On the Dynamics of Leverage, Liquidity, and Risk[1]

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February 4, 2010

[1]PRELIMINARY DRAFT, COMMENTS WELCOME. We gratefully acknowledge financial support from the Bankard Fund for Political Economy, the National Science Foundation (grant SES-0649442), the National Centre of Competence in Research "Financial Valuation and Risk Management" (NCCR FINRISK), and the Swiss Finance Institute.

Abstract

The recent financial crisis has highlighted the key role of leveraged financial institutions as liquidity providers. We incorporate leveraged financial institutions into a dynamic general equilibrium portfolio choice model in order to analyze the dynamics of risk, leverage, liquidity and asset prices. We particularly emphasize the role of self-fulfilling changes in expectations that can lead to sudden large shifts in risk, liquidity and leverage. This can take the form of a financial panic with a big drop in asset prices. Such panics become much more severe when taking place against the backdrop of leveraged institutions that are in weak financial health. We show that the model can account for the main features of the current crisis, both during the panic and pre-panic stages of the crisis.

1 Introduction

The recent financial crisis is pushing macroeconomists to incorporate financial intermediation in their analyses. While some traditional aspects of financial intermediation have already received much attention in the past[1], new elements have been identified. In particular, the role of highly leveraged financial institutions such as hedge funds, investment banks and brokers and dealers has been emphasized as their significance has increased with the growth of market-based finance. Instead of intervening directly in the lending process, like commercial banks, these institutions' main activity is to intervene directly in financial markets. They typically play the role of arbitrageurs and provide liquidity to the markets, thereby reducing volatility. However, when they suffer large financial losses, their role is diminished. This can lead to a significant decline in market liquidity, which results in increased risk, higher margin requirements and deleveraging.

This role of leveraged institutions has been documented both at an empirical level and by a growing theoretical literature.[2] The theoretical literature has shed light on the various links between the capital of leveraged institutions, market liquidity, risk and leverage. It has also documented a variety of amplification mechanisms associated with leveraged institutions. However, it has not focused much on dynamics. By contrast, the recent crisis has exhibited very rich dynamics. During the first year of the crisis we saw a relatively modest drop in equity prices and increase in risk, while leverage actually continued to rise. This was followed by a financial panic in the Fall of 2008. Over a short span of time, market liquidity and leverage collapsed, volatility soared (VIX index tripled) and equity prices dropped by 50%.

The aim of this paper is to shed light on the dynamics of risk, leverage, liquidity and asset prices by incorporating leveraged financial institutions into a dynamic general equilibrium model. The purpose is both to understand at a general level what drives these dynamics and to shed light on the recent crisis. We particularly

[1]For example, one can think of the seminal contributions of Diamond and Dybvig (1983), Bernanke and Gertler (1989), or Kyiotaki and Moore (1995).

[2]For the empirical literature see for example recent contributions by Adrian and Shin (2009, 2010) and Adrian at.al. (2009). For the theoretical literature, see Adrian and Shin (2008), Brunnermeier and Pedersen (2009), Danielsson et al. (2009), Gromb and Vayanos (2002,2008), He and Krishnamurthy (2008a, b), Kyle and Xiong (2001) and Xiong (2001).

emphasize the role of self-fulfilling changes in expectations that can lead to sudden large shifts in risk, liquidity, and leverage. This can take the form of a financial panic with a big drop in asset prices. Such panics become much more severe when taking place against the backdrop of leveraged institutions that are in weak financial health.

The model is a simple dynamic portfolio choice model with two types of agents, “leveraged institutions” and “investors”, that trade bonds and equity. The only difference between these agents is that leveraged institutions are less risk-averse, leading to a fraction of wealth allocated to equity that is larger than 1. They therefore borrow from investors to finance holdings of equity. While we consider different types of shocks, we focus mostly on financial shocks that redistribute wealth between leveraged institutions and investors, motivated by the losses that leveraged institutions experienced in the sub-prime mortgage market during the recent crisis.

In contrast with most of the literature, leveraged institutions do not face financing constraints, such as value at risk constraints or borrowing constraints.[3] Such constraints have valid micro foundations, and can be important for addressing policy questions, but they are not central to shedding light on the overall macro dynamics of risk, liquidity, leverage and asset prices. We also abstract from other features that could plausibly generate financial panics, such as Diamond-Dybvig type bank runs, Knightian uncertainty and complexity externalities.[4] While these aspects may certainly have played an important role during the recent crisis, we argue that occasional financial panics naturally occur in almost any dynamic portfolio choice model, even when these additional features are not present. The type of shock is not key to this either. We show that financial panics can also develop with technology shocks that are a standard staple of macro models.

A key aspect of the model is that the financial health of leveraged institutions plays two distinct roles. On the one hand a decline in the capital of leveraged financial institutions has implications as an economic fundamental. It shrinks their

[3]See Adrian et al. (2009) and Danielsson et al. (2008)) for models with value at risk constraints. Borrowing constraints are often introduced through margin requirements. Examples are Gromb and Vayanos (2002, 2008) and Brunnermeier and Pederson (2009). Others, such as He and Krishnamurthy (2009), derive the constraint from an agency problem.

[4]See Caballero and Krishnamurthy (2008) for a model with Knightian uncertainty and Caballero and Simsek (2009) for a model with complexity externalities.

balance sheets, which leads to a decline in market liquidity. This effect is further amplified as reduced liquidity implies a rise in asset price volatility, which leads to deleveraging that further shrinks the balance sheets of leveraged institutions. But completely separate from this role as a fundamental, the health of leveraged financial institutions can play another role by affecting perceived risk in a self-fulfilling way. It may act as a "sunspot-like" variable by providing a coordination mechanism that shifts perceptions of risk over time.[5] Such self-fulfilling changes in perceived risk are possible because risk is closely connected to the volatility of future risk. We show that risk and the volatility of risk can change jointly in a self-fulfilling way that is consistent with market equilibrium.

In addition to sunspot-like effects, but closely related, the model generates multiple equilibria. A key source of multiplicity is related to the dual role played by the financial health of leveraged institutions. There is a low risk equilibrium, where the capital of leveraged institutions only plays the role of a fundamental. In addition there are high risk equilibria where the financial state of leveraged institutions plays the additional role of a sunspot that generates self-fulfilling shifts in risk. Entirely separate from this, there is another source of multiple equilibria that is associated with the circular relationship between risk, leverage and liquidity. This latter source of multiplicity has also been emphasized by Brunnermeier and Pedersen (2009) and can be found in limited participation models such as Pagano (1989), Allen and Gale (1994), or Jeanne and Rose (2002).

The presence of multiple equilibria naturally leads to the possibility of financial panics in the form of a switch between high and low risk states. We show this by solving for switching equilibria that allow for switches between high and low risk states based on a Markov process. We find that the magnitude of a financial panic is larger the weaker the financial health of leveraged institutions at the time of the panic. When leveraged institutions are in bad financial shape, a switch from a low to a high risk state implies a sharp drop in asset prices, leverage and liquidity and a large increase in risk. In terms of the 2007-2008 financial crisis we can think of

[5]Manuelli and Peck (1992) have introduced the concept of sunspot-like equilibria in a dynamic OLG model. The basic idea is that there are circumstances where fundamental shocks play the role of extrinsic or sunspot shocks by affecting expectations in a self-fulfilling way. In the limiting case where fundamental uncertainty goes to zero, these equilibria converge to pure sunspot equilibria.

the period prior to the Lehman Brothers collapse as the pre-panic period and the subsequent months as a panic period. We provide an illustration to show that the model can account for the dynamics of risk, liquidity, leverage and assets prices during both the pre-panic and panic periods.

The model obviously does not provide a full explanation for what happened during the recent financial crisis. First, as already emphasized above, we abstract from aspects such as financial constraints on leveraged institutions, bank-runs (through the repos market) and complexity issues, which likely all played an important role.[6] Second, we take the losses of leveraged financial institutions as given, modeled as a wealth redistribution between leveraged institutions and investors. We make no attempt to account for the large losses in the securitized subprime mortgage market. Third, we only focus on the asset price implications of leverage and do not analyze its real implications.[7] While some of these elements can be introduced through extensions, a key message from the paper is that a simple bare bones portfolio choice model is sufficient to deliver very rich dynamics that is broadly consistent with what we saw during the crisis.

The model is related to some recent papers in the finance literature on the role of leveraged financial institutions. A key difference is our focus on dynamics. In some papers dynamics is limited by the focus on models with a limited number of periods (e.g. Brunnermeier and Pederson (2009)). Others take the process of risk as exogenous (e.g. Fostel and Geneakoplos (2008a,b)). There are some papers in which risk does change endogenously over time. Perhaps most closely related is Xiong (2001), who also considers a dynamic portfolio choice model. However, he does not consider the possibility of multiple equilibria. He and Krishnamurthy (2008a) calibrate a model that allows risk to evolve endogenously over time. They find little time variation. This is consistent with our finding that outside of a financial panic the impact of shocks on risk is limited. Finally, Danielsson et al.(2009) consider endogenously time-varying risk in a framework where investment by leveraged institutions is driven by a value at risk criterion. In that case effective

[6]Bank-runs cannot occur in our model as we rule out the possibility of bankruptcy.

[7]It would be interesting to link our analysis to the recent research in macroeconomics that has highlighted the role of uncertainty. For example Bloom (2009) shows that stock market volatility is highly correlated with other measures of uncertainty and that it has a significant impact on industrial production in the US.

risk aversion also varies over time. None of these papers consider the possibility of financial panics.

The remainder of the paper is organized as follows. The next section describes the model, its quadratic approximation and its solution. Section 3 describes the equilibria and analyzes the mechanisms at work. Section 4 examines to what extent the model is consistent with the recent crisis. Section 5 examines several extensions, showing in particular that most of our results also hold with aggregate shocks, such as persistent productivity shocks or endowment shocks. Section 6 concludes.

2 A Dynamic Model with Leveraged Institutions

The model is a simple infinite horizon closed economy portfolio choice model with both leveraged and non-leveraged institutions. It leads to a market clearing condition for equity that will be the focus of all the subsequent analysis. We first describe the building blocks of the model, followed by a discussion of the solution method.

2.1 Description of the Model

2.1.1 Overview

We consider an infinite horizon closed economy populated by two-period overlapping generations of agents. There are three types of agents: leveraged institutions, investors and entrepreneurs. Investors and leveraged institutions optimally allocate their portfolio between claims on capital (equity) and a risk-free bond. They only differ in their level of relative risk aversion: γ_L for leveraged institutions and γ_I for investors. Leveraged institutions are less risk averse, so that $\gamma_L < \gamma_I$.[8] The

[8] Garleanu and Pedersen (2009) and Longstaff and Wang (2008) also assume that leveraged institutions are less risk averse than other investors. Other types of heterogeneity that have been introduced to separate leveraged institutions from other agents include assumptions that leveraged institutions have a specialized skill to choose risky assets (Brunnermeier and Sannikov (2009), Gertler and Kiyotaki (2009)), are more productive (Kiyotaki (1998)) or are more optimistic (Geanakoplos (2003)).

only role of entrepreneurs is to generate an elastic bond supply. They borrow from investors and leveraged institutions to operate their firms.

An important simplification that contributes to the tractability of the model is the simple overlapping generations structure. If instead we assumed that agents have infinite lives, the wealth levels of leveraged institutions and investors would be additional state variables. This would complicate the analysis in that we would no longer be able to graphically represent the multiple equilibria or even be sure that we know what all the equilibria are. However, most of the economic mechanisms present with infinite lives are also present in the OLG model. The only missing aspect is that with long-lived agents a change in asset prices feeds back to the wealth of the agents (and therefore the capital of leveraged institutions). In an extension in Section 5 we will consider the impact of this feedback effect.

In the model there are both standard productivity shocks and innovations that we will refer to as financial shocks. A financial shock redistributes wealth between investors and leveraged institutions, leaving the aggregate endowment constant.[9] This shock is motivated by the recent mortgage crisis. The biggest losers were clearly financial institutions with significant exposure to the sub-prime mortgage market. We take this shock as exogenously given.

2.1.2 Assets

There is a constant supply of capital K that generates a random output of $A_t K$ of a single consumption good. We will describe the process of productivity A_t below. Equity is a claim on the output of each unit of capital. The equity supply is therefore K as well. The price of capital, and therefore the value of one equity claim, is Q_t. This is measured in terms of the consumption good. The dividend on each equity is the output generated by each unit of capital, which is A_t. The return on capital from t to $t+1$ is then

$$R_{K,t+1} = \frac{A_{t+1} + Q_{t+1}}{Q_t} \tag{1}$$

Agents face uncertainty both about the dividend A_{t+1} and about next period's equity price Q_{t+1}.

[9] We assume incomplete markets so that agents cannot insure against these shocks.

There is also a one-period risk-free bond with a return R_{t+1} from t to $t+1$. While it is risk-free from the perspective of time t, it varies endogenously over time. The aggregate supply of the risk-free bond is zero. In equilibrium, leveraged institutions are in general short in the bond while investors are long in the bond. One can therefore think of investors as lending to leveraged institutions, allowing them to leverage their capital when investing in equity.

2.1.3 Wealth of Investors and Leveraged Institutions

Investors and leveraged institutions born at time t receive endowments $W_{I,t}$ and $W_{L,t}$. This is the wealth that they invest in equity and bonds. We describe the process of the endowments below. We only consider shocks that redistribute the endowments between investors and leveraged institutions, which are the financial shocks mentioned above.

The wealth of these same investors and leveraged institutions next period will depend on asset returns. However, these returns are fully consumed and agents are replaced by a new generation of investors and leveraged institutions. Letting I and L denote respectively investors and leveraged institutions, the wealth at $t+1$ of agents born at time t is

$$W_{i,t+1} = R^p_{i,t+1} W_{i,t} \tag{2}$$

for $i = I, L$. Here $R^p_{i,t+1}$ is the portfolio return

$$R^p_{i,t+1} = \alpha_{i,t} R_{K,t+1} + (1 - \alpha_{i,t}) R_{t+1} \tag{3}$$

where $\alpha_{i,t}$ is the share invested in equity. Agents simply consume their wealth: $C_{i,t+1} = W_{i,t+1}$.

2.1.4 Portfolio Allocation

Agents i ($i = I, L$) maximize expected utility

$$E_t \frac{C_{i,t+1}^{1-\gamma_i}}{1-\gamma_i} \tag{4}$$

where γ_i is the rate of relative risk-aversion. Our solution method and assumptions about shocks discussed below imply that the portfolio return is log-normally distributed. In that case maximizing expected utility is equivalent to maximizing

$$E_t r^p_{i,t+1} - \frac{1}{2}(\gamma_i - 1)var(r^p_{i,t+1}) \tag{5}$$

where $r^p_{i,t+1} = ln(R^p_{i,t+1})$ is the log portfolio return. Portfolio allocation therefore takes the form of a standard mean-variance tradeoff.

We adopt the following continuous time approximation of the log portfolio return (see Appendix A for a derivation):

$$r^p_{i,t+1} = \alpha_{i,t}(R_{K,t+1} - 1) + (1 - \alpha_{i,t})(R_{t+1} - 1) - 0.5\alpha^2_{i,t}var(R_{K,t+1}) \tag{6}$$

The same approach is adopted by Campbell and Viceira (1999, 2002), Campbell, Chan and Viceira (2003) and in a general equilibrium framework by Evans and Hnatkovska (2007).[10] Campbell and Viceira (2002) provide a detailed motivation of this approximation. An important property is that it rules out bankruptcy, even when investors hold leveraged positions, as the log portfolio return is always finite. This approximates the continuous time case in which bankruptcy cannot occur because, as Campbell and Viceira (2002) emphasize, "losses can always be stemmed by rebalancing before they lead to bankruptcy".

Maximization of (5) then implies a standard mean variance portfolio:

$$\alpha_{i,t} = \frac{E_t R_{K,t+1} - R_{t+1}}{\gamma_i var_t(R_{K,t+1})} \tag{7}$$

It depends on the expected excess return, divided by the variance of the excess return times the rate of risk-aversion.

As in equilibrium the expected excess return on equity will be positive, it implies that leveraged institutions invest a larger fraction in equity ($\gamma_L < \gamma_I$). When $\alpha_{L,t} > 1$ leveraged institutions are truly leveraged. Leverage is equal to the ratio of assets to capital (wealth). When $\alpha_{L,t} > 1$, the only assets on the balance sheet of leveraged institutions are equity and the leverage ratio is simply the portfolio share $\alpha_{L,t}$ invested in equity.

[10]The only difference is that these authors express the approximation in terms of the log equity and bond returns rather than their levels. An alternative approach to solving discrete time portfolio problems is to adopt the order method developed by Tille and van Wincoop (2010) and Devereux and Sutherland (2008). However, this works only for pure fundamentals equilibria. For the other equilibria in the model the asset price depends on shocks with coefficients that have components of negative orders.

2.1.5 Entrepreneurs

The only role of entrepreneurs is to generate an elastic net supply of bonds. Otherwise it would be impossible for agents to shift between stocks and bonds in the aggregate. The price of stock would then be constant. One could alternatively introduce interest rate elastic saving.[11]

Entrepreneurs born in period t receive an endowment W_E and produce goods in period $t+1$ with the production function $Y_{t+1} = \left[\nu K^E_{t+1} - \frac{1}{2}\left(K^E_{t+1}\right)^2\right]/\eta$. The capital good used for production by the entrepreneurs is the same as the consumption good. In order to purchase K^E_{t+1} units of capital, an entrepreneur issues $K^E_{t+1} - W_E$ risk-free bonds. In period $t+1$ the entrepreneur consumes the income from production minus the repayment of the debt: $Y_{t+1} - R_{t+1}(K^E_{t+1} - W_E)$. The optimal supply of bonds by entrepreneurs is then

$$\nu - W_E - \eta R_{t+1} \tag{8}$$

2.1.6 Market Clearing

There are two market clearing conditions, for equity and for bonds.[12] We first impose aggregate asset market clearing, by setting aggregate demand for stocks plus bonds equal to aggregate asset supply, and then separately impose equity market clearing.

Taking the sum of the market clearing conditions for bonds and capital, the wealth of investors and leveraged institutions, $W_t = W_{L,t} + W_{I,t}$, is equal to the sum of equity and bond supply:

$$W_t = Q_t K + \nu - W_E - \eta R_{t+1} \tag{9}$$

This implies a simple positive linear relationship between the equity price and the interest rate on bonds.

[11] In the finance literature it is generally assumed that the net bond supply is perfectly elastic at a constant interest rate. It is attractive though to endogeneize the interest rate, both from a theoretical perspective (to keep the model general equilibrium) and an empirical perspective (substantial drop in the T-bill rate during the panic in the Fall of 2008).

[12] There is a third market clearing condition for goods, but we can drop it as a result of Walras' Law.

Equilibrium in the equity market is:

$$\alpha_{L,t} W_{L,t} + \alpha_{I,t} W_{I,t} = Q_t K \tag{10}$$

Using (7) and expression (1) for the equity return, we can rewrite the equity market clearing condition as

$$\frac{E_t(A_{t+1} + Q_{t+1}) - R_{t+1} Q_t}{var_t(Q_{t+1} + A_{t+1})} \tilde{W}_t = K \tag{11}$$

On the left hand side the numerator is the expected excess payoff on one equity, the denominator the variance of the excess payoff, and

$$\tilde{W}_t = \frac{W_{L,t}}{\gamma_L} + \frac{W_{I,t}}{\gamma_I} \tag{12}$$

is a risk-aversion weighted measure of wealth. The demand for assets is driven by this risk-weighted wealth rather than a simple aggregate of wealth.[13] This is because the lower risk aversion makes equity demand by leveraged institutions more responsive to changes in wealth than that of investors. If for example leveraged institutions have a leverage ratio of 50, while investors allocate 50% to stock, a $1 shift from investors to leveraged institutions raises demand for equity by $49.5.

2.1.7 Shocks

We choose a specification that implies a constant variance of A_{t+1} and $\tilde{W}_{t+1}$ in response to technology and financial shocks. This guarantees that the time-varying risk in the model, associated with Q_{t+1}, is entirely endogenous.

For productivity we assume

$$A_{t+1} = \bar{A} e^{a_{t+1} - 0.5 a_{t+1}^2} \tag{13}$$

where

$$a_{t+1} = \rho_a a_t + \epsilon_{t+1}^a \tag{14}$$

and ϵ_{t+1}^a has a $N(0, \sigma_a^2)$ distribution. Our solution method discussed below uses a quadratic approximation of the model. A quadratic approximation of A_{t+1} is $\bar{A}(1 + a_{t+1})$, which has a constant variance.

[13] An alternative interpretation of $\widetilde{W}_t$ is that it represents the wealth-weighted average risk aversion in the market. Thus, a decline in $\widetilde{W}_t$ represents an increase in market risk aversion.

With regards to the redistributive financial shocks we assume

$$W_{L,t} = \bar{\theta} e^{-\widetilde{\theta}_{Lt}} W \qquad \text{with} \qquad \widetilde{\theta}_{Lt} = \frac{\theta_t}{\bar{\theta}} + \frac{1}{2}\left(\frac{\theta_t}{\bar{\theta}}\right)^2 \tag{15}$$

$$W_{I,t} = (1-\bar{\theta}) e^{-\widetilde{\theta}_{It}} W \qquad \text{with} \qquad \widetilde{\theta}_{It} = -\frac{\theta_t}{1-\bar{\theta}} + \frac{1}{2}\left(\frac{\theta_t}{1-\bar{\theta}}\right)^2 \tag{16}$$

where

$$\theta_{t+1} = \rho_\theta \theta_t + \epsilon^\theta_{t+1} \tag{17}$$

and $\epsilon^\theta_{t+1} \sim N(0, \sigma^2_\theta)$. A rise in θ_t represents a redistribution away from leveraged institutions to investors. We assume that ϵ^θ_{t+1} is uncorrelated with ϵ^a_{t+1}.

Quadratic approximations used in the solution method give $W_{L,t} = (\bar{\theta} - \theta_t)W$ and $W_{I,t} = (1 - \bar{\theta} + \theta_t)W$. The variance of $\tilde{W}_t$ is then constant. Specifically, the quadratic approximation gives

$$\tilde{W}_t = \bar{W} - m\theta_t \tag{18}$$

where

$$\bar{W} = W\left(\frac{\bar{\theta}}{\gamma_L} + \frac{(1-\bar{\theta})}{\gamma_I}\right) \tag{19}$$

$$m = W\left(\frac{1}{\gamma_L} - \frac{1}{\gamma_I}\right) \tag{20}$$

The aggregate endowment, after the quadratic approximation, is the constant W.

2.2 Interaction Between Risk, Leverage and Liquidity

Before turning to a solution of the model, it is worth discussing how risk, leverage and liquidity are linked in the model. Risk is defined as the variance of the equity payoff $A_{t+1} + Q_{t+1}$. Leverage is the portfolio share $\alpha_{L,t}$ allocated to equity by leveraged institutions.

Liquidity is a more abstract concept and requires a bit more discussion. In the literature market liquidity is generally associated with how much the asset price, or its expected excess payoff, needs to adjust to clear the market in response to exogenous asset demand or supply shocks.[14] When a shock requires a large

[14]See Amihud et al. (2005) or Vayanos and Wang (2009) for surveys on liquidity and asset prices.

change in the price or expected excess payoff, liquidity is considered to be low. We define liquidity in terms of how much the expected excess payoff on equity needs to change to generate equilibrium as this connects closely to the empirical measure of liquidity in Pastor and Shambough (2003) that will be used in section 4. Defining it alternatively by how much the price needs to change to generate equilibrium gives very similar results since a larger change in the expected excess payoff requires a larger change in the price.

In the model $\tilde{W}_t$, which depends on θ_t, is the source of asset demand shocks. We therefore define liquidity in the model as

$$\frac{\partial E_t(A_{t+1} + Q_{t+1} - R_{t+1}Q_t)}{\partial \tilde{W}_t} \tag{21}$$

This is generally negative as an increase in $\tilde{W}_t$ leads to an increase in asset demand, which implies a lower equilibrium expected excess payoff on equity. The larger the drop in the expected excess payoff, and therefore the more negative (21), the lower liquidity. Taking a derivative with respect to the market clearing condition (11) for equity, liquidity is equal to

$$-\frac{K var_t(Q_{t+1} + A_{t+1})}{\tilde{W}_t^2} + \frac{K}{\tilde{W}_t}\frac{\partial var_t(Q_{t+1} + A_{t+1})}{\partial \tilde{W}_t} \tag{22}$$

Liquidity depends on three variables: risk, risk-weighted wealth, and the derivative of risk with respect to risk-weighted wealth. High risk implies that equity is unattractive and therefore leverage is low. Leveraged institutions, as well as investors, then have less exposure to the equity market and respond less to changes in the expected excess payoff on equity. Larger changes in the equilibrium expected excess payoff are then necessary to clear the equity market, so that liquidity is low. Similarly, lower wealth means that less money is on the line in the equity market and therefore larger changes in the price and expected excess payoff are needed to clear the market. Liquidity will be low. Finally, the last term in (22) shows that liquidity also depends on the derivative of risk with respect to risk-weighted wealth. This term is generally absent in the literature. It shows up here as risk is endogenously time-varying. When a drop in wealth raises risk, it requires an even larger increase in the expected excess payoff to clear the market.

The model implies a circular relationship between risk, leverage and liquidity. Higher risk implies lower leverage, so that the equity market will be thinner and liquidity lower. In turn a drop in liquidity, when persistent, implies an increase in risk as future financial shocks have a larger price impact. We develop further insights on the link between these variables in the next section, when discussing the nature of the equilibria.

2.3 Solution Method

Most of the analysis focuses on the redistributive financial shocks. When doing so, we set $\rho_a = 0$ in order to minimize the number of state variables and therefore facilitate the graphical representation of equilibria. We describe here the solution method for that case. We take up the case of persistent productivity shocks in Section 5.

Substituting the equilibrium interest rate R_{t+1} as a function of Q_t from (9), the market clearing condition can be written as[15]

$$\left(\bar{A} + E_t Q_{t+1} - \frac{1}{\eta}(\nu - W - W_E)Q_t - \frac{K}{\eta}Q_t^2\right)(\bar{W} + m\theta) = Kvar(Q_{t+1} + A_{t+1}) \tag{23}$$

The solution to this equation involves a mapping from θ_t to Q_t.

We will find a solution that is accurate up to a quadratic approximation. We start by conjecturing

$$q_t \equiv ln(Q_t) = \tilde{q} - v\theta_t - V\theta_t^2 \tag{24}$$

where $\tilde{q}$, v and V are parameters that need to be solved. We substitute this into the equity market clearing condition and verify that it holds up to quadratic terms in θ_t. In other words, we impose equality between the left and right hand side of the market clearing condition for constant terms, terms that are linear in θ_t and terms that are quadratic in θ_t. This leads to three non-linear equations in the unknowns $\tilde{q}$, v and V that need to be solved jointly.

We leave most details of the algebra to Appendix B, but it is useful to briefly go through some of the steps. The terms in the market clearing condition that are linear or quadratic in Q_t are approximated as a quadratic function of θ_t by using the conjecture (24) and expanding around $\theta_t = 0$.

[15] Here we have also used that $E_t A_{t+1} = \bar{A}$ when using the quadratic approximation of A_{t+1}. We also use that a quadratic approximation of W_t is W.

In computing the expectation and variance of Q_{t+1}, we first write

$$Q_{t+1} = \tilde{Q} e^{-v\theta_{t+1} - V\theta_{t+1}^2}$$

where $\tilde{Q} = e^{\tilde{q}}$. We then obtain a quadratic approximation of this expression around $\theta_{t+1} = 0$ and substitute $\theta_{t+1} = \rho_\theta \theta_t + \epsilon_{t+1}$. In addition we use the continuous time approximation $(\epsilon_{t+1}^\theta)^2 = \sigma_\theta^2$. This yields

$$Q_{t+1} = \tilde{Q}\left(1 - \omega\sigma_\theta^2 - v\rho\theta_t - \omega\rho^2\theta_t^2 - (v + \omega 2\rho\theta_t)\epsilon_{t+1}^\theta\right) \tag{25}$$

where $\omega = V - 0.5v^2$. This is used to compute the expectation and variance of Q_{t+1}.

It is noteworthy that the variance is in general time varying. We have

$$var_t(Q_{t+1}) = \tilde{Q}^2 \left(v + 2\omega\rho_\theta\theta_t\right)^2 \sigma_\theta^2 \tag{26}$$

which is a quadratic function of θ_t. The variance is constant only in the case where $\rho_\theta = 0$. In that case the state variable θ_t does not affect the equilibrium tomorrow and therefore has no implication for tomorrow's equity price and its associated risk.

We substitute the expressions for Q_t, Q_t^2, $E_t Q_{t+1}$ and $var_t(Q_{t+1})$ as quadratic functions of θ_t into the market clearing condition. Dropping terms that are cubic in θ_t, we obtain an expression of the form

$$Z_0 + Z_1\theta_t + Z_2\theta_t^2 = 0 \tag{27}$$

where Z_0, Z_1, and Z_2 are functions of $\tilde{Q}$, v, and V. We find the solution by setting $Z_0 = 0$, $Z_1 = 0$, and $Z_2 = 0$.

We take the following approach to represent the equilibria graphically. Define $\tilde{V} = \tilde{Q}V$. In the Appendix we show that $Z_0 = 0$ implies

$$\tilde{V} = \alpha_1 + \alpha_2 v^2 \tag{28}$$

where α_1 and α_2 are functions of $\tilde{Q}$. After substituting this into the expressions associated with $Z_1 = 0$, and $Z_2 = 0$, we obtain

$$h_1 + h_2 v + h_3 v^2 + h_4 v^3 = 0 \tag{29}$$

$$g_1 + g_2 v + g_3 v^2 + g_4 v^3 + g_5 v^4 = 0 \tag{30}$$

where h_i and g_i are functions of $\tilde{Q}$.

These are third and fourth order polynomials that we solve numerically. (29) and (30) represent two schedules that map a given $\tilde{Q}$ into v, with possibly multiple solutions. We then plot these two schedules and find out where they cross, which represents an equilibrium $\tilde{Q}$ and v. $\tilde{V}$, and therefore V, then follows from (28). Once we visually inspect where the equilibria are, we compute the exact equilibria numerically. This is done by solving a fixed point problem in $\tilde{Q}$ and v from (29)-(30), using a starting point close to the location of an equilibrium through visual inspection.

3 Multiple Equilibria

The model generates multiple equilibria. We first illustrate this by graphically showing the equilibria for a particular parameterization. In order to provide insight on what drives these multiple equilibria, we will consider two special cases. After that we return to the general case and argue that θ_t plays a dual role in the model, both as a fundamental that affects wealth and a sunspot that generates self-fulfilling shifts in perceived risk. We finish the section by discussing switching equilibria that allow for occasional self-fulfilling shifts between low and high risk states.

3.1 Graphic Representation of Equilibria

Figure 1 represents the schedules (29) and (30) for a particular parameterization. Schedule (29) is represented by the red lines and (30) by the blue triangular shape. The parameterization (at the bottom of the Figure) is not chosen to match any data, but simply to generate a picture that illustrates the multiplicity of equilibria. The Figure shows 4 equilibria. This is typical for a broad range of parameter choices in the model. There are also parameterizations for which there are just 2 equilibria (an example follows below) or no equilibria.[16]

While we will investigate the nature of all equilibria, we focus mostly on equilibria 1 and 2 where $v > 0$. In that case an increase in $\tilde{W}_t$ raises the equity price

[16] Although we cannot rule it out for sure, we have not found parameterizations where there are either 3 equilibria or just 1 equilibrium.

(when evaluated at $\theta_t = 0$). Equilibrium 1 is the low risk equilibrium as v and V are close to 0 and the equity price is therefore almost constant. Equilibrium 2 is the high risk equilibrium where v and V are respectively 0.9 and 3.1. The standard deviation of q_{t+1} is respectively 0.01 and 0.36 in equilibria 1 and 2 (evaluated at $\theta_t = 0$).

Two factors drive the multiplicity of equilibria. The first is associated with self-fulfilling beliefs about the magnitude of liquidity. It results from the circular relationship between risk, leverage and liquidity discussed in section 2.2. When agents believe that liquidity is low, they believe that wealth shocks have a big impact on the equity price. This implies that risk is high. High risk implies low leverage, which in turn implies a thin market and therefore low liquidity. The perceived low liquidity is therefore self-fulfilling.

There is a second, more subtle, factor generating multiplicity of equilibria that is associated with a dual role of θ_t. In equilibrium 1, θ_t plays the standard role of a fundamental that affects wealth. But we will show that in the other equilibria θ_t has the additional role of a sunspot that leads to self-fulfilling changes in perceived risk.

In order to help better understand these sources of multiple equilibria, we now turn to two special cases. In the first case we set $\rho_\theta = 0$. In this case only the first type of multiple equilibria is present because risk is constant in each equilibrium. The second special case is where $\gamma_I = \gamma_L$, so that $m = 0$ and θ_t does not affect $\tilde{W}_t$. In that case θ_t does not enter the model anywhere and therefore represents an extrinsic sunspot variable. For both of these special cases we keep the other parameters unchanged relative to that in Figure 1.

3.2 Special Case I: Circular Relationship between Risk, Leverage and Liquidity

The first special case is shown in Figure 2. There are just two equilibria. Equilibrium 1 is again the low risk equilibrium and equilibrium 2 the high risk equilibrium. The multiplicity that arises here is reminiscent of models with limited participation. Examples of limited participation models with multiple equilibria are Pagano (1989), Allen and Gale (1994) and Jeanne and Rose (2002). In these models there are fewer agents in the market in the high risk equilibrium, reducing liquidity and therefore generating higher risk. The opposite is the case in the low risk equilibrium. The difference in our model is that the number of agents does not decrease when risk is high, but rather agents reduce their exposure. Similar to a smaller number of agents in the market, lower leverage implies a thinner market (low liquidity), which in turn justifies the high risk beliefs.

This first source of multiple equilibria results from the circular relationship between risk, leverage and liquidity. It does not rely on the dynamic nature of the model. It applies similarly in static models, as in most of the limited participation models. Even though our model is dynamic, risk is constant over time within each of the equilibria in this special case.

3.3 Special Case II: Sunspot Equilibria with Self-fulfilling Shifts in Risk

The second special case, where θ_t is a sunspot, is shown in Figure 3. In this case there are again 4 equilibria. Equilibrium 1 is the only fundamentals equilibrium. In this equilibrium, θ_t has no impact on the equity price. The equity price is constant as $v = V = 0$. The other three equilibria are all sunspot equilibria as either V, or both v and V, are non-zero. Equilibria 2 and 4 are essentially the same as only the sign of v differs. This amounts simply to replacing the sunspot θ_t with the sunspot $-\theta_t$. They follow the same process.

In order to understand these sunspot equilibria, consider equilibrium 3 in Figure 3. In this case $v = 0$ and $V > 0$. Denoting $Risk_t$ as the variance of Q_{t+1} from the perspective of time t, from (26) we have

$$Risk_t = 4\rho_\theta^2 \tilde{Q}^2 V^2 \sigma_\theta^2 \theta_t^2 \tag{31}$$

In contrast to the first special case, risk is now time-varying. A change in θ_t from zero, in either direction, leads to an increase in risk. The quadratic approximation of the equity price in this case is

$$Q_t = \tilde{Q} - V\tilde{Q}\theta_t^2 \tag{32}$$

Using (31), this can also be written as

$$Q_t = \tilde{Q} - \lambda Risk_t \tag{33}$$

where $\lambda = 1/[4\rho_\theta^2 \tilde{Q} V \sigma_\theta^2] > 0$.

The equity price only fluctuates in this case due to self-fulfilling changes in risk that are generated by changes in θ_t. How is this possible? An increase in risk by definition means more uncertainty about Q_{t+1}. In this case $Q_{t+1} = \tilde{Q} - \lambda Risk_{t+1}$, so that Q_{t+1} is only affected by risk at $t+1$. An increase in risk about Q_{t+1} therefore implies an increase in uncertainty about future risk $Risk_{t+1}$. In other words, the level of risk and the volatility of future risk must change simultaneously. This is indeed the case here:[17]

$$var_t(Risk_{t+1}) = 64\rho_\theta^6 \tilde{Q}^4 V^4 \sigma_\theta^6 \theta_t^2 \tag{34}$$

This source of multiple equilibria has nothing to do with the circular relationship between risk, leverage and liquidity. It is a bit unusual to even think of what liquidity means in this context as θ_t does not generate asset demand shocks through wealth.

It is maybe surprising that these types of sunspot equilibria associated with self-fulfilling joint shifts in risk and the volatility of risk have not been analyzed before in the economics literature. They naturally occur with infinite horizon in any market where demand depends on risk. While this is surely the case in asset markets, it could apply to goods and labor markets as well. To see the generality of the argument, consider a market where demand is a negative linear function of both the price p_t and risk $var_t(p_{t+1})$. Equating this to supply (which may be constant or depend positively on the price), the equilibrium price is

$$p_t = \phi_1 - \phi_2 var_t(p_{t+1}) \tag{35}$$

Assume that S_t is a sunspot variable that follows an AR process with persistence ρ_s and variance of its innovation of σ_s^2. Then it is easily verified that $p_t = \phi_1$ and $p_t = \bar{p} - S_t^2/[4\phi_2\rho_s^2\sigma_s^2]$ are both solutions (with $\bar{p}$ a constant). The former is a fundamental equilibrium and the latter a sunspot equilibrium.[18]

[17] We use (31) at $t+1$ and $\theta_{t+1}^2 = \rho_\theta^2\theta_t^2 + 2\rho_\theta\theta_t\epsilon_{t+1}^\theta + (\epsilon_{t+1}^\theta)^2$. We again adopt the continuous time approximation σ_θ^2 for the last term.

[18] In this case no approximation is needed to solve for the sunspot equilibrium. When we allow demand to depend negatively on p_t as well as p_t^2, it is easily verified that there are also sunspot equilibria that depend both on S_t and S_t^2. However, in that case an approximation is needed that drops cubic and higher order terms.

3.4 The General Case: Dual Role of θ_t

We now return to the general case where both ρ_θ and m are non-zero, as in the example of Figure 1. In that case both sources of multiple equilibria are present simultaneously and are impossible to disentangle.

In this general case θ_t plays two separate roles. Its first role is that of a fundamental. As long as $m > 0$, an increase in θ_t (lower relative wealth of leveraged institutions) reduces equity demand and its price. In addition we saw in section 2.2 that a contraction of the wealth of leveraged institutions, which lowers $\tilde{W}_t$, lowers market liquidity. This in turn increases risk, which lowers asset demand and the equity price even more. All of this is further amplified as the higher risk reduces leverage, which reduces liquidity even further. With all of these amplification mechanisms it may be surprising that nonetheless the sensitivity of the asset price to θ_t is so small in the fundamental equilibrium 1 in Figure 1 (v and V are both close to 0). The reason for this is that there is a counteracting force. The drop in the equity price raises the expected excess return on equity, which increases leverage. This in turn raises liquidity and reduces risk.

The second role of θ_t is that of a sunspot that leads to self-fulfilling shifts in risk. This additional role occurs in equilibria 2, 3 and 4 of Figure 1. In order to see this, start from $m = 0$, where θ_t is a pure sunspot. As we raise m slightly above 0, so that θ_t is no longer a pure sunspot and affects wealth, equilibria 2, 3 and 4 remain very close to the sunspot equilibria in Figure 3. As we let $m \to 0$, these equilibria converge to the sunspot equilibria. Introducing a fundamental role for θ_t therefore does not remove its sunspot role. Only equilibrium 1 is a pure fundamental equilibrium that converges to the fundamental equilibrium 1 of Figure 3 when m goes to zero.

In the context of a very different model, Manuelli and Peck (1992) also find equilibria that converge to pure sunspot equilibria as the fundamental component of a shock vanishes. They call these sunspot-like equilibria. They write: "There are two ways that random fundamentals can influence economic outcomes. First, randomness affects resources which intrinsically affects prices and allocation. Second, the randomness can endogenously affect expectations or market psychology, thereby leading to excessive volatility." In their model this dual role is played by aggregate endowment shocks. In our model it is played by θ_t.[19]

3.5 Switching Equilibria

Beyond the equilibria that we have already discussed, there are additional equilibria that allow for a switch between high and low risk states. As we will see, this is particularly relevant when trying to explain sudden panics in financial markets. The reason is that when we switch to the high risk state, θ_t suddenly takes on the additional role of a sunspot generating a self-fulfilling increase in perceived risk.

Let state 1 be the low risk state and state 2 the high risk state. Let $p_1 > 0.5$ be the probability that we remain in a low risk state next period when we are in a low risk state today. Similarly, $p_2 > 0.5$ is the probability that we remain in a high risk state next period when we are in a high risk state today. The switch between states therefore follows a simple Markov process. It is driven by a sunspot that is external to the model.[20]

We conjecture that the log equity price in state i is

$$q_t = q_i - v_i\theta_t - V_i\theta_t^2 \tag{36}$$

We now need to solve for 6 unknown parameters of the equity price (3 for each state). This is done by imposing equity market equilibrium as before (up to quadratic terms in θ_t. This needs to be done separately for both states. In computing the expectation and variance of Q_{t+1} we now need to take into account that a switch to a different state is possible. We leave the algebra to Appendix C.

We focus on a switch between the low and high risk states represented by equilibria 1 and 2 of Figure 1. However, we do not literally switch between equilibria 1 and 2. We switch between high and low risk states. When p_1 is extremely close to 1, the low risk state is very close to equilibrium 1. But when p_1 drops further below 1, a switch to the high risk state becomes more likely. The possibility of

[19]Spears, Srivastava and Woodford (1990) also present a model with sunspot-like equilibria. They point out that "...a sharp distinction between "sunspot equilibria" and "non-sunspot equilibria" is of little interest in the case of economies subject to stochastic shocks to fundamentals." Indeed, as we raise m slightly above 0, it is technically no longer a pure sunspot equilibrium, but operates just like one.

[20]Notice that the sunspot shock leading to a switch in equilibria is totally different from the sunspot role played by *theta* shocks.

switching to a high risk state makes the low risk state itself riskier, and more risky than equilibrium 1.

As an illustration, Figure 5 shows the values of Q_i, v_i and V_i in the low and high risk states for the case where $p_1 = p_2$. When these probabilities are equal to 1, the two states correspond exactly to equilibria 1 and 2 of Figure 1. A couple of points are worth making. First, switching equilibria only exist when the probability of remaining in the same state is high enough. In this example the high and low risk equilibria become the same equilibrium when $p_1 = p_2$ is less than 0.7. Second, in this illustration it is mainly the low risk state that is affected by the probabilities. The lower the probability of staying in the low risk state, the higher the risk in the low risk state (higher values of v and V).

The increase in risk when we switch to the high risk state leads to a drop in asset demand and therefore the equity price. It also reduces leverage and therefore market liquidity. When the magnitude of these changes is very large we can speak of a financial panic. While such a panic occurs unexpectedly and suddenly in the model, the magnitude of the panic depends critically on the financial health of leveraged institutions.

Consider for example the equity price (a similar argument applies to risk, leverage and liquidity). The change in the log equity price from the low to the high risk state is

$$\tilde{q}_2 - \tilde{q}_1 - (v_2 - v_1)\theta_t - (V_2 - V_1)\theta_t^2 \tag{37}$$

Since $v_2 - v_1$ and $V_2 - V_1$ are positive (see Figure 5), the price impact is larger when θ_t is more positive and leveraged institutions are therefore in weaker financial shape. For example, when $p_1 = p_2 = 0.75$ the switch leads to a drop in the equity price by 12% when starting from the neutral level of $\theta_t = 0$, but a drop by 85% when θ_t is two standard deviations above its unconditional mean of 0.

The main reason for this result is that after the switch to the high risk equilibrium θ_t takes on the additional role of a sunspot that generates a self-fulfilling increase in risk. Completely separate from its fundamental role through the impact on wealth, it becomes a variable around which agents suddenly coordinate their expectations and perceptions of risk. The weaker the financial health of leveraged institutions (the higher θ_t), the bigger this effect. In addition the increase in risk also strengthens the fundamental impact of the weak health of leveraged institutions. This is because the higher risk reduces liquidity, which amplifies the price impact of the drop in asset demand that took place prior to the panic due to financial losses of leveraged institutions.

It is therefore very well possible that the negative implications of the financial losses of leveraged institutions are mainly felt after a switch to the high risk equilibrium, with a much more modest impact prior to that. We will further explore this in the next section.

4 Dynamics of Risk, Leverage and Liquidity during the Recent Financial Crisis

As pointed out in the Introduction, our aim here is not to explain the ultimate source of the recent financial crisis. In particular, we take accumulating financial losses of leveraged institutions as given and focus on the implications for the dynamics of risk, leverage, liquidity and asset prices. The model is also too simple to consider a precise calibration to the data. Nonetheless we show that it generates dynamics that is qualitatively consistent with what happened during the 2007-2008 crisis.

The recent crisis should be broken into two parts. The first part is the relatively calm period from the beginning of 2007 until September of 2008. The second part is the financial panic that started in September of 2008 and lasted at least till the end of 2008. Using data for the United States, we focus on what happened with regards to the following set of variables: (1) stock prices, (2) T-bill rate, (3) equity price risk, (4) volatility of risk, (5) net worth of leveraged institutions, (6) leverage, and (7) market liquidity. Stock prices are measured by the DJ U.S. total stock market index. Risk is measured as the CBOE SPX volatility VIX index. Volatility of risk is the standard deviation of the VIX index over the past 30 days. Net worth and leverage are based on U.S. brokers and dealers as reported by the Federal Reserve Flow of Funds. Market liquidity is from Pastor and Shambaugh (2003) and measures the impact of order flow on the expected excess return. This variable is the most difficult to measure in the data as it is a theoretical concept that does not have a straightforward empirical counterpart.

The dynamics of the variables during the crisis are illustrated in Figure 5. The vertical line represents the collapse of Lehman Brothers on September 15, 2008, which we consider to be the start of the financial panic. The charts can be summarized as follows. After a modest decline in stock prices and increase in risk during the tranquil period of the crisis, stock prices suddenly crashed and risk spiked in September 2008. Volatility of risk also shot up, after showing no trend before that. A flight to quality during the panic lead to a drop in the T-bill rate to near zero. Net worth gradually declined after mid 2007 until the third quarter 2008, to quickly recover after the crisis. Financial leverage rose significantly during the tranquil period of the crisis and then fell sharply during the panic stage of the crisis. Finally, liquidity fell sharply during the financial panic after a much more modest drop prior to that.[21]

Model Simulation

In order to illustrate the dynamics of the variables in the model, and relate them to the recent crisis, we consider a two-state switching equilibrium as described in section 3.5. As we discuss further below, the qualitative implications of the model are not very sensitive to parameterization. The parameterization that we choose to illustrate the dynamics (see bottom of Figure 6) is nonetheless different from that in Section 3. While nice in terms of illustrating graphically the multiple equilibria, the parameterization in Section 3 is less attractive in terms of illustrating the dynamics. For example, it implies extreme interest rate and stock price volatility and a leverage ratio below 1 in the high risk equilibrium.[22]

We assume that the probability of staying in the low risk state once we are in a low risk state is 0.95. This implies that a switch to a high risk state happens infrequently. The probability of remaining in the high risk state once we enter the high risk state is set at 0.7. This implies that the high risk state is generally of much shorter duration than the low risk state.

We simulate the model over 16 periods, which we interpret as quarters. We do not make any attempt to match the process of financial losses in the data. Rather,

[21] In the pre-panic period there is one sharp negative outlier in Q1, 2008. This may be a data measurement problem as Pastor and Stambaugh (2003) do not have actual data on order flow to estimate liquidity (they use a proxy that depends on the volume of trade signed by the direction of the price change).

[22] At the same time, the parameterization we choose here is less attractive for the purpose of graphically illustrating all the multiple equilibria. For example, equilibrium 4 occurs at extremely negative values for v.

to facilitate understanding of what is driving the model, we consider a simple step function for θ_t. In period 2, θ_t rises from 0 to 0.3. It then stays there until the end of period 10. In period 11, θ_t drops back to zero, for example because of recapitalization of leveraged institutions by the government. The wealth of leveraged institutions then follows an analogous downward step function, illustrated in the first chart of Figure 6. It follows the overall pattern seen in the data in Figure 5, although obviously the deterioration of the capital of leveraged institutions was more gradual in the data.

Apart from the shock to θ_t, the model is also hit by a switch to the high risk state. This is indicated by the shaded area in the charts of Figure 6. The economy is in the high risk state from period 8 to period 14.

Interpreting periods as quarters, with the second period being Q1, 2007, the shocks that we consider are as follows. First, in Q1, 2007, there is large shock to θ_t that represent financial losses that reduce the capital (wealth) of leveraged institutions. At this time the economy is still in the low risk state that we consider to be the tranquil part of the crisis. This low risk state lasts through Q2, 2008. In Q3, 2008 there is a switch to the high risk state (financial panic). In Q2, 2009, θ_t is restored to its original level of zero. Leveraged institutions are recapitalized to the level before the crisis. Finally, in Q1, 2010, the economy returns to the low risk state.

These dates are obviously not meant to match the exact length of the panic or the period of financial weakness of leveraged institutions. Rather, we use this simple on-off analysis to highlight the separate roles of the financial health of leveraged institutions and the panic state. During different time intervals we consider all possible combinations of the financial health of leveraged institutions (normal versus bad) and the state (low risk, high risk) in order to evaluate the separate contribution of both elements.

Figure 6 illustrates the impact of the simulation on risk, leverage, liquidity, and asset prices. We use the following measures for the variables in Figure 6. The stock price and gross interest rate are Q_t and R_{t+1}. The stock price is normalized to 100 before the shocks. Risk is measured as the standard deviation of Q_{t+1}/Q_t. The volatility of risk is the standard deviation at time t of our risk measure at $t+1$. Leverage is equal to $\alpha_{L,t}$. Finally, liquidity is the measure (21) discussed in section 2.2, which stays close to the Shambaugh and Pastor (2003) measure in the data.

The main findings from Figure 6 can be summarized as follows. There is a large impact on asset prices, risk, and liquidity only when the economy is simultaneously hit by a panic (high risk state) and the financial wealth of leveraged institutions is weak. Either one of them alone has a far more modest negative impact. For example, the large deterioration of the financial health of leveraged institutions alone, during the tranquil part of the crisis, leads to only a modest increase in risk and drop in the equity price. Only when combined with a financial panic in Q3, 2008, do we see a large spike in risk and crash in the equity price and market liquidity.

But the panic by itself does not account for this large change in equity price, risk and liquidity either. This is illustrated by the fact that in Q2, 2009 (period 11) these variables are largely restored to pre-panic levels. At that point we are still in the panic state (high risk state) but the leveraged institutions are recapitalized. In other words, a large financial panic with a sharp increase in risk and a steep fall in the equity price and market liquidity only occurs when accompanied by a weak state of the financial health of leveraged institutions.

A couple of points are worth making about financial leverage and the volatility of risk. During the tranquil state of the crisis leverage more than doubles as a result of the large financial losses of leveraged institutions. Two opposing forces are at work here. First, the drop in the equity price raises the expected excess return on equity, which leads to increased leverage. Second, the increase in risk reduces leverage. We find that the first force almost always dominates in the model during the tranquil part of the crisis. When we are hit by the financial panic in Q3, 2008, risk spikes, causing a sharp drop in demand for equity. This leads to sharp deleveraging. Leverage returns to nearly its pre-crisis level.

With regards to the volatility of risk, we see that it is very little affected by the large financial losses of leveraged institutions alone. Only when combined with the panic does the volatility of risk spike (it increases about tenfold). However, in contrast to risk itself, the volatility of risk remains quite high even when leveraged institutions are recapitalized. While at this point the level of risk is only modestly high, the uncertainty about future risk remains high as risk is very sensitive to the financial condition of leveraged institutions in the panic state. This is the result of its sunspot role.

While the simple exercise we have conducted here is not meant to match precise data, the overall pattern in these variables is broadly in line with that in the data in Figure 5. During the pre-panic state of the crisis we saw that the impact on the equity price, risk, and liquidity was quite modest, as is the case in the model. The volatility of risk did not change at all, again in line with the model. The substantial increase in financial leverage during this period is also consistent with that in the model.[23] Then, during the switch to the panic state the model accounts for the sharp drop in the equity price, financial leverage, and market liquidity and the sharp increase in risk and the volatility of risk.

These results still hold qualitatively when considering different parameterizations. In the Technical Appendix we illustrate this point by considering large changes in each of the individual parameters relative to the values chosen for the illustration in Figure 6. Similarly, we show that the results are also not sensitive to the step function for θ_t. A gradual deterioration of the financial health of leveraged institutions leads to very similar charts as those in Figure 6.

5 Extensions

An important question is whether the results from our simple framework still hold in a more general context. In particular, what happens with other sources of shocks? And how would the results be affected if the initial wealth of agents is affected by asset prices? In this section, we examine these two questions.

There are several other types shocks that have an impact similar to θ shocks. This can be seen from the market clearing condition (11). For example, shocks to the (linearized) total endowment W would have the same impact on $\tilde{W}_t$ as θ shocks. Similarly shocks to the supply of capital $1/K$ would have a similar impact as shocks to $\tilde{W}_t$. We could also introduce noise traders who would have an impact similar to supply shocks. On the other hand, it is not straightforward to see the impact of persistent technology shocks. We examine this issue in the first subsection.

The second extension generalizes the benchmark model by making the wealth

[23]The model does not account for the drop in the interest rate prior to the panic as that is largely related to monetary policy. It also does not account for the negative spike in liquidity in Q1, 2007. But as already pointed out, this may well be due to measurement problems.

of investors and leveraged institutions explicitly depend on the asset price. We do so through a very simple mechanism that keeps the tractability of the model intact. The purpose of this extension is to consider the impact of an additional feedback mechanism that has been widely discussed in the context of the recent crisis. A drop in asset prices reduces the wealth of financial institutions further, which could serve to amplify the impact of the original financial losses.

5.1 Persistent Technology Shocks

So far we have focused on the case where $\rho_a = 0$, so that technology shocks have no persistence. In that case technology shocks only add a constant term to both the expected value and the variance of the excess payoff on equity. In that case, and assuming $\rho_\theta > 0$, the only state variable is θ_t. Instead, when technology shocks are persistent, a_t becomes a state variable. We now consider the case where $\rho_a > 0$ and where financial shocks are absent ($\sigma_\theta = 0$), so that a_t is the only state variable. The conjectured log asset price then becomes

$$q_t = \tilde{q} + va_t - Va_t^2 \tag{38}$$

The solution method is analogous to that for the case of financial shocks and we leave the details to the Technical Appendix.

Panel A of Figure 7 provides a picture of the equilibria for the same parameterization as in Figure 1. The only change is that we replaced $\rho_\theta = 0.4$ with $\rho_a = 0.4$ and set $\sigma_\theta = 0$. Figure 7 looks remarkably similar to Figure 1. There are again four equilibria and the shapes of the curves are also broadly similar. This happens even though the shock is now totally different (technology shocks versus wealth redistribution shocks).

In the case of persistent financial shocks we saw that there are two sources of multiplicity of equilibria. The first is due to self-fulfilling beliefs about the magnitude of liquidity. The second is associated with the possibility of sunspot equilibria. With technology shocks however, only the second source of multiple equilibria is present. In the case of no persistence of the state variable, for which we previously illustrated the first type of multiple equilibria, it is easy to check that now there is only a single equilibrium where $v = V = 0$. Moreover, the concept of market liquidity is without meaning when there are only technology shocks.

The multiplicity of equilibria is now only driven by the sunspot role of the state variable a_t. Equilibria 2, 3 and 4 are all sunspot-like equilibria. This can be seen by replacing the technology variable (13) with

$$A_{t+1} = \bar{A}e^{ma_{t+1}-0.5ma_{t+1}^2}$$

where $m \geq 0$. As $m \to 0$, equilibria 2, 3 and 4 converge to sunspot equilibria. When $m = 0$ the technology variable a_t has no direct impact on the model. But there are still 3 sunspot equilibria where a_t impacts the asset price. This is illustrated in panel B of Figure 7, which looks very similar to Figure 3 for the case of financial shocks.

It is also again possible to show that there are switching equilibria that allow for a switch between high and low risk states. As before, the impact of a switch to the bad state is larger the weaker the fundamental (in this case the lower a_t).

5.2 Endowment in Capital

We now return to the benchmark model with financial shocks. While we have seen that the model has a variety of amplification mechanisms that enhance the impact of the deteriorating health of leveraged institutions, we have abstracted from one mechanism that has been frequently discussed. The financial losses of leveraged institutions will be amplified by the resulting drop in asset prices. The model abstracts from this by adopting an OLG structure, where agents are born with a goods endowment. The OLG assumption is made for tractability as in an infinite horizon framework the wealth of investors and leveraged institutions are additional state variables. In that case it would be impossible to visually inspect the multiplicity of equilibria and hard to know that multiple equilibria even exist.

Here we consider an extension that keeps intact the tractability generated by the OLG structure, while at the same time allowing for wealth to be a function of the asset price. We do so by assuming that newborn agents receive an endowment of capital in addition to their endowment in goods. Moreover, we introduce depreciation of capital equal to the capital endowment, so that the aggregate capital stock is unchanged. Assume that a fraction δ of the capital stock depreciates each period. The return on capital is then $R_{K,t+1} = (A_{t+1} + (1-\delta)Q_{t+1})/Q_t$. Leveraged institutions receive $\delta_L K$ and investors receive $\delta_I K$, with $\delta_L + \delta_I = \delta$. The value of these additional endowments is respectively $\delta_L K Q_t$ and $\delta_I K Q_t$. Agents receive this on top of their non-asset endowments that depend on θ_t as before. A drop in the asset price Q_t therefore has an additional negative wealth effect that was not previously present. Otherwise the model is the same as before. We again leave all the algebra to the Technical Appendix and only discuss the qualitative impact of this additional mechanism.

Introducing a capital endowment has two major implications regarding volatility and the impact of shocks. First, the percentage change in wealth due to a shock to θ_t is smaller. This reduces the impact of θ shocks and reduces volatility. The second, and more interesting, effect is that the capital endowment amplifies the impact of price changes, thereby increasing volatility. We find that the second effect dominates in the case of sunspot-like equilibria, so that volatility is amplified, but the first effect dominates in the fundamental equilibrium, so that volatility is dampened.

In order to illustrate these points, consider a numerical example and assume that $\delta_I = \delta_L = 0.25$. The precise values of these numbers do not matter for the qualitative impact of this extension. If we keep the parameterization otherwise the same as in Figure 1, we find the following. First, the price impact of shocks in the fundamental equilibrium 1 is indeed slightly dampened. The values of v and V change from respectively 0.034 and 0.076 under the benchmark parameterization to 0.025 and 0.038 with the additional feedback mechanism. On the other hand, the price impact of shocks is significantly amplified in the three sunpot-like equilibria. For example, in equilibrium 2 the values of v and V change from respectively 0.891 and 3.142 under the benchmark to 1.924 and 8.802 with the additional feedback mechanism. This means that the impact of financial shocks on the asset price is more than doubled. This is a result of the sunspot role of θ_t. Financial losses lead to a self-fulfilling increase in risk, and the volatility of risk, in equilibrium 2. This higher risk reduces asset demand, which lowers the asset price. The lower asset price in turn reduces the wealth of investors and leveraged institutions. This further reduces asset demand and therefore the asset price. The larger price impact in equilibrium 2 also implies that the magnitude of financial panics (in switching equilibria) is amplified by this additional mechanism.

6 Conclusion

This paper has examined the impact of leveraged institutions, as less risk-averse investors, on the dynamics of asset prices. Their more aggressive behaviour in presence of expected excess returns gives leveraged investors a stabilizing role that reduces market volatility. But this stabilizing role tends to fluctuate over time thereby increasing volatility and contributing to time-varying risk. There are two main reasons behind the fluctuations in this stabilizing role. First, their wealth can vary over time. For example, a drop in their wealth gives them a smaller role which increases price risk. The second reason is the perception of risk. Since risk is time varying, its perception also changes over time. In particular it is possible to a have large changes in the perception of risk due to the presence of multiple equilibria.

While the core of our analysis focuses on shocks to the wealth of leveraged institutions, we also show that the source of shocks is not crucial. What matters is the arbitraging role of leveraged institutions when confronted by shocks. In all the cases we have investigated, we found multiple equilibria, with high-volatility sunspot-like equilibria. Shocks have a dual role. On the one hand they affect market fundamentals. On the other hand they affect the perception of risk. This second role dominates in the sunspot-like equilibria.

The two stages of the recent financial crisis are difficult to explain by traditional models. While the crisis started in mid-2007, it only turned into a dramatic situation in the fall 2008. We show, however, that our framework is consistent with this evolution and with the major features of the crisis. A decline in the wealth of leveraged institutions followed by a jump in perceived risk can explain the evolution of asset prices, volatility, liquidity, leverage, and the volatility of risk. We should note, however, that our simplified framework is not meant to capture all the aspects of the recent financial crisis and should be seen as complementary to the existing analysis.

The simplicity of our framework has allowed us to identify various mechanisms at work, in particular in the context of multiple equilibria. Now that these mechanisms have been identified, it would be interesting to extend the analysis in various directions, especially in a macroeconomic perspective. Our future research will explore several of these directions. First, we need to explore the real effects by introducing a more interesting production and lending process. Second, we want to explore the international dimensions. Since leveraged institutions are mainly based in the US, it will be interesting to investigate the implications of this asymmetry for international capital flows. Third, we want to explore policy issues. An obvious concern is whether monetary policy should take into account asset price dynamics in setting interest rates. Fourth, we want to analyze empirically the interactions between leverage, liquidity, risk and asset prices.

Appendix

A Deriving the Optimal Portfolio

Optimal portfolio shares can be derived by using the continuous time approximation of the log portfolio return using Ito's Lemma. To see how this is done, here follows a brief digression into continuous time algebra. Let Z_t be the cumulative return of investment in equity and Z_t^B the cumulative return of investment in the riskfree bond. Assume, in continuous time notation

$$dZ_t = Z_t(\mu_t dt + \sigma_t d\omega_t) \tag{39}$$

$$dZ_t^B = Z_t^B \mu_t^B dt \tag{40}$$

Note that the corresponding discrete time analogs are

$$R_{K,t+1} - 1 = \frac{dZ_t}{Z_t} = \mu_t dt + \sigma_t d\omega_t \tag{41}$$

$$R_{t+1} - 1 = \frac{dZ_t^B}{Z_t^B} = \mu_t^B dt \tag{42}$$

Now consider the overall portfolio return. Let Z_t^p be the cumulative portfolio return in continuous time. We have

$$dZ_t^p = Z_t^p \left(\alpha_t \frac{dZ_t}{Z_t} + (1 - \alpha_t) \frac{dZ_t^B}{Z_t^B} \right) \tag{43}$$

Therefore

$$\frac{dZ_t^p}{Z_t^p} = \alpha_t(\mu_t dt + \sigma_t d\omega_t) + (1 - \alpha_t)\mu_t^B dt \tag{44}$$

This again corresponds to discrete time:

$$R_{t+1}^p - 1 = \frac{dZ_t^p}{Z_t^p} \tag{45}$$

We are now in a position to apply Ito's Lemma. For a function $f(Z_t^p)$ we have

$$df\left(Z_t^p\right) = \left[Z_t^p \left[\alpha_t \mu_t + (1 - \alpha_t)\mu_t^B \right] f'\left(Z_t^p\right) + \frac{\left(Z_t^p\right)^2}{2} \alpha_t^2 \sigma_t^2 f''\left(Z_t^p\right) \right] dt + f'\left(Z_t^p\right) Z_t^p \alpha_t \sigma_t d\omega_t \tag{46}$$

If f is the log function we have:

$$dln(Z_t^p) = \left[\left[\alpha_t \mu_t + (1-\alpha_t)\mu_t^B\right] - \frac{1}{2}\alpha_t^2 \sigma_t^2\right] dt + \alpha_t \sigma_t d\omega_t \tag{47}$$

There are now two alternative, but closely related, ways to proceed to obtain a continuous time approximation of the log portfolio return. One can either relate the log portfolio return to the log return of equity and bonds or relate the log portfolio return to the level returns on equity and bonds. The first approach is followed by Campbell. We will follow the second approach.

The expression for $dln(Z_t^p)$ is the continuous time equivalent of the log portfolio return $ln(R_{t+1}^p)$. Consider the Campbell approach first. To that end we need to have expressions for the log equity and bond returns. The relation between continuous and discrete time is as follows:

$$r_{K,t+1} = dln(Z_t) \tag{48}$$

$$r_{t+1} = dln(Z_t^B) \tag{49}$$

Applying Ito's Lemma we have

$$dln(Z_t) = \mu_t dt + \sigma_t d\omega_t - 0.5\sigma_t^2 dt \tag{50}$$

$$dln(Z_t^B) = \mu_t^B dt \tag{51}$$

This implies

$$\begin{aligned} ln(R_{t+1}^p) = dln(Z_t^p) = \alpha_t dln(Z_t) + (1-\alpha_t) dln(Z_t^B) + 0.5\alpha_t(1-\alpha_t)\sigma_t^2 dt = \\ \alpha_t r_{K,t+1} + (1-\alpha_t) r_{t+1} + 0.5\alpha_t(1-\alpha_t) var(r_{K,t+1}) \end{aligned} \tag{52}$$

This is exactly the expression for the log portfolio return used by Campbell (and of course also Evans and Hnatkovska).

Before showing the second method, it is useful to show the implied portfolio. Assume that agents maximize

$$E_t \frac{1}{1-\gamma} \left(R_{t+1}^p\right)^{1-\gamma} = E_t \frac{1}{1-\gamma} e^{(1-\gamma) r_{t+1}^p} \tag{53}$$

If the portfolio return is log normal, this becomes

$$\frac{1}{1-\gamma} e^{(1-\gamma) E_t r_{t+1}^p + 0.5(1-\gamma)^2 var(r_{t+1}^p)} \tag{54}$$

Maximizing this is equivalent to maximizing

$$E_t r_{t+1}^p + 0.5(1-\gamma)var(r_{t+1}^p) \tag{55}$$

From the continuous time approximation of the log portfolio return, this last expression becomes

$$\alpha_t E_t r_{K,t+1} + (1-\alpha_t) r_{t+1} + 0.5\alpha_t(1-\alpha_t)var(r_{K,t+1}) + 0.5(1-\gamma)\alpha_t^2 var(r_{K,t+1}) \tag{56}$$

Maximizing with respect to α_t gives the familiar portfolio

$$\alpha_t = \frac{E_t r_{K,t+1} - r_{t+1} + 0.5var(r_{K,t+1})}{\gamma var(r_{K,t+1})} \tag{57}$$

Campbell then uses the linear approximation of the log equity return as a function of the log asset price and log dividend.

The second method is really quite similar. Again using (47), we have

$$\begin{aligned} &ln(R_{t+1}^p) = dln(Z_t^p) = \alpha_t \frac{dZ_t}{Z_t} + (1-\alpha_t)\frac{dZ_t^B}{Z_t^B} - 0.5\alpha_t^2\sigma_t^2 dt = \\ &\alpha_t(R_{K,t+1}-1) + (1-\alpha_t)(R_{t+1}-1) - 0.5\alpha_t^2 var(R_{K,t+1}) \end{aligned} \tag{58}$$

We again maximize (55), which now becomes

$$\begin{aligned} &\alpha_t E_t(R_{K,t+1}-1) + (1-\alpha_t)(R_{t+1}-1) - 0.5\alpha_t^2 var(R_{K,t+1}) + 0.5(1-\gamma)\alpha_t^2 var(R_{K,t+1}) = \\ &\alpha_t E_t(R_{K,t+1}-1) + (1-\alpha_t)(R_{t+1}-1) - 0.5\gamma\alpha_t^2 var(R_{K,t+1}) \end{aligned} \tag{59}$$

Maximization again gives a familiar portfolio:

$$\alpha_t = \frac{E_t R_{K,t+1} - R_{t+1}}{\gamma var(R_{K,t+1})} \tag{60}$$

B Solving the Model

From subsection 3.2, we find:

$$\begin{aligned} Z_0 \quad &= \bar{W}\left(\bar{A} + \tilde{Q} + \tilde{Q}(-V + 0.5v^2)\sigma_\theta^2 - \frac{1}{\eta}(\nu - E)\tilde{Q} - \frac{1}{\eta}K\tilde{Q}^2\right) \\ &-K\bar{A}^2\sigma_a^2 + \tilde{Q}^2 K v^2 \sigma_\theta^2 \end{aligned} \tag{61}$$

$$
\begin{aligned}
Z_1 \quad &= m\left(\bar{A}+\tilde{Q}+\tilde{Q}(-V+0.5v^2)\sigma_\theta^2-\frac{1}{\eta}(\nu-E)\tilde{Q}-\frac{1}{\eta}K\tilde{Q}^2\right)+ \\
&\bar{W}\tilde{Q}(v\rho-v\frac{1}{\eta}(\nu-E)-\frac{1}{\eta}2K\tilde{Q}v)-4K\tilde{Q}^2v(-V+0.5v^2)\rho\sigma_\theta^2 \qquad (62)
\end{aligned}
$$

$$
\begin{aligned}
&Z_2 = m\tilde{Q}(v\rho-v\frac{1}{\eta}(\nu-E)-2\frac{1}{\eta}K\tilde{Q}v)+ \\
&\bar{W}\tilde{Q}\left[(-V+0.5v^2)\rho^2-\frac{1}{\eta}(\nu-E)(-V+0.5v^2)-\frac{1}{\eta}2K\tilde{Q}(-V+v^2)\right] \\
- \quad &4K\tilde{Q}^2(V-0.5v^2)^2\rho^2\sigma_\theta^2 \qquad (63)
\end{aligned}
$$

The strategy is to solve $\tilde{Q}V$ from (61) as a quadratic function of v and substitute the result in (62) and (63). This gives respectively a third and fourth order polynomial in v that needs to be solved numerically.

From (61) we can solve

$$\tilde{Q}V = \alpha_1 + \alpha_2 v^2 \qquad (64)$$

where

$$\alpha_1 = \frac{1}{\sigma_\theta^2}\left(\bar{A}+\tilde{Q}-\frac{1}{\eta}(\nu-E)\tilde{Q}-\frac{1}{\eta}K\tilde{Q}^2\right)-\frac{K\bar{A}^2\sigma_a^2}{\bar{W}\sigma_\theta^2} \qquad (65)$$

$$\alpha_2 = 0.5\tilde{Q}-\frac{\tilde{Q}^2K}{\bar{W}} \qquad (66)$$

From (62) we have

$$\beta_1+\beta_2v+\beta_3v^2+\beta_4v^3+\beta_5[\tilde{Q}V]+\beta_6[\tilde{Q}V]v \qquad (67)$$

where

$$\beta_1 = m\left(\bar{A}+\tilde{Q}-\frac{1}{\eta}(\nu-E)\tilde{Q}-\frac{1}{\eta}K\tilde{Q}^2\right) \qquad (68)$$

$$\beta_2 = \bar{W}\tilde{Q}\left(\rho-\frac{1}{\eta}(\nu-E)-\frac{1}{\eta}2\tilde{Q}K\right) \qquad (69)$$

$$\beta_3 = 0.5m\tilde{Q}\sigma_\theta^2 \qquad (70)$$

$$\beta_4 = -2K\tilde{Q}^2\rho\sigma_\theta^2 \qquad (71)$$

$$\beta_5 = -m\sigma_\theta^2 \qquad (72)$$

$$\beta_6 = 4K\tilde{Q}\rho\sigma_\theta^2 \qquad (73)$$

Finally, (63) can be written as

$$\lambda_1 v + \lambda_2 v^2 + \lambda_3 v^4 + \lambda_4[\tilde{Q}V] + \lambda_5[\tilde{Q}V]^2 + \lambda_6[\tilde{Q}V]v^2 = 0 \tag{74}$$

where

$$\lambda_1 = m\tilde{Q}\left(\rho - \frac{1}{\eta}(\nu - E) - \frac{1}{\eta}2K\tilde{Q}\right) \tag{75}$$

$$\lambda_2 = 0.5\bar{W}\tilde{Q}\rho^2 - 0.5\bar{W}\tilde{Q}\frac{1}{\eta}(\nu - E) - \frac{1}{\eta}2\bar{W}K\tilde{Q}^2 \tag{76}$$

$$\lambda_3 = -K\tilde{Q}^2\rho^2\sigma_\theta^2 \tag{77}$$

$$\lambda_4 = -\bar{W}\left(\rho^2 - \frac{1}{\eta}(\nu - E) - \frac{1}{\eta}2\tilde{Q}K\right) \tag{78}$$

$$\lambda_5 = -4K\rho^2\sigma_\theta^2 \tag{79}$$

$$\lambda_6 = 4K\tilde{Q}\rho^2\sigma_\theta^2 \tag{80}$$

Substituting (64) into (67), we have

$$h_1 + h_2 v + h_3 v^2 + h_4 v^3 = 0 \tag{81}$$

where

$$h_1 = \beta_1 + \beta_5\alpha_1 \tag{82}$$

$$h_2 = \beta_2 + \beta_6\alpha_1 \tag{83}$$

$$h_3 = \beta_3 + \beta_5\alpha_2 \tag{84}$$

$$h_4 = \beta_4 + \beta_6\alpha_2 \tag{85}$$

Substituting (64) into (74), we have

$$g_1 + g_2 v + g_3 v^2 + g_4 v^4 = 0 \tag{86}$$

where

$$g_1 = \lambda_4\alpha_1 + \lambda_5\alpha_1^2 \tag{87}$$

$$g_2 = \lambda_1 \tag{88}$$

$$g_3 = \lambda_2 + \lambda_4\alpha_2 + 2\lambda_5\alpha_1\alpha_2 + \lambda_6\alpha_1 \tag{89}$$

$$g_4 = \lambda_3 + \lambda_5\alpha_2^2 + \lambda_6\alpha_2 \tag{90}$$

We solve the polynomial numerically in Gauss.

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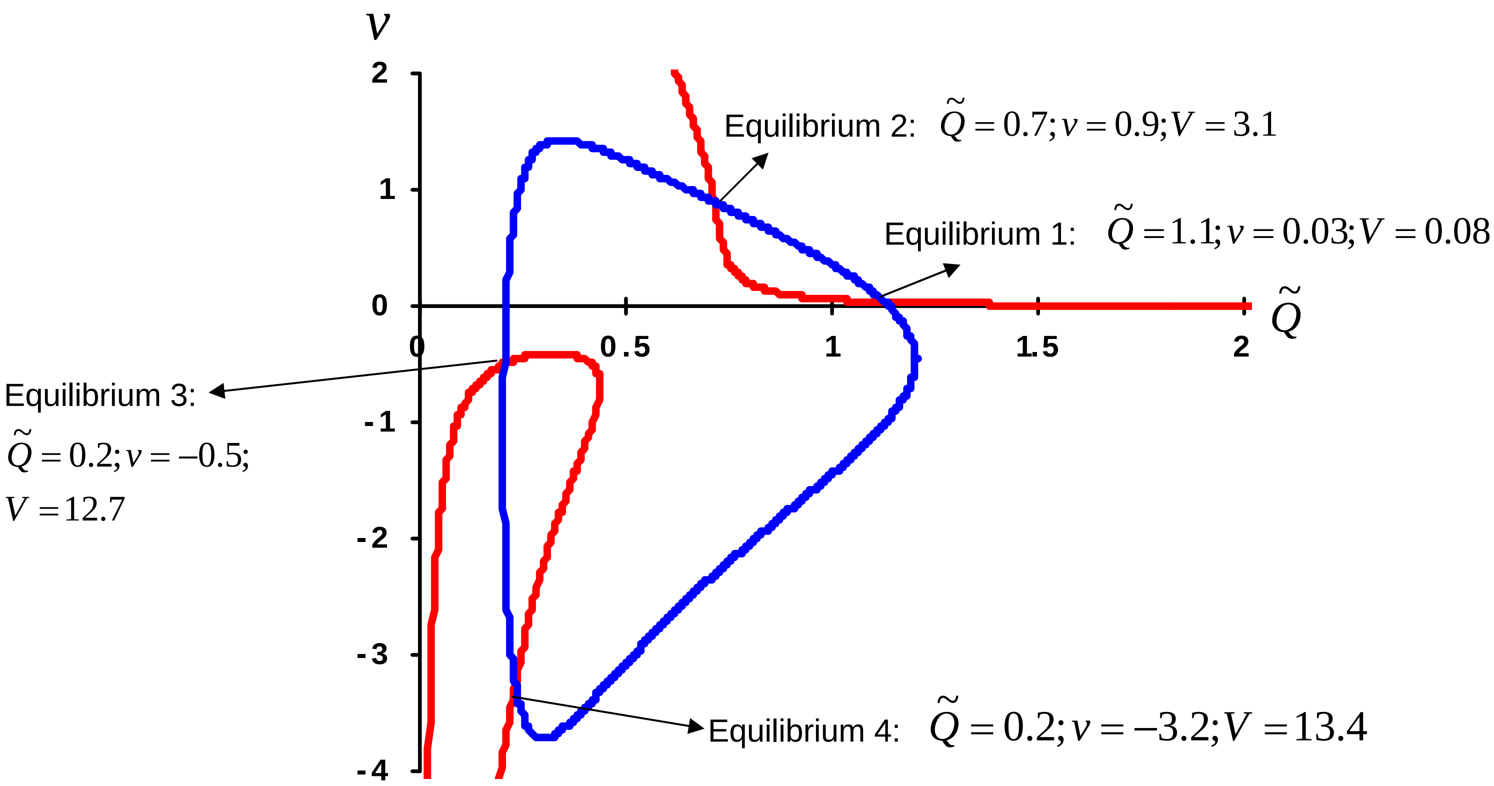
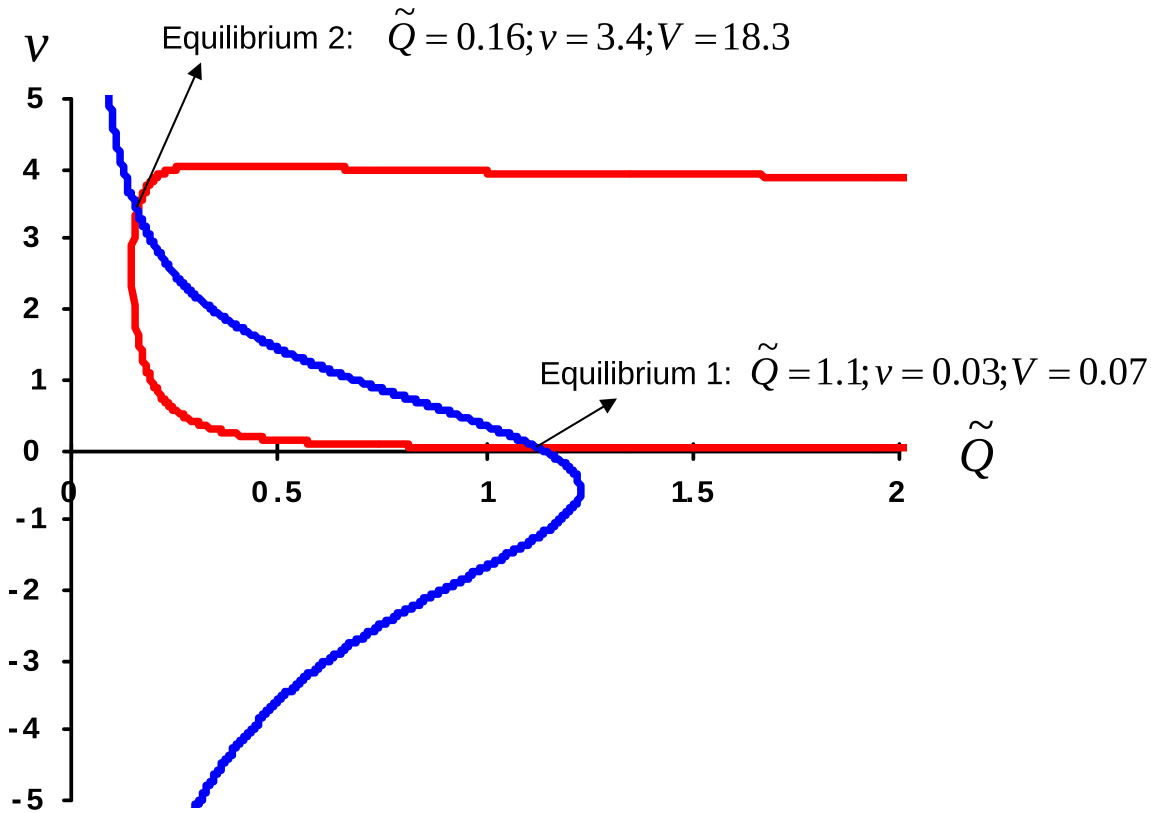
Figure 1 Multiple Equilibria*



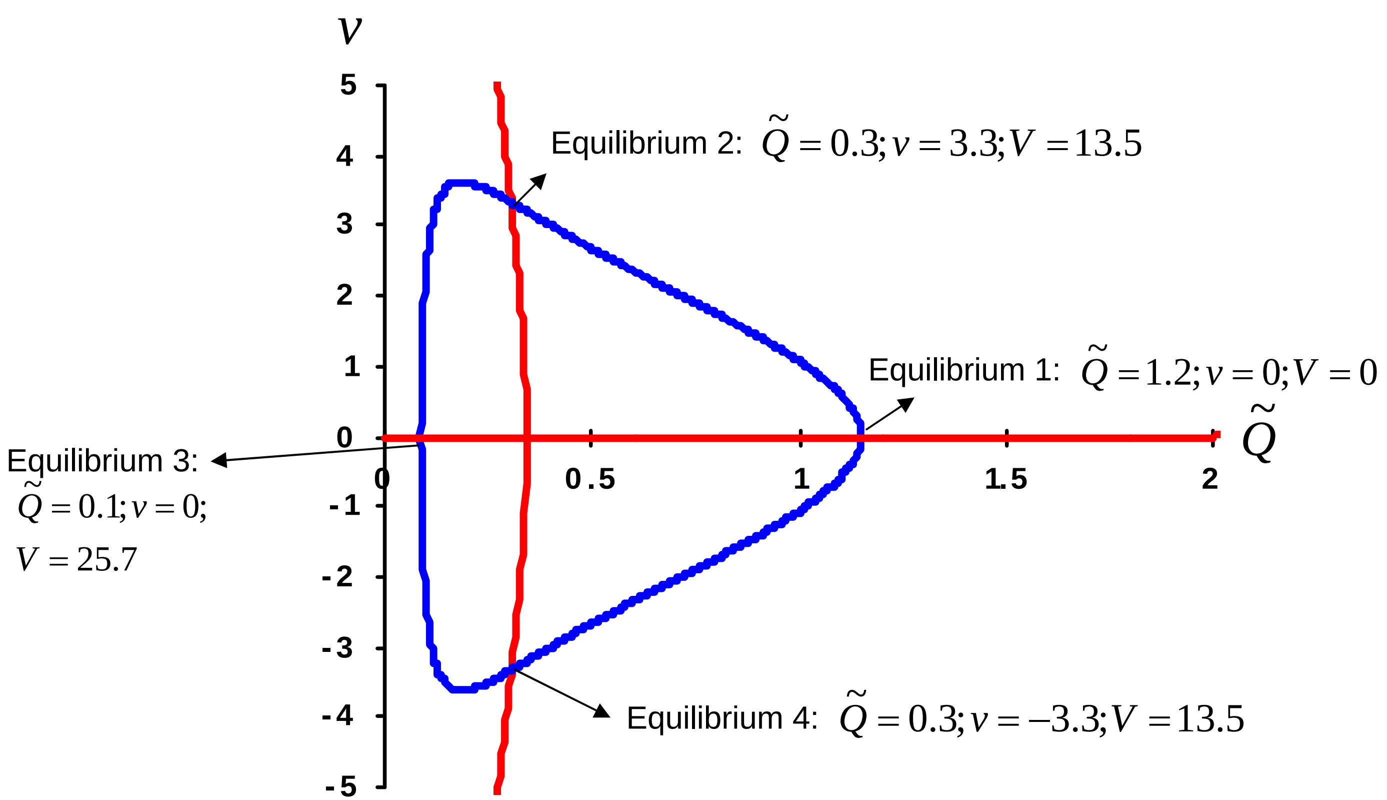
$$* \ \overline{A} = 0.3; \upsilon - W - W_E = 0.1; \eta = 1; \sigma_a = 0.5; \sigma_\theta = 0.4; \rho_\theta = 0.4; \gamma_I = 10; \gamma_L = 1; W = 2; \overline{\theta} = 0.3$$

Figure 2 Special Case 1: No Persistence*



$$* \; \overline{A} = 0.3; \upsilon - W - W_E = 0.1; \eta = 1; \sigma_a = 0.5; \sigma_\theta = 0.4; \rho_\theta = 0; \gamma_I = 10; \gamma_L = 1; W = 2; \overline{\theta} = 0.3$$

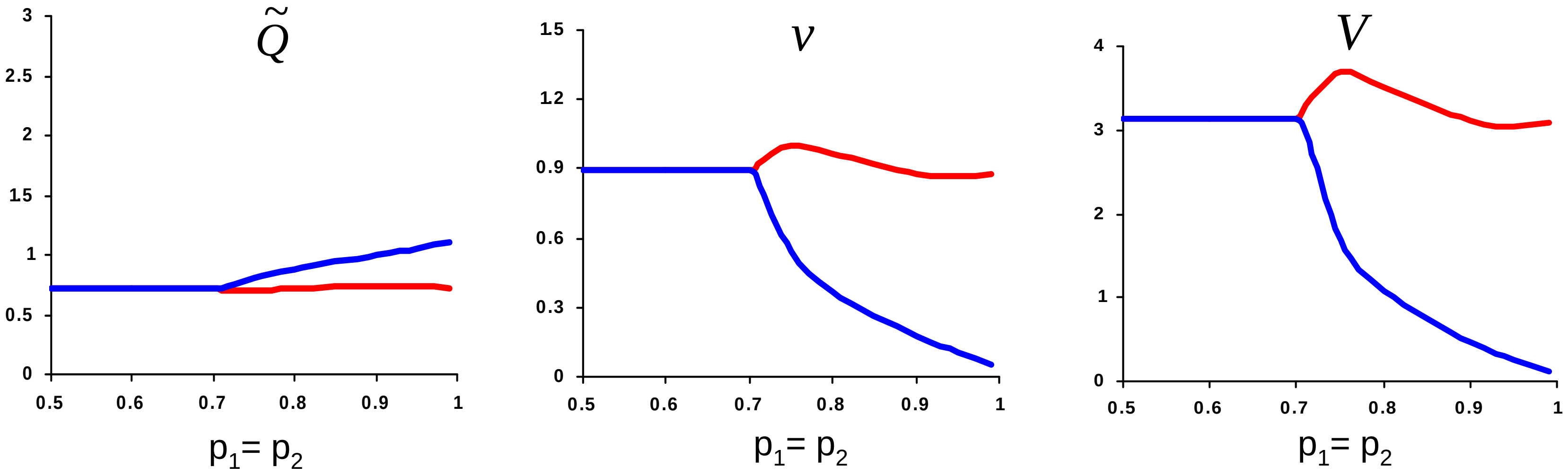
Figure 3 Special Case 2: Sunspot Equilibria*



* $\overline{A} = 0.3; \upsilon - W - W_E = 0.1; \eta = 1; \sigma_a = 0.5; \sigma_\theta = 0.4; \rho_\theta = 0.4; \gamma_I = 1; \gamma_L = 1; W = 2; \overline{\theta} = 0.3$

Figure 4 Switching Equilibria*

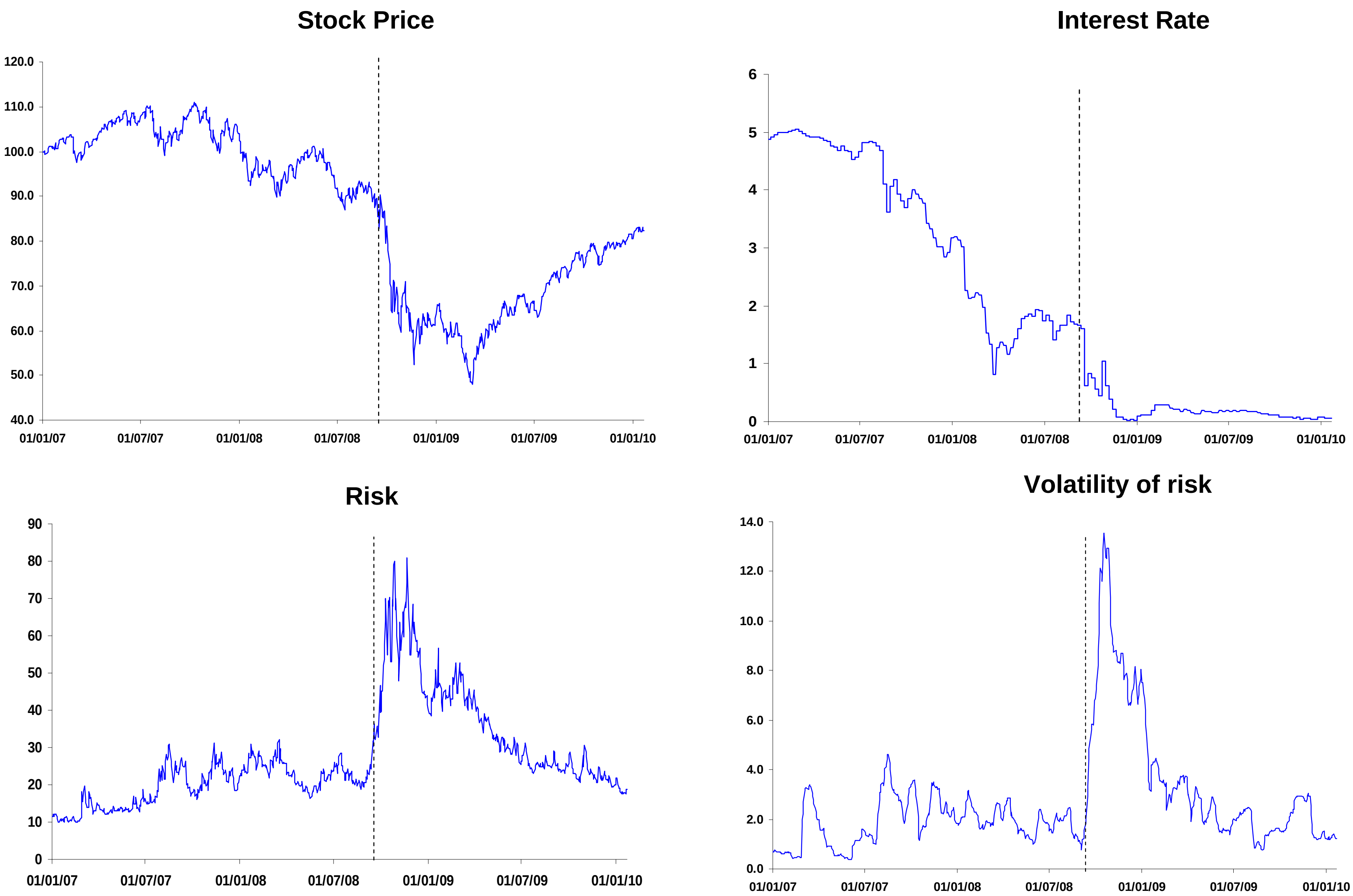
blue=low risk state; red=high risk state



* This is based on the parameters of Figure 1. When p_1=p_2=1, the high and low risk states correspond exactly to equilibria 1 and 2 in Figure 3.

Figure 5a Stock prices, interest rate, and risk

vertical lines = Lehman Brothers bankruptcy (Sept. 15, 2008)



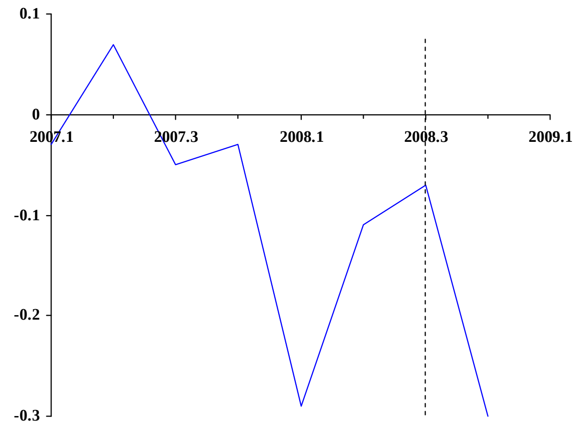
Source: Datastream, daily data. Stock prices are the DJ U.S. total market price index (January 1, 2007 = 100). The interest rate is the U.S. 3 month Treasury bill. The risk measure is the CBOE SPX volatility VIX index. The volatility of risk is the 30 days standard deviation of the VIX index.

Figure 5b Leverage and liquidity

vertical lines = Q3 2008

Net worth brokers and dealers

Liquidity

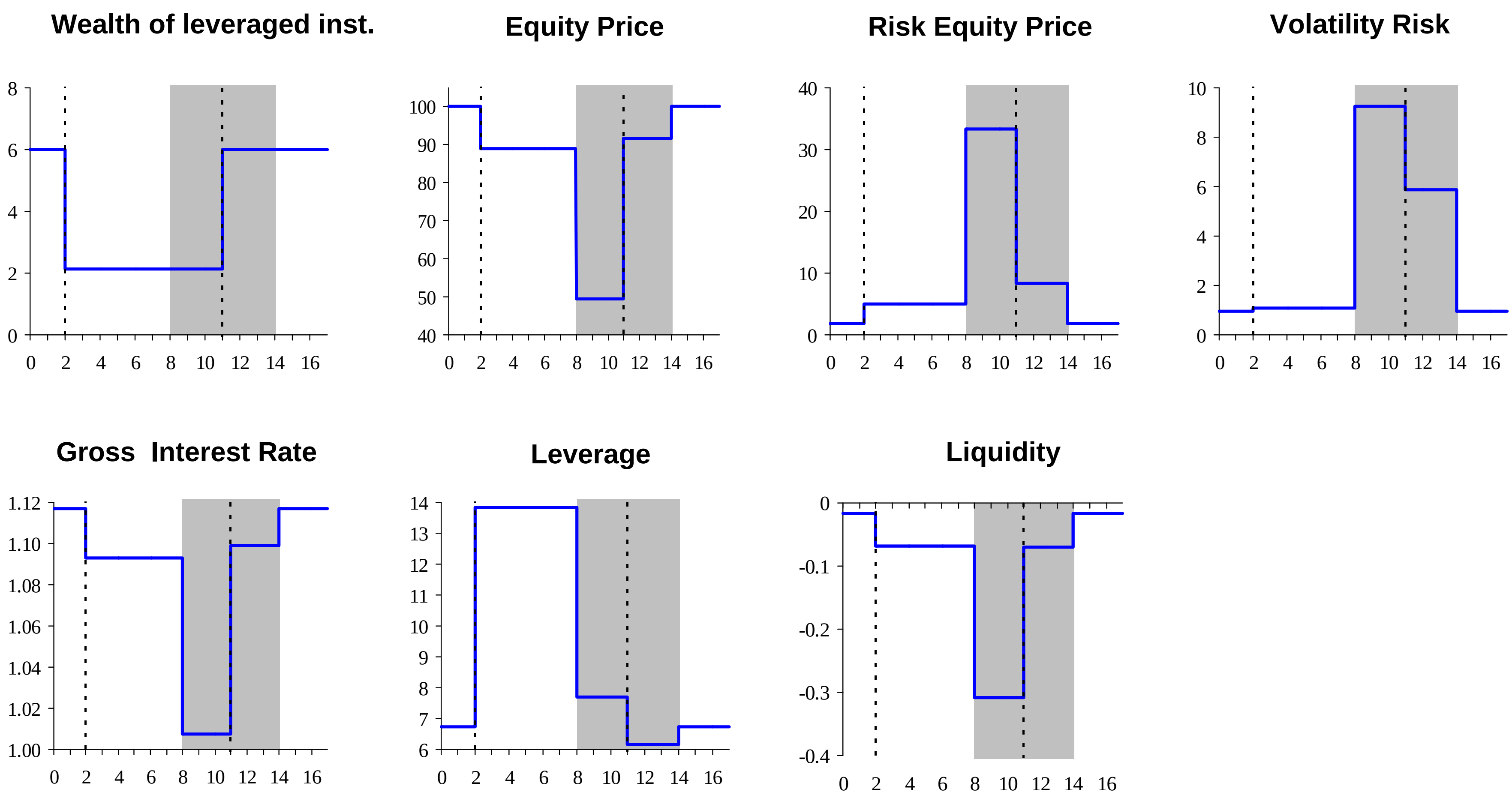


Leverage brokers and dealers

Source: Data on brokers and dealers from the Fed's Flow of Funds (L.129); net worth is assets minus liabilities, billion US $; leverage is net worth divided by assets. The liquidity measure is from Pastor and Shambaugh (2009).

Figure 6 Model Simulation

shaded area = high risk equilibrium; vertical lines = endowment shock

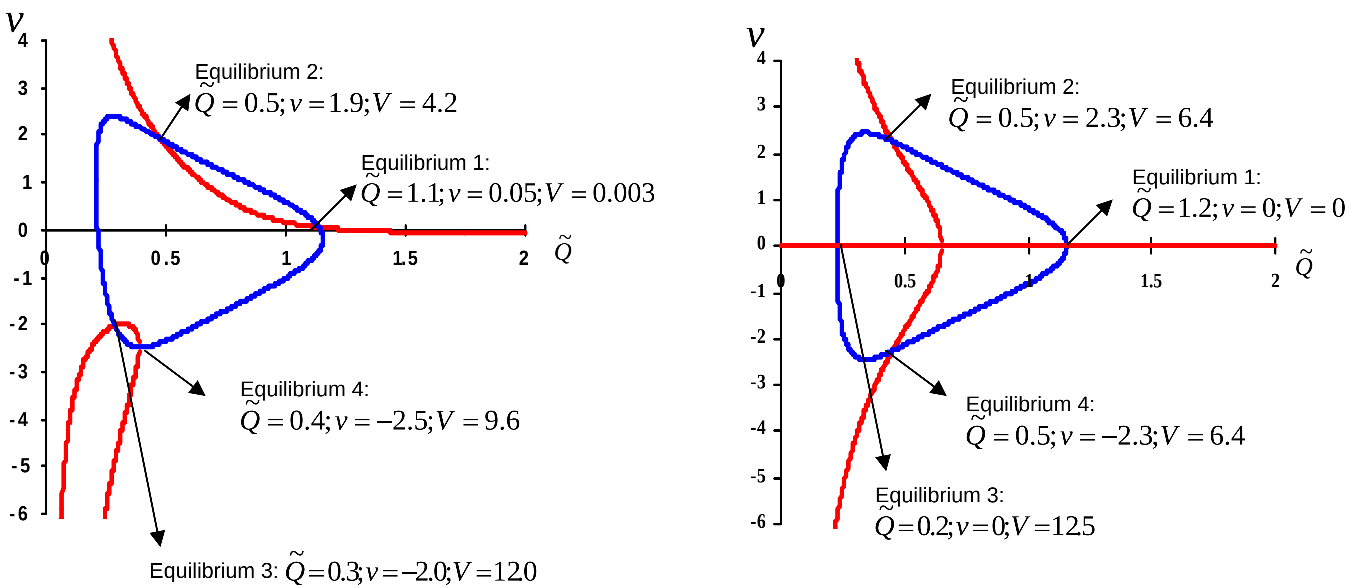


The economy starts in the low risk equilibrium. At time 2 the share of the endowment of leveraged institutions to total endowment shifts from 0.4 to 0.1. The economy stays in the low risk equilibrium until time 8, at which point is shifts to the high risk equilibrium. At time 11 endowments shift back towards the initial allocation. The economy remains in the high risk equilibrium until time 14, at which points it shifts back to the low risk equilibrium.

$$\overline{A} = 0.3; \upsilon - W - W_E = 180; \eta = 200; \sigma_a = 0.1; \sigma_\theta = 0.1; \rho_\theta = 0.7; \gamma_I = 20; \gamma_L = 1; W = 15; \overline{\theta} = 0.4; k = 20;\ p_1 = 0.95;\ p_2 = 0.7$$

Figure 7 Multiple Equilibria with Technology Shocks*

Panel A Technology as Fundamental Panel B Technology as Pure Sunspot



* Panel A shows the equilibria with persistent technology shocks. We set $\sigma_{\square}$=0 (no financial shocks) and ρ_a=0.4. Otherwise the parameters are the same as in Figure 1. Panel B considers pure sunspot equilibria by removing the technology variable a_t from the model. It then becomes a pure sunspot. Otherwise the parameterization is the same as in panel A.