Turbulence, Training and Unemployment: Do we need higher training subsidies?

Pascal Belan[*] Arnaud Chéron[†]

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Abstract

This paper develops a matching model where firms invest in transferable human capital training and where workers endowed with heterogenous abilities face a risk of human capital loss during unemployment spell. We show that despite a rise of the probability of human capital loss (turbulence) accounts for increasing unemployment, the first best policy does not necessarily consists in implementing higher training subsidies.

1 Introduction

For a decade, Ljungqvist and Sargent (LS hereafter) emphasize in various papers the primarily role played by turbulence in rising unemployment for economies with generous unemployment benefit systems (see among others Ljungqvist and Sargent [1998,2007]). The interaction of increasing probability of human capital loss during unemployment spell with high unemployment compensations lead to a sharp rise of European unemployment. This paper aims at examining the first-best design of training subsidies in the context of a frictional labor market search model with turbulence. It is our contention that, despite the fact that a rise in the probability of human capital loss, called hereafter turbulence, increases unemployment, the first best policy does not necessarily consist in implementing higher training subsidies.

Indeed, since Ljungqvist and Sargent [1998], the following set of observations is well established:

[*]Université de Nantes (LEMNA)

[†]Université du Maine (GAINS-TEPP) and EDHEC

- Long-tenured displaced workers experienced large and enduring earning losses. This features workers' skill depreciation during unemployment spells.[1]
- The 1980's was accompanied by an increase in the dispersion of earnings and the intertemporal volatility of individual earnings.

Then, considering exogenous human capital sticking partly to the worker and partly to the job, LS show in frictional labor market models that the interaction of higher skill depreciation probability with generous unemployment benefit system accounts for the rise in European unemployment.

From this analysis, it remains however several open issues. In particular, face to skill depreciation, should we implement training subsidies? If yes, in the context of higher turbulence, should we subsidy more training costs? Our answer to these two questions are: yes and not necessarily.

From a first best perspective, the use of training subsidies is obvious in the context of externality and inefficiencies. At the end of the 1990's, Acemoglu [1997], Acemoglu and Pischke [1998, 1999a, 1999b] and Acemoglu and Shimer [1999] put emphasize on this point by focusing on inefficient training in the context of frictional labor markets. More precisely, Acemoglu [1997] points out the possibility of an externality between the worker and his future employers in the context of frictional labor market ("poaching externality"). In competitive equilibrium, the worker obtains 100% of the increase in productivity due to training in general human capital, and was therefore willing to pay the cost through wage cuts (see Becker [1964]).[2] On the contrary, a frictional labor market may explain the willingness of employers to bear part of the costs of general training (bargaining power of firms, asymmetric information). But training investment may benefit to future employers that is, with some probability, an unknown party (the future

[1] As emphasized by Neal [1995], subsequent earnings of displaced workers can be thought of as corresponding to an indicator of human capital surviving beyond the old match.

[2] By distinguishing general and specific training Becker [1964] emphasizes that a worker should pay for any general training which allows him to use the new skills when employed by other firms. In a competitive framework, workers pay for training in general human capital. In this context, inefficiency in training investment is the consequence of incomplete contracts or credit market imperfections between the worker and his current employer (as stressed by Becker [1964] and Grout [1984]). Contractual arrangements, as exit penalties for workers who quit their firm, can deal with such inefficiency. But in this theory, government intervention should be mostly limited to improving loan markets and subsidies to training are unnecessary.

employer) is getting a proportion of the training benefit when the worker is displaced. This results in underinvestment because the rents accruing to this third party do not feature in the calculations of the worker and his current employer. It is then obvious that the size of these externalities should be related to the probability of human capital depreciation, hence turbulence as defined by LS. A first contribution of our approach is to examine the interaction between turbulence and poaching externalities, by enlightening its impact on the optimal training subsidies design.

Beyond this, our analysis adds another externality related to the social unemployment gain of training. Indeed, training can also increase the probability of leaving unemployment and contributes to increasing steady-state employment, hence output. This issue has not been addressed by subsequent works of Acemoglu and co-authors (see Leuven [2005] for a survey). Once again, firms do not take this "steady-state unemployment externality" into account when they evaluate training investments. This results in lower training investments than the one associated to the first-best allocation.

The immediate consequence is that both externalities (poaching and steady-state unemployment) make training subsidies efficient. Nevertheless, we point out that the way turbulence interacts with training subsidies differ according to the externality that plays the dominant role. Otherwise stated, it can be the case that despite higher turbulence leads to increase unemployment, the first best policy consists in implementing lower training subsidies.

Section 2 of this paper develops a frictional labor market model with heterogenous workers, where firms can invest in up-to-date knowledge training that raises workers' productivity. Wages are determined according to Nash-bargaining. Turbulent times is characterized by the possibility for workers to face depreciation of their transferable knowledge during unemployment spell. We then show that the equilibrium is featured by a decrease in individual earnings in case of job displacement, and an increasing (decreasing) relationship between unemployment rate (share of trained workers) and the probability of human capital loss. Section 3 is devoted to the characterization and the analysis of the efficient training system. We show that the higher the turbulence, the lower the size of the poaching externality, and this decreases the efficient subsidy rate of training costs. On the contrary, the higher the turbulence, the higher the size of steady-state unemployment externality, so that it requires a higher subsidy rate of training costs. In Section 4, we extend our benchmark model in different ways to account: *(i)* for holdup problems, *(ii)* distorsive taxation instruments to balance the training budget, and *(iii)* endogenous training intensity. Finally, Section 5 concludes.

2 Benchmark Model

2.1 Environment and labor market flows

Time is continuous. The population of workers is a continuum of unit mass. Workers look for jobs and are randomly matched with employers looking for workers to fill vacant units of production. A productive unit is the association of one worker and one firm. Workers are heterogeneous with respect to exogenous ability a which determines their productivity on the job. Ability is distributed on the interval $[\underline{a}, \bar{a}]$ according to p.d.f. $f(a)$.

Firms can pay for a fixed training cost γ_F[3] in order to provide worker up-to-date knowledge which rises its productivity from a to $(1+\Delta)a$, with $\Delta > 0$. This brings for workers transferable (portable) job skills which can be used in any future occupation. For the sake of simplicity, it is assumed that workers cannot accumulate skills according to tenure and experience: either the worker has up-to-date knowledge which improves its efficiency on the job according to its ability (with an amount Δa), or nothing. During unemployment spell, with instantaneous probability π, the worker may loose the benefits of past training, and will then need a new formation to recover its up-to-date knowledge when matched with a new job. This assumption introduces obsolescence of human capital as a result of what Ljunqvist and Sargent [1998] have called turbulence. It embodies the possibility of substantial human capital destruction after job loss (Jacobson & al. [1993], Farber [1997,2005]). At this stage, training policy of the firm therefore simply consists in determining a threshold ability $\tilde{a}$ above which workers are trained if the latter faced depreciation of their knowledge.

It follows that any unemployed individual belongs to one of the three following categories: (1) type-0 individuals: unable for training ($a \leq \tilde{a}$); (2) type-1 individuals: able enough for training ($a \geq \tilde{a}$), previously trained and still highly productive; (3) type-2 individuals: able enough for training ($a \geq \tilde{a}$), but with obsolete knowledge or not previously trained. In steady state, type-2 unemployed is always an individual whom general human capital has became obsolete.

We consider exogenous contact rates for workers[4] and assume that un-

[3]In the context of Nash bargaining of wages no matter who pay the cost of training until it is assumed that no party can renege the wage contract once the match is formed (no hold-up); proof available upon request. See Section 4.1 for a related discussion on the impact of hold-up on the policy design.

[4]Endogenous recruitment decisions would add some complexities without modifying

employed people who do not have access to training (and consequently to up-to-date knowledge) face a lower probability to get a job

$$p(a) = \begin{cases} p_0 & \text{if } a \leq \tilde{a} \\ p & \text{if } a > \tilde{a} \end{cases}$$

with $p_0 < p$. All jobs have the same separation rate δ. We denote respectively employment and unemployment levels of a-ability population by $e(a)$ and $u(a)$. Since, in steady state, inflow into unemployment $\delta(f(a) - u(a))$ is equal to outflow $p_0 u(a)$, for $a \leq \tilde{a}$ and $pu(a)$, for $a > \tilde{a}$, one obtains

$$u(a) = f(a) \times \begin{cases} \frac{\delta}{\delta+p_0} & \text{if } a \leq \tilde{a} \\ \frac{\delta}{\delta+p} & \text{if } a > \tilde{a} \end{cases}$$

For $a \geq \tilde{a}$, unemployment $u(a)$ is divided between type-1 individuals who are still highly productive, and type-2 individuals who have endured obsolescence of their general human capital: $u(a) = u_1(a) + u_2(a)$, for $a \geq \tilde{a}$. In steady state, inflow into type-2 unemployment $\pi u_1(a)$ is equal to outflow $pu_2(a)$. Thus,

$$u_1(a) = f(a) \times \frac{\delta}{\delta+p} \times \frac{p}{\pi+p}, \quad u_2(a) = f(a) \times \frac{\delta}{\delta+p} \times \frac{\pi}{\pi+p}$$

Lastly, the overall unemployment rate, $u = \int_{\underline{a}}^{\overline{a}} u(a)da$, is

$$u = \frac{\delta}{\delta+p} + F(\tilde{a})\frac{\delta(p-p_0)}{(\delta+p)(\delta+p_0)} \tag{1}$$

where F is the c.d.f. associated to f. Unemployment is negatively related to training: if more workers have access to training (lower $\tilde{a}$) the unemployment rate is reduced. Then, as will be shown hereafter, since economic turbulence has an impact on firms' training decision, it may account for higher unemployment.

2.2 Behaviors

2.2.1 Training decision

For a firm, the intertemporal value of a filled job depends on the ability a and of the worker's type. We denote this value by $J_i(a)$, $i \in \{0, 1, 2\}$. If the worker is of type 0 ($a \leq \tilde{a}$), instantaneous production is a. If the worker is of

our main conclusions.

type 1 or 2 ($a \geq \tilde{a}$), training will increase his productivity and instantaneous production becomes $(1+\Delta)\,a$. The corresponding Bellman equations write

$$\begin{aligned} rJ_0(a) &= a - w_0(a) - \delta\,(J_0(a) - V) \\ rJ_i(a) &= (1+\Delta)a - w_i(a) - \delta\,(J_i(a) - V)\,, \quad i \in \{1,2\} \end{aligned}$$

where V is the intertemporal value of a vacancy, r the interest rate, and $w_i\,(a)$, $i \in \{0,1,2\}$, the wage rate paid on job i.

The training policy simply consists here on determining the ability threshold $\tilde{a}$ above which it is the interest of the firm to train the workers whenever it comes from type-2 unemployment. The critical ability value $\tilde{a}$ is defined by[5]

$$J_2(\tilde{a}) = J_0(\tilde{a}) + \gamma_F$$

which implies

$$\Delta\tilde{a} = w_2(\tilde{a}) - w_0(\tilde{a}) + (r+\delta)\gamma_F \tag{2}$$

This condition states that firm trains the workers if and only if the present value of the productivity gain $\Delta a/\,(r+\delta)$ is, at least, as high as the present value of the wage gap plus the training cost. Since, we assume that efficiency of training increases with workers' ability (Δa is increasing in a), it can be the case that low-ability workers will never be trained.

2.2.2 Nash bargaining

We consider bilateral negotiation of wages according to the conventional Nash bargaining sharing rule. It is obvious that there exists an unenforceable problem because workers have ex-post incentives to renege this contract. This issue is addressed in section 4.1.

The respective intertemporal values of employment and unemployment are denoted by $E_i\,(a)$ and $\mathcal{U}_i(a)$, $i \in \{0,1,2\}$. For unable individuals ($a \leq \tilde{a}$), steady-state Bellman equations write

$$r\mathcal{U}_0\,(a) = b + p_0\,(E_0\,(a) - \mathcal{U}_0\,(a)) \tag{3}$$

$$rE_0\,(a) = w_0\,(a) - \delta\,(E_0\,(a) - \mathcal{U}_0\,(a)) \tag{4}$$

where b represents home production. For individuals with ability $a \geq \tilde{a}$, Bellman equations turn out to be

$$\begin{aligned} r\mathcal{U}_1\,(a) &= b + p\,(E_1\,(a) - \mathcal{U}_1\,(a)) + \pi\,(\mathcal{U}_2\,(a) - \mathcal{U}_1\,(a)) && (5) \\ r\mathcal{U}_2\,(a) &= b + p\,(E_2\,(a) - \mathcal{U}_2\,(a)) && (6) \\ rE_1\,(a) &= w_1\,(a) - \delta\,(E_1\,(a) - \mathcal{U}_1\,(a)) && (7) \\ rE_2\,(a) &= w_2\,(a) - \delta\,(E_2\,(a) - \mathcal{U}_1\,(a)) && (8) \end{aligned}$$

[5] Section 4.3 allows for endogenous training intensity.

Notice that any type-1 and type-2 worker who is losing his job becomes type-1 unemployed since he still has up-to-date general human capital. Nevertheless, his knowledge may become obsolete with instantaneous probability π, which leads him to fall in type-2 unemployment.

At this stage, wages are the solutions of the following sharing rules

$$\beta J_0(a) = (1-\beta)(E_0(a) - \mathcal{U}_0(a)) \tag{9}$$
$$\beta J_1(a) = (1-\beta)(E_1(a) - \mathcal{U}_1(a)) \tag{10}$$
$$\beta(J_2(a) - \gamma_F) = (1-\beta)(E_2(a) - \mathcal{U}_2(a)) \tag{11}$$

It should be noticed that the threat point of type-2 workers is type-2 unemployment value $\mathcal{U}_2(a)$ since it is assumed that training would only occur after wage bargaining.

2.3 Equilibrium wages and training rule

Proposition 1. *Job displacement implies a decrease in individual earnings.*

Proof. Let $x = \frac{\beta(r+\delta+p)}{r+\delta+\beta p}$ and $x_0 = \frac{\beta(r+\delta+p_0)}{r+\delta+\beta p_0}$. Wage equations solve (see Appendix 7.1)

$$w_0(a) = x_0 a + (1-x_0)b \tag{12}$$
$$w_1(a) = x(1+\Delta)a + (1-x)\left[b - \pi(\mathcal{U}_1(a) - \mathcal{U}_2(a))\right] \tag{13}$$
$$w_2(a) = x\left[(1+\Delta)a - (r+\delta)\gamma_F\right] + (1-x)\left[b - \delta(\mathcal{U}_1(a) - \mathcal{U}_2(a))\right] \tag{14}$$

with

$$\mathcal{U}_1(a) - \mathcal{U}_2(a) = \frac{\beta p}{r+\pi+\beta p}\gamma_F$$

Furthermore, since $\mathcal{U}_1(a) - \mathcal{U}_2(a) = \frac{p(w_1(a)-w_2(a))}{(r+\delta)(r+\pi+p)}$, it is straightforward to see that $\mathcal{U}_1(a) - \mathcal{U}_2(a) > 0$ is consistent with $w_1(a) > w_2(a)$. □

As usual, wages correspond to a weighted average of worker's contribution for firms and reservation wages. It should be emphasized that, for type-1 and type-2 workers, the latter is negatively related to the unemployment gain of being previously trained, $\mathcal{U}_1(a) - \mathcal{U}_2(a)$. On the one hand, type-1 unemployed workers are expecting to face depreciation of their human capital formation with probability π if staying in unemployment (equation (13)). On the other hand, employment is more worthwhile for unemployed workers of type-2 due to the expected increase in unemployed value after being trained (equation (14)).

Furthermore, this expected unemployed surplus related to training is increasing (decreasing) with respect to workers' bargaining power β (to the rate of human capital depreciation π). The latter reflects the fact that higher turbulence reduces the comparative advantage of those workers previously trained. In turn, higher bargaining power for workers means that workers internalize a higher wage cut due to the cost of training (see equation (14)). Hence, the relative value of having up-to-date knowledge (type-1 unemployed worker) is higher.

Proposition 2. *The equilibrium training ability threshold is characterized by*

$$\Delta\tilde{a} = \gamma_F \left[\frac{r(r+\delta+\pi+\beta p)+\delta\pi}{r+\pi+\beta p} \right] + (\tilde{a}-b)\left(\frac{x-x_0}{1-x}\right) \tag{15}$$

Proof. Substitute out for $w_0(a)$ and $w_2(a)$ from (12) and (14) in (2) one gets the equilibrium definition of $\tilde{a}$. □

Corollary 1. *If $r \to 0$, the equilibrium training ability threshold is characterized by*

$$\Delta\tilde{a} = \gamma_F \frac{\delta\pi}{\pi+\beta p} + (\tilde{a}-b)\,\beta\left(\frac{p-p_0}{\delta+\beta p}\right) \tag{16}$$

Property 1. *A higher turbulence reduces the share of trained workers and raises the unemployment rate.*

Proof. >From Proposition 2, it is straightforward to see that $\tilde{a}$ increases with π. Moreover, equation (1) implies that u is positively related to the fraction of workers, $(1-F(\tilde{a}))$, who have access to training. This concludes the proof. □

Because higher turbulence (higher π) reduces the relative value of training, it increases the reservation wage of type-2 workers, and consequently raises the ability threshold $\tilde{a}$. Otherwise stated, turbulence discourages firms to train by increasing threat points of type-2 workers who look for jobs.[6] Then, because the fraction of workers who never have access to up-to-date knowledge and face a lower probability to exit unemployment ($p_0 < p$) is increasing, the overall unemployment rate turns out to be higher.

Property 2. *A lower probability of unemployment exit for untrained workers reduces the share of trained workers.*

[6]It should be emphasized that this result can be consistent with an average wage that decreases with turbulence. Indeed, on the one hand, more workers do not have access to training and are paid $w_0(a)$ and, on the other hand, wage of type-1 workers is negatively related to turbulence (see equation (13)).

Proof. From Proposition 2, it is straightforward to see that $\tilde{a}$ decreases with p_0. □

This property actually stems on the fact that the threat point of workers is as much important as the probability of exiting unemployment is large. Then, a lower probability to exit unemployment for untrained workers reduces their wage so that, as a consequence, the additional wage cost for firms associated with training is increased. This reduces firms incentives to train, by raising the ability threshold of workers.

3 The design of training subsidy

Turbulence is found to increase unemployment by reducing training. How does this affect the labor market policy design? Our objective is to address this issue from a first best perspective by first showing what could be an optimal training subsidy system. Then, we would like to highlight the role played by turbulence on this efficient training system design.

3.1 Efficient training

We consider that the problem of the planner consists in maximizing the steady-state average output value net of turn over costs by choosing the optimal ability threshold below which workers are not trained (*i.e.* throughout we consider $r \to 0$). This problem can be stated as follows

$$\max_{a^\star} \int_{\underline{a}}^{a^\star} a(f(a) - u(a))da + (1+\Delta) \int_{a^\star}^{\bar{a}} a(f(a) - u(a))da$$
$$+b \int_{\underline{a}}^{\bar{a}} u(a)\, da - \gamma_F p \int_{a^\star}^{\bar{a}} u_2(a)\, da$$

subject to the equilibria between inflows and outflows in unemployment, or equivalently

$$u(a) = f(a) \times \begin{cases} \frac{\delta}{\delta+p_0} & \text{if } a \leq a^\star \\ \frac{\delta}{\delta+p} & \text{if } a > a^\star \end{cases}, \; u_2(a) = f(a) \frac{\delta}{\delta+p} \frac{\pi}{\pi+p} \text{ for } a > a^\star$$

Proposition 3. *The efficient training ability threshold is defined by*

$$\Delta a^\star = \gamma_F \frac{\delta\pi}{\pi+p} - (a^\star - b)\frac{\delta}{p}\left(\frac{p-p_0}{\delta+p_0}\right) \tag{17}$$

Corollary 2. *Let us consider $p_0 = p$ and $r \to 0$, the efficient training ability threshold satisfies*

$$\Delta a^\star = \gamma_F \frac{\delta\pi}{\pi + p} < \Delta\tilde{a} \tag{18}$$

This first suggests that it could be socially optimal to never give access to training for workers endowed with low abilities, so does if $\Delta\underline{a} < \gamma_F \frac{\delta\pi}{\pi+p}$. Nevertheless, at the optimum, more workers should have access to training than would be at equilibrium.

This result is consistent with the one for instance derived by Acemoglu [1997]: firms do not internalize the social gain for future employers related to their own training decision. Training not only increases productivity in the current firm but also may increase productivity of the worker in his future jobs whenever worker exogenously exits that firm, does not loose its up-to-date knowledge during the unemployment spell, and is hired by an other firm. Only workers may internalize this social gain of training. Nevertheless, since $\beta < 1$, when workers move to another job, they only get a fraction of the additional productivity related to training. Thus, the relative value of having up-dated-knowledge ($\mathcal{U}_1 - \mathcal{U}_2$) is lower than the additional productivity related to training. Hence, the reservation wage of type-2 workers is too high and keeps too much people out of training. Of course, if $\beta = 1$, the bargaining power of the workers would internalize the total gain of training and equilibrium training would be optimal.

Corollary 3. *Let us consider $p_0 = 0$ and $r \to 0$. The efficient training ability threshold satisfies*

$$\Delta a^\star = \gamma_F \frac{\delta\pi}{\pi + p} - (a^\star - b) \tag{19}$$

The case $p_0 = 0$ highlights that a lower contact probability for workers who do not benefit from up-dated knowledge reduces the efficient ability threshold $a^\star$. Hence, it increases the fraction of workers who have access to training. On the contrary, recall that $\tilde{a}$ is as much important as $p - p_0$ is large.[7] Therefore, the gap between equilibrium and optimum increases.

In particular, this reflects the fact that firms do not internalize the impact of training on (un)employment rate. When $p_0 = 0$, keeping a worker of ability a out of training leads to a social output loss that amounts to $a - b$. Firms do not valuate this loss, whereas the planner does.[8] Then, obviously, the size

[7] Actually we consider $p_0 = 0$ in Corollary 3, but it is obvious that the result would also be obtained for any value of $p_0 < p$.

[8] For $0 < p_0 < p$, the externality consists in having not internalized the fact that workers who are not trained face a longer unemployment spell than those who have access to up-to-date knowledge.

of the externality is as much important as the difference between market and home productions, *i.e.* the social cost of a rise in unemployment increases with $\tilde{a} - b$.

Property 3. *Consider $r \to 0$. For any $p_0 \leq p$, the equilibrium unemployment is higher than its efficient value.*

Proof. Straightforward from $a^\star < \tilde{a}$ and the positive relation between u and $\tilde{a}$ as defined by equation (1). □

Otherwise stated, since firms do not train enough workers, too much workers face a low probability of exit from unemployment.

3.2 Turbulence and the optimal training subsidies

We consider that the government can subsidy training at the time of job creation, by paying a fraction s of the training cost γ_F. A lump-sum tax T paid by firms on each job allows to balance government budget.[9] The latter solves

$$T = s\gamma_F \frac{p \int_{\tilde{a}}^{\overline{a}} u_2(a)da}{\int_{\underline{a}}^{\overline{a}}(1 - u(a))da} \tag{20}$$

Then, in such an environment, the equilibrium ability threshold $\tilde{a}$ decreases with respect to the subsidy rate s (assuming $r \to 0$)

$$\Delta\tilde{a} = \frac{\pi\delta}{\pi + \beta p}\gamma_F(1 - s) + (\tilde{a} - b)\,\beta\left(\frac{p - p_0}{\delta + \beta p}\right) \tag{21}$$

so that s can be designed to restore efficiency of training and reach optimal unemployment. Let us now deal successively with the two externalities discussed earlier.

Proposition 4. *Assuming $p = p_0$ and $r \to 0$, the optimal rate of training subsidy is*

$$s^\star = (1 - \beta)\left(\frac{p}{\pi + p}\right)$$

Proof. Straightforward by equalizing $\tilde{a}$ and $a^\star$ in equations (18) and (21) (assuming $r \to 0$ and $p = p_0$). □

Property 4. *If $p_0 = p$, the higher the turbulence, the lower the subsidy rate of training cost.*

[9] At this stage, we do not consider any distortive taxes. See section 4.2 for a detailed discussion on this issue.

In such circumstances, unless $\beta = 1$, it is efficient to subsidy the training cost. More importantly, Proposition 4 states that the optimal subsidy rate decreases with turbulence. Indeed, the higher turbulence is, the lower the size of the externality is. When the turbulence is high, it is less likely that a firm would benefit from training decision of other firms. Otherwise stated, the social return to training converges to its private return. Ultimately, if $\pi \to \infty$, training investment collapses to job-specific human capital, so that both equilibrium training policy and unemployment rate are optimal and no subsidy is required. Otherwise stated, from that point of view, we need less training subsidies when turbulence is higher.

Proposition 5. *Consider $r \to 0$, $p_0 = 0$ and $\beta = 1$. The optimal rate of training subsidy is*

$$s^\star = \left(\frac{1}{1+\Delta}\right)\left(1 + \frac{p}{p+\delta}\right)\left(1 - \frac{b}{\gamma_F}\Delta\left(\frac{\pi+p}{\pi\delta}\right)\right)$$

Proof. Straightforward by equalizing $\tilde{a}$ and $a^\star$ in equations (19) and (21), assuming $r \to 0$, $\beta = 1$ and $p_0 = 0$. □

Property 5. *If $p_0 = 0$ and $\beta = 1$, the higher the turbulence, the higher the subsidy rate of training costs.*

By assuming $p_0 = 0 < p$ and $\beta = 1$, Proposition 5 abstracts from the poaching externality and deals with the other externality. It turns out that a higher turbulence require to implement a higher training subsidy rate to restore efficiency. Indeed, in a context of higher economic turbulence, training ability thresholds tend to increase. Then, the output loss $\tilde{a} - b$ due to the rise in unemployment also increases. The social planner internalizes this movement, whereas firms do not.[10]

Therefore, the overall impact of turbulence on the subsidy rate of training cost is found to be ambiguous. Indeed, to enter in more details, the marginal aggregate output loss is increasing with the productivity threshold as implied by higher turbulence ($a^\star$ increases with π). This means that the size of the externality neglected by firms is increased by higher turbulence. This introduces an opposite force to the reduction of inefficiencies associated with higher job-specificity of human capital acquisition when turbulence is higher.

It is thus unclear whether higher turbulence requires to implement a higher subsidy rate of training cost.

[10]Recall that when $p_0 = p$, unemployment rate u no longer depends on $\tilde{a}$ so that higher turbulence has no incidence on the unemployment rate.

4 Extensions and Robustness

4.1 Revisiting training subsidies with hold-up

Since $w_2(a) < w_1(a)$, workers have ex-post a clear incentive to renege the initial wage agreement. Without unforceable contractual arrangement of the type developed by Malcomson [1997], a hold-up problem arises. In this event, the sharing rule then rewrites as $\beta J_2(a) = (1-\beta)(E_2 - \mathcal{U}_2)$, so that the equilibrium with hold-up is characterized by

$$w_1(a) = w_2(a) = x(1+\Delta)a + (1-x)b \;\text{ and }\; \mathcal{U}_1 = \mathcal{U}_2$$

For the sake of simplicity, let now abstract from heterogenous contact probabilities to focus on the role of hold-up on the design of training subsidies.

Proposition 6. *Let $p_0 = p$ and $r \to 0$. The equilibrium ability training threshold is $\Delta\tilde{a} = (r+\delta)\gamma_F$, so that the efficiency subsidy rate turns out to be*

$$s^{\star\star} = \frac{p}{p+\pi} > s^{\star}$$

This clearly states that higher training subsidies are required to offset the hold-up distortion.

4.2 The optimal training system with heterogenous distortive taxes

Let us consider that to balance its training budget, the government can use a lump sum tax T_i and a proportional tax t_i which might depend on worker's type. More precisely, we assume $t_1 = t_2 = t$ as well as $T_1 = T_2 = T$, but pay a special attention to the question of taxing workers who never benefit from the training policy by imposing no specific restriction on T_0 and t_0.

Values functions and sharing rules can then be re-stated as follows (with $r \to 0$)

$$\begin{aligned}
\delta J_0(a) &= a - T_0 - w_0(a)(1+t_0) && (22)\\
\delta J_i(a) &= (1+\Delta)a - T - w_i(a)(1+t) \quad i \in \{1,2\} && (23)\\
\beta J_0(a) &= (1-\beta)(E_0(a) - \mathcal{U}_0(a))(1+t_0) && (24)\\
\beta J_1(a) &= (1-\beta)(E_1(a) - \mathcal{U}_1(a))(1+t) && (25)\\
\beta(J_2(a) - \gamma_F(1-s)) &= (1-\beta)(E_2(a) - \mathcal{U}_2(a))(1+t) && (26)
\end{aligned}$$

The equilibrium with such a tax policy is then characterized by

$$\Delta\tilde{a} = T - T_0 + b(t - t_0) + \frac{\pi\delta}{\pi+\beta p}\gamma_F(1-s)$$

It is obvious that this threshold does not depend neither on T nor t if the taxation system is homogenous, that is $T_0 = T$ and $t_0 = t$. This reflects the fact that the gap in wage costs $((1+t)w_i(a) + T,\ i \in \{0,2\})$ between type-2 and type-0 jobs does not depend on fiscal instruments.

One may also assume that only workers with ability $a > \tilde{a}$ are elgible for tax because they have access at a moment time to training, while those with ability $a \leq \tilde{a}$ have not. This consists for instance in assuming $T_0 = t_0 = 0$, $T_{1,2} = T$ and $t_{1,2} = 0$. In this case, one obtains

$$\begin{aligned} \Delta\tilde{a} &= T + \frac{\pi\delta}{\pi + \beta p}\gamma_F(1-s) && (27) \\ T &= \frac{\pi\delta}{\pi + p} s\gamma_F && (28) \end{aligned}$$

Then, assuming again $p_0 = p$, it appears that if $T_0 = t_0 = 0$, the overall training cost has to be subsidy ($s = 1$) to restore equilibrium efficiency. Considering $s = 1$ in (27) and (28), one gets indeed $\Delta\tilde{a} = \Delta a^\star$ when $p_0 = p$.

4.3 Endogenous Training Intensity

A worker with a given ability a can be either trained or not, and if trained the training intensity maximizes the value of the job for the firm or equivalently for the worker (from the assumption of Nash bargaining). Let now assume $\gamma_F = \gamma + c(\Delta)$, so that the training cost for the firm depends both on a fixed cost and a variable cost according to the training intensity, with $c' > 0, c'' \geq 0$, implying decreasing returns with training intensity. Let introduce two subsidy instruments, so that the subsidized training cost writes as $\gamma(1 - s_0) + c(\Delta)(1 - s_\Delta)$. Values of jobs can now be stated as follows

$$\begin{aligned} J_0(a) &= \frac{a - w_0(a) - T}{r + \delta} \\ J_1(a) &= \frac{a' - w_1(a') - T}{r + \delta} \\ J_2(a) &= \max_{\Delta} \left\{ \frac{a' - w_2(a') - T}{r + \delta} - [\gamma(1 - s_0) + c(\Delta)(1 - s_\Delta)] \right\} \\ & \quad\ s.t. \quad a' = a(1 + \Delta) \end{aligned}$$

Firms' decisions are both characterized by a training intensity $\Delta(a)$ and an ability threshold $\tilde{a}$ which solves $J_0(\tilde{a}) = J^\star(\tilde{a})$.

Proposition 7. *Let us assume* $c(\Delta) = \frac{\Delta^2}{2}$ *and* $r \to 0$. *The equilibrium training policy is defined by the set* $\{\tilde{a}, \tilde{\Delta}(a)\}$ *which solves*

$$\begin{aligned}
\tilde{a} &= \left(\frac{\delta\pi}{\pi + \beta p}\right)\sqrt{2\gamma\left(\frac{1 - s_0}{1 + s_\Delta}\right)} \\
\tilde{\Delta}(a) &= \frac{a}{1 - s_\Delta}\left(\frac{\pi + \beta p}{\delta\pi}\right)
\end{aligned}$$

Proof. See appendix 7.3 for details. □

Without fixed cost ($\gamma = 0$), all workers would have access to training, that is $\tilde{a} = 0$, even though the intensity would still continuously increase from $\underline{a}\left(\frac{\pi+\beta p}{\delta\pi}\right) \to \overline{a}\left(\frac{\pi+\beta p}{\delta\pi}\right)$.

Proposition 8. *The optimal training subsidy system is given by*

$$\begin{aligned}
s_\Delta^\star &= (1 - \beta)\frac{p}{\pi + p} > 0 \\
s_0^\star &= 1 - (1 + s_\Delta^\star)\left(\frac{\pi + \beta p}{\pi + p}\right)^2 > 0
\end{aligned}$$

Proof. Consider $p_0 = p$, one should first notice that the efficient training policy is characterized by the set $\{a^\star, \Delta^\star(a)\}$

$$\begin{aligned}
a^\star &= \left(\frac{\delta\pi}{\pi + p}\right)\sqrt{2\gamma} \\
\Delta^\star(a) &= a\left(\frac{\pi + p}{\delta\pi}\right)
\end{aligned}$$

It is then straightforward to derive $s_0^\star$ and $s_\Delta^\star$. Some studious calculus allow to insure that $s_0^\star > 0$. □

5 Conclusion

To be completed

6 References

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7 Appendix

7.1 Wages and filled-job intertemporal values

For $a < \tilde{a}$, from equations (3) and (4) it comes that

$$(r+\delta+p_0)\left(E_0(a)-\mathcal{U}_0(a)\right)=w_0(a)-b$$

so that (9) implies

$$\beta(r+\delta+p_0)\left(a-w_0(a)(1+t_0)-T_0\right)=(1-\beta)(r+\delta)\left(w_0(a)-b\right)(1+t_0)$$

Then, from the Nash bargaining assumption, we know that $w_0(a)$ maximizes $\left(E_0(a)-\mathcal{U}_0(a)\right)^{\beta}\left(J_0(a)\right)^{1-\beta}$, which implies

$$(r+\delta+\beta p_0)\left(a-w_0(a)(1+t_0)-T_0\right)=(1-\beta)(r+\delta)(a-b)(1+t_0)$$

hence

$$\begin{aligned} w_0(a) &= \frac{a-T_0}{1+t_0}-\frac{(1-\beta)(r+\delta)(a-b)}{r+\delta+\beta p_0} \\ &= \frac{\beta(r+\delta+p_0)\left(\frac{a-T_0}{1+t_0}\right)+(1-\beta)(r+\delta)b}{r+\delta+\beta p_0} \end{aligned}$$

and

$$J_0(a)=\frac{(1-\beta)\left(a-T_0-b(1+t_0)\right)}{r+\delta+\beta p_0} \tag{29}$$

Consider now $a \geq \tilde{a}$, equations (5), (6), (7) and (8) imply

$$\begin{aligned} (r+\delta+p)\left(E_1(a)-\mathcal{U}_1(a)\right) &= w_1(a)-b+\pi\left(\mathcal{U}_1(a)-\mathcal{U}_2(a)\right) \\ (r+\delta+p)\left(E_2(a)-\mathcal{U}_2(a)\right) &= w_2(a)-b+\delta\left(\mathcal{U}_1(a)-\mathcal{U}_2(a)\right) \\ (r+\pi)\left(\mathcal{U}_1(a)-\mathcal{U}_2(a)\right) &= p\left[\left(E_1(a)-\mathcal{U}_1(a)\right)-\left(E_2(a)-\mathcal{U}_2(a)\right)\right] \end{aligned}$$

so that we have

$$\mathcal{U}_1(a)-\mathcal{U}_2(a)=\frac{p\left(w_1(a)-w_2(a)\right)}{(r+\delta)(r+\pi+p)} \tag{30}$$

Furthermore, from the Nash bargaining assumption, $w_i(a)$ maximizes $\left(E_i(a)-\mathcal{U}_i(a)\right)^{\beta}\left(J_i(a)\right)^{1-\beta}$, for $i=1,2$. Thus sharing rules write

$$\begin{aligned} \beta J_1(a) &= (1-\beta)(E_1(a)-\mathcal{U}_1(a))(1+t) \\ \beta(J_2(a)-\gamma_F) &= (1-\beta)(E_2(a)-\mathcal{U}_2(a))(1+t) \end{aligned}$$

so that it comes

$$\begin{aligned} & \beta(r+\delta+p)((1+\Delta)a-T-w_1(a)(1+t)) \\ = & (1-\beta)(r+\delta)[w_1(a)-b+\pi(\mathcal{U}_1(a)-\mathcal{U}_2(a))](1+t) \end{aligned}$$

$$\begin{aligned} & \beta(r+\delta+p)(1+\Delta)a-T-w_2(1+t)(a)-(r+\delta)\gamma_F \\ = & (1-\beta)(r+\delta)[w_2(a)-b+\delta(\mathcal{U}_1(a)-\mathcal{U}_2(a))](1+t) \end{aligned}$$

We then find

$$\begin{aligned} w_1(a) &= \frac{\beta(r+\delta+p)}{r+\delta+\beta p}\frac{(1+\Delta)a-T}{1+t} \\ &\quad +\frac{(1-\beta)(r+\delta)}{r+\delta+\beta p}[b-\pi(\mathcal{U}_1(a)-\mathcal{U}_2(a))] \end{aligned}$$

$$\begin{aligned} w_2(a) &= \frac{\beta(r+\delta+p)}{r+\delta+\beta p}\left[\frac{(1+\Delta)a-(r+\delta)\gamma_F-T}{1+t}\right] \\ &\quad +\frac{(1-\beta)(r+\delta)}{r+\delta+\beta p}[b-\delta(\mathcal{U}_1(a)-\mathcal{U}_2(a))] \end{aligned}$$

This implies

$$\begin{aligned} w_1(a)-w_2(a) &= \frac{\beta(r+\delta+p)}{r+\delta+\beta p}(r+\delta)\frac{\gamma_F}{1+t} \\ &\quad -\frac{(1-\beta)(r+\delta)}{r+\delta+\beta p}(\pi-\delta)(\mathcal{U}_1(a)-\mathcal{U}_2(a)) \end{aligned}$$

Combining this with (30), one gets:

$$w_1(a)-w_2(a)=\frac{\beta(r+\delta)}{r+\pi+\beta p}(r+\pi+p)\frac{\gamma_F}{1+t}$$

and then

$$\mathcal{U}_1(a)-\mathcal{U}_2(a)=\frac{\beta p}{r+\pi+\beta p}\frac{\gamma_F}{1+t}$$

Lastly, wage equations are given by:

$$
\begin{aligned}
w_1(a) &= x\left(\frac{(1+\Delta)a-T}{1+t}\right)+(1-x)\left[b-\pi\left(\mathcal{U}_1(a)-\mathcal{U}_2(a)\right)\right] \\
w_2(a) &= x\left[\frac{(1+\Delta)a-(r+\delta)\gamma_F-T}{1+t}\right]+(1-x)\left[b-\delta\left(\mathcal{U}_1(a)-\mathcal{U}_2(a)\right)\right]
\end{aligned}
$$

where

$$
\mathcal{U}_1(a)-\mathcal{U}_2(a)=\frac{\beta p}{r+\pi+\beta p}\frac{\gamma_F}{1+t}
$$

Job values then turn out to be defined by:

$$
J_1(a)=\frac{1-\beta}{r+\delta+\beta p}\left[(1+\Delta)a-T-[b-\pi\left(\mathcal{U}_1(a)-\mathcal{U}_2(a)\right)](1+t)\right]
$$

$$
\begin{aligned}
J_2(a)-\gamma_F &= \frac{1-\beta}{r+\delta+\beta p}\left[(1+\Delta)a-(r+\delta)\gamma_F-T\right] \\
&\quad -\frac{1-\beta}{r+\delta+\beta p}[b-\delta\left(\mathcal{U}_1(a)-\mathcal{U}_2(a)\right)](1+t) \qquad (31)
\end{aligned}
$$

7.2 Equilibrium with taxes

Wages equations are be defined by

$$
\begin{aligned}
w_0(a) &= x\left(\frac{a-T}{1+t}\right)+(1-x)b \\
w_1(a) &= x\left[\frac{(1+\Delta)a-T}{1+t_0}\right]+(1-x)\left(b-\pi\left(\mathcal{U}_1-\mathcal{U}_2\right)\right) \\
w_2(a) &= x\left[\frac{(1+\Delta)a-T-\delta\gamma_F(1-s)}{1+t}\right]+(1-x)\left(b-\delta\left(\mathcal{U}_1-\mathcal{U}_2\right)\right)
\end{aligned}
$$

with $\mathcal{U}_1-\mathcal{U}_2=\frac{\beta p}{\pi+\beta p}\frac{\gamma_F}{1+t}$. The productivity threshold solves

$$
J_2(\tilde{a})=J_0(\tilde{a})+\gamma_F(1-s)
$$

with $J_2(a)$ and $J_0(a)$ as defined by equations (29) and (31). Thus

$$
\Delta\tilde{a}=(1+t)\left[w_2(\tilde{a})-w_0(\tilde{a})\right]+\delta\gamma_F
$$

7.3 Endogenous training intensity

Let denote $\tilde{\Delta}(a)$ the optimal training intensity and $J_2^{\star}(a)$ the associated value, the wage setting process is characterized by the following rules

$$\begin{aligned} \beta J_0(a) &= (1-\beta)(E_0(a) - \mathcal{U}_0(a)) \\ \beta J_1(a) &= (1-\beta)(E_1(a) - \mathcal{U}_1(a)) \\ \beta\left[J_2^{\star}(a) - \gamma(1-s_0) - c(\tilde{\Delta}(a))(1-s_{\Delta})\right] &= (1-\beta)(E_2(a) - \mathcal{U}_2(a)) \end{aligned}$$

which implies the following wage equations

$$\begin{aligned} w_0(a) &= xa + (1-x)b \\ w_1(a) &= x\left(1+\tilde{\Delta}(a)\right)a \\ &\quad + (1-x)\left[b - \frac{\pi\beta p}{r+\pi+\beta p}\left[\gamma(1-s_0) + c(\tilde{\Delta}(a))(1-s_{\Delta})\right]\right] \\ w_2(a) &= x\left\{\left(1+\tilde{\Delta}(a)\right)a - (r+\delta)\left[\gamma(1-s_0) + c(\tilde{\Delta}(a))(1-s_{\Delta})\right]\right\} \\ &\quad + (1-x)\left[b - \frac{\delta\beta p}{r+\pi+\beta p}\left[\gamma(1-s_0) + c(\tilde{\Delta}(a))(1-s_{\Delta})\right]\right] \end{aligned}$$

By substituting out for the expression of $w_2(a)$ in $J_2(a)$, one gets the optimal training intensity $\tilde{\Delta}(a)$ maximizes

$$(1+\Delta)a - b - [\gamma(1-s_0) + c(\Delta)(1-s_{\Delta})]\left[r+\delta - \frac{\delta\beta p}{r+\pi+\beta p}\right]$$

with respect to Δ. The first order condition is

$$\left[\frac{r(r+\delta+\pi+\beta p)+\delta\pi}{r+\pi+\beta p}\right](1-s_{\Delta})c'(\Delta) = a$$

which defines optimal training $\tilde{\Delta}(a)$. From the convexity of the cost function ($c'' \geq 0$), it comes that *(i)* training intensity increases with ability a, and *(ii)* decreases (increases) with π (βp). The latter reflects the fact that the relative value of having updated knowledge in case of unemployment is lower which translates into higher reservation wages for type-2 workers.