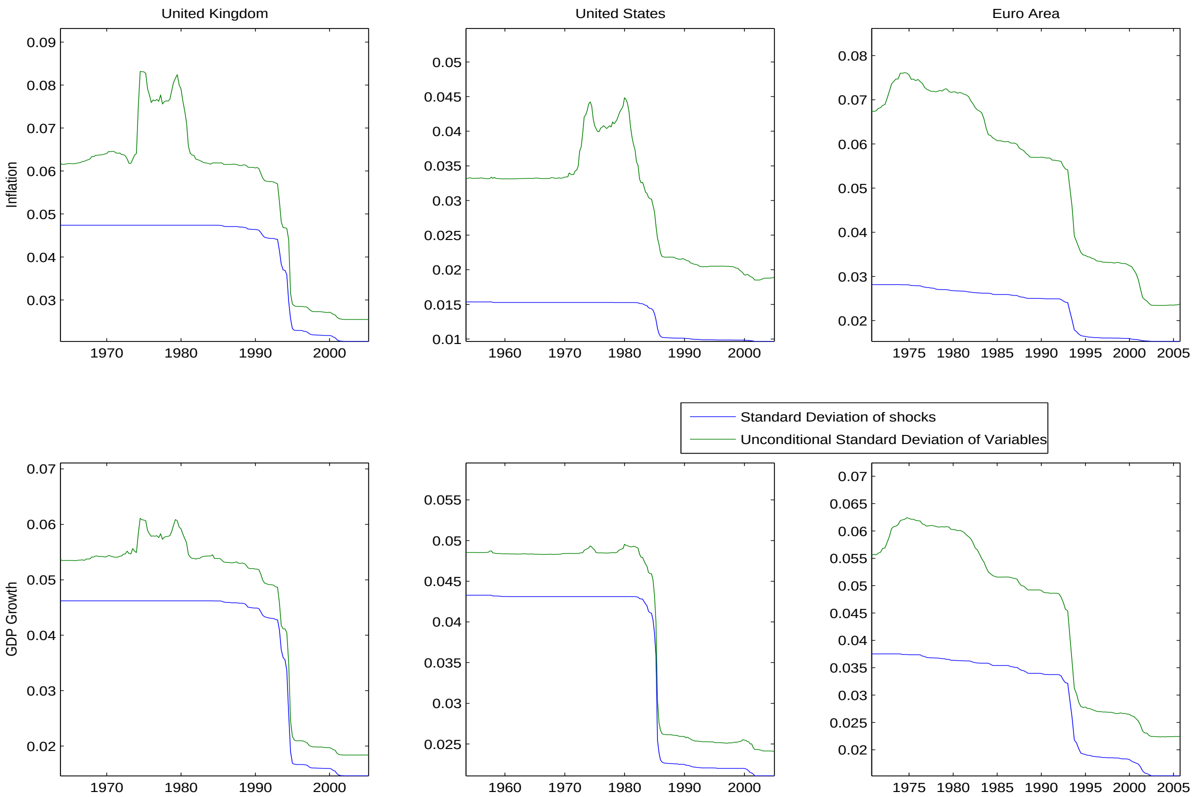
just above 2%.[11] The results are similar for the US, with the high trend inflation of the 1970s and the early 1980s declining after the Volcker experiment of the early 1980s. In the Euro area the high trend inflation of the 1970s and early 1980s declines more gradually than in the other economies, although towards the starting date of the European monetary union (EMU) Euro area trend inflation appears to start to decline at a higher pace.[12]

Figure 3 presents evidence on inflation persistence and plots the normalised spectral densities of UK, US and Euro area inflation. These are calculated for each regime as

$$\frac{S_\pi(\varpi, s)}{\int_\varpi S_\pi(\varpi, s)} \tag{26}$$

[11]Note that this is not equal to the current 2% inflation target in the United Kingdom, as this target refers to the consumer price index. Hence, differences in composition, e.g. trade and services have different weights in the GDP deflator than in the consumer price index, means that the numerical trend levels of the two will be different.

[12]The dynamic pattern (but not the level) of Euro area trend inflation in Figure 2 looks remarkably similar to the low frequency component that Assenmacher-Wesche and Gerlach (2006) extract from the same Euro area GDP deflator inflation series with frequency domain techniques.



The blue lines show the square roots of the inflation and GDP growth error variance from the UK, the US and the Euro area estimates of MS-VAR (19) with $p = 2$ and $M = 2$, i.e. the square root of the diagonal elements of inflation and GDP growth in the estimated Ω_S in (19) with $p = 2$ and $M = 2$. The green lines show a weighted average of the regime-specific square roots for the inflation and GDP growth elements of (28) with the weights derived from the draw of the state variable s_t based on the UK, US and Euro area estimates of MS-VAR (19) with $p = 2$ and $M = 2$.

where

$$S_\pi(\varpi, s) = \frac{1}{2\pi}\left(I - \tilde{A}_s e^{-i\varpi}\right)^{-1} \hat{\Omega}_S \left(I - \tilde{A}'_s e^{i\varpi}\right)^{-1} \tag{27}$$

and ϖ denotes the frequency, I is a conformable identity matrix, $\tilde{A}_s$ denotes the companion matrix formed at the posterior mean and $\hat{\Omega}_S$ is the posterior mean estimate of the covariance matrix. Figure 3 reports the weighted average of the normalised spectrum across the regimes for the three economies. The normalised spectrum for the UK confirms recent evidence presented in Benati (2004). The inflation targeting period has been characterised by very low inflation persistence. For the US, the post-Volcker period saw a sharp fall in inflation persistence. Cogley and Sbordone (2005) find similar results in their TVP model but report less time-variation than depicted in figure 3. Finally, Euro area inflation persistence exhibits a sharp decrease in the early 1990s, but note that both before and after this persistence decrease, the level of low frequency Euro area inflation persistence is quite a bit higher than in the other economies.

In their analysis of volatility breaks in a large number of US macroeconomic time series, Sensier and Van Dijk (2004) find for the bulk of those series that the observed decrease in volatility seemed more likely to be related to break in the error (or shock) variance than due to a break in the mean of the series. Is this also the case for our data? In Figure 4 we show the evolution of the standard deviation of the shocks to inflation and GDP growth as well as the standard deviation of inflation and real GDP growth. In the former, this is the square root of the corresponding diagonal term in the two-regime disturbance covariance matrix Ω_S of the UK, US and Euro area estimates of MS-VAR (19), whereas the latter equals the square root of the corresponding element of $\hat{V}$, where

$$\hat{V} = vec\left(E\left(Y_t'Y_t\right)\right) = \left(I - \tilde{A}_s \otimes \tilde{A}_s\right) \times vec\left(\hat{\Omega}_S\right) \tag{28}$$

based on estimates of (19). For the UK, the main decline in volatilities coincides with the introduction of inflation targeting at the end of 1992 and Bank of England independence in 1997, whereas for the Euro area the big volatility decline takes in 1993 after the break-up of ERM I. For the US, this occurred with the start of the post-Volcker era. Interestingly, for all economies these volatility reductions seems largely related to less volatile shocks, which is in line with Sensier and Van Dijk (2004). Note, however, that the sharp increase in (mainly inflation) volatility and subsequent decrease of the 1970s and early 1980s was not related to a change in the volatility of shocks, which suggests that shifts in the long-run means of the series are more likely to have caused this phenomenon.

4 Structural estimation results

In order to get estimates of the structural parameters that underly our general NKPC equation (5), we will use an estimate of the MS-VAR model in π_t, Δy_t, mc_t and R_t as the auxiliary model on which we impose the cross-equation restrictions that correspond with (5). In this section we will generalise the approach outlined in Section 2.3 to the case where one uses a MS-VAR model like (19) as the auxiliary model instead of a standard linear VAR model, and report estimation and test results regarding the structural parameters in NKPC equation (5). Details regarding estimation of and inference on the structural parameters using the MS-VAR auxiliary model can be found in Section 4.1, and the results of this approach are reported in Section 4.1.1.

4.1 Structural estimation using the MS-VAR model

As a starting point write MS-VAR model (19) in MS-VAR(1) form, i.e.

$$\tilde{Y}_t = \tilde{\mu}_s + \tilde{A}_s \tilde{Y}_{t-1} + \tilde{\Omega}_S \tilde{\nu}_t \tag{29}$$

with $\tilde{Y}_t = (Y_t' \; Y_{t-1}' \cdots Y_{t-p+1}')'$, $\tilde{\mu}_s = (\mu_s' \; 0 \cdots 0)'$, $\tilde{\nu}_t = (\nu_t' \; 0 \cdots 0)'$ and the $4p \times 4p$ companion matrix

$$\tilde{A}_s = \begin{pmatrix} A_{s,1} & A_{s,2} & \cdots & A_{s,p-1} & A_{s,p} \\ I & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & I & & \mathbf{0} & \mathbf{0} \\ \vdots & & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & I & \mathbf{0} \end{pmatrix}$$

Table 2: Bounds of the grid points for the structural parameter estimation.

Parameter	α	ϱ	θ
Bounds	$\{0,1\}$	$\{0,1\}$	$\{1,50\}$

This MS-VAR set-up is then used with the moment conditions (17) to estimate regime-specific structural parameters for (5)

$$\min_{\psi_{1,s}} F'(\tilde{\mu}_s, \bar{\pi}_s, \bar{R}_s, \Delta\bar{y}_s, \bar{mc}_s, \tilde{A}_s, \psi_{1,s}) F(\tilde{\mu}_s, \bar{\pi}_s, \bar{R}_s, \Delta\bar{y}_s, \bar{mc}_s, \tilde{A}_s, \psi_{1,s}); \quad s = 1, \ldots, M \tag{30}$$

with $\psi_{1,s} = (\alpha_s \quad \theta_s \quad \varrho_s)'$ using the two-step estimation approach described in Section 2.3.

As explained in Section 3.1, we employ Gibbs sampling to estimate s_t, μ_s and $A_{1,s}, \ldots, A_{p,s}$ in MS-VAR model (19). The reduced form results in Section 3.3 are then based on the moments of the retained draws from the joint posterior distribution for s_t, μ_s and $A_{1,s}, \ldots, A_{p,s}$[13]. In each of these draws from this joint posterior distribution, we also estimate ψ_1, s by minimising (30) for $s = 1, \ldots, M$ ψ_1, s for $s = 1, \ldots, M$: $\psi_1, s, g, \ s = 1, \ldots, M, \ g = 1, \ldots, \tilde{M}$, where $\tilde{M}$ denotes the number of retained draws.

The minimum distance estimation uses a simple grid search procedure where the objective function is evaluated over 20 grid points for each of the deep parameters: α, which is the fraction of firms who do not receive a signal to re-optimise prices, the indexation parameter ϱ, and the substitution elasticity across goods θ. The bounds of the grid points are specified in Table 2.

4.1.1 UK Estimates

Table 3 reports the moments of the structural estimates of the NKPC equation (5) for the UK across the two trend inflation regimes (top two panels) and for the entire sample (last panel)[14]. In addition to reporting medians and means we capture uncertainty by reporting the median absolute deviation of the structural estimates across the Gibbs draws.

Estimates of ϱ and α appear to be more precise than the estimate of θ that has a large median absolute devation in both regimes. The median estimated value of θ indicates that the mark-up has varied between 4% and 5 % for the UK over the two regimes. Our estimate for α in the low inflation regime is 0.25.

We find a significant degree of indexation across the two trend inflation regimes. In addition, the degree of indexation appears to have declined with the estimate of ϱ ranging from 0.8 in regime 1 to 0.55 in regime 2. The reduction in ϱ across regimes appears to be the only noticeable indication of structural change. However, the uncertainty surrounding the regime 2 estimate of ϱ is large enough to cast doubt on this as conclusive evidence for instability. Similarly, the distribution of the time-invariant estimate of ϱ (10^{th} and 90^{th} percentiles 0.3 and 0.95) includes

[13]Here, the number of retained draws refers to MCMC draws after the burn in period that satisfy the identification restrictions.

[14]Note that, in computing time-invariant structural estimates we still allow trend values of the endogenous variables to vary across regimes.

Table 3: Structural estimates for the United Kingdom

	Median	Mean	MAD
		Regime 1	
θ	20.61	24.66	17.15
ϱ	0.80	0.65	0.15
α	0.20	0.30	0.15
		Regime 2	
θ	23.06	26.18	17.15
ϱ	0.55	0.52	0.30
α	0.25	0.34	0.15
		Both Regimes	
θ	25.51	27.49	17.15
ϱ	0.80	0.71	0.15
α	0.20	0.26	0.10

Notes: The table reports the mean, median, and the median absolute deviations ('MAD') of the regime-specific estimates of the structural parameters underlying (5) across the retained draws from the Gibbs sampler for the UK MS-VAR model (19) with $p = 2$ and $M = 2$.

the regime specific median estimates. Although these bounds do not represent moments of the posterior distribution, they indicate that the evidence for instability in ϱ is not clear cut. We explore this further by computing the probability of an increase (or decrease) in this parameter across regimes.

The second column of Table 4 shows that the probability of a decrease in ϱ from the high to low trend inflation regime is larger than the probability of an increase in this parameter. However both probabilities are close enough to 0.5 to cast doubt on the possibility of a systematic change.

The median point estimates of θ varies little across the two regimes in Table 3. The probabilities in Table 4 indicate little evidence for an increase or decrease in this parameter across the two regimes.

The estimation results in Table 3 suggests that the degree of price stickiness, as determined by α (i.e. the fraction of firms that are allowed to re-optimise their prices), is lower in the high trend inflation regime prevalent in the mid-1970s and the early 1980s and rises slightly in the low trend inflation regime. Remember that our NKPC specification is based on Calvo (1983) pricing. Both the Calvo (1983) and Taylor (1980) pricing schemes assume that there is exogenous staggering of price changes across firms, either because a firms sets its price randomly (Calvo) or after N periods (Taylor), and thus the fraction of price adjusting firms is constant over time. This time-dependent pricing is in contrast to state-dependent pricing: firms face a 'menu cost'

Table 4: Probability of structural change for the United Kingdom

	θ	ϱ	α
Regime 2 $<$ Regime 1	0.41	0.59	0.38
Regime 2 $>$ Regime 1	0.59	0.41	0.62

Notes: The probabilities in row 'Regime 2 $<$ Regime 1' ('Regime 2 $>$ Regime 1') for each structural parameter is computed as the ratio of the retained draws from the Gibbs sampler for the UK MS-VAR model (19) with $p = 2$ and $M = 2$, for which a structural parameter is larger (smaller) in Regime 2 than in Regime 1.

on adjusting prices, but as monetary shocks and inflation variability become larger it becomes increasingly costly to keep prices fixed (e.g. Ball *et al.* (1989) and Dotsey *et al.* (1999)). Thus, the fraction of firms that adjust prices varies over time depending on the size of inflationary shocks and over different inflation regimes. Our result of an increase in price stickiness in the low trend inflation regime *vis-à-vis* the high trend inflation regime seems therefore at first sight more in line with state-dependent pricing than with the time-dependent pricing assumption that underpins our NKPC model.

However, there is not much evidence for a *significant* change in α. Firstly, the probabilities in the last column of Table 4 are not very far from 0.5. Secondly, the distribution of the time-invariant estimates of these parameters includes the regime specific point estimates (10^{th} and 90^{th} percentiles 0.05 and 0.60). Therefore, as in the case of the other structural parameters ϱ and θ, there is no evidence that the Calvo α has varied with shifts in trend inflation. It also suggest that once one account for shifts in trend inflation, there is no significant evidence that time-dependent pricing is empirically inappropriate. This is in accordance with some micro-data studies across firms in different economies that establish that time-dependent pricing across firms dominates state-dependent pricing under regular economic conditions; see, for example, Klenow and Kryvtsov (2005) for the United States and Stahl (2005) for Germany.

Overall we find limited variation in the structural parameters across the equilibrium inflation regimes. However, this variation is not large or systematic and does not, in our view, represent conclusive evidence for structural instability.

4.1.2 US Estimates

Estimates for the US are reported in Table 5. Our estimates for θ in the US imply a mark-up of around 4.5% to 5.8% across the two regimes. This combined with an estimate for α that is slightly smaller than the the one reported in Cogley and Sbordone (2005) suggests that prices have been less flexible in the US than suggested by the results in Cogley and Sbordone (2005) as well as compared to the UK.

We find very different values for ϱ in the US from those reported in Cogley and Sbordone

Table 5: Structural estimates for the United States

	Median	Mean	MAD
		Regime 1	
θ	18.16	23.95	14.70
ϱ	0.80	0.67	0.15
α	0.35	0.41	0.25
		Regime 2	
θ	23.06	26.52	17.15
ϱ	0.55	0.51	0.30
α	0.45	0.46	0.25
		Both Regimes	
θ	30.40	29.49	17.15
ϱ	0.85	0.74	0.10
α	0.30	0.35	0.15

Notes: See the notes in Table 3.

(2005). Our median estimate of the indexation parameter is large in both regimes (see Table 5). This finding is in sharp contrast to Cogley and Sbordone (2005) who report a value of zero. With respect to that finding, we find based on our structural estimates that the probability that $\varrho \approx 0$ slightly increases when trend inflation is low, but remains small, i.e. from just under 10% in regime 1 to 19% in regime 2. Given these small probabilities for $\varrho \approx 0$ it seems that, as in the UK, irrespective of the level of equilibrium inflation inflation indexation seems to be entrenched in firms' pricing behaviour.

As in the case of the UK, the (median) estimate of ϱ in Table 5 falls across the two regimes where the low equilibrium inflation regime is associated with lower indexation. However, when we compute the probabilities whether the indexation parameter has been different across the trend inflation regimes, see Table 6, this suggests that the change in ϱ is unlikely to be systematic.

Table 6: Probability of structural change for the United States

	θ	ϱ	α
Regime 2 $<$ Regime 1	0.38	0.59	0.41
Regime 2 $>$ Regime 1	0.62	0.41	0.59

Notes: See the notes in Table 4.

Table 7: Structural estimates for the Euro Area

	Median	Mean	MAD
		Regime 1	
θ	18.16	23.02	12.25
ϱ	0.70	0.60	0.15
α	0.45	0.44	0.30
		Regime 2	
θ	20.61	25.38	14.70
ϱ	0.45	0.48	0.30
α	0.45	0.46	0.25
		Both Regimes	
θ	23.06	26.38	14.70
ϱ	0.75	0.66	0.20
α	0.35	0.39	0.20

Notes: See the notes in Table 3.

In addition, the bounds of the time-invariant estimates (10^{th} percentiles 0.3 and 0.95) again include the point estimates in both regimes.

We have a similar picture for the other 'deep' parameters that underly our NKPC relationship, i.e. Calvo α and θ. The median estimates of θ increases slightly in regime 2 but remains close to the time-invariant estimate in the last panel of the Table 5 and the computed probabilities for differences in θ across regimes in Table 6 back this up. The slight increase in α in Table 5 across regimes again suggests that price flexibility may have moved more in line with a predominance of state-dependent pricing in the economy. However, the results in Table 6 caution against interpreting this as a systematic shift in this parameter.

On balance, our results suggest little conclusive evidence against the hypothesis that there exists for the US a Calvo pricing-based NKPC trade-off relationship that has remained structurally invariant under shifts in equilibrium inflation rates.

4.1.3 Euro Area Estimates

Finally, the Euro area estimates of the structural parameters underlying NKPC (5) are reported in Table 7. The estimates show not much variation in the structural parameters across the two trend inflation regimes, except for the inflation indexation parameter ϱ. This is not unlike the findings for the other economies, and the question is, again, whether this observed decline in ϱ is structural or not? It appears that it is not, given the reported probabilities of different degrees if inflation indexation across the regimes in Table 8.

For the remaining structural parameters, θ and the Calvo α (see also (5)), similar probabilities as for ϱ indicate that these parameters have also not been subjected to any structural

Table 8: Probability of structural change for the Euro Area

	θ	ϱ	α
Regime 2 < Regime 1	0.41	0.56	0.44
Regime 2 > Regime 1	0.59	0.44	0.56

Notes: See the notes in Table 4.

changes across the different Euro area trend inflation regimes; see Table 8. Therefore, the lower panel in Table 7 reports for the structural parameters under the assumption that they have not changed across the different trend inflation regimes. As in the case of the other economies, these estimates suggest that even if one allows for shifts in trend inflation, inflation indexation still significantly affects firms' price setting behaviour in the Euro area. Also, given the median θ estimate, which suggests an Euro area mark-up of 4.5%, and a median α estimate that is the highest across our three economies, Euro area price flexibility is lower than in the UK or the US.

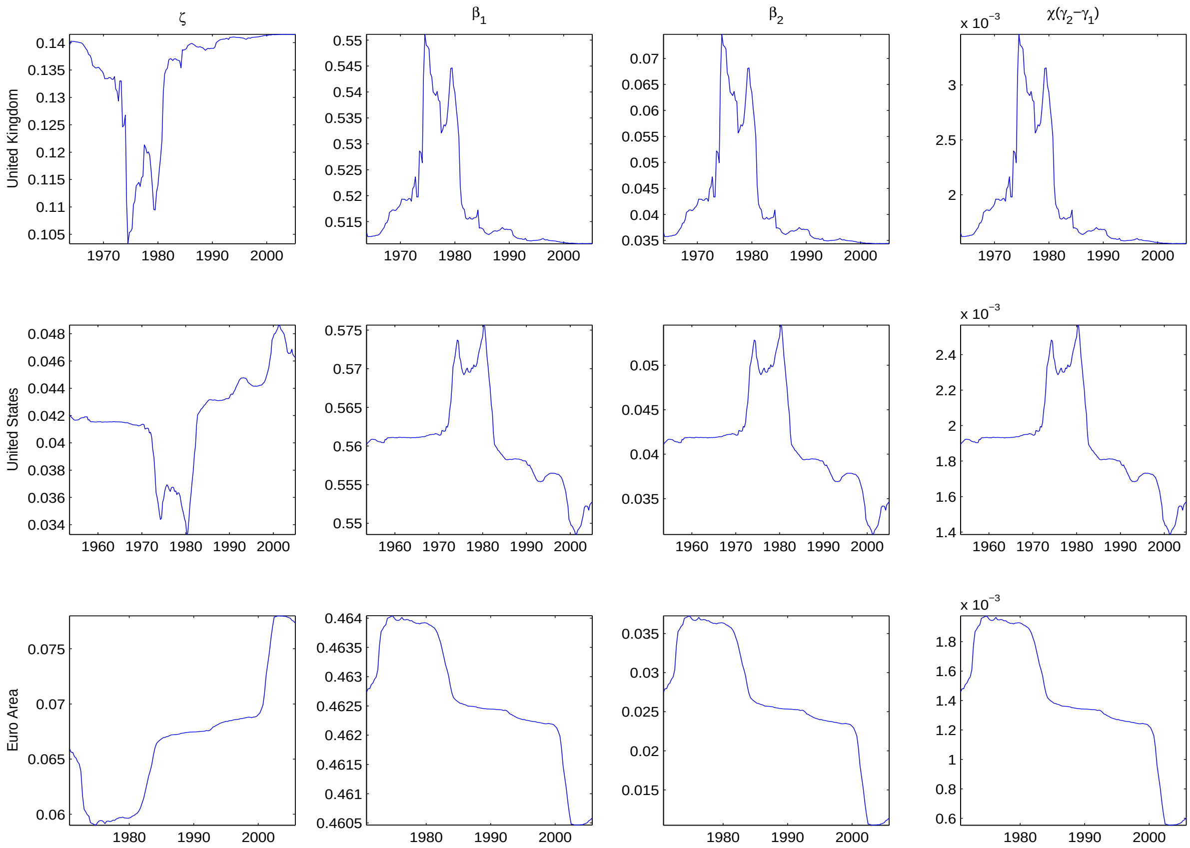
Hence, as for the other economies, the NKPC for the Euro area seems to be structurally invariant to shifts in trend inflation, and thus changes in the monetary policy regime.

One feature of our structural estimates across all three economies is that even when we allow for changes in trend inflation, inflation indexation (ϱ in (5) is still a significant factor in the process that drives inflation. This is in contrast with the theoretical arguments made by Sahuc (2006), namely that the empirical success of the 'hybrid' NKPC, log-linearised around 0, is due to the fact that these specifications ignore that trend inflation has been greater than 0 and time varying. It also contrast with the Cogley and Sbordone (2005) estimates of an US NKPC that allows for postitive and time-varying trend inflation: they claim that inflation indexation becomes insignificant. Hence, our estimation results, which are similar across different economies, indicate that staggered pricing is not enough to explain the trade-off between inflation and real activity satisfactorily, given the slightly the 'ad hoc' micro foundation for inflation indexation. Possibly additional, real, rigidities are needed for this, e.g. a Calvo pricing-based NKPC with real wage rigidity added to it, as in the case of Blanchard and Galí (2005).

4.2 Coefficients of the non-zero trend inflation NKPC

What are the implications of these structural estimates for the time variation in the coefficients of the NKPC in (5)? We examine this question by considering the evolution of the reduced form coefficients ζ, b_1, b_2 and $\chi(\gamma_2 - \gamma_1)$. An increase in equilibrium inflation, *ceteris paribus*, will affect the New Keynesian inflation-real costs relationship as follows: *(i)* The impact of current real costs on current inflation decreases, and *(ii)* the impact of both one-quarter ahead and higher order inflation expectations on current inflation increases. What is the intuition for this? Firms face uncertainty about when in the future they will be able to re-optimise their prices again and therefore the higher equilibrium inflation is, the larger the risk for firms that future profits will be eroded by inflation. As a consequence, firms attach more weight to future risks

Figure 5: Coefficients of the NKPC under shifting non-zero trend inflation



The figure plots the median values of β_1, β_2 and $\chi(\gamma_2 - \gamma_1)$ for (5) across the Gibbs sampling replications for MS-VAR model (19) with $p = 2$ and $M = 2$, where in each replication we construct a weighted average of the regime-specific long-run values $\tilde{\varrho}$, b_1, b_2 and $\chi(\gamma_2 - \gamma_1)$ (using the regime-specific estimates of α, θ and ϱ) with the weights derived from the draw of the state variable s_t. Except for equilibrium inflation rate $\bar{\pi}$, we fix the remaining structural parameters across the two trend inflation regimes.

and less weight to current costs.

Figure 5 plots the estimated values of these parameters for the UK, the US and the Euro area. Note that the results in Figure 5 assume that the structural parameters are constant at their full sample estimates and are constructed using the posterior means of the MS-VAR parameters. As we argue above, there is little evidence for signifcant structural change in these parameters.

Across all economies, b_1 is the largest coefficient. In contrast, the higher order terms $\chi\,(\gamma_2 - \gamma_1)$ are relatively small in magnitude. Although our estimate for ζ, b_2 and $\chi\,(\gamma_2 - \gamma_1)$, for the US, are close to those in Cogley and Sbordone (2005) in terms of magnitude, our estimate of b_1 is smaller.

As predicted by the theory, we find that the weight on current real costs decreases when trend inflation increases, while the weight on inflation expectation terms increase during these periods. All economies seem to be currently in low trend inflation regimes–our estimates indicate that the weight placed by firms on current costs is higher in this regime, while inflation expectations are not as important for price-setting as they were in the high inflation scenarios of the 1970s and the early 1980s.

5 Concluding remarks

The current paper estimates a Calvo (1983) pricing-based NKPC for the UK, the US and the Euro area under the assumption of non zero trend inflation. We characterise the reduced form dynamics of inflation, economic growth and real marginal costs via Markov Switching VAR models that allow for independent switching int the coefficients and the covariance matrix. The structural parameters are then estimated via a minimum distance estimator applied in each regime.

Reduced form results from the VAR models confirm recent observations on the 'Great Moderation'. In particular, we find that the recent period has been characterised by low levels of trend inflation, low inflation persistence and less volatile inflation and GDP growth both for the UK, the US and the Euro area.

In contrast, for all economies we find only weak evidence for structural change in the 'deep' parameters of the NKPC. Therefore our results largely confirm Cogley and Sbordone (2005). That is, using a different empirical methodology we find that structural parameters of the NKPC have been largely invariant to shifts in trend inflation. Note, however, that in contrast to Cogley and Sbordone (2005) we find for the UK, the US and the Euro area that there still is a significant degree of inflation indexation after one corrects for non-zero, shifting trend inflation. This may indicate that in addition to staggered pricing, other forms of (real) rigidities also has to be taken into account when modelling the inflation-real activity trade-off.

Appendices

A Gibbs sampling algorithm for estimating the MS-VAR model

The Gibbs sampler cycles through the following steps.

1. **Sampling the Covariance states** S_t :

Given starting values for the VAR parameters and covariances, the unobserved state variable for the two covariance regimes S_t is drawn using Multi-Move Gibbs sampling to draw from the joint conditional density $f\left(S_t|Y_t, \mu_s, A_{1,s}, \ldots, A_{p,s}, \tilde{P}, \tilde{Q}\right)$. Kim and Nelson (1999a, Chapter 9) show that the markov property of S_t implies that

$$f\left(S_t|Y_t\right) = f\left(S_T|Y_T\right)\prod_{t=1}^{T-1} f\left(S_t|S_{t+1}, Y_t\right) \tag{A.1}$$

where we have suppressed the conditioning arguments. This density can be simulated in two steps:

1. (a) Calculating $f\left(S_T|Y_T\right)$: The Hamilton (1989) filter provides $f\left(S_t|Y_t\right), t = 1, ...T$. The last iteration of the filter provides $f\left(S_T|Y_T\right)$.
   (b) Calculating $f\left(S_t|S_{t+1}, Y_t\right)$: Kim and Nelson (1999a, Chapter 9) show that

$$f\left(S_t|S_{t+1}, Y_t\right) \propto f\left(S_{t+1}|S_t\right) f\left(S_t|Y_t\right) \tag{A.2}$$

   where $f\left(S_{t+1}|S_t\right)$ is the transition probability and $f\left(S_t|Y_t\right)$ is obtained via Hamilton (1989) filter in step a. Kim and Nelson (1999a) (pp 214) show how to sample S_t from (A.2).

2. Sampling the covariance matrices Ω_S :

The covariance matrix in each regime $S = 1, 2$ is drawn from an inverse Wishart distribution. That is

$$\Omega_S^{-1} \sim W\left(\bar{S}_S^{-1}, v_S\right)$$

where the scale matrix $\bar{S}_1 = I\left(S_t = 1\right)\left(\bar{E}_t'\bar{E}_t\right)$ and $\bar{S}_2 = I\left(S_t = 2\right)\left(\bar{E}_t'\bar{E}_t\right)$ where $E_t = Y_t - \mu - \sum_{j=1}^{p} A_j Y_{t-j}$ and the 'bars' denote the average across $s_t = 1..M$ with the weights given by s_t. $I\left(.\right)$ is an indicator function that selects the observations for $S = 1, 2$. v_S is set equal to the number of observations in each regime[15].

3. **Sampling the coefficient states** s_t :

Given Ω_S, we re-write the model as

$$Y_t^* = \mu_s + \sum_{j=1}^{p} A_{j,s} Y_{t-i}^* + \breve{V}_t \tag{A.3}$$

[15]Note that we require each (coefficient and variance) regime to have at least $N(N \times P + 1) + 5$ observations.

where $Y_t^* = I(S_t = 1)\left[\Omega_1^{-1/2} Y_t\right] + I(S_t = 2)\left[\Omega_2^{-1/2} Y_t\right]$ and $E\left(\breve{V}_t' \breve{V}_t\right) = \tilde{I}_4$ where $\tilde{I}_4$ is a 4×4 identity matrix. This is an MSVAR model with a switching intercept and autoregressive parameters but a homoscedastic covariance matrix. We again use Multi-Move Gibbs sampling to draw $s_t, t = 1, 2, ...T$ from the joint conditional density $f(s_t | Y_t^*, \mu_s, A_{1,s}, \ldots, A_{p,s}, P)$ using the methods detailed in Kim and Nelson (1999a).

4. **Sampling** $\mu_s,\ A_{1,s}, \ldots, A_{p,s}$:

Conditional on a draw for s_t the model in equation A.3 is simply a sequence of Bayesian VAR models (with an identity covariance matrix). Collecting the VAR coefficients for regime $s = i$ into the $(N \times (N \times P + 1))$ vector Υ_s and the RHS (i.e. lags and the intercept terms) of equation 19 into the matrix X_t and letting $\hat{\Upsilon}_s$ denote the OLS estimates of the VAR coefficients the conditional posterior distributions are given by (see Uhlig (2005)):

$$\Upsilon_s \tilde{} N\left(\bar{\Upsilon}_s, \tilde{I}_4 \otimes \hat{V}_s\right)$$

where

$$\begin{aligned}
\bar{\Upsilon}_s &= \left(N_0 + X_t^{s\prime} X_t^s\right)^{-1}\left(N_0 \Upsilon_0 + X_t^{s\prime} X_t^s \hat{\Upsilon}_s\right) \\
\hat{V}_s &= \left(N_0 + X_t^{s\prime} X_t^s\right)^{-1}
\end{aligned}$$

and Υ_0 and N_0 denote the prior mean and variance. X_t^s for $s = 1, ..M$ denotes observations for a particular regime. In specifying the prior mean, we loosely follow Sims and Zha (1998). We assume that Υ_0 implies an AR(1) structure (with the intercept equal to zero) for each endogenous variable. As our variables are already in growth rates we center the prior at the OLS estimates of the AR(1) coefficient for each variable (rather than 1, i.e. a random walk). As in Sims and Zha (1998), the variance of the prior distribution is specified by a number of hyperparameters that control the variation around the prior. Our choice for these hyperparameters implies a fairly loose prior for the autoregressive coefficients in the VAR. The prior on the intercept terms is tighter and this choice ensures that trend values of the endogenous variables are more precisely estimated within each regime[16]. We do not consider 'unit root' or 'cointegration' priors.

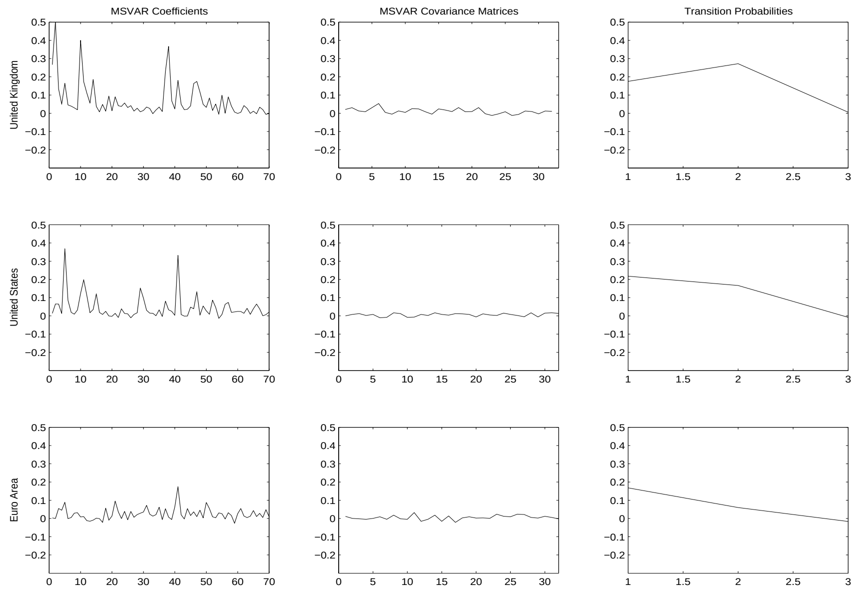
5. **Sampling** $\tilde{P}$ and $\tilde{Q}$:

The prior for the elements of the transition probability matrix p_{ij} and q_{11}, q_{12} is of the following form

$$\begin{aligned}
p_{ij}^0 &= D(u_{ij}) \\
q_{11}^0 &= D(u_{11}), q_{12}^0 = D(u_{12})
\end{aligned} \tag{A.4}$$

[16]Letting μ denote the hyperparameters, we set $\mu_0 = 1, \mu_1 = 0.5, \mu_2 = 1, \mu_3 = 1$ and $\mu_4 = 0.01$. The diagonal elements of the prior covariance matrix N_0 (relating to the autoregressive coefficients) are given as $\left(\frac{\mu_0 \mu_1}{\sigma_j p^{\mu_3}}\right)^2$ where σ_j denotes the variance of the error from an AR regression for the j^{th} variable and $p = 1..P$ denotes the lags in the VAR. The intercept terms in the VAR are controlled by the term $(\mu_0 \mu_4)^2$.

Figure A.1: 20^{th} order autocorrelations of the Gibbs draws



where $D(.)$ denotes the Dirichlet distribution and $u_{ij} = 20$ if $i = j$ and $u_{ij} = 1$ if $i \neq j$. This choice of u_{ij} implies that the regimes are fairly persistent. The posterior distribution is:

$$\begin{aligned} p_{ij} &= D\left(u_{ij} + \eta_{ij}\right) \\ q_{11} &= D\left(u_{11} + \bar{\eta}_{11}\right), q_{12} = D\left(u_{11} + \bar{\eta}_{12}\right) \end{aligned} \tag{A.5}$$

where η_{ij} denotes the number of times regime i is followed by regime j. $\bar{\eta}_{11}$ and $\bar{\eta}_{12}$ denote the same quantities for the variance regimes.

Two issues arise in the Gibbs sampling algorithm outlined above. First, as mentioned in the text, normalisation restrictions need to be placed on the draws of the VAR coefficients. We implement this in a straight forward manner by imposing the condition that $\bar{\pi}_{i+1} < \bar{\pi}_i$ where $i = 1, 2, ..M$. This normalisation is imposed via rejection sampling. The second issue concerns draws where one of the regimes is not visited. As noted by Sims and Zha (2006), this implies that in the next step of the sampler, the data are not informative for the redundant regime. We deal with such draws in the following way: If a redundant (coefficient or variance) state is encountered in step 1, we discard this draw and keep on re-drawing s_t until all regimes are reached or the number of these intermediate draws exceeds 1000. In the latter case we use the intial conditions to evaluate step 2 and step 3 but do not retain the draw[17].

Figure A.1 plots 20^{th} order autocorrelations of the retained draws. The low autocorrelations provide some evidence of convergence to the ergodic distribution.

[17]For the main MSVAR models, this upper limit is reached rarely.

B Constructing Labour Share Data

There are a number of pitfalls in constructing consistent labour share data, eg the influence of the government sector and the number of self-employed earners in an economy, and thus one has to be careful in constructing this approximation of the marginal costs. Of course, data availability plays a role too. Given this we use slightly different measures of the labour share for the economies under consideration.

United Kingdom

The UK labour share that we use in this paper is constructed in an identical manner to the one used in Batini *et al.* (2000) using the same data sources, and we both took the measure back to 1963.I as well as extended it to 2005.II. Basically, the Batini *et al.* (2000) labour share measure takes the ratio of total compensation of employees to nominal GDP at factor costs and corrects it for both the presence of the number of selfemployed jobs as well as the government sector; for more details about the construction and the data sources see Batini *et al.* (2000).

United States

We construct the US labour share directly using the neoclassical growth framework, suggesting constant long-run shares of capital and labour inputs in production

$$Y_t = \Lambda_t N_t^{(1-\kappa)} K_t^{\kappa}; 0 < \kappa < 1 \tag{B.1}$$

which is the Cobb-Douglas production function (23). Following Cooley and Prescott (1995, Section 4) we use the income categories in the US Bureau of Economic Analysis' National Income and Product Accounts (NIPA) to establish the share of *unambiguous* labour income of income generated by the private sector. In constructing total private sector labour income one needs to take a stand how much of the more ambiguous income categories, such as proprietors' income as well as supplements to wages and salaries, should be allocated to private sector labour income. Analogous to Cooley and Prescott (1995, Section 4) we allocate these ambiguous income categories to private labour income according to the share of labour income in measured GDP. Following their line of reasoning, we can define now the labour share (or unit labour costs) as (for notational convenience we suppress the time indexes):

$$\frac{WN}{PY} = \frac{\text{Unambiguous Labour Income} + \text{Private Share Supplements}}{\text{GDP} - \text{Ambiguous Labour Income} - \text{Government Labour Income}} \tag{B.2}$$

where

- *Unambiguous Labour Income* equals 'wages and salary accruals, other', i.e. row B203RC1, Table 1.12 in NIPA.
- *Private Share Supplements* equals 'supplements to wages and salaries' (row A038RC1, Table 1.12 in NIPA) times the ratio of 'wages and salary accruals, other' (row B203RC1, Table 1.12 in NIPA) to 'wages and salary accruals' (row A034RC1, Table 1.12 in NIPA).
- *GDP* is nominal GDP from row A191RC1, Table 1.1.5 in NIPA.

- *Ambiguous Labour Income* equals the sum of
  - 'Proprietors' income with IVA and CCAdj' (row A041RC1, Table 1.12 in NIPA).
  - The difference between nominal GDP (row A191RC1, Table 1.1.5 in NIPA) and national income (row A032RC1, Table 1.12 in NIPA).
- *Government Labour Income* equals the sum of
  - 'Wages and salary accruals, government' (row A553RC1, Table 1.12 in NIPA).
  - 'Supplements to wages and salaries' (row A038RC1, Table 1.12 in NIPA) times the ratio of 'wages and salary accruals, government' (row A553RC1, Table 1.12 in NIPA) to 'wages and salary accruals' (row A034RC1, Table 1.12 in NIPA).

Euro Area

The quality of the Euro area data equivalents that feed into the construction of labour share series is not as good as for the other economies. In particular, from the AWM data base it is not possible to fully adjust labour share for the claim of the government sector on resources of the economy. What we can do is to approximate labour share *(i)* at factor costs (i.e. correct for the total of taxes and subsidies that the government has levied or paid on the production factors), and *(ii)* corrected for self-employed earners.

So, basically, we construct total nominal labour income minus the part of government taxes/subsidies on production factors allocated to labour divided by nominal GDP at factor costs, i.e.

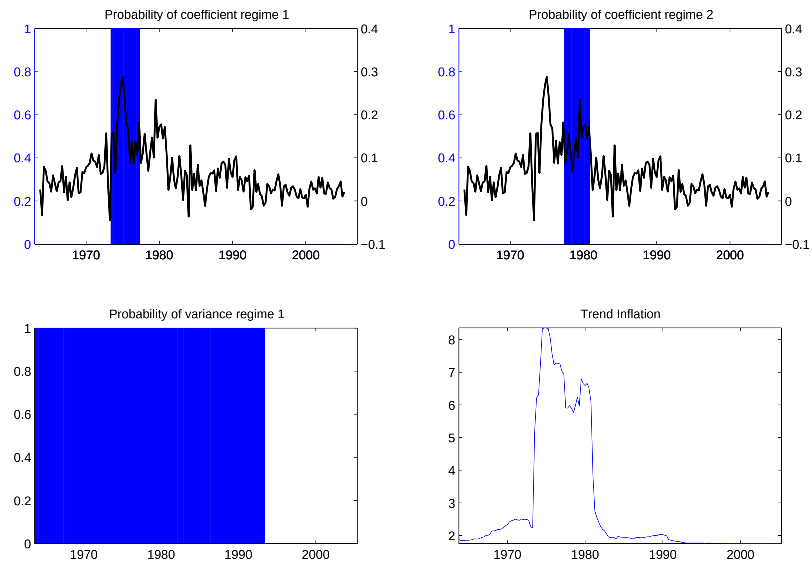
$$\frac{WN}{PY} = \frac{\text{Labour Income} - \frac{2}{3} \times (\text{GDP at market prices} - \text{GDP at factor costs})}{\text{GDP at factor costs}} \tag{B.3}$$

where

- *Labour Income* equals the product of
  - The ratio of 'Compensation to employees' ('WIN' in the AWM database) over 'Employees (in persons)' ('LEN' in the AWM database).
  - 'Total employment (in persons)' ('LNN' in the AWM database).

  which rescales total nominalwage income for the presence of self-employed earners.
- $\frac{2}{3}$ × (GDP at market prices − GDP at factor costs) allocates that part of the total net amount of government taxes/subsidies on production factors to labour according to the assumed long-run production share of labour within a Cobb-Douglas production function.

Figure C.2: Reduced form results



C Three regime MS-VAR model for the UK

In this appendix, we summarise the results for an MS(3)-VAR(2) model for the UK that produces a very similar marginal likelihood to the one estimated for a two regime model.

Figure C.2 summarised the reduced form results from this model. The top two panels show that the first two regimes pick up the two peaks in inflation in the mid-1970s and the early 1980s. In the two regime model, both are picked up by regime 1 (see Figure 1). The estimate of trend inflation is very similar to that shown in Figure 2.

Table C.1 presents estimates of the structural parameters in each regime. The median estimate of θ in the first two regimes is identical and is similar to the estimates reported in Table 3. The median value of θ increases in regime 3 but this estimate is fairly imprecise. The probabilities in Table C.2 suggest little evidence of systematic change in this parameter.

Table C.2 suggests similar conclusions about ϱ. An increase or decrease in this parameter has been equally likely. Note also that there is little difference in this parameter across the first two regimes. As before, ϱ falls over the regimes. regime 1 is fairly large and close to the upper bound. As before, α increases and is highest in the regime associated with lowest trend inflation. However, the estimates in regime 2 and regime 3 are fairly imprecise and this suggests that evidence for a significant shift in this parameter is weak.

Overall, these results suggest that our overall conclusions on parameter stability are preserved in this three regime model. In addition, it is unclear (from the reduced form and structural estimates) that there are significant differences across the first two regimes.

Table C.1: Structural estimates for the United Kingdom with three regimes

	Median	Mean	MAD
		Regime 1	
θ	18.16	23.35	12.25
ϱ	0.80	0.70	0.15
α	0.10	0.23	0.05
		Regime 2	
θ	18.16	23.99	12.25
ϱ	0.70	0.60	0.25
α	0.25	0.36	0.20
		Regime 3	
θ	27.95	27.54	19.60
ϱ	0.50	0.54	0.30
α	0.35	0.41	0.25
		All Regimes	
θ	20.61	25.64	12.25
ϱ	0.90	0.79	0.05
α	0.25	0.27	0.10

Notes: See the notes in Table 3.

Table C.2: Probability of structural change for the United Kingdom with three regimes

	θ	ϱ	α
Regime 3 $<$ Regime 2	0.40	0.51	0.41
Regime 2 $<$ Regime 1	0.44	0.51	0.29
Regime 3 $>$ Regime 2	0.6	0.49	0.59
Regime 2 $>$ Regime 1	0.56	0.49	0.71

Notes: See the notes in Table 4.

References

Albert, J. H. and S. Chib, 1993, Bayes Inference via Gibbs Sampling of Autoregressive Time Series Subject to Markov mean and variance shifts, *Journal of Business & Economic Statistics* **11**, 1–15.

Ascari, G., 2004, Staggered Prices and Trend Inflation: Some Nuisances, *Review of Economic Dynamics* pp. 642–667.

Assenmacher-Wesche, K. and S. Gerlach, 2006, Interpreting Euro Area Inflation at Low and High Frequencies, *CEPR Discussion Paper 5632*, Centre for Economic Policy Research.

Backus, D., P. Kehoe and F. Kydland, 1992, International Real Business Cycles, *Journal of Political Economy* **101**, 745–775.

Bakshi, H., P. Burriel-Llombart, H. Khan and B. Rudolf, 2003, Endogenous Price Stickiness, Trend Inflation, and the New Keynesian Phillips Curve, *Working Paper 191*, Bank of England.

Ball, Laurence M., N. Gregory Mankiw and David H. Romer, 1989, The New Keynesian Economics and the Output-Inflation Trade-off, *NBER Reprints 1111*, National Bureau of Economic Research, Inc. available at http://ideas.repec.org/p/nbr/nberre/1111.html.

Batini, N., B. Jackson and S. Nickell, 2000, Inflation Dynamics and the Labour Share in the UK, *Discussion Paper 2*, Bank of England External MPC Unit.

Batini, N., B. Jackson and S. Nickell, 2005, An Open-Economy New Keynesian Phillips Curve for the U.K., *Journal of Monetary Economics* **52**, 1061–1071.

Benati, L., 2004, Evolving post-World War II U.K. Economic Performance, *Journal of Money, Credit, and Banking* **36**, 691–718.

Blanchard, O. and J. Galí, 2005, Real Wage Rigidities and the New Keynesian Model, *NBER Working Paper 11806*, National Bureau of Economic Research.

Calvo, G., 1983, Staggered Prices in a Utility-Maximization Framework, *Journal of Monetary Economics* **12**, 983–998.

Chamberlain, G., 1982, Multivariate Regression Models for Panel Data, *Journal of Econometrics* **18**, 5–46.

Christiano, L., M. Eichenbaum and C. Evans, 2005, Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy, *Journal of Political Economy* **113**, 1–45.

Cogley, T. and A. M. Sbordone, 2005, A Search for a Structural Phillips Curve, *Staff Report 203*, Federal Reserve Bank of New York.

Cogley, T. and T. J. Sargent, 2002, Evolving Post World War II U.S. Inflation Dynamics, *in* B. S. Bernanke and K. Rogoff (editors), *NBER Macroeconomics Annual 2001*, MIT Press, Cambridge, U. S.

Cogley, T. and T. J. Sargent, 2005, Drifts and Volatilities: Monetary Policies and Outcomes in the Post WWII U.S., *Review of Economic Dynamics* **8**, 262–302.

Cooley, T. F. and E. C. Prescott, 1995, Economic Growth and Business Cycles, *in* T. F. Cooley (editor), *Frontiers of Business Cycle Research*, Princeton University Press, Princeton, New Jersey.

Dotsey, M., R. King and A. Wolman, 1999, State-Dependent Pricing and the General Equilibrium Dynamics of Money and Output, *Quarterly Journal of Economics* **114**, 655–690.

Fagan, G., J. Henry and R. Mestre, 2001, An Area-Wide Model (AWM) for the Euro Area, *Working Paper 42*, European Central Bank.

Galí, J. and M. Gertler, 1999, Inflation Dynamics: A Structural Econometric Analysis, *Journal of Monetary Economics* pp. 195–222.

Gelfand, A. E. and D. K. Dey, 1994, Bayesian Model Choice: Asymptotics and Exact Calculations, *Journal of the Royal Statistical Society (Series B)* **56**, 501–514.

Hamilton, J. D., 1989, A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle, *Econometrica* **57**, 357–384.

Hamilton, J. D., 1994, *Time Series Analysis*, Princeton University Press, Princeton.

Hansen, L. P. and T. J. Sargent, 1981, Linear Rational Expectations Model for Dynamically Interrelated Variables, *in* R. E. Lucas and T. J. Sargent (editors), *Expectations and Econometric Practice*, University of Minnesota Press, Minneapolis.

Kim, C-J. and C. R. Nelson, 1999a, *State-Space Models with Regime Switching*, MIT Press, Cambridge, Massachusetts.

Kim, Chang-Jin and Charles R. Nelson, 1999b, Has The U.S. Economy Become More Stable? A Bayesian Approach Based On A Markov-Switching Model Of The Business Cycle, *The Review of Economics and Statistics* **81**(4), 608–616.

Klenow, P. J. and O. Kryvtsov, 2005, State-Dependent or Time-Dependent Pricing: Does it Matter for Recent U.S. Inflation?, *NBER Working Paper 11043*, National Bureau of Economic Research.

Kodde, D. A., F. C. Palm and G. A. Pfann, 1990, Asymptotic Least Squares Estimation Efficiency Considerations and Applications, *Journal of Applied Econometrics* **5**, 229–243.

Levin, A. T. and J. M. Piger, 2004, Is Inflation Persistence Intrinsic in Industrial Economies?, *Working Paper 334*, European Central Bank.

McConnell, M and G Perez-Quiros, 2000, Output Fluctuations in the United States: What has Changed Since the Early 1980's, *American Economic Review* **90**, 1464 – 1476.

Sahuc, J. -G., 2006, Partial Indexation, Trend Inflation, and the Hybrid Phillips Curve, *Economics Letters* **90**, 42–50.

Sbordone, A. M., 2002, Prices and Unit Labour Costs: A New Test of Price Stickiness, *Journal of Monetary Economics* **49**, 265–292.

Sensier, M. and D. J. C. Van Dijk, 2004, Testing for Volatility Changes in U.S. Macroeconomic Time Series, *Review of Economics and Statistics* **86**, 833–839.

Sims, C. A. and T. Zha, 2006, Were there Regime Switches in US Monetary Policy?, *American Economic Review* **96**, 54–81.

Sims, Christopher A and Tao Zha, 1998, Bayesian Methods for Dynamic Multivariate Models, *International Economic Review* **39**(4), 949–68. available at http://ideas.repec.org/a/ier/iecrev/v39y1998i4p949-68.html.

Stahl, H., 2005, Time-Dependent or State-Dependent Price Setting? Micro-evidence from German Metal-Working Industries, *Working Paper 534*, European Central Bank.

Taylor, J. B., 1980, Aggregate Dynamics and Staggered Contracts, *Journal of Political Economy* **88**, 1–23.

Uhlig, H., 2005, What are the Effects of Monetary Policy on Output? Results from an Agnostic Identification Procedure, *Journal of Monetary Economics* **52**, 381–419.