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Globalization, variety trade and rising inequality in the nation
(preliminary)

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Abstract

This paper analyses the theoretical possibility about the role played by extensive margins for wage inequality in the nation. Supposing that the creation of new varieties is more skill intensive relative to the creation of intensive margins (already existing goods), the model naturally predicts the wage divergence between two working classes, skilled and unskilled. In addition allowing the two-country dynamic stochastic general equilibrium structure, the relationship between trade and inequality is analyzed. It is shown that intra-industry variety trade along the logic of home market effect would account for the inequality.

1 Introduction

A rising inequality is at the center of political debate today for industrialized countries. The economists have extensively discussed as a motif of rising inequality 1) Stolper-Samuleson mechanism, 2) different pace of productivity growth among factors and and 3) different factor supply. Here I add the role played by extensive margin in the above list. The expansion of trade flow is mainly driven by extensive margins (Mayer and Ottaviaono (2008)). The creation of new varieties is supposed to need more skill intensive workers relative to the production of already existing goods (intensive margins). Variety creation and expansion of trade in terms of extensive relative to intensive margins means immediately a higher demand for the labor service which is used intensively in the variety creation activity.

In this paper there are two working classes, skilled and unskilled. The model is inspired by Ghironi and Melitz (2005) so as them he has the feature of two-country full dynamic stochastic general equilibrium model. Traditionally in the international trade theory we have the HOS model to address this question of inequality An increased occasion of trade, by changing the relative price among sectors, creates the inequality of reward among the factors needed for but with different intensity by the sectors. (Stolper-Samuelson theorem). Indeed the model presented in this paper may be considered as "one" sector model (but with multiple varieties in it) in contrast to HOS model which contains multiple sectors and give the Stolper-Samuelson effect As it is well known allowing the multiple sectors and its sector specific product, the remuneration of factors which are used with different intensity is strongly connected to the relative price among these sector specific products (hence the relative demand for them). This statement is the so called "factor price equalization theorem" which is the base of the Stolper-Samuelson. Here the model is considered to have only one sector but with multiple varieties (hence it is considered one representative average product is traded). In particular there are "two activities": creation of intensive and extensive margins in which they use different factor intensity. So the motif of the difference of factor remuneration is driven by the degree of extensive margins induced by Home market effect mechanism.

However the Stolper-Samuelson force is eliminated from the construction these two points of view are complementary As inter-industrial trade would become a source of inequality the intra-industrial trade would do also. Because of the DSGE feature of the model it is relatively easy to see, (numerically) the prediction about macroeconomic variables including wage rates after such as productivity growth, endowment supply, deregulation policy and trade liberalization.

(more complete literature review to be added)

The structure of the paper is as follows: in the next section I present the model. Here the key concept is the different factor intensity between intensive and extensive margin creation. In section 3 the principle mechanism which creates the wage rates divergence between two working classes is highlighted In section 4 the model is calibrated and the IRF is given for the economy wide TFP increase. In the end a brief conclusion is given.

2 The model

There are two countries, Home and Foreign, two factors of production, skilled and unskilled labor and one industry but two activities, creation in terms of intensive and extensive margin. As it will be precise it is supposed that the creation of extensive margin is skill intensive. Only skilled household have a chance to smooth the consumption by bond or share holding. There is no international trade of financial asset so the trade is balanced.

2.1 Households

There are two types of household who provide one unit of skilled or unskilled labor exogenously The number of each household providing skilled or unskilled is exogenously given, $L_{s.t}$ and $L_{us.t}$. For the type $j = s$, us of representative household she maximizes:

$$U_j = E\sum_{t=0}^{\infty}\beta^t\frac{C_{j.t}^{1-\gamma}}{1-\gamma} \tag{1}$$

where $C_{j..t}$ is the consumption at t of skilled and unskilled household respectively. This is defined as follows:

$$C_{j.t} = \left[C_{j.H.t}^{1-\frac{1}{\omega}} + C_{j.F.t}^{1-\frac{1}{\omega}} \right]^{\frac{1}{1-\frac{1}{\omega}}} \tag{2}$$

and

$$C_{j.H.t} = \left[\int_0^{N_t} c_{j.t}(h)^{1-\frac{1}{\sigma}} dh \right]^{\frac{1}{1-\frac{1}{\sigma}}}, \qquad C_{j.F.t} = \left[\int_0^{N_t^*} c_{j.t}(f)^{1-\frac{1}{\sigma}} df \right]^{\frac{1}{1-\frac{1}{\sigma}}} \tag{3}$$

where $j = s$, us. $C_{j.t}$ is the total demand. $C_{j.H.t}$ and $C_{j.F.t}$ are the demand of Home produced and that of Foreign produced goods (import). $c_{j.t}(h)$ and $c_{j.t}(f)$ are the demand for individual variety indexed by h for home originated and f for Foreign originated. The corresponding price indices are:

$$P_t = \left[P_{H.t}^{1-\omega} + P_{F.t}^{1-\omega} \right]^{\frac{1}{1-\omega}} \tag{4}$$

and

$$P_{H.t} = \left[\int_0^{N_t} p_t(h)^{1-\sigma} dh \right]^{\frac{1}{1-\sigma}}, \qquad P_{F.t} = \left[\int_0^{N_t^*} p_t(f)^{1-\sigma} df \right]^{\frac{1}{1-\sigma}} \tag{5}$$

The similar expression holds for Foreign.

2.1.1 Budget constraint

Only skilled households have a chance to save with bond or share holding. The financial assets are only domestically hold. The budget constraint of skilled household (in unit of home consumption goods) is given by:

$$C_{s.t} + B_{t+1} + s_{h.t+1} x_{h.t} (N_t + N_{E.t}) = (1 + r_t) B_t + w_{s.t} + s_{h.t} N_t (x_{h.t} + d_{h.t}) \tag{6}$$

where B_{t+1} is real bond, $s_{h.t+1}$ is the share holding into t+1, $x_{h.t}$ is the real share price of Home originated firm. They finance all existing firm at t N_t and new entrants $N_{E.t}$. As it is discussed later the death shock takes place at the very end of the period after holding decision. r_t is the real interest rate. $w_{s.t}$ is the real wage of skilled worker. $d_{h.t}$ is the dividend of Home originated firm.

The unskilled household consume just what they earn. That of unskilled is given by:

$$C_{us.t} = w_{us.t} \tag{7}$$

2.1.2 The first order conditions

The optimal consumption of Home produced and Foreign produced goods is given by:

$$C_{j.H.t} = \left(\frac{P_{H.t}}{P_t}\right)^{-\omega} C_{j.t}, \qquad C_{j.F.t} = \left(\frac{P_{F.t}}{P_t}\right)^{-\omega} C_{j.t} \tag{8}$$

and using symmetry among varieties, that of individual firms are:

$$c_{j.h.t} = \left(\frac{p_{h.t}}{P_{H.t}}\right)^{-\sigma} C_{j.H.t}, \qquad c_{j.f.t} = \left(\frac{p_{h.t}}{P_{H.t}}\right)^{-\sigma} C_{j.F.t} \tag{9}$$

In addition for skilled household, the Euler bond holds:

$$C_{s.t}^{-\gamma} = \beta E_t \left(1 + r_{t+1}\right) C_{s.t+1}^{-\gamma} \tag{10}$$

and the Euler share holds:

$$x_{h.t} = \beta \left(1 - \delta\right) E_t \left(\frac{C_{s.t+1}}{C_{s.t}}\right)^{-\gamma} \left(x_{h.t+1} + d_{h.t+1}\right) \tag{11}$$

For notational convenience I use the following definition of relative prices. (counted in terms of Home and Foreign goods)

$$\rho_{H.t} = \frac{P_{H.t}}{P_t}, \qquad \rho_{F.t} = \frac{P_{F.t}}{P_t}, \qquad \rho_{h.t} = \frac{p_{h.t}}{P_t}, \qquad \rho_{f.t} = \frac{p_{f.t}}{P_t}$$

$$\rho^*_{H.t} = \frac{P^*_{H.t}}{P^*_t}, \qquad \rho^*_{F.t} = \frac{P^*_{F.t}}{P^*_t}, \qquad \rho^*_{h.t} = \frac{p^*_{h.t}}{P^*_t}, \qquad \rho^*_{f.t} = \frac{p^*_{f.t}}{P^*_t}$$

2.2 Firms

2.2.1 The creation of intensive margins

Firms are monopolistically competitive. Once entered in the market after paying a fixed cost (we will discuss this point later) for the production of goods, they need to hire the effective labor unit which is composed of skilled and unskilled labor. I suppose that the total cost function is given by:

$$TC_t^{IM} \equiv w_{s.t} l_{s.t}^{IM} + w_{us.t} l_{us.t}^{IM} = \frac{w_{s.t}^{\zeta} w_{us.t}^{1-\zeta}}{z_t} y_{h.t} \tag{12}$$

where ζ $(1-\zeta)$ is the weight of skilled (unskilled) labor which range $0 < \zeta < 1$. Applying Hoteling's lemma, the factor demand of skilled and unskilled labor in the intensive margins are:

$$l_{s.t}^{IM} = \zeta \frac{y_{h.t}}{z_t} \left(\frac{w_{us}}{w_s}\right)^{1-\zeta}, \qquad l_{us.t}^{IM} = \left(1 - \zeta\right) \frac{y_{h.t}}{z_t} \left(\frac{w_s}{w_{us}}\right)^{\zeta} \tag{13}$$

Dividends are given by:

$$d_{h.t} = \left(\rho_{h.t} - \frac{w_{s.t}^{\zeta} w_{us.t}^{1-\zeta}}{z_t} \right) y_{h.t} \tag{14}$$

where

$$y_{h.t} = L_{s.t} c_{sh.t} + L_{us.t} c_{ush.t} + (1+\tau_t) \left(L_{s.t}^{*} c_{sh.t}^{*} + L_{us.t}^{*} c_{ush.t}^{*} \right) \tag{15}$$

τ_t is the iceberg type transportation cost. Knowing the demands firms freely choose the optimal price in each period as the consequence of maximization of current dividends. This gives standard pricing: marginal cost over markup:

$$\rho_{h.t} = \frac{\sigma}{\sigma - 1} \frac{w_{s.t}^{\zeta} w_{us.t}^{1-\zeta}}{z_t} \tag{16}$$

Note because of the iceberg type transportation cost:, the exporting price $\rho_{h.t}^{*}$ (in Foreign consumption unit) is given by $\rho_{h.t}^{*} = Q_t^{-1} (1+\tau_t) \rho_{h.t}$. Using these optimal pricing and demands functions the dividends can be expressed as:

$$d_{h.t} = \frac{\rho_{h.t}^{1-\omega}}{\sigma} \left(\frac{1}{N_t} \right)^{\frac{\sigma-\omega}{\sigma-1}} \left[C_t + \phi_t Q_t^{\omega} C_t^{*} \right] \tag{17}$$

where $\phi_t \equiv (1+\tau_t)^{1-\omega}$. This is very intuitive expression. An increase of real price decrease (increase) the real dividends under $\omega > 1$ ($\omega < 1$). This is because with the symmetry among firms from same origin what is crucial for dividend is the elasticity of substitution between Home and Foreign goods. Under higher elasticity they can attract more demand with a lower price. Note that because all variables are "real", an increase of total number of varieties competing in Home increases the real price $\rho_{h.t.}$ However which countries' variety increases is also important. Because the Home firms compete more closely with Home originated firms than Foreign (remember $\sigma > \omega$), an increase of Home number of firms induces an additional competing effect. This effect is captured in $\left(\frac{1}{N_t}\right)^{\frac{\sigma-\omega}{\sigma-1}}$. Other terms in the expression is standard.

2.2.2 The creation of extensive margins

Firms need to pay the entry cost to enter in the market in terms of effective labor. The total cost used for one firm establishment (extensive margin) is given by:

$$TC_t^{EM} \equiv w_{s.t} l_{s.t}^{EM} + w_{us.t} l_{us.t}^{EM} = \frac{w_{s.t}^{\eta} w_{us.t}^{1-\eta}}{z_{E.t}} f_{E.t} \tag{18}$$

where f_E is the number of effective labor needed which supposed to be exogenous. η $(1-\eta)$ is the weight of skilled (unskilled) labor which range $0 < \eta < 1$.

For the same token the factor demand of skilled and unskilled labor in the extensive margin are:

$$l_{s.t}^{EM}=\eta\frac{f_{E.t}}{z_{E.t}}\left(\frac{w_{us}}{w_s}\right)^{1-\eta},\qquad l_{us.t}^{EM}=(1-\eta)\frac{f_{E.t}}{z_{E.t}}\left(\frac{w_s}{w_{us}}\right)^{\eta}\tag{19}$$

2.2.3 Factor intensity

The activity of the creation of extensive margin is supposed to use more skilled labor intensive technology relative to that of intensive margins. Defining the each labor demand for one unit of each margin as $a_{k.j}$ where $k=IM, EM$ and $j=s, us$, this means:

$$\frac{a_{EM.s}}{a_{EM.us}}>\frac{a_{IM.s}}{a_{IM.us}}\tag{20}$$

Using the factor demand function this relationship immediately means:

$$\eta>\zeta\tag{21}$$

As we will see later this is a crucial hypothesis. This allows to diverge the wage between skilled and unskilled in function of new entry.

2.2.4 The motion of the firms

The motion of the firms is identical to Ghironi and Melitz (2005):

$$N_t=(1-\delta)\left(N_{t-1}+N_{E.t-1}\right)\tag{22}$$

Firms don't produce immediately after entry. They need one period of time-to build and they start their production from the next. Beside the mechanism by free entry which determines the new entry (or new exit) hence the number of the firms, there exists a natural mortality rate of firms embodied in the ecnomomy. This is the analogy of the capital depreciation of the standard RBC serving principally to bring back the economy to the steady state.

2.3 Aggregate accounting, labor market clearings and balanced trade

Noting $\int_0^{L_{s.t}}B_t=0$ and $\int_0^{L_{s.t}}x_{h.t}=1$, the aggregation of the budget constraint of both among skilled and unskilled we have:

$$L_{s.t}C_{s.t}+N_{E.t}x_{h.t}=L_{s.t}w_{s.t}+N_td_{h.t},\qquad L_{us.t}C_{us.t}=L_{us.t}w_{us.t}\tag{23}$$

The total aggregation gives Home's aggregated identity:

$$C_t+N_{E.t}x_{h.t}=w_t+N_td_{h.t}\tag{24}$$

where $C_t = L_{s.t}C_{s.t} + L_{us.t}C_{us.t}$ and $w_t = L_{s.t}w_{s.t} + L_{us.t}w_{us.t}$. This says the aggregated consumption plus investment must equal to aggregated labor income plus dividends.

The labor market clearing conditions of both type of labor are given by:

$$L_{s.t} = N_t l_{s.t}^{IM} + N_{E.t} l_{s.t}^{EM}, \qquad L_{us.t} = N_t l_{us.t}^{IM} + N_{E.t} l_{us.t}^{EM} \tag{25}$$

Note that the scale of production of each firm can be expressed as:

$$y_{h.t} = \sigma \frac{d_{h.t}}{\rho_{h.t}} = (\sigma - 1)\frac{d_{h.t}}{w_{s.t}^{\zeta} w_{us.t}^{1-\zeta}} z_t \tag{26}$$

Plugging the factor demands () and () and using the free entry and above expression of individual intensive margin, the labor market clearing conditions become:

$$L_{s.t} = \zeta(\sigma - 1)\frac{N_t d_{h.t}}{w_{s.t}} + \eta \frac{N_{E.t} x_{h.t}}{w_{s.t}}, \qquad L_{us.t} = (1-\zeta)(\sigma - 1)\frac{N_t d_{h.t}}{w_{us.t}} + (1-\eta)\frac{N_{E.t} x_{h.t}}{w_{us.t}} \tag{27}$$

The similar expressions hold for Foreign.

Finally the trade is supposed to be balanced.

$$Q_t^{2\omega-1} \rho_{h.t}^{1-\omega} N_t^{\frac{1-\omega}{1-\sigma}} C_t^* = \rho_{f.t}^{*1-\omega} N_t^{*\frac{1-\omega}{1-\sigma}} C_t \tag{28}$$

2.4 Summery

The system is resumed in the following table:

Table 1 The summery of the system

Pricing:

$$\rho_{h.t} = \frac{\sigma}{\sigma - 1}\frac{w_{s.t}^{\zeta} w_{us.t}^{1-\zeta}}{z_t}, \qquad \rho_{f.t}^* = \frac{\sigma}{\sigma - 1}\frac{w_{s.t}^{*\zeta} w_{us.t}^{*1-\zeta}}{z_t^*} \tag{29}$$

(Exported price is mill pricing.

$$\rho_{h.t}^* = Q_t^{-1}(1 + \tau_t)\rho_{h.t}, \qquad \rho_{f.t} = Q_t(1 + \tau_t)\rho_{f.t}^* \tag{30}$$

)

Price Indices (which are numeraire and redundant)

$$1 = N_t^{\frac{1-\omega}{1-\sigma}} \rho_{h.t}^{1-\omega} + \phi_t N_t^{*\frac{1-\omega}{1-\sigma}} Q_t^{1-\omega} \rho_{f.t}^{*1-\omega} \tag{31}$$

$$1 = N_t^{*\frac{1-\omega}{1-\sigma}} \rho_{f.t}^{*1-\omega} + \phi_t N_t^{\frac{1-\omega}{1-\sigma}} Q_t^{\omega-1} \rho_{h.t}^{1-\omega} \tag{32}$$

Dividends:

$$d_{h.t} = \frac{\rho_{h.t}^{1-\omega}}{\sigma} \left(\frac{1}{N_t}\right)^{\frac{\sigma-\omega}{\sigma-1}} \left[C_t + \phi_t Q_t^{\omega} C_t^*\right] \tag{33}$$

$$d_{f.t}^* = \frac{\rho_{f.t}^{*1-\omega}}{\sigma} \left(\frac{1}{N_t^*}\right)^{\frac{\sigma-\omega}{\sigma-1}} \left[C_t^* + \phi_t Q_t^{-\omega} C_t\right] \tag{34}$$

Free entry:

$$x_{h.t} = \frac{w_{s.t}^{\eta} w_{us.t}^{1-\eta}}{z_{E.t}} f_{E.t}, \qquad x_{f.t}^* = \frac{w_{s.t}^{*\eta} w_{us.t}^{*1-\eta}}{z_{E.t}^*} f_{E.t}^* \tag{35}$$

The motion of the firms:

$$N_t = (1-\delta)\left(N_{t-1} + N_{E.t-1}\right) \tag{36}$$

$$N_t^* = (1-\delta)\left(N_{t-1}^* + N_{E.t-1}^*\right) \tag{37}$$

Euler bonds (for skilled and unskilled)

$$C_{s.t}^{-\gamma} = \beta E_t \left(1 + r_{t+1}\right) C_{s.t+1}^{-\gamma} \tag{38}$$

$$C_{s.t}^{*-\gamma} = \beta E_t \left(1 + r_{t+1}^*\right) C_{s.t+1}^{*-\gamma} \tag{39}$$

Euler shares:

$$x_{h.t} = \beta (1-\delta) E_t \left(\frac{C_{s.t+1}}{C_{s.t}}\right)^{-\gamma} \left(x_{h.t+1} + d_{h.t+1}\right) \tag{40}$$

$$x_{f.t}^* = \beta (1-\delta) E_t \left(\frac{C_{s.t+1}^*}{C_{s.t}^*}\right)^{-\gamma} \left(x_{f.t+1}^* + d_{f.t+1}^*\right) \tag{41}$$

Aggregated budget constraints for both type of households:

$$L_{s.t} C_{s.t} + N_{E.t} x_{h.t} = L_{s.t} w_{s.t} + N_t d_{h.t}, \qquad L_{us.t} C_{us.t} = L_{us.t} w_{us.t} \tag{42}$$

$$C_t = L_{s.t} C_{s.t} + L_{us.t} C_{us.t} \text{ and } w_t = L_{s.t} w_{s.t} + L_{us.t} w_{us.t} \tag{43}$$

$$L_{s.t}^* C_{s.t}^* + N_{E.t}^* x_{f.t}^* = L_{s.t}^* w_{s.t}^* + N_t^* d_{f.t}^*, \qquad L_{us.t}^* C_{us.t}^* = L_{us.t}^* w_{us.t}^* \tag{44}$$

$$C_t^* = L_{s.t}^* C_{s.t}^* + L_{us.t}^* C_{us.t}^* \text{ and } w_t^* = L_{s.t}^* w_{s.t}^* + L_{us.t}^* w_{us.t}^* \tag{45}$$

Labor market clearing:

$$L_{s.t} = \zeta (\sigma - 1) \frac{N_t d_{h.t}}{w_{s.t}} + \eta \frac{N_{E.t} x_{h.t}}{w_{s.t}}, \qquad L_{us.t} = (1-\zeta)(\sigma - 1) \frac{N_t d_{h.t}}{w_{us.t}} + (1-\eta) \frac{N_{E.t} x_{h.t}}{w_{us.t}} \tag{46}$$

$$L^*_{s.t} = \zeta (\sigma - 1) \frac{N^*_t d^*_{f.t}}{w^*_{s.t}} + \eta \frac{N^*_{E.t} x^*_{h.t}}{w^*_{s.t}}, \qquad L^*_{us.t} = (1-\zeta)(\sigma - 1) \frac{N^*_t d^*_{f.t}}{w^*_{us.t}} + (1-\eta) \frac{N^*_{E.t} x^*_{f.t}}{w^*_{us.t}} \tag{47}$$

Trade balance:

$$Q_t^{2\omega-1} \rho_{h.t}^{1-\omega} N_t^{\frac{1-\omega}{1-\sigma}} C^*_t = \rho^{*1-\omega}_{f.t} N_t^{*\frac{1-\omega}{1-\sigma}} C_t \tag{48}$$

3 Entry, relative wage and welfare transmission between skilled and unskilled

The most important feature of the model is the new entry which requires skill intensive effective labor. This means that the new entry account for the wage divergence between skilled and unskilled. The mechanism is hightlighted in the following equations. Log-linearlising Home labor market clearing conditions for two types of labor and using that of free entry condition, I can write:

$$\begin{aligned} &[\zeta (1-\zeta)(\sigma - 1) S_D + \eta (1-\eta) S_I] (\widehat{w}_{s.t} - \widehat{w}_{us.t}) \\ &\quad = (\eta - \zeta) S_I \left(\widehat{N}_{E.t} + \widehat{f}_{E.t} - \widehat{z}_{E.t} \right) + \zeta S_{W_{us}} \widehat{L}_{us.t} - (1-\zeta) S_{W_s} \widehat{L}_{s.t} \end{aligned} \tag{49}$$

Given $\eta > \zeta$ and other things equals, new entry increases the wage of skilled relatively. This is because the new entry requires relatively more skilled labor service than unskilled. However wage divergence decreases, other things equals, when a decrease of firm setting up cost (by deregulation, an decrease of $f_{E.t}$ or a TFP shock, an increase of $z_{E.t}$) occuers This is because if $\widehat{N}_{E.t} = 0$, hence $\widehat{x}_{h.t} = 0$ to newly satisfy the free entry condition the remuneration of the unskilled workers should go up more proportionally than skilled. These mechanism disappears when $\eta = \zeta$ meaning that the two wage rates moves in the same way as if only one factor of production exists in the economy. Note also the relative factor supply affects the relative wage between two working classes. For example a higher number of unskilled workers $L_{us.t}$ increases the wage gap.

New entry accounts for the wage inequality (precisely *real* wage meaning including welfare consistent variety effect). However this doesn't automatically mean welfare divergence between skilled and unskilled. There is a sort of welfare transmission from skilled to unskilled by investment. As we have seen the unskilled doesn't have any chance to "smooth" their consumption (no saving).

They eat exactly what they earn. Only the skilled does postpone their consumption by bond or investment. However the gain from this investment is not only limited for skilled households. The expansion of newly available variety as a consequence of investment by rich is *for every one*. So being obliged to eat just what they earn, the consumption basket of unskilled household expands with new varieties improving their welfare.

4 Calibration

The parameters are calibrated as follows:

Table
parameters

γ	elasticity of the consumption marginal utility	2
β	discount factor	0.99
σ	elasticity of substitution among varieties	3.8
ω	Armingthon elasticity	2
τ	transportation cost	0.3
δ	death shock	0.025
ϕ	(calculated value)	0.77
ζ	weight of skilled in intensive margins sector	0.5
η	weight of skilled in extensive margins sector	0.9

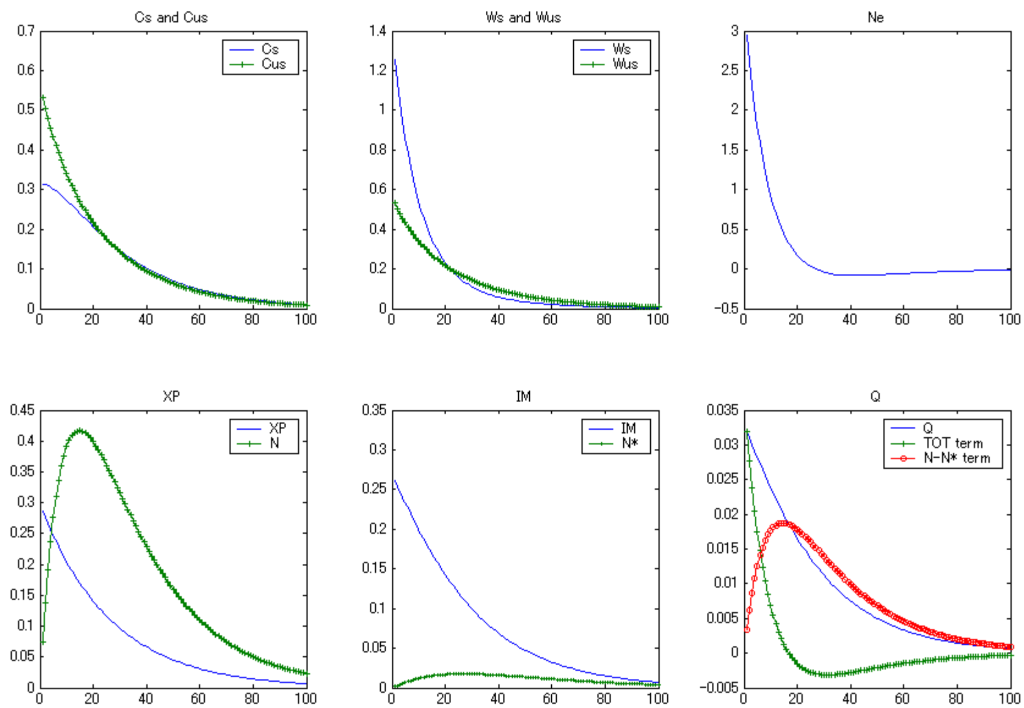
The large part of the parameters in the above table is taken from the literature. Relatively small elasticity of substitution among varieties (so relatively high markup) is justified by the fact that firms should pay sunk entry cost before producing (GM). What I need to be precise is the last two parameters, ζ and η. They should be estimated based on same empirical relationship. However here I choose them arbitrary leaving this task for the future version of the paper. With above parameters the steady state aggregated wage and consumption of two working classes relative to the aggregate consumption becomes as follows (see the appendix for calculation):

$S_{W_s} = 0.54$	$S_{W_{us}} = 0.39$
$S_{Cs} = 0.61$	$S_{Cus} = 0.39$

With these values I make a IRF exercise in the next section.

4.1 Economy wide TFP increase

To see the functioning of the model here I discuss the case of economy wide productivity increase (a simultaneous 1 % TFP increase of intensive and extensive margin creation, z_t and $z_{E.t}$ so as if there is only one aggregated productivity in the economy) and see how majors variables moves. The autocorrelation of the shock is set to 0.9. Because my focus is on the inequality and trade expansion due to extensive margin, I present only the concerning variables here. All the other variables' IRF are found in Appendix including Foreign. All variables are counted by consumption goods, hence welfare consistent variables.



(50)

The effect of such economy wide productivity gains appears on impact as an investment boom, an increased number of new entrants at Home. Precisely new entry takes place to newly pins down the free entry conditions shaken by the shock at each point of the time. This rising number of Home located firms induced by an increased economic size (here by productivity gains), which is well known in international economics as Home market effect, appears also in general equilibrium in terms of wage appreciation for Home (a higher aggregated income W_t relative to Foreign)[1].

This wage increase, however, is not equal between two working classes. As we have seen in the previous section new entry becomes the source of inequality

[1] The degree of new entry depends on the financial structure as pointed out in CMP(2007). Intuitevely more the skilled houshold have a chance to make a risk sharing by internatinally traded assets, more investment (new entry) would take place in the country which has had the productivity gains.

of earning. Skilled workers are better off transitory in terms of income. However this is not the whole of the story. Skilled workers do invest one part of their income reflecting their consumption smoothing motivation (a hump-shaped pass of skilled workers' consumption observed as in the standard RBC model with capital accumulation). So unskilled workers realize transitory a higher consumption variation relative to skilled. In addition the realized gains from this investment (realized new entrants) are also consumed by unskilled workers.

At the same time, the volume of trade expands reflecting this increased size of Home economy. Remembering that in this benchmark case the trade is balanced, the Home export increase is mainly driven by the increased number of Home variety as we can see in the graphic. For adjustment intensive margins shrinks in the export. For import, Home import more Foreign varieties transitory but she imports relatively a lot in terms of intensive margin to realize a balanced trade. To be honest this excessive increase of extensive margin in Home export is on the one hand due to the fact that all varieties are immediately "tradables" contrast to GM where there exists selection process to export. However on the other hand this reflects the facts of love for variety of Foreigners.

At the end I would like to discuss about the (welfare consistent) real exchange rate variation to deeply understand the functioning of the model. From the price index, the log-deviation of the real exchange rate is expressed as:

$$\widehat{Q}_t = -\frac{1-\phi}{1+\phi}\widehat{TOT}_t + \frac{1-\phi}{1+\phi}\frac{1}{\sigma-1}\left(\widehat{N}_t - \widehat{N}_t^*\right) \tag{51}$$

The terms of trade depreciation (a decrease of $\widehat{TOT}_t$) works to appreciate the real exchange rate because of Home bias in the consumption (here expressed with trade cost). In addition to this standard term, a larger variation of Home originated variety relative to Foreign works to depreciates it (also this is true only in the presence of Home biased consumption). The last graphic represents the real exchange variation and its components as above expression. The terms of trade depreciates on impact reflecting the strong productivity increase relative to the wage appreciation in the marginal cost of intensive margins This "terms of labor depreciation (GM)" is reversed across the time because a strong wage appreciation effect starting to appear due to entry. However the variety effect is always relatively strong and the welfare based real exchange rate depreciates until he goes back to the initial steady state level (no reversal in the dynamic).

5 Conclusion

(To be completed)

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A Symmetric steady state ratio

All symmetric steady state ratios (which are expressed without time and country index) are defined relative to total consumption:

$$S_D \equiv \frac{Nd}{C} = \frac{1}{\sigma} \tag{52}$$

$$S_I \equiv \frac{N_E x}{C} = \frac{1}{\sigma} \frac{\beta\delta}{1 - \beta(1-\delta)} \tag{53}$$

$$S_W \equiv \frac{w}{C} = \frac{\sigma - 1}{\sigma} + \frac{1}{\sigma} \frac{\beta\delta}{1 - \beta(1-\delta)} \tag{54}$$

Also from labor market clearing

$$\begin{aligned} S_W &= S_{W_s} + S_{W_{us}} \equiv \frac{L_s w_s}{C} + \frac{L_{us} w_{us}}{C} && (55) \\ &= [\zeta(\sigma-1) S_D + \eta S_I] + [(1-\zeta)(\sigma-1) S_D + (1-\eta) S_I] && (56) \end{aligned}$$

From aggregated identity:

$$1 = S_{Cs} + S_{Cus} \equiv \frac{L_s C_s}{C} + \frac{L_{us} C_{us}}{C} \tag{57}$$

from aggregated identity among unskilled:

$$\frac{L_{us} C_{us}}{C} = \frac{L_{us} w_{us}}{C} = (1-\zeta)(\sigma-1) S_D + (1-\eta) S_I \tag{58}$$

then the part of steady state consumption by skilled is given by:

$$S_{Cs} \equiv \frac{L_s C_s}{C} = 1 - (1-\zeta)(\sigma-1) S_D + (1-\eta) S_I \tag{59}$$

At the end, noting $p_h = p_f^*$ $(p_f = p_h^*)$ because of the transportation cost,

$$\rho_H^{1-\omega} = \rho_h^{1-\omega} \left(\frac{1}{N}\right)^{\frac{1-\omega}{\sigma-1}} = \frac{1}{1 + \left(\frac{P_F}{P_H}\right)^{1-\omega}} = \frac{1}{1 + \left(\frac{N^{*\frac{1}{\sigma-1}}(1+\tau)p_f^*}{N^{\frac{1}{\sigma-1}} p_h}\right)^{1-\omega}} = \frac{1}{1+\phi} \tag{60}$$

also in the same way

$$\rho_F^{1-\omega} = \rho_f^{1-\omega} \left(\frac{1}{N^*}\right)^{\frac{1-\omega}{\sigma-1}} = \frac{1}{\left(\frac{P_H}{P_F}\right)^{1-\omega} + 1} = \frac{\phi}{1+\phi} \tag{61}$$

at the symmetric steady state we have $\rho_H = \rho_F^*$ and $\rho_F = \rho_H^*$

B Log-linearised system

From the symmetric steady state

Pricing:

$$\widehat{\rho}_{h.t} = \zeta \widehat{w}_{s.t} + (1-\zeta)\,\widehat{w}_{us.t} - \widehat{z}_t, \qquad \widehat{\rho}^*_{f.t} = \zeta \widehat{w}^*_{s.t} + (1-\zeta)\,\widehat{w}^*_{us.t} - \widehat{z}_t^{\,*} \quad (62)$$

Price Index (which are numeraire and redundant)

$$0 = \frac{1}{1+\phi}\left[(1-\omega)\,\widehat{\rho}_{h.t} - \frac{1-\omega}{\sigma-1}\widehat{N}_t\right] + \frac{\phi}{1+\phi}\left[\widehat{\phi}_t + (1-\omega)\,\widehat{Q}_t + (1-\omega)\,\widehat{\rho}^*_{f.t} - \frac{1-\omega}{\sigma-1}\widehat{N}^*_t\right] \quad (63)$$

$$0 = \frac{1}{1+\phi}\left[(1-\omega)\,\widehat{\rho}^*_{f.t} - \frac{1-\omega}{\sigma-1}\widehat{N}^*_t\right] + \frac{\phi}{1+\phi}\left[\widehat{\phi}_t - (1-\omega)\,\widehat{Q}_t + (1-\omega)\,\widehat{\rho}_{h.t} - \frac{1-\omega}{\sigma-1}\widehat{N}_t\right] \quad (64)$$

Dividends:

$$\widehat{d}_{h.t} = (1-\omega)\,\widehat{\rho}_{h.t} - \frac{\sigma-\omega}{\sigma-1}\widehat{N}_t + \frac{1}{1+\phi}\widehat{C}_t + \frac{\phi}{1+\phi}\widehat{\phi}_t + \frac{\phi}{1+\phi}\omega\widehat{Q}_t + \frac{\phi}{1+\phi}\widehat{C}^*_t \quad (65)$$

$$\widehat{d}^*_{f.t} = (1-\omega)\,\widehat{\rho}^*_{f.t} - \frac{\sigma-\omega}{\sigma-1}\widehat{N}^*_t + \frac{1}{1+\phi}\widehat{C}^*_t + \frac{\phi}{1+\phi}\widehat{\phi}_t - \frac{\phi}{1+\phi}\omega\widehat{Q}_t + \frac{\phi}{1+\phi}\widehat{C}_t \quad (66)$$

Free entry:

$$\widehat{x}_{h.t} = \eta\widehat{w}_{s.t} + (1-\eta)\,\widehat{w}_{us.t} + \widehat{f}_{E.t} - \widehat{z}_{E.t}, \qquad \widehat{x}^*_{f.t} = \eta\widehat{w}^*_{s.t} + (1-\eta)\,\widehat{w}^*_{us.t} + \widehat{f}^*_{E.t} - \widehat{z}^*_{E.t} \quad (67)$$

The motion of the firms

$$\widehat{N}_t = (1-\delta)\,\widehat{N}_{t-1} + \delta\widehat{N}_{E.t-1} \quad (68)$$

$$\widehat{N}^*_t = (1-\delta)\,\widehat{N}^*_{t-1} + \delta\widehat{N}^*_{E.t-1} \quad (69)$$

Euler bonds

$$\gamma E_t \widehat{C}_{s.t+1} = \gamma \widehat{C}_{s.t} + \widehat{r}_{t+1} \quad (70)$$

$$\gamma E_t \widehat{C}^*_{s.t+1} = \gamma \widehat{C}^*_{s.t} + \widehat{r}^*_{t+1} \quad (71)$$

Euler shares:

$$\gamma\left(E_t\widehat{C}_{s.t+1}-\widehat{C}_{s.t}\right)=\beta\left(1-\delta\right)E_t\widehat{x}_{h.t+1}-\widehat{x}_{h.t}+\left[1-\beta\left(1-\delta\right)\right]E_t\widehat{d}_{h.t+1} \quad (72)$$

$$\gamma\left(E_t\widehat{C}^*_{s.t+1}-\widehat{C}^*_{s.t}\right)=\beta\left(1-\delta\right)E_t\widehat{x}^*_{f.t+1}-\widehat{x}^*_{f.t}+\left[1-\beta\left(1-\delta\right)\right]E_t\widehat{d}^*_{f.t+1} \quad (73)$$

Aggregated budget constraints for both type of households:

$$S_{C_s}\left(\widehat{L}_{s.t}+\widehat{C}_{s.t}\right)+S_I\left(\widehat{N}_{E.t}+\widehat{x}_{h.t}\right)=S_{W_s}\left(\widehat{L}_{s.t}+\widehat{w}_{s.t}\right)+S_D\left(\widehat{N}_t+\widehat{d}_{h.t}\right),$$
$$S_{C_{us}}\left(\widehat{L}_{us.t}+\widehat{C}_{us.t}\right)=S_{W_{us}}\left(\widehat{L}_{us.t}+\widehat{w}_{us.t}\right) \quad (74)$$

$$\widehat{C}_t=S_{C_s}\left(\widehat{L}_{s.t}+\widehat{C}_{s.t}\right)+S_{C_{us}}\left(\widehat{L}_{us.t}+\widehat{C}_{us.t}\right) \text{ and } \widehat{w}_t=S_{W_s}\left(\widehat{L}_{s.t}+\widehat{w}_{s.t}\right)+S_{W_{us}}\left(\widehat{L}_{us.t}+\widehat{w}_{us.t}\right) \quad (75)$$

$$S_{C_s}\left(\widehat{L}^*_{s.t}+\widehat{C}^*_{s.t}\right)+S_I\left(\widehat{N}^*_{E.t}+\widehat{x}^*_{f.t}\right)=S_{W_s}\left(\widehat{L}^*_{s.t}+\widehat{w}^*_{s.t}\right)+S_D\left(\widehat{N}^*_t+\widehat{d}^*_{f.t}\right),$$
$$S_{C_{us}}\left(\widehat{L}^*_{us.t}+\widehat{C}^*_{us.t}\right)=S_{W_{us}}\left(\widehat{L}^*_{us.t}+\widehat{w}^*_{us.t}\right) \quad (76)$$

$$\widehat{C}^*_t=S_{C_s}\left(\widehat{L}^*_{s.t}+\widehat{C}^*_{s.t}\right)+S_{C_{us}}\left(\widehat{L}^*_{us.t}+\widehat{C}^*_{us.t}\right) \text{ and } \widehat{w}^*_t=S_{W_s}\left(\widehat{L}^*_{s.t}+\widehat{w}^*_{s.t}\right)+S_{W_{us}}\left(\widehat{L}^*_{us.t}+\widehat{w}^*_{us.t}\right) \quad (77)$$

Labor market clearing:

$$S_{W_s}\left(\widehat{L}_{s.t}+\widehat{w}_{s.t}\right)=\zeta\left(\sigma-1\right)S_D\left(\widehat{N}_t+\widehat{d}_{h.t}\right)+\eta S_I\left(\widehat{N}_{E.t}+\widehat{x}_{h.t}\right),$$
$$S_{W_{us}}\left(\widehat{L}_{us.t}+\widehat{w}_{us.t}\right)=\left(1-\zeta\right)\left(\sigma-1\right)S_D\left(\widehat{N}_t+\widehat{d}_{h.t}\right)+\left(1-\eta\right)S_I\left(\widehat{N}_{E.t}+\widehat{x}_{h.t}\right) \quad (78)$$

$$S_{W_s}\left(\widehat{L}^*_{s.t}+\widehat{w}^*_{s.t}\right)=\zeta\left(\sigma-1\right)S_D\left(\widehat{N}^*_t+\widehat{d}^*_{f.t}\right)+\eta S_I\left(\widehat{N}^*_{E.t}+\widehat{x}^*_{f.t}\right),$$
$$S_{W_{us}}\left(\widehat{L}^*_{us.t}+\widehat{w}^*_{us.t}\right)=\left(1-\zeta\right)\left(\sigma-1\right)S_D\left(\widehat{N}^*_t+\widehat{d}^*_{f.t}\right)+\left(1-\eta\right)S_I\left(\widehat{N}^*_{E.t}+\widehat{x}^*_{f.t}\right) \quad (79)$$

Trade balance

$$\left(2\omega-1\right)\widehat{Q}_t+\left(1-\omega\right)\widehat{\rho}_{h.t}+\frac{1-\omega}{1-\sigma}N_t+\widehat{C}^*_t=\left(1-\omega\right)\widehat{\rho}^*_{f.t}+\frac{1-\omega}{1-\sigma}N^*_t+\widehat{C}_t \quad (80)$$

C Dividend

Using the optimal price, the dividend can be written as:

$$d_{i.t} = \frac{\rho_{i.t}}{\sigma} y_{i.t} \tag{81}$$

where $i = h, f$ and $I = H, F$. Noting that the production is determined by the demand from each country,

$$d_{i.t} = \frac{\rho_{i.t}}{\sigma} \left[\rho_{i.t}^{-\sigma} \rho_{I.t}^{\sigma-\omega} C_t + (1 + \tau_t) \rho_{i.t}^{*-\sigma} \rho_{I.t}^{*\sigma-\omega} C_t^* \right] \tag{82}$$

Finally using the definition of relative price and mill pricing condition, it becomes ().

D Balanced trade condition

It must be with balanced trade,

$$IMP_t = XP_t \tag{83}$$

where

$$IMP_t \equiv \rho_{f.t} \phi_t N_t^* c_{f.t} \text{ and } XP_t \equiv Q_t \rho_{h.t}^* \phi_t N_t c_{h.t}^* \tag{84}$$

using $c_{f.t} = \left(\frac{p_{f.t}}{P_{F.t}} \right)^{-\sigma} \left(\frac{P_{F.t}}{P_t} \right)^{-\omega} C_t = \rho_{f.t}^{-\sigma} \rho_{F.t}^{\sigma-\omega} C_t$ and $c_{h.t}^* = \left(\frac{p_{h.t}^*}{P_{H.t}^*} \right)^{-\sigma} \left(\frac{P_{H.t}^*}{P_t^*} \right)^{-\omega} C_t^* = \rho_{h.t}^{*-\sigma} \rho_{H.t}^{*\sigma-\omega} C_t^*$, it becomes ().

E Empirically consistent variable

All the variables counted in terms of CES consumption goods are not empirically consistent because it includes variety effect. For any such variable X_t the empirically consistent variable is given by the following formura:

$$\frac{P_t X_t}{\widetilde{P}_t} \tag{85}$$

where $\widetilde{P}_t$ is the average price properly weighted. So the log-deviation of above variable is found as follows:

$$\widehat{\left(\frac{P_t X_t}{\widetilde{P}_t} \right)} = \widehat{X}_t + \widehat{P}_t - \widehat{\widetilde{P}}_t \tag{86}$$

also

$$\widehat{P}_t = \frac{1}{1+\phi}\widehat{P}_{H.t} + \frac{\phi}{1+\phi}\widehat{P}_{F.t} \tag{87}$$

and

$$\widehat{P}_{H.t} = \frac{1}{1-\sigma}\widehat{N}_t + \widehat{p}_{h.t}, \qquad \widehat{P}_{F.t} = \frac{1}{1-\sigma}\widehat{N^*}_t + \widehat{p}_{f.t} \tag{88}$$

So I can write:

$$\widehat{P}_t = \widehat{\widehat{P}}_t - \frac{1}{\sigma-1}\left(\frac{1}{1+\phi}\widehat{N}_t + \frac{\phi}{1+\phi}\widehat{N^*}_t\right) \tag{89}$$

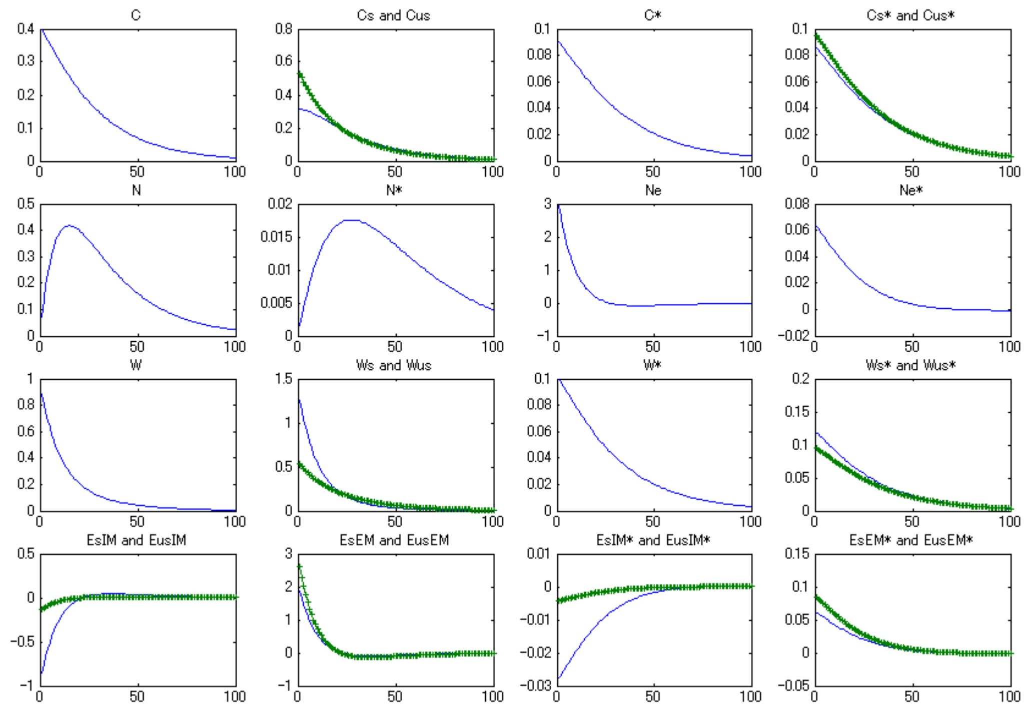
where $\widehat{\widehat{\widetilde{P}}}_t = \frac{1}{1+\phi}\widehat{p}_{h.t} + \frac{\phi}{1+\phi}\widehat{p}_{f.t}$ (Average price variation is the weighted variation of home and foreign produced goods at Home).

Finally

$$\left(\frac{\widehat{P_t X_t}}{\widetilde{P}_t}\right) = \widehat{X}_t - \frac{1}{\sigma-1}\left(\frac{1}{1+\phi}\widehat{N}_t + \frac{\phi}{1+\phi}\widehat{N^*}_t\right) \tag{90}$$

This says that to construct empirically consistent value the variety effect weighted by the elasticity about love for variety and by its steady state consumption shares should be eliminated.

F IRF of economy wide productivity growth



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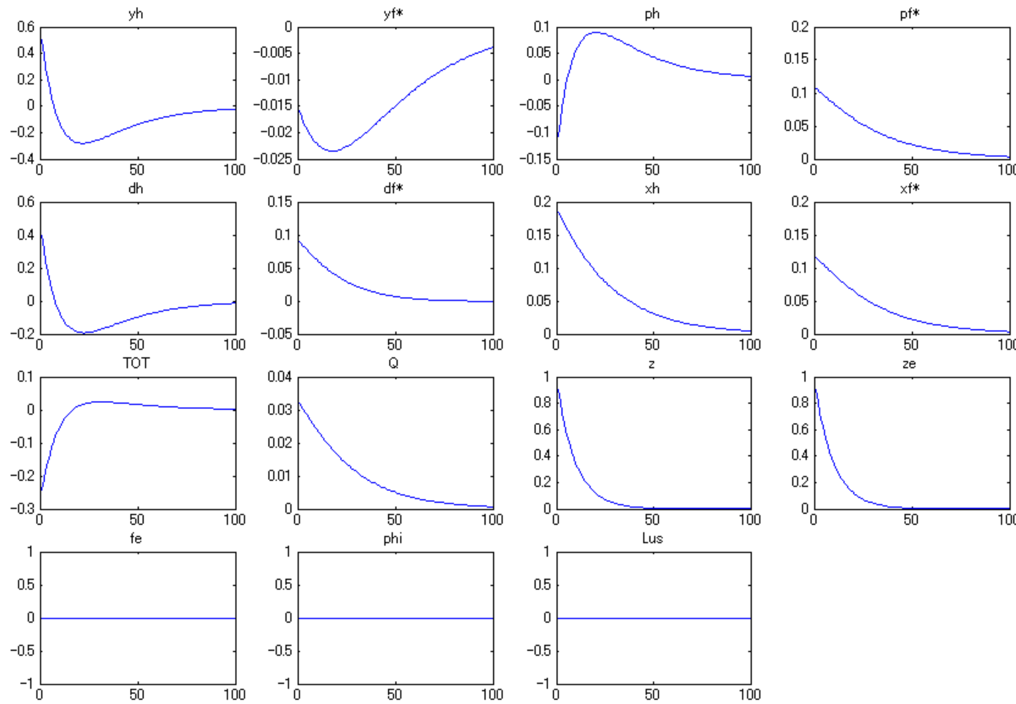

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