

Public debt optimality: transitional issues

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11th February 2008

Abstract

This paper reconsiders the effect of public debt when taking the transition into account. The theoretical framework we adopt is in line with the works of Aiyagari and McGrattan (1998) and Floden (2001). We find that taking into account transition effects raises the welfare gains associated to a higher public debt. The additional debt issuing allows for a temporary reduction in the tax rate, which raises the disposable income. Moreover, it turns out that part of transition-adjusted welfare gains stems from the temporary increase in labor supply which accounts for the increase in the interest rate. The distributional properties of a public debt increase share some common features with the steady state comparisons: an increase in public debt mainly benefits rich and high productivity agents.

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1 Introduction

The analysis of the impact of the fiscal policy has given rise to a rich theoretical literature. According to the Ricardian view (Ricardo [1951a, 1951b], Barro [1979]), a change in the timing of taxes has no effect at all. In an environment where the pure Ricardian equivalence would hold, an increase in disposable income, resulting from a reduction of (lump-sum) taxation, would be entirely saved, and would exactly match the increase in the net debt issuing. Yet, many studies cast doubt on the validity of the Ricardian view. The Ricardian equivalence holds if households are linked by intergenerational altruism, taxes are lump-sum and capital markets are perfect. It turns out that the timing of taxes matters if taxes are not lump-sum (Auerbach and Kotlikoff [1987], Trostel [1993]). A temporary reduction in capital (*resp.* labor) income tax rate will lead agents to save (*resp.* consume) more in the short run. Moreover, the assumption of perfectly altruistic households comes up against many criticisms. A fraction of households do not bequeath (Laitner [1979], Feldstein [1988]) and not all transfers are motivated by altruism (Davies [1981], Bernheim, Shleifer and Summers [1985]). In these models the Ricardian equivalence theorem stops holding. Furthermore, the analysis of Poterba and Summers [1987] offers an empirical criticism of the Ricardian equivalence theorem. Part of the tax burden is reduced by shifting to future generations. Finally, financial markets are far from being perfect. The presence of frictions on the financial markets is partly due to the existence of liquidity constrained agents (Hayashi [1985], Zeldes, [1989], Japelli [1990], Grant [2003]). Hubbard and Judd [1986a, 1986b] have shown the quantitative importance of liquidity constraints for short run issues. As inefficiencies in financial markets prevent households from borrowing, a temporary tax cut financed by debt leads them to consume more (Daniel [1993], Heathcote [2005]).

Another strand of the literature focus on the long run effect of public debt. Number of theses studies suggest that higher public debt is not desirable on account of its crowding out effect on the physical capital accumulation (Diamond [1965], Bernheim [1989]). However, these different analyses rest on the assumption that insurance markets are complete. Woodford [1990], the first, departed from the complete markets assumption. He shows that agents who are liquidity constrained—that is agents who are unable to borrow against their future income—benefit from higher public debt. The quantitative analysis of Aiyagari and McGrattan [1998] confirms the result of Woodford [1990]. They reveals that the optimum quantity of public debt is positive when insurance markets are incomplete and agents face borrowing constraints. A higher public debt enhances the liquidity by

providing an additional means of smoothing consumption and loosens the borrowing constraint. That is why public debt is welfare enhancing. However, the welfare gains of a higher public debt are very low. Floden [2001] emphasizes that the analysis of Aiyagari and McGrattan [1998] is based on the utilitarian welfare criterion. The latter hides the risk sharing improvement associated to higher public debt. Moreover, his analysis reveals that positive public debt effects vanish if transfers are used optimally. However, the liquidity constraint loosening effect of public debt pertains to the short run since it is at work only at the dates when the public debt is increased. The extra debt issuing allows for a temporary reduction in the income tax rate which raises the disposable income. As market imperfections are such that some households in the economy would like to borrow however cannot find credit, then these households are likely to increase their consumption in response to the temporary reduction in income tax rate. Therefore, the short run effects of higher public debt cannot be fully apprehended by steady state comparisons of Aiyagari and McGrattan [1998] and Floden [2001].

We here intend to reassess the welfare gains of a higher public debt when taking into account the short run effects, captured by the transition, in order to shed light on the liquidity constraint loosening effect of public debt. In other words, we ask (i) whether the short run effect that the transitional dynamic simulations allow to capture are likely to increase noticeably the welfare gains associated to a higher public debt? (ii) what are the distributional characteristics of public debt on the transitional path?

The theoretical framework that we adopt is in line with the papers of Aiyagari and McGrattan (1998) and Floden (2001). We depart from the framework of Heathcote [2005] because we do not assume that the tax rate is stochastic¹. We consider an economy with incomplete markets and a large number of *ex ante* identical infinitely-lived agents who face idiosyncratic labor income shocks. As private insurance markets are incomplete, agents, who cannot borrow, save for a precautionary motive (Aiyagari (1994)). Labor supply is assumed to be endogenous. Households are subject to a proportional income tax.

According to the usual steady state optimality criterion (utilitarian welfare criterion), welfare is maximized for a ratio of debt to GDP, equal to 200%, which is noticeably higher than is benchmark level, equal to 2/3. The consumption gain of being at the optimal public debt over GDP ratio instead of the benchmark public debt over GDP ratio is equal to 0.298%. It turns out that the transition-adjusted welfare gains associated to a higher ratio of public debt to GDP² are almost five times as high as the welfare gain based on

¹This assumption implies that households face aggregate as well as idiosyncratic risk.

²We compute the adjusted-transition welfare gain and welfare gain based on steady state comparisons

steady state comparisons. When the calculation of welfare effects of public debt is based on the steady state comparisons, the poorest 5% of agents undergo a consumption loss. In the long run, the increase in the income tax rate and in wage associated to higher public debt reduce the disposable income. However, when the transition is taken into account, namely the temporary income tax rate cut, the poorest 5% of agents experience a consumption gain. The richest 5% of agents continue to experience welfare gain. The characterization of the transitional path of the interest rate, the labor supply, the GDP and consumption reveals that the transition-adjusted welfare gains associated to a higher public debt are not only due to the temporary decrease in income tax rate. It turns out that the increase in labor supply which accounts for the temporary and substantial increase in the interest rate plays a substantial role. Moreover, we show that the transition-adjusted welfare gains associated to a higher public debt depend on the type of fiscal adjustment chosen. If we assume that transfers, instead of the income tax rate, adjust temporarily, the transition-adjusted welfare gains associated to a higher public debt are much greater. The temporary increase in transfers benefits the poorest agents because it shifts resources from wealthy agents to poor agents.

The paper is organized as follows. Section 2 presents the model and the transitional dynamics. In section 3 we discuss the calibration. In section 4, we first determine the optimum quantity of public debt by comparing steady states then we characterize the transitional path and compute the transition-adjusted welfare gains of a higher ratio of public debt to GDP.

2 The model

This economy is populated by a continuum of households, a representative firm and the government.

2.1 The firms

Each period, one good is produced by a representative firm which has a Cobb-Douglas production function. There is no aggregate risk and the production function writes:

$$F(K_t, Z_t N_t) = K_t^\alpha (Z_t N_t)^{1-\alpha}$$

for a particular transitional path. More precisely, we calculate it for the transitional path characterized by an initial ratio of public debt to GDP equal to 2/3 and a final ratio of public debt to GDP equal to 2. It requires to define the number of periods during which the government operates the increase in the public debt over GDP ratio.

where K_t , N_t , Z_t respectively denote the aggregate stock of physical capital, the detrended aggregate labor supply in efficiency units and the labor productivity. Z grows at the exogenous rate g , so we will write $Z_t = (1 + g)^t$, given that the initial level of labor productivity is set to unity. Factor markets are competitive, so each factor earns its own marginal productivity:

$$\begin{aligned} r_t &= F_K(K_t, Z_t N_t) - \delta \\ w_t &= F_N(K_t, Z_t N_t) \end{aligned}$$

Given the homogeneity of degree 1 of the production function, a balanced growth path is possible, where N_t and r_t are both constant, and where K_t and w_t grow at the rate g .

2.2 Households

There is a continuum of infinitely-lived agents of unit mass. Each period, households receive capital, bonds and labor incomes. Households face an idiosyncratic risk on their individual labor productivity. This shock is governed by a Markov chain. Agents cannot borrow and there is no insurance markets against this risk. In order to smooth consumption, agents can only self insure, that is, accumulate (*resp.* deplete) assets when their labor productivity is high (*resp.* low). Agents choose on how much to consume and how much time to spend working (and hence, following the budget constraint, on how much to save). The gross labor income can then be decomposed into the product of the current level of productivity, the current level of the wage per efficiency unit, and the fraction of time at work. This leads to the following asset evolution equation:

$$a_{t+1} = a_t + (r_t a_t + e_t w_t l_t) (1 - \tau_t) + T r_t - c_t$$

where a_t , e_t , l_t , $T r_t$, τ_t and c_t respectively denote the current level of financial wealth, the level of the productivity shock, the labor supply, lump-sum transfers from the government, the proportional income tax rate and consumption. Agents preferences are represented by a time additive intertemporal utility function, U :

$$U_0 = \sum_{t=0}^{\infty} \beta^t u(c_t, l_t)$$

β is the time discount factor and u is the instantaneous utility function which form is:

$$u(c_t, l_t) = \frac{c_t^{1-\rho} \exp[-(1-\rho)\zeta l_t^{1+\eta}]}{1-\rho}$$

with ζ a constant, ρ and $1/\eta$ respectively the relative risk aversion and the labor-supply elasticity.

The recursive formulation of the agent program is:

$$V_t(a_t, e_t) = \underset{c_t, a_{t+1}, l_t}{Max} \{u(c_t, l_t) + \beta E[V_{t+1}(a_{t+1}, e_{t+1})/(a_t, e_t)]\}$$

subject to:

$$\begin{cases} c_t + a_{t+1} = a_t + (r_t a_t + e_t w_t l_t) (1 - \tau_t) + Tr_t \\ a_{t+1} \geq 0 \\ c_t \geq 0 \end{cases}$$

From here on, we detrend the relevant variables by the growth rate g . Although, in the true dynamic version of the intertemporal equilibrium, the time-dependence will obviously be kept, this operation enables us to stationarize the balanced growth path, which we may regard as a 'terminal condition' for the path of the economy under rational expectations. Let us denote with a 'hat' any variable which is detrended in the following way: $\hat{x}_t = \frac{x_t}{(1+g)^t}$. Specifically, we also impose: $\hat{V}_t(\hat{a}_t, e_t) = \frac{V_t(a_t, e_t)}{(1+g)^{(1-\rho)t}$. The detrended program then writes:

$$\hat{V}_t(\hat{a}_t, e_t) = \underset{\hat{c}_t, \hat{a}_{t+1}, l_t}{Max} \{u(\hat{c}_t, l_t) + \beta(1+g)^{(1-\rho)} E[\hat{V}_{t+1}(\hat{a}_{t+1}, e_{t+1})/(\hat{a}_t, e_t)]\} \quad (1)$$

subject to:

$$\begin{cases} \hat{c}_t + \hat{a}_{t+1}(1+g) = \hat{a}_t + (r_t \hat{a}_t + e_t \hat{w}_t l_t) (1 - \tau_t) + \widehat{Tr}_t \\ \hat{a}_{t+1} \geq 0 \\ \hat{c}_t \geq 0 \end{cases}$$

In the stationary equilibrium, the various inputs in the agent program, namely $\hat{w}_t, r_t, \tau_t, \widehat{Tr}_t$ are all constant, so that the time dependence can be dropped. The decision rules will be denoted $\hat{c}_t(\hat{a}_t, e), l_t(\hat{a}_t, e)$ and $\hat{a}_{t+1}(\hat{a}_t, e)$ (*resp.* $\hat{c}(\hat{a}, e), l(\hat{a}, e)$ and $\hat{a}'(\hat{a}, e)$) in the dynamic version (*resp.* in the stationary one).

2.3 The government

The government issues public debt, taxes labor and capital income and operates transfers. His expenses - government spendings, transfers and the interest payment of the current debt- match his revenues- taxes collection and the net public debt issuing:

$$G_t + Tr_t + r_t B_t = B_{t+1} - B_t + T_t$$

where G_t, Tr_t, B_t, T_t are respectively the public consumption, lump-sum transfers, the current stock of public debt³ and the tax revenues, defined as follows:

$$T_t = \tau_t (N_t w_t + r_t A_t) = \tau_t (N_t w_t + r_t (K_t + B_t))$$

By assumption, the policy instruments which the government chooses to impose, at every period, are the ratio of each of the previous aggregates with respect to output Y_t ⁴:

$$\frac{G_t}{Y_t} = \gamma_t, \frac{Tr_t}{Y_t} = \chi_t, \frac{B_t}{Y_t} = b_t$$

The detrended version of the government budget constraint writes:

$$\widehat{G}_t + \widehat{Tr}_t + r_t \widehat{B}_t = (1 + g) \widehat{B}_{t+1} - \widehat{B}_t + \widehat{T}_t$$

2.4 Definition of the equilibria

2.4.1 Stationary equilibrium

In this economy, we will denote as stationary equilibrium an equilibrium where detrended variables are constant over time. Given a policy $\{\chi, \gamma, b\}$, the stationary equilibrium consists of the vector $\{\widehat{c}(\widehat{a}, e), \widehat{a}'(\widehat{a}, e), l(\widehat{a}, e), \Lambda(\widehat{a}, e), \widehat{K}, N, \widehat{Y}, r, \widehat{w}, \tau\}$ where $\Lambda(\widehat{a}, e)$ is the probability measure of agents over the space state, or equivalently, the cross sectional distribution of agents. The stationary equilibrium is attained when this vector is such that:

- Given r, \widehat{w}, τ and $\widehat{Tr} = \chi \widehat{Y}$, the decision rules $\{\widehat{c}(\widehat{a}, e), \widehat{a}'(\widehat{a}, e), l(\widehat{a}, e)\}$ are solutions of the stationary version of program (1),
- $\Lambda(\widehat{a}, e)$ is the unique stationary distribution consistent with the previous decision rules,
- The labor and the capital market clear:

$$\widehat{K} + \widehat{B} = \sum_{e \in E} \int \widehat{a} \Lambda(\widehat{a}, e) d\widehat{a}$$

$$N = \sum_{e \in E} \int e l(\widehat{a}, e) \Lambda(\widehat{a}, e) d\widehat{a}$$

³The time subscript for the stock of public debt refers to the date when debt will be paid back, and not to the date of issuing.

⁴The ratios are time-dependent simply because we will implement the transition of one equilibrium, characterized by certain values for these instruments, to another one. This notation then allows for any adjustment of the policy over time.

- Factor prices verify:

$$r = F'_{\widehat{K}}(\widehat{K}, N) - \delta$$

$$\widehat{w} = \frac{F'_N(K, Z_t N)}{Z_t} = F'_N(\widehat{K}, N)$$

- The Government budget is balanced:

$$(r - g) \widehat{B} = \widehat{T} - (\widehat{G} + \widehat{T}r) \iff (r - g) b = \tau \left(1 - \delta \frac{\widehat{K}}{\widehat{Y}} + rb \right) - (\gamma + \chi)$$

2.4.2 Transitional dynamics

As we will compute the transition from one steady state to another, we need a theoretical characterization of more complex dynamics. Basically, we need, as a starting point, initial conditions, which fully describe the state of the economy -that is, the state of all the heterogeneous agents, and that of the government- at date $t = 0$. Given these initial conditions, the transition consists of a path of all relevant variables, namely that of $r_t, \widehat{w}_t, \widehat{Y}_t, N_t, \widehat{K}_t, \widehat{B}_t, \widehat{T}r_t, \tau_t$. The transition rests on these paths being consistent with the agents' decision rules, which are simply the time-variable solutions of program (1) -note that this program is not only relevant for stationary environments, but also for time-varying ones. In practice, not all of the variables above are mutually independent. The core of the transition can be summarized by only two vectors, $(r_t)_{t \geq 0}, (N_t)_{t \geq 0}$. The paths of $\widehat{w}_t, \widehat{K}_t, \widehat{Y}_t, \widehat{B}_t, \widehat{T}r_t$ can then be obtained. This leads us to the following definition of a transitional path:

Given initial conditions $\{\Lambda_0(\cdot, \cdot), \widehat{B}_0\}$, and given the time-vector of exogenous policy instruments $\{b_t, \chi_t, \gamma_t\}$, a dynamic rational expectation equilibrium consists of $\{\widehat{c}_t(\widehat{a}, e), \widehat{a}'_t(\widehat{a}, e), l_t(\widehat{a}, e), \Lambda_t(\widehat{a}, e), r_t, \widehat{w}_t, \tau_t, \widehat{Y}_t\}$ such that:

- at any date T , given the vectors $\left\{ (r_t)_{t \geq T}, (\widehat{w}_t)_{t \geq T}, (\tau_t)_{t \geq T}, \left(\widehat{T}r_t = \chi_t \widehat{Y}_t \right)_{t \geq T} \right\}$, $\widehat{c}_T(\widehat{a}, e), \widehat{a}_{T+1}(\widehat{a}, e), l_T(\widehat{a}, e)$ are the decision rules obtained from program (1),
- at any date T , Λ_{T+1} is derived from Λ_T and the above decision rules,
- at any date T , the labor and the capital markets clear, that is:

$$N_T = \sum_{e \in E} \int e l_T(\widehat{a}, e) \Lambda_T(\widehat{a}, e) d\widehat{a}$$

$$\widehat{K}_T + \widehat{B}_T = \sum_{e \in E} \int \widehat{a} \Lambda_T(\widehat{a}, e) d\widehat{a}$$

and

$$\begin{aligned} r_T &= F_{\widehat{K}}(\widehat{K}_T, N_T) - \delta \\ \widehat{w}_T &= F'_N(\widehat{K}_T, N_T) \end{aligned}$$

- The law of motion of the stock of public debt is:

$$(\gamma_T + \chi_T)\widehat{Y}_T + r_T\widehat{B}_T = (1 + g)\widehat{B}_{T+1} - \widehat{B}_T + \tau_T(\widehat{Y}_T - \delta\widehat{K}_T + \widehat{B}_T)$$

3 Calibration

The calibration of the model is the same as that of Floden (2001) except for the productivity process. The model period is the year. Following Floden (2001), we resort to the procedure of Tauchen (1986) to approximate the productivity process. Floden (2001) assumes that wages are made up of two components, one permanent ability level and one temporary component. Unlike Floden (2001), we assume that there is no difference in permanent ability level. Therefore, the productivity process is defined as follows:

$$\log(e_t) = \psi \log(e_{t-1}) + \varepsilon_t$$

where ψ is the degree of persistence of shocks and $\varepsilon \sim N(0, \sigma_\varepsilon)$. As Floden (2001), we set ψ to 0.9 and σ_ε to 0.21. We have calibrated the time discount factor to obtain a capital/output ratio equal to 2.5. Table (1) summarizes our calibration choices. In the Appendix section, we briefly present our numerical strategy.

Table 1: parameter values

ρ	β	δ	γ	α	η	χ	b	g	ζ
1.5	0.988551	0.075	0.217	0.3	2	0.082	$\frac{2}{3}$	0.0185	9.1449

4 Steady state optimality

In this section, we briefly present the results from stationary equilibrium simulations. Our baseline corresponds to a debt/GDP ratio equal to 2/3. In the simulations, a single policy parameter is modified, namely the ratio $b = \frac{\widehat{B}}{\widehat{Y}}$. Figure (1) below presents the major statistics of interest.

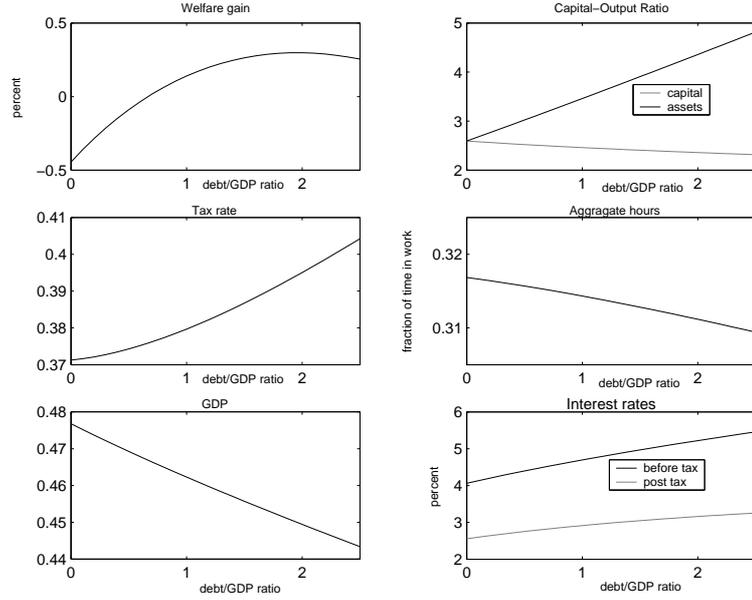


Figure 1: Steady State Simulation: aggregates variables

In particular, the welfare is computed using the utilitarian criterion, as follows:

$$W = \sum_{e \in E} \int \widehat{V}(\widehat{a}, e) \Lambda(\widehat{a}, e) d\widehat{a}$$

For the gains or losses to be interpretable, we compute the percentage of consumption x that agents in the baseline would need, in order to be indifferent with the policy change. We assume that the relative change in consumption leaves the leisure choice unchanged. For a given agent, with an expected utility $\widehat{V}(\widehat{a}, e)$, this implies that:

$$\begin{aligned} \widehat{V}(\widehat{a}, e) &= E_0 \sum_{t \geq 0} \left[\beta (1+g)^{(1-\rho)t} \right] u(\widehat{c}_t, l_t) \\ \implies E_0 \sum_{t \geq 0} \left[\beta (1+g)^{(1-\rho)t} \right] u((1+x)\widehat{c}_t, l_t) &= (1+x)^{(1-\rho)} \widehat{V}(\widehat{a}, e) \end{aligned}$$

Noting A the baseline case and B the policy change, one obtains:

$$\implies x = \frac{\left[\sum_{e \in E} \int \widehat{V}_B(\widehat{a}, e) \Lambda_B(\widehat{a}, e) d\widehat{a} \right]^{\frac{1}{(1-\rho)}}}{\left[\sum_{e \in E} \int \widehat{V}_A(\widehat{a}, e) \Lambda_A(\widehat{a}, e) d\widehat{a} \right]^{\frac{1}{(1-\rho)}}} - 1$$

Quantitatively, the optimal level of public debt is much larger than that obtained by Aiyagari and McGrattan (1998), and somewhat larger than that of Floden (2001).

That our optimal level of debt should be higher than Aiyagari and McGrattan's (1998) comes as no surprise: our calibration is identical to that of Aiyagari and McGrattan's (1998), except for the productivity shock process. As our calibration of the process, taken from Floden (2001), generates considerably more income risk, the proportion of liquidity-constrained agents is higher here. Therefore, and, using the explanation offered by the authors, public debt improves the self-insurance by increasing the liquidity: more risk makes liquidity more beneficial, so the optimal level increases. Our optimal level of debt is somewhat higher than Floden's (2001) : a possible reason would be that public debt has poor redistributive virtues: it would not be worth looking for too high levels of debt in Floden (2001), because his distribution features more inequality than ours.

The increase in the stock of public debt tends to raise the equilibrium interest rate, and, at the same time, reduces the stock of physical capital, as the crowding out of capital takes place. Globally, debt increases the liquidity available to households. The optimal debt/GDP ratio, worth 200%, yields a 0.298% welfare gain. The welfare differs according to the agents classes. We sort agents by their intertemporal welfare, and focus on the bottom 5% and top 5% percentiles of this distribution. We will call the former the richest 5%, and the latter, the poorest 5%⁵. While the richest 5% of agents experience a high increase in consumption which amounts to 6.02%, the poorest 5% of agents undergo a decrease in consumption of 2.95%.

The distributional properties of public debt are illustrated in Figure (2), which plots the individual consumption-equivalent variation in percentage points, when comparing the baseline ($b = 2/3$) with the optimal steady state ($b = 200\%$). Each individual state is characterized by the current level of financial wealth -on the horizontal axis- and by the level of productivity -one curve for each level. This figure reveals that the welfare changes are increasing in both the stock of assets and the productivity shock. Public debt affects the individual welfare levels through 3 channels: it increases the tax rate, it reduces the wage and it increases the interest rate. The first two effects act negatively on the individual well-being, and the third one positively. The increasing pattern of the curve derives simply from the fact that the gain of a higher interest rate depends positively on the current stock of wealth.

Surprisingly, the welfare variations are positive only for asset stocks above a threshold

⁵Our classification, in terms of intertemporal welfare, does not correspond exactly to the distribution of asset holdings. Indeed, the welfare also depends on the current productivity level of the agent. Calling the bottom 5% 'the poorest 5%' is therefore not fully correct, but we use this expression for an obvious reason of simplicity.

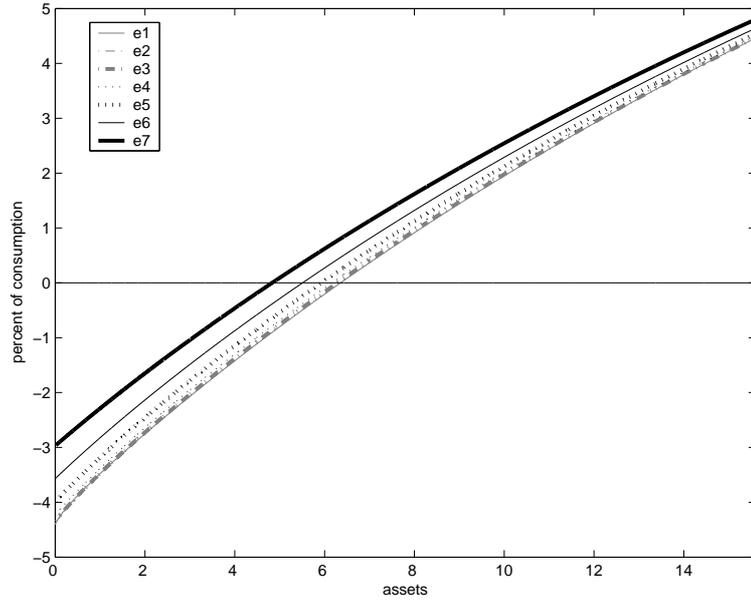


Figure 2: Consumption-equivalent variation by productivity level

(between 4.83 and 6.36, depending on the productivity level considered) and an analysis of the distribution of agents in the baseline economy (not reproduced here) indicates that more than 90% of the households are located below these thresholds. The main channel through which an increase in public debt is welfare-enhancing then appears to be a composition effect: the distribution of agents is modified, as agents hold on average a larger amount of wealth in the $b = 2$ scenario. One might then be tempted to claim that this composition effect neglects a transitional saving effort required to move from one distribution to the other, and that the gains, adjusted for this transitional effort, might turn negative. This would however be a serious mistake, as the debt increase, in itself, enhances the liquidity held by households, without requiring an additional net saving effort. In the pure Ricardian equivalence case, this is exactly what happens: to make up for the reduction in current taxes, and the expected increase in future ones, agents save the additional disposable income triggered by the tax reduction. Here, a crowding out of private capital takes place, which means that the agents will not, on average, in the long run, save the entire increase in public debt: the move from the first distribution to the second one will in fact generate some dissavings. Steady state welfare comparisons therefore do not lead to a conclusive message: the long-run gains are rather small, seem due to a composition effect, and dissaving is expected to occur between the steady states.

5 Optimality in the transition

5.1 Public debt and self-insurance

We here intend to simulate the transition of the economy from a steady state, characterized by a given set of policy instruments, to another. Our main motivation is to cast light on the liquidity-constraint relaxing effect of public debt. Aiyagari and McGrattan's (1998) argument indeed contains 2 separate components regarding the impact of public debt on the ability households have to self-insure, and only one of them seems to apply to steady states analysis. The authors state that a higher public debt over GDP ratio both (i) enhances the liquidity available in the economy, and (ii) relaxes the borrowing constraint. While the second suggested mechanism may seem the more obvious one, it happens that only the first one is really suited for steady state comparisons.

The first mechanism rests on the behavior of households, facing an uninsurable idiosyncratic income risk, a borrowing constraint and whose preferences feature prudence. As is now common in such a setup, the equilibrium interest rate is lower than the rate of time preference, for the capital supply to be bounded. The aggregate -or average- level of wealth is a measure of the available liquidity. Obviously, the higher the aggregate wealth, the more efficient is self-insurance. If agents are richer on average, they will less often be stuck on the borrowing limit. Yet, the aggregate capital supply is endogenous, and depends on the incentive agents have to save. This is where public debt comes into the story. By increasing the interest rate, public debt makes households less reluctant to increase their savings, even though the interest rate remains below the rate of time preference. The utilitarian criterion is then affected, and, depending on the initial size of public debt, it may increase. As for the second mechanism, it needs be acknowledged that the proportion of liquidity-constrained agents diminishes, as public debt increases. However, those who have reached the liquidity-constraint, or are about to, are likely to suffer from the debt increase: all they see is that they have exhausted their asset buffer, and the higher public debt further reduces their disposable income, by bearing on taxes. In the end, the intuition that may lead us to think of public debt as a means of relaxing the borrowing constraint invariably brings us back to transitional issues: the additional debt issuing allows for a temporary reduction in the tax rate, which raises the disposable income. Of course, this mechanism only operates at the times when the debt is increased, and ends as soon as debt stabilizes. This does not conflict with Aiyagari and McGrattan's (1998) formal proof of the equivalence between public debt and a loosening of the borrowing constraint. In their stylized environment (lump-sum taxation and inelastic labor supply),

everything happens as if the borrowing limit had been increased. Yet, the agents that hit this limit are worse-off, because they now have to face the interest-payment of this new liability. The transitional mechanisms, on the opposite, take the temporary additional income -which exactly amounts to the increase in the borrowing limit- into account.

These remarks suggest that the 2 components of the impact of public debt on the ability households have to self-insure can be associated with different time-horizons. The liquidity-enhancing effect acts on average. For a given household, characterized by a given current level of financial wealth, this effect pertains to the long run: only in the long run does a household expect his welfare to be equal to the utilitarian criterion, because it takes time before the future productivity shocks will make the future anticipated state of the household independent from its current one. The liquidity-constraint relaxing effect, clearly, pertains to the short-run, as it works only at the dates when public debt is increased. Consequently, the welfare effects of public debt cannot be entirely apprehended by steady state comparisons. This, in turn, leads to the following questions, which could find an answer in the transitional dynamics simulations: (i) what is the order of magnitude of the short run effect, omitted by steady state comparisons, (ii) what are the distributional characteristics of public debt on the transitional path, as compared to their steady state counterpart, and (iii) does the transition-adjusted welfare effects considerably alter the steady state message regarding the optimal level of public debt?

5.2 Simulations of the transitional dynamics

The pre-reform (*resp.* the post-reform) public debt over GDP ratio will be denoted b_{init} (*resp.* b). Although the model itself has already been fully described in the second section, some assumptions regarding the timing of the policy change are necessary. Precisely, we assume that the change pertains to the ratio b , and is operated in T_{policy} periods, from date $t = 0$ to date $t = T_{policy} - 1$. In most of our simulations, $T_{policy} = 1$, and the final debt/GDP ratio will be attained as soon as $t = 1$. Yet, in the first simulations which are presented, a very large increase in this ratio is implemented, so that a single adjustment would not seem appropriate. Consequently, we assume that, whenever the policy adjustment takes more than one period to be performed, the debt/GDP ratio is chosen to evolve linearly towards its final level:

$$b_t = b_{init} + \frac{t}{T_{policy} - 1} (b - b_{init}), \quad \text{for } t < T_{policy} \quad (2)$$

$$b_t = b, \quad \text{for } t \geq T_{policy} \quad (3)$$

Given the above assumption, another fiscal adjustment is required, to ensure that the government budget constraint is balanced. Among the many possibilities that this modelling offers, we are naturally inclined to favor the tax rate τ_t . We therefore assume that, along the whole transitional paths, the ratios γ_t and χ_t are kept unchanged, so that the tax rate verifies the following equation:

$$\begin{aligned}\tau_0 &= \frac{\gamma_0 \widehat{Y}_0 + \chi_0 \widehat{Y}_0 + (1 + r_0) \widehat{B}_0 - (1 + g) b \widehat{Y}_1}{(\widehat{Y}_0 - \delta \widehat{K}_0 + \widehat{B}_0)} \\ \tau_t &= \frac{(\gamma_t + \chi_t) \widehat{Y}_t + (1 + r_0) b_t \widehat{Y}_t - (1 + g) b_{t+1} \widehat{Y}_{t+1}}{(\widehat{Y}_t - \delta \widehat{K}_t + \widehat{B}_t)} \quad \text{for } t \geq 1\end{aligned}$$

The difference between the two equations derives from the fact that, for $t \geq 1$, the level of debt is given by $\widehat{B}_t = b_t \widehat{Y}_t$. It is however not the case at date $t = 0$; indeed, the level of debt represents the fraction b_0 of the initial GDP \widehat{Y}_{-1} , but the GDP is not predetermined (because of the endogenous labor supply), which means that $\widehat{Y}_0 \neq \widehat{Y}_{-1}$.

Clearly, when public debt is increased, this generates a reduction in the tax rate. Besides, from date $t = T_{policy}$ on, the government achieves to reach the final levels for its policy instruments expressed in *relative* terms (the ratios b , χ , γ), but not in *absolute* ones. Indeed, given the evolution of the stock of physical capital, and the adjustment in the aggregate level of labor supply, the GDP will take time to reach its final detrended level.

The transition-adjusted welfare change is, here again, presented in percentage points of consumption. The baseline economy A is chosen to be the initial steady state, and it is compared to the welfare, measured at date $t = 0$, of the whole population, on the transitional path. Since this utilitarian criterion hides the diversity of the individual welfare adjustments, we also include the proportion of agents for whom the welfare is increased by the policy change in table (3) (those for whom the individual percentage change of consumption is positive).

We first present simulations in which the debt/GDP ratio is switched from its initial level (67%) to its long-run optimal one (200%). Obviously, a policy shock operated in one single period would make little sense. Therefore, we have set T_{policy} at different values, ranging from 8 to 20. The table (2) below presents the results⁶.

⁶We have not calculated the proportion of agents favoring the policy shock through steady state comparisons, simply because different steady states imply different distribution of agents. For example, the poorest 5% for $b_{init} = 2/3$ are not the same as the poorest 5% when $b = 200\%$. More than that, comparing steady states means that we only consider long-run equilibria, and in the long-run, all agents are ex ante alike.

Table 2: Transition-adjusted welfare change

$b_{init}=\frac{2}{3} \rightarrow b = 2.0$				
Steady State				
	Average	Top 5%	Bottom 5%	% of agents
	0.298%	6.02%	-2.95%	/
Transition				
	Average	Top 5%	Bottom 5%	% of agents
$T_{policy} = 8$	1.348%	4.988%	0.211%	90.36%
$T_{policy} = 12$	1.383%	4.491%	0.534%	96.55%
$T_{policy} = 16$	1.382%	4.101%	0.715%	97.97%
$T_{policy} = 20$	1.365%	3.775%	0.827%	99.73%

Note: The increase in b is operated in T periods

To have an idea of the effects of a more moderate public debt increase, we have performed a debt/GDP ratio switch from 67% to 80%. In this simulation, as in all those which will follow, the adjustment is operated in a single period: $T_{policy} = 1$. Table (3) below presents the associated results.

Table 3: Transition-adjusted welfare change

$b_{init}=\frac{2}{3} \rightarrow b = 0.8$	Steady State	Transition
average	0.061%	0.191%
top 5%	0.638%	0.656%
bottom 5%	-0.294%	0.049%
% of agents	/	91.30%

The answer to the first question which we have asked immediately derives from the previous tables: gains are much larger than what steady state comparisons seemed to suggest. The quantitative difference is such that it almost leads to a qualitative difference: Aiyagari and McGrattan (1998) results lead to so low welfare variations that they implied that the current level of debt, as far as only self-insurance was concerned, hardly needed be changed. This is definitely not the case here. The next section addresses the second question: a particular transition is more thoroughly analyzed, and the distributional properties of a public debt increase are brought into light.

5.3 Zooming in on a particular transitional path

We here present more detailed results for the transitional path characterized by an initial debt GDP ratio $b_{init} = 2/3$, and a debt increase $b_t = b = 0.8$ for $t \geq 1$, with $T_{policy} =$

1. The previous results have shown that a non-negligible gain, in terms of utilitarian criterion, is to be expected from such a policy change. We intend to better understand the macroeconomic adjustment in the short run and the distributional impact of the policy shock. In particular, we wish to single out the liquidity-constraint relaxing effect, and possibly other short run effects which were not thought of in the first place. The graphs 3 below plot the paths of the stock of capital, the interest rate, the tax rate, the labor supply, the GDP, the consumption and the wage.

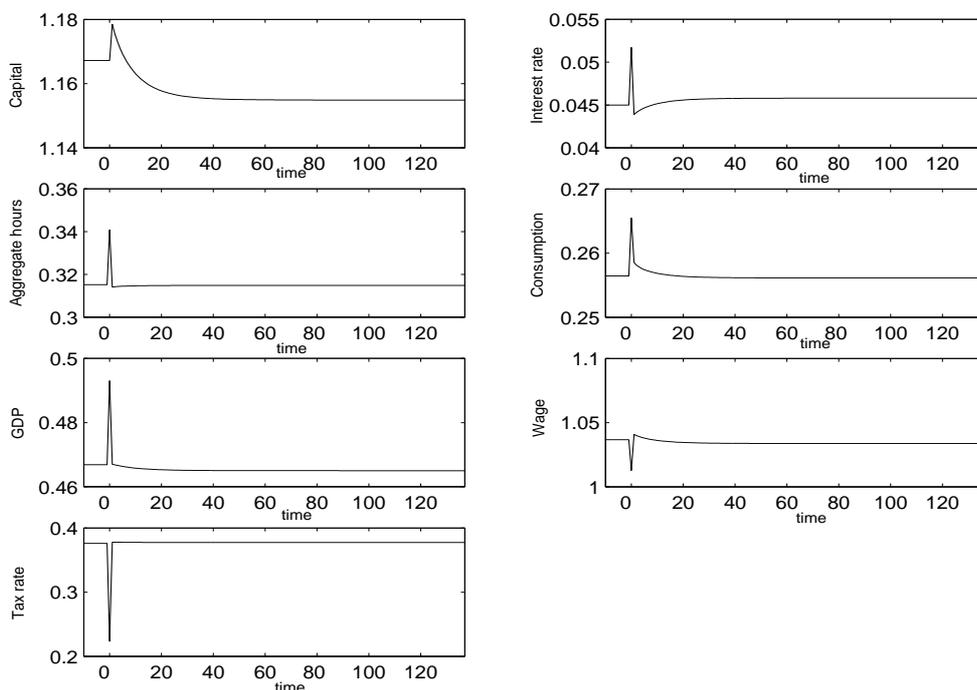


Figure 3: the dynamics of the economy in the baseline model

The path of the interest rate is non-monotonic: it first increases substantially at date $t = 0$ (by 0.67 percentage points), then falls sharply at date $t = 1$, and progressively re-increases to reach its final steady-state level. The instantaneous increase in the interest rate is due to the labor supply adjustment: since hours worked are less heavily taxed, the intertemporal substitution effect accounts for the large increase in labor supply at date $t = 0$. This increase in labor supply positively affects the return on capital⁷.

⁷Note that the increase in public debt is only effective at the end of date 0 (with our convention, it refers to date $t = 1$). Therefore, both the beginning-of-period financial wealth of households, and the public debt B_0 , are predetermined. With the convention adopted, all income flows are end-of-period,

To understand the following path of the interest rate, let us, first, recall what would happen in an environment where the pure Ricardian equivalence would hold. There, the increase in disposable income, resulting from a reduction of (lump-sum) taxation, would be entirely saved, and would exactly match the increase in the net debt issuing. Among the many departures from this world, the endogenous labor supply certainly plays a major role. As can be observed from the graph, labor supply instantly jumps from 0.3153 to 0.3408 (which amounts to a 8.1% increase). This generates a considerable increase in date $t = 0$ GDP of 5.60%. Therefore, the households savings increase is driven not only by the reduction in taxes (which amounts to 40.53%), but also by the increase in gross income. From $t = 1$ onwards, labor supply is brought back to a considerably lower level, close to its final one. Therefore, the main determinant of the interest rate at date $t = 1$ is the household capital supply, that is, their financial wealth, net of public debt issuing. The increase in savings at the end of date $t = 0$ then explains why the interest rate drops so much at date 1 (the decrease amounts to 0.78 percentage points): it is consistent with the path of capital, which shows that capital indeed increases from $t = 0$ to $t = 1$, by 0.965%, and then reverts back to its new long run level. The discontinuity of the path of capital is exclusively due to the labor supply being endogenous. To give more credit to this assertion, we have built an inelastic-labor version of the model, and the results, presented in section 5.4.1, indeed ensure us that the path of capital is smooth in this case.

Consumption at date $t = 0$ increases from its initial level, and then diminishes. It can be observed that consumption does not seem fully continuous between dates 0 and 1 (although the deviation from a continuous graph is very modest). This is due to the fact that the consumption and labor supply choices are mutually dependent: the higher the labor supply, the higher the marginal utility of consumption, as the equation below shows:

$$u'_{c_t} = c_t^{-\rho} \exp[-(1 - \rho)\zeta l_t^{1+\gamma}] \implies \frac{\partial u'_{c_t}}{\partial l_t} = - \underbrace{(1 + \gamma)(1 - \rho)}_{>0} l_t^\gamma c_t^{-\rho} \exp[-(1 - \rho)\zeta l_t^{1+\gamma}]$$

This property rests on the relative risk aversion being greater than unity. In such a case, the consumption and labor supply choices seem complementary: the more agents work, the more willing they are to consume. Since the labor supply is much higher at date $t = 0$ than at any other date, it follows that there is an additional motive to consume⁸.

which means that the capital supply is totally predetermined at date $t = 0$. The increase in the interest rate is then fully driven by the increase in the labor supply.

⁸Note that, as we have illustrated just above, this does not prevent households from saving considerably at the end date $t = 0$.

When labor supply is inelastic, consumption is perfectly smooth (see section 5.4.1).

The differentiated impact of the policy change on the heterogeneous households is presented on Figure (4), which plots the welfare change, measured at date $t = 0$, as a function of the current asset holdings, for the 7 different levels of productivity. The asset axis has been scaled so that its maximum corresponds to the richest agents in the initial stationary distribution. Yet, half (resp. 75%) of the households hold less than 0.88 (resp. 2.11) units of assets.

Unlike the analysis based on the steady state comparisons, the welfare gains are characterized by a U-shape, for all productivity levels. For productivity shocks $e1$ and $e2$, the welfare variation is first positive for those whose financial assets level is respectively less than 0.02 and 0.06 on assets, then negative, and positive again (for those whose financial assets level is respectively higher than 2.60 and 2.20). The other productivity shocks experience a welfare gain whatever the level of financial assets. It was not the case when the calculation of the welfare gains did not take into account the transition.

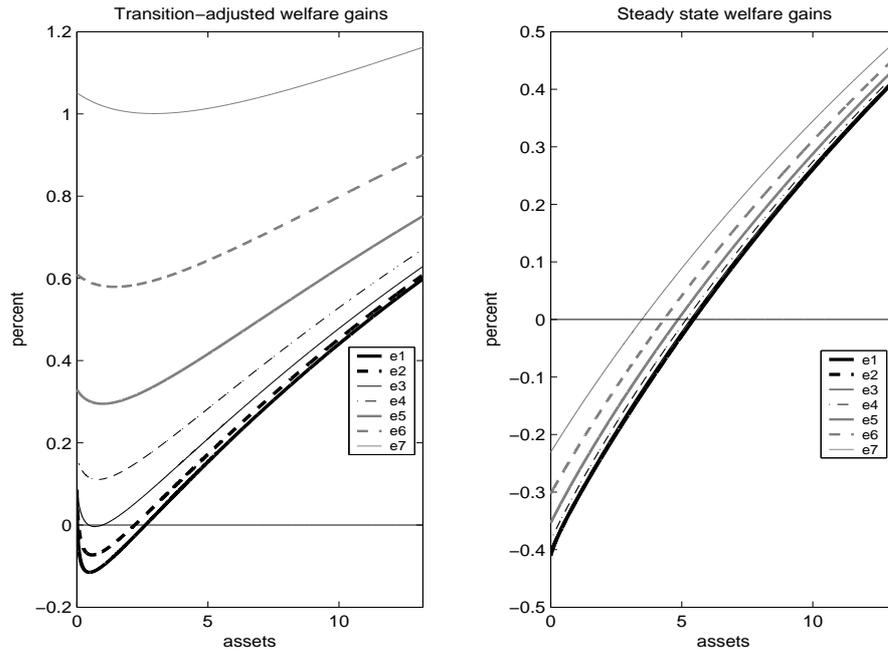


Figure 4: Equivalent consumption variation by productivity level at date $t = 0$

The time path of the tax rate, the interest rate and the wage account for the differentiated welfare impact of the reform. Precisely, a large gain is derived from the initial tax reduction. What makes its impact difficult to single out, is that it bears both on labor and capital income, and obviously depends on the labor supply. The adjustment in

pre-tax factor prices is also likely to play a significant role in the shaping of the welfare changes.

Other things being equal, the decrease in the initial tax rate should benefit more (i) households who are close to the liquidity constraint, provided that they are willing to work more, (ii) households who are currently hit by a high productivity shock, (iii) households who supply a large quantity of labor and (iv) households who possess a large amount of financial assets. The first channel tends to make the gain larger, the poorer the household. The second channel is accounted for by the fact that the curves are indeed ranked as expected: the gains are larger, the higher the productivity. The third one makes the gain a decreasing function of asset holdings, as the labor supply, for any given productivity level, is decreasing in financial wealth. The fourth one works just about the opposite way: the gain in after-tax capital income is larger, the larger the financial wealth. The subsequent time-profile for the tax rate is extremely flat, as it almost reaches its final value from date $t = 1$ on: the associated distributional impact should be, at most, modest. As for the wage and the interest rate, the previous graphs showed that they experience non monotonous dynamics. Consequently, their overall impact among the heterogeneous households, measured as of date $t = 0$, is not straightforward.

At this stage, we can notice that the pure liquidity-relaxing effect (channel (i)), which pertains to the welfare of poor households with low productivity, is rather modest. The welfare gain of households with no wealth and the lowest productivity amounts to a small 0.063%. When they hold a small but positive stock of assets, low productivity households even undergo a welfare decrease. Channel (ii) appears very clearly in the graph, as, for any given level of assets, the curve is higher, the higher the productivity. Channel (iii) should tend to make the curves decreasing, which all are, at least for moderate asset holdings. Channel (iv) tends to make each curve increasing, which is the case for large asset holdings and for all productivity levels. In the next section, we perform additional simulations, which will help disentangle, up to a certain point, these various mechanisms and make each one's impact clearer.

5.4 Other settings

The previous simulations have shown that the short run adjustments are rather complex, and that the distributional implications of the debt increase are not trivial at all. Many channels pertaining to the tax reduction seem to be operating simultaneously. To clarify all these remaining issues, we have performed 3 other simulations, which will now be

presented.

5.4.1 Inelastic labor supply

In the previous subsection, we argued that the discontinuity of the path of capital is due to the labor supply adjustment when the ratio of debt to GDP switches from $2/3$ to 0.8 . To give more credit to this assertion, we build an inelastic-labor version of the model. All model parameters are kept unchanged, apart from the labor supply, which is here assumed to be identical for all agents populating the economy, and equal to the average labor supply in the baseline model⁹. For the initial equilibrium, characterized by a debt/GDP ratio of $2/3$, to share the same macroeconomic features as that of the baseline, we have re-calibrated the discount factor, β , so that the initial capital-output ratio is worth 2.5 . We obtain $\beta = 0.99141$.

The figure (5) below presents the dynamics of the economy, for a debt/GDP adjustment from $b = 2/3$ to $b = 0.8$ and $T_{policy} = 1$. As was expected, and already mentioned in the baseline simulations, the path of capital, the interest rate, GDP and the wage all are continuous, confirming the intuition that the discontinuity in the baseline case is wholly driven by the considerable increase of labor supply at date $t = 0$.

The increase in the ratio of debt to GDP from $2/3$ to 0.8 raises the liquidity available to households. As a result, the interest rate increases. The crowding-out effect of physical capital appears quite clearly, although its magnitude is smaller than in the baseline case. Consumption increases at date $t = 0$ on account of the decrease in the income tax rate which occurs at date $t = 0$ ¹⁰. However, the increase in consumption at date $t = 0$ is lower than in the baseline case because households cannot work more. The increase in consumption only amounts to 0.58% whereas it is as high as 3.5% when labor supply is elastic.

The transition-adjusted welfare change are presented in table (4) when the labor supply is inelastic.

It appears that the transition-adjusted welfare gain are lower when labor supply is inelastic. The 5% poorest agents even experience a welfare loss, although its magnitude is extremely small.

The welfare impact is illustrated in Figure 6. Each graph pertains to a productivity

⁹The level of the labor supply corresponds to the average level of the labor supply when the ratio of debt to GDP is equal to $2/3$ which is equal to 0.315255 .

¹⁰The decrease in the income tax rate is the counterpart of the increase in the ratio of debt to GDP from $2/3$ to 0.8 .

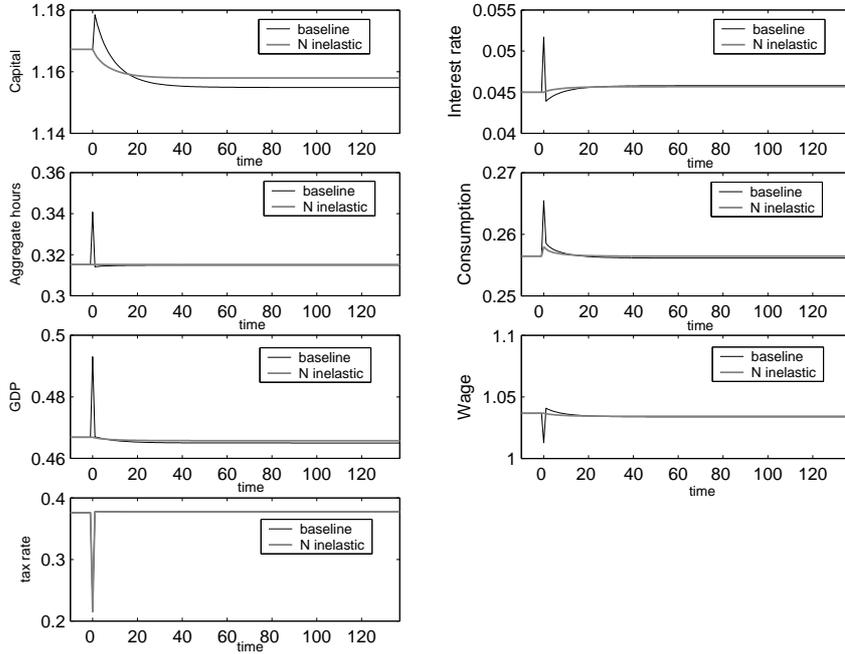


Figure 5: the dynamics of the economy when the labor supply is inelastic

level; it plots the welfare variation in this simulation, and that corresponding to the baseline model. Results are qualitatively similar to the ones derived from the baseline model: all curves display a 'U' pattern. Gains in the inelastic labor setting almost always lie below those of the baseline economy. A possible -although unverifiable- explanation for this could be that, whatever the individual state considered, a significant fraction of the welfare gains in the baseline model is driven by the ability households have to increase their labor supply at date $t = 0$, to take advantage of an unusually high wage. This is of course no more possible here.

5.4.2 Transfer adjustment at date 0

For anyone who has in mind the potentially powerful liquidity-relaxing effect of public debt, the previous results may not be so compelling, and may well be attributable to the type of fiscal adjustment chosen at date $t = 0$. Instead of reducing the tax rate, the budgetary surplus can also be redistributed to agents in many other ways. Among them, lump-sum transfers should operate a significant redistribution. Therefore, we here consider the case where the initial tax rate τ_0 remains equal to its pre-reform value, and

Table 4: Transition-adjusted welfare change when labor supply is inelastic

$b_{init} = \frac{2}{3} \rightarrow b = 0.8$	Baseline		N inelastic	
	Steady State	Transition	Steady State	Transition
average	0.061%	0.191%	0.066%	0.155%
top 5%	0.638%	0.656%	0.559%	0.563%
bottom 5%	-0.294%	0.049%	-0.220%	-0.001%
% of agents	/	91.30%	/	86.53%

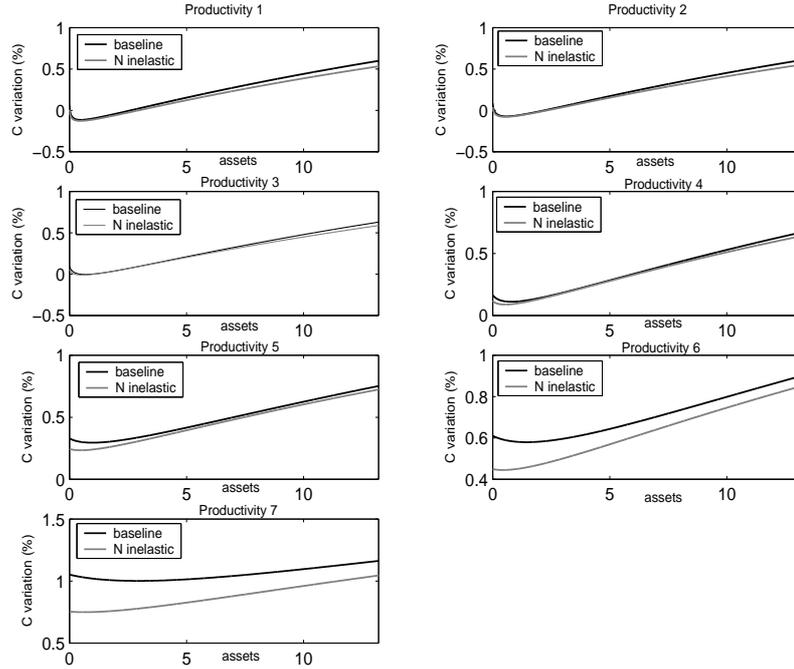


Figure 6: Welfare gains by productivity level when the labor supply is inelastic

where initial transfers endogenously adjust. The government budget constraint at date 0 then implies:

$$\widehat{T}r_0 = (1 + g)b\widehat{Y}_1 - (1 + r_0)\widehat{B}_0 + \tau_0 \left(\widehat{Y}_0 - \delta\widehat{K}_0 + \widehat{B}_0 \right) - \gamma\widehat{Y}_0$$

The transition bears on a debt/GDP increase from $b_{init} = 2/3$ to $b = 0.8$, as was the case in the previous section. The figure (7) below plots the time-profile of the various variables of interest.

The time-profiles are qualitatively quite different from their baseline model counterparts: the path of capital and that of the interest rate are both smooth, and the labor supply slightly diminishes at date 0, but remains smooth afterwards. Consumption does

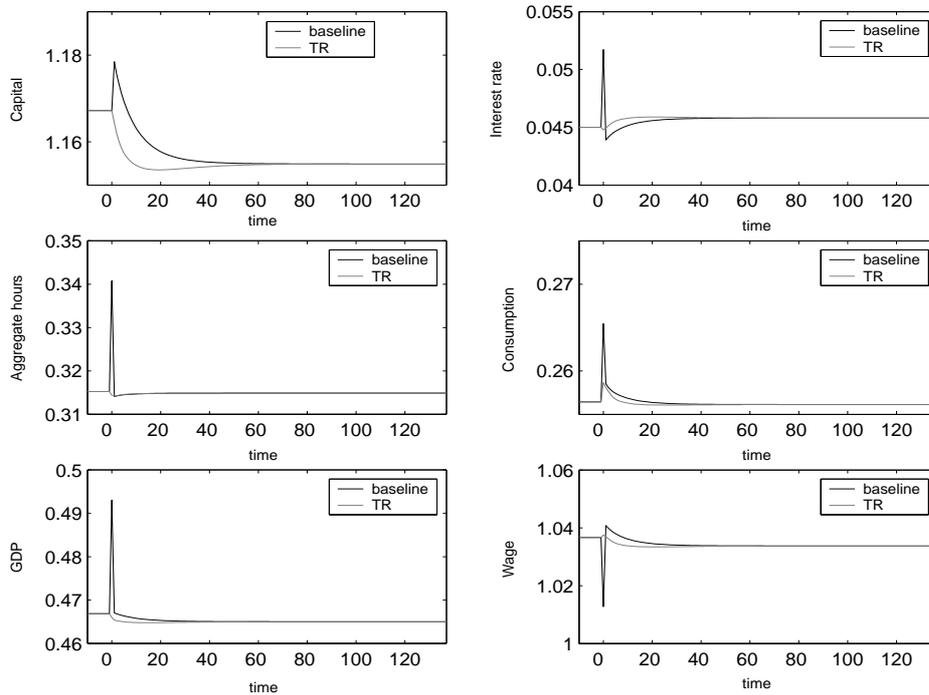


Figure 7: The dynamics of the economy when a lump-sum transfers is operated

not exhibit the slight kink which has been previously discussed over. This is consistent with the fact that the average labor supply does not experience any significant change at date $t = 0$. The dynamics of consumption enable clearly to reveal the aggregate gains from a tax reduction, which stimulates the intertemporal substitution: consumption remains significantly higher in the baseline model during the first 40 periods. Another illustration of the same mechanism lies in the differentiated time-profiles for the stock of capital, which is also higher in the baseline version.

The transition-adjusted welfare change are presented in table (5) when the budgetary surplus is redistributed in the form of lump-sum transfers.

The welfare gains are different when the budgetary surplus due to the increase in public debt is redistributed in the form of lump-sum transfers. The richest agents experience a lower welfare gain: it is almost divided by two. On the contrary, the welfare gain of the poorest agents increases substantially: it is higher than 1%, while it is barely equal to 0.05% when the government adjusts the tax rate τ_0 instead of the lump-sum transfers.

The differentiated welfare effects are shown on Figure 8. In the transfer-adjusted simulation, all curves are initially downward sloping: the transfer is identical in absolute

Table 5: Transition-adjusted welfare change when there is a lump-sum transfer

$b_{init} = \frac{2}{3} \rightarrow b = 0.8$	Baseline		Lump-sum transfer	
	Steady State	Transition	Steady State	Transition
average	0.061%	0.191%	0.061%	0.316%
top 5%	0.638%	0.656%	0.638%	0.341%
bottom 5%	-0.294%	0.049%	-0.294%	1.058%
% of agents	/	91.30%	/	100%

term, but the gain is computed as a percentage variation in consumption. Consequently, richer households, characterized by a higher initial intertemporal consumption, will tend to gain less from this policy shock. What may seem puzzling is the fact that the slopes turn upward for higher asset holdings. Yet, the larger gain cannot be attributed to a higher capital income at date $t = 0$, because the return on capital does not jump, and because the tax rate is initially constant. The only remaining explanation lies in the whole time-profile of the interest rate, that is, the interest rate at dates $t \geq 1$ and which is indeed increasing. The gains resulting from an increase in the interest rate along the whole path, and of course in the long run, are far from negligible, as they amount to 0.5% for the median productivity and for relatively large asset holdings. They are smaller than in the baseline case, but of the same order of magnitude. However, for wealthy households, higher productivity levels are characterized by a larger difference in welfare gains in the two models: when the initial tax rate is reduced (baseline model), the gain derived from an increased labor supply is larger, the higher the current wage. When transfers are adjusted at $t = 0$, labor supply remains roughly constant. This comparison corroborates the intuition that the type of fiscal adjustment at date $t = 0$ has a considerable impact on how poor agents might, or might not, benefit from the public debt increase.

5.4.3 Fixed factor prices

The non-monotonous dynamics of pre-tax factor prices and their distributional properties can be analyzed by comparing the baseline model with that of an economy where factor prices are exogenous. This corresponds to the small open economy assumption; the simulation is performed, not so much for its own results, as for a means to isolate the contribution of the factor prices adjustments.

For the comparison to be as straightforward as possible, we assume that the initial equilibrium is identical in the two versions. In other words, the initial equilibrium in the small open economy version is characterized by the same before tax interest rate.

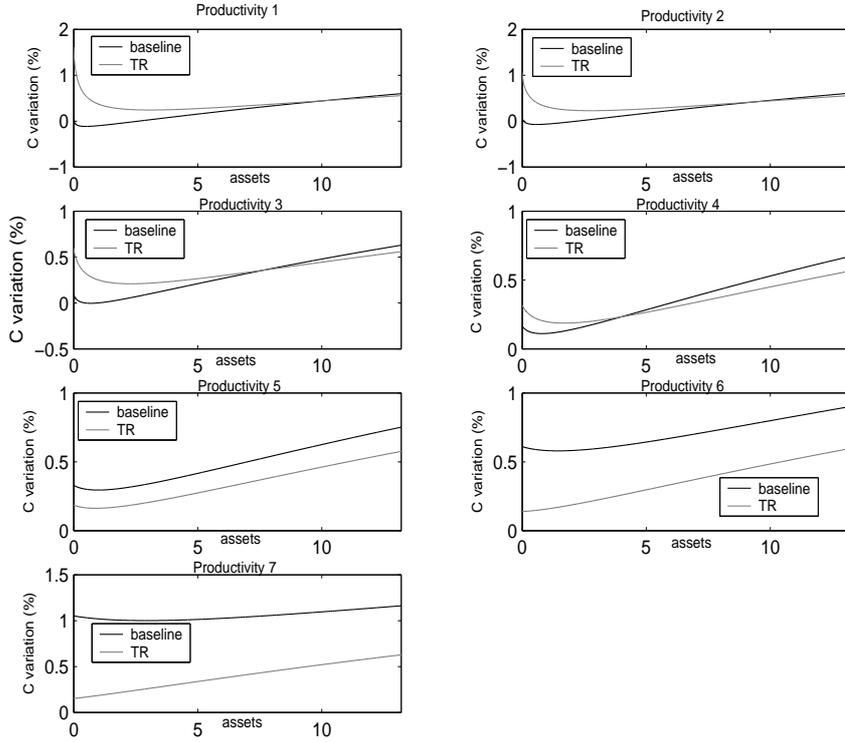


Figure 8: Welfare change by productivity level when a lump-sum transfers is operated

From date $t = 0$ on, the GDP and the domestic demand are disconnected, and the same applies to the capital demand and the capital supply. The policy instruments are still defined as fractions of the domestic GDP; taxes, however, bear on the income of residents. While, in the general equilibrium version, this income exactly amounts to the GDP (net of the capital depreciation) and the interest payment of the public debt, it here simply corresponds to the global wage bill and the interest payment of total asset holdings.

Figures (9) present the aggregate results, and also plot, for comparative purposes, the baseline time-profiles.

The initial increase in asset holdings is almost identical, but the subsequent decrease is much more pronounced in the fixed-price case: the difference is due to the additional incentive to save triggered by the interest rate adjustment, in the baseline model. Labor supply experiences similar dynamics in the two models. The tax rate dynamics is also qualitatively similar to that of the baseline case, but it tends to converge towards a higher level in the long run. Indeed, in this version, taxes apply only to residents' income. As the public debt increases, the domestic asset supply -firms' capital and the government debt- considerably increases, but the domestic asset demand does not match this increase,

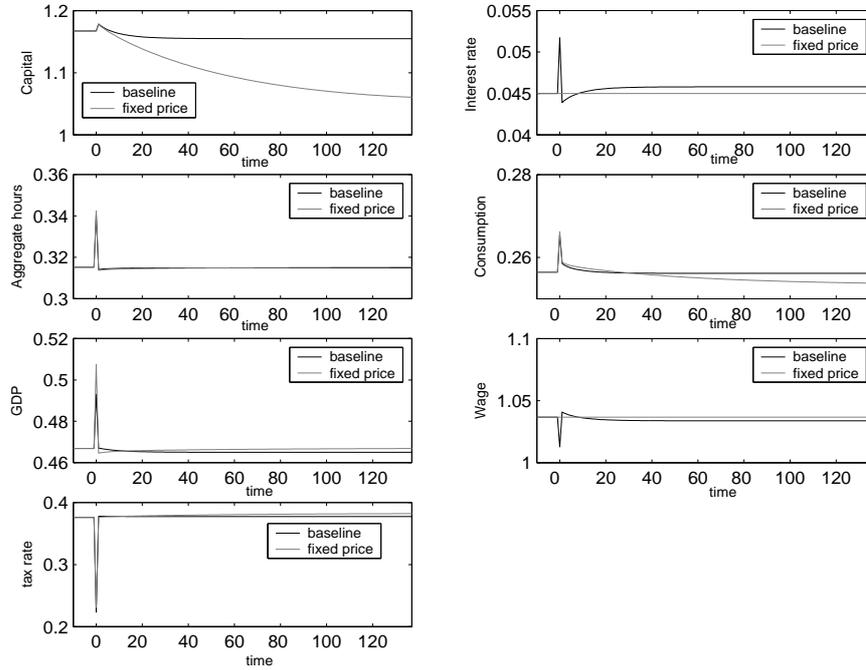


Figure 9: The dynamics of the economy when factor prices are fixed

because the interest rate is exogenous¹¹. While the government expenses are still proportional with respect to the GDP -transfers, spending, and the interest payment of the debt which is still defined as a fraction of the GDP-, taxes apply to a base smaller than the GDP. The behavior of the tax rate, in turn, explains the dynamics of consumption: it converges toward a level significantly below its baseline counterpart. Finally, the fact that the various adjustments take a much longer time to converge is noteworthy; it is not so surprising, once we notice that the households asset holdings are subject to a much larger variation here than in the other simulations..

The transition-adjusted welfare changes are presented in table (6) when the factor prices are fixed.

The welfare gains are very different when factor prices are fixed. On average, agents suffer from the increase in public debt. The richest agents experience a lower welfare gain. The latter is almost divided by 6. Here, the poorest agents undergo a welfare loss. Finally, only 26% of the agents favor the increase in public debt.

Figure (10) presents the welfare impact and confirms the previous results (Table 6).

¹¹This means that a fraction of domestic assets will be acquired by foreign households.

Table 6: Transition-adjusted welfare change when there factor prices are fixed

$b_{init} = \frac{2}{3} \rightarrow b = 0.8$	Baseline		Fixed Prices	
	Steady State	Transition	Steady State	Transition
average	0.061%	0.191%	0.061%	-0.058%
top 5%	0.638%	0.656%	0.638%	0.11%
bottom 5%	-0.294%	0.049%	-0.294%	-0.092%
% of agents		91.30%		26.19%

It appears that the fixed prices tend to make households worse off. This directly follows from the long-run increase in the tax rate (it climbs from 37.6% to 38.2%). For the lowest 4 productivity levels, the curves still feature a 'U' shape, although the increasing part is much less pronounced than in the baseline simulation. While these 4 productivity levels are characterized by welfare losses, the highest 3 productivities are associated with welfare gains. Indeed, the gain derives from the initial reduction in the tax rate, that is, the initial increase in labor and capital incomes. The after-tax wage increase at date $t = 0$ is even larger here than in the baseline simulation, because the before-tax wage at date $t = 0$ remains constant here, while it significantly decreased in the baseline. Clearly, this effect mostly benefits agents with high wages, which explains why the gain is larger for higher productivity levels. Besides, the after-tax interest rate increase at date $t = 0$ positively affects rich households: other things being equal, it tends to make the curves increasing in the stock of assets. This can be observed only for the lowest 4 productivity levels. The reason is that the differentiated impact of the policy shock with respect to asset holdings results from two opposing mechanisms: this one, and the differentiated response of labor supply. Whatever the productivity, the labor supply is decreasing in asset holdings, therefore the absolute gain will be smaller for richer agents. It tends to make the benefit of a higher after-tax wage decreasing in the stock of financial assets.

5.5 Transition-adjusted optimality and equilibrium

The previous simulations have shown, among others, that (i) a progressive transition towards the long-run optimal level yields significant welfare gains and that (ii) considerable transitional adjustments operate in the short run. We could then wonder whether the transitional gains arising when debt is increased could not make further increases of the debt/GDP ratio, above its long-run optimal level, beneficial. It is clear that long-run costs will emerge, but it is much less clear whether they should necessarily dominate the short run gains. As we will see in the simulations, intertemporal welfare gains generated by debt

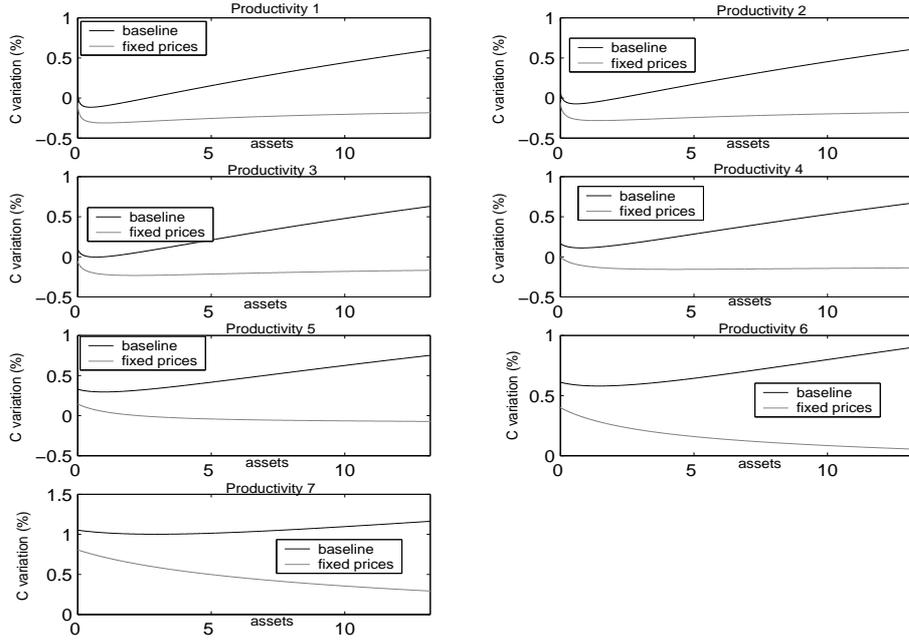


Figure 10: Welfare gains by productivity level when factor prices are fixed

increases continue to occur, even when the initial debt/GDP ratio is above its long-run optimal level. This raises the question of the relevance of the long-run welfare criterion, and invites to re-think the optimality through searching for an optimal transitional path, and not just a steady state optimal level of public debt.

This model, however, can only perform transitions where the path of the debt/GDP ratio is known in advance. For given initial conditions, it could be that the optimal path (given these initial conditions) be associated with a time-varying debt/GDP ratio. This would imply considering an enormous variety of possible paths for the policy instrument b_t . The dynamics of debt is also unavoidably associated with the problem of the commitment credibility, or the time-consistency of debt policies (see, for instance, Krusell, Quadrini and Rios-Rull, 1997).

To handle these two difficulties simultaneously -finding the optimal time-profile of public debt, which at the same time would be time-consistent-, a solution could be to consider a recursive formulation of this problem: each period, the government chooses the ratio b that maximizes the intertemporal welfare criterion, knowing that, next period, it will do so as well. To describe entirely the equilibrium, we would need to build a reaction function representing, for any state of the economy, the current choice of the government, and a law of motion for the state variables. For technical reasons, the description of

this dynamic game is unfortunately out of the scope of the present paper. However, once this has been acknowledged, a more simple concept of optimality can be singled out and performed. Indeed, rather than assuming that the government can choose its policy instruments at each period, we will assume that the changes apply for an arbitrarily long time. Under this assumption, the reaction function associated with the next choice becomes useless: this reaction would alter the welfare in a distant future, so distant that, at the limit, this choice would not affect the current intertemporal welfare and, in turn, the current choice.

However long the periodicity, maximizations are assumed to occur repeatedly. The initial conditions need therefore be those of a steady state associated with a given debt/GDP ratio, as a previous optimization has occurred a long time ago. An equilibrium is reached when the government chooses today the same debt/GDP ratio as the one chosen previously. For a transition-adjusted optimum to be reached, the condition simply imposes that, given the initial debt/GDP ratio, no once-and-for-all deviation yields welfare gains.

Similarly, one could consider a politico-economic equilibrium, where the outcome of a vote is the choice of the majority. Such an equilibrium would be formally similar to the dynamic game described above: the only difference would be that, at each period, a voting rule -the majority rule, for instance- would replace the welfare criterion as the choice function. Again, the recursive formulation of this similar problem cannot be implemented here, but, provided that we impose the same restriction on the periodicity of the choice, a formally identical solution emerges. Precisely, a majority voting equilibrium with an arbitrarily long voting periodicity is such that, given an initial state of the economy characterized by a debt/GDP ratio b , no once-and-for-all deviation is preferred by a majority of households. The restriction on the periodicity of the choice is certainly here less stringent than in the welfare-maximizing case: institutional arrangements do not permit too frequent nation-wide elections that would entail such drastic macroeconomic policy change, while welfare maximization, per se, is orthogonal to these institutional features.

To perform this exercise, we have simulated a 10% increase, starting with steady states associated with various debt/GDP ratios. Whenever a clear majority favors the debt increase (resp. whenever the policy shock generates gains in the utilitarian criterion), the initial debt/GDP ratio is not that of the politico-economic equilibrium (resp. it does not correspond to the highest utilitarian criterion).

Table 7 below presents, for various initial debt/GDP ratios b_{init} , the welfare change implied by a 10 percentage points increase in the debt/GDP ratio, and the proportion of

households for whom the intertemporal utility increases.

Table 7: Transition-adjusted welfare change when there factor prices are fixed

change in public debt	Utilitarian welfare criterion	proportion of households
$b_{init} = 1 \rightarrow b = 1.1$	0.128%	85.12%
$b_{init} = 2 \rightarrow b = 2.1$	0.084%	75.94%
$b_{init} = 3 \rightarrow b = 3.1$	0.053%	57.03%
$b_{init} = 4 \rightarrow b = 4.1$	0.031%	50.22%
$b_{init} = 4.1 \rightarrow b = 4.2$	0.0287%	49.96%
$b_{init} = 4.5 \rightarrow b = 4.6$	0.022%	49.45%
$b_{init} = 5 \rightarrow b = 5.1$	0.013%	48.92%
$b_{init} = 5.9 \rightarrow b = 6.0$	0.00047%	48.16%
$b_{init} = 6.0 \rightarrow b = 6.1$	-0.001037%	48.12%

It appears that the majority of agents favors debt increases until it reaches the level of 410%. In terms of the utilitarian criterion, gains cease to occur for a level as high as 600%. The equilibrium level of debt derived from the majority voting criterion is significantly lower than its utilitarian counterpart. The agents that suffer from debt increases, as has been stressed previously, are those with moderate and low asset holdings, while those who gain are rich agents, and/or agents currently hit by a high productivity shock. We have also seen that the gains obtained by rich agents can be quite high, while the losses (the minimum on the 'U' curves) are lower, in absolute value. Therefore, when the majority criterion equilibrium has been reached (410%), the proportions of winners and losers are equal, by the gain of the winners are higher than the losses of the losers; this explains why further increases induce intertemporal welfare gains.

These figures are obviously very high, and the intertemporal gains one can expect from a debt increase (utilitarian criterion) become very small, as the initial debt/GDP ratio considered increases. The absence of a true recursive formulation, enabling for a period-by-period re-optimization, could be responsible for this result. However, even in a more sophisticated setup where repeated welfare maximizations (or repeated votes) would be apprehended, these transitional gains would remain. Time-consistent behaviors could very probably temper the results, but, at the very least, these simulations clearly stress the significant temptation to deviate toward higher levels of public debt, in order to capture short-run gains.

6 Conclusion

In this paper, we have slightly modified and extended Floden's economy (2001) to describe explicitly the transitional dynamics resulting from a public debt increase. It appears that gains, measured with the utilitarian criterion, are considerably higher than what steady state simulations suggested. As an example, a progressive increase of the debt/GDP ratio, from its current 67%, to the long run optimum of 200%, generates as much as 1.3% consumption gains, while it is only equal to 0.3% in steady state comparisons. During the transition, and as could be expected, the largest temporary effect pertains to the labor supply adjustment at date $t = 0$, which is responsible for a capital supply overshooting at date $t = 1$. In distributional terms, the liquidity-relaxing effect of public debt can indeed be singled-out, but the welfare gains of liquidity-constrained households are quite modest. The largest gains clearly benefit rich households with current high productivity. This partly corroborates the steady state results, where gains depend mainly and positively on current asset holding, but little on the current wage. Finally, be it in utilitarian terms, or from a majority voting perspective, the transitional dynamics gives rise to short-run gains that create a strong temptation to deviate towards higher levels of public debt, much higher in fact than the steady state optimum. Although the transition-adjusted gains become very small, and although a much more severe long-run cost will be borne later, this result emphasizes the high temptation to benefit from short-lived effects. This result, however, rests on a particular assumption, namely that no further public choice -utilitarian criterion maximization, or majority voting- ever occurs. Whether this result would carry to an economy where the public choice is repeatedly implemented, remains an open issue.

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Numerical strategy

The state space simply consists of the individual asset holdings and the current productivity shock. We have used a non-uniform grid with 5000 points to discretize the interval $[0; a_{\max}]$. With $a_{\max} = 60$, no agent comes as near as two thirds of this bound. The productivity process is approximated by a 7-value Markov chain.

The steady state equilibrium computation requires to find the equilibrium interest rate, and average labor supply, that enter the agents' program as inputs. We therefore look for values of these two endogenous variables that guarantee the equilibrium conditions, simply by initiating the computation with an initial guess, and then updating the guess by a relaxation method. We have performed two nested loops: for a given interest rate, we look for the labor supply which, imposed *ex ante*, is obtained *ex post*, and the larger loop updates the interest rate. With a relaxation parameter for the labor supply equal to 0.9, the convergence is rather fast with respect to this variable. The relaxation parameter bearing on the interest rate updating is, however, much lower, and the convergence, or divergence, differs significantly, depending on the imposed debt-GDP ratio. This is why this relaxation parameter is re-calculated from the second iteration on, to both ensure the convergence and increase the convergence speed. If, for instance, the convergence criterion, measured in terms of the relative difference between the *ex ante* and the *ex post* interest rates, evolves very slowly, the relaxation parameter is suitably increased (its own updating depending on the updating of the relative difference). This greatly improves the convergence speed.

The individual decision rules are computed by finding the zero of the Euler equation. The asset accumulation is not restricted to belong to the grid, and is computed by a simple linear interpolation between the two consecutive grid points for which the Euler equation residual switches from a negative value to a positive one (if this switch occurs; otherwise, the agent is liquidity-constrained).

For the transitional dynamics computation, the algorithm rests on a fixed-point of the paths of the interest rate and the labor supply. We used 140 periods, to ensure that the model has converged to its final steady state. The initial condition corresponds to the initial distribution of agents, which is the steady state distribution for given policy parameters. The terminal condition consists of the value functions for the final steady state. Therefore, before computing the transition itself, we need to find these two stationary equilibria. To compute the transition, we then simply guess a path for the interest rate and the aggregate labor supply, compute its associated decision rules on the transition,

iterate on the initial distribution of agents to obtain the implied dynamics, and update the two paths, again by a relaxation method. Here, the relaxation parameter is much lower, and does not exceed 0.15. The convergence criterion corresponds to the maximal relative difference between the ex ante and ex post path for the interest rate and the aggregate labor supply, and is equal to $10^{-4}\%$.

The computation has been performed in C++. It takes something like an hour and a half, with a 1.6 GHz processor.