Currency Misalignments and Optimal Monetary Policy: A Reexamination

Charles Engel
University of Wisconsin

November 25, 2008

Preliminary Draft

Abstract

This paper examines optimal monetary policy in an open-economy two-country model with sticky prices. We find that optimal policy must target not only inflation and the output gap, but also the currency misalignment. However the interest rate reaction function that supports this targeting rule involves only the CPI inflation rate. This result highlights how examination of interest-rate reaction functions may hide important trade-offs facing policymakers. The model is a modified version of Clarida, Gali, and Gertler's (JME, 2002). The key change is that we allow local-currency pricing and consider the policy implications of currency misalignments. Besides highlighting the importance of the currency misalignment, our model also gives a rationale for targeting CPI inflation, rather than PPI inflation as in Clarida, Gali, and Gertler.

Email: cengel@ssc.wisc.edu. Address: Department of Economics, 1080 Observatory Drive, Madison, WI, 53706-1393. I am indebted very much to Mick Devereux, Jon Faust, Lars Svensson and Mike Woodford for comments and suggestions that motivated this work.

It is widely understood that purchasing power parity does not hold in the short run. Empirical evidence points to the possibility of "local-currency pricing" (LCP).[1] That is, exporting firms may price discriminate among markets, and set prices in the buyers' currencies. If nominal prices respond sluggishly to market conditions but exchange rates are flexible, the possibility arises of exchange-rate misalignments. A currency could be overvalued if the consumer price level is higher at home than abroad when compared in a common currency, or undervalued if the relative price level is lower at home.

There is frequent public discussion of the importance of controlling currency misalignments. For example, on November 3, 2008, Robert Rubin (former U.S. Secretary of the Treasury) and Jared Bernstein (of the Economic Policy Institute) co-authored an op-ed piece in the New York Times that argued, "Public policy…has been seriously deficient [because of] false choices, grounded in ideology." One of the principles they argue that all should agree upon is "we need to work with other countries toward equilibrium exchange rates." Yet there is little support in the modern New Keynesian literature on monetary policy for the notion that central banks should target exchange rates. Specifically, if policymakers are already optimally responding to inflation and the output gap, is there any reason to pay attention to exchange-rate misalignments?

Our answer is yes. In a simple, familiar framework, this paper draws out the implications for monetary policy when currency misalignments are possible. Currency misalignments lead to inefficient allocations for reasons that are analogous to the problems with inflation in a world of staggered price setting. When there are currency misalignments, households in the Home and Foreign countries may pay different prices for the identical good. A basic tenet of economics is that violations of the law of one price are inefficient – since the good's marginal cost is the same irrespective of where the good is sold, it is not efficient to sell the good at different prices. We find that optimal monetary policy trades off this currency misalignment with inflation and output goals.

The literature has indeed previously considered models with local-currency pricing. Some of these models are much richer than the model considered here. To understand the contribution of this paper, it is helpful to place it relative to four sets of papers:

1. Clarida, Gali, and Gertler (JME, 2002) develop what is probably the canonical model for open-economy monetary policy analysis in the New Keynesian framework. Their paper assumes

[1] Many studies have found evidence of violations of the law of one price for consumer prices. Two prominent studies are Engel (1999) and Atkeson and Burstein (2008). The literature is voluminous – these two papers contain many relevant citations.

that firms set prices in the producer's currency (PCP, for "producer-currency pricing.") Their two-country model also assumes Home and Foreign households have identical preferences. These two assumptions lead to the conclusion that purchasing power parity holds at all times – the consumption real exchange rate is constant.

This paper introduces local-currency pricing into CGG's model. We derive simple rules for monetary policy that are similar to CGG's. While the model is not rich relative to sophisticated models in the literature (models that introduce capital, working capital, capacity utilization, habits in preferences, etc.), the simple model is helpful for developing intuition because the model can be solved analytically, an explicit second-order approximation can be derived, explicit "target criteria" for policy can be derived, and explicit interest rate reaction functions can be derived. As we shall quickly explain, we learn a lot from these relationships in the simple model.

The paper also allows Home and Foreign households to have different preferences. They can exhibit a home bias in preferences – a larger weight on goods produced in a household's country of residence. This generalization does not change the optimal target criteria at all in the CGG framework, but as we now explain, is helpful in developing a realistic LCP model.

2. Devereux and Engel (2003) explicitly examine optimal monetary policy in a two-country framework with LCP. Corsetti and Pesenti (2005) extend the analysis in several directions. However, neither of these studies is suited toward answering the question posed above: is currency misalignment a separate concern of monetary policy, or will the optimal exchange-rate behavior be achieved through a policy that considers inflation and the output gap?

These models have a couple of crucial assumptions that make them unsuited to answering this question. First, like CGG, they assume identical preferences in both countries. This assumption (as we show below) leads to the outcome that currency misalignments are the only source of CPI inflation differences between the two countries in the LCP framework. Eliminating inflation differences eliminates currency misalignments and vice-versa.

Second, price stickiness is the only distortion in the economy in these papers. In contrast, CGG introduce "cost-push shocks", so that policymakers face a tradeoff between the goals of zero inflation and zero output gap. In Devereux and Engel (2003), the optimal monetary policy under LCP sets inflation to zero in each country, thus eliminating any currency misalignment.

By introducing home bias in preferences, the tight link between relative inflation rates and currency misalignments is broken. A more realistic model for inflation results in which relative CPI inflation rates depend not only on currency misalignments, but also on the internal relative price of imported to domestically-produced goods. Moreover, we follow CGG in allowing for cost-push shocks.[2]

This paper also derives optimal policy in a framework that is consonant with the bulk of New Keynesian models of monetary policy analysis. Devereux and Engel (2003) and Corsetti and Pesenti (2005) assume price setting is synchronized, with prices set one period in advance. Here we adopt the standard Calvo price-setting technology, which allows for asynchronized price setting. This change is important, because it emphasizes the point that the cost of inflation under sticky is misaligned relative prices. Also, the previous papers assumed that the money supply was the instrument of monetary policy. This paper follows CGG and most of the modern literature in assuming that the policymakers directly control the nominal interest rate in each country.[3]

3. Monacelli (2005) has considered optimal monetary policy under LCP in a simple small-country model. But a small-country model is not capable of addressing the global misallocation of resources arising from violations of the law of one price. In such a model, import prices are exogenous for the Home country, and the welfare of the rest of the world is ignored. Hence, such a framework is not designed to consider the problems of currency misalignments.

4. Many papers have considered whether it is beneficial to augment the interest rate reaction function of central banks with an exchange-rate variable. They ask the question: if the Taylor rule has the interest rate reacting to inflation and the output gap, is there any gain from adding the exchange rate? An important recent paper in this vein that considers a sophisticated two-country model with LCP is that of Adolfson et. al. (2007). Earlier papers have taken a similar approach in rich models of small open economies (e.g., Smets and Wouters (2002), Kollmann (2002), Leitemo and Soderstrom (2005).) Typically these studies find little or no evidence of welfare gains from adding the exchange rate to the Taylor rule.

The question posed this way is misleading. To understand this point, it is helpful first to return to the optimal policy analysis in CGG. That paper finds (recall, under their assumption of PCP) that optimal monetary policy can be characterized by a pair of "target criteria" or "targeting

---

[2] The contribution of Sutherland (2005) merits attention. His two-country model allows for imperfect pass-through, and for differences in Home and Foreign preferences. His model is static, and he derives a welfare function in which the variance of the exchange rate appears. However, the other terms in the welfare function are prices, so it is not clear how this function relates to standard quadratic approximations that involve output gaps and inflation levels. Moreover, Sutherland does not derive optimal monetary policy in his framework.

[3] While the model of this paper adheres strictly to the set-up of CGG, changing only the assumptions of identical preferences and LCP instead of PCP price setting, the model is very similar to that of Benigno's (2004). Woodford (2007) also considers the LCP version of CGG (though not for optimal monetary policy analysis) and makes the connection to Benigno's paper.

rule": $\tilde{y}_t + \xi\pi_{Ht} = 0$ and $\tilde{y}_t^* + \xi\pi_{Ft}^* = 0$. In these equations, $\tilde{y}_t$ refers to the output gap of the Home country – the percentage difference between the actual output level and its efficient level. π_{Ht} is the producer-price inflation rate in the Home country. Analogously, $\tilde{y}_t^*$ is the Foreign output gap and π_{Ft}^* the Foreign PPI inflation rate.[4] These equations describe the optimal tradeoff of the output gap and inflation for the policymaker. It will be desirable to allow inflation to be positive if the output gap is negative, for example. CGG then derive optimal interest rate rules that will deliver these optimal policy tradeoffs. They find the optimal interest rate reaction functions (assuming discretionary policy) are: $r_t = \overline{rr}_t + b\pi_{Ht}$ and $r_t^* = \overline{rr}_t^* + b\pi_{Ft}^*$. r_t is the Home nominal interest rate, and $\overline{rr}_t$ is the "Wicksellian" or efficient real interest rate. The response of the interest rate to inflation, *b*, is a function of model parameters.[5]

The key point to be made here is that CGG's model shows that optimal policy must trade off the inflation and output goals of the central bank. But the optimal interest rate reaction function does *not* necessarily include the output gap. That is, adding the output gap to the interest rate rule that already includes inflation will not improve welfare.

An analogous situation arises in the LCP model concerning currency misalignments. We can characterize the "target criteria" in this model with two rules, as in the CGG model. The first is $\tilde{y}_t + \tilde{y}_t^* + \xi(\pi_t + \pi_t^*) = 0$. This rule, at first glance, appears to be simply the sum of the two "target criteria" in the CGG model. It is, except that the inflation rates that appear in this tradeoff (π_t and π_t^*) are CPI inflation rates, rather than PPI as in CGG's model. The second target criterion is $\frac{1}{\sigma}\tilde{q}_t + \xi(\pi_t - \pi_t^*) = 0$, where q_t is the real exchange rate (defined as Foreign prices relative to Home prices expressed in a common currency.) $\tilde{q}_t$ is the deviation of the real exchange rate from its efficient level. We define the parameters in this equation below. The important point is that the tradeoff described here relates real exchange rates and relative CPI inflation rates. For example, even if inflation is low in the Home country relative to the Foreign country, optimal policy may under some circumstances still call for a tightening of the monetary policy stance in the Home country if the Home currency is sufficiently undervalued.

Like CGG, we can derive the optimal interest rate rules that support these targeting rules. We find these interest rate reaction functions are $r_t = \overline{rr}_t + b\pi_t$ and $r_t^* = \overline{rr}_t^* + b\pi_t^*$. They are identical to the ones derived in CGG (the parameter *b* is the same), except they target CPI

---

[4] ξ is a preference parameter defined below.

[5] Specifically, we show below that $b = \rho + (1-\rho)\sigma\xi$. We define the parameters below.

inflation rather than PPI inflation as in CGG. The conclusion is that while the target criteria include currency misalignments, the currency misalignment is not in the optimal interest rate reaction function. If we focus on only the latter, we miss this tradeoff the policymaker faces.

Previous studies have found little welfare gain from adding an exchange rate variable to the Taylor rule. Properly speaking, these studies should be interpreted as studying the effects of simple targeting rules under commitment. Our results describe the target criteria and optimal interest rate reaction functions under discretionary policymaking. But our results here suggest, at least, that even if there is no role for the currency misalignment in a simple targeting rule, exchange rate concerns may still be important in terms of welfare. This point is brought out in the context of a relatively simple model that can be solved analytically (with approximations), but is obscured in larger models that are solved numerically.

As in CGG, we consider only optimal policy under discretion. Furthermore, we consider only optimal cooperative policy. There are two reasons why we examine only cooperative policy. First, as we describe below, there are some complex problems that arise under non-cooperative policy in our setting which are sometimes ignored in the literature. Second, and more important, it is difficult to conceive of a central bank today taking into account the exchange rate in its policy decisions in a non-cooperative way.[6] It seems likely that if central banks are going to move toward policies that explicitly target exchange rates, they will do so cooperatively.

1. Model Set-up

The model we examine is nearly identical to CGG's. We consider two countries of equal size, while CGG allow the population of the countries to be different. Since the population size plays no real role in their analysis, we simplify along this dimension. But we make two significant generalizations. First, we allow for different preferences in the two countries. Home agents may put a higher weight in utility on goods produced in the Home country. Home households put a weight of $\frac{\nu}{2}$ on Home goods and $1-\frac{\nu}{2}$ on Foreign goods (and vice-versa for Foreign households.) This is a popular assumption in the open-economy macroeconomics literature, and can be considered as a short-cut way of modeling "openness". That is, a less open country puts less weight on consumption of imported goods, and in the limit the economy becomes closed if it imports no goods. The second major change we allow, as already noted, is we allow LCP instead of PCP.

[6] For example, under the rules of the WTO, countries may not deliberately devalue their currencies.

The reader familiar with CGG may simply skip the layout of the model in this section because our model and notation is otherwise identical.

The model assumes two countries, each inhabited with a continuum of households, normalized to a total of one in each country. Households have utility over consumption of goods and disutility from provision of labor services. In each country, there is a continuum of goods produced, each by a monopolist. Households supply labor to firms located within their own country, and get utility from all goods produced in both countries. Each household is a monopolistic supplier of a unique type of labor to firms within its country. Wages adjust continuously, but households exploit their monopoly power by setting a wage that incorporates a mark-up over their utility cost of work. We assume that there is trade in a complete set of nominally-denominated contingent claims

Monopolistic firms produce output using only labor, subject to technology shocks. We compare two pricing scenarios for firms: PCP vs. LCP. Under PCP, each monopolistic firm sets a price in its own currency for sale to both countries. Under LCP, each firm sets a price in local currency for local sales, and in the importer's currency for sales abroad. In both models, firms face a Calvo price-setting constraint.

Government is assumed to have only limited fiscal instruments. The government can set a constant output subsidy rate for monopolists, which will achieve an efficient allocation in the non-stochastic steady state. But unfortunately, the mark-up charged by workers is time-varying because the elasticity of demand for their labor services is assumed to follow a stochastic process. These shocks are sometimes labeled "cost-push" shocks, and give rise to the well-known tradeoff in CGG's work between controlling inflation and achieving a zero output gap.

1.a. Households

The representative household in the home country maximizes

(1) $$U_t(h) = E_t\left\{\sum_{j=0}^{\infty}\beta^j\left[\frac{1}{1-\sigma}C_{t+j}(h)^{1-\sigma} - \frac{1}{1+\phi}N_{t+j}(h)^{1+\phi}\right]\right\},\ \sigma > 0, \phi \geq 0$$

$C_t(h)$ is the consumption aggregate. We assume Cobb-Douglas preferences:

(2) $$C_t(h) = \left(C_{Ht}(h)\right)^{\frac{\nu}{2}}\left(C_{Ft}(h)\right)^{1-\frac{\nu}{2}},\ 0 \leq \nu \leq 2\,.$$

If $\nu = 1$, Home and Foreign preferences are identical as in CGG. There is home bias in preferences when $\nu > 1$.

In turn, $C_{Ht}(h)$ and $C_{Ft}(h)$ are CES aggregates over a continuum of goods produced in each country:

(3) $$C_{Ht}(h) = \left(\int_0^1 C_{Ht}(h,f)^{\frac{\xi-1}{\xi}} df \right)^{\frac{\xi}{\xi-1}} \quad \text{and} \quad C_{Ft}(h) = \left(\int_0^1 C_{Ft}(h,f)^{\frac{\xi-1}{\xi}} df \right)^{\frac{\xi}{\xi-1}}.$$

$N_t(h)$ is an aggregate of the labor services that the household sells to each of a continuum of firms located in the home country:

(4) $$N_t(i) = \int_0^1 N_t(i,f)df .$$

Households receive wage income, $W_t(h)N_t(h)$, aggregate profits from home firms, Γ_t. They pay lump-sum taxes each period, T_t. Each household can trade in a complete market in contingent claims (arbitrarily) denominated in the home currency. The budget constraint is given by:

(5) $$P_tC_t(h) + \sum_{s^t \in \Omega_t} Z(s^{t+1} \mid s^t)D(h,s^{t+1}) = W_t(h)N_t(h) + \Gamma_t - T_t + D(h,s^t),$$

where $D(h,s^t)$ represents household *h*'s payoffs on state-contingent claims for state s^t. $Z(s^{t+1} \mid s^t)$ is the price of a claim that pays one dollar in state s^{t+1}, conditional on state s^t occurring at time *t*.

In this equation, P_t is the exact price index for consumption, given by:

(6) $$P_t = k^{-1}P_{Ht}^{\nu/2}P_{Ft}^{1-(\nu/2)}, \quad k = (1-(\nu/2))^{1-(\nu/2)}(\nu/2)^{\nu/2}.$$

P_{Ht} is the Home-currency price of the Home aggregate good and P_{Ft} is the Home currency price of the Foreign aggregate good. Equation (6) follows from cost minimization. Also, from cost minimization, P_{Ht} and P_{Ft} are the usual CES aggregates over prices of individual varieties:

(7) $$P_{Ht} = \left(\int_0^1 P_{Ht}(f)^{1-\xi} df \right)^{\frac{1}{1-\xi}}, \text{ and } P_{Ft} = \left(\int_0^1 P_{Ft}(f)^{1-\xi} df \right)^{\frac{1}{1-\xi}}.$$

Households are monopolistic suppliers of their unique form of labor services. Household *h* faces demand for its labor services given by:

(8) $$N_t(h) = \left(\frac{W_t(h)}{W_t} \right)^{-\eta_t} N_t,$$

where

(9) $$W_t = \left(\int_0^1 W_t(h)^{1-\eta_t} dh \right)^{\frac{1}{1-\eta_t}}.$$

Foreign households have analogous preferences and face an analogous budget constraint.

Because all Home households are identical, we can drop the index for the household and use the fact that aggregate per capita consumption of each good is equal to the consumption of each good by each household. The first-order conditions for consumption are given by:

(10) $$P_{Ht}C_{Ht} = \frac{\nu}{2} P_t C_t \, ,$$

(11) $$P_{Ft}C_{Ft} = \left(1 - \frac{\nu}{2}\right) P_t C_t \, ,$$

(12) $$C_{Ht}(f) = \left(\frac{P_{Ht}(f)}{P_{Ht}}\right)^{-\xi} C_{Ht} \quad \text{and} \quad C_{Ft}(f) = \left(\frac{P_{Ft}(f)}{P_{Ft}}\right)^{-\xi} C_{Ft} \, ,$$

(13) $$\beta\left(C(s^{t+1})/C(s^t)\right)^{-\sigma}(P_t/P_{t+1}) = \ddot{Z}(s^{t+1} \mid s^t) \, .$$

In equation (13), we explicitly use an index for the state at time t for the purpose of clarity. $\ddot{Z}(s^{t+1}, s^t)$ is the normalized price of the state contingent claim. That is, it is defined as $Z(s^{t+1} \mid s^t)$ divided by the probability of state s^{t+1} conditional on state s^t.

Note that the sum of $Z(s^{t+1} \mid s^t)$ across all possible states at time t + 1 must equal $1/R_t$, where R_t denotes the gross nominal yield on a one-period non-state-contingent bond. Therefore, taking a probability-weighted sum across all states of equation (13), we have the familiar Euler equation:

(14) $$\beta R_t E_t\left[\left(C(s^{t+1})/C(s^t)\right)^{-\sigma}(P_t/P_{t+1})\right] = 1 \, .$$

Analogous equations hold for Foreign households. Since contingent claims are (arbitrarily) denominated in Home currency, the first-order condition for Foreign households that is analogous to equation (13) is:

(15) $$\beta\left(C^*(s^{t+1})/C^*(s^t)\right)^{-\sigma}(E_t P_t^*/E_{t+1}P_{t+1}^*) = \ddot{Z}(s^{t+1} \mid s^t) \, .$$

The first-order condition for household h's choice of labor supply is given by:

(16) $$\frac{W_t(h)}{P_t} = (1+\mu_t^W)(C_t(h))^{\sigma}(N_t(h))^{\phi} \, , \text{ where } \mu_t^W \equiv \frac{1}{\eta_t - 1} \, .$$

The optimal wage set by the household is a time-varying mark-up over the marginal disutility of work (expressed in consumption units.)

Because all households are identical, we have $W_t = W_t(h)$ and $N_t = N_t(h)$.

1.b. Firms

Each Home good, $Y_t(f)$ is made according to a production function that is linear in the labor input. These are given by:

(17) $Y_t(f) = A_t N_t(f)$.

Note that the productivity shock, A_t, is common to all firms in the Home country. $N_t(f)$ is a CES composite of individual home-country household labor, given by:

(18) $$N_t(f) = \left(\int_0^1 N_t(h, f)^{\frac{\eta_t - 1}{\eta_t}} dh \right)^{\frac{\eta_t}{\eta_t - 1}},$$

where the technology parameter, η_t, is stochastic and common to all Home firms.

Profits are given by:

(19) $\Gamma_t(f) = P_{Ht}(f) C_{Ht}(f) + E_t P_{Ht}^*(f) C_{Ht}^*(f) - (1 - \tau_t) W_t N_t(f)$.

In this equation, $P_{Ht}(f)$ is the home-currency price of the good when it is sold in the Home country. $P_{Ht}^*(f)$ is the foreign-currency price of the good when it is sold in the Foreign country. Following the unfortunate notation of CGG, E_t is the nominal exchange rate, defined as the Home currency price of Foreign currency, and should not be mistaken for the conditional expectations operator. $C_{Ht}(f)$ is aggregate sales of the good in the home country:

(20) $C_{Ht}(f) = \int_0^1 C_{Ht}(h, f) dh$.

$C_{Ht}^*(f)$ is defined analogously. It follows that $Y_t(f) = C_{Ht}(f) + C_{Ht}^*(f)$.

There are analogous equations for $Y_t^*(f)$, with the foreign productivity shock given by A_t^*, the foreign technology parameter shock given by η_t^*, and foreign subsidy given by τ_t^*.

We will consider three different scenarios for firm behavior. In the first, prices can be adjusted freely. In the second, the PCP scenario that CGG analyze, firms set prices in their own country's currency and face a Calvo pricing technology. In the third, when firms are allowed to change prices according to the Calvo pricing rule, they set a price in their own currency for sales in their own country and a price in the other country's currency for exports. This is the LCP scenario.

We adopt the following notation. For any variable X_t:

$\dot{X}_t$ is the value under flexible prices.

$\overline{X}_t$ is the value of variables under globally efficient allocations. In other words, this is the value for variables if prices were flexible, and optimal subsidies to monopolistic suppliers of labor and monopolistic producers of goods were in place. This includes a time-varying subsidy to suppliers of labor to offset the time-varying mark-up in wages in equation (16).

$\tilde{X}_t$ is the gap: the value of the variable under PCP or LCP relative to $\overline{X}_t$.

We will treat the PCP and LCP cases separately, so there will be no need to use notation to distinguish variables under PCP versus LCP.

Flexible Prices

Home firms maximize profits given by equation (19), subject to the demand curve (12). They optimally set prices as a mark-up over marginal cost:

(21) $$\dot{P}_{Ht}(f) = \dot{E}_t \dot{P}^*_{Ht}(f) = (1-\tau_t)(1+\mu^P)\dot{W}_t / A_t, \text{ where } \mu^P \equiv \frac{1}{\xi - 1}.$$

When optimal subsidies are in place:

(22) $$\overline{P}_{Ht}(f) = \overline{E}_t \overline{P}^*_{Ht}(f) = \overline{W}_t / A_t.$$

From (16), (21), and (22), it is apparent that the optimal subsidy satisfies

(23) $$(1-\tau_t)(1+\mu^P)(1+\mu^W_t) = 1.$$

Note from (21) that all flexible price firms are identical and set the same price. Because the demand functions of Foreign residents have the same elasticity of demand for Home goods as Home residents, firms set the same price for sale abroad:

(24) $$E_t \dot{P}^*_{Ht} = \dot{P}_{Ht} \text{ and } E_t \overline{P}^*_{Ht} = \overline{P}_{Ht}$$

PCP

We assume a standard Calvo pricing technology. A given firm may reset its prices with probability $1-\theta$ each period. We assume that when the firm resets its price, it will be able to reset its prices for sales in both markets. The PCP firm sets both prices in its own currency – that is, the Home firm sets both $P_{Ht}(f)$ and $P^{**}_{Ht}(f) \equiv E_t P^*_{Ht}(f)$ in Home currency. (As will become apparent, the firm optimally chooses the same price for both markets, $P_{Ht}(f) = P^{**}_{Ht}(f)$.)

The firm's objective is to maximize its value. Its value is equal to the value of its entire stream of dividends, valued at state-contingent prices. Given equation (13), it is apparent that the firm that selects its prices at time *t*, chooses its reset prices, $P^0_{Ht}(f)$ and $P^{0**}_{Ht}(f)$, to maximize

(25) $$E_t \sum_{j=0}^{\infty} \theta^j Q_{t,t+j} \left[P^0_{Ht}(f) C_{Ht+j}(f) + P^{0**}_{Ht} C^*_{Ht+j}(f) - (1-\tau_t) W_{t+j} N_{t+j}(f) \right],$$

subject to the sequence of demand curves given by equation (12) and the corresponding Foreign demand equation for Home goods. In this equation, we define

(26) $$Q_{t,t+j} \equiv \beta^j \left(C_{t+j}/C_t\right)^{-\sigma} (P_t / P_{t+j}).$$

The solution for the optimal price for the Home firm for sale in the Home country is given by:

(27) $$P_{Ht}^0(z) = \frac{E_t \sum_{j=0}^{\infty} \theta^j Q_{t,t+j}(1-\tau_t) W_{t+j} P_{Ht+j}^{\xi} C_{Ht+j} / A_{t+j}}{E_t \sum_{j=0}^{\infty} \theta^j Q_{t,t+j} P_{Ht+j}^{\xi} C_{Ht+j}}.$$

For sale in the foreign market, we have:

(28) $$P_{Ht}^{0**}(z) = \frac{E_t \sum_{j=0}^{\infty} \theta^j Q_{t,t+j}(1-\tau_t) W_{t+j} (E_{t+j} P_{Ht+j}^{*})^{\xi} C_{Ht+j}^{*} / A_{t+j}}{E_t \sum_{j=0}^{\infty} \theta^j Q_{t,t+j} (E_{t+j} P_{Ht+j}^{*})^{\xi} C_{Ht+j}^{*}}.$$

LCP

The same environment as the PCP case holds, with the sole exception that the firm sets its price for export in the importer's currency rather than its own currency when it is allowed to reset prices. The Home firm, for example, sets $P_{Ht}^{*}(f)$ in Foreign currency. The firm that can reset its price at time *t* chooses its reset prices, $P_{Ht}^{0}(f)$ and $P_{Ht}^{0*}(f)$, to maximize

(29) $$E_t \sum_{j=0}^{\infty} \theta^j Q_{t,t+j} \left[P_{Ht}^0(z) C_{Ht+j}(f) + E_t P_{Ht}^{0*} C_{Ht+j}^{*}(f) - (1-\tau_t) W_{t+j} N_{t+j}(f) \right].$$

The solution for $P_{Ht}^0(z)$ is identical to (27). We find for export prices,

(30) $$P_{Ht}^{0*}(z) = \frac{E_t \sum_{j=0}^{\infty} \theta^j Q_{t,t+j}(1-\tau_t) W_{t+j} (P_{Ht+j}^{*})^{\xi} C_{Ht+j}^{*} / A_{t+j}}{E_t \sum_{j=0}^{\infty} \theta^j Q_{t,t+j} E_{t+j} (P_{Ht+j}^{*})^{\xi} C_{Ht+j}^{*}}.$$

1.4 Equilibrium

Goods market clearing conditions in the Home and Foreign country are given by:

(30) $$Y_t = C_{Ht} + C_{Ht}^{*} = \frac{\nu}{2}\frac{P_t C_t}{P_{Ht}} + \left(1-\frac{\nu}{2}\right)\frac{P_t^{*} C_t^{*}}{P_{Ht}^{*}} = k^{-1}\left(\frac{\nu}{2} S_t^{1-(\nu/2)} C_t + \left(1-\frac{\nu}{2}\right)(S_t^{*})^{-\nu/2} C_t^{*}\right),$$

(31) $$Y_t^{*} = C_{Ft} + C_{Ft}^{*} = \left(1-\frac{\nu}{2}\right)\frac{P_t C_t}{P_{Ft}} + \frac{\nu}{2}\frac{P_t^{*} C_t^{*}}{P_{Ft}^{*}} = k^{-1}\left(\frac{\nu}{2}(S_t^{*})^{1-(\nu/2)} C_t^{*} + \left(1-\frac{\nu}{2}\right) S_t^{-\nu/2} C_t\right).$$

We have used S_t and S_t^{*} to represent the price of imported to locally-produced goods in the Home and Foreign countries, respectively:

(32) $S_t = P_{Ft} / P_{Ht}$,

(33) $S_t^* = P_{Ht}^* / P_{Ft}^*$.

Equations (13) and (15) give us the familiar condition that arises in open-economy models with a complete set of state-contingent claims when PPP does not hold:

(34) $$\left(\frac{C_t}{C_t^*}\right)^{\sigma} = \frac{E_t P_t^*}{P_t} = \frac{E_t P_{Ht}^*}{P_{Ht}} (S_t^*)^{-\nu/2} S_t^{(\nu/2)-1}.$$

Total employment is determined by output in each industry:

(35) $$N_t = \int_0^1 N_t(f)df = A_t^{-1}\int_0^1 Y_t(f)df = A_t^{-1}\left(C_{Ht}V_{Ht} + C_{Ht}^* V_{Ht}^*\right).$$

where

(36) $$V_{Ht} \equiv \int_0^1 \left(\frac{P_{Ht}(f)}{P_{Ht}}\right)^{-\xi} df \text{ , and } \qquad V_{Ht}^* \equiv \int_0^1 \left(\frac{P_{Ht}^*(f)}{P_{Ht}^*}\right)^{-\xi} df \text{ .}$$

Since all households are identical, we have from equation (16):

(37) $W_t / P_{Ht} = (1+\mu_t^W) C_t^{\sigma} N_t^{\phi} S_t^{1-(\nu/2)}$.

As in CGG, we will assume that subsidies to monopolists are not set at their optimal level except in steady-state. That is, instead of the efficient subsidy given in equation (23), we have:

(38) $(1-\tau)(1+\mu^P)(1+\mu^W) = 1$.

Here, μ^W is the steady-state level of μ_t^W. We have dropped the time subscript on the subsidy rate τ_t because it is not time-varying.

Flexible Prices

As we have already noted, all flexible price firms set the same price. From (23), using (38), we have:

(39) $\dot{P}_{Ht} = \dot{E}_t \dot{P}_{Ht}^* = \dot{W}_t / ((1+\mu^W)A_t)$ and $\dot{P}_{Ft}^* = \dot{E}_t^{-1} \dot{P}_{Ft} = \dot{W}_t^* / ((1+\mu^{*W})A_t^*)$.

We can conclude:

(40) $\dot{S}_t^{*-1} = \dot{S}_t$.

Because $\dot{P}_{Ht}(f)$ is identical for all firms, (35) collapses to

(41) $\dot{Y}_t = A_t \dot{N}_t$.

Equations (30), (31), (34), (37) and its foreign counterpart, (39), and (41) and its foreign counterpart determine $\dot{C}_t$, $\dot{C}_t^*$, $\dot{Y}_t$, $\dot{Y}_t^*$, $\dot{N}_t$, $\dot{N}_t^*$ $\dot{S}_t$ and the real product wages, $\dot{W}_t / \dot{P}_{Ht}$ and $\dot{W}_t^* / \dot{P}_{Ft}^*$ as functions of productivity shocks, A_t and A_t^*, and mark-up shocks, μ_t^W and μ_t^{*W}. In section 2 below, we present log-linearized versions of these nine equations, and solve them.

PCP

Under the Calvo price setting mechanism, a fraction θ of prices remain unchanged from the previous period. From equation (7), we can write:

(42) $$P_{Ht} = \left[\theta (P_{Ht-1})^{1-\xi} + (1-\theta)(P_{Ht}^{0})^{1-\xi} \right]^{1/(1-\xi)},$$

(43) $$P_{Ht}^{*} = \left[\theta (P_{Ht-1}^{*})^{1-\xi} + (1-\theta)(P_{Ht}^{0*})^{1-\xi} \right]^{1/(1-\xi)}$$

Taking equations (27), (28), (42), and (43), we see that the law of one price holds under PCP. That is, $P_{Ht}(f) = E_t P_{Ht}^{*}(f)$ for all *f*, hence $P_{Ht} = E_t P_{Ht}^{*}$. Hence, under PCP we have $S_t^{*-1} = S_t$.

LCP

Equations (42) and (43) hold in the LCP case as well. However, the law of one price does not hold.

2. Log-linearized Model

In this section, we present log-linear approximations to the models presented above. Our approach to the optimal policy is to consider a second-order approximation of the welfare function around the efficient steady state (the steady state in which the optimal subsidy given by equation (38) is imposed.) The derivation of the welfare function itself requires a second-order approximation of the welfare function itself, but in the course of the derivation will actually require second-order approximations to some of the equations of the model. However, for many purposes, the first-order approximations are useful: the constraints in the optimization problem need only be approximated to the first order; the optimality conditions for monetary policy – the "target criteria" – are linear; and, we will analyze the dynamics under the optimal policy in the linearized model.

In our notation, lower case letters refer to the log of the corresponding upper case letter less its deviation from steady state.

Equations (30)-(38) are common to all three models – flexible price, PCP, and LCP. We will present the log-linearized versions of those first. We will make use of the fact that in all three models, to a first-order approximation, the relative price of Foreign to Home goods is the same in both countries, so $s_t^{*} = -s_t$. That relationship is obvious in the flexible price and PCP models, but will require some explanation in the LCP case. We will postpone that explanation until later.

We define the log of the deviation from the law of one price as:

(44) $\Delta_t \equiv e_t + p^*_{Ht} - p_{Ht}$.

In the flexible-price and PCP models, $\Delta_t = 0$. In the LCP model, because $s^*_t = -s_t$, we have that the law of one price deviation is the same for both goods: $\Delta_t \equiv e_t + p^*_{Ft} - p_{Ft}$

In all three models, to a first order, $\ln(V_{Ht}) = \ln(V^*_{Ht}) = \ln(V_{Ft}) = \ln(V^*_{Ft}) = 0$. That allows us to approximate equation (35) and its foreign counterpart as:

(45a) $n_t = y_t - a_t$, and

(45b) $n^*_t = y^*_t - a^*_t$.

The market-clearing conditions, (30) and (31) are approximated as:

(46a) $$y_t = \frac{\nu(2-\nu)}{2} s_t + \frac{\nu}{2} c_t + \frac{2-\nu}{2} c^*_t,$$

(46b) $$y^*_t = \frac{-\nu(2-\nu)}{2} s_t + \frac{\nu}{2} c^*_t + \frac{2-\nu}{2} c_t.$$

The condition arising from complete markets that equates the marginal utility of nominal wealth for Home and Foreign households, equation (34), is given by:

(47) $$\sigma c_t - \sigma c^*_t = \Delta_t + (\nu - 1) s_t.$$

The real Home and Foreign product wages, from equation (37), are given by:

(48a) $$w_t - p_{Ht} = \sigma c_t + \phi n_t + \frac{2-\nu}{2} s_t + \mu^W_t,$$

(48b) $$w^*_t - p^*_{Ft} = \sigma c^*_t + \phi n^*_t - \left(\frac{2-\nu}{2}\right) s_t + \mu^{*W}_t.$$

For use later, it is helpful to use equations (46)-(48) to express c_t, c^*_t, s_t, $w_t - p_{Ht}$, and $w^*_t - p^*_{Ft}$ in terms of y_t and y^*_t and the exogenous disturbances, a_t, a^*_t, μ^W_t, and μ^{*W}_t:

(49a) $$c_t = \frac{D+\nu-1}{2D} y_t + \frac{D-(\nu-1)}{2D} y^*_t + \frac{\nu(2-\nu)}{2D} \Delta_t$$

(49b) $$c^*_t = \frac{D+\nu-1}{2D} y^*_t + \frac{D-(\nu-1)}{2D} y_t - \frac{\nu(2-\nu)}{2D} \Delta_t$$

(50) $$s_t = \frac{\sigma}{D}(y_t - y^*_t) - \frac{(\nu-1)}{D} \Delta_t,$$

(51a) $$w_t - p_{Ht} = \left(\frac{\sigma(1+D)}{2D} + \phi\right) y_t + \frac{\sigma(D-1)}{2D} y^*_t + \frac{D-(\nu-1)}{2D} \Delta_t - \phi a_t + \mu^W_t,$$

(51b) $$w^*_t - p^*_{Ft} = \left(\frac{\sigma(1+D)}{2D} + \phi\right) y^*_t + \frac{\sigma(D-1)}{2D} y_t + \frac{\nu-1-D}{2D} \Delta_t - \phi a^*_t + \mu^{*W}_t,$$

where $D \equiv \sigma\nu(2-\nu) + (\nu-1)^2$.

Flexible Prices

We can solve for the values of all the real variables under flexible prices by using equations (45a), (45b), (49a), (49b), (50), (51a), and (51b), as well as the price-setting conditions, from (39):

(52a) $\dot{w}_t - \dot{p}_{Ht} = a_t$,

(52b) $\dot{w}_t^* - \dot{p}_{Ft}^* = a_t^*$.

We will not solve out the flexible-price values, since we do not need that information. Note that the values of the variables under the globally efficient allocation are equal to the flexible-price values, but with a coefficient of zero on the mark-up shocks. Thus we have, for example, from (51)-(52):

(53a) $$a_t = \overline{w}_t - \overline{p}_{Ht} = \left(\frac{\sigma(1+D)}{2D} + \phi\right)\overline{y}_t + \frac{\sigma(D-1)}{2D}\overline{y}_t^* - \phi a_t ,$$

(53b) $$a_t^* = \overline{w}_t^* - \overline{p}_{Ft}^* = \left(\frac{\sigma(1+D)}{2D} + \phi\right)\overline{y}_t^* + \frac{\sigma(D-1)}{2D}\overline{y}_t - \phi a_t^* .$$

PCP

Log-linearization of equations (27) and (42) gives us the familiar New Keynesian Phillips curve for an open economy:

(54) $$\pi_{Ht} = \delta(w_t - p_{Ht} - a_t) + \beta E_t \pi_{Ht+1} ,$$

where $\delta = (1-\theta)(1-\beta\theta)/\theta$.

We can rewrite this equation as:

$$\pi_{Ht} = \delta(\tilde{w}_t - \tilde{p}_{Ht}) + \beta E_t \pi_{Ht+1} ,$$

or, using (51a) and (53a):

(55) $$\pi_{Ht} = \delta\left[\left(\frac{\sigma(1+D)}{2D} + \phi\right)\tilde{y}_t + \frac{\sigma(D-1)}{2D}\tilde{y}_t^*\right] + \beta E_t \pi_{Ht+1} + u_t ,$$

where $u_t = \delta\mu_t^W$.

Similarly for foreign producer-price inflation, we have:

(56) $$\pi_{Ft}^* = \delta\left[\left(\frac{\sigma(1+D)}{2D} + \phi\right)\tilde{y}_t^* + \frac{\sigma(D-1)}{2D}\tilde{y}_t\right] + \beta E_t \pi_{Ft+1}^* + u_t^* .$$

LCP

Equations (54) holds in the LCP model as well. But in the LCP model, the law of one price deviation is not zero. We have:

(57) $$\pi_{Ht} = \delta\left[\left(\frac{\sigma(1+D)}{2D}+\phi\right)\tilde{y}_t + \frac{\sigma(D-1)}{2D}\tilde{y}_t^* + \frac{D-(\nu-1)}{2D}\Delta_t\right] + \beta E_t\pi_{Ht+1} + u_t\,,$$

(58) $$\pi_{Ft}^* = \delta\left[\left(\frac{\sigma(1+D)}{2D}+\phi\right)\tilde{y}_t^* + \frac{\sigma(D-1)}{2D}\tilde{y}_t + \frac{\nu-1-D}{2D}\Delta_t\right] + \beta E_t\pi_{Ft+1}^* + u_t^*\,.$$

In addition, from (30) and (43), we derive:

(59) $$\pi_{Ht}^* = \delta(w_t - p_{Ht}^* - e_t - a_t) + \beta E_t\pi_{Ht+1}^* = \delta(w_t - p_{Ht} - \Delta_t - a_t) + \beta E_t\pi_{Ht+1}^*$$

We can rewrite this as

(60) $$\pi_{Ht}^* = \delta\left[\left(\frac{\sigma(1+D)}{2D}+\phi\right)\tilde{y}_t + \frac{\sigma(D-1)}{2D}\tilde{y}_t^* - \left(\frac{D+\nu-1}{2D}\right)\Delta_t\right] + \beta E_t\pi_{Ht+1}^* + u_t\,.$$

Similarly, we can derive:

(61) $$\pi_{Ft} = \delta\left[\left(\frac{\sigma(1+D)}{2D}+\phi\right)\tilde{y}_t^* + \frac{\sigma(D-1)}{2D}\tilde{y}_t + \frac{D+\nu-1}{2D}\Delta_t\right] + \beta E_t\pi_{Ft+1} + u_t^*\,.$$

From (57)-(58) and (60)-(61), we see $\pi_{Ft} - \pi_{Ht} = \pi_{Ft}^* - \pi_{Ht}^*$. Assuming a symmetric initial condition, we conclude $s_t^* = -s_t$ as we noted above. That is, the relative price of Foreign to Home goods is the same in both countries. We emphasize that this is true in general for a first-order approximation.

3. Loss Functions and Optimal Policy

We derive the loss function for the cooperative monetary policy problem. We examine the cooperative problem because there are difficult problems with examining the non-cooperative policy problem under complete markets. We comment on those below. In addition, the symmetry in this problem allows us to derive a loss function for the cooperative problem that is relatively simple, but the loss function for the non-cooperative problem is rather intractable. We follow CGG in considering policy under discretion.

The loss function is derived from a second-order approximation to households' utility functions. Loss is measured relative to the efficient allocations. The efficient allocations cannot be obtained with monetary policy alone because of the sticky-price externality, and because we assume the policymaker does not have access to fiscal instruments aside from setting a constant subsidy rate to firms.

The policymaker wishes to minimize

(62) $$E_t\sum_{j=0}^{\infty}\beta^j \mathrm{X}_{t+j}\,.$$

The policymaker has home and foreign nominal interest rates as instruments. As is standard in the literature, we can model the policymaker as directly choosing output gaps, inflation levels, and (in the LCP case) deviations from the law of one price, subject to constraints. From the first-order conditions, we can back out the optimal choice of nominal interest rates using a log-linearized version of equation (14) and its foreign counterpart, given by:

(63) $$r_t - E_t \pi_{t+1} = \sigma(E_t c_{t+1} - c_t),$$

(64) $$r_t^* - E_t \pi_{t+1}^* = \sigma(E_t c_{t+1}^* - c_t^*).$$

In these equations, π_t and π_t^* refer to Home and Foreign consumer price inflation, respectively:

(65) $$\pi_t = \frac{\nu}{2}\pi_{Ht} + \frac{2-\nu}{2}\pi_{Ft},$$

(66) $$\pi_t^* = \frac{\nu}{2}\pi_{Ft}^* + \frac{2-\nu}{2}\pi_{Ht}^*.$$

There are three technical problems that arise if we try to examine non-cooperative policy. One problem is discussed in detail in Devereux and Engel (2003). To consider policies in the non-cooperative framework, we want to look at the effects of one country changing its policies, holding the other countries policies constant. In a complete markets world, to evaluate all possible alternative policies, we need to calculate prices of state-contingent claims under alternative policies. In particular, we cannot assume equation (34), which was derived assuming equal initial Home and Foreign wealth, holds under all alternative policies that the competitive policymaker considers. Policies can change state-contingent prices and therefore change the wealth distribution. It is not uncommon for studies of optimal policy in open economies to treat equation (34) as if it is independent of the policy choices, but it is not. There is a special case in which it holds in all states, which is the set-up in CGG. When the law of one price holds, when Home and Foreign households have identical preferences and there are no preference shocks, and when preferences over Home and Foreign aggregates are Cobb-Douglas, equation (34) holds in all states. This well-known outcome arises because in all states of the world, the terms of trade change in such a way as to leave Home/Foreign wealth unchanged.

Even if we could surmount this technical challenge,[7] there are a couple of other technical challenges that appear in our framework that did not plague CGG. First, in the LCP model, we do not have $S_t^{*-1} = S_t$, that the relative price of Foreign to Home goods is identically the same in both countries. This relationship holds up to a first-order log-linear approximation in the LCP

[7] The Appendix of Devereux and Engel (2003) demonstrates how this problem can be handled.

model (as long as we assume equal speeds of price adjustment for all goods,[8] but only up to a first-order. Messy first-order relative price terms (s_t and s_t^*) appear in the objective function of the non-cooperative policymaker. However, those wash out in the objective function under cooperation. Second, CGG neatly dichotomize the choice variables in their model – the Home policymaker sets Home PPI inflation and the Home output gap taking the Foreign policy choices as given, and vice-versa for the Foreign policymaker. Such a neat dichotomy is not possible in the model with currency misalignments – we cannot just assign the exchange rate to one of the policymakers.

In a sense, all of these technical problems are related to the real world reason why it is more reasonable to examine policy in the cooperative framework when currency misalignments are possible. The non-cooperative model assumes that central banks are willing and able to manipulate currencies to achieve better outcomes. However, in practice both WTO rules and implicit rules of neighborliness prohibit this type of policy. Central banks have been unwilling to announce explicit targets for exchange rates without full cooperation of their partners.

Even if the cooperative policy analysis is not a realistic description of actual policy decision-making, we can consider this approach as giving us a measure of what could be achieved under cooperation.

<u>PCP</u>

The Appendix shows the steps for deriving the loss function for the PCP model with home bias in preferences. It is worth pointing out one aspect of the derivation. In closed economy models with no investment or government, consumption equals output. That is an exact relationship, and therefore we have the deviation of consumption from the efficient level equals the deviation of output from the efficient level to any order of approximation: $\tilde{c}_t = \tilde{y}_t$. In the open economy, the relationship is not as clear-cut. When preferences of Home and Foreign agents are identical, and markets are complete, then the consumption aggregates in Home and Foreign are always equal (up to a constant of proportionality equal to relative wealth.) But that is not true when preferences are not the same. Equation (34) shows that we do not have $C_t = C_t^*$ under complete markets, even if the law of one price holds for both goods. Because of this, we do not have that $\tilde{c}_t + \tilde{c}_t^*$ equals $\tilde{y}_t + \tilde{y}_t^*$, except to a first-order approximation. Since we are using a second-order approximation of the utility function, we need to account for the effect of different preferences (or the effects of the terms of trade) in translating consumption gaps into output gaps.

We find:

[8] See Benigno (2004) and Woodford (2007) on this point.

(67) $$\mathrm{X}_t \propto \frac{\nu(2-\nu)\sigma(\sigma-1)}{4D}(\tilde{y}_t - \tilde{y}_t^*)^2 - \left(\frac{\sigma+\phi}{2}\right)((\tilde{y}_t)^2 + (\tilde{y}_t^*)^2) - \frac{\xi}{2\delta}\left((\pi_{Ht})^2 + (\pi_{Ft}^*)^2\right).$$

This loss function extends the one derived in CGG to the case of home bias in preferences, or nontraded goods (i.e., $\nu \geq 1$ rather than $\nu = 1$.)

The policymaker chooses values for $\tilde{y}_t$, $\tilde{y}_t^*, \pi_{Ht}$ and π_{Ft}^* to maximize (62), with X_t given by (67), subject to the Phillips curves (55) and (56). Under discretion, the policymaker takes past values of $\tilde{y}_t$, $\tilde{y}_t^*, \pi_{Ht}$ and π_{Ft}^* as given, and also does not make plans for future values of these variables understanding that future incarnations of the policymaker can alter any given plan. The policymaker at time *t* cannot influence $E_t\pi_{Ht+1}$ and $E_t\pi_{Ft+1}^*$ because future inflation levels are chosen by future policymakers and there are no endogenous state variables that can limit the paths of future inflation levels. Hence, the policymaker's problem is essentially a static one – to maximize (67) subject to (55) and (56), taking $E_t\pi_{Ht+1}$ and $E_t\pi_{Ft+1}^*$ as given.

Even though we have introduced home bias in consumption, the optimal policy rules are the same as in CGG. The first-order conditions are given by:

(68) $$\tilde{y}_t + \tilde{y}_t^* + \xi(\pi_{Ht} + \pi_{Ft}^*) = 0$$

(69) $$\tilde{y}_t - \tilde{y}_t^* + \xi(\pi_{Ht} - \pi_{Ft}^*) = 0.$$

These two "target criteria" can be rewritten as:

(70) $$\tilde{y}_t + \xi\pi_{Ht} = 0, \quad \text{and} \quad \tilde{y}_t^* + \xi\pi_{Ft}^* = 0.$$

The criteria given in (70) are identical to those that arise in the closed-economy version of this model. There is a tradeoff between the goals of eliminating the output gap and driving inflation to zero, and the elasticity of substitution among goods produced in the country determines the weights given to output gaps and inflation.

It is worth emphasizing that equation (70) indicates the optimal policy entails a tradeoff between the output gap and the *producer price* inflation level. In a closed economy with no intermediate goods, there is no distinction between producer and consumer prices. But in an open economy there is an important distinction. The policies described in (70) imply that policymakers should not give any weight to inflation of imported goods. In conjunction with the Phillips curves, (55) and (56), equation (70) allows us to solve for the Home and Foreign output gaps and π_{Ht} and π_{Ft}^* as functions of current and expected future cost-push shocks, u_t and u_t^*. With the output gap determined by optimal policy, the terms of trade must adjust to insure goods market clearing. But the terms of trade adjust freely in the PCP world, because nominal exchange rate changes translate directly into import price changes. In essence, the import sector is like a flexible-price sector, so policymakers can ignore inflation in that sector, as in Aoki (2001).

LCP

The Appendix shows the derivation of the loss function in the LCP case. Two aspects of the derivation merit attention. First, in examining the first-order dynamics of the model, we can make use of the first-order approximation $s_t^* = -s_t$. That is an exact equation when the law of one price holds, but in the LCP model this relationship does not in general hold to a second-order approximation. The derivation of the loss function must take this into account. The second point to note is that, as is standard in this class of models, price dispersion leads to inefficient use of labor. But, to a second-order approximation, this loss depends only on the cross-section variances of p_{Ht}, p_{Ht}^*, p_{Ft}, and p_{Ft}^*, and not their comovements (which would play a role in a third-order approximation.)

The loss function is given by:

$$\begin{aligned} \mathrm{X}_t \propto & \frac{\nu(2-\nu)\sigma(\sigma-1)}{4D}(\tilde{y}_t - \tilde{y}_t^*)^2 - \left(\frac{\sigma+\phi}{2}\right)((\tilde{y}_t)^2 + (\tilde{y}_t^*)^2) - \left(\frac{\nu(2-\nu)}{4D}\right)\Delta_t^2 \\ & - \frac{\xi}{2\delta}\left(\frac{\nu}{2}(\pi_{Ht})^2 + \frac{2-\nu}{2}(\pi_{Ft})^2 + \frac{\nu}{2}(\pi_{Ft}^*)^2 + \frac{2-\nu}{2}(\pi_{Ht}^*)^2\right) \end{aligned} \quad (71)$$

The loss function is similar to the one under PCP. The main point to highlight is that squared deviations from the law of one price matter for welfare, as well as output gaps and inflation rates. Deviations from the law of one price are distortionary and are a separate source of loss in the LCP model.

Why does the currency misalignment appear in the loss function? That is, if both Home and Foreign output gaps are zero, and all inflation rates are zero, what problem does a misaligned currency cause? From equation (50), if the currency is misaligned, then internal relative prices (s_t) must also differ from their efficient level if the output gap is zero. The Home and Foreign countries could achieve full employment, but the distribution of the output between Home and Foreign households is inefficient. For example, suppose $\Delta_t > 0$, which from (50) implies we must have $\tilde{s}_t < 0$ if both output gaps are eliminated. On the one hand, $\Delta_t > 0$ tends to lead to overall consumption in Home to be high relative to Foreign consumption (see equations (48)-(49).) That occurs because financial markets pay off to Home residents when their currency is weak. But Home residents have a home bias for Home goods. That would lead to overproduction in the Home country, were it not for relative price adjustments – which is why $\tilde{s}_t < 0$.

The policymaker under discretion seeks to maximize (62) with X_t given by (71) subject to the constraints of the Phillips curves, (57)-(58) and (60)-(61). There is an additional constraint in the LCP model. Note that $\pi_{Ft} - \pi_{Ht} = s_t - s_{t-1}$. But from equation (50), we have

(72) $$s_t - s_{t-1} = \frac{\sigma}{D}\left(y_t - y_t^* - (y_{t-1} - y_{t-1}^*)\right) - \frac{(\nu-1)}{D}(\Delta_t - \Delta_{t-1}).$$

Using (72) in conjunction with (57) and (61), we derive:

(73) $$\begin{aligned}&\frac{\sigma}{D}\left(y_t - y_t^* - (y_{t-1} - y_{t-1}^*)\right) - \frac{(\nu-1)}{D}(\Delta_t - \Delta_{t-1}) = \\ &\delta\left[\left(\frac{\sigma}{D} + \phi\right)(\tilde{y}_t - \tilde{y}_t^*) - \frac{(\nu-1)}{2D}\Delta_t\right] + \beta E_t(\pi_{Ht+1} - \pi_{Ft+1}) + u_t - u_t^*\end{aligned}.$$

This constraint arises in the LCP model but not in the PCP model precisely because import prices are sticky and subject to a Calvo price-adjustment mechanism, rather than free to respond via nominal exchange-rate changes.

Another contrast with the PCP model is that there are four sticky prices in the LCP model, so non-zero inflation rates for each of the four matter for welfare. Indeed, we note that we can rewrite the loss function as:

(74) $$\begin{aligned}\mathrm{X}_t \propto\ &\frac{\nu(2-\nu)\sigma(\sigma-1)}{4D}(\tilde{y}_t - \tilde{y}_t^*)^2 - \left(\frac{\sigma+\phi}{2}\right)((\tilde{y}_t)^2 + (\tilde{y}_t^*)^2) - \left(\frac{\nu(2-\nu)}{4D}\right)\Delta_t^2 \\ &- \frac{\xi}{2\delta}\left((\pi_t)^2 + (\pi_t^*)^2 + \frac{\nu(2-\nu)}{2}(s_t - s_{t-1})^2\right)\end{aligned}.$$

Under this formulation, the loss function is seen to depend on the aggregate CPI inflation rates, π_t and π_t^*, and the change in the terms of trade, $s_t - s_{t-1}$, rather than the four individual inflation rates given in equation (71). This formulation is particularly useful when we consider a simplification below, under which $s_t - s_{t-1}$ is independent of policy.

The LCP optimization problem under discretion becomes very messy and difficult because of the additional constraint given by equation (73). That is because there now are endogenous state variables – the choices of Home output gap relative to the Foreign output gap and the deviation from the law of one price puts constraints on the evolution of future output gaps, inflation rates and deviations from the law of one price. In the LCP case, the dynamic game between current and future policymakers is non-trivial.

But inspection of equation (73) reveals a special case in which the policy decision under uncertainty can be settled under the same simple conditions as in the PCP model. When $\phi = 0$, so utility is linear in labor, equation (73) simplifies considerably. Indeed, we can rewrite it as:

(75) $$s_t - s_{t-1} = -\delta \tilde{s}_t + \beta E_t(s_{t+1} - s_t) + u_t - u_t^*.$$

With $\phi = 0$, we have $\tilde{s}_t = s_t - \bar{s}_t = s_t - (a_t - a_t^*)$. So we can write (75) as a second-order expectational difference equation:

(76) $$s_t = \frac{1}{1+\delta+\beta} s_{t-1} + \frac{\beta}{1+\delta+\beta} E_t s_{t+1} + \frac{1}{1+\delta+\beta} \vartheta_t,$$

where $\vartheta_t = \delta(a_t - a_{t*}) + u_t - u_t^* = \delta \dot{s}_t$.

The point here is that equation (76) determines the evolution of s_t independent of policy choices. So while s_t is a state variable, it is not endogenous for the policymaker. One nice thing about considering this special case is that the parameter ϕ does not appear in either the target criteria or the optimal interest rate rule in the CGG model, so we can compare the criteria and rules directly to the LCP model. We also note that Devereux and Engel (2003) make the same assumption on preferences.

We can derive the first-order conditions for the policymaker, which do not depend on any assumption about the stochastic process for s_t. We replace equation (73) with equation (50), expressed in "gap" form, as a constraint on the choice of optimal values by the policymaker.

In fact, in this case the policy problem can be simplified more by using version (74) of the loss function. A useful way to rewrite (74) when $\phi = 0$ is:

(77) $$\mathrm{X}_t \propto \frac{-\sigma}{4D}(\tilde{y}_t^R)^2 - \frac{\sigma}{4}(\tilde{y}_t^W)^2 - \left(\frac{\nu(2-\nu)}{4D}\right)\Delta_t^2 - \frac{\xi}{4\delta}\left((\pi_t^R)^2 + (\pi_t^W)^2 + \nu(2-\nu)(s_t - s_{t-1})^2\right)$$

where we are using the R superscript to represent Home relative to Foreign. That is, $\pi_t^R = \pi_t - \pi_t^*$, $u_t^R = u_t - u_t^*$, etc. Likewise, the W superscript refers to the sum of Home and Foreign variables: $\pi_t^W = \pi_t + \pi_t^*$, $u_t^W = u_t + u_t^*$, etc. Since $s_t - s_{t-1}$ is independent of policy, we can express the policymaker's problem as choosing relative and world output gaps, $\tilde{y}_t^R$ and $\tilde{y}_t^W$, relative and world CPI inflation rates, π_t^R and π_t^W, and the currency misalignment, Δ_t to maximize (77) subject to the "gap" version of equation (50) and the linear combination of the Phillips curves that give us equations for CPI inflation in each country (which are derived here under the assumption that $\phi = 0$):

(78) $\tilde{s}_t = \frac{\sigma}{D}(\tilde{y}_t - \tilde{y}_t^*) - \frac{(\nu-1)}{D}\Delta_t$

(79) $\pi_t^R = \delta\left[\frac{\sigma(\nu-1)}{D}\tilde{y}_t^R + \frac{\sigma\nu(2-\nu)}{D}\Delta_t\right] + \beta E_t \pi_{t+1}^R + (\nu-1)u_t^R$

(80) $\pi_t^W = \delta\sigma\tilde{y}_t^W + \beta E_t \pi_{t+1}^W + u_t^W$

The first condition seems quite similar to the first condition in the PCP case, (70):

(81) $\tilde{y}_t^W + \xi\pi_t^W = 0$.

The condition calls for a tradeoff between the world output gap and the world inflation rate, just as in the PCP case. But there is a key difference – here in the LCP model, it is the CPI, not the PPI, inflation rates that enter into the policymaker's tradeoff.

The second condition can be written as:

(82) $\frac{1}{\sigma}\tilde{q}_t + \xi\pi_t^R = 0$.

Here, q_t is the consumption real exchange rate, defined as:

(83) $q_t = e_t + p_t^* - p_t$.

$\tilde{q}_t$ is the deviation of the real exchange rate from its efficient level, and we have used the relationship:

(84) $\tilde{q}_t = \frac{\nu-1}{D}\tilde{y}_t^R + \frac{\nu(2-\nu)}{D}\Delta_t$.

Equation (82) represents the second of the target criteria as a tradeoff between misaligned real exchange rates and relative CPI inflation rates. In the LCP model, where exchange-rate misalignments are possible, we can see from (84) this optimal policy involves trading off relative output gaps, relative CPI inflation rates, and the currency misalignment.

Optimal Policy under PCP vs. LCP

It is helpful to compare the target criteria under PCP and LCP. We will compare conditions (68) and (69) under PCP to conditions (81) and (82) under LCP.

First, compare (68) to (81). Both involve the tradeoff between the world output gap and world inflation. But under PCP, producer price inflation appears in the tradeoff. However, world producer price inflation is equal to world consumer price inflation under PCP. To see this,

(85) $\pi_t^W = \frac{\nu}{2}\pi_{Ht} + \frac{2-\nu}{2}\pi_{Ft} + \frac{\nu}{2}\pi_{Ft}^* + \frac{2-\nu}{2}\pi_{Ht}^* = \pi_{Ht} + \pi_{Ft}^*$.

The second equality holds because the relative prices are equal in Home and Foreign under PCP (and, for that matter, to a first-order approximation under LCP) so $\pi^*_{Ht} = \pi^*_{Ft} + \pi_{Ht} - \pi_{Ft}$.

This tradeoff is the exact analogy to the closed economy tradeoff between the output gap and inflation, and the intuition of that tradeoff is well understood. On the one hand, with asynchronized price setting, inflation leads to misalignment of relative prices, so any non-zero level of inflation is distortionary. On the other hand, because the monopoly power of labor is time-varying due to the time-varying elasticity of labor demand, output levels can be inefficiently low or high even when inflation is zero. Conditions (68) or (81) describe the terms of that tradeoff. Inflation is more costly when the higher is the elasticity of substitution among varieties of goods, ξ, because a higher elasticity will imply greater resource misallocation when there is inflation.

The difference in optimal policy under PCP versus LCP comes in the comparison of condition (69) with (82). Under PCP, optimal policy trades off Home relative to Foreign output gaps with Home relative to Foreign PPI inflation. Under LCP, the tradeoff is between the real exchange rate and Home relative to Foreign PPI inflation.

First, it is helpful to consider equation (82) when the two economies are closed, so that $\nu = 2$. Using (84), under this condition, $D = 1$, and (82) reduces to $\tilde{y}^R_t + \xi\pi^R_t = 0$. Of course, when $\nu = 2$, there is no difference between PPI and CPI inflation, and so in this special case the optimal policies under LCP and PCP are identical. That is nothing more than reassuring, since the distinction between PCP and LCP should not matter when the economies are closed.

When $\nu \neq 2$, understanding these conditions is more subtle. It helps to consider the case of no home bias in preferences, so $\nu = 1$. Imagine that inflation rates were zero, so that there is no misallocation of labor within each country. Further, imagine that the world output gap is zero. There are still two possible distortions. First, relative Home to Foreign output may not be at the efficient level. Second, even if output levels are efficient, the allocation of output to Home and Foreign households may be inefficient if there are currency misalignments.

When $\nu = 1$, it follows from equations (46a) and (46b) that relative output levels are determined only by the terms of trade. We have $\tilde{y}^R_t = \tilde{s}_t$. On the other hand, from equation (47) when $\nu = 1$, relative consumption is misaligned when there are currency misalignments, $\tilde{c}^R_t = \frac{1}{\sigma}\Delta_t$. Moreover, when $\nu = 1$, the deviation of the real exchange rate from its inefficient level is entirely due to the currency misalignment: $\tilde{q}_t = \Delta_t$.

Under PCP, the law of one price holds continuously, so there is no currency misalignment. In that case, $\Delta_t = 0$, and relative Home to Foreign consumption is efficient. In that case, policy can influence the terms of trade in order to achieve the optimal tradeoff between relative output gaps and relative inflation, as expressed in equation (69). Policy can control the terms of trade under PCP because the terms of trade can adjust instantaneously and completely through nominal exchange-rate adjustment. That is, $s_t = p_{Ft} - p_{Ht} = e_t + p^*_{Ft} - p_{Ht}$. While p^*_{Ft} and p_{Ht} do not adjust freely, the nominal exchange rate e_t is not sticky, so the terms of trade adjust freely.

Under LCP, the nominal exchange rate does not directly influence the consumer prices of Home to Foreign goods in either country. For example, in the Home country, $s_t = p_{Ft} - p_{Ht}$. Because prices are set in local currencies, neither p_{Ft} and p_{Ht} adjust freely to shocks. In fact, as we have seen, when $\phi = 0$, monetary policy has no control over the internal relative prices.

But under LCP, there are currency misalignments, and monetary policy can control those. Recall from (44) $\Delta_t \equiv e_t + p^*_{Ht} - p_{Ht} = e_t + p^*_{Ft} - p_{Ft}$, so the currency misalignment adjusts instantaneously with nominal exchange rate movements. Because policy cannot influence the relative output distortion (when $\nu = 1$) but can influence the relative consumption distortion, the optimal policy puts full weight on the currency misalignment. When $\nu = 1$, we can write (82) as $\frac{1}{\sigma}\Delta_t + \xi\pi^R_t = 0$. When $\Delta_t > 0$, so that the Home currency is undervalued, and $\pi_t - \pi^*_t > 0$, the implications for policy are obvious. Home monetary policy must tighten relative to Foreign. But the more interesting case to consider is when Home inflation is running high, so that $\pi_t - \pi^*_t > 0$, but the currency is overvalued, so that $\Delta_t < 0$. Then equation (82) tells us that the goals of maintaining low inflation and a stable currency are in conflict. Policies that improve the inflation situation may exacerbate the currency misalignment. Equation (82) parameterizes the tradeoff.

<u>The Interest-Rate Reaction Function</u>

We can derive an interest rate rule that will support the optimal policies given by equations (81) and (82).

Substituting equation (82) into equations (57)-(58) and (60)-(61), and using the definitions of CPI inflation given in equations (65) and (66), we can derive:

(85) $$\pi^R_t = \frac{\beta}{1+\delta\sigma\xi}E_t\pi^R_{t+1} + \frac{\nu - 1}{1+\delta\sigma\xi}u^R_t,$$

We will assume that u_t and u_t^* are each AR(1) processes, independently distributed, with a serial correlation coefficient given by ρ. Under these assumptions, we can solve (85) as:

(86) $$\pi_t^R = \frac{\nu - 1}{1 + \delta\sigma\xi - \beta\rho} u_t^R .$$ [9]

Similarly, under the optimal monetary policy, we can get a solution for "world" inflation:

(87) $$\pi_t^W = \frac{\beta}{1 + \delta\sigma\xi} E_t \pi_{t+1}^W + \frac{1}{1 + \delta\sigma\xi} u_t^W ,$$

Under the assumption that the mark-up shocks follow AR(1) processes, we find:

(88) $$\pi_t^W = \frac{1}{1 + \delta\sigma\xi - \beta\rho} u_t^W$$

Substituting these equations into the Euler equations for the Home and Foreign country, given by equations (63) and (64), making use of the consumption equations (49a) and (49b), we find:

(89) $$r_t^R = (\rho + (1-\rho)\sigma\xi)\pi_t^R + \frac{\sigma(\nu - 1)}{D}(E_t \bar{y}_{t+1}^R - \bar{y}_t^R) = (\rho + (1-\rho)\sigma\xi)\pi_t^R + \overline{rr}_t^R ,$$

(90) $$r_t^W = (\rho + (1-\rho)\sigma\xi)\pi_t^W + \sigma(E_t \bar{y}_{t+1}^W - \bar{y}_t^W) = (\rho + (1-\rho)\sigma\xi)\pi_t^W + \overline{rr}_t^W .$$

Here, $\overline{rr}_t$ represents the real interest rate in the efficient economy. We can use (89) and (90) to write:

(91) $$r_t = (\rho + (1-\rho)\sigma\xi)\pi_t + \overline{rr}_t$$

(92) $$r_t^* = (\rho + (1-\rho)\sigma\xi)\pi_t^* + \overline{rr}_t^* .$$

Surprisingly, these interest-rate reaction functions are identical to the ones derived in CGG for the PCP model, except that the inflation term that appears on the right-hand-side of each equation is CPI inflation here, while it is PPI inflation in CGG.

This finding starkly highlights the difference between monetary rules expressed as "target criteria" and monetary rules expressed as interest-rate reaction functions. The optimal interest-rate reaction functions presented in equations (91) and (92) appear to give no role for using monetary policy to respond to law of one price deviations. However, the "target criteria" show the optimal tradeoff does give weight to the law of one price deviation. The key to understanding this apparent conflict is that the reaction functions, such as (91) and (92) are not only setting inflation rates. By setting Home relative to Foreign interest rates, they are also prescribing a

[9] See the Appendix for the complete solutions of the model under optimal policy.

relationship between Home and Foreign output gaps, the law of one price deviation, and Home relative to Foreign inflation rates.

If central bankers really did mechanically follow an interest rate rule, their optimal policy rules would have the nominal interest rate responding only to CPI inflation. In practice, however, central bankers set the interest rates to achieve their targets. We have shown that the optimal target criteria involves tradeoffs among the goals of achieving zero inflation, driving the output gaps to zero, and eliminating the law of one price gap.

4. Conclusions

Policymakers do not in general adhere to simple interest-rate reaction functions. Instead, as Svensson (1999) and Svensson and Woodford (2005) have argued, they set targets for key economic variables. It has generally been believed, especially in light of CGG, that the key tradeoffs in an open economy are the same as in a closed economy. That is, policymakers should target a linear combination of inflation and the output gap. This paper shows that in fact, when our model is rich enough to allow for currency misalignments, the tradeoffs should involve not only inflation and the output gap but also the exchange rate misalignment. However, the interest-rate reaction functions rule that supports this policy has the nominal interest rate reacting only to CPI inflation.

A technical contribution of this paper is the derivation of the loss function when there is home bias in consumption and deviations from the law of one price. Currency misalignments may arise in some approaches for reasons other than local-currency-pricing. But future work can make use of the loss function derived here, or at least of the steps used in deriving the loss function. Indeed, future work will explore the role of nominal wage stickiness. For reasons discussed in Engel (2006), LCP itself might be more plausible in a framework in which there is nominal wage stickiness and local distribution costs of imports.

References

Adolfson, Malin; Stefan Laseen; Jesper Linde; and, Mattias Villiani. 2007. "Bayesian Estimation of an Open-Economy DSGE Model with Incomplete Pass-Through." Journal of International Economics 72:2, 481-511.

Aoki, Kosuke. 2001. "Optimal Monetary Policy Responses to Relative Price Changes." Journal of Monetary Economics 48:1, p. 55-80.

Atkeson, Andrew and Ariel Burstein. 2008. "Pricing to Market, Trade Costs, and International Relative Prices." American Economic Review, forthcoming.

Benigno, Gianluca. 2004. "Real Exchange Rate Persistence and Monetary Policy Rules." Journal of Monetary Economics 51:3, 473-502.

Betts, Caroline and Michael B. Devereux. 1996. "The Exchange Rate in a Model of Pricing to Market." European Economic Review 40:3-5, 1007-1021.

Betts, Caroline and Michael B. Devereux. 2000. "Exchange Rate Dynamics in a Model of Pricing to Market." Journal of International Economics 50:1, 215-244.

Clarida, Richard; Jordi Gali; and, Mark Gertler. 2002. "A Simple Framework for International Monetary Policy Analysis." Journal of Monetary Economics 49:5, 879-904.

Corsetti, Giancarlo and Paolo Pesenti. 2005. "International Dimensions of Optimal Monetary Policy." Journal of Monetary Economics 52:2, 281-305.

Devereux, Michael B., and Charles Engel. 2003. "Monetary Policy in the Open Economy Revisited: Exchange Rate Flexibility and Price Setting Behavior." Review of Economic Studies 70:4, 765-783.

Engel, Charles. 1999. "Accounting for U.S. Real Exchange Rate Changes." Journal of Political Economy 107:3, 507-538.

Engel, Charles. 2006. "Equivalence Results for Optimal Pass-Through, Optimal Indexing to Exchange Rates, and Optimal Choice of Currency for Export Pricing." Journal of the European Economics Association 4:6, 1249-1260.

Gali, Jordi. 2008. Monetary Policy, Inflation, and the Business Cycle (Princeton University Press.)

Kollmann, Robert. 2002. "Monetary Policy Rules in the Open Economy: Effects on Welfare and Business Cycles." Journal of Monetary Economics 49:5, 989-1015.

Leitemo, Kai and Ulf Soderstrom. 2005. "Simple Monetary Policy Rules and Exchange Rate Uncertainty." Journal of International Money and Finance 24:3, 481-507.

Monacelli, Tommaso. 2005. "Monetary Policy in a Low Pass-Through Environment." Journal of Money, Credit and Banking 37:6, 1047-1066.

Smets, Frank and Raf Wouters. 2002. "Openness, Imperfect Exchange-Rate Pass-Through, and Monetary Policy." Journal of Monetary Economics 49:5, 947-981.

Sutherland, Alan. 2005. "Incomplete Pass-Through and the Welfare Effects of Exchange Rate Variability." Journal of International Economics 65:2, 375-399.

Svensson, Lars. 1999. "Inflation Targeting as a Monetary Policy Rule." Journal of Monetary Economics 43:9, 607-654.

Svensson, Lars and Michael Woodford. 2005. "Implementing Optimal Monetary Policy through Inflation-Forecast Targeting." In Ben S. Bernanke and Michael Woodford (eds.), The Inflation Targeting Debate (University of Chicago Press.)

Woodford, Michael. 2003. Interest and Prices (Princeton University Press.)

Woodford, Michael. 2007. "Globalization and Monetary Control." In Jordi Gali and Mark Gertler, eds., International Dimensions of Monetary Policy (National Bureau of Economic Research), forthcoming.

Appendix

A.I Derivation of Welfare Function in Clarida-Gali-Gertler model with Home Bias in preferences

The object is to rewrite the welfare function, which is defined in terms of home and foreign consumption and labor effort into terms of the squared output gap and squared inflation. We derive the joint welfare function of home and foreign households, since we will be examining cooperative monetary policy.

Most of the derivation requires only 1st-order approximations of the equations of the model, but in a few places, 2nd-order approximations are needed. If the approximation is 1st-order, I'll use the notation "$+o\left(\left\|a^2\right\|\right)$" to indicate that there are 2nd-order and higher terms left out, and if the approximation is 2nd-order, I will use "$+o\left(\left\|a^3\right\|\right)$". ($a$ is notation for the log of the productivity shock)

From equation (1) in the text, the period utility of the planner is given by:

(A.1) $$u_t \equiv \frac{1}{1-\sigma}(C_t^{1-\sigma} + C_t^{*1-\sigma}) - \frac{1}{1+\phi}(N_{t+j}^{1+\phi} + N_{t+j}^{*1+\phi}).$$

Take a second-order log approximation around the non-stochastic steady state. We assume the optimal subsidies, given by equation (38), hold in steady state. In the non-stochastic steady state, under symmetry, we have $C^{1-\sigma} = C^{*1-\sigma} = N^{1+\phi} = N^{*1+\phi}$. The fact that $C^{1-\sigma} = N^{1+\phi}$ follows from the fact that in steady state, $C = N$ from market clearing and symmetry, and $C^{-\sigma} = N^{\phi}$ from the steady-state version of (16) given $W/P = 1$ in steady state.

We get:

(A.2) $$\begin{aligned} u_t &= 2\left(\frac{1}{1-\sigma} - \frac{1}{1+\phi}\right)C^{1-\sigma} + C^{1-\sigma}\left(c_t - c + c_t^* - c\right) + \frac{1-\sigma}{2}C^{1-\sigma}\left((c_t - c)^2 + (c_t^* - c)^2\right) \\ &-C^{1-\sigma}\left(n_t - n + n_t^* - n\right) - \frac{1+\phi}{2}C^{1-\sigma}\left((n_t - n)^2 + (n_t^* - n)^2\right) + o\left(\left\|a^3\right\|\right) \end{aligned}.$$

Since we can equivalently maximize an affine transformation of (A.2), it is convenient to simplify that equation to get:

(A.3) $$u_t = c_t + c_t^* - h_t - h_t^* + \frac{1-\sigma}{2}\left(c_t^2 + c_t^{*2}\right) - \frac{1+\phi}{2}\left(h_t^2 + h_t^{*2}\right) + o\left(\left\|a^3\right\|\right).$$

Utility is maximized when consumption and employment take on their efficient values:

(A.4) $$u_t^{\max} = \overline{c}_t + \overline{c}_t^* - \overline{n}_t - \overline{n}_t^* + \frac{1-\sigma}{2}\left(\overline{c}_t^2 + \overline{c}_t^{*2}\right) - \frac{1+\phi}{2}\left(\overline{n}_t^2 + \overline{n}_t^{*2}\right) + o\left(\left\|a^3\right\|\right).$$

In general, this maximum may not be attainable because of distortions. We can write $x_t = \overline{x}_t + \tilde{x}_t$, where $\tilde{x}_t \equiv x_t - \overline{x}_t$. So, we have:

(A.5) $$\begin{aligned} u_t &= \left[\overline{c}_t + \overline{c}_t^* - \overline{n}_t - \overline{n}_t^* + \frac{1-\sigma}{2}\left(\overline{c}_t^2 + \overline{c}_t^{*2}\right) - \frac{1+\phi}{2}\left(\overline{n}_t^2 + \overline{n}_t^{*2}\right)\right] \\ &+\tilde{c}_t + \tilde{c}_t^* - \tilde{n}_t - \tilde{n}_t^* + \frac{1-\sigma}{2}\left(\tilde{c}_t^2 + \tilde{c}_t^{*2} + 2\overline{c}_t\tilde{c}_t + 2\overline{c}_t^*\tilde{c}_t^*\right) - \frac{1+\phi}{2}\left(\tilde{n}_t^2 + \tilde{n}_t^{*2} + 2\overline{n}_t\tilde{n}_t + 2\overline{n}_t^*\tilde{n}_t^*\right) \\ &+o\left(\left\|a^3\right\|\right) \end{aligned}$$

or,

(A.6) $$u_t - u_t^{\max} = \tilde{c}_t + \tilde{c}_t^* - \tilde{n}_t - \tilde{n}_t^* + \frac{1-\sigma}{2}\left(\tilde{c}_t^2 + \tilde{c}_t^{*2}\right) - \frac{1+\phi}{2}\left(\tilde{n}_t^2 + \tilde{n}_t^{*2}\right) \\ +(1-\sigma)\left(\overline{c}_t\tilde{c}_t + \overline{c}_t^*\tilde{c}_t^*\right) - (1+\phi)\left(\overline{n}_t\tilde{n}_t + \overline{n}_t^*\tilde{n}_t^*\right) + o\left(\left\|a^3\right\|\right).$$

The object is to write (A.t) as a function of squared output gaps and squared inflation if possible. We need a second-order approximation of $\tilde{c}_t + \tilde{c}_t^* - \tilde{n}_t - \tilde{n}_t^*$. But for the rest of the terms, since they are squares and products, the 1st-order approximations that have already been derived will be sufficient.

Recalling that $\Delta_t = 0$ in the PCP model, we can write equations (49a)-(49b) as:

(A.7) $c_t = c_y y_t + (1-c_y)y_t^* + o\left(\left\|a^2\right\|\right)$,

(A.8) $c_t^* = (1-c_y)y_t + c_y y_t^* + o\left(\left\|a^2\right\|\right)$,

where $c_y \equiv \dfrac{D+\nu-1}{2D}$.

It follows from (A.7) and (A.8) that:

(A.9) $\overline{c}_t = c_y\overline{y}_t + (1-c_y)\overline{y}_t^* + o\left(\left\|a^2\right\|\right)$,

(A.10) $\overline{c}_t^* = (1-c_y)\overline{y}_t + c_y\overline{y}_t^* + o\left(\left\|a^2\right\|\right)$,

(A.11) $\tilde{c}_t = c_y\tilde{y}_t + (1-c_y)\tilde{y}_t^* + o\left(\left\|a^2\right\|\right)$,

(A.12) $\tilde{c}_t^* = (1-c_y)\tilde{y}_t + c_y\tilde{y}_t^* + o\left(\left\|a^2\right\|\right)$.

Next, we can easily:

(A.13) $\tilde{n}_t = \tilde{y}_t + o\left(\left\|a^2\right\|\right)$, and

(A.14) $\tilde{n}_t^* = \tilde{y}_t^* + o\left(\left\|a^2\right\|\right)$.

These follow as in (45a)-(45b) because $n_t = y_t - a_t + o\left(\left\|a^2\right\|\right)$ and $\overline{n}_t = \overline{y}_t - a_t$ (and similarly in the Foreign country.)

We need expressions for $\overline{n}_t$ and $\overline{n}_t^*$. We have from (53a)-(53b):

$$a_t = \left(\frac{\sigma(1+D)}{2D} + \phi\right)\overline{y}_t + \frac{\sigma(D-1)}{2D}\overline{y}_t^* - \phi a_t,$$

$$a_t^* = \left(\frac{\sigma(1+D)}{2D} + \phi\right)\overline{y}_t^* + \frac{\sigma(D-1)}{2D}\overline{y}_t - \phi a_t^*.$$

Using $a_t = \overline{y}_t - \overline{n}_t$ and $a_t^* = \overline{y}_t^* - \overline{n}_t^*$, we can write these as

(A.15) $\overline{n}_t = \dfrac{1-\sigma}{1+\phi}\left[(\nu-1)c_y + \dfrac{2-\nu}{2}\right]\overline{y}_t + \dfrac{1-\sigma}{1+\phi}\left[(1-\nu)c_y + \dfrac{\nu}{2}\right]\overline{y}_t^* + o\left(\left\|a^2\right\|\right)$, and

(A.16) $\overline{n}_t^* = \dfrac{1-\sigma}{1+\phi}\left[(\nu-1)c_y + \dfrac{2-\nu}{2}\right]\overline{y}_t^* + \dfrac{1-\sigma}{1+\phi}\left[(1-\nu)c_y + \dfrac{\nu}{2}\right]\overline{y}_t + o\left(\left\|a^2\right\|\right)$.

Turning attention back to the loss function in equation (A.6), we focus first on the terms $\dfrac{1-\sigma}{2}\left(\tilde{c}_t^2 + \tilde{c}_t^{*2}\right) - \dfrac{1+\phi}{2}\left(\tilde{n}_t^2 + \tilde{n}_t^{*2}\right) + (1-\sigma)\left(\overline{c}_t\tilde{c}_t + \overline{c}_t^*\tilde{c}_t^*\right) - (1+\phi)\left(\overline{n}_t\tilde{n}_t + \overline{n}_t^*\tilde{n}_t^*\right)$. As noted above, these involve only squares and cross-products of $\tilde{c}_t$, $\tilde{c}_t^*$, $\overline{c}_t$, $\overline{c}_t^*$, $\tilde{n}_t$, $\tilde{n}_t^*$, $\overline{n}_t$, and $\overline{n}_t^*$. We can substitute from equations (A.9)-(A.16) into this expression. It is useful provide a few lines of algebra since it is a bit messy:

$$
\begin{aligned}
&\frac{1-\sigma}{2}\left(\tilde{c}_t^2+\tilde{c}_t^{*2}\right)-\frac{1+\phi}{2}\left(\tilde{n}_t^2+\tilde{n}_t^{*2}\right)+(1-\sigma)\left(\bar{c}_t\tilde{c}_t+\bar{c}_t^*\tilde{c}_t^*\right)-(1+\phi)\left(\bar{n}_t\tilde{n}_t+\bar{n}_t^*\tilde{n}_t^*\right)\\
&=\left(\frac{1-\sigma}{2}\right)\left(2c_y^2-2c_y+1\right)\left(\tilde{y}_t^2+\tilde{y}_t^{*2}\right)-(1-\sigma)\left(2c_y^2-2c_y\right)\tilde{y}_t\tilde{y}_t^*-\left(\frac{1+\phi}{2}\right)\left(\tilde{y}_t^2+\tilde{y}_t^{*2}\right)\\
&+(1-\sigma)\left(2c_y^2-2c_y\right)\left(\bar{y}_t-\bar{y}_t^*\right)\left(\tilde{y}_t-\tilde{y}_t^*\right)+(1-\sigma)\left(\bar{y}_t\tilde{y}_t+\bar{y}_t^*\tilde{y}_t^*\right)\\
&-(1-\sigma)\left(\frac{\nu}{2}\bar{c}_t+\left(\frac{2-\nu}{2}\right)\bar{c}_t^*\right)\tilde{y}_t-(1-\sigma)\left(\frac{\nu}{2}\bar{c}_t^*+\left(\frac{2-\nu}{2}\right)\bar{c}_t\right)\tilde{y}_t^*\\
&=\left(\frac{1-\sigma}{2}\right)\left(2c_y^2-2c_y+1\right)\left(\tilde{y}_t^2+\tilde{y}_t^{*2}\right)-(1-\sigma)\left(2c_y^2-2c_y\right)\tilde{y}_t\tilde{y}_t^*-\left(\frac{1+\phi}{2}\right)\left(\tilde{y}_t^2+\tilde{y}_t^{*2}\right)\\
&+2(1-\sigma)\left(c_y^2-c_y\right)\left(\bar{y}_t-\bar{y}_t^*\right)\left(\tilde{y}_t-\tilde{y}_t^*\right)+(1-\sigma)\left(\bar{y}_t\tilde{y}_t+\bar{y}_t^*\tilde{y}_t^*\right)\\
&-(1-\sigma)\left[\left((\nu-1)c_y-\frac{\nu}{2}\right)\left(\bar{y}_t-\bar{y}_t^*\right)\left(\tilde{y}_t-\tilde{y}_t^*\right)+\bar{y}_t\tilde{y}_t+\bar{y}_t^*\tilde{y}_t^*\right]\\
&=\left(\frac{1-\sigma}{2}\right)\left(2c_y^2-2c_y+1\right)\left(\tilde{y}_t^2+\tilde{y}_t^{*2}\right)-(1-\sigma)\left(2c_y^2-2c_y\right)\tilde{y}_t\tilde{y}_t^*-\left(\frac{1+\phi}{2}\right)\left(\tilde{y}_t^2+\tilde{y}_t^{*2}\right)\\
&+(1-\sigma)\left(2(c_y^2-c_y)-(\nu-1)c_y+\frac{\nu}{2}\right)\left(\bar{y}_t-\bar{y}_t^*\right)\left(\tilde{y}_t-\tilde{y}_t^*\right)+o\left(\left\|a^3\right\|\right)
\end{aligned}
\tag{A.17}
$$

Now return to the $\tilde{c}_t+\tilde{c}_t^*-\tilde{n}_t-\tilde{n}_t^*$ term in equation (A.6) and do a 2nd-order approximation. Start with equation (30), dropping the k^{-1} term because it will not affect the approximation, and noting that in the PCP model, $S_t^*=S_t^{-1}$:

(A.18) $Y_t=\frac{\nu}{2}S_t^{(2-\nu)/2}C_t+\left(\frac{2-\nu}{2}\right)S_t^{\nu/2}C_t^*$

Then use equation (34), but using the fact that $S_t^*=S_t^{-1}$ and there are no deviations from the law of one price:

(A.19) $C_t^*=C_tS_t^{\frac{1-\nu}{\sigma}}$.

Substitute in to get:

(A.20) $Y_t=\frac{\gamma}{2}S_t^{\frac{2-\nu}{2}}C_t+\left(\frac{2-\nu}{2}\right)S_t^{\frac{\nu}{2}+\frac{1-\nu}{\sigma}}C_t$.

Solve for C_t:

(A.21) $C_t=Y_t\left(\frac{\gamma}{2}S_t^{\frac{2-\nu}{2}}+\left(\frac{2-\nu}{2}\right)S_t^{\frac{\nu}{2}+\frac{1-\nu}{\sigma}}\right)^{-1}$, or,

(A.22) $c_t=y_t-\ln\left(\frac{\nu}{2}e^{\left(\frac{2-\nu}{2}\right)s_t}+\left(\frac{2-\nu}{2}\right)e^{\left(\frac{\nu}{2}+\frac{1-\nu}{\sigma}\right)s_t}\right)$.

Take first and second derivatives, evaluated at the non-stochastic steady state:

(A.23) $\frac{\partial c_t}{\partial s_t}\Big|_{s=0}=\left(\frac{\nu-2}{2}\right)\left(\nu+\frac{1-\nu}{\sigma}\right)$

(A.24) $\frac{\partial^2 c_t}{\partial s_t^2}\Big|_{s=0}=\left(\frac{\nu-2}{2}\right)\frac{\nu}{2}(\nu-1)^2\left(\frac{\sigma-1}{\sigma}\right)^2$.

Then we get this 2nd-order approximation:

(A.25) $c_t = y_t + \left(\frac{\nu-2}{2}\right)\left(\nu + \frac{1-\nu}{\sigma}\right)s_t + \frac{1}{2}\left(\frac{\nu-2}{2}\right)\frac{\nu}{2}(\nu-1)^2\left(\frac{\sigma-1}{\sigma}\right)^2 s_t^2 + o\left(\left\|a^3\right\|\right)$.

Symmetrically,

(A.26) $c_t^* = y_t^* - \left(\frac{\nu-2}{2}\right)\left(\nu + \frac{1-\nu}{\sigma}\right)s_t + \frac{1}{2}\left(\frac{\nu-2}{2}\right)\frac{\nu}{2}(\nu-1)^2\left(\frac{\sigma-1}{\sigma}\right)^2 s_t^2 + o\left(\left\|a^3\right\|\right)$.

Since we are only interested in $\tilde{c}_t + \tilde{c}_t^*$, we can add these together to get:

(A.27) $c_t + c_t^* = y_t + y_t^* + \left(\frac{\nu-2}{2}\right)\frac{\nu}{2}(\nu-1)^2\left(\frac{\sigma-1}{\sigma}\right)^2 s_t^2 + o\left(\left\|a^3\right\|\right)$.

Now we can take a 1st-order approximation for s_t to substitute out for s_t^2. From equation (50), we have:

(A.28) $s_t == \frac{\sigma^2}{D^2}(y_t^* - y_t)^2 + o\left(\left\|a^3\right\|\right)$.

Substituting into equation (A.28), we can write:

(A.29) $c_t + c_t^* = y_t + y_t^* + \Omega(y_t^* - y_t)^2 + o\left(\left\|a^3\right\|\right)$,

where $\Omega = \frac{\nu(\nu-2)}{4}\left(\frac{(\nu-1)(\sigma-1)}{D}\right)^2$.

Evaluating (A.29) at flexible prices, we have:

(A.30) $\overline{c}_t + \overline{c}_t^* = \overline{y}_t + \overline{y}_t^* + \Omega(\overline{y}_t^* - \overline{y}_t)^2 + o\left(\left\|a^3\right\|\right)$.

It follows from the fact that $\tilde{c}_t + \tilde{c}_t^* = c_t + c_t^* - (\overline{c}_t + \overline{c}_t^*)$ that

(A.31) $\tilde{c}_t + \tilde{c}_t^* = \tilde{y}_t + \tilde{y}_t^* + \Omega(\tilde{y}_t^2 + 2\overline{y}_t\tilde{y}_t + \tilde{y}_t^{*2} + 2\overline{y}_t^*\tilde{y}_t^* - 2\overline{y}_t\tilde{y}_t^* - 2\overline{y}_t^*\tilde{y}_t - 2\tilde{y}_t\tilde{y}_t^*) + o\left(\left\|a^3\right\|\right)$.

See section A.III below for the second-order approximations for $\tilde{n}_t$ and $\tilde{n}_t^*$:

(A.32) $\tilde{n}_t = \tilde{y}_t + \frac{\xi}{2}\sigma_{pt}^2 + o\left(\left\|a^3\right\|\right)$

(A.33) $\tilde{n}_t^* = \tilde{y}_t^* + \frac{\xi}{2}\sigma_{p^*t}^2 + o\left(\left\|a^3\right\|\right)$.

Substitute expressions (A.31)-(A.33) along with (A.17) into the loss function (A.t):

(A.34)
$$\begin{aligned} u_t - u_t^{\max} &= \left[\Omega + (1-\sigma)(c_y^2 - c_y)\right](\tilde{y}_t - \tilde{y}_t^*)^2 - \left(\frac{\sigma+\phi}{2}\right)\left(\tilde{y}_t^2 + \tilde{y}_t^{*2}\right) \\ &\quad - \frac{\xi}{2}\left(\sigma_{pt}^2 + \sigma_{p^*t}^2\right) + 2\Omega\left(\overline{y}_t - \overline{y}_t^*\right)\left(\tilde{y}_t - \tilde{y}_t^*\right) \\ &\quad + (1-\sigma)\left(2(c_y^2 - c_y) - (\nu-1)c_y + \frac{\nu}{2}\right)\left(\overline{y}_t - \overline{y}_t^*\right)\left(\tilde{y}_t - \tilde{y}_t^*\right) + o\left(\left\|a^3\right\|\right) \end{aligned}$$

Some tedious algebra demonstrates that

(A.35) $2\Omega + (1-\sigma)\left(2(c_y^2 - c_y) - (\nu-1)c_y + \frac{\nu}{2}\right) = 0$.

So, finally we can write:

$$
\text{(A.36)}\quad
\begin{aligned}
u_t - u_t^{\max} &= \left[\Omega + (1-s)(c_y^2 - c_y)\right](\tilde{y}_t - \tilde{y}_t^*)^2 - \left(\frac{\rho+\phi}{2}\right)\left(\tilde{y}_t^2 + \tilde{y}_t^{*2}\right) - \frac{\xi}{2}\left(\sigma_{pt}^2 + \sigma_{p^*t}^2\right) \\
&= \left(\frac{\nu(\nu-2)\sigma(1-\sigma)}{4D}\right)(\tilde{y}_t - \tilde{y}_t^*)^2 - \left(\frac{\sigma+\phi}{2}\right)\left(\tilde{y}_t^2 + \tilde{y}_t^{*2}\right) - \frac{\xi}{2}\left(\sigma_{pt}^2 + \sigma_{p^*t}^2\right) + o\left(\left\|a^3\right\|\right)
\end{aligned}.
$$

This expression reduces to CGG's when there is no home bias ($\gamma = 1$). To see this from their expression at the top of p. 903, multiply their utility by 2 (since they take average utility), and set their γ equal to ½ (so their country sizes are equal.

A.II Derivation of Welfare Function under LCP with Home Bias in Preferences

The second-order approximation to welfare in terms of logs of consumption and employment of course does not change, so equation (A.6) still holds. As before, we break down the derivation into two parts. We use first-order approximations to structural equations to derive an approximation to the quadratic term

$$
\frac{1-\sigma}{2}\left(\tilde{c}_t^2 + \tilde{c}_t^{*2}\right) - \frac{1+\phi}{2}\left(\tilde{n}_t^2 + \tilde{n}_t^{*2}\right) + (1-\sigma)\left(\bar{c}_t\tilde{c}_t + \bar{c}_t^*\tilde{c}_t^*\right) - (1+\phi)\left(\bar{n}_t\tilde{n}_t + \bar{n}_t^*\tilde{n}_t^*\right).
$$

Then we use second order approximations to the structural equations to derive an expression for $\tilde{c}_t + \tilde{c}_t^* - \tilde{n}_t - \tilde{n}_t^*$.

The quadratic term involves squares and cross-products of $\tilde{c}_t$, $\tilde{c}_t^*$, $\bar{c}_t$, $\bar{c}_t^*$, $\tilde{n}_t$, $\tilde{n}_t^*$, $\bar{n}_t$, and $\bar{n}_t^*$. Expressions (A.9)-(A.10) still gives us first-order approximations for $\bar{c}_t$ and $\bar{c}_t^*$; equations (A.13)-(A.14) are first-order approximations for $\tilde{n}_t$ and $\tilde{n}_t^*$; and, (A.15)-(A.16) are first-order approximations for $\bar{n}_t$ and $\bar{n}_t^*$. But we need to use equations (49a)-(49b) and (A.11)-(A.12) to derive:

$$
\text{(A.37)}\quad \tilde{c}_t = c_y\tilde{y}_t + (1-c_y)\tilde{y}_t^* + \frac{\nu(2-\nu)}{2D}\Delta_t + o\left(\left\|a^2\right\|\right),
$$

$$
\text{(A.38)}\quad \tilde{c}_t^* = (1-c_y)\tilde{y}_t + c_y\tilde{y}_t^* - \frac{\nu(2-\nu)}{2D}\Delta_t + o\left(\left\|a^2\right\|\right).
$$

With these equations, we can follow the derivation as in equation (A.17). After tedious algebra, we arrive at the same result, with the addition of the terms $\frac{(1-\sigma)\nu^2(2-\nu)^2}{4D^2}\Delta_t^2$ and $\frac{(1-\sigma)\nu(2-\nu)(\nu-1)}{2D^2}\Delta_t(y_t - y_t^*)$. Note that the last term involves output levels, not output gaps. That is, we have:

$$
\text{(A.39)}\quad
\begin{aligned}
&\frac{1-\sigma}{2}\left(\tilde{c}_t^2 + \tilde{c}_t^{*2}\right) - \frac{1+\phi}{2}\left(\tilde{n}_t^2 + \tilde{n}_t^{*2}\right) + (1-\sigma)\left(\bar{c}_t\tilde{c}_t + \bar{c}_t^*\tilde{c}_t^*\right) - (1+\phi)\left(\bar{n}_t\tilde{n}_t + \bar{n}_t^*\tilde{n}_t^*\right) \\
&= \left(\frac{1-\sigma}{2}\right)\left(2c_y^2 - 2c_y + 1\right)\left(\tilde{y}_t^2 + \tilde{y}_t^{*2}\right) - (1-\sigma)\left(2c_y^2 - 2c_y\right)\tilde{y}_t\tilde{y}_t^* - \left(\frac{1+\phi}{2}\right)\left(\tilde{y}_t^2 + \tilde{y}_t^{*2}\right) \\
&+ (1-\sigma)\left(2(c_y^2 - c_y) - (\nu-1)c_y + \frac{\nu}{2}\right)\left(\bar{y}_t - \bar{y}_t^*\right)\left(\tilde{y}_t - \tilde{y}_t^*\right) + \frac{(1-\sigma)\nu^2(2-\nu)^2}{4D^2}\Delta_t^2 \\
&+ \frac{(1-\sigma)\nu(2-\nu)(\nu-1)}{2D^2}\Delta_t(y_t - y_t^*) + o\left(\left\|a^2\right\|\right)
\end{aligned}
$$

The derivation of $\tilde{c}_t + \tilde{c}_t^* - \tilde{n}_t - \tilde{n}_t^*$ is similar to the PCP model. However, one tedious aspect of the derivation is that we cannot make use of the equality $S_t^* = S_t^{-1}$ that holds under PCP and flexible prices. We write out the equilibrium conditions for home output, and its foreign equivalent, from equations (30) and (31):

(A.40) $Y_t = \frac{\nu}{2} S_t^{1-(\nu/2)} C_t + \left(1 - \frac{\nu}{2}\right)(S_t^*)^{-\nu/2} C_t^*$,

(A.41) $Y_t^* = \frac{\nu}{2} (S_t^*)^{1-(\nu/2)} C_t^* + \left(1 - \frac{\nu}{2}\right) S_t^{-\nu/2} C_t$.

We directly take second-order approximations of these equations around the efficient non-stochastic steady state:

$$
\begin{aligned}
y_t + \frac{1}{2} y_t^2 &= \frac{\nu}{2} c_t + \left(\frac{2-\nu}{2}\right) c_t^* + \frac{\nu}{2}\left(\frac{2-\nu}{2}\right) s_t - \frac{\nu}{2}\left(\frac{2-\nu}{2}\right) s_t^* \\
&+ \frac{1}{2}\left\{ \frac{\nu}{2} c_t^2 + \left(\frac{2-\nu}{2}\right) c_t^{*2} + \frac{\nu}{2}\left(\frac{2-\nu}{2}\right)^2 s_t^2 + \left(\frac{\nu}{2}\right)^2 \left(\frac{2-\nu}{2}\right) s_t^{*2} \right\} \\
&+ \frac{\nu}{2}\left(\frac{2-\nu}{2}\right) s_t c_t - \frac{\nu}{2}\left(\frac{2-\nu}{2}\right) s_t^* c_t^* + o\left(\left\|a^3\right\|\right)
\end{aligned} \tag{A.42}
$$

$$
\begin{aligned}
y_t^* + \frac{1}{2} y_t^{*2} &= \frac{\nu}{2} c_t^* + \left(\frac{2-\nu}{2}\right) c_t + \frac{\nu}{2}\left(\frac{2-\nu}{2}\right) s_t^* - \frac{\nu}{2}\left(\frac{2-\nu}{2}\right) s_t \\
&+ \frac{1}{2}\left\{ \frac{\nu}{2} c_t^{*2} + \left(\frac{2-\nu}{2}\right) c_t^2 + \frac{\nu}{2}\left(\frac{2-\nu}{2}\right)^2 s_t^{*2} + \left(\frac{\nu}{2}\right)^2 \left(\frac{2-\nu}{2}\right) s_t^2 \right\}. \\
&+ \frac{\nu}{2}\left(\frac{2-\nu}{2}\right) s_t^* c_t^* - \frac{\nu}{2}\left(\frac{2-\nu}{2}\right) s_t c_t + o\left(\left\|a^3\right\|\right)
\end{aligned} \tag{A.43}
$$

Note that in a second-order approximation, we cannot impose $s_t = -s_t^*$. However, we can impose $s_t^2 = s_t^{*2}$. Then adding (A.42) and (A.43) together, we find:

(A.44) $y_t + y_t^* + \frac{1}{2}(y_t^2 + y_t^{*2}) = c_t + c_t^* + \frac{1}{2}\left\{ c_t^2 + c_t^{*2} + \nu\left(\frac{2-\nu}{2}\right) s_t^2 \right\} + o\left(\left\|a^3\right\|\right)$.

Next, we can use equations (49a), (49b), and (50) to get approximations for c_t^2, c_t^{*2}, and s_t^2. These equations are linear approximations for c_t, c_t^*, and s_t, but since we are looking to approximate the squares of these variables, that is sufficient. With some algebra, we find:

$$
\begin{aligned}
c_t^2 + c_t^{*2} + \nu\left(\frac{2-\nu}{2}\right) s_t^2 &= y_t^2 + y_t^{*2} + \nu\left(\frac{2-\nu}{2}\right)\left(\frac{(1-\nu)(1-\sigma)}{D}\right)^2 (y_t - y_t^*)^2 \\
&+ \nu\left(\frac{2-\nu}{2}\right)\frac{1}{D^2}\Delta_t^2 + \nu\left(\frac{2-\nu}{2}\right)\left(\frac{2(1-\nu)(1-\sigma)}{D^2}\right)\Delta_t (y_t - y_t^*) + o\left(\left\|a^3\right\|\right)
\end{aligned} \tag{A.45}
$$

Then, substituting (A.45) into (A.44) and rearranging, we find:

$$
\begin{aligned}
c_t + c_t^* &= y_t + y_t^* - \frac{\nu}{2}\left(\frac{2-\nu}{2}\right)\left(\frac{(1-\nu)(1-\sigma)}{D}\right)^2 (y_t - y_t^*)^2 \\
&- \frac{\nu}{2}\left(\frac{2-\nu}{2}\right)\frac{1}{D^2}\Delta_t^2 - \frac{\nu}{2}\left(\frac{2-\nu}{2}\right)\left(\frac{2(1-\nu)(1-\sigma)}{D^2}\right)\Delta_t (y_t - y_t^*) + o\left(\left\|a^3\right\|\right)
\end{aligned} \tag{A.46}
$$

Note that if set $\Delta_t = 0$ in (A.46), we would arrive at the second-order approximation for $c_t + c_t^*$ from the PCP model.

Then following the derivations as in the PCP model derivation of (A.31), we can write:

(A.47)
$$\begin{aligned}\tilde{c}_t + \tilde{c}_t^* &= \tilde{y}_t + \tilde{y}_t^* + \Omega(\tilde{y}_t^2 + 2\overline{y}_t\tilde{y}_t + \tilde{y}_t^{*2} + 2\overline{y}_t^*\tilde{y}_t^* - 2\overline{y}_t\tilde{y}_t^* - 2\overline{y}_t^*\tilde{y}_t - 2\tilde{y}_t\tilde{y}_t^*)\\ &\quad -\frac{\nu}{2}\left(\frac{2-\nu}{2}\right)\frac{1}{D^2}\Delta_t^2 - \frac{\nu}{2}\left(\frac{2-\nu}{2}\right)\left(\frac{2(1-\nu)(1-\sigma)}{D^2}\right)\Delta_t(y_t - y_t^*) + o\left(\left\|a^3\right\|\right)\end{aligned}$$

As shown in section A.III, we can make the following second-order approximation:

(A.48) $$\tilde{n}_t + \tilde{n}_t^* = \tilde{y}_t + \tilde{y}_t^* + \frac{\xi}{2}\left[\frac{\nu}{2}\sigma_{p_H t}^2 + \frac{2-\nu}{2}\sigma_{p_H * t}^2 + \frac{\nu}{2}\sigma_{p_F * t}^2 + \frac{2-\nu}{2}\sigma_{p_F t}^2\right] + o\left(\left\|a^3\right\|\right).$$

We then can substitute (A.47), and (A.48), along with (A.39) into the loss function (A.6). Notice the cancellations that occur. The cross product terms on $\Delta_t(y_t - y_t^*)$ in equations (A.39) and (A.47) cancel. The other cross product terms involving output gaps and efficient levels of output also cancel, just as in the PCP model, when we used equation (A.35). Hence, we can write:

(A.49)
$$\begin{aligned}u_t - u_t^{\max} &= \left(\frac{\nu(\nu-2)\sigma(1-\sigma)}{4D}\right)(\tilde{y}_t - \tilde{y}_t^*)^2 - \left(\frac{\sigma+\phi}{2}\right)\left(\tilde{y}_t^2 + \tilde{y}_t^{*2}\right) - \left(\frac{\nu(2-\nu)}{4D}\right)\Delta_t^2\\ &\quad -\frac{\xi}{2}\left[\frac{\nu}{2}\sigma_{p_H t}^2 + \frac{2-\nu}{2}\sigma_{p_H * t}^2 + \frac{\nu}{2}\sigma_{p_F * t}^2 + \frac{2-\nu}{2}\sigma_{p_F t}^2\right] + o\left(\left\|a^3\right\|\right)\end{aligned}.$$

A.III Derivations of Squared Inflation Terms in Loss Functions

In the PCP case, we can write

(A.50) $$A_t N_t = A_t\int_0^1 N_t(f)df = Y_t\int_0^1\left(\frac{P_{Ht}(f)}{P_{Ht}}\right)^{-\xi} df = Y_t V_t,$$

where $V_t \equiv \int_0^1\left(\frac{P_{Ht}(f)}{P_{Ht}}\right)^{-\xi} df$. Taking logs, we can write:

(A.51) $a_t + n_t = y_t + v_t$.

We have $v_t \equiv \ln\left(\int_0^1 e^{-\xi\hat{p}_{Ht}(f)}df\right)$,

where we define

(A.52) $\hat{p}_{Ht}(f) \equiv p_{Ht}(f) - p_{Ht}$.

Following Gali (2008), we note

(A.53) $$e^{(1-\xi)\hat{p}_{Ht}(f)} = 1 + (1-\xi)\hat{p}_{Ht}(f) + \frac{(1-\xi)^2}{2}\hat{p}_{Ht}(f)^2 + o\left(\left\|a^3\right\|\right).$$

By the definition of the price index P_{Ht}, we have $\int_0^1 e^{(1-\xi)\hat{p}_{Ht}(f)}df = 1$. Hence, from (A.53),

(A.54) $$\int_0^1 \hat{p}_{Ht}(f)df = \frac{\xi-1}{2}\int_0^1 \hat{p}_{Ht}(f)^2 df + o\left(\left\|a^3\right\|\right).$$

We also have

(A.55) $$e^{-\xi\hat{p}_{Ht}(f)} = 1 - \xi\hat{p}_{Ht}(f) + \frac{\xi^2}{2}\hat{p}_{Ht}(f)^2 + o\left(\left\|a^3\right\|\right)$$

It follows, using (A.54):

(A.56) $$\int_0^1 e^{-\xi\hat{p}_{Ht}(f)}df = 1 - \xi\int_0^1 \hat{p}_{Ht}(f)df + \frac{\xi^2}{2}\int_0^1 \hat{p}_{Ht}(f)^2 df + o\left(\left\|a^3\right\|\right) = 1 + \frac{\xi}{2}\int_0^1 \hat{p}_{Ht}(f)^2 df + o\left(\left\|a^3\right\|\right)$$

Note the following relationship:

(A.57) $$\int_0^1 \hat{p}_{Ht}(f)^2 df = \int_0^1 (p_{Ht}(f) - E_f(p_{Ht}(f))^2 df + o\left(\left\|a^3\right\|\right) = \text{var}(p_{Ht}) + o\left(\left\|a^3\right\|\right).$$

Using our notation for variances, $\sigma_{p_H t}^2 \equiv \text{var}(p_{Ht})$, and taking the log of (A.56) we arrive at

(A.58) $v_t = \frac{\xi}{2}\sigma^2_{p_H t} + o\left(\left\|a^3\right\|\right)$.

Substituting this into equation (A.51), and recalling that $\bar{y}_t = \bar{n}_t + a_t$, we arrive at equation (A.32). The derivation of (A.33) for the Foreign country proceeds identically.

For the LCP model, we will make use of the following second-order approximation to the equation $Y_t = C_{Ht} + C^*_{Ht}$:

(A.59) $y_t = \frac{\nu}{2}c_{Ht} + \left(\frac{2-\nu}{2}\right)c^*_{Ht} + \frac{1}{2}\left(\frac{\nu}{2}\right)\left(\frac{2-\nu}{2}\right)(c^2_{Ht} + 2c_{Ht}c^*_{Ht} + c^{*2}_{Ht}) + o\left(\left\|a^3\right\|\right)$.

In the LCP model, we can write:

(A.60) $A_t N_t = A_t\int_0^1 N_t(f)df = C_{Ht}\int_0^1\left(\frac{P_{Ht}(f)}{P_{Ht}}\right)^{-\xi} df + C^*_{Ht}\int_0^1\left(\frac{P^*_{Ht}(f)}{P^*_{Ht}}\right)^{-\xi} df = C_{Ht}V_{Ht} + C^*_{Ht}V^*_{Ht}$,

where the definitions of V_{Ht} and V^*_{Ht} are analogous to that of V_t in the PCP model. Taking a second-order log approximation to (A.60), we have:

(A.61)
$$\begin{aligned} a_t + n_t &= \frac{\nu}{2}(c_{Ht} + v_{Ht}) + \left(\frac{2-\nu}{2}\right)(c^*_{Ht} + v^*_{Ht}) \\ &+ \frac{1}{2}\left(\frac{\nu}{2}\right)\left(\frac{2-\nu}{2}\right)\left((c_{Ht} + v_{Ht})^2 + 2(c_{Ht} + v_{Ht})(c^*_{Ht} + v^*_{Ht}) + (c^*_{Ht} + v^*_{Ht})^2\right) + o\left(\left\|a^3\right\|\right) \end{aligned}$$

We can follow the same steps as in the PCP model to conclude:

(A.62) $v_{Ht} = \frac{\xi}{2}\sigma^2_{p_H t} + o\left(\left\|a^3\right\|\right)$

(A.63) $v^*_{Ht} = \frac{\xi}{2}\sigma^2_{p_H * t} + o\left(\left\|a^3\right\|\right)$

Substituting these expressions into (A.61) and cancelling higher order terms, we find:

(A.64)
$$a_t + n_t = \frac{\nu}{2}(c_{Ht} + \frac{\xi}{2}\sigma^2_{p_H t}) + \left(\frac{2-\nu}{2}\right)(c^*_{Ht} + \frac{\xi}{2}\sigma^2_{p_H * t}) + \frac{1}{2}\left(\frac{\nu}{2}\right)\left(\frac{2-\nu}{2}\right)(c^2_{Ht} + 2c_{Ht}c^*_{Ht} + c^{*2}_{Ht}) + o\left(\left\|a^3\right\|\right)$$

Then using equation (A.59), we can write:

(A.65) $a_t + n_t = y_t + \frac{\xi}{2}\left(\frac{\nu}{2}\sigma^2_{p_H t} + \left(\frac{2-\nu}{2}\right)\frac{\xi}{2}\sigma^2_{p_H * t}\right) + o\left(\left\|a^3\right\|\right)$.

Keeping in mind that $\bar{y}_t = \bar{n}_t + a_t$, we can write:

(A.66) $\tilde{n}_t = \tilde{y}_t + \frac{\xi}{2}\left(\frac{\nu}{2}\sigma^2_{p_H t} + \left(\frac{2-\nu}{2}\right)\frac{\xi}{2}\sigma^2_{p_H * t}\right) + o\left(\left\|a^3\right\|\right)$.

Following analogous steps for the Foreign country,

(A.67) $\tilde{n}^*_t = \tilde{y}^*_t + \frac{\xi}{2}\left(\frac{\nu}{2}\sigma^2_{p_F * t} + \left(\frac{2-\nu}{2}\right)\frac{\xi}{2}\sigma^2_{p_F t}\right) + o\left(\left\|a^3\right\|\right)$.

Adding (A.66) and (A.67) gives us equation (A.48).

Finally, to derive the loss functions for policymakers (equation (67) for the PCP model and (71) for the LCP model), we note that the loss function is the present expected discounted value of the period loss functions derived here (equation (A.36) for the PCP model and (A.49) for the LCP model.) That is, the policymaker seeks to minimize $E_t\sum_{j=0}^{\infty}\beta^j(u_{t+j} - u^{\max}_{t+j})$.

Following Woodford (2003, chapter 6), we can see that, in the PCP model, if prices are adjusted according to the Calvo price mechanism given by equation (42) for P_{Ht} that

(A.68) $\sum_{j=0}^{\infty}\beta^{j}\sigma_{p_H t+j}^{2}=\frac{\beta}{(1-\beta\theta)(1-\theta)}\sum_{j=0}^{\infty}\beta^{j}\pi_{Ht+j}^{2}$.

Analogous relationships hold for P_{Ft}^{*} in the PCP model, and for P_{Ht}, P_{Ft}^{*}, P_{Ft}, and P_{Ht}^{*} in the LCP model. We can then substitute this relationship into the present value loss function, $E_t\sum_{j=0}^{\infty}\beta^{j}(u_{t+j}-u_{t+j}^{\max})$, to derive the loss functions of the two models presented in the text.

B. Solutions for Endogenous Variables under Optimal Policy Rules

We assume shocks follow the processes (we have dropped the W superscript on the wage mark-up shocks):

$a_t=\rho a_{t-1}+\varepsilon_{at}$

$a_t^{*}=\rho a_{t-1}^{*}+\varepsilon_{a^*t}$

$\mu_t=\rho\mu_{t-1}+\varepsilon_{\mu t}$

$\mu_t^{*}=\rho\mu_{t-1}^{*}+\varepsilon_{\mu^*t}$

s_t is determined by :

$$s_t=\theta s_{t-1}+\frac{\theta\delta}{1-\beta\theta\rho}\left[(a_t-a_t^{*})+(\mu_t-\mu_t^{*})\right].$$

The solutions for the variables that appear in the loss function under the various policies described in the text are:

$$\pi_t^{R}=\frac{\delta(\nu-1)}{1+\sigma\xi\delta-\beta\rho}(\mu_t-\mu_t^{*})$$

$$\pi_t^{W}=\frac{\delta}{1+\sigma\xi\delta-\beta\rho}(\mu_t+\mu_t^{*})$$

$$\tilde{y}_t^{W}=-\xi\pi_t^{W}$$

$$\tilde{y}_t^{R}=\nu(2-\nu)(s_t-(a_t-a_t^{*}))-(\nu-1)\pi_t^{R}$$

$$\tilde{y}_t=\frac{1}{2}(\tilde{y}_t^{W}+\tilde{y}_t^{R})$$

$$\tilde{y}_t^{*}=\frac{1}{2}(\tilde{y}_t^{W}-\tilde{y}_t^{R})$$

$$\Delta_t=-\xi\sigma\pi_t^{R}-(\nu-1)(s_t-(a_t-a_t^{*}))$$

$$\pi_{Ht}=\frac{1}{2}(\pi_t^{W}+\pi_t^{R})-\left(\frac{2-\nu}{2}\right)(s_t-s_{t-1})$$

$$\pi_{Ft}=\frac{1}{2}(\pi_t^{W}+\pi_t^{R})+\frac{\nu}{2}(s_t-s_{t-1})$$

$$\pi_{Ft}^{*}=\frac{1}{2}(\pi_t^{W}-\pi_t^{R})+\left(\frac{2-\nu}{2}\right)(s_t-s_{t-1})$$

$$\pi_{Ht}^{*}=\frac{1}{2}(\pi_t^{W}-\pi_t^{R})-\frac{\nu}{2}(s_t-s_{t-1})$$