Monetary Policy in a Currency Union with Heterogeneous Limited Asset Markets Participation

Fabian Eser [*][†]

30th November 2009

Abstract

I develop a tractable model of a New Keynesian currency union made up of small open economies with heterogeneous rates of limited asset markets participation (LAMP) to study monetary policy under strongly asymmetric monetary policy transmission and imperfect risk-sharing

Firstly, the heterogeneity of asset markets participation (AMP) can be captured by one parameter, the coefficient of variation (CV). While monetary policy can guarantee equilibrium determinacy by following an active or passive rule depending on the sign of the interest-elasticity of output, the CV can change the sign and the size of the latter.

Secondly, due to the determinacy for union-aggregates does not guarantee determinacy in every member country. However, the more open a country is in trade terms, the greater the rate of LAMP for which the country still displays determinacy. For complete openness, determinacy is guaranteed.

Thirdly, considering the optimal union-wide targeting rule, a higher mean of LAMP and dispersion of AMP increase the desired inflation volatility and decrease the desired output volatility.

Keywords: Monetary Union, Limited Asset Markets Participation, Heterogeneity, (Optimal) Monetary Policy, Real (In)determinacy, Sticky Prices

JEL Classification: E52, F41, E44

Contents

1   Introduction

Imperfect risk sharing and asymmetric monetary transmission channels are real world challenges for monetary policy in a currency union. This paper examines in a theoretical context the implication of these features for monetary policy in a currency union. More precisely, I examine monetary policy in a currency union whose member countries exhibit heterogenous rates of limited asset markets participation (LAMP). Due to this heterogenous LAMP, perfect risk sharing across member countries fails and the monetary transmission mechanism can differ dramatically across countries. Potentially, countries could exhibit elasticities of output to the union-wide nominal interest rate of opposite sign. This provides an extreme test of the implications of structural asymmetry and imperfect risk sharing for monetary policy in a currency union.

Limited asset markets participation, i.e. the fact that not all households in an economy participate in asset markets, is an acknowledged feature of real world economies.[1] The concern of this paper is what LAMP means for monetary policy in a currency union, in particular when it differs across countries. Tables 1 and 2 present data for stockownership and homeownership for the two biggest currency unions, the US and the euro area. Both are imperfect indicators of asset markets participation (AMP). Homeownership may often require financing, but not always. Similarly, while stock ownership may be sufficient to indicate AMP, it is not necessary. What Tables 1 and 2 show is that, depending on the indicator, LAMP can be high; homeownership for instance is more prevalent than stock ownership. More than that, it differs markedly across countries, although the euro area exhibits considerably more heterogeneity than the US, which also has a greater level of AMP according to both measures.

---

[1]LAMP proves difficult to quantify. On the one hand, this is due to the variety of asset market participation, be it through stock ownership, corporate or government bonds, housing finance, or simply a bank account. On the other hand, in many models asset markets are simply a set of state-contingent securities, which makes it difficult to say what the best real world equivalent for them is. Thus, while one can argue over the best indicator for LAMP and its correct level, its significance is clear. Campbell and Mankiw (1989) attempt to quantify LAMP based on aggregate time series. They find that around 0.4 to 0.5 of the US population consume their current income and interpret this as the rate of LAMP. Mulligan and Sala-i-Martin (2000) present data from the 1989 US Survey of Consumer Finances according to which 59% of the population held no interest-bearing assets, while 25% did not even have a checking account. More recently, Vissing-Jørgensen (2002) reports PSID data according to which 21.75% of US population hold shares and 31.40% own bonds. Similarly, based on 1999 PSID data, Caner and Wolff (2004) classify, according to net worth, 25.9% of the population as asset poor. Excluding home equity from net worth even 41.7% can be considered asset-poor. Meanwhile, Bilbiie and Straub (2006) estimate the level of LAMP to be 0.44 during the pre-Volcker era and 0.24 during the Volcker-Greenspan period. Coenen and Straub (2005), by contrast, investigate the European Monetary Union and estimate the level in the euro area to lie between 0.24 and 0.37.

Table 1: Dispersion of Euro Area Asset Markets Participation

Country	Stock Ownership Rate	Home Ownership Rate
Austria	0.10	0.58
Belgium	0.38	0.71
Finland	-	0.65
France	0.43	0.57
Germany	0.24	0.43
Greece	0.10	0.80
Ireland	-	0.75
Italy	0.10	0.70
Luxembourg	-	0.75
Netherlands	0.24	0.57
Portugal	-	0.75
Spain	0.11	0.86
Statistic		
Mean	0.21	0.68
Standard Deviation	0.13	0.12
Coefficient of Variation $CV_{1-\lambda}$	0.62	0.18
$CV^2_{1-\lambda}$	0.39	0.03

Notes: The Stock Ownership data are taken from table 1 in Christelis et al. (2009) who obtain it from SHARE data. The Home Ownership rate data aren taken from table 1 in European Central Bank (2009), where it appears as 'Owner-Occupancy Rate'.

Table 2: Dispersion of US Asset Markets Participation

Region	Stock Ownership Rate	Home Ownership Rate
Midwest	0.55	0.81
Northeast	0.55	0.71
South	0.43	0.78
West	0.52	0.77
Statistic		
Mean	0.51	0.77
Standard Deviation	0.06	0.04
Coefficient of Variation $CV_{1-\lambda}$	0.11	0.06
$CV^2_{1-\lambda}$	0.01	0.003

Notes: The data are taken from table 1 in Christelis et al. (2009) who obtain it from SHARE data.

In this paper, by contrast, we allow for heterogenous LAMP. That is, in every country there is a rate of LAMP and these rates can differ across countries. Heterogenous LAMP directly leads to an asymmetric transmission mechanism of monetary policy. Thus, the first question of interest is what this asymmetry in the monetary transmission mechanism means for the union-wide interest rate policy. As a modelling tool LAMP is interesting in that, firstly, it allows us to investigate asymmetries in the transmission mechanism with just two statistics, its mean rate and its dispersion across other countries. Secondly, we can study with LAMP not least for the most extreme test of structural heterogeneity, where the interest-elasticity of output in one country may be negative while in another it is positive. Thus, a currency union model with heterogenous LAMP can serve as a laboratory to explore how much heterogeneity in the monetary transmission mechanism is feasible in a currency union. In this sense LAMP can proxy for other heterogeneities in labour, product, housing or financial markets which are certainly important.[2] Another convenient feature of LAMP is that it allows us to investigate the importance of financial market integration for a currency union. As Non-Ricardian agents have no access to asset markets, the rate of LAMP in a country directly determines its extent of financial-risk sharing with the rest of the currency union.

To investigate the importance of heterogenous LAMP in a currency union, I introduce LAMP as modelled in Bilbiie (2008) into the currency union model of Galí and Monacelli (2008). The latter build a model of a monetary union made up of a continuum of small open economies each of which has mass zero. This implies that each member country by itself does not affect any other member country or the union as a whole. While Galí and Monacelli (2008) allow for asymmetric shocks across countries, all members of the currency union are structurally symmetric. There is no LAMP, so that in all countries all households are Ricardian and complete financial markets ensure perfect international risk sharing across countries.[3]

---

[2]Cecchetti (1999) identifies differences in legal structures, size and concentration of banking in firm finance as well as industrial structure. Maclennan et al. (1999) identify significant differences in financial market capitalisations relative to GDP which are based on different, pension, banking, corporate finance systems. Furthermore differences in tax systems and credit availability change the effects of housing wealth on consumption, which is also documented more recently by Muellbauer (2007) comparing the US and UK.

[3]Next to Galí and Monacelli (2008) there are more conventional two-country models of a currency union, such as the one by Beetsma and Jensen (2005) who build on Benigno (2004). There are, however, several advantages to the framework of Galí and Monacelli (2008). Two-country models are less suitable to analyse a small open economy within a currency union. Furthermore, Galí and Monacelli (2008) allows to derive a fully quadratic loss function avoiding multiplicative terms in different variables as obtained by Beetsma and Jensen (2005). The presence of more than two countries also makes the mean and dispersion of variables, which are the focus of the present analysis, more meaningful. Importantly, while Beetsma and Jensen (2005)

Bilbiie (2008) extends the benchmark New Keynesian model summarised for instance in Clarida et al. (1999), Woodford (2003) or Galí (2008).[4] Bilbiie (2008) shows that LAMP leads to a bifurcation of dynamics. If the rate of LAMP remains below a certain threshold value, the dynamics of the benchmark New Keynesian model are preserved. Indeed the effectiveness of interest rate policy increases relative to the standard case. If the rate of LAMP, however, exceeds a certain threshold then the IS relation changes sign. Behind this result lies the interrelation between labour and asset markets. The intuition is the following:[5] if Non-Ricardian households exceed a certain threshold proportion, then a fall of the interest rate has the following effect. Ricardian households plan to save less and consume more. With a high proportion of Non-Ricardian agents this increase in demand increases marginal cost so much that Ricardian agents anticipate so high a fall in potential profits that they end up cutting back consumption rather than increasing. This way the fall in the interest rate can lead to an overall fall of consumption and output. When the interest-elasticity of output is negative, this in turn has the policy implication that the Taylor principle is also reversed in the sense that determinacy requires a passive policy in the sense that in a Taylor-type instrument rule the coefficient on inflation is smaller than one.

The main results of the present paper are the following. I show how the heterogeneity of LAMP can be captured solely in terms of the mean rate of LAMP and the dispersion of LAMP. The heterogeneity of LAMP turns out to matter crucially for dynamics and determinacy. Disregarding dispersion will be quantitatively and can be qualitatively inaccurate, as for a given mean rate of LAMP, greater dispersion can make the sign of the union-wide IS relation change. Depending on the sign of this elasticity, monetary policy can then achieve union-wide equilibrium determinacy by following either an active or passive Taylor rule.

As every country technically has a measure of zero, the determinacy conditions identified for the union as a whole are insufficient to guarantee that in a particular country the equilibrium path is determinate. Strictly speaking no country has mass zero, this assumption may simply be as inapplicable to the real world. The most important dimension of

---

assume that purchasing-power-parity (PPP) holds at all times, the introduction of home-bias in Galí and Monacelli (2008) allows for deviations from PPP. This turns out to be particularly important when risk sharing among countries is not perfect.

[4]Galí et al. (2004), too, study the implications of Non-Ricardian agents in a model with capital and non-separable utility. These assumptions, however, preclude analytical results, making Bilbiie (2008) the more convenient reference point.

[5]For a more detailed account see Bilbiie (2008).

the measure zero assumption is that the interest rate set at the union-level does not systematically react to the conditions of any particular country. In the euro area, for instance, country weights entering the HICP price index are calculated based on its share of private domestic consumption expenditure in the euro area. Inflation of the HICP index is in turn the variable which the European Central Bank (ECB) targets as part of its inflation targeting strategy. While in 2006 Germany has the largest weight with 28.7%, the values for Portugal, Ireland and Luxembourg are only 2.2%, 1.3% and 0.2% respectively. Out of the 12 earliest member of the European Monetary Union, only France, German, Italy and Spain have a weight in the union of more than 5%. Based on their small weight, the assumption that the union-wide interest rate policy does not react systematically to the conditions in the smaller countries is plausible and deserves scrutiny.

For country-level determinacy I thus demonstrate that, if the rate of LAMP in a country exceeds a certain threshold, equilibrium in that country is indeterminate. This is essentially due to a failure of risk sharing, as risk with the rest of the union is only shared by the Ricardian consumers. While the union as a whole can react to a negatively sloping IS relation by adopting a passive interest-rate rule, an individual country cannot do so and runs into difficulties if its rate of LAMP exceeds a certain critical value. However, we also find that the more open a country is, i.e. the less home bias it exhibits, the higher the rate of LAMP for which equilibrium in that country is determinate. Indeed, in the absence of home bias a country can guarantee equilibrium determinacy. This emphasises the importance of risk sharing and trade-integration in a currency union. This further lends support to McKinnon (1963) who argued that openness with potential currency-area partners increases the appeal of a fixed exchange-rate between them.

Finally, I identify the optimal policy which minimises the union-wide quadratic social loss. Optimal inflation volatility increases in the mean and dispersion of LAMP while optimal output volatility decreases in those parameters. An implied optimal Taylor-rule recommends, subject to the satisfaction of the Taylor principle, that the higher the mean and the dispersion of LAMP, the less aggressive should be the response of the nominal interest-rate to inflation.

The structure of the paper is as follows. Section 2 discusses the microfoundations in detail. Section 3 derives structural IS relations and Phillips curves for the union as a whole, as well as a representative country. Section 4 discusses the requirements for equilibrium

determinacy, with 4.1 discussing the union as a whole. However, because a particular country c has measure zero within the union, it could be indeterminate without affecting the rest of the union. Thus, section 4.2 turns to the conditions for which a particular country c exhibits equilibrium determinacy. Section 5 identifies the optimal monetary policy which minimises the union-wide quadratic social loss.

2 Microfoundations

2.1 Households

The general form of notation is summarised in appendix 7.1. The model of the currency union follows Galí and Monacelli (2008). The currency union is made up of a continuum of small open economies. Each country's economy is indexed by $c \in [0, 1]$. As each country c has measure zero, its domestic policy does not have any impact on the other union members. All union members have the same preferences, technology and market structure. However, they can be subject to imperfectly correlated shocks.

Here I go beyond Galí and Monacelli (2008) by assuming that asset markets participation is limited. More than that, the degree of asset markets participation is allowed to vary across countries. I thus assume that in each country a fraction $(1 - \lambda^c)$ of the population are asset market participants who can smooth their consumption over time through asset markets. These consumers are called 'Ricardian' consumers. The variables pertaining to these households will be superscripted by R. The remaining fraction of the population λ^c consists of households not participating in asset markets. These consumers are called 'Non-Ricardian' as they are unable to smooth their consumption across time through asset markets participation. Their variables carry the superscript N.

Let $Z \in [N, R]$ denote the type of household. $C_t^{Z,c}$ is the total consumption by all households of type Z in country c. Both household types have the same utility function

$$U \equiv E_t \sum_{i=0}^{\infty} \beta^i \left[\ln C_{t+i}^{Z,c} - \zeta \frac{(N_{t+i}^{Z,c})^{1+\eta}}{1+\eta} \right]. \tag{1}$$

$\beta \in (0, 1)$ is the subjective discount factor common to all households, N_t is the supply of labour in terms of hours worked, η is the inverse of the labour supply elasticity, while ζ determines the weight on the disutility of labour supply. As in Bilbiie (2008) the log/constant-

relative-risk-aversion-utility function is chosen as it delivers constant hours worked for Non-Ricardian agents. This brings analytical convenience without being necessary for any of the results.

2.1.1 Consumption and Price Indices

Aggregation across Consumer Types In any country c the consumption of Ricardian consumers $C_t^{R,c}$ and the consumption of Non-Ricardian consumers $C_t^{N,c}$ is aggregated as follows:

$$C_t^c \equiv (1 - \lambda^c)C_t^{R,c} + \lambda^c C_t^{N,c}. \tag{2}$$

The model will be log-linearised around the non-stochastic steady state, where (2) becomes

$$\hat{c}_t^c = (1 - \lambda^c)\hat{c}_t^{R,c} + \lambda^c \hat{c}_t^{N,c}. \tag{3}$$

Similarly, labour supply aggregates as

$$N_t^c \equiv (1 - \lambda^c)N_t^{R,c} + \lambda^c N_t^{N,c}, \text{ or } \quad \hat{n}_t^c = (1 - \lambda^c)\hat{n}_t^{R,c}. \tag{4}$$

For the last equality note that $\hat{n}_t^{N,c} = 0$, as hours worked by non-Ricardian agents are constant.

Aggregation of Domestic and Imported Consumption For every household type Z let lower-scripted country-variables denote the place of production and upper-scripted country-variables the place of consumption. Total consumption of consumer Z is composed of consumption of goods produced domestically, $C_{c,t}^{Z,c}$, and goods which are produced in the rest of the union $C_{F,t}^{Z,c}$. For type Z, the composite consumption index $C_t^{Z,c}$ is

$$C_t^{Z,c} \equiv \frac{(C_{c,t}^{Z,c})^{1-\alpha}(C_{F,t}^{Z,c})^{\alpha}}{(1-\alpha)^{(1-\alpha)}\alpha^{\alpha}}, \tag{5}$$

where $\alpha \in [0,1]$ is the weight on imported goods in the utility function. For $\alpha < 1$ there is a home bias in private consumption. For $\alpha = 0$, the country is essentially a closed economy. α can thus be seen as a parameter capturing the openness of a country towards the rest of the union. α is assumed to be the same for all countries.[6]

---

[6]In section 4.3 I consider the case of α varying across countries.

There is a continuum $j \in [0,1]$ of differentiated consumption goods, where $\theta > 1$ is the elasticity of substitutions between any two goods produced within a country. $C_{c,t}^{Z,c}$ is the CES aggregator of consumption defined as $C_{c,t}^{Z,c} \equiv \left(\int_0^1 C_{c,t}^{Z,c}(j)^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}}$. $C_{F,t}^{Z,c}$, the index of country c's consumption of imported goods, is given by $C_{F,t}^{Z,c} \equiv \exp \int_0^1 c_{f,t}^{Z,c} \, df$, where $c_{f,t}^{Z,c} \equiv \ln C_{f,t}^{Z,c}$ is the log of an index of the quantity of goods consumed in country c which are imported from country f. This index is defined as $C_{f,t}^{Z,c} \equiv \left(\int_0^1 C_{f,t}^{Z,c}(j)^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}}$. $P_t^f(j)$ is the price of good j produced in country f, expressed in units of the single currency. The demand functions in a given country c for domestic goods and goods imported from f are respectively

$$ C_{c,t}^{Z,c}(j) = \left(\frac{P_t^c(j)}{P_t^c} \right)^{-\theta} C_{c,t}^{Z,c} \qquad ; \qquad C_{f,t}^{Z,c}(j) = \left(\frac{P_t^f(j)}{P_t^f} \right)^{-\theta} C_{f,t}^{Z,c}. \tag{6} $$

The price index of domestically produced goods for countries c and f are:

$$ P_t^c \equiv \left(\int_0^1 P_t^c(j)^{1-\theta} \, dj \right)^{\frac{1}{1-\theta}} \qquad ; \qquad P_t^f \equiv \left(\int_0^1 P_t^f(j)^{1-\theta} \, dj \right)^{\frac{1}{1-\theta}}. \tag{7} $$

The demand functions (6) and prices indices (7) aggregate as $\int_0^1 P_t^c(j) C_{c,t}^{Z,c}(j) \, dj = P_t^c C_{c,t}^{Z,c}$ and $\int_0^1 P_t^f(j) C_{f,t}^{Z,c}(j) \, dj = P_t^f C_{f,t}^{Z,c}$. Let the union-wide price index be $P_t^* \equiv \exp \int_0^1 p_t^f \, df$, where $p_t^f = \ln P_t^f$. For any individual country c, P_t^* also represents the price index for all imported goods. Further, the optimal allocation of expenditures on imported goods by country of origin implies $P_t^f C_{f,t}^{Z,c} = P_t^* C_{F,t}^{Z,c}$. Using this, we can write total expenditures on imported goods as $\int_0^1 P_t^f C_{f,t}^{Z,c} \, df = P_t^* C_{F,t}^{Z,c}$. Let the consumer price index (CPI) in country c be

$$ P_t^{cpi,c} \equiv (P_t^c)^{1-\alpha} (P_t^*)^\alpha. \tag{8} $$

Then the optimal allocation of expenditures between domestic and imported goods in that country is given by:

$$ P_t^c C_{c,t}^{Z,c} = (1-\alpha) P_{c,t}^c C_t^{Z,c} \qquad ; \qquad P_t^* C_{F,t}^{Z,c} = \alpha P_{c,t}^c C_t^{Z,c}. \tag{9} $$

Combining the previous results, the total expenditures by the households in country c can be written as

$$ P_t^c C_{c,t}^{Z,c} + P_t^* C_{F,t}^{Z,c} = P_t^{cpi,c} C_t^{Z,c}. \tag{10} $$

8

Terms of Trade The bilateral terms of trade between countries c and f, i.e. the price of country f's domestically produced goods in terms of those of country c are

$$S_{f,t}^c \equiv \frac{P_t^f}{P_t^c}. \tag{11}$$

The effective terms of trade of country c are thus

$$S_t^c \equiv \frac{P_t^*}{P_t^c}, \tag{12}$$

which can be written as $S_t^c = \exp \int_0^1 (p_t^f - p_t^c) \, df = \exp \int_0^1 s_{f,t}^c \, df$, where $s_{f,t}^c \equiv \ln S_{f,t}^c, p_t^f \equiv \ln P_t^f, p_t^c \equiv \ln P_t^*$. Further, in terms of logs, $s_t^c = \int_0^1 s_{f,t}^c \, df$. The CPI index defined in (8) and the domestic price level are, in logs, related as $p_t^{cpi,c} = p_t^c + \alpha s_t^c$.

Define domestic inflation as the rate of change of the price index of domestically produced goods, i.e. $\pi_t^c \equiv p_t^c - p_{t-1}^c$. Then domestic producer inflation and CPI inflation are related as follows:

$$\pi_t^{cpi,c} = \pi_t^c + \alpha \Delta s_t^c. \tag{13}$$

Hence, the difference between CPI inflation and domestic inflation is proportional with coefficient α to the percentage change in the terms of trade.

2.1.2 Ricardian Consumers

Financial markets are assumed to be complete, while the economy is cashless. The monetary authority defines a unit of account in which all assets are denoted. In terms of this unit of account, $B_t^{R,c}$ denotes the agent's end-of-period portfolio holdings of all state-contingent assets except shares. $A_t^{R,c}$ is the wealth excluding shares at the beginning of the period. $\Omega_t^{R,c}$ $V_t^{R,c}$ is the average market value of shares in intermediate good firms at time t. $D_t^{R,c}$ are real dividend payoff of these shares. The nominal wage is W_t and wage income $W_t N_t$ represents all non-financial income. Due to full arbitrage, arbitrage opportunities are absent. Thus, the current and the future asset portfolio, which is random, are related through a unique stochastic discount factor, $Q_{t,t+1}$, which is implied by

$$B_t^{R,c} = E_t[Q_{t,t+1} A_{t+1}^{R,c}]. \tag{14}$$

For shares, absence of arbitrage opportunities implies

$$V_t^{R,c} = E_t[Q_{t,t+1}(V_{t+1}^{R,c} + P_{t+1}^{cpi,c}D_{t+1}^{R,c})]. \tag{15}$$

Iterated forward, the Euler condition gives the fundamental pricing condition $V_t^{R,c} = E_t\left[\sum_{i=1}^{\infty} Q_{t,t+i}P_{t+i}^{cpi,c}D_{t+i}^c\right]$. In terms of the stochastic discount factor, the one-period short-term riskless nominal interest rate, i_t, is given by $\frac{1}{1+\imath_t} = E_t Q_{t,t+1}$.

The representative agent's flow budget constraint is given by:

$$\int_0^1 P_t^c(j)C_{c,t}^{R,c}(j)\ dj + \int_0^1 \left\{\int_0^1 P_t^f(j)C_{f,t}^{R,c}(j)\ dj\right\}\ df + B_t^{R,c} + \Omega_{t+1}^{R,c}V_t^{R,c}$$
$$\leq A_t^{R,c} + \Omega_t^{R,c}(V_t^{R,c} + P_t^{cpi,c}D_t^{R,c}) + W_t^c N_t^{R,c}.$$

Using (10) and conditioning on the optimal allocation of household expenditures, the period budget constraint can be written as $P_t^{cpi,c}C_t^{R,c} + B_t^{R,c} + \Omega_{t+1}^{R,c}V_t^{R,c} \leq A_t^{R,c} + \Omega_t^{R,c}(V_t^{R,c} + P_t^{cpi,c}D_t^{R,c}) + W_t^c N_t^{R,c}$, which log-linearised around the steady state is $\hat{c}_t^{R,c} = w_t^c - p_t^{cpi,c} + \hat{n}_t^{R,c} + \frac{1}{1-\lambda}d_t^c$.

Imposing the usual no-Ponzi condition for each state and using the arbitrage relations (14) and (15) as well as the relations on prices (10), we obtain the intertemporal budget constraint conditional on optimal expenditure allocation:

$$\sum_{i=0}^{\infty} E_t\left[Q_{t,t+i}P_{t+i}^{cpi,c}C_{t+i}^{R,c}\right] \leq A_t^{R,c} + V_t^{R,c} + E_t\left[Q_{t,t+i}(W_{t+i}^c N_{t+i}^{R,c})\right]. \tag{16}$$

The problem of the representative asset market participant is to maximise (1) subject to (16). This gives rise to the static optimality condition characterising labour supply

$$\frac{W_t^c}{P_t^{cpi,c}} = \zeta C_t^{R,c}(N_t^{R,c})^\eta, \qquad \text{or} \qquad w_t^c - p_t^{cpi,c} = \hat{c}_t^{R,c} + \eta\hat{n}_t^{R,c}. \tag{17}$$

The intertemporal consumption Euler equation, with $\hat{\imath}_t = \ln\frac{1+\imath_t}{1+\imath}$, is

$$\frac{1}{1+\imath_t} = \beta E_t\left[\frac{C_t^{R,c}}{C_{t+1}^{R,c}}\frac{P_t^{cpi,c}}{P_{t+1}^{cpi,c}}\right], \qquad \text{or} \qquad \hat{c}_t^{R,c} = E_t\hat{c}_{t+1}^{R,c} - (\hat{\imath}_t - E_t\pi_{t+1}^c). \tag{18}$$

2.1.3 Non-Ricardian Consumers

Non-Ricardian consumers consume their wage income in each period. Thus, the period budget constraint conditional on optimal expenditure allocation can be written as

$$P_t^{cpi,c} C_t^{N,c} \leq W_t^c N_t^{N,c}, \text{or} \qquad w_t^c - p_t^{cpi,c} = \hat{c}_t^{N,c}. \tag{19}$$

Non-Ricardian Consumers maximise utility (1) subject to the constraint (19). This is solved by the static optimality condition

$$\frac{W_t^c}{P_t^{cpi,c}} = \zeta C_t^{N,c} (N_t^{N,c})^\eta, \qquad \text{or} \qquad w_t^c - p_t^{cpi,c} = \hat{c}_t^{N,c}, \tag{20}$$

where again use is made of $\hat{n}_t^{N,c} = 0$.

2.1.4 International Risk Sharing

The first order condition (18) also holds for Ricardian consumers in each foreign country f:

$$\frac{1}{1 + \iota_t} = \beta E_t \left[\frac{C_t^{R,f}}{C_{t+1}^{R,f}} \frac{P_t^{cpi,f}}{P_{t+1}^{cpi,f}} \right]. \tag{21}$$

We can combine (18) and (21) to obtain $C_t^{R,c} = \vartheta^c C_t^{R,f} (S_{f,t}^c)^{1-\alpha}$, for all $c, f \in [0,1]$ and for all t. The constant ϑ_c depends on initial conditions regarding relative net asset holdings, Galí and Monacelli (2008). We thus can assume initial conditions such that $\vartheta^c = \vartheta = 1$ for all $c \in [0,1]$. Then $C_t^{R,c} = C_t^{R,f} (S_{f,t}^c)^{1-\alpha}$, which we can log-linearise and aggregate over all countries f to obtain

$$\hat{c}_t^{R,c} = \hat{c}_t^{R,*} + (1-\alpha) s_t^c, \tag{22}$$

where the union-wide consumption by Ricardian households is given by $c_t^{R,*} \equiv \int_0^1 c_t^{R,f} \, df$.

Note that (22) implies that only Ricardian consumers with access to asset markets can share risk internationally. In the presence of Non-Ricardian consumers, perfect international risk sharing therefore breaks down. This break-down in risk sharing has important effects on dynamics, as discussed below.

2.2 Firms

A representative firm of final good producers uses a constant elasticity of substitution (CES) production function

$$Y_t^c = \left(\int_0^1 Y_t^c(j)^{\frac{\theta-1}{\theta}} \, dj \right)^{\frac{\theta}{\theta-1}}, \tag{23}$$

which aggregates a continuum of intermediate goods indexed by $j \in [0,1]$. Demand for each intermediate good is given by $Y_t^c(j) = \left(\frac{P_t^c(j)}{P_t^c} \right)^{-\theta} Y_t^c$, while the price index is given in (7).

Each intermediate good is produced by a monopolist indexed by j using technology given by

$$Y_t^c(j) = A_t^c N_t^c(j) - F^c \quad \text{for} \quad N_t^c(j) > F^c, \tag{24}$$

and 0 otherwise. The fixed cost F^c is assumed to be common to all firms in country c.

The country wide production function, integrating (24) across all j is given by

$$Y_t^c \Delta_t^c = A_t^c N_t^c - F^c, \tag{25}$$

where price dispersion is defined as $\Delta_t^c \equiv \int_0^1 \left(\frac{P_t^c(j)}{P_t^c} \right)^{-\theta} dj$. Note that the existence of price-dispersion is contingent on sticky prices, which are introduced below. Under flexible prices, there is no price-dispersion. It is straightforward to show that equilibrium variations in $\ln \Delta_t$ are of second order. Thus, log-linearised to first order, (25) becomes

$$\hat{y}_t^c = (1 + \mu)\hat{n}_t^c + (1 + \mu)a_t^c. \tag{26}$$

Real profits of firm j are given by $D_t^c(j) = \frac{P_t^c(j)}{P_t^c} Y_t^c(j) - \frac{W_t^c}{P_t^c} N_t^c(j)$. Profits aggregated across all firms are $\int_0^1 D_t^c(j)dj = D_t^c = \int_0^1 \frac{P_t^c(j)}{P_t^c} Y_t^c(j) - \frac{W_t^c}{P_t^c} N_t^c(j) = Y_t^c \int_0^1 \left(\frac{P_t^c(j)}{P_t^c} \right)^{1-\theta} - \frac{W_t^c}{P_t^c} \int_0^1 N_t^c(j)dj$. Aggregate real profits are given by $D_t^c = \left(1 - \frac{MC_t^c}{P_t^c}\Delta_t^c \right) Y_t^c - \frac{MC_t^c}{P_t^c} F^c$. These aggregate profits are distributed to asset holds as dividends.

Cost Minimisation Cost minimisation by the firm, taking wages as given, implies a nominal marginal cost of $MC_t^c = \frac{W_t^c}{A_t^c}$. Following Clarida et al. (1999), we assume that a cost-push shock $e^{\psi^{-1}u_t^c}$ enters the marginal cost relation. Marginal costs then are

$$MC_t^c = \frac{W_t^c}{A_t^c} e^{\psi^{-1}u_t^c}, \qquad \text{or} \qquad \hat{mc}_t^c = w_t^c - a_t^c + \psi^{-1}u_t^c. \tag{27}$$

Inflation Dynamics with Calvo Price-Setting The derivation of a relation between inflation and marginal cost requires an assumption about price-setting. Here this follows the discrete-time variant of Calvo (1983). In this set-up the opportunity of firms to adjust prices follows an exogenous Poisson process. Independent of history, there is a constant probability of $(1 - \omega)$ that a firm can adjust its price, so that each period a fraction ω of firms leaves the prices of their product unchanged. The expected waiting time for the next price adjustment is therefore $(1 - \omega)^{-1}$. Based on this assumption, the New Keynesian relation between inflation and marginal cost takes the form

$$\pi_t^c = \beta E_t \pi_{t+1}^c + \psi(mc_t^c - p_t^c), \tag{28}$$

where $\psi \equiv \frac{(1-\omega)(1-\beta\omega)}{\omega}$ and mc_t^c is the nominal marginal cost of supply.[7]

3 Equilibrium Dynamics

In this section we derive the economy-wide relations from the micro-founded optimisation problems solved by consumers and firms. The model derived nests the closed economy of Bilbiie (2008) for a complete home bias of $\alpha = 0$. The model also reduces to the symmetric currency union of Galí and Monacelli (2008) when rate of LAMP is set to zero in all countries.

3.1 Aggregation

Goods market clearing requires (3) to hold. Labour market clearing requires (4). Further, as markets are complete and agents trading these assets are identical, state-contingent assets are in zero net supply. By Walras' Law, the remaining market must clear, too. Market clearing in equities implies that the share holdings of each asset holder are $\Omega_{t+1}^c = \Omega_t^c = \frac{1}{1-\lambda^c}$.

3.2 Steady State

To study dynamics, I take a log-linear approximation around the non-stochastic steady state. I find the steady state by considering the optimality conditions when all variables are constant and all shocks absent. The Euler equation (18) relates the steady state riskless one period net nominal interest rate to the subjective discount factor $1 + \iota_t = \beta^{-1}$.

---

[7]For details see e.g. Woodford (2003) or Galí (2008).

Within each country, the steady state mirrors the closed-economy version case considered in Bilbiie (2008). In steady state all firms are symmetric and apply a gross mark-up $\frac{\theta}{\theta-1}$ of over nominal marginal cost. Define the steady state net markup which producers charge as $\mu \equiv (\theta-1)^{-1}$ and the share of fixed cost in steady state output as $F_Y \equiv \frac{F^c}{Y^c}$. Assume that the steady state net markup which producers charge is the same across all countries. As shown in Bilbiie (2008) and detailed in appendix 7.2, the steady state share of real profits is given by $\frac{D^c}{Y^c} = \frac{\mu - F_Y^c}{1+\mu}$, while the steady state share of real earnings is $\frac{\frac{W^c}{P^c}N^c}{Y^c} = \frac{1+F_Y^c}{1+\mu}$. If we let the steady state net mark-up equal the output share of the fixed cost, i.e. $\mu = F_Y^c$, the steady state share of real profits in output is zero, which makes the share of real earnings in output be equal to one. By implication, also the consumption shares or Ricardian and Non-Ricardian agents are equal to each other and equal to one: $\frac{C^{R,c}}{Y^c} = \frac{C^{N,c}}{Y^c} = 1$. This simplifies the algebra while not being necessary for any of the results.

Due to the absence of profits in steady state, both types of consumers have the same steady state budget constraint: $C^{Z,c} = W^c/P^c N^{Z,c}$, for $z \in [N, R]$. Similarly, the intra-temporal optimality conditions are in steady state $C^{Z,c} = \frac{1}{\zeta(N^{Z,c})^\eta} \frac{W^c}{P^c}$. This allows to solve for the steady state consumption and types as $C^{Z,c} = \zeta^{-\frac{1}{1+\eta}} \frac{W^c}{P^c}$, while hours are $N^{Z,c} = \zeta^{-\frac{1}{1+\eta}}$. The above also implies that for both consumer types steady state consumption equals total consumption and output $C^{N,c} = C^{R,c} = Y^c$.

Regarding steady states across the union, the risk sharing condition (22) becomes $C^{R,c} = C^{R,f}(S_{f,t}^c)^{1-\alpha}$. For simplicity, I consider a symmetric steady state for which $S_f^c = 1$. This implies $P_t^f = P_t^c$, which we can normalise to one. The symmetric steady state $S_f^c = 1$ implies that consumption of Ricardian consumers in different countries are equal $C^{R,c} = C^{R,f}$. As above, we see that in each country steady state consumption of both types of consumers is equal to each other and to country output; we also have $Y^c = Y^f$ for all $c, f \in [0, 1]$.

3.3 Goods Market Clearing

Market clearing of good j in country c requires

$$Y_t^c(j) = C_{c,t}^c(j) + \int_0^1 C_{c,t}^f(j) \, df = \left(\frac{P_t^c(j)}{P_t^c}\right)^{-\theta} \left(C_{c,t}^c + \int_0^1 C_{c,t}^f \, df\right).$$

Using the condition for the optimal allocation of expenditures between domestic and imported goods (9), we obtain $Y_t^c(j) = \left(\frac{P_t^c(j)}{P_t^c}\right)^{-\theta} \left[(1-\alpha)\left(\frac{P_{c,t}^c}{P_t^c}\right)C_t^c + \alpha \int_0^1 \left(\frac{P_{c,t}^f}{P_t^c}\right)C_t^f \, df\right]$. With

the definitions of bilateral and effective terms of trade given in (11) and (12) we obtain an expression for $Y_t^c(j)$, which, plugged into the definition of country c's aggregate output (23), gives the following aggregate goods market condition

$$Y_t^c = (1 - \alpha)(S_t^c)^\alpha C_t^c + \alpha(S_t^c)^\alpha \int_0^1 (S_{f,t}^c)^{1-\alpha} C_t^f \, df. \tag{29}$$

As shown in appendix 7.3, a log-linear first order approximation of (29) around the symmetric steady state yields

$$\hat{y}_t^c = (2 - \alpha)\alpha s_t^c + (1 - \alpha)\hat{c}_t^c + \alpha\hat{c}_t^*, \tag{30}$$

where $c_t^* \equiv \int_0^1 c_t^c \, dc \equiv \int_0^1 c_t^f \, df$ is the consumption index for the union as a whole.

Departing from the definition of aggregate consumption, we can substitute out the consumption of non-Ricardian agents using their labour supply relation (20). Using in turn the labour supply-relation of Ricardian consumers, (17), as well as the definition of aggregate labour, (4), we find a relation linking aggregate consumption, consumption by Ricardian consumers and aggregate labour supply

$$\hat{c}_t^c = \hat{c}_{R,t}^c + \frac{\lambda^c}{1 - \lambda^c}\frac{\eta}{1 + \mu}\hat{y}_t^c - \eta\frac{\lambda^c}{1 - \lambda^c}a_t^c. \tag{31}$$

$$\hat{c}_t^* = \hat{c}_{R,t}^* + \int_0^1 \frac{\lambda^c}{1 - \lambda^c}\frac{\eta}{1 + \mu}\hat{y}_t^c \, dc - \int_0^1 \eta\frac{\lambda^c}{1 - \lambda^c}a_t^c \, dc. \tag{32}$$

Combining (31), (32) with (30) and the risk sharing condition for Ricardian consumers, (22), we find the goods market clearing condition for country c as

$$\left(1 + \alpha\frac{\eta}{1 + \mu}\frac{\lambda^c}{1 - \lambda^c}\right)\hat{y}_t^c = \alpha s_t^c + \hat{c}_t^c + \alpha\frac{\eta}{1 + \mu}\int_0^1 \frac{\lambda^c}{1 - \lambda^c}\hat{y}_t^c \, dc + \alpha\eta\frac{\lambda^c}{1 - \lambda^c}a_t^c - \alpha\eta\int_0^1 .\frac{\lambda^c}{1 - \lambda^c}a_t^c \, dc. \tag{33}$$

In the absence of LAMP, risk sharing across countries is perfect, as in Galí and Monacelli (2008). In that case the goods market clearing condition reduces to $\hat{y}_t^c = \alpha s_t^c + \hat{c}_t^c$. For a closed economy the goods market clearing condition is the familiar $\hat{y}_t^c = \hat{c}_t^c$. For an open economy with LAMP, however, risk is not shared perfectly across countries. As a result additional output and technology terms, of both country and the union, enter (33).

Note that we can integrate (33) over c to obtain the union-wide resource constraint, which takes the familiar form

$$\hat{y}_t^* = \hat{c}_t^*, \tag{34}$$

15

where we make use of the fact that $\int_0^1 s_t^c \, dc = 0$.

3.4 Phillips Curve

Assuming Calvo price-setting, the New Keynesian relation between inflation and marginal cost is given by (28). Combing this with the result from cost-minimisation (27), the effective terms of trade (12), the labour-supply relation of Ricardian consumers (17), condition (31) as well as the market clearing condition (33), we obtain the following Phillips curve for country c, expressed directly in terms of gap variables

$$\pi_t^c = \beta E_t \pi_{t+1}^c + \psi \chi^c \tilde{y}_t^c - \psi \alpha \frac{\eta}{1+\mu} \int_0^1 \frac{\lambda^c}{1-\lambda^c} \tilde{y}_t^c \, dc + u_t^c, \tag{35}$$

where $\chi^c \equiv \left(1 + \frac{\eta}{1+\mu} + \alpha \frac{\eta}{1+\mu} \frac{\lambda^c}{1-\lambda^c}\right)$.

A union-wide Phillips curve can be obtained by integrating (35) over all countries c,

$$\pi_t^* = \beta E_t \pi_{t+1}^* + \kappa^* \tilde{y}_t^* + u_t^*, \tag{36}$$

where $\kappa^* \equiv \psi \chi^*$ and $\chi^* \equiv 1 + \eta/(1+\mu)$. Note that at the union level, the Phillips curve is independent of the degree of asset markets participation.[8]

Unlike the union-wide Phillips curve, the Phillips curve for country c (35) is not invariant to the rate of LAMP. This stems from the failure of perfect international risk sharing, as noted in (22) and discussed in section 3.3. As a further implication, union-wide output enters the Phillips curve of country c. Seen from the country perspective union-wide output, however, is an exogenous shock.

3.5 IS relation

Combining the consumption Euler equation of Ricardian consumers (18) with the condition linking the domestic producer and CPI inflation rates to the terms of trade (13), the condition (31) as well as the market clearing for country c, we obtain what looks like a dynamic IS relation for country c which here is expressed directly in terms of deviations from flexible

---

[8]As discussed in Bilbiie (2008) for the closed economy, this is due to the assumption that steady state consumption shares are equal across consumer types. Bilbiie (2005) shows that the presence of Non-Ricardian consumers modifies the elasticity of marginal cost to movements in the output gap if the steady state consumption shares are not equal.

price values:

$$\delta^c \tilde{y}_t^c = \delta^c E_t \tilde{y}_{t+1}^c - (\hat{\imath}_t - E_t \pi_{t+1}^c - r_t^{n,c}) - \frac{\alpha\eta}{1+\mu} \int_0^1 \frac{\lambda^c}{1-\lambda^c} \Delta E_t \tilde{y}_{t+1}^c \, dc, \qquad (37)$$

where $\delta^c \equiv 1 - (1-\alpha)\frac{\eta}{1+\mu}\frac{\lambda^c}{1-\lambda^c}$ and $r_t^{n,c} \equiv (1+\mu)(1-\delta^c)\Delta E_t a_{t+1}^c + \alpha\eta \int_0^1 \frac{\lambda^c}{1-\lambda^c}\Delta E_t a_{t+1}^c \, dc - \frac{\alpha\eta}{1+\mu}\int_0^1 \frac{\lambda^c}{1-\lambda^c}\Delta E_t \hat{y}_{t+1}^{n,c} \, dc$ is the natural real rate of interest obtaining under flexible prices. Note that from the perspective of country c, the union-wide interest rate $\hat{\imath}_t$ is exogenous, in this sense 37 is not an IS relation in the true sense.

For $\alpha = 0$ (37), reduces to the IS relation of a closed economy.[9] The closed economy case provides some context for the currency union results I present below. In the closed economy, δ^c simplifies to $\delta = 1 - \frac{\eta}{1+\mu}\frac{\lambda}{1-\lambda}$. Now, $\delta > 0$ if and only if $\lambda < 1/[1 + \eta/(1+\mu)]$. For levels of LAMP greater than the RHS, the IS relation essentially switches sign. When the IS relation switches sign, interest rate increases have an expansionary effect on output.

In the standard economy without LAMP, an increase in the interest rate leads to a fall in demand, which leads to lower output, lower consumption and lower real wages. If there is LAMP, but its level stays below the critical level so that δ remains positive, the fall in demand will be even greater. This is due to Non-Ricardian agents consuming their real wage every period. For them a lower real wage directly means lower consumption, adding to the fall in demand by Ricardian agents.

If LAMP rises to a level so that $\delta < 0$, these dynamics change. The reason lies in the interrelation of labour and asset markets. The real wage fall reduces marginal cost and thus has a positive effect on profits. If LAMP is high and the elasticity of labour supply low, the profit increase leads to a positive income effect for Ricardian agents. The dividend income each Ricardian agent is $\frac{1}{1-\lambda}$ and thus increases non-linearly in the rate of Non-Ricardian agents. The positive income effect can overturn the usually contractionary effects of the interest rate increase. This way the interest rate increase leads to an expansion of output.

With LAMP in the currency union, the flip in the IS relation described can also occur in the union. To find the union-wide IS relation we integrate (37) over all countries

$$\int_0^1 \left(1 - \frac{\eta}{1+\mu}\frac{\lambda^c}{1-\lambda^c}\right)\tilde{y}_t^c \, dc = \int_0^1 \left(1 - \frac{\eta}{1+\mu}\frac{\lambda^c}{1-\lambda^c}\right)E_t\tilde{y}_{t+1}^c \, dc - (\hat{\imath}_t - E_t\pi_{t+1}^* - r_t^{n,*}), \tag{38}$$

where $r_t^{n,*} = \int_0^1 r_t^{n,c} \, dc = \eta \int_0^1 \frac{\lambda^c}{1-\lambda^c}\Delta E_t a_{t+1}^c \, dc - \frac{\alpha\eta}{1+\mu}\int_0^1 \frac{\lambda^c}{1-\lambda^c}\Delta E_t \hat{y}_{t+1}^{n,c} \, dc$ is the union wide

---

[9]The closed economy is considered in detail in Bilbiie (2008).

natural real rate of interest. The next section shows how to treat the integrals over the product of LAMP rates and output.

3.6 Terms of Trade

Combining the risk sharing condition, (22), with (31) and (32), we can write the terms of trade exclusively as a function of output, written here directly in terms of deviations of output from its flexible-price natural rates

$$s_t^c = p_t^* - p_t^c = \delta^c \tilde{y}_t^c - \int_0^1 \delta^c \tilde{y}_t^c \ dc, \tag{39}$$

where δ^c is the inverse of the elasticity of output with regard to the effective terms of trade

$$\delta^c \equiv 1 - (1 - \alpha)\frac{\eta}{1 + \mu}\frac{\lambda^c}{1 - \lambda^c}. \tag{40}$$

Note that δ^c increases in the openness parameter α and in the labour-supply elasticity η^{-1} so that the terms of trade elasticity of output in country c is decreasing in α and η^{-1}. Thus, the better integrated a country is into the union and the more elastic is labour supply, the less is its domestic output affected by variations in the terms of trade.

(39) is a crucial equilibrium condition in the present model, relating the effective terms of trade of country c to the level of output in country c as well as the union-wide output. More specifically, from the perspective of country c the union-aggregate terms are exogenous. So what turns out important for the dynamics within the country is the relation of domestic prices to the level of domestic output in (39).

Note that if there is no LAMP, then (39) simplifies to $s_t^c = \tilde{y}_t^c - \tilde{y}_t^*$, guaranteeing a one-to-one negative relationship between prices and the level of output in country c. In the presence of LAMP the terms of trade-elasticity of output is not necessarily equal to one. Indeed, the elasticity of output with regard to the terms of trade is positive if and only if

$$\lambda^c < \frac{1}{1 + (1 - \alpha)\frac{\eta}{1+\mu}} \equiv \overline{\lambda^c}. \tag{41}$$

Note that if (41) is not fulfilled, we have a negative relation between the terms of trade and domestic output. As the union-wide price level is exogenous from the point of view of a particular country c what is more important than the terms of trade as such is that there

18

is a negative relation between domestic output and domestic prices. This is assured as long as (41) applies.

δ^c also appears in the country-level IS relation, (37). The terms of trade elasticity is thus equal to the negative of the elasticity of output with regard to the union-wide interest rate. They thus switch sign at the same time. As discussed above, however, because the nominal interest rate is exogenous from the country viewpoint, an IS relation in the true sense exists only at the union-level. The determination of output at the country level is instead governed by the terms of trade relation (39).

Generally, we expect a rise in the price level to lead to fall of demand and output. As long as (41) is fulfilled this is also the case. For $\lambda^c < \overline{\lambda^c}$ this is not only the case; the presence of Non-Ricardian consumers increases the fall in demand. For them the rise in the price level reduces the real wage, which is their sole source of income. As a result, demand falls by even more than in a full-participation economy. However, the same way the IS relation can flip in the closed economy, as discussed in section 3.5 and Bilbiie (2008), the terms of trade elasticity changes sign for $\lambda^c > \overline{\lambda^c}$. Again the intuition lies in the interaction of the labour and asset markets. If the labour supply elasticity is low and the rate of LAMP high, the rise in the price level can lead to an expansion of output. The price level rise reduces real wages, which in turn lowers marginal costs to producers. The increase in profits generates an income effect, which leads to an overall expansion of output.

3.7 Union-Wide Mean Ratio of LAMP

The relation of the terms of trade with output (39) and the union-wide IS relation all contain $\int_0^1 \frac{\lambda^c}{1-\lambda^c} \tilde{y}_t^c \, dc$, a term multiplicative in the rate of LAMP and output. This can be expressed as

$$\int_0^1 \frac{\lambda^c}{1-\lambda^c} \tilde{y}_t^c \, dc = \int_0^1 \frac{\lambda^c}{1-\lambda^c} \, dc \int_0^1 \tilde{y}_t^c \, dc + Cov[\frac{\lambda^c}{1-\lambda^c}, \tilde{y}_t^c], \tag{42}$$

where $Cov[\frac{\lambda^c}{1-\lambda^c}, \tilde{y}_t^c]$ is the covariance of the LAMP parameters with output across the member countries. However, it is difficult to think of a systematic cross-country relationship between these parameters and the output gap. As a result, I assume that the LAMP mean ratio of LAMP is independent of output so that $Cov[\frac{\lambda^c}{1-\lambda^c}, \tilde{y}_t^c] = 0$. We can then approximate

$$\int_0^1 \frac{\lambda^c}{1-\lambda^c} \tilde{y}_t^c \, dc \simeq \int_0^1 \frac{\lambda^c}{1-\lambda^c} \, dc \int_0^1 \tilde{y}_t^c \, dc = E\left[\frac{\lambda}{1-\lambda}\right] \tilde{y}_t^*, \tag{43}$$

where

$$E\left[\frac{\lambda}{1-\lambda}\right] \equiv \int_0^1 \frac{\lambda^c}{1-\lambda^c} \, dc \tag{44}$$

is the union-wide mean ratio of Non-Ricardian to Ricardian agents.

With (44), the inverse of the interest-elasticity of output is $\delta^* = \left(1 - \frac{\eta}{1+\mu} E\left[\frac{\lambda}{1-\lambda}\right]\right)$. This is positive if and only if $E\left[\frac{\lambda}{1-\lambda}\right] < \frac{1+\mu}{\eta}$. However, (44) is not directly observable, as it is non-linear in the rate of LAMP; more precisely it is convex in the rate of LAMP.

As detailed in appendix 7.4, we can approximate[10] $E\left[\frac{\lambda}{1-\lambda}\right]$ around the expected level of Non-Ricardian agents in the economy, $E[\lambda]$, in order to relate the union-wide mean ratio to the computable statistics of the mean and the dispersion of LAMP in the union:

$$E\left[\frac{\lambda}{1-\lambda}\right] = \frac{E[\lambda]}{1-E[\lambda]} + \frac{E[(\lambda - E[\lambda])^2]}{(1-E[\lambda])^3} + O|r|^4, \tag{45}$$

where $O|r|^4$ represents residuals of order 4 and higher. This approximation is accurate to fourth order if we restrict attention to a symmetric distribution of LAMP. If we interpret denominators of both fractions on the right-hand-side as weights on the numerators, we see that the weight on the mean rate of LAMP, $(1-E[\lambda])^{-1}$, is smaller than the weight on the variance of AMP, $(1-E[\lambda])^{-3}$. Due to the cubic term the weight on the variance of AMP increases significantly in the mean rate of LAMP. Thus, the higher the mean rate of LAMP, the more its variance matters.

In place of the variance we can rewrite (45) in terms of the coefficient of variation. The coefficient of variation is defined as the ratio of the standard-deviation to the mean: $CV_{1-\lambda} \equiv \frac{\sqrt{E[(\{1-\lambda\}-\{1-E[\lambda]\})^2]}}{(1-E[\lambda])}$. The advantage of the coefficient of variation is that it provides a normalised unit-less measure of the dispersion of LAMP. Furthermore, the coefficient of variation is often used for distributions with high variance, which applies even more to the squared coefficient of variation. Using the coefficient of variation, we can thus write the union-wide mean ratio of LAMP in terms of the mean and the dispersion of LAMP as

$$E\left[\frac{\lambda}{1-\lambda}\right] = \frac{\lambda^* + CV_{1-\lambda}^2}{1-\lambda^*} + O|r|^4, \tag{46}$$

where $\lambda^* \equiv E[\lambda]$ is the mean rate of LAMP in the union.

Only the union-wide Phillips curve is unaffected by the considerations of the previous

---

[10]I am indebted to Bruno Strulovici for suggesting this approximation.

section, as it is invariant to LAMP. I repeat it here for convenience. However, we can use the approximation to the expected mean ratio (46) to simplify the union-wide IS relation (38) as

$$\tilde{y}_t^* = E_t \tilde{y}_{t+1}^* - (\delta^*)^{-1}(\hat{\imath}_t - E_t \pi_{t+1}^* - r_t^{n,*}), \tag{47}$$

where $\delta^* \equiv \left(1 - \frac{\eta}{1+\mu} \frac{\lambda^* + CV_{1-\lambda}^2}{1-\lambda^*}\right)$. Furthermore, the terms of trade equation (49) is now

$$s_t^c = p_t^* - p_t^c = \delta^c \tilde{y}_t^c - \delta^* \tilde{y}_t^*. \tag{48}$$

Differencing (48), we obtain a relation of the inflation differentials to output differentials, which proves useful in the optimal policy problem

$$\pi_t^* - \pi_t^c = \delta^c \Delta \tilde{y}_t^c - \delta^* \Delta \tilde{y}_t^*. \tag{49}$$

4 Determinacy

(36) and (47), together with either an optimal targeting rule or a specified Taylor rule for the nominal interest rate constitute a complete system of the union-aggregates, output, inflation and the interest-rate as a function of union-wide cost-push and natural-interest rate shocks.

Within the union, each individual country c is a small open economy with mass zero. Due to this mass zero assumption, the determinacy conditions of the union-aggregates and a single member country c are separate. As discussed in more detail in section 4.2, even if we have a unique equilibrium for union-wide aggregates, we have to examine under what conditions equilibrium determinacy obtains in a member country within the union.

4.1 Union-Wide Determinacy

Let us assume that the monetary authority follows a Taylor rule of the form

$$\hat{\imath}_t = \varphi_\pi E_t \pi_{t+1}^*. \tag{50}$$

Plugging (50) into (47) gives, together with (36), a dynamic system of two endogenous non-predetermined variables $m_t^* \equiv [\tilde{y}_t^*, \pi_t^*]'$ with exogenous shocks $v_t \equiv [r_t^{n,*}, u_t^*]'$:

$$E_t z_{t+1}^* = \Gamma^* z_t^* + \Psi^* v_t^*, \tag{51}$$

with the coefficient matrices are $\Gamma^* = \begin{bmatrix} 1 - \beta^{-1}(\delta^*)^{-1}\kappa^*(\varphi_\pi - 1) & \beta^{-1}(\delta^*)^{-1}(\varphi_\pi - 1) \\ -\beta^{-1}\kappa^* & \beta^{-1} \end{bmatrix}$ and

$\Psi^* = \begin{bmatrix} -(\delta^*)^{-1} & -\beta^{-1}(\delta^*)^{-1}(\varphi_\pi - 1) \\ 0 & -\beta^{-1} \end{bmatrix}$. As shown in appendix 7.6, we can then prove the following conditions for the existence of a locally unique rational expectations equilibrium.

PROPOSITION 1 [EQUILIBRIUM DETERMINACY IN THE UNION] *Under the forward-looking Taylor rule $\hat{\imath}_t = \varphi_\pi E_t \pi_{t+1}^*$, the necessary and sufficient conditions for equilibrium determinacy at the level of the union are the following.*

Case	Interest-Elasticity of Output	Taylor coefficient	Equilibrium
I^*	$(\delta^*)^{-1} > 0$	$\varphi_\pi \in [1, 1 + \delta^* \frac{2(1+\beta)}{\kappa^*}]$	determinate
II^*	$(\delta^*)^{-1} < 0$	$\varphi_\pi \in [1 + \delta^* \frac{2(1+\beta)}{\kappa^*}, 1] \cap [0, \infty]$	determinate
III^*	$0 > (\delta^*)^{-1} > -\frac{2(1+\beta)}{\kappa^*}$	$\varphi_\pi = 0$	determinate

In case I^*, the monetary authority can achieve determinacy by following an instrument rule which is called active, i.e. with coefficient $\varphi_\pi > 1$. As (50) reacts to expected future variables, the usual upper bound[11] on φ_π applies. Case I^* is another instance where the Taylor principle applies, which requires increases in the real interest in response to rising inflation.

In case II^*, where the inverse of the interest-elasticity of output is positive, a Taylor rule which is passive, i.e. with a coefficient smaller than one, can achieve determinacy. For a passive rule the upper bound discussed becomes a lower bound. A passive rule means that, while nominal interest is increased in the face of expected future inflation, real rates fall. As in case II^*, increases in the real interest-rate are contractionary, this achieves determinacy. As the point of determinacy is to rule out self-fulfilling dynamics, let us consider briefly how a passive monetary policy rule can achieve this in the present case. Imagine there is a non-

---

[11]Cf. e.g. Bullard and Mitra (2002), Bernanke and Woodford (1997).

fundamental fall in expected inflation. Under a passive interest rate rule, nominal interest rates are set lower, but by less than the reduction in expected inflation, which causes the real interest rate to increase. Given $\delta^* < 0$, the rising real interest rate leads to an increase in output. This in turn creates inflation, contradicting the initial non-fundamental decrease in inflation. Thus, the passive monetary policy rule achieves determinacy, ruling out self-fulfilling dynamics.

While cases I^* and II^* show that for all values of LAMP the monetary authority can achieve determinacy if reacting with the interest rate to union-wide endogenous conditions, case III^* shows that determinacy can even obtain if the interest-rate is pegged, in the sense of not responding to endogenous variables due to $\varphi_\pi = 0$. In the absence of LAMP case III^* cannot arise. As discussed by Bilbiie (2008), the practical relevance of case III^* is limited in the presence of a monetary authority which does react with the interest rate to endogenous conditions. However, in the monetary union member setting, we will see that an analogue of case III^* reappears at the country-level. As the union-wide interest-rate reacts to union-wide aggregates and is thus exogenous from the perspective of any individual country, and as countries are related to other union members through a currency-peg, case III^* has more relevance at the level of member-countries, as discussed in more detail in section 4.2.

Note that the conditions summarised in proposition 1 are analogous to those applying to the closed economy examined in Bilbiie (2008). There is one crucial addition here, however, as I allow for heterogeneity of LAMP across the union.

COROLLARY 1 [DISPERSION] *The dispersion of asset markets participation enters dynamics and the conditions for determinacy. The higher the dispersion of AMP across the union, the lower the mean rate of LAMP for which the interest-elasticity of output is positive.*

$(\delta^*)^{-1}$ increases in both the mean and the dispersion of LAMP. Therefor there is a trade-off between mean and dispersion in determining the sign of the interest-elasticity of output. More precisely, the interest-elasticity of union-wide output is negative if and only if

$$\lambda^* < \frac{1}{1 + \frac{\eta}{1+\mu}} - \frac{\frac{\eta}{1+\mu}}{1 + \frac{\eta}{1+\mu}} CV_{1-\lambda}^2 \equiv \overline{\lambda^*} \tag{52}$$

and positive otherwise. If the rate of LAMP is homogenous across the union, i.e. $CV_{1-\lambda} = 0$,

(52) reduces to $\lambda^* < \frac{1}{1+\frac{\eta}{1+\mu}}$, which is the condition in the closed economy, as discussed in section 3.5. This critical value for zero dispersion defines an upper bound for the critical value we obtain when there is dispersion in (52). Ignoring dispersion would not only lead to quantitatively incorrect values for the interest-elasticity of output; by (52) this could also lead to the qualitatively wrong conclusion about the sign of the union-wide interest-elasticity of output.

The fact that the dispersion of LAMP matters is essentially due to the non-linearity in λ of the expected mean ratio $E[\frac{\lambda}{1-\lambda}]$. Due to this strong convexity, higher rates of LAMP in some countries make the expected mean ratio disproportionately larger than lower rates of LAMP in others.

The coefficient and effect of dispersion increases in the inelasticity of labour supply η and decreases in the steady state net markup μ. The higher is η, the more are small variations in hours and output associated with large variations in the real wage and thus consumption by Non-Ricardian agents. Hence, consumption of Non-Ricardian agents can vary more across union-members so that the dispersion of LAMP has greater effects.

A similar reason applies to the effect of μ. From the production function $\hat{y}_t = (1+\mu)a_t + (1+\mu)\hat{n}_t$, we see that for a given change in output, hours worked vary more the larger μ. The more hours vary, the less does the real wage and consumption by Non-Ricardian agents change; so the dispersion of AMP matters less. As the mean of LAMP is also multiplicative in $\frac{\eta}{1+\mu}$, these two parameters amplify the effects of the mean in the same way.

Tables 3 and 4 illustrate the quantitative importance of taking the dispersion of asset markets participation into account. Table 3 lists the critical values $\overline{\lambda^*}$ for which case I^* occurs and the IS relation is negatively sloped, as in the standard case. For illustrative purposes the threshold values are calculated for different values of the inverse of the elasticity of labour supply η and for different values of the coefficient of variation of AMP, $CV_{1-\lambda}$. The values in Table 4 instead show the reduction in the critical value of $\overline{\lambda^*}$ which is due to $CV_{1-\lambda}$.

Tables 3 and 4 illustrate two things in particular. Firstly, small values of $CV_{1-\lambda}$ reduce the critical value of LAMP only by small amounts. If, however, $CV_{1-\lambda}$ is high, the threshold value for LAMP is reduced considerably. Secondly, even for relatively small values of AMP, the reduction in the threshold values can be significant if the elasticity of labour supply is small. For instance, considering the evidence on LAMP quoted in the introduction and the

Table 3: Critical Values for Limited Asset Markets Participation

Coeff. of Var. $CV^2_{1-\lambda}$	Inverse Labour Supply Elasticity η				
	0.5	1	2	3	5
0.03	0.70	0.53	0.43	0.26	0.17
0.1	0.68	0.50	0.39	0.21	0.11
0.3	0.62	0.41	0.28	0.07	0
0.5	0.56	0.32	0.17	0	0
1	0.41	0.09	0	0	0

Notes: The values in the table are the critical values $\overline{\lambda^*}$ $= 1/[(1+\eta/(1+\mu)] - (\eta/(1+\mu)/[1+\eta/(1+\mu)] \, CV^2_{1-\lambda}$ in (52).

Table 4: Reduction in Critical Values for LAMP due to Dispersion of AMP

Coeff. of Var. $CV^2_{1-\lambda}$	Inverse Labour Supply Elasticity η				
	0.5	1	2	3	5
0.03	0.01	0.01	0.02	0.02	0.03
0.1	0.03	0.05	0.06	0.07	0.08
0.3	0.09	0.14	0.19	0.21	0.24
0.5	0.15	0.23	0.31	0.24	0.4
1	0.29	0.45	0.63	0.71	0.81

Notes: The values are $(\eta/(1+\mu)/[1+\eta/(1+\mu)] \, CV^2_{1-\lambda}$ in (52).

values for 3calculated in Tables 1 and 2 it is easy to see that, firstly, the critical values in Table 3 are of empirically relevant size and that ignoring the associated values in 3 could be quantitatively and qualitatively inaccurate.

The role of dispersion for dynamics and determinacy identified here for heterogeneous LAMP applies more generally to structural heterogeneity in a monetary union. It is quantitatively and qualitatively important to account for non-linearities in parameters when aggregating them across countries. As shown by the approximation detailed in appendix 7.4, for any function of parameters the union-wide mean of the function of parameters will not equal the function of the union-wide mean of the parameters when the function is non-linear. For convex functions dispersion adds to the function of the mean, while for concave functions dispersion subtracts from the function of the mean.

4.2 Country Determinacy

As mentioned above, determinacy of the union-wide equilibrium does not by itself guarantee that a country c with measure zero exhibits itself equilibrium determinacy. This section follows the approach outlined in Galí and Monacelli (2005) to derive conditions of determinacy

in a specific country c.

The nominal interest rate is exogenous from the perspective of country c. $\hat{\imath}_t$ responds only systematically to union-wide aggregates and not country-specific output and inflation. That exogeneity of the interest-rate leads to indeterminate equilibria, is, however, a standard-result in the monetary policy literature, as discussed for instance in Woodford (2005), Galí (2009).

Indeed, if we compute the roots for the system of country-level Phillips curve (35) and country-level IS relation (37), we find the usual result that for interest-rate pegs equilibrium is indeterminate; except for the odd-ball case $0 > (\delta^c)^{-1} > -\frac{2(1+\beta)}{\kappa^c}$, which corresponds directly to case III^* describing a union-wide interest rate peg. However, as discussed in Galí and Monacelli (2005), considering the roots of (35) and (37) is, on the one hand, mistaken as the country-level IS relation (37) is not an equilibrium relation in a currency-union; only at the union-level is there an IS-relation in the proper sense. On the other hand, it would mean disregarding an additional equilibrium condition, namely the terms of trade relation (39) according to which the level of output is a function of the terms of trade. In other words, as there is only a union-wide IS-relation, (39) determines how domestic output is affected by the terms of trade. Indeed, as the union-wide price-level is exogenous from the perspective of a particular country, the most important aspect of (39) is the relation between the domestic level of output and the domestic price level.

In order to obtain valid conditions for the determinacy of a specific country c, we can difference the Phillips curve of country c (35) and the union (36) and combine it with the condition relating the terms of trade and country c output (39). We then obtain a second order difference equation in the terms of trade:

$$\beta\varpi^c E_t s_{t+1}^c - s_t^c + \varpi^c s_{t-1}^c = -\varpi^c \left(\frac{\kappa^c}{\delta^c} \int_0^1 \delta^c \, dc - \int_0^1 \kappa^c \, dc \right) \tilde{y}_t^* - \varpi^c(u_t^c - u_t^*), \qquad (53)$$

where

$$\varpi^c \equiv \left(1 + \beta + \frac{\kappa^c}{\delta^c} \right)^{-1}. \qquad (54)$$

The coefficient on union-wide output is a shock to the effective terms of trade in country c. However, this is the case if and only if the rate of LAMP in country c differs from that in the rest of the union. If the rate of LAMP is positive but homogenous across all countries, the coefficient on $\tilde{y}_t^*$ in (53) is zero.

Stability, however, depends on the roots of the second order difference equation of the effective terms of trade, (53). If this equation has a unique stationary solution, then it is

$$s_t^c = \varsigma^c s_{t-1}^c + \varsigma^c \sum_{i=0}^{\infty} (\beta \varsigma^c)^i (E_t u_{t+i}^c - E_t u_{t+i}^*), \tag{55}$$

where determinacy obtains if out of the two roots of (55) one lies outside and one lies inside the unit circle. As proven in appendix 7.7, we can establish the following.

PROPOSITION 2 [DETERMINACY IN COUNTRY c] *Determinacy in country c depends on the following conditions.*

Case	LAMP	Terms of Trade Elasticity of Output	Equilibrium
I^c	$0 < \lambda^c < \frac{1}{1+(1-\alpha)\frac{\eta}{1+\mu}}$	$(\delta^c)^{-1} > 0$	*determinate smooth*
II^c	$\frac{1}{1+(1-\alpha)\frac{\eta}{1+\mu}} < \lambda^c < \frac{1+\frac{\psi\frac{\eta}{1+\mu}}{2(1+\beta)+\psi}}{1+(1-\alpha)\frac{\eta}{1+\mu}}$	$0 > -\frac{2(1+\beta)}{\kappa^c} > (\delta^c)^{-1}$	*determinate oscillatory*
III^c	$\frac{1+\frac{\psi\frac{\eta}{1+\mu}}{2(1+\beta)+\psi}}{1+(1-\alpha)\frac{\eta}{1+\mu}} < \lambda^c < 1$	$0 > (\delta^c)^{-1} > -\frac{2(1+\beta)}{\kappa^c}$	*indeterminate*

COROLLARY 2 [INDETERMINACY UNDER LIMITED ASSET MARKETS PARTICIPATION] *Under heterogeneous limited asset markets participation equilibrium determinacy of union-wide aggregates does not guarantee determinacy in every country of measure 0.*

COROLLARY 3 [COMPLETE OPENNESS GUARANTEES DETERMINACY] *The less home bias a country exhibits, the higher the rate of LAMP for which the desirable case I^c occurs. If a country is completely open, determinacy is guaranteed regardless of the level of limited asset markets participation, the elasticity of labour supply and other parameters.*

To illustrate these results Figure 1 plots the rates of LAMP of an individual country for which the three different cases arise, as a function of the openness parameter $\alpha \in [0, 1]$, where for $\alpha = 0$ a country is a closed economy, while for $\alpha = 1$ a country exhibits no home bias in consumption at all. In each of the two plots, the lower line represents $\lambda^c = \frac{1}{1+(1-\alpha)\frac{\eta}{1+\mu}}$, while the upper line plots $\lambda^c = \frac{1+\frac{\psi\frac{\eta}{1+\mu}}{2(1+\beta)+\psi}}{1+(1-\alpha)\frac{\eta}{1+\mu}}$. For the net markup I assume $\mu = 0.2$ as in Bilbiie (2008), while for $\psi = 0.66$ (Rotemberg and Woodford (1997)). Next to the openness parameter, the second key parameter determining which of the cases arises is the elasticity of labour supply η. For more elastic labour supply, higher values of LAMP are consistent

Figure 1: Determinacy Regions as Function of Openness and LAMP
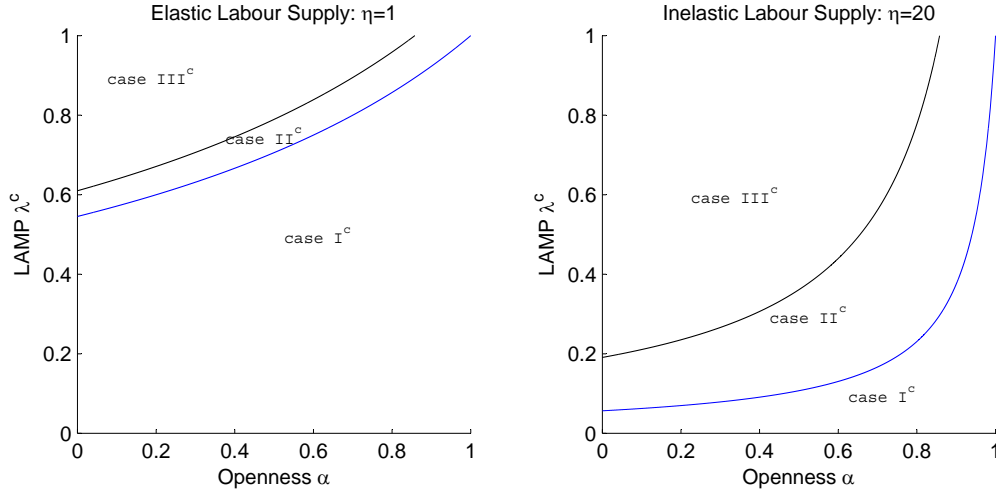
with case I^c. This can be seen from comparing the left graph for elastic labour supply with the right graph with inelastic labour supply.

In case I^c the terms of trade-elasticity is positive and the equilibrium is determinate with smooth dynamics. Indeed, as the union-wide price-level is exogenous from the perspective of a country, what matters is that there is a negative relation between domestic prices in country c and output in country c.

In case II^c the terms of trade elasticity is negative and very large in absolute value, making domestic output virtually inelastic to the domestic price-level. While equilibrium determinacy obtains, dynamics are oscillatory. Note, however, that case II^c generally applies only to a very narrow range of λ^c.

Case III^c applies to even larger values for LAMP which are are consistent with virtually the entire range of negative terms of trade elasticities. Apart from the intermediate case II^c we thus obtain equilibrium determinacy for a positive terms of trade elasticity and equilibrium indeterminacy for negative terms of trade elasticity.

The key to understanding these results is that risk in country c is shared with the rest of the union only by the Ricardian agents, as seen from the risk sharing condition (22). Obviously, the larger the share of Ricardian agents, the more a country can share risk internationally. Furthermore, we see that the correlation of Ricardian consumption in country c with that in the union is higher, the more open the country is.

A different way to see the role of openness is that home bias leads to deviations from purchasing-power-parity (PPP). This in turn is due to the fact that home-bias attributes to

28

consumption produced in the domestic country a higher weight in total consumption than its weight within the union. The greater the deviation from PPP, the lower the level of risk sharing for which equilibrium determinacy in country c can be sustained.

The country determinacy conditions can be interpreted as conditions on the determinate transmission of policy which set at the union-level into each country. Monetary policy is set in response to union-wide aggregates. Proposition 1 summarises the conditions on monetary policy to achieve a unique rational expectations equilibrium for union-wide aggregates. Proposition 2 then shows conditions under which the currency peg to the union transmits policy so that there is a unique rational expectations equilibrium in each country. Under imperfect risk sharing, the latter needs to be taken into account in addition to the union-wide conditions.

4.3 Heterogenous Openness

So far I have assumed that the parameter determining a country's openness, α, is the same for all countries in the union. However, I imply in the discussion of proposition 2 that what matters for a country is primarily its own openness towards other countries, rather than the openness of the remaining countries towards itself. In this section I allow α to vary across countries by indexing it by c, as it is done with λ^c in the rest of the paper. This way I will show that, indeed, what matters for the determinacy in country c is its own openness towards the others, i.e. α^c, rather than the openness of the other countries $\int_0^1 \alpha^c dc = \alpha^*$.

In section 4.2 we see that non-oscillatory determinacy obtains if and only if case I^c occurs. Case I^c, in turn, only occurs if and only if the relation between domestic output $\tilde{y}_t^c$ and domestic prices p_t^c is negative. Here I show that if we allow α to vary across country, i.e. $\alpha^c \neq \alpha^f$, then for any country c case I^c obtains if and only if

$$0 < \lambda^c < \frac{1}{1 - (1 - \alpha^c)\frac{\eta}{1+\mu}}. \tag{56}$$

As the home bias parameter α does not appear in the union-wide relations (47) and (36), these are unaffected by the generalisation of α. However, the country Phillips curve (35) and the terms of trade relation (49) no longer apply. As shown in appendix 7.8 for heterogenous

openness, the Phillips curve is

$$
\begin{aligned}
\pi_t^c =& \beta E_{t+1}\pi_t^c + \psi\left(\left[\frac{\eta}{1+\mu} + \frac{1+\alpha^*\frac{\eta}{1+\mu}\frac{\lambda^c}{1-\lambda^c}}{1-\alpha^c+\alpha^*}\right]\tilde{y}_t^c - \frac{\alpha^*\frac{\eta}{1+\mu}}{1-\alpha^c+\alpha^*}\int_0^1\frac{\lambda^c}{1-\lambda^c}\tilde{y}_t^c dc\right) \\
&- \psi\left(\frac{(1-\alpha^c)\alpha^c+\alpha^*-(1-\alpha^*)\alpha^*}{1-\alpha^c+\alpha^*}p_t^* + \frac{(1-\alpha^c)\alpha^c+\alpha^*-(1-\alpha^c)\alpha^*}{1-\alpha^c+\alpha^*}p_t^c\right) + u_t^c,
\end{aligned}
\tag{57}
$$

while the terms of trade equation becomes

$$
\begin{aligned}
\left(1-(1-\alpha^c)\frac{\eta}{1+\mu}\frac{\lambda^c}{1-\lambda^c}\right)\tilde{y}_t^c =& -(1-\alpha^c+\alpha^*)p_t^c + (1+\alpha^c(\alpha^*-\alpha^c))p_t^* \\
&+ \int_0^1\left(1-\alpha^c+\alpha^*-(1-\alpha^c)\frac{\eta}{1+\mu}\frac{\lambda^c}{1-\lambda^c}\right)\tilde{y}_t^c\ dc.
\end{aligned}
\tag{58}
$$

Note that (57) and (58) reduce to (35) and (49) for $\alpha^* = \alpha^c = \alpha$. What matters for determinacy in (58) is that $\tilde{y}_t^c$ and p_t^c are of opposite sign. This is the case if and only if (56) applies.

5 Optimal Policy

We find the optimal policy by minimising the quadratic loss function subject to linear constraints, thus providing a valid approximation to the non-linear problem. The quadratic loss function is found as a second order approximation to the households' utility, as shown in appendix 7.5. As policy is set by the union-wide monetary authority, we also solve the union-wide optimal policy problem.

As in the closed economy of Bilbiie (2008), country-level utility is given by a weighted sum of the utility of Ricardian and Non-Ricardian agents. The weight in this sum is given by the mass of Ricardian versus Non-Ricardian agents in the population, i.e. λ^c. Under the assumption that the steady state is efficient and equitable, where the latter means that consumption of Ricardian and Non-Ricardian consumers is equalised, we obtain a loss function as a second order approximation to country-level utility. The union-wide loss function is then obtained by integrating over the utility of all countries. As demonstrated

in appendix 7.5, the social loss function is thus given by:

$$\sum_{t=0}^{\infty} \beta^t \int_0^1 U_t^c \, dc = -\frac{1}{2} U_C C \frac{\theta}{\psi} \sum_{t=0}^{\infty} \beta^t L_t + t.i.p. + O\|a\|^3,$$

with

$$L_t \equiv \int_0^1 (\pi_t^c)^2 + \phi^c (\tilde{y}_t^c)^2 \, dc, \tag{59}$$

where $\phi^c \equiv \frac{1+\eta}{1-\lambda^c} \frac{\psi}{\theta}$. This section considers the optimal interest rate policy under discretion. Regarding the union-wide problem, the method follows Galí and Monacelli (2008). The monetary authority sets the optimal path for the nominal interest rate $\{\imath_t\}_{t=0}^{\infty}$ to minimise the union-wide social loss (59). The optimal nominal interest rate path can be obtained residually from the union-wide IS relation (47), as a function of optimal paths for union-wide inflation and output. Thus, in a first-stage of optimisation the monetary authority sets $\{\pi_t^c\}_{t=0}^{\infty}$ and $\{\tilde{y}_t^c\}_{t=0}^{\infty}$, $\forall c \in [0,1]$ to minimise (59) subject to several constraints: firstly, for each country the Phillips curve (35); secondly, the terms of trade (49), as these govern how policy is transmitted from the union-level to each individual country; and thirdly two aggregation constraints

$$\pi_t^* = \int_0^1 \pi_t^c \, dc; \qquad \tilde{y}_t^* = \int_0^1 \tilde{y}_t^c \, dc. \tag{60}$$

Formally, the policymaker's problem thus is

$$
\begin{aligned}
max_{\pi_t^c, \tilde{y}_t^c, \pi_t^*, \tilde{y}_t^*} &-\frac{1}{2} \sum_{t=0}^{\infty} \beta^t \int_0^1 (\pi_t^c)^2 + \phi^c (\tilde{y}_t^c)^2 \, dc \\
&- 2\Lambda_t^{c,pc} \left\{ \beta E_t \pi_{t+1}^c + \kappa^c \tilde{y}_t^c - \psi \frac{\alpha\eta}{1+\mu} \int_0^1 \frac{\lambda^c}{1-\lambda^c} \, dc \, \tilde{y}_t^* - \pi_t^c \right\} \\
&- 2\Lambda_t^{c,tot} \left\{ \delta^c \Delta \tilde{y}_t^c - \int_0^1 \delta^c \, dc \, \Delta \tilde{y}_t^* - (\pi_t^* - \pi_t^c) \right\} \\
&- 2\Lambda_t^{*,\pi} \left\{ \pi_t^* - \int_0^1 \pi_t^c \, dc \right\} - 2\Lambda_t^{*,y} \left\{ \tilde{y}_t^* - \int_0^1 \tilde{y}_t^c \, dc \right\},
\end{aligned}
\tag{61}
$$

where $\Lambda_t^{c,pc}, \Lambda_t^{c,tot}, \Lambda_t^{*,\pi}, \Lambda_t^{*,y}$ are Lagrange-multipliers on the Phillips curve (35), the terms of trade (49) and the two aggregation constraints (60). The introduction of the two aggregation constraints requires not only finding first order conditions to π_t^c and $\tilde{y}_t^c$ but also π_t^* and $\tilde{y}_t^*$.

The first order conditions for π_t^c, $\tilde{y}_t^c$, π_t^* and $\tilde{y}_t^*$ are

$$\pi_t^c - \Delta\Lambda_t^{c,pc} + \Lambda_t^{c,tot} + \Lambda_t^{*,\pi} = 0 \tag{62}$$

$$-\phi^c\tilde{y}_t^c + \kappa^c\Lambda_t^{c,pc} + \psi\int_0^1 \frac{\alpha\eta}{1+\mu}\frac{\lambda^c}{1-\lambda^c}\ dc\ \Lambda_t^{c,tot} - \Lambda_t^{*,y} = 0 \tag{63}$$

$$-\Lambda_t^{c,tot} + \Lambda_t^{*,\pi} = 0 \tag{64}$$

$$-\psi\int_0^1 \frac{\alpha\eta}{1+\mu}\frac{\lambda^c}{1-\lambda^c}\ dc\ \Lambda_t^{c,pc} - \int_0^1 \delta^c\ dc\ \Lambda_t^{c,tot} + \Lambda_t^{*,y} = 0. \tag{65}$$

Integrating (62)-(65) and combining these equations, we obtain the optimality condition $\kappa^*\pi_t^* = -\int_0^1 \phi^c\tilde{y}_t^c\ dc$. Again assuming independence between parameter ϕ^c and output, we can write this optimal targeting rule under discretion as

$$\pi_t^* = -\frac{\phi^*}{\kappa^*}\tilde{y}_t^*, \tag{66}$$

where

$$\phi^* = (1+\eta)\frac{\psi}{\theta}\frac{1+CV_{1-\lambda}^2}{1-\lambda^*}. \tag{67}$$

This optimal targeting rule displays the usual lean-against-the-wind property: when inflation is above the natural rate, it is optimal to contract output below target. How aggressively output should be reduced, depends positively on the inflation elasticity of output κ^* and inversely on the weight on output in the loss function ϕ^*. ϕ^* increases in both the mean rate of LAMP λ^* and the dispersion of AMP $CV_{1-\lambda}$. Higher mean and variance of LAMP thus tend to make lower output movements more desirable.

That higher levels of both mean and variance of LAMP increase optimal inflation relative to output volatility can also be seen from the closed form solutions for inflation and output. Assuming the cost-push shock follows an AR(1) process $E_t u_{t+1}^* = \rho_u u_t$, with $\rho_u \in (0,1)$, we can combine (66) and the Phillips curve, (36), to obtain

$$\pi_t^* = \frac{\phi^*}{(\kappa^*)^2 + \phi^*(1-\beta\rho_u))}u_t^*; \qquad \tilde{y}_t^* = -\frac{\kappa^*}{(\kappa^*)^2 + \phi^*(1-\beta\rho_u))}u_t^*. \tag{68}$$

We thus can establish the following property of the optimal targeting-rule in a heterogeneous union.

PROPOSITION 3 [OPTIMAL TARGETING RULE UNDER HETEROGENOUS LIMITED ASSET

MARKETS PARTICIPATION] *Under the optimal targeting rule in the presence of heterogeneity, the desired inflation volatility increases in the mean of LAMP and the dispersion of AMP, while the desired output volatility decreases in the mean of LAMP and the dispersion of AMP.*

Plugging the reduced form solutions (68) into the IS relation (47) we can solve for an optimal Taylor-rule in expected inflation as a function of the optimal targeting rule[12]

$$\hat{\imath} = \varphi_\pi^{opt} E_t \pi_{t+1}, \tag{69}$$

where the optimal Taylor-rule coefficient is given by

$$\varphi_\pi^{opt} \equiv 1 + \delta^* \frac{\kappa^*}{\phi^*} \frac{1 - \rho_u}{\rho_u}. \tag{70}$$

Let us examine φ_π^{opt} for $\delta^* > 0$. As δ^* decreases in both the mean and dispersion of LAMP and ϕ^* increases in both the mean and dispersion of LAMP, we see that the optimal Taylor-rule coefficient decreases in both the mean and dispersion of LAMP. Thus, the higher the mean of LAMP and dispersion of AMP, the softer should be the response of the nominal interest rate to expected inflation.

6 Conclusion

This paper builds a tractable model of a currency union characterised by heterogenous limited asset markets participation (LAMP) for the analysis of monetary policy. Heterogenous LAMP leads to the failure of perfect risk sharing and asymmetric transmission channels across member countries, two important features of currency unions in the real world.

If the rate of LAMP increases beyond a threshold value aggregate demand dynamics become non-standard, with nominal interest rate rises expanding output. I show how to capture the heterogeneity of asset markets participation in a single statistic of dispersion, the coefficient of variation of asset markets participation. Dispersion becomes a key parameter in union-wide dynamics. This is essentially due to the fact that LAMP has a strongly non-linear effect on the dynamics of individual countries. As a particular rate of LAMP

---

[12]While (69) delivers determinacy, implied optimal Taylor-rules are neither unique nor necessarily determinate, as discussed by Jensen (2002).

is specific to individual countries, its distribution across the union becomes key. I show that greater dispersion reduces the threshold value of LAMP for which standard aggregate dynamics, i.e. a negative interest-elasticity of output, obtain.

I demonstrate that if the interest-elasticity of output is negative, the monetary authority should follow an active Taylor rule to achieve equilibrium determinacy. If, by contrast, the interest elasticity of output is positive, the monetary authority should follow a passive Taylor rule. As the threshold mean value of LAMP in the union for which the interest-elasticity of union-wide output changes sign is lower, the higher is the dispersion of asset markets participation, ignoring dispersion can lead to both qualitatively and quantitatively wrong conclusions about aggregate demand dynamics and the required interest rate policy in a currency union.

Countries are modelled as small open economies which haver measure zero within the union. Thus, the interest rate which is set in response to union-wide aggregates does not respond explicitly to the conditions in a specific countries. In this sense, the union-wide interest rate is exogenous from the point of view of the member countries. As a result, we do not only have to consider the determinacy conditions for union-wide aggregates; we also have to check for determinacy in ever single country. The key link between the country and the union here are a relation between a country's terms of trade and output. Fundamentally, determinacy in a specific country requires the terms of trade to be stationary. With this condition I show that a member country can exhibit equilibrium determinacy if its rate of LAMP exceeds a certain threshold value. The reason is two-fold. Firstly, a high rate of LAMP can lead to non-standard aggregate dynamics in the country. Secondly, the rate of LAMP determines the degree of risk sharing of a country with the rest of the union. If dynamics are non-standard and too little risk is shared, indeterminacy can obtain.

However, the more open a country is in trade terms towards the rest of the union, the larger the range of LAMP for which a country can maintain equilibrium determinacy. If a country has no home bias at all and is completely open to the rest of the union, determinacy obtains regardless of the rate of LAMP. The results on country-level determinacy, that a lower level of LAMP and a higher degree of openness are paramount to assuring determinacy shows how important financial and trade integration are for the effective function of a currency union.

I also characterise optimal union-wide monetary policy in the presence of heterogeneity.

Again, a greater level of dispersion of asset markets participation has fundamentally the same effect as a lower mean rate of LAMP across the union. We thus find that under the optimal targeting rule in the presence of heterogeneity, the desired inflation volatility increases in the mean of LAMP and the dispersion of AMP, while the desired output volatility decreases in the mean of LAMP and the dispersion of AMP. Solving for an implied optimal Taylor rule as a function of expected inflation, we see that the nominal interest rate should respond more aggressively to expected inflation the lower the mean rate of LAMP and the lower the dispersion of AMP.

7 Appendix

7.1 Notation

For any variable X_t, let $x_t \equiv \ln X_t$, that is lower case variables denote logs, unless otherwise noted. The steady state of X_t carries no time subscript and is written X. The value of X_t under flexible prices is called its natural rate, i.e. X_t^n. In this paper we will make assumptions ensuring that the natural rate is also efficient. The log-deviation of X_t is defined and written as $\hat{x}_t = x_t - x$. Gap variables, in turn, are defined as the difference between the log-deviation of the current rate of X_t from steady state and the log-deviation of the natural rate from steady state, i.e. $\tilde{x}_t = \hat{x}_t - \hat{x}_t^n$. All country variables are superscripted with c, as in x_t^c. Union wide variables are defined as $x_t^* \equiv \int_0^1 x_t^c \, dc$.

7.2 Steady State Relations

Profit Function   Real profits of firm j are given by $D_t^c(j) = \frac{P_t^c(j)}{P_t^c} Y_t^c(j) - \frac{W_t^c}{P_t^c} N_t^c(j)$. Profits aggregated across all firms are $\int_0^1 D_t^c(j) dj = D_t^c = \int_0^1 \frac{P_t^c(j)}{P_t^c} Y_t^c(j) - \frac{W_t^c}{P_t^c} N_t^c(j) = Y_t^c \int_0^1 \left(\frac{P_t^c(j)}{P_t^c}\right)^{1-\theta} - \frac{W_t^c}{P_t^c} \int_0^1 N_t^c(j) dj$. Aggregate hours are $\int_0^1 N_t^c(j) \, dj = \frac{\int_0^1 Y_t^c(j) \, dj + F^c}{A_t^c} = \frac{Y_t^c}{A_t^c} \Delta_t^c + \frac{F}{A_t^c}$, so that aggregate real profits are given by

$$D_t^c = \left(1 - \frac{MC_t^c}{P_t^c} \Delta_t^c\right) Y_t^c - \frac{MC_t^c}{P_t^c} F^c.$$

Log-linearised this becomes $d_t^c = -(\hat{mc}_t^c - p_t^c) + \frac{\mu}{1+\mu} \hat{y}_t^c$, where d_t^c is defined as a share of output, $d_t^c \approx \ln D_t^c - \ln Y^c$ to allow for zero steady state profits.

Steady State Profit Share in Output   In steady state price-dispersion is absent: $\Delta_t^c = 1$. Then the steady state share of profits in output is $\frac{D^c}{Y^c} = \left(1 - \frac{MC^c}{P^c}\right) - \frac{MC^c}{P^c} \frac{F^c}{Y^c} = \frac{\mu - F_Y^c}{1+\mu}$.

Real Earnings Share in Output   In steady state, we have $P^c = (1+\mu)MC^c = (1+\mu)\frac{W^c}{A^c}$. Recalling the production function $Y^c = A^c N^c - F^c$ this becomes $\frac{N \frac{W^c}{P^c}}{Y^c + F^c}(1+\mu) = 1$, which in turn can be rearranged to give an expression for the steady state share of the real earnings in output $\frac{\frac{W^c}{P^c} N^c}{Y^c} = \frac{1+F_Y^c}{1+\mu}$.

Consumption Shares in Output   Using (10) and conditioning on an optimal allocation of household expenditures, the period budget constraint can be written as: $P_t^{cpi,c} C_t^{R,c} +$

36

$B_t^{R,c} + \Omega_{t+1}^{R,c} V_t^{R,c} \le A_t^{R,c} + \Omega_t^{R,c}(V_t^{R,c} + P_t^{cpi,c} D_t^{R,c}) + W_t^{R,c} N_t^{R,c}$. Assume that the steady state hours of both types of agents are the same, i.e. $N^{R,c} = N^{N,c} = N^c$. In steady state we have $PC^{R,c} = \frac{1}{1-\lambda^c} P^c D^c + W^c N^{R,c}$. Using the production function $Y^c + A^c N^c - F^c$, this becomes $C^{R,c} = \frac{1}{1-\lambda^c} D^c + \frac{Y^c + F^c}{1+\mu}$. Using the expression for the steady state share of profits, the steady state share of the consumption of Ricardian households in output is $\frac{C^{R,c}}{Y^c} = \frac{1}{1-\lambda^c} \frac{\mu - F_Y^c}{1+\mu} + \frac{1+F_Y^c}{1+\mu}$.

The budget constraint of non-Ricardian consumers (19) in steady state is $P^c C^{N,c} = W^c N^{N,c}$. Using the result on the steady state share of real earnings in output this can be written as $\frac{C^{N,c}}{Y^c} = \frac{1+F_Y^c}{1+\mu}$.

7.3 Goods Market Clearing

This section shows how to obtain (30) from (29). Approximate

$$\int_0^1 (S_{f,t}^c)^{1-\alpha} C_t^f \, df \approx \left(\frac{P^f}{P^c}\right)^{1-\alpha} \int_0^1 C^f \frac{C_t^f - C^f}{C^f} \, df + (1-\alpha)\frac{C^f (P^f)^{-\alpha}}{(P^c)^{1-\alpha}} \int_0^1 P^f \frac{P_t^f - P^f}{P^f} \, df$$
$$- (1-\alpha)C^f \left(\frac{P^f}{P^c}\right)^{1-\alpha} (P^c)^{-1} P^c \frac{P_t^c - P^c}{P^c}$$
$$= C^f \int_0^1 c_t^f \, df + (1-\alpha)C^f \int_0^1 (p_t^f - p_t^c) \, df$$
$$= C^f [c_t^* + (1-\alpha)s_t^c].$$

Using this in the log-liberalisation of (29), we obtain

$$Y\frac{Y_t^c - Y}{Y} = (1-\alpha)C^c(\alpha s_t^c + c_t^c) + \alpha C^f[\alpha s_t^c + c_t^* + (1-\alpha)s_t^c].$$

For the symmetric steady state this simplifies to (30).

7.4 Union-Wide Mean Ratio

Let

$$f(x) \equiv \frac{x}{1-x}. \tag{71}$$

Taking a Taylor-expansion of $f(x)$ around $E[x]$, gives

$$f(x) = f(E[x]) + f'(E[x])(x - E[x]) + \frac{f''(E[x])}{2}(x - E[x])^2 + \frac{f'''(E[x])}{6}(x - E[x])^3 + \ldots$$

Taking expectations of the previous expression we obtain

$$E[f(x)] = f(E[x]) + f'(E[x]) \underbrace{(E[x] - E[x])}_{=0} + \frac{f''(E[x])}{2} E[(x - E[x])^2] + \frac{f'''(E[x])}{6} E[(x - E[x])^3] + \dots$$

Note that for symmetric distributions $\frac{f'''(E[x])}{6} E[(x - E[x])^3] = 0$. This is assumed here. Given the functional form of (71), we obtain (46).

7.5 Loss Function

The loss function approximated here is the general CRRA utility function

$$U(C_t, N_t) = \frac{C_t^{1-\gamma}}{1-\gamma} - \zeta \frac{(N_{t+i}^c)^{1+\eta}}{1+\eta}. \tag{72}$$

The loss function used in the text is a general case of this with γ. The general case is considered for convenience without affecting any result.

Assume that the steady state is efficient and that steady state shares of consumption are equal across agents and countries.[13] Then in steady state we have

$$(C^R)^\gamma (N^R)^\eta = (C^N)^\gamma (N^N)^\eta = \frac{W}{P} = \frac{Y}{N} = 1,$$

where $N^{R,c} = N^{N,c} = N = Y$ and $C^{R,c} = C^{N,c} = C = Y$.

Assume that the social planner maximises the union-wide utility, where in each country the utility is a convex combination of the utilities of Ricardian and Non-Ricardian consumers. The weights in turn are determined by the mass of the type of agents in each country, i.e. λ^c:

$$U_t^c[C_t^c, N_t^c] \equiv \lambda^c U^{N,c}[C_t^{N,c}, N_t^{N,c}] + (1 - \lambda^c) U^{R,c}[C_t^{R,c}, N_t^{R,c}].$$

Let Z denote the type of consumer, i.e. $Z \in [N, R]$. Then for each type Z the second order

---

[13]As discussed inter alia in Woodford (2003), this is ensured by a assuming a fiscal authority taxing sales at a constant rate τ and redistributing the proceeds through lump-sum payments T so that in steady state prices are at marginal cost and profits are zero. Then the profit function is $D_t(j) = (1-\tau)\frac{P_t(j)}{P_t} Y_t(j) - \frac{MC_t}{P_t} N_t(j) + T_t$. For $T_t = \tau P_t(j) Y_t(j)$ the budget is balanced. Efficiency is achieved by setting $\tau = -\mu$. Then the flexible price equilibrium is efficient with $P_t^e(j) = MC_t^e$ and $D_t^e(j)$, where the variables with superscript e denote the efficient flexible price values.

approximation of utility around the efficient flexible price equilibrium is:

$$
\begin{aligned}
U_t^{Z,c} &\equiv U[C_t^{Z,c}, N_t^{Z,c}] - U[C_Z^{Z,c,n}, N_t^{Z,c,n}] \\
&= U_C C_Z \left\{ \tilde{c}_t^{Z,c} + \frac{1-\gamma}{2}(\tilde{c}_t^{Z,c})^2 + (1-\gamma)\hat{c}_t^{Z,c,n}\tilde{c}_t^{Z,c} \right\} \\
&\quad - V_N N_Z \left\{ \tilde{n}_t^{Z,c} + \frac{1+\eta}{2}(\tilde{n}_t^{Z,c})^2 + (1+\eta)\hat{n}_t^{Z,c,n}\tilde{n}_t^{Z,c} \right\} + t.i.p. + O\|a\|^3.
\end{aligned}
\tag{73}
$$

Noting that $U_C C_R = U_C C_N = U_C C = V_N N_R = V_N N_R = V_N N$ and that[14] $\hat{c}_t^{R,n} = \hat{c}_t^{N,n} = \hat{c}_t^n$, (73) can be aggregated to

$$
\begin{aligned}
U_t^c &= U_C C \left\{ \tilde{c}_t^c + (1-\gamma)\hat{c}_t^{c,n}\tilde{c}_t^c + \frac{1-\gamma}{2}\left(\lambda^c(\tilde{c}_t^{N,c})^2 + (1-\lambda^c)(\tilde{c}_t^{R,c})^2 \right) \right\} \\
&\quad - U_C C \left\{ \tilde{n}_t^c + (1+\eta)\hat{n}_t^{c,n}\tilde{n}_t^c + \frac{1+\eta}{2}\left(\lambda^c(\tilde{n}_t^{N,c})^2 + (1-\lambda^c)(\tilde{N}_t^{N,c})^2 \right) \right\} + t.i.p. + O\|a\|^3.
\end{aligned}
\tag{74}
$$

We can develop the linear terms

$$
\tilde{c}_t^c + (1-\gamma)\hat{c}_t^{c,n}\tilde{c}_t^c - \tilde{n}_t^c + (1+\eta)\hat{n}_t^{c,n}\tilde{n}_t^c.
\tag{75}
$$

From the production function we find

$$
\tilde{n}_t^c = \tilde{y}_t^c + \ln \Delta_t^c.
\tag{76}
$$

Substituting (76) into (75) for $\tilde{n}_t^c$, and substituting (33) into (75) for $\tilde{c}_t^c$, we obtain

$$
\underbrace{\tilde{y}_t^c - \tilde{y}_t^c}_{=0} + \underbrace{(1-\gamma)\hat{c}_t^{c,n}\tilde{y}_t^c - (1-\gamma)\hat{c}_t^{c,n}\tilde{y}_t^c}_{=0} + \Theta_t^c - \ln \Delta_t^c,
\tag{77}
$$

where $\Theta_t^c \equiv -\alpha s_t^c + \alpha \frac{\eta}{1+\mu}\frac{\lambda^c}{1-\lambda^c}\tilde{y}_t^c - \alpha\frac{\eta}{1+\mu}\int_0^1 \frac{\lambda^c}{1-\lambda^c}\tilde{y}_t^c \, dc$ $+ (1-\gamma)\hat{c}_t^{c,n}\left(-\alpha s_t^c + \alpha\frac{\eta}{1+\mu}\frac{\lambda^c}{1-\lambda^c}\tilde{y}_t^c - \alpha\frac{\eta}{1+\mu}\int_0^1 \frac{\lambda^c}{1-\lambda^c}\tilde{y}_t^c \, dc \right)$. Importantly, note that $\int_0^1 \Theta_t^c \, dc = 0$. As the linear terms in (75) boil down to $\Theta^c - \ln \Delta_t^c$, (74) simplifies to

$$
\begin{aligned}
U_t^c &= U_C C \left\{ \Theta^c + \frac{1-\gamma}{2}\left(\lambda^c(\tilde{c}_t^{N,c})^2 + (1-\lambda^c)(\tilde{c}_t^{R,c})^2 \right) \right\} \\
&\quad - U_C C \left\{ \frac{1+\eta}{2}\left(\lambda^c(\tilde{n}_t^{N,c})^2 + (1-\lambda^c)(\tilde{n}_t^{R,c})^2 + \ln \Delta_t^c \right) \right\} + t.i.p. + O\|a\|^3.
\end{aligned}
\tag{78}
$$

In the case of log-utility, to which at this stage we restrict attention in the main text $\gamma = 1$. Then second order terms in consumption drop out and the labour supply of Non-Ricardian

---

[14]This is due to the fact that in the flexible-price equilibrium profits are zero.

agents is zero. Thus, (78) simplifies to

$$U_t^c = -U_C C \frac{1+\eta}{2} \left(-\Theta^c + (1-\lambda^c)(\tilde{n}_t^{R,c})^2 + \ln \Delta_t^c \right) + t.i.p. + O\|a\|^3. \tag{79}$$

Approximating (25) to second order around the efficient steady state, we obtain

$$\tilde{n}_t^c = \tilde{y}_t^c + \ln \Delta_t^c. \tag{80}$$

Using (80) together with the fact that in the log-utility case $\hat{n}_t^{N,c} =$ in (79), we obtain

$$U_t^c = -U_C C \frac{1}{2} \frac{1+\eta}{1-\lambda^c} \left(-\Theta^c + (\tilde{y}_t^c)^2 + \ln \Delta_t^c \right) + t.i.p. + O\|a\|^3. \tag{81}$$

For the price dispersion term it can be shown that

$$\sum_{t=0}^{\infty} \beta^t \ln \Delta_t^c = \frac{1}{2} \frac{\theta}{\psi} \sum_{t=0}^{\infty} \beta^t (\pi_t^c)^2. \tag{82}$$

This makes use of Lemma 1 in Galí and Monacelli (2008), which demonstrates that $\ln \Delta_t \simeq \frac{\theta}{2} Var_j p_t(j)$ and Woodford (2003), ch.6, showing that $\sum_{t=0}^{\infty} \beta^t Var_j p_t(j) = \psi^{-1} \sum_{t=0}^{\infty} \beta^t \pi_t +$ $t.i.p. + O\|a\|^3$. Applying the approximations (81) and (82) to (78), we obtain the loss function for any country c as

$$\sum_{t=0}^{\infty} \beta^t U_t^c = \sum_{t=0}^{\infty} \beta^t - \frac{1}{2} U_C C \left[\frac{\theta}{\psi} (\pi_t^c)^2 + \frac{1+\eta}{1-\lambda^c} (\tilde{y}_t^c)^2 \right] + \Theta_t^c + t.i.p. + O\|a\|^3. \tag{83}$$

The union-wide loss function is then obtained by integrating (83) over all countries c:

$$\sum_{t=0}^{\infty} \beta^t \int_0^1 U_t^c \, dc = -\frac{1}{2} U_C C \frac{\theta}{\psi} \sum_{t=0}^{\infty} \beta^t L_t + t.i.p. + O\|a\|^3,$$

where

$$L_t \equiv \int_0^1 (\pi_t^c)^2 + \phi^c (\tilde{y}_t^c)^2 \, dc, \tag{84}$$

and where $\phi^c \equiv \frac{1+\eta}{1-\lambda^c} \frac{\psi}{\theta}$. Note that the term c which is present in (83) drops out of the union-wide loss function as $\int_0^1 \Theta_t^c \, dc = 0$.

7.6 Union-Wide Determinacy

This section proves Proposition 1.

Proof. By Blanchard and Kahn (1980) determinacy of (51) requires both of the roots of Γ^* to lie outside the unit circle. By Woodford (2003), appendix E, this requires as necessary and sufficient condition that either all the conditions (1), i.e. (1.a) $\det \Gamma^* > 1$, (1.b)$\det \Gamma^* + \mathrm{tr}\,\Gamma^* + 1 > 0$, (1.c) $\det \Gamma^* - \mathrm{tr}\,\Gamma^* + 1 > 0$, or all the conditions (2), i.e. (2.a) $\det \Gamma^* + \mathrm{tr}\,\Gamma^* + 1 < 0$, (2.b) $\det \Gamma^* - \mathrm{tr}\,\Gamma^* + 1 < 0$ are fulfilled. We have $\det \Gamma^* = \beta^{-1}$ and $\mathrm{tr}\,\Gamma^* = 1 + \beta^{-1} - \beta^{-1}(\delta^*)^{-1}\kappa^*(\phi_\pi - 1)$. It is straightforward to show that Condition (2) never holds. (1a) is trivially fulfilled. For $\delta^* > 0$, (1b) requires $\phi_\pi < 1 + \delta^* \frac{2(1+\beta)}{\kappa^*}$ and (1c) requires $(\phi_\pi > 0)$, which gives case I^*. For $\delta^* < 0$, (1b) requires $\phi_\pi < 1$ while (1c) requires $\phi_\pi > 1 + \delta^* \frac{2(1+\beta)}{\kappa^*}$, which gives case II^*. For $(\delta^* < 0)$ if $\phi_\pi = 0$ (1b) and (1c) are also fulfilled if and only if $1 + \delta^* \frac{2(1+\beta)}{\kappa^*} < 0$, which gives case III^*. $\qquad\square$

7.7 Country Determinacy

This section proves Proposition 2. (55) has the two roots $\varsigma_1^c = \frac{1+\sqrt{1-4(\varpi^c)^2\beta}}{2\varpi^c\beta}$ and $\varsigma_2^c = \frac{1-\sqrt{1-4(\varpi^c)^2\beta}}{2\varpi^c\beta}$, where ϖ^c is defined in (54).

LEMMA 1 *For $\kappa^c/\delta^c > 0$ and $\kappa^c/\delta^c < -2(1+\beta)$ the roots ς_1^c and ς_2^c are real. For $0 > \kappa^c/\delta^c > -2(1+\beta)$ the roots ς_1^c and ς_2^c are complex conjugates.*

Proof. $\varsigma_{1,2}^c$ are real if and only if $1 - 4(\varpi^c)^2\beta > 0$. This requires that $(1-\beta)^2 + (1+\beta+\kappa^c/\delta^c)^2 > (1+\beta)^2$. For usual values for the subjective discount rate β we have $(1-\beta)^2 \simeq 0$. Thus, the roots are real if and only if $(1 + \beta + \kappa^c/\delta^c)^2 > (1+\beta)^2$. $\qquad\square$

LEMMA 2 *For $\kappa^c/\delta^c > 0$ and $\kappa^c/\delta^c < -2(1+\beta)$ the root $|\varsigma_1^c| > 1$.*

Proof. Consider whether it can be true that $\varsigma_1^c = \frac{1+\sqrt{1-4|\varpi^c|^2\beta}}{2|\varpi^c|\beta} < 1$. This would require the following to hold

$$\sqrt{1 - 4|\varpi^c|^2\beta} < 2|\varpi^c|\beta - 1. \tag{85}$$

Consider the RHS for $\kappa^c/\delta^c > 0$. For these values the RHS becomes $\frac{2\beta}{1+\beta+\kappa^c/\delta^c} - 1 > 0 \Leftrightarrow \beta - 1 > \kappa^c/\delta^c$. As $\beta - 1 < 0$ and we have assumed $\kappa^c/\delta^c > 0$, this yields a contradiction. Thus, for $\kappa^c/\delta^c > 0$ ς_1^c is outside the unit circle. Consider the RHS for $\kappa^c/\delta^c < -2(1+\beta)$. To this end let $\kappa^c/\delta^c \equiv -2(1+\beta) - \epsilon$ for an arbitrarily small $\epsilon > 0$. Then the RHS of (85) requires $\frac{2\beta}{|1+\beta-[2(1+\beta)+\epsilon]|} - 1 > 0$. This is equivalent to requiring $2\beta > |-(1+\beta+\epsilon)| \Leftrightarrow \beta - 1 > \epsilon$. As $\beta - 1 < 0$ and we have assumed $\epsilon > 0$, we obtain a contradiction. Thus, for $\kappa^c/\delta^c < -2(1+\beta)$, ς_1^c is outside the unit circle. $\qquad\square$

LEMMA 3 *For $\kappa^c/\delta^c > 0$, the root $0 < \varsigma_2^c < 1$. For $\kappa^c/\delta^c < -2(1+\beta)$, the root $0 > \varsigma_2^c > -1$.*

Proof. Consider whether it can be true that $\varsigma_2^c = \frac{1-\sqrt{1-4|\varpi^c|2\beta}}{2|\varpi^c|\beta} < 1$. This can be rewritten as $1 - 2|\varpi^c|\beta < \sqrt{1 - 4|\varpi^c|^2\beta}$. From the previous section we know that both for $\kappa^c/\delta^c > 0$ and $\kappa^c/\delta^c < -2(1+\beta)$ the LHS is great than zero, as is the RHS. Thus, it is valid to compare the squares of both sides. Simplifying, we obtain the condition

$$|\varpi^c| < \frac{1}{1+\beta}. \tag{86}$$

(86) is satisfied if and only if either $\kappa^c/\delta^c > 0$, in which case $1 > \varsigma_2^c > 0$ or $\kappa^c/\delta^c < -2(1+\beta) \Leftrightarrow 1 + \delta^c\frac{2(1+\beta)}{\kappa^c} < 0$ in which case $0 > \varsigma_2^c > -1$. $\square$

LEMMA 4 *For $0 > \kappa^c/\delta^c > -2(1+\beta)$, the roots $|\varsigma_1^c|, |\varsigma_2^c| > 1$.*

Proof. As seen above, for $0 > \kappa^c/\delta^d > -2(1+\beta)$ the roots $\varsigma_{1,2}^c = \frac{1\pm\sqrt{1-4(\varpi^c)2\beta}}{2\varpi^c\beta}$ are complex conjugates which can be written as $\varsigma_1^c = \mathfrak{a} + \mathfrak{b}\imath$ and $\varsigma_2^c = \mathfrak{a} - \mathfrak{b}\imath$, where $\mathfrak{a} \equiv \frac{1}{2\varpi^c\beta}$, $\mathfrak{b} \equiv \frac{\sqrt{1-4(\varpi^c)2\beta}}{2\varpi^c\beta}$, and $\imath \equiv \sqrt{-1}$. Complex conjugate roots always have the same absolute value. Consider whether both complex root lie inside the unit circle. This is the case if and only if $\sqrt{\mathfrak{a}^2 + \mathfrak{b}^2} < 1$. This requires $\left(\frac{1}{2\varpi^c\beta}\right)^2 + \left(\frac{\sqrt{1-4(\varpi^c)2\beta}}{2\varpi^c\beta}\right)^2 < 1$. With ϖ^c from (54) this can be simplified to $2\frac{\kappa^c}{\delta^c}(1+\beta) + (\frac{\kappa^c}{\delta^c})^2 < \beta^2 - 1$. For usual values of β, the RHS can be safely assumed to be $\beta^2 - 1 \simeq 0$ so that we require

$$2\frac{\kappa^c}{\delta^c}(1+\beta) + (\frac{\kappa^c}{\delta^c})^2 < 0. \tag{87}$$

The LHS equals zero for $\frac{\kappa^c}{\delta^c} = 0$ and $\frac{\kappa^c}{\delta^c} = -2(1+\beta)$. The quadratic function in $\frac{\kappa^c}{\delta^c}$ is strictly convex so that (87) is fulfilled if and only if $0 > \frac{\kappa^c}{\delta^c} > -2(1+\beta)$. $\square$

7.8 Heterogenous Openness

The composite consumption index (5) now is $C_t^{Z,c} \equiv \frac{(C_{c,t}^{Z,c})^{1-\alpha^c}(C_{F,t}^{Z,c})^{\alpha^c}}{(1-\alpha^c)^{(1-\alpha^c)}(\alpha^c)^{\alpha^c}}$. The CPI (8) is now defined as $P_t^{cpi,c} \equiv (P_t^c)^{1-\alpha^c}(P_t^*)^{\alpha^c}$. The optimal allocation of expenditures (9) between domestic and imported goods is now given by $P_t^c C_{c,t}^{Z,c} = (1 - \alpha^c)P_{c,t}^c C_t^{Z,c}$ and $P_t^* C_{F,t}^{Z,c} = \alpha P_{c,t}^{Z,c} C_t^{Z,c}$ so that CPI and domestic price levels are related, in logs, as $p_t^{cpi,c} = p_t^c + \alpha^c s_t^c$. International risk sharing takes now place through $C_t^{R,c} = C_t^{R,f}\frac{(P_t^f)^{1-\alpha^f}}{(P_t^c)^{1-\alpha^c}}$. Log-linearising

and integrating over countries the international risk sharing condition is

$$\hat{c}_t^{R,c} = \hat{c}_t^{R,*} + (1 - \alpha^*)p_t^* - (1 - \alpha^c)p_t^c, \tag{88}$$

where, in analogy to the assumption $Cov[\frac{\lambda^c}{1-\lambda^c}, \tilde{y}_t^c]$ in section 3.7, I make the assumption that $Cov[\alpha^c, p_t^c] = 0$. Note that due to $\alpha^c \neq \alpha^f$ we cannot write (88) in terms of the terms of trade $s_t^c = p_t^* - p_t^c$ directly; the different α introduce a wedge into the terms of trade, so we have to write the condition in terms of the price-level.

To find the goods market clearing condition we can follow the steps in section 3.3. Making the additional assumption that $Cov[\alpha^c, C_t^c] = 0$, equation (29) becomes $Y_t^c(j) = \left(\frac{P_t^c(j)}{P_t^c}\right)^{-\theta}\left[(1 - \alpha^c)\left(\frac{P_{c,t}^c}{P_t^c}\right)C_t^c + \alpha^* \int_0^1 (S_t^c)^{\alpha^f}(s_{f,t}^c)^{1-\alpha^f}C_t^f \ df\right]$. Then, the equivalent of equation (30) is

$$\hat{y}_t^c = (1 - \alpha^c)\alpha^c s_t^c + (1 - \alpha^c)\hat{c}_t^c + \alpha^* s_t^c + \alpha^* \hat{c}_t^*. \tag{89}$$

To obtain (57) and (58) we can then use (88) and (89) and otherwise just follow exactly the steps outlined in the text to derive (35) and (49).

Bibliography

Beetsma, R. and Jensen, H. (2005). Monetary and fiscal policy interactions in a micro-founded model of a monetary union, *Journal of International Economics* **67**(2): 320–352.

Benigno, P. (2004). Optimal monetary policy in a currency area, *Journal of International Economics* **63**(2): 293–320.

Bernanke, B. and Woodford, M. (1997). Inflation forecasts and monetary policy, *Journal of Money, Credit and Banking* **29**(4): 653–684.

Bilbiie, F. (2005). Limited asset markets participation, monetary policy and (inverted) Keynesian logic, *Working Paper 09/2005*, Nuffield College, Oxford.

Bilbiie, F. (2008). Limited asset markets participation, monetary policy and (inverted) Keynesian logic, *Journal of Economic Theory* **140**(1): 162–196.

Bilbiie, F. and Straub, R. (2006). Asset-market participation, monetary policy rules, and the great inflation, *IMF Working Paper WP/06/200*, International Monetary Fund.

Blanchard, O. and Kahn, C. (1980). The solution of linear difference models under rational expectations, *Econometrica* **48**(5): 1305–1311.

Bullard, J. and Mitra, K. (2002). Learning about monetary policy rules, *Journal of Monetary Economics* **49**(6): 1105–1129.

Calvo, G. (1983). Staggered prices in a utility maximizing framework, *Journal of Monetary Economics* **12**(3): 383–398.

Campbell, J. and Mankiw, N. G. (1989). Consumption, income and interest rates: Reinterpreting the time series evidence, *in* O. Blanchard and S. Fisher (eds), *NBER Macroeconomics Annual*, MIT Press, pp. 185–216.

Caner, A. and Wolff, E. (2004). Asset poverty in the United States, 1984-99: Evidence from the panel study of income dynamics, *Review of Income and Wealth* **50**(4): 493–518.

Cecchetti, S. (1999). Legal structure, financial structure, and the monetary policy transmission mechanism, *Working Paper 7151*, National Bureau of Economic Research.

Christelis, D., Georgakos, D. and Haliassos, M. (2009). Economic integration and mature portfolios, *mimeo*, Goethe University Frankfurt.

Clarida, R., Galí, J. and Gertler, M. (1999). The science of monetary policy: A New Keynesian perspective, *Journal of Economic Literature* **37**(4): 1661–1707.

Coenen, G. and Straub, R. (2005). Does government spending crowd in private consumption?, *International Finance* **8**(3): 435–470.

European Central Bank (2009). Housing finance in the euro area, *European Central Bank Structural Issues Report* .

Galí, J. (2008). *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework*, Princeton University Press.

Galí, J. (2009). Constant interest rate projections without the curse of indeterminacy: A note, *International Journal of Economic Theory* **5**: 61–68.

Galí, J., López-Salido, D. and Vallés, J. (2004). Rule-of-thumb consumers and the design of interest rate rules, *Journal of Money, Credit, and Banking* **36**(4): 739–764.

Galí, J. and Monacelli, T. (2005). Optimal monetary and fiscal policy in a currency union, *Working Paper 11815*, National Bureau of Economic Research.

Galí, J. and Monacelli, T. (2008). Optimal monetary and fiscal policy in a currency union, *Journal of International Economics* **76**: 116–132.

Jensen, H. (2002). Monetary policy frameworks and real equilibrium determinacy, *mimeo*, University of Copenhagen.

Maclennan, D., Muellbauer, J. and Stevens, M. (1999). Asymmetries in housing and financial market institutions and EMU, *Discussion Paper 2062*, CEPR.

McKinnon, R. (1963). Optimal currency areas, *American Economic Review* **53**(4): 717–725.

Muellbauer, J. (2007). Housing, credit and consumer expenditure, *mimeo*, University of Oxford.

Mulligan, C. and Sala-i Martin, X. (2000). Extensive margins and the demand for money at low interest rates, *Journal of Political Economy* **108**(5): 961–991.

Rotemberg, J. and Woodford, M. (1997). An optimization-based econometric model for the evaluation of monetary policy, *in* B. Bernanke and J. Rotemberg (eds), *NBER Macroeconomics Annual 1997*, MIT Press, pp. 297–346.

Vissing-Jørgensen, A. (2002). Limited asset market participation and the elasticity of intertemporal substitution, *Journal of Political Economy* **110**(4): 825–853.

Woodford, M. (2003). *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton University Press, Princeton and Oxford.

Woodford, M. (2005). Central bank communication and policy effectiveness, *Working Paper 11898*, National Bureau of Economic Research.