Sales and monetary policy*

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Abstract

This paper analyses the effect of monetary policy in a model where sales occur in equilibrium. Some consumers are loyal to a particular brand of a good, others are bargain hunters. As a result, producers find it optimal to sell their goods at a "normal" price through some retailers and at a "sale" price through others. We calibrate the model and find that if both "normal" and "sale" prices are sticky but producers can adjust the amount "on sale", money is close to neutral: a monetary shock has a small (but positive) impact on output. The effects of monetary policy depend crucially on the reason for why there are sales in equilibrium.

KEYWORDS: sales; monetary policy.

JEL CLASSIFICATION: E3, E5.

1 Introduction

The real effects of monetary policy depend on how sticky prices are, that is, how long it takes for firms to react to shocks to the money supply by readjusting their prices. Attempts to measure the frequency of price adjustment using micro data have found that how sales are treated is of tremendous importance. With no distinction drawn between sales and regular price changes, Bils and Klenow (2004) obtain estimates of the median duration of a price spell between 4 and 4.5 months. On the other hand, Nakamura and Steinsson (2007), excluding sales from the sample, obtain estimates of between 8 and 11 months.

Sales are especially important in some sectors. Figure 1 shows the weekly retail and wholesale prices for Bass Ale at Dominick's supermarkets.[1] We see that prices change frequently, but usually return to the same level after a price cut. The figure also shows that sales are often coordinated between producers and retailers: retail and wholesale prices

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[1] Prices are for a single six-pack, taken from Goldberg and Hellerstein (2007).

frequently move in the same direction and at the same time. This is consistent with evidence from the marketing literature (e.g., Walters, 1989). Monetary policy is of course not the principal driving force behind those sales. However, we expect that decisions about putting products on sale and about the duration of sales react to fluctuations in demand for goods. But this means that those decisions must also react to monetary policy shocks.

What we do in this paper is to examine the effects of monetary policy in a model where sales are an equilibrium phenomenon. The margin provided by adjusting the fraction of goods on sale turns out to be of crucial importance. After calibrating our model, we find that an unexpected monetary shock has only a small positive effect on output: an increase of 1% in the money supply increases output by around 0.1% and the price level by around 0.9%. So, when examining the effects of monetary policy, prices that behave similarly to Figure 1 are best treated as flexible.

We analyse the effect of monetary policy in a model with a continuum of producers that sell their goods through retailers. The price of a particular good may not be the same across all retailers. This is consistent with evidence showing that sales are not strongly correlated across retailers (Hosken and Reiffen, 2001). Producers are therefore allowed to sell their good at a "normal" price through some retailers and at a "sale" price through others.

To begin with, we consider producers who have two exogenously fixed prices. When the monetary shock is revealed, they can adjust the proportion of sales they offer. In this economy, when wages are fixed, as long as the fraction of sales is strictly between zero and one, monetary policy has no first-order effect on output. Following an increase in the money supply, producers offer a smaller fraction of goods on sale, so the price level increases.[2] Strikingly, this margin wipes out any possible effect on output in spite of price and wage rigidity. If wages are flexible, money turns out to be countercyclical (for all reasonable calibrations). The decrease in the amount of sales following a monetary expansion leads to a price increase, which raises wages. As labour costs rise, the firm chooses to cut its production further (reducing sales by even more), so output actually falls and the price level rises by a greater amount.

The previous exercise presupposes that firms have chosen two prices at some earlier stage. But why would firms actually want to have multiple prices for the same good? We have not so far explained why they would want this, and the implications for monetary

[2] By price-level we mean the effective cost of the basket of goods purchased by consumers. This may or may not correspond to official statistical indices depending on how sales are treated when these are compiled.

policy are likely to depend on the rationale for having two prices in equilibrium. We build a model with an important new feature: customers can now choose between a range of brands for each type of product they buy.

The model assumes some consumers are loyal to a particular brand of a good, while others are bargain hunters. This generates a demand curve where marginal revenue is non-monotonic (a similar feature is found when there is a kink in demand). In this environment, there can be an equilibrium where producers sell their goods through different retailers at different prices. We focus on economies where there is an equilibrium in two prices: a normal price (high) and a sale price (low). This model introduces into macroeconomics the price-discrimination motive for sales that has been analysed by Varian (1980) and Sobel (1984). We develop a tractable general equilibrium model of sales with predictions that allow it to be calibrated to match the micro evidence on pricing that has become available in recent years.

One important implication of the model is that there are decreasing returns to sales. When many brands of the same type of product are offered on sale in a supermarket, other producers of this type of product are more reluctant to put their own brand on sale. But if no-one is selling on sale, producers have much more incentive to sell their own brand at a lower price and capture all the bargain-hunting customers.

This finding has consequences for the effects of monetary policy. Following a positive monetary policy shock, producers reduce their fraction of sales. But as they do this, it becomes more attractive for others to target the bargain hunters by offering more sales. The net effect is that sales fall by a smaller amount leading to a smaller increase in the price level so monetary policy has a relatively larger effect on output than otherwise.

Our calibrated model can be used to assess the quantitative impact of a monetary policy shock. In our preferred specification with flexible wages, 10% of the instantaneous effect of monetary policy impacts on output and 90% impacts on prices. Robustness checks shows that for reasonable parameter ranges, the model predicts that money is not countercyclical and also that its effect on output is no more than 25% of the monetary shock.With fixed wages, around 75% of the monetary shock goes to output and 25% goes to prices in our baseline calibration. Robustness checks demonstrate that the effect on output should be approximately between 70% and 80%.

The results demonstrate the importance of modelling the reasons behind producers' sales decisions: even for the analysis of monetary policy where conventional wisdom suggests they are essentially orthogonal. Our analysis indicates that the prices of goods which are subject to frequent slaes are best treated as flexible.

1.1 Related Literature

TBW

2 The model with exogenous sales

There is a continuum of consumers (measure 1); a continuum of producers (measure 1); a continuum of retailers (measure 1).

Consumers maximize utility that depends on consumption and labour supply. Each consumer shops at a single retailer.

Each producer produces one good and goods are imperfect substitutes.

Producers sell their goods to retailers, from whom consumers buy. We abstract from the potentially complicated bargaining between retailers and producers. Assuming the bargaining is efficient, prices are set to maximize the joint surplus. We consider that a good assumption because prices of transactions between producers and retailers can be conditional on quantities and there is no reason to believe they would choose a suboptimal mix of final price and quantities.

Crucially, the producer can sell its good through some retailers at price P_N and through some others at the sales price, P_S. In this section, we will not explain why the producer would do so, we will take it as given. It will be endogeneized later.

We add the assumption that producers set their prices in advance.

2.1 The consumer's problem

Consumers have Dixit-Stiglitz preferences. Their utility is given by:

$$\begin{aligned} U(C,L) &= \frac{C^{1-\rho}-1}{1-\rho} - \frac{L^{\omega+1}}{\omega+1} \\ C &= \left(\int_i C_i^{(\varepsilon-1)/\varepsilon} di\right)^{\varepsilon/(\varepsilon-1)} \end{aligned}$$

$\varepsilon > 1$, $\rho > 0$, $\omega > 0$.

If the price paid for good i is P_i, we have that the price index P faced by that consumer is:

$$P = \left(\int_i P_i^{1-\varepsilon} di\right)^{1/(1-\varepsilon)}$$

The budget constraint can be written as:

$$PC \leq WL$$

where W is the nominal wage.

Goods market equilibrium implies $C = Y$.

M is the money supply. A cash in advance constraint implies:

$$Y = \frac{M}{P}$$

So demand for good i is given by:

$$C_i = \left(\frac{P_i}{P}\right)^{-\varepsilon} Y$$

2.2 The producer's problem

In order to produce and distribute y_i units of good i, labour is needed. More specifically

$$y_i = L_i^{\alpha}$$

Which implies:

$$L_i = y_i^{1/\alpha}$$

Labour is homogeneous.

The producer sells a proportion s of goods at price P_S through some retailers and a proportion $(1-s)$ at price P_N through other retailers.

The real profits associated with production of good i are given by:

$$\pi = s\left(\frac{P_S}{P}\right)^{1-\varepsilon} Y + (1-s)\left(\frac{P_N}{P}\right)^{1-\varepsilon} Y - \frac{W}{P}\left[s\left(\frac{P_S}{P}\right)^{-\varepsilon} Y + (1-s)\left(\frac{P_N}{P}\right)^{-\varepsilon} Y\right]^{1/\alpha} \quad (1)$$

In a symmetric equilibrium, where all firms have the same P_S and P_N, all firm choose the same s and y. However, the quantity produced by the firm (y) is not exactly the same as the aggregate output (Y). The relationship between y and Y is given by:

$$y = \delta Y \quad \text{where} \quad \delta = s\left[\frac{P_S}{P}\right]^{-\varepsilon} + (1-s)\left[\frac{P_N}{P}\right]^{-\varepsilon}$$

For $s = 0$ and $s = 1$, $\delta = 1$.

2.3 Monetary policy — fixed wages

After the producer has chosen P_N and P_S, and wages are set at W, the money supply (M) is revealed.

The producer can then adjust the amount (s) sold at sale price.

The following proposition establishes the main result of this section:

Proposition 1 *Given prices P_N and P_S, as long as s is between 0 and 1, the firm's output (y_i) is not affected by monetary shocks.*

Proof. *The first order condition with respect to s implies:*

$$\left(\frac{P_S}{P}\right)^{1-\varepsilon} Y - \left(\frac{P_N}{P}\right)^{1-\varepsilon} Y = \frac{W}{\alpha P}(y_i)^{\frac{1-\alpha}{\alpha}} \left[\left(\frac{P_S}{P}\right)^{-\varepsilon} Y - \left(\frac{P_N}{P}\right)^{-\varepsilon} Y\right]$$

Which yields:

$$y_i = \left[\frac{\alpha}{W}\left(\frac{P_S^{1-\varepsilon} - P_N^{1-\varepsilon}}{P_S^{-\varepsilon} - P_N^{-\varepsilon}}\right)\right]^{\alpha/(1-\alpha)} \tag{2}$$

The equation shows that the quantity produced, y_i, does not depend on any nominal variable (P or Y), which proves the claim. ■

Monetary policy causes small changes in Y because of the way Y is aggregated to form the consumption basket. But because y and Y are very similar, Y is approximately constant — indeed the value of Y is the same in the cases $s \to 0^+$ and $s \to 1^-$.

A positive shock to M leads producers to sell fewer of their goods on sale. As Y is approximately constant, an increase in M has to be followed by a corresponding increase in P. The prices P_S and P_N are sticky; the proportion sold on sale (s) is responsible for the adjustment.

Notice that the above result holds for *any* P_S and P_N as long as $s \in (0, 1)$. The producer can mimic any price increase by adjusting s.

The intuition is the following: higher s means that: (i) revenues are higher because at the sale price the quantity sold is higher and the price is lower, but as $\varepsilon > 1$, the quantity effect dominates; (ii) costs are higher because quantity sold is higher and, on the top of that, (iii) as marginal cost is increasing, the marginal cost of production also increases. The producer chooses s so that the marginal increment in revenue from increasing sales compensates the marginal increase in cost due to higher quantity plus the marginal increment in cost.

Now, if prices and wages are fixed, an increase in the price level multiplies the demand for goods at both prices by the same factor — $P^{-\varepsilon}$ — and an increase in Y also multiplies the demand for goods at both prices by the same factor — Y. So, as long as the marginal cost of production is constant, changes in P and Y do not affect the optimal choice of s, because such changes do not affect the ratio of the difference between the marginal revenues and the difference between marginal costs at prices P_S and P_N, respectively.

But changes in the marginal cost of production affect the balance between these two effects. If marginal cost is too high, it is worth while for the producer to cut production and reduce s (the opposite case is marginal cost being too low). With monetary shocks, the optimal decision of the producer is to keep marginal costs constant. Any change in P or Y affects demand and the producer adjusts s to keep the quantity produced constant.

In this model with two exogenously fixed prices, the possibility of adjusting the amount on sale is enough to translate an increase in M into an increase in P with no effect on the quantity produced by the firm.

2.4 Monetary policy — flexible wages

Now consider what happens when wages adjust following the monetary shock. In equilibrium they are equal to:

$$W = PY^{\rho}y^{\omega/\alpha} \tag{3}$$

The real wage in equal to ratio between marginal disutility of work and marginal utility of consumption.

The following proposition establishes the main result of this section:

Proposition 2 *Given prices P_N and P_S, as long as s is between 0 and 1, neglecting changes in δ (which are quantitatively small) the effect of monetary policy on Y and P is given by:*

$$\frac{\partial P}{\partial M}\frac{M}{P} = \frac{\rho\alpha + 1 + \omega - \alpha}{\rho\alpha + 1 + \omega - 2\alpha} \quad , \quad \frac{\partial Y}{\partial M}\frac{M}{Y} = \frac{-\alpha}{\rho\alpha + 1 + \omega - 2\alpha}$$

Proof. *The first order condition (Equation 2) and the equation for the wage (Equation 3) yield:*

$$\begin{aligned} PY^{\rho}y^{\omega/\alpha}y_i^{(1-\alpha)/\alpha} &= \alpha\left(\frac{P_S^{1-\varepsilon} - P_N^{1-\varepsilon}}{P_S^{-\varepsilon} - P_N^{-\varepsilon}}\right) \\ \log(P) + \left(\rho + \frac{\omega + 1 - \alpha}{\alpha}\right)\log(Y) + \left(\frac{\omega + 1 - \alpha}{\alpha}\right)\log(\delta) &= \log\left[\alpha\left(\frac{P_S^{1-\varepsilon} - P_N^{1-\varepsilon}}{P_S^{-\varepsilon} - P_N^{-\varepsilon}}\right)\right] \end{aligned}$$

Differentiating with respect to $\log(M)$*:*

$$\frac{\partial P}{\partial M}\frac{M}{P} = -\left(\frac{\rho\alpha + \omega + 1 - \alpha}{\alpha}\right)\frac{\partial Y}{\partial M}\frac{M}{Y} - \left(\frac{\omega + 1 - \alpha}{\alpha}\right)\frac{\partial \delta}{\partial M}\frac{M}{\delta}$$

Neglecting δ and using the fact that the elasticities of Y and P must sum to 1, the claim is obtained. ■

As before, with monetary shocks, the optimal decision of the producer is to keep marginal cost constant. But now marginal cost changes with changes in P or Y because of their effect on wages. An increase in Y raises marginal costs via higher wages (elasticity $\rho + \omega/\alpha$) and via higher output, hence lower marginal production (elasticity $(1 - \alpha)/\alpha$).

It also increases P, via higher wage (elasticity 1). The optimal decision of the producer implies quantities and prices such that nominal marginal cost is kept constant.

For any reasonable calibration, the effect of Y on marginal costs is higher than the effect of Y on prices, which implies that a positive monetary shock increases prices and decreases output. A positive shock to the money supply is followed by a decrease in sales, which increases P. But if the producer acts as before, keeping fixed the quantity it produces, the increase in P means its nominal marginal cost will actually increase. To keep marginal cost constant, the producer makes an additional reduction in sales, leading to a further increase in prices and a, consequently a decline in output. In equilibrium, the impact of the decrease in output on marginal cost compensates tje impact of the increase in price. Surprisingly, money is counter-cyclical.

2.4.1 Calibration

The impact of monetary policy depends on ρ, ω and α. Setting $\rho = 3$ (relative risk aversion coefficient), $\omega = 1.4$ (inverse of the Frisch elasticity) and $\alpha = 2/3$ (the elasticity of output with respect to labor), we obtain:

$$\frac{\partial Y}{\partial M}\frac{M}{Y} = -21.7\%$$

Sensitivity with respect to ρ: with log utility ($\rho = 1$), the effect on output equals -38.5%, and with $\rho = 5$, the effect on output is: -15.2%.

Sensitivity with respect to ω: with $\omega = 0.5$, the effect on output equals -30.8%. With $\omega = 3$, the effect on output is: -18.2%.

Summing up, in this model, monetary policy is significantly countercyclical: a 1% increase in M leads to a contraction of output of around 0.2%.

3 The model with sales in equilibrium

The above model consider sales but does not explain why there are sales in the first place. In this section, we will provide a rationale for sales. There is a continuum of types of goods and, for each type, there is a continuum of brands. Each produces manufactures one particular brand for a given type. Producers face two types of consumers: the loyal customers, who always buy their preferred brand, and the bargain-hunters, who will seek out the best deals, taking into account their own preferences. In equilibrium, producers may choose to sell at different prices.

The model is set up so that although different consumers are loyal to brands of different goods, in the aggregate everything is symmetric. So, all brands have the same fraction of loyal costumers and all costumers are loyal to a brand for the same fraction of goods.

3.1 The consumer's problem

There is a continuum of consumers with measure 1. They buy the whole range of different types of goods, but buy only one brand for each type.

For the sake of tractability and clarity, we want to separate the choice of brand and the amount chosen for consumption.

As before, regardless of the chosen brand chosen, the demand for good type i is given by:

$$c_i = \left(\frac{P_i}{P}\right)^{-\varepsilon} Y$$

Except for the choice of the brand, the consumer's problem is the same as that studied at section 2.1.

3.1.1 The choice of brand

Each consumer is loyal to a particular brand for a proportion λ of the goods. So, each brand is chosen by a proportion λ of the consumers, irrespective of its price.

For the remaining $(1-\lambda)$ types of goods, the consumer chooses among n brands. We refer to this type of consumers as bargain-hunters. So, each brand is also considered by n times the measure of bargain-hunters.

A bargain-hunter has preferences factors $z_1, z_2, ..., z_n$ for each of the brands of the type of good he is considering. The chosen brand j is such that:

$$j = \arg\max \{z_1 - p_1, z_1 - p_2, ..., z_n - p_n\}$$

where $p_j = \log(P_j)$.

The preference factors z_j are drawn from a distribution $f()$, with standard deviation σ and cumulative distribution function $F()$.

3.2 The producer's problem

The producer faces a demand function with non-monotonic marginal revenue. As marginal costs are increasing, for some parameter values, in equilibrium the producer will choose to have a fraction s of its goods sold at price P_S through some retailers, and a fraction $(1-s)$ of its goods sold at price P_N through other retailers.

We now consider an equilibrium in which the producer sells its good at price P_S through s retailers and at price P_N through $(1-s)$ retailers.

A producer faces a measure λ of loyal consumers. Moreover, there is a measure $n(1-\lambda)$ that may buy its brand depending on prices and preferences.

Denote by $v_S(1-\lambda)$ the amount sold to bargain hunters if the good is on sale (price P_S) and by $v_N(1-\lambda)$ the amount sold to bargain hunters if the good is offered at price P_N. v_S and v_N are between 0 and n.

The parameters λ and n and the variables s, v_S and v_N determine the amount a producer sells at P_S, denoted by γ_S, and the amount sold at P_N, denoted by γ_N. Brand j will be chosen by all loyal costumers irrespective of price (λ) and by a fraction of the bargain hunters. We obtain:

$$\begin{aligned}\gamma_S &= s(\lambda+(1-\lambda)v_S) \\ \gamma_N &= (1-s)\left(\lambda+(1-\lambda)v_N\right)\end{aligned}$$

The profit of a producer can thus be written as:

$$\pi=\gamma_S\left(\frac{P_S}{P}\right)^{1-\varepsilon}Y+\gamma_N\left(\frac{P_N}{P}\right)^{1-\varepsilon}Y-W\left[\gamma_S\left(\frac{P_S}{P}\right)^{-\varepsilon}Y+\gamma_N\left(\frac{P_N}{P}\right)^{-\varepsilon}Y\right]^{1/\alpha} \tag{4}$$

Equation (4) is similar to Equation (1), but instead of s and $(1-s)$, we have γ_S and γ_N, respectively. As we will see, that difference will play a key role in our analysis.

3.2.1 The decision problem for bargain hunters

Consider a particular producer that offers its good at price P_1. A bargain hunter who considers buying that brand compares it against another $n-1$ brands, with prices $P_2, P_3, ..., P_n$. The consumer's choice depends on prices and on her idiosyncratic preference factors, $z_1, z_2, ..., z_n$.

The probability that the producer's brand will be chosen is:

$$\begin{aligned}& \int_{z_1} f(z_1)\Pr\left(z_1-p_1\geq z_2-p_2, z_1-p_1\geq z_3-p_3,...,z_1-p_1\geq z_n-p_n\right)dz_1 \\ =& \int_{z_1} f(z_1)\Pr\left(z_2\leq z_1-p_1+p_2, z_3\leq z_1-p_1+p_3,...,z_n\leq z_1-p_1+p_n\right)dz_1 \\ =& \int_{z_1} f(z_1)F\left(z_1-p_1+p_2\right)F\left(z_1-p_1+p_3\right)...F\left(z_1-p_1+p_n\right)dz_1\end{aligned}$$

In equilibrium, some producers will sell at price P_N and others will sell at price P_S. Let $p_S=\log(P_S)$ and $p_N=\log(P_N)$.

If that are k goods on sale, that probability can be written as:

$$= \int_{z_1} f(z_1) F\left(z_1 - p_1 + p_S\right)^k F\left(z_1 - p_1 + p_N\right)^{n-1-k} dz_1$$

The probability that k goods are on sale among the n considered by the consumer is:

$$\binom{n-1}{k} s^k (1-s)^{n-1-k}$$

Combining both equations, we find that the probability that the producer's brand is chosen is:

$$\begin{aligned} & s^k(1-s)^{n-1-k} \int_{z_1} f(z_1) F\left(z_1 - p_1 + p_S\right)^k F\left(z_1 - p_1 + p_N\right)^{n-1-k} dz_1 \\ = & \int_{z_1} f(z_1) \left[\sum_{k=0}^{n-1} \binom{n-1}{k} \left[sF\left(z_1 - p_1 + p_S\right)\right]^k \left[(1-s)F\left(z_1 - p_1 + p_N\right)\right]^{n-1-k} \right] dz_1 \end{aligned}$$

Applying the Binomial Theorem, the expression becomes:

$$\int_{z_1} f(z_1) \left[sF\left(z_1 - p_1 + p_S\right) + (1-s)F\left(z_1 - p_1 + p_N\right)\right]^{n-1} dz_1$$

Denote by $v_1(1-\lambda)$ the amount sold to bargain hunters of a brand at price P_1. Then, v_1 is n times the probability a bargain hunter chooses that brand:

$$v_1 = n \int_{z_1} f(z_1) \left[sF\left(z_1 - p_1 + p_S\right) + (1-s)F\left(z_1 - p_1 + p_N\right)\right]^{n-1} dz_1 \tag{5}$$

v_S and v_N are obtained from the above expression when $p_1 = p_S$ and $p_1 = p_N$, respectively.

Equation (5) shows that for any price P_1, v_1 depends positively on p_S and p_N. If the other producers charge a higher price, the measure of bargain hunters that choose brand 1 is larger.

Moreover, if $p_N > p_S$, then v_1 is decreasing in s. A higher proportion of brands offered at discount implies a smaller measure of bargain hunters choosing brand 1.

Thus, v_1 depends not only on the producer's decision but also on the decision of the other producers in the economy. This affects the equilibrium values of γ_S and γ_N and that is the key feature of this model of endogenous sales.

The elasticity of v_1 with respect to P_1 is given by:

$$\begin{aligned} \eta_1 = & \frac{\partial v_1}{\partial P_1} \frac{P_1}{v_1} = \frac{n(n-1)}{v_1} \int_{z_1} f(z_1) \left[sF\left(z_1 - p_1 + p_S\right) + (1-s)F\left(z_1 - p_1 + p_N\right)\right]^{n-2} \times \\ & \times \left[f\left(z_1 - p_1 + p_S\right) - f\left(z_1 - p_1 + p_N\right)\right] dz_1 \end{aligned}$$

η_S and η_N are obtained from the above expression when $p_1 = p_S$ and $p_1 = p_N$, respectively.

3.2.2 The producer's choice

We now analyse the producer's choice of P_N, P_S and s, taking as given that all other producers are selling $(1-s)$ goods at price P_N and s goods at price P_S.

The values of v_S and v_N depend on the individual producer's own prices, prices of the others brands and the aggregate s, but does not depend on the individual producer's own choice of s.

Taking derivative of Equation (4) with respect to s, we get the following first order condition:

$$y_i = \left[\frac{A^{\frac{1}{1-\alpha}}}{W} \left(\frac{(\lambda + (1-\lambda)v_S)P_S^{1-\varepsilon} - (\lambda + (1-\lambda)v_N)\, P_N^{1-\varepsilon}}{(\lambda + (1-\lambda)v_S)P_S^{-\varepsilon} - (\lambda + (1-\lambda)v_N)\, P_N^{-\varepsilon}} \right) \right]^{\alpha/(1-\alpha)} \tag{6}$$

which is similar to the first order condition obtained in the case with exogenous sales (Equation (2)). The difference is that the terms $(\lambda + (1-\lambda)v_S)$ and $(\lambda + (1-\lambda)v_N)$ multiply prices. Here, the amount produced is not independent of the others' choice regarding sales.

Define the mark-ups μ_S and μ_N as

$$\mu_S = \frac{P_S}{xP} \quad , \quad \mu_N = \frac{P_N}{xP}$$

where x is the real marginal cost. Manipulating the first order conditions with respect to P_N and P_S, we get:

$$\begin{aligned} \mu_S &= \frac{\varepsilon + \left(\frac{(1-\lambda)v_S}{\lambda+(1-\lambda)v_S}\right)\eta_S}{(\varepsilon-1) + \left(\frac{(1-\lambda)v_S}{\lambda+(1-\lambda)v_S}\right)\eta_S} \\ \mu_N &= \frac{\varepsilon + \left(\frac{(1-\lambda)v_N}{\lambda+(1-\lambda)v_N}\right)\eta_N}{(\varepsilon-1) + \left(\frac{(1-\lambda)v_N}{\lambda+(1-\lambda)v_N}\right)\eta_N} \end{aligned}$$

And equation 6 can be written as:

$$\frac{\mu_N - 1}{\mu_S - 1} = \frac{\lambda + (1-\lambda)v_S}{\lambda + (1-\lambda)v_N} \left(\frac{\mu_S}{\mu_N}\right)^{-\varepsilon}$$

The above three equations, combine with the definitions of η_S, η_N, v_S and v_N, yield the equilibrium values of μ_S, μ_N and s.

3.2.3 Equilibrium

The other equilibrium conditions are:

$$
\begin{aligned}
x &= \left[s(\lambda + (1-\lambda)v_S)\mu_S^{1-\varepsilon} + (1-s)\left(\lambda + (1-\lambda)v_N\right)\mu_N^{1-\varepsilon}\right]^{1/(\varepsilon-1)} \\
\delta &= \left[s(\lambda + (1-\lambda)v_S)\mu_S^{-\varepsilon} + (1-s)\left(\lambda + (1-\lambda)v_N\right)\mu_N^{-\varepsilon}\right] x^{-\varepsilon} \\
Y &= \left(\frac{(\alpha x)^{\alpha}}{\delta^{1-\alpha+\omega}}\right)^{1/[(1-\alpha)+\alpha\rho+\omega]} \\
y_i &= \delta Y \\
\frac{W}{P} &= Y^{\rho} y^{\omega/\alpha} \\
P &= \frac{M}{Y}
\end{aligned}
$$

3.2.4 Decreasing return to sales

In contrast to the case of exogenous sales, here the individual decision about production depends on s, the amount of sales, through v_S and v_N. A higher s implies that it is more difficult for a producer to sell to a bargain hunter. That provides more incentives for them to target the loyal customers, thus reducing sales of their own goods.

As seen earlier, a monetary shock induces companies to reduce the amount sold at the lower price (P_S). But with a lower s, v_S and v_N increase: the chances of selling to a bargain hunter is higher. In other words, as producers reduce their sales, incentives to target bargain hunters increase. So the reduction in sales (increase in P) following the monetary shock is less pronounced.

4 The impact of monetary policy

The impact of monetary policy is analysed by studying the effect of small perturbations in M after P_S, P_N have already been chosen. We calibrate the model and calculate the impact on output and prices of an unexpected small increase in the money supply.

We first solve for the steady-state equilibrium. Starting from an initial guess of (s, P_S, P_N), we calculate the macroeconomic aggregates and relevant equilibrium variables (Y, P, y, W, x). Then we find the best response of a producer (her choices of s, P_S and P_N), using the Nelder-Mead (Simplex) algorithm. Iterating on (s, P_S, P_N), we obtain the equilibrium in this economy.

For some parameter values, the producer charges a unique price ($P_S = P_N$). In other cases, in equilibrium, producers find it optimal to sell their goods at the normal price through some retailers and at the sales price through others.

4.1 Calibration and results

There are 7 parameters to calibrate: $\rho, \omega, \alpha, \lambda, \sigma, \varepsilon, n$.

The calibration of the first 3 parameters follow the literature: $\rho = 3$ (relative risk aversion coefficient), $\omega = 1.4$ (inverse of the Frisch elasticity) and $\alpha = 2/3$ (labour share). Those effect of monetary policy is only mildly affected by changes in ρ and almost unaffected by changes in ω and α.

The other 4 parameters are chosen to match some facts about sales and price-setting:

- P_S/P_N . Nakamura and Steinsson (2007) find that the median difference between $\log(P_S)$ and $\log(P_N)$ is 0.295.

$$\log(P_N) - \log(P_S) = 0.295 \Rightarrow P_S/P_N = 0.745$$

  and that is the value we choose for P_S/P_N .

- Equilibrium s: Nakamura and Steinsson (2007) report that the fraction of price quotes with sales, weighted by the number of observations, is 12.1%. We use this number in our exercise. If weighted by expenditure, the fraction of price quotes with sales in their sample goes down to 7.4%. We also experiment with this number.

- The ratio between the quantity sold in a particular retailer at the sale and normal prices. In our model, this corresponds to $(\lambda + (1 - \lambda)v_S) \div (\lambda + (1 - \lambda)v_N)$. Using the findings from Blattberg and Neslin (1990), Walters (1990) and Gupta (1988), we estimate that ratio should be around 5 to 7.

- The average mark-up in the economy. The literature often uses values from 1.2 to 1.4 (Rotemberg and Woodford, 1993).

It turns out that, in equilibrium, when producers choose to charge two prices, the sales price is a few percentage points above the marginal cost and the normal price is not far from $\varepsilon/(\varepsilon - 1)$. So, requiring two different prices in equilibrium and matching 3 of the above facts usually leaves little margin for matching the fourth. We choose to prioritize matching the first two, for which estimates are more reliable.

The results are shown in the attached tables. In our baseline calibration, a 1% money shock increases output by 0.107%. The effect increases to 0.25% if we consider $s = 7.5\%$. This contrasts with the model with exogenous sales, where such a shock would lead to a *fall* in output of 0.22%.

With fixed wages, the effect on output is stronger. In our baseline calibration, we obtain an increase in Y of 0.74%. The effect increases to 0.82% with $s = 7.5\%$. That is in sharp contrast with the model with exogenous sales where money is neutral.

Those differences between the effect of monetary policy stem from the decreasing returns from sales, which is a result of the reasons for why producers sell their goods at different prices through different retailers.

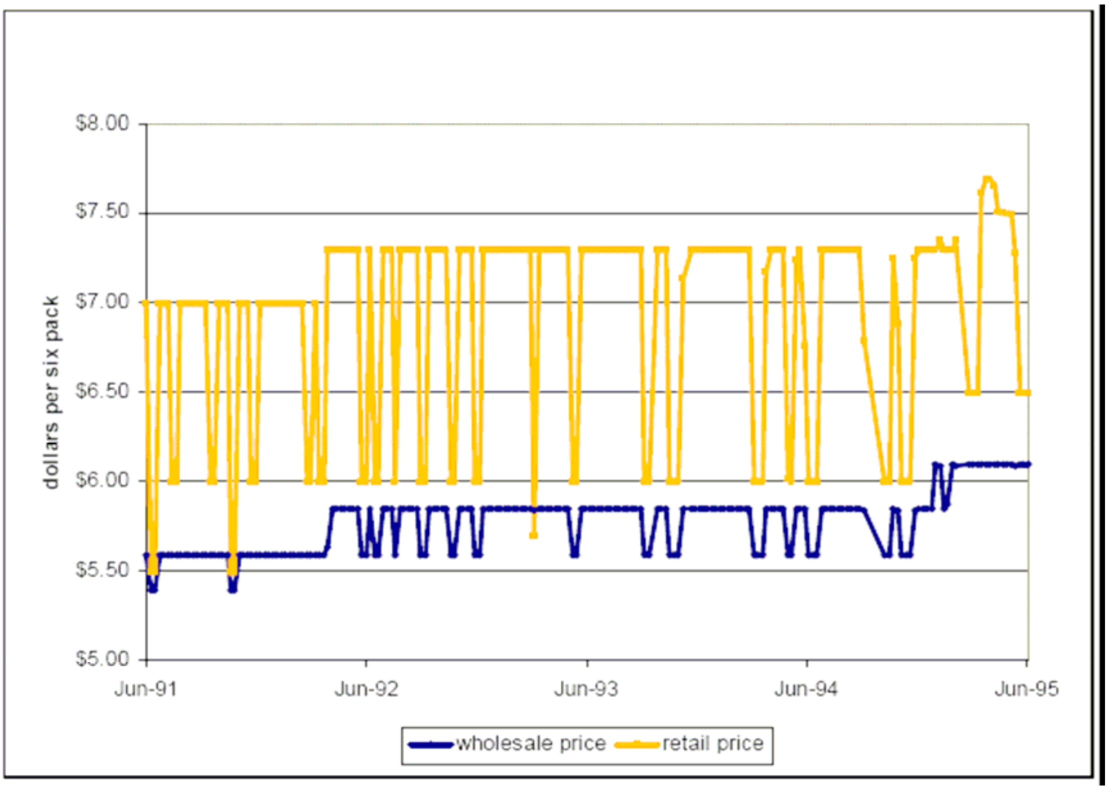
5 Dynamics

TBW

6 Concluding remarks

TBW

Figure 1



Weekly retail and wholesail price for Bass Ale, prices are for a single six-pack.
Source: Dominick’s.

Baseline calibration

$\boldsymbol{n}$	125	$\boldsymbol{P_S/P_N}$	0.743
σ	0.08	s	12.1%
ε	3.3	quantity ratio	6.55
λ	0.85	average mark-up	1.34

Impact on $\boldsymbol{Y}$ of a small monetary shock: **10.7%.**

Sensitivity: λ

λ	**0.85**	**0.75**	**0.80**	**0.90**	**0.95**
$\Delta Y/Y$	**10.7%**	**-2.3%**	**3.7%**	**19.4%**	**31.6%**
P_S/P_N	0.743	0.736	0.739	0.750	0.767
s	12.1%	18.4%	15.1%	9.2%	6.4%
q_S/q_N	6.55	7.72	7.20	5.70	4.37
mark-up	1.34	1.29	1.31	1.37	1.40
n	125	125	125	125	125
σ	0.08	0.08	0.08	0.08	0.08
ε	3.3	3.3	3.3	3.3	3.3

Sensitivity: n

n	**125**	**40**	**60**	**200**	**400**
$\Delta Y/Y$	**10.7%**	**4.3%**	**7.1%**	**12.2%**	**13.9%**
P_S/P_N	0.743	0.765	0.756	0.738	0.732
s	12.1%	20.4%	16.5%	10.3%	8.4%
q_S/q_N	6.55	4.50	5.22	7.42	8.67
mark-up	1.34	1.32	1.33	1.35	1.35
λ	0.85	0.85	0.85	0.85	0.85
σ	0.08	0.08	0.08	0.08	0.08
ε	3.3	3.3	3.3	3.3	3.3

Sensitivity: ε

ε	**3.3**	**2.8**	**4**
$\Delta Y/Y$	**10.7%**	**13.2%**	**6.9%**
P_S/P_N	0.743	0.686	0.801
s	12.1%	9.5%	16.7%
q_S/q_N	6.55	8.22	5.00
mark-up	1.34	1.44	1.25
λ	0.85	0.85	0.85
σ	0.08	0.08	0.08
n	125	125	125

Sensitivity: σ

σ	**0.08**	**0.05**	**0.10**
$\Delta Y/Y$	**10.7%**	**15.1%**	**7.0%**
P_S/P_N	0.743	0.728	0.757
s	12.1%	7.1%	16.7%
q_S/q_N	6.55	9.93	5.17
mark-up	1.34	1.35	1.33
λ	0.85	0.85	0.85
ε	3.3	3.3	3.3
n	125	125	125

Sensitivity: quantity ratio ***(q_S/q_N)***

q_S/q_N	**6.55**	**4.38**	**7.29**	**9.34**
$\Delta Y/Y$	**10.7%**	**16.9%**	**8.6%**	**3.8%**
P_S/P_N	0.743	0.744	0.746	0.745
s	12.1%	12.1%	12.1%	12.1%
mark-up	1.34	1.42	1.32	1.29
n	125	50	115	80
σ	0.08	0.08	0.07	0.05
ε	3.3	3	3.4	3.5
λ	0.85	0.91	0.83	0.78

Sensitivity: ***s***

s	**12.1%**	**9.7%**	**7.5%**
$\Delta Y/Y$	**10.7%**	**18.8%**	**24.6%**
P_S/P_N	0.743	0.744	0.744
q_S/q_N	6.55	5.52	5.55
mark-up	1.34	1.38	1.40
n	125	100	130
σ	0.08	0.08	0.08
ε	3.3	3.2	3.2
λ	0.85	0.90	0.92