

Multiple-Bank Lending, Creditor Rights and Information Sharing

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Abstract: When a customer can borrow from several competing banks, multiple lending raises default risk. If creditor rights are poorly protected, this contractual externality can generate novel equilibria with strategic default and rationing, in addition to equilibria with excessive lending or non-competitive rates. Information sharing among banks about clients' past indebtedness lowers interest and default rates, improves access to credit (unless the value of collateral is very uncertain) and may act as a substitute for creditor rights protection. If information sharing also allows banks to monitor their clients' subsequent indebtedness, the credit market may achieve full efficiency.

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1 Introduction

In most countries, firms tend to borrow from several banks: less than 15 percent of European firms have only a single bank as source of credit (Ongena and Smith, 2000), and even small and medium-sized firms patronize several lenders (Detragiache, Garella and Guiso, 2000, and Farinha and Santos, 2002). This pattern is found in the United States as well, as is documented by Petersen and Rajan (1994). Multiple-bank lending can affect credit market performance by generating a contractual externality between lenders, as each bank's lending may increase the default risk for the others. This externality may be mitigated by information-sharing arrangements, such as private credit bureaus or public credit registries, which in most countries enable lenders to verify the overall debt commitment of their credit applicants.

The objective of this paper is to establish the link between the contractual externalities arising from multi-bank lending and the design of information sharing arrangements. First, we show that, if there is no information sharing, this externality will often lead banks to ration credit (denying credit to some applicants), and borrowers to default strategically. Second, we demonstrate that introducing information sharing always reduces interest rates and default rates and may eliminate rationing by preventing strategic defaults.

We frame the analysis in a competitive credit market, where each customer can borrow from several banks. Borrowers can invest either in a small project or in a larger but less profitable one, whose returns they can partially appropriate at the expense of lenders. The fraction of the returns they can appropriate depends on the degree of creditor protection. Borrowers post risky collateral, so that they may default when the value of collateral is low. Lenders cannot observe which project will actually be carried out by a borrower, which gives rise to common-agency moral hazard.¹

Contractual externalities may take two different forms. First, a credit applicant may attempt to play lenders one against another by accepting multiple loan offers so increasing their default risk, this externality depends on the availability of multiple loan offers by competing banks. Second, each bank must worry that outside lenders may behave opportunistically, extending

¹Bernheim and Whinston (1986a, 1986b) offer the first general treatment of this class of models. Kahn and Mookherjee (1998) specialize the analysis to the case of insurance contracts, but consider a model with sequential offers. Segal and Whinston (2003), Bisin and Guaitoli (2004) and Martimort (2004) consider a more general contracting space by introducing latent contracts and menus. Calzolari and Pavan (2006) and Martimort and Stole (2003) consider closely related issues in an adverse selection setting.

extra credit to their common customer while protecting their own claims via high interest rates. And customers may wish avail themselves of this credit in order to undertake larger projects and so extract greater private benefits at lenders' expense. Our model encompasses both types of externality and brings out their implications for credit market equilibrium.

The danger of opportunistic behavior by outside lenders can generate various equilibrium outcomes. When creditor rights protection is at intermediate levels, two types of equilibria may emerge: the current lender may discourage entry either by granting an inefficiently large loan (an overlending result in the spirit of Bizer and DeMarzo, 1992) or by charging non-competitive rates to make its own borrowers unattractive to opportunistic competitors (a result that echoes Parlour and Rajan, 2001).

If creditor protection is poor, a novel type of equilibrium with rationing and strategic default emerges, in which only a fraction of the entrepreneurs applying for credit at the going rates are funded, and some of them get credit from several banks. In these equilibria, interest rates exceed the competitive level, but entry of new lenders is blocked by the fear of funding overindebted entrepreneurs, although all active banks make zero profits. Moreover, greater volatility of collateral value – a typical feature of economic downturns – lowers the fraction of entrepreneurs who manage to obtain credit and invest. This rationing equilibrium necessarily features multi-bank lending, in contrast to the equilibria described in the previous paragraph, which are compatible with exclusive contractual relationships.

In the parameter region where creditor rights are very poorly protected and the collateral values are highly volatile, only equilibria with rationing or complete market collapse survive. In this region, credit market segmentation also emerges: in the rationing equilibrium, different groups of lenders offer credit at different interest rates, some charging usury rates, even higher than the monopoly level.²

Our second set of new results concerns the effects of information sharing on the contractual externalities. Information sharing mitigates them by allowing banks to condition their loans on the borrowers' contractual history, so as to better guard against opportunistic lending. More precisely, information sharing expands the region where lending is offered at competitive rates

²This segmentation appears fully consistent with a pattern featured by credit markets of developing countries, where borrowers apply either to the formal credit market (banks) or to informal markets, and the latter charge rates much higher.

and efficiency prevails, and it eliminates rationing whenever the entrepreneurs' collateral is not very volatile. This brings out the "bright side" of information sharing under multi-bank lending: by allowing banks to adjust loan offers to the credit history of their applicants, it enables them to rule out strategic defaults and expand the availability of credit.

If the value of collateral is very uncertain, however, information sharing among banks induces a unique equilibrium, in which the credit market fully collapses: banks charging usury rates can exploit the additional information to better target creditworthy customers, and thereby make the market unviable for non-usurious lenders. This reveals a potential "dark side" of information sharing. But, in this area lenders will never share information unless forced to do so, so that voluntary information sharing always increases social surplus.

In most of this paper, the analysis assumes that defaulted claims are liquidated pro rata, and that information sharing only concerns past obligations. However, we do make two extensions to consider the case where information sharing allows liquidation according to seniority, and that in which banks monitor the future indebtedness of their current clients. In both instances, the benefits of information sharing are amplified.³

Taken together, our model's results have three empirically testable implications. First, in the absence of information sharing, entrepreneurs whose collateral value is more volatile have a poorer chance of being funded if credit protection is low; and this rationing is associated with high interest rates and default rates, consistent with the evidence from developing countries (Mookherjee et al. 2000). Second, information sharing should reduce default and interest rates, as it allows banks to detect borrowers who attempt to overborrow and thus mitigates strategic default; these predictions square with a number of studies based on cross-country aggregate data (Djankov, McLiesh and Shleifer, 2007, Jappelli and Pagano, 2002, Pagano and Jappelli, 1993) and on firm-level data (Brown, Jappelli and Pagano, 2008; Galindo and Miller, 2001). Third, information sharing should increase access to funds by eliminating rationing, except for situations where creditor rights are poorly protected and collateral values are very uncertain; thus, in most parameter regions, information sharing may substitute for creditor rights protection, consistent

³This highlights that full information sharing is not equivalent to exclusivity: even when banks can fully condition their contracts on all the terms of the contracts that their customers have signed and will sign with other banks, it may be impossible for a bank to protect its claims from competing claims by later lenders. For comparison of the efficiency of exclusive and non-exclusive lending, itself against Bisin and Guaitoli (2004) and Attar, Campioni and Piaser (2006), among others.

with the evidence in Djankov et al. (2007) and Brown et al. (2008).

Our analysis can also help to interpret the evidence in Herzberg, Liberti and Paravisini (2008), that the extension of Argentina's public credit register to loans below the \$200,000 threshold resulted in lower lending and higher default rates. They find that these effects are due to firms that borrowed from multiple lenders, and that after the reform firms tended to borrow from fewer banks. This evidence is surprising in light of the literature just cited, but it accords with the comparative statics of our rationing equilibrium if an information-sharing mechanism were introduced after loan contracts were signed but before investment projects have been completed. In this case, lenders discovering that their client has taken multiple loans would call them back, both reducing and concentrating credit. This unexpected cutback might also cause a transitory increase in defaults, by forcing some firms to breach obligations with their suppliers. However, the long-run effect should be the opposite as we have seen: our model predicts that information sharing will eventually reduce defaults.

Moreover, our analysis complements the earlier models of information sharing in credit markets, which invariably assume exclusive lending. These models show that sharing data on defaults and customers' characteristics enables banks to lend more safely, overcoming adverse selection (Pagano and Jappelli, 1993), or by promoting greater effort to repay loans (Padilla and Pagano, 1997 and 2000).⁴ Unlike the present paper, however, these models do not explain why banks should also share information about their customers' indebtedness, which is crucial to the activity of many credit bureaus and registries.

Finally, our paper also relates to the vast literature on the determinants of credit rationing (e.g., Stiglitz and Weiss (1981), Besanko and Thakor (1985), Bester (1987), Aghion and Bolton (1997), Piketty (1997), Carlin and Robb (2008) among others), in which there is a common feature: rationing arises because the interest rate charged by banks is "too low" to enable the credit market to clear but no bank attempts to raise it, fearing the adverse effect that this would have on the quality of loan applicants. In contrast, in our model banks react to the danger of opportunistic lending both by rationing and raising their lending rates above the competitive level, in some cases up to or even beyond the monopoly level.

The paper is structured as follows. Section 2 lays out the model. Section 3 introduces

⁴In a sequential common agency game with adverse selection Calzolari and Pavan, 2006, also analyze the conditions under which information sharing between principals may enhance efficiency.

the notions of incentive compatibility relevant for the characterization of equilibria. Section 4 presents the equilibria in the regime with no information sharing. Section 5 analyzes how equilibria change when banks can condition lending on their customers' past indebtedness. Section 6 extends the analysis to the case in which banks can condition also on customers' subsequent borrowing and that in which repayments in case of default are according to seniority when information sharing is in place. Section 7 concludes. All proofs are in the Appendix.

2 The model

We consider a countably infinite set of banks $B = \{1, 2, 3, \dots\}$ that compete by offering credit to a set of risk-neutral entrepreneurs $E = \{1, 2, \dots, \bar{e}\}$. For simplicity, the interest rate at which banks raise funds is standardized to zero. Each entrepreneur can undertake a small project or a large one, requiring an investment x or $2x$. The two projects have non-stochastic revenues y_S and y_L , with $y_L > y_S$, so that the net surplus is $v_S \equiv y_S - x$ or $v_L \equiv y_L - 2x$. Due to decreasing returns, the surplus of the small project exceeds that of the large one: $v_S > v_L$. Due to limited managerial capacity, each entrepreneur can undertake at most one project.

At the investment stage, entrepreneurs have no resources, so they must borrow. Banks offer loans for which entrepreneurs can apply sequentially. A credit contract $c_b = (l_b, r_b)$ issued by bank $b \in B$ consists of a loan l_b and a repayment rate r_b .

The contractual environment is shaped by the following assumptions:

- (A1) *Hidden action*: Lenders cannot verify the actual size of the borrower's project, so that an entrepreneur with a loan of size x can borrow an additional x and undertake the large project.
- (A2) *Limited enforcement*: Borrowers are protected by limited liability and can appropriate a fraction $\phi \in (0, 1]$ from the revenue of the large project, which cannot be seized by lenders in case of default.
- (A3) *Uncertain future wealth*: After the investment stage, each entrepreneur has a stochastic wealth \tilde{w} that is equal either to $\bar{w} + \sigma$ or to $\bar{w} - \sigma$ with equal probability.
- (A4) *Costly state verification*: The realization of future wealth \tilde{w} is unverifiable except in case

of default.

Assumptions (A1) and (A2), together with multiple-bank lending, create a moral hazard problem: after borrowing an amount x , the entrepreneur may want to borrow an additional x and undertake the large project, so as to appropriate a share ϕ of its revenue. This can damage lenders, since the large project yields less than the small one and its return can be partially appropriated by the entrepreneur. Assumption (A3) states that at the repayment stage the borrower's wealth is stochastic, with standard deviation $\sigma \in [0, 1]$ and expected value \bar{w} , which we normalize to 1 with no loss of generality. The future wealth \tilde{w} can be interpreted as the value of the entrepreneur's personal assets or as a random component of the firm's profits.⁵ Assumption (A4) rules out financing contracts contingent on future wealth, and implies pure debt financing: verifying borrowers' wealth is so costly as to be worthwhile only if default occurs.⁶ Taken together, (A3) and (A4) generate scope for strategic default.

We assume that the investment necessary for the large project ($2x$) exceeds the entrepreneur's expected endowment (1). As we shall see, this parametric restriction allows for the existence for equilibria where the large and inefficient project is funded.

2.1 Information-sharing regimes

In most of the paper we study two alternative regimes of communication between banks:

- under *no information sharing*, banks lending to the same borrower cannot verify either the total indebtedness of the borrower or their own seniority;
- under *information sharing*, banks can verify the total indebtedness of an entrepreneur at the loan application stage.

Most credit reporting systems allow lenders to condition their lending on applicants' total debt. Banks can interrogate a credit reporting agency about the exposure of a prospective client, but can ordinarily file the request only when they receive a loan application.⁷

⁵The model is easily extended to allow for a certain wealth endowment at the investment stage, as long as it is not sufficient to finance the minimal investment x .

⁶This assumption is also made by Bizer and DeMarzo (1992) and by Bisin and Rampini (2006). It also rules out insurance contracts with which entrepreneurs can hedge against their wealth risk.

⁷In real world credit markets, this provision prevents banks from exploiting information-sharing systems as a marketing device, soliciting applicants after learning about their loan exposure.

In Section 6 we consider a more extensive form of information sharing, whereby banks can request credit reports also after the loan application stage, in order to monitor subsequent changes in clients' exposure. This enables lenders to use covenants, so as to make repayments contingent on subsequent borrowing.

All information-sharing regimes are initially analyzed under the assumption that defaulted debts are liquidated *pro rata*, as often occurs for unsecured lending. In Section 5, we consider the case in which information-sharing arrangements allow seniority-based repayment.

2.2 The game

We represent market interactions as a game in which entrepreneurs visit banks and apply for credit sequentially (time line in Figure 1). Each bank b can offer a single type of loan contract denoted by $c_b = (l_b, r_b)$. The uncertainty about entrepreneurs' endowments is resolved at the final stage $\bar{\tau}$. The contracting process between 0 and $\bar{\tau}$ includes an infinite number of stages in which banks post loan offers, entrepreneurs apply for credit and banks decide whether to grant it. During this process, the loans granted cannot be neither invested or consumed. Once the contracting process ends at $\bar{\tau}$, entrepreneurs decide whether to go on to the investment phase at $\bar{\tau} + 1$ or to return the money to their lenders.⁸

[Insert Figure 1]

Banks can verify that funded entrepreneurs do undertake an investment project, but cannot observe which one.⁹ The return on the investment project chosen and the final value of wealth \tilde{w} are realized at the final stage $\bar{\tau} + 2$, where loans are repaid in full or the borrower defaults.

At every stage $\tau \in \mathbb{N}$ between 0 and $\bar{\tau}$, the sequence of events is as follows: (i) the bank with the corresponding index ($b = \tau$) posts a contract c_τ ; (ii) all entrepreneurs can visit it and apply for c_τ ;¹⁰ (iii) bank τ accepts or rejects applications.¹¹ At any $\tau \in \mathbb{N}$ the action of a generic entrepreneur e is $a_e(\tau) = 1$ if he files a loan application, or $a_e(\tau) = 0$ if he does not.

⁸This assumption guarantees effective competition between lenders, since it enables entrepreneurs to opt out from a loan if they find a better offer.

⁹Thus we rule out a further moral hazard problem, which would arise if borrowers could consume the funds lent for investment.

¹⁰The case in which they do not apply is captured as an application for the null contract $c_0 \equiv (l_0 = 0, r_0 = 0)$.

¹¹These assumptions imply that each bank is active in only two stages, but this entails no loss of generality.

Bank τ 's action $(c_\tau, \beta(\tau))$ is a vector including the contract posted c_τ and a sequence of replies $\beta(\tau) = (\beta_1(\tau), \dots, \beta_{\bar{e}}(\tau))$ to the applicants, where $\beta_e(\tau) \in \{\text{yes}, \text{no}\}$ denotes the reply of bank $b = \tau$ to entrepreneur e . A contract between entrepreneur e and bank τ is signed at stage τ if and only if $a_e(\tau) = 1$ and $\beta_e(\tau) = \text{yes}$.

The ‘‘history’’ h_e^τ known to entrepreneur e and the ‘‘history’’ h_b^τ known to bank b at each date τ is what each have observed up to that date. Without information sharing, each entrepreneur knows his own applications and outcomes; each bank τ the applications received, its own acceptance decisions and the contracts $M(\tau) = (c_1, c_2, \dots, c_{\tau-1})$ offered by banks before stage τ .¹²

Under information sharing, the bank receiving a loan application at stage τ also observes the entrepreneur’s indebtedness up to that stage; entrepreneurs observe the same histories as without information sharing. We denote the indebtedness of entrepreneur e at stage τ as the total repayment he has pledged up to that stage, that is, $R(\tau) = \sum_{j \leq \tau} r_j$, where r_τ is the repayment obligation under contract c_τ . Hence, when information is shared, a bank lending at stage τ knows the history $h_\tau^\tau = \{M(\tau), R(\tau)\}$.

Entrepreneurs’ and banks’ strategies are mappings from their set of possible histories to the set of actions: loan applications, investment and repayment decisions by entrepreneurs, and loan offers and acceptances by banks.

Let $h^\tau = \{h_e^\tau, h_b^\tau\}_{e \in E, b \in B}$ be the full description of market interactions between entrepreneurs and banks, and let H^τ be the set of possible histories up to τ . To introduce the players’ payoffs, let us define the final indebtedness as the representative entrepreneur’s total repayment obligation $R(h^\tau)$:

$$R(h^\tau) = \sum_{\tau \leq \bar{\tau}} \hat{r}_\tau,$$

¹²Formally, at each date $\tau \in \mathbb{N}$, entrepreneur e knows the history $h_e^\tau = \{M(\tau); a_e^\tau, \beta_e^\tau\}$ where $a_e^\tau = \{a_e(1), a_e(2), \dots, a_e(\tau-1)\}$ is the sequence of his applications and $\beta_e^\tau = \{\beta_e(1), \beta_e(2), \dots, \beta_e(\tau-1)\}$ is the corresponding sequence of acceptance decisions by banks. Similarly, for each $\tau \in \mathbb{N}$, the bank lending at stage τ knows the history $h_b^\tau = \{M(\tau), a(\tau)\}$, where $a(\tau) = (a_1(\tau), a_2(\tau), \dots, a_{\bar{e}}(\tau))$ are the loan applications received at τ . If instead bank b is inactive up to τ (which happens if $b < \tau$), it only knows the set of loan offers by all banks, that is, $h_b^\tau = \{M(\tau)\}$.

where $\widehat{r}_\tau = r_\tau$ if bank τ and the entrepreneur agree to a loan contract c_τ , and $\widehat{r}_\tau = 0$ if they do not. Similarly, let the total nominal loan value $D(h^\tau)$ of the representative entrepreneur be

$$D(h^\tau) = \sum_{\tau \leq \bar{\tau}} \widehat{l}_\tau,$$

where $\widehat{l}_\tau = l_\tau$ if the bank and the entrepreneur agree to a loan c_τ , and $\widehat{l}_\tau = 0$ if they do not.

For any history h^τ such that agreed contracts imply total repayment obligation $R(h^\tau)$, an entrepreneur with project $n \in \{S, L\}$ and final wealth \widetilde{w} obtains the final payoff:

$$x_n(h^\tau, \widetilde{w}) = \phi_n y_n + \max \{0, (1 - \phi_n) y_n + \widetilde{w} - R(h^\tau)\},$$

where $\phi_S = 0$ and $\phi_L = \phi$.

Thus, the representative entrepreneur maximizes

$$\mathbf{E}_{\widetilde{w}}[x_n(h^\tau, \widetilde{w})] = \frac{1}{2}[x_n(h^\tau, \bar{w} - \sigma) + x_n(h^\tau, \bar{w} + \sigma)].$$

Finally, the profit that a bank expects by lending to an entrepreneur undertaking a project of size $n \in \{S, L\}$ is

$$\pi_\tau^n(h^\tau) = \mathbf{E}_{\widetilde{w}}[r_\tau^n(h^\tau, \widetilde{w}) - l_\tau],$$

where $r_\tau^n(h^\tau, \widetilde{w})$ is the actual repayment contingent on the contracting history h^τ , the wealth realization \widetilde{w} and an investment of size $n \in \{S, L\}$. Since defaulted loans are repaid *pro rata*, the loan repayment is

$$r_\tau^n(h^\tau, \widetilde{w}) = \min \left\{ r_\tau, \frac{l_\tau}{D(h^\tau)} [(1 - \phi_n) y_n + \widetilde{w}] \right\}.$$

Due to the lumpiness of investment choices, we can focus only on strategies whereby entrepreneurs borrow either x or $2x$ and sign contracts with at most two banks. So, with minor abuse of notation we shall denote by $u(c, c_\emptyset)$ the expected utility of an entrepreneur who signs only contract $c = (x, r)$ and undertakes the small project, and by $u(c, c')$ that of an entrepreneur who signs contracts $c = (x, r)$ and $c' = (x, r')$ and undertakes the large project. Symmetrically, we denote the expected profit of a bank offering contract c by $\pi(c, c_\emptyset)$ when the borrower takes no

other loan, and by $\pi(c, c')$ where he also signs contract c' with another bank.¹³

Throughout the paper, we characterize the Perfect Bayesian Equilibria (PBE) of this game under the following tie-breaking assumption:

(A5) Tie-breaking: Whenever a bank is indifferent between issuing two loan contracts, it offers the one preferred by the entrepreneur, i.e., the one with the lower rate.

3 Incentive compatibility

In our setting, efficiency requires all entrepreneurs to undertake the small project. This outcome would be ensured by exclusive lending, as banks can costlessly prevent their customers from borrowing from other lenders. In our model, however, exclusivity is not enforceable: once they have received a loan to carry out the small project, entrepreneurs may try to borrow more and switch to the large project, so as to appropriate a fraction ϕ of its revenue. An entrepreneur may do so by taking two small loans, each issued to fund a single small project: if he has no incentive to do so, we say that his *individual* incentive constraint is satisfied. Alternatively, an entrepreneur who already obtained a small loan may fund the large project by taking an additional loan deliberately designed by another bank to this purpose. If this deviation of the entrepreneur and outside bank together is unprofitable, we shall say that their *joint* incentive constraint is satisfied.

Formally, if C is a given set of loans available on the market, the contract $c = (x, r) \in C$ is individually incentive-compatible within C if

$$u(c, c_\emptyset) \geq u(c, c') \quad \forall c' = (x, r') \in C, \quad (1)$$

possibly with $r' = r$, where c_\emptyset is the null contract. Note that individual incentive compatibility does not require the c' to make non-negative profits when taken in conjunction with c .

However, there are situations in which an outside bank offering an additional loan c' can profit from it, even though it increases the entrepreneur's indebtedness and thus his default risk. The outside bank may gain if the entrepreneur pays a rate high enough to compensate for the additional implied risk. Such a gain would come at the expense of the initial lender,

¹³The formal definitions for these expressions are provided in the Appendix.

thereby creating a negative contractual externality as in Bizer and DeMarzo (1992). Joint incentive compatibility holds if this opportunistic behavior by the entrepreneur and outside bank is unprofitable.

Formally, let $C(c)$ denote the set of all contracts that yield non-negative profits if offered to an entrepreneur who already signed contract c . Joint incentive compatibility requires at least one of the two following conditions to be satisfied:

$$C(c) = \emptyset \text{ or } u(c, c_\emptyset) > \max_{c' \in C(c)} u(c, c'). \quad (2)$$

The set $C(c)$, which contains all the contracts $c' = (x, r')$ such that $\pi(c', c) \geq 0$, is non-empty if and only if the expected maximal profit to the bank issuing c' is non-negative, that is

$$\mathbb{E}_{\tilde{w}} \left[\max \left\{ (1 - \phi) y_L + \tilde{w} - r, \frac{(1 - \phi) y_L + \tilde{w}}{2} \right\} \right] - x \geq 0, \quad (3)$$

where $(1 - \phi) y_L + \tilde{w} - r$ is the maximum amount that a bank offering c' can obtain in case of no default after the bank offering $c = (x, r)$ has been repaid, and $[(1 - \phi) y_L + \tilde{w}]/2$ is its repayment in case of default.

Notice that individual incentive compatibility is a stronger notion than joint incentive compatibility, as the latter applies only to a subset of the contracts relevant for the former. The reason for distinguishing these two notions is that they involve different types of equilibria. Condition (1) must hold in any equilibrium where several banks are active (i.e., offer loan contracts and accept some applications). For example, a zero-profit equilibrium where several banks offer the perfectly competitive contract $c^{PC} = (x, x)$ can exist only if $u(c^{PC}, c_\emptyset) \geq u(c^{PC}, c^{PC})$. The weaker notion of joint incentive compatibility becomes relevant for equilibria where only one bank is active, as it guarantees that the contract offered by this bank is robust to deviations that are jointly profitable for the entrepreneur and an outside bank.

4 Equilibria with no information sharing

Three different types of equilibria can emerge in the regime without information sharing among banks, depending on parameters: (i) efficient equilibria, where all entrepreneurs implement the

small project, possibly borrowing at non-competitive rates; (ii) inefficient overlending equilibria, where all entrepreneurs undertake the large project, and (iii) inefficient rationing equilibria, where funding goes only to a fraction of entrepreneurs, some of which get credit from several banks and strategically default.

To illustrate the regions in which these different equilibria arise, we focus on two key parameters: ϕ , the fraction of the revenues that can be appropriated from the large project, and σ , the riskiness of entrepreneurs' wealth. Poor creditor protection (a large ϕ) heightens the entrepreneurs' temptation to overborrow and select the large project. Similarly, wealth volatility (a high σ) gives outside banks an incentive to induce overborrowing, since limited liability allows them to shift the implied extra default risk onto the initial lender. In short, while higher values of ϕ increase borrowers' private incentives to behave opportunistically against all lenders, higher values of σ increase outside banks' gains from opportunistic lending.

Accordingly, we characterize the equilibrium outcomes for different parameter regions in the Cartesian plane (σ, ϕ) . As illustrated in Figure 2, the admissible parameter space is formed by the square $[0, 1]^2$, since both ϕ and σ range between 0 and 1.

[Insert Figure 2]

In region *A*, where either ϕ or σ is low, the efficient project is funded at the competitive rate: the negative externality due to multi-bank lending is tenuous, since the individual incentive constraint (1) holds when several banks offer the competitive loan contract c^{PC} .

In region *B*, where ϕ or σ is larger than in region *A*, the individual incentive constraint is never satisfied, although the joint constraint is satisfied if the inside bank charges a sufficiently high non-competitive rate r , as in Parlour-Rajan (2001). On the one hand, the violation of the individual incentive constraint prevents undercutting: r is such that if an outside bank offered a loan x at a lower rate r' , the entrepreneur would take both contracts, and the undercutting bank would make a loss. On the other hand, as the joint incentive constraint is met, there is no danger that an outside bank can induce the entrepreneur to switch to the large project, so that the inside bank can safely charge a non-competitive rate. Indeed it is precisely by charging a high enough rate that the inside bank reduces the surplus available to outside banks and entrepreneurs, deterring opportunistic lending.

This equilibrium has the rather unrealistic feature that a single lender must supply the entire market. But in a subset of region B a rationing equilibrium also exists, where several banks are active and only some entrepreneurs receive credit. Here borrowers seek multiple loans whenever possible (since the individual incentive constraint is violated), but rationing reduces the probability of getting two loans, so as to undertake the large project and default. Equilibrium obtains when interest rates and rationing ensure that no inactive bank is willing to enter the market, for fear of attracting too many applicants already laden with debt.

In regions C and D of Figure 2, the joint incentive constraint is not satisfied. In region C , where ϕ is not too high, the large project is viable and the only possible equilibrium involves funding this project at the competitive rate; in equilibrium, no bank can profitably induce the entrepreneur to switch to the small project, for fear of further opportunistic lending. But, in region D , where both ϕ and σ are highest, not even the large project is viable, so there is no equilibrium in which all entrepreneurs are funded: *at most a fraction of entrepreneurs obtain credit, or none*. In this region, even outside opportunistic lenders are “unsafe”: entrepreneurs can expropriate such a large fraction ϕ of the large project’s revenue that they will seek loans from any lender, at whatever rate, and then default on all of them. Therefore, even opportunistic lenders must ration credit in order to break even.

4.1 Equilibrium characterization

In this section, we formally characterize the equilibria described intuitively so far.

4.1.1 Region A

In region A , all entrepreneurs invest in the efficient project and borrow at the competitive rate $r^{PC} = x$, and several banks are active. For this equilibrium to exist, the individual incentive constraint must hold when the contract $c^{PC} = (x, x)$ is offered by several banks; that is,

$$y_S - r + 1 \geq \phi y_L + E_{\tilde{w}} [\max \{0, (1 - \phi)y_L + \tilde{w} - r - r'\}], \quad (4)$$

for

$$r = r' = x. \quad (5)$$

The expression on the left-hand side of (4) is the borrower's payoff from executing the small project and repaying r ; while the right-hand side is the payoff that he would get by switching to the large project. Substituting (5) into (4) yields the boundary:

$$\phi \leq \underline{\phi}(\sigma) = \min \left\{ \frac{v_S + 1}{y_L}, \frac{1 + \Delta v + v_S - \sigma}{y_L} \right\}. \quad (6)$$

Since by construction the set of contracts that satisfy individual incentive compatibility is smaller than that of contracts that satisfy joint incentive compatibility, $\phi \leq \underline{\phi}(\sigma)$ guarantees that in Region A there are no profitable deviations from the perfectly competitive contract.

The magnitude of this region is inversely related to the excess value generated by the efficient project, $\Delta v = v_S - v_L$: the greater this difference, the weaker the temptation to switch to the large and inefficient project. The function $\underline{\phi}(\sigma)$ is decreasing in σ , because when their wealth is riskier, entrepreneurs gain more by overborrowing and defaulting in the bad state. Thus, efficiency always prevails for Δv sufficiently large and σ sufficiently small. To summarize:

Proposition 1 *In region A there is a unique equilibrium in which all entrepreneurs undertake the efficient project and borrow at competitive rates. This region is non-empty, and its area is increasing in Δv and decreasing in σ .*

4.1.2 Region B

We have seen that points to the right of the $\underline{\phi}(\sigma)$ locus violate the individual incentive constraint under perfect competition. Lemma 1 in the Appendix shows that the same applies to any loan with a rate larger than the perfectly competitive one, because larger rates reduce the payoff from the small project. Therefore, in region B no contract $c = (x, r)$ with $r \geq x$ satisfies the individual incentive constraint $u(c, c) > u(c, c_0)$ and as a consequence the only possible equilibria are either non-competitive, with only one bank active, or inefficient, with at least some entrepreneurs undertaking the large project.

As a first step, we verify that in region B the monopoly contract $c^M = (x, y_S)$ satisfies the joint incentive constraint, because no outside bank can break even when lending to the

entrepreneur, i.e. $C(c^M) = \emptyset$ as required by (2). This requires

$$E_{\tilde{w}} \left[\max \left\{ (1 - \phi) y_L + \tilde{w} - y_S, \frac{(1 - \phi) y_L + \tilde{w}}{2} \right\} \right] - x \leq 0,$$

which implicitly defines the frontier of the set $C(c^M)$:

$$\phi_m(\sigma) = \frac{v_L + 1}{y_L} + \max \left\{ 0, \frac{\sigma - 2v_S}{3y_L} \right\}. \quad (7)$$

Figure 2 shows that this function is constant at $\hat{\phi} = (v_L + 1)/y_L$ for σ sufficiently low, and then becomes linearly increasing in σ , since greater collateral volatility increases the profits of opportunistic entrants.

Thus, for any $\phi \geq \phi_m(\sigma)$ the joint incentive constraint is satisfied at the monopoly rate. But this constraint may also hold for $\phi < \phi_m(\sigma)$: there can be a contract c' on which the outside bank would break even when offering additional lending; but this contract will require a rate so high that the entrepreneur is not willing to accept it. Formally, $u(c, c_\emptyset) > \max_{c' \in C(c)} u(c, c')$ as required by (2). This amounts to:

$$u(c, c_\emptyset) - u(c, c') = y_S - r + 1 - (\phi y_L + \frac{1}{2} \max \{0, (1 - \phi) y_L + 1 + \sigma - r - r'\}) \geq 0, \quad (8)$$

which guarantees that the entrepreneur will not want to switch to the large project if the deviant bank offers c' , where c' satisfies the outside bank's zero-profit condition

$$\pi(c', c) = \frac{1}{2} \left\{ \frac{(1 - \phi) y_L + 1 - \sigma}{2} + r' \right\} - x = 0. \quad (9)$$

These two conditions yield the boundary

$$\phi \leq \underline{\phi}'(\sigma) = \min \left\{ \frac{v_S + 1}{y_L}, \frac{1 + 4\Delta v + v_L - \sigma}{y_L} \right\}.$$

To summarize, joint incentive compatibility is satisfied either for $\phi \geq \phi_m(\sigma)$ or for $\phi \leq \min \{ \phi_m(\sigma), \underline{\phi}'(\sigma) \}$. Region B is the area where one of these two conditions is satisfied but the individual incentive constraint does not hold, that is, $\phi > \underline{\phi}(\sigma)$.

In region B joint incentive compatibility, which is a necessary condition for an efficient

non-competitive equilibrium, turns out to be also sufficient for its existence. In this type of equilibrium, the rate offered by the only active bank, say $\hat{c} = (x, \hat{r})$, is determined not only by the need to prevent opportunistic lending by an outside bank, but also by the need to avoid undercutting aimed at poaching clients. In the appendix we show that this rate \hat{r} is the lower between the monopoly rate and the highest rate that prevents undercutting.

But, as already noted, a subset of region B also has rationing equilibria, where several banks are active and some entrepreneurs are denied credit. These equilibria exist if the large project is not viable, since the borrower's pledgeable wealth $y_L - y_L\phi + 1$ falls short of the project's cost $2x$. Since the individual incentive compatibility constraint is violated, entrepreneurs always apply to all active banks, hoping to find at least two lenders. As banks accept applications randomly, in equilibrium some entrepreneurs will not receive credit, some will manage to borrow the per-capita amount x and others $2x$. An active bank earns the profit $\hat{r} - x$ on each client who is granted a single loan, and loses money on those who get two loans of size x and default in the bad state. Therefore, each bank's expected profit is decreasing in the fraction of contracts offered by competitors. The fraction of loan applications accepted by each bank is such that it breaks even: no inactive bank has the incentive to enter and accept additional applicants, since the average creditworthiness of its customers would be too low to break even. In region B , this rationing equilibrium is the only one consistent with multiple active banks.

These results are summarized in the following proposition:

Proposition 2 *There are two classes of equilibria where all active banks charge a non-competitive rate and offer a loan of size x :*

- (i) *in region B , there exists an equilibrium without rationing, where a single bank is active and funds the efficient project by offering the loan x to all entrepreneurs. In the subset of region B where the large project is viable, this equilibrium is unique.*
- (ii) *in the subset of region B where the large project is not viable, there are zero-profit symmetric equilibria with rationing, where only a subset of banks is active and each bank offers only a single loan contract to a fraction α of entrepreneurs.*

4.1.3 Region C

In region C , neither the individual nor the joint incentive compatibility conditions are satisfied. So, as in region B , there can be no equilibrium in which all entrepreneurs get a loan x and undertake the small project. However, in this area ϕ is sufficiently small that the large project is financially viable, since the pledgeable wealth of an entrepreneur who undertakes this project, $(1 - \phi)y_L + 1$, is greater than the cost of the project, $2x$, that is,

$$\phi < \hat{\phi} = \frac{y_L - 2x + 1}{y_L} = \frac{v_L + 1}{y_L}. \quad (10)$$

Here there is a unique equilibrium in which the large project is funded at competitive rates $r = 2x$: no bank can profitably deviate by offering a small loan to fund the small and efficient project, since the joint incentive compatibility condition is not satisfied.

As $\hat{\phi}$ is an increasing function of $y_L/2x$, the size of region C is increasing in the profitability of the large project. To summarize,

Proposition 3 *In region C, there is a unique equilibrium in which all entrepreneurs undertake the inefficient project and borrow at the competitive rate. This region is non-empty if and only if $\Delta v < \min(1/4, x/2)$.*

4.1.4 Region D

Region D is where moral hazard problems are most severe: the fraction of surplus that borrowers can appropriate is very large, so that the inefficient project is not financially viable ($\phi > \max\{\hat{\phi}, \underline{\phi}'(\sigma)\}$), and collateral value is very volatile, so that the joint incentive compatibility constraint is violated ($\phi < \phi_m(\sigma)$). Therefore, deterring opportunistic behavior by competitors becomes impossible. As a result, no equilibrium exists in which all entrepreneurs obtain credit. Here, either there is rationing, in the sense that not all credit applicants obtain a loan, or else the credit market collapses.

The rationing equilibrium in this region differs markedly from that in region B : at least two different contracts must be offered, one at “usury rate” r^U above the monopoly level $r^M = y_S$ and the other at the monopoly rate. The “usury rate” r^U is the maximum rate that an entrepreneur who has already borrowed at the monopolistic rate can pledge without defaulting in the good

state. As shown in the Appendix, this rate is $r^U = (1 - \phi)y_L + 1 + \sigma - y_S > r^M = y_S$. Therefore:

Proposition 4 *In region D, there exist both a competitive equilibrium with rationing and an equilibrium with market collapse. In the equilibrium with rationing, banks accept a fraction of loan applications such that each of them makes zero expected profits. Some banks offer the monopoly contract $c^M = (x, r^M)$ and others the usurious contract $c^U = (x, r^U)$. This region is non-empty if and only if $\Delta v < (2 - 3v_S)$ and $v_S < 1/2$.*

In the rationing equilibrium, entrepreneurs apply for both the monopoly and the usury loan: some get no loan, some get one at the monopoly rate, others get both the monopoly and the usurious contract, and the rest take two contracts at the usury rate. A bank issuing a monopoly loan earns profits on the clients who take no other loan, and makes losses on those who do take any other loan. A bank lending at usury rates makes profits on clients who signed the monopoly contract with a competitor, and makes losses those with another usury contract.

The reason why there must be some banks offering loans at usury rates is that in this region the value of collateral is so volatile that even the monopolistic contract does not satisfy the joint incentive constraint, and creditor protection is so poor that an outside bank lending to an entrepreneur who has already taken a monopoly contract, must charge more than the monopoly rate. Entrepreneurs are willing to take loans at such a high rate because the usury loan allows them to appropriate part of the large project's return, while by defaulting they avoid paying these high rates in the bad state. In equilibrium, users receive more applications from entrepreneurs who are more likely to default.

4.2 Empirical predictions

Before introducing information sharing in the model, let us discuss some testable implications of the foregoing results concerning the effects of creditor rights protection. Improving creditor protection corresponds to a lower ϕ in Figure 2. In a country with low variability of borrowers' wealth (low σ), strengthening creditor rights may shift the economy from region B to region A . If in region B the credit market features rationing, the shift to the competitive equilibrium of region A implies easier access to credit and lower default rates. If instead in region B the market features a non-competitive equilibrium, then shifting to region A implies more intense banking

competition and lower interest rates. In a country with high variability of borrowers' wealth (high σ), the same reform occurs may shift the economy from region D to B , which could also eliminate of rationing.

In summary, the model predicts that a creditor-friendly reform increases the availability of credit, and reduce default rates and interest rates by increasing banking competition and reducing strategic defaults. These predictions are consistent with cross-country data and with U.S. data on interstate differences in bankruptcy law. La Porta et al. (1997) and Djankov et al. (2007) show that the breadth of credit markets is positively correlated across countries with measures of creditor rights protection. Along the same lines, Gropp, Scholz and White (1997) find that households living in states with comparatively high exemptions are more likely to be turned down for credit, borrow less and pay higher interest rates; and White (2006) shows that debt forgiveness in bankruptcy harms future borrowers by reducing credit availability and raising interest rates.

5 Equilibria with information sharing

We now turn to the case where banks share information on entrepreneurs' borrowing histories, and in particular on their total exposure. This form of information sharing, which is widespread in credit markets, helps banks to guard against the risk of default, by conditioning loan offers on the applicants' financial exposure.

As in the previous section, we continue to assume that creditors recover on a *pro-rata* basis in case of default. Natural enough in markets with no information sharing, where no lender knows customers' past indebtedness, such an assumption may be questionable when information sharing makes each lender aware of his seniority. Nevertheless, *pro-rata* liquidation occurs even in countries where credit bureaus are widespread. For instance, in the U.S. consumer credit market, the assets of defaulting borrowers (above a minimum threshold) are liquidated *pro-rata* under Chapter 7 (Parlour and Rajan, 2001). Retaining *pro-rata* liquidation also facilitates the comparison with the no information sharing case analyzed so far. The extension to seniority-based liquidation is left to Section 6.2.

The effects of information sharing are illustrated in Figure 3. Comparing it with Figure 2 shows that information sharing changes the equilibrium configuration by shifting the boundaries

between regions from the dashed lines (the boundaries in Figure 2) to the solid ones. Moreover, information sharing eliminates rationing equilibria.

[Insert Figure 3]

Specifically, the efficient and competitive region expands, from region A in Figure 2 to A' in Figure 3. Now in the area between the dashed line $\underline{\phi}(\sigma)$ and the solid line $\underline{\phi}'(\sigma)$, borrowers willing to switch to the large project can no longer obtain an additional loan at the competitive rate, because banks can refuse lending to borrowers who have already taken a loan and because outside banks cannot profit from opportunistic lending (joint incentive compatibility being satisfied in this area). As they no longer fear entrepreneurs playing them one against another, banks are now willing to offer loans of size x at the competitive rate in equilibrium.

What is more, in area A' all non-competitive equilibria disappear because information sharing allows outside lenders to safely undercut incumbents: starting from an equilibrium candidate with non-competitive rates, any bank can offer a better rate to any entrepreneur who is not yet indebted.

A second effect of information sharing is that rationing equilibria disappear in area B . To see why, recall that absent information sharing, in region B some entrepreneurs take two loans at the monopolistic rate and default. But with information sharing, instead, no bank will give a second loan, since it would anticipate that the double borrower would default and inflict losses on all his lenders.

The two effects just described – expansion of the competitive and efficient region, and removal of credit rationing – underscore the positive side of information sharing, its tendency to enhance efficiency by mitigating the contractual externalities of non-exclusivity. To summarize:

Proposition 5 *Under information sharing, the region with a unique, efficient and competitive equilibrium expands from A to A' . This region is non-empty and for $\Delta v > x/2$ coincides with the whole parameter space $[0, 1]^2$. In region $B' \subset B$ there is a unique non-competitive equilibrium with no rationing. The region where only the inefficient project is funded is the same as without information sharing: $C' = C$.*

Information sharing may also have a side however. This emerges in region D' (which coincides with region D in Figure 2). Here too, rationing disappears, but in this region information sharing

induces a unique equilibrium with market collapse. Indeed, in this region neither the individual nor the joint incentive constraints are met, so that already funded entrepreneurs are willing to take additional loans at the expenses of the no- usury rates, and outside banks are willing to offer them credit (as they expect to recover their money at the expense of the non-usury lenders). Absent information sharing, even usurers must worry about the risk of lending to a customer already indebted with another usurer: in this area the large project is not viable, so that two usurers dealing with the same client lose money. In equilibrium this limits lending at usury rates, but with information sharing usurers can easily target all clients not indebted with other usurers. In so doing, though, they make lending unprofitable for any bank charging lower rates, and thereby cause the entire loan market to collapse :

Proposition 6 *In region D' there is a unique equilibrium with market collapse.*

It may seem paradoxical that in region D' information sharing reduces efficiency even though it mitigates contractual externalities. Actually, however, in this region contractual externalities between usurers were beneficial in the absence of information sharing: banks lending at usury rates had to worry about customers playing them one against the other, which kept them from competing too aggressively against non-usury lenders. Information sharing dispenses them from this concern, but their more aggressive lending then kills off the market.

5.1 Empirical predictions: effects of information sharing

Our results furnish a number of testable predictions on how information sharing about past indebtedness should affect credit market performance. In most parameter regions, the effect is an increase in banking competition or the removal of credit rationing. In either case, the availability of credit should increase and interest rates should fall, and in the second case default rates should also decline. These are the same effects that would be triggered by better legal protection of creditors, as illustrated in Section 4.2. Hence, introducing a public credit register can be regarded as a substitute for strengthening creditor rights, consistent with the evidence of Djankov et al. (2007) and Brown et al. (2008).

However, this substitutability does not hold in region D' of Figure 3, where borrowers' collateral value is very volatile (σ large). Here information sharing induces market collapse,

whereas a sufficiently great improvement in creditor protection would ease access to credit and reduce default and interest rates, by shifting the economy into region C' . So, in region D' information sharing – far from substituting for investor protection – worsens credit market performance. The empirical counterpart here may be the situation experienced in poor rural areas of several developing countries, where potential borrowers are farmers with very risky wealth and limited risk sharing opportunities, while lenders often charge usury rates. In such environments, information sharing would enhance the users' ability to target clients, and so disrupt the viability of lending at non-usury rates. However, in this region lenders would never want an information sharing system to be introduced, as it would put them all out of business.

6 Extensions

So far, our analysis has proceeded so far under two simplifying assumptions: that in the information sharing regime banks learn their customers' indebtedness at the contracting stage only; and that liquidation of defaulted loans is *pro rata*. Now, we show that when either assumption is relaxed, our main qualitative results survive, and indeed that the beneficial effects of information sharing are amplified.

6.1 Full information sharing and loan covenants

The information sharing system described in Section 5 does not allow banks to monitor the subsequent credit exposure of its customers. However, one may envisage a situation in which banks use a credit register to check exposures even after lending: we call this a regime of “full information sharing”. In this regime, banks can impose loan covenants setting limits on borrowers' total indebtedness. That is, their loan can include – in addition to the interest rate and the amount – a callability clause that forces early liquidation and repayment if the customer's total indebtedness exceeds a specified threshold at $\bar{\tau}$ – just before the investment is made. If this early liquidation entails no costs, callability clauses effectively enable banks to enforce exclusivity, so to perfectly protect themselves against opportunistic borrowing by customers as well as opportunistic lending by competitors:¹⁴

¹⁴This proposition is self-evident and therefore its proof is omitted.

Proposition 7 *With full information sharing and no liquidation costs, for all ϕ there is a unique efficient and competitive equilibrium.*

The effectiveness of the protection afforded by the callability clause is inversely related to the cost of exercising the call. In reality, these costs are often non-negligible. For instance, banks may incur judicial costs if borrowers refuse to comply. Alternatively, if information about total indebtedness were to reach the lender after the entrepreneur has already invested in the project, liquidation costs may reduce the recoverable amounts, making such covenants less effective. Thus, full information sharing will eliminate the contractual externalities from multiple-bank lending only if information about indebtedness is timely. If it comes too late to stop the investment in the large project, full information sharing is equivalent to the regime described in Section 5.

6.2 Information sharing and seniority

To this point, defaulted debts were assumed to be liquidated *pro rata*. But the creation of an information sharing mechanism (such as a credit register) may facilitate the seniority ranking of creditors, thus allowing the enforcement of seniority-based liquidation in case of default. For instance, the land registries used to record mortgage claims – the ancestors of modern information sharing arrangements – served the dual purpose of enabling lenders to verify the residual collateral of credit applicants and of documenting the seniority of their claims. Accordingly, now we assume that when information sharing is introduced, the liquidation of defaulted claims is based on seniority. As we shall see, the beneficial effects of information sharing are strengthened.

Assuming that defaulted debts are repaid according to seniority changes the definition of banks' payoffs: for each history $h^{\bar{\tau}}$ and each realization \tilde{w} , bank τ 's expected profit

$$\pi_{\tau}^n(h^{\bar{\tau}}, \tilde{w}) = E_{\tilde{w}}[r_{\tau}^n(h^{\bar{\tau}}, \tilde{w}) - l_{\tau}]$$

from the loan contract c_{τ} is now defined by the effective repayment:

$$r_{\tau}^n(h^{\bar{\tau}}, \tilde{w}) = \min \{ r_{\tau}, \max \{ 0, (1 - \phi_n)y_n + \tilde{w} - R^{\tau}(h^{\bar{\tau}}) \} \}.$$

This expression takes the seniority of bank τ into account, as the resources it can get in case of

liquidation are net of the repayments to more senior lenders, $R^\tau(h^\tau) = \sum_{\tau' < \tau} \hat{r}_{\tau'}$. For instance, with only two lenders $R^2(h^\tau) = \hat{r}_1$, where \hat{r}_1 is the repayment to the senior bank 1. Clearly, the junior bank 2 appropriates a smaller share of the borrower's assets than under *pro rata* liquidation, which weakens the temptation to lend opportunistically. As a result, the area where the senior bank can safely fund the small project at the competitive rate expands from region A' in Figure 3 to A'' in Figure 4. Moreover, the $\phi_m(\sigma)$ locus shifts downward and to the right compared to Figure 3: region B' , where banks charge non-competitive rates, expands to B'' in Figure 4 at the expense of region D' . This reflects the fact that with seniority-based liquidation a senior bank offering a small loan at the monopoly rate can protect its claim against outside banks better, so that non-competitive equilibria occur in a larger region of the parameter space $[0, 1]^2$. Finally, the $\hat{\phi}$ horizontal locus, above which the large project is not viable, remains unaffected, its height being determined only by the technological parameters y_L and x .

[Insert Figure 4]

Formally, the new locus $\underline{\phi}''(\sigma)$ in Figure 4, below which all entrepreneurs invest in the efficient project and get credit at the competitive rate x , is again defined by the joint incentive constraint. Now, however, while condition (8) remains unaffected, the repayment r' appearing in (9) must enable the junior entrant to break even, taking into account its lower seniority rights. Thus, the zero profit condition of the junior bank becomes:

$$\mathbb{E}_{\tilde{w}}[\pi(r', \tilde{w})] = \mathbb{E}_{\tilde{w}}[\min \{r', \max \{0, (1 - \phi)y_L + \tilde{w} - x\}\}] - x = 0, \quad (11)$$

where the term in square brackets indicates that in case of default the junior bank gets what is left over once the senior bank has been repaid.¹⁵ The $\underline{\phi}''(\sigma)$ locus is found by substituting the zero-profit condition (11) into the incentive constraint (8).

For the reason discussed above, $\underline{\phi}''(\sigma) > \underline{\phi}'(\sigma)$ for $\Delta v < x/2$. This condition is essential for the problem at hand to be interesting, since it is what ensures that the efficient and competitive region does not embrace the entire admissible parameter space, as seen in Proposition 5. The $\underline{\phi}''(\sigma)$ locus is decreasing in σ and has the same slope $1/y_L$ as the locus $\underline{\phi}'(\sigma)$ in Figure 3.

¹⁵The junior bank never finds it profitable to charge a rate higher than the maximum that it can be repaid in the good state.

The expansion of the competitive and efficient region comes at the expense of region C' in Figure 3, which shrinks to region C'' in Figure 4: in this area, funding the large project at competitive rates is still the unique equilibrium and is supported by the same strategies described in the analysis of the regime with no information sharing.

Finally, the region where information sharing triggers market collapse shrinks from D' to D'' , because of a rightward shift of the locus $\phi_m(\sigma)$, above which no bank is willing to provide additional lending if the incumbent is charging the monopoly rate $r_m = y_S$. The new locus $\phi'_m(\sigma)$ is thus defined by the zero profit condition of the junior bank funding an entrepreneur with indebtedness x . Since now it can at most expect the repayment $\max\{0, (1 - \phi)y_L + 1 - \sigma - y_S\}$, a junior bank obtains expected profit

$$\pi'(\phi) = E_{\tilde{w}}[\max\{0, (1 - \phi)y_L + \tilde{w} - y_S\}] - x. \quad (12)$$

The condition $\pi'(\phi) = 0$ implicitly defines the function $\phi'_m(\sigma)$. This condition differs from (3) in that the maximal repayment that the junior bank gets from defaulting debtor is lower with seniority than in the *pro rata* regime. This also explains why $\phi'_m(\sigma)$ lies below $\phi_m(\sigma)$: senior banks can protect themselves against opportunistic lenders in situations where poor creditor protection would not allow them to do so under *pro rata* liquidation.

Summarizing, these shifts enlarge the region where the small and efficient project can be funded (from $A' \cup B'$ to $A'' \cup B''$), and also the region where it can be funded at competitive rates (from A' to A''). Conversely, region C' where the large and inefficient project is funded shrinks to C'' . This reflects the idea that seniority-based liquidation mitigates the contractual externality arising from opportunistic lending; that is, it expands the area where the joint incentive constraint is met – a result that echoes Bisin and Rampini (2005). Interestingly, this extension indicates that the benefit for credit market performance is greater when information sharing is used jointly with seniority-based rather than *pro-rata* liquidation.

7 Concluding remarks

When people can borrow from several competing banks, lending of each contributes to the customer's default risk. We show that the magnitude of this contractual externality among

banks depends on the degree of creditor rights protection and on the volatility of the value of collateral. When creditor rights are well protected, this externality is absent or tenuous, so that banks can lend at competitive rates without fearing that their customers will take additional loans. When creditor protection is in an intermediate range, this externality generates equilibria with excessive lending – as in Bizer and DeMarzo (1992) – or with *de facto* monopoly – as in Parlour and Rajan (2001). Finally, when creditor rights are poorly protected, a novel type of rationing equilibrium emerges, where not all entrepreneurs have access to credit; when the value of borrowers' collateral is sufficiently volatile, this equilibrium is unique and involves credit market segmentation and the charging of usury rates by some lenders. A testable implication of the model is that better creditor rights protection invariably reduces default rates but has a non-monotonic effect on the volume of lending.

Information sharing among banks is the other key institutional variable that affects the equilibrium configuration. Information sharing mitigates the contractual externalities arising from multiple-bank lending by allowing banks to condition their loans on the borrower's contractual history, so to guard themselves against opportunistic lending by competitors. As a result, information sharing expands the region where loans are offered at competitive rates and efficiency prevails; and it eliminates the credit rationing that occurs when collateral volatility takes intermediate values. But if the value of collateral is very volatile, information sharing has a drawback. Banks charging usury rates can exploit the additional information to better target creditworthy customers, which makes the market unviable for non-usury lenders. The outcome is credit market collapse.

Most of the paper deals with a minimal form of information sharing, by which lenders can only learn the total past indebtedness of their credit applicants. Extending the model, however, we also analyze richer institutional arrangements whereby banks can also condition their lending policy on customers' future indebtedness or can enforce seniority claims against their competitors. In both of these extensions, the benefits of information sharing systems are amplified, because communication allows banks to better deter the opportunistic behavior that can stem from multi-bank lending.

Appendix: Proofs

In characterizing credit market equilibria we restrict banks' strategy set by positing that loans are only of size x or $2x$. Under this assumption, an entrepreneur will find it optimal to take credit from at most two banks (even if he may well find it convenient to apply to many banks, given the sequential nature of our negotiation process).

Consistent with the notation used, the utility that an entrepreneur obtains if he signs both the loan contracts $c = (x, r)$ and $c' = (x, r')$ is equal to:

$$u(c, c') = \phi y_L + E_{\tilde{w}} [\max \{0, (1 - \phi) y_L + \tilde{w} - r - r'\}],$$

and the expected profit of a bank offering the loan contract c to an entrepreneur who also signs c' with another bank is equal to:

$$\pi(c, c') = E_{\tilde{w}} [\min \{r, r(\tilde{w})\}] - x,$$

where for $\tilde{w} \in \{\bar{w} - \sigma, \bar{w} + \sigma\}$:

$$r(\tilde{w}) = \max \left\{ \frac{(1 - \phi) y_L + \tilde{w}}{2}, (1 - \phi) y_L + \tilde{w} - r' \right\}.$$

Finally, $c^{PC} = (x, x)$ and $c^m = (x, y_S)$ denote the contracts offering the perfectly competitive and the monopolistic rate, respectively.

The following lemmas are preliminary to the proofs of the paper's main results.

Lemma 1 *The following properties hold:*

- (i) *For any pair of contracts $c = (x, r)$ and $c' = (x, r')$, with $r' \geq r \geq x$, $(1 - \phi) y_L + 1 - \sigma - r - r' < 0$ is a necessary condition for $u(c, c') > u(c, c_\emptyset)$.*
- (ii) *For any contract $c = (x, r)$ such that $r \geq x$ and $2r > (1 - \phi) y_L + 1 - \sigma$, $u(c, c) \geq u(c, c_\emptyset)$ (resp. $<$) if $\phi \geq \underline{\phi}(\sigma)$ (resp. $<$), with:*

$$\underline{\phi}(\sigma) = \max \left\{ \frac{y_S + 1 - r}{y_L}, \frac{1 + \Delta v + v_S - \sigma}{y_L} \right\}.$$

Proof. The proof of part (i) follows immediately from the fact that the NPV of the small project exceeds that of the large one, this proof is thus omitted. As for part (ii), straightforward calculations imply that for $\phi = \underline{\phi}(\sigma)$,

$$\begin{aligned} u(c, c) - u(c, c_\emptyset) &= \phi y_L + E_{\tilde{w}} [\max \{0, (1 - \phi) y_L + \tilde{w} - 2r\}] - (y_S + 1 - r) = \\ &= \phi y_L + \frac{1}{2} \max \{0, (1 - \phi) y_L + 1 + \sigma - 2r\} - (y_S + 1 - r) = 0, \end{aligned}$$

which immediately implies the result, since $u(c, c) - u(c, c_\emptyset)$ is monotonically increasing in ϕ . ■

Lemma 2 *Entrepreneurs' and banks' payoff functions satisfy the following properties:*

- (i) *For any pair of contracts $c = (x, r)$ and $c' = (x, r')$ such that $\pi(c', c) > 0$, $(1 - \phi)y_L + 1 + \sigma < r + r'$ and $r < y_S$, there exists a contract $\hat{c} = (x, \hat{r})$ such that $\pi(\hat{c}, c) \geq \pi(c', c)$ and $(1 - \phi)y_L + 1 + \sigma \geq r + \hat{r}$.*
- (ii) *The contract $c = (x, r)$ satisfies the joint incentive compatibility constraint if and only if one of the following inequalities holds:*

$$\phi > \phi_m(\sigma), \quad (\text{A1})$$

$$\phi \leq \underline{\phi}'(\sigma), \quad (\text{A2})$$

where

$$\phi_m(\sigma) = \hat{\phi} + \max \left\{ 0, \frac{\sigma - 2(r - x)}{3y_L} \right\}$$

for $r = y_S$, and

$$\underline{\phi}'(\sigma) = \max \left\{ \frac{y_S + 1 - r}{y_L}, \frac{1 + 4\Delta v + y_L - x - r - \sigma}{y_L} \right\}$$

for $r = x$.

Proof. The proof of part (i) follows immediately from the continuity of banks' expected profits and is accordingly omitted. As for part (ii), let $C(c)$ denote the set of all contracts yielding non-negative profits if offered to an entrepreneur who has already signed contract c . Joint incentive compatibility then requires at least one of the two following conditions to hold:

$$C(c) = \emptyset \text{ or } u(c, c_\emptyset) > \max_{c' \in C(c)} u(c, c').$$

The set $C(c)$ contains all the contracts $c' = (x, r')$ such that $\pi(c', c) \geq 0$. It is empty if and only if the expected maximal profit to the bank issuing c' is negative, that is

$$\mathbb{E}_{\tilde{w}} \left[\max \left\{ (1 - \phi)y_L + \tilde{w} - r, \frac{(1 - \phi)y_L + \tilde{w}}{2} \right\} \right] - x < 0, \quad (\text{A3})$$

where $(1 - \phi)y_L + \tilde{w} - r$ is the maximum amount that a bank offering c' can obtain in case of no default after the bank offering $c = (x, r)$ has been repaid, while $[(1 - \phi)y_L + \tilde{w}]/2$ is its repayment in case of default. It is then immediate to verify that (A3) holds as long as (A1) is met. Otherwise, in the parameter region where (A1) does not hold, for c to satisfy joint incentive compatibility one must have $u(c, c_\emptyset) > \max_{c' \in C(c)} u(c, c')$. Using (8) and (9) one can immediately show that this is true as long as (A2) is met. ■

In what follows we shall denote by $\underline{c} = (x, \underline{r})$, with $\underline{r} = 2x - ((1 - \phi)y_L + 1 - \sigma)/2$, the contract whose rate solves equation (9), which represents the zero-profit condition of the outside

bank for $\phi < \phi_m(\sigma)$, in the definition of the joint incentive compatibility constraint. This contract, which exists by Lemma 2, earns zero profit if accepted together with any $\hat{c} = (x, \hat{r})$, with $\hat{r} \geq x$. Moreover, we shall also denote by c^* the contract whose rate solves $u(c^*, \underline{c}) = u(\underline{c}, c_\emptyset)$, and by \tilde{c} that solving $u(\tilde{c}, c^{PC}) = u(c^{PC}, c_\emptyset)$.

Proof of Proposition 1

Consider a candidate equilibrium $*$ where (i) all banks offer the contract c^{PC} and extend credit to all applicants irrespective of the history they have observed; (ii) if only c^{PC} is offered, each entrepreneur applies to a non-empty subset of active banks, signs c^{PC} with one of them, and undertakes the efficient project; otherwise, he applies for all contracts and undertakes his preferred project; in addition, when indifferent between two loan contracts, he randomizes with equal probability among the banks offering them; (iii) banks' and entrepreneurs' beliefs are such that a bank offering $c = (x, r)$ with $r > x$ accepts all applications.

For $\phi < \underline{\phi}(\sigma)$, (i)-(iii) identify a PBE. Indeed, no entrepreneur can profitably deviate since the individual incentive constraint is satisfied as shown in Lemma 1, so that $u(c^{PC}, c_\emptyset) > u(c^{PC}, c^{PC})$; and no bank can profitably deviate since $\underline{\phi}(\sigma) < \underline{\phi}'(\sigma)$ implies that the joint incentive constraint is also satisfied in the region under consideration. Indeed, given the off-equilibrium beliefs specified in (iii), it is sequentially rational for bank τ to offer c^{PC} after observing a deviation $c_{\tau'} = (x, r)$, with $r_{\tau'} > x$, by a bank $\tau' < \tau$.

In order to prove uniqueness, we need to show that for $\phi < \underline{\phi}(\sigma)$ there exists no equilibrium where either $c' = (2x, r')$ or $c'' = (x, r'')$, with $r' \geq 2x$, and $r'' > x$, is signed by any entrepreneur. The condition $\phi \leq \underline{\phi}(\sigma)$, together with the continuity of the entrepreneurs' expected utility, implies $u(c_\varepsilon, c_\emptyset) > u(c_\varepsilon, c')$ for $c_\varepsilon = (x, r = x + \varepsilon)$, with $\varepsilon > 0$ and sufficiently small, $c = (x, r')$, and $r' > x + \varepsilon$. As a consequence, if c^{PC} is not offered, and some bank is earning zero profits in equilibrium, it can profitably deviate by offering c_ε . Therefore a necessary condition for contracts charging non-competitive rates to be signed in equilibrium is that all banks earn positive profits. We conclude the proof by showing that under **A5** there exists no equilibrium where more than one bank earns profits for $\phi \leq \underline{\phi}(\sigma)$: first, in equilibrium all entrepreneurs will take either the contract at the lowest rate or the two contracts at the two lowest rates; second, each active bank issuing $c = (r, x)$ with $r > x$ makes positive profit and therefore will accept all applications; third, if two or more banks offer $c = (x, r)$, each of their competitors will find it profitable to undercut this offer by offering $c = (x, r - \varepsilon)$. ■

Proof of Proposition 2

Part (i) The proof of this part is developed in two steps.

Step 1 From lemma 1, it follows that $r^* > \underline{r} > x$ for $\phi > \underline{\phi}'(\sigma) > \underline{\phi}(\sigma)$, since in the parameter region under consideration the individual incentive compatibility is not met, that is $u(c, c) > u(c, c_\emptyset)$ for any $c = (x, r)$, and $u(c, \underline{c}) - u(\underline{c}, c_\emptyset)$ is decreasing in r . We now show that for $\phi > \max\{\underline{\phi}'(\sigma), \phi_m(\sigma)\}$ and $\phi > \hat{\phi}$, there exists an unique PBE where all entrepreneurs sign contract $\hat{c} = (x, \hat{r})$, with $\hat{r} = \min\{r^*, y_S\}$ with bank 1, and the equilibrium strategies are

as follows: (i) Bank 1 offers $\widehat{c} = (x, \widehat{r})$, and extends credit to all applicants. (ii) Bank 2 offers \underline{c} and accepts all applications if bank 1 has offered $c_1 = (x, r_1)$, with $r_1 > r^*$, and remains inactive otherwise. (iii) Bank $\tau > 2$ issues \underline{c} and accepts all applications whenever bank τ' , for $\tau' < \tau$, offers $c_{\tau'} = (x, r_{\tau'})$ with $r_{\tau'} < \underline{r}$, and no other bank τ'' , for $\tau < \tau'' < \tau'$, has offered \underline{c} ; it remains inactive otherwise. (iv) If only \widehat{c} is offered by bank 1, each entrepreneur borrows from this bank and undertakes the small project; otherwise he applies for all contracts and undertakes his preferred project; if indifferent between two loans, he randomizes between them, and if indifferent between the large and the small project he chooses the large one. (v) If bank τ deviates from its equilibrium strategy by offering $c = (x, r)$ with $r \neq x$, all the other players believe that it will reject all applications for this contract.

First, we show that bank 1 cannot profitably deviate by increasing its rate above \widehat{r} . This is obvious for $\widehat{r} = y_S$. If instead $\widehat{r} < y_S$, setting $r_1 > \widehat{r} = r^*$ would induce bank 2 to offer \underline{c} while all other banks would remain inactive. From the definition of c^* , it follows that all entrepreneurs prefer to borrow from bank 2 only, so that bank 1 would earn zero profits. To prove that bank τ , with $\tau > 1$, cannot profitably deviate, one needs to show that it cannot attract clients by offering $c = (x, r)$, with $x \leq r \leq r^*$. First, for any $c = (x, r)$ with $r \in (\underline{r}, r^*)$, $u(c^*, c) > u(c, c_\emptyset)$, since $u(c^*, c) - u(c, c_\emptyset)$ is increasing in r and $u(c^*, \underline{c}) = u(\underline{c}, c_\emptyset)$. As a consequence, c , if offered, is signed by all entrepreneurs together with c^* . Moreover, c makes losses, because the large project is not viable, implying that at least one contract between c^* and c must make losses, and $r^* > r$. Second, if bank τ issues $c = (x, r)$ with $r < \underline{r}$, and bank $\tau + 1$ responds by issuing \underline{c} and accepting all applications for this contract, entrepreneurs take both c and \underline{c} since in the parameter region under consideration $u(c', c) - u(c, c_\emptyset) > 0$ by Lemma 2. Moreover, bank $\tau + 1$ earns zero profits on each entrepreneur signing both \underline{c} and c as the zero-profit condition (9) is satisfied for $\phi > \phi_m(\sigma)$. Since the large project is not viable, offering c is not profitable. Finally, bank τ 's strategy satisfies sequential rationality. Indeed, given the beliefs specified in (v), it is profitable for this bank to offer \underline{c} whenever bank $\tau - 1$ offers $c_{\tau-1} = (x, r_{\tau-1})$, with $r_{\tau-1} < \underline{r}$, given the strategy of bank $\tau + 1$.

We can now prove that under **A5** in the region under consideration there is no PBE where all entrepreneurs obtain credit at a rate different from \widehat{r} . The proof is by contradiction. Suppose first that there exists a PBE where a single bank lends to all entrepreneurs at the rate $r > r^*$. By construction, any inactive bank can profitably offer \underline{r} . Second, if $r \in (\underline{r}, r^*)$, the same logic developed in the first part of the proof implies that the incumbent can increase its rate up to r^* without inducing any reaction by competing banks. Third, if $r < \underline{r}$, by construction of \underline{r} there exists a contract (x, r') that can be profitably offered by a deviant bank. Moreover, for $\phi > \phi'(\sigma) > \phi(\sigma)$, entrepreneurs will find it rational to sign both (x, r) and (x, r') , thereby inducing the incumbent bank to make losses. Finally, it is straightforward to verify that for $\phi > \phi_m(\sigma) \geq \widehat{\phi}$ there cannot exist either an equilibrium where some entrepreneurs undertake the large project, or an equilibrium where all entrepreneurs are served and more than one bank is active.

Step 2 We will show that for $\phi \geq \max\{\underline{\phi}(\sigma), \widehat{\phi}\}$ and $\phi \leq \underline{\phi}'(\sigma)$, there exists an efficient, non-competitive PBE where all entrepreneurs undertake the small project. Consider the following strategy profile: (i) Bank 1 offers $\widehat{c} = (x, \widehat{r})$, with $\widehat{r} = \min\{\widetilde{c}, y_S\}$, where \widetilde{c} solves $u(\widetilde{c}, c^{PC}) = u(c^{PC}, c_\emptyset)$, and extends credit to all applicants. (ii) Bank τ , with $\tau > 1$, offers c^{PC} and accepts all applications for this contract whenever bank τ' , with $\tau' < \tau$, offers $c_{\tau'} = (x, r_{\tau'})$ with $r_{\tau'} > \widetilde{r}$ and no other bank τ'' with $\tau' < \tau'' < \tau$; it remains inactive otherwise. (iii) If only \widehat{c} is offered by bank 1 each entrepreneur borrows from this bank and undertakes the small project; otherwise he applies for all contracts and undertakes his preferred project; if indifferent between two loans, he randomizes between them, and if indifferent between the large and the small project he chooses the large one. (iv) If bank τ deviates from its equilibrium strategy by offering $c = (x, r)$ with $r \neq x$, all the other players believe that it will reject all applications for this contract.

Note first that by Lemma 1 $\widetilde{r} > x$ for $\phi \geq \underline{\phi}(\sigma)$. We begin the existence proof by showing that bank 1 cannot profitably deviate by increasing its rate above \widehat{r} . This is obvious for $\widehat{r} = y_S$; if $\widehat{r} < y_S$, setting $r_1 > \widehat{r} = \widetilde{r}$ would induce bank 2 to offer c^{PC} while all other banks would remain inactive. By definition of \widetilde{c} , $u(c_1, c^{PC}) < u(c^{PC}, c_\emptyset)$ and all entrepreneurs prefer to borrow from bank 2 only, so that bank 1 would earn zero profits. To prove that bank τ , with $\tau > 1$, cannot profitably deviate, recall that the contract c^{PC} satisfies the joint incentive constraint in the region under consideration. Hence, bank τ cannot attract any client by offering $c = (x, r)$ with $r > \widetilde{r}$, because according to equilibrium strategies bank $\tau + 1$ would respond by offering the zero-profit contract c^{PC} , the entrepreneurs would take both contracts and, by definition of joint incentive compatibility, $c = (x, r)$ would make losses. Thus, for existence we need to show that no bank can profitably deviate by offering $c = (x, r)$ with $x \leq r \leq \widetilde{r}$. For any $c = (x, r)$ with $r \in [x, \widetilde{r}]$ it must be $u(\widetilde{c}, c) \geq u(c, c_\emptyset)$ because $u(\widetilde{c}, c) - u(c, c_\emptyset)$ is increasing in r and $u(\widetilde{c}, c^{PC}) = u(c^{PC}, c_\emptyset)$; this, in turn, implies that, if taken by some entrepreneurs, c makes losses because the large project is not viable; hence at least one contract in the set $\{c, \widetilde{c}\}$ must make losses and $\widetilde{r} > r$. The same argument used in step 1 allows to verify that banks' strategies are sequentially rational.

Proving uniqueness requires the same arguments developed above and relies on the fact that the contract c^{PC} satisfies the joint incentive constraint in the parameter region under consideration.

Finally, the same type of arguments used above allow to show that in the subset of region B where $\phi \geq \underline{\phi}(\sigma)$ and $\phi \leq \min\{\underline{\phi}'(\sigma), \widehat{\phi}\}$ there exists a unique efficient non-competitive equilibrium where all entrepreneurs undertake the small project and borrow at the rate $\widehat{r} = \min\{\widetilde{r}, y_S - (y_L - 2x)\}$. Indeed, since in this region the large project is viable, the equilibrium rate must satisfy the condition that entrepreneurs must not gain by borrowing $2x$ from a deviant bank and undertake the large project, that is, $y_S - r \geq y_L - 2x$. Notice also that in the parameter region under consideration, equilibria where some entrepreneurs are excluded from credit cannot exist. This is because for $\phi \leq \widehat{\phi}$ banks can profitably fund the large project by offering a loan of size $2x$ and attract all rationed entrepreneurs.

Part (ii) As before, consider first the parameter region where $\phi \geq \max \{ \phi_m(\sigma), \phi'(\sigma) \}$, and a bank τ offering contract $\hat{c} = (x, \hat{r})$ with $\hat{r} > x$ and accepting a fraction α of the applications received. The expected profit of this bank is

$$\alpha H(\alpha) = \alpha \left((1 - \alpha)^{\hat{\tau}-1} \pi(\hat{c}, c_\emptyset) + \pi(\hat{c}, \hat{c}) \sum_{k=1}^{\hat{\tau}-1} \frac{(\hat{\tau}-1)!}{k! (\hat{\tau}-1-k)!} \frac{2\alpha^k}{k} (1 - \alpha)^{\hat{\tau}-1-k} \right),$$

if $\hat{\tau} - 1$ other banks offer the contract \hat{c} and accept a fraction α of applications (while other banks remain inactive). The first term in parenthesis is the bank's expected profit if no other bank happens to accept the entrepreneur's applications, while the second is the bank's expected profit if at least one other bank does accept. As $H(\cdot)$ is continuous and monotone in α , $H(0) > 0$ and $H(1) < 0$ (because the large project is unviable so that $\pi(\hat{c}, \hat{c}) < 0$), it must be $H(\alpha^*) = 0$ for some $\alpha^* \in (0, 1)$.

We now show that the following conditions identify a PBE: (i) For any possible observable history, h^τ , with $\tau \leq \hat{\tau}$, bank τ issues $\hat{c} = (x, \hat{r})$, with $\hat{r} = \min \{ y_S, r^* \}$; moreover, if \hat{c} is also issued by all banks $\tau \in \{1, 2, \dots, \tau - 1\}$, bank τ accepts the fraction $\alpha^* \in (0, 1)$ of its applications by choosing randomly among applicants; and, for all possible histories such that only $\tau - 1 - n$ banks moving before τ issue \hat{c} , bank τ accepts the fraction $(1 + n)\alpha^*(h^{\bar{\tau}})$ of applications. (ii) Bank $\hat{\tau} + 1$, issues \hat{c} and accepts the fraction $(1 + n)\alpha^*(h^{\bar{\tau}})$ of applications for any possible history such that \hat{c} is the only contract issued before $\hat{\tau} + 1$, and only $\tau - 1 - n$ banks issue \hat{c} before $\hat{\tau} + 1$. Otherwise, it issues \underline{c} and accepts all applications for this contract, if at least one of the banks moving before $\hat{\tau} + 1$ has issued $c_\tau = (x, r_\tau)$ with $r_\tau > r^*$. (iii) Bank $\hat{\tau} + 2$ issues \underline{c} and accepts all its applications, if at least one bank moving before $\hat{\tau} + 2$ issued $c = (x, r)$ with $r < \underline{r}$. (iv) If \hat{c} is the only contract issued by active banks, each entrepreneur chooses the sequence of applications and undertakes the large project whenever possible. Otherwise, he applies for all contracts and undertakes his project; when indifferent between two loans he randomizes between them. (v) If bank τ deviates from its equilibrium strategy by offering $c = (x, r)$ with $r \neq x$, all the other players believe that it will reject all applications for such a contract.

Since by Lemma 1 the individual incentive compatibility is not met in the region under consideration, so that $u(\hat{c}, \hat{c}) > u(\hat{c}, c_\emptyset)$, no entrepreneur can profitably deviate. Indeed $\hat{r} \leq y_S$ and $u(\hat{c}, \hat{c}) > u(\hat{c}, c_\emptyset)$ imply that entrepreneurs will find it rational to undertake the small project whenever they get only one loan, and the large project when they get two loans. Moreover, no active bank can profitably deviate by changing the fraction of accepted applications for \hat{c} because $E\pi_\tau(\alpha_\tau) = \alpha_\tau H(\alpha^*) = 0$. Finally, exactly the same type of reasoning used in the proof of part (i) implies that no active bank can profitably deviate by offering $c \neq \hat{c}$ and that banks' strategies are sequentially rational.

To complete the existence proof one must verify that if offering contract $c = (x, r)$ is not a profitable deviation for an active bank, then it is not profitable for any bank. It is easy to see that, given the specified equilibrium strategies of the first $\hat{\tau} + 1$ banks, any bank issuing contract c and accepting a fraction α of its applications earns the same expected profit irrespective of whether such a bank is active or inactive in equilibrium.

Finally, the same arguments developed above allow to show that in the parameter region where $\phi \geq \max\{\underline{\phi}(\sigma), \widehat{\phi}\}$ and $\phi \leq \underline{\phi}'(\sigma)$ there exists a zero-profit PBE where each bank $\tau \leq \widehat{\tau}$ issues the contract $\widehat{c} = (x, \widehat{r})$, with $\widehat{r} = \min\{\widetilde{r}, y_S\}$, and accepts the fraction $\widehat{\alpha} \in (0, 1)$ of applications for \widehat{c} , choosing randomly among applicants. ■

Proof of Proposition 3

Consider the following candidate equilibrium: (i) Bank 1 posts $c^L = (2x, 2x)$ and supplies credit to all applicants. (ii) Bank $\tau > 1$ posts c^L and offers credit to all applicants as long as each bank τ' , for $\tau' < \tau$, posted $c' = (2x, r')$ with $r' \geq 2x$, and offers \underline{c} if bank τ' posted $\widehat{c} = (x, \widehat{r})$ with $\widehat{r} \geq x$ and no other bank τ'' , for $\tau' < \tau'' < \tau$, issued \underline{c} . (iii) If only c^L is offered, each entrepreneur applies to a non-empty subset of active banks, signs c^L with one of them and undertakes the large project. Otherwise, he applies for all contracts and undertakes his preferred project; when indifferent between two loans he randomizes between them. (iv) If bank τ deviates from its equilibrium strategy by offering $c = (x, r)$ with $r > x$, all the other players believe that it will reject all applications for this contract, and that if bank τ offers $c = (2x, r)$ with $r > 2x$, it will accept all applications for this contract.

For the large project to be fundable, the pledgeable income of an entrepreneur undertaking this project must be positive, that is, $(1 - \phi)y_L + 1 - 2x \geq 0$. Hence $\phi \leq \widehat{\phi}$ is a necessary condition for $c^L = (2x, 2x)$ to be offered. Since c^L is the only contract issued in the equilibrium candidate described by (i)-(iii), to prove that these conditions identify an equilibrium one must only check that no bank can profitably deviate. To this end, note first that bank τ cannot profitably deviate by offering $c_1 = (2x, r_1)$, with $r_1 > 2x$, since according to (ii) c_1 is undercut by bank $\tau + 1$. Moreover, since in the parameter region under consideration the joint incentive constraint is not met, Lemma 2 implies that an entrepreneur accepting $\widehat{c} = (x, \widehat{r})$ with $\widehat{r} \geq x$ necessarily signs two contracts, say \widehat{c} and c' in equilibrium. Moreover, $c' = (x, r')$ must be such that $r' \geq \underline{r}$ since $\underline{c} = (x, \underline{r})$ earns zero profits if taken together with \widehat{c} . This in turn implies $\widehat{r} \geq \underline{r}$ since two banks offering \underline{c} earn zero profit on an entrepreneur signing this contract with both of them, and the profit that each bank earns on this entrepreneur is weakly decreasing in the rate charged by the other. One can then easily verify that signing both \widehat{c} (with $\widehat{r} \geq \underline{r}$) and \underline{c} makes the entrepreneur strictly worse off than in the equilibrium candidate, where his utility is $v_L + 1$. Hence, no bank can profitably deviate. Finally, notice that banks' strategies are sequentially rational, exactly by the same arguments used in the proof of Propositions 1 and 3.

For uniqueness, note first that for $\phi \leq \widehat{\phi}$ the large project is viable and can be safely funded at the competitive rate. **A5** then implies that neither an equilibrium where some banks fund this project at non-competitive rates nor one where a subset of entrepreneurs are excluded from credit can exist. Moreover, by Lemma 2 the joint incentive constraint is not met for $\phi \leq \widehat{\phi} \leq \phi_m(\sigma)$ and $\phi > \underline{\phi}'(\sigma)$. This, together with $\phi \leq \widehat{\phi}$, implies that there is no equilibrium where a subset of entrepreneurs undertake the small project.

It then remains to show that in region C there is no equilibrium where: (i) all entrepreneurs are served and undertake the large project; (ii) a fraction α of them sign two loans of size x while the fraction $1 - \alpha$ sign one loan; (iii) all active lenders make zero profits, while those

offering a loan of size x offer non-competitive rates and ration their credit applicants. For an equilibrium to feature these properties, it must be the case that the entrepreneurs who take two loans default only in the bad state, since the large project can be safely funded for $\phi \leq \widehat{\phi}$. Since the joint incentive constraint is not met in region C , by Lemma 2 each bank offering a loan of size x must charge a rate $r \geq \underline{r}$ to earn non-negative profits. Once more, one can then easily show that signing two contracts $c = (x, r)$ with $r \geq \underline{r}$ makes the entrepreneur worse off than signing $c = (2x, 2x)$. Hence, a PBE cannot feature properties (i)-(iii).

To conclude the proof, we show the parameter region where $\phi \in (\underline{\phi}'(\sigma), \widehat{\phi}]$, is non empty for $\Delta v < 1/4$: since $\phi'(\sigma)$ is decreasing in σ , a sufficient condition for this is that $\underline{\phi}'(\sigma) < \widehat{\phi}$ at $\sigma = 1$, that is

$$\widehat{\phi} - \underline{\phi}'(\sigma)|_{\sigma=1} = \frac{v_L + 1}{y_L} - \frac{v_L + 4\Delta v}{y_L} \geq 0,$$

which immediately yields $\Delta v < 1/4$. ■

Proof of Proposition 4

Before describing the PBE in this region, consider an equilibrium outcome where:

(I) The set of active banks contains n banks issuing $c^m = (x, r^m = y_S)$ and n' banks issuing $c^u = (x, r^u)$, with $r^u = (1 - \phi)y_L + 1 + \sigma - y_S$ being the maximal rate that does not induce default in the good state by an entrepreneur who accepts both c^m and c^u .

(II) Each bank issuing c^m (respectively c^u) signs this contract with the fraction α^m (respectively α^u) of applicants, and selects applications randomly.

Let $\alpha^c H(c, \mathbf{c}, \boldsymbol{\alpha})$ be the profit obtained by a bank issuing the contract c and providing credit to the fraction α^c of applicants when the vector of contracts $\mathbf{c} = (c_1, c_2, \dots, c_n)$ is also issued and the acceptance policies of the banks issuing these contracts are summarized by the vector $\boldsymbol{\alpha} = (\alpha_1, \alpha_2, \dots, \alpha_n)$.

Let $\tilde{\mathbf{c}}$ and $\tilde{\boldsymbol{\alpha}}$ denote two $n - 1 + n'$ dimensional vectors having their first $n - 1$ components equal to c^m and α^m respectively, and the remaining components equal to c^u and α^u , respectively. Under conditions (I) and (II) above, the expected profit that a bank issuing c^m earns on each unit of this contract is

$$H(c^m, \tilde{\mathbf{c}}, \tilde{\boldsymbol{\alpha}}) = f(\alpha^m)\pi(c^m, c^m) + g(\alpha^m, \alpha^u)\pi(c^m, c^u) + (1 - f(\alpha^m) - g(\alpha^m, \alpha^u))\pi(c^m, c_0),$$

where

$$f(\alpha^m) = \sum_{k=1}^{n-1} \frac{(n-1)!}{k!(n-1-k)!} \frac{2}{k} (\alpha^m)^k (1 - \alpha^m)^{n-1-k}$$

and

$$g(\alpha^m, \alpha^u) = (1 - f(\alpha^m)) \sum_{k=1}^{n'} \frac{n'!}{k!(n'-k)!} (\alpha^u)^k (1 - \alpha^u)^{n'-k}$$

are respectively the probability that at least two banks accept an entrepreneur's application for c^m provided one bank accepts it, and the probability that an entrepreneur who has successfully applied for c^m with a bank obtains no acceptance from any of the other banks offering this

contract and at least one acceptance from the banks offering the contract c^u .

Similarly, under conditions (I) and (II) the expected profit that each bank issuing c^u earns on each unit of this contract is:

$$\alpha^u H(c^u, \mathbf{c}', \boldsymbol{\alpha}') = f'(\alpha^m, \alpha^u) \pi(c^u, c^m) + g'(\alpha^m, \alpha^u) \pi(c^u, c^u)$$

where \mathbf{c}' and $\boldsymbol{\alpha}'$ are $n - 1 + n'$ dimensional vectors having their first n components equal to c^m and α^m respectively, and the remaining components equal to c^u and α^u , respectively; and

$$f'(\alpha^m, \alpha^u) = (1 - \alpha^m)^{n-1} \alpha^m \sum_{k=1}^{n'-1} \frac{(n'-1)!}{k! (n'-1-k)!} \frac{2}{k} (\alpha^u)^{k-1} (1 - \alpha^u)^{n'-1-k}$$

and

$$g'(\alpha^m, \alpha^u) = (1 - \alpha^m)^n \sum_{k=1}^{n'-1} \frac{(n'-1)!}{k! (n'-1-k)!} \frac{2}{k} (\alpha^u)^{k-1} (1 - \alpha^u)^{n'-1-k}.$$

Since $\phi < \phi_m(\sigma)$, we have $\pi(c^u, c^m) > 0$. This, together with the fact that the large project is not viable, implies $\pi(c^m, c^u) < 0$, $\pi(c^m, c^m) < 0$ and $\pi(c^u, c^u) < 0$.

As a first step to prove the existence of a PBE satisfying conditions (I) and (II) we show that the system

$$\alpha^m H(c^m, \tilde{\mathbf{c}}, \tilde{\boldsymbol{\alpha}}) = 0; \quad \alpha^u H(c^u, \mathbf{c}', \boldsymbol{\alpha}') = 0, \quad (\text{A4})$$

has a solution. To this end, note first that $H(c^u, \mathbf{c}', \boldsymbol{\alpha}') > 0$ for $\boldsymbol{\alpha}'$ such that $\alpha'_{n+k} = \alpha^u = 0$ for $0 < k < n'$ and $\alpha'_k = \alpha^m$ close to 1 for $k < n$, $H(c^u, \mathbf{c}', \boldsymbol{\alpha}') < 0$ for $\boldsymbol{\alpha}'$ such that $\alpha'_k = \alpha^m$ close to 0 for $k < n$, and $H(c^u, \mathbf{c}', \boldsymbol{\alpha}')$ continuous in α^m , imply that, for $\alpha'_{n+k} = \alpha^u = 1$, with $0 < k < n'$, there exists $\bar{\alpha}^m \in (0, 1)$, such that $H(c^u, \mathbf{c}', \boldsymbol{\alpha}') = 0$. Moreover, $\partial H(c^u, \mathbf{c}', \boldsymbol{\alpha}') / \partial \alpha'_k > 0$ for $k \leq n$ and $\partial H(c^u, \mathbf{c}', \boldsymbol{\alpha}') / \partial \alpha'_k < 0$ for $k > n$, imply, by the implicit function theorem, that for all $\alpha^m < \bar{\alpha}^m$ there exists a differentiable, increasing function $\alpha^u(\alpha^m)$, such that $H(c^u, \mathbf{c}', \boldsymbol{\alpha}') = 0$, for $\alpha'_k = \alpha^m$ for $k < n$ and $\alpha'_k = \alpha^u(\alpha^m)$ for $k > n$, and $\alpha^u(\alpha^m)$ close to 0 for α^m close to 0. Finally, $H(c^m, \tilde{\mathbf{c}}, \tilde{\boldsymbol{\alpha}})$ is continuous in $\tilde{\alpha}_k$ for all $k < n + n'$, it is positive for $\tilde{\alpha}_k = \alpha^m$ for $k \leq n$, $\tilde{\alpha}_k = \alpha^u(\alpha^m)$ for $k > n$ whenever α^m and $\alpha^u(\alpha^m)$ are both sufficiently small, and it is negative for $\alpha^m = \bar{\alpha}^m$ and $\alpha^u(\alpha^m) = 1$ since $1 - f(\alpha^m) - g(\alpha^m, \alpha^u) = 0$ for $\alpha^u = 1$. Hence there exists $\hat{\alpha}^m < \bar{\alpha}^m$ such that $H(c^m, \tilde{\mathbf{c}}, \tilde{\boldsymbol{\alpha}}) = 0$. This in turn implies the existence of a solution of (A4).

Let $\underline{c} = (x, \underline{r})$ be the contract such that $\pi(\underline{c}, c^m) = 0$, in the region where the joint incentive constraint is not satisfied, and let $\mathbf{c}^\tau = (c_1, c_2, \dots, c_{\tau-1})$ be the set of contracts issued by banks moving before stage τ . Finally, denote $(\mathbf{c}^{\tau*}, \boldsymbol{\alpha}^{\tau*})$ the vector of contracts and acceptance policies up to stage τ , defined by conditions (I) and (II) above. We now show that for $\phi > \max\{\hat{\phi}, \phi(\sigma)\}$, $\phi < \phi_m(\sigma)$ and $v_S < 1/2$, there exists a PBE where $(\mathbf{c}^\tau, \boldsymbol{\alpha}^\tau) = (\mathbf{c}^{\tau*}, \boldsymbol{\alpha}^{\tau*})$ for all τ supported by following strategy profile: (i) Bank $\tau = 1$ issues the monopoly contract c^m and extends credit to all applicants. (ii) Bank τ , for $\tau \in \{2, \dots, n\}$, issues c^m and accepts a fraction α^m of applications if all contracts in \mathbf{c}^τ are issued at rates equal to $r^m = y_S$; it issues \underline{c} and accepts all applications for this contract if $c_{\tau-k} = (x, r > r^m)$ for $2 < k < \tau - 1$ and $c_{\tau'} \neq (x, \underline{r})$ for

$\tau - k < \tau' < \tau - 1$, while it remains inactive if $c_{\tau+1-k} = \underline{c}$ for some $k \in \{1, 2, \dots, \tau - 1\}$. (iii) Bank τ , for $\tau \in \{n + 1, \dots, n + 1 + n'\}$ issues c^u and accepts a fraction α^u of applications if and only if n of the banks moving before it offered rates lower than or equal to r^u , and all the others offered contracts with rates equal to or higher than r^u . Whenever this is not the case, and there exists c such that $H(c, \mathbf{c}^{\tau-1}, \boldsymbol{\alpha}^{\tau-1}) = 0$, bank τ offers c or remains inactive. (iv) Bank τ , with $\tau > n + 1 + n'$, remains inactive as long as each bank $k' \in \{1, \dots, n\}$ has issued c^m and each bank $\tau'' \in \{n + 1, \dots, n + 1 + n'\}$ has issued c^u . Whenever this is not the case, and there exists a zero profit contract c such that $H(c, \mathbf{c}^{\tau*}, \boldsymbol{\alpha}^{\tau*}) = 0$, bank τ offers c or remains inactive. (v) Entrepreneurs apply for all contracts and, given the acceptances, undertake their preferred project. (vi) If bank τ deviates from its equilibrium strategy by offering $c = (x, r)$ with $r \neq x$, all the other players believe that it will reject all applications for this contract.

We start by considering possible deviations of the banks moving in the first n stages. First, none of these banks can profitably deviate by offering a contract $c = (x, r)$ with $r > y_S$. Indeed, according to equilibrium strategies this deviation would induce bank $\tau + 1$ to offer the contract \underline{c} and to accept all applications for this contract. As a consequence, if bank τ , for $\tau < n$, were the first to deviate by offering $c = (x, r > y_S)$, it would find it rational to refuse all applications for this contract, implying that the deviation to c would not yield any profit. Thus, none of the first n banks will find it profitable to deviate. In addition, sequential rationality is satisfied as \underline{c} does not make losses in the case where no application for c is accepted.

Moreover, none of the banks offering c^m in the equilibrium candidate can gain by lowering its rate. Indeed, suppose bank τ offers $c = (x, r < y_S)$. Since no bank moving after τ changes its equilibrium behavior after observing the issuance of $c = (x, r)$ with $r < y_S$, bank τ will end up making losses, unless it refuses all applications for this contract. This implies that $c = (x, r < y_S)$ is not a profitable deviation. Again, since it is rational for bank τ not to accept any application for $c = (x, r < y_S)$ given the equilibrium strategies of the other banks, banks' equilibrium choices are sequentially rational.

Consider now the banks moving after $\tau = n$. None of them can profitably deviate by offering a rate $r' > r^u$, since according to the equilibrium strategies no subsequent bank deviates after having observed that bank τ offers $c' = (x, r')$, implying that bank τ earns zero profits by offering c' . This is true because, for given strategies of competitors and entrepreneurs, a bank earns the same profit by offering either c^u or c' . Similarly, the profit of all the other banks remains unchanged if one of the banks offering c^u switches to c' . This makes it sequentially rational for any bank $\tau' > \tau$ to continue to play its equilibrium action after having observed that bank τ offers c' instead of c^u .

Second, consider the case where bank τ , for $n + 1 \leq \tau \leq n + 1 + n'$, deviates by offering $c' = (x, r')$ with $r' < r^u$. It is straightforward to verify that, since entrants' profits are decreasing in the fraction of usury contracts and in the number of banks offering them, $H(c, \mathbf{c}^{\tau*}, \boldsymbol{\alpha}^{\tau*}) < H(c, \mathbf{c}^{\tau-1*}, \boldsymbol{\alpha}^{\tau-1*})$ for all τ such that $n + 1 \leq \tau \leq n + n' + 1$. From this, it follows that there exists a contract $c = (x, r)$ such that $H(c, \mathbf{c}^{\tau-1}, \boldsymbol{\alpha}^{\tau-1}) = 0$. Thus, according to equilibrium strategies, by issuing c' bank τ induces bank $\tau + 1$ to react by offering c and accepting all

entrepreneurs' applications. To evaluate whether contract c' earns profit once c is introduced, two cases must then be considered: the one in which $H(c', \mathbf{c}^{\tau-1*}, \boldsymbol{\alpha}^{\tau-1*}) > 0$ and that where $H(c', \mathbf{c}^{\tau-1*}, \boldsymbol{\alpha}^{\tau-1*}) \leq 0$.

In the former case, it must be either $r \geq \underline{r}$, since all banks offering before τ charge rates at least equal to the monopoly rate, or $r' > r$ since $H(c', \mathbf{c}^{\tau-1*}, \boldsymbol{\alpha}^{\tau-1*}) > H(c, \mathbf{c}^{\tau-1*}, \boldsymbol{\alpha}^{\tau-1*}) = 0$. Now let $(\hat{\mathbf{c}}^\tau, \hat{\boldsymbol{\alpha}}^\tau) = ((\mathbf{c}^{\tau-1*}, c), (\boldsymbol{\alpha}^{\tau-1*}, \alpha^\tau = 1))$; since $r' > r$ and $\alpha^\tau = 1$, all the entrepreneurs who obtain a monopoly contracts will accept that contract in conjunction with c , while none of them will receive credit from the bank offering c' . Thus,

$$H'(c, \hat{\mathbf{c}}^\tau, \hat{\boldsymbol{\alpha}}^\tau) = \varphi(c', c^u)\pi(c', c^u) + \varphi(c', c)\pi(c', c),$$

where $\varphi(c', c^u)$ and $\varphi(c', c)$ are the probabilities that an entrepreneur will obtain credit from bank τ and bank τ' , and from bank τ and a bank issuing c^u , respectively. Since $\pi(c', c) = \pi(c', c^u) < 0$, because an entrepreneur taking c' and c necessarily defaults in both states, it follows that $H'(c, \hat{\mathbf{c}}^\tau, \hat{\boldsymbol{\alpha}}^\tau) < 0$.

Otherwise, in the case where $H(c', \mathbf{c}^{\tau-1*}, \boldsymbol{\alpha}^{\tau-1*}) < 0$, one necessarily has $r' < r$. Since $H(c', \mathbf{c}^\tau, \boldsymbol{\alpha}^\tau) \leq H(c', \mathbf{c}^{\tau-1*}, \boldsymbol{\alpha}^{\tau-1*})$ whenever $r' < r$, it follows that $H(c', \hat{\mathbf{c}}^\tau, \hat{\boldsymbol{\alpha}}^\tau) < 0$.

Thus, in both cases where $H(c', \mathbf{c}^{\tau*}, \boldsymbol{\alpha}^{\tau*}) > 0$ and $H(c', \mathbf{c}^{\tau*}, \boldsymbol{\alpha}^{\tau*}) \leq 0$, the bank offering c' will make losses unless it refuses all the applications for c' after having issued this contract. therefore, no bank offering the usury contract can profitably deviate. Moreover, it is sequentially rational for bank τ to offer the zero-profit contract c , instead of introducing a contract making positive profits conditional on past offers, since, whenever bank τ does not offer c , bank $\tau + 1$ will do it according to equilibrium strategies.

Finally, consider the case where bank τ , for $\tau > n + 1 + n'$, deviates by offering $c' = (x, r')$. Straightforward calculations, which are left to the reader, show that there exists a contract c such that $H(c, \mathbf{c}^{\tau*}, \boldsymbol{\alpha}^{\tau*}) = 0$. Therefore, one can use exactly the same argument developed in evaluating the profitability of deviations of banks offering c^u in equilibrium to prove that no inactive bank can profitably deviate.

To complete the proof, we need to show that in the parameter region under consideration there exists no equilibrium where all entrepreneurs are funded, while there is an equilibrium with market collapse.

Showing that in this region not all entrepreneurs can be funded in equilibrium is simple. For $\phi > \hat{\phi}$ the large project is not financially viable, hence it cannot be funded in equilibrium. Moreover, there is no equilibrium where each entrepreneur receives a loan equal to x at a rate $r \in [x, y_S]$ and undertakes the small project. Indeed, by construction for $\phi > \hat{\phi}(\sigma)$ an entrepreneur is better off signing both (x, r) with $r \in [x, y_S]$ and (x, r') , instead of (x, r) only. Moreover, for $\phi < \phi_m(\sigma)$ an outside bank earns strictly positive profits by offering (x, r') to an entrepreneur signing this contract in conjunction with (x, r) . Hence, starting from any equilibrium candidate where some entrepreneur signs (x, r) with $r \in [x, y_S]$, any inactive bank can profitably deviate.

That in region D there exists an equilibrium with market collapse follows from the fact that in this area the joint incentive constraint is not met even when the inside bank charges the monopoly rate. Consider the following strategy profile: (i) Bank 1 offers \underline{c} and extends credit to all applicants. (ii) Each bank $\tau > 1$ remains inactive as long as only \underline{c} has been offered up to stage τ , while it offers \underline{c} and accepts all applications for this contract if $c = (x, r)$ with $r < \underline{r}$, is offered by some bank $\tau' < \tau$ and no other bank has offered \underline{c} before stage τ . (iii) Entrepreneurs apply for all contracts and chose their preferred project type. (iv) Banks' and entrepreneurs' beliefs are such that if bank τ deviates from its equilibrium strategy by offering $c = (x, r)$ with $r > x$, it will reject all applications for such a contract.

An equilibrium with market collapse is immediately identified by (i)-(iv). Indeed, bank 1 has no incentive to change its rate since $\underline{c} > y_S$ for $\phi > \hat{\phi}$ and the joint incentive constraint is not met in the region under consideration. For the same reason, inactive banks have no incentive to undercut \underline{c} . It is also quite straightforward to verify that the same conditions satisfy the sequentiality requirement.

Finally, using the expression for $\phi_m(\sigma)$ in (7) and that for $\hat{\phi}$ in (10), one can easily verify that that region D is non-empty if and only if $v_S < 1/2$. ■

Proof of Proposition 5

The proof is developed in four steps.

Step 1. We first show that under information sharing a unique, competitive and efficient equilibrium exists in region A' , that is, in the parameter region where $\phi \leq \underline{\phi}'(\sigma)$. Consider the following strategy profile: (i) Each bank issues the contract c^{PC} and extends credit to all entrepreneurs who did not sign any other contract before applying for c^{PC} . (ii) If only c^{PC} is offered, each entrepreneur applies to a non-empty subset of active banks, signs c^{PC} with one of them and undertakes the small project. Otherwise, he applies for all contracts and undertakes his preferred project, and when indifferent between two loans, he randomizes between them. (iii) If bank τ deviates from its equilibrium strategy by offering $c = (x, r)$ with $r \neq x$, all the other players believe that it will reject all applications for this contract.

Showing that neither entrepreneurs nor banks can profitably deviate from this strategy profile if c^{PC} satisfies the individual incentive constraint (that is, in the subregion of A' where $\phi \leq \underline{\phi}(\sigma)$) requires the same arguments used in the proof of Proposition 1. It then remains to show the result only for the subregion of A' where $\phi \in [\underline{\phi}(\sigma), \underline{\phi}'(\sigma)]$. We first verify that in this region no bank can profitably deviate. Suppose, indeed, that bank τ issues a contract $c_\tau = (x, r_\tau)$, with $r_\tau > x$, instead of c^{PC} . Then, given the strategy profile, entrepreneurs will not apply for c_τ since they can gain by signing c^{PC} with bank $\tau + 1$. In this case, bank τ would be left with no customers, hence it cannot profitably deviate. Moreover, since in the region under consideration c^{PC} satisfies the joint incentive constraint, offering c^{PC} is sequentially rational for bank $\tau + 1$. Showing that entrepreneurs cannot deviate is straightforward.

For uniqueness, note that neither a PBE where some banks fund the large project nor one where some entrepreneurs are excluded from credit can exist in region A' . This is straightforward

since the NPV of the small project exceeds that of the large one and, in the region under consideration, the joint incentive compatibility constraint holds. We then need to show that under **A5** there exists no equilibrium where the small project is funded at a rate $r > x$. Exactly the same argument developed in the first part of the proof, together with the continuity of the entrepreneurs' expected utility, implies that if the non-competitive contract $c = (x, r = x + \varepsilon)$ is offered, any bank can profitably (safely) deviate by offering a contract $c' = (x, r)$ such that $x < r' < x + \varepsilon$ to all entrepreneurs who have not signed c previously.

Finally, in order to show that in the information-sharing regime the efficiency region expands one can immediately verify that $\underline{\phi}'(\sigma) > \underline{\phi}(\sigma)$, so that $A \subset A'$.

Step 2. Showing that in the parameter region where $\phi \in [\underline{\phi}'(\sigma), \widehat{\phi}]$ there exists a PBE where the large project is funded at the competitive rate requires exactly the same arguments used in the proof of Proposition 3. Uniqueness, instead, follows from Lemma 2 and **A5**: it is easy to verify that for $\phi > \underline{\phi}'(\sigma)$ no contract $c = (x, r)$ with $r \geq x$ satisfies the joint incentive constraint, hence there is no equilibrium where some entrepreneurs undertake the small project. Finally, since in the region under consideration the large project is financially viable, also an equilibrium where some entrepreneurs are excluded from credit cannot exist.

Step 3. Showing that in region C' there exists a PBE such that all entrepreneurs sign a contract $\widehat{c} = (x, \widehat{r})$ where $\widehat{r} = \min\{r^*, y_S\}$ with a single active bank follows the same arguments used in the proof of Proposition 2, and is thus omitted.

Step 4. Finally, we show that in region C' there cannot exist equilibria where some entrepreneurs are excluded from credit. The proof is by contradiction. Suppose that some entrepreneurs are excluded from credit in equilibrium. Then, in the region under consideration, any bank τ can profitably deviate by offering a contract $c_\tau = (x, r_\tau)$, with $r_\tau = \min\{y_S, r^*\}$, to all entrepreneurs that did not sign any other contract before stage τ . By definition of r^* this contract satisfies the joint incentive constraint, and it would be accepted by entrepreneurs excluded from credit as it yields a non-negative utility. ■

Proof of Proposition 6

We first show that in region D there exists a PBE with market collapse. Consider the following strategy profile: (i) Each bank τ remains inactive (i.e., issues the null contract) as long as each bank τ' , with $\tau' < \tau$, is inactive; otherwise, it issues $\underline{c} = (x, \underline{r})$ and extends credit only to entrepreneurs who have previously signed $c_{\tau'} = (x, r' < \underline{r})$ with bank τ' . (ii) Each entrepreneur applies for all contracts and undertakes his preferred project, and when he is indifferent between two loans, he randomizes between them. (iii) If bank τ deviates from its equilibrium strategy by offering $c = (x, r)$ with $r \neq x$, all the other players believe that it will reject all applications for this contract.

For the above strategy profile to be a PBE, one only needs to verify that banks have no profitable deviations. First, in the parameter region under consideration funding the large project is not profitable since this is not financially viable. Moreover, issuing a contract $c_\tau =$

(x, r_τ) with $x \leq r_\tau < \underline{r}$ is also not profitable for any bank τ : given the strategy profile (i)-(iii), if bank τ deviates by issuing c_τ , bank $\tau + 1$ will profitably issue \underline{c} and extend credit to all entrepreneurs who signed c_τ . Since in the parameter region under consideration no contract satisfies the joint incentive constraint, Lemma 2 implies that $u(c_\tau, \underline{c}) > u(c, c_\emptyset)$ and $\pi(\underline{c}, c) = 0$. Therefore, bank τ makes losses since $\phi > \widehat{\phi}$ and $\underline{r} > y_S$. Moreover, the fact that $\pi(\underline{c}, c) = 0$ implies that offering \underline{c} only to those entrepreneurs who previously signed a contract $c = (x, r)$ with bank τ' is sequentially rational for any bank τ such that $\tau > \tau'$. This shows that there exists a PBE featuring no trade in region D .

For uniqueness, we need to show that there are no other equilibria with lending. First, the same logic used in the proof of Proposition 4 together with **A5** implies that there is no PBE where all entrepreneurs are funded in region D . We must then show that rationing equilibria too cannot exist. The proof is by contradiction. Suppose that there does exist an equilibrium where some agents are rationed. By the proof of Proposition 5 it follows that, if such an equilibrium exists in the parameter region under consideration, it must be true that: (i) some banks offer usury rates $r^u = (1 - \phi) y_L + 1 + \sigma - y_S$ and accept only a fraction α^u of applications, while others charge the monopoly rate $r^m = y_S$ and accept only a fraction α^m of applications; (ii) a fraction of entrepreneurs borrow from two banks and undertake the large project while a fraction end up only with the monopoly contract $c^m = (x, r^m)$ and undertake the small project; (iii) active banks make zero expected profits. But when banks share information about past indebtedness this configuration cannot be an equilibrium, because any bank can always profitably attract the fraction of entrepreneurs who signed only c^m , by offering them the usury contract $c^u = (x, r^u)$. Indeed, as shown in the proof of Proposition 5, $\phi > \max\{\underline{\phi}'(\sigma), \widehat{\phi}\}$ implies $u(c^u, c^m) > u(c^m, c_\emptyset)$, and $\phi < \phi_m(\sigma)$ yields $\pi(c^u, c^m) > 0$. ■

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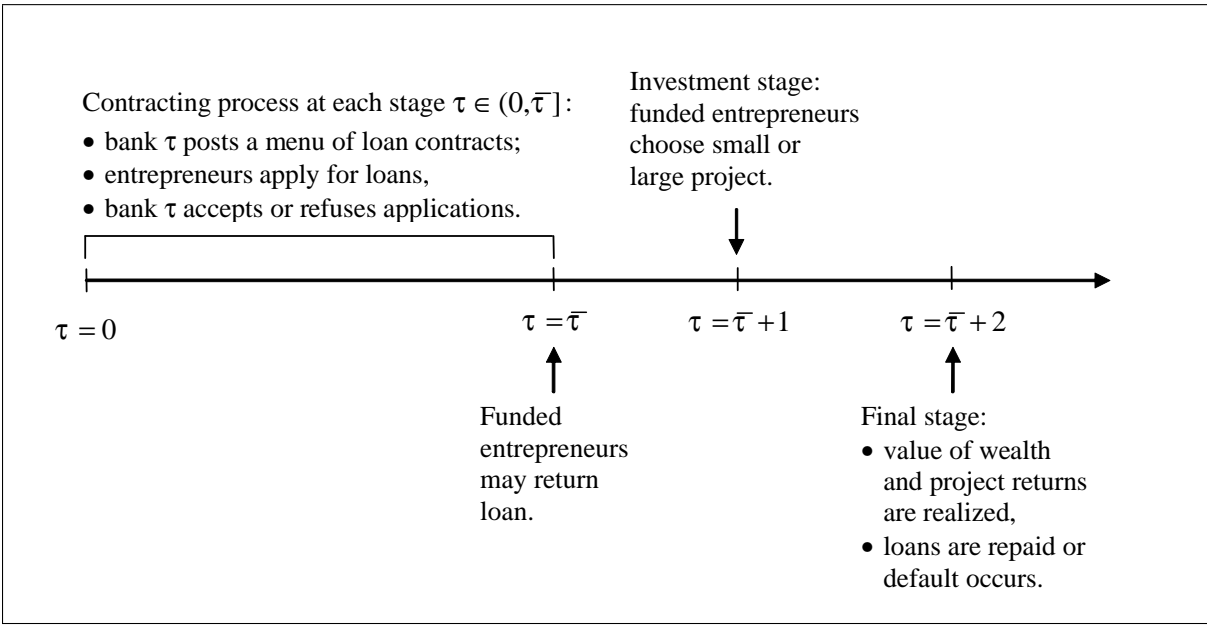


FIGURE 1. TIME LINE

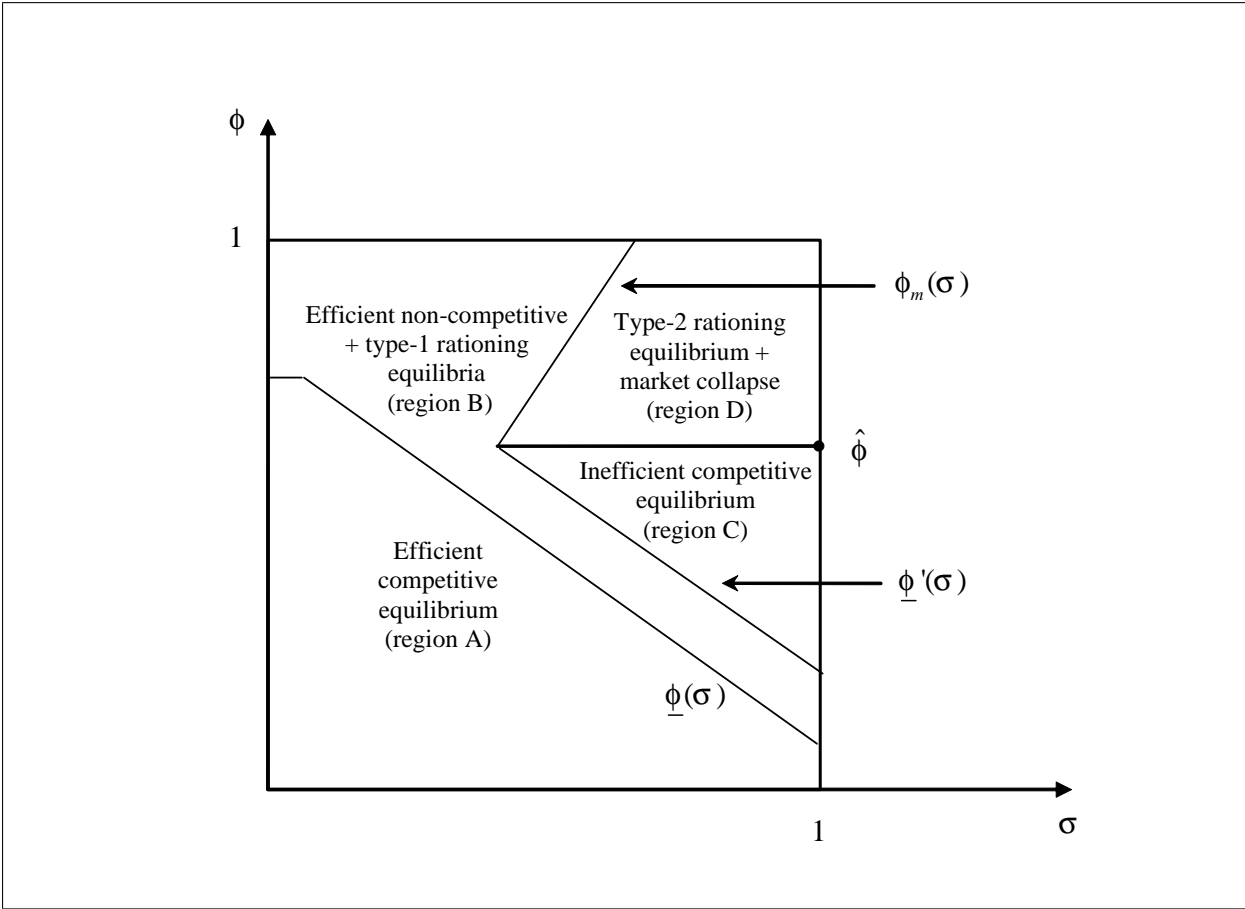


FIGURE 2. EQUILIBRIA WITH NO INFORMATION SHARING

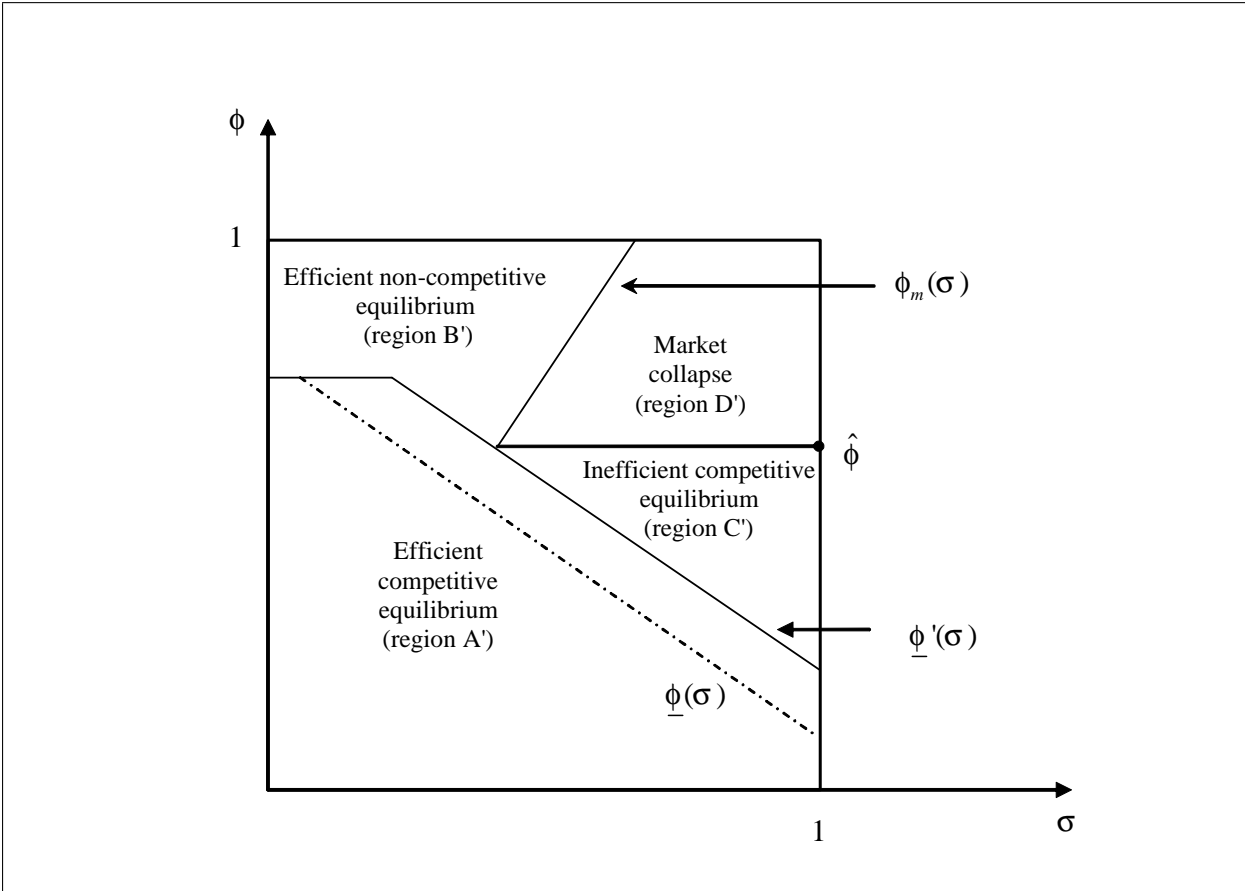


FIGURE 3. EQUILIBRIA WITH INFORMATION SHARING AND PRO-RATA LIQUIDATION

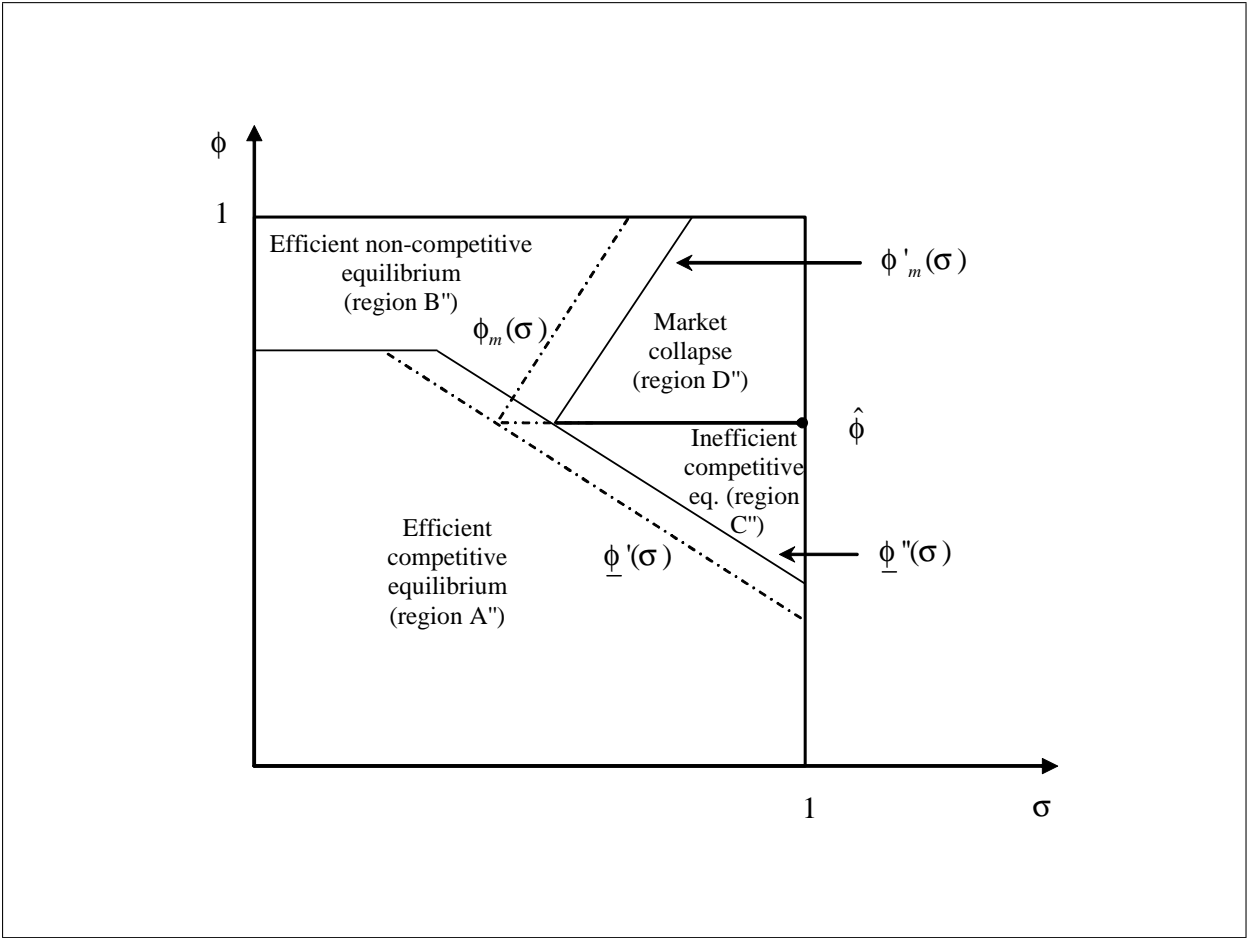


FIGURE 4. EQUILIBRIA WITH INFORMATION SHARING AND SENIORITY-BASED LIQUIDATION