Transformation of the Family under Rising Land Pressure: A Theoretical Essay

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1 Introduction

We have today a solid grasp of why and how land tenure rules evolve at community level. More precisely, we understand the conditions under which a shift occurs from corporate ownership of land (possibly including the granting of long-term use rights to individual households) to individualized forms of tenure ranging from less to more complete private property rights (Demsetz, 1967; Ault and Rutman, 1979; Hayami and Kikuchi, 1981; Feder and Feeny, 1981; Feder and Noronha, 1987; Baland and Platteau, 1998; Platteau, 1996 and 2000: Chaps. 3-4). However, the organizational features of the landholding unit itself evolve over space and time and these variations are far from being understood. What we argue in this paper is that the same force that drives the individualization of land tenure, leading to better internalization of externalities and stronger incentives to conserve and improve land, also drives the individualization of the family unit possessing and managing the land. This force is the growing scarcity of land that results from population growth and/or market integration.

Individualization at the farm-cum-family level occurs when either of the two following circumstances arise: (i) the head of a collective farm decides to grant individual plots to members of the household who are entitled to keep for themselves the entire proceeds therefrom while being simultaneously required to work on the collective, family fields; (ii) the head agrees to split the stem household, implying that some members leave with a portion of the land in order to form separate, autonomous branch households based on the nuclear family. It also seems clear that (ii) leads to a more individualized farm unit than (i), yet the order in which these two forms should succeed each other as land pressure rises is far from evident.

To elucidate these questions, a theoretical framework is needed in which the logic of behaviour of different actors (the head and the members) and their strategic interactions are specified. So far, economists have proposed few theories of the evolution of the farm-cum-family structure, and these theories aim at explaining either the shift from the collective farm to the mixed form in which individual and collective fields coexist, or the breakup of the collective farms into individual units. Fafchamps (2001) offers an example of the former by developing a theoretical model to explain the decision of the household head to allocate individual plots to family members. At the core of his model is a problem of commitment that leads the family head to reward other family members for their labor on the collective field by giving them individual fields. However, Fafchamps himself recognizes that the commitment problem only exists if the short-term gain of deviating from cooperation (which means here reneging on the promise to reward the workers for their effort on the collective field) exceeds the long-term flow of benefits ensuing from a smooth relationship between the household head and the working members. Since within the family the game played is by definition of a long (and indeterminate) duration, and the discount rate of future benefits typically low (future cooperation among close relatives matters a lot), the above condition appears to be restrictive. Moreover, even assuming that Fafchamps' hypothesis is valid, it remains unclear why there should be a tendency over time for collective farms to transform themselves into mixed farms. Finally, Fafchamps does not consider a potential break-up of the household accompanied by a (partial or complete) division of the extended family's landholding.

The second issue is addressed by Foster and Rosenzweig (2002), who propose a structural model to explain household-cum-landholding division. They do not allow for individual plots, as for them co-residence implies collective farming only. In their model an extended family is composed of several claimants to the land who may decide to split if the benefit of sharing public goods by co-residing is smaller than the loss of efficiency due to decreasing returns to scale in production. They do not explicitly model the moral-hazard-in-team problem which plagues collective production. Their framework allows two possible answers to the question of the increasing incidence of individual farms: (i) growing disinterest of younger generations in the sort of public goods produced on the collective farm, and (ii) rising importance of decreasing returns to scale as a result of the shift to more land-intensive agricultural techniques. Linked up with the latter approach is the work of Boserup (1965) which lies outside the field of economics yet, with its distinctly economic flavor, has had a large resonance among development economists (Binswanger and Rosenzweig, 1986; Binswanger and McIntire, 1987; Pingali and Binswanger, 1986; Pingali et al., 1987; Binswanger et al., 1989; Hayami and Otsuka, 1985). Grounded in incentive considerations the incidence of which is assumed to change with ecological conditions, it is especially relevant in the context of this paper because it attributes the rise of peasant farms to growing land scarcity. As land pressure increases, so the argument runs, farmers are induced to shift to more intensive forms of land use, which implies that they adopt increasingly land-saving and labour-using techniques. An important characteristic of these techniques is that labour quality, which is costly to monitor, assumes growing importance. Given the incentive problems associated with care-intensive activities (sometimes labeled "management diseconomies of scale"), the small family or peasant farm in which a few co-workers (spouses and their children) are residual claimants, appears to be the most efficient farm structure.

It is puzzling to observe, however, that even in conditions of technological stagnation the individualization of the farm-cum-family structure may occur. Thus in the San-Koutiala-Sikasso (S-K-S) region in Mali, although there is no clear evidence of technological change, collective farms coexist with mixed farms and small farms born of the break-up of large family farms. It is also evident that the latter two forms have become more widespread as time elapses. From our structured interviews with household heads in thirty villages in the region, two major explanations for this evolution emerge: land pressure and increasing individual consumption needs, particularly among the younger generations. Until quite recently land in the region was still rather abundant, and it was possible for new settlers into a village to be given land by the village authorities. This has changed as the population of the region increased, and there is nowadays no idle land left within villages that could be attributed to newcomers. As land becomes scarce, family heads find it increasingly difficult to ensure the subsistence of the extended family from the traditional collective field. They claim that this new situation leaves them with no other choice than to let some family members acquire more autonomy through the ability to cultivate individual plots or to form separate branch households.

The second commonly heard explanation is what senior villagers call the advent of "modernity" whose origin they date back to the cotton boom. The rhetoric is that, nowadays, young people have greater needs, they want to own a motorbike, nice clothes, sometimes even a cellular phone... As the head is allegedly unable to meet the needs of all family members from the proceeds of the collective field, he may accept to give out individual fields, or to allow some children to leave the main household with some land. Note the close formal analogy between these two explanations since the latter -the extent of needs to be satisfied from a given amount of land increases- appears to be the converse of the former -the amount of land available to satisfy a given extent of needs diminishes. In practice, however, the two outcomes are caused by different forces: rising numbers, on the one hand, and increased market integration, on the other hand.

Given that the classical Boserupean framework based on induced technological change cannot explain the individualization of farm structures occurring in the S-K-S region, we want to propose an alternative theory that is susceptible of explaining such a move in conditions of rising land scarcity and technological stagnation, or accounting for the coexistence of the three above farm-cum-family structures in a static environment characterized by heterogeneous land endowments at farm levels. The idea is, therefore, to write a static model, as parsimonious as possible, in which these different regimes are featured, and to check through comparative statics whether and in which sense increasing land scarcity (or growing consumption needs of members) leads to individualization of the farm unit. Here is an evolution that has obvious implications for efficiency in areas where land markets are almost nonexistent so that any change in land allocation is the outcome of a decision regarding the organization of the family farm.

We explicitly model the moral-hazard-in-team problem on the collective field and, in our framework, the family head decides how to share the collective produce between himself and the household members. The moral-hazard-in-team problem is, therefore, compounded by the disincentive effect of the share system of labour remuneration. Since (male) members have an outside option, thehead must ensure that they reach their reservation utility lest they should stop working on the collective field. Male members include the younger brothers of the head and his sons and nephews in age of working.[1] The main advantage of giving out individual fields or letting some (male) members set up their own household is that, since there is no incentive problem on individual plots, production is more efficient there than on the collective field. (This aspect of the problem, the issue of labour allocation between individual and collective fields, has also been addressed by economists in the context of the analysis of agricultural cooperatives (Putterman, 1983; Meyer, 1989.)) Consequently, it may be in the interest of the household head to allow male members to secure most of their subsistence needs from individualized land portions. There is an obvious trade-off, however, because the head's income entirely comes from the collective produce which is bound to fall as a result of the competition between the family field and the individual plots for the allocation of labor.[2] In the case of a family split, the size of the collective field along with the total labor force available for work on this field decrease. However, the proceeds of the collective field have now to be shared among fewer members, and it is no more incumbent on the household head to provide for the needs of the departed members. Depending on the

[1] Traditionally, when the head passes away, the eldest surviving brother takes the head of the family, and if there is no surviving brother on the farm, the eldest son succeeds to his father. As a result customary inheritance patterns exclude females. We thus focus on the transmission of land rights among male members of the family.

[2] In line with empirical evidence, we assume the head cannot extract rent from individual plots.

relative importance of all these effects, the father may prefer a mixed regime with individual fields to the collective regime, or he may choose to split the family.

The paper is structured as follows. In Section 2, we briefly present empirical evidence from Southern Mali that is directly relevant for our topic. Then we turn to the theoretical analysis that is the core of the paper in section 3. We set up the model, define each regime and explore the forces at play when choosing across regimes. In Section 4, we derive analytical results regarding the role of land pressure and reservation utility in regime choice. In Section 5, we present simulation results to illustrate in a more complete manner the way the three regimes occur in a reservation utility-land endowment space. Under conditions of heterogenous land endowment at farm level and static technology, their coexistence is shown to be possible. Section 6 concludes.

2 Farm-cum-family structures in Southern Mali

In 2007 we conducted a systematic household survey on a random sample of 301 households in Southern Mali. On the basis of this dataset, we report descriptive evidence regarding the simultaneous presence of the three above-described farm-cum-family structures in the study area, and we comment on the appropriateness of a principal-agent framework to analyze this phenomenon theoretically.

To begin with, it is important to note that land markets are almost non-existent in the region: 80% of the parcels in the sample were inherited (post or pre-mortem), 10% were cleared by the owner a few decades ago when there was still land available in the open access zones, and 9% have been borrowed by the interviewed households.[3] A second

[3]Land lending is not synonymous of renting. We carefully asked both to borrowers and lenders whether there was any type of cash payment, or goods and services exchanged for the land, and the answer was always negative. Often the land is borrowed over several generations. With increasing land pressure, however, conflicts between owners and borrowers have become more common, frequently because the family which borrowed land a generation ago is reluctant to return it to the owner.

observation is that both types of transformations of the farm-cum-family coexist and are not mutually exclusive. A household head who has let some members depart may choose to grant individual plots to those remaining on the farm. Individual plots are allotted to male members living on the farm in about one fourth of the households (23.3%), and in most cases, when there are individual plots on the farm, all male members above a certain age are granted a plot. Members who cultivate an individual plot keep the production for their own consumption, or that of their children. Only 6% of them "helped" the household head in the previous year with either cash or crop, and in those cases, the transfers were very small. Furthermore, a large majority of household heads admit that members who possess an individual plot have no obligation to transfer income to them, either in cash or in kind. Also revealing is the fact that most household heads consider that, when individual plots are granted, they are no more responsible for the financing of marriage-related expenditures including brideprice payments. They now befall the holders of individual plots.[4]

The practice of giving out individual plots seems to be spreading. In fact, when asked whether male members had individual plots when they were cultivating under the authority of the former head, family heads answered "yes" in only 17.3% of the cases. In semi-opened interviews with a small sub-sample of household heads, we asked those who did not have access to individual plots while cultivating under the authority of the former head, why they have now chosen to grant such plots to male members of their own households. Most of them referred to growing land pressure and the consequent need of the household head to discharge the financial burden weighing on him.[5] The same motive emerges from the answers given by household heads who did not choose to grant individual plots of land. The most common reason adduced by them to explain their refusal is the lack of workforce available within the family, and the harsh competition that would arise between the collective field

[4] In terms of the model developed later, granting individual plots relaxes the participation constraint.

[5] A few of our respondents mentioned the decline of cotton prices that make it difficult to cover the subsistence needs of the whole family from the collective field only.

and the individual plots, should the latter be granted.

This explanation stands strongly confirmed by our data. More precisely, households which have granted individual plots to (male) members turn out to have significantly less land per male member (3.01 ha) than those running pure collective farms (3.67 ha). When the influence of a host of other factors is duly controlled for in a regression analysis, the land endowment variable appears to have a strongly significant negative effect on the probability for a household to have individual plots coexisting with collective fields. Moreover, this result is extremely robust to alternative models and lists of explanatory variables, both in terms of statistical significance and size of the coefficient of that variable.[6]

Land pressure thus seems to play a key role in the emergence of a mixed regime and the resulting individualization of land tenure and consumption decisions. As mentioned in the introduction, the other explanation given by village elders - who were interviewed during a community survey - is that these plots allow the satisfaction of the growing consumption needs of the young generation. In this light, it is not surprising that the financial autonomy that accompanies the possession of individual plots is cited by the beneficiaries as one of the most important advantages resulting from their exploitation.

On the other hand, the main shortcoming of individual plots is clearly that members with individual plots tend to neglect the collective field. This is actually the reason most often mentioned by household heads to explain the occurrence of higher yields on individual fields. For example, one of them said that "more effort is put on individual field and when the workers arrive on the collective plot, they are tired". Another one complained that when they work on the collective field, his sons "are prone to keep energy in reserve for their individual plots".[7] This sort of statements suggest that the granting of individual plots exacerbates the problem of moral-hazard-in-team on the collective field.[8][9]

[6]The results of our regression analyses are reported in a research report that is available from the authors.
[7]In French, "ils se reservent".
[8]In terms of the model developed later, it correspond to a worsening of the moral-hazard-in-team problem.
[9]Another shortcoming of individual plots which has been frequently cited by our respondents, especially

To evaluate the prevalence of family splits, we use information about the histories of current family heads. There are three main ways by which the interviewed family head could have reached that authority position. First, there is the way of custom: at the death of the family head, his eldest living brother or his eldest son (living on the farm) succeeds him to exert authority over the family and the farm. This is the case for 59.1% of interviewed household heads. The second method consists for members to split from the stem household so as to form branch households while the head of the former is still alive. In most cases they receive an "equitable share" of the family's land endowment: for example, if there are four brothers and the father agrees to their breaking away from the main household, he gives them each one fourth of the total endowment. Of the current family heads about one fourth (24.3%) belong to this category.

Finally, the last possibility occurs when the family separates at the death of the head giving rise to several branch households, typically because brothers do not get on well enough to operate together in the absence of the father. Often in those cases, brothers of the same mother stay together. About 17% of the sample household heads fall in that category. While separation of the household is then accompanied by the division of the family land, splitting is not the outcome of a decision of the household head to let some members leave with a portion of that land. As pointed out above, it is usually the end result of serious intra-family conflicts that can no more be settled by a respected authority figure. In our theoretical approach, which assumes that decisions are made by the household head, a split corresponds to the second above-described situation in which the household head decides to let some

by household heads or by members belonging to families where there is no individual plot, is the risk of intra-family tensions and conflicts arising from the coexistence of collective and individual activities. Such risk is evidently linked to the moral-hazard-in-team problem since manifestations of labour shirking may easily prompt accusations of misbehaviour among family members. This is not the only cause of conflicts, however, since jealous feelings may also arise from varying performances on the individual plots. These two reasons may of course be interdependent in so far as those who perform less well on their individual plot may allege that other members perform better because they shirk in their work on the collective field. Interestingly, 16% of the living heads believe that, after their death, their sons will not stay together in an integrated household.

(male) members go with a certain amount of land.[10]

The main reasons given by the sample household heads to explain why they themselves broke away from the stem household are rising land pressure in the stem household (34% of interpretable answers), and the eruption of conflicts within the family, most often with their brothers or uncles (again 34%).[11] Other reasons include low production in the stem household, and the existence of special needs that could not be satisfied had the member stayed with the whole family (expensive medicine to cure a wife, for example). In our theoretical approach, attention is focused on the first eventuality in which land pressure is the primary cause of family breakups. We are aware that intra-family conflicts are a significant additional cause of such breakups. However, we do not feel that we have much to say about them in the type of analytical framework used in this paper, except if conflicts are fanned by growing land scarcity.

The theoretical model we have written to account for the existence of various farm-cum-family structures is based on a principal-agent relationship between the head of the household and male members of the same. Such a model seems appropriate to deal with the patriarchal society of southern Mali in which the authority of a household head is rarely challenged. Our survey provides some clues attesting to the importance of the authority exercised by him. For example, when asked whether members of their family seek their approval before taking a loan, hardly 6% of the household heads answered "no". We then asked them whether in the past they have sometimes opposed such a demand, and more than 87% answered "yes". Moreover, when queried about their underlying motives, they nearly always argued that they consider themselves responsible for the family in general, and for repayment of defaulted loans taken by family members, in particular. Hence their perceived

[10]When asked whether male members had individual plots while they were still cultivating under the authority of the former head, 23% of those who split (under the second scenario) answered yes. In terms of our model, this evidence suggests that their former head chose a split-cum-individual-plots regime. However, it is worth stressing that the majority of recorded splits occurred in families which did not have individual plots before the split (especially in the district of San).

[11]These percentages are based on answers given to open questions that we later classified into categories.

right to decide if members may borrow.

We also asked family heads whether household members may seek individual plots of land without asking for their approval: only 8% answered "yes". The two main reasons adduced by them to explain that members need their approval are the following: (1) as an authority figurehead, they can decide "everything", so that not consulting them amounts to a lack of respect (55.4%); (2) "free" decisions by members are likely to cause conflicts within the family (29.5%). There is nonetheless one domain in which household heads admit that their power is limited. This is with respect to consumption choices made by children who have independent incomes (from individual plots) and claim the right to spend them according to their own preferences. The assumption of patriarchy implies not only that the household head decides whether part of the family land will be earmarked for individual plots or not, but also whether some members (and how many) will be allowed to leave the stem household and form separate branch households by using a share of the family land. We cannot exclude the possibility that a (male) member decides to part with the family against the wish of the living head, yet the price to pay for such a rebellious decision is disinheritance. Since it is not the outcome of the head's decision, or at least an agreement with him, we do not model this kind of eventuality which in any event appears to be rare in our study area. It also bears noting that our model does not aim at explaining inheritance patterns since there is no choice between different ways of bequeathing land to members. It is centered around the question of the optimal farm-cum-family structure that the household head wants to establish or maintain, given that he has to make the best possible living from it while satisfying the reservation utilities of the members.

3 A simple model of family farm structure

3.1 The general framework

A household head has N male family members of whom n live and farm with him, and $N-n$ have formed independent households. The male members who left each received an equitable share of the father's total land endowment, $\overline{A}$. This area, $\frac{\overline{A}}{N}$, can be seen as a pre-mortem inheritance transfer. Thus, when the father chooses to let $N-n$ members leave the extended family to form their own separate households, the area remaining for the extended family farm is $A = \frac{n\overline{A}}{N}$. The agricultural production function is $f(a,l)$, where a is land and l is labor. We assume that f exhibits constant returns to scale and is twice continuously differentiable in both arguments, with $\frac{\partial f}{\partial i} \geq 0$, $\frac{\partial f^2}{\partial^2 i} \leq 0$, for $i = a$ and $i = l$. An individual's utility is $x - v(l)$, where x is the production that the individual consumes and l the level of labour he exerts. The function $v(l)$ is the disutility of labor and we assume that v is twice differentiable, with $\frac{\partial v}{\partial l} > 0$ and $\frac{\partial v^2}{\partial^2 l} > 0$.

Labor on the stem household's farm is supplied by male members who have stayed with the head. The head allocates available land A between a collective field, where the male members work together, and individual fields, where each works individually and for his own benefit. We assume that members operating inside the extended family farm receive an equal treatment with respect to both the division of the produce of the collective field and the apportioning of the land earmarked for individual farming. Therefore, each member receives a share $\frac{1-\alpha}{n}$ of the production on the collective field, where α represents the father's share, and is awarded an individual plot of size $A^I \leq \frac{A}{n}$, if the father so decides.[12]

The assumption of harvest sharing according to a fixed proportion in the collective field requires some explanation. To begin with, it must be pointed out that, under the alternative

[12]The assumption of equal treatment is in line with field observations. Family heads justify this equal treatment by fear of the conflicts that would otherwise arise.

fixed-rent system, the mixed farm structure cannot be a Nash equilibrium. When the rent is fixed, the head's best action is to set it equal to the entire production of the collective field, and let the members reach their reservation utility from the production obtained on their individual plot alone. This implies that members are better off not applying any effort on the collective field, thus leaving the father with zero income. In other words, under a fixed-rent contract, the household head will always choose a pure collective system. Such a model thus fails to account for the existence of mixed farm structures. Abstracting from risk considerations, the best way to explain the choice of the share contract of labour remuneration in spite of its well-known disadvantage of encouraging labour shirking has actually been suggested by Eswaran and Kotwal (1985). They frame the problem as a trade-off between incentive considerations and use of the landowner's management skills. In the context of a patriarchal family, the head, indeed, is not a passive actor, but a manager who makes important decisions such as choosing the type of crop to grow, fixing the calendar of agricultural operations, setting the days of the week and the hours of the day when members have to work on the collective fields. In these circumstances, the share contract may dominate both the fixed-rent contract (in which the head's management skills are poorly used) and the fixed time wage rate system (in which the workers have no incentive to apply any effort).[13]

Another essential assumption of our model is that members consume the whole production of their individual fields, implying that the father's entire consumption is obtained from his share of the output produced on the collective field, $A - nA^I$. When $A^I = 0$, we say that the farm structure or regime is a pure collective farm, whereas if $A^I > 0$, it is mixed. In congruence with our field observations again, we thus assume that there is no possibility of income transfer from household members to the head. One plausible explanation for such a restriction is that there exists a serious risk of output under-reporting by members when

[13]Since there are no price fluctuations in our model, and considerations of product quality do not play any role, the share system of labour remuneration is strictly equivalent to the piece-wage-rate system. Yet, because production is collective, the moral-hazard-in-team problem compounds the disincentive effect caused by these equivalent contracts.

the household is large and the individual plots are scattered over long distances.

One unit of labor, whether applied on the collective field or on the individual plot, causes the same disutility . Therefore, member's j utility can be written as $x_j - v(l_j^C + l_j^I)$, where x_j is the sum of the share received from the collective field and the production from his individual plot, l_j^C is the level of effort applied to the collective field, and l_j^I the level of effort applied to the individual field. Since the head does not observe individual labor contributions, a moral-hazard-in-team problem arises on the family field. Finally, members have an outside option that provides them utility $\underline{u}$, giving rise to a participation constraint.

The problem can be seen as a two-stage game. In the first stage, the head chooses α, A^I and n. In the second stage, members observe these choices and individually decide how much effort to apply to the collective field and their individual plot. Since members have identical preferences and are treated equally, they behave similarly so that we are solving for a symmetric Nash equilibrium in the second stage. This allows us to solve for a single pair (l^C, l^I), and to write the whole problem as follows:

$$\begin{aligned}
\text{Max }_{n,\alpha,A^I} R &= \alpha f\left(A - nA^I, nl^C\right) && (1)\\
\text{s.t.: } \{l^C, l^I\} &= \text{Argmax }_{l_j^C, l_j^I} \frac{1-\alpha}{n} f\left(A - nA^I, l_j^C + (n-1)l^C\right) + f\left(A^I, l_j^I\right) - v(l_j^C + l_j^I) && (2)\\
l^C &\geq 0 \text{ and } l^I \geq 0 && (3)\\
\underline{u} &\leq \frac{1-\alpha}{n} f\left(A - nA^I, nl^C\right) + f\left(A^I, l^I\right) - v(l^C + l^I) && (4)\\
0 &\leq \alpha \leq 1 && (5)\\
0 &\leq nA^I \leq A && (6)\\
A &= \frac{n\overline{A}}{N} && (7)
\end{aligned}$$

In the incentive compatibility constraint, total labour on the collective field is written as $l_j^C + (n-1)l^C$, since each member takes the behavior of other male members as given when choosing own level of effort.

3.2 Giving out individual fields?

A first question to ask is the following: under which conditions does a household head finds it optimal to distribute part of the family land to male members for private use, when n members remain on the farm to cultivate A? The problem is not trivial since there are two forces working in opposite directions. On the one hand, unlike the collective field, individual plots are used efficiently, owing to the lack of any incentive problem. As a consequence, a smaller amount of land has to be dedicated to meeting the members' reservation utility $\underline{u}$ under a mixed system than under a pure collective regime. As a result, the head is able to extract a larger rent from the area left for collective farming. On the other hand, incentives to work on the collective field decrease when there is competition between collective production on the family field and individual production on private plots. This is because the worker is a full residual claimant on the latter whereas on the former, he suffers from both the moral-hazard-in-team problem and the disincentive effect of the share system of labour remuneration. Efficiency on the land wherefrom the father derives his income is therefore impaired.

To find the conditions under which individual fields exist, we solve the problem sequentially. First we determine the optimal α for a given A^I, $\alpha^*(A^I)$ (section 3.2.1). Thereafter we examine how the value function of this degenerate problem changes when A^I changes (section 3.2.2). If $\frac{\partial V}{\partial A^I}(\alpha^*(A^I)) < 0$ for all A^I such that $0 < A^I \leq \frac{A}{n}$, the head will not allocate individual fields, while if, $\frac{\partial V}{\partial A^I}(\alpha^*(A^I)) > 0$ over some range, the head may choose to allocate individual fields.

3.2.1 The family head's problem when A^I is fixed

To solve this type of principal-agent problem, it is convenient to use a first-order approach that consists of replacing the maximization problem of the agent with its first-order conditions. For such an approach to be valid, however, the optimization problem needs to be concave and the solution to the first-order conditions unique. When there exist both a collective field and individual plots, members apply a positive amount of effort in the two locations, and these conditions are satisfied. Therefore, we can replace the male members' maximization problem with the first-order conditions with respect to l^C and l^I:

$$
\begin{aligned}
0 &= \frac{1-\alpha}{n} f_L(A - nA^I, l^C + L) - v'(l^C + l^I) && (8)\\
&= \frac{1-\alpha}{n} f_L(A - nA^I, nl^C) - v'(l^C + l^I) && (9)\\
0 &= f_L(A^I, l^I) - v'(l^C + l^I) && (10)
\end{aligned}
$$

Total labor on the collective field in the incentive constraint is first written $l^C + L$ to stress that each member takes the behavior of others as given when deciding how much effort to apply to that field. We then replace $l^C + L$ by nl^C because the head knows that members behave identically, implying that they all apply the same effort, l^C. Moreover, he uses the same information when he puts them at their reservation utilities.

If the head chooses not to give out individual fields, there is a corner solution which is not a solution to the first-order condition (10).[14] In this case, $l^I = 0$ and we may replace the Argmax constraint by:

$$0 = \frac{1-\alpha}{n} f_L(A, nl^C) - v'(l^C)$$

Since the first order conditions differ depending on whether $A^I = 0$ or $A^I > 0$, we analyze the mixed regime and the pure collective regime separately.

[14] Since $A^I = 0 \Rightarrow f_L(A^I, l^I) = 0$, but for all l^I, $v'(l^C + l^I) > 0$.

In the mixed regime, for a given $0 < A^I \leq \frac{A}{n}$, the head's rent is $R^*(A^I)$ which is defined by the following system:

$$
\begin{aligned}
R^*(A^I) &= \text{Max }_{\alpha, l^C, l^I} \alpha f\left(A - nA^I, nl^C\right) \\
\text{s.t.: } 0 &= \frac{1-\alpha}{n} f_L(A - nA^I, nl^C) - v'(l^C + l^I) && (11) \\
0 &= f_L(A^I, l^I) - v'(l^C + l^I) && (12) \\
\underline{u} &\leq \frac{1-\alpha}{n} f\left(A - nA^I, nl^C\right) + f\left(A^I, l^I\right) - v(l^C + l^I) && (13) \\
0 &< \alpha < 1 && (14)
\end{aligned}
$$

In the pure collective regime ($A^I = 0$), the head chooses α so as to solve[15]:

$$
\begin{aligned}
\text{Max }_{\alpha, l} \quad & \alpha f\left(A, nl\right) \\
\text{s.t.: } 0 \quad &= \quad \frac{1-\alpha}{n} f_L(A, nl) - v'(l) && (15) \\
\underline{u} \quad &\leq \quad \frac{1-\alpha}{n} f\left(A, nl\right) - v(l) && (16) \\
0 \quad &< \quad \alpha < 1 && (17)
\end{aligned}
$$

The moral-hazard-in-team problem and the disincentive effect of the share system of labour remuneration are captured by the incentive compatibility constraints on the collective field, equations (11) in the mixed regime and (15) in the pure collective regime. To apply the first-best level of effort, the agent should receive the full benefit of his labor on the margin, $f_L(A - nA^I, nl^C)$. However, given the contractual form, he only receives $\frac{1-\alpha}{n} f_L(A - nA^I, nl^C)$. As a consequence, he works less than the Pareto optimal level.[16] We now turn to the question

[15] In the appendix, section A.1, analytical expressions for the lagrangian multipliers corresponding to that case are derived.

[16] As explained above, the formulation for the mixed regime (equations (11) to (14)) does not accommodate the corner solution corresponding to the absence of individual fields. It is clear from equation (12), however, that when A^I tends to 0, l^I tends to zero, so that inequality (13) tends to inequality (16), and the father's rent under the mixed regime approaches its level under the pure collective regime. The continuity of $R(A^I)$ near 0 is useful to characterize the solution of the head's problem of choosing A^I.

of how the father's rent is modified when the size of the collective field changes.

3.2.2 The family head's problem of choosing A^I

As pointed out above, whether or not the mixed regime will be chosen depends on how $R^*(A^I)$ changes with A^I.[17] In order to analyze the sign of $\frac{\partial R}{\partial A^I}$, we apply the envelop theorem to the solution of the previous problem in the mixed regime. For a given A^I, the Lagrangian for the mixed regime is:

$$\begin{aligned} L\left(l^C, l^I, \alpha\right) &= \alpha f\left(A - nA^I, nl^C\right) - \lambda\left(v'(l^C + l^I) - \frac{1-\alpha}{n} f_L(A - nA^I, nl^C)\right) \\ &- \mu\left(v'(l^C + l^I) - f_L\left(A^I, l^I\right)\right) \\ &- \nu\left(\underline{u} - \frac{1-\alpha}{n} f\left(A - nA^I, nl^C\right) - f\left(A^I, l^I\right) + v(l^C + l^I)\right) \end{aligned}$$

The envelop theorem implies:

$$\begin{aligned} \frac{\partial V}{\partial A^I} = \frac{\partial L}{\partial A^I} &= -n\alpha f_A^C - \lambda(1-\alpha) f_{LA}(A - nA^I, nl^C) + \mu f_{LA}\left(A^I, l^I\right) \\ &- \nu(1-\alpha) f_A(A - nA^I, nl^C) + \nu f_A\left(A^I, l^I\right) \end{aligned} \tag{18}$$

To understand the underlying logic of our model, it is useful to interpret each term of the above expression. As A^I increases (by one unit), the size of the collective field decreases (by n units), and the first term indicates how, everything else being constant, the family head's rent declines with the size of the field from which it is extracted. The second term captures the lower incentives for male members to work on the collective field as A^I increases (we show in appendix, section A.2 and A.2.2, that λ is positive). For a given amount of effort, indeed, the marginal product of labour falls when land becomes smaller. The third term

[17] If $\frac{\partial R^*}{\partial A^I} < 0$ for all A^I, then the pure collective regime will be the preferred one. This is true since the head's rent in the mixed regime tends to the rent in the collective regime when A^I tends to zero.

reflects the negative impact on R caused by the enlarged size of the individual plots: the sons have more incentive to spend effort on their individual plot since the marginal productivity of labour has increased for a given amount of effort. As a result, the cost of their effort on the collective field is now higher (we show in appendix, section A.2 and A.2.2, that μ is negative).

The last two terms of equation 18 indicate how a change in A^I modifies the participation constraint, and how this affects the head's utility (bear in mind that $\nu \geq 0$ since the head's rent increases if the participation constraint is relaxed). Other things being equal (the distribution of labour efforts being constant), reallocation of land from the collective field to individual plots has the effect of enhancing the ability to produce $\underline{u}$ on the latter and simultaneously decreasing the ability to do so on the former. Measured by the marginal productivity of land in the two locations, this combined effect is positive overall because incentive problems exist on the collective field but not on the individual plots.[18]

We cannot, therefore, exclude the possibility that, over some range of A^I values, $\frac{\partial R^*}{\partial A^I} > 0$, implying that the household head may prefer the mixed regime over the pure collective regime. This is because, to sum up, an increase in the size of individual plots has opposite effects on the head's rent. By decreasing incentives to work on the collective field, it reduces the overall production on this portion of the farm, thereby reducing the base from which the father obtains his income. At the same time however, it relaxes the participation constraint of all the male members, as a result of which the head may dedicate a smaller share of collective production to meeting their reservation utility.[19]

[18] Indeed, assuming constant returns to scale, we have that $f_L(A^I, l^I) < f_L(A - nA^I, nl^C)$ and $f_A(A^I, l^I) > f_A(A - nA^I, nl^C)$. The latter inequality implies, a fortiori, that: $-\nu(1-\alpha) f_A(A - nA^I, nl^C) + \nu f_A\left(A^I, l^I\right) > 0$.

[19] In fact, if there exists an interior solution to the father's problem, it occurs at a point where the participation constraint binds. Indeed if the sons are able to achieve their reservation utility by just relying on the production of their individual fields, $\nu = 0$, and the head's rent is unambiguously decreasing in the size of individual plots. This case is treated in appendix, section A.2.

3.3 Splitting the family

Rather than keeping the family whole with or without individual fields, the head may choose to split it and divide the land so that the departing members can form separate branch households on a portion of the family land assets. It may appear puzzling that the head would accept such an evolution of the family structure: wouldn't he be better off by simply letting some members achieve their outside option instead of giving them part of the family land? When directly questioned on this point during interviews, household heads often explain that from their viewpoint the worst situation occurs when male family members leave the village. Letting them go without land is synonym of "loosing" them. Although possible reasons are not difficult to figure out, it is outside the scope of the present analysis to model why heads are ready to give up land in order to keep their brothers and their sons close to them. We just assume implicitly that the head would face some large lumpsum cost if he would let male members leave without land, since this would mean that they will then leave the village and opt out of the local social network.

Recall that, when a male member leaves, he receives a fraction $\frac{1}{N}$ of the total land endowment of the family, $\overline{A}$. What are the costs and benefits of splitting the family? When is it the preferred regime? To understand the effects of splitting the family, we examine the effects of a unit increase in the number of sons who stay within the extended family.

Whether in the pure collective regime ($A^I = 0$) or in the mixed regime ($A^I > 0$), if the head decides to keep one more member with him, the impact on his rent is formally defined as follows:

$$\frac{\partial R}{\partial n} = \left(\frac{\partial A}{\partial n} - A^I - n\frac{\partial A^I}{\partial n} \right) \alpha f_A + l^C \alpha f_L + n\frac{\partial l^C}{\partial n} \alpha f_L + \frac{\partial \alpha}{\partial n} f \tag{19}$$

The first term is the *land endowment effect*. When one more member stays on the farm area, the total farm is bigger (in fact $\frac{\partial A}{\partial n} = \frac{\overline{A}}{N}$), but the collective field is not necessarily larger since the additional member receives A^I and the size of the individual fields may be adjusted by the head. The second term is the *labour endowment effect*: the increase in the labour force working on the collective field has a positive direct effect on total production. The last two terms are linked to incentives and are less straightforward to sign. We label the third term the *labour incentive effect*, and the fourth term the *incentive compensation effect*. The third term indicates how the individual incentive to work on the collective field changes when an additional member stays on the farm, thereby accentuating the moral-hazard-in-team problem. We show in appendix (section A.3 for the case of a split occurring in the collective regime, and section A.3.2 for the case of a split occurring in the mixed regime) that, as expected, this term is negative. The fourth term depicts how the head adapts his share to the change in family size. As proven in appendix, this term is also negative, indicating that he makes up for the poorer work incentives by allowing male members to keep a greater share of the collective field's production.

Reasoning in the converse way, an important lesson to draw from the ambiguous sign of $\frac{\partial R}{\partial n}$ is that, by inducing a son to leave the stem household and form a branch household, the family head is not certain to increase his own income. This is in spite of the fact that he does not have anymore to provide for the consumption needs of the departing son and that the incentives to work on the collective field improve for the members who stay on the farm. There are, indeed effects working in the opposite direction: the departing son stops working on the collective field, and is moreover offered a share of the family land assets as he leaves the main household.

4 Analytical results: the effects of land scarcity and increasing consumption needs

Recall that one of the main reasons given by local elders for the increasing prevalence of extended family farms with individual fields, and of family splits, is the increase in land pressure. In terms of our model, such increase may be measured by a decrease of the land endowment, for a given family size.[20] The other main reason is that (male) members have greater consumption needs than in the past. This change may be captured by an increase in the reservation utility, $\underline{u}$, which these members require from the head in order to continue to work and stay with him. It is because they perceive to have better outside opportunities, typically in the form of migration to Malian cities or neighboring countries that they feel able to demand a higher level of welfare. Improved communication and increased mobility have no doubt contributed to these enhanced perceptions of potential employment opportunities outside the native village.

In this section, we test whether the above explanations can be supported by our theoretical framework. We examine first how the head's incentive to give out individual plots changes with the family land endowment and the members' reservation utility. We then examine how the head's incentive to split the family is affected by the same variables. In each case we summarize our results in a proposition, and we briefly explain how they were obtained, while referring the reader to the appendix for a presentation of the complete formal proofs. These results, it must be noted, are derived by using specific forms for the production function, the Cobb-Douglas function ($f(a,l) = a^{\varepsilon} l^{1-\varepsilon}$) and for the cost of effort, the polynomial function ($v(l) = \omega l^2$).

The effect of land endowment on the choice between the mixed and the pure collective regimes

Proposition 1 *When land is very abundant, the head always prefer a pure collective farm to a mixed structure where male member have individual plots that they cultivate for their own benefit. In this circumstance, the participation constraints of members are not binding.*

[20] Conversely, we could consider an increase in family size for a given land endowment. However, a change in family size leads to more complicated analytical expressions than a change in land endowment, since n does not only measure land scarcity but also the intensity of the moral-hazard-in-team problem.

As land become scarce, however, the mixed structure may become more attractive.

Suppose, in particular, that the head of a collective farm is just indifferent between operating the farm as a pure collective unit or as a mixed unit. A marginal decrease in land endowment induces him to strictly prefer the mixed regime over the collective regime. Conversely, a marginal increase in land endowment induces him to strictly prefer the pure collective regime. As $\overline{A}$ goes from $+\infty$ to 0, either the collective farm remains superior over the full range of land endowments, or the mixed farm dominates below a critical level of land endowment.

The finding that participation constraints of family members are unbinding when land is very abundant reflects the fact that the head can earn a higher rent by giving to these members incentives to work (on the collective field) that allow them to be better off than by taking up their outside income opportunity.

The effect of reservation utility on the choice between the mixed and the pure collective regimes

Proposition 2 *When the workers' reservation utility is very low, the head always prefer a pure collective farm to a mixed structure where male member have individual plots that they cultivate for their own benefit. In this circumstance, the participation constraints of members are not binding. As the reservation utility increases, however, the mixed structure may become more attractive.*

Suppose, in particular, that the head of a collective farm is just indifferent between operating the farm as a pure collective unit or as a mixed unit. A marginal increase in the reservation utility induces him to strictly prefer the mixed regime over the collective regime. As $\underline{u}$ goes from 0 to $+\infty$, either the collective farm remains superior over the full range of land endowments, or the mixed farm dominates above a critical level of reservation utility.

The effect of land endowment on the choice between splitting the family and keeping it whole

Proposition 3 *When land is very abundant, the head of a purely collective farm will not accept to let some male members leave with a portion* $1/N$ *of the land endowment: he wants to keep the family whole. Conversely, when land is very scarce, the head of a purely collective farm or a mixed farm will choose to split the family and let some members leave with a portion* $1/N$ *of the land. Furthermore, there exists a unique level of land endowment* $0 < \overline{A} < +\infty$ *that makes the head of a purely collective farm just indifferent between letting some male members leave with a portion 1/N of the family land, and keeping the family whole.*

The effect of reservation utility on the choice between splitting the family and keeping it whole

Proposition 4 *When the members' reservation utility is very low, the head of a pure collective farm will not accept to let some male members leave with a portion 1/N of the land: he wants to keep the family whole. Conversely, when the members' utility is very high, the head of a purely collective farm or a mixed farm will choose to split the family and let some members leave with a portion 1/N of the land. Furthermore, there exists a unique level of reservation utility* $0 < \underline{u} < +\infty$ *that makes the head of a purely collective farm just indifferent between letting some male members leave with a portion 1/N of the family land, and keeping the family whole.*

The role of land endowment and reservation utility when the father chooses across the three regimes

If we assume a Cobb-Douglas production function, and a polynomial cost of effort, we know that:

- For large $\overline{A}$ or small $\underline{u}$, the pure collective regime dominates the mixed regime and the head will not split the family.
- For small $\overline{A}$ or large $\underline{u}$, the mixed regime may dominate the collective regime, and whichever of these two regimes prevails, some splitting will occur.

5 Simulation Results

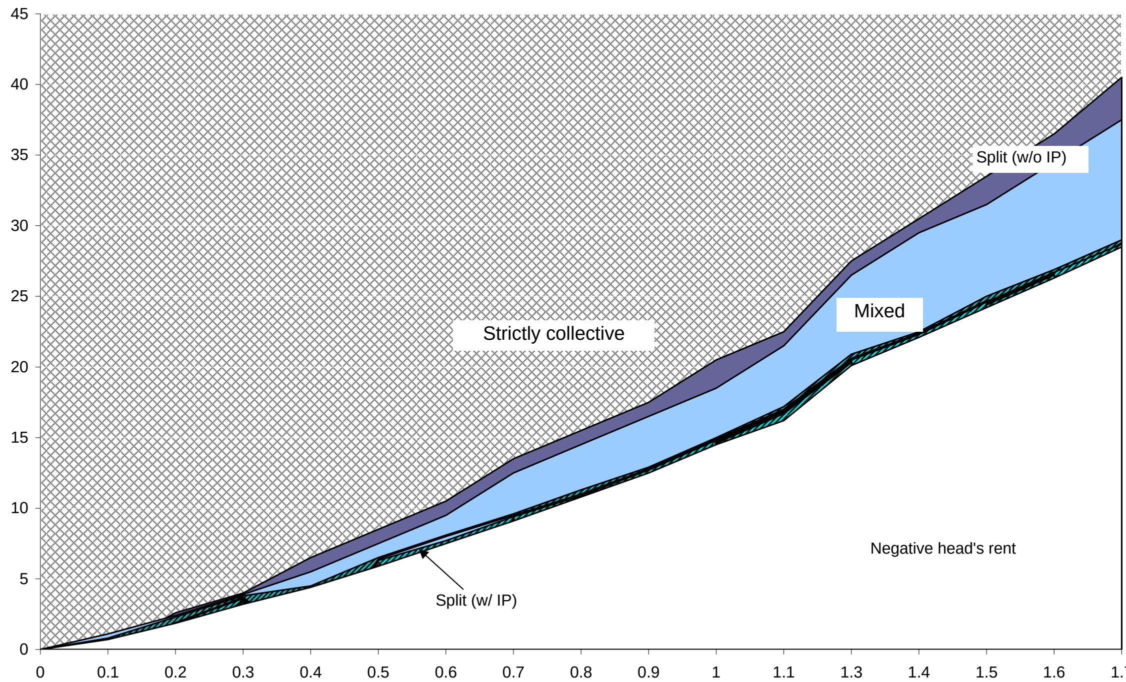

Figure 1: Partition of the land endowment - reservation utility space into regimes.

5.1 The principal-agent model

As is evident from the synthesis presented under section 4, our analytical exploration of the role of the farm land endowment and of the members' reservation utility does not yield a complete set of predictions. For example, we cannot be sure that for small $\overline{A}$, a family head operating in the pure collective regime will not choose to shift to the mixed regime instead of splitting the family. Hence the need to resort to simulation in order to obtain results that allow for a comparison of all the possible regimes simultaneously, and to examine whether in a $(\overline{A}, \underline{u})$ space, the mixed, the split and the pure collective regimes actually coexist.[21]

Our simulation work is summarized in Figure 1 where the family land endowment is measured along the vertical axis and the members' reservation utility along the horizontal axis. What are the main results emerging from this figure?

To begin with, the results analytically obtained in the previous section stand confirmed. First, the pure collective regime appears to be superior to all the other regimes in the upper left portion corresponding to small values of $\underline{u}$. Second, the area corresponding to the pure collective regime (squared area) lies above the areas corresponding to both the mixed regime (in light grey) and collective farming-cum-splitting (shaded area, labeled "split w/o IP", or split without individual plots). Moreover, the triangle-like shape of the squared zone indicates that the smaller the reservation utility $\underline{u}$, the lower the threshold value of $\overline{A}$ above which the pure collective regime dominates the alternative regimes (the squared zone expands in size as we move to the left in the upper part of the graph). In other words, pure collective farms may subsist even in conditions of land scarcity but only provided that exit opportunities for members are sufficiently bad. Conversely, they may withstand the pressure of rising outside opportunities if land is sufficiently abundant.

Third, when the head operates his farm under the pure collective regime and $\overline{A}$ becomes

[21]The simulation is conducted using the software Mathematica. For a given $(\overline{A}, \underline{u}, N)$, for all $1 \leq n \leq N$, we numerically solve for the head's share when he does not give out individual plots and for decreasing sizes of the collective field. Practically, in the case of the results presented below, we decrease the size of the collective field by steps of 0.25. In Mathematica we use the command "FindRoot", to obtain α when the participation constraint binds. For each n, we then compute the head's rent for each size of the collective field and compare it to his rent in the collective regime. For that n we thus know whether the head will choose to give out individual plots, and the maximum rent the head can obtain when he keeps n members on the family farm. Comparing the head's rent over the range of n, we determine whether the head prefers to split the family ($n < N$), or not ($n = N$). The parameters used are: $N = 10$, $\varepsilon = 0.7$ and $\omega = 0.5$.

sufficiently small (or $\underline{u}$ sufficiently large), he chooses to split the family (shift down from the squared zone to the dark grey area, labeled "split (w/o IP)). And when the head operates a mixed farm and $\overline{A}$ becomes sufficiently small (or $\underline{u}$ sufficiently large), he chooses also to split the family (shift down from the light grey to the shaded area, labeled "split w/IP", or split with individual plot). Fourth, the mixed regime emerges as the optimal farm structure when the reservation utility is not too small and the farmland area is not too large.

The use of simulation also brings to light a number of results that cannot be derived analytically, and therefore add to the knowledge acquired in the previous section. The main finding here concerns the sequence in which optimal regimes succeed each other, as we vary the values of $\overline{A}$ or $\underline{u}$. As $\overline{A}$ is marginally lowered, or $\underline{u}$ is marginally raised, a head operating a pure collective farm may split the family while clinging to collective farming in the remaining portion of the stem household. When $\overline{A}$ is lowered, or $\underline{u}$ raised, to a larger extent still, the head may instead choose the mixed farm in which all members stay in the stem household but obtain access to individual plots. And when the change in $\underline{u}$ or $\overline{A}$ values is made even greater, splitting the family while granting individual plots to the members who stay with the head becomes the optimal regime. It is noticeable that the area corresponding to the last regime is quite thin in comparison to the area corresponding to the mixed regime. This is also true, yet to a smaller extent, of the area depicting the split-cum-collective farming regime.

Finally, and rather unexpectedly, the split-cum-individual plots regime does not appear to be resistant against the sporadic invasion of pockets of dominance of the (pure) mixed regime (the shaded area contains thin areas of light grey). The complex pattern described in Figure 1 has much to do with the fact that the split option actually conceals numerous possibilities corresponding to the departure of any discrete number of family members. Whenever some members remain in the stem household, moreover, individual plots may or may not be granted by the head. Each of these possibilities is considered in turn when comparing the available regimes during the simulation procedure.

Why is it that as land becomes more scarce, or as exit options of family members improve beyond a point, split-cum-collective farming becomes preferable to collective farming even before the mixed regime - which, on the face of it, is a less individualized form - becomes optimal? Here is an apparently intriguing result produced by our simulation work, yet what we show is that a partial, not a complete split of the farm-cum-family may prove superior to the mixed farm structure. Indeed, the split regime evinces greater flexibility inasmuch as the head chooses how many members to let go. Both regimes entail a reduction of the farm area devoted to the collective field so that (some) members can produce on their own plot to meet (part of) their needs. Correspondingly, a portion of the workforce ceases to be available for the collective field. In the split-cum-collective farming regime, this decrease takes on the form of a reduced number of workers with the attendant result that the moral-hazard-in-team problem is mitigated. But this is not the case under the mixed regime. There is thus an obvious tradeoff between the size of the work force available to work on the collective field (larger under the mixed regime) and the extent of the moral-hazard-in-team problem (also greater under the mixed regime). What our results indicate is that the latter, adverse effect outweighs the former beneficial effect when land is not too scarce (or the reservation utility is not too high), while the reverse is true when land scarcity (or the reservation utility) exceeds a certain threshold.

The endogenously generated values of the decision variables chosen by the head, n, A^I, and α are reported in Appendix C. A systematic feature emerges from the table: whenever a reduction of the land endowment causes a shift from the split regime (without individual plots in the stem household) to the mixed regime, the father's share undergoes a sharp increase. This is because under the split regime, the members remaining in the stem household devote their entire working time to the collective field. Under the mixed regime, by contrast, if all the members work on the collective field, they devote only a part of their time to it. Hence the need for the head to make up for the ensuing income shortfall by raising his share.[22]

5.2 An alternative framework with an altruistic family head

Relaxing the assumption of a strictly selfish patriarch at the head of the farm and simultaneously removing the participation constraints provide interesting insights into the functioning of our model. More precisely, it reveals that the key feature driving the comparative static results obtained lies in the participation constraints. It is, indeed, the tightening of these constraints under conditions of improved outside opportunities for members, or of growing land scarcity, that induces the family head to put more weight on efficiency considerations so as to be able to satisfy them. As we know, this implies a transformation of the farm-cum-family structure toward more individualized forms. Consider now the alternative framework in which there is no participation constraint, but the head has an altruistic utility function. Altruism can be construed as meaning that the head attaches a positive weight to the members' welfare while making his allocative decisions or alternatively, that members exert pressure on him to the effect that he takes their interests into account. In the former case, the weight put on the members' welfare reflects the head's degree of altruism while in the latter case it reflects the bargaining power of the members.

When we work out the numerical solutions to this newly defined problem, we find that the three farm-cum-family structures may again arise, yet it is only for relatively high levels of altruism (or members' bargaining strength) that individualized forms are preferred by the head. The second finding, however, contradicts the comparative-static results obtained under the initial model: the farm-cum-family structure chosen is insensitive to variations in

[22] Interestingly, the relationship between growing land scarcity and the father's share, α, is never monotonic under the mixed regime. Thus $\underline{u}$ being given, as land endowment $\overline{A}$ is reduced and the size of the individual fields is increased, α, may rise or fall but only up to a point beyond which it starts moving in the opposite direction. And this change itself may just precede another reversal. In words, the head is not always in a position to (partly) make up for a reduction in his income base (a fall in the size of the collective field) by increasing his share of the smaller collective produce. Whether he can do it or not depends on the precise configuration of the parameters.

land pressure for the members. This is because, when conditions become more stringent, the head now has the ability to transfer part of the welfare loss to the members whereas he had to operate under binding participation constraints in the base model. Thus if he is sufficiently selfish to prefer the pure collective farm structure, he will stick to it under conditions of increasingly severe land pressure. Efficiency gains are thereby lost in conditions where they matter much, yet this is not the main concern of the head since by accepting a reduction of the collective field, his income loss would be greater than when the farm remains purely collective. In other words, in the absence of participation constraints, an increase in land pressure does not affect the outcome of the trade-off between efficiency in production and the head's ability to extract incomes.

6 Conclusion

On the basis of a stylized representation of a patriarchal family farm, and in a context of absent land markets, it is possible to use a simple analytical structure to account for possible transformations of a collectively operated farm based upon an extended family unit. More precisely, as land scarcity increases, or as exit options available to family members improve (say, as a result of growing market integration), the pure collective farm will unavoidably become inferior to alternative farm structures from the standpoint of the family head who draws his entire income from a share of the collectively produced harvest. One of these alternative forms is a mixed farm structure combining a collective field with individual plots of land. Another one is a regime in which branch households are formed as a result of the decision of the patriarch to allow the split of the stem household and the concomitant division of the extended family's assets. In the remaining part of the stem household, collective cultivation may be combined with individual fields, but this is not a necessity. As the number of (male) members leaving the stem household may be any number between zero and the total number of them in that household, there is a large variety of alternative forms to the pure collective farm, and each of them needs to be considered in a comparison between possible farm structures.

In spite of the analytical simplicity of the basic farm structure contemplated in our model, a complete comparison turns out to be quite complex, and we had to resort to the simulation technique in order to obtain a complete mapping of regime choice into a reservation utility/land endowment space. The most significant result is the following: as land scarcity increases (or as exit options for members improve), splitting the main household while sticking to the pure collective mode of operation in its remaining portion appears to be the first alternative farm organization able to supersede the pure collective farm. It is only at higher levels of scarcity (or exit option levels) that the mixed farm structure becomes the optimal organization from the patriarch's standpoint. And it is at still higher levels that splitting combined with individual plots in the remaining stem household emerges as the best solution.

The above result critically hinges on the existence of participation constraints. In the absence of such constraints, indeed, an increase in land pressure does not affect the outcome of the trade-off between efficiency in production and the head's ability to extract incomes. This is evident, for example, when we assume that the family head is altruistic. Other variants of our model have less significant consequences. Thus, assuming that members with individual plots can make income transfers in favour of the family head would, for obvious reasons, make the pure collective farms less appealing than the alternative forms. Furthermore, if we assume that disutility of effort is greater on the collective field than on the individual plots, the case of individualization is again strengthened. In the other way around, the presence of scale economies in the production of the collective field and in the consumption of the collective produce would enhance the advantages of the collective farm and enlarge the region of its feasibility. Likewise, the presence of fixed costs, such as storage costs, increases the advantage of mixed farms over branch households as a way of individualizing the collective farm structure as land becomes more scarce. Finally, allowing for dynamic considerations of the sort considered by Boserup could only reinforce our conclusion that rising land pressure leads to more individualized farm-cum-family structures.[23] One of the main merits of our model is actually to show that individualization of farm units can result from land scarcity even in the absence of induced technical change.

[23] In a dynamic setup of the model, one might wish to assume that allowing the departure of members to form separate branch households is a more irreversible step than granting individual plots to staying members. This would obviously reinforce the case for the mixed farm structure.

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Appendix

A Analytical framework

A.1 Optimization in the pure collective regime

In this section we formally derive the Lagrangian multipliers to the problem described by Equations 15 to 17. The Lagrangian for this problem is:

$$L = \alpha f(A, nl) - \beta\left(\frac{1-\alpha}{n} f_L(A, nl) - v'(l)\right) - \gamma\left(u - \frac{1-\alpha}{n} f(A, nl) + v\right) \quad (20)$$

The FOC are (we ignore the arguments of the various functions):

$$\begin{aligned}
\frac{\partial L}{\partial \alpha} &= f + \frac{\beta}{n} f_L - \frac{\gamma}{n} f = 0 \\
\frac{\partial L}{\partial l} &= \alpha n f_L - \beta(1-\alpha) f_{LL} + \beta v'' + \gamma(1-\alpha) f_L - \gamma v' = 0
\end{aligned}$$

The first equality implies: $\gamma = n + \beta \frac{f_L}{f}$. The second equality can thus be rewritten:

$$\alpha n f_L - \beta(1-\alpha) f_{LL} + \beta v'' + (1-\alpha) f_L n + \beta \frac{f_L^2}{f}(1-\alpha) - n v' - \beta \frac{f_L v'}{f} = 0$$

Replacing v' with $\frac{1-\alpha}{n} f_L$ and solving for β, we obtain:

$$\beta = \frac{f_L(n-1+\alpha)}{(1-\alpha) f_{LL} - v'' - \frac{f_L^2}{f}(1-\alpha)(1-\frac{1}{n})}$$

Finally:

$$\gamma = n - \frac{1}{\frac{v'' f}{f_L^2(n-1+\alpha)} + \frac{(1-\alpha)(-f_{LL}) f}{f_L^2(n-1+\alpha)} + \frac{(1-\alpha)(1-\frac{1}{n})}{n-1-\alpha}}$$

A.2 Optimization in the mixed regime, for a given A^I

In this section we formally derive the Lagrangian multipliers to the problem described by Equations 11 to 14. These multipliers have different expressions depending on whether the participation constraint binds.

A.2.1 Unbinding participation constraint

In the case where A^I is large enough so that sons can meet their reservation utility from their individual field alone ($f\left(A^I, l^I\right) - v(l^C + l^I) \geq \underline{u}$). In this case $\nu = 0$ and the FOC are:

$$\frac{\partial L}{\partial \alpha} = f(A - nA^I, nl^C) - \frac{\lambda}{n} f_L(A - nA^I, nl^C) = 0 \quad (21)$$

$$\frac{\partial L}{\partial l^C} = \alpha n f_L(A - nA^I, nl^C) - \lambda\left(v''(l^C + l^I) - (1-\alpha) f_{LL}(A - nA^I, nl^C)\right) - \mu v''(l^C + l^I) = 0 \quad (22)$$

$$\frac{\partial L}{\partial l^I} = -\lambda v''(l^C + l^I) - \mu\left(v''(l^C + l^I) - f_{LL}\left(A^I, l^I\right)\right) = 0 \quad (23)$$

In the following we use the subscript C for the production function on the collective field and I to designate the production function on individual plots. The first equation implies: $\lambda = \frac{nf^C}{f_L^C}$. Substituting λ in the last equation yields: $\mu = \frac{-v''\frac{nf^C}{f_L^C}}{v'' - f_{LL}^I}$. Since λ is unambiguously positive while μ is unambiguously negative, $\frac{\partial V}{\partial A^I} = -\alpha n f_A^C - \lambda \frac{1-\alpha}{n} f_{LA}^C + \mu \frac{1}{m} f_{LA}^C$ is negative, so that until the participation constraint binds, it is always optimal for the father to decrease the size of the individual plots, and thereby increase the size of the collective field.

A.2.2 Binding participation constraint

The FOC of the maximization problem in this case are:

$$\frac{\partial L}{\partial \alpha} = f(A-nA^I, nl^C) - \lambda\frac{1}{n}f_L(A-nA^I, nl^C) - \nu\frac{1}{n}f(A-nA^I, nl^C) = 0 \tag{24}$$

$$\frac{\partial L}{\partial l^C} = \alpha n f_L(A-nA^I, nl^C) - \lambda\left(v''(l^C+l^I) - (1-\alpha)f_{LL}(A-nA^I, nl^C)\right) - \mu v''(l^C+l^I) \tag{25}$$

$$-\nu\left(-(1-\alpha)f_L(A^C, ml^C) + v'(l^C+l^I)\right) = 0 \tag{26}$$

$$\frac{\partial L}{\partial l^I} = 0 \tag{27}$$

$$= -\lambda v''(l^C+l^I) - \mu\left(v''(l^C+l^I) - f_{LL}\left(A^I, l^I\right)\right) - \nu\left(-f_L\left(A^I, l^I\right) + v'(l^C+l^I)\right) \tag{28}$$

Equation 28 implies: $\mu = -\lambda\frac{v''}{v''-f^I_{LL}}$, since $-f_L\left(A^I, l^I\right) + v'(l^C+l^I) = 0$. Equation 24 implies: $\nu = n - \lambda\frac{f^C_L}{f^C}$. Replacing μ and λ in equation 25 by these expressions yields:

$$\begin{aligned}
& \alpha n f^C_L - \lambda(v'' - (1-\alpha)f^C_{LL}) + \lambda\frac{v''^2}{v''-f^I_{LL}} - n(-(1-\alpha)f^C_L + v') + \lambda\frac{f^C_L}{f^C}(-(1-\alpha)f^C_L + v') = 0 \\
\Leftrightarrow\ & \alpha n f^C_L + (m-1)(1-\alpha)f^C_L + \lambda\left(-v'' + (1-\alpha)f^C_{LL} + \frac{v''^2}{v''-f^I_{LL}} + \frac{(f^C_L)^2}{f^C}(1-\alpha)(-1+\frac{1}{n})\right) = 0 \\
\Leftrightarrow\ & \lambda = -\frac{(n-1-\alpha)f^C_L}{-v'' + (1-\alpha)f^C_{LL} + \frac{v''^2}{v''-f^I_{LL}} + \frac{(f^C_L)^2}{f^C}(1-\alpha)(-1+\frac{1}{n})} \\
\Leftrightarrow\ & \lambda = -\frac{(n-1-\alpha)f^C_L}{(1-\alpha)f^C_{LL} + \frac{v''f^I_{LL}}{v''-f^I_{LL}} + \frac{(f^C_L)^2}{f^C}(1-\alpha)(-1+\frac{1}{n})}
\end{aligned}$$

This implies $\lambda > 0$, $\mu < 0$. We also know: $\nu > 0$ (property of the Lagrangian multiplier of an inequality). We derive the expression for ν (needed below):

$$\begin{aligned}
\nu &= n + \frac{(f^C_L)^2(n-1+\alpha)}{\frac{v''f^I_{LL}f^C}{v''-f^I_{LL}} + (1-\alpha)f^C_{LL}f^C + (f^C_L)^2(1-\alpha)(-1+\frac{1}{n})} \\
\Leftrightarrow \nu &= n - \frac{1}{\frac{v''(-f^I_{LL})}{v''-f^I_{LL}}\frac{f^C}{(f^C_L)^2(n-1+\alpha)} + \frac{(1-\alpha)(-f^C_{LL})f^C}{(f^C_L)^2(n-1+\alpha)} + \frac{(1-\alpha)(1-\frac{1}{n})}{n-1+\alpha}}
\end{aligned}$$

A.3 Splitting: signing the incentive effects

In this section we show that the labor incentive effect $\frac{\partial l^C}{\partial n}$ and the incentive compensation effect $\frac{\partial \alpha}{n}$ in equation 19 are both negative. The analytical expression for $\frac{\partial l^C}{\partial n}$ and $\frac{\partial \alpha}{\partial n}$ depends on whether we are considering split in a pure collective regime or split in a mixed regime.

A.3.1 Splitting under the pure collective regime

We apply the Cramer's rule to the system of equations from which the optimal values for l^C and α are implicitly obtained:

$$\begin{cases} F_1 = \frac{1-\alpha}{n} f_L(A, nl^C) - v'(l^C) = 0 \\ F_2 = \frac{1-\alpha}{n} f\left(A, nl^C\right) - v(l^C) - \underline{u} = 0 \end{cases} \tag{29}$$

Assuming that f is homogeneous of degree 1 (or that we have constant returns to scale) we obtain:

$$\begin{aligned} \frac{\partial l^C}{\partial n} &= -\frac{\det\begin{pmatrix} \frac{\partial F_1}{\partial \alpha} & \frac{\partial F_1}{\partial n} \\ \frac{\partial F_2}{\partial \alpha} & \frac{\partial F_2}{\partial n} \end{pmatrix}}{\det\begin{pmatrix} \frac{\partial F_1}{\partial \alpha} & \frac{\partial F_1}{\partial l^C} \\ \frac{\partial F_2}{\partial \alpha} & \frac{\partial F_2}{\partial l^C} \end{pmatrix}} \\ &= -\frac{-\frac{1-\alpha}{n^2} f_L \frac{f}{n}}{-\frac{f_L}{n}((1-\alpha)f_L - v') + \frac{f}{n}((1-\alpha)f_{LL} - v'')} \end{aligned}$$

This expression is unambiguously negative (recall that $v' = \frac{1-\alpha}{n} f_L$ so that $(1-\alpha)f_L - v' > 0$). Note that assumption of the constant returns to scale has greatly simplified the expressions for $\frac{\partial F_1}{\partial n}$ and $\frac{\partial F_2}{\partial n}$. It implies that f is homogeneous of degree 1 and f_L of degree 0, so that by virtue of Euler's theorem: $f = n\frac{\overline{A}}{N}f_A + nlf_L$ and $n\frac{\overline{A}}{N}f_{LA} + nlf_{LL} = 0$. Thus:

$$\begin{aligned}\frac{\partial F_1}{\partial n} &= -\frac{1-\alpha}{n^2}f_L + \frac{1-\alpha}{n}\frac{\overline{A}}{N}f_{LA} + l\frac{1-\alpha}{n}f_{LL} \\ &= \frac{1-\alpha}{n^2}(-f_L + Af_{LA} + nlf_{LL}) \\ &= \frac{1-\alpha}{n^2}(-f_L + 0) = -\frac{1-\alpha}{n^2}f_L\end{aligned}$$

$$\begin{aligned}\frac{\partial F_2}{\partial n} &= -\frac{1-\alpha}{n^2}f + \frac{1-\alpha}{n}\frac{\overline{A}}{N}f_A + l\frac{1-\alpha}{n}f_L \\ &= \frac{1-\alpha}{n^2}(-f + Af_A + nlf_L) \\ &= \frac{1-\alpha}{n^2}(-f + f) = 0\end{aligned}$$

Similarly we obtain:

$$\begin{aligned}\frac{\partial \alpha}{\partial n} &= -\frac{\det\begin{pmatrix}\frac{\partial F_1}{\partial n} & \frac{\partial F_1}{\partial l^C} \\ \frac{\partial F_2}{\partial n} & \frac{\partial F_2}{\partial l^C}\end{pmatrix}}{\det\begin{pmatrix}\frac{\partial F_1}{\partial \alpha} & \frac{\partial F_1}{\partial l^C} \\ \frac{\partial F_2}{\partial \alpha} & \frac{\partial F_2}{\partial l^C}\end{pmatrix}} \\ &= -\frac{-\frac{f_L}{n^2}(f_L(1-\alpha) - v')}{-\frac{f_L}{n}((1-\alpha)f_L - v') + \frac{f}{n}((1-\alpha)f_{LL} - v'')}\end{aligned}$$

A.3.2 Splitting under the mixed regime

For a given A^I, α, l^C and l^I are the (implicit) solution to the following system:

$$\begin{cases} E_1 = \frac{1-\alpha}{n}f_L((\frac{\overline{A}}{N} - A^I)n, l^C + L) - v'(l^C + l^I) = 0 \\ E_2 = f_L(A^I, l^I) - v'(l^C + l^I) = 0 \\ E_3 = \frac{1-\alpha}{n}f\left((\frac{\overline{A}}{N} - A^I)n, nl^C\right) + f\left(A^I, l^I\right) - v(l^C + l^I) - \underline{u} = 0 \end{cases} \tag{30}$$

Like in the case of the pure collective regime, we can use the system of equations that implicitly define α, l^C and l^I in order to find expressions for $\frac{\partial \alpha}{\partial n}$ and $\frac{\partial l^C}{\partial n}$:

$$\begin{aligned}
\frac{\partial \alpha}{\partial n} &= -\frac{\det\begin{pmatrix} \frac{\partial E_1}{\partial n} & \frac{\partial E_1}{\partial l^C} & \frac{\partial E_1}{\partial l^I} \\ \frac{\partial E_2}{\partial n} & \frac{\partial E_2}{\partial l^C} & \frac{\partial E_2}{\partial l^I} \\ \frac{\partial E_3}{\partial n} & \frac{\partial E_3}{\partial l^C} & \frac{\partial E_3}{\partial l^I} \end{pmatrix}}{\det\begin{pmatrix} \frac{\partial E_1}{\partial \alpha} & \frac{\partial E_1}{\partial l^C} & \frac{\partial E_1}{\partial l^I} \\ \frac{\partial E_2}{\partial \alpha} & \frac{\partial E_2}{\partial l^C} & \frac{\partial E_2}{\partial l^I} \\ \frac{\partial E_3}{\partial \alpha} & \frac{\partial E_3}{\partial l^C} & \frac{\partial E_3}{\partial l^I} \end{pmatrix}} = -\frac{\text{NUM1}}{\text{DEN}} \\
\text{NUM1} &= \frac{1}{n^2}(-f_L^C(1-\alpha))(-f_L^C(1-\alpha)+v')(f_{LL}^I - v'') \\
\text{DEN} &= \frac{1}{n}\left((1-\alpha)((f_L^C)^2 - f^C f_{LL}^C)f_{LL}^I + v''(-(1-\alpha)(f_L^C)^2 + f^C((1-\alpha)f_{LL}^C + f_{LL}^I)\right) \\
&+ \frac{1}{n}\left(f_L^C v'(-f_{LL}^I + v'')\right)
\end{aligned}$$

To obtain this expression, we used again the fact that f is homogeneous of degree 1 and f_L homogeneously of degree 0. Both the numerator and denominator are unambiguously negative, so that $\frac{\partial \alpha}{\partial n} < 0$.

$$\begin{aligned}
\frac{\partial l^C}{\partial n} &= -\frac{\det\begin{pmatrix} \frac{\partial E_1}{\partial \alpha} & \frac{\partial E_1}{\partial n} & \frac{\partial E_1}{\partial l^I} \\ \frac{\partial E_2}{\partial \alpha} & \frac{\partial E_2}{\partial n} & \frac{\partial E_2}{\partial l^I} \\ \frac{\partial E_3}{\partial \alpha} & \frac{\partial E_3}{\partial n} & \frac{\partial E_3}{\partial l^I} \end{pmatrix}}{\det\begin{pmatrix} \frac{\partial E_1}{\partial \alpha} & \frac{\partial E_1}{\partial l^C} & \frac{\partial E_1}{\partial l^I} \\ \frac{\partial E_2}{\partial \alpha} & \frac{\partial E_2}{\partial l^C} & \frac{\partial E_2}{\partial l^I} \\ \frac{\partial E_3}{\partial \alpha} & \frac{\partial E_3}{\partial l^C} & \frac{\partial E_3}{\partial l^I} \end{pmatrix}} = -\frac{\text{NUM2}}{\text{DEN}} \\
\text{NUM2} &= \frac{1}{n^3}\left(f^C f_L^C(1-\alpha)(f_{LL}^I - v'')\right)
\end{aligned}$$

NUM2 is unambiguously negative so that $\frac{\partial l^C}{\partial n} < 0$.

It is evident that $\frac{\partial R}{\partial n}$ has an ambiguous sign.

B Analytical results

B.1 Proof of proposition 1

We first show that if $\overline{A}$ tends to $+\infty$ then the collective regime dominates the mixed regime. We then examine the influence of $\overline{A}$ on the head's propensity of giving out individual fields when he is just indifferent across both regimes.

When $\overline{A}$ tends to $+\infty$, the participation constraint becomes unbinding in the collective regime. If the participation is also unbinding in the mixed regime for all A^I, we know that the mixed regime will never be chosen over the collective one (ADD SECTION REF). If the participation constraint binds in the mixed regime at the optimal A^I, we can show that sons work more overall in the collective regime (where they obtain more than their reservation utility) than in the mixed regime, suggesting that there is no incentive advantage to the mixed regime, which is then never optimal. THIS IS NO FORMAL PROOF, HOW TO SHOW IT FORMALLY??

Suppose that the head is just indifferent across regime. Let us compute the impact of an increase in $\overline{A}$ on the head's rent under each regime and compare the expressions obtained. In the mixed regime, for a given A^I:

$$\frac{\partial R}{\partial \overline{A}} = \frac{\partial L}{\partial \overline{A}} = \alpha \frac{n}{N} f_A^C + \lambda \frac{1-\alpha}{n} \frac{n}{N} f_{LA}^C + \nu \frac{1-\alpha}{n} \frac{n}{N} f_A^C$$

Since $\nu = n - \lambda \frac{f_L^C}{f^C}$ (Section A.2.2), we can write:

$$\frac{\partial R}{\partial \overline{A}} = \frac{n}{N} \alpha f_A^C + \lambda \frac{1-\alpha}{N} f_{LA}^C + (n - \lambda \frac{f_L^C}{f^C}) \frac{1-\alpha}{N} f_A^C$$

Or:

$$\frac{\partial R}{\partial \overline{A}} = \frac{n}{N} f_A^C + \lambda \frac{1-\alpha}{N} f_{LA}^C \left(1 - \tau_{LA}\right)$$

where $\tau_{LA} = \frac{f_A f_L}{f f_{LA}}$ is the substitution elasticity of production factors. Because $\tau_{LA} = 1$ in the case of the Cobb-Douglas function, the above expression reduces to:

$$\frac{\partial R}{\partial \overline{A}} = \frac{n}{N} f_A^C$$

A unit increase in the total family endowment increases the area on farm by $\frac{n}{N}$ and the impact on the head's rent is equal to $\frac{n}{N}$ times the marginal productivity of land. The same holds in the collective regime:

$$\begin{aligned}
\frac{\partial R}{\partial \overline{A}} &= \frac{\partial L}{\partial \overline{A}} = \alpha \frac{n}{N} f_A - \beta \frac{1-\alpha}{N} f_{LA} + \gamma \frac{1-\alpha}{N} f_A \\
&= \frac{n}{N} f_A + \frac{1-\alpha}{N} \beta \left(f_{LA} - \frac{f_L f_A}{f} \right) \\
&= \frac{n}{N} f_A
\end{aligned}$$

Consider a household head who is indifferent between the mixed and the pure collective regimes. If the land endowment decreases marginally, his rent decreases by $\frac{n}{N} f_A$ in the pure collective regime and by $\frac{n}{N} f_A^C$ in the mixed regime (holding A^I constant). Since in the mixed regime there is less labor applied by unit of collective land (due to competition of individual fields), $f_A^C < f_A$, so that the head's rent decreases to a smaller extent in the mixed regime. This holds a fortiori if the father is free to change A^I. If the land endowment increases marginally, the rent increases more in the pure collective regime than in the mixed regime, and this holds for all $0 < A^I < \frac{A}{n}$. As a result, even if the household head chooses a new A^I, the mixed regime becomes strictly less favorable than the pure collective regime.

B.2 Proof of proposition 2

We first show that if $\underline{u}$ tends to 0 then the collective regime dominates the mixed regime. We then examine the influence of $\underline{u}$ on the head's propensity of giving out individual fields when he is just indifferent across both regimes.

When $\underline{u}$ tends to zero, the participation constraint becomes unbinding in the mixed regime for all A^I. To prove it, let us show that if the incentive constraints are satisfied, then the participation constraint is automatically satisfied for $\underline{u}$ very close to zero. With a Cobb-Douglas production function ($f(a,l) = a^{\varepsilon} l^{1-\varepsilon}$), and a polynomial cost of effort ($v(l) = \omega l^2$), the incentive constraints are:

$\frac{1-\alpha}{n}(1-\varepsilon)(A-nA^I)^{\varepsilon}(ml^C)^{-\varepsilon} = 2\omega(l^C + l^I)$ and $(1-\varepsilon)(A^I)^{\varepsilon}(l^I)^{-\varepsilon} = 2\omega(l^C + l^I)$.

We thus have:

$$\begin{aligned}
\frac{1-\alpha}{n}(1-\varepsilon)(A-nA^I)^{\varepsilon}(nl^C)^{-\varepsilon} + (1-\varepsilon)(A^I)^{\varepsilon}(l^I)^{-\varepsilon} &= 4\omega(l^C + l^I) \\
\Leftrightarrow (l^C + l^I)\left(\frac{1-\alpha}{n}(1-\varepsilon)(A-nA^I)^{\varepsilon}(nl^C)^{-\varepsilon} + (1-\varepsilon)(A^I)^{\varepsilon}(l^I)^{-\varepsilon}\right) &= 4\omega(l^C + l^I)^2 \\
\Leftrightarrow \frac{1-\alpha}{n}(1-\varepsilon)(A-nA^I)^{\varepsilon}n^{-\varepsilon}(l^C)^{1-\varepsilon} + (1-\varepsilon)(A^I)^{\varepsilon}(l^I)^{1-\varepsilon} & \\
+l^I\frac{1-\alpha}{n}(1-\varepsilon)(A-nA^I)^{\varepsilon}(nl^C)^{-\varepsilon} + l^C(1-\varepsilon)(A^I)^{\varepsilon}(l^I)^{-\varepsilon} &= 2\omega(l^C + l^I)^2 + l^C 2\omega(l^C + l^I) \\
& \quad +l^I 2\omega(l^C + l^I) \\
\Leftrightarrow \frac{1-\alpha}{n}(1-\varepsilon)(A-nA^I)^{\varepsilon}n^{-\varepsilon}(l^C)^{1-\varepsilon} + (1-\varepsilon)(A^I)^{\varepsilon}(l^I)^{1-\varepsilon} &= 2\omega(l^C + l^I)^2 \\
\Rightarrow \frac{1-\alpha}{n}(A-nA^I)^{\varepsilon}n^{1-\varepsilon}(l^C)^{1-\varepsilon} + (1-\varepsilon)(A^I)^{\varepsilon}(l^I)^{1-\varepsilon} &> \omega(l^C + l^I)^2
\end{aligned}$$

We have just shown that when the incentive constraints are satisfied, then the participation constraint is automatically satisfied for $\underline{u}$ very close to zero. We know that the mixed regime never dominates if the participation constraint is unbinding for all A^I, since the father's rent is then monotonically decreasing in A^I. As a result, when $\underline{u}$ tends to zero the head of a collective farm never finds it optimal to grant individual plot.

We now turn to the case where the head is just indifferent between the pure collective and the mixed regime. In this case, the solutions to the mixed and the all collective problem are such that: $\alpha^m (f^C)^m = \alpha^s f^s$ where the superscripts m and s refers to the mixed and the pure collective regime respectively, and the arguments of the production function are ignored for brevity. Since both the area of the collective field and for the same α, the son's incentive to work on this field are greater in the pure collective regime, we know that unless $\alpha^m > \alpha^s$, the father's rent is greater in the strictly collective regime. This implies $(f^C)^m < f^s$, and $(f_L^C)^m > f_L^s$ (since f is increasing and concave in labor).

The envelop theorem implies that marginal increase in $\underline{u}$ decreases the father's rent by γ in the pure collective regime and by ν in the mixed regime (since we know that the optimal A^I in the mixed regime is such that the participation constraint binds, cf footnote 3.2.2.), where the Lagrangian multipliers have a parallel expression:

$$\gamma = n - \frac{1}{\frac{v'' f^s}{(f_L^s)^2(n-1+\alpha^s)} + \frac{(1-\alpha^s)(-f_{LL}^s) f^s}{(f_L^s)^2(n-1+\alpha^s)} + \frac{(1-\alpha^s)(1-\frac{1}{n})}{n-1+\alpha^s}} \quad (31)$$

$$\nu = n - \frac{1}{\left(\frac{-f_{LL}^I}{v''-f_{LL}^I}\right) \frac{v''(f^C)^n}{((f_L^C)^n)^2(n-1+\alpha^m)} + \frac{(1-\alpha^m)(-(f_{LL}^C)^m)(f^C)^m}{((f_L^C)^m)^2(n-1+\alpha^m)} + \frac{(1-\alpha^m)(1-\frac{1}{n})}{n-1+\alpha^m}} \quad (32)$$

With a Cobb-Douglas production function ($f(A, l) = A^{\varepsilon} l^{1-\varepsilon}$) and a polynomial cost of effort ($v(l) - \omega l^2$), we have: $\frac{-f f_{LL}}{f_L^2} = \frac{\varepsilon}{1-\varepsilon}$ and $\frac{f v''}{f_L^2} = \frac{1-\alpha}{1-\varepsilon}$. Using these relationships, these expressions become:

$$\gamma = n - \frac{1}{\frac{(1-\alpha^s)}{(1-\varepsilon)(n-1+\alpha^s} + \frac{(1-\alpha^s)\varepsilon}{(1-\varepsilon)(n-1+\alpha^s)} + \frac{(1-\alpha^s)(1-\frac{1}{n})}{n-1+\alpha^s}}$$

$$\nu = n - \frac{1}{\left(\frac{-f_{LL}^I}{v''-f_{LL}^I}\right) \frac{(1-\alpha^m)}{(1-\varepsilon)(n-1+\alpha^m)} + \frac{(1-\alpha^m)\varepsilon}{(1-\varepsilon)(n-1+\alpha^m)} + \frac{(1-\alpha^m)(1-\frac{1}{n})}{n-1+\alpha^m}}$$

With $\alpha^m > \alpha^s$, we thus have $\gamma > \nu$. At a point of indifference, a marginal increase in the reservation utility has thus a greater (negative) impact in the pure collective regime than in the mixed regime. The head would thus strictly prefer the mixed regime.

We cannot conclude about the father's preference in the reverse situation of a *marginal decrease in $\underline{u}$*. If A^I is constrained to remain constant, we know the pure collective regime would become more desirable, however, since the father may change A^I, we do not know about his choice.

B.3 Proof of proposition 3

To analyze how a marginal change in $\underline{u}$ change incentives to split the family, we examine the conditions under which $\frac{\partial R}{\partial n} > 0$. We need to separate between the case of the pure collective and the mixed regime, since they differ in terms of the expression for $\frac{\partial R}{\partial n}$.

B.3.1 The reservation utility and the decision to split in the pure collective regime

We know that as $\underline{u}$ tends to zero, the participation constraint becomes unbinding. It is easy to show that if the participation constraint does not bind, then $\frac{\partial R}{\partial n}$ is unambiguously positive (PROOF IN CAHIER), meaning it is never desirable for the head to let one son leave the farm with some land. This implies that when $\underline{u}$ tends to zero, the head will never choose to split the family.

Conversely when $\underline{u}$ tends to $+\infty$, the participation constraint is binding and we can show that the family head will choose to split the family. To obtain an expression for $\frac{\partial R}{\partial n}$ we replace $\frac{\partial \alpha}{\partial n}$ and $\frac{\partial l}{\partial n}$ in equation 19 by the expression obtained in section A.3.1, we have:

$$\begin{aligned}\frac{\partial R}{\partial n} &= \alpha(\frac{\overline{A}}{N} f_A + l f_L) - \frac{f \alpha f_L \frac{1-\alpha}{n^2} f_L + f \frac{f_L}{n^2}(f_L(1-\alpha) - v')}{\frac{f_L}{n}((1-\alpha) f_L - v') - \frac{f}{n}((1-\alpha) f_{LL} - v'')} \\ &= \frac{\alpha}{n} f - \frac{f f_L^2 \frac{1-\alpha}{n^2}(\alpha + 1 - \frac{1}{n})}{\frac{f_L}{n}((1-\alpha) f_L - v') - \frac{f}{n}((1-\alpha) f_{LL} - v'')}\end{aligned}$$

Thus:

$$\begin{aligned}
\frac{\partial R}{\partial n} &> 0 \\
\Leftrightarrow \frac{\alpha}{n} &> \frac{f_L^2 \frac{1-\alpha}{n^2}(\alpha + 1 - \frac{1}{n})}{\frac{f_L}{n}((1-\alpha)f_L - \frac{1-\alpha}{n} f_L) - \frac{f}{n}((1-\alpha)f_{LL} - v'')} \\
&> \frac{\frac{1}{n}(\alpha + 1 - \frac{1}{n})}{(1-\frac{1}{n}) - \frac{f f_{LL}}{f_L^2} + \frac{f v''}{f_L^2(1-\alpha)}}
\end{aligned}$$

With the Cobb-Douglas production function and the polynomial cost of effort function, we again have: $\frac{-f f_{LL}}{f_L^2} = \frac{\varepsilon}{1-\varepsilon}$ and $\frac{f v''}{f_L^2} = \frac{1-\alpha}{1-\varepsilon}$. This considerably simplifies the previous expression since we now obtain the following condition:

$$\begin{aligned}
\frac{\partial R}{\partial n} &> 0 \\
\Leftrightarrow \alpha &> \frac{\alpha + 1 - \frac{1}{n}}{\frac{2}{1-\varepsilon} - \frac{1}{n}} \\
\Leftrightarrow \alpha &> \frac{1 - \frac{1}{n}}{\frac{1+\varepsilon}{1-\varepsilon} - \frac{1}{n}}
\end{aligned}$$

This condition is increasingly difficult to satisfy as n increases (for all n: $\frac{\partial \psi}{\partial n} > 0$, with $\psi = \frac{1-\frac{1}{n}}{\frac{1+\varepsilon}{1-\varepsilon} - \frac{1}{n}}$), which is intuitive. Once the family is really large, it becomes less interesting for the head to keep it whole.

Furthermore, when $\underline{u}$ gets very large, then α tends to 0 (proof in section B.5) and we have:

$$\begin{aligned}
\frac{\partial R}{\partial n} &< 0 \\
\Leftrightarrow 0 &> \frac{1 - \frac{1}{n}}{\frac{1+\varepsilon}{1-\varepsilon} - 1}
\end{aligned}$$

This last inequality holds for all $n > 1$, and suggests that the family head will always split the family if $\underline{u}$ is infinitely large

We have just shown that the head of a collective farm chooses to split the family when $\underline{u}$ tends to $+\inf$ while he prefers to keep the family whole when $\underline{u}$ tends to 0. Since the father's rent is monotonically decreasing in $\underline{u}$, there must exist a unique level of $\underline{u}$ where the head is just indifferent between splitting and not.

B.3.2 The reservation utility and the decision to split the family in the mixed regime

We know that if $\underline{u}$ tends to zero, then the collective regime always dominates the mixed regime and the section above treats the case of the collective regime. If $\underline{u}$ tends to $+\infty$, the participation constraint binds and $\frac{\partial R}{\partial n}$ is as follows. Replacing $\frac{\partial \alpha}{\partial n}$ and $\frac{\partial l}{\partial n}$ in equation 19 by the expressions obtained in section A.3.2, we have:

$$\frac{\partial R}{\partial n} = \frac{\alpha}{n} f^C + \frac{f^C f_L^C (1-\alpha)(f_L^C - v')(f_{LL}^I - v'')}{n\left((1-\alpha)((f_L^C)^2 - f^C f_{LL}^C) f_{LL}^I + v''(-(1-\alpha)(f_L^C)^2 + f^C((1-\alpha) f_{LL}^C + f_{LL}^I) + f_L^C v'(-f_{LL}^I + v'')\right)}$$

Thus:

$$\begin{aligned} \frac{\partial R}{\partial n} &> 0 \\ \Leftrightarrow \alpha &> \frac{f_L^C (1-\alpha)(f_L^C - v')(f_{LL}^I - v'')}{(1-\alpha)((f_L^C)^2 - f^C f_{LL}^C) f_{LL}^I + v''(-(1-\alpha)(f_L^C)^2 + f^C((1-\alpha) f_{LL}^C + f_{LL}^I) + f_L^C v'(-f_{LL}^I + v'')} \end{aligned}$$

With the Cobb-Douglas production function and the polynomial cost of effort, we again have: $\frac{-ff_{LL}}{f_L^2} = \frac{\varepsilon}{1-\varepsilon}$ and $\frac{fv''}{f_L^2} = \frac{1-\alpha}{1-\varepsilon}$. As a result:

$$\begin{aligned}
\frac{\partial R}{\partial n} &> 0 \\
\Leftrightarrow \alpha &> \frac{f_L^C(1-\alpha)(f_L^C - v')(f_{LL}^I - v'')}{f_{LL}^I (f_L^C)^2 \frac{1-\alpha}{1-\varepsilon} - v''(f_L^C)^2 \frac{1-\alpha}{1-\varepsilon} + v'' f^C f_{LL}^I + (f_L^C)^2 \frac{1-\alpha}{n}(-f_{LL}^I + v'')} \\
&> \frac{(f_L^C)^2(1-\alpha)(1-\frac{1-\alpha}{n})(f_{LL}^I - v'')}{(f_L^C)^2 \frac{1-\alpha}{1-\varepsilon}(-f_{LL}^I + v'')\left(-\frac{1}{1-\varepsilon} + \frac{1}{n}\right) + v'' f^C f_{LL}^I} \\
&> \frac{1 - \frac{1-\alpha}{n}}{\frac{1}{1-\varepsilon}\left(\frac{1}{1-\varepsilon} - \frac{1}{n}\right) + \frac{v'' f^C f_{LL}^I}{(f_L^C)^2(1-\alpha)(f_{LL}^I - v'')}} \\
&> \frac{1 - \frac{1-\alpha}{n}}{\frac{1}{1-\varepsilon}\left(\frac{1}{1-\varepsilon} - \frac{1}{n}\right) + \frac{1}{\frac{(1-\alpha)(f_L^C)^2}{f^C v''} - \frac{(1-\alpha)(f_L^C)^2}{f_{LL}^I f^C}}} \\
&> \frac{1 - \frac{1-\alpha}{n}}{\frac{1}{1-\varepsilon}\left(\frac{1}{1-\varepsilon} - \frac{1}{n}\right) + \frac{1}{(1-\varepsilon) - \frac{f^I}{f^C} \frac{\frac{n^2}{1-\alpha}(f_L^I)^2}{f_{LL}^I f^I}}} \\
&> \frac{1 - \frac{1-\alpha}{n}}{\frac{1}{1-\varepsilon}\left(\frac{1}{1-\varepsilon} - \frac{1}{n}\right) + \frac{1}{(1-\varepsilon) + \frac{f^I}{f^C} \frac{n^2}{1-\alpha} \frac{1-\varepsilon}{\varepsilon}}}
\end{aligned}$$

When $\underline{u}$ is very large, then α tends to 0 (proof in section B.5.2) and we have:

$$\begin{aligned}
\frac{\partial R}{\partial n} &< 0 \\
\Leftrightarrow 0 &< \frac{1 - \frac{1}{n}}{\frac{1}{1-\varepsilon}\left(\frac{1}{1-\varepsilon} - \frac{1}{n}\right) + \frac{1}{(1-\varepsilon) + \frac{f^I}{f^C} n^2 \frac{1-\varepsilon}{\varepsilon}}}
\end{aligned}$$

This last inequality holds for all $n > 1$. Therefore conclude that, when the reservation utility is very large, then, in the mixed regime, the father will choose to split the family.

B.4 Proof of proposition 4

To analyze how a marginal change in $\overline{A}$ changes incentives to split the family, we use the same arguments as for the role of $\underline{u}$. Let us first examine the case of the collective farm.

When land is very abundant we know that the participation constraint does not bind. Then we have shown that it is never optimal to split the family. Conversely when land is very scarce, then the participation constraint binds and we have shown that:

$$\begin{aligned} \frac{\partial R}{\partial n} &> 0 \\ \Leftrightarrow \alpha &> \frac{1-\frac{1}{n}}{\frac{1+\varepsilon}{1-\varepsilon}-\frac{1}{n}} \end{aligned}$$

When $\overline{A}$ gets very small, then α tends to 0 (proof in section B.5) and we have:

$$\begin{aligned} \frac{\partial R}{\partial n} &< 0 \\ \Leftrightarrow 0 &> \frac{1-\frac{1}{n}}{\frac{1+\varepsilon}{1-\varepsilon}-1} \end{aligned}$$

This last inequality holds for all $n > 1$, and suggests that the family head will always split the family if $\overline{A}$ is close to zero.

Since the father's rent is monotonically increasing in $\overline{A}$, there must exist a level of $\overline{A}$, where he is just indifferent between splitting the family or keeping it whole.

The case of the mixed regime is interesting only if $\overline{A}$ is small enough for the participation constraint to bind (otherwise we know that the collective regime always dominates). In that case we know that:

$$\begin{aligned} \frac{\partial R}{\partial n} &> 0 \\ \Leftrightarrow \alpha &> \frac{1-\frac{1-\alpha}{n}}{\frac{1}{1-\varepsilon}\left(\frac{1}{1-\varepsilon}-\frac{1}{n}\right)+\frac{1}{(1-\varepsilon)+\frac{f^I}{f^C}\frac{n^2}{1-\alpha}\frac{1-\varepsilon}{\varepsilon}}} \end{aligned}$$

When $\overline{A}$ tends to 0, α tends to 0 (proof in section B.5) and as before we have:

$$\begin{aligned} \frac{\partial R}{\partial n} &< 0 \\ \Leftrightarrow 0 &< \frac{1-\frac{1}{n}}{\frac{1}{1-\varepsilon}\left(\frac{1}{1-\varepsilon}-\frac{1}{n}\right)+\frac{1}{(1-\varepsilon)+\frac{f^I}{f^C}n^2\frac{1-\varepsilon}{\varepsilon}}} \end{aligned}$$

This last inequality holds for all $n > 1$. Therefore conclude that, when the land is very scarce, then, in the mixed regime, the father will choose to split the family.

B.5 Limit of α when $\overline{A}$ or $\underline{u}$ tend to zero or $+\infty$

Again we distinguish between the pure collective and the mixed regime.

B.5.1 Strictly collective case

We want to show:

- When $\overline{A}$ tends to 0, α tends to 0.
- When $\underline{u}$ tends to $+\infty$, α tends to 0.
- When $\underline{u}$ tends to 0, α tends to $\frac{1+\varepsilon}{2}$.

Recall that, when the participation constraint binds, α and l are the solution to the following system:

$$\begin{cases} G_1 = 0 = \frac{1-\alpha}{m} f_L(\frac{n\overline{A}}{N}, ml) - v'(l) \\ G_2 = 0 = \frac{1-\alpha}{m} f\left(\frac{n\overline{A}}{N}, ml\right) - v(l) - \underline{u} \end{cases} \tag{33}$$

As $\overline{A}$ decreases or $\underline{u}$ increases, the participation constraint becomes tighter and eventually binds. To establish the first proposition, we proceed as follows. First we show that α is monotonically decreasing in $-\overline{A}$ (or: $\frac{\partial \alpha}{\partial \overline{A}} > 0$). This implies that α tends to its minimal value when $\overline{A}$ tends to zero. Then we assume a Cobb-Douglas production function and a polynomial cost of effort and we show that for all $\alpha > 0$, there exists a land endowment such that the system of equations is satisfied. This implies that the limit of α when $\overline{A}$ tends to 0 cannot be strictly positive, it has to be 0. Let's analyze the sign of $\frac{\partial\alpha}{\partial\overline{A}}$. Applying Cramer's rule to the first two equations yields:

$$\begin{aligned}
\frac{\partial\alpha}{\partial A} &= -\frac{\det\begin{pmatrix} \frac{\partial G_1}{\partial l} & \frac{\partial G_1}{\partial A} \\ \frac{\partial G_2}{\partial l} & \frac{\partial G_2}{\partial A} \end{pmatrix}}{\det\begin{pmatrix} \frac{\partial G_1}{\partial l} & \frac{\partial G_1}{\partial \alpha} \\ \frac{\partial G_2}{\partial l} & \frac{\partial G_2}{\partial \alpha} \end{pmatrix}} \\
&= -\frac{n}{N}\frac{\frac{(1-\alpha)^2}{n} f_{LL} f_A - \frac{n}{N}\frac{1-\alpha}{n} f_A v'' - \frac{1-\alpha}{n} f_{LA}\left((1-\alpha)f_L - v'\right)}{-\frac{1}{n} f\left((1-\alpha)f_{LL} - v''\right) + \frac{1}{n} f_L\left((1-\alpha)f_L - v'\right)}
\end{aligned}$$

This expression is unambiguously positive. Now we assume the same functional forms for the production function and the cost of effort function as in section 4. Then we can replace the first equation in the system with: $l = A^{\frac{\varepsilon}{1+\varepsilon}} \frac{(1-\alpha)^{\frac{1}{1+\varepsilon}}(1-\varepsilon)^{\frac{1}{1+\varepsilon}}}{n(2\omega)^{\frac{1}{1+\varepsilon}}}$

For all $\alpha > 0$, we can then find $\overline{A}$ such that the system is satisfied. Indeed the second equation can be written:

$$\underline{u} = \left(\frac{n}{N}\overline{A}\right)^{\frac{2\varepsilon}{1+\varepsilon}} \left(\frac{1-\alpha}{n} \left(\frac{(1-\alpha)^{\frac{1}{1+\varepsilon}}(1-\varepsilon)^{\frac{1}{1+\varepsilon}}}{(2\omega)^{\frac{1}{1+\varepsilon}}} \right)^{1-\varepsilon} - \omega \left(\frac{(1-\alpha)^{\frac{1}{1+\varepsilon}}(1-\varepsilon)^{\frac{1}{1+\varepsilon}}}{(2\omega)^{\frac{1}{1+\varepsilon}}} \right)^2 \right)$$

We can thus set $\overline{A}$:

$$\overline{A} = \frac{N}{n}\underline{u}^{\frac{1+\varepsilon}{2\varepsilon}} \left(\frac{1-\alpha}{n} \left(\frac{(1-\alpha)^{\frac{1}{1+\varepsilon}}(1-\varepsilon)^{\frac{1}{1+\varepsilon}}}{(2\omega)^{\frac{1}{1+\varepsilon}}} \right)^{1-\varepsilon} - \omega \left(\frac{(1-\alpha)^{\frac{1}{1+\varepsilon}}(1-\varepsilon)^{\frac{1}{1+\varepsilon}}}{(2\omega)^{\frac{1}{1+\varepsilon}}} \right)^2 \right)^{-\frac{1+\varepsilon}{2\varepsilon}}$$

To prove the second proposition we just need to show $\frac{\partial\alpha}{\partial\underline{u}} < 0$, and then the argument developed above applies.

$$\frac{\partial\alpha}{\partial\underline{u}} = -\frac{\det\begin{pmatrix}\frac{\partial G_1}{\partial l} & \frac{\partial G_1}{\partial\underline{u}}\\ \frac{\partial G_2}{\partial l} & \frac{\partial G_2}{\partial\underline{u}}\end{pmatrix}}{\det\begin{pmatrix}\frac{\partial G_1}{\partial l} & \frac{\partial G_1}{\partial\alpha}\\ \frac{\partial G_2}{\partial l} & \frac{\partial G_2}{\partial\alpha}\end{pmatrix}} = -\frac{-\left((1-\alpha)f_{LL}-v''\right)}{-\frac{1}{n}f\left((1-\alpha)f_{LL}-v''\right)+\frac{1}{n}f_L\left((1-\alpha)f_L-v'\right)}$$

This last expression is unambiguously negative.

To prove the third proposition, we first show that when $\underline{u}$ tends to 0 the participation constraint becomes unbinding and then derive an expression for α in that case. Let's first show that if (α, l) satisfy the incentive constraint, then, for $\underline{u}$ very close to zero, $\frac{1-\alpha}{n}f - v > 0$ so that the participation constraint is automatically satisfied. With a Cobb-Douglas production function ($f(a,l) = a^{\varepsilon}l^{1-\varepsilon}$), and a polynomial cost of effort ($v(l) = \omega l^2$), the incentive constraint is:

$$\begin{aligned}\frac{1-\alpha}{n}(1-\varepsilon)A^{\varepsilon}(nl)^{-\varepsilon} - 2\omega l = 0\\ \Rightarrow \frac{1-\alpha}{n}(1-\varepsilon)A^{\varepsilon}n^{\varepsilon}l^{1-\varepsilon} - 2\omega l = 0\\ \Rightarrow \frac{n}{1-\varepsilon}\frac{1-\alpha}{n}(1-\varepsilon)\left(A\right)^{\varepsilon}n^{\varepsilon}l^{1-\varepsilon} > \frac{1-\alpha}{n}(1-\varepsilon)A^{\varepsilon}n^{\varepsilon}l^{1-\varepsilon} > 2\omega l^2 > \omega l^2\\ \Rightarrow \frac{1-\alpha}{n}A^{\varepsilon}(nl)^{1-\varepsilon} - \omega l^2 > 0\end{aligned}$$

The FOC of the optimization problem in the pure collective regime when $\gamma = 0$ (unbinding participation constraint) reduce to: $\alpha f_L + \frac{ff_{LL}}{f_L}(1-\alpha) - \frac{f}{f_L}v'' = 0$. With the Cobb-Douglas production function and the polynomial cost of effort function, we have again: $\frac{-ff_{LL}}{f_L^2} = \frac{\varepsilon}{1-\varepsilon}$

and $\frac{fv''}{f_L^2} = \frac{1-\alpha}{1-\varepsilon}$. As a result:

$$\begin{aligned}\alpha &= \frac{fv'' - ff_{LL}}{f_L^2 - ff_{LL}} = (1-\varepsilon)\frac{fv'' - ff_{LL}}{f_L^2} = 1 - \alpha + \varepsilon \\ \alpha &= \frac{1+\varepsilon}{2}\end{aligned}$$

Thus, when $\underline{u}$ tends to 0, α tends to $\frac{1+\varepsilon}{2}$ and we have:

B.5.2 Mixed case

We want to show that in the mixed regime, for a given A^I:

- When $\underline{u}$ tends to $+\infty$, α tends to 1.
- When $\underline{u}$ tends to 0, α tends to 0.

To prove the propositions, we use the same arguments as in the pure collective case. We show below that $\frac{\partial \alpha}{\partial \underline{u}} < 0$. We assume that the same functional forms as previously. For all $\alpha < 1$, we can find $\underline{u}$ such that the system defining l^C, l^I and α holds:

$$\begin{cases} E_1 = \frac{1-\alpha}{n} f_L((\frac{\overline{A}}{N} - A^I)n, nl^C) - v'(l^C + l^I) = 0 \\ E_2 = f_L(A^I, l^I) - v'(l^C + l^I) = 0 \\ E_3 = \frac{1-\alpha}{n} f\left((\frac{\overline{A}}{N} - A^I)n, nl^C\right) + f\left(A^I, l^I\right) - v(l^C + l^I) = \underline{u}ineu \end{cases} \tag{34}$$

From E_2 we can extract $l^I(l^C, \alpha)$.[24] Then E_1 defines $l^C(\alpha)$.[25] As a result we can write $l^I(\alpha)$ and $l^C(\alpha)$ and plug these expression in E_3. Finally E_3 defines $\underline{u}(\alpha)$: for all $\alpha > 0$ there is a $\underline{u}$ such that α is solution to the system. This combined to the fact that $\frac{\partial \alpha}{\partial \underline{u}} < 0$ implies that the limit of α when $\underline{u}$ tends to $+\infty$ can only be 0 and conversely the limit of α when $\underline{u}$

[24] $l^I(l^C, \alpha) = nl^C(\frac{1-\alpha}{n})^{-\frac{1}{\varepsilon}} \frac{A^I}{\overline{A} - nA^I}$

[25] $l^C = \left(\frac{(1-\alpha)(A - nA^I)^{\varepsilon} n^{-\varepsilon-1}}{2\omega(1 + n(\frac{1-\alpha}{n})^{-\frac{1}{\varepsilon}} \frac{A^I}{A - nA^I})}\right)^{\frac{1}{1+\varepsilon}}$

tends to 0 can only be 1 (since $\frac{\partial\alpha}{\partial\underline{u}} < 0$ implies that α tends asymptotically to its limits and the upper limit cannot be strictly smaller than 1, while the lower limit cannot be strictly larger than 0). Let's show that $\frac{\partial\alpha}{\partial\underline{u}} < 0$.

$$\frac{\partial\alpha}{\partial\underline{u}} = -\frac{\det\begin{pmatrix} \frac{\partial E_1}{\partial\underline{u}} & \frac{\partial E_1}{\partial l^C} & \frac{\partial E_1}{\partial l^I} \\ \frac{\partial E_2}{\partial\underline{u}} & \frac{\partial E_2}{\partial l^C} & \frac{\partial E_2}{\partial l^I} \\ \frac{\partial E_3}{\partial\underline{u}} & \frac{\partial E_3}{\partial l^C} & \frac{\partial E_3}{\partial l^I} \end{pmatrix}}{\det\begin{pmatrix} \frac{\partial E_1}{\partial\alpha} & \frac{\partial E_1}{\partial l^C} & \frac{\partial E_1}{\partial l^I} \\ \frac{\partial E_2}{\partial\alpha} & \frac{\partial E_2}{\partial l^C} & \frac{\partial E_2}{\partial l^I} \\ \frac{\partial E_3}{\partial\alpha} & \frac{\partial E_3}{\partial l^C} & \frac{\partial E_3}{\partial l^I} \end{pmatrix}} = -\frac{\text{NUM3}}{\text{DEN}}$$

$$\begin{aligned} \text{NUM3} &= -f^C_{LL} f^I_{LL}(1-\alpha) + ((1-\alpha)f^C_{LL} + f^I_{LL})v'' \\ \text{DEN} &= \frac{1}{n}\left((1-\alpha)((f^C_L)^2 - f^C f^C_{LL})f^I_{LL} + v''(-(1-\alpha)(f^C_L)^2 + f^C((1-\alpha)f^C_{LL} + f^I_{LL})\right) \\ &+ \frac{1}{n}\left(f^C_L v'(-f^I_{LL} + v'')\right) \end{aligned}$$

$NUM3$ as well as DEN are unambiguously negative, so that $\frac{\partial\alpha}{\partial\underline{u}} < 0$.

We could not prove similar propositions about $\overline{A}$. We cannot use the same arguments as in the collective case, because there is no explicit expression for $\overline{A}(\alpha)$ with the chosen functional form.

C Simulation results

Table 1: Simulation results: key parameters and main endogenous variables

$\underline{u}$	$\overline{A}$	n	A	A_i	α	R	regime	$\underline{u}$	$\overline{A}$	n	A	A_i	α	R	regime
0.8	11	1	1.1	0.60	0.03	0.01	split + CI	1.5	25	10	25	2.40	0.36	0.08	CI
0.8	12	10	12	1.05	0.47	0.16	CI	1.5	26	10	26	2.40	0.61	0.21	CI
0.8	13	10	13	1.05	0.63	0.32	CI	1.5	27	10	27	2.40	0.7	0.32	CI
0.8	14	10	14	1.00	0.48	0.45	CI	1.5	28	10	28	2.35	0.54	0.45	CI
0.8	15	8	12	0.00	0.1	0.73	split	1.5	29	10	29	2.35	0.6	0.57	CI
0.8	16	10	16	0.00	0.12	1.12	strict coll	1.5	30	10	30	2.35	0.64	0.69	CI
0.8	17	10	17	0.00	0.16	1.52	strict coll	1.5	31	10	31	2.30	0.56	0.82	CI
0.9	13	10	13	1.25	0.54	0.06	CI	1.5	32	5	16	0.00	0.1	0.95	split
0.9	14	9	12.6	1.23	0.59	0.19	split+CI	1.5	33	7	23.1	0.01	0.09	1.16	split
0.9	15	10	15	1.20	0.5	0.33	CI	1.5	34	10	34	0.00	0.09	1.45	coll
0.9	16	10	16	1.20	0.59	0.48	CI	1.5	35	10	35	0.00	0.11	1.79	coll
0.9	17	5	8.5	0.00	0.12	0.67	split	1.6	27	7	18.9	2.63	0.68	0.06	split + CI
0.9	18	10	18	0.00	0.1	1.02	strict coll	1.6	28	10	28	2.60	0.51	0.19	CI
1	15	7	10.5	1.43	0.58	0.07	split + CI	1.6	29	10	29	2.60	0.63	0.31	CI
1	16	10	16	1.40	0.48	0.21	CI	1.6	30	10	30	2.55	0.5	0.42	CI
1	17	10	17	1.40	0.61	0.35	CI	1.6	31	10	31	2.55	0.56	0.55	CI
1	18	10	18	1.40	0.68	0.48	CI	1.6	32	10	32	2.55	0.61	0.67	CI
1	19	5	9.5	0.00	0.1	0.64	split	1.6	33	10	33	2.50	0.54	0.79	CI
1	20	8	16	0.00	0.1	0.92	split	1.6	34	10	34	2.50	0.58	0.91	CI
1	21	10	21	0.00	0.11	1.28	strict coll	1.6	35	6	21	0.00	0.09	1.07	split
1.1	17	10	17	1.60	0.33	0.08	CI	1.6	36	6	21.6	0.02	0.11	1.27	split
1.1	18	10	18	1.60	0.59	0.23	CI	1.6	37	10	37	0.00	0.09	1.57	strict coll
1.1	19	10	19	1.60	0.69	0.35	CI	1.7	29	9	26.1	2.84	0.66	0.05	split + CI
1.1	20	10	20	1.55	0.53	0.49	CI	1.7	30	9	27	2.83	0.68	0.16	split + CI
1.1	21	10	21	1.55	0.59	0.63	CI	1.7	31	10	31	2.80	0.54	0.29	CI
1.1	22	5	11	0.00	0.12	0.81	split	1.7	32	10	32	2.80	0.63	0.41	CI
1.1	23	10	23	0.00	0.09	1.13	strict coll	1.7	33	10	33	2.75	0.52	0.52	CI
1.3	21	10	21	2.00	0.5	0.1	CI	1.7	34	10	34	2.75	0.57	0.64	CI
1.3	22	10	22	2.00	0.69	0.22	CI	1.7	35	10	35	2.75	0.61	0.76	CI
1.3	23	10	23	1.95	0.49	0.35	CI	1.7	36	10	36	2.70	0.55	0.88	CI
1.3	24	10	24	1.95	0.57	0.48	CI	1.7	37	10	37	2.70	0.58	1	CI
1.3	25	10	25	1.95	0.63	0.61	CI	1.7	38	5	19	0.00	0.11	1.16	split
1.3	26	10	26	1.90	0.54	0.73	CI	1.7	39	5	19.5	0.00	0.13	1.35	split
1.3	27	5	13.5	0.00	0.11	0.91	split	1.7	40	8	32	0.00	0.11	1.68	split
1.3	28	10	28	0.00	0.08	1.16	strict coll	1.7	41	10	41	0.00	0.1	1.99	strict coll
1.4	23	10	23	2.20	0.47	0.09	CI	1.8	31	10	31	3.05	0.22	0.02	CI
1.4	24	10	24	2.20	0.67	0.22	CI	1.8	32	10	32	3.05	0.64	0.15	CI
1.4	25	10	25	2.15	0.48	0.33	CI	1.8	33	10	33	3.05	0.7	0.25	CI
1.4	26	10	26	2.15	0.56	0.47	CI	1.8	34	10	34	3.00	0.54	0.38	CI
1.4	27	10	27	2.15	0.62	0.59	CI	1.8	35	10	35	3.00	0.61	0.5	CI
1.4	28	10	28	2.10	0.54	0.71	CI	1.8	36	10	36	3.00	0.65	0.61	CI
1.4	29	10	29	2.10	0.58	0.84	CI	1.8	37	10	37	2.95	0.57	0.73	CI
1.4	30	7	21	0.00	0.09	1.06	split	1.8	38	10	38	2.95	0.6	0.84	CI
1.4	31	10	31	0.00	0.09	1.31	strict coll	1.8	39	10	39	2.95	0.63	0.96	CI
1.5	25	10	25	2.40	0.36	0.08	CI	1.8	40	10	40	2.90	0.58	1.08	CI
1.5	26	10	26	2.40	0.61	0.21	CI	1.8	41	5	20.5	0.00	0.11	1.25	split
1.5	27	10	27	2.40	0.7	0.32	CI	1.8	42	8	33.6	0.01	0.09	1.47	split
1.5	28	10	28	2.35	0.54	0.45	CI	1.8	43	10	43	0.00	0.09	1.75	strict coll