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ABSTRACT

Anticipated Growth and Business Cycles in Matching Models*

Positive news about future productivity growth causes a contraction in most neoclassical business cycle models, which is counterfactual. We show that a business cycle model that incorporates the standard matching framework can generate an expansion. Although the wealth effect of an increase in expected productivity induces workers to reduce their labour supply, the matching friction has the opposite effect leaving labour supply roughly unaffected. Employment increases because the matching friction also induces firms to post more vacancies. This translates into additional resources, which makes it possible for both consumption and investment to increase in response to positive news about future productivity growth before the actual increase in productivity materializes.

JEL Classification: E24, E32 and J41
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1 Introduction

Economists have long recognized the importance of expectations in explaining economic fluctuations. As early as 1927, Pigou postulated that "*the varying expectations of business men ... constitute the immediate cause and direct causes or antecedents of industrial fluctuations*."[1] A recent episode where many academic and non-academic observers attribute a key role to expectations is the economic expansion of the 1990s. During the 1990s, economic agents observed an increase in current productivity levels, but also became more optimistic regarding future growth rates of productivity. In fact, there was a strong sense of moving towards a new era, the "new economy", of higher average productivity growth rates for the foreseeable future. With the benefit of hindsight it is easy to characterize the optimism about future growth rates as "unrealistic". At the time, however, the signals about future productivity were in fact remarkable, and the view that a new era was about to begin was shared by many experts, including economic policy makers such as Alan Greenspan.[2] Similarly, the question arises whether the downward adjustment of these high expectations about future growth rates did not at least magnify, if not cause, the economic downturn that took place at the beginning of the new millennium.

More formal empirical evidence that business cycles are caused by anticipated changes in future productivity is provided by Beaudry and Portier (2005a). They use changes in stock prices to identify that fraction of future changes in productivity that is anticipated, and argue that this fraction is actually quite large. They show that innovations in technology are small but initiate substantial future increases in productivity.[3] Moreover, this

[1] Pigou (1927, p. 29).

[2] See, for example, the following quote in Greenspan (2000): "... there can be little doubt that not only has productivity growth picked up from its rather tepid pace during the preceding quarter-century but that the growth rate has continued to rise, with scant evidence that it is about to crest. In sum, indications ... support a distinct possibility that total productivity growth rates will remain high or even increase further."

[3] The standard assumption that productivity follows an AR(1) with an autoregressive coefficient close to or equal to one is not consitent with this empirical evidence. For an AR(1) process, a positive shock implies that expected productivity growth *decreases* when the autoregressive coefficient is less than one and remains unchanged when the autoregressive coefficient is equal to one.

expectation shock leads to a boom in output, consumption, investment, and hours worked before the anticipated productivity growth actually materializes.

Beaudry and Portier (2005b) analyze whether existing neo-classical models can generate Pigou cycles. In a Pigou cycle, output, consumption, investment, and hours worked jointly increase in response to an anticipated increase in productivity and these variables decline when the anticipated increase fails to materialize. They consider a large class of models and show that the answer is no.[4] Instead, the typical response is an increase in consumption but a decrease in investment and hours worked. The reason is that the wealth effect induces agents to increase consumption and leisure. It is not difficult to generate an increase in investment, because the anticipated increase in productivity also causes the expected return on capital to go up.[5] The problem is, however, that higher levels of investment are typically financed by a reduction in consumption, not by an increase in hours worked. The real challenge is therefore to build a model in which hours worked increase in response to anticipated productivity growth.

Perhaps, it should not come as a surprise that an anticipated increase in productivity does not lead quite naturally to a boom in existing models. In most business cycle models, aggregate productivity is an exogenous process, and agents get the increase in productivity "for free". As a result, the aggregate economy behaves the way an individual agent behaves if he finds out about a windfall to be received in the near future. He goes on a spending spree (i.e., consumption increases), takes a vacation (i.e., employment decreases), and finances this indulgence by dissaving (i.e., investment decreases).

Recently, some models have been developed where an increase in expected productivity generates a business cycle boom even though productivity improvements still fall like manna from heaven. Exemplary papers are Beaudry and Portier (2004), Beaudry and Portier (2005b), Christiano, Motto, and Rostagno (2006), and Jaimovich and Rebelo (2006). In Beaudry and Portier (2004), Beaudry and Portier (2005b), and Jaimovich and

[4] Cochrane (1994) and Danthine, Donaldson, and Johnsen (1998) have made the same observation for more specific models.

[5] If the elasticity of intertemporal substitution is high enough, then the substitution effect dominates the wealth effect, and investment increases.

Rebelo (2006), the positive co-movement of investment and consumption is generated by making it too costly for variables to move in the "wrong" direction. This can be accomplished by complementarities in the production technology or particular forms of capital adjustment costs. Christiano, Motto, and Rostagno (2006) assume that nominal wages are sticky and show that this implies an expansionary monetary policy when expected future productivity increases. The reason is that the increase in the real wage caused by the expansion brings about a reduction in inflation when nominal wages are sticky. The reduction in inflation in turn leads to a reduction in interest rates when the central bank follows a Taylor rule.

In this paper, we approach the challenge to build a model that can generate Pigou cycles from a different angle. The key idea is that increases in aggregate productivity are not free, at least not to everybody, and that in order to benefit from the anticipated increase in productivity agents have to invest resources. To show the strength of this argument, we build a model in which the productivity process is still exogenous, and an increase in productivity is free to everybody already engaged in productive activities. The increase in productivity does not come for free, however, to firms and workers that are not already engaged in market production. Instead, forming a productive relationship takes time and requires resources. As a result, firms start investing in new projects immediately and do not delay looking for additional workers when expected productivity growth increases. Similarly, there is an incentive for workers to enter the labor force as soon as expectations about future productivity increase. As employment increases, consumption and investment can increase before actual productivity goes up.

In particular, we incorporate a standard labor market matching framework into a real business cycle (RBC) model.[6] The labor market matching model is becoming (or is) the benchmark model to explain fluctuations in aggregate employment. Because of the matching friction it is well suited to model the idea that not everybody automatically benefits from productivity increases. To see whether this model can generate Pigou cycles,

[6]Several papers have incorporated the matching framework into a real business cycle model and examined the effect of *realized* increases in productivity. Examples are Merz (1995), Andolfatto (1996), and den Haan, Ramey, and Watson (2000).

we study the transition from a low-growth-*low*-expectations regime to a low-growth-*high*-expectations regime. This transition does not affect actual productivity, but it does affect the probability of switching to a regime with high productivity growth rates. Expected future productivity, thus, increases. We will show that this increase in expected productivity generates an economic expansion even though productivity levels themselves have not yet gone up.

In the standard matching framework, the mass of workers that is either employed or searching for a job, that is, the total labor force, is fixed. In contrast, in the standard RBC model, labor supply is determined by a labor/leisure decision. An anticipated productivity increase then generates a reduction in employment, because the wealth effect increases the demand for leisure. The standard matching model—by keeping the mass of potential workers fixed—does not allow for this channel to operate and consequently makes it easier to generate Pigou cycles. We show, however, that the model can still generate Pigou cycles if we allow the wealth effect to affect labor supply just as in standard RBC models.[7]

In our model, just as in the standard RBC model, the wealth effect associated with an increase in expected productivity growth has a downward effect on labor supply. Nevertheless, labor supply only displays a very modest decline in our framework. The reason is that—because of the matching friction—the increase in the expected productivity growth rate increases the benefits for workers of being in a productive relationship, just like it increases the benefits for firms. Due to the higher number of vacancies being posted, the small decrease in the labor force goes together with an increase in the employment rate and a reduction in the unemployment rate.

The paper is structured as follows. In Section 2 we discuss the model. In Section 3, we discuss standard summary statistics and we explain why our model generates a countercyclical unemployment rate, whereas other models with endogenous labor force participation do not. In Section 4, we document that the model can generate a typical expansion in response to an increase in the anticipated productivity growth rate. The last

[7] In the matching literature, it is more common to model changes in the labor supply by means of endogenous search intensity. The advantage of our approach to endogenize the labor supply is that there is a clear empirical counterpart, which facilitates the calibration of the model.

section concludes.

2 Model

There are three types of agents: a representative household, entrepreneurs, and workers. The representative household takes the decision how much to consume, how much to save, and how much labor to supply. Entrepreneurs decide in how many new projects to invest and how much capital to rent for existing projects. In the planning phase, each new project requires a periodic fixed investment until production starts. Starting a new project also entails posting a vacancy. The number of vacancies and the number of workers searching for a job determine—using a standard matching function—the number of new productive relationships. Exogenous separation occurs with probability ρ^x. Productivity is high enough so that endogenous separation does not occur. At the end of any given period, all the agents in the economy distribute their net earnings to the representative household.

2.1 Production

Production takes place within a relationship consisting of a worker and an entrepreneur. The production technology is given by

$$y_t = Z_t k_t^{\alpha}, \tag{1}$$

where Z_t is an aggregate productivity, y_t firm output, and k_t firm capital. Capital is rented by the firm at rate R_t and the firm pays the worker a wage W_t. Each period the worker and the entrepreneur divide revenues net of capital payments

$$\overline{p}_t = Z_t k_t^{\alpha} - R_t k_t. \tag{2}$$

The wage process is given by

$$W_t = (1 - \omega_1)\omega_0 \mathrm{E}\left[\overline{p}_t\right] + \omega_1 \omega_0 \overline{p}_t, \tag{3}$$

where ω_0 and ω_1 are fixed parameters and $\mathrm{E}[\overline{p}_t]$ is the unconditional expectation of $\overline{p}_t$. The parameter ω_1 controls how the wage rate responds to changes in net revenues. If

$\omega_1 = 0$, wages are fixed, whereas if $\omega_1 = 1$, wages are proportional to net revenues. The average wage rate, $\mathrm{E}[W_t]$, is equal to $\omega_0 \mathrm{E}[\overline{p}_t]$. Thus, ω_0 determines the fraction of net revenues the worker receives on average.

The firm chooses the capital stock that maximizes (2). Thus,

$$k_t = \left(\frac{Z_t}{R_t}\right)^{1/(\alpha-1)}. \tag{4}$$

2.2 Productivity process

The process of aggregate productivity is given by

$$\ln Z_t = G_t + \rho \ln Z_{t-1} + \sigma\varepsilon_t. \tag{5}$$

The innovation, ε_t, has a standard Normal distribution. The drift term, G_t, can take a low value, G^{low}, and a high value, G^{high}. There are two regimes in which $G_t = G^{low}$. If, in period t, the economy is in the *low-growth-low-expectations* regime (or regime 1), then $G_t = G^{low}$ and it is impossible that $G_{t+1} = G^{high}$. If the economy is in the *low-growth-high-expectations* regime (or regime 2), then $G_t = G^{low}$ but $G_{t+1} = G^{high}$ is possible. Moving from regime 1 to regime 2, therefore, corresponds to a situation where expectations increase but current productivity levels remain unchanged.[8] There is only one regime in which $G_t = G^{high}$. This is the *high-growth* regime (or regime 3). The probability of moving from regime i to regime j is denoted by π_{ij}. The key restriction on the transition probabilities is that $\pi_{23} > \pi_{13}$.

2.3 New projects

Entrepreneurs decide wether they want to start new projects. In the planning phase, projects require an investment equal to ψ each period. If the plan turns out to be successful, production can start. In the planning phase, entrepreneurs also search for a worker. The number of entrepreneurs with projects in the planning phase is determined by the free-entry condition, that is, the cost, ψ, has to equal the value of a successful project times the probability of being successful.

[8] This regime change corresponds to the experiment considered in Beaudry and Portier (2005b).

Profits of successful projects, p_t, are equal to net revenues minus wage payments, i.e., $p_t = \overline{p}_t - W_t$. The value of a successful project to the entrepreneur is simply the discounted value of profits, taking into account that the project is subject to the possibility of exogenous destruction in subsequent periods. Thus,

$$V_t = \beta \mathrm{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} (p_{t+1} + (1 - \rho^x) V_{t+1}) \right], \tag{6}$$

where $(C_{t+1}/C_t)^{-\gamma}$ is the marginal rate of substitution. The free-entry condition can then be written as

$$\psi = \lambda_t^f V_t, \tag{7}$$

where λ_t^f is the probability that a project in the planning phase is successful and a suitable worker is found. In Fujita (2003) planning of the project and searching for a worker are modelled separately. For parsimony, we adopt here the standard convention, subsume planning and searching under one phase, and assume that the probability λ_t^f describes success on both counts.

2.4 Matching market

On the matching market, entrepreneurs post vacancies and search for a worker. The number of matches, M_t, is determined by the number of searching workers, i.e., the unemployed, N_t^s, and the number of vacancies, N_t^v, which is equal to the number of projects in the planning phase. The matching process is modelled with the standard constant returns to scale matching function, unless matches are less than the number of workers searching for a job. That is,

$$M_t = \min\{\mu_0 N_t^s \left(\frac{N_t^v}{N_t^s} \right)^{\mu_1}, N_t^s\}, \tag{8}$$

$$\lambda_t^w = \frac{M_t}{N_t^s}, \text{ and } \lambda_t^f = \frac{M_t}{N_t^v}. \tag{9}$$

We allow the number of matches to exceed the number of vacancies,which would correspond with a value of λ_t^f bigger than 1.[9] Note that it is logically not impossible that a firm manages to get more than one worker by posting only one vacancy.

[9]For the functional formof the matching function, the equilibrium choice for N^v is bounded away from zero as long as $\psi < V_t$. If one would impose that $M_t \leq N_t^v$ (i.e., $\lambda_t^f \leq 1$), then a sudden jump to

2.5 The household

The household chooses consumption, C_t, total labor supply, and next period's beginning-of-period capital stock, K_{t+1}. Labor supply is equal to the sum of employed workers, N_t^w, and workers searching for a job, N_t^s. Capital earns a rate of return R_t and depreciates at rate δ.

Next period's beginning of period employment consists of those workers that have not experienced exogenous separation, $(1-\rho^x)N_t^w$, and those workers that are matched during the current period, $\lambda_t^w N_t^s$. Thus,

$$N_{t+1}^w = \lambda_t^w N_t^s + (1-\rho^x)N_t^w. \tag{10}$$

The household trades off the benefits of being engaged in market activity with benefits of activities such as leisure and home production. Searching is assumed to be a full-time activity. Consequently, the time spent on leisure and home production, L_t, is equal to $N^* - N_t^s - N_t^w$. The utility of current-period leisure is given by $\phi L_t^{1-\kappa}/(1-\kappa)$.

The household's maximization problem is as follows.

$$\max_{\{C_{t+j},N_{t+j}^s,N_{t+j+1}^w\}_{j=0}^\infty} \mathrm{E}_t \sum_{j=0}^{\infty}\left[\beta^j \frac{C_{t+j}^{1-\gamma}-1}{1-\gamma} + \phi\frac{\left(N^* - N_{t+j}^s - N_{t+j}^w\right)^{1-\kappa}}{1-\kappa}\right], \tag{11}$$

s.t.

$$N_{t+j+1}^w = \lambda_{t+j}^w N_{t+j}^s + (1-\rho^x)N_{t+j}^w, \tag{12}$$

$$C_{t+j} + K_{t+j+1} = W_{t+j}N_{t+j}^w + R_{t+j}K_{t+j} + (1-\delta)K_{t+j} + P_{t+j} - N_{t+j}^v\psi. \tag{13}$$

Here, $P_t = p_t N_t^w$ are the profits of the entrepreneurial sector.

This specification of the utility function for the representative agent assumes that there is perfect risk sharing, not only in terms of consumption, but also in terms of leisure.[10] An alternative would be to use the lottery setup of Rogerson (1998), where agents use lotteries

$N^v = 0$ would occur if V_t would drop below ψ. By allowing $M_t > N_t^v$ whenever $\psi > V_t$, one avoids this discontinuity. This simplifies the computational procedure considerably even though it is rare that $\psi > V_t$.

[10] A similar approach is followed by Hornstein (1998), Shi and Wen (1999), and Tripier (2003).

to insure consumption against unfavorable labor market outcomes.[11] This approach seems less suitable for a model with labor force participation, since it implies that not only being unemployed, but also not being in the labor force is a random outcome. Moreover, Ravn (2006) shows that in a matching model the implied linear utility function leads to a relationship between aggregate consumption and labor market tightness, N_t^v/N_t^s, that is inconsistent with the empirical properties of smooth aggregate consumption on one hand and volatile tightness on the other for reasonable parameter values. In Appendix A.3, we show that our specification avoids Ravn's consumption-tightness puzzle.

Let η_t^m be the Lagrange multiplier of the constraint of the law of motion of N_t^w. This multiplier represents the shadow price for a worker of being in a match. The first-order conditions are as follows.

$$C_t^{-\gamma} = E_t\left[\beta C_{t+1}^{-\gamma}\left(r_{t+1} + (1-\delta)\right)\right], \tag{14}$$

$$\phi L_t^{-\kappa} = \lambda_t^w \eta_t^m, \tag{15}$$

$$\eta_t^m = \beta \mathrm{E}\left[W_{t+1}C_{t+1}^{-\gamma} - \phi L_{t+1}^{-\kappa} + (1-\rho^x)\eta_{t+1}^m\right]. \tag{16}$$

Equation (14) is the standard intertemporal Euler equation. Equation (15) is the first-order condition of leisure. The left-hand side of this equation is the disutility of entering the labor market, i.e., the disutility of searching, and the right-hand side is the expected benefit of searching, $\lambda_t^m \eta_t^m$, that is, the worker gets η_t^m with probability λ_t^m. Equation (16) specifies the expected benefit of leaving period t employed, η_t^m. First, a matched worker obtains a wage payment worth $W_{t+1}C_{t+1}^{-\gamma}$. Second, the worker has to put in effort, generating disutility of $-\phi L_{t+1}^{-\kappa}$. Finally, in case of no separation, the worker gets the expected benefits of leaving period $t+1$ employed, η_{t+1}^m.

[11] The utility of leisure would then be given by $\phi\left[(N^* - N_t^s - N_t^w)\times 1^{1-\kappa} + (N_t^s + N_t^w)\times 0^{1-\kappa}\right]/(1-\kappa)$, which is equal to $\phi(N^* - N_t^s - N_t^w)/(1-\kappa)$, i.e., utility is linear in leisure.

2.6 Recursive equilibrium

Equilibrium on the market for rental capital requires that total demand for capital is equal to the available aggregate capital stock.

$$N_t^w k_t = K_t. \tag{17}$$

The state variables of the model, s_t, consist of Z_t, the growth regime, K_t, and N_t^w. An equilibrium is a set of functions $C(s_t)$, $K'(s_t)$, $N^s(s_t)$, $N^w(s_t)$, $N^v(s_t)$, $V(s_t)$, $\eta_t^m(s)$, $R(s_t)$, $\lambda_t^w(s)$, $\lambda_t^f(s)$, and $k(s_t)$ that are consistent with

- household optimization, that is, the first-order conditions (14), (15), and (16), the budget constraint (13), and the law of motion for matched workers (12),
- the free-entry condition (7),
- firm optimization, that is, the first-order condition (4),
- the value of a successful project to the entrepreneur given by (6),
- the definition of the matching probabilities, and
- the capital market clearing condition (17).

2.7 Allocation friction

We also consider a version of the model where substitution between consumption and investment is costly. The idea is that different technologies are used to produce consumption and investment, and that moving away from the steady state ratio of consumption to investment is therefore costly. In this modified version of the model, the aggregate budget constraint is given by

$$\begin{aligned} \left[\eta_c \left(C_t^{\nu_c}\right)^{\xi} + \eta_i \left(I_t^{\nu_i}\right)^{\xi}\right]^{1/\xi} &= W_t N_t^w + R_t K_t + P_t - N_t^v \psi, && (18) \\ K_{t+1} &= I_t + (1-\delta) K_t. && (19) \end{aligned}$$

If $\eta_c = \eta_i = \nu_c = \nu_i = \xi = 1$, the model is identical to the benchmark model. As the value of ξ increases above 1, it becomes costlier to substitute consumption and investment. We choose η_c and η_i such that the steady state of the modified model is equal to the steady state of the benchmark model. We choose ν_c and ν_i such that the volatility of consumption and investment are not affected by the magnitude of the allocation costs.

2.8 Calibration

Parameter values are either set to standard values or calibrated to match key characteristics of empirical data. The model period is one month. Parameter values are given in Table 1. This table also reports either the source for the parameter value or the empirical moment that is most important for the identification of the parameter value. Data sources used to estimate the empirical moments are given in Appendix A.1.

Preferences. Using a standard annual discount rate of 4% implies for a monthly model a value of β equal to 0.9966. The coefficient of relative risk aversion, γ, plays a key role in the model and we will consider several values. The benchmark value is 0.475. The reason for this choice will become clear in Section 4.4. The scaling factor of the utility of leisure, ϕ, is chosen so that the steady state labor force, $N^s + N^w$, is equal to 1. To ensure that labor force participation, $(N^s + N^w)/N^*$, is equal to the observed value of 0.6274, we set $N^* = 1.5938$. The curvature parameter in the utility function of leisure, κ, is chosen to ensure that the model matches the volatility of labor force participation. The calibrated value of κ implies an elasticity of labor supply with respect to the expected benefit of being matched, $\lambda^w \eta^m$, equal to 0.24.[12] This is slightly higher than 0.15, which is the typical value of the Frisch elasticity used in New-Keynesian models.[13]

Production technology. The standard annual depreciation of 10% corresponds to a value of δ equal to 0.0084 on a monthly basis. The value for α is chosen so that the labor share is equal to the standard value of two thirds. The remaining one third is divided between capital providers, who get a share α of total output, and entrepreneurs, who get

[12] The elasticity of labor supply with respect to the expected benefit of being matched is equal to $(N^*/(N^s + N^w) - 1)/\kappa$.

[13] See, for example, Ball and Romer (1989).

$\omega_0(1-\alpha)$. Thus, $\alpha + \omega_0(1-\alpha) = 1/3$. The calibrated value for ω_0 is equal to 0.9725 (see discussion below). Thus, $\alpha = 0.315$. This implies a steady state ratio of physical capital to output, k, equal to 2.22 on an annual basis. The ratio of total capital to output $(N^w k + N^w V)/N^w y$ is equal to 2.27 on an annual basis, which is fairly close to the typical value of 2.5.

Productivity process. Edge, Laubach, and Willams (2004) report real-time estimates of long-run productivity growth and find that estimates of the Congressional Budget office vary from a low of 1.2% in 1996 to a high of 2.7% in 2001. In our monthly model, we set $G^{high} = -G^{low} = \left(\frac{1}{12}\right) 0.5\%$. This means that on an annual basis the difference between the low-growth and the high-growth regime is equal to one percentage point. Such a change seems reasonable, so our results are not driven by unrealistic changes in expectations.

To examine whether our model can generate Pigou cycles, we need to specify the transition matrix such that the expected growth rate in regime 2 exceeds the expected growth rate in regime 1. An easy way to do this is to set $\pi_{13} = 0$. For parsimony, we set $\pi_{21} = \pi_{32} = 0$. Finally, we set the transition probabilities such that the unconditional probability of being in the high-growth regime is equal to 1/2, so that $\mathrm{E}[G_t] = \mathrm{E}[\ln(Z_t)] = 0$. Given these restrictions, the transition matrix Π is fully determined by the expected duration of staying in regime 2, ζ_2, and the expected duration of staying in regime 3, ζ_3, and Π can be written as

$$\Pi = \begin{bmatrix} 1-(\zeta_3-\zeta_2)^{-1} & (\zeta_3-\zeta_2)^{-1} & 0 \\ 0 & 1-\zeta_2^{-1} & \zeta_2^{-1} \\ \zeta_3^{-1} & 0 & 1-\zeta_3^{-1} \end{bmatrix}. \tag{20}$$

We set ζ_3 equal to 120, so that the expected duration of staying in the high-growth regime is ten years. We set ζ_2 equal to 12 months.

A switch from regime 2 to regime 3 increases the value of productivity, Z_t, in the first period by $G^{high} - G^{low}$ percentage points. In each subsequent period the economy remains in regime 3, the value of Z_t continues to increase. The higher the value of ρ, the more persistent the effects of the regime change on the growth rate. The idea of our regime change is a persistent change in the growth rate so ρ should be high, but for computational reasons we consider values less than 1. In particular, we set ρ equal to 0.98. We have also considered $\rho = 0.99$ and $\rho = 0.95$ and found that the results are qualitatively very similar.

The value for the volatility of the innovation, σ, is chosen as follows. The standard process for quarterly productivity in the real business cycle literature is $\ln(\widetilde{Z}_t) = 0.95 \ln(\widetilde{Z}_{t-1}) + 0.007\widetilde{\varepsilon}_t$, where $\widetilde{\varepsilon}_t$ is a standard normal.[14] We set the value of σ so that the volatility of HP-filtered $\widetilde{Z}_t$ is equal to the volatility of our process Z_t, as specified in Equation (5), after the monthly observations of Z_t have been transformed into quarterly data and then HP-filtered.

Wage process. The parameter ω_1 controls the sensitivity of wages to changes in net revenues, $Z_t k_t^{\alpha} - R_t k_t$. We calibrate ω_1 to match the volatility of wages relative to the volatility of labor productivity. The value of ω_0 represents the share of net revenues that workers receive. A smaller value of $1 - \omega_0$ implies that firm value, V_t, is more responsive to changes in productivity and implies a higher level of employment volatility. We choose the value of ω_0 to match the volatility of the employment ratio, N_t^w/N^*, relative to the volatility of labor productivity, implying a value for ω_0 of 0.9725. This value and our value for α imply that workers obtain 66.67% of value added, providers of capital receive 31.45%, and entrepreneurs receive 1.89%.

Matching technology. The matching elasticity with respect to labor market tightness, μ_1, is taken from Petrongolo and Pissarides (2001). The values of μ_0, ψ, and ρ^x are chosen to match (i) a steady state matching probability for the worker equal to the empirical average of 45.4%, (ii) a steady state matching probability for the firm equal to 33.8%, and (iii) a steady state unemployment rate equal to the empirical average of 5.7%.[15]

[14] See, for example, Cooley and Prescott (1995).

[15] A monthly matching probability for the firm equal to 33.8% implies that the probability of not being matched within any given quarter is equal to 29%, which corresponds to the value reported in van Ours and Ridder (1992).

Model without labor force participation. The parameters for the model without endogenous labor force participation are identical to those of the model with endogenous labor force participation, except that $\phi = 0$, that is, there is no disutility of labor, and the time endowment, N^*, is rescaled so that the mass of the labor force, $N_t^s + N_t^w$, remains equal to the steady state value in the benchmark model, i.e., $N_t^s + N_t^w = 1$.

3 Summary statistics

3.1 Standard business cycle and labor market statistics

Table 2 reports standard business cycle as well as labor market statistics for the model with and without endogenous labor force participation. Generated volatilities for consumption, investment, and output have the standard ordering. That is, consumption is less volatile and investment is more volatile relative to output.[16] HP-filtered output is 42% more volatile than total factor productivity in the model with labor force participation and 28% more volatile in the model without labor force participation, so endogenous labor supply is helpful in magnifying shocks.

Output and labor productivity are not quite as volatile as in the data. Just as in most models, shocks are not sufficiently magnified. However, the volatilities of labor market statistics relative to the volatility of labor productivity, such as the volatility of tightness and the volatilities of the matching probabilities, do look very good. As pointed out by Shimer (2005), standard matching models cannot generate sufficient volatility in those statistics. In our model, the share that accrues to the entrepreneur is—as in Hagedorn and Manovskii (2006)— relatively small, inducing volatile profits, which in turn generate sufficiently volatile labor market statistics. One labor market statistic that the model does not fit well is the volatility of the labor share. In the standard RBC model, the labor share

[16] The model underestimates the relative volatility of consumption. The same is true for standard business cycle models. See, for example, Cooley and Prescott (1995). Employment in our model fluctuates less than hours in the standard RBC model, which explains why the marginal productivity of aggregate capital and the rental rate are less volatile as well. This in turn explains why investment in our model is somewhat less volatile than in the standard RBC model.

does not fluctuate at all and is equal to $1 - \alpha$ in every period. Similarly, in our model the combined share that goes to the worker and the entrepreneur is fixed and equal to $1 - \alpha$. This fixed share, however, is divided in non-constant proportions, so that in our model the labor share does fluctuate. The standard deviation of the labor share, relative to the standard deviation of labor productivity is, however, still only 25% of its empirical counterpart.

A look at the Beveridge Curve, i.e., the co-movement between unemployment and vacancies, reveals a difference between the model with and without labor force participation. The data display a very strong negative correlation. Both models can generate a strong negative correlation, although not as high as the empirical counterpart. In the model with endogenous labor force participation, however, the stronger incentive to enter the labor market when economic conditions improve induces a lower correlation between unemployment and vacancies. In particular, the correlation between unemployment and vacancies is equal to -0.40 in the model with endogenous labor force participation, -0.72 in the model without endogenous labor force participation, and -0.93 in the data.

3.2 Countercyclical unemployment rate

Tripier (2003) argues that RBC models with both a matching framework and endogenous labor force participation cannot generate a countercyclical unemployment rate. Veracierto (2004) reaches the same conclusion. Our model, however, does generate a countercyclical unemployment rate. In the data the correlation between the unemployment rate and output is equal to -0.86 and we find correlation coefficients equal to -0.80 and -0.89 for the model with and without endogenous labor force participation, respectively. The correlation between unemployment and labor productivity is equal to -0.61 in the model with endogenous labor force participation, even stronger than the empirical counterpart, which is equal to -0.33.

The question arises why a model as simple as ours can generate a countercyclical unemployment rate whereas other models with endogenous labor force participation cannot.[17] The difficulty of generating a countercyclical unemployment rate is related to the challenge for matching models to generate sufficient volatility in tightness, N_t^v/N_t^s. If tightness and therefore the matching probability are not very responsive to a positive productivity shock, then an increase in labor force participation leads to an increase in the unemployment rate. In our model, profits and therefore vacancies do respond strongly to an increase in productivity. The sharp increase in vacancies allows the model to generate an increase in labor force participation *and* a reduction in the unemployment rate at the same time.

4 Pigou cycles: Expectation driven real business cycles

In this section, we document that our model can generate Pigou cycles. In particular, we document that an increase in the probability of moving to the high-growth regime can generate an increase in output, consumption, investment, and employment, i.e., the features of a typical boom.[18]

4.1 Impulse response functions

Output response when expectations increase. Figure 1 plots the impulse response function of output if the economy moves into the low-growth-high-expectations regime. Since the model is nonlinear, we calculate the impulse response function at different points in the state space.[19] The graph plots the average response and the response corresponding to the 10^{th} and 90^{th} percentile. From Panel A we can see that output does not change in the month when the economy moves from regime 1 to regime 2. The reason is that

[17] Haefke and Reiter (2006) develop a much more intricate model with heterogeneity in home production that also generates a countercyclical unemployment rate.

[18] In terms of our driving process this means a movement from the low-growth-low-expectations regime to the low-growth-high-expectations regime.

[19] The impulse response function measures the effect of moving to regime 2 compared to staying in regime 1. To determine the set of initial points in the state space to consider, we simulate the economy for 100,000 periods, and then use the values of the state variables in the periods when the economy switches from regime 1 to regime 2.

capital and employment are predetermined. Panel A however demonstrates that output does increase in the second month even though current productivity levels still have not changed. Panel B plots the response of output net of the investment costs associated with new projects. Initially, the investment in new projects reduces the amount of remaining available resources. After already two months, however, this output measure displays a positive response even when the responses for the 10^{th}-percentile are used, which are below the mean responses.

The figure also makes clear the dependence on initial conditions, because the impulse response function of the 10th percentile lies substantially below the response of the 90th percentile.[20] Although it matters for the quantitative results in which state of the world the economy switches, the nonlinearity does not affect the qualitative results. We will, therefore, for the sake of expositional clarity, from now on only report the mean responses.

Responses of consumption, investment, and hours worked when expectations increase. Panel C of Figure 1 plots the impulse response function of employment and Panel D plots the responses of consumption, investment, and investment excluding the investment in new projects. Our timing assumption implies that projects started in period t can at best be productive in period $t + 1$. Consequently, employment only starts to increase in the period after the shock. As documented in the graph, it takes several periods before the increase in employment settles down.

Because both capital and employment are predetermined in the period that the shock occurs, either consumption or total investment must decrease in the first period. For the chosen value of the elasticity of intertemporal substitution $(1/\gamma)$ the value of consumption increases, thus, total investment decreases in the period of the shock. In the second period, however, when employment and resources are higher, both the consumption and the total investment response are positive.

In the period of the shock, investment in new projects sharply increases by 15%. Since total investment decreases, this means that the investment in existing projects decreases by

[20] The switch to regime 2 has the strongest effects on economic activity when the initial levels of employment, capital, and productivity are the lowest.

more than total investment. In the first period, investment in existing projects decreases by 1.4% compared with a 0.35% decrease in total investment. Investment in existing projects quickly recovers, however, and displays a positive response after one quarter.

An expected increase in productivity versus an actual increase. Panel A of Figure 2 plots the (mean) impulse response function of output when the economy moves to regime 2 and the impulse response function when the economy moves to regime 3 after having been in regime 2 for 120 months.[21] The figure makes clear that just a change in expectations can generate a response that is initially substantial relative to the actual response if the growth rate does increase. In particular, the output response when the economy moves into regime 2 relative to the response when the economy moves into regime 3 is equal to 39% and 12% after six and twelve periods, respectively. After some time, however, the increase of output if the economy continues to be in regime 3 naturally overwhelms the response in regime 2, because the growth rate is assumed to be persistent.

Panel B therefore plots the cyclical component of moving into regime 2 and into regime 3. The graph shows that the cyclical response of output when expected productivity increases is substantial relative to the cyclical response of output when an actual increase in productivity occurs. In fact, in the first 18 months the cyclical response of output when moving to regime 2 exceeds the cyclical response when moving to regime 3. In both cases, the cyclical component is initially negative, which is due to the increase in the trend.[22] The cyclical component becomes positive much quicker when the economy moves into regime 2 than when it moves into regime 3, which is due to a smaller increase in the trend in regime 2. When the economy moves into regime 2, the maximum cyclical response is equal to 42% of the maximum response observed when the economy moves into regime 3.

[21] The responses when the economy moves to regime 3 are hardly affected by the number of periods the economy has spent in regime 2. Below, we calculate the trend component of the response using a two-sided filter and by letting the economy be in regime 2 for such a long time period we ensure that the trend component of the response of moving to regime 2 is—at least initially—not affected by the subsequent shift to regime 3.

[22] The trend is calculated using the HP filter, which is a two-sided filter. Higher future values, thus, raise the current trend value.

4.2 The role of labor force participation

Figure 3 plots the output response of switching to regime 2 for both the economy with and without endogenous labor force participation. The response is slightly bigger in the economy without endogenous labor force participation.

In standard RBC models, endogenous labor supply is an important reason why the model cannot generate Pigou cycles. In particular, the increase in consumption reduces the marginal benefit of working, resulting in lower labor supply. This channel is operating in our matching model with endogenous labor supply as well. Moreover, since wages are related to current-period profits, wages actually decrease in regime 2, further reducing labor supply.

In our model, in contrast to standard RBC models with endogenous labor supply, there is, however, a force that pushes labor supply up. The anticipated increase in productivity implies an increase in expected future wages and, thus, the benefits of being employed. Workers therefore enter the labor force for the same reason as entrepreneurs start new projects. Moreover, the increase in vacancies increases the matching probability for the worker, further pushing labor force participation up. This channel almost offsets the substitution effect of the current-period wage reduction and the wealth effect. Consequently, the labor force declines very little in regime 2, which explains why the response of employment and output in the economy with endogenous labor force participation are so similar to the responses in the economy without endogenous labor force participation.

4.3 Beaudry-Portier puzzle

As discussed in the introduction, Beaudry and Portier (2005b) show that for a large class of neoclassical models it is not possible for consumption, investment, and hours worked to jointly increase if productivity remains constant. There are two parts to this puzzle. First, investment has to increase. Second, the increase of investment must be financed by higher employment not by less consumption. As pointed out by Beaudry and Portier

(2004), obtaining the first is easy, but obtaining the second is less straightforward.[23] In our model, however, the forward-looking behavior of firms starting new projects and workers deciding to enter the labor force does lead to an increase in resources. Moreover, for our benchmark parameter values this increase in resources is used for both an increase in consumption and investment. The result that both consumption and investment increase is, however, sensitive to the value of the elasticity of intertemporal substitution, $1/\gamma$, which will be discussed in the next section.

Beaudry and Portier (2005b) use a local approach and investigate whether consumption, investment, and hours worked can simultaneously and *instantaneously* increase in response to positive news. Our model cannot generate such an instantaneous co-movement, because—given our timing assumption—capital and employment are predetermined.[24] The instantaneous or first-period response, however, is not that interesting. What really matters is whether consumption, investment, and hours worked move up together in the periods following the increase in agents' expectations. If the reader insists on a positive co-movement in the first period he can always think of the first period as the first quarter which corresponds to the first three periods in our model.

4.4 The role of the elasticity of intertemporal substitution

So far we have shown that both the model with and the model without endogenous labor force participation can generate a Pigou boom, i.e., consumption, investment, and hours worked jointly increase when expectations about future productivity increase. Although

[23] If agents have an infinite elasticity of intertemporal substitution then the higher expected rental rate of capital induces more investment, which is made possible by lowering consumption.

[24] Because capital and employment are predetermined, total resources are initially constant. Thus, if consumption increases then investment must decrease and vice versa. In an earlier version of this paper, we used an alternative timing assumption where matching takes place at the beginning of the period and the creation of productive relationships can therefore take place within the period. Resources can then increase "instantaneously". In the current version, we adopt the standard timing assumption in the literature, because the standard timing makes it easier to calibrate the model. For example, with the alternative timing assumption there are agents who within a given period are both searching, i.e., unemployed, and working.

the increase in employment is a robust result, the result that both consumption and investment increase is not and only holds for a small range of values for the intertemporal substitution, $1/\gamma$. In particular, the responses of consumption and investment are both positive only when γ takes on values in the range from 0.45 to 0.5 (0.425 to 0.55) when we consider the responses starting in the second(third) period. Our benchmark value of γ was chosen to be in this range. For smaller values of γ, investment increases, but consumption—at least initially—decreases, whereas for higher values of γ we find that consumption increases, investment in new projects increases, but investment in existing projects decreases. Moreover, as investment in existing projects decreases, at some point the reduction in physical capital more than offsets the increase in employment and output decreases. For example, when $\gamma = 1$, the output response turns negative after 50 months and output minus investment in new projects turns negative after 24 months. In the next section, we show how allocation costs can make the predictions of the model much more robust with respect to changes in γ.

4.5 Allocation costs

In the benchmark model, the household can costlessly allocate available resources between consumption and investment. Let Y_t be equal to output net of investment in new projects. Consumption, C_t, and investment in existing projects, I_t, have to satisfy

$$C_t + I_t = Y_t. \tag{21}$$

The implicit assumption is that the price of one unit of investment is always equal to one unit of consumption, which is not very realistic. In the version of the model with allocation costs, consumption and investment have to satisfy

$$\left[\eta_c \left(C_t^{\nu_c}\right)^{\xi} + \eta_i \left(I_t^{\nu_i}\right)^{\xi}\right]^{1/\xi} = Y_t \tag{22}$$

with $\xi \geq 1$. By increasing ξ, the allocation friction becomes bigger, and if ξ approaches infinity, then consumption and investment are a fixed fraction of each other. Given a value for ξ, the values of η_c and η_i are chosen to ensure that steady state values are the same as in the model without allocation costs ($\xi = 1$). By varying ν_c and ν_i we can furthermore ensure that the volatilities of consumption and investment are not affected.

Table 3 reports the range of values for γ for different values of ξ such that both consumption and total investment increases in regime 2. Since there is an initial decrease in at least one of the two variables, we report the range of values economy when consumption and investment are positive from the second period onwards, from the third period onwards, and when consumption and investment are eventually positive. As one allows for a longer time for the consumption and the investment response to become positive, the range, of course, increases. The table makes clear that as the value of ξ increases, the presence of Pigou cycles becomes a much more robust outcome of the model, that is, consumption and investment jointly increase for a larger set of values of γ. For example, when ξ is equal to 3, then both consumption and investment are positive from the third period onwards when γ is in the range from 0.400 to 1.950. The presence of Pigou cycles becomes even more robust as ξ takes on yet higher values.[25]

For computational convenience, we set $\nu_c = \nu_i = 1$ in the numerical experiments in Table 3. Therefore, as ξ increases, the volatilities of consumption and investment change and become more similar. By adjusting ν_c and ν_i we can correct for this change. This is documented in the last two columns of Table 2. The column under Model III reports summary statistics when $\nu_c = \nu_i = 1$ (and $\xi = 1.25$). The column under Model III* reports summary statistics when ν_c and ν_i are adjusted to obtain the same consumption and investment volatilities as when ξ is equal to 1. When we compare the results under Model III and Model III* we can see that the statistics are very similar, except, of course, for the volatility of consumption and investment. When we compare the results for Model III*, which has $\xi = 1.25$, with Model II, which has $\xi = 1$, then we see that all the summary statistics are very similar. Thus, the introduction of allocation costs can make the results

[25] As the value of ξ increases, the model becomes more nonlinear and more difficult to solve, so we do not report a systematic set of results for higher values of ξ. We have, however, solved several models with higher values of ξ, and find that the presence of Pigou cycles really does seem to become more robust. For example, when $\xi = 7$ and $\gamma = 3$, then consumption, employment, and total investment are all positive in the second month.

much more robust in the sense that Pigou cycles can be obtained for a much wider range of values of γ, without affecting the other properties of the model.

5 Conclusion

The standard RBC model together with the basic matching framework is successful in generating an increase in resources in anticipation of an increase in productivity. It is less successful in generating a Pigou cycle. That is, although there are values of the intertemporal substitution, $1/\gamma$, for which both consumption and investment increase, the range of values is small. We show, however, that by introducing allocation costs, the model can generate Pigou cycles for a wide range of values for γ. There are likely to be other mechanisms that can generate a positive co-movement between consumption and investment when resources increase. For example, if a large enough fraction of the economy are "rule-of-thumb" consumers that simply consume a fraction of net resources, then, by construction consumption and investment increase if net resources increase.[26]

In our model, existing productive relationships automatically benefit from higher productivity growth. That is, technological progress is assumed to be disembodied. It may very well be the case that in reality existing relationships also have to invest some resources in terms of investment or additional workers in order to benefit from productivity increases. This could reinforce the mechanism highlighted in this paper.

[26] Recently, rule-of-thumb consumers have also been used in New Keynesian models. See, for example, Gali, López-Salido, and Vallés (2004).

A Appendix

A.1 Data Sources

In this section, we discuss the data used for the calibration of the model and for the calculation of the summary statistics in Table 2.

National Income Series

- Real gross domestic product, GDPC96
- Real gross private domestic investment, GPDIC96
- Real personal consumption expenditures (nondurable goods), PCNDGC96

These series were downloaded from Federal Reserve Economic Data (FRED).

Job finding and separation probabilities

- Job finding probability
- Job separation probability

The "probabilities" are obtained from the continuous-time "rates" using

$$probability = 1 - \exp(-rate).$$

The rates were downloaded from http://home.uchicago.edu/~shimer/data/flows/. See Shimer (2005) and Shimer's home page for additional details.

Current Population Survey

- Unemployment rate, LNS14000000Q
- Employment population ratio, LNS12300000Q
- Civilian labor force participation rate, LNS11300000Q

Vacancies

- Index of Help Wanted Advertising in Newspapers, HELPWANT

These data were downloaded from Federal Reserve Economic Data (FRED). These series, together with the unemployment rate, are used to construct a measure of labor market tightness.

Productivity and technology statistics

- Output, PRS85006043
- Current \$ output, PRS85006053
- Employment, PRS85006033
- Nominal compensation, PRS85006063
- Labor share, PRS85006173

These series were downloaded from the Bureau of Labor Statistics. As is standard in the literature, these series are for the non-farm business sector. Real output and employment are used to construct labor productivity. The wage rate was calculated using

$$wage\ rate = \frac{Compensation}{Employment} \times \frac{Output}{Current\ \$\ Output}. \tag{23}$$

The labor share index was turned into an actual labor share series by rescaling the index so that the observed value in 2002Q3 is equal to 78%, the value reported in Gomme and Rupert (2004).

A.2 Numerical Solution

The model is solved with standard projection techniques; Chebyshev nodes are used to construct the grid, and third-order Chebyshev polynomials are used to approximate the conditional expectations. Integrals are calculated using Hermite-Gaussian quadrature. Details of the numerical algorithm are given in Table A.1.

One problem with rectangular grids is that they may lead to combinations of the state variables that never actually materialize, and where the approximation, or perhaps even the model itself, is not well defined. We have encountered this problem for combinations of low values of Z_t and K_t and high values of N_t^w. At these points in the state space, the desired amount of leisure combined with the value of employment, which is a state variable, result in negative numbers of agents searching, N_t^s. When simulating, the economy never gets close to these problematic points in the state space. We deal with this problem by simply not using the problematic grid points in the projection step.

A.3 Avoiding the consumption-tightness puzzle

A.3.1 Consumption and tightness in Ravn (2006)

Ravn (2006) highlights a problematic relationship between aggregate consumption and labor market tightness in DSGE models with labor market matching and endogenous labor force participation. Ravn (2006) assumes that agents can use financial markets to ensure that consumption does not depend on their labor market outcome, but that their labor market status does affect their utility. The budget set of the agent is not convex, because labor market choices are indivisible. To deal with the indivisibility, Ravn (2006) uses lotteries—as in Hansen (1985) and Rogerson (1998)—to determine which agents engage in what kind of activity.[27] This assumption corresponds to a current-period utility function for the representative household of the following form

$$U(C_t, N_t^s, N_t^w) = \frac{C_t^{1-\gamma} - 1}{1-\gamma} + N_t^s H(1-\phi_s) + N_t^w H(1-\phi_w) + (N^* - N_t^s - N_t^w) H(1). \quad (24)$$

Here, $H(\cdot)$ is the function that measures the utility of current-period leisure. Its argument is equal to $1-\phi_s$ when searching, $1-\phi_w$ when working, and 1 when not doing either. Since these values are constant, this utility specification can be written in the following

[27] The lottery approach seems less convincing with a labor force participation choice, because it implies that a random draw determines whether a worker is searching for a job or not. Another awkward aspect of the lottery approach in a matching framework is that a key aspect of the matching model is workers being fixed in a relationship. The idea that lotteries each period determine whether a worker is searching, working, or unemployed seems inconsistent with the matching friction.

linear form

$$U(C_t, N_t^s, N_t^w) = \frac{C_t^{1-\gamma} - 1}{1-\gamma} - N_t^s \overline{\phi}_s - N_t^w \overline{\phi}_w. \tag{25}$$

Ravn (2006) shows that the matching framework combined with the linear utility specification and Nash bargaining implies that the volatility of the marginal utility of consumption is proportional to the volatility of labor market tightness. In particular, the following relationship has to hold

$$\overline{\phi}_s = \frac{\eta_1}{(1-\eta_1)} \theta_t \psi C_t^{-\gamma}, \tag{26}$$

where η_1 denotes the bargaining weight of the worker. From this equation we obtain

$$d \ln \theta_t = \gamma d \ln C_t. \tag{27}$$

It follows that tightness should be highly correlated with consumption and that—for reasonable values of γ—tightness cannot be too volatile. Ravn (2006) shows that the first property is satisfied in the data, but that the second is not. Since observed tightness is much more volatile than consumption, this relationship can only be satisfied for values of γ that are generally considered implausible.

A.3.2 Consumption and tightness in our model

In our model, the volatility of labor market tightness is not constrained by the volatility of aggregate consumption in this manner. In fact, in our benchmark model the ratio of the standard deviation of labor market tightness relative to the standard deviation of consumption is equal to 42. This is even higher than the empirical counterpart, which is equal to 22.8. In this section, we demonstrate how our model avoids the problematic restriction discovered by Ravn (2006), because we do not use Nash bargaining and because we use a different utility function. First, we analyze the link between tightness and consumption when our wage setting rule is used. Second, we analyze the link between tightness and consumption when our utility function is used.

No Nash bargaining. Suppose that, as in Ravn (2006), the utility function is given by Equation (25). Then the first-order condition is equal to

$$\overline{\phi}_s = \lambda_t^w \mathrm{E}_t \left[\sum_{j=0}^{\infty} \overline{\beta}^j \left\{ \beta \left(W_{t+j+1} C_{t+j+1}^{-\gamma} - \overline{\phi}_w \right) \right\} \right], \tag{28}$$

where $\overline{\beta} = \beta(1-\rho^x)$. This is equivalent to

$$\overline{\phi}_s + \frac{\lambda_t^w \beta \overline{\phi}_w}{1-\overline{\beta}} = \lambda_t^w \mathrm{E}_t \left[\sum_{j=0}^{\infty} \overline{\beta}^j \left\{ \beta W_{t+j+1} C_{t+j+1}^{-\gamma} \right\} \right]. \tag{29}$$

The free-entry condition for the firm can be written as

$$\psi C_t^{-\gamma} = \lambda_t^f \mathrm{E}_t \left[\sum_{j=0}^{\infty} \overline{\beta}^j \beta \left(\overline{p}_{t+j+1} - W_{t+j+1} \right) C_{t+j+1}^{-\gamma} \right]. \tag{30}$$

Redefining coefficients, we can write the wage rule as

$$W_t = \overline{\omega}_0 + \overline{\omega}_1 \overline{p}_t. \tag{31}$$

Then Equations (29) and (30) can be written as

$$\overline{\phi}_s + \frac{\lambda_t^w \beta \overline{\phi}_w}{1-\overline{\beta}} = \lambda_t^w \left(\Omega_t + \overline{\omega}_1 \mathrm{E}_t \left[\sum_{j=0}^{\infty} \overline{\beta}^j \beta \overline{p}_{t+j+1} C_{t+j+1}^{-\gamma} \right] \right), \text{ and} \tag{32}$$

$$\psi C_t^{-\gamma} = \lambda_t^f \left(-\Omega_t + (1-\overline{\omega}_1) \mathrm{E}_t \left[\sum_{j=0}^{\infty} \overline{\beta}^j \beta \overline{p}_{t+j+1} C_{t+j+1}^{-\gamma} \right] \right), \tag{33}$$

$$\text{where } \Omega_t = \overline{\omega}_0 \mathrm{E}_t \left[\sum_{j=0}^{\infty} \overline{\beta}^j \beta C_{t+j+1}^{-\gamma} \right]. \tag{34}$$

Combining Equations (32) and (33) we obtain

$$\overline{\phi}_s + \frac{\lambda_t^w \beta \overline{\phi}_w}{1-\overline{\beta}} = \lambda_t^w \left(\frac{\Omega_t}{1-\overline{\omega}_1} + \frac{\overline{\omega}_1}{(1-\overline{\omega}_1)} \frac{\lambda_t^w}{\lambda_t^f} \psi C_t^{-\gamma} \right). \tag{35}$$

This equation is the counterpart of Equation (26). Just like Equation (26) it is based on a utility function that is linear in N_t^s and N_t^w, but is based on our wage rule instead of

Nash bargaining. In addition to consumption and labor market tightness it also contains Ω_t. More importantly, it does not impose such a tight constraint as the one discovered by Ravn (2006) for Nash bargaining. To see the link between the restriction implied by Equation (26) and the (lack of a) restriction in Equation (35), consider the wage setting rule in which agents simply obtain constant fractions of revenues, that is, $\omega_0 = 0$, and $0 < \omega_1 < 1$.[28] Then Equation (35) can be written as

$$\overline{\phi}_s + \mu_0 \theta_t^{\mu_1} \frac{\beta \overline{\phi}_w}{1-\overline{\beta}} = \frac{\overline{\omega}_1}{(1-\overline{\omega}_1)} \theta_t \psi C_t^{-\gamma}. \tag{36}$$

This equation is similar to Equation (26), but there is one additional time-varying term on the left-hand side. So it no longer is the case that $\ln \theta_t - \gamma \ln C_t$ has to be equal to a constant.

If wages are completely sticky then Equation (35) can be written as

$$\overline{\phi}_s + \frac{\lambda_t^w \overline{\phi}_w}{1-\beta} = \lambda_t^w \Omega_t. \tag{37}$$

Now we have a relationship between tightness and Ω_t.[29] Again this relationship is less problematic than the one derived in Ravn (2006). To see this, first consider the case where $\overline{\phi}_w = 0$, in which case the left-hand side is constant. Then we obtain a relationship similar to the one derived in Ravn (2006), but with the marginal utility replaced by the discounted sum of marginal utilities. Unless changes in the discount factor are quantitatively important, the discounted sum of marginal utilities may very well be less volatile than the current marginal utility. This specification would then also induce the consumption-tightness puzzle. But when $\overline{\phi}_w$ is positive then a change in labor market tightness increases both the left-hand and the right-hand side of Equation (37) and a smaller change in Ω_t is therefore needed.

Non-linear utility of leisure. More important than the wage setting rule is the utility specification. Our nonlinear specification makes it easier to avoid Ravn's consumption-tightness puzzle. In particular, for our specification the equivalent of Equation (26) would

[28] This implies that $\Omega_t = 0$.

[29] λ_t^w is a function of tightness only.

be

$$\phi L_t^{-\kappa} + \lambda_t^w \mathrm{E}_t \left[\sum_{j=0}^{\infty} \overline{\beta}^j \phi \beta L_t^{-\kappa} \right] = \lambda_t^w \left(\frac{\Omega_t}{1-\overline{\omega}_1} + \frac{\overline{\omega}_1}{(1-\overline{\omega}_1)} \frac{\lambda_t^w}{\lambda_t^f} \psi C_t^{-\gamma} \right). \tag{38}$$

There is another time-varying term on the left-hand side. Moreover, if labor force participation is fairly inelastic, i.e., κ is high, then this additional term is quite volatile even if leisure is not.

As in Ravn (2006), we assume that there is complete insurance of the consumption stream, but in our framework leisure is insured as well. With the lottery approach, the unemployed agent is best off, since he receives the same amount of consumption and more leisure. In our model agents are equally well off, either because agents that do not work spend more time on household chores, or because agents truly care about the overall amount of leisure of the joint household.

Both the representative agent of Ravn (2006) and ours are abstractions. Part of the motivation of a particular abstraction lies in the properties of the model it generates. Ravn (2006) shows that the linear specification leads to a "consumption-tightness" puzzle. We have shown that the consumption-tightness puzzle can be avoided by using a nonlinear utility function and an alternative wage setting rule.

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Table 1: Parameter Values (Monthly Model)

Parameter	Value	Target / Source
Discount factor, β	0.9966	Standard annual value = 0.96
Relative risk aversion, γ	0.475	Range of values considered
Scaling utility of leisure, ϕ	0.44	$N^s + N^w = 1$
Curvature utility of leisure, κ	2.5	$sd((N^s+N^w)/N^*)$ / $sd(Y/N^w) = 0.182$
Time endowment, N^*	1.5938	$(N^s + N^w)/N^* = 0.6274$
Curvature production function, α	0.315	$\alpha + \omega_0 (1- \alpha) = 1/3$
Depreciation rate, δ	0.0084	Standard annual value = 0.10
Drift term, $g^{high} = - g^{low}$	0.005/12	Congressional Budget Office
ζ_2	12	Expected duration of staying in regime 2
ζ_3	120	Expected duration of staying in regime 3
Persistence parameter, ρ	0.98	See discussion in the main text
Innovation standard deviation, σ	0.0042	Match standard deviation of $\ln(Z_t) = 0.95\ln(Z_{t-1}) + 0.007\varepsilon_t$
Wage sensitivity, ω_1	0.7547	$sd(W)$ / $sd(Y/N^w) = 0.755$
Share of entrepreneur, ω_0	0.9725	$sd(N^w/N^*)$ / $sd(Y/N^w) = 0.437$
Match elasticity, μ_1	0.50	Petrongolo and Pissarides (2001)
Match scaling, μ_0	0.39	$\lambda^w = 45.4\%$
Period entry cost, ψ	0.94	$\lambda^f = 33.8\%$
Exogenous destruction rate, ρ^x	0.027	$N^s / (N^s + N^w) = 5.7\%$

Table 2: Summary Statistics

	Data	Model I	Model II	Model III	Model III*
Variable labor force participation		Yes	No	No	No
Allocation costs		No	No	Yes	Yes
Used for calibration of Model I:					
sd($(N^s+N^w)/N^*$) / sd($\ln(Y/N^w)$)	0.182	0.189	-	-	-
sd(N^w/N^*) / sd($\ln(Y/N^w)$)	0.437	0.437	0.282	0.255	0.296
sd($\ln W$) / sd($\ln(Y/N^w)$)	0.755	0.755	0.755	0.755	0.755
Not used for calibration:					
sd($\ln Y$)	0.0158	0.0126	0.0114	0.0110	0.0114
sd($\ln I$) / sd($\ln Y$)	4.560	3.733	3.753	1.924	3.757
sd($\ln C$) / sd($\ln Y$)	0.696	0.305	0.301	0.637	0.299
sd($\ln N^w$)/ sd($\ln Y$)	0.466	0.448	0.342	0.317	0.355
sd($\ln Y$) / sd($\ln z$)		1.420	1.280	1.238	1.281
sd($\ln(Y/N^w)$)	0.0132	0.0076	0.0081	0.0080	0.0080
sd($N^w W/Y$) / sd($\ln(Y/N^w)$)	0.644	0.163	0.163	0.164	0.163
sd(N^v/N^s) / sd($\ln(Y/N^w)$)	18.98	21.49	20.79	19.47	21.23
sd(λ^w) / sd($\ln(Y/N^w)$)	2.644	3.426	3.342	3.156	3.379
Correlation($N^s/(N^s+N^w)$, $\ln Y$)	-0.86	-0.80	-0.89	-0.89	-0.88
Correlation($N^s/(N^s+N^w)$, $\ln(Y/N^w)$)	-0.33	-0.61	-0.76	-0.78	-0.73
Correlation($N^s/(N^s+N^w)$, $\ln N^v$)	-0.93	-0.40	-0.72	-0.72	-0.71

Notes: US Data are quarterly data that are HP-filtered using a smoothing parameter of 1600. Model data are Monthly data that are transformed into Quarterly data and then HP-filtered. All variables that are not expressed as a rate are logged.

Table 3: Admissible range for γ

Allocation costs	$\zeta = 1$	$\zeta = 1.25$	$\zeta = 1.5$	$\zeta = 3$
Using total investment				
Output, consumption, employment, investment $\uparrow$				
from second period	[0.450,0.500]	[0.650,0.825]	[0.750,1.000]	[1.025,1.900]
from third period	[0.425,0.550]	[0.575,0.875]	[0.600,1.100]	[0.400,1.950]
eventually	[0.000,0.675]	[0.000,1.000]	[0.000,1.175]	[0.000,2.000]
Using only investment in Physical Capital (by existing projects)				
Output, consumption, employment, investment $\uparrow$				
from third period	[0.425,0.450]	[0.575,0.650]	[0.600,0.700]	[0.400,0.925]
eventually	[0.000,0.550]	[0.000,0.775]	[0.000,0.875]	[0.000,1.175]

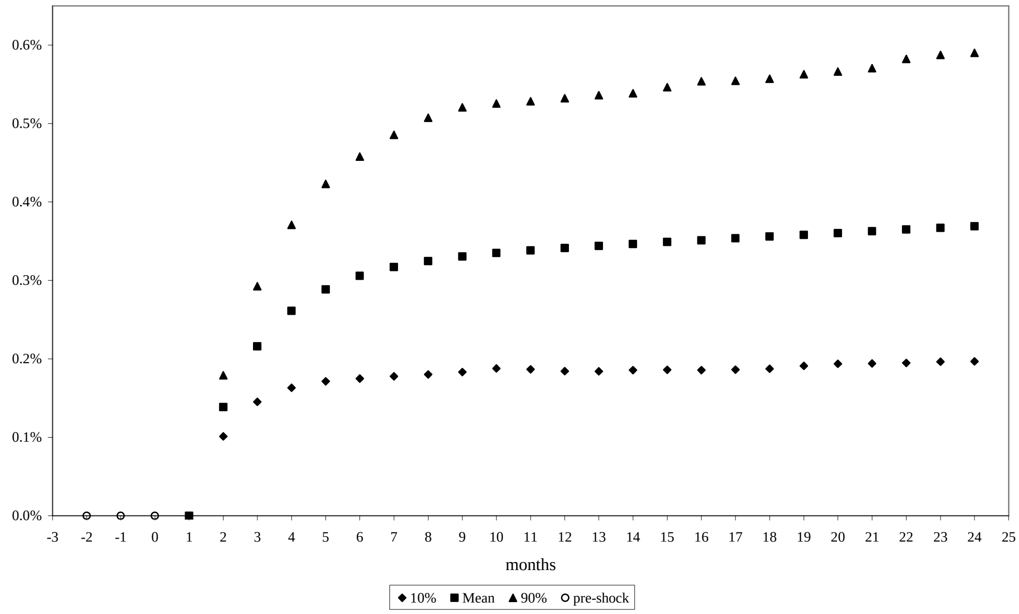
Notes: This table reports the range of values of γ for which both the consumption and the investment response are positive eventually as well as the range of values this occurs from the second period onwards and the third period onwards.

Table A.1: Details of numerical procedure

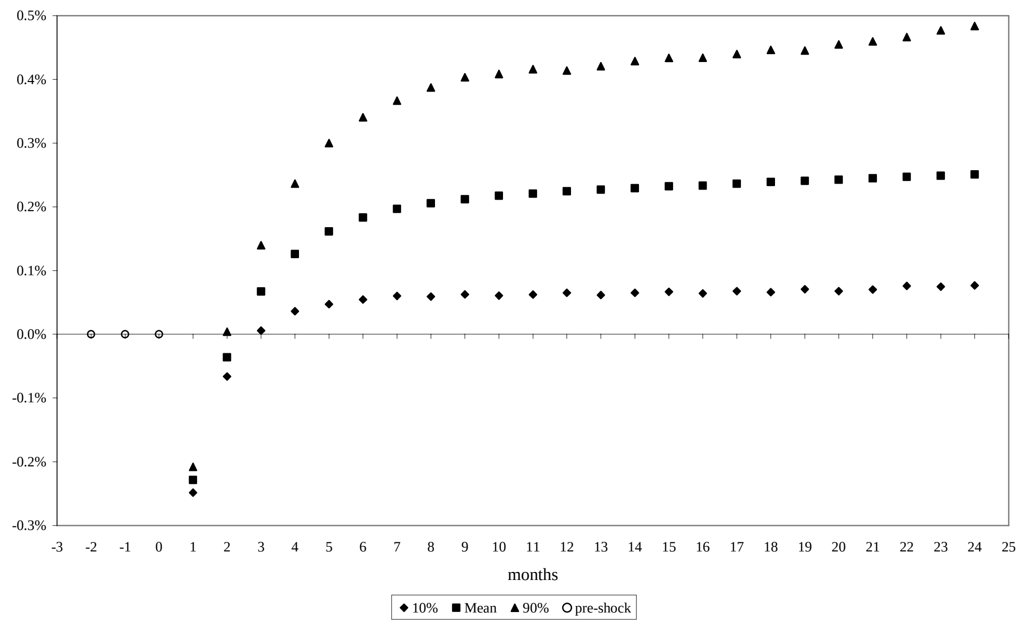
# Chebyshev nodes	
K	4
N^w	4
Z	12
Bounds on grid	
K	[95,130]
N^w	[0.82,0.98]
Z	[0.89,1.11]
Order of Polynomial	
K	3
N^w	3
Z	3
# quadrature nodes	3

Figure 1: Response to anticipated productivity increase

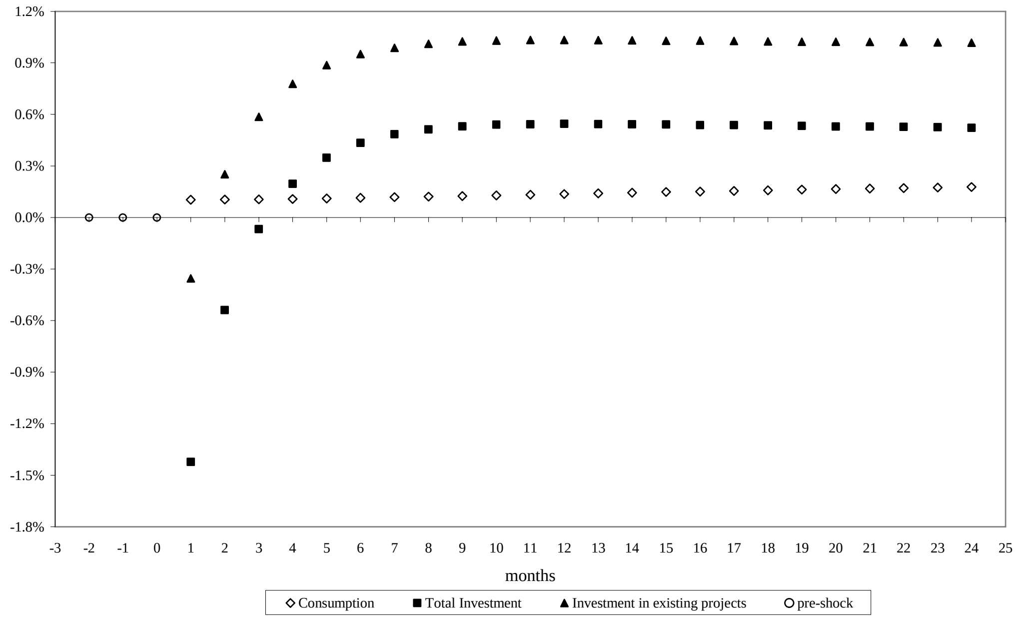
A. Output



B. Output net of investments in new projects



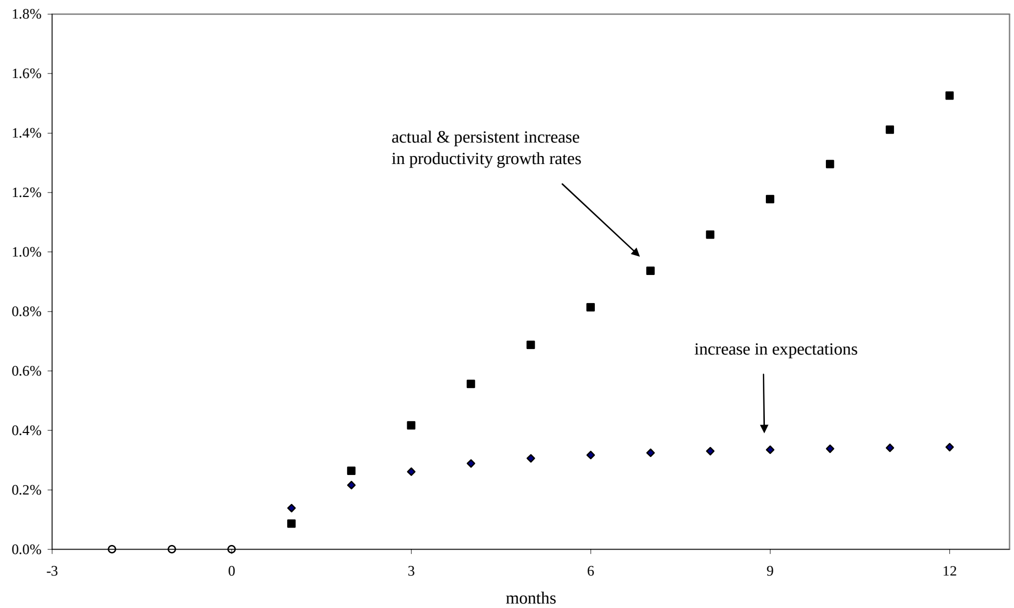
C. Consumption and Investment



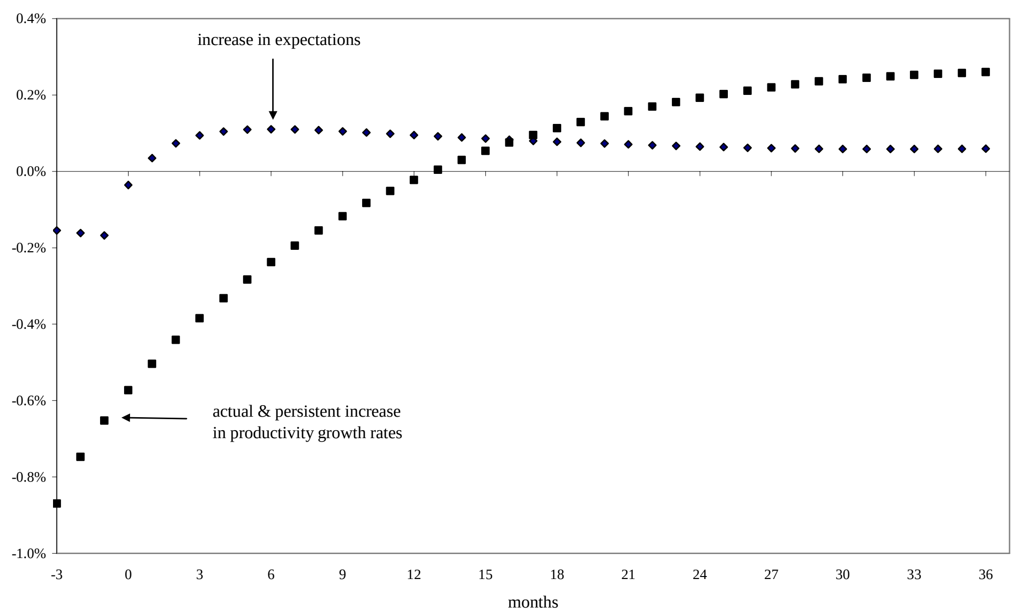
Notes: These graphs plot the responses when the economy moves from regime 1 to regime 2. Because of nonlinearities in the model, responses are calculated at different initial conditions. Panel A plots the mean response as well as the 10^{th} and the 90^{th} percentile. Panels B and C only plot the mean response.

Figure 2: An expected versus an actual increase in productivity

A. Actual output responses



B: Cyclical output responses



Notes: These graphs plot the average responses when the economy moves from regime 1 to regime 2 and when the economy moves from regime 2 to regime 3. Panel A calculates the *actual* responses when the economy moves first into regime 2 and (after 120 periods) into regime 3. Panel B calculates the corresponding cyclical responses when the HP filter is used to calculate the trend. In both cases, period 1 corresponds to the first period when the actual output response is positive.

Figure 3: Output with and without endogenous labor force participation