STOCK PRICES AND MONETARY POLICY SHOCKS: A GENERAL EQUILIBRIUM APPROACH

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Preliminary

Abstract. Recent empirical literature documents that unexpected changes in the nominal interest rates have a significant effect on stock prices: a 25-basis point increase in the Fed funds rate is associated with an immediate decrease in broad stock indices that may range from 0.6 to 1.7 percent, followed by a gradual decay as stock prices revert towards their long-run expected value. In this paper, we assess the ability of a general equilibrium New Keynesian asset-pricing model to account for these facts. The model we consider allows for staggered price and wage setting, as well as time-varying risk aversion through habit formation. We find that the model predicts a stock market response to policy shocks that matches empirical estimates, both qualitatively and quantitatively. However, the model seems to underestimate the contribution of time-varying expected excess returns in generating the current response of *ex post* excess returns to the shock. Our findings are robust to a range of variations and parameterizations of the model.

Keywords: Monetary policy; Asset prices; New Keynesian general equilibrium model
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1. Introduction

The reaction of the stock market to monetary policy shocks has been the subject of much empirical research in recent years. In particular, this literature documents that an unexpected change in the nominal interest rates has significant and persistent effects on stock prices. Papers focusing on the instant stock market response to such a shock report that a 25-basis point increase in the Fed funds rate is associated with an immediate decrease in broad US stock indices that ranges from 0.6 to 1.7 percent, depending on the sample and estimation method being used (e.g., Bernanke and Kuttner, 2005; Rigobon and Sachs, 2004; Craine and Martin, 2004). Moreover, various authors document the dynamic effects of policy shocks and report a gradual mean reversion of stock prices and returns following the shock (e.g., Lastrapes, 1998; Rapach, 2001; Neri, 2004).

Such estimated reactions of the stock market to policy shocks are of potential interest for macroeconomists for two reasons. First, they convey important information on the transmission channels of monetary policy, since policy shocks affect financial variables directly and immediately, while they only have delayed and indirect effect on macroeconomic variables. Second, these estimates provide raw stylized facts against which the quantitative predictions of alternative theoretical frameworks can be evaluated. In this paper, we assess the ability (and potential limitations) of a simple New Keynesian asset-pricing model to account for such empirical regularities. In particular, we address the impact and dynamic adjustment of the stock market following a nominal interest rate shock within a quantitative general equilibrium framework that makes the necessary assumptions, but no more, to account for the evidence that we have just summarized.

The first required property of the model is that money should be non-neutral and consequently monetary policy shocks affect real variables. We generate this feature through the common assumption that goods prices are set in a staggered fashion by monopolistically competitive firms (rather than being fully flexible and taken as given by competitive firms). This assumption classifies our model as New Keynesian, various versions of which have already been extensively used to account for the documented effects of policy shocks on macroeconomic variables (e.g., Christiano et al., 2005; Amato and Laubach, 2003; Woodford, 2003). While the sticky-price framework has occasionally been used to study some asset pricing issues such as the term structure of interest rates, its ability to account for the effects of monetary policy shocks on the stock market has not yet been assessed, as far as we are aware. This is rather surprising, given that the asset pricing version of this model seems to be the most natural framework within which the quantitative impact of interest rate shocks on the stock market can fruitfully be examined.

Nevertheless, the basic sticky price model suffers from one unfortunate implication, which relates to its predicted labour market adjustments following a monetary policy shock. For example, after a contractionary shock, e.g. an increase in the interest rate set by the Central Bank, firms' labour demand falls. If nominal wages are fully flexible, this translates into a sharp fall in the real wage, which in turn lowers the production cost of firms and ultimately raises the firms' profits paid out as dividends. Both implications are clearly counterfactual, since profits and dividends are procyclical, while the real wage is only mildly procyclical (see Christiano et al., 2005; Bernanke and Kuttner, 2005). The simplest explanation for the mild documented drop in the real wage and implied increase in profits and dividends following the shock is that nominal wages are also sticky and set in a staggered fashion. In this paper, we follow much of the literature in assuming staggered wage setting by households, modelled as monopolistic suppliers of labour services who face specific constraints on nominal wage adjustment (e.g., Erceg et al., 2000; Christiano et al., 2005).

Finally, both the evidence on stock market volatility in general (e.g. Campbell and Shiller, 1988, Campbell, 2003) and that on the specific impact of monetary policy shock (e.g. Bernanke and Kuttner, 2005) point to the fact that expected excess returns are time-varying and that such variations contribute substantially to the volatility of stock prices and ex post excess returns, just as dividends and real interest rates do. We therefore introduce an active role for time-varying expected excess returns in the stock market reaction to policy shocks by assuming that households form consumption habits, with a specification for habit formation that generates time-variations in households' risk aversion. Thus, the working assumption here is that the stock market response to monetary policy shocks can be explain within a general equilibrium asset pricing model where excess asset returns variations need not be accounted for by behavioral departures from rational expectations.

As it turns out, these three assumptions (staggered price setting, staggered wage setting and time-varying risk aversion) are sufficient to explain, both qualitatively and quantitatively, the response of stock prices to a monetary policy shock documented by empirical studies. Thus, while adding other realistic features to our baseline model (such as the introduction of lags in information processing as in Amato and Laubach, 2003, or capital formation with capital adjustment costs as in Christiano et al., 2005) would probably refine our results, they do not strictly appear as necessary to account for the empirical evidence summarized above. In more detail, we find that, using a parameterization that is in line with the business cycle facts, the predicted impact multiplier is well inside the range of available empirical estimates; moreover this number is robust to a variety of parameterizations and simple versions of the model. Our results suggest that the baseline New Keynesian model provides a potential general equilibrium explanation for the observed stock market reaction to monetary policy shocks.

Our work relates to various strands of the literature. We have already mentioned the empirical papers on which our quantitative investigation is based (more details are provided in section 2). We have also referred to some contributions that study empirically the effect of monetary policy shocks on macroeconomic variables; an extensive survey of this work can be found in Christiano et al. (1999). Of course, there is also a long tradition in assessing the asset pricing implications of dynamic macroeconomic models, particularly within the Real Business Cycle tradition (see Alvarez, 1998, Boldrin et al., 2001 and Lettau, 2003, for recent contributions). Within the New Keynesian tradition, Blanchard (1981) provides an early theoretical analysis of the stock market response to a monetary shock using a rational expectations model with sticky goods prices and flexible asset prices. While in Blanchard (1981) sticky prices are modelled in an ad hoc fashion, Svensson (1986) rationalizes price stickiness by introducing pre-determined pricing by monopolistically competitive firms. Both papers are dynamic asset-pricing models with their own form of staggered price adjustment; however these are theoretical contributions which, unlike our work, do not aim in assessing the ability of the corresponding models to match empirical stock price multipliers. In a similar vein, some papers have studied the implications of sticky prices and non-neutral monetary policy for the shape of the yield curve and the information that can be extracted from it. An early contribution along this line of research is Fuhrer and Moore (1992), who study the indicator properties of long interest rates and exchange rates in a dynamic model with staggered price setting. More recently, Bekaert et al. (2005) have offered a New Keynesian analysis of the yield curve with the goal of tracing the properties of the yield curve to a variety of underlying macroeconomic shocks. The analysis offered here differs from these studies in that exclusively focuses on the stock market effects of policy surprises, leaving aside their effects on the term structure of interest rates and exchange rates.

The rest of the paper is organized as follows. Section 2 presents the empirical findings in more detail. Section 3 introduces the macro block of our basic New Keynesian model. Section 4 derives in detail the asset pricing block of the model. In section 5 we explain and then implement the solution procedure we use to compute and decompose the stock-price multiplier. Section 6 presents the results. In section 7 we summarize our findings and provide some concluding remarks.

2. Empirical Evidence

Table 1 reports the main pieces of recent evidence relating to the impact effects of unanticipated monetary policy shock. For each study we refer to, we only report the baseline estimates of the reaction of broad stock market indexes (NYSE and SP500), leaving aside results based on robustness checks or on less representative indices (e.g., the NASDAQ). The figures reported in the last column are the reaction of the stock market value or index return following a one percentage point surprise increase in the Fed funds rate (the two measures are nearly identical since price changes govern returns changes at high frequency). The exact value of the multiplier may vary across specifications, depending on the particular empirical methodology being implemented or the underlying data being used (e.g. the exact stock market index whose variation is measured, or the specific futures rate used to extract markets expectations and isolate the surprise component of policy shocks). However, despite these variations the overall picture that emerges from these numbers is consistent across papers, with a monetary policy shock having a significant impact on the stock market and estimated multipliers ranging from -2.55% to -6.81%. To summarize, a 0.25 basis point increase in the Fed funds rate is associated with a proportional fall in the stock market of about 1%.

Apart from their immediate impact on stock market indices, monetary policy shocks are also shown to have different and persistent effects on financial asset prices. For example, Patelis (1997) shows that monetary policy indicators such as the Fed funds rate or the term spread help forecast future excess returns. Other papers have used identified VARs to recover the dynamic adjustment of stock prices to policy shocks. For example, Lastrapes documents that the reversion of stock prices following a money supply shock is of comparable speed as that of macroeconomic variables in a number of OECD countries. In a related paper, Rapach (2001) extends and confirms this observation of a gradual decay of stock prices following a monetary policy shock. Such impulse-response patterns suggest that stock-price variables share much of the dynamic properties of other economic aggregates (at least at the quarterly frequency that we are considering here)and that they can consequently be modelled using similar macroeconomic models.

	Market Index	Sample	Multiplier
Bernanke and Kuttner (2005)	NYSE	1989-2002	-2.55 to -4.68
Rigobon and Sachs (2004)	NYSE	1994-2001	-4.85 to -5.16
	SP5000	1994-2001	-5.78 to -6.81
Craine and Martin (2004)	NYSE	1988-2001	-2.80 to -4.92

Table 1: Stock price or daily return response to a 1 percent surprise increase in the Fed funds rate.

3. A Basic New Keynesian Model

We now introduce our baseline model, the asset-pricing implications of which we derive in Section 4. The model is essentially a stripped-down version of the New Keynesian framework, based on Woodford (2003) and Amato and Laubach (2003). Time is discrete. The economy is populated by monopolistically competitive firms and households that adjust nominal prices and wages in a staggered fashion and where households form consumption habits. There is also a monetary authority that sets the nominal interest rates according to a Taylor rule. The aggregate capital stock is constant and normalized to one.

3.1. Varieties and aggregators. There is a continuum of households of measure one, indexed by $\iota \in [0,1]$and a continuum of firms of measure one, indexed by $h \in [0,1]$. Each household is the monopolistic supplier of a specific variety of labour service demanded by all firms and consumes all varieties of the consumption good, each of which is produced by a monopolistic firm. Individual varieties of the consumption good and labour service contribute towards households' instantaneous utility and firms' production according to the following constant-elasticity-of-substition (CES) aggregators:

$$C_t(\iota) = \left[\int_0^1 C_t(h)^{\frac{\theta_p - 1}{\theta_p}} dh\right]^{\frac{\theta_p}{\theta_p - 1}},\ N_t(h) = \left[\int_0^1 N_t(\iota)^{\frac{\theta_w - 1}{\theta_w}} d\iota\right]^{\frac{\theta_w}{\theta_w - 1}}, \tag{1}$$

where $\theta_p, \theta_w > 1$ are the cross partial elasticity of substitution between varieties of the consumption good and labour service, respectively. Let $P_t(h)$ denote the nominal price of consumption variety h and $W(\iota)$ the nominal wage of labour service variety ι. From (1), the optimal shares of each variety in the relevant aggregator are given by:

$$C_t(h)/C_t(\iota) = (P_t(h)/P_t)^{-\theta_p},\ N_t(\iota)/N_t(h) = (W_t(\iota)/W_t)^{-\theta_w}, \tag{2}$$

where P_t and W_t are the following conformable CES price and wage indices:

$$P_t = \left[\int_0^1 P_t(h)^{1-\theta_p} dh\right]^{\frac{1}{1-\theta_p}},\ W_t = \left[\int_0^1 N_t(\iota)^{1-\theta_w} d\iota\right]^{\frac{1}{1-\theta_w}}.$$

For the remainder of the paper we focus on the symmetric equilibrium with full consumption insurance, where all households end up consuming the same quantity of the consumption aggregator. Since we do not assume capital accumulation (and thus there is no investment demand), the consumption aggregator will be $C_t(\iota) = C_t = Y_t$, for all $\iota \in [0,1]$. However, both aggregators in (1) will in general be composed of unequal shares of individual varieties, due to the different prices of varieties in (2).

3.2. Firms. All monopolistically competitive firms follow with the same production function $Y_t(h) = \exp(\hat{z}_t) N_t(h)$, where $Y_t(h)$ is the output of firm h, $N_t(h)$ is the use of the labour aggregator by firm $h$and $\hat{z}_t$ is an aggregate productivity shock obeying the following AR(1) process:

$$\hat{z}_t = \alpha \hat{z}_{t-1} + u_t, \ u_t \sim N\left(0, \sigma_u^2\right). \tag{3}$$

Firms maximize the present value of monopolistic profits that are paid out to their owners (i.e. the households) in the form of dividends. The real dividend paid out by firm h at date t is the receipts from selling one unit of good h minus its production cost, i.e.

$$D_t(h) = (P_t(h) Y_t(h) - W_t N_t(h)) / P_t. \tag{4}$$

A firm h sets the selling price of its produced variety taking as given aggregate demand C_t, the general price and wage levels P_t and W_t, the production function $Y_t(h) = e^{\hat{z}_t} N_t(h)$, the demand curve for its own variety in (2)and the exogenous constraints on price setting it faces.

The price adjustment mechanism assumed here is similar to that in Christiano et al. (2005). Specifically, in each period there is an instantaneous probability $1-\psi_p \in (0,1)$ that a firm optimally resets the nominal price it charges. Non-optimized prices grow at the rate of last period's price inflation, which occurs with probability ψ_p. It may then be shown that the dynamics of the price level can be first-order approximated by the following 'New Keynesian Phillips Curve' (see Woodford, 2003, chap. 3):

$$\pi_t = \frac{1}{1-\beta} \pi_{t-1} + \frac{\beta}{1-\beta} E_t(\pi_{t+1}) + \kappa_p (\hat{\omega}_t - \hat{z}_t), \tag{5}$$

where π_t denotes the level-deviation of the inflation rate $P_t/P_{t-1} - 1$ from its steady state, $\hat{\omega}_t$ is the log-deviation of the real wage W_t/P_t from its steady state, $\beta \in (0,1)$ is the subjective discount factor of the representative household and $\kappa_p = \left(1 - \beta\psi_p\right)\left(1 - \psi_p\right)/\psi_p$. In (5), current inflation increases with the real unit production cost $\hat{\omega}_t - \hat{z}_t$, because of the markup pricing rule followed by monopolistically competitive firms. It also depends on past inflation through the indexation of non-optimized prices, as well as on future inflation, since re-optimizing firms set the price that will best keep their own selling price in line with the future general price level.[1]

3.3. Households. The instantaneous utility of household ι is given by:

$$u(C_t, H_t) - \upsilon(N_t(\iota)) = \frac{(C_t - H_t)^{1-\sigma}}{1-\sigma} - \frac{N_t(\iota)^{1+\eta}}{1+\eta}, \quad \sigma > 0, \eta > 0,$$

where C_t is the consumption aggregator (defined in (1)), $N_t(\iota)$ is labour supply and H_t is an external habit term that only depends on past aggregate consumption, i.e.

$$H_t = b\bar{C}_{t-1}, \ b \in (0,1),$$

[1] As we will discuss later, considering plausible forms of partial indexation (rather than full indexation) on past inflation hardly makes any difference for our quantitative results; for this reason, our benchmark analysis is based on (5) for describing the aggregate price dynamics.

where $\bar{C}_{t-1}$ is the aggregate past consumption and $C_t = \bar{C}_t$ in equilibrium. The type of habit formation posited here is similar to that in Alvarez (1998) and Boldrin et al. (2001), with the difference that the habit stock affects households' utility externally rather than internally.

We adopt the habit formation assumption essentially for two reasons. First, habits typically introduce sluggishness in the endogenous response of output to policy shocks. This is in line with empirical evidence (e.g. see Fuhrer, 2000) and is also relevant for asset prices through the way output fluctuations affect monopoly profits and thus the dividends paid out by firms. Second, specifying that habits enter as a difference (rather than as a ratio) in the households' utility function generates time-varying risk aversion; this will be an important ingredient for our analysis since it will naturally affect asset prices through changes in the expected excess returns at which dividends are discounted. The simplest form of habit formation that satisfies these properties is one with one lag only and habits being external to the representative household (i.e., the 'catching-up-with-the-Joneses' specification).

In every period, household ι chooses consumption, labour supply and asset holdings, taking goods prices and asset prices as given, so as to maximize:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(u\left(C_t, H_t\right) - \upsilon\left(N_t\left(\iota\right)\right)\right), \tag{6}$$

Households can transfer wealth across periods using both one-period nominal bonds and infinitely-lived shares, which are claims to the dividend flows paid out by firms. Nominal bonds are in zero net supply (since all households are identical ex ante) and the number of shares of each firm h is normalized to one. Households face the following budget constraint:

$$C_t + \frac{B_t}{P_t} + \int_0^1 S_t\left(h\right) Q_t\left(h\right) dh = \frac{W_t N_t}{P_t} + \frac{I_{t-1} B_{t-1}}{P_t} + \int_0^1 S_{t-1}\left(h\right)\left(Q_t\left(h\right) + D_t\left(h\right)\right) dh. \tag{7}$$

In (7), B_t and $S_t(h)$ denote the holdings of nominal bonds and shares of firm h by the representative household at the end of period t, respectively. I_{t-1} is the gross interest rate on nominal bonds from date $t-1$ to date t, and $Q_t(h)$ and $D_t(h)$ are the real price of a share of firm h and the dividend paid out by firm h, respectively, both expressed in terms of aggregate consumption units.

Let $\Lambda_t \equiv (C_t - H_t)^{-\sigma}$ denote the households' marginal utility of current consumption. Using expressions (6)-(7) to compute the optimal demands of households for bonds and share and then imposing the clearing of both asset markets, we find that asset prices must obey the following Euler equations:

$$\beta E_t \left(\frac{\Lambda_{t+1}}{\Lambda_t} \frac{I_t P_t}{P_{t+1}} \right) = 1, \tag{8}$$

$$\beta E_t \left[\frac{\Lambda_{t+1}}{\Lambda_t} \left(\frac{Q_{t+1}\left(h\right) + D_{t+1}\left(h\right)}{Q_t\left(h\right)} \right) \right] = 1, \text{ for all } h \in [0,1]. \tag{9}$$

Then, linearizing the bond Euler equation (8) around steady state and using the fact that $C_t = Y_t$ in equilibrium, yields the following New Keynesian IS curve

$$\hat{y}_t = \left(\frac{b}{1+b} \right) \hat{y}_{t-1} + \left(\frac{1}{1+b} \right) E_t\left(\hat{y}_{t+1}\right) - \left(\frac{1-b}{\sigma\left(1+b\right)} \right) E_t\left(i_t - \pi_{t+1}\right), \tag{10}$$

where i_t is the level-deviation of the nominal interest rate $I_t - 1$ from its steady state and $\hat{y}_t$ denotes the log-deviation of current output from its steady state. Equation (10) summarizes the determinant of current aggregate demand, which is affected by the real interest rate through intertemporal substitution in consumption, future aggregate demand due to consumption smoothing and past aggregate demand due to habit formation.

Household ι has monopolistic market power over the supply of labour variety $\iota$and sets the wage charged so as to maximize (6), taking as given his budget set (7), the general price and wage levels P_t and W_t, the demand curve for labour variety ι in (2) and the exogenous constraints on nominal wage adjustment. The assumed wage adjustment mechanism is similar to that of prices: households optimally reset nominal wages with probability $1 - \psi_w \in (0, 1)$ and let nominal wages grow at the rate of last period's wage inflation with probability ψ_w. The aggregate price dynamics is then first-order approximated by the following wage Phillips curve (see Woodford (2003) again for details):

$$\pi_t^w = \frac{1}{1-\beta}\pi_{t-1}^w + \frac{\beta}{1-\beta}E_t(\pi_{t+1}^w) + \kappa_w\left(\hat{s}_t - \hat{\omega}_t\right), \tag{11}$$

where π_t^w is the wage inflation rate, $\hat{s}_t$ is the log-deviation of the average marginal rate of substitution between leisure and consumption $S_t = \upsilon'\left(N_t\right)/u_1\left(C_t, H_t\right)$ from its steady state and

$$\kappa_w = \frac{\left(1-\psi_w\right)\left(1-\beta\psi_w\right)}{\left(1+\eta\theta_w\right)\psi_w}.$$

In equilibrium, we have $C_t = Y_t$, $H_t = C_{t-1}$ and $N_t = Y_t/Z_t$, so that $\hat{s}_t$ is given by:

$$\hat{s}_t = \left(\frac{\sigma}{1-b} + \eta\right)\hat{y}_t - \left(\frac{b\sigma}{1-b}\right)\hat{y}_{t-1} - \eta\hat{z}_t. \tag{12}$$

The explanations for expressions (11)–(12) are the same as that for (5). With monopolistically competitive labour markets, optimizing households wish to keep their wage markup intact and thus raise the wage charged in response to an increase in the consumption-leisure MRS relative to the current real wage (see Erceg et al., 2001). Past wage inflation indexes non-optimized wages and thus affects current inflation. The attempt by optimizing households to keep their wage in line with the (anticipated) general wage level generates a feedback from future to current wage inflation. Finally, the dynamics of the log-real wage, $\hat{\omega}_t$, are given by:

$$\hat{\omega}_t = \hat{\omega}_{t-1} + \pi_t^w - \pi_t. \tag{13}$$

3.4. Monetary Authority. The model is closed by specifying the way the central bank provides nominal anchor. In our baseline specification, we assume that the central bank reacts to current inflation and current output according to the following Taylor rule:

$$i_t = \gamma i_{t-1} + (1-\gamma)\left(\rho_\pi \pi_t + \rho_y \hat{y}_t\right) + \varepsilon_t, \tag{14}$$

where ρ_π and ρ_y are positive reaction coefficients, $\gamma \in (0, 1)$ reflects the degree of interest-rate smoothing by the central bank and $\varepsilon_t \sim N\left(0, \sigma_\varepsilon^2\right)$ is a nominal interest rate innovation, which by definition is unanticipated by private agents.

3.5. Dynamics of the Macro Variables. Equations (3), (5) and (10) – (14) define a backward-forward expectational dynamic system with seven equations, seven endogenous variables,($\hat{y}_t$, i_t, π_t, π_t^w, $\hat{s}_t$, $\hat{\omega}_t$, $\hat{z}_t$) and two exogenous shocks (u_t and ε_t) that fully characterize the dynamics of the macroeconomic aggregates. These expressions are summarized below for quick reference.

$$
\begin{aligned}
\hat{c}_t &= \hat{y}_t = \left(\frac{b}{1+b}\right)\hat{y}_{t-1} + \left(\frac{1}{1+b}\right)E_t\left(\hat{y}_{t+1}\right) - \left(\frac{1-b}{\sigma\left(1+b\right)}\right)E_t\left[i_t - \pi_{t+1}\right], && \text{(M1)}\\
i_t &= \gamma i_{t-1} + (1-\gamma)\left(\rho_\pi \pi_t + \rho_y \hat{y}_t\right) + \varepsilon_t, \quad \varepsilon_t \sim N\left(0, \sigma_\varepsilon^2\right), && \text{(M2)}\\
\pi_t &= \left(\frac{1}{1+\beta}\right)\pi_{t-1} + \left(\frac{\beta}{1+\beta}\right)E_t\left(\pi_{t+1}\right) + \kappa_p\left(\hat{\omega}_t - \hat{z}_t\right), && \text{(M3)}\\
\pi_t^w &= \left(\frac{1}{1+\beta}\right)\pi_{t-1}^w + \left(\frac{\beta}{1+\beta}\right)E_t\left(\pi_{t+1}^w\right) + \kappa_w\left(\hat{s}_t - \hat{\omega}_t\right), && \text{(M4)}\\
\hat{s}_t &= \left(\frac{\sigma}{1-b} + \eta\right)\hat{y}_t - \left(\frac{b\sigma}{1-b}\right)\hat{y}_{t-1} - \eta \hat{z}_t, && \text{(M5)}\\
\hat{\omega}_t &= \hat{\omega}_{t-1} + \pi_t^w - \pi_t, && \text{(M7)}\\
\hat{z}_t &= \alpha \hat{z}_{t-1} + u_t,\ u_t \sim N\left(0, \sigma_u^2\right). && \text{(M8)}
\end{aligned}
$$

4. Financial Markets and Asset Pricing

We now turn to the asset-pricing implications of the New Keynesian model just described. We are mostly interested in the impact and dynamic adjustments of stock prices following a policy shock, rather than in the stochastic properties of stock prices and returns per se. Therefore, we cannot simply recover unconditional first and second moments of those variables by subjecting our economy to a repeated sequence of policy and technology shocks. Rather, we must keep track of the households' information set in every period, since this information set is used by households to form *conditional* expectations for all future values of the variables relevant for the determination of stock prices (i.e. dividends, real interest rates and expected excess returns).

The most tractable way of doing this is to use the households' rational expectations of those values that are based on the VAR representation of the model's linearized dynamics. The linearized dynamic system remains valid as long as fluctuations around the deterministic steady state are sufficiently small. However, the standard way of applying this approach has a major drawback: by simply linearizing the dynamic system around its steady state, we essentially lose second-order information that enters expected returns and may significantly affect the reaction of stock prices to policy shocks. This point is particularly relevant here since our habit formation specification precisely allows for variations in equilibrium expected excess returns.

The approach we propose consists of combining these linear and nonlinear elements in the following way. First, we consider a usual first-order approximation of dividends and real interest rates around their steady states, which allows these variables to enter the vector of endogenous state variables along with the macro variables used in the previous section. Then, we combine the lognormal framework (e.g. Campbell, 2003) with a particular linear approximation of the stochastic discount factor that allows us to express expected excess returns as a linear function of the state vector and preserve some of the second order information relevant for the determination of asset prices. Finally, we use the VAR dynamics of the state vector to compute rational forecasts of dividends, real interest rates and expected excess returns, which can then be inserted into a log-linear present value formula to recover current equilibrium stock prices.[2] Each of these steps are described in detail in the following subsections.

4.1. Dividends and the risk-free rate. Substituting the demand for variety h from (2) into (4) and using the fact that $N_t(h) = e^{-\hat{z}_t} Y_t(h)$, we may rewrite the dividend paid out by firm h at date t as:

$$D_t(h) = \left(\frac{P_t(h)}{P_t} - e^{-\hat{z}_t} \frac{W_t}{P_t} \right) \left(\frac{P_t(h)}{P_t} \right)^{-\theta_p} Y_t.$$

Log-linearizing this yields the following

$$\hat{d}_t(h) = \hat{y}_t + (1 - \theta_p)(\hat{\omega}_t - \hat{z}_t) \equiv \hat{d}_t. \tag{15}$$

Note that the relative prices of varieties $P_t(h)/P_t$, only have second-order effects on firms' profits and thus disappear from the linearized dividend equation (15). This property, i.e. that all firms approximately pay out the same dividend stream, together with the fact any dividend stream is valued using a single pricing kernel (thanks to full consumption insurance), will allows us later on to straightforwardly aggregate firms' share prices into a single broad stock market index.

Let us now turn to the determination of the real interest rate of this economy. In principle, the real interest rate on a risk-free one-period bond that pays out one unit of the consumption good can be decomposed into the contributions of the nominal bond rate, expected inflation and a correction term reflecting the negative compensation for not bearing the inflation risk associated with holding nominal one-period bonds. In this paper, however, we take a first-order approximation to this risk-free interest rate and thus write its log-deviation from steady state as:

$$\hat{r}^f_{t+1} = r^f_{t+1} + \ln \beta \approx i_t - E_t(\pi_{t+1}), \tag{16}$$

where r^f_{t+1} is the log risk-free rate and $r^f = -\ln \beta$ is its value at the deterministic steady state (see (8)). Note that taking into account the second-order properties of the risk-free rate is straightforward but cumbersome and does not affect our quantitative results significantly.

4.2. Expected excess returns. Let $M_{t+1} = \beta \Lambda_{t+1}/\Lambda_t$ be the households' marginal rate of intertemporal substitution or the unique stochastic discount factor (SDF) of this economy.[3] Also, let

$$R^e_{t+1}(h) = (Q_{t+1}(h) + D_{t+1}(h))/Q_t(h) \tag{17}$$

be the return on holding a share of firm h from date t to date $t+1$. Then, (9) may equivalently be written as

$$E_t\left(M_{t+1} R^e_{t+1}(h)\right) = 1.$$

[2] Alvarez (1998) and Lettau (2003) also suggest ways in which linear and nonlinear elements can jointly be used to derive closed-form solutions to general equilibrium asset-pricing models. However, these approaches cannot be applied here. In Alvarez (1998), perfect competition allows for identifying ex post asset returns with the marginal product of capital, which can then be directly extracted from the linearised macroeconomic block of the model. This is impossible here since we are pricing a stream of pure profits, so that we must go instead from prices (given by the present value formula) to ex post returns (which are a weighted sum of stock price growth and dividend growth). Lettau's (2003) approach, on the other hand, can only be used when expected excess returns are constant.

[3] The two are identical due to full consumption insurance.

We now make several standard assumptions about the distribution of asset returns and aggregate consumption (those will turn out to be approximately true in equilibrium, provided that asset price fluctuations are small enough for our approximate asset pricing formula to be valid). In particular, we assume that ex post asset returns, $R^e_{t+1}(h)$and future consumption, C_{t+1}, are jointly conditionally homoskedastic and log-normally distributed. This implies that the SDF is approximately log-normally distributed. Its conditional heteroskedasticity, that results from the assumed utility function, generates a time varying price of risk and affects equilibrium excess returns and prices. Using the fact that both stochastic shocks in the economy are log-normal, the above expression can be rewritten in logs as:

$$E_t\left(m_{t+1}\right) + E_t\left(r^e_{t+1}(h)\right) + \frac{1}{2}\left(\sigma^2_h + \sigma^2_{m,t} + 2\sigma_{hm,t}\right) = 0, \tag{18}$$

where $m_{t+1} \equiv \ln M_{t+1}$ is the log SDF, $r^e_{t+1}(h) \equiv \ln R^e_{t+1}(h)$ is the log-stock return on share of firm h, $\sigma^2_{m,t} \equiv \mathrm{var}_t\left(m_{t+1}\right)$ is the conditional variance of the log SDF, $\sigma^2_h \equiv \mathrm{var}_t\left(r^e_{t+1}(h)\right)$ is the conditional variance of the log stock return and $\sigma_{hm,t} \equiv \mathrm{cov}\left(r^e_{t+1}(h), m_{t+1}\right)$ the conditional covariance of the log-stock return with the log SDF. σ^2_h is constant by assumption and thus not indexed by t.

From (18), the expected log-excess return on a share of firm h is then given by (see Campbell, 2003):

$$E_t(r^e_{t+1}(h) - r^f_{t+1}) = -\sigma_{hm,t} - \frac{\sigma^2_h}{2}. \tag{19}$$

Apart from the role of precautionary savings, which foster aggregate savings and thus lower excess returns (captured by the term $\sigma^2_h/2$), expression (19) reflects the usual pricing of systematic payoff risk in complete markets general equilibrium economies. For example, an asset payoff that is highly correlated with aggregate consumption provides a poor hedge against consumption fluctuations and thus commands high expected excess returns; this effect is reflected by the negative correlation between future marginal utility of consumption and the asset return and thus a high value of $-\sigma_{hm,t}$ in (19).

Let $\hat{r}^e_{t+1}$ be the deviation of the log expected return *from the deterministic steady state* where all shocks are set to zero at all times. Along this steady state, there is no risk premium and we have $r^e_{t+1}(h) = r^f = -\ln\beta$; we may then rewrite (19) in terms of deviations from steady state as follows:

$$E_t(\hat{r}^e_{t+1}(h) - \hat{r}^f_{t+1}) = -\sigma_{hm,t} - \frac{\sigma^2_h}{2}. \tag{20}$$

Excess equity returns in (20) affect asset prices through the discounting of dividend streams. Thus we need to determine the two components of the right-hand-side of (20) in order to analyze their effects on stock prices.

We start by deriving an expression for $\sigma_{hm,t}$. As explained earlier, going from excess returns, real interest rates and dividends to equilibrium stock prices requires forming VAR-based forecasts of all future values of these underlying determinants. We thus aim at expressing the time-varying covariance term in (20) as a function of variables that can be forecasted from the macroeconomic block of the model, while at the same time capturing the role played by time-varying risk aversion.

For this purpose, first let

$$\Lambda_{t+1} = (C_{t+1} - bC_t)^{-\sigma} \equiv \Lambda(C_{t+1}, C_t)$$

and

$$\Theta_t = -\frac{C_t u_{11}(C_t, C_{t-1})}{u_1(C_t, C_{t-1})} = -\frac{C_t \Lambda_1(C_t, C_{t-1})}{\Lambda(C_t, C_{t-1})} = \frac{\sigma C_t}{C_t - bC_{t-1}} = \frac{\sigma}{1 - be^{-\Delta \hat{c}_t}} \tag{21}$$

be the households' risk aversion coefficient at date t. Taking a first-order Taylor expansion of $\Lambda(C_{t+1}, C_t)$ around any point (X, Y) that is sufficiently close to (C_{t+1}, C_t) we obtain

$$\Lambda(C_{t+1}, C_t) \approx \Lambda(X, Y) + \Lambda_1(X, Y)(C_{t+1} - X) + \Lambda_2(X, Y)(C_t - Y).$$

Provided that consumption is sufficiently smooth, so that C_t is sufficiently close to C_{t-1}, we may take $(X, Y) = (C_t, C_{t-1})$ as the point around which we linearize.[4] Then, we can rearrange this to get:

$$\frac{\Lambda(C_{t+1}, C_t) - \Lambda(C_t, C_{t-1})}{\Lambda(C_t, C_{t-1})} \approx \frac{\Lambda_1(C_t, C_{t-1}) C_t}{\Lambda(C_t, C_{t-1})} \left(\frac{C_{t+1} - C_t}{C_t} \right) + \frac{\Lambda_2(C_t, C_{t-1}) C_{t-1}}{\Lambda(C_t, C_{t-1})} \left(\frac{C_t - C_{t-1}}{C_{t-1}} \right).$$

This expression essentially approximates marginal utility growth (left hand side) with an appropriate weighted sum of current and past consumption growth (right hand side). We can now rewrite the marginal utility growth as:

$$\Delta \ln \Lambda(C_{t+1}, C_t) \approx -\Theta_t (\Delta \hat{c}_{t+1} - \Delta \hat{c}_t) - \sigma \Delta \hat{c}_t. \tag{22}$$

The effect of consumption growth on risk aversion follows from our assumed utility function; for example, when consumption falls relative to past consumption, so that $\Delta \hat{c}_t < 0$, then the local curvature of the utility function increases, thereby making households more risk averse. Equation (22) implies that innovations to the stochastic discount factor can be approximately written as

$$m_{t+1} - E_t m_{t+1} = \Delta \ln \Lambda(C_{t+1}, C_t) - E_t(\Delta \ln \Lambda(C_{t+1}, C_t)) = -\Theta_t (\hat{c}_{t+1} - E_t \hat{c}_{t+1}).$$

We can therefore approximately express the conditional covariance between the log SDF and the log-stock return as:

$$\sigma_{hm,t} \approx E_t \left[-\Theta_t (\hat{c}_{t+1} - E_t(\hat{c}_{t+1})) (r^e_{t+1}(h) - E_t r^e_{t+1}(h)) \right] = -\sigma_{hc} \Theta_t,$$

where σ_{hc} is the *constant* conditional covariance of log-consumption and log-asset returns (recall that the two variables are jointly conditionally homoskedastic by assumption).

We will next need to determine the second component of the right-hand-side of (20), i.e. $\sigma^2_h/2$. We defer this derivation to a later section, where we explain how to retrieve σ_{hc} and σ^2_h jointly. Substituting our approximated $\sigma_{hm,t}$ into (20), we find that expected excess returns, in terms of log-deviations from the deterministic steady state, are approximately given by:

$$E_t(\hat{r}^e_{t+1}(h) - \hat{r}^f_{t+1}) = \sigma_{hc} \Theta_t - \frac{\sigma^2_h}{2}, \tag{23}$$

[4] This approximation is in fact more accurate than linearising $\Lambda(C_{t+1}, C_t)$ around steady state, since consumption persistence implies that C_t is at least as close to C_{t-1} as it is to its steady state value.

which is only a function of $\Delta\hat{c}_t$ (see (21)). In short, (23) states that rising current risk aversion, Θ_t, raises expected excess returns and therefore it increases the premium required for holding risky shares. The overall effect is scaled by the consumption risk associated with holding share h, i.e. the covariance of ex post returns with next period's consumption σ_{hc}.

4.3. Stock prices. Having derived expressions for all the underlying determinants of stock prices (i.e., dividends, risk-free rates and expected excess returns), we may now turn to the implied equilibrium stock prices. This may be done by using the log-linear present value model of Campbell and Shiller (1988). More specifically, linearizing (17) around the deterministic steady state and using (15), we may write ex post log-stock returns as follows:

$$\hat{r}^e_{t+1}(h) = \beta\hat{q}_{t+1}(h) + (1-\beta)\hat{d}_{t+1} - \hat{q}_t(h), \tag{24}$$

where $\hat{q}_t(h)$ denotes the log-deviation of firm hs share price from the deterministic steady state. Note that the unconditional means of $\hat{r}^e_t(h)$ and $\hat{q}_t(h)$ are different from zero here, since holding risky shares requires a positive average returns premium (i.e. $E\left(\hat{r}^e_{t+1}(h)\right) > 0$) that depresses average stock prices (i.e. $\hat{q}_t(h) < 0$), provided that the portfolio risk effect in (23) dominates the precautionary savings effect (i.e. $\sigma_{hc}\Theta_t - \sigma^2_h/2 > 0$). However, the approximation in (24) will remain valid as long as fluctuations are sufficiently small, that is as long as $E\left(\hat{r}^e_{t+1}(h)\right)$ is sufficiently close to $E(\hat{r}^f_{t+1}) = 0$. On average, we have $E\left(\hat{r}^e_{t+1}(h)\right) = -(1-\beta)E(\hat{q}_t(h))$ since $E(\hat{d}_{t+1}) = 0$ in (24).

Solving (24) for $\hat{q}_t(h)$, substituting it into (20) and applying the expectation operator on both sides, we get

$$\hat{q}_t(h) = \beta E_t(\hat{q}_{t+1}(h)) + (1-\beta)E_t\hat{d}_{t+1} - \hat{r}^f_{t+1} - \sigma_{hc}\Theta_t + \frac{\sigma^2_h}{2}. \tag{25}$$

Finally, iterating (25) and rearranging under the condition that no rational bubble occurs (i.e., $\lim_{n\to\infty}\beta^n\hat{q}_{t+n}(h) < \infty$), the share price of firm h may be written as

$$\hat{q}_t(h) = \frac{\sigma^2_h/2 - \sigma_{hc}\bar{\Theta}}{1-\beta} + (1-\beta)\sum_{j=0}^{\infty}\beta^j E_t(\hat{d}_{t+1+j}) - \sum_{j=0}^{\infty}\beta^j E_t(\hat{r}^f_{t+1+j}) - \sigma_{hc}\sum_{j=0}^{\infty}\beta^j E_t(\hat{\Theta}_{t+j}), \tag{26}$$

where $\bar{\Theta} = \sigma/(1-b)$ is the mean risk aversion coefficient and $\hat{\Theta}_t = \Theta_t - \bar{\Theta}$ its level-deviation from the mean. Equation (26) is intuitive: stock prices increase with future dividends (second term), but decrease with current and future risk-free rates (third term) and expected excess returns (fourth term). The constant (first term) just reflects the difference between the average stock price along the stochastic equilibrium and its value at the deterministic steady state, around which the linearization was taken. For example, a greater covariance between consumption and returns, σ_{hc}, makes asset h more risky and thus lowers its average value, relative the deterministic steady state; but higher return risk fosters precautionary savings, which tends to raise asset demand and prices, relative to the deterministic steady state. All summation terms are centered around their unconditional mean. The corresponding centered asset-price variable is simply $\tilde{q}_t(h) \equiv \hat{q}_t(h) - \left(\sigma^2_h/2 - \sigma_{hc}\bar{\Theta}\right)/(1-\beta)$.

Note that expression (26) is not quite yet operative because stock prices actually appear on both sides of it: the covariance term σ_{hc} determines how time-variations in risk aversion affect prices, but σ_{hc} is not a deep parameter of the model but an endogenous parameter that depends on equilibrium asset prices. Similarly, both σ_h^2 and σ_{hc} enter the constant term while they are endogenously determined in equilibrium. In perfectly competitive economies, the ex post return on stocks would be given by the marginal product of capital and its first and second moments could be directly extracted from the macroeconomic block of the model (e.g., Alvarez, 1998). This cannot be done in our imperfectly competitive model, so we must recover ex post return from dividends and prices using (15), (24) and (26). However, we show in the next section that under certain assumptions, there is only one possible combination of σ_h^2 and σ_{hc} that is consistent with (26). This can be recovered from (26) and the VAR representation of the macro dynamics of the model.

Finally, note that since dividends and risk-free rates in (26) are identical across firms in equilibrium, so are the parameters σ_h^2 and $\sigma_{hc}$and the implied prices $\hat{q}_t(h)$. We may thus aggregate share prices into a single price index, i.e., $\hat{q}_t = \hat{q}_t(h)$, for all $h \in [0,1]$.

4.4. Dynamics of the Financial Variables. We summarize the expressions that determine the approximate equilibrium evolution of the financial variables:

$$\hat{q}_t = \frac{\sigma_h^2/2 - \sigma_{hc}\bar{\Theta}}{1-\beta} + (1-\beta)\sum_{j=0}^{\infty}\beta^j E_t\hat{d}_{t+1+j} - \sum_{j=0}^{\infty}\beta^j E_t\hat{r}^f_{t+1+j} - \sigma_{hc}\sum_{j=0}^{\infty}\beta^j E_t\left(\hat{\Theta}_{t+j}\right), \tag{F1}$$

$$\hat{r}^f_{t+1} = i_t - E_t(\pi_{t+1}), \tag{F2}$$

$$\hat{d}_t = \hat{y}_t + (1-\theta_p)\hat{\omega}_t - (1-\theta_p)\hat{z}_t, \tag{F3}$$

$$\hat{r}^e_{t+1} = \beta\hat{q}_{t+1} + (1-\beta)\hat{d}_{t+1} - \hat{q}_t. \tag{F4}$$

5. Solution Strategy

Our goal is to compute the reaction of stock prices to an unexpected policy shock, where the three channels emphasized above (dividends, real interest rates, excess returns) play an active role in generating this reaction. We thus proceed as follows. First, we compute all the first and second moments of the variables required for computing asset prices, assuming that only technology shocks occur in every period, while interest rate shocks are completely shut down; in particular, second moments can be computed directly from the VAR representation of the model's macro block, (M1)-(M8). Then, we compute and simulate the impulse responses of all relevant variables to a once-occurring, unexpected shock to the nominal interest rate (i.e. an ε_t shock), in an economy where technology shocks are expected to occur in every period. Since technology shocks are the only shocks that repeatedly influence prices, consumption growth is perfectly correlated with asset returns and we have $\sigma_{hc} = \sigma_h\sigma_c$. This assumption is particularly important when we determine the endogenous parameters σ_{hc} and σ_c.

The first step is to compute the joint dynamics of all variables that appear in the systems (M1)-(M8) and (F2)-(F3). These variables are collected into a vector

$$\chi_t = [\hat{y}_t,\ i_t,\ \pi_t,\ \pi_t^w,\ \hat{s}_t,\ \hat{w}_t,\ \hat{d}_t,\ \hat{r}^f_{t+1},\ \hat{z}_{1t},\ \hat{z}_{2t}]',$$

where $\hat{z}_{1t} = \hat{z}_t$ and $\hat{z}_{2t} = 0\hat{z}_{2t-1} + \varepsilon_t$. Note that $\hat{r}^f_{t+1} = \hat{\imath}_t - E_t(\pi_{t+1})$ is the ex ante real interest rate and is thus known at date t. Then, for given values of all the deep parameters of the model we can solve the linear dynamic system (M1)-(M8) and (F2)-(F3) by applying the method of undetermined coefficients. The solution of the system can be written in the following VAR representation:[5]

$$\chi_t = F\chi_{t-1} + L\epsilon_t, \tag{27}$$

where F and L are conformable matrices and where:

$$\epsilon_t = \begin{bmatrix} u_t \\ \varepsilon_t \end{bmatrix} \sim N(0, \Sigma), \ \Sigma = \begin{bmatrix} \sigma_u^2 & 0 \\ 0 & \sigma_\varepsilon^2 \end{bmatrix}.$$

Next, we want to use (27) to derive an expression for the stock price as a function of present and past values of χ. At this stage, all sequences that enter the summation terms in (26) can be forecasted using (27), apart from $\hat{\Theta}_t$ which is a nonlinear function of $\Delta\hat{c}_t$ (see (21)). However, linearizing (21) and using the fact that $\hat{c}_t = \hat{y}_t$, we can write the centered absolute risk aversion coefficient as:

$$\hat{\Theta}_t \approx -\left(\frac{\bar{\Theta}b}{1-b}\right)\Delta\hat{y}_t,$$

which can now also be extracted from (27).

Now let $\mathbf{e}_k$ denote a column indicator vector that picks generic variable k in the vector χ_t, that is a vector such that $k_t = \mathbf{e}_k'\chi_t$. Expectations of future dividends, risk-free rates and risk aversion coefficients are then given by

$$\begin{aligned} E_t(\hat{d}_{t+1+j}) &= \mathbf{e}_d' F^{j+1}\chi_t, \\ E_t(\hat{r}^f_{t+1+j}) &= \mathbf{e}_{r^f}' F^j \chi_t, \\ E_t(\hat{\Theta}_{t+1+j}) &= \mathbf{e}_y' \left(F^{j+1} - F^j\right)\chi_t, \end{aligned}$$

for $j = 0, 1, \ldots$. Then, substituting these sequences into (26), we can now rewrite the value of the stock market index only as a function of constants and the current and last period's value of the state vector χ:

$$\begin{aligned} \hat{q}_t = & \underbrace{\frac{\sigma_h^2/2 - \bar{\Theta}\sigma_h\sigma_c}{1-\beta}}_{constant} + \underbrace{(1-\beta)\,\mathbf{e}_d'\,(1-\beta F)^{-1} F\chi_t}_{dividends\ contribution} \\ & - \underbrace{\mathbf{e}_{r^f}'\,(1-\beta F)^{-1}\chi_t}_{real\ interest\ rates\ contribution} - \underbrace{\frac{b\bar{\Theta}\sigma_h\sigma_c}{(1-b)}\mathbf{e}_y'\left[\chi_{t-1} - (1-\beta)(1-\beta F)^{-1}\chi_t\right]}_{excess\ returns\ contribution}. \end{aligned} \tag{28}$$

[5] Another way of expressing the log-linear macro block (e.g., Christiano, 2002) is

$$\tilde{\chi}_t = A\tilde{\chi}_{t-1} + B\tilde{z}_t$$

where $\tilde{\chi}_t$ is χ_t but without $\hat{z}_{1t}$, $\hat{z}_{2t}$ and $\tilde{z}_t = P\tilde{z}_{t-1} + \epsilon_t$ is the vector of autoregressive shocks. Here however, it is more convenient to write the decomposition in terms of white noise shocks, so that we can work with the expectations more easily. This can be done by writing

$$\chi_t = \begin{pmatrix} \tilde{\chi}_t \\ \tilde{z}_t \end{pmatrix} = \underbrace{\begin{pmatrix} A & BP \\ \mathbf{O}_{2\times 7} & P \end{pmatrix}}_{=F} \begin{pmatrix} \tilde{\chi}_{t-1} \\ \tilde{z}_{t-1} \end{pmatrix} + \underbrace{\begin{pmatrix} B \\ I_2 \end{pmatrix}}_{=L} \begin{pmatrix} u_t \\ \varepsilon_t \end{pmatrix}$$

The last step in computing equilibrium stock prices is to determine σ_c and σ_h. First note that we can easily retrieve σ_c^2 from the linearized macro block by rewriting $\hat{c}_t = \hat{y}_t = \mathbf{e}_y'\chi_t$ and using (27) to get

$$\sigma_c^2 = E_t\left[\left(\hat{y}_{t+1} - E_t\hat{y}_{t+1}\right)^2\right] = \mathbf{e}_y' L\Sigma L'\mathbf{e}_y = L_{11}^2\sigma_u^2, \tag{29}$$

where L_{11} is the $(1,1)$ element of matrix L, corresponding to the elasticity of output with respect to the technology innovation. We have used the fact that σ_ε^2 is zero; given the experiment we are performing this is legitimate, since there is only a once occurring monetary shock. Regarding σ_h^2, we first rewrite (28) as

$$\hat{q}_t = \tau_0 + \tau_1'\chi_t + \tau_2'\chi_{t-1},$$

where

$$\begin{aligned}
\tau_0 &= \frac{\sigma_h^2/2 - \bar{\Theta}\sigma_h\sigma_c}{1-\beta}, \\
\tau_1' &= (1-\beta)\frac{b\bar{\Theta}\sigma_h\sigma_c}{1-b}\mathbf{e}_y'\left(I-\beta F\right)^{-1} - \mathbf{e}_{rf}'\left(I-\beta F\right)^{-1} + (1-\beta)\,\mathbf{e}_d'\left(I-\beta F\right)^{-1}F, \\
\tau_2' &= -\frac{b\bar{\Theta}\sigma_h\sigma_c}{1-b}\mathbf{e}_y'.
\end{aligned}$$

Then, from (24) ex post returns innovations are given by:

$$\begin{aligned}
r_{t+1}^e(h) - E_t\left(r_{t+1}^e(h)\right) &= \beta\left(\hat{q}_{t+1} - E_t\hat{q}_{t+1}\right) + (1-\beta)\left(\hat{d}_{t+1} - E_t(\hat{d}_{t+1})\right) \\
&= \left(\beta\tau_1 + (1-\beta)\,\mathbf{e}_d\right)'\left(\chi_{t+1} - E_t\chi_{t+1}\right).
\end{aligned}$$

Taking the conditional variance operator on both sides of the latter equation, we find that σ_h^2 is given by

$$\sigma_h^2 = \left(\beta\tau_1 + (1-\beta)\,\mathbf{e}_d\right)' L\Sigma L'\left(\beta\tau_1 + (1-\beta)\,\mathbf{e}_d\right). \tag{30}$$

Since τ_1 is linear in σ_h, the above is a linear equation in σ_h^2 and can thus be easily solved to retrieve σ_h^2, once we have evaluated the matrices F and L from the rest of the parameter values.

All terms in (28) except for χ_t are now pinned down by (21), the matrices F and L in (27)and the expressions for σ_h^2 and σ_c^2 given by (29) and (30). Moreover, χ_t is endogenously determined by the exogenous shock vector through (27). We thus have all the elements necessary for the computation of the impact and propagation of an interest rate shock on the stock market, as well as for its decomposition into the relative contributions of the three underlying stock price determinants. The experiment we make is thus the following. We start off from a point in time $t = T-1$ where all variables are at their unconditional mean, so that $\chi_{T-1} = 0$ and thus $\hat{q}_{T-1} = \left(\sigma_h^2/2 - \bar{\Theta}\sigma_h\sigma_c\right)/(1-\beta)$. At date T, a once occurring unexpected policy shock ε_T occurs that raises the *level* of the interest rate by 1%, i.e. a shock in (14), that generates $\Delta i_T = 1$.[6]

We then compute the instantaneous stock price growth triggered by this policy change. We also plot the dynamic adjustment of stock prices, $\hat{q}_{T+j}$, $j = 1, 2, ...$, as well as that of the rest of the

[6] The size of this shock can be determined once the parameter values are set (see next section).

IES coefficient	$1/\sigma =$	1.00	Responsiveness to inflation	$\rho_\pi =$	1.50
Labour supply coef.	$\eta =$	0.00	Responsiveness to output	$\rho_y =$	0.60
Habit persistence	$b =$	0.80	Fraction of unchanged prices	$\psi_p =$	0.60
Discounting	$\beta =$	0.99	Fraction of unchanged wages	$\psi_w =$	0.90
Interest rate persistence	$\gamma =$	0.85	Elast. of demand for goods	$\theta_p =$	4.00
Tech shock persistence	$\alpha =$	0.99	Elast. of demand for labour	$\theta_w =$	4.00
Shock for technology	$\sigma_u =$	$\lvert\sigma_c/L_{11}\rvert$	Phillips	$\kappa_p =$	see (5)
Variance of r_t^e	$\sigma_h^2 =$	see (30)	Wage Phillips	$\kappa_w =$	see (11)
Variance of c_t	$\sigma_c^2 =$	see (29)			

Table 2: Baseline parameterisation.

variables of interest and finally decompose $\Delta\hat{q}_T$ into its three components in (28). This is given by

$$\mathcal{M}_q(\varepsilon_T) \equiv \Delta\hat{q}_T = \underbrace{(1-\beta)\,\mathbf{e}'_d\,(I-\beta F)^{-1}\,FL\begin{bmatrix}0\\ \varepsilon_T\end{bmatrix}}_{\textit{dividend contribution}} - \underbrace{\mathbf{e}'_{rf}\,(I-\beta F)^{-1}\,L\begin{bmatrix}0\\ \varepsilon_T\end{bmatrix}}_{\textit{risk free rate contribution}}$$
$$\underbrace{+\frac{\bar{\Theta}\sigma_h\sigma_c b}{(1-b)}\mathbf{e}'_y\,(1-\beta)\,(I-\beta F)^{-1}\,L\begin{bmatrix}0\\ \varepsilon_T\end{bmatrix}}_{\textit{risk aversion contribution}}. \tag{31}$$

The multipliers obtained in this way are consistent with the way the evidence is reported, since the latter documents the effect of a level-variation of the Central Bank's nominal interest rate (e.g., a 25 basis points increase) on the growth of the stock market index (e.g., a fall of the index by 1%).

6. Results

6.1. Baseline Parameterization. We assume a quarterly specification for the parameters of the model. Our baseline parameterization is put forth in Table 2; we discuss each of these parameters in turn. The parameter σ is typically assumed to vary between 1 and 5 in most of the macroeconomics literature. We choose $\sigma = 1$ which is more in line with the business cycle literature. Next, we set the parameter η to be 0, which is a common assumption. As it will turn out, the choice of η does not significantly affect the results. For the discounting we choose $\beta = 0.99$ which is typical for quarterly calibrations. The habit parameter is set to $b = 0.8$ following existing literature such as Alvarez (1998). Turning to the parameters of the Taylor rule, for the Volker-Greenspan era, a robust estimate for the US is around $\gamma = 0.85$. For example, Clarida, Gali and Gertler (2000) calculate $\gamma \in [0.73,\ 0.88]$ depending on which sample/measure is used. Judd and Rudebush (1998) suggest $\gamma \in [0.56,\ 0.73]$, Amato and Laubach (1999) give $\gamma \in [0.78,\ 0.92]$ and Kozicki (1999) gives $\gamma \in [0.75,\ 0.82]$.

Conventional estimates for the response parameters in the Taylor rule are $\rho_\pi \approx 1.5$ and $\rho_y < 1.0$, but estimates may vary substantially from one paper to the other. For example Judd and Rudebush (1998) estimate $\rho_\pi \in [1.46,\ 1.69]$ and $\rho_y \in [0.36,\ 0.99]$, Clarida et al. (2000) give $\rho_\pi \in [1.97,\ 2.15]$ and $\rho_y \in [0.55,\ 1.49]$ and Kozicki (1999) gives $\rho_\pi \in [1.05,\ 1.66]$ and $\rho_y \in [0.42,\ 0.52]$. We choose $\rho_\pi = 1.5$ and $\rho_y = 0.6$ in our benchmark experiment.

The elasticities of demand for goods and labour are set to $\theta_p = \theta_w = 4$. In the literature, these parameters vary between 3 and 10, although the estimates of Christiano et al. (2005) have a larger variation. Finally, we set the degree of price rigidity ψ_p to 0.6 and the degree of wage rigidity ψ_w to 0.9. Highly rigid wages ensure that firm profits and thus dividends are procyclical.

There are four more parameters to be determined, namely α, σ_u, σ_c and σ_h^2. As already explained, σ_h and σ_c are inferred from expressions (30)–(29). For a given α, we may either fix σ_u and calculate σ_c or vice versa. We choose to fix σ_c and match it to a number from the data, so that we can calculate the corresponding variance for the technology shock. We use $\sigma_c = 0.0045$, which is the implied quarterly standard deviation for US data, for the period 1970.1-1998.3 (see Campbell, 2003, table 4). What remains to be determined is a combination of α and σ_u that gives $\sigma_c = 0.0045$. As it will turn out, the size of these parameters is insignificant for the analysis that we are interested in here and only matters to the extent that they should be sensible and also yield a sensible value for σ_h. One such combination is given by setting $\alpha = 0.99$. Given the rest of our parameterization, the standard deviation of the technology is given by $\sigma_u = 0.0279$. This is within reasonable limits: the literature reports numbers for σ_u between 0.008 and 0.04 (see Wouters and Smets, 2003, Danthine and Kurman, 2004, Collard and Dellas, 2005 and Rabanal and Rubio-Ramirez, 2005). The persistence parameter is somewhat higher than the usually reported numbers around 0.96, but nevertheless, it generates a σ_h that is of the same order of magnitude as the one observed in the data. σ_h^2 can then be computed using (30) and the implied σ_h is 0.0220. This is about one third of the one observed in the data (Campbell, 2003, table 4).

6.2. Results. In order to simulate the reaction of stock prices to a one percentage point decrease in the central bank rate and generating impulse-response functions, we first need to calculate the size of the shock ε_T that would generate such a change. We have that

$$1 = \Delta i_T = \mathbf{e}_i' L \epsilon_T = L_{22} \varepsilon_T \Longrightarrow \varepsilon_T = 1/L_{22},$$

where L_{22} is the elasticity of the nominal interest rate with respect to the monetary innovation. Moreover, we want the effect of this increase to the stock prices to be $-x\%$, where $x \in [2.55, 6.81]$. For this purpose we use as a measure of the impact the stock price multiplier calculated earlier, evaluated at $\varepsilon_T = 1/L_{22}$. This is because the percentage change in the stock price is

$$x\% \approx \Delta \ln Q_T = \Delta \hat{q}_T = \mathcal{M}_q \left(1/L_{22}\right).$$

Figure 1 provides the impulse response functions of all variables of interest following a one percentage point increase in the nominal interest rate. Table 3 gives the proportional change in stock prices and ex post excess returns following this shock, as well as the breakdown of those in the three channels. The corresponding effect on ex post excess returns is obtained as a weighted average of price and dividend changes, i.e. $\Delta \hat{r}_T^e (h) = \beta \Delta \hat{q}_T + (1 - \beta) \Delta \hat{d}_T$.

The dynamic adjustment of macroeconomic variables to an interest rate shock is roughly consistent with empirical impulse-responses (e.g. Christiano et al., 2005). The nominal interest rate rise is contractionary, with the consequence of lowering both price and wage inflation, the overall implication of both being a mildly procyclical real wage adjustment. Importantly, staggered wage adjustment generates procyclical profits and dividends, as is consistent with the data; with fully flexible wages, labour market adjustments in the face of a falling labour demand would cause real wages and thus firms production costs to shoot down, thereby generating countercyclical profits

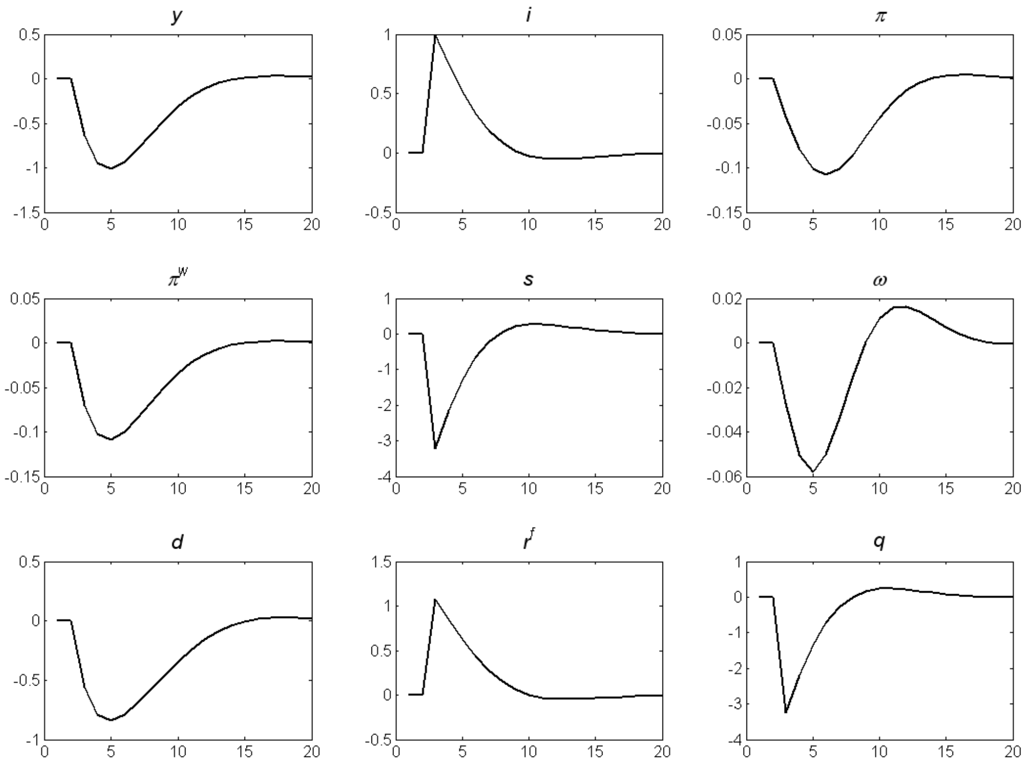

Figure 1: Dynamic responses of key variables to a one percentage point increase in the nominal interest rate.

and dividend payments.

Our baseline calibration generates a stock market impact multiplier of -3.2322, which is well inside the range of available empirical estimates, which range between -2.55 and -6.81. Although this range may appear to be large and easy to fall into quantitatively, recall that our parameters were chosen to be in line with the business cycle literature and were thus not designed to match the empirical value of the stock price multiplier. This result suggests that our baseline New Keynesian model provides a potential general equilibrium explanation for the observed stock market reaction to monetary policy shocks.

This conclusion deserves one cautious note, however. The decomposition of ex post excess returns following the policy shock that we obtain from the model (second line of Table 3) gives a surprisingly small role to variations in ex ante excess returns and a comparatively large one to changes in real interest rates. This prediction is in contrast to the VAR-based decomposition of empirical returns proposed by Bernanke and Kuttner (2005), which suggests that ex post excess returns variations following a nominal interest rate shock work predominantly through variations in ex ante excess returns, with a small contribution of real interest rate changes. However, Bernanke and Kuttner's result of a small real interest rate contribution naturally follows from their very quick estimated decay of the real interest rate following the policy shock: real rates deviations from the mean have a half-life of no more than two months and have completely died out after four. With

	Total Impact	Dividend Contr.	Real Int. Rate Contr.	Excess Returns Contr.
$\mathcal{M}_q\,(1/L_{22})$	-3.2322	-0.0473	-3.1848	-0.0001
$\mathcal{M}_{er}\,(1/L_{22})$	-3.2055	-0.0524	-3.1529	-0.0001

Table 3: Impact of 1 percent increase in the nominal interest rate on stock prices and excess returns.

such a rapid reversion of real rates, these are bound to have little effect on stock prices since the latter ultimately depend on the infinite sequences of future real rates, dividends and excess returns. Although this speed of adjustment is not necessarily inconsistent with previous estimates based on monthly data (see Bernanke and Mihov, 1998), the quarterly macroeconomic evidence on which our model builds typically documents a much slower reversion of real interest rates following an exogenous policy shock and thus a potentially larger role for such rates in explaining the stock market response to the shock (e.g., Christiano et al., 2005; Amato and Laubach, 2003, Boivin and Giannoni, 2002).

Bearing this in mind, there is one feature of our model suggesting that it may still be underestimating the role of ex ante excess returns when computing the reaction of ex post excess returns to a policy shock. This feature relates to the effect of habit formation on the aggregate consumption path. On one hand, habit formation causes time-variations in the local curvature of utility function, which generates countercyclical changes in risk aversion and thus procyclical changes in expected excess returns (at least with the difference specification adopted here); those in turn tend to magnify the stock market response to the shock. On the other hand, the very nature of habit formation limits the consumption response to exogenous shocks when consumption is optimally chosen by households, rather than being exogenously given as in pure exchange economies (e.g., Lettau and Uhlig, 2000). This limited endogenous response of consumption tends to smooth out the risk aversion reaction to the policy shock in (21), thereby lowering the implied reaction of expected excess returns, relative to the one that would prevail in a partial equilibrium setting.

6.3. Sensitivity Analysis. We have performed various robustness checks. After having varied all parameters of the model, we found our measure $\mathcal{M}_q\,(1/L_{22})$ to be very robust to changes in most of them. However, $\mathcal{M}_q\,(1/L_{22})$ is somewhat sensitive to some of them, notably the utility parameters σ and b and the Taylor rule parameters γ, ρ_π and ρ_y. Unsurprisingly, these are the parameters that have a direct effect on the behavior of consumption (utility parameters) and the real interest rates (Taylor rule parameters through their effects on nominal interest rates), i.e. the two variables that are relevant for understanding the breakdown of the impact of the shock on stock prices. Table 4 provides the ranges of variation of $\mathcal{M}_q\,(1/L_{22})$, as well as the variation of the size of the three channels, when we vary these parameters within admissible ranges. Even if plausible variations of the deep parameters in Table 4 may significantly affect the predicted value of the price multiplier, it turns out that most implied values of it stay within the interval consistent with empirical studies (with the exception of somewhat extreme values of ρ_y or γ). Similarly, such parameter changes do not alter the broad features of our impact decomposition, thus confirming the main conclusions drawn from the baseline specification. Finally, when calculating the relative contributions of each component to $\mathcal{M}_q\,(1/L_{22})$ we find that these change very little, reinforcing our claim that our main result is robust to parameter changes.

Parameters	$\mathcal{M}_q\,(1/L_{22})$	Dividends	Real Interest rate	Excess Returns
Baseline	-3.2322	-0.0473	-3.1848	-0.0001
$\sigma \in [1, 5]$	-3.2322 to -4.5453	-0.0473 to -0.0113	-3.1848 to -4.4891	-0.0001 to -0.0000
$b \in [0.5, 0.9]$	-3.1125 to -3.4646	-0.0436 to -0.0497	-3.0689 to -3.4148	-0.0000 to -0.0002
$\rho_\pi \in [1.05, 1.65]$	-3.3281 to -3.2063	-0.0500 to -0.0466	-3.2715 to -3.1597	-0.0066 to -0.0000
$\rho_y \in [0.3, 1.50]$	-3.8938 to -2.4148	-0.0722 to -0.0261	-3.7893 to -2.3886	-0.0000 to -0.0001
$\gamma \in [0.55, 0.95]$	-1.5329 to -7.1387	-0.0112 to -0.1618	-1.5217 to -6.9709	-0.0000 to -0.0060

Table 4: Sensitivity Analysis.

We have also performed some sensitivity check with respect to some structural assumptions of the model. For example, and as we mentioned earlier, considering partial rather than full indexation of non-optimized prices and wages in (5) and (11) turns out to affect our baseline results insignificantly. Similarly, considering a form of long-memory habit, leaves the results practically unaffected. Finally, the same applies to using several variations of the Taylor rule, including forward looking versions.

7. Concluding Remarks

The motivation behind our work comes from recent literature that documents the effects of unexpected monetary policy on the stock market. We ask and assess whether a basic DSGE model with New Keynesian features can account for the now well documented response of the stock market to changes in the nominal interest rate by the Central Bank, both qualitatively and quantitatively. The model we consider is the simplest possible version of a New Keynesian framework that may have the ability to explain such facts: Building on the basic New Keynesian model of Woodford (2003), first we assume that both prices and wages are sticky (the latter ingredient is required to ensure procyclical dividends) and second, we assume that households form habits (this is required to generate time varying risk aversion and equity premia, an important element of our analysis). The model is then augmented in a natural way with a financial market, which we analyze in detail in order to address our asset pricing questions. The model is parameterized in line with the business cycle literature, i.e. so that it generates commonly accepted dynamics for the main macroeconomic aggregates.

We view our findings as somewhat mixed. On one hand, the model succeeds in matching the main empirical fact that we wish to capture, i.e. that an unexpected contractionary increase of the nominal interest rate of 25 basis points leads to (approximately) one percent immediate drop in the stock market; moreover, this result is very robust to simple variations and parameterizations of the model. One the other hand, when attempting to break down the impact of unexpected monetary policy on the stock price to the three relevant channels (i.e. dividends, real interest rates and ex-ante excess returns), we find that the relative contribution of real interest rates to the total impact on stock prices is much larger than what some empirical studies have documented. We attribute this to two reasons: first, to the slow mean reversion of real interest rates predicted by New Keynesian models and second to the smoothness of the endogenous consumption process of our general equilibrium setting.

What can we learn from this analysis? First, to our knowledge, our paper is the first attempt to understand this interesting asset pricing fact in the context of a general equilibrium business cycle model. Given the general difficulty in reconciling the business cycle and asset pricing literatures, we believe that this first step goes a rather long way in understanding the links and interactions between monetary policy and the stock market. Our analysis thus provides a platform for further research that would seek to improve our understanding of how different factors may affect these links.

Second, an interesting by-product of our analysis is that the methodology for deriving present value expressions for the asset prices preserves some of the valuable second order information that is usually lost when linearizing dynamic systems. Although the methodology described here is particular to our New Keynesian framework, we conjecture that it can be easily applied to other settings.

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