The Great Diversification and its Undoing*

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Abstract

We investigate the hypothesis that macroeconomic fluctuations are primitively the results of many microeconomic shocks, and show that it has significant explanatory power for the evolution of macroeconomic volatility. We define the "fundamental" volatility as the volatility that would arise from an economy made entirely of idiosyncratic microeconomic shocks, occurring primitively at the level of sectors or firms. In its empirical construction, motivated by a simple model, the share of sales of various sectors varies over time (in a way we directly measure), while the volatility of those sectors remains constant. We find that fundamental volatility accounts for the swings in macroeconomic volatility in the USA and the other major world economies in the past half century. It accounts for the "great moderation" and its undoing. Controlling for our measure of fundamental volatility, there is no break in output volatility. The initial great moderation is due to a decreasing share of manufacturing between 1975 and 1985. The recent rise of macroeconomic volatility is due to the increase of the size of the financial sector. As the origin of aggregate shocks can be traced to identifiable microeconomic shocks, we may better understand the origins of aggregate fluctuations. (JEL: E32, E37)

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1 Introduction

This paper explores the hypothesis that changes in the microeconomic composition of the economy during the post-war period can account for the "great moderation" and its unraveling, both in the USA and in the other major world economies. We call "fundamental volatility" the volatility that would come only from microeconomic shocks. If aggregate shocks come in large part from microeconomic shocks (augmented by amplification mechanisms), then aggregate volatility should track fundamental volatility. To operationalize this idea, the key quantity we consider (which constitutes one departure from other studies) is the following definition of "fundamental volatility":

$$\sigma_F(t) = \sqrt{\sum_{i=1}^{n} \left(\frac{S_{it}}{\mathrm{GDP}_t} \right)^2 \sigma_i^2}$$

where S_{it} is the gross output (not just value added) of sector i, and σ_i is the standard deviation of the total factor productivity (TFP) in the sector. Note that in this measure the weights do not add up to one. Those are the "Domar weights" that research in productivity studies (Domar 1961, Hulten 1978) has identified as the proper weights to study the impact of microeconomic shocks.

Figure 1 plots $\sigma_F(t)$ for the USA. We see a local peak around 1975, then a fall (due to the decline of manufacturing), then a new rise (which we will relate to the rise of finance). This looks tantalizingly like the evolution of US volatility. Indeed, we show statistically that the volatility of the innovations to GDP are well explained by the fundamental volatility $\sigma_F(t)$. In particular, our measure explains the great moderation: the existence of a break in the volatility of US GDP around 1984. After controlling for fundamental volatility, there is no break in GDP volatility. Our measure also accounts for the rise of GDP volatility, as finance became large from the mid-1990s onward, creating an increase in fundamental volatility.

In Figure 3 we present a similar analysis for the major economies for which we could get disaggregated data about shares and TFP movements: Japan, Germany, France and the United Kingdom. The results are also that GDP volatility tracks fundamental volatility.

Our conclusion is that fundamental volatility appears to be a quite useful explanatory construct. It provides an operational way to understand the evolution of volatility, and understand more about its origins.

Hence, our paper may bring us closer to a concrete understanding of the origin of macroeconomic shocks. What causes aggregate fluctuations? It has proven convenient to think of aggregate productivity shocks, but their origin is mysterious: what is the common high-frequency productivity shock that affects Wal-Mart and Boeing? This is why various economists have progressively developed the hypothesis that macroeconomic fluctuations can be traced back to microeconomic fluctuations. This literature includes Long and Plosser (1983), who proposed a baseline model where sectors have constant sizes. Its implementation is relatively complex, as it requires sector sizes that are constant over time (unlike the evidence we rely on), and use of input-output matrices. Horvath (1998, 2000) perhaps made the greatest strides toward developing these ideas empirically, in the context of a rich model with dynamic linkages. The richness of the model perhaps makes it difficult to see what drives its empirical features, and certainly prevents the use of a simple concept like the concept of fundamental volatility.

Dupor (1999) disputes that the origins of shocks can be microeconomic, on the grounds of the central limit theorem: if there are a large number of sectors, aggregate volatility should vanish, proportionally to the square root of the number of sectors. Hence Horvath's result would stem from poorly disaggregated data.[1] Gabaix (2009) points out that "sectors" may be arbitrary constructs, and the fat-tailed, Zipf distribution of firms (or perhaps very disaggregated sectors) necessarily leads to a high amount of aggregate volatility coming from microeconomic shocks, something he dubs the "granular" hypothesis. In that view, microeconomic fluctuations create a fat-tailed distribution of firms or sectors (Simon 1955, Gabaix 1999, Luttmer 2007). In turn, the large firms or sectors coming from that fat-tailed distribution create GDP flucutations. Gabaix also highlights the conceptual usefulness of the notion of fundamental volatility.[2] Carvalho (2009) shows how viewing the links between narrowly defined sectors as a network gives extra flesh to the way microeconomic fluctuations affect

[1] Interesting other conceptual contributions for the micro origins of macro shocks include Bak *et al.* (1993), Jovanovic (1987), Durlauf (1993) and Nirei (2006).

[2] In this paper we use 2 to 3 digit sectors, rather than more disaggregated sectors or firms. This is because of data availability. The major data sets at the firm level (Compustat, Amadeus) have the worldwide sales of Daimler-Benz or General Motors, not domestic production; the latter is what the economic framework requires. In addition, globalization has increased the importance of worldwide compared to domestic production in datasets. Hence, it is difficult to use the existing firm-level dataset for our objective on low-frequency trends in microeconomic composition (even though they might be useful for yearly frequency shocks). This is why we rely on sectors: statistical agencies have indeed computed measures of gross output and value added for basic sectors. We conjecture that going to more disaggregated data would enrich the economic understanding of the microeconomic developement (e.g., the big productivity growth of the retail sector was due to Wal Mart, rather than a mysterious shock affecting a whole sector), but data availability prevent us from pursuing that idea in this paper.

aggregate volatility.

Against this backdrop, we use a simple way to cut through the complexity of the situation, and rely on a simple, transparent microeconomic construct, fundamental volatility, to predict an important macroeconomic quantity, GDP volatility.

By bringing fundamental volatility into the picture, we contribute to the literature on the origins of the "great moderation", a term coined by Stock and Watson (2002): the decline in the volatility of US output growth around 1984, up until about 2007 and the financial crisis. The initial contributions (McConnell and Perez-Quiros 2000, Blanchard and Simon 2001) diagnosed the decline in volatility, and conjectured that some basic explanations (including sectoral shifts – we will come back to that) did not seem promising. Perhaps better inventory management or better monetary policy were prime candidates. However, given the difficulty of relating those notions to data, much of the discussion was conjectural. Later, more full-fledged theories of the great moderation have been advanced. Arias, Hansen and Ohanian (2007) attribute the changes in volatility to changes in TFP volatility, within a one-sector model. Our work sheds light on the observable, microeconomic origins of this change in TFP volatility. Justiniano and Primiceri (2008) document that much of the great moderation could be traced back to a change in the volatility of the investment demand function. Gali and Gambetti (2009) document a change in the both the volatility of initial impulses, and in the impulse-response mechanism. Compared to those studies, we use much more disaggregated data, which allows us to calculate the fundamental volatility of the economy. Because we use richer disaggregated data, we can obviate some of the more heavy artillery of dynamic stochastic general equilibrium (DSGE) models, and have a parsimonious toolkit to think about volatility. We defer the discussion of Jaimovich and Siu (2009) to section 4.5. We view our effects as complementary to Jaimovich and Siu's (2009) effects of time-varying labor supply elasticity.

Finally, we relate to the literature on technological diversification and its effects on aggregate volatility. Imbs and Wacziarg (2003) and Koren and Tenreyro (2007) have shown that cross-country variation in the degree of diversification in the menu of technologies being used helps explain cross-country variation in GDP volatility and its relation with the level of development of an economy. Here we concentrate on the time series dimension of this mechanism and how it can generate the observed long swings in volatility for a given economy. We also differ in that we build our measures of diversification and sectoral level volatility from microeconomic TFP accounting rather than from unspecified sectoral shocks. Koren and Tenreyro (2009) and Moro (2009) on the other hand provide models of, respectively, technological diversification (through an expanding number of input varieties) and structural change in a two-sector economy. Their respective calibrations show that it is possible to build models that generate a decline of volatility with the level of development/structural change that is consistent with empirical evidence. However they do not contemplate the possibility of a reversal of these patterns. We instead show, through our fundamental volatility construct, that such a reversal is present and is key to explaining the recent period of higher volatility.

The methodological principle of this paper is to use as simple and transparent an approach as possible. In particular, we find a way to obviate the use of the input-output matrix, which has no claim to be stable over time, and is not necessary in our framework. We examine the economics through a very simple two-period model, rather than an infinite-horizon DSGE model. Useful as they are for a host of macroeconomic questions, those models have many free parameters, and we find it instructive to simply focus our attention on a zero-free parameter construct, the fundamental volatility of the economy. This said, a potentially fruitful next step is to build a DSGE model on the many sectors in the economy.

Section 2 presents a very simple framework that motivates our concept of fundamental volatility, and its implementation. Section 3 presents the basic empirical results. Section 4 expands on a variety of points, particularly the role of correlation in the business cycle, and why previous analyses had become pessimistic about the role of microeconomic shocks. It contains also a model that may help to clarify the size of the productivity multiplier and of comovement. Section 5 concludes on the role of policy and the use of fundamental volatility as an early warning system. The appendices provide a full account of the data and procedures we employ, as well the proofs.

2 Framework

In this section we present a very simple model that exposes the basic ideas, and motivates our empirical work. Section 4.2 presents a fuller model. Consider a shock dA_{it}/A_{it} to the productivity in sector i. Hulten (1978) shows that aggregate TFP growth $(d\Lambda/\Lambda)$ can then be written as:

$$\frac{d\Lambda}{\Lambda} = \sum_{i=1}^{N} \frac{S_{it}}{Y_t} \cdot \frac{dA_{it}}{A_{it}}$$

where S_{it} is the dollar value of sales (gross output) in sector i, and Y_t is GDP. S_{it}/Y_t is called the "Domar" weight. Note that the sum of the weights $\sum_{i=1}^{N} S_{it}/Y_t$, can be greater than 1. This is a well-known and important feature of models of complementarity. If on average the value added to sales ratio is 2, and each sector has a TFP increase of 1%, the aggregate TFP increase is 2%.[3] This effect is discussed further in section 4.2.

Consider the baseline case where productivity shocks dA_{it}/A_{it} are uncorrelated across i's, and unit i has a standard deviation of shocks $\sigma_i^2 = var\left(dA_{it}/A_{it}\right)/dt$. Then, we have $\sigma_\Lambda = \sigma_F$, where we define:

$$\sigma_F(t) = \sqrt{\sum_{i=1}^{N} \left(\frac{S_{it}}{Y_t}\right)^2 \sigma_i^2} \tag{1}$$

This defines the "fundamental" volatility, which comes from microeconomic shocks. Gabaix (2009) calls this the "granular" volatility.

To see the changes in GDP, we use a simple static model. Consider that GDP is: $Y = \Lambda K^{1-\alpha} L^{\alpha}$, the representative agent's utility is $C - L^{1+1/\varphi}$ (the unit prefactors are innocuous normalizations), and capital can be rented at a price r. The agent's consumption is $Y - rK$. The competitive outcomes implement the planner's problem, which is to maximize the agent utility, subject to the resource constraint:

$$\max_{K,L} C - L^{1+1/\varphi} \text{ subject to } C = \Lambda K^{1-\alpha} L^{\alpha} - rK.$$

The solution obtains by standard methods detailed in the proof of Proposition 1.

$$Y = k \Lambda^{\frac{1+\varphi}{\alpha}},$$

for an unimportant constant k. Taking logs, $\ln Y = \frac{1+\varphi}{\alpha} \ln \Lambda + \ln k$, and a change in TFP $d\Lambda/\Lambda$ creates a change in GDP equal to

$$\frac{dY}{Y} = \frac{1+\varphi}{\alpha} \frac{d\Lambda}{\Lambda}.$$

Given that the volatility of TFP is the fundamental volatility $\sigma_F(t)$, the volatility of GDP is $\sigma_{GDP}(t) = \frac{1+\varphi}{\alpha} \sigma_F(t)$. We summarize the situation in the next Proposition.

[3] To see this effect, consider an economy with a production function which, at the level of the representative firm, is $Q = A\left(L/b\right)^b \left(X/\left(1-b\right)\right)^{1-b}$, where X is the intermediary inputs. GDP is $Y = \max_X A\left(L/b\right)^b \left(X/\left(1-b\right)\right)^{1-b} - X$. Solving for X yields $Y = A^{1/b} L$. Though TFP is A at the firm level, it is $A^{1/b}$ at the aggregate level. In addition, the ratio of total sales (i.e., gross output) to GDP is $1/b$.

Proposition 1 *The volatility of GDP is*

$$\sigma_{GDP}(t) = \mu \cdot \sigma_F(t), \tag{2}$$

where the fundamental volatility $\sigma_F(t)$ is given by (1), and the productivity multiplier μ is equal to

$$\mu = \frac{1+\varphi}{\alpha} \tag{3}$$

Here α is the labor share and φ is the Frisch elasticity of labor supply.

Our hypothesis is that indeed, $\sigma_F(t)$ explains a substantial part of GDP volatility, as motivated by (2). We construct $\sigma_F(t)$ as in equation (1), keeping σ_i time-independent. We do this for two reasons. First, as we will see later, the volatility of TFP at the micro level does not exhibit any marked trend. It is very similar in the first and second parts of the sample. Second, by using a constant σ_i, we highlight that the changes in fundamental volatility come only from changes in the shares of the largest sectors in the economies, rather than their volatility (which would make the explanation run the risk of being circular). We explain time-varying GDP volatility solely with time-varying shares of economic activity within the economy.

To interpret the results, it is useful to have a word about the calibration. We interpret the elasticity of labor supply broadly, including not only changes in hours worked per employed worker, but also changes in employment, and changes in effort. Using this notion, recent research (e.g. summarized in Hall 2009a,b) is consistent a value $\varphi = 2$, in part because of the large reaction of employment and effort (as opposed to simply hours worked per employed worker) to business cycle conditions. Using these values and the labor share of $\alpha = 2/3$, we obtain a multiplier of $\mu = 4.5$.

Figure 1 shows the fundamental volatility graphs from 1960 to 2008. We see that fundamental volatility and GDP volatility track each other rather well. By 2008 we're already at mid-80s levels.

There is an extension we will sometimes consider. Call $\rho_{ij} = corr\left(dA_i/A_i, dA_j/A_j\right)$ the cross-correlation between sector i and j. We can also define:

$$\sigma_F^{Full}(t) = \sqrt{\sum_{i,j=1\ldots N} \left(\frac{S_{it}}{Y_t}\right)\left(\frac{S_{jt}}{Y_t}\right)\rho_{ij}\sigma_i\sigma_j} \tag{4}$$

Note that $\sigma_F^{Full}(t)$ should be, essentially by construction, the volatility of TFP. The advantage of this construct, though, will be to do the following thought experiment. Suppose that the

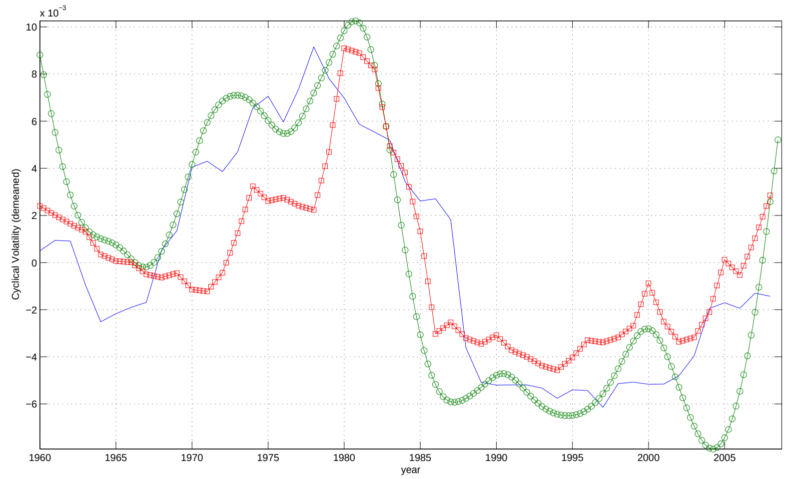

Figure 1: Fundamental Volatility and GDP volatility. The red-square line gives the fundamental volatility ($4.6\sigma_{Ft}$, also demeaned). The solid and circle lines are annualized (and demeaned) estimations of GDP volatility. The solid blue line depicts rolling window estimates of standard deviation of GDP volatility. The green solid-circle line depicts the HP trend of instantaneous standard deviation.

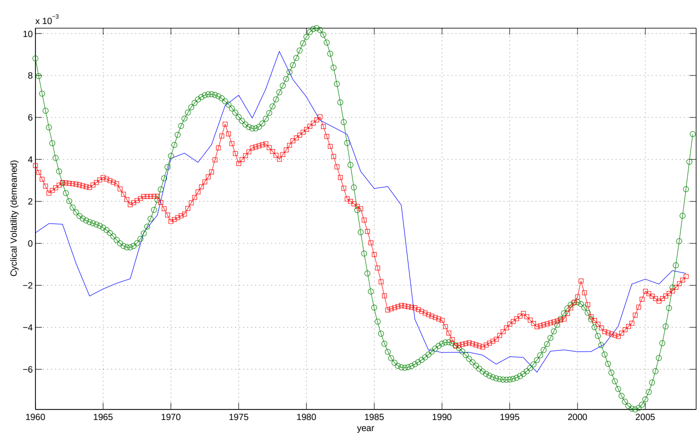

Figure 2: Fundamental Volatility (full matrix) and GDP volatility. The red-square line gives the fundamental volatility drawn from full variance-covariance matrix of TFP ($4.6\sigma_{Ft}^{Full}$, also demeaned). The solid and circle lines are annualized (and demeaned) estimations of GDP volatility. The solid blue line depicts rolling window estimates of standard deviation of GDP volatility. The green solid-circle line depicts the HP trend of instantaneous standard deviation.

shares S_{it}/Y_t change, and the variance-covariance matrix $\left(\rho_{ij}\sigma_i\sigma_j\right)$ doesn't change, how much should GDP volatility change? Figure 2 shows the "fundamental volatility" graph including the full covariance matrix, i.e. accounting for cross terms. Cross terms help to get the level right but at the cost of decreasing any dramatic increases in the 2000s.

3 Fundamental Volatility and Low Frequency Movements in GDP Volatility

3.1 US Evidence

3.1.1 Basic Facts

As a baseline measure of cyclical volatility we first obtain deviations from the HP trend of log quarterly real GDP (smoothing parameter 1600; source FRED database; sample is 1947:Q1 till 2009:Q4). We then compute the standard deviation at quarter t using a rolling window of 10 years (41 quarters, centered around quarter t) In order to extend the period to the latest recession, from 2005:Q1 till 2009:Q4 we use uncentered (i.e. progressively more one-sided) windows for this period. We refer to his measure as $\sigma_{Yt}^{\text{Roll}}$.

As a robustness check we also consider a different measure of cyclical volatility, namely the instantaneous quarterly standard deviation as computed by McConnell and Quiros (2000) For this measure we start by fitting an AR(1) model to real GDP growth rates (1960:Q1 until 2008:Q4):

$$\Delta y_t = \psi + \phi \Delta y_{t-1} + \epsilon_t \tag{5}$$

where y_t is log GDP. We obtain as estimates $\widehat{\psi} = 0.006$ ($t = 6.78$) and $\widehat{\phi} = 0.292$ ($t = 4.20$). As is well known, an unbiased estimator of the annualized standard deviation is given by $2\sqrt{\frac{\pi}{2}}|\widehat{\epsilon}_t|$, where the factor 2 converts quarterly volatility into annualized volatility, and the $\sqrt{\frac{\pi}{2}}$ comes from the fact if $\varepsilon \sim N\left(0, \sigma^2\right)$, then $\sigma = E\left[\sqrt{\frac{\pi}{2}}\left|\varepsilon\right|\right]$. We refer to

$$\sigma_{Yt}^{\text{Inst}} \equiv 2\sqrt{\frac{\pi}{2}}|\widehat{\epsilon}_t| \tag{6}$$

as the measure of the "instantaneous" measure of GDP volatility. We shall also use σ_{Yt}^{HP}, the Hodrick-Prescott smoothing of the instantaneous volatility $\sigma_{Yt}^{\text{Inst}}$.

Figure 1 plots the familiar great moderation graphs depicting the halving of volatility in the mid-80s. Interestingly, both measures also point to a significant increase in volatility from the early 2000s on, mostly as a result of the recent crisis. It also depicts the sample fit of our fundamental volatility measure,σ_{Ft}, for the annual case given a baseline value of $b = 4.6$. In particular it again shows the two cyclical measures above (annualized and demeaned), adding a third (red-square line) which is given by $4.6\sigma_{Ft}$ (demeaned).

We run least square regressions of the type:

$$\sigma_{Yt} = a + b\sigma_{Ft} + \eta_t$$

where σ_{Ft} is our measure of fundamental volatility and σ_{Yt} is one of the measures of volatility described above: $\sigma_{Yt}^{\text{Roll}}$, for the rolling window estimate, $\sigma_{Yt}^{\text{Inst}}$ for the instantaneous standard deviation measure. We also consider the alternative measure of fundamental volatility, σ_{Ft}^{Full}, which includes TFP covariance terms. Notice finally that these measures are only available on an annual basis. As such we pursue two different strategies: i) annualizing the standard deviation measure by averaging the left hand side over four quarters ("annual" below) or, ii) linearly interpolating our measures of fundamental volatility in order to obtain quarterly frequency data.

Table 1: GDP Volatility and Fundamental Volatility

	Annual Data- $\sigma_{Yt}^{\text{Roll}}$		Annual Data- $\sigma_{Yt}^{\text{Inst}}$		Quarterly Data- $\sigma_{Yt}^{\text{Roll}}$		Quarterly Data- $\sigma_{Yt}^{\text{Inst}}$	
	σ_{Ft}	σ_{Ft}^{Full}	σ_{Ft}	σ_{Ft}^{Full}	σ_{Ft}	σ_{Ft}^{Full}	σ_{Ft}	σ_{Ft}^{Full}
$\widehat{a}$	-0.029 $(-5.53;0.005)$	-0.041 $(-7.13;0.006)$	-0.0459 $(-3.53;0.013)$	-0.0459 $(-4.27;0.013)$	-0.031 $(-12.26;0.003)$	-0.041 $(-14.46;0.003)$	-0.0478 $(-4.10;0.012)$	-0.067 $(-4.88;0.014)$
$\widehat{b}$	4.614 (8.39;0.574)	3.981 (9.74;0.409)	6.741 (4.70;1.434)	5.727 (4.70;1.085)	5.056 (18.18;0.281)	4.046 (19.66;0.206)	6.955 (5.40;1.288)	5.919 (5.98;0.989)
R^2	0.60	0.67	0.33	0.38	0.63	0.67	0.13	0.16

Notes: Regression of GDP volatility on fundamental volatility. We regress $\sigma_{Yt} = a + b\sigma_{Ft} + \eta_t$ on fundamental volatility. In parenthesis are t-statistics and standard errors.

Table 1 summarize the results, at respectively annual and quarterly frequency. We find a good statistical and economy significance of σ_F. [4] It is the sole regressor, and its R^2 is around

[4] Our model predicts an intercept $a = 0$. Simple variants could predict a positive a, or a negative a, as we find here empirically. A positive a is generated by adding other shocks to GDP. A negative a is generated if the multiplier μ is increasing in σ_F rather than constant, i.e. if the economy's technologies are more flexible when the environment is more volatile.

63% for the rolling estimate of volatility.[5] This shows that σ_{Ft} explain a good fraction of the historical evolution on GDP volatility.

Note that in our regressions, all the movements come from the sizes of sectors: their volatilities are fixed in our construction of σ_F. We do it for parsimony's sake, and also because it is warranted by the evidence: the average volatility of sectoral-level microeconomic volatility did not have noticeable trends in the sample. Indeed, the average sectoral-level volatility is 3.69% in the 1960-1984 period, and 3.68% in the 1984-2005 period. Hence our construction of σ_F allows to isolate the impact of the changes in microeconomic composition of the economy.

3.1.2 Accounting for the Break in US GDP Volatility

A common way of encoding the great moderation is to test the null hypothesis of a constant level in GDP volatility

$$\sigma_{Yt} = a + \eta_t$$

against an alternative representation featuring a break in the level

$$\sigma_{Yt} = a + cD_t + \eta_t$$

where D_t is a dummy variable assuming a value of 1 from period for $t \geq T$, for an estimated break date T. Following McConnell and Quiros (2000), we take σ_t to be given by instantaneous volatility measure σ_{Yt}^{Inst} and test for the presence of a break in level using Andrews' (1993) $F-$test statistic (the *SupF* statistic).[6] In what follows, we look for a single break date T, where we assume T lies in a range $[T_1, T_2]$ where $T_1 = 0.2n$ and $T_2 = 0.8n$ and n is the total number of observations (i.e. the trimming percentage is set at 20% of the sample[7])

To assure comparability and since our sample period does differ, we start by reconfirming the findings in McConnell and Quiros (2000). We do find strong support for a level break with SupF statistic of 32.326 (the 5% critical value 8.752). The estimated break date T is 1984:1 and is estimated with a 90% confidence interval given by 1981:2–1986:4. The estimated value

[5]As a two-step OLS can be slightly inefficient econometrically, we have also performed an ARCH-type maximum-likelihood estimation, based on the joint system (5) and $\sigma_t = \alpha + \beta\sigma_{Ft} + \eta_t$. Its results are very similar to those in Table 1.

[6]We use code made available by Qu and Perron (2007).

[7]Bai and Perron (2006) find that serial correlation can induce significant size distortions when low values of the trimming percentage are used and recommend values of 15% or higher.

of c is -0.0104 ($t = -5.50$) implying a permanent percentage point decrease in aggregate volatility after this date.

Table 2: Break Tests with Fundamental Volatility

	Break test with or without Fundamental Volatility on the RHS			
	Without	With		
	H_0: No break in a	H_0: No Break in a	H_0: No Break in b	H_0: No Break in a, b
	(i)	(ii)	(iii)	(iv)
SupF test	32.33	8.60	8.96	9.33
Null of no break	Reject	Accept	Accept	Accept
Est. break date	1984:1	None	None	None

Notes: We perform a break test for equation $\sigma_{Yt}^{Inst} = a + \eta_t$ (column i) and $\sigma_{Yt}^{Inst} = a + b\sigma_{Ft} + \eta_t$ the regression of instantaneous GDP volatility on fundamental volatility (columns ii-iv). Column (i) confirms that, without conditioning on fundamental volatility, there is a break in GDP volatility (the great moderation). Next, column (ii) performs a test on the a coefficient. We cannot reject the null hypothesis of no break. This means that, once we control for fundamental volatility, there is no break in GDP volatility. The subsequent tests for breaks in b and (a, b) are extra robustness checks (columns iii-iv); they confirm the conclusion that controlling for fundamental volatility there is no break in GDP volatility. From Andrews (1993) the 10% asymptotic critical values for *SupF* statistic are 11.35 for (ii) and (iii) and 9.54 for (iv).

We now present our main test. We test the hypothesis that once our fundamental volatility measure is accounted for in the dynamics of $\sigma_{Yt}^{\text{Inst}}$, there is no such level break in aggregate volatility. That is we test for the null of no break in the intercept in the following equation:

$$\sigma_{Yt}^{\text{Inst}} = a + b\sigma_{Ft} + \eta_t$$

To rule out the additional possibility that the break in aggregate volatility is the result of a break in its link with fundamental volatility, we also consider testing the null of no break on the slope parameter b and the joint null of no break in both a and b [8]. The results are in Table 2. We cannot reject the null of no break in any of these settings. We conclude that, after controlling for the time series behavior fundamental volatility, there is no break in GDP volatility.

3.2 International Evidence

We now extend the previous analysis to the four other major economies: France, Japan, Germany and the UK. As is well known (see Stock and Watson 2005), these countries have exhibited quite different low-frequency dynamics of GDP volatility throughout the last half-century. Under the hypothesis of this paper, it should be the case that the evolution of our measure of fundamental volatility is also heterogeneous across these economies.

Relative to the US, we face greater data limitations, both on the time series and cross-sectional dimensions. We are able to construct the Domar weight measures from 1970 to 2005 (from 1973 for Japan). Though we are able to get considerable sectoral detail for nominal measures, sector-specific price indexes are only available for half or less of the sectors in each country[9]. This renders impossible an accurate weighting by sectoral TFP volatility. Therefore we choose to consider a special case of our fundamental volatility measure given by

$$\overline{\sigma}_F(t) = \overline{\sigma}\sqrt{\sum_{i=1}^{N}\left(\frac{S_{it}}{Y_t}\right)^2} \tag{7}$$

where $\overline{\sigma}$ is the average standard deviation of sectoral TFP over the entire sample period, in the country under consideration. The values of $\overline{\sigma}$ are 2.3% for France, 2.0% for Germany, 3.2% for Japan, 2.2% for the UK, and 3.4% for the USA. Finally, motivated by our discussion above we consider a multiplier $\mu = 4.5$ to obtain the volatility of GDP implied by our fundamental volatility measure, i.e. $\sigma_Y(t) = 4.5\overline{\sigma}_F(t)$. As in the US case, we take as a baseline measure of cyclical volatility the 10-year rolling window standard deviation of HP filtered quarterly real GDP (smoothing parameter 6.5). Figure 3 compares the evolution of these measures (where again, we demean both measures).

[8] As discussed above the original σ_{Ft} is measured on an annual frequency while σ_{Yt}^{Inst} is quarterly. To maximize the total number of observations we opt to use a quarterly interpolation of our fundamental volatility measure.

[9] See Appendix A for more details on the sources, description and construction of these measures.

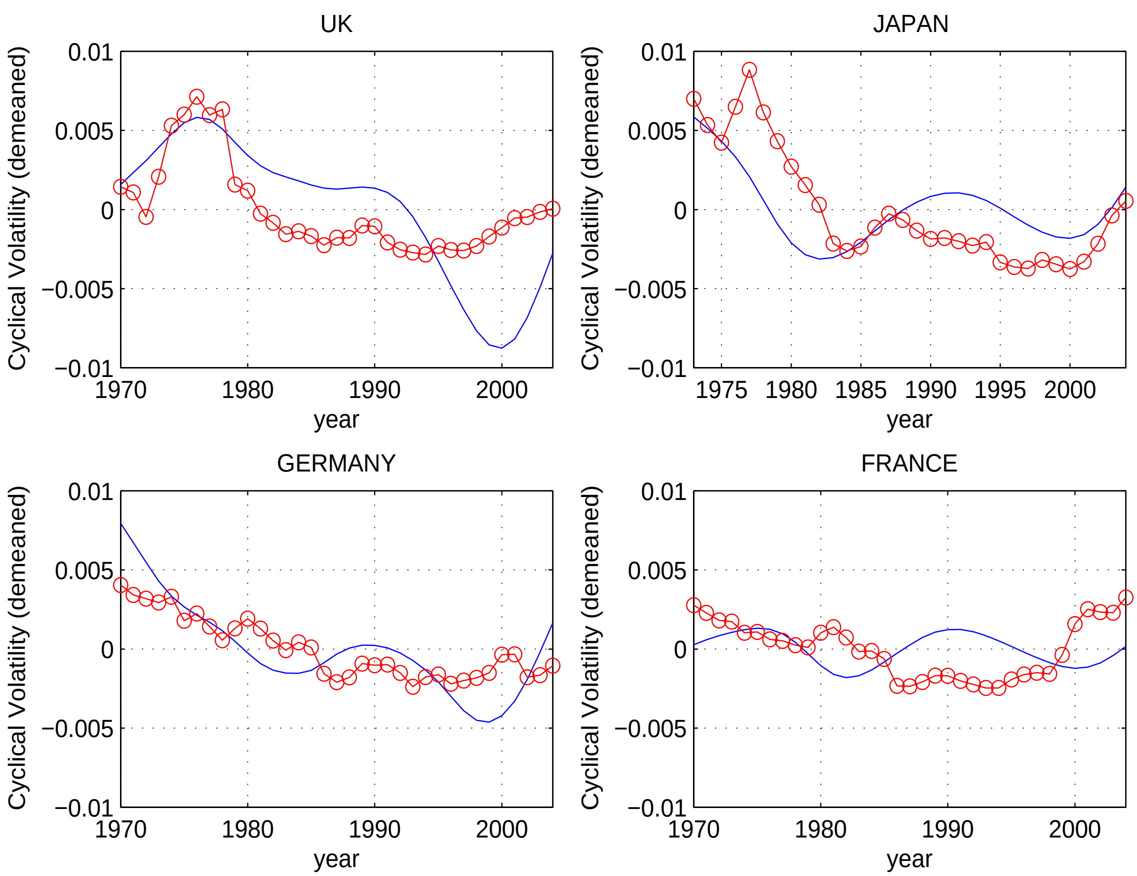

Figure 3: GDP volatility and Fundamental volatility in four OECD countries. Blue solid line: Smoothed rolling window standard deviation of deviations from HP-trend of quarterly real GDP. Red circle line: Fundamental volatility measure, $\sigma_Y(t) = 4.5\overline{\sigma}_F(t)$. Both measures are demeaned. We report results for the four large countries for which we have enough disaggregated data.

As in the US case, our proposed measure seems to account well for the (different) low-frequency movements in GDP volatility in this set of countries. In the UK it captures the strong reduction in volatility in the late 70s and its leveling off up until the mid 1990s. We cannot account for the short lived drop in UK volatility around 2000 (but notice that by 2005 the levels of our measure and the data are again very close). As Stock and Watson (2005) noticed, Germany provides a different picture, that of a large but gradual decline. Again, our measure also displays a much smoother negative trend. For Japan, fundamental volatility tracks well the fall in GDP volatility in the late 70s and early 80s as well as the often noted reversal occurring in the mid-eighties. For France, our measure displays no discernible trend, overing around its mean throughout the sample period. This agrees with the muted low frequency dynamics of French GDP volatility.

Table 3: GDP Volatility and Fundamental Volatility: International Evidence

	$\sigma_{ct}(OLS)$	$\sigma_{ct}(OLS)$	$\sigma_{ct}(IV)$	$\sigma_{ct}(IV)$
$\widehat{\beta}$	3.193 (6.68;0.478)	1.971 (3.41;0.578)	1.954 (4.12;0.475)	1.094 (3.07;0.356)
χt	No	Yes	No	Yes
Observations	172	172	170	170

Notes: We run the regression $\sigma_{Yct} = \alpha_c + \chi t + \beta \overline{\sigma}_{Fct} + \varepsilon_{ct}$, where σ_{Yct} is the country volatility using a rolling window measure, $\overline{\sigma}_{Fct}$ is the fundamental volatility of the country defined in (7), α_c a country fixed effect and χt a linear time trend. The IV instruments by the lagged value of $\overline{\sigma}_{Fct}$. t-statistics and standard errors in parenthesis.

To complement this we consider running panel regressions

$$\sigma_{Yct} = \alpha_c + \chi t + \beta \overline{\sigma}_{Fct} + \varepsilon_{ct}$$

where σ_{Yct} is our rolling window measure of cyclical volatility for country c in year t and $\overline{\sigma}_{Fct}$ are the country-specific fundamental volatility measures. We include the US along with the four other economies mentioned above. To preserve comparability we construct and use the $\overline{\sigma}_{Fct}$ for the US (rather than σ_{Ft} described in the previous section) We use country fixed effects α_c and consider running the above panel with and without a common linear time trend χt. We view the specification without a time trend as the cross-country analog of the regressions run above for the US alone. The specification with a time trend allows us to control for potential common factors affecting volatility in all countries and therefore identifies β through cross-country timing differences in the evolution fundamental volatility. While this specification renders the value of β not comparable to the values obtained for the simple US regression, it strengthens our results by minimizing possible spurious regression type problems in our baseline specification. As a first-pass control for potential endogeneity issues with our fundamental volatility measure we also consider instrumenting it by its lagged value[10].

Table 3 reports the results. All results are significant at the 1% level. Again, we confirm the existence of a tight link between aggregate volatility and our fundamental volatility measure. Notice that for the no-fixed effects case our measure is quantitatively similar in a cross-country case (a value of 4.5 is well within the 95% confidence multiplier). Its significance survives when we allow for common time fixed effects and when instrumented by its own lag.

4 Extensions and Discussion

The previous section has shown that fundamental volatility correlates well with GDP volatility. In this section, we study a few residual issues. We start with a brief history of the causes of changes in fundamental volatility.

4.1 A Brief History of Fundamental Volatility in the US

We now dig deeper and provide an account of the evolution of our fundamental volatility measure in the last half-century. We find it useful to break this account into three separate questions: i) what accounts for the "long and large decline" of fundamental volatility from the 1960s to the early 90s?; ii) what accounts for the interruption of this trend from the mid-70s to the early 80s? and iii) what is behind the reversal of fundamental volatility dynamics observed around the mid-90s and its subsequent increase up until 2008?

Our answers are the following: i) the long and large decline of fundamental volatility from the 1960s to the early 90s is due to the smaller size of a handful of heavy manufacturing sectors. ii) The growth of the oil sector (which itself can be traced to the rise of the oil sector) accounts for the burst of volatility in the mid-1970s. iii) The rise the size of the financial

[10] We report heteroskedastic and autocorrelation robust standard errors by using a Newey-West estimator with 2 lags.

sector is an important determinant of the increase in fundamental volatility. We now detail our answers.

We find that the low-frequency decline in fundamental volatility observed from 1960 to 1990 can be accounted for almost entirely by the demise of a handful of heavy manufacturing sectors: Construction, Primary Metals, Fabricated Metal Products, Machinery (excluding computers) and Motor Vehicles[11]. These sectors, while only moderately large in the value-added sense in 1960 – in total they accounted for 18% of total value added in 1960 - are both relatively more intensive intermediate inputs users and relatively more volatile, thus accounting for a disproportionately large fraction of aggregate fundamental volatility in 1960 (30% of σ_F^2).[12] In this sense, the relatively high aggregate volatility in the early 1960s was the result of an undiversified technological portfolio, loading heavily on a few heavy manufacturing industries Their demise, starting around the early 70s and accelerating around 1980, meant that by 1990 they accounted for only 10% of aggregate volatility.

Another way to see this is to compute a counterfactual fundamental volatility measure, where we fix the Domar weights of these sectors to their sample average. This enables us to ask what would have happened to fundamental volatility had these sectors not declined during the period of analysis. We find that in this counterfactual economy the level of fundamental volatility would have barely changed from the early 1960s to the early 90s. At the same time, it is also clear that the dynamics of these sectors does not account either for the spike in fundamental volatility around 1980 nor do they play a role in its continued rise from the mid-90s onwards.

Instead, we find that the spectacular rise and precipitous decline of fundamental volatility from the early 70s to the mid-80s is largely accounted by the dynamics of two energy-related sectors: Oil and gas extraction and Petroleum and coal products. By 1981, these two sectors accounted for 41% of fundamental volatility, a four-fold increase from the average over the remainder of the sample. The rise and fall of these two sectors account for 94% of the increase in aggregate volatility from 1971 to 1980 and 86% of its decline during the 1980-1986 period.

[11] A simple way to assess the sources of the movements in $\sigma_F(t)$ is to decompose aggregate changes according to the (volatility-weighted) contribution of each individual technology.

$$\sigma_F^2(t) - \sigma_F^2(t-j) = \sum_{i=1}^{N} \left[\left(\frac{S_{it}}{Y_t} \right)^2 - \left(\frac{S_{it-j}}{Y_{t-j}} \right)^2 \right] \sigma_i^2$$

[12] The share of σ_F^2 due to sector i is defined as $\left(\frac{S_{it}}{Y_t}\right)^2 \sigma_i^2/\sigma_F^2$. Those shares add up to 1.

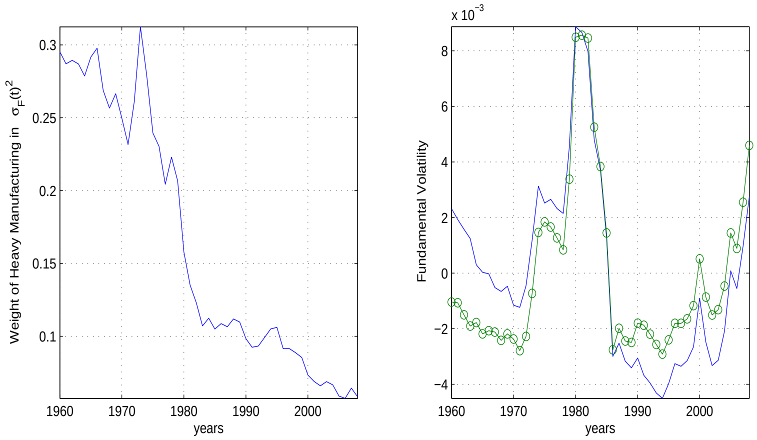

Figure 4: **Left:** Weight of heavy manufacturing sectors in $\sigma_F^2(t)$. **Right**: Blue continuous line is the baseline fundamental volatility measure ($4.5\sigma_F(t)$ demeaned). Green circled line gives counterfactual volatility measure (also demeaned) where weights of heavy manufacturing sectors are fixed at their sample average.

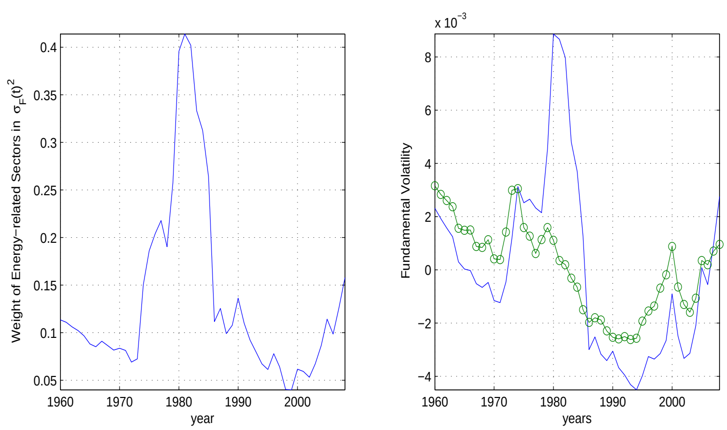

Figure 5: **Left:** Weight of Energy-related sectors in $\sigma_F^2(t)$. **Right**: Continuous line is the baseline fundamental volatility measure ($4.5\sigma_F(t)$ demeaned). Circled line gives counterfactual volatility measure (also demeaned) where weights of energy-related sectors are fixed at their sample average.

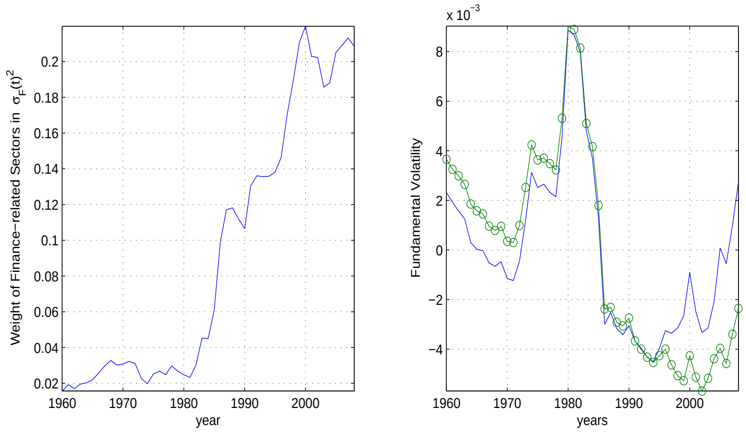

Figure 6: **Left:** Weight of Finance-related sectors in $\sigma_F^2(t)$. **Right**: Blue continuous line is the baseline fundamental volatility measure ($4.5\sigma_F(t)$ demeaned). Green circled line gives counterfactual volatility measure (also demeaned) where weights of finance-related sectors are fixed at their sample average.

The importance of these two energy sectors for the volatility spike around 1980 can also be gauged by resorting to the same type of counterfactual exercise: what would have happened to fundamental volatility had the weight of these sectors remained constant throughout the sample? The answer, not surprisingly, is that there would have been no heightened volatility in the late 70s. Rather, what emerges is a steady decline in fundamental volatility from the early 70s to the early 90s, being led by the dynamics of the manufacturing sectors described above.

Also apparent from the graphs above is that while energy-related sectors have contributed somewhat to the increase in fundamental volatility from 2000 onwards, they cannot account for the mid-90s turning point in fundamental volatility nor for its heightened level by 2008. That is, even when we fix the weights of these sectors to their sample mean, our counterfactual volatility measure still points to a marked increase in fundamental volatility over the last decade and a half, undoing most of the (low frequency) volatility declines of the previous three decades[13]. We find that the answer for this lies elsewhere, and is given by the rise of

[13] The left panel tells why: while on the increase from the 2000s onwards, the structural transformation of the US economy has rendered it less dependent on energy sectors. As such, the weight of these energy sectors in aggregate fundamental volatility is about a third of that it was in 1980. This is reminiscent of the analysis in Blanchard and Gali (2008) who show that, despite the unprecedented price increases in this period (larger

finance.

Next, we build on the Philippon's (2008) analysis of the evolution of the GDP share of the financial sector, but revisit it through the metric of fundamental volatility. We find that the combined contribution of three finance-related sectors – Depository Institutions, Non-Depository Financial Institutions (including Brokerage Services and Investment Banks), and Insurance – to fundamental volatility increased tenfold from the early 1980s to the 2000s, with the latest of these sharp movements occurring in the mid-1990s and coinciding with the rise of our fundamental volatility measure. From the late 1990s onward, these three sectors have accounted for roughly 20% of fundamental volatility.

In a counterfactual economy where the weights of these sectors are held fixed, fundamental volatility would have prolonged its trend decline up until the early 2000s. As discussed above, while the renewed exposure to energy-related sectors would have reversed this trend somewhat, the implied level of fundamental volatility at the end of the sample would have been lower, in line with that observed in the early 1990s (and not that of the early 1960s and 70s as our baseline measure implies). The rise of finance is thus the key to explaining the undoing of the great moderation from the mid-1990s onwards: as the U.S. economy loaded more and more on these sectors, fundamental volatility rose, reflecting a return to a relatively undiversified portfolio of sectoral technologies.

4.2 A Model That Helps Think About Comovement

We presented earlier a simple model that gave a microfoundation for the multiplier μ, but it was not tightly integrated to the micro level activity. We next extend the model with explicitly modeled microeconomic units. The cost is that it is more specific – it relies on a CES production function.[14] The benefit is that we can trace into much more cleanly the structure of this economy. It gives rise to a very large comovement of output within the economy, even though primitive shocks are uncorrelated. We will also see how the mismeasurement of inputs

than those observed in the 1970s), the impact of oil shocks on aggregate activity has been reduced three-fold.

[14]In the CES world we parametrize (with positive elasticity of substitution), a positive TFP shocks increases a sector's size. This is not necessarily a good thing. When a sector is very new (say, electronic gadgets), size of that sector grows as the sector becomes more productive (as more products as invented). However, perhaps in a long run sense, some sectors shrink as they become very productive (e.g., agriculture). Following the macro tradition, we eschew here a calibrated modelling of this heterogeneity in the link between productivity and size.

creates the appearance of a comovement in TFP across sectors, even though TFP shocks are primitively uncorrelated.

4.2.1 Model Setup

The previous section has shown how idiosyncratic sectoral shocks might explain a significant portion of aggregate fluctuations. This section considers whether they can also plausibly create the strong comovement between the various firms or sectors of the economy, as observed, for instance, by Long and Plosser (1983), Horvath (1998, 2000), Shea (2002), Foerster, Sarte and Watson (2008) and Carvalho (2009).[15] We present a simplified version of those models that can be worked out in full detail. Its main virtue is that it is solvable in closed form, so that the mechanisms are fairly transparent.

After a shock to unit i, all the other firms adjust instantaneously, rather than over time through the input-output matrix. There is an aggregate good. Each intermediate good unit i uses L_i, K_i, X_i of labor, capital and aggregate good, to produce:

$$Q_i = \frac{1}{\kappa} A_i \left(L_i^{\alpha} K_i^{1-\alpha} \right)^b X_i^{1-b} \tag{8}$$

with $\kappa = b^b (1-b)^{1-b}$. GDP is production net of the intermediate inputs, the X_i's:

$$Y = \left(\sum_i Q_i^{1/\psi} \right)^{\psi} - \sum_i X_i \tag{9}$$

with $\psi > 1$. b is the share of intermediate inputs, and will also be the ratio of value added to sales, both at the level of the unit, and of the economy. The transformation from the goods Q_i to the final good $\left(\sum_i Q_i^{1/\psi} \right)^{\psi}$ and the intermediary inputs is made by a competitive fringe of firms.

The representative agent's utility function is $U = Y - L^{1+1/\varphi}$. Capital can be rented at a rate r. Thus, the social planner's program is: $\max_{\{K_i, L_i, X_i\}} Y - L^{1+1/\varphi} - rK$ subject to $\sum K_i = K; \sum L_i = L$. We assume that the prices equal marginal cost. This could be caused by competition, or by an input subsidy equal to ψ for the intermediary firms.

[15] Long and Plosser (1983) impose a Cobb-Douglas structure, which imposes zero idiosyncratic movement in the sales per employee and dollar sales.

The model gives:

$$\text{GDP}: Y = \Lambda L^{\alpha} K^{1-\alpha} \tag{10}$$

$$\text{TFP}: \Lambda = \left(\sum_i A_i^{1/(\psi-1)} \right)^{(\psi-1)/b} \tag{11}$$

$$\text{Sum of sales}: H = \sum p_i Q_i = \frac{Y}{b} \tag{12}$$

$$\frac{\text{Sales}_i}{\text{GDP}}: \frac{p_i Q_i}{Y} = \frac{1}{b} \left(\frac{A_i}{\Lambda^b} \right)^{1/(\psi-1)} \tag{13}$$

The result is standard, except for the b term in Eq. (11), which indicates that $1/b$ is a "productivity multiplier". If all firms increase their productivity A_i by 1%, TFP increases by $1/b$ %. This effect comes from the fact that a Hicks-neutral productivity shock increases gross output (sales), not just value added, and has been analyzed by Domar (1961), Hulten (1978) and Jones (2009).

4.2.2 Comovement in Output

We assume that we start from a steady state equilibrium, and that, in the short run, labor but not capital is reallocated across firms.

Models such as (8) always deliver a Sales / Employees ratio that is independent of the unit's productivity. The reason for this almost surely counterfactual prediction, is that labor is assumed to be costlessly adjustable. To capture the realistic case of labor adjustment costs, we assume that a fraction $1-\nu$ of labor is a quasi-fixed factor, in the sense of Oi (1962). Technically, we represent $L_i = L_{V,i}^{\nu} L_{F,i}^{1-\nu}$, where $L_{V,i}$ and $L_{F,i}$ are respectively the variable part labor and the quasi-fixed part of labor. After a small shock, only $L_{V,i}$ adjusts. The disutility of labor remains $L^{1+1/\varphi}$, where $L = L_V^{\nu} L_F^{1-\nu}$ is aggregate labor. We assume that capital and intermediary inputs are flexible. The appendix relaxes that assumption.

One can now study the effect of a productivity shock $\widehat{A}_i$ to each unit i. We call $S_i = p_i Q_i$ the dollar sales of unit i. The next proposition, whose proof is in the Appendix, describes how the economy reacts to microeconomic shocks. We use the "hat" notation to indicate a proportional change: $\widehat{Z} = dZ/Z$.[16]

[16] The rules are well-known, and come from taking the logarithm and differentiating. For instance, $\widehat{X^{\alpha} Y^{\beta} Z^{\gamma}} = \alpha \widehat{X} + \beta \widehat{Y} + \gamma \widehat{Z}$.

Proposition 2 *Suppose that each unit i receives a productivity shock $\widehat{A}_i$. Macroeconomic variables change according to:*

$$TFP: \ \widehat{\Lambda} = \sum \frac{S_i}{Y} \widehat{A}_i = \sum \frac{Sales_i}{GDP} \widehat{A}_i \tag{14}$$

$$GDP: \ \widehat{Y} = \frac{1+\varphi}{\alpha} \widehat{\Lambda} \tag{15}$$

$$Employment: \widehat{L} = \frac{\varphi}{1+\varphi} \widehat{Y} \tag{16}$$

$$Wage: \widehat{w} = \frac{1}{1+\varphi} \widehat{Y} \tag{17}$$

and microeconomic-level variables change according to:

$$\text{Dollar sales and dollar value added}: \ \widehat{S}_i = \widehat{Y}_i = \beta \widehat{A}_i + \overline{\Phi} \widehat{Y} \tag{18}$$

$$Production: \widehat{Q}_i = \psi \beta \widehat{A}_i + (1 - \psi \Phi) \widehat{Y} \tag{19}$$

$$Price: \widehat{p}_i = -(\psi - 1) \beta \widehat{A}_i + (\psi - 1) \beta \Phi \widehat{Y} \tag{20}$$

$$Employment: \widehat{L}_i = \nu \beta \widehat{A}_i + (\xi - \nu \Phi) \widehat{Y} \tag{21}$$

$$\text{Use of intermediary inputs and capital: } \widehat{X}_i = \widehat{K}_i = \widehat{S}_i \tag{22}$$

where

$$\beta = \frac{1}{\psi - 1 + b\alpha (1 - \nu)} \tag{23}$$

$$\Phi = \frac{\beta b \alpha \varphi}{1+\varphi}, \qquad \overline{\Phi} = 1 - \Phi \tag{24}$$

Equation (14) is Hulten's (1978) equation. TFP is entirely the sum of idiosyncratic unit-level shocks. Otherwise equations (15)-(17) are standard. GDP growth is TFP growth, multiplied by an amplification mechanism, labor supply.

The new results are the sector-level changes, in equations (18)-(22). The economy behaves like a one-factor model, with an "aggregate shock", the GDP shock $\widehat{Y}$. Again, this shock stems from a multitude of idiosyncratic shocks. The "aggregate shock" causes all sector-level quantities to comove. Economically, when sector i has a positive shock, it makes the aggregate economy more productive (equations 11 and 14), and affects the other firms in three different ways. First, other sectors can use more intermediary inputs produced by sector i, hence increasing their production. Second, sector i demands more inputs from the other firms (equation 18), which leads their production to increase. Third, given sector i commands a large share of output, it will use more of the inputs of the economy, which tends to reduce the other sectors' output. The net effect depends on the magnitudes of the elasticities.

All the variables in Proposition 2 have a positive loading on the GDP factor $\widehat{Y}$, i.e. they all comove positively with GDP. We conclude that the above model may be a useful benchmark to understand comovement of the business cycle.

Equation (15) yields $\sigma_Y = \mu\sigma_{TFP}$ with the "multiplier" $\mu = \frac{1+\varphi}{\alpha}$, as in equation (2)-(3) We calibrate the model using conventional parameters to the extent possible, with parameters summarized in Table 4.

Table 4: Calibration. The first part of the Table shows the postulated values. The second part shows the resulting values for a few quantities.

Calibrated Values	
Labor share	$\alpha = 2/3$
One minus share of intermediate inputs	$b = 1/2$
Elasticity of labor supply	$\varphi = 2$
Product differentiation parameter	$\psi = 1.2$
Share of labor that is variable in the short run	$\nu = 1/2$
Resulting Values	
Elasticity of output to productivity β	$\beta = 1.81$
Multiplier	$\mu = 4.5$
Elasticitity Φ	$\Phi = 0.2$
Fraction of GDP variance attributed to comovement	$f = 0.93$

To quantify the size of comovement, researchers (e.g. Shea 2002) often do the following decomposition. Call Y_i the value added of sector i, total GDP change is: $dY = \sum_i dY_i$, hence, in growth rate, $\widehat{Y} = \sum_i \frac{Y_i}{Y}\widehat{Y}_i$. Hence, the variance of GDP growth can be decomposed as , with

$$\sigma_Y^2 = D + N, \qquad D = \sum_i \left(\frac{Y_i}{Y}\right)^2 var\left(\widehat{Y}_i\right), \qquad N = \sum_{i \neq j} \left(\frac{Y_i}{Y}\right)\left(\frac{Y_j}{Y}\right) cov\left(\widehat{Y}_i, \widehat{Y}_j\right)$$

The term D represents the diagonal terms in GDP growth, while the term N represents the non-diagonal terms, hence the terms that come from linkages in the economy. The following ratio $f = N/\sigma_Y^2$ captures how much of GDP variance is due to comovement.

Proposition 3 *Call f the fraction of GDP variance attributed to comovement in a variance accounting sense. Its value is given in equation (44) of the appendix. In the empirically-relevant benchmark where, at the micro level, most shocks are idiosyncratic ($\beta^2\sigma_A^2 \gg \overline{\Phi}^2\sigma_Y^2$), we have*

$$f = 1 - \frac{b^2\beta^2}{\mu^2} \tag{25}$$

However, economically, all the shocks are primitively idiosyncratic.

Of course, as μ increases the fraction attributed to comovement increases. Using our calibration, we find $f = 93\%$. This is to say that even though primitive shocks are purely idiosyncratic in our model, linkages create such a large comovement that, in a volatility accounting sense, 93% are mechanically attributed to comovement. This measure is congruent with the empirical findings of Shea (2002), who finds estimates of f in the range 80%-95%.

Other economic mechanisms generate a high multiplier. For instance, Bernanke and Gerler's (1989) financial accelerator creates a potentially large amplification. Also, expectations (e.g., Lorenzoni 2009) can be a plausible channel. Under that view, agents form their perceptions based on shocks to the largest segments of the economy, which leads them to put more weight on those microeconomic shocks than they would if they had access to all information in the economy. Developing that view further, based on the observable idiosyncratic shocks, would be interesting.

4.2.3 Comovement in Measured TFP

The data show some positive correlation in measured TFP innovations across sectors. The previous model generates positive comovement from independent TFP shocks. Hence, if there is perfect measurement of TFP, it will generate no comovement of TFP. Hence, we interpret the data in the following way. We say that a fraction θ in the change in the effective number of hours is not measured. For instance, a secretary will work harder when there's lots of work to do, and less intensely when there's less work. Still, the total number of hours that are counted is the same, say 40 hours per week. In that case, $\theta = 1$. If she does some overtime, so that some of her extra efforts appear in the labor supply statistics, then $\theta < 1$.

For simplicity, we assume that only labor is mismeasured. The measured number of hours is:

$$\widehat{L}_i^m = (1-\theta)\,\widehat{L}_i$$

where the superscript m denotes the $m$easured quantities. Measured TFP growth will be:

$$\begin{aligned}\widehat{TFP}_i^m &= \widehat{Q}_i - b\alpha\widehat{L}_i^m - b(1-\alpha)\,\widehat{K}_i - (1-b)\,\widehat{X}_i \\ &= \left[\widehat{Q}_i - b\alpha\widehat{L}_i - b(1-\alpha)\,\widehat{K}_i - (1-b)\,\widehat{X}\right] + \theta b\alpha\widehat{L}_i\end{aligned}$$

hence:

$$\widehat{TFP}_i^m = \widehat{TFP}_i + \theta b\alpha\widehat{L}_i \tag{26}$$

In other terms, the measured TFP is the true TFP, plus the increase in effective labor $(\widehat{L}_i)$ times labor share in output-cum-intermediary-inputs $(b\alpha)$, times the mismeasurement factor θ. Using (21), we have:

$$\begin{aligned}\widehat{TFP}_i^m &= (1+\theta b\alpha\nu\beta)\,\widehat{A}_i + \theta b\alpha\,(\xi-\nu\Phi)\,\widehat{Y} \\ &\equiv c_A\widehat{A}_i + c_Y\widehat{Y}\end{aligned}$$

To analyze this economy, take the limit where σ_A is much bigger than σ_Y, which is validated empirically, and yields clean expression. We first consider the measured diagonal matrix.

$$\sigma_F^m = c_A\sigma_F.$$

The measured change in productivity is (using $\sum_i A_i/Y = 1/b$)

$$\begin{aligned}\widehat{\Lambda}^m &= \sum_i \frac{A_i}{Y}\widehat{TFP}_i^m = \sum_i \frac{A_i}{Y}\left(c_A\widehat{A}_i + c_Y\widehat{Y}\right) = c_A\widehat{\Lambda} + \frac{c_Y}{b}\widehat{Y} \\ &= \left(c_A + \frac{c_Y}{b}\mu\right)\widehat{\Lambda}\end{aligned}$$

so the volatility of measured TFP, $\sigma_F^{Full,m} = Var\left(\widehat{\Lambda}^m\right)^{1/2}$, is:

$$\sigma_F^{Full,m} = \left(c_A + \frac{c_Y}{b}\mu\right)\sigma_F$$

The ratio between the full TFP volatility and the part coming from the diagonal of the TFP variance-covariance matrix is:

$$\frac{\sigma_F^{Full,m}}{\sigma_F^m} = 1 + \frac{c_Y\mu}{c_A b} \tag{27}$$

By this equation (and solving for θ), the empirically-measured ratio of $\sigma_F^{Full,m}/\sigma_F^m \simeq 1.4$ corresponds to $\theta = 24\%$ of the labor input being undermeasured. That seems reasonable to us. The corresponding value of σ_F^m/σ_F is simply $c_A = 1.03$. So, mismeasurement of inputs affects a lot the apparent comovement between sectors (as it is cause of comovement, and the productivity multiplier is large), but only little the measurement of sectoral-level productivity (idiosyncratic factors generally dominate aggregate factors at the microeconomic level). In our model, all primitive shocks come from idiosyncratic microeconomic shocks, but there is comovement in output because of production linkages. In terms, there is positive comovement in measured TFP because statistical agencies do not control well for unmeasured increases in labor inputs, i.e. "effort."

4.3 Time-Varying Tail Risk in the Aggregate Economy

Recent events have forced economists and policy makers to reassess the mid-2000s belief of an ever more stable economy and to update the probability of large fluctuations, or tail events, in aggregate GDP growth. For example, in a recent IMF position paper, Blanchard et al. (2010) state that "the great moderation led too many (including policy makers and regulators) to understate macroeconomic risk, ignore, in particular, tail risk and take positions [...] which turned out to be much riskier after the fact." In this section we ask whether, from the vantage point of our fundamental volatility construct, this understatement of tail risk was warranted.

To understand this exercise, first recall the basic framework introduced in section 2. Aggregate GDP growth follows aggregate TFP growth up to a constant multiplier. In turn, aggregate TFP growth is given by Hulten's formula, the Domar weighted average of micro-level TFP growth:

$$\frac{dY}{Y} = \frac{1+\varphi}{\alpha}\frac{d\Lambda}{\Lambda}, \qquad \frac{d\Lambda}{\Lambda} = \sum_{i=1}^{N} \frac{S_{it}}{Y_t} \cdot \frac{dA_{it}}{A_{it}}.$$

Now, think about feeding the time series of observed Domar weights into this basic framework while drawing sectoral TFP growth as independent draws from sectoral level univariate distributions. More specifically, for each time period, we feed the corresponding Domar weight vector and perform 10,000 draws of independent, sectoral TFP shocks, normally distributed with mean zero and where the variance of each sector's TFP growth is fixed at its full sample empirical estimate. Throughout, we use the calibration $\frac{1+\varphi}{\alpha} = 4.5$. We then compute the probability of having that year's GDP growth below $-1.64\sigma_{GDP}$, where σ_{GDP} is the model implied average volatility of aggregate GDP for the full sample period. The value 1.64 is chosen so that if the volatility is constant, the tail probability is 5% ($P(X \leq -1.64) = 0.05$ if X is a standard normal). We then repeat this for all time periods and obtain "tail risk probabilities" from 1960-2008.

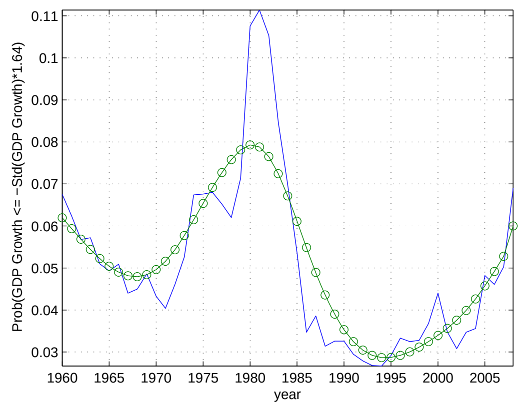

Figure 7: Tail Risk Probability. This Figure plots for each year the probability of negative GDP growth in excess of 1.64 standard deviations, given the level of fundamental volatility.

Figure 7 reports the result. The overall sample mean of tail risk is 0.051, very close to a prior of 0.05. Not surprisingly, tail risk follows the dynamics of our fundamental volatility measure. Through the lenses of the model, tail risk was reduced fourfold during the great moderation, from the high water mark of 0.11 in 1980 (implying a large fluctuation every 9 years) to a low of 0.028 in 1994 (a large fluctuation every 36 years). Taking the early 60s as a benchmark implies a halving of tail risk. Notice however that tail risk has been on the rise since the mid-90s, reaching 0.07 in 2008 (a large fluctuation every 14 years), roughly the same probability observed in the late 1970s. We conclude that while the great moderation did imply a marked decrease of tail risk in GDP growth by the mid 2000s, larger (negative and positive) events were to be expected.

Notice also that by virtue of averaging over many shocks for each sector-year, this tail-risk measure holds true for *typical* sectoral TFP disturbances. However, it does not inform us of the fragility of aggregate growth to *tail events in particular sectors* of the economy. To explore this alternative notion of fragility, or *conditional* tail risk, we consider what would have happened - throughout the sample period - if a given sector had suffered a two-standard-deviation shock to its TFP growth rate. To be precise, we implement the following procedure.

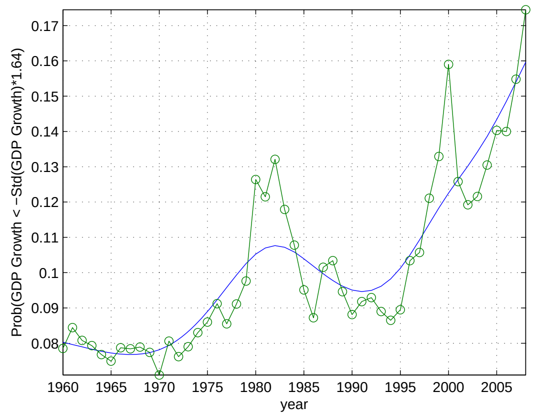

Figure 8: Tail Risk Probability and Fragility to the Non-Depository Financial Sector. This Figure plots for each year the probability of negative GDP growth in excess of 1.64 standard deviations, given two standard devation shocks to the financial sector.

Define Σ to be a diagonal matrix with non-zero entries given by each sector's (full-sample) variance of TFP growth. Let $B = \Sigma^{1/2}$, and define $G_t = BX_t$, i.e. $G_{it} = \sum_j B_{ij} X_{jt}$. Let j^* be the index of the particular sector we're interested in exploring. Then, the procedure is: simulate X_{jt} i.i.d. with mean zero and unit standard deviation for $j \neq j^*$, and set $X_{j*t} = -2$ (for a shock equal 2 standard deviations). Then, compute $G_{it} = BX_t$, and then TFP. We repeat this for every t from 1960-2008, feeding in the actual Domar Weights.

Given the discussion at the end of the last subsection and the recent focus on aggregate risks posed by the so-called "shadow banking system" we focus on a particular finance sector in our sample: non-depository financial institutions (including security and commodity brokerage services and investment banks). Consistent with our earlier results, Figure 8 shows a dramatic rise in the exposure of the aggregate economy to tail events in this sector. By 2008, the probability of a large negative event in GDP conditional on a negative tail event in this sector reaches a peak of 17.3% (or one every six years). While, by definition, the conditioning event is a rare one, we note that the fragility of the aggregate economy to any such event in this sector had been unprecedentedly high since the late 1990s: the 1998-2008 average is 14%.

4.4 Discussion: Link with previous analyses

Previous papers have analyzed sectoral composition, and have concluded that it was not very important to explain the great moderation (Blanchard and Simon 2001, McConnell and Perez-Quiros 2000, Stock and Watson 2002). One meaningful difference is that our study considers the "Domar weights" (sales-weighted) rather than value added weights, which makes an important difference for the evolution of volatility (see the Appendix for more evidence on these). Also, the previous literature finds that volatility of output falls in all sectors, hence concludes that some common factor must be at play. However, in the model of section 4.2, the original shock comes purely from disaggregated shocks, but affects all firms and sectors.[17] Hence, it will appear to an econometrician that there is a common shock, but this common shock itself comes purely from microeconomic shocks.

4.5 Time-Varying Elasticity of Labor Supply

Jaimovich and Siu (JS, 2009) have proposed that changes in the composition of the labor force account for part of the movement in GDP volatility across the G7 countries. Their idea is that the young and the old have a more elastic labor supply, hence a high fraction of young and old in the population should mean a higher elasticity of labor supply. The JS variable is about the amplification of primitive shocks, while our variable is about the primitive shocks themselves.

In terms of our model in (2)-(3), this corresponds to having a time-varying Frisch elasticity of labor supply $\varphi(t)$, i.e.

$$\sigma_{GDP}(t) = \frac{1+\varphi(t)}{\alpha}\sigma_F(t) \tag{28}$$

The JS composition of the workforce effect is in the term $\varphi(t) = A + B \cdot JS_t$, while the effect we focus on in this paper is the $\sigma_F(t)$ term. To investigate (28), we run the regression:

$$\sigma_{Yct} = \alpha_c + \beta\overline{\sigma}_{Fct} + \gamma JS_{ct} + \delta\overline{\sigma}_{Fct} \times JS_{ct} + \chi t + \varepsilon_{ct}$$

Both coefficients β and γ are positive and significant. The cross-term δ is not, perhaps because of a lack of power. We conclude that the JS labor supply elasticity and the fundamental volatility are both relevant to

[17] A more analytical note on this point is available upon request.

5 Conclusion

We have investigated the explanatory power of "fundamental volatility" to understand the swings in macroeconomic volatility, and found it to be quite good. Fundamental volatility explains the great moderation, and its undoing. It is directly observable, with no parameter adjustment.

Our findings do support the view that the key to macroeconomic volatility might be found in microeconomic shocks. Of course, they need to be enriched by some "propagation mechanisms". Their identification might be made easier if we think that microeconomic shocks are the primary factors that are propagated. For instance, we do not deny that monetary shocks may be important. However, they may largely be part of the response to other shocks (e.g., real shocks caused by oil or finance), rather than really primitive shocks.

Our findings pose the welfare consequences of the microeconomic composition of an economy. In models with financial frictions, a rise in volatility is typically welfare-reducing. Perhaps finance was too big and created too much volatility in the 2000s? Perhaps the oil-dependent industries where too big and created too much volatility in the 1970s?

In addition, fundamental volatility can serve as an "early warning system" to measure future volatility. In retrospect, the surge in the size of finance in the 2000s could have been used to detect a great source of new macroeconomic volatility.

In any case, we think that fundamental microeconomic volatility is a useful tool to consider when thinking about the causes and consequences of aggregate fluctuations.

References

Arias, Andres, Gary Hansen and Lee Ohanian. 2007. "Why Have Business Cycle Fluctuations Become Less Volatile." *Economic Theory*, 32(1): 43-58.

Bak, Per, Ken Chen, José Scheinkman, and Michael Woodford. 1993. "Aggregate Fluctuations from Independent Sectoral Shocks: Self-Organized Criticality in a Model of Production and Inventory Dynamics." *Ricerche Economiche*, 47(1): 3-30.

Basu, John Fernald, John Fernald, Jonas Fisher, and Miles Kimball. 2009. "Sector-Specific Technical Change." mimeo.

Blanchard, Olivier, and John Simon. 2001. "The Long and Large Decline in U.S. Output Volatility." *Brookings Papers on Economic Activity*, 165-71.

Buera, Francisco and Joseph Kaboski. 2009. "The Rise of the Service Economy." UCLA Working Paper.

Carvalho, Vasco. 2009. "Aggregate Fluctuations and the Network Structure of Intersectoral Trade." CREI Working Paper.

Domar, Evsey. 1961. "On the Measurement of Technological Change." *Economic Journal*, 71: 709-29.

Durlauf, Steven. 1993. "Non Ergodic Economic Growth." *Review of Economic Studies*, 60(2): 349-66.

Gabaix, Xavier. 1999. "Zipf's Law for Cities: An Explanation." *Quarterly Journal of Economics*, 114(3): 739–767.

Gabaix, Xavier. 2009. "The Granular Origins of Aggregate Fluctuations." National Bureau of Economic Research Working Paper 15286.

Gali, Jordi and Luca Gambetti. 2009. "On the Sources of the Great Moderation," *American Economic Journal: Macroeconomics*, 1(1): 26–57

Hall, Robert. 2009a. "Reconciling Cyclical Movements in the Marginal Value of Time and the Marginal Product of Labor," *Journal of Political Economy*, 117(2): 281-323.

Hall, Robert. 2009b. "By How Much Does GDP Rise if the Government Buys More Output?" *Brookings Papers on Economic Activity*, 2: 183-231.

Horvath, Michael. 1998. "Cyclicality and Sectoral Linkages: Aggregate Fluctuations from Sectoral Shocks," *Review of Economic Dynamics*, 1: 781-808.

Horvath, Michael. 2000. "Sectoral Shocks and Aggregate Fluctuations," *Journal of Monetary Economics*, 45(1): 69-106.

Imbs and Wacziarg. 2003. "Stages of Diversification", *American Economic Review*, 93: 63-86.

Jaimovich, Nir and Henry Siu. 2009. "The Young, the Old, and the Restless: Demographics and Business Cycle Volatility" *American Economic Review*, 99(3): 804-26.

Jones, Charles I. 2009. "Intermediate Goods and Weak Links: A Theory of Economic Development," Stanford University Working Paper.

Jorgenson, Ho, and Stiroh. 2008. "A Retrospective Look at the U.S. Productivity Growth Resurgence," *Journal of Economic Perspectives*, 22(1): 3-24.

Jovanovic, Boyan. 1987. "Micro Shocks and Aggregate Risk," *Quarterly Journal of Economics*, 102(2): 395-409.

Justiniano, Alejandro and Giorgio E. Primiceri. 2008. "The Time-Varying Volatility of Macroeconomic Fluctuations," *American Economic Review*, 98(3): 604-41.

Koren, Miklós, and Silvana Tenreyro. 2007. "Volatility and Development," *Quarterly Journal of Economics*, 122(1): 243-87.

Koren, Miklós, and Silvana Tenreyro. 2009. "Technological Diversification," CEU and LSE Working paper.

Long, John, and Charles Plosser. 1983. "Real Business Cycles," *Journal of Political Economy*, 91(1): 39–69.

Lorenzoni, Guido. 2009. "A Theory of Demand Shocks," *American Economic Review*, 99(5): 2050-84.

Luttmer, Erzo G. J. 2007. "Selection, Growth, and the Size Distribution of Firms." *Quarterly Journal of Economics*, 122(3): 1103–1144.

McConnell, Margaret, and Gabriel Perez-Quiros. 2000. "Output Fluctuations in the United States: What has Changed Since the Early 1980's," *American Economic Review*, 90(5): 1464-76.

Moro, Alessio. 2009. "The Structural Transformation between Manufacturing and Services and the Decline in US GDP Volatility," Universidad Carlos III Working Paper.

Nirei, Makoto. 2006. "Threshold Behavior and Aggregate Critical Fluctuations," *Journal of Economic Theory*, 127: 309-22.

Philippon, Thomas. 2008. "The Evolution of the US Financial Industry from 1860 to 2007," New York University Working Paper.

Qu, Zhongjun and Pierre Perron. 2007. "Estimating and testing structural changes in multivariate regressions," *Econometrica*, 75(2): 459–502.

Shea, John. 2002. “Complementarities and Comovements,” *Journal of Money, Credit and Banking*, 42(2): 412-33.

Simon, Herbert. 1955. “On a Class of Skew Distribution Functions.” *Biometrika*, 42(3-4): 425–440.

Stiroh, Kevin. 2009. “Volatility Accounting: A Production Perspective on Increased Economic Stability,” *Journal of the European Economic Association*, 7(4): 671-96.

Stock, James and Mark Watson. 2002. “Has the Business Cycle Changed and Why?” *NBER Macroeconomics Annual*, 17: 159-218

Stock, James and Mark Watson. 2005. “Understanding Changes in International Business Cycle Dynamics,” *Journal of the European Economic Association* 3(5): 968-1006.

Timmer, M., T. van Moergastel, E. Stuivenwold, G. Ypma, M. O’Mahony and M. Kangasniemi. 2007. EU KLEMS Growth and Productivity Accounts Version 1.0, Part I, Methodology.

A Data Appendix

USA data. The main data source for this paper was constructed by Dale Jorgenson and associates[18] and gives a detailed breakdown of the entire US economy into 88 sectors. The data is annual and covers the period between 1960 and 2005. The original sources are input-output tables and industry gross output data compiled by the Bureau of Labor Statistics and the Bureau of Economic Analysis The data is organized according to the KLEM methodology reviewed in Jorgenson, Gallop and Fraumeni (1987) and Jorgenson, Ho and Stiroh (2005). In particular, the input data incorporates adjustments for quality and composition of capital and labor. To the best of our knowledge this is the most detailed (balanced) panel coverage of US sectors available, offering a unified dataset for the study of sectoral productivity.[19]

From this dataset, and for each year-industry pair, we observe nominal values of sectoral gross output, capital, labor and material inputs supplied by the 88 sectors (plus non-competing imports) as well as the corresponding price deflators. Following Jorgenson, Ho and Stiroh (2008) and Basu et al. (2009) we concentrate on private sector output, thus excluding services produced by the government (but including purchases of private-sector goods and services by the government). We also exclude from the analysis the imputed service flow from owner-occupied housing.[20] This yields a panel of 77 sectors which forms the basis of all computations below (see appendix for their definitions).

International data. For UK, Japan, Germany and France we resort to the EUKLEMS database (see Timmer et al, 2007 for a full account of the dataset). As the name indicates, this database is again organized according the KLEM methodology proposed by Jorgenson and associates. To preserve comparability we focus on private sector accounts, thus excluding publicly provided goods and services. We thus exclude sectors under the heading 'non-market services' in the dataset. These also include real estate services as the database does not make a distinction between real estate market services and the service flow from owner occupied residential buildings (see Timmer et al, 2007, Appendix Table 3 for definitions and discussion).

[18] The dataset is available for download at Dale Jorgenson's Dataverse website, `http://dvn.iq.harvard.edu/dvn/dv/jorgenson`.

[19] The NBER-CES manufacturing database provides further detail but only includes manufacturing industries. As it will be made clear in the paper it is crucial to account for the growth of service sectors when looking at cross-sectoral diversification and the great moderation.

[20] Finally we drop two sectors for which there is no data in the original dataset, "Uranium Ore" and "Renting of Machinery".

For each country we obtain a panel of nominal sectoral gross output and value added at the highest level of disaggregation possible. For the UK we end up with 66 sectors, Japan, 58, Germany 50 and France 46. For roughly half of these sectors we can compute TFP growth from the gross output perspective. We then compute the average (across sectors) standard deviation of TFP growth during the entire sample period. For UK, Germany and France the resulting panel runs from 1970-2005. For Japan it starts in 1973, due to the unavailability of earlier data.

B Proof Appendix

Proof of Proposition 1 The planner's problem is $\max_{K,L} \Lambda K^{1-\alpha} L^{\alpha} - L^{1+1/\varphi} - rK$. The first order conditions with respect to K and L give: $(1-\alpha)\frac{Y}{K} = r$, $\alpha \frac{Y}{L} = (1+1/\varphi) L^{1/\varphi}$, so that $K = (1-\alpha) Y/r$, $L = \left(\frac{\alpha}{1+1/\varphi} Y\right)^{\frac{\varphi}{1+\varphi}}$, and

$$Y = \Lambda K^{1-\alpha} L^{\alpha} = \Lambda \left((1-\alpha)\frac{Y}{r}\right)^{1-\alpha} \left(\frac{\alpha}{1+1/\varphi} Y\right)^{\frac{\alpha\varphi}{1+\varphi}} = (1-\alpha)^{1-\alpha} \left(\frac{\alpha}{1+1/\varphi}\right)^{\frac{\alpha}{1+\varphi}} \Lambda Y^{1-\frac{\alpha}{1+\varphi}}$$

Finally $Y = k\Lambda^{\frac{1+\varphi}{\alpha}}$ with $k = \left[(1-\alpha)^{1-\alpha}\left(\frac{\alpha}{1+1/\varphi}\right)^{\frac{\alpha}{1+\varphi}}\right]^{\frac{1+\varphi}{\alpha}}$.

Proof of Proposition 2 We found it useful to state a general proposition with an arbitrary number of fixed and variable factors. Consider:

$$Q_i = A_i \left(\prod_f L_{if}^{\alpha_{if}}\right)^b X_i^{1-b}$$

where L_{if} is the share of factor f in sector i. For instance, in our economy there are three factors:

$$(L_f)_{f=1...3} = (\text{Labor,Capital,Intermediary inputs}) = (L, K, X)$$

and their weights are:

$$(\gamma_f)_{f=1...3} = (\alpha b, (1-\alpha) b, 1-b).$$

Call $\gamma_f = b\alpha_{if}$ for f a primitive factor, and $\gamma_f = (1-b)$ for $f =$the intermediary input.

We define the output gross of intermediary inputs by:

$$H = A\left(\frac{L^{\alpha}K^{1-\alpha}}{b}\right)^b \left(\frac{X}{1-b}\right)^{1-b} = kA\prod L_f^{\gamma_f}$$

and we define output net of intermediary inputs as:

$$
\begin{aligned}
Y &= \max_X H - X \\
&= \max_X A\left(\frac{L^\alpha K^{1-\alpha}}{b}\right)^b \left(\frac{X}{1-b}\right)^{1-b} - X
\end{aligned}
$$

This works out to be:

$$Y = A^{1/b} L^\alpha K^{1-\alpha} = A^{1/b} \prod_{f \neq 3} L_f^{\gamma_f} \tag{29}$$

So TFP is $\Lambda = A^{1/b}$.

Each resource L_f has a cost L_f^{1/ξ_f}. In the case of labor, $1/\xi_L = 1+1/\varphi$, i.e. $\xi_L = \varphi/(1+\varphi)$. On the other hand, as the cost of the intermediary good X is linear in X, $\xi_3 = 1$. If capital is elastic in the short run, $\xi_2 = 1$, while if it is completely inelastic, $\xi_2 = 0$. We start with a general Proposition.

Proposition 4 *(General case) Suppose that each unit i receives a productivity shock $\widehat{A}_i$. Macroeconomic variables change according to:*

$$\mathit{TFP}: \ \widehat{\Lambda} = \sum \frac{S_i}{Y} \widehat{A}_i = \sum \frac{\mathit{Sales}_i}{\mathit{GDP}} \widehat{A}_i \tag{30}$$

$$\mathit{GDP}: \ \widehat{Y} = \frac{1}{1 - \sum_f \alpha_f \xi_f} \widehat{\Lambda} \tag{31}$$

$$\textit{Employment of factor } f: \ \widehat{L}_f = \xi_f \widehat{Y} \tag{32}$$

$$\textit{Wage of factor } f: \widehat{w}_f = \left(1 - \xi_f\right) \widehat{Y} \tag{33}$$

Microeconomic-level variables change according to:

$$\textit{Dollar sales}: \ \widehat{S}_i = \beta \widehat{A}_i + \overline{\Phi} \widehat{Y} \tag{34}$$

$$\textit{Production}: \widehat{Q}_i = \psi \beta \widehat{A}_i + (1 - \psi \Phi) \widehat{Y} \tag{35}$$

$$\textit{Price}: \widehat{p}_i = -(\psi - 1) \beta \widehat{A}_i + (\psi - 1) \beta \Phi \widehat{Y} \tag{36}$$

$$\textit{Employment of factor } f: \widehat{L}_{if} = \nu_f \beta \widehat{A}_i + \left(\xi_f - \nu_f \Phi\right) \widehat{Y} \tag{37}$$

$$\textit{Use of intermediary input}: \widehat{X}_i = \widehat{S}_i \tag{38}$$

where

$$
\begin{aligned}
\beta &= \frac{1}{\psi - \sum_f \gamma_f \nu_f} = \frac{1}{\psi - (1-b) - b \sum_f \alpha_f \nu_f} \\
&= \frac{1}{\psi - \sum_{f \in F \cup X} \textit{Share of factor } f \times \textit{Flexibility ratio of factor } f}
\end{aligned}
$$

and

$$\Phi = \beta\left(1-\sum_f \gamma_f \xi_f\right) = \beta b\left(1-\sum_f \alpha_f \xi_f\right)$$
$$= \beta\left(1-\sum_{f\in F\cup X} \textit{Share of factor } f \times \textit{Adjusted supply elasticity of factor } f\right)$$

where $f \in F \cup X$ denotes the primitive factors (labor, capital) and also the intermediary input X.

Proof of Proposition 4 *Step 1. Frictionless equilibrium.* We define:

$$H = \left(\sum_i Q_i^{1/\psi}\right)^{\psi} \tag{39}$$

The price of unit i is: $p_i = \frac{\partial H}{\partial Q_i}$, so: $S_i/H = p_i Q_i/H = Q_i \frac{\partial H}{\partial Q_i}/H$, and

$$\frac{S_i}{H} = \left(\frac{Q_i}{H}\right)^{1/\psi} \tag{40}$$

Because H is homogenous of degree 1, $H = \sum \frac{\partial H}{\partial Q_i} Q_i = \sum S_i$. H is the sum of sales in the economy.

Unit i solves: $\max_{L_{if}} p_i Q_i - \sum_f w_f L_{if}$, which gives: $L_{if} = S_i \gamma_f / w_f \propto S_i$. We use $\propto$ to mean that the variables are proportional, up to a factor that does not depend on i. So, $S_i^{\psi} \propto Q_i \propto A_i S_i$ by (8), so $S_i \propto A_i^{1/(\psi-1)}$. Calling $B = \sum A_i^{1/(\psi-1)}$, and using the adding up constraint $\sum L_{if} = L_f$, we find the constant of proportionality: $L_{if} = L_f A_i^{1/(\psi-1)}/B$. Plugging this in (39), we get:

$$H = B^{\psi-1}\left(\frac{L^{\alpha}K^{1-\alpha}}{b}\right)^{b}\left(\frac{X}{1-b}\right)^{1-b}$$

Now, we solve for X, i.e. solve $\max_X H - X$. The solution is: $Y = H - X = B^{(\psi-1)/b} L^{\alpha} K^{1-\alpha}$, i.e. the announced relation.

Also, as $Y = H - \sum X_i$, and $X_i = (1-b) S_i$, $Y = H - \sum (1-b) S_i = bH$.

Step 2. Changes, assuming $\nu_f = 1$. To keep the proof streamlined, we first consider the case $\nu_f = 1$, i.e. the case with no frictions in the adjustment of labor, and with the possibility that $\sum_f \gamma_f$ is not 1. We now look at changes. TFP growth comes from (11), and is also Hulten's formula. $Y = bH$ gives $\widehat{Y} = \widehat{H}$. The optimal use of factor f maximizes $Y - \sum C_f L_f^{1/\xi}$, so $\gamma_f Y/L_f = L_f^{1/\xi_f - 1}$, and $L = Y^{\xi_f}$ times a constant, and

$$\widehat{L_f} = \xi_f \widehat{Y} \tag{41}$$

This implies that

$$\widehat{Y} = \widehat{\Lambda} + \sum_f \gamma_f \widehat{L}_f = \widehat{\Lambda} + \left(\sum_f \gamma_f \xi_f\right) \widehat{Y}$$

and

$$\widehat{Y} = \frac{\widehat{\Lambda}}{1 - \sum_f \gamma_f \xi_f}$$

The wage is $w_f = \frac{1}{\xi} L^{1/\xi_f - 1}$, so: $\widehat{w}_f = \left(\frac{1}{\xi_f} - 1\right) \widehat{L}_f$, hence

$$w_f = \left(1 - \xi_f\right) \widehat{Y}. \tag{42}$$

It is convenient that one can solve for changes in the macroeconomic variables without revisiting the firms' decision problems.

We now turn to the unit-level changes. Optimization of the demand for labor gives $w_f L_{if} = \gamma_f S_i$, so

$$\widehat{L}_{if} = \widehat{S}_i - \widehat{w_f} = \widehat{S}_i - \left(1 - \xi_f\right) \widehat{Y}$$

We have, from (8),

$$\begin{aligned} \widehat{Q}_i &= \widehat{A}_i + \sum_f \gamma_f \widehat{L}_{if} = \widehat{A}_i + \sum_f \gamma_f \left(\widehat{S}_i - \left(1 - \xi_f\right) \widehat{Y}\right) \\ &= \widehat{A}_i + \left(\sum_f \gamma_f\right) \widehat{S}_i - \sum_f \gamma_f \left(1 - \xi_f\right) \widehat{Y} \end{aligned}$$

Eq. (40) gives $\widehat{Q}_i = \psi \widehat{S}_i + (1 - \psi) \widehat{H}$, and using $\widehat{Y} = \widehat{H}$,

$$\psi \widehat{S}_i + (1 - \psi) \widehat{Y} = \widehat{Q}_i = \widehat{A}_i + \left(\sum_f \gamma_f\right) \widehat{S}_i - \sum_f \gamma_f \left(1 - \xi_f\right) \widehat{Y}$$

which gives

$$\begin{aligned} \widehat{S}_i &= \frac{\widehat{A}_i + \left[\psi - 1 - \sum_f \gamma_f \left(1 - \xi_f\right)\right] \widehat{Y}}{\psi - \sum_f \gamma_f} = \frac{\widehat{A}_i}{\psi - \sum_f \gamma_f} + \left(1 - \frac{1 - \sum_f \gamma_f \xi_f}{\psi - \sum_f \gamma_f}\right) \widehat{Y} \\ &= \beta \widehat{A}_i + \overline{\Phi} \widehat{Y} \end{aligned}$$

where

$$\beta = \frac{1}{\psi - \sum_f \gamma_f}, \qquad \Phi = \beta \left(1 - \sum_f \gamma_f \xi_f \right)$$

and as $\widehat{Q}_i = \psi \widehat{S}_i + (1-\psi)\widehat{Y}$, we obtain the announced expressions for $\widehat{S}_i$ and $\widehat{Q}_i$. $\widehat{L}_{if}$ comes from $\widehat{L}_{if} = \widehat{S}_i - \widehat{w}_f$. S_i was defined as $S_i = p_i Q_i$, which gives $\widehat{p}_i = \widehat{S}_i - \widehat{Q}_i$.

Step 3. With a general $\lambda \in [0,1]$. After the changes $\widehat{A}_i$, only $L_{V,i}$ can adjust. The planner optimizes the variable part of labor supply: $\max_{L_{Vf}} A \prod_f \left(L_{Vf}^{\nu_f} L_{Ff}^{1-\nu_f} \right)^{\gamma_f} - \left(L_{Vf}^{\nu_f} L_{Ff}^{1-\nu_f} \right)^{1/\xi_f}$. Note that this is isomorphic to optimizing the total labor supply, defining $L_f = L_{Vf}^{\nu_f} L_{Ff}^{1-\nu_f}$. Hence, we have (41) and (42).

For the unit-level variables, one replaces $(\alpha_f, \gamma_f, \xi_f)$ by $(\alpha'_f, \gamma'_f, \xi'_f) = (\alpha_f \nu_f, \gamma_f \nu_f, \xi_f/\nu_f)$, which delivers

$$\beta = \frac{1}{\psi - \sum_f \gamma_f \nu_f}, \qquad \Phi = \beta \left(1 - \sum_f \gamma_f \xi_f \right).$$

The expression for employment becomes:

$$\widehat{L}_{V,i} = \nu_f \left[\beta \widehat{A}_i + \left(\frac{\xi_f}{\nu_f} - \Phi \right) \widehat{Y} \right] = \nu_f \beta \widehat{A}_i + (\xi_f - \nu_f \Phi) \widehat{Y}$$

This concludes the proof of Proposition 4.

Proof of Proposition 2 We apply the results from Proposition 4. We particularize it to the case of flexible capital ($\xi_K = 1, \nu_K = 1$), and flexible intermediary inputs ($\xi_X = 1, \nu_X = 1$), while labor is less flexible ($\xi_L = \xi, \nu_L = \nu$ can be less than 1). Then, using $\xi = \varphi / (1+\varphi)$,

$$\widehat{Y} = \frac{1}{1-(1-\alpha)-\alpha\xi_L}\widehat{\Lambda} = \frac{1}{\alpha(1-\xi)}\widehat{\Lambda} = \frac{1+\varphi}{\alpha}\widehat{\Lambda}$$

$$\widehat{L} = \xi \widehat{Y} = \frac{\varphi}{1+\varphi}\widehat{Y}, \qquad \widehat{w} = (1-\xi) = \frac{\widehat{Y}}{1+\varphi}.$$

Also,

$$\beta^{-1} = \psi - \sum_f \gamma_f \nu_f = \psi - b\alpha\nu - b(1-\alpha) - (1-b) = \psi - 1 + b\alpha(1-\nu)$$

$$\Phi = \beta \left(1 - \sum_f \gamma_f \xi_f \right) = \beta (1 - b\alpha\xi - b(1-\alpha) - (1-b)) = \beta b \alpha (1-\xi) = \beta b \alpha \frac{\varphi}{1+\varphi}.$$

Proof of Proposition 3 In this model, value added is proportional to sales, $S_i = bS_i$ (this comes from the first order condition with respect to X_i). So

$$
\begin{aligned}
D &\equiv \sum_i \left(\frac{Y_i}{Y}\right)^2 var\left(\frac{dY_i}{Y_i}\right) = \sum_i b^2 \left(\frac{S_i}{Y}\right)^2 var\left(\frac{dS_i}{S_i}\right) \\
&= b^2 \sum_i \left(\frac{S_i}{Y}\right)^2 var\left(\beta \widehat{A}_i + \overline{\Phi}\widehat{Y}\right) \qquad (43)
\end{aligned}
$$

and $f = N/\sigma_Y^2 = 1 - D/\sigma_Y^2$. Consider first the case where at the microeconomic level most shocks are idiosyncratic, i.e. $\beta^2\sigma_A^2 \gg \overline{\Phi}^2\sigma_Y^2$. Then,

$$
f \simeq 1 - \frac{b^2 \sum_i \left(\frac{S_i}{Y}\right)^2 \beta^2 \sigma_A^2}{\sigma_Y^2} = 1 - \frac{b^2\beta^2\sigma_F^2}{\mu^2\sigma_F^2} = 1 - \frac{b^2\beta^2}{\sigma_F^2}.
$$

In the general case, give (14) implies $cov\left(\widehat{A}_i, \widehat{Y}\right) = \frac{S_i}{Y}\mu\sigma_A^2$, (43) gives:

$$
\begin{aligned}
f &= 1 - \frac{b^2 \sum_i \left(\frac{S_i}{Y}\right)^2 \left[\beta^2\sigma_A^2 + \overline{\Phi}^2\mu^2\sigma_F^2 + 2\beta\overline{\Phi}\left(\frac{S_i}{Y}\right)\mu\sigma_A^2\right]}{\mu^2\sigma_F^2} \\
&= 1 - \frac{b^2\beta^2}{\mu^2} - \overline{\Phi}^2 b^2 \left(\sum_i \left(\frac{S_i}{Y}\right)^2\right) - \frac{2b^2\beta\overline{\Phi}\sigma_A^2}{\mu\sigma_F^2} \sum_i \left(\frac{S_i}{Y}\right)^3
\end{aligned}
$$

so

$$
f = 1 - \frac{b^2\beta^2}{\mu^2} - \overline{\Phi}^2 b^2 \sum_i \left(\frac{S_i}{Y}\right)^2 - \frac{2b^2\beta\overline{\Phi}}{\mu} \frac{\sum_i \left(\frac{S_i}{Y}\right)^3}{\sum_i \left(\frac{S_i}{Y}\right)^2}. \qquad (44)
$$

C Technological Diversification Patterns

Recall that in the construction of our fundamental volatility measure, the only time varying element that we allow for is the sum of squared Domar weights, $H_t^D = \sum_{i=1}^N \left(S_{it}/Y_t\right)^2$, where S_{it} is sector i nominal gross output in year t and Y_t gives the total (nominal) value added for the private sector economy in year t. While Domar weights do not sum to one - as gross output at the sector level exceeds sectoral value added by the amount of intermediate input consumed by that sector - this measure is akin to the more usual Herfindahl indexes of concentration. In particular, looking at the cross-sectional (uncentered) second moment to characterize dispersion/concentration in technology loadings is still valid. The graph below shows the evolution of this measure for the US.

From the peak in 1960 to the through in 1997, there is a 33% drop in the H_t^D measure. These dynamics are key for explaining the evolution of our fundamental volatility measure.

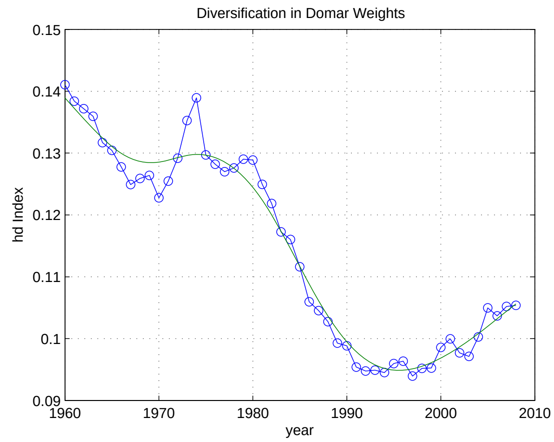

Figure 9: Evolution of H_t^D, 1960-2008

As such, it is important to perform a number of robustness checks and confirm that the same pattern obtains: i) in more disaggregated data, and across different classification systems; ii) for different dispersion/concentration measures and iii) for value added shares.

C.1 More Disaggregated Data

First we look at the raw BLS data underlying much of the construction of Jorgenson's dataset. This data is both defined at a more disaggregated level and according to different classification systems. Namely, we source two different vintages of BLS "interindustry relationships" data(i.e. Input-Output data). The first is based on SIC classification (a mixture of two and three digit SIC sectors) running from 1977 to 1995 for a total of 185 sectors. The second is the latest vintage produced by the BLS, based on the newer NAICS classification system and runs from 1993-2008 (for a total of 200 sectors).

Based on this data we calculate the implied herfindahl-like measure. Notice that given the difference in underlying classification systems the levels are not comparable, neither among themselves nor with the ones reported above. Nevertheless the dynamics seem to be in broad agreement with those above: fall in technological concentration in late 70s and early 80s and if anything a stronger reversal of this pattern from late 90s onwards.

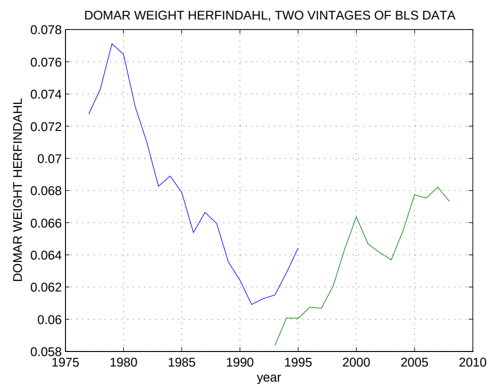

Figure 10: Herfindahl measure for Domar weights computed from two vintages of raw BLS data (Green line :1977-1995 SIC data; Blue line 1993-2008 NAICS data)

C.2 Checking against other dispersion measures

Though directly relevant to the story of this paper, herfindahl are not the only dispersion measure. However, looking at alternative measures of concentration, such as the Gini Index, the broad story is unchanged: decline in concentration for both measures up until the early 90s followed by re-concentration from mid to late 90s, where the latter now shows up much more strongly.

The patterns above are robust when we consider other measures of dispersion: cross sectional standard deviation; coefficient of variation; max-min spread and max-median spread.

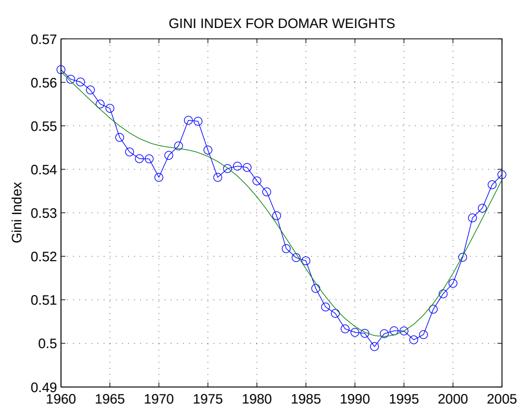

Figure 11: Gini Index for Sectoral Domar Weights.

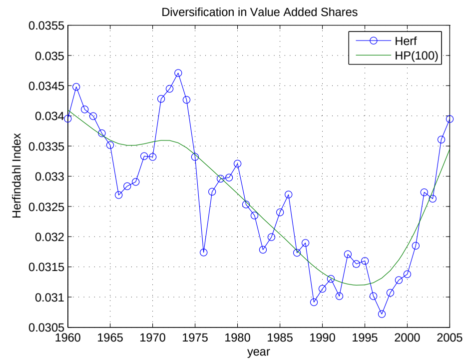

Figure 12: Herfindahl index of sectoral concentration from value added shares data from 1960-2005.

An alternative is to look into diversification patters in sectoral value added shares, where the corresponding herfindahl at year t, H_t^{VA}, is defined as $H_t^{VA} = \sum_{i=1}^{N} (Y_{it}/Y_t)^2$, where Y_{it} is nominal value added in sector i at year t. Figure 12 depicts the evolution of this index and the corresponding HP-filtered series.

Again, looking at value-added shares leads to the same U-shaped pattern. We also see that, quantitatively, the peak to through movement in the value added Herfindahl is smaller. This is as it should be: manufacturing technologies are relatively more intermediate input intensive. As such the gradual move away from manufacturing and into services implies that the Domar weights-based measure of concentration fell relatively more than the value added one: not only has economic activity relied more on what were initially small - in a value added sense - sectors but these latter sectors have relatively lower gross output to value added ratios.[21]

[21] A way to confirm this is to regress the average growth rate of each sector's value added share on the average weight of intermediate input purchases in sectoral gross output (average over the entire sample period 1960-2005). We obtain a negative and significant slope coefficient.