TI-games I:
An Exploration of Type Indeterminacy in Strategic Decision-making

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Abstract

In this paper we explore an extention of the Type Indeterminacy model of decision-making to strategic decision-making. A 2X2 game is investigated. We first show that in a one-shot simultaneaous move setting the TI-model is equivalent to a standard incomplete information model. We then let the game be preceded by a cheap-talk promise exchange game. We show in an example that in the TI-model the promise stage can have impact on next following behavior while the standard classical model predicts no impact whatsoever. The TI approach differs from other behavioral approaches in identifying the source of the effect of cheap-talk promises in the intrinsic indeterminacy of the type.

Keywords: quantum indeterminacy, type, strategic decision-making, game

1 Introduction

This paper belongs to a very recent and rapidly growing literature where formal tools of Quantum Mechanics are proposed to explain a variety of behavioral anomalies in social sciences and in psychology (see e.g. [1, 2, 4, 5, 7, 9, 10, 14, 18, 19]).

The use of quantum formalism in game theory was initiated by Eisert et al. [8] who propose that models of quantum games can be used to study how the extension of classical moves to quantum ones can

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affect the analysis of a game.[1] Another example is La Mura [17] who investigates correlated equilibria with quantum signals in classical games. In this paper we introduce some features of an extension of the Type Indeterminacy (TI) model of decision-making [16] from simple decisions to strategic decisions. We study, in two different settings, a 2x2 game with options, to cooperate and to defect and we refer to it as a Prisoner Dilemma, PD[2]. In the first setting, the players move simultaneously and the game is played once. In the second setting, the simultaneous move PD game is preceded by a promise exchange game. Our aim is to illustrate how the TI approach can provide an explanation as to why cheap talk promises matter.[3] There exists a substancial litteratur on cheap talk communication games (see for instance [15] for a survey). The approach in our paper does *not* belong to the litteratur on communication games. The cheap talk promise exchange stage is used to illustrate the possible impact of a move with no payoff implication. Various behavioral theories have also been proposed to explain the impact of cheap talk promises when theory predicts that there is none. They most often rely on very specific assumptions amounting to adding ad-hoc elements to the utility function (a moral cost for breaking promises) or emotional communication [11]. Our approach provides an explanation relying on a fundamental structure of the model i.e., the quantum indeterminacy of players' type. An advantage of our approach is that the type indeterminacy hypothesis also explains a variety of other so called behavioral anomalies such as framing effects, cognitive dissonance [16], the disjunction effect [3] or the inverse fallacy [10].

A main interest with TI-game is that the Type Indeterminacy hypothesis can modify quite significantly the way we think about games. Indeed, a major implication of the TI-hypothesis is to extend the field of strategic interactions. This is because actions impact not only on the payoffs but also on the profile of types, i.e., on who the players *are*. In a TI-model, players do not have a deterministic (exogenously given) type. The types change along the game together with the chosen actions (which are modelled as measurements of the type). We provide an example showing that an initially non-cooperative player can be (on average) turned into a rather cooperative one by confronting him with a tough player in a cheap talk promise exchange game.

Not surprisingly we find that there exists no distinction in terms of predictions between the standard Bayesian and Type Indeterminacy approaches in a simultaneous move context. The two models yield distinct predictions under the following conditions: i. at least one player makes more than one move; ii. those moves correspond to non-commuting Game Situations[4]; iii. a first-coming move separates between

[1] From a game-theoretical point of view the approach consists in changing the strategy spaces, and thus the interest of the results lies in the appeal of these changes.

[2] This is for convenience, as we shall see that the game is not perceived as a true PD by all possible types of a player.

[3] Cheap talk promises are promises that can be broken at no cost.

[4] A Game Situation is an operator that measures the type of a player, see below.

"potential" types that would otherwise interfere in the determination of the outcome of a next-coming interaction. We show that under those conditions a move with no informational content or payoff relevance still impacts on the outcome of the game.

2 A TI-model of strategic decision-making

Generals

In the TI-model a simple decision situation is represented by an *observable*[5] called a *DS*. A decision-maker is represented by his state or *type*. A type is a vector $|t_i\rangle$ in a Hilbert space. The measurement of the observable corresponds to the act of choosing. Its outcome, the chosen item, actualizes an *eigentype*[6] of the observable (or a *superposition*[7] of eigentypes if the measurement is coarse). It is information about the preferences (type) of the agent. For instance consider a model where the agent has preferences over sets of three items, i.e. he can rank any 3 items from the most preferred to the least preferred. Any choice experiment involving three items is associated with six eigentypes corresponding to the six possible rankings of the items. If the agent chooses a out of $\{a, b, c\}$ his type is projected onto some superposition of the rankings $[a > b > c]$ and $[a > c > b]$. The act of choosing is modelled as a measurement of the (preference) type of the agent and it impacts on the type i.e., it changes it (for a detailed exposition of the TI-model see [16]). How does this simple scheme change when we are dealing with strategic decision-making?

We denote by *GS* (for Game Situation) an observable that measures the type of an agent in a strategic situation, i.e. in a situation where the outcome of the choice, in terms of the agent's utility, depends on the choice of other agents as well. The interpretation of the outcome of the measurement is that the chosen action is a *best reply* against the opponents' expected action. This interpretation parallels the one in the simple decision context. There, we interpret the chosen item as the *preferred one* in accordance with an underlying assumption of (basic) rationality i.e., the agent maximizes his utility (i.e., chooses what he prefers). The notion of revealed preferences and a fortiori of revealed best-reply is problematic however. A main issue here is that a best reply is a response to an *expected* play. When the expected play involves subjective beliefs there may be a problem as to the measurability of the preferences. This is

[5] An observable is a linear operator.

[6] The eigentypes are the types associate with the eigenvalues of the observable i.e., the possible outcomes of the measurement of the *DS*.

[7] A superposition is a linear combination of the form $\sum \lambda_i |t_i\rangle$; $\sum \lambda_i^2 = 1$.

in particular so if subjective beliefs are quantum properties.[8] But in the context of maximal information games (which means that the initial types are pure types)we are dealing objective probabilities so it is warranted to talk about revealed best-reply.

TI-games are game with type indeterminate players, i.e., games characterized by uncertainty. In particular, players do not know the payoff of other players. The standard (classical) approach to incomplete information in games is due to Harsanyi. It amounts to transforming the game into a game of imperfect information where Nature moves at the beginning of the game and selects, for each player, one among a multiplicity of possible types (payoff functions). A player's own type is his private information. But in a TI-game the players may not even know their *own payoff*. This is true even in TI-game of *maximal information* where all players are represented by pure types.[9] In this paper we focus on TI-games of maximal information. Can the Harsanyi approach be extended to TI-games? We shall argue that the TI-paradigm gives even more content to Harsanyi's approach. What is a fictitious Nature's move in Harsanyi's setting becomes a real move (a measurement) with substantial implications. And the theoretical multiplicity of types of a player becomes a real multiplicity of "selves".

Types and eigentypes

We use the term *type* to refer to a *quantum pure state* of a player. A pure type is maximal information about the player i.e., about his payoff function.[10] But because of (intrinsic) indeterminacy, the type is *not* complete information about the payoff function in all games simultaneously not even to the player himself.

In a TI-game we also speak about the *eigentypes* of any specific game M, these are *complete information* about the payoff functions *in a specific static game* M. Any eigentype of a player knows his own M-game payoff function but he may not know that of the other players. The eigentypes of a TI-game M are identified with their payoff function in that game.

So we see that while the Harsanyi approach only uses a single concept, i.e., that of type and it is identified both with the payoff function and with the player. In any specific TI-game M we must distinguish between the type which is identified with the player and the eigentype which is identified with the payoff function in game M. A helpful analogy is with multiple-selves models (see e.g., [20] and [12]). In multiple-selves models, we are most often dealing with two "levels of identity". These two levels are identified with short-run impulsive selves on the one side and a long-run "rational self" on the other side. In our context we have two levels as well: the level of the player (the type) and the level of the selves

[8] If subjective beliefs and preferences are quantum properties that do not commute then they cannot be measured simultaneously.

[9] For a discussion about pure and mixed types (states) see Section 3.2 in [5].

[10] The payoff of a player is a function of all the players' actions.

(the eigentypes) which are to be viewed as potential incarnations of the player *in a specific game.*[11]

A central assumption that we make is that the reasoning leading to the determination of the best-reply is performed at the level of the eigentypes of the game. This key assumption deserves some discussion. What we have in mind is very much in line with quantum computing. What is happening in the head of a player is some form of parallel reasoning, all the active (with non-zero coefficient of superposition) eigentypes perform their own strategic thinking. Another way to put it is that we assume that the player is able to reason from different perspectives. Note that this is not as demanding as it may at first appear. Indeed we are used in standard game theory to the assumption that players are able to put themselves "in the skin" of other players to think out how those will play in order to be able to best-respond to that.

As in the basic TI-model, the outcome of the act of choosing, here a *move,* is information about the (actualized) type of the player. The act of choosing changes the type from some initial type to the actualized one. We call *GO* or Game Operator, a complete collection of (commuting) *GS* (each defined for a specific opponent). The outcome of a *GO* is an eigentype of the game, it gives information about how a player plays against any possible opponent in a specific game. Each player is an independent system i.e., there is no entanglement between players.[12]

We next investigate an example of a maximal information (see below for precise definition) two-person game. The objective is to introduce some basic features of TI-games in a simple context and to illustrate an equivalence and some distinctions between the Bayes-Harsanyi approach and the TI-approach.

A single interaction

Consider a 2X2 symmetric game, *M*, and for concreteness we call the two possible actions cooperate (C) and defect (D) (as in a Prisoner's Dilemma game but as we shall see below for certain types, it is a coordination game) and we define the preference types of game M also called the M-eigentypes as follows:

θ_1 : prefers to cooperate whatever he expects the opponent to do;

θ_2 : prefers to cooperate if he expects the opponent to cooperate with probability $p > q$ (for some $q \leq 1$) otherwise he prefers to defect;

θ_3 : prefers to defect whatever he expects the opponent to do.

An example of these types is in the payoff matrices below where we depict the row player's payoff:

[11] A superposition is a linear combination of the form $\sum \lambda_i |t_i\rangle$; $\sum \lambda_i^2 = 1$.

[12] In future research we intend to investigate the possibility of entenglement between players.

$$\theta_1 : \begin{pmatrix} & C & D \\ C & 10 & 5 \\ D & 0 & 0 \end{pmatrix}, \quad \theta_2 : \begin{pmatrix} & C & D \\ C & 10 & 0 \\ D & 6 & 8 \end{pmatrix}, \quad \theta_2 : \begin{pmatrix} & C & D \\ C & 0 & 0 \\ D & 10 & 5 \end{pmatrix}$$

Note that these types are complete characterization in the sense that they give the player's payoff for any action of the opponent.

We shall now proceed to investigate this simultaneous move TI-game. We note immediately that θ_1 and θ_3 are non-strategic while θ_2 is, i.e., his best-reply will depend on what he expects the opponent to do. The initial types are generally not eigentypes of the game under consideration. Let player 1 be described by the superposition:

$$|t_1\rangle = \lambda_1 |\theta_1\rangle + \lambda_2 |\theta_2\rangle + \lambda_3 |\theta_3\rangle, \sum \lambda_i^2 = 1.^{13} \tag{1}$$

We shall first be interested in the optimal play of player 1 when he interacts with a player 2 of different eigentypes. Suppose he interacts with a player 2 of eigentype θ_1. Using the definitions of the eigentypes θ_i above and (1), we know by Born's rule[14] that with probability $\lambda_1^2 + \lambda_2^2$ player 1 plays C (because θ_2's best-reply to θ_1 is C) and he collapses on the (superposed) type $|t_1'\rangle = \frac{\lambda_1}{\sqrt{\lambda_1^2+\lambda_2^2}} |\theta_1\rangle + \frac{\lambda_2}{\sqrt{\lambda_1^2+\lambda_2^2}} |\theta_2\rangle$. With probability λ_3^2 he plays D and collapses on the eigentype θ_3. If instead player 1 interacts with a player 2 of type θ_3 then with probability λ_1^2 he plays C and collapses on the eigentype θ_1 and since $\theta_2's$ best-reply to θ_3 is *D,* with probability $\lambda_2^2 + \lambda_3^2$ he plays D and collapses on type $|t"_1\rangle = \frac{\lambda_2}{\sqrt{\lambda_3^2+\lambda_2^2}} |\theta_2\rangle + \frac{\lambda_3}{\sqrt{\lambda_3^2+\lambda_2^2}} |\theta_3\rangle$.

We note that the probabilities for player 1's moves depends on the opponent's type and corresponding expected play - as usual. More interesting is that, as a consequence, the *resulting type* of player 1 also depends on the type of the opponent. This is because in a TI-model the act of choice is a measurement of the own type and the act of choice modofies it. We interpret the resulting type as the initial type modified by the measurement. In a one-shot context, this is just an interpretation since formally it cannot be distinguished from a classical informational interpretation where the resulting type captures our revised beliefs about player 1.

We now consider a case when player 2's type is indeterminate as well, it is given by

$$|t_2\rangle = \gamma_1 |\theta_1\rangle + \gamma_2 |\theta_2\rangle + \gamma_3 |\theta_3\rangle, . \sum \gamma_i^2 = 1. \tag{2}$$

From the point of view of the eigentypes of a player (the θ_i), the situation can be analyzed as a standard situation of incomplete information. We consider two examples:

[14] The calculus of probability in Quantum Mechanics is done according to Born's rule which defines the probability for the different eigentypes is given by the square of the coefficients of superposition.

Example 1 *Let* $\lambda_1^2 \geq q$*, implying that the eigentype type* θ_2 *of player 2 cooperates and let* $\gamma_1^2 + \gamma_2^2 \geq q$ *so the eigentype* θ_2 *of player 1 cooperates as well.*

Example 2 *Let* $\lambda_1^2 \geq q$ *so the eigentype* θ_2 *of player 2 cooperates but now let* $\gamma_1^2 + \gamma_2^2 < q$ *so here the eigentype* θ_2 *of player 1 prefers to defect.*

In Example 1 the types θ_1 and θ_2 of both players pool to cooperate. So in particular player 1's resulting type is a superposition of $|\theta_1\rangle$ and $|\theta_2\rangle$ with probability $\left(\lambda_1^2 + \lambda_2^2\right)$ and it is the eigentype $|\theta_3\rangle$ with probability λ_3^2. In Example 2, player 1's eigentypes θ_2 and θ_3 pool to defect so player 1's resulting type is a superposition of $|\theta_2\rangle$ and $|\theta_3\rangle$ with probability $\lambda_2^2 + \lambda_3^2$ and $|\theta_1\rangle$ with probability λ_1^2. So we see again how the resulting type of player 1 varies with the initial (here superposed) type of his opponent.

When both players play a best reply to each other we have an equilibrium more precisely:

Definition

A static TI-equilibrium of a game M is

i. A profile of strategies such that each one of the M−eigentypes of each player maximizes his expected utility given the (superposed) type of his opponent and the strategies played by the opponent's eigentypes:

$$s_1^*\left(\theta_{iM}^1\right) = \arg\max_{s_1'.\in S} \sum_{\theta_{iM}^2} p\left(\theta_{iM}^2 \middle| \theta_2\right) u_{iM}\left(s_1', s_2\left(\theta_{iM}^2\right), \left(\theta_{iM}^1, \theta_{iM}^2\right)\right) \text{ for all } \theta_{iM}^1$$

and similarly for player 2.

ii. A corresponding profile of resulting types, one for each player and each action.

$$\left|\theta_1^{t=1}\right| a_1\rangle = \sum_{\theta_{iM};s_1^*\left(\theta_{iM}^1\right)=a_1} \frac{\lambda_{iM}}{\sqrt{\sum \lambda_{jM}^2\left(s_1^*\left(\theta_{jM}^1\right)=a_1\right)}} \left|\theta_{iM}; s_1^*\left(\theta_{iM}^1\right)=a_1\right\rangle$$

similarly for $\left|\theta_1^{t=1}\right| a_2\rangle$, $\left|\theta_2^{t=1}\right| a_1\rangle$ *and* $\left|\theta_2^{t=1}\right| a_2\rangle$.

For concreteness we shall now solve for the TI-equilibrium of this game in a numerical example. Suppose the initial types are

$$|t_1\rangle = \sqrt{.7}\,|\theta_1\rangle + \sqrt{.2}\,|\theta_2\rangle + \sqrt{.1}\,|\theta_3\rangle, \tag{3}$$

$$|t_2\rangle = \sqrt{.2}\,|\theta_1\rangle + \sqrt{.6}\,|\theta_2\rangle + \sqrt{.2}\,|\theta_3\rangle. \tag{4}$$

Given the payoff matrices above, the threshold probability q that rationalizes the play of C for the eigentype θ_2 is $q = .666$. For the ease of presentation, we let $q = .7$. We know that the θ_2 of player 2 cooperates since $\lambda_1^2 = .7 \geq q$ and so does the θ_2 of player 1 since $\gamma_1^2 + \gamma_2^2 = .8 > q$.

In the TI-equilibrium of this game player 1 plays C with probability .9 and collapses on $|t_1'\rangle = \frac{\sqrt{.7}}{\sqrt{.7+.2}}|\theta_1\rangle + \frac{\sqrt{.2}}{\sqrt{.7+..2}}|\theta_2\rangle$ and with probability .1 player 1 plays D and collapses on $|\theta_3\rangle$. Player 2 plays C with probability .8 and collapses on $|t_2'\rangle = \frac{\sqrt{.4}}{\sqrt{.4+.4}}|\theta_1\rangle + \frac{\sqrt{.4}}{\sqrt{.4+.4}}|\theta_2\rangle$ and with probability .2, he plays D and collapses on $|\theta_3\rangle$.

We note that the *mixture actually played* by player 1 (.9C, .1D) is *not* the best reply of any of his eigentypes. The same holds for player 2. The eigentypes are the "real players" and they play pure strategies.

We end this section with a comparison of the TI-game approach with the standard incomplete information treatment of this game where the square of the coefficients of superposition in (1) and (2) are interpreted as players' beliefs about each other. The sole substantial distinction is that in the Bayes-Harsanyi setting the players privately learn their own type *before* playing while in the TI-model they learn it in the process of playing. A player is thus in the same informational situation as his opponent with respect to his own play. However under our assumption that all the reasoning is done by the eigentypes, the classical approach and the TI-approach are indistinguishable. They yield the same equilibrium outcome. The distinctions are merely interpretational.

Statement 1

The TI-model of a simultaneous one-move game is equivalent to a Bayes-Harsanyi model.

A formal proof of Statement 1 can be found in our companion paper "TI-game 2".

This central equivalence result should be seen as an achievement which provides support for the hypotheses that we make to extend the basic TI-model to strategic decision-making. Indeed, we do want the non-classical model to deliver the same outcome in a simultaneous one-move context.[15] We next move to a setting where one of the players is involved in a sequence of moves. This is the simplest setting in which to introduce the novelty brought about by the type indeterminacy hypothesis.

A multi-stage TI-game

In this section we introduce a new interaction involving player 1 and a third player, a promise game. We assume that the GS representing the promise game do not commute with the GS representing the game M (described in the previous section).[16] Player 1 and 3 play a promise game where they choose between either making a non-binding promise to cooperate with each other in game M or withholding from making

[15] Indeed we know that quantum indeterminacy cannot be distinguished from incomplete information in the case of a single measurement. A simultaneous one-move game corresponds to two single measurements performed on two non-entangled systems.

[16] To each game we associate a collection of GS each of which measures the best reply a possible type of the opponent.

such a promise. Our objective is to show that playing a promise exchange game - with a third player - can increase the probability for cooperation (decrease the probability for defection) between the player 1 and 2 in a next following game *M.* Such an impact of cheap-talk promises is related to experimental evidence reported in Frank (1988)

We shall compare two situations called respectively protocol I and II. In protocol 1 player 1 and 2 play game *M.* In protocol II we add a third player, 3, and we have the following sequence of events:

step 1 Player player 1 and 3 play a promises exchange game N, described below.

step 2 Player 1 and 2 play M.

step 3 Player 1 and 3 play M.[17]

The promise exchange game

At *step 1*, player 1 and 3 have to simultaneously select one of the two announcements: "I promise to play cooperate", denoted, P, and "I do not promise to play cooperate" denoted $no - P$. The promises are cheap-talk i.e., breaking them in the next following games has no implications for the payoffs i.e., at step 2 or step 3.

There exists three eigentypes in the promise exchange game:

τ_1 : prefers to never make cheap-talk promises - let him be called the "honest type";

τ_2 : prefers to make a promise to cooperate if he believes the opponent cooperates with probability $p \geq q$ (in which case he cooperates whenever he is of type θ_2 or θ_1 or any superposition of the 2). Otherwise he makes no promises - let him be called the "sincere type";

τ_3 : prefers to promise that he will cooperate whatever he intends to do - he can be viewed as the "opportunistic type".

Information assumptions

Before moving further to the analysis of the behavior in protocol II we have to make clear the information that the players have at the different stages of the game. Specifically we assume that:

i. All players know the statistical correlations (conditional probabilities) between the eigentypes of the two (non-commuting) games.[18]

ii. At *step 2,* player 2 knows that player 1 has interacted with player 3 but he does not know the outcome of the interaction.

[17] The reason why we have the interaction at *step 3* is essentially to motivate the promise exchange game. Our main interest will focus on the interaction at *step 2.*

[18] So in particular they can compute the correlation between the *plays* in the different games.

We note that ii. implies that we are not dealing with an issue of strategic commmunication between player 1 and 2. No message is being received by player 2.

The classical model

We first establish that in the classical setting we have the same outcome in protocol I and at *step 2* of protocol II. We already know from Statement 1 that the analysis of a TI model of game M is fully equivalent with the classical Bayes-Harsanyi analysis of the corresponding incomplete information game.

We investigate in turn how the interaction between player 1 and 3 at *step 1* affects the incentives and/or the information of player 1 and 2 at *step 2*. Let us first consider the case of player 1. In a classical setting, player 1 knows his own type, so he learns nothing from the promise exchange stage. Moreover the announcement he makes is not payoff relevant to his interaction with player 2. So the promise game has no direct implication for his play with player 2. As to player 2, the question is whether he has reason to update his beliefs about player 1. Initially he knows $|t_1\rangle$ from which he derives his beliefs about player 1's equilibrium play in game M. By our informational assumption (i) he also knows the statistical correlations between the eigentypes of the two games from which he can derive the expected play conditional on the choice at the promise stage. He can write the probability of e.g., the play of D using the conditional probability formula:

$$p(D) = p(P)\, p(D|\, P) + p(no - P)\, p(D|\, no - P)\,. \tag{5}$$

He knows that player 1 interacted with 3 but he does not know the outcome of the interaction. Therefore he has no new element from which to update his information about player 1. We conclude that the introduction of the interaction with player 3 at *step 1* leaves the payoffs and the information in the game M unchanged. Hence, expected behavior at *step 2* of protocol II is the same as in protocol I.

The TI-model

Recall that the GS representing the promise game do not commute with the GS representing the game M. We now write eq. (1) and (2) in terms of the eigentypes of game *N,* i.e., of the promise stage eigentypes:

$$|t_1\rangle = \lambda'_1 |\tau_1\rangle + \lambda'_2 |\tau_2\rangle + \lambda'_3 |\tau_3\rangle \text{ and } |t_3\rangle = \gamma'_1 |\tau_1\rangle + \gamma'_2 |\tau_2\rangle + \gamma'_3 |\tau_3\rangle\,.$$

Each one of the $N-$eigentype can in turn be expressed in terms of the eigentypes of game M :

$$\begin{aligned} |\tau_1\rangle &= \delta_{11}|\theta_1\rangle + \delta_{12}|\theta_2\rangle + \delta_{13}|\theta_3\rangle \\ |\tau_2\rangle &= \delta_{21}|\theta_1\rangle + \delta_{22}|\theta_2\rangle + \delta_{23}|\theta_3\rangle \\ |\tau_3\rangle &= \delta_{31}|\theta_1\rangle + \delta_{32}|\theta_2\rangle + \delta_{33}|\theta_3\rangle \end{aligned} \tag{6}$$

where the δ_{ij} are the elements of the basis transformation matrix (see the last subsection below). Assume that player 3 is (initially) of type θ_3 with probability close to 1, we say he is a "tough" type. We shall investigate the choice of between P and *no-P* of player 1 i.e., the best response of the eigentypes τ_i of player 1.

By definition of the τ_i type, we have that τ_1 always plays *no-P* and τ_3 always play *P.* Now by assumption, player 3 is of type θ_3 who never cooperates. Therefore, by the definition of τ_2, player 1 of type τ_2 chooses not to promise to cooperate, he plays *no*$-P$.

This means that at *step 1* with probability $\lambda_1'^2 + \lambda_2'^2$ player 1 plays *no*$-P$ and collapses on $|\widehat{t_1}\rangle = \frac{\lambda_1'}{\sqrt{(\lambda_1^{2\prime}+\lambda_2^{2\prime})}}|\tau_1\rangle + \frac{\lambda_2'}{\sqrt{(\lambda_1^{2\prime}+\lambda_2^{2\prime})}}|\tau_2\rangle$. With probability $\lambda_3'^2$ he collapses on $|\tau_3\rangle$.

We shall next compare player 1's propensity to defect in protocol I with that propensity in protocol II. For simplicity we shall assume the following correlations: $\delta_{13} = \delta_{31} = 0$, meaning that the honest type τ_1, never systematically defects and that the opportunistic guy τ_3 never systematically cooperate.

Player 1's propensity to defect in protocol I

We shall consider the same numerical example as before i.e., given by (3) and (4) so in particular we know that θ_2 of player 1 cooperates so $p(D\,||t_1\rangle) = \lambda_3^2$. But our objective in this section is to account for the indeterminacy due to the fact that in protocol I the promise game is *not* played. We have

$$|t_1\rangle = \lambda_1'|\tau_1\rangle + \lambda_2'|\tau_2\rangle + \lambda_3'|\tau_3\rangle$$

and using the formulas in (6) we substitute for the $|\tau_i\rangle$

$$\begin{aligned} |t_1\rangle &= \lambda_1'(\delta_{11}|\theta_1\rangle + \delta_{12}|\theta_2\rangle + \delta_{13}|\theta_3\rangle) + \lambda_2'(\delta_{21}|\theta_1\rangle + \delta_{22}|\theta_2\rangle + \delta_{23}|\theta_3\rangle) \\ &\quad + \lambda_3'(\delta_{31}|\theta_1\rangle + \delta_{32}|\theta_2\rangle + \delta_{33}|\theta_3\rangle). \end{aligned}$$

Collecting the terms we obtain

$$\begin{aligned} |t_1\rangle &= \left(\lambda_1'\delta_{11} + \lambda_2'\delta_{21} + \lambda_3'\delta_{31}\right)|\theta_1\rangle + \left(\lambda_1'\delta_{12} + \lambda_2'\delta_{22} + \lambda_3'\delta_{32}\right)|\theta_2\rangle + \\ &\quad \left(\lambda_{13}'\delta + \lambda_2'\delta_{23} + \lambda_3'\delta_{33}\right)|\theta_3\rangle. \end{aligned}$$

We know from the preceding section that both $|\theta_1\rangle$ and $|\theta_2\rangle$ choose to cooperate so

$$p(D\,||t_1\rangle) = p(|\theta_3\rangle\,||t_1\rangle).$$

Using $\delta_{13} = 0$, we obtain the probability for player 1's defection in protocol I*:*

$$p\left(D\left||t_1\right\rangle\right)_M = \left(\lambda_2'\delta_{23} + \lambda_3'\delta_{33}\right)^2 = \lambda_2^{2\prime}\delta_{23}^2 + \lambda_3^{2\prime}\delta_{33}^2 + 2\lambda_2'\delta_{23}\lambda_3'\delta_{33}. \tag{7}$$

Player 1's propensity to defect in protocol II

When the promise game is being played, i.e. the measurement N is performed, we can (as in the classical setting) use the conditional probability formula to compute the probability for the play of D

$$p\left(D\left||t_1\right\rangle\right)_{MN} = p\left(P\right)p\left(D|\,P\right) + p\left(no - P\right)p\left(D|\,no - P\right). \tag{8}$$

Let us consider the first term: $p\left(P\right)p\left(D|\,P\right)$. We know that $p\left(P\right) = p\left(\left|\tau_3\right\rangle\right) = \lambda_3^{2\prime}$. We are now interested in $p\left(D|\,P\right)$ or $p\left(D\left|\tau_3\right\rangle\right)$. $\left|\tau_3\right\rangle$ writes as a superposition of the θ_i with θ_1 who never defects, θ_3 who always defect while θ_2's propensity to defect depends on what he expects player 2 to do. We cannot take for granted that player 2 will play in protocol II as he plays in protocol I. Instead we assume for now that eigentype θ_2 of player 2 chooses to cooperate (as in protocol I) because he expects player 1's propensity to cooperate to be no less than in protocol I. We below characterize the case when this expectation is correct. Now if θ_2 of player 2 chooses to cooperate so does θ_2 of player 1 and $p\left(D\left|\tau_3\right\rangle\right) = \delta_{33}^2$ so

$$p\left(P\right)p\left(D|\,P\right) = \lambda_3^{2\prime}\delta_{33}^2$$

We next consider the second term of (8). The probability for $p\left(no - P\right)$ is $\left(\lambda_1^{2\prime} + \lambda_2^{2\prime}\right)$ and the type of player 1 changes, he collapses on $\left|\widehat{t}_1\right\rangle = \frac{\lambda_1'}{\sqrt{\left(\lambda_1^{2\prime}+\lambda_2^{2\prime}\right)}}\left|\tau_1\right\rangle + \frac{\lambda_2'}{\sqrt{\left(\lambda_1^{2\prime}+\lambda_2^{2\prime}\right)}}\left|\tau_2\right\rangle$. Since we consider a case when θ_2 of player 1 cooperates, the probability for defection of type $\left|\widehat{t}_1\right\rangle$ is $\left(\frac{\lambda_1'}{\sqrt{\left(\lambda_1^{2\prime}+\lambda_2^{2\prime}\right)}}\right)^2\delta_{13}^2 + \left(\frac{\lambda_2'}{\sqrt{\left(\lambda_1^{2\prime}+\lambda_2^{2\prime}\right)}}\right)^2\delta_{23}^2$. Recalling that $\delta_{13} = 0$, we obtain that $p\left(no - P\right)p\left(D|\,no - P\right)$ is equal to

$$\left(\lambda_1^{2\prime} + \lambda_2^{2\prime}\right)\left(\frac{\lambda_2'}{\sqrt{\left(\lambda_1^{2\prime}+\lambda_2^{2\prime}\right)}}\right)^2\delta_{23}^2 = \lambda_2^{2\prime}\delta_{23}^2$$

which gives

$$p\left(D\left||t_1\right\rangle\right)_{MN} = \lambda_2^{2\prime}\delta_{23}^2 + \lambda_3^{2\prime}\delta_{33}^2 \tag{9}$$

Comparing formulas in (7) and (9) :

$$p\left(D\left||t_1\right\rangle\right)_{MN} - p\left(D\left||t_1\right\rangle\right)_M = -2\lambda_2'\delta_{23}\lambda_3'\delta_{33} \tag{10}$$

which can be negative or positive because the interference terms only involves amplitudes of probability i.e., the square roots of probabilities. The probability to play defect decreases (and thus the probability for cooperation increases) when player 1 plays a promise stage whenever $2\lambda_2'\delta_{23}\lambda_3'\delta_{33} < 0$. In that case the expectations of player 2 are correct and we have that the θ_2 type of both players cooperate which we assumed in our calculation above.[19]

Result 1: *When player 1 meets a tough player 3 at step 1, the probability for playing defect in the next following M game is not the same as in the M game alone,* $p(D\,||t_1\rangle)_M - p(D\,||t_1\rangle)_{MN} \neq 0$.

It is interesting to note that $p(D\,||t_1\rangle)_{MN}$ is the same as in the classical case, it can be obtained from the same conditional probability formula.

In order to better understand our *Result 1,* we now consider a case when player 1 meets with a "soft" player 3, i.e., a θ_1 type, at *step 1.*

The soft player 3 case

In this section we show that if the promise stage is an interaction with a soft player 3 there is no effect of the promise stage on player 1's propensity to defect and thus no effect on the interaction at *step 2.*.

Assume that player 3 is (initially) of type θ_1 with probability close to 1. What is the best reply of the N-eigentypes of player 1, i.e., how do they choose between P and *no-P*? By definition we have that τ_1 always plays *no-P* and τ_3 always play *P.* Now by the assumption we just made player 3 is of type θ_1 who always cooperates so player 1 of type τ_2 chooses to promise to cooperate, he plays P.

This means that at *t=1* with probability $\lambda_1'^2$ he collapses on $|\tau_1\rangle$ and with probability $\lambda_2'^2 + \lambda_3'^2$ player 1 plays P and collapses on $\left|\widehat{t}_1\right\rangle = \frac{\lambda_2'}{\sqrt{\lambda_2^{2\prime}+\lambda_3^{2\prime}}}\,|\tau_2\rangle + \frac{\lambda_3'}{\sqrt{\lambda_2^{2\prime}+\lambda_3^{2\prime}}}\,|\tau_3\rangle$. We shall compute the probability to defect of that type.[20] We first the type vector $\left|\widehat{t}_1\right\rangle$ in terms of the M-eigentypes,

$$\begin{aligned}\left|\widehat{t}_1\right\rangle &= \left(\frac{\lambda_2'}{\sqrt{\lambda_2'^2+\lambda_3'^2}}\right)(\delta_{21}\,|\theta_1\rangle + \delta_{22}\,|\theta_2\rangle + \delta_{23}\,|\theta_3\rangle) \\ &\quad + \left(\frac{\lambda_3'}{\sqrt{\lambda_2'^2+\lambda\prime_3^2}}\right)(\delta_{31}\,|\theta_1\rangle + \delta_{32}\,|\theta_2\rangle + \delta_{33}\,|\theta_3\rangle)\end{aligned}$$

As we investigate player 1's M-eigentypes' best reply, we again have to make an assumption about player 2's expectation. And the assumption we make is that he believes that player 1's propensity to defect is unchanged, so as in protocol I the θ_2 of both players cooperate and only θ_3 defects. We have

$$p\left(D\,\left|\left|\widehat{t}_1\right\rangle\right.\right)_{MN} = \left[\frac{\lambda_2'}{\sqrt{\lambda_2'^2+\lambda_3'^2}}\delta_{23} + \frac{\lambda_3'}{\sqrt{\lambda_2'^2+\lambda_3'^2}}\delta_{33}\right]^2$$

[19] For the case the best reply of the θ_2 types changes with the performence of the promise game, the comparison between the two protocols is less straightforward.

[20] Recall that τ_1 never defects.

$$p\left(D\left|\left|\widehat{t}_1\right.\right\rangle\right)_{MN} = \frac{1}{\lambda_2^{2\prime} + \lambda_3^{2\prime}}\left[\lambda\prime_2^2\delta_{23}^2 + \lambda\prime_3^2\delta_{33}^2 + 2\lambda_2^\prime\lambda_3^\prime\delta_{23}\delta_{33}\right]$$

The probability for defection is thus

$$p\left(D\left|\left|t_1\right.\right\rangle\right)_{MN} = P\left(\tau_1\right)p\left(D\left|\left|\tau_1\right.\right\rangle\right) + P\left(\widehat{t}_1\right)p\left(D\left|\left|\widehat{t}_1\right.\right\rangle\right) =$$

$$0 + \left(\lambda_2^{2\prime} + \lambda_3^{2\prime}\right)\frac{1}{\lambda_2^{2\prime} + \lambda_3^{2\prime}}\left[\lambda_2^{2\prime}\delta_{23}^2 + \lambda_3^{2\prime}\delta_{33}^2 + 2\lambda_2^\prime\lambda_3^\prime\delta_{23}\delta_{33}\right] = \lambda_2^{2\prime}\delta_{23}^2 + \lambda_3^{2\prime}\delta_{33}^2 + 2\lambda_2^\prime\lambda_3^\prime\delta_{23}\delta_{33}.$$

Comparing with eq. (7) of protocol I we see that here

$$p\left(D\left|\left|t_1\right.\right\rangle\right)_M = p\left(D\left|\left|t_1\right.\right\rangle\right)_{MN}$$

There is NO effect of the promise stage. This is because the interference effects are still present. We note also that player 2 was correct in his expectation about player 1's propensity to defect.

Result 2

If player 1's move at step 1 does not separate between the N-eigentypes that would otherwise interfere in the determination of his play of D at step 2 then $p\left(D\left|\left|t_1\right.\right\rangle\right)_M = p\left(D\left|\left|t_1\right.\right\rangle\right)_{MN}$.

Let us try to provide an intuition for our two results. In the absence of a promise stage (protocol I) both the sincere and opportunistic type coexist in the mind of player 1. Both these two types have a positive propensity to defect. When they coexist they interfere positively(negatively) to reinforce(weaken) player 1's propensity to defect. When playing the promise exchange game the two types may either separate or not. They separate in the case of a tough player 3. Player 1 collapses either on a superposition of the honest and sincere type (and chooses *no-P*) or on the opportunistic type (and chooses P). Since the sincere and the opportunistic types are separated (by the first measurement, game N) there is no more interference. In the case of a soft player 3 case, the play of the promise game does not separate the sincere from the opportunistic guy, they both prefer P. As a consequence the two Neigentypes interfere in the determination of outcome of the next following M game as they do in protocol I.

In this example we demonstrated that in a TI-model of strategic interaction, a promise stage does make a difference for players' behavior in the next following performance of game *M.* The promise stage makes a difference because it may destroy interference effects that are present in protocol I.

Quite remarkably the distinction between the predictions of the classical and the TI-game only appears in the *absence* of the play of a promise stage (with a tough player). Indeed the probability formula that applies in the TI-model for the case the agent undergoes the promise stage (9) is the same as the conditional probability formula that applies in the standard classical setting.

A few words about the structure of the example

In the example above we are dealing with a type space Θ which has six elements. These elements go in two families corresponding to the two games i.e., M: $\{\theta_1, \theta_2, \theta_3\}$ and the promise game, N: $\{\tau_1, \tau_2, \tau_3\}$. So for instance the strategic type θ_1 is defined as a mapping from the simplex of the opponent possible types into actions $\theta_1 : \Delta(\{\theta_1, \theta_2, \theta_3\}) \to A$ where A is the set of actions, $A = \{C, D\}$. It is interpreted as the best reply of player 1 against player 2 in the M game. Similarly τ_1 is defined by a mapping $\tau_1 : \Delta(\{\tau_1, \tau_2, \tau_3\}) \to A'$, where $A' = \{P, no - P\}$ is interpreted as the best reply of player 1 to player 3 in the promise exchange game. The corresponding GS are indexed by the type of the opponent.

Our type space is a three dimensional Hilbert space where, $\{|\theta_1\rangle, |\theta_2\rangle, |\theta_3\rangle\}$ and $\{|\tau_1\rangle, |\tau_2\rangle, |\tau_3\rangle\}$ are two alternative basis. So in contrast with a standard Harsanyi type space where all types are alternatives (orthogonal) to each other, here $|\theta_1\rangle \perp |\theta_2\rangle$ and $|\tau_1\rangle \perp |\tau_2\rangle$ but $|\theta_i\rangle$ is not orthogonal to $|\tau_i\rangle$, $i = 1, 2, 3$. The two games are incompatible measurements of the type of a player. A basis transformation matrix links the eigentypes of the two GO M and N :

$$\begin{pmatrix} \langle\tau_1|\,\theta_1\rangle = \delta_{11} & \langle\tau_1|\,\theta_2\rangle = \delta_{12} & \langle\tau_1|\,\theta_3\rangle = \delta_{13} \\ \langle\tau_2|\,\theta_1\rangle = \delta_{21} & \langle\tau_2|\,\theta_2\rangle = \delta_{22} & \langle\tau_2|\,\theta_3\rangle = \delta_{23} \\ \langle\tau_3|\,\theta_1\rangle = \delta_{31} & \langle\tau_3|\,\theta_2\rangle = \delta_{32} & \langle\tau_3|\,\theta_3\rangle = \delta_{33} \end{pmatrix}.$$

Since there are three eigentypes and only two actions, two of the eigentypes must pool in their choice. The corresponding GS are coarse measurements of the type.

3 Concluding remarks

In this paper we have explored an extension of the Type Indeterminacy model of decision-making to strategic decision-making in a maximal information context. We did that by means of an example of a 2X2 game that we investigate in two different settings. In the first setting the game is played directly. In the second setting the game is preceded by a promise exchange game. We first find that in a one-shot setting the TI-model is equivalent to the standard Bayes-Harsanyi approach to games of incomplete information. This is no longer true in the sequential move setting. We give an example of circumstances under which the predictions of the two models are not the same. We show that the TI-model can provide an explanation for why a cheap-talk promises matter. The promise game can separates between types and destroys interference effects that otherwise contribute to the determination of the propensity to defect in the next following game.

Last we want to emphasis the very explorative character of this paper. A companion paper TI-game 2 develops the basic concepts and solutions of TI-games. We believe that this avenue of research has a rich potential to explain a variety of puzzles in (sequential) interactive situations and to give new impulses to game theory.

References

[1] Busemeyer J.R., Wang, Z. and Townsend J.T. (2006) "Quantum Dynamics of Human Decision-Making" *Journal of Mathematical Psychology* 50, 220-241.

[2] Busemeyer J. R. (2007) "Quantum Information Processing Explanation for Interaction between Inferences and Decisions." *Proceedings of the Quantum Interaction Symposium AAAI Press.*

[3] Busemeyer, J.R. Matthew, M., and Wang Z. (2006) "An information processing explanation of the Disjuction effect". In Sun and Miyake (Eds), 131-135.

[4] Busemeyer JR, Santuy E. ant A. Lambert-Mogiliansky (2008) "Distinguishing quantum and markov models of human decision making" in Proceedings of the the second interaction symposium (QI 2008), 68-75.

[5] Danilov V. I. and A. Lambert-Mogiliansky (2008) "Measurable Systems and Behavioral Sciences". *Mathematical Social Sciences 55,* 315-340.

[6] Danilov V. I. and A. Lambert-Mogiliansky. (2008) "Decision-making under non-classical uncertainty" in Proceedings of the the second interaction symposium (QI 2008), 83-87.

[7] Deutsch D. (1999) "Quantum Theory of Propability and Decisions". *Proc. R. Soc. Lond. A* 455, 3129-3137.

[8] Eisert J., M. Wilkens and M. Lewenstein (1999), “Quantum Games and Quantum Strategies” *Phys. Rev. Lett.* 83, 3077.

[9] Franco R., (2007) "The conjunction Fallacy and Interference Effects" *arXiv:0708.3948v1*

[10] Franco R. (2008) " The inverse fallacy and quantum formalism" in Proceedings of the the second interaction symposium (QI 2008), 94-98.

[11] Frank H. R. 1988, *Passion within Reason*, W.W. Norton & company, New York - London

[12] Fudenberg D. and D.. Levine (2006) "A Dual Self Model of Impuls Control" American Economic Review 96 (2006) 1446-1476.

[13] Fudenberg D. and J. Tirole (1991) *Game Theory*, MIT Press.

[14] Khrennikov A. Yu (2007) "A Model of Quantum-like decision-making with application to psychology and Cognitive Sciences" *http://arhiv.org/abs/0711.1366*

[15] Koesller F. et F. Forges 2008 " Transmission Strategique de l'Information et Certification" Annales d'Economie et de Statistiques.

[16] Lambert-Mogiliansky A., S. Zamir, and H. Zwirn. (2007) "Type-indeterminacy - A Model for the KT-(Kahneman and Tversky)-man", available on *ArXiv:physics/0604166* forthcoming in the *Journal of Mathematical Psychology* 2009.

[17] La Mura P. (2003) "Correlated Equilibria of Classical Strategic Games with Quantum Signals" Game Theory and Information 0309001 *EconWPA*.

[18] La Mura P. (2005) "Decision Theory in the Presence of Risk and Uncertainty". *mimeo.* Leipzig Graduate School of Business.

[19] La Mura P. (2008) "Prospective expected utility" in Proceedings of the the second interaction symposium (QI 2008), 87-94.

[20] R.H. Strotz (1956) "Myopya and Time Inconsistency in Dynamic Utility Maximization" *Review of Economic Studies* Vol 23/3 165-180.