Do Matching Frictions Explain Unemployment? Not in Bad Times.

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Abstract

This paper proposes a model of the labor market that integrates two sources of unemployment: matching frictions and job rationing. To examine how these two sources interact over the business cycle, I decompose unemployment into a cyclical component—caused by job rationing—and a frictional component—caused by matching frictions. Formally, I define the cyclical component of unemployment as the part that would prevail if recruiting costs were zero, and the frictional component as additional unemployment due to positive recruiting costs. I prove that during recessions cyclical unemployment increases, driving the rise in total unemployment, whereas frictional unemployment decreases. Intuitively, in bad times, there are too few jobs, the labor market is slack, recruiting is inexpensive, and matching frictions contribute little to unemployment. I specify a model in which job rationing stems from a small amount of wage rigidity and diminishing marginal returns to labor. In the model calibrated with U.S. data, I find that when unemployment is below 5%, it is only frictional; but when unemployment reaches 9%, frictional unemployment amounts to less than 2% of the labor force, and cyclical to more than 7%.

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1 Introduction

Large fluctuations in unemployment frequently recur across the U.S. and Europe, most recently in 2009, and remain a major concern for policymakers. To determine optimal unemployment-reducing policies, it is critical to identify the main sources of unemployment. This paper integrates search-and-matching and job-rationing theories of unemployment to propose a framework that accommodates two important sources of unemployment: matching frictions, and a possible shortage of jobs. The paper then studies how these two sources interact over the business cycle to shed new light on the mechanics of unemployment fluctuations.

Specifically, the paper develops a tractable model that distinguishes between two components of unemployment: (i) cyclical unemployment, which is caused by job rationing; and (ii) frictional unemployment, which is caused by matching frictions. Formally, I define the cyclical component of unemployment as the part that would prevail if recruiting costs were zero, and the frictional component as additional unemployment due to positive recruiting costs. By studying these components, I make four contributions to our understanding of unemployment fluctuations. First, I propose a condition under which cyclical unemployment is positive. The second and main result of the paper is that during a recession, cyclical unemployment increases, driving the rise in total unemployment, while frictional unemployment decreases. Third, I find that in a model calibrated with U.S. data: (i) frictional unemployment remains below 5.2%; (ii) in steady state, frictional unemployment amounts to 4.3% of the labor force, and cyclical to 1.5%; (iii) when total unemployment reaches 9.2%, as in the U.S. in 2009:Q2, frictional unemployment drops to 1.6% of the labor force, and cyclical reaches 7.6%; and (iv) cyclical unemployment is more than twice as volatile as frictional unemployment. Fourth, I show that even a small amount of wage rigidity, such as that estimated in microdata with earnings of newly hired workers, is sufficient to amplify realistic labor productivity shocks as much as observed in the data.

This paper builds on Mortensen and Pissarides's (1994) search-and-matching model by relaxing two of its key assumptions: completely flexible wages and constant marginal returns to labor. These assumptions are critical because either implies that unemployment would disappear in the absence of matching frictions. To relax these assumptions, I develop a dynamic stochastic general equilibrium model in which large, monopolistic firms face a labor market with matching frictions, as in Blanchard and Galí (2008). All household members are in the labor force at all times, either working or searching for a job. Firms set prices and hire new workers each period in response to exogenous job destruction and productivity shocks. Recruiting is costly because of matching frictions, especially in expansions when firms post many vacancies and the pool of unemployed workers is small. In a frictional labor market there is no compelling theory of wage determination, which prompts the choice of a general wage schedule. Instead of deriving results for a particular wage-setting mechanism, I find conditions on the wage schedule for my results to hold. Furthermore, this generality allows me to nest as special cases various influential models of the search-and-matching literature, which provide valuable points of comparison.

Central to my analysis is job rationing. I assume that the marginal profit from hiring labor gross of recruiting expenses (the gross marginal profit) decreases with employment and could be exhausted before all workers are employed. Under this assumption, jobs are rationed when productivity is low enough: even if recruiting costs were zero, workers could not all be profitably employed and some unemployment, which I call cyclical unemployment, would remain. This is because profit-maximizing firms expand employment to the point where the gross marginal profit from hiring labor has fallen to the marginal cost of recruiting; in particular, firms do not hire past the point at which gross marginal profit is nil. In recessions marginal profitability falls and job rationing is more acute. Therefore cyclical unemployment increases, raising total unemployment. Frictional unemployment decreases simultaneously. Intuitively, when there are many unemployed workers and few vacancies, each vacancy is filled rapidly and at low cost in spite of matching frictions. Since recruiting costs barely raise the marginal cost of labor, profit-maximizing monopolistic firms barely reduce production and employment compared to the levels prevailing with no recruiting cost. Consequently matching frictions contribute little to unemployment, and frictional unemployment is low in recessions.

The model in this paper encompasses three standard search-and-matching models as special cases: the canonical model with Nash bargaining, its variant with rigid wages, and its variant with large firms and intrafirm bargaining. However, there is no job rationing in these models; in other words, there is no unemployment without positive recruiting costs. The canonical model features atomistic firms in which the marginal product of labor remains above the value of unemployment for workers (for example, Mortensen and Pissarides 1994, Pissarides 2000). Once search costs are sunk, matches always generate a positive surplus, which is shared between firm and worker by Nash bargaining over wages. When recruiting costs converge to zero, the net profit from a match is positive for any level of employment. Consequently, firms enter the labor market until all the labor force is employed. The property that unemployment disappears when recruiting costs converge to zero also holds when rigid wages are introduced into the model (for example, Shimer 2004, Hall 2005a). This is because rigid wages are solely a way to divide the surplus between firms and workers; thus, they always lie between the marginal product of labor, which is independent of employment, and the value of unemployment for workers. For the same reason, this property also applies in large-firm models with rigid wages in which (i) production functions exhibit constant marginal returns to labor (for example, Blanchard and Galí 2008); or (ii) production functions exhibit diminishing marginal returns to labor but capital adjusts immediately to employment (for example, Gertler and Trigari 2009). Lastly, this property holds in large-firm search-and-matching models with diminishing marginal returns to labor (for example, Cahuc and Wasmer 2001, Elsby and Michaels 2008). This is because these models use Stole and Zwiebel's (1996a) wage-setting mechanism, so the wage is derived from Nash bargaining over surplus from the marginal worker-firm match. Therefore, without recruiting costs, the wage remains below the marginal product of labor for any level of employment.[1] To conclude, neither wage rigidity nor diminishing marginal returns to labor alone suffices to introduce job rationing into the model.

The absence of job rationing in existing search-and-matching models is critical because without it, all unemployment is frictional. The absence of cyclical unemployment has several important implications for the impact of labor market policies on unemployment: (i) policies improving matching are likely to always reduce unemployment; (ii) direct job creation by the government is likely to have no effect on unemployment; (iii) policies reducing the search effort of the unemployed are likely to always increase unemployment. This paper offers a more nuanced theory of unemployment over the business cycle: cyclical unemployment is caused by job rationing, and composes most of unemployment in recessions; frictional unemployment is caused by matching frictions, and composes all of unemployment in expansions. These results suggest that the effectiveness of labor market policies depends on the state of the labor market: (i) policies improving matching reduce unemployment in expansions but not in recessions; (ii) direct job creation by the government has no effect on unemployment in expansions but reduces unemployment in recessions; (iii) policies reducing the search effort of the unemployed, such as a generous unemployment insurance, increase unemployment in expansions but have no effect on unemployment in recessions. From a normative standpoint, these results imply that policymakers should adapt labor market policies to the state of the labor market.

Quantifying the fluctuations of cyclical and frictional unemployment over the business cycle

[1] This result also holds in the model proposed by Rotemberg (2008)—a variant of the search-and-matching model in which large, monopolistic firms Nash-bargain wages with individual workers.

is necessary for assessing the economic relevance of the theory, as well as for developing policy recommendations. To do so I consider a special case of the general model, in which the combination of diminishing marginal returns to labor and some wage rigidity yields job rationing. These assumptions are appealing for four reasons. First, both have been used (but not combined) in the search-and-matching literature. Second, both are empirically relevant. At business cycle frequency, some production inputs may be slow to adjust. Thus, short-run production functions are likely to exhibit diminishing marginal returns to labor. There are also substantial ethnographic and empirical literatures documenting wage rigidity.[2] Third, both assumptions are standard in the broader macroeconomic literature.[3] Fourth, this specification of job rationing can be calibrated with readily available data. Diminishing marginal returns to labor can be estimated using aggregate data on labor share and the response of wages to productivity shocks has been estimated with microdata on individual wages.

Calibrating the model and imposing labor productivity shocks estimated in U.S. data produces moments for labor market variables that are close to their empirical counterparts. In particular, even high estimates of wage flexibility, such as those obtained by Haefke et al. (2008) using earnings of new hires, are sufficient to amplify productivity shocks. The simulated elasticity of labor market tightness with respect to labor productivity is 15, higher than the estimated elasticity of 9.[4] I also compare actual unemployment with the unemployment series simulated from actual labor productivity. Model-generated unemployment matches actual unemployment closely. These results suggest that in spite of its simplicity, the model fits the data notably well, lending support to the quantitative analysis of unemployment and its components.

Exploiting this calibrated model, I can decompose historical U.S. unemployment into cyclical and frictional series. These series suggest that as long as total unemployment is below 5.2%, it can all be attributed to matching frictions. In steady state, total unemployment amounts to 5.8%

[2]For instance, see Doeringer and Piore (1971), Blinder et al. (1998), Campbell and Kamlani (1997) and Bewley (1999) for ethnographic evidence. See Kramarz (2001) for a survey of studies based on wage microdata, as well as Dickens et al. (2007) and Elsby (2009) for more recent evidence.

[3]First, there is a long tradition of macroeconomic models featuring short-run production functions with labor as the only variable input, and with diminishing marginal returns to labor (for example, Solow and Stiglitz 1968, Lindbeck and Snower 1994, Benigno and Woodford 2003). Second, wage rigidity features in the many general-equilibrium models that use Taylor's (1979) and Calvo's (1983) staggered wage-setting mechanisms—Christiano et al. (2005) and Blanchard and Galí (2007) argue that wage rigidity is important for improving realism of general-equilibrium models.

[4]Shimer (2005) and Costain and Reiter (2008) previously noted that Mortensen and Pissarides (1994) model may not amplify labor productivity shocks sufficiently, compared to empirical evidence. Mortensen and Nagypál (2007) survey the different approaches that have been used to increase amplification. Pissarides (2009) suggests that the amount of wage rigidity estimated in microdata using the wages of new hires may be insufficient to amplify shocks in a search-and-matching model.

of the labor force, frictional unemployment to 4.3%, and cyclical unemployment to 1.5%. But in the second quarter of 2009, when total unemployment reached 9.2%, cyclical unemployment increased to 7.6%, while frictional unemployment decreased to 1.6%. Next, I simulate moments for unemployment and its components. I find that cyclical unemployment is more than twice as volatile as frictional unemployment. Lastly, the impulse response functions of unemployment and its components highlight a mechanism through which unemployment lags productivity in downturns: firms intertemporally substitute recruiting from the future to the present immediately after a negative productivity shock. By doing so, they take advantage of a slack labor market to recruit at low cost now, instead of recruiting in a tighter labor market in the future.

This paper contributes to the unemployment literature by integrating two major strands of research: the search-and-matching literature, which has become the standard theoretical framework for analyzing labor market fluctuations, and the job-rationing literature. The search-and-matching model has been used widely in macroeconomics and related disciplines; it has been embedded into real business cycle models (for example, Merz 1995, Andolfatto 1996), trade models (for example, Helpman et al. 2008) and dynamic stochastic general equilibrium models with wage and price rigidities (for example, Gertler et al. 2008). The job-rationing literature includes work on efficiency-wage models (Stiglitz 1976, Solow 1979), gift-exchange models (Akerlof 1982), insider-outsider models (Lindbeck and Snower 1988), and social-norm models (Solow 1980, Akerlof 1980).[5] This paper shows that unemployment is best described as a combination of frictional and cyclical unemployment. It also implies that search-and-matching theory describes the labor market well in normal and good times; and job-rationing theory describes the labor market well in bad times; but only the integration of both theories adequately explains unemployment over the entire business cycle. A second contribution to the literature is to formalize, study theoretically, and quantify a decomposition of unemployment into cyclical and frictional components. Although the concepts of frictional and cyclical unemployment have long existed, to the best of my knowledge, this analysis has not previously been conducted.

The analysis begins with an elementary model of the labor market in Section 2, to provide intuition for the results of the paper. Section 3 presents the general model on which my analysis rests. Section 4 solves for the equilibrium, and Section 5 analytically studies unemployment and

[5]My definition of job rationing rules out labor-turnover models (Stiglitz 1974), or shirking models (Shapiro and Stiglitz 1984). This choice is motivated by the observation that when a simple shirking model is used as a wage-setting mechanism in the canonical search-and-matching model, wages are extremely procyclical. In a calibrated model, wages fall by 40% when the unemployment rate climbs from 5% to 10%. Therefore, plausible shocks cannot generate fluctuations in unemployment and vacancy of the magnitude observed in the data.

its components. Section 6 presents special cases of the general model, including several influential models from the search-and-matching literature, and a specific model of job rationing. Section 7 calibrates the specific model of job rationing and Section 8 analyzes cyclical and frictional unemployment quantitatively. Section 9 concludes and discusses optimal unemployment-reducing policies in light of the results of the paper. All proofs are in Appendix A.

2 Intuition from an Elementary Model

This section develops the simplest model of unemployment embodying two elements of the labor market that I consider to be essential: frictions hindering matching of jobseekers with firms, and a possible shortage of jobs given the number of workers in the labor force.[6] In this model, both matching frictions and job rationing prevent full employment. This elementary model provides the key insight that when there are fewer jobs than workers and the number of jobs decreases further, total and cyclical unemployment increase, but frictional unemployment decreases.

2.1 Set-up

This is a discrete-time model. K jobs are matched with L workers. At the beginning of each period, there are N worker-job matches, V vacant jobs, and U unemployed workers. Jobs can either be vacant or filled with exactly one worker: $V + N = K$. Workers can either be unemployed or hired in exactly one job: $U + N = L$.

The stocks of vacancies and unemployed workers evolve as the result of a continuous process of match creation and destruction. Each period, unemployed workers apply to vacant jobs. There are frictions in the matching process; therefore, not all applicants can find a job at once. At the end of each period, a fraction s of the N existing worker-job matches are destroyed. When a match is destroyed, the worker becomes unemployed and the job becomes vacant.

2.2 Matching function

Each jobseeker applies randomly to one vacancy. A fraction $\omega \leq 1$ of workers are suitable for each job. If at least one suitable application is received for a vacant job, the job is filled; otherwise, it remains vacant. If several suitable workers apply to the same job, one of them gets the job; the

[6] This model resembles that of Petrongolo and Pissarides (2001) and Blanchard and Diamond (1989).

others remain unemployed. Since applications are random, $1/V$ is the probability that a worker applies to a given job. $(1 - 1/V)^{\omega U}$ is the probability that a given vacancy does not receive any of the ωU suitable applications. The expected number of matches during a period is therefore described by a matching function $h(U, V)$ given by

$$h(U, V) = V \cdot \left[1 - \left(1 - \frac{1}{V}\right)^{\omega U}\right].$$

2.3 Stationary equilibrium

The economy settles at a stationary equilibrium determined by the job destruction rate, the effectiveness of the matching process, and the number of jobs (K) and workers (L). The two endogenous variables are the unemployment rate $u \equiv U/L$ and the vacancy rate $v \equiv V/L$. The equilibrium is determined by two equations, parameterized by the job-worker ratio in the economy $\Theta \equiv K/L$. First, an accounting identity imposes the condition that the number of employed workers equals the number of filled jobs:

$$u = (1 - \Theta) + v. \tag{1}$$

Second, stationarity of the stock of matches implies that each period, the number of matches destroyed equals the number of new matches created:

$$s(1 - u) = h(u, v). \tag{2}$$

I assume that the number of vacancies is large, so the matching function is given by:[7]

$$h(u, v) = v \cdot \left(1 - e^{-\frac{\omega \cdot u}{v}}\right). \tag{3}$$

The system (1)-(2), combined with the approximation of the matching function (3), yields comparative statics for unemployment and vacancies in equilibrium.

LEMMA 1. *In any stationary equilibrium parameterized by a job-worker ratio $\Theta \in \mathbb{R}^{++}$:*

(i) $\nabla_\Theta u < 0$;

(ii) $\nabla_\Theta v > 0$.

[7] I use the result that for a large V, $(1 - 1/V)^{\omega U}$ is well approximated by $\exp(-\frac{\omega \cdot U}{V})$.

$\nabla_\Theta u$ denotes the partial derivative of u with respect to Θ. Lemma 1 shows that unemployment naturally increases when the number of jobs in the economy decreases relative to the size of the labor force, whereas the vacancy rate decreases at the same time. Intuitively, when the job-worker ratio Θ decreases, employment decreases. Therefore, a smaller number of new matches $h(u, v)$ are sufficient to balance job destruction to maintain the number of productive matches. Given that there are more unemployed workers looking for jobs, fewer vacancies are required to obtain the same number of matches each period. Combining both effects, it is clear that there are fewer vacancies at a stationary equilibrium in which the job-worker ratio is lower. The lemma implies that for a fixed labor force, the unemployment rate and the vacancy rate move in opposite directions as the number of jobs in the economy fluctuates: the points (u, v) describe a downward-sloping Beveridge curve.

2.4 Cyclical and frictional unemployment

I focus on the case in which there are more workers than jobs ($\Theta < 1$). Some workers remain unemployed because there are fewer jobs than workers ($K < L$), and because of matching frictions. Without matching frictions, all jobs would be filled at all times: there would always be K matches and $L - K$ unemployed workers; hence, the unemployment rate would be

$$u^C = 1 - \Theta. \tag{4}$$

I define u^C as cyclical unemployment. When matching frictions are at play, some jobs remain vacant. The number V of vacant jobs corresponds to the number of workers who are unemployed because of frictions; hence, the additional unemployment caused by frictions is

$$u^F = v. \tag{5}$$

I define u^F as frictional unemployment.[8] Equation (1) implies that $u = u^C + u^F$. I can now prove the key result of this section, which prefigures the main results of the paper.

PROPOSITION 1. *In any stationary equilibrium parameterized by a job-worker ratio $\Theta \in (0, 1)$:*

(i) $\nabla_\Theta u^C < 0$;

[8] Absent matching frictions, the number of matches is determined by the side of the labor market in shorter supply: $N = \min\{K, L\}$. If there are more jobs than workers ($\Theta \geq 1$), then there is no unemployment because $N = L \leq K$. Hence, when there are more jobs than workers, all unemployment is frictional.

(ii) $\nabla_{\Theta} u^F > 0$.

Proposition 1 shows that when the number of jobs decreases relative to the size of the labor force, cyclical unemployment increases, but frictional unemployment decreases. This result is intuitive. First, with a constant labor force, cyclical unemployment mechanically increases when there are fewer jobs, because it is defined as the difference between the number of jobs and the number of workers. Second, matching frictions require some jobs to remain vacant in order to attract applications and generate new matches—these new matches balance job destructions in a stationary equilibrium. Therefore, frictions increase unemployment by reducing the number of productive jobs. When there are fewer jobs, there are more jobseekers; thus, each vacancy is more likely to receive at least one suitable application and generate a match; hence, fewer vacancies are needed in equilibrium and frictional unemployment decreases.

Figure 1 illustrates these findings. I choose $s = 0.095$, which is the weekly separation rate in the U.S. over the 2001–2009 period. I then pick $\omega = 0.20$ to obtain an unemployment rate $u = 5.6\%$ for a vacancy-unemployment ratio $v/u = 0.45$, in line with U.S. data over the 2001–2009 period.[9] The top graph shows that the probability that a vacancy generates a match is decreasing with the job-worker ratio $\Theta = K/L$. The bottom graph shows that frictional unemployment is increasing with the job-worker ratio, when the ratio is less than 1. The rest of the paper finds that the results derived in this elementary model hold in a dynamic stochastic general equilibrium framework.

3 A General Model

This section presents a model that builds on the standard Neo-Keynesian model by adding matching frictions as in Blanchard and Galí (2008). The model is kept very simple to preserve tractability and portability. It is kept standard and relatively general to convince the reader that the results do not depend on specific assumptions, or on specific functional forms. As illustrated in Section 6, it nests influential search-and-matching models as special cases.

In this economy, all household members are in the labor force at all times, either working or searching for a job. Each period, the household spends all its income across differentiated goods; this determines the demand faced by firms. Large, monopolistic firms set prices and hire workers

[9] I focus on this time period because it is the longest period for which good data on vacancies and worker-firm separations are available in the U.S. These data are collected by the Bureau of Labor Statistics (BLS) with the Job Openings and Labor Turnover Survey (JOLTS).

from a frictional labor market, in response to exogenous job destruction and productivity shocks. Firms' hiring decisions depend on current and expected recruiting costs, and on expected profits from a match.

3.1 Source of fluctuations

This is a discrete-time model. Fluctuations are driven by labor productivity, which follows a stochastic process $\{a_t\}_{t=0}^{+\infty}$. Firms and household make decisions whose time t components are functions of the history of realizations of productivity $a^t = (a_0, a_1, \ldots, a_t)$, and of the initial employment level in the economy N_{-1}.[10]

3.2 Households

The representative household is composed of a mass 1 of members. The household ranks consumption streams according to

$$\mathbb{E}_0 \left[\sum_{t=0}^{+\infty} \delta^t \cdot C_t \right], \tag{6}$$

where $\delta \in (0, 1)$ is the discount rate, and $\mathbb{E}_0$ denotes the mathematical expectation conditioned on time 0 information. C_t is the Dixit-Stiglitz composite consumption index defined by:

$$C_t = \left(\int_0^1 C_t(i)^{(\epsilon-1)/\epsilon} di \right)^{\epsilon/(\epsilon-1)},$$

where $\epsilon \in (1, +\infty)$, and $C_t(i)$ is the quantity of good $i \in [0, 1]$ consumed in period t. The price of good i is $P_t(i)$ and the aggregate price index is

$$P_t = \left(\int_0^1 P_t(i)^{\epsilon-1} di \right)^{1/(\epsilon-1)}.$$

All household members participate in the labor market, and supply labor inelastically. The household has employed workers in all firms, and unemployed workers searching for a job. As in Merz (1995), the representative household construct gives rise to perfect risk sharing. Household

[10] All firms are assumed to initially have the same size, so that N_{-1} determines initial employment in each firm.

members pool their income before choosing consumption. They face a budget constraint:

$$\int_0^1 P_t(i) \cdot C_t(i) di = P_t \cdot W_t \cdot N_t + P_t \cdot \pi_t. \tag{7}$$

W_t denotes the average real wage paid by firms, π_t denotes aggregate real profit made by firms, $P_t \cdot W_t \cdot N_t$ is total wage income, and $P_t \cdot \pi_t$ is aggregate nominal profit. I assume that the household owns all firms, and that firms redistribute all their profits to the household. The household is risk-neutral and consumes all income each period.[11]

DEFINITION 1 (Household problem)**.** The household chooses a stochastic processes $\{C_t(i), C_t\}_{t=0}^{+\infty}$ to maximize (6) subject to the sequence of budget constraints (7), taking as given prices, wage, profits, and employment $\{P_t, P_t(i), W_t, \pi_t, N_t\}_{t=0}^{+\infty}$. The time t element of household's choice must be measurable with respect to $\left(a^t, N_{-1}\right)$.

Given aggregate consumption C_t, the household's optimal demand for good i is:

$$C_t(i) = C_t \cdot \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon}. \tag{8}$$

Then, the budget constraint can be rewritten to determines total consumption C_t:

$$C_t = W_t \cdot N_t + \pi_t.$$

3.3 Labor Market

Workers can be hired by a continuum of firms indexed by $i \in [0, 1]$. At the end of period $t-1$, a fraction s of the N_{t-1} existing worker-job matches are exogenously destroyed. Workers who lose their job can apply for a new job immediately. At the beginning of period t, a pool U_{t-1} of

[11] In their general equilibrium model, Blanchard and Galí (2008) introduce risk-averse agents that can save to smooth consumption because their focus in on the design of optimal monetary policy. In my case, savings are not relevant, and I simplify the exposition by abstracting from them. Introducing risk-aversion and allowing for savings would not change the theoretical predictions of the model. I explored the quantitative implications of this extension with a calibrated model in which (i) the household has log utility and (ii) can purchase state-contingent securities to smooth consumption. The dynamics of the model are scarcely modified. For instance, in response to a negative productivity shocks, the impulse response functions (IRFs) of the (log-linearized) models with risk-neutrality and with risk-aversion are nearly identical. On impact, labor market tightness, recruiting, and output fall lower, but consumption remains higher with risk-aversion. The intuition is that firms recruit less to increase profits today, even if they incur lower profits in the future. The reason for this intertemporal substitution is that the future is more heavily discounted when risk-averse agents expect an increasing stream of consumption. However, the largest relative difference between IRFs in the risk-neutrality and risk-aversion case remain very low—below 1% for labor market tightness, or below 0.5% for consumption and output.

unemployed workers are looking for a job:

$$U_{t-1} = 1 - (1 - s) \cdot N_{t-1}. \tag{9}$$

Search frictions in the labor market require firms to spend resources to recruit new workers. V_t is the number of vacancies opened by firms at the beginning of period t, and $\theta_t \equiv V_t / U_{t-1}$ is the labor market tightness. The number of matches made in period t is given by a constant-returns matching function $h(U_{t-1}, V_t)$, which is differentiable and increasing in both arguments. An unemployed worker finds a job with probability

$$f(\theta_t) = \frac{1}{U_{t-1}} \cdot h(U_{t-1}, V_t) = h(1, \theta_t),$$

and a vacancy is filled with probability

$$q(\theta_t) = \frac{1}{V_t} \cdot h(U_{t-1}, V_t) = h\left(\frac{1}{\theta_t}, 1\right) = \frac{f(\theta_t)}{\theta_t}.$$

Labor market tightness θ_t summarizes labor market conditions. In a tight market, it is easy for jobseekers to find new jobs—the job-finding probability $f(\theta_t)$ is high—and difficult for firms to hire workers—the job-filling probability $q(\theta_t)$ is low .

To simplify the firm's problem, I assume no randomness at the firm level, so that a firm posting n vacancies gets $q(\theta_t) \cdot n$ workers. $c \in (0, +\infty)$ is the per-period cost of a vacancy (e.g., advertising cost), expressed in units of composite consumption. Therefore, a firm spends

$$R(\theta_t, c) = \frac{c}{q(\theta_t)} \tag{10}$$

to recruit a worker immediately. When the labor market becomes tighter, firms must post more vacancies to attract new hires (e.g., advertise the same job in many more newspapers), and recruiting becomes more costly.

In this setting, firm i decides the number $H_t(i) \geq 0$ of workers to hire at the beginning of period t. The aggregate number of recruits is $H_t = \int_0^1 H_t(i) di$. The aggregate number of new hires H_t, labor market tightness θ_t, and unemployment U_{t-1} are related by the job-finding probability:

$$f(\theta_t) = \frac{H_t}{U_{t-1}}. \tag{11}$$

Upon hiring, $N_t(i) = (1-s)N_{t-1}(i) + H_t(i)$ workers are employed in firm i. The aggregate number of employed workers is $N_t = \int_0^1 N_t(i)di$. Production occurs once firms have hired workers.

3.4 Wage schedule

Wages are set once employment has been determined. Hence, hiring costs are sunk at the time of wage setting. As argued by Hall (2005a), there is no compelling theory of wage determination in this context, and many wage schedules may be consistent with equilibrium. I assume that the wage schedule is additively separable in three components, each influenced by a different source of wage fluctuation:

$$W_t(i) = \mathbb{E}_t\left[W(N_t(i), \theta_t, \theta_{t+1}, a_t)\right] \equiv S(N_t(i), a_t) + X(\theta_t, c) + \mathbb{E}_t\left[Z(\theta_{t+1}, c)\right], \tag{12}$$

where $W_t(i)$ is the wage paid by firm i to all its workers at time t, and $\mathbb{E}_t$ denotes the mathematical expectation conditioned on time t information.

This wage schedule has a natural interpretation. Since labor productivity (a_t) and employment ($N_t(i)$) determine current marginal productivity in the firm, they are likely to affect wages paid to workers. Labor market tightness in the current period (θ_t) and in the next (θ_{t+1}) determine outside opportunities of firms and workers, and are likely to affect wages as well.[12] In fact, the term $S(N_t(i), a_t)$ captures the influence of productivity and employment on wages; the term $X(\theta_t, c)$ captures the influence of current labor market conditions; and the term $\mathbb{E}_t\left[Z(\theta_{t+1}, c)\right]$ captures the influence of the labor market conditions expected next period.

This wage schedule is not completely general, but as shown in Section 6, it nests as special cases the schedules from a large class of wage-setting mechanisms: the generalized Nash bargaining (for example, Mortensen and Pissarides 1994); Stole and Zwiebel's (1996a) intrafirm bargaining (for example, Cahuc et al. 2008); and reduced-form rigid wages (for example, Shimer 2004, Blanchard and Galí 2008). In this setting, firms have some monopsony power: they can affect wages via their choice of employment.[13]

I make the following assumptions on the wage schedule.

ASSUMPTION 1. $S : \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}$, $X : \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}$, and $Z : \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}$ are continuous

[12] Expectations about next period's state of the labor market matter because workers will be on the labor market next period if bargaining negotiations break down, if they quit, or if they are dismissed.

[13] Manning (2003) offers an exhaustive study of labor markets in which firms have some monopsony power.

and differentiable in all arguments.

ASSUMPTION 2. For all $\theta \in \mathbb{R}^+$, $X(\theta, 0) = 0$ and $Z(\theta, 0) = 0$. For all $c \in \mathbb{R}^+$, $X(0, c) = 0$ and $Z(0, c) = 0$.

ASSUMPTION 3. For all $(\theta, c) \in \mathbb{R}^+ \times \mathbb{R}^+$, $\nabla_\theta(X + Z) \geq 0$.

Assumption 2 ensures that when recruiting costs or labor market tightness are nil, labor market conditions do not influence wages. Assumption 3 imposes that in a stationary environment, wages increase with labor market tightness. Intuitively, when the labor market is tighter, it is more costly for firms to recruit but easier for workers to find jobs; thus, labor market conditions are more favorable to workers, which will increase wages. Assumptions 1, 2, and 3 are satisfied for all the specific schedules studied in Section 6.

3.5 Firms

The firm's expected sum of discounted real profits is:

$$\mathbb{E}_0 \left[\sum_{t=0}^{+\infty} \delta^t \cdot \pi_t(i) \right], \tag{13}$$

where $\pi_t(i)$ is the real profit of firm i in period t:

$$\pi_t(i) = Y_t(i) \cdot \frac{P_t(i)}{P_t} - W_t(i) \cdot N_t(i) - R(\theta_t, c) \cdot H_t(i).$$

$Y_t(i)$ is the demand firm i faces, $\frac{P_t(i)}{P_t}$ is the relative price it sets, and $W_t(i)$ is the average real wage it pays. Aggregate real profit satisfies $\pi_t = \int_0^1 \pi_t(i) di$.

Firm i's production function $F(N_t(i), a_t)$ is differentiable and increasing in both arguments. Firm i faces a production constraint:

$$Y_t(i) \leq F(N_t(i), a_t). \tag{14}$$

It also faces a constraint on the number of workers employed in period t:

$$N_t(i) \leq (1 - s) \cdot N_{t-1}(i) + H_t(i). \tag{15}$$

DEFINITION 2 (Firm problem)**.** The firm chooses a stochastic processes $\{H_t(i), P_t(i)\}_{t=0}^{+\infty}$ to maximize (13) subject to the sequence of production constraints (14) and recruitment constraints (15), taking as given the wage schedule (12), as well as aggregate price, labor market tightness, and labor productivity $\{P_t, \theta_t, a_t\}_{t=0}^{+\infty}$. The time t element of a firm's choice must be measurable with respect to $\left(a^t, N_{-1}\right)$.

In equilibrium, endogenous layoffs never occur. Therefore, firms recruit some workers each period, and the Lagrangian for the firm problem is simply:

$$\begin{aligned}\mathcal{L} =& \mathbb{E}_0 \sum_{t=0}^{+\infty} \delta^t \left\{ Y_t \cdot \left(\frac{P_t(i)}{P_t}\right)^{1-\epsilon} - \left[S(N_t(i), a_t) + X(\theta_t, c) + Z(\theta_{t+1}, c)\right] \cdot N_t(i) \right.\\ & \left. -R(\theta_t, c) \cdot \left[N_t(i) - (1-s) \cdot N_{t-1}(i)\right] + \nu_t \cdot \left[F(N_t(i), a_t) - Y_t \cdot \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon}\right] \right\},\end{aligned}$$

where ν_t is the Lagrange multiplier associated with the production constraints and reflects the marginal profit from producing one more item. The first-order condition with respect to $P_t(i)$ yields

$$\frac{P_t(i)}{P_t} = \mathcal{M} \cdot \nu_t, \tag{16}$$

where $\mathcal{M} \equiv \frac{\epsilon}{\epsilon-1}$ is the markup charged by the monopoly. First-order condition (16) also implies that the monopolist sets its relative price as a markup over the marginal cost of producing one more item. The first-order condition with respect to $N_t(i)$ yields

$$\nu_t \cdot \nabla_N F(N_t(i), a_t) = W_t + R(\theta_t, c) + N_t(i) \cdot \nabla_N S(N_t(i), a_t) - \delta \cdot (1-s) \cdot \mathbb{E}_t\left[R(\theta_{t+1}, c)\right]. \tag{17}$$

First-order condition (17) says that firm i hires labor until marginal profit from hiring equals marginal cost. The marginal profit is the product of the marginal profit from producing one more item (ν_t) and the marginal product of labor ($\nabla_N F(N_t(i), a_t)$). Marginal cost is the sum of the wage (W_t), the recruiting cost ($R(\theta_t, c)$), the change in the wage bill from increasing employment marginally ($N_t(i) \cdot \nabla_N S(N_t(i), a_t)$), minus the discounted cost of recruiting a worker next period ($\delta \cdot (1-s) \cdot \mathbb{E}_t\left[R(\theta_{t+1}, c)\right]$).

3.6 Resource constraint

All production in the economy is constrained to be either consumed or allocated to recruiting:

$$Y_t = \int_0^1 C_t(i)di + R(\theta_t, c) \cdot H_t, \tag{18}$$

where Y_t is total output in period t:

$$Y_t = \int_0^1 Y_t(i)di. \tag{19}$$

4 Equilibrium

This section defines and specifies an equilibrium for the model. It starts by characterizing the equilibrium condition that private efficiency of worker-firm matches be respected at all times. This condition implies that no inefficient worker-firm separations occur in equilibrium; that is, worker-firm matches generating a positive surplus are never destroyed.

4.1 No-inefficient-separation condition

In the model, existing employment relationships generate a positive surplus because there is a cost to matching a firm with a worker (Hall 2005a). A worker-firm match is privately efficient as long as it maintains a positive surplus for both parties: in this case there is no opportunity for mutual improvement. Any wage schedule that ensures the private efficiency of existing relationships at all times is consistent with equilibrium. In fact, equilibrium requires that neither workers nor firms endogenously break an existing match since any match generates some surplus.[14] Workers do not have any endogenous incentive to quit; therefore, the sole restriction on the wage schedule is that it remains low enough to prevent endogenous layoffs. A firm's optimal hiring behavior is detailed in Lemma 2.

LEMMA 2. *Let the price $\hat{P}_t(i)$ be defined $\forall t \geq 0$ by*

$$Y_t \cdot \left(\frac{\hat{P}_t(i)}{P_t}\right)^{-\epsilon} = F((1-s) \cdot N_{t-1}(i), a_t).$$

[14] Equivalently, the only separations observed in equilibrium are the exogenous destructions of a fraction s of all jobs each period.

Then, let the marginal profit $\hat{\nu}_t(i)$ be defined by

$$\hat{\nu}_t(i) = \frac{1}{\mathcal{M}} \cdot \frac{\hat{P}_t(i)}{P_t}.$$

There exist marginal costs $\nu_t^H(i) > \nu_t^L(i)$ such that:

(i) if $\hat{\nu}_t(i) < \nu_t^L(i)$, firm i lays workers off;

(ii) if $\hat{\nu}_t(i) \in [\nu_t^L(i), \nu_t^H(i)]$, firm i freezes hiring;

(iii) if $\hat{\nu}_t(i) > \nu_t^H(i)$, firm i hires workers.

$\hat{P}_t(i)$ is the highest price that firm i can charge without any layoffs. If it charged a higher price, its demand would fall, it would reduce production, and would eventually lay some workers off. Thus $\hat{\nu}_t(i)$ is the highest marginal profit that firm i can obtain with no layoffs. The firm's marginal cost function is discontinuous at the beginning-of-period employment level $(1-s)\cdot N_{t-1}(i)$ because hiring new workers is costly, whereas freezing hiring or laying workers off is costless. $\nu_t^L(i)$ is the limit of the marginal cost function from below. It is the highest marginal cost that the firm possibly faces if it lays some workers off, and the marginal cost it faces if it freezes hiring. $\nu_t^H(i) > \nu_t^L(i)$ is the limit of the marginal cost function from above. It is the lowest marginal cost that the firm possibly faces if it hires some workers. The optimal decision of a monopolist is characterized by the equality of marginal costs and marginal revenues.[15] If $\hat{\nu}_t(i) < \nu_t^L(i)$, firm i must reduce its workforce to increase its gross marginal profit and reduce its marginal costs, which implies layoffs. Conversely, if $\hat{\nu}_t(i) > \nu_t^H(i)$, firm i must hire more workers to reduce its gross marginal profit and increase its marginal cost until both are equal. If $\hat{\nu}_t(i) \in [\nu_t^L(i), \nu_t^H(i)]$, firm i optimally freezes hiring.

ASSUMPTION 4. Let $\{N_t\}_{t=0}^{+\infty}$ and $\{\theta_t\}_{t=0}^{+\infty}$ be stochastic processes for aggregate employment and labor market tightness. I assume that the wage schedule satisfies $\forall t \geq 0$:

$$\frac{\mathcal{M}}{\nabla_N F((1-s)\cdot N_{t-1}, a_t)} \{(1-s)\cdot N_{t-1}\cdot \nabla_N S((1-s)\cdot N_{t-1}, a_t) + S((1-s)\cdot N_{t-1}, a_t) + \mathbb{E}_t\left[Z(\theta_{t+1}, c)\right] - \delta\cdot(1-s)\cdot \mathbb{E}_t\left[R(\theta_{t+1}, c)\right]\} \leq 1. \tag{20}$$

[15]To ensure uniqueness of the solution to the firm's optimization program, I assume that the marginal cost function increases with employment.

Using Lemma 2 and the actual characterization of thresholds ν^H and ν^L, Proposition 2 offers a condition on the wage schedule such that private efficiency of worker-firm matches is respected at all times.

PROPOSITION 2 (No-inefficient-separation condition)**.** *Let $\{N_t\}_{t=0}^{+\infty}$ be the stochastic process for aggregate employment in a symmetric equilibrium. Let $\{\theta_t\}_{t=0}^{+\infty}$ be the corresponding process for labor market tightness, defined from aggregate employment using* (9) *and* (11)*. Then hiring freezes occur with probability zero. A necessary and sufficient condition for inefficient worker-firm separations not to occur is that the wage schedule satisfies Assumption 4.*

In a symmetric equilibrium, if no firm recruits, $\theta_t = 0$ and $R(0, c) = 0$. Thus, once the symmetric behavior of firms is aggregated, the marginal cost function is continuous in employment and there are no hiring freezes. Condition (20) ensures that the productivity-dependent component of the wage $S((1-s) \cdot N_{t-1}, a_t)$ falls sufficiently relative to the decrease in marginal product of labor $\nabla_N F$ in response to an adverse productivity shock. In Section 6.4, I propose a specific model with job rationing and derive a condition on the primitives of the model—production function, wage schedule, and stochastic process for labor productivity—such that (20) holds.

4.2 Definition and characterization of the symmetric equilibrium

I normalize the aggregate price level P_t to remain constant over time.

DEFINITION 3 (Symmetric equilibrium)**.** Given initial employment N_{-1} and a stochastic process $\{a_t\}_{t=0}^{+\infty}$ for labor productivity, a symmetric equilibrium is a collection of stochastic processes

$$\{C_t, N_t, Y_t, H_t, \theta_t, U_t, W_t\}_{t=0}^{+\infty}$$

that solve the household and firm problems, satisfy the law of motion for unemployment (9), the law of motion for labor market tightness (11), the wage schedule (12), the resource constraint (18), and respect the no-inefficient-separation condition (20).

A symmetric equilibrium satisfies the following conditions:

- Law of motion for employment:

$$N_t = (1-s) \cdot N_{t-1} + H_t$$

- Law of motion for unemployment:

$$U_{t-1} = 1 - (1-s) \cdot N_{t-1}$$

- Law of motion for labor market tightness:

$$f(\theta_t) = \frac{H_t}{U_{t-1}}$$

- Resource constraint:

$$Y_t = C_t + R(\theta_t, c) \cdot H_t$$

- Production constraint:

$$Y_t = F(N_t, a_t)$$

- Wage rule:

$$W_t = S(N_t, a_t) + X(\theta_t, c) + \mathbb{E}_t \left[Z(\theta_{t+1}, c) \right]$$

- Firm's Euler equation:

$$\frac{1}{\mathcal{M}} \cdot \nabla_N F(N_t, a_t) = N_t \cdot \nabla_N S(N_t, a_t) + W_t + R(\theta_t, c) - (1-s) \cdot \delta \cdot \mathbb{E}_t \left[R(\theta_{t+1}, c) \right] \tag{21}$$

- No-inefficient-separation condition:

$$\frac{\mathcal{M}}{\nabla_N F((1-s) \cdot N_{t-1}, a_t)} \{(1-s) \cdot N_{t-1} \cdot \nabla_N S((1-s) \cdot N_{t-1}, a_t) + S((1-s) \cdot N_{t-1}, a_t) - (1-s) \cdot \delta \cdot \mathbb{E}_t \left[R(\theta_{t+1}), c \right] \} \leq 1$$

5 Analytical Characterization of Unemployment Components

In this section, I introduce job rationing to define cyclical and frictional unemployment. Then, I study the properties of unemployment and its components.

5.1 Job rationing

DEFINITION 4 (Gross marginal profit). For all $(N_t, a_t) \in (0, 1] \times \mathbb{R}^{++}$, I define the *gross marginal profit* as

$$J(N_t, a_t) \equiv \frac{1}{\mathcal{M}} \nabla_N F(N_t, a_t) - S(N_t, a_t) - N_t \cdot \nabla_N S(N_t, a_t). \tag{22}$$

$J(N_t, a_t)$ represents the marginal profit from an additional match gross of the marginal cost imposed by labor market frictions. This marginal cost is the sum of a recruiting cost $R(\theta_t, c)$, a cost $X(\theta_t, c) + \mathbb{E}_t [Z(\theta_{t+1}, c)]$ imposed indirectly through the wage schedule, minus the opportunity cost of hiring a worker next period $(1 - s)\delta \cdot \mathbb{E}_t [R(\theta_{t+1}, c)]$. In a symmetric equilibrium, a firm's Euler equation (21) can be rewritten as

$$J(N_t, a_t) = R(\theta_t, c) + X(\theta_t, c) + \mathbb{E}_t [Z(\theta_{t+1}, c)] - (1 - s)\delta \cdot \mathbb{E}_t [R(\theta_{t+1}, c)], \tag{23}$$

which imposes that the gross marginal profit equals the marginal cost associated with matching frictions in equilibrium.

ASSUMPTION 5. For all $a \in \mathbb{R}^{++}$, $\lim_{N \to 0} J(N, a) > 0$.

By Assumption 5, the gross marginal profit is always positive for the first worker hired by the firm. Combined with Assumption 2, steady-state production and employment are always positive. I now impose conditions on the gross marginal profit function $J : (0, 1] \times \mathbb{R}^{++} \to \mathbb{R}$ that yield job rationing.

ASSUMPTION 6.

(i) For all $(N, a) \in (0, 1] \times \mathbb{R}^{++}$, $\nabla_N J(N, a) < 0$.

(ii) There exists $(N, a) \in (0, 1] \times \mathbb{R}^{++}$, $J(N, a) < 0$.

LEMMA 3. *Under Assumptions 5 and 6, there exists a non-empty, open interval $\mathcal{I} \subseteq \mathbb{R}^+$ such that for any $a \in \mathcal{I}$, the equation $J(N, a) = 0$ admits a unique solution $N^C(a) \in (0, 1)$. Let $\mathcal{A} = \cup \mathcal{I}$ be the union of all such open intervals. I shall refer to $\mathcal{A}$ as the* interval of rationing.

Since the gross marginal profit $J(N, a)$ is decreasing in employment, worker-firm matches made when employment is above $N^C(a)$ yield a negative marginal profit. The profit from these matches is even more negative once the additional costs due to matching frictions are accounted for. In this sense, the number of jobs is rationed: no more than $N^C(a)$ jobs are created by profit-maximizing firms.

By assumption, when recruiting cost $c = 0$, the right-hand side of (23) is nil, because the marginal cost from matching frictions decreases to zero. Thus, without recruiting costs, equilibrium condition (23) becomes

$$J(N_t, a_t) = 0. \tag{24}$$

With productivity in the interval of rationing, (24) admits a solution $N^C(a_t) < 1$, which can be interpreted as employment when there are no recruiting costs. The key implication of Assumption 6 is that the economy may remain below full-employment even when there are no recruiting costs. Adding recruiting costs leads firm to curtail employment further. Hence, both job rationing and search frictions cause unemployment.

5.2 Recessions

Productivity does not affect the recruiting cost function (R), or components of the wage schedule dependent on labor market conditions (X, Z). On the other hand, productivity does influence the gross marginal profit J, because both the production function F and the component S of the wage schedule depend on productivity. The following assumption specifies the form of this influence, which drives fluctuations in the model.

ASSUMPTION 7. For all $(N, a) \in (0, 1] \times \mathbb{R}^{++}$, $\nabla_a J(N, a) > 0$.

ASSUMPTION 8. For all $(n, a) \in [0, N^C(a)) \times \mathcal{I}$, $\nabla_{n,a} J(N^C(a) - n, a) \leq 0$.

Assumption 7 implies that when labor productivity falls, gross marginal profits fall. Assumption 8 implies that when labor productivity falls, gross marginal profit—seen as a function of employment—does not become "too flat". These assumptions allow me to characterize the interval of rationing $\mathcal{A}$ and fluctuations in N^C, as described in Lemma 4.

LEMMA 4. *Under Assumptions 5, 6, and 7:*

(i) $\mathcal{A} = \mathbb{R}^{++}$ *or there exists* $a^C \in \mathbb{R}^{++}$, $\mathcal{A} = (0, a^C)$*;*

(ii) $N^C : \mathcal{A} \to (0, 1)$ *is continuous, differentiable, and for* $a \in \mathcal{A}$*:* $\nabla_a N^C(a) > 0$.

Lemma 4 states that after a fall in labor productivity, $N^C(a)$ falls and the constraint on employment becomes more stringent.[16] Therefore, the shortage of jobs becomes more pronounced when the economy slides into a recession.

5.3 Cyclical and frictional unemployment

Assumption 6 introduces job rationing, which allows me to decompose equilibrium unemployment into cyclical and frictional components.

DEFINITION 5 (Cyclical and frictional unemployment)**.** Let $\{a_t\}$ be the stochastic process for labor market productivity, and $\{U_t\}$ be the stochastic process for equilibrium unemployment. Under Assumption 6, I can construct two stochastic processes $\{U_t^C, U_t^F\}$. If $a_t \in \mathcal{A}$, their time t elements are defined by:

$$U_t^C = 1 - N^C(a_t) \tag{25}$$

$$U_t^F = U_t - U_t^C. \tag{26}$$

If $a_t \notin \mathcal{A}$, $U_t^C = 0$ and $U_t^F = U_t$. U_t^C is *cyclical unemployment* at time t, and U_t^F is *frictional unemployment* at time t.

When productivity a_t is in the interval of rationing $\mathcal{A}$, employment is bounded above by $N^C(a_t)$. Even if search costs are zero, employment equals $N^C(a_t) < 1$ and the economy is below full-employment. Cyclical unemployment $U_t^C = 1 - N^C(a_t)$ represents unemployment caused by job rationing; it reflects the lack of jobs in the economy, independently of matching frictions.

$U_t^F = U_t - U_t^C$ can be expressed as a function of employment N_t and $N^C(a_t)$:

$$U_t^F = s \cdot N_t + \left[N^C(a_t) - N_t\right]. \tag{27}$$

The first term in (27) is unemployment due to job destruction during period t. It reflects the inflow of separated workers into unemployment at the end of period t, following end-of-period job destructions.[17] The second term in (27) is additional unemployment caused by matching frictions. In fact, $N^C(a_t)$ would be the prevailing employment if there were no matching frictions. Once

[16] Assumption 8 is satisfied by a large class of functions, because a sufficient condition for it to hold is $\nabla^2_N J(N,a) \geq 0$ and $\nabla_{N,a} J(N,a) \geq 0$.

[17] This component would vanish in a continuous-time model and it is not central to understanding the unemployment dynamics studied in the next sections.

recruiting costs are taken into account, the marginal cost of labor increases and monopolistic firms reduce employment to $N_t < N^C(a_t)$. The difference between these two employment levels is additional unemployment caused by matching frictions.

5.4 Fluctuations of unemployment and its components over the business cycle

To analytically characterize the behavior of unemployment and its components in an environment with aggregate shocks, I make the following approximation.

ASSUMPTION 9 (Stochastic equilibrium)**.** Flows into employment and flows out of employment are equal:

$$f(\theta_t) \cdot U_{t-1} = s \cdot N_t. \tag{28}$$

This approximation is motivated by the observation that rates of job destruction and job creation are very large, while the amplitude of productivity shocks is small; thus, unemployment rapidly converges to a stochastic equilibrium in which inflows to and outflows from employment are balanced; hence, the stochastic equilibrium of unemployment is a good approximation to the dynamic path of unemployment. Empirically, Hall (2005b) shows that actual unemployment scarcely deviates from its stochastic-equilibrium level in U.S. data over the 1948–2001 period; Rotemberg (2008) conducts a similar analysis to show that the stochastic equilibrium for unemployment tracks actual unemployment closely.[18] Finally, Section 8 numerically studies a model in which unemployment is not constrained to remain at its stochastic-equilibrium level, to confirm the robustness of the theoretical findings.

Assumption 9 allows an important simplification by linking unemployment U_t to labor market tightness θ_t through a Beveridge Curve

$$U_t = \frac{s}{s + (1-s) \cdot f(\theta_t)}, \tag{29}$$

which can be depicted as a downward-sloping curve in the vacancy-unemployment plane (see Figure 2). If productivity follows a first-order Markov process, then in equilibrium at each date $t \geq 0$, unemployment U_t, employment N_t, and labor market tightness θ_t solely depend on the realization of productivity a_t (and not on the history of shocks a^t). These equilibrium values

[18] Equation (28) does not hold exactly all the time since unemployment varies. But flows into and out of employment are close enough most of the time to legitimately abstract from the adjustment dynamics of unemployment, and work under Assumption 9.

are determined by a system of three equations: (9), (23), and (29). In particular, I can define $U : \mathbb{R}^{++} \to [0,1]$ such that $U(a)$ is the level of unemployment in equilibrium when the realization of productivity is a. With definition 5 I can define two other functions, $U^C : \mathbb{R}^{++} \to [0,1]$ and $U^F : \mathbb{R}^{++} \to [0,1]$, such that $U^C(a)$ and $U^F(a)$ are the levels of cyclical and frictional unemployment in equilibrium when the realization of productivity is a. Proposition 3 states the fundamental result of this paper, which describes how unemployment and its components fluctuate with productivity.

PROPOSITION 3 (Decomposition of unemployment)**.** *Consider an economy with job rationing (Assumption 6), in which the derivatives of gross marginal profit satisfy some regularity conditions (Assumption 7 and 8). Assume that flows in and out of unemployment are equal (Assumption 9). Assume further that the stochastic process $\{a_t\}$ for labor productivity follows a random walk; that $\theta(\cdot)$ is sufficiently linear (Assumption A1); and that the variance of the labor productivity process is small enough (Assumption A3). Then $\forall\ a \in \mathcal{A}$:*

(i) $\nabla_a U < 0$;

(ii) $\nabla_a U^C < 0$;

(iii) $\nabla_a U^F > 0$.

This proposition shows that when jobs are rationed, total and cyclical unemployment are decreasing with productivity, whereas frictional unemployment is increasing. That is, when the economy enters a recessions, cyclical unemployment rises, driving the rise in total unemployment; at the same time, frictional unemployment falls.

It is reasonable to assume that productivity follows a random walk: in Section 7.1, I construct a quarterly labor productivity series using data from the Bureau of Labor Statistics (BLS) to find that (log) productivity is quite autocorrelated; I also repeat the analysis with the quarterly utilization-adjusted TFP series from Fernald (2009) to find that (log) TFP is highly autocorrelated; and Basu et al. (2006) find that yearly purified total factor productivity (TFP) is nearly a random walk. The random-walk assumption and the assumptions following it in the text of Proposition 3 imply that firms avoid substituting too much recruiting intertemporally, which ensures that unemployment decreases with productivity.[19, 20]

5.5 Intuition in an environment with no aggregate shocks

Without aggregate shocks, the economy is stationary, and the equilibrium on the labor market can be described by three endogenous variables: unemployment U, employment N, and labor market tightness θ. They are determined by three equations: the definition of unemployment (9); the Beveridge curve (29), which clearly holds in a stationary environment; and a firm's Euler equation in a stationary environment:

$$J(N,a) = X(\theta,c) + Z(\theta,c) + [1-(1-s)\cdot\delta]\cdot R(\theta,c), \tag{30}$$

which is the key equation of the system. The left-hand side of this equation is gross marginal profit from hiring labor, which is strictly decreasing in employment. The right-hand side is marginal cost caused by matching frictions: recruiting costs $R(\theta,c)$; plus the component of the wage depending on the state of the labor market $[X(\theta,c)+Z(\theta,c)]$; minus the opportunity cost of recruiting $(1-s)\cdot\delta\cdot R(\theta,c)$. The right-hand side is strictly increasing in labor market tightness θ, and therefore strictly increasing in employment, which ensures uniqueness of the equilibrium. Corollary 1 translates Proposition 3 into comparative-static results around steady state.

COROLLARY 1 (Comparative statics)**.** *In an economy without aggregate shocks such that $a_t = a \in \mathcal{A}$ $\forall t \geq 0$:*

(i) $\nabla_a U < 0$;

(ii) $\nabla_a U^C < 0$;

(iii) $\nabla_a U^F > 0$.

[19] Assume that a state is characterized by low productivity and a high probability of transitioning to a high-productivity state, and another state is characterized by medium productivity but lower probability of transitioning to a high-productivity state. Recruiting could be higher and unemployment lower in the low-productivity state than in the medium-productivity state. In the former low-productivity state, the opportunity cost of recruiting is low because recruiting is expected to be expensive next period. In the latter medium-productivity state, the opportunity cost of recruiting is low because recruiting is expected to be cheap next period. If fluctuations in opportunity cost supersede those in marginal product of labor, unemployment may not be decreasing in productivity.

[20] Appendix A contains the proof of proposition 3, describes precisely all the conditions required for it to hold, and offers a more general lemma which holds when labor productivity is AR(1) (Lemma A1). Appendix B proves a similar result in a two-state economy under more general conditions on the stochastic process followed by labor productivity.

Corollary 1 implies that around any steady-state at which jobs are rationed, we have the following comparative-static results: when labor productivity a decreases, total unemployment increases, cyclical unemployment increases, but frictional unemployment decreases.

This results can be illustrated with a simple diagram. Expressing both labor market tightness θ and employment N as functions of unemployment U in (30), I can represent the steady-state equilibrium condition on a plane with unemployment on the x-axis and marginal profit on the y-axis. This simple diagram is shown in Figure 3. The upward-sloping, solid line is gross marginal profit $J(N, a)$. The downward-sloping, dotted line is the marginal cost imposed by matching frictions $X(\theta, c) + Z(\theta, c) + [1 - (1 - s) \cdot \delta] \cdot R(\theta, c)$. Cyclical unemployment is unemployment prevailing when the recruiting cost c is zero: it is obtained at the intersection of the gross marginal profit curve with the x-axis.[21] Total unemployment is obtained at the intersection of the gross marginal profit and marginal cost curves, and frictional unemployment is the difference between total and cyclical unemployment.

When productivity decreases, the upward-sloping, gross marginal profit curve shifts to the right. At the current employment level, gross marginal profit falls below the marginal cost of matching frictions. Thus, firms reduce hiring to increase gross marginal profit. At the aggregate level, lower recruiting efforts by firms reduce labor market tightness, which reduces the marginal cost of matching frictions. This corresponds to a movement along the downward-sloping marginal cost curve. The adjustment continues until gross marginal profit equals search-friction-related marginal cost. Then the economy reaches a new equilibrium, with high unemployment and lower labor market tightness.

Since the gross marginal profit curve shifts to the right, the constraint imposed by job rationing on employment is tighter, and cyclical unemployment is higher. Since there are fewer jobs, the labor market is slacker and the marginal cost of matching frictions is lower. In particular, recruiting is less expensive in a slack labor market: many jobseekers apply to few vacancies, and each vacancy can be filled rapidly, at low cost. Hence, a smaller reduction in employment, from the level prevailing when the recruiting cost c is zero, suffices to bring the economy to equilibrium. Consequently frictions contribute little to unemployment, and frictional unemployment falls.

[21] When $c = 0$, the marginal cost imposed by matching frictions $X(\theta, c) + Z(\theta, c) + [1 - (1 - s) \cdot \delta] \cdot R(\theta, c)$ falls to 0.

6 Special Cases of the General Model

This section first shows that the standard Mortensen-Pissarides (MP) model, its variant with rigid wages, and large-firm models with intrafirm bargaining are nested in the general labor market model presented in Section 3. Critically, these models have no job rationing, and cyclical unemployment is nil. In contrast, I abandon the assumption that all workers are employed when recruiting costs converge to zero. I develop a specific model in which job rationing stems from diminishing marginal returns to labor and some real wage rigidity.

6.1 Standard Mortensen-Pissarides model

In this section, I specialize my general model to the standard MP framework (for example, Pissarides 2000, Shimer 2005).[22] To do so, I make the following assumptions.

ASSUMPTION 10 (Perfect competition)**.** $\mathcal{M} = 1$.

ASSUMPTION 11 (Constant returns to labor)**.** $F(N_t, a_t) = a_t \cdot N_t$.

ASSUMPTION 12. There exists $\beta \in (0,1)$ such that the wage $W_t(i)$ paid by firm i in period t is given by (12) where:

(i) $S(N_t(i), a_t) = 0$;

(ii) $X(\theta_t, c) = c \cdot \dfrac{\beta}{1-\beta} \cdot \dfrac{1}{q(\theta_t)}$;

(iii) $Z(\theta_{t+1}, c) = c \cdot \dfrac{\beta}{1-\beta} \cdot \delta \cdot (1-s) \cdot \left(\theta_{t+1} - \dfrac{1}{q(\theta_{t+1})}\right)$.

LEMMA 5 (Equivalence with Nash bargaining)**.** *Assume that wages are bargained each period, and that the wage $W_t(i)$ in period t in firm i is determined by the generalized Nash bargaining solution. Let β be a worker's bargaining power. Then $W_t(i)$ is given by Assumption 12.*

The bargaining solution divides surplus from the match between the worker and firm, with the worker keeping a fraction $\beta \in (0,1)$ of the surplus. In this setting, a firm's surplus from an established relationship is simply given by the hiring cost $c/q(\theta_t)$, since a firm can always immediately replace a worker at that cost during the matching period. When labor market tightness θ_t is high,

[22] Similar results could be obtained with search-and-matching models using alternative bargaining procedure to divide the surplus between firm and worker (for example, Hall and Milgrom 2008).

many firms compete for few unemployed workers. Unemployed workers can find a job quickly, but it takes time for a firm to find a worker. Since a worker's outside option improves relative to a firm's outside option, the wage offered to workers increases. Even though this model is not specified exactly like the canonical model and the wage equation does not take the standard form, their labor market equilibria are virtually identical.[23]

Using the notation defined in Section 4, gross marginal profit becomes

$$J(N_t, a_t) = a_t \tag{31}$$

because $S(N_t, a_t) = 0$. Since the gross marginal profit satisfies neither the first nor the second condition of Assumption 6, all workers would be employed if there were no recruiting costs.

PROPOSITION 4 (Full employment in MP model)**.** *Under Assumptions 10, 11, and 12, when* $c \to 0$*,* $N_t \to 1, \ \forall t \geq 0$*.*

Marginal product of labor is independent of employment, and always greater than the value of unemployment for workers. Even without recruiting costs, matches always generate a positive surplus, which is divided between firm and worker by Nash bargaining over wages. Without matching frictions, the firm faces no costs in creating a match, which implies that the net profit from a match is always positive. As a consequence, firms enter the labor market until all the labor force is employed. As a direct consequence of the absence of job rationing, all unemployment is frictional in the MP model.

6.2 Mortensen-Pissarides model with sticky real wages

I now show that introducing wage rigidity in the MP model does not suffice to introduce job rationing. Keeping Assumptions 10 and 11, I replace the Nash-bargaining assumption by a wage rule that only partially adjusts to productivity shocks. Shimer (2004) and Hall (2005a) study this type of MP model with sticky wages (MPS model) to demonstrate that rigid wages help amplify

[23]The equilibrium condition arising from a firm's Euler equation in this model can be written as:

$$\frac{1}{q(\theta_t)} + \beta\delta(1-s)\mathbb{E}_t\left[\theta_{t+1}\right] = (1-\beta)\frac{a_t}{c} + \delta(1-s)\mathbb{E}_t\left[\frac{1}{q(\theta_{t+1})}\right],$$

which is comparable to equation (6) in Shimer (2005) since I assume $\lambda = 1$ (an aggregate shock occurs each period), $z = 0$, $1 + r + s \approx 1$, and $\mathbb{E}_t\left[\theta_{t+1}\right] \approx \theta_t$ when a_t follows a random walk.

shocks.[24] The wage schedule borrows Blanchard and Galí's (2008) specification.

ASSUMPTION 13 (Partially rigid wage)**.** There exists $\gamma \in [0,1]$ and $w_0 \in \mathbb{R}^+$ such that the wage $W_t(i)$ paid by firm i in period t is given by (12) where:

(i) $S(N_t(i), a_t) = w_0 \cdot a_t^\gamma$;

(ii) $X(\theta_t, c) = 0$;

(iii) $Z(\theta_{t+1}, c) = 0$.

With $\gamma = 0$, wages are completely rigid, which corresponds to Shimer's (2004) specification. Under these assumptions, the gross marginal profit becomes

$$J(N_t, a_t) = a_t - w_0 \cdot a_t^\gamma. \tag{32}$$

As for the MP model, the gross marginal profit is independent of employment, and does not satisfy Assumption 6 for any $\gamma \in [0,1]$. As a direct consequence of the absence of job rationing, all unemployment is frictional in the MPS model.

PROPOSITION 5 (Full employment in MPS model)**.** *Assume that* $w_0 \leq a_t^{1-\gamma}, \ \forall t \geq 0$. *Under Assumptions 10, 11, and 13, when* $c \to 0$, $N_t \to 1, \ \forall t \geq 0$.

When a worker and a firm meet, they match if the wage is between the two parties' reservation levels. When recruiting costs converge to 0, the firm's reservation level is the marginal product of labor, which is independent of employment and greater than the rigid wage level by assumption. Hence, if one job is profitable, infinitely many jobs would be profitable, and the economy would operate at full employment.

This results can be illustrated with a simple diagram shown in Figure 4. This diagram represents the steady-state equilibrium condition on a plane with unemployment on the x-axis and marginal profit on the y-axis. The gross marginal profit $J(N, a)$ is independent of employment and is represented by the horizontal, solid line. The downward-sloping, dotted line is the marginal cost of hiring $(1 - (1 - s)\delta)R(\theta, c)$. Total unemployment is obtained at the intersection of the gross marginal profit and marginal hiring cost curves. The gross marginal profit is positive for

[24] Similar results could be obtained in large-firm models with rigid wages in which (i) production functions exhibit constant marginal returns to labor (for example, Blanchard and Galí 2008); or (ii) production functions exhibit diminishing marginal returns to labor but capital adjusts immediately to employment (for example, Gertler and Trigari 2009).

any employment level, and any productivity such that $w_0 \leq a_t^{1-\gamma}$. When recruiting cost $c = 0$, the marginal hiring cost converges to 0 for any positive employment level, while the gross marginal profit is positive. Therefore, firms keep on hiring as long as unemployment is positive. Hence, there is no job rationing and no cyclical unemployment in this model. To conclude, wage rigidity does not suffice to introduce job rationing.

6.3 Large-firm model with intrafirm bargaining

In this section, I show that introducing diminishing marginal returns to labor in the MP model does not suffice to introduce job rationing. I specialize my general model to a large-firm, search-and-matching model with the intrafirm bargaining procedure of Stole and Zwiebel (1996a) and Stole and Zwiebel (1996b).[25] I assume perfect competition (Assumption 10), and make two assumptions about the production function and wage-setting.

ASSUMPTION 14 (Diminishing marginal returns to labor)**.** $F(N_t, a_t) = a_t \cdot N_t^{\alpha}$.

ASSUMPTION 15. There exists $\beta \in (0, 1)$ such that the wage $W_t(i)$ paid by firm i in period t is given by (12) where:

(i) $S(N_t(i), a_t) = \beta \dfrac{\alpha \cdot a_t \cdot N_t(i)^{\alpha-1}}{1 - \beta(1-\alpha)}$;

(ii) $X(\theta_t, c) = 0$;

(iii) $Z(\theta_{t+1}, c) = c \cdot (1-s)\delta \cdot \beta \cdot \theta_{t+1}$.

LEMMA 6 (Equivalence with Stole and Zwiebel's (1996a) bargaining)**.** *Assume that wages are bargained each period, and that the wage $W_t(i)$ in period t in firm i is determined by Stole and Zwiebel's (1996a) bargaining solution. Let β be a worker's bargaining power. Then $W_t(i)$ is given by Assumption 15.*

The gross marginal profit becomes

$$J(N_t, a_t) = \left[\frac{1-\beta}{1-\beta(1-\alpha)}\right] a_t \cdot \alpha \cdot N_t^{\alpha-1},$$

and satisfies the first condition in Assumption 6 since $\nabla_N J < 0$. However, the gross marginal profit $J(N_t, a_t)$ always remains positive in spite of diminishing marginal returns to labor. This is

[25] The model presented here shares features with large-firm models studied in Cahuc and Wasmer (2001), Cahuc et al. (2008), and Elsby and Michaels (2008).

because the wage falls sufficiently when employment increases and the marginal product of labor decreases. Hence, there is no job rationing and all unemployment is frictional.

PROPOSITION 6 (Full employment in SZ model)**.** *Under Assumptions 10, 14, and 15, when* $c \to 0$, $N_t \to 1, \ \forall t \geq 0$.

Intrafirm bargaining implies that the wage is derived from Nash bargaining over the surplus from the marginal worker-firm match. When recruiting costs are zero, the wage remains below the marginal product of labor for any employment level, and it is profitable for firms to continue hiring until everybody is employed. Thus, introducing downward-sloping demand for labor is not sufficient for obtaining job rationing and positive cyclical unemployment.

6.4 A specific model of job rationing

There are a variety of models with job rationing. Here, I present only one possible source of job rationing: the combination of real wages that only partially adjust to productivity shocks (Assumption 13) with diminishing marginal returns to labor (Assumption 14). The introduction of wage rigidity into the model follows the reduced-form approach of the literature.[26] Under these assumptions, gross marginal profit becomes

$$J(N_t, a_t) = \frac{1}{\mathcal{M}} a_t \cdot \alpha \cdot N_t^{\alpha-1} - w_0 \cdot a_t^{\gamma}. \tag{33}$$

It satisfies Assumption 6 because it decreases with employment, and $J(1, a_t) < 0$ when productivity a_t is low enough. Intuitively, when the firm expands employment, the marginal product of labor falls while wages do not adjust; thus, gross marginal profit falls and is exhausted when employment is high enough. Moreover, gross marginal profit satisfies Assumption 7, because it increases with productivity. Intuitively, when productivity falls, the marginal product of labor falls while real wages adjust only partly to productivity shocks; thus, the marginal profitability of monopolistic firms falls. Exploiting the specific functional form of the gross marginal profit, I can now repeat the analysis of Section 5 and propose a particular economic interpretation.

[26] Microfounded models of wage rigidity remain too complex to be embedded in macroeconomic models. Consequently, most macroeconomic models in the search literature use reduced-form approaches to wage rigidity. For instance, Shimer (2004), Hall (2005a), Krause and Lubik (2007), Blanchard and Galí (2008), Sveen and Weinke (2008), and Faia (2008) assume simple rigid-wage schedules. Thomas (2008) or Gertler and Trigari (2009) assume that wages can only be renegotiated at distant time intervals (Calvo (1983) wage setting).

First, the interval of rationing is $\mathcal{A} = (0, a^C)$, where

$$a^C = \left(\frac{\mathcal{M} \cdot w_0}{\alpha}\right)^{\frac{1}{1-\gamma}}. \tag{34}$$

The interval of rationing is wider when the markup $\mathcal{M}$ is higher, the steady-state wage w_0 is higher, and the production function parameter α is lower.

Second, I solve the equation $J(N, a) = 0$ with $a \in \mathcal{A}$ to find cyclical unemployment:

$$U_t^C = 1 - \left(\frac{\alpha}{\mathcal{M} \cdot w_0}\right)^{\frac{1}{1-\alpha}} \cdot a_t^{\frac{1-\gamma}{1-\alpha}}. \tag{35}$$

Whereas canonical search-and-matching models only highlight the role of the matching process on unemployment, this model also considers other factors. For instance, improving product market competition would reduce the markup that monopolistic firms charge, and lower cyclical unemployment. Introducing these factors deepens our understanding of unemployment and suggests different ways to tackle unemployment. Frictional unemployment is implicitly determined from (27) and the firm's Euler equation:

$$J(N_t, a_t) = R(\theta_t, c) - (1-s)\delta \cdot \mathbb{E}_t \left[R(\theta_{t+1}, c)\right]. \tag{36}$$

Finally, I specialize the results of Proposition 3 to this model of job rationing.

COROLLARY 2. *Assume that the stochastic process $\{a_t\}$ for labor productivity follows a random walk. Assume that Assumptions 9, A1, and A3 hold. Then sufficient conditions for the results of Proposition 3 to hold are*

$$1 - (1-\alpha)\frac{\ln(1-s)}{\ln(1-2.5 \cdot \sigma)} \leq \gamma \leq \frac{1}{2-\alpha}. \tag{37}$$

Condition (37) states that for a given production function parameter α, wages need to be rigid enough to obtain sufficient fluctuations in cyclical unemployment, and also need to be flexible enough to avoid layoffs with high probability. Using the calibrated parameter values derived in Section 7, (37) imposes $0.62 \leq \gamma \leq 0.79$, which is satisfied by my calibration of $\gamma = 0.7$.

7 Calibration

The model developed in Section 6.4 provides an intuitive understanding of unemployment and its components, and yields analytical results. This tractability and portability come at the cost of realism. Therefore, I follow the tradition of Kydland and Prescott (1982) and calibrate the model using micro and macro evidence. I calibrate all parameters at a weekly frequency, which is a good approximation for the continuous-time nature of unemployment flows.[27] Table 1 summarizes the calibrated parameters.

7.1 Stochastic process for labor productivity

I estimate the log of labor productivity as a residual $\log(a_t) = \log(Y_t) - \alpha \cdot \log(N_t)$. This measure of labor productivity corresponds more closely to the concept of labor productivity defined in the model—in which there is no capital—and is commonly used in the literature (for example, Shimer 2005, Gertler and Trigari 2009). Y_t and N_t are seasonally-adjusted, quarterly real output and employment in the nonfarm business sector, respectively, and are constructed by the BLS Major Sector Productivity and Costs (MSPC) program. The sample period is 1964:Q1–2009:Q2. To emphasize business-cycle-frequency fluctuations, I take the difference between log labor productivity and a low frequency trend—a Hodrick-Prescott (HP) filter with a smoothing parameter 10^5, as in Shimer (2005). I estimate the stochastic process followed by detrended log productivity as an AR(1) process with mean zero: $\log(a_{t+1}) = \rho \log(a_t) + z_{t+1}$, where $z \sim N(0, \sigma^2)$. I obtain an autocorrelation of 0.897 and a conditional standard deviation of 0.0087. At weekly frequency, this requires setting $\rho = 0.991$ and $\sigma = 0.0026$.[28]

7.2 Preferences

I calibrate the markup at $\mathcal{M} = 1.11$, using Christiano et al.'s (2005) estimation of a general-equilibrium model with flexible prices. This markup corresponds to an elasticity of substitution

[27] I consider a week as 1/12 of a quarter and 1/4 of a month. The relevant measure of unemployment in the model is beginning-of-period unemployment, which determines labor market tightness and recruiting costs. As discussed in Section 5.3, part of beginning-of-period frictional unemployment comes mechanically from the discrete inflow of labor into unemployment at the end of each period, caused by job destructions. This component of frictional unemployment is an artifact of the discrete-time structure of the model, which can be minimized by calibrating the model at weekly frequency. Models are commonly calibrated at such frequency in the literature (for example, Hagedorn and Manovskii 2008, Elsby and Michaels 2008).

[28] Non-detrended data are more persistent. When I repeat the estimation with non-detrended quarterly productivity, I obtain an autocorrelation of 1.0043 and a conditional standard deviation of 0.0090.

across goods of $\epsilon = 9$.

7.3 Labor market

I first estimate the recruiting cost as a fraction of the wage bill (c), the job destruction rate (s), and the matching function (ω, η). To estimate the separation rate s, I use the seasonally-adjusted, monthly time series for Total Separations in all nonfarm industries, computed by the Bureau of Labor Statistics (BLS) from the Job Openings and Labor Turnover Survey (JOLTS) for the period from December 2000 to June 2009.[29] The average separation rate is 0.038. At weekly frequency, the separation rate is 0.0095.

For the recruiting cost, I use the microeconomic evidence gathered by Barron et al. (1997) and find that on average, the flow cost of opening a vacancy amounts to 0.098 of a worker's wage.[30] These numbers account only for the labor costs of recruiting. Silva and Toledo (2006) argue that recruiting could also involve advertising, agency fees or even travel costs for applicants. Using data collected by PricewaterhouseCooper, they report that 0.42 of a worker's monthly wage could be spent on each hire. Unfortunately, they do not report recruiting times. Using the average job-filling rate of 1.3 in JOLTS, 2000–2009, the flow cost of recruiting would be 0.54 of a worker's wage, which seems large as it amounts to five times the labor costs reported by Barron et al. (1997). I calibrate flow recruiting costs as 0.32 of a worker's wage, the midpoint between the two previous estimates.[31]

Following the literature (for example, Hall 2005a), I specify the matching function as

$$h(U, V) = \omega U^{\eta} V^{1-\eta}, \tag{38}$$

and pick $\eta = 0.5$, which is reasonable in light of empirical results surveyed by Petrongolo and Pis-

[29] December 2000 to June 2009 is longest period for which time series from JOLTS are available. Comparable data were unfortunately not available before December 2000.

[30] Using the 1980 Employment Opportunity Pilot Project survey (2,994 observations) they find that employers spend on average 5.7 hours per offer, make 1.02 offers per hired worker, and that it takes employers 13.4 days to fill a position. Hence the flow cost of maintaining a vacancy open is $5.7/8 \times 1.02/13.4 \approx 0.054$ of a worker's wage. Adjusting for the possibility that hiring is done by supervisors who receive above-average wages (as in Silva and Toledo (2006)), the flow cost of keeping an open vacancy is $c = 0.071$ of a worker's wage. With the 1982 Employment Opportunity survey (1,270 observations), the corresponding numbers are 10.4 hours, 1.08 offers, 17.2 days, and the flow cost is $c = 0.106$. Finally, with the 1993 survey conducted by the authors for the W. E. Upjohn Foundation for Employment Research (210 observations), the numbers are 18.8 hours, 1.16 offers, 30.3 days, and the flow cost is $c = 0.117$.

[31] Using the average unemployment rate and labor market tightness in JOLTS, I find that $c = 0.32$ corresponds to 0.89% of the total wage bill being spent on recruiting. My estimate is average compared to others found in the literature: for example, 0.213 in Shimer (2005), 0.357 in Pissarides (2009), or 0.433 in Hall and Milgrom (2008).

sarides (2001). To estimate the matching efficiency ω, I use seasonally-adjusted, monthly series for the number of hires and vacancies from JOLTS, 2000–2009. I use the seasonally-adjusted, monthly unemployment level computed by the BLS from the Current Population Survey (CPS) over the same period. For each month i, I calculate θ_i as the ratio of vacancies to unemployment and the job-finding probability f_i as the ratio of hires to unemployment. I compute the least-squares estimate of ω, which minimizes $\sum_i (f_i - \omega \theta_i^{1-\eta})^2$:

$$\hat{\omega} = \frac{\sum_i \theta_i^{1-\eta} f_i}{\sum_i \theta_i^{2(1-\eta)}}.$$

The resulting estimate is $\hat{\omega} = 0.93$. My estimate at weekly frequency is therefore 0.23.

Finally, I calibrate the wage w_0 to obtain a steady-state unemployment of 5.8%, which is the average of a low frequency trend—an HP filter with smoothing parameter 10^5—for unemployment over the period 1964–2009.[32]

7.4 Diminishing marginal returns to labor: estimates and evidence

Estimate from labor share. In steady-state, the labor share $\overline{ls} \equiv (\overline{w} \cdot \overline{n}) / \overline{y}$ is:

$$\overline{ls} = \frac{\alpha}{\mathcal{M}} - [1 - \delta(1-s)] \cdot \frac{c}{q(\overline{\theta})} \cdot \overline{n}^{1-\alpha}.$$

I target a steady-state labor share of $\overline{ls} = 0.66$ and a steady-state unemployment rate of $\overline{u} = 5.8\%$. Using the calibration of the labor market above, these targets imply steady-state employment $\overline{n} = 0.951$, and steady-state labor market tightness $\overline{\theta} = 0.45$.[33] Finally, I estimate the production function parameter α at 0.74, which is larger than the labor share because of monopolistic rents and recruiting costs.

Evidence on production function. At business cycle frequency, production inputs do not adjust fully to change in employment. Capital is especially slow to adjust, and is assumed to be constant in my production function. Since my model aims to shed new light on cyclical fluctuations in unemployment, it is not concerned by long-term fluctuations in the stock of capital. In this context,

[32] The unemployment series used is a quarterly average of monthly unemployment rates constructed by the BLS. The average unemployment rate over the same period is nearly identical, at 5.9%.

[33] Refer to Appendix C for a complete description of the steady-state of the general-equilibrium model.

assuming a short-run production function with diminishing marginal returns is reasonable.[34] At longer horizon, the production function may exhibit diminishing marginal returns to labor if some production inputs such as land or managerial talent are in fixed supply.

7.5 Wage rigidity: estimates and evidence

Estimate from aggregate wage data. Table 2 estimates the elasticity of aggregate wages with respect to labor productivity. I use average real hourly earning in the nonfarm business sector as the wage series. I estimate $\gamma = 0.44$ $(s.e. = 0.07)$, in line with previous studies (for example, Hagedorn and Manovskii 2008).[35]

I also perform robustness checks, as detailed in Table 2, which confirm that aggregate wage data exhibit mild procyclicality. In particular, I use as wage series the measure of total compensation in private industries constructed by the BLS as part of the Employment Cost Index (ECI).[36] This index measures change in the cost of labor, controlling for employment shifts among occupations and industries over the business cycle. Thus, this wage measure is not prone to the composition bias previously exhibited in other aggregate wage data by Solon et al. (1994). I find $\gamma = 0.28$ $(s.e. = 0.10)$. This estimate does not suggest a stronger procyclicality of wages once composition bias is controlled for.

Microevidence. I now present estimates of wage rigidity obtained in the literature using microdata on workers' individual wages. Panel data on individual workers usually show more cyclicality than aggregate data because they are less prone to composition effects. Surveying studies such as Bils (1985), Solon et al. (1994), or Shin and Solon (2008), Pissarides (2009) estimates the productivity-elasticity of wages for job stayers in the 0.3–0.5 range for the U.S.[37]

However, Pissarides (2009) argues that wages of job movers may actually be more cyclical. The

[34] Elsby and Michaels (2008) make this argument as well.

[35] The wage series is seasonally-adjusted, average hourly earning in the nonfarm business sector, constructed by the BLS Current Employment Statistics (CES) program. It is deflated by the seasonally-adjusted Consumer Price Index (CPI) for all urban households constructed by BLS. Average hourly earning is a quarterly series, and CPI is a quarterly average of monthly series. The quarterly labor productivity series used is the one presented in Section 7.1. Wage and productivity series are detrended using an HP filter with smoothing parameter 10^5. The sample period is 1964:Q1–2009:Q2.

[36] Compensation of private industry workers is a seasonally-adjusted, quarterly series that I deflate using the CPI. The quarterly labor productivity series used is that presented in Section 7.1. Wage and productivity series are detrended using an HP filter with smoothing parameter 10^5. The sample period used is 2001:Q1–2009:Q2

[37] The studies surveyed by estimate unemployment-elasticities. Therefore,Pissarides (2009) estimates a relationship between productivity and unemployment. He then multiplies unemployment-elasticities by -0.34 to convert them to productivity-elasticities.

task of estimating wage rigidity for newly hires is arduous. As noted by Gertler and Trigari (2009), there are obvious composition effects among jobs newly created over the business cycle, which are difficult to control for. For instance, it is possible that workers hired in recessions and booms differ, and that the types of jobs created and destroyed differ as well. In particular, workers may accept lower-paid jobs in recessions ("stopgap jobs"), and move to better jobs during expansions. Martins et al. (2009) are one of the first studies to estimate wage flexibility for new hires, controlling for these composition effects. They use Portuguese employer-employee longitudinal data over the period 1982–2007.[38] Surprisingly, their estimates of wage cyclicality for job movers in line with those of Solon et al. (1994) for all job stayers in the U.S.—an unemployment-elasticity of -1.5, which corresponds to a productivity-elasticity of around 0.5. This suggests that the cyclicality of entry wages may not be higher than that of wages paid to continuing workers.

A recent study by Haefke et al. (2008) estimates the productivity-elasticity of job movers using panel data for U.S. workers. They do not control for composition bias in the type of jobs accepted by workers over the cycle because of data limitations. As expected, their estimate is higher than that of Martins et al. (2009). For a sample of production and supervisory workers over the period 1984–2006, they obtain a productivity-elasticity of total earnings of 0.7. I use an elasticity of $\gamma = 0.7$ in my calibration, and show in Section 8 that this estimate suffices to deliver large fluctuations in total and cyclical unemployment over the business cycle.

Ethnographic evidence. I have calibrated the wage schedule to be consistent with empirical evidence. I now present ethnographic evidence in support of this particular functional form.

This wage schedule does not depend on the marginal product of labor in the firm, and does not respond to labor market conditions directly.[39] The disconnect between wages and both marginal productivity and labor market conditions can be explained by the rise of the personnel management movement after World War I, which led to a widespread adoption of internal labor markets within firms (Jacoby 1984, James 1990). Doeringer and Piore (1971) documented that in these structures, which are motivated by concerns for equity within firms, wages are tied to job description, and are therefore insensitive to labor market and marginal productivity conditions. Galuscak et al. (2008) provide recent evidence on the major role played by internal labor markets (and not

[38] The authors argue that their results are not driven by specificities of the Portuguese labor market, since wages tend to exhibit more cyclicality in Portugal than in the U.S.

[39] If a firm decides to increase employment, marginal product of labor falls but wages remain constant. If labor market conditions change independently of productivity (e.g., if separation rate or recruiting costs vary), wages remain constant.

external labor markets) to explain wages paid.

Labor market institutions could also hamper downward wage adjustments, even in the face of a slack labor market. For instance, the National Industry Recovery Act of 1933 is often blamed for persistent high real wages during the Great Depression (Temin 1990, Cole and Ohanian 2004). More recently, unions adamantly opposed nominal pay cuts during the Finnish Depression of 1991-1993, in spite of rampant unemployment (Gorodnichenko et al. 2009).

Lastly, managerial best practices oppose pay cuts. Detailed interviews of compensation managers by Bewley (1999) provide evidence that employers avoid pay cuts even in bad times because they believe pay cuts antagonize workers and ultimately reduce productivity and profitability. Bewley's findings are confirmed by surveys of human resource officers (Blinder et al. 1998, Campbell and Kamlani 1997), and by the study of workers' reactions to pay cuts in natural experiments (Krueger and Mas 2004, Mas 2006).

8 Quantitative Characterization of Unemployment Components

Having calibrated the model with matching frictions and job rationing, I now study the quantitative properties of cyclical and frictional unemployment in this model.

8.1 Beveridge Curve

To provide some intuition, I represent steady-state total, cyclical, and frictional unemployment as functions of labor market tightness. These curves in the (θ, u) plane are Beveridge curves, and shifts in labor productivity induce movements along these curves.

Figure 2 depicts the standard Beveridge curve (solid line), which relates total unemployment to labor market tightness, and its decomposition into curves for frictional (dotted line) and cyclical (dashed line) unemployment. When total unemployment rises above 5%, which corresponds to a labor market tightness below 0.6, some cyclical unemployment prevails. In this case, productivity a is in the interval of rationing $\mathcal{A} = (0, a^C)$. As productivity falls further, labor market tightness falls and cyclical unemployment increases. When labor market tightness is above 0.6, jobs are not rationed ($a \geq a^C$), and all unemployment is frictional. In this regime, frictional unemployment increases as labor market tightness decreases. When labor market tightness falls below 0.6, jobs are rationed ($a < a^C$) and frictional unemployment decreases as labor market tightness decreases.

It is clear that steady-state frictional unemployment is bounded above and reaches its maximum for $a = a^C$. With my calibration, it never rises above 5%.

8.2 Impulse response functions in log-linearized model

To understand how labor market variables respond to a productivity shock, I compute impulse response functions (IRFs) in a log-linearized model.[40] I perturb the log-linearized model with an adverse shock to labor productivity of one standard deviation (-0.0026). The IRFs are shown in Figure 5. On impact, output, consumption, employment, labor market tightness, the number of hires, and wages fall discretely.

The drop in labor market tightness is about 15 times the drop in labor productivity. This implies an elasticity of labor market tightness with respect to productivity of 15. The empirical counterpart of this elasticity is the coefficient obtained in an OLS regression of log labor market tightness on log productivity. This coefficient can be derived from Table 3 that presents moments in U.S. data for the period 1964–2009: $\rho(\theta, a) \times \sigma(\theta)/\sigma(a) = 0.479 \times 0.344/0.019 = 8.67$. The simulated elasticity is higher than its empirical counterpart. Therefore, a small amount of wage rigidity (as observed in microdata for new hires) is sufficient to generate fluctuations in labor market tightness in response to labor productivity shocks of a magnitude observed in the data.

This result contributes to a large literature on the role of wage rigidity in explaining unemployment fluctuations and confirms a comparative-static exercise presented in Hall and Milgrom (2008). Following Shimer's (2005) critique of the standard search-and-matching model, several studies used variants of the standard model involving higher wage rigidity to generate greater fluctuations in unemployment.[41] This line of research has been criticized for exaggerating the rigidity of wages in spite of empirical evidence suggesting that wages for new hires are more flexible than that of existing workers (for example, Pissarides 2009, Haefke et al. 2008). Calibrating my model with an estimate of wage cyclicality from microdata of new hires, I show that even a small amount of rigidity is sufficient to amplify productivity shocks as much as in the data.

Since there are no endogenous separations, unemployment behaves as a state variable, and it does not jump on impact. Instead, it slowly builds, peaking around 4 months after the productivity shock. This result is in line with the empirical findings of Stock and Watson (1999),

[40] For further details on the log-linearization, please see Appendix C.

[41] For instance, Hall (2005a) studies the effect of real wage rigidity, Hall and Milgrom (2008) propose a different bargaining mechanism that delivers more rigid wages, and Gertler and Trigari (2009) introduce staggered real-wage setting.

which suggest that employment lags the business cycle by approximately one quarter in the U.S., whereas productivity slightly leads the cycle.

Finally, Figure 6 shows how cyclical and frictional unemployment respond to a negative productivity shock. Cyclical unemployment jumps up on impact. Frictional unemployment, on the other hand, jumps down. This simulation result confirms the theoretical results derived in Section 5: when productivity is in the interval of rationing, which is the case at steady state, and an adverse productivity shock hits the economy, total and cyclical unemployment rise, while frictional unemployment falls.

8.3 Simulated moments

Before delving further into a quantitative analysis of unemployment and its components, I verify that the model provides a sensible description of reality by comparing important simulated first and second moments to their empirical counterparts. A comparison of simulated and empirical moments suggests that in spite of its simplicity, this model performs well at replicating labor market fluctuations.

First moments. The average unemployment rate for the period 2000–2009 is $\overline{u} = 5.3\%$. Using estimates of the job destruction rate and matching function, as well as equation (A16) that relates steady-state unemployment and labor market tightness, I infer that steady-state labor market tightness $\overline{\theta} = 0.54$. Its empirical counterpart, average labor market tightness from JOLTS over the period 2000–2009, is 0.58. The similarity of these two values suggests that the matching paradigm, together with my calibration, describes mechanics of the labor market well.

Second moments. I now focus on second moments of the unemployment rate U, the vacancy rate V, labor market tightness $\theta = V/U$, real wage W, output Y, and labor productivity a. The moments in U.S. data during 1964–2009 are presented in Table 3. Unemployment U is a quarterly average of the monthly unemployment series constructed by the BLS. Output Y is real output in the nonfarm business sector. Labor productivity a is constructed in Section 7.1. The quarterly real wage series is average hourly wage in nonfarm business sector. To construct a series of vacancies over the period, I merge the job openings data from JOLTS for 2001–2009, with the Conference Board help-wanted advertising index, measured as the number of help-wanted advertisements in major newspapers, for 1964–2001. This dataset is a standard proxy for vacancies (for example, Shimer 2005). JOLTS began only in December 2000, and the Conference Board data become less relevant after 2000 due to the major role taken by the Internet as a source of job advertising, which made the merger of both datasets necessary. I construct labor market tightness θ as the ratio of vacancies to unemployment, constructed by the BLS from CPS. All variables are seasonally-adjusted and expressed in logs as deviations from trend obtained by applying an HP filter with smoothing parameter 10^5 to the quarterly data.

I generate a series of productivity shock (z_t) with $z_t \sim N(0, 0.0026)$ for all t, with which I perturb the log-linearized system. I obtain weekly series of log-deviations for all the variables. I then record values every 12 weeks for the series (Y_t), (a_t), and (W_t), which have quarterly frequency in the data. I record values every 4 weeks and then take quarterly averages for the series (U_t), (V_t), and (θ_t), which have monthly frequency and are averaged to quarterly series in the data. I discard the first 1,200 weeks of simulation to remove the effect of initial conditions. I have simulated a total of 200 samples of 182 quarters (2,184 weeks), corresponding to quarterly data from 1964:Q1 to 2009:Q2. Each sample gives me an estimate of the means of the model-generated data. I compute standard deviations of estimated means across model-generated samples, which indicate the precision of model predictions. Simulated moments are presented in Table 4.

Simulated and empirical moments for productivity are similar because I calibrate the productivity process to match the data. All other simulated moments are outcomes of the mechanics of the model. For unemployment, vacancies, and labor market tightness, simulated standard deviations are close, but lower than empirical moments. Simultaneously, simulated correlation of these variables with productivity is close to 1, but empirical correlations are below 0.5. This implies that in the data, fluctuations in labor market variables are driven in part by productivity, and in part by other shocks. Because my simple model only considers productivity shocks, it cannot achieve the degree of volatility observed in the data. However, as seen with IRFs, amplification is at least as strong as in the data. The simulated correlation of unemployment with vacancies is -0.92, very close to the empirical value of -0.89.

The behavior of output is similar in the model and the data. But aggregate wages vary twice as much in the data as in the model. Nominal factors are one source of discrepancy.[42] In addition, wage and unemployment are too closely correlated in the model, because rigid wages are the only

[42]The correlation of wages to productivity is only 0.646 in the data. Wages have been documented to exhibit a significant amount of nominal rigidity (for example, Akerlof et al. 1996). Adding log price level as a regressor in a regression of log wage on log productivity increases the R^2 from 0.19 to 0.65. My preferred estimate for the coefficient on price is around -0.35 (s.e.=0.03). The coefficient on productivity falls to 0.30 (s.e.=0.06).

channel through which productivity shocks lead to unemployment fluctuations. In reality, other shocks and channels are at play.

Comparison with simulated moments in the MP, MPS, and SZ model. I now briefly compare the empirical performance of my model to that of the other search-and-matching models presented in Section 6.[43] Tables 5, 6, 7, and 8 display the simulated moments in the MP model, MPS model with $\gamma = 0$, MPS model with $\gamma = 0.7$, and SZ model. These models do not match empirical evidence as well the model with job rationing.

In the MPS model with $\gamma = 0$, a high degree of wage rigidity produces too much amplification. The productivity-elasticity of labor market tightness is $0.809 \times 4.708/0.018 = 211$, which is more than 20 times the elasticity observed in U.S. data. In the MPS model with $\gamma = 0.7$, wages are as flexible as in my model. But the gross marginal profit is extremely small because firms are perfectly competitive, and it is independent of employment. Thus, a small shock to productivity is much more amplified in the MPS model with $\gamma = 0.7$ than in the job-rationing model.

In the MP and SZ models productivity shocks are not sufficiently amplified. As highlighted by Shimer (2005), the elasticity of labor market tightness with respect to productivity is close to 1 ($0.975 \times 0.018/0.018 \approx 1$). In the general equilibrium model presented in Section 3, the value to the household of having a member unemployed is nil: unemployed workers search for jobs, and they neither have time for leisure nor for home production; moreover, I abstract from any intervention by the government, so that there is no unemployment insurance. Since unemployment is a costly experience, bargained wages are low except if workers have a lot of bargaining power. In practice, targeting a stead-state unemployment rate of 5.8% requires setting a high bargaining power in both models. Therefore, wages in the calibrated model are very flexible, and labor market variable are very stable. To increase the amplification of productivity shocks, the value of unemployment must be increased—unemployment should be a more pleasant experience.[44] However, this assumption contradicts empirical evidence that shows that a spell of unemployment has a large and long-lasting negative impact on future health and professional outcomes (for example, von Wachter et al. 2007, Sullivan and von Wachter 2009).

[43] I calibrate the MP model (presented in Section 6.1), the MPS model (presented in Section 6.2), and the SZ model (presented in Section 6.3) in Appendix D.

[44] For instance, Hagedorn and Manovskii (2008) show that the canonical MP model matches empirical moments if the value of time of an unemployed worker is 95.5% that of an employed worker.

8.4 Actual and model-generated unemployment

This section compares model-generated unemployment with U.S. post-war unemployment. For this analysis, I cannot use a log-linearized model because cyclical and frictional unemployment are nonlinear. When the economy departs from steady-state, cyclical unemployment falls to zero in booms; frictional unemployment is increasing with productivity in the interval of rationing, but is decreasing outside it; this makes the log-linear model a poor approximation for this exercise. Instead I use the nonlinear model.

First, I approximate the AR(1) stochastic process for labor productivity estimated in Section 7 as a 200-state Markov chain (Tauchen 1986, Tauchen and Hussey 1991). I detrend the labor productivity series constructed in Section 7.1 from U.S. data, using an HP filter with smoothing parameter 10^5. I discretize detrended productivity in the state space of the Markov chain for labor productivity. This discretization yields a series of state realizations that I use to stimulate the model.

Second, I assume that flows into and out of employment balance each other (Assumption 9). This assumption, together with the assumption that productivity shocks follow a Markov process, allow me to express implicitly equilibrium labor market tightness, employment, and unemployment as a function of labor productivity. Then solving the nonlinear, rational-expectation model boils down to solving a system of nonlinear equations with as many equations as states of productivity, which can easily be done numerically.

Third, since each state of labor productivity is associated with a given unemployment rate, I can associate each observation of quarterly productivity in U.S. data with a model-generated unemployment rate. Comparing simulated and actual unemployment indicates how much of unemployment fluctuations can be explained by the model.[45] The two series are shown on the top graph in Figure 7. Both have the same standard deviation of 0.010. While not perfect, the match is remarkably good given the simplicity of the model: the correlation of the two series is 0.55 and even higher on the first half of the sample.

Labor productivity is not adjusted for variable factor utilization. Therefore, fluctuations in labor productivity may be partly endogenous. To address this issue, I construct another series of model-generated unemployment using the quarterly, utilization-adjusted total factor productivity series (TFP) from Fernald (2009) as the model driving force. Actual and model-generated unemployment are shown on the bottom graph in Figure 7. The fit of the model remains good.

[45] More precisely, I detrend quarterly unemployment using an HP filter with smoothing parameter 10^5, in order to make it comparable to simulated unemployment obtained with a HP-filtered productivity series.

8.5 Historical decomposition of unemployment

The preceding sections suggest that the model matches empirical data quite well. I can now examine how the model decomposes U.S. post-war unemployment into cyclical and frictional components. I pursue the exercise from Section 8.4, and associate each observation of quarterly productivity in U.S. data with model-generated cyclical and frictional unemployment rates computed with (35) and (36). The top left graph in Figure 8 shows the resulting decomposition of model-generated unemployment. In this framework, unemployment is solely frictional below 5.2%. Above 5.2%, there is some cyclical and some frictional unemployment. Moreover, it is clear that frictional unemployment falls when cyclical unemployment rises. Indeed, spikes in unemployment are accompanied by sharp drops in frictional unemployment, and steep rises in cyclical unemployment. Based on the actual productivity series for the U.S., the model predicts that unemployment should have been highest in the 1981–1982 recession. It predicts that unemployment should have peaked at 9.2%, with frictional unemployment falling to 1.6% of the labor force, and cyclical unemployment reaching 7.6%. However, actual (detrended) unemployment only reached 8.5% during this recession. On the other hand, the model underestimates unemployment in the current recession: unemployment reached 9.2% (as of 2009:Q2), but the model only predicts a peak of unemployment at 8.5%, with frictional unemployment at 2.3% and cyclical at 6.2%. These discrepancies suggest that factors other than productivity drove unemployment fluctuations during these periods.

I repeat the decomposition exercise using utilization-adjusted TFP series from Fernald (2009) as the driving force in the model. This decomposition is presented on the top right graph in Figure 8. Results are very similar to those obtained with labor productivity as driving force.

Finally, I approach the decomposition exercise from another angle. I determine the productivity series such that model-generated unemployment matches actual unemployment exactly. Then, I infer cyclical and frictional unemployment rates from this productivity series.[46] The decomposition is shown on the bottom graph in Figure 8. Current events illustrate how the composition of unemployment drastically changes over the business cycle. In 2007:Q2, actual unemployment was at 4.9%, all of which was frictional. In 2008:Q2, actual unemployment was at 5.8%, of which 4.3% was frictional and 1.5% was cyclical. Finally, in 2009:Q2, actual unemployment reached 9.2%,

[46]This model-generated productivity does not exactly match actual productivity, just as model-generated unemployment cannot match actual unemployment if I stimulate the model with actual productivity (see Section 8.4). This limitation notwithstanding, the exercise provides another useful illustration of the theoretical results of the paper.

frictional unemployment fell to 1.6%, and cyclical unemployment increased drastically to 7.6%.

8.6 Simulated moments

Table 9 reports the moments of total, cyclical, and frictional unemployment obtained by simulating the calibrated model. I use the same framework as in the two previous sections (Sections 8.4 and 8.5). I simulate a weekly series of labor productivity using the Markov-chain approximation described in Section 8.4. I obtain a weekly series for all the variables in the model. I record values every 12 weeks to obtain quarterly series for all variables. I discard the first 1,200 weeks of simulation in order to remove the effect of initial conditions. I have simulated a total of 200 samples of 182 quarters, corresponding to quarterly data from 1964:Q1 to 2009:Q2. I compute means of the model-generated data, and standard deviations of estimated means across model-generated samples.

On average, frictional unemployment is about twice as high as cyclical unemployment (3.9% versus 2.0%). But cyclical unemployment is more than twice as volatile as frictional unemployment; quarterly standard deviations are 0.021 and 0.008 respectively. Cyclical unemployment is also nearly twice as volatile as total unemployment, whose quarterly standard deviation is 0.013.

8.7 Robustness checks

IRFs in the nonlinear model. I use Fair and Taylor's (1983) shooting algorithm with perfect foresight to compute exact impulse response functions (IRFs) to large negative labor productivity shocks in the exact nonlinear model. Figure 9 displays the responses of labor market tightness, total, cyclical, and frictional unemployment. The first observation from these exact IRFs is that after a negative productivity shock, cyclical unemployment rises while frictional unemployment decreases, consistent with the IRFs in the log-linearized model.

The second observation from the decomposition of unemployment is a mechanism through which unemployment lags productivity in downturns. For large adverse shocks, frictional unemployment may even become negative on impact. When frictional unemployment becomes negative, matching frictions actually reduce unemployment. In the periods immediately following a drop in productivity, firms intertemporally substitute recruiting from future periods to the present. Firms take advantage of a slack labor market to recruit at low cost now, instead of recruiting in a tighter labor market in the future. These intertemporal substitution effects caused by matching frictions slows the growth of unemployment in the short run, and delay the spike of unemployment by about a quarter.

No-inefficient-separation condition. When I compute the responses shown in Figure 9 with the shooting algorithm, I allow firms to lay workers off if it is profitable to do so. Labor market tightness, however, always remains positive; therefore, it is never optimal for firms to lay workers off under the calibrated wage schedule, even after very large productivity shocks.

9 Concluding Remarks and Extensions

By modeling unemployment as the result of matching frictions and job rationing, I develop a tractable, general model of the labor market in which unemployment can be decomposed into cyclical and frictional components. I find that when adverse economic shocks occur, total unemployment increases, cyclical unemployment increases, but frictional unemployment decreases. Thus, matching frictions account for unemployment in good and normal times and job rationing accounts for nearly all unemployment in bad times. The degree of wage rigidity and diminishing marginal returns to labor observed in the data predict some cyclical unemployment in the average state and generate fluctuations in cyclical unemployment that are more than twice as large as those of total and frictional unemployment.

These fluctuations in cyclical and frictional unemployment suggest that optimal unemployment-reducing policies should adapt to the changing state of the labor market. In a work in progress, I show that when job rationing generates inefficiently high unemployment, labor market policies can improve welfare significantly. Specifically, I evaluate three labor market policies—direct employment, placement services, and wage subsidies—over the business cycle. I compute state-dependent fiscal multipliers (the increase in social welfare obtained by spending one dollar on a policy) to find that placement services are the most effective policy in normal and good times, but direct employment and wage subsidies are more effective in bad times. Intuitively, in bad times, frictional unemployment is low; placement services aim to further reduce this component, and are therefore ineffective. The effectiveness of direct employment is a function of how much it crowds private employment out; in bad times, competition for workers is weak and crowding out is limited; thus, this policy is very effective.

This paper is a first attempt at providing a unified framework to study unemployment, and it has limitations that will have to be addressed in future research. Most importantly, although I propose a simple wage rule that yields job rationing, I do not propose an associated wage-setting mechanism. Insights from ethnographic studies of the workplace and empirical evidence suggest that job rationing is a reality of the labor market. Yet it cannot be generated by standard wage-setting mechanisms. An important agenda for future research is to design a tractable wage-setting mechanism explaining the wage rigidity observed in the data, to improve our understanding of job rationing. [47]

Second, flows out of employment are countercyclical, in particular because layoffs are quite countercyclical (for example, Davis et al. 2006, Fujita and Ramey 2007, Elsby et al. 2009). My model abstracts from this issue and assumes a constant, exogenous rate of job destruction. Understanding these job destructions and their interaction with job rationing should be explored in future research.

Third, the model is simplistic in that there are only productivity shocks. There is growing evidence that other types of shocks are likely to affect the labor market. For instance, future work could explore how demand shocks or financial disturbances affect the behavior of unemployment and its components.

To conclude, the model presented in this paper represents an improvement over the current unemployment literature by bringing together two strands of research. It presents many promising avenues that will develop our theoretical understanding of the causes of unemployment, and also offer novel policy insights.

[47] Microfounded models of wage rigidity have recently been developed to improve realism of the wage setting. However, these models remain too complex to be analytically tractable in macroeconomic models. For instance, Kennan (2006) uses asymmetric information in a search model; Rudanko (2009) builds a model in which long-term contracting and insurance motives between risk-neutral firms and risk-averse agents yield wage rigidity; Elsby (2009) builds a dynamic model of downward nominal wage rigidity based on loss aversion.

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A Proofs

Proof of Lemma 1. Using the approximation of the matching function given by (3), I get the following partial derivatives:

$$\nabla_u h = \omega \cdot e^{-\frac{\omega u}{v}}$$
$$\nabla_v h = 1 - \left(1 + \frac{\omega u}{v}\right) e^{-\frac{\omega u}{v}}.$$

Given that $g : x \mapsto (1+x)e^{-x}$ is decreasing on $\mathbb{R}^{+}$ and $g(0) = 1$, then for any $(u, v) \in \mathbb{R}^{++} \times \mathbb{R}^{++}$, $\nabla_u h > 0$ and $\nabla_v h > 0$. Equations (1) and (2) define $u,\ v$ as implicit functions of Θ. For any $\Theta > 0$, the system admits a unique solution. The Implicit Function Theorem applies, and $u : \mathbb{R}^{++} \to [0, 1]$ and $v : \mathbb{R}^{++} \to \mathbb{R}^{++}$ are both continuous and differentiable functions of Θ. Differentiating (1) and (2) yields:

$$\nabla_\Theta u(-s - \nabla_u h) = \nabla_v h \cdot \nabla_\Theta v \tag{A1}$$
$$\nabla_\Theta u = -1 + \nabla_\Theta v. \tag{A2}$$

From (A1), I infer that for any $\Theta \in \mathbb{R}^{++}$, $\nabla_\Theta u \cdot \nabla_\Theta v < 0$. From (A2), I infer that for any $\Theta \in \mathbb{R}^{++}$, $\nabla_\Theta u < \nabla_\Theta v$. This proves the lemma.

Proof of Proposition 1. The proposition follows from equations (4), (5), and Lemma 1.

Proof of Lemma 2. I first define the Lagrangian for firm i's problem, taking into account possibilities of layoffs:

$$\begin{aligned}\mathcal{L} = & \mathbb{E}_0 \sum_{t \geq 0} \delta^t \Bigg\{ Y_t \left(\frac{P_t(i)}{P_t}\right)^{1-\epsilon} - N_t(i) \cdot [S(N_t(i), a_t) + X(\theta_t, c) + Z(\theta_{t+1}, c)] \\ & - \mathbf{1}\{N_t(i) > (1-s)N_{t-1}(i)\} R(\theta_t, c) \cdot H_t(i) + \nu_t \left[F(N_t(i), a_t) - Y_t \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} \right] \Bigg\}.\end{aligned}$$

The firm faces a production constraint. Let $\hat{P}_t(i)$ be such that:

$$Y_t \left(\frac{\hat{P}_t(i)}{P_t}\right)^{-\epsilon} = F((1-s)N_{t-1}(i), a_t).$$

The maximum marginal profit that the firm can extract without laying workers off is

$$\hat{\nu}_t(i) = \frac{1}{\mathcal{M}} \cdot \frac{\hat{P}_t(i)}{P_t}.$$

Next, I define $\forall\ t \geq 0$:

$$\nu_t^L(i) = \frac{(1-s)N_{t-1}(i)\nabla_N S((1-s)N_{t-1}(i), a_t) + W_t - \delta \mathbb{E}_t\left[\nabla_{N_t}\mathcal{L}_{t+1}\right]}{\nabla_N F((1-s)N_{t-1}(i), a_t)}$$
$$\nu_t^H(i) = \frac{(1-s)N_{t-1}(i)\nabla_N S((1-s)N_{t-1}(i), a_t) + W_t + R(\theta_t, c) - \delta \mathbb{E}_t\left[\nabla_{N_t}\mathcal{L}_{t+1}\right]}{\nabla_N F((1-s)N_{t-1}(i), a_t)},$$

where I define $\forall\ t \geq 0$:

$$\mathcal{L}_{t+1} = \sum_{\tau \geq t+1} \delta^{\tau-(t+1)} \Bigg\{ Y_\tau \left(\frac{P_\tau(i)}{P_\tau}\right)^{1-\epsilon} - N_\tau(i)\cdot\left[S(N_\tau(i), a_\tau) + X(\theta_\tau, c) + Z(\theta_{\tau+1}, c)\right]$$
$$-\mathbf{1}\left\{N_\tau(i) > (1-s)N_{\tau-1}(i)\right\} R(\theta_\tau, c)\cdot H_\tau(i) + \nu_\tau\left[F(N_\tau(i), a_\tau) - Y_\tau\left(\frac{P_\tau(i)}{P_\tau}\right)^{-\epsilon}\right]\Bigg\}.$$

Computing $\nu_t^L(i)$ and $\nu_t^H(i)$ requires computing $\mathbb{E}_t\left[\nabla_{N_t}\mathcal{L}_{t+1}\right]$. Let $\mathcal{F}$ be the $\sigma-$algebra generated by future realizations of the stochastic process $\{a_\tau, \tau \geq t+1\}$, taking as given $\mathcal{I}_t$, the information set at time t. I partition $\mathcal{F}$ as follows:

$$\mathcal{F} = \mathcal{F}^+ \cup \mathcal{F}^- \cup_{h=1}^{+\infty} \mathcal{F}^h. \tag{A3}$$

$\mathcal{F}^+$ is the subset of future realizations of $\{a_t\}$ such that there is hiring next period. $\mathcal{F}^-$ is the subset such that there are layoffs next period. Last, for $h \geq 1$, $\mathcal{F}^h$ is the subset such that there is a hiring freeze for the h next periods. Let $p^+ = \mathbb{P}(\mathcal{F}^+)$, $p^- = \mathbb{P}(\mathcal{F}^-)$, and $p^h = \mathbb{P}(\mathcal{F}^h)$ be the measure of these subsets. Using the law of total probability over this partition:

$$\mathbb{E}_t\left[\nabla_{N_t}\mathcal{L}_{t+1}\right] = p^+ \times \mathbb{E}_t\left[\nabla_{N_t}\mathcal{L}_{t+1}|\mathcal{F}^+\right] + p^- \times \mathbb{E}_t\left[\nabla_{N_t}\mathcal{L}_{t+1}|\mathcal{F}^-\right] + \sum_{h=1}^{+\infty} p^h \times \mathbb{E}_t\left[\nabla_{N_t}\mathcal{L}_{t+1}|\mathcal{F}^h\right].$$

It is easy to show that:

$$\mathbb{E}_t\left[\nabla_{N_t}\mathcal{L}_{t+1}|\mathcal{F}^+\right] = (1-s)\mathbb{E}_t\left[R(\theta_{t+1}, c)|\mathcal{F}^+\right]$$
$$\mathbb{E}_t\left[\nabla_{N_t}\mathcal{L}_{t+1}|\mathcal{F}^-\right] = 0$$
$$\mathbb{E}_t\left[\nabla_{N_t}\mathcal{L}_{t+1}|\mathcal{F}^h\right] = \mathbb{E}_t\Bigg[\sum_{j=t+1}^{t+h} \delta^{j-(t+1)}(1-s)^{j-t}\Bigg\{\frac{\nabla_N F((1-s)^{j+1-t}N_{t-1}(i), a_j)}{\mathcal{M}}$$
$$\times\left(\frac{F((1-s)^{j+1-t}N_{t-1}(i), a_j)}{Y_j}\right)^{-1/\epsilon} - \nabla_N S((1-s)^{j+1-t}N_{t-1}(i), a_j) - W_j\Bigg\}|\mathcal{F}^h\Bigg].$$

Therefore, $\nu_t^L(i)$ and $\nu_t^H(i)$ are well defined, and depend on future realizations of $\{\theta_\tau, \tau \geq t+1\}$, as well as on employment at the beginning of period t: $(1-s)N_{t-1}(i)$. I assume that marginal cost is strictly increasing in $N_t(i)$, so that the firm's optimization has a unique solution (the marginal profit function strictly decreases with $N_t(i)$). $\nu_t^L(i)$ is the lowest marginal cost that the firm can achieve by keeping all its workforce. This is achieved by freezing hiring. $\nu_t^H(i) > \nu_t^L(i)$ is the lowest marginal cost the firm can achieve, while recruiting workers. It is achieved by recruiting an infinitely small amount of workers. Then, the optimal decision of the firm is obtained by comparing $\nu_t^L(i), \nu_t^H(i)$, and $\hat{\nu}_t(i)$.

Proof of Proposition 2. In symmetric environment, if a firm freezes hiring, all firms do so, $\theta_t = 0$, $R(\theta_t, c) = 0$, and for all i, $\nu_t^L(i) = \nu_t^H(i)$. This means that the hiring freezes occur with probability 0. Either all firms recruit, or they all lay workers off. Moreover, in symmetric environment, all firms set the same price. Using Lemma 2, we know that at a symmetric equilibrium, the employment decision of firms is determined by the value of:

$$G(N_{t-1}, a_t) = \left\{ \frac{\mathcal{M}}{\nabla_N F((1-s)N_{t-1})} \right\} \{(1-s)N_{t-1} \cdot \nabla_N S + S((1-s)N_{t-1}, a_t) + \mathbb{E}_t [Z(\theta_{t+1}, c)] - (1-s)\delta \cdot \mathbb{E}_t \left[R(\theta_{t+1}, c) | \mathcal{F}^+ \right] \cdot p^+ \}.$$

$p^+ \in [0, 1]$ is the measure of future states of the world in which there is recruiting in equilibrium in the next period (see proof of Lemma 2). I assume that $G(N, a)$ is strictly increasing in N so that the symmetric equilibrium (if it exists) is unique. Then, recruiting occurs in period t in a symmetric equilibrium if and only if $G(N_{t-1}, a_t) < 1$. Therefore, a necessary and sufficient condition to avoid layoffs is $\forall t \geq 0$,

$$\left\{ \frac{\mathcal{M}}{\nabla_N F((1-s)N_{t-1})} \right\} \{(1-s)N_{t-1} \nabla_N S + S((1-s)N_{t-1}, a_t) + \mathbb{E}_t [Z(\theta_{t+1}, c)] - (1-s)\delta \mathbb{E}_t [R(\theta_{t+1}, c)] \} \leq 1.$$

I use $\mathbb{E}_t [R(\theta_{t+1}, c)] = \mathbb{E}_t [R(\theta_{t+1}, c) | \mathcal{F}^+] \cdot p^+$ because $\mathbb{E}_t [R(\theta_{t+1}, c) | \mathcal{F}^-] = 0$, and $p^h = 0$ for all h (using partition defined by (A3)).

Proof of Lemma 3. Assume that $\exists (N^*, a^*) \in (0, 1] \times \mathbb{R}^{++},\ J(N^*, a^*) < 0$. I apply the Intermediate-Value theorem, given that $J(., a^*) : (0, 1] \to \mathbb{R}$ is continuous, $\lim_{N \to 0} J(N, a^*) > 0$, and $J(N^*, a^*) < 0$. This implies that the equation $J(N, a^*) = 0$ admits at least one solution, whose uniqueness derives from the strict monotonicity of $J(., a^*)$ (Assumption 6). Given that J is continuously differentiable, and $\nabla_N J \neq 0$, the Implicit Function Theorem indicates that there exist an open interval $\mathcal{I}$ centered at a^* such that for all $a \in \mathcal{I}$, $J(N, a) = 0$ admits a unique solution. The function $N^C : \mathcal{I} \to (0, 1)$ such that $J(N^C(a), a) = 0$ is therefore well-defined.

Proof of Lemma 4. Note that $\cup \mathcal{I} \neq \emptyset$ because there exists at least one open interval that satisfies Lemma 3.

Case 1: I assume $\lim_{a \to +\infty} J(1, a) < 0$. Obviously, $\cup \mathcal{I} \subseteq \mathbb{R}^{++}$, since all $\mathcal{I} \subseteq \mathbb{R}^{++}$. Next, let $a^+ \in \mathbb{R}^{++}$. Then $J(1, a^+) < 0$ because $\lim_{a \to +\infty} J(1, a) < 0$ and $\nabla_a J > 0$. I can apply the Intermediate-Value theorem as in the proof of Lemma 3, to show that $J(N, a^+) = 0$ admits a unique solution. Since $\mathbb{R}^{++}$ is an open interval, I conclude that $\mathbb{R}^{++} \subseteq \cup \mathcal{I}$. Finally, $\mathbb{R}^{++} = \cup \mathcal{I} = \mathcal{A}$.

Case 2: I assume $\lim_{a \to +\infty} J(1, a) \geq 0$. We also assumed that $\exists (N^*, a^*) \in (0, 1] \times \mathbb{R}^{++},\ J(N^*, a^*) < 0$. Since $\nabla_N J < 0$, $J(1, a^*) < 0$. Applying the Intermediate-Value theorem to the continuous function $J(1, .)$, I conclude that there exists a unique $a^C \in \{\mathbb{R}^{++}, +\infty\},\ J(1, a^C) = 0$. I can repeat the proof of Lemma 3 with any $a^+ \in (0, a^C)$ to show that $J(N, a^+) = 0$ admits a unique solution, because $\nabla_a J > 0$ so that $J(1, a^+) < 0$. Thus $(0, a^C)$ is one such $\mathcal{I}$ described by Lemma 3, and $(0, a^C) \subseteq \cup \mathcal{I}$.

It is also obvious that any $a > a^C$ satisfies $\forall N \in (0,1],\ \ J(N,a) > 0$ (because $\nabla_N J < 0$ and $\nabla_a J > 0$), and cannot belong to any $\mathcal{I}$. Therefore, a belongs to all $\overline{\mathcal{I}}$, and $[a^C, +\infty) \subseteq \cap\overline{\mathcal{I}} = \overline{\cup\mathcal{I}}$. The set $\overline{X}$ is the complement of set X in $\mathbb{R}^{++}$. Accordingly, $\cup\mathcal{I} \subseteq \overline{[a^C, +\infty)} = (0, a^C)$. To conclude, $(0, a^C) = \cup\mathcal{I}$

Pursuing argument from the proof of Lemma 3, the Implicit Function Theorem tells us that $N^C : \mathcal{A} \to (0,1)$ is continuously differentiable, and that by combining Assumptions 6 and 7

$$\nabla_a N^C = \frac{-\nabla_a J}{\nabla_N J} < 0.$$

Proof of Proposition 3.

ASSUMPTION A1. $D \equiv \sup_{a\in\mathcal{A}} |\nabla_a \theta| < +\infty$, and $\nabla_a^2 \theta$ is small enough to be neglected in my local approximations.

ASSUMPTION A2. Assume that the stochastic process $\{a_t\}$ for labor productivity follows an AR(1) process: $a_{t+1} = \rho a_t + z_{t+1}$, with $z \sim N\left(0, \sigma^2\right)$. Assume that ρ is close enough to 1 for my local approximations to be valid.

ASSUMPTION A3. I assume that σ^2 is small enough for all the k^{th} moments of $N\left(0, \sigma^2\right)$, $k \geq 4$, to be small enough for my local approximations to be valid. If $\left(\nabla_\theta^3 Z - \delta(1-s)\nabla_\theta^3 R\right) < 0$, I also assume that $\forall \theta \in \mathbb{R}^+$:

$$-\frac{(1-\delta(1-s))\,\nabla_\theta R + \nabla_\theta X + \nabla_\theta Z}{\left(\nabla_\theta^3 Z - \delta(1-s)\nabla_\theta^3 R\right)\cdot D^2} > \sigma^2.$$

LEMMA A1. *Under Assumptions A2, 6, 7, 8, 9, A1, A3, and $\forall\ a \in \mathcal{A}$:*

(i) $\nabla_a U < 0$;

(ii) $\nabla_a U^C < 0$;

(iii) $\nabla_a U^F > 0$.

PROOF. Combining (9) and (29), which link employment to labor market tightness under Assumption 9, I can express N_t as $N_t = N(\theta_t)$. $N : \mathbb{R}^+ \to [0,1]$ is continuous and differentiable, and $N(\theta)$ is given by:.

$$N(\theta) = \frac{f(\theta)}{s + (1-s)f(\theta)}. \tag{A4}$$

In particular $\nabla_\theta N > 0$. The equilibrium in period t is determined by firm's Euler equation (21), which I rewrite here for convenience:

$$R(\theta_t, c) + X(\theta_t, c) + \mathbb{E}_t\left[Z(\theta_{t+1}, c)\right] - \delta(1-s)\mathbb{E}_t\left[R(\theta_{t+1}, c)\right] = J(N(\theta_t), a_t). \tag{A5}$$

Given that the stochastic process $\{a_t\}_{t=0}^{+\infty}$ for productivity is an AR(1) process, I can rewrite $\mathbb{E}_t\left[.\right] \equiv \mathbb{E}\left[.|a^t\right]$ as $\mathbb{E}\left[.|a_t\right]$. Thus, I can write θ_t as a function of a_t: $\theta_t = \theta(a_t)$. I assume that the equilibrium exists and is unique. Therefore, I assume that $\theta : \mathbb{R}^+ \rightarrow \mathbb{R}^+$ is uniquely defined.[48] I also assume that θ is continuous and differentiable on $\mathbb{R}^+$. Thus, I can linearize $\theta(\cdot)$ (using x' to denote variable x in the next period):

$$R(\theta(a'),c) = R(\theta(\rho a + z),c) = R(\theta(\rho a),c) + z \cdot \nabla_\theta R \cdot \nabla_a \theta + z^2 \cdot \left\{\nabla^2_\theta R \cdot (\nabla_a\theta)^2 + \nabla_\theta R \cdot \nabla^2_a \theta\right\} + o(z^2).$$

Moreover, $\mathbb{E}\left[z\right] = 0$, $\mathbb{E}\left[z^2\right] = \sigma^2$, $\mathbb{E}\left[z^3\right] = 0$, and neglecting the fourth and higher moments of $N\left(0,\sigma^2\right)$ (which boils down to neglecting σ^4):

$$\mathbb{E}\left[R(\theta(a'),c)|a\right] = \int_{-\infty}^{+\infty} R(\theta(\rho a + z),c)\phi(z)dz \approx R(\theta(\rho a),c) + \left\{\nabla^2_\theta R \cdot (\nabla_a\theta)^2 + \nabla_\theta R \cdot \nabla^2_a \theta\right\}\sigma^2.$$

Next, neglecting the second-order term $\nabla^2_a\theta$:

$$\begin{aligned}\mathbb{E}\left[R(\theta(a'),c)|a\right] &\approx R(\theta(\rho a),c) + \left\{\nabla^2_\theta R \cdot (\nabla_a\theta)^2\right\}\sigma^2 \\ &\approx R(\theta(a),c) - (1-\rho)a\nabla_\theta R \cdot \nabla_a\theta + \left\{\nabla^2_\theta R \cdot (\nabla_a\theta)^2\right\}\sigma^2 + o((1-\rho)a).\end{aligned}$$

To be precise, I neglect the term factored by $(1-\rho)\sigma^2$ that would arise if I wrote the Taylor approximation of $\nabla^2_\theta R$ around $\nabla^2_\theta R(\theta(a),c)$, and evaluated it at $\rho\theta(a)$. Neglecting this term, and second-order (and above) terms $\nabla^2_a\theta$ yields:

$$\sigma^2\left\{\nabla^2_\theta R \cdot (\nabla_a\theta)^2\right\}|_{\theta=\rho\theta(a)} \approx \sigma^2\left\{\nabla^2_\theta R \cdot (\nabla_a\theta)^2\right\}|_{\theta=\theta(a)}.$$

Following the same procedure, I approximate:

$$\mathbb{E}\left[Z(\theta(a'),c)|a\right] \approx Z(\theta(a),c) - (1-\rho)a\nabla_\theta Z \cdot \nabla_a\theta + \left\{\nabla^2_\theta K \cdot (\nabla_a\theta)^2\right\}\sigma^2 + o((1-\rho)a).$$

These approximations allow me to rewrite the equilibrium condition (A5) for any $a \in \mathcal{A}$. ρ is close enough to 1 so that I can abstract from all terms in $(1-\rho)$. Taking derivative with respect to a and neglecting the second-order term $\nabla^2_a\theta$:

$$\left[(1-\delta(1-s))\nabla_\theta R + \nabla_\theta X + \nabla_\theta Z + \left\{\nabla^3_\theta Z - \delta(1-s)\nabla^3_\theta R\right\}\cdot(\nabla_a\theta)^2\sigma^2 - \nabla_N J \nabla_\theta N\right]\nabla_a\theta = \nabla_a J.$$

We have $\nabla_\theta R > 0, \nabla_\theta(X+Z) > 0, \nabla_N J < 0, \nabla_a J > 0$, and $\nabla_\theta N < 0$. Using Assumption A3, this implies:

$$\left[(1-\delta(1-s))\nabla_\theta R + \nabla_\theta X + \nabla_\theta Z + \left\{\nabla^3_\theta Z - \delta(1-s)\nabla^3_\theta R\right\}\cdot(\nabla_a\theta)^2\sigma^2 - \nabla_N J \nabla_\theta N\right] > 0.$$

Then I can conclude that for any $a \in \mathbb{R}^+$, $\nabla_a\theta > 0$. Stepping back to:

$$\left[(1-\delta(1-s))\nabla_\theta R \nabla_\theta X + \nabla_\theta Z + \left\{\nabla^3_\theta Z - \delta(1-s)\nabla^3_\theta R\right\}\cdot(\nabla_a\theta)^2\sigma^2\right]\nabla_a\theta = \nabla_a J\left(N(\theta(a)),a\right).$$

[48] Mortensen and Nagypál (2007) prove this result formally in a model in which J does not depend on θ—the standard Mortensen and Pissarides (1994) model with constant marginal returns to labor. However, this type of proof based on a fixed-point theorem and Blackwell's sufficient conditions for a contraction would not work here.

I conclude that in this stochastic environment, and for any $a \in \mathbb{R}^+$, $\nabla_a J\left(N(\theta(a)), a\right) > 0$.

Next, rewriting the gross marginal profit J for any $a \in \mathcal{A}$:

$$\begin{aligned} J(N(a), a) =& J(N^C(a) - N^F(a), a) - J(N^C(a), a) \\ =& \int_{N^C(a)}^{N^C(a)-N^F(a)} \nabla_N J(N, a) dN \\ =& \int_0^{N^F(a)} \nabla_n J(N^C(a) - n, a) dn, \end{aligned}$$

where the function $N^F(\cdot)$ is simply $N^F(\cdot) = N^C(\cdot) - N(\cdot)$. Differentiating with respect to a:

$$\nabla_a J(N(a), a) = \int_0^{N^F(a)} \nabla_{n,a} J(N^C(a) - n, a) dn - \nabla_a N^F \cdot \nabla_N J(N(a), a).$$

Using $\nabla_a J(N(a), a) > 0$, $\nabla_{n,a} J(N^C(a) - n, a) \leq 0$, and $\nabla_N J(N, a) < 0$, it follows that $\nabla_a N^F > 0$. We also proved that $\nabla_a N > 0$, and we know that $\nabla_a N^C < 0$. To conclude, it suffices to notice that $U(a) = 1 - (1-s)N(a)$, $U^F(a) = s \cdot N(a) + N^F(a)$, and $U^C(a) = 1 - N^C(a)$. □

Proof of Lemma 5. Let $\mathbb{L}_t$ denote the value to the representative household of having a marginal member employed after the matching process in period t, expressed in consumption units. Let $\mathbb{U}_t$ denotes the value to the representative household of having a marginal member unemployed.

$$\begin{aligned} \mathbb{L}_t &= W_t + \delta \mathbb{E}_t \left[\{1 - s(1 - f(\theta_{t+1}))\} \mathbb{L}_{t+1} + s\left(1 - f(\theta_{t+1})\right) \mathbb{U}_{t+1} \right] \\ \mathbb{U}_t &= \delta \mathbb{E}_t \left[(1 - f(\theta_{t+1})) \mathbb{U}_{t+1} + f(\theta_{t+1}) \mathbb{L}_{t+1} \right]. \end{aligned}$$

These continuation values are the sum of current payoffs, plus the discounted expected continuation values. Combining both conditions yields the household's surplus from an established job relationship:

$$\mathbb{L}_t - \mathbb{U}_t = W_t + \delta \mathbb{E}_t \left[(1-s)(1 - f(\theta_{t+1})) \left(\mathbb{L}_{t+1} - \mathbb{U}_{t+1} \right) \right].$$

In this setting, the firm's surplus from an established relationship is simply given by the hiring cost $c/q(\theta_t)$, since a firm can immediately replace a worker at that cost during the matching period. Assume that wages are continually renegotiated. Since the bargaining solution divides the surplus of the match between the worker and firm with the worker keeping a fraction $\beta \in (0, 1)$ of the surplus, the worker's surplus each period is related to the firm's surplus:

$$\mathbb{L}_t - \mathbb{U}_t = \frac{\beta}{1-\beta} \frac{c}{q(\theta_t)}.$$

Thus, the solution of the bargaining game is

$$W_t = c \frac{\beta}{1-\beta} \left\{ \frac{1}{q(\theta_t)} - \delta(1-s) \mathbb{E}_t \left[\frac{1}{q(\theta_{t+1})} - \theta_{t+1} \right] \right\}.$$

Proof of Lemma 6. The wage schedule $W(N_t)$ is determined by Nash bargaining over the marginal surplus from a match. I assume that the wage that solves the bargaining problem does not generate layoffs. This simplifies the analysis. I verify at the end of the derivation that the solution actually satisfies this condition. As in the proof of Lemma 5, the surplus to the representative household of having a marginal member employed in an established job relationship is:

$$\mathbb{L}_t - \mathbb{U}_t = W_t + \delta \mathbb{E}_t \left[(1-s)(1-f(\theta_{t+1})) (\mathbb{L}_{t+1} - \mathbb{U}_{t+1}) \right]. \tag{A6}$$

Following the derivations in Section 3.5, the marginal profit to the firm of having an additional worker, once the relationship is established, is:

$$\mathbb{J}_t = \nabla_N F - W_t - N_t \nabla_N W + (1-s)\delta \mathbb{E}_t \left[\frac{c}{q(\theta_{t+1})} \right]. \tag{A7}$$

This marginal profit corresponds to the surplus of the established relationship accruing to the firm. Note that firm maximizes profit taken the wage rule as given, and that the first-order conditions derived in Section 3.5 (by assumption, $\mathcal{M} = 1$, i.e. $\epsilon \to +\infty$, so that $P_t(i) = P_t$ and $\nu_t = 1$) imply that

$$\mathbb{J}_t = \frac{c}{q(\theta_t)}. \tag{A8}$$

Since the bargaining solution divides the surplus of the match between the worker and firm with the worker keeping a fraction $\beta \in (0, 1)$ of the surplus, the worker's marginal surplus each period is related to the firm's marginal surplus:

$$\mathbb{L}_t - \mathbb{U}_t = \frac{\beta}{1-\beta} \mathbb{J}_t. \tag{A9}$$

Combining (A6)-(A9), I can derive a differential equation in the wage schedule:

$$W(N_t) + \beta N_t \nabla_N W = \beta \left[\nabla_N F + c(1-s)\delta \mathbb{E}_t \left[\theta_{t+1} \right] \right].$$

With $F(N_t, a_t) = a_t N_t^{\alpha}$, the solution of the above equation is:

$$W(N_t) = \beta \left[\frac{\alpha \cdot a_t \cdot N_t^{\alpha-1}}{1-\beta(1-\alpha)} + c(1-s)\delta \mathbb{E}_t \left[\theta_{t+1} \right] \right].$$

Proof of Proposition 4. Plugging the wage schedule assumed in Assumption 12 into the equilibrium condition (21) derived in the general case yields:

$$\frac{c}{q(\theta_t)} + c\delta(1-s)\beta \mathbb{E}_t \left[\theta_{t+1} \right] = (1-\beta) a_t + c\delta(1-s)\mathbb{E}_t \left[\frac{1}{q(\theta_{t+1})} \right]. \tag{A10}$$

No aggregate shock. Without aggregate shocks, the following equilibrium condition determines implicitly θ as a function of c:

$$c \left\{ (1-\delta(1-s)) \frac{1}{q(\theta(c))} + \delta(1-s)\beta\theta(c) \right\} = (1-\beta)a.$$

Assume that $\exists L \in \mathbb{R}^+$, $\theta(c) < L$ for all c. Then (since $1/q(\cdot)$ is increasing in θ):

$$0 < \left\{(1-\delta(1-s))\frac{1}{q(\theta(c))} + \delta(1-s)\beta\theta(c)\right\} < \left\{(1-\delta(1-s))\frac{1}{q(L)} + \delta(1-s)\beta L\right\} \equiv \lambda$$

and for $0 < c < \frac{1}{\lambda}(1-\beta)a$, the equilibrium condition cannot hold. Therefore:

$$\lim_{c \to 0} \theta(c) = +\infty.$$

With aggregate shocks. I assume that the stochastic process $\{a_t\}_{t=0}^{+\infty}$ is a Markov process. Then Mortensen and Nagypál (2007) show that there exists a unique function $\theta : \mathbb{R}^+ \to \mathbb{R}^+$, continuous and differentiable, that solves this sequence of equations (A10) . I use the standard specification for the matching function $h(U_t, V_t) = \omega U_t^\eta V_t^{1-\eta}$, such that $c/q(\theta_t) = c \cdot \omega \cdot \theta_t^\eta$. Given $\{a_t\}$, the stochastic process for equilibrium labor market tightness $\{\theta_t\}_{t=0}^{+\infty}$ is unique (Mortensen and Nagypál 2007). I now show that $\left\{\hat{\theta}_t\right\}_{t=0}^{+\infty}$ whose time t elements are measurable with respect to a^t, and are defined for all $t \geq 0$ by

$$\hat{\theta}_t = \frac{1-\beta}{c\delta(1-s)\beta} \times \frac{a_{t-1}}{\mathbb{E}_{t-1}\left[a_t\right]} \times a_t$$

satisfies this equation when recruiting costs $c \to 0$ (I noted $a_{-1} = 1, \mathbb{E}_{-1}\left[a_0\right] = 1$). With such a stochastic process for labor market tightness:

$$\begin{aligned} c/q(\theta_t) &\sim c^{1-\eta} \cdot \psi_1(a^t) \\ \mathbb{E}_t\left[c/q(\theta_{t+1})\right] &\sim c^{1-\eta} \cdot \psi_2(a^t) \\ c\mathbb{E}_t\left[\theta_{t+1}\right] &\sim \frac{1-\beta}{\delta(1-s)\beta} \cdot a_t. \end{aligned}$$

Thus, when $c \to 0$:

$$\begin{aligned} c/q(\theta_t) &\to 0 \\ \mathbb{E}_t\left[c/q(\theta_{t+1})\right] &\to 0 \\ c\mathbb{E}_t\left[\theta_{t+1}\right] &\to \frac{1-\beta}{\delta(1-s)\beta} \cdot a_t. \end{aligned}$$

It is clear that as $c \to 0$, this process for labor market tightness does solve the equilibrium condition. Therefore, when $c \to 0$, $\theta_t \to +\infty$, $f(\theta_t) \to 1$, $U_t \to s$, $N_t \to 1$, and $W_t \to (1-\beta)a_t$.

Proof of Proposition 5. I assume that the stochastic process $\{a_t\}_{t=0}^{+\infty}$ is a Markov process. Plugging the wage schedule assumed in Assumption 13 into the general equilibrium condition (21) yields:

$$\frac{c}{q(\theta_t)} = a_t - w_0 + c\delta(1-s)\mathbb{E}_t\left[\frac{1}{q(\theta_t+1)}\right]. \tag{A11}$$

The stochastic process $\{\theta_t\}_{t=0}^{+\infty}$ that solves the sequence of equations (A11) satisfies $\theta_t = \theta(a_t, c)$ for all t, where $\theta : \mathbb{R}^+ \times \mathbb{R}^{++} \to \mathbb{R}^+$ is defined implicitly by:

$$\frac{1}{q(\theta(a,c))} = \mathbb{V}(a,c)$$

and the value function $\mathbb{V}$ is defined recursively by

$$\mathbb{V}(a_t, c) = \frac{a_t - w_0}{c} + \delta(1-s)\mathbb{E}\left[\mathbb{V}(a_{t+1}, c)|a_t\right].$$

For all $(a, c) \in \mathbb{R}^+ \times \mathbb{R}^{++}$:

$$\mathbb{V}(a,c) \geq \frac{a - w_0}{c}.$$

Therefore, for all $a \in \mathbb{R}^+$, $\lim_{c\to 0} \mathbb{V}(a, c) = +\infty$. Using the standard matching function specification: $1/q(\theta) = \omega \cdot \theta^\eta$. Therefore, for all $a \in \mathbb{R}^+$:

$$\lim_{c\to 0} \theta(a,c) = +\infty.$$

To conclude, equilibrium employment and unemployment can be written as $N_t = N(a_t, c)$ and $U_t = U(a_t, c)$, using equations (29) and (9). Then, for all $a \in \mathbb{R}^+$, $\lim_{c\to 0} N(a, c) = 1$ and $\lim_{c\to 0} U(a, c) = s$.

Proof of Proposition 6. Plugging the wage rule assumed in Assumption 15 into the general equilibrium condition (21) yields:

$$\frac{c}{q(\theta_t)} + c(1-s)\beta\delta\mathbb{E}_t\left[\theta_{t+1}\right] = \left[\frac{1-\beta}{1-\beta(1-\alpha)}\right] a_t \cdot \alpha \cdot N_t^{\alpha-1} + c\delta(1-s)\mathbb{E}_t\left[\frac{1}{q(\theta_t+1)}\right].$$

The proof is similar to that of Proposition 4, but replacing $(1-\beta)a_t$ by $\left[\frac{1-\beta}{1-\beta(1-\alpha)}\right] a_t \cdot \alpha$, defining N_t as a function of θ_t using (29) and (9), and noting that as $\theta_t \to +\infty$, $N_t \to 1$.

Proof of Corollary 2. I need to determine a sufficient condition for Assumption 8. Using (33):

$$J(N^C(a) - n, a) = \frac{\alpha}{\mathcal{M}} a \cdot \left[N^C(a) - n\right]^{\alpha-1} - w_0 \cdot a^\gamma$$
$$\nabla_n J(N^C(a) - n, a) = \frac{\alpha \cdot (1-\alpha)}{\mathcal{M}} \cdot \left[a^{1/(\alpha-2)} \cdot N^C(a) - a^{1/(\alpha-2)} \cdot n\right]^{\alpha-2}.$$

Since $(2-\alpha) \geq 0$ and $N^C(a) - n \; \forall n \in [0, N^C(a)] \geq 0$, part (ii) holds if and only if

$$\nabla_a \left[a^{1/(\alpha-2)} \cdot N^C(a) - a^{1/(\alpha-2)} \cdot n\right] \geq 0.$$

A sufficient condition is

$$\nabla_a \left[a^{1/(\alpha-2)} \cdot N^C(a)\right] \geq 0,$$

because $-\nabla_a \left[a^{1/(\alpha-2)} \cdot n\right] \geq 0$. Since

$$a^{1/(\alpha-2)} \cdot N^C(a) = \left(\frac{\alpha}{\mathcal{M} \cdot w_0}\right)^{\frac{1}{1-\alpha}} \cdot a^{\frac{1-\gamma}{1-\alpha} - \frac{1}{2-\alpha}},$$

a sufficient condition is

$$\frac{1-\gamma}{1-\alpha} - \frac{1}{2-\alpha} \geq 0,$$

which implies $\gamma \leq \frac{1}{2-\alpha}$. Next, I determine a condition on the stochastic process for productivity, as well as the parameters of the model, such that endogenous layoffs do not occur. Assume that no such layoffs occurred at time t. Using the approximation developed in the proof of Proposition 3, the equilibrium condition becomes:

$$\frac{\alpha}{\mathcal{M}} a_t N_t^{\alpha-1} - w_0 a_t^\gamma = (1 - \delta(1-s))R(\theta_t, c) - \sigma^2 \nabla_\theta^2 R \cdot (\nabla_a \theta)^2 .$$

Notice that $\nabla_\theta^2 R < 0$, so that I can infer:

$$N_t^{\alpha-1} \geq \frac{\mathcal{M} w_0}{\alpha} a_t^{\gamma-1}. \tag{A12}$$

A necessary and sufficient condition to avoid endogenous layoffs in period $t+1$ is:

$$\frac{\alpha}{\mathcal{M}} a_t (1-s)^{\alpha-1} N_t^{\alpha-1} - w_0 a_t^\gamma + \mathbb{E}_{t+1}\left[R(\theta_{t+2}, c)\right] \geq 0.$$

Since $R \geq 0$, a sufficient condition is

$$\frac{\alpha}{\mathcal{M}} a_t (1-s)^{\alpha-1} N_t^{\alpha-1} - w_0 a_t^\gamma \geq 0.$$

From (A12), and using $a_{t+1} = a_t + z_t$, I find a sufficient condition on the productivity shock in period t:

$$z_t \geq a_t \cdot \left[(1-s)^{\frac{1-\alpha}{1-\gamma}} - 1\right].$$

Let $\Phi(\cdot)$ be the cumulative distribution function of the $N(0,1)$ distribution. Given that z_t is normally distributed with variance σ^2, I infer that layoffs occur with probability below:

$$\Phi\left(\frac{a_t}{\sigma} \cdot \left[(1-s)^{\frac{1-\alpha}{1-\gamma}} - 1\right]\right).$$

Since $a_t \geq 0.93$ in practice (for instance once I discretize the AR(1) process using a 200-state Markov chain), imposing $\left[1 - (1-s)^{\frac{1-\alpha}{1-\gamma}}\right] > 2.5 \cdot \sigma$ ensures that endogenous layoffs occur with probability below 1%.

B Extension to a Two-State Markov-Chain Productivity Process

I assume that labor productivity follows a 2-state Markov chain with state space $\{a_L, a_H\}$ and an ergodic transition matrix $\mathbf{\Lambda}$. I assume that $0 < a_L < a_H$, and that there is job rationing in both states, so that $U_L^C > 0$ and $U_H^C > 0$. Under Assumption 9, since productivity follows a Markov process, all labor market variables at time t solely depend on the realization of productivity in the current period. I note $\theta_i,\ N_i,\ U_i,\ U_i^C,$ and U_i^F the value of these variables when productivity $a_t = a_i$.

ASSUMPTION A4 (FOSC). Consider a Markov chain with state space $\{a_1, \ldots, a_n\} \in \mathbb{R}^n$ and $a_1 < \ldots < a_n$. Then $\forall i, j = 1 \ldots n$, $i > j$:

$$\mathbb{P}\left\{.|a_i\right\} \succeq_{FOSC} \mathbb{P}\left\{.|a_j\right\}, \tag{A13}$$

where $\mathbb{P}\left\{.|a\right\}$ is the conditional transition probability in state a, and $\succeq_{FOSC}$ indicates first-order stochastic dominance.

This assumption implies that the matrix $\mathbf{\Lambda}$ preserves ordering of vectors.

LEMMA A2. *Let $\mathbf{\Lambda}$ be the $n \times n$ transition matrix of a Markov chain that satisfies Assumption A4. Let $\mathbf{X} = [X_i]_{i=1,\ldots,n}$ be such that $X_1 < \ldots < X_n$. Let $\mathbf{Y} = (Y_i)_{i=1,\ldots,n}$ be defined by $\mathbf{Y} = \mathbf{\Lambda X}$. Then the ordering of vector $\mathbf{X}$ is preserved after multiplication by the transition matrix $\mathbf{\Lambda}$: $Y_1 < \ldots < Y_n$.*

PROOF. Assume that $\mathbf{\Lambda} = [\Lambda_{i,j}]_{i,j=1,\ldots,n}$ satisfies Assumption A4. Then by definition of first-order stochastic dominance, for $i > j$, and for any $s = 1, \ldots, n$:

$$\sum_{q \geq s} \Lambda_{i,q} \geq \sum_{q \geq s} \Lambda_{j,q}. \tag{A14}$$

I denote $X_0 \equiv 0$. For $i = 1, \ldots, n$:

$$\begin{aligned} Y_i &= \sum_{s=1}^{n} \Lambda_{i,s} X_s \\ &= \sum_{s=1}^{n} \left(\sum_{q=s}^{n} \Lambda_{i,q} \right) (X_s - X_{s-1}). \end{aligned}$$

Therefore for $i > j$:

$$Y_i - Y_j = \sum_{s=1}^{n} \left[\left(\sum_{q=s}^{n} \Lambda_{i,q} \right) - \left(\sum_{q=s}^{n} \Lambda_{j,q} \right) \right] (X_s - X_{s-1}).$$

The terms in brackets are always nonnegative by assumption (see (A14)). The terms in parenthesis are nonnegative because of the ordering of $\mathbf{X}$. Hence, $Y_i > Y_j$. □

ASSUMPTION A5. For all $(\theta, c) \in \mathbb{R}^+ \times \mathbb{R}^+$, $\nabla_\theta X > 0$.

This assumption, which is satisfied for all the specific wage schedules studied in this paper (Section 6) implies that wages are higher when the current labor market is tighter. This is a natural assumption given that workers' position is more favorable when the labor market is tight. In a two-state world, the main result of the paper (Proposition 3) obtains for any stochastic process for productivity satisfying Assumption A4 .

PROPOSITION A1. *Under Assumption A4:*

(i) $U_H < U_L$;

(ii) $U_H^C < U_L^C$;

(iii) $U_H^F > U_L^F$.

PROOF. In this world, equilibrium condition (23) is:

$$[R(\theta_i, c)] = [J(N_i, a_i)] - [X(\theta_i, c)] - \mathbf{\Lambda}\,[Z(\theta_i, c)] + \delta(1-s)\mathbf{\Lambda}\,[R(\theta_i, c)]\,,$$

where $[X_i]$ is the column vector stacking up the X_i, $i = L,\ H$. Iterating forward:

$$[R(\theta_i, c)] = \left(\sum_{j=0}^{+\infty} \delta^j (1-s)^j \mathbf{\Lambda}^j\right) \{[J(N_i, a_i)] - [X(\theta_i, c)] - \mathbf{\Lambda}\,[Z(\theta_i, c)]\}\,.$$

Let

$$\mathbf{L} \equiv (1 - \delta \cdot (1-s))^{-1} \left(\sum_{j=0}^{+\infty} \delta^j (1-s)^j \mathbf{\Lambda}^j\right).$$

Notice that $\mathbf{L}$ is well defined because $\mathbf{\Lambda}$ is ergodic. Thus, all eigenvalues are in the unit circle, and $\sum_{j=0}^{+\infty} \delta^j (1-s)^j \mathbf{\Lambda}^j$ converges. Since $\mathbf{\Lambda}$ satisfies Assumption A4, for all $i \geq 0$, $\mathbf{\Lambda}^i$ satisfies Assumption A4 as well (note that $\mathbf{\Lambda}^i$ is also a transition matrix). Thus, $\mathbf{L}$ also satisfies Assumption A4 and preserves ordering of vectors as described in Lemma A2.

I now reason by contradiction. Assume that $\theta_H < \theta_L$. Then $N_H < N_L$ and $J(N_H, a_H) > J(N_L, a_L)$. Moreover, $X(\theta_H, c) < X(\theta_L, c)$, and $(X + Z)(\theta_H, c) < (X + Z)(\theta_L, c)$. Noting $X_i \equiv X(\theta_i, c),\ Z_i \equiv Z(\theta_i, c)$, I can rewrite:

$$\begin{aligned}
X_H + \mathbf{\Lambda}\,[Z_i] &= \lambda_{H,H}\,(X_H + Z_H) + (1 - \lambda_{H,H})(X_L + Z_L) + (1 - \lambda_{H,H})(X_H - X_L) \\
X_L + \mathbf{\Lambda}\,[Z_i] &= (1 - \lambda_{L,L})(X_H + Z_H) + \lambda_{L,L}\,(X_L + Z_L) + (1 - \lambda_{L,L})(X_L - X_H).
\end{aligned}$$

By Assumption A4: $\lambda_{H,H} > 1 - \lambda_{L,L}$. Thus, $X_H + \mathbf{\Lambda}\,[Z_i] < X_L + \mathbf{\Lambda}\,[Z_i]$. Since L preserves ordering of vectors, I infer that $R(\theta_H, c) > R(\theta_L, c)$ which implies $\theta_H > \theta_L$. I reach a contradiction. Thus, $a_H > a_L \Rightarrow \theta_H > \theta_L$. This means that $X_H + Z_H > X_L + Z_L$, $X_H > X_L$. Therefore, it must be that $J(N_H, a_H) > J(N_L, a_L)$, otherwise I would reach the contradiction that $\theta_H < \theta_L$, as in the first part of the proof. Proceeding as in the end of the proof of Proposition 3 yields the results. □

However, this result does not generalize to a world in which productivity follows a n-state Markov chain. Assume that productivity follows a 3-state Markov chain with transition matrix:

$$\mathbf{\Lambda} = \begin{bmatrix} 0.99 & 0.01 & 0 \\ 0 & 0.01 & .99 \\ 0 & 0 & 1 \end{bmatrix} \tag{A15}$$

with $a_1 < a_2 < a_3$, and a_3 sufficiently larger than a_2, and a_1 very close to a_2, and $a_i \in \mathcal{A}$ for $\forall i$. Proposition A1 may not hold in this case. In state 2, firms recruit a lot more than in state 1, even though productivity levels are close in both states, because they anticipate that they are very likely to be in state 1 next period. In state 3, productivity is high, the labor market will be tight and recruiting costly. Therefore, firms substitute future recruiting to the current period, in which the labor market is more slack. Because the recruiting activity is a lot higher in state 2 than in state 1, total unemployment is much lower, even though cyclical unemployment in both states are close. Consequently, frictional unemployment must be much lower in state 2 than in state 1.

C Complete Log-Linear Model

I first characterize the steady state of the model, and then describe the log-linearized equilibrium conditions around this steady state. $\overline{x}$ denotes the steady-state value of variable X_t. The symmetric steady-state equilibrium $\{\overline{c}, \overline{n}, \overline{y}, \overline{h}, \overline{\theta}, \overline{u}, \overline{w}\}$ is characterized by the following equations:

$$\overline{u} = \frac{s}{s + (1-s)f(\overline{\theta})} \tag{A16}$$

$$\overline{n} = \frac{1-\overline{u}}{1-s} \tag{A17}$$

$$\overline{h} = s \cdot \overline{n} \tag{A18}$$

$$\overline{c} = \overline{n}^{\alpha} - \frac{c \cdot s}{q(\overline{\theta})}\overline{n} \tag{A19}$$

$$\overline{y} = \overline{n}^{\alpha} \tag{A20}$$

$$\overline{w} = w_0 \tag{A21}$$

$$0 = \frac{\alpha}{\mathcal{M}}\overline{n}^{\alpha-1} - \overline{w} - [1 - \delta(1-s)]\frac{c}{q(\overline{\theta})} \tag{A22}$$

$$\overline{a} = 1 \tag{A23}$$

$\check{x}_t \equiv d\ln(X_t)$ denotes the logarithmic deviation of variable X_t. The equilibrium is described by the following system of log-linearized equations:

- Definition of labor market tightness:

$$1 - \eta \cdot \check{\theta}_t = \check{h}_t - \check{u}_{t-1}$$

- Definition of unemployment:

$$\check{u}_{t-1} + \frac{1-\overline{u}}{\overline{u}}\check{n}_{t-1} = 0$$

- Law of motion of employment:

$$\check{n}_t = (1-s)\check{n}_{t-1} + s \cdot \check{h}_t$$

- Resource constraint:

$$\check{y}_t = (1-s_1)\check{c}_t + s_1\left(\check{h}_t + \eta \cdot \check{\theta}_t\right),$$

with $s_1 = \frac{c \cdot s}{q(\overline{\theta})} \overline{n}^{1-\alpha}$.

- Production constraint:
$$\check{y}_t = \check{a}_t + \alpha \check{n}_t$$
- Wage rule:
$$\check{w}_t = \gamma \cdot \check{a}_t$$
- Firm's Euler equation:
$$-\check{a}_t + (1-\alpha) \cdot \check{n}_t + s_2 \cdot \check{w}_t + s_3 \cdot \eta \cdot \check{\theta}_t + (1 - s_2 - s_3)\mathbb{E}_t \left[\eta \cdot \check{\theta}_{t+1}\right] = 0$$
with $s_2 = \overline{w} \cdot \frac{\mathcal{M}}{\alpha} \cdot \overline{n}^{1-\alpha}$ and $s_3 = \frac{c}{q(\overline{\theta})} \cdot \frac{\mathcal{M}}{\alpha} \cdot \overline{n}^{1-\alpha}$.
- Productivity shock:
$$\check{a}_t = \rho \cdot \check{a}_{t-1} + z_t$$

D Calibration of the MP, MPS, and SZ Models

D.1 MP model

In steady-state, since $c = 0.32 \times \overline{w}$:
$$\frac{1 - \delta(1-s)}{q(\overline{\theta})} = \frac{1 - \overline{w}}{0.32 \times \overline{w}}.$$
I target $\overline{u} = 5.8\%$, or equivalently $\overline{\theta} = 0.45$. This pins down $\overline{w} = 0.990$, and $c = 0.32$. Then, in steady state
$$\frac{1 - \delta \cdot (1-s)}{q(\overline{\theta})} + \beta \cdot \delta \cdot (1-s)\overline{\theta} = (1-\beta)\frac{1}{c},$$
which pins down the bargaining power $\beta = 0.86$.

D.2 MPS model

In steady-state, $\overline{w} = w_0$ and $c = 0.32 \times \overline{w}$, so
$$\frac{1 - \delta(1-s)}{q(\overline{\theta})} = \frac{1 - w_0}{0.32 \times w_0}.$$
I target $\overline{u} = 5.8\%$, or equivalently $\overline{\theta} = 0.45$. This pins down $w_0 = 0.990$, and $c = 0.32$.

D.3 SZ model

Let $\kappa = \dfrac{\alpha}{1-\beta\cdot(1-\alpha)}$. The steady-state wage equation, firm's Euler equation, and definition of the labor share are

$$\overline{w} = \beta\left[\kappa\cdot\overline{n}^{\alpha-1} + c\cdot(1-s)\cdot\delta\cdot\overline{\theta}\right] \tag{A24}$$

$$(1-\beta)\cdot\kappa\cdot\overline{n}^{\alpha-1} = \left[1-\delta(1-s)\right]\frac{c}{q(\overline{\theta})} + c\cdot(1-s)\cdot\delta\cdot\beta\cdot\overline{\theta} \tag{A25}$$

$$\overline{ls} = \overline{w}\cdot\overline{n}^{1-\alpha}. \tag{A26}$$

Combining (A24), (A25), and (A26), and using $c = 0.32\times\overline{w}$ yields:

$$\kappa = \left[(1-\delta\cdot(1-s))\frac{0.32}{q(\overline{\theta})} + 1\right]\overline{ls} \tag{A27}$$

$$\overline{ls} = \overline{w}\cdot\overline{n}^{1-\alpha} \tag{A28}$$

$$\overline{w} = \beta\left[\kappa\cdot\overline{n}^{\alpha-1} + c\cdot(1-s)\cdot\delta\cdot\overline{\theta}\right]. \tag{A29}$$

Equation (A27) identifies $\kappa = 0.67$, given that I target $\overline{ls} = 0.66$ and $\overline{\theta} = 0.45$. Equation (A28) then determines $\overline{w} = 69$, given that I target $\overline{n} = 0.95$. Finally, (A29) determines $\beta = 0.86$. I can then calculate $\alpha = \dfrac{\kappa-\kappa\beta}{1-\kappa\beta} = 0.21$. To compute the moments from the SZ model, I need to log-linearize it. Only two equations differ between the job-rationing model and the SZ model:

- Wage rule:

$$\check{w}_t = s_1\left[\check{a}_t + (\alpha-1)\cdot\check{n}\right] + (1-s_1)\mathbb{E}_t\left[\check{\theta}_{t+1}\right],$$

  with $s_1 = \dfrac{\beta\cdot\alpha}{1-\beta(1-\alpha)}\cdot\dfrac{\overline{n}^{\alpha-1}}{\overline{w}}$.

- Firm's Euler equation:

$$\check{a}_t + (\alpha-1)\cdot\check{n}_t - s_2\cdot\eta\cdot\check{\theta}_t + \left[s_2\cdot(\eta-1)\cdot\delta\cdot(1-s) - 1 + s_2\right]\mathbb{E}_t\left[\check{\theta}_{t+1}\right] = 0,$$

  with $Q = \dfrac{(1-\beta)\cdot\alpha}{1-\beta\cdot(1-\alpha)}\cdot\overline{n}^{\alpha-1}$ and $s_2 = \dfrac{c}{q(\overline{\theta})}\dfrac{1}{Q}$.

E Figures and Tables

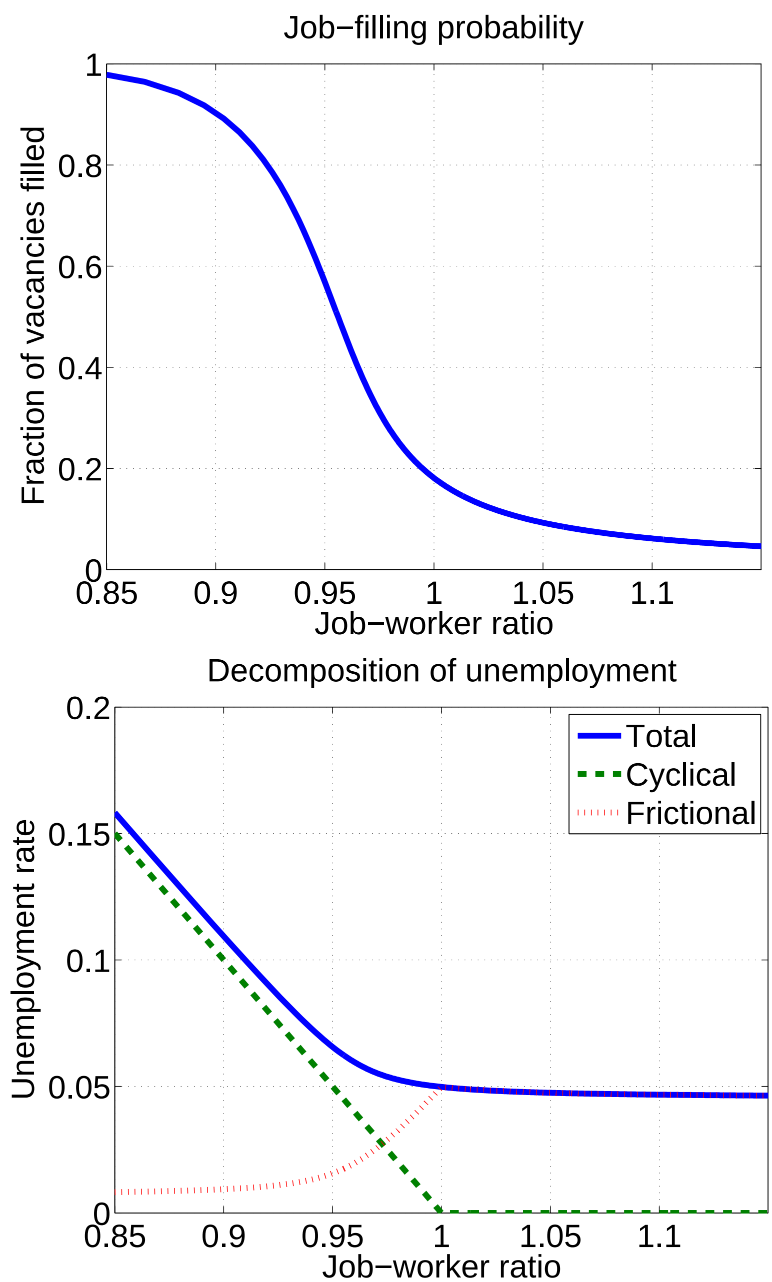


FIGURE 1: STATIONARY EQUILIBRIUM IN THE ELEMENTARY MODEL

Notes: I choose $s = 0.095$, which is the weekly separation rate estimated in Section 7. I then pick $\omega = 0.20$, which yields an unemployment rate $u = 5.6\%$ for a vacancy-unemployment ratio $v/u = 0.45$, in line with U.S. data over the period 2001–2009. I first vary the job-worker ratio $\Theta = K/L$ in the range $[0.85, 1.15]$ to compute the corresponding equilibrium unemployment rate u and vacancy rate v from the system of equations (1)-(2). Then I compute the job-filling probability with (3), and the decomposition of unemployment with (4) and (5).

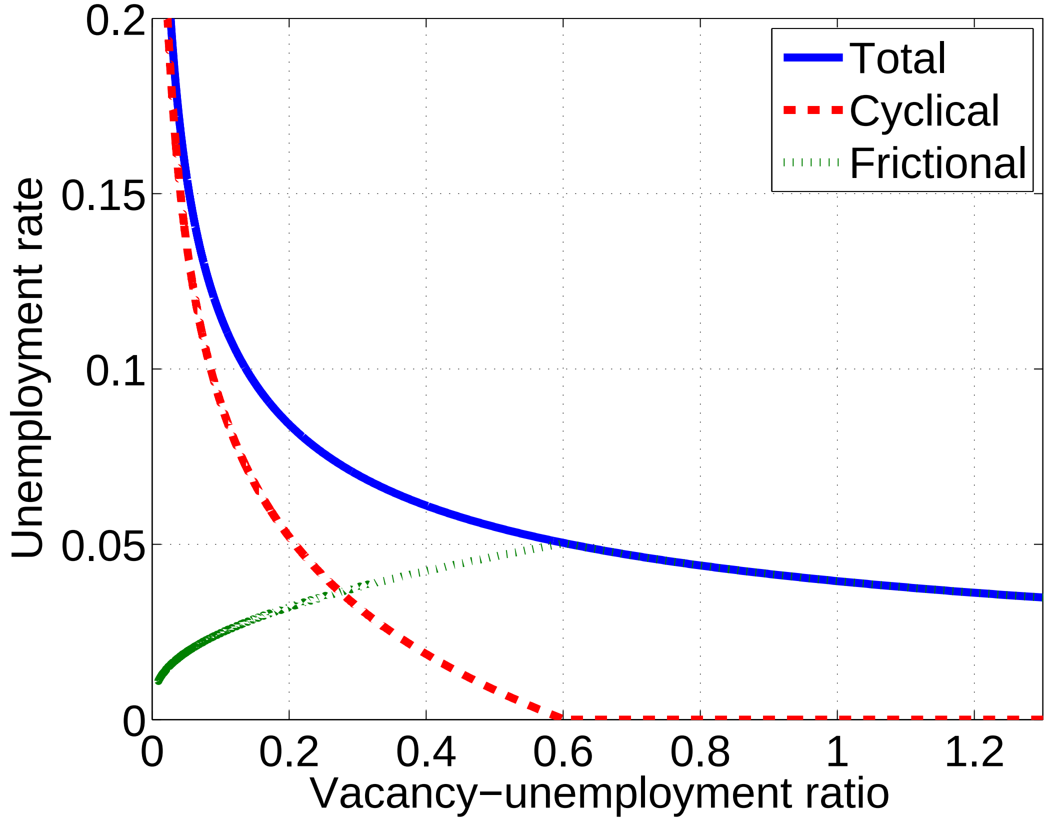


FIGURE 2: BEVERIDGE CURVE FROM CALIBRATED MODEL

Notes: This graph is obtained by computing a continuum of steady-state equilibria in the labor market, associated with a continuum of realizations of productivity. I solve for total unemployment from a system of three equations: (9), (29), and the steady-state version of (36). I can determine cyclical unemployment from (35), and obtain frictional unemployment from the difference between these two series.

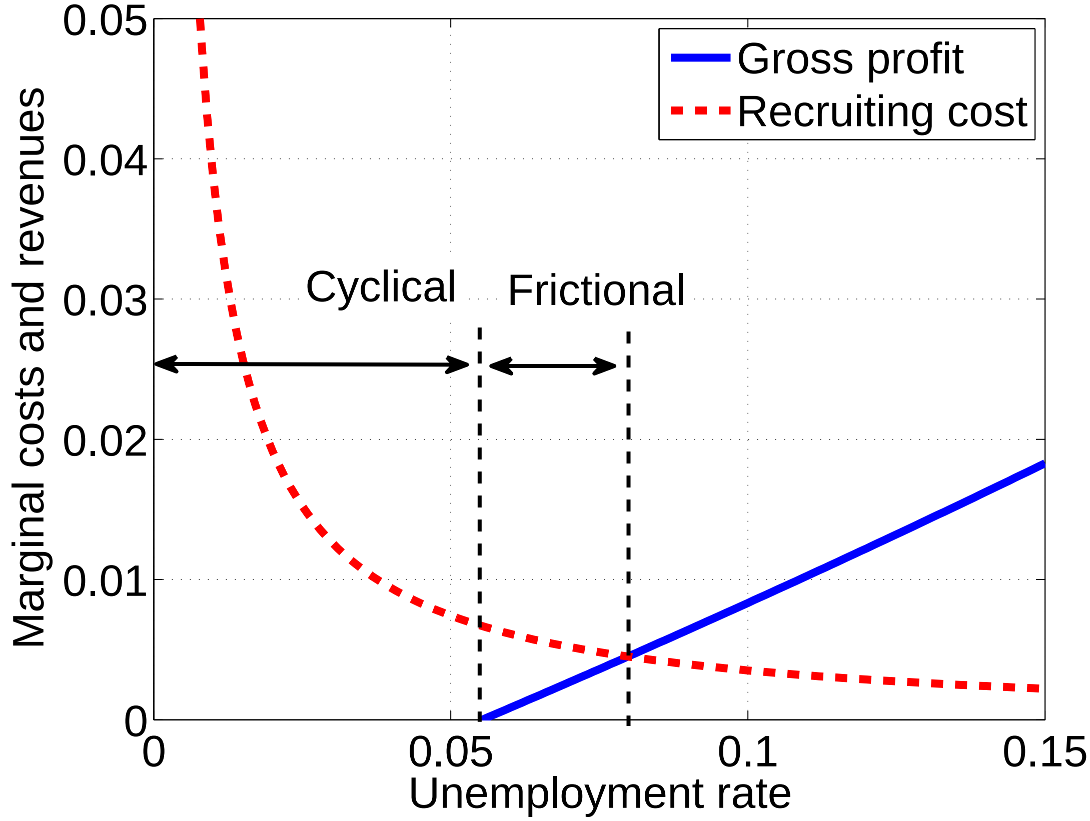


FIGURE 3: STEADY-STATE EQUILIBRIUM IN A MODEL WITH JOB RATIONING

Notes: This graph describes a steady-state equilibrium in the model of job rationing presented in Section 6.4. It is obtained by plotting the recruiting cost $R(\theta, c)$ and the gross marginal profit (33) for a continuum of unemployment rates.

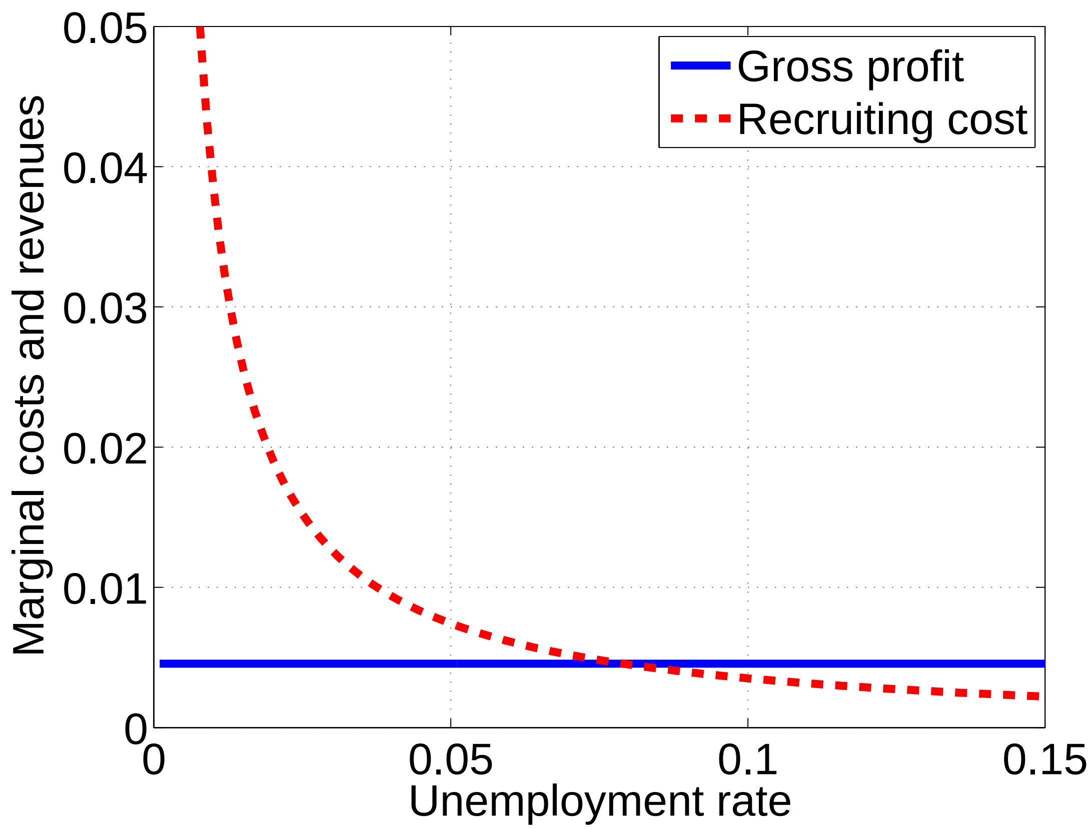


FIGURE 4: STEADY-STATE EQUILIBRIUM IN THE MPS MODEL

Notes: This graph describes a steady-state equilibrium in the MPS model presented in Section 6.2. It is obtained by plotting the recruiting cost $R(\theta, c)$ and the gross marginal profit (32) for a continuum of unemployment rates.

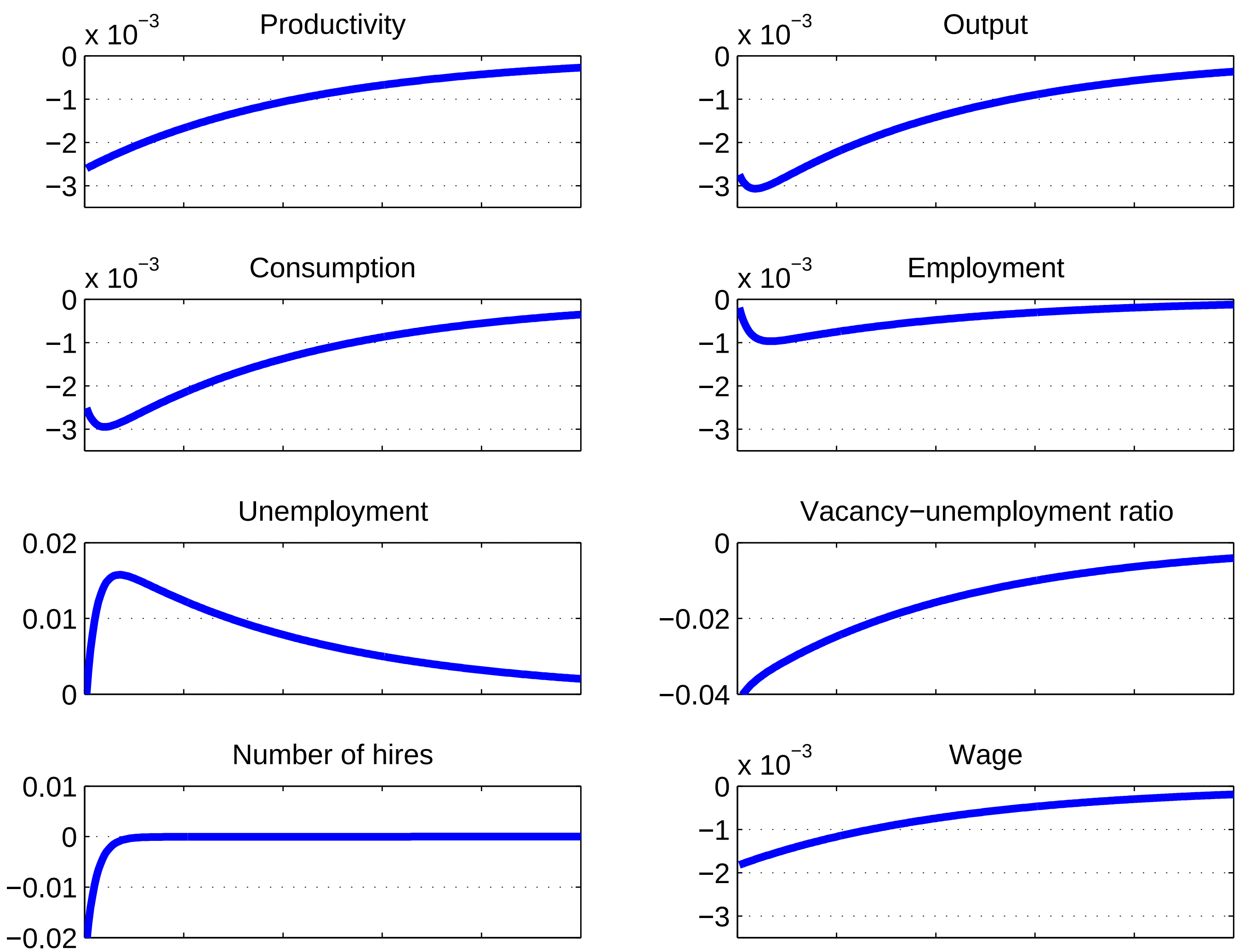


FIGURE 5: IRFS TO NEGATIVE PRODUCTIVITY SHOCKS OF ONE STANDARD DEVIATION

Notes: Impulse response functions (IRFs) represent the log-deviation from steady-state for each variable. IRFs are obtained by log-linearizing the model, as detailed in Appendix C. The total time period displayed on the x-axis is 250 weeks. The shock imposed to productivity is $-\sigma = -0.0026$.

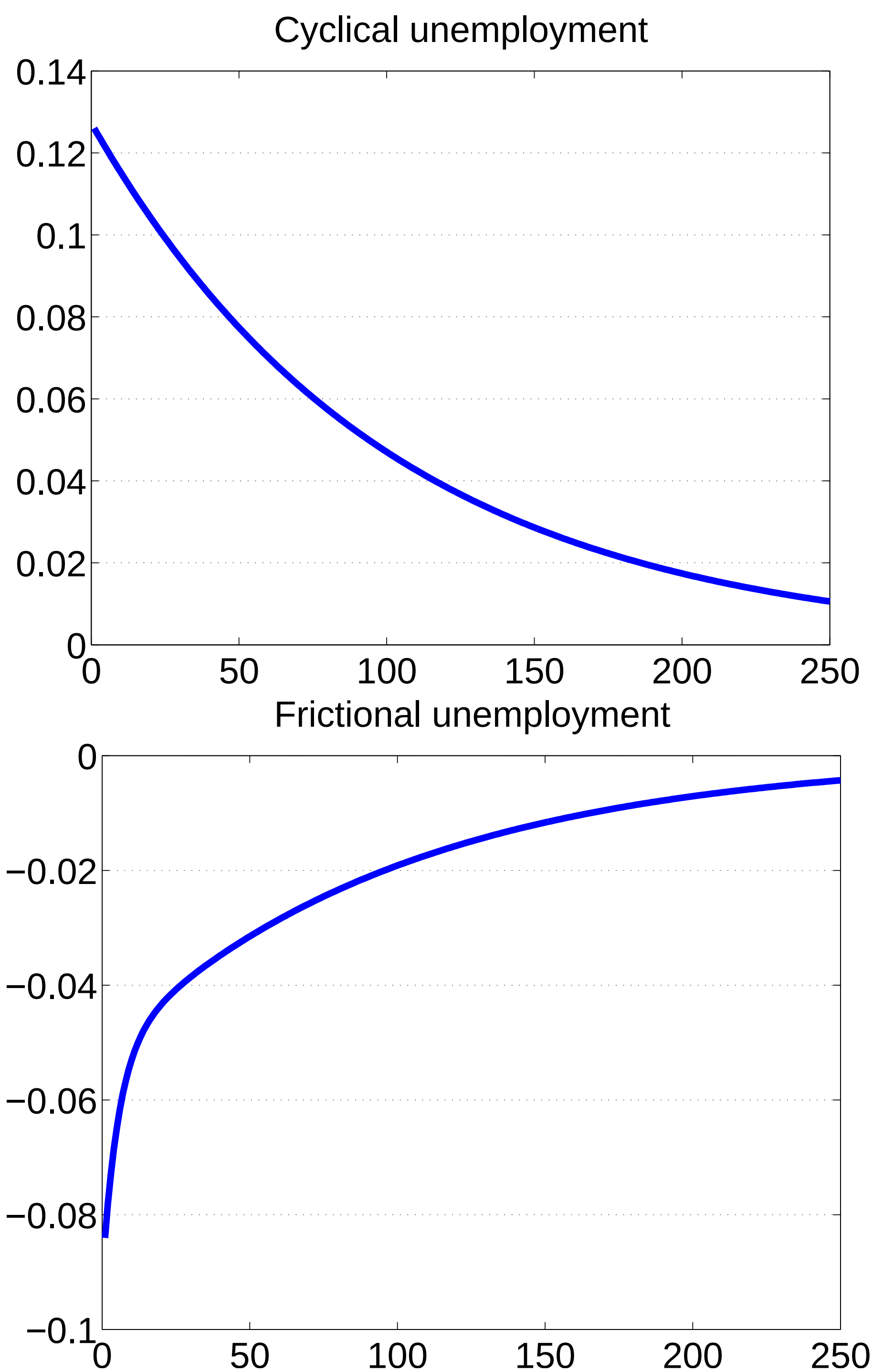
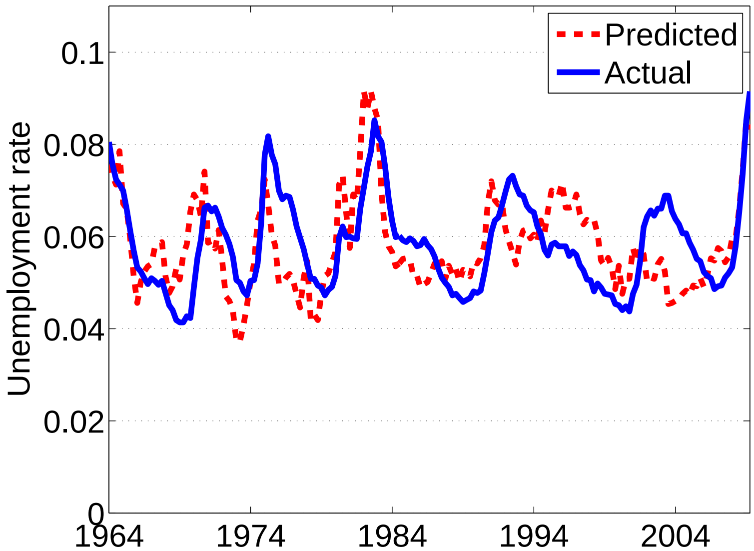


FIGURE 6: IRFS TO NEGATIVE PRODUCTIVITY SHOCK OF ONE STANDARD DEVIATION

Notes: Impulse response functions (IRFs) represent the log-deviation from steady-state for each variable. IRFs are obtained by log-linearizing the model, as detailed in Appendix C. The time period on the x-axis is a week. The shock imposed to productivity is $-\sigma = -0.0026$.



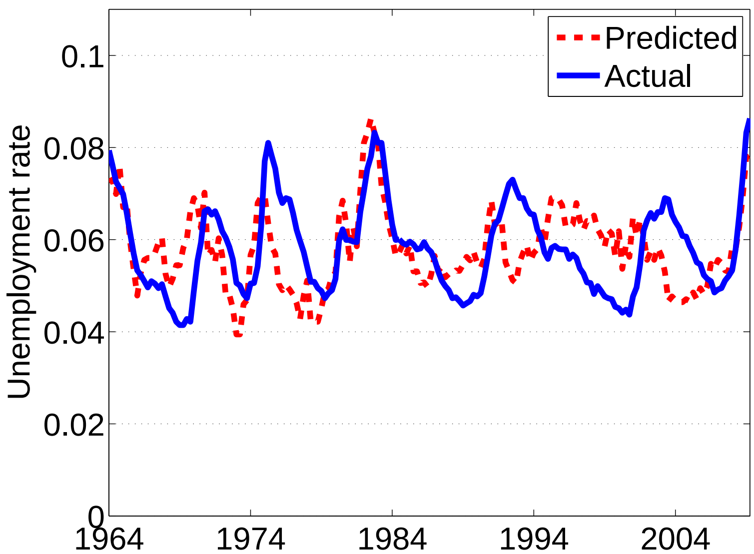


FIGURE 7: ACTUAL UNEMPLOYMENT, AND MODEL-GENERATED UNEMPLOYMENT UNDER ACTUAL PRODUCTIVITY SHOCKS (TOP) AND ACTUAL TFP SHOCKS (BOTTOM) FROM U.S. DATA, 1964–2009

Notes: Actual unemployment is the quarterly average of seasonally-adjusted monthly series constructed by the BLS from the CPS. The top graph compares actual unemployment with the unemployment series generated when the nonlinear model is stimulated by the quarterly labor productivity series constructed in Section 7.1 using output and employment data provided by the BLS. The bottom graph compares actual unemployment with the unemployment series generated when the nonlinear model is stimulated by the quarterly, utilization-adjusted TFP series constructed by Fernald (2009). Productivity, TFP, and actual unemployment are detrended with a HP filter with smoothing parameter 10^5. The time period is 1964:Q1–2009:Q2. The construction of model-generated unemployment is detailed in Section 8.4.

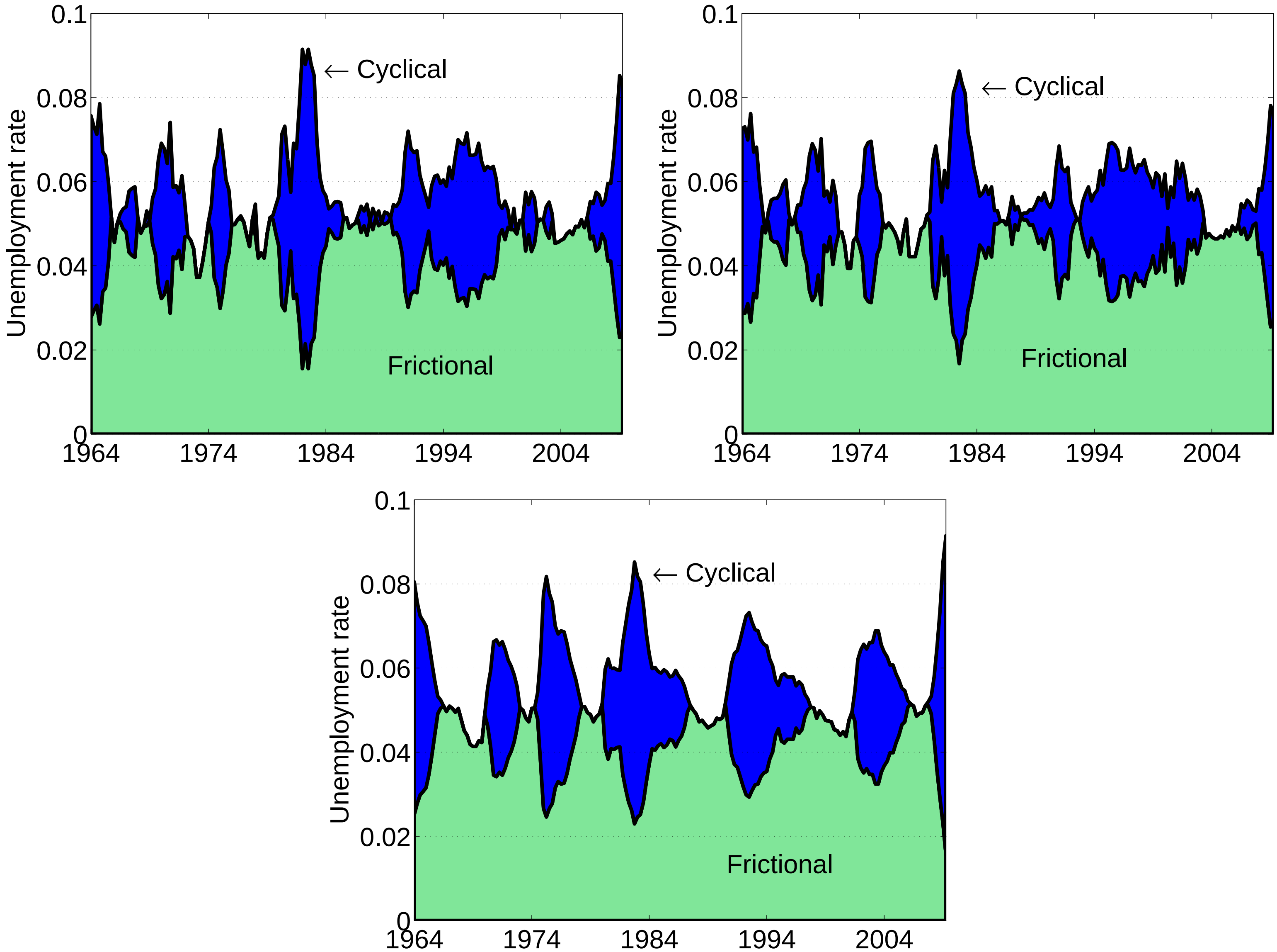


FIGURE 8: DECOMPOSITION OF MODEL-GENERATED (TOP) AND ACTUAL (BOTTOM) U.S. UNEMPLOYMENT, 1964–2009

Notes: The top left graph decomposes the unemployment series generated when the nonlinear model is stimulated by the quarterly labor productivity series constructed in Section 7.1 using output and employment data provided by the BLS. The top right graph decomposes the unemployment series generated when the nonlinear model is stimulated by the quarterly, utilization-adjusted TFP series constructed by Fernald (2009). The bottom graph decomposes actual unemployment, which is the quarterly average of seasonally-adjusted, monthly series constructed by the BLS from the CPS. Productivity, TFP, and actual unemployment are detrended with an HP filter with smoothing parameter 10^5. The time period is 1964:Q1–2009:Q2. The construction and decomposition of model-generated unemployment, as well as the decomposition of actual unemployment, are detailed in Sections 8.4 and 8.5.

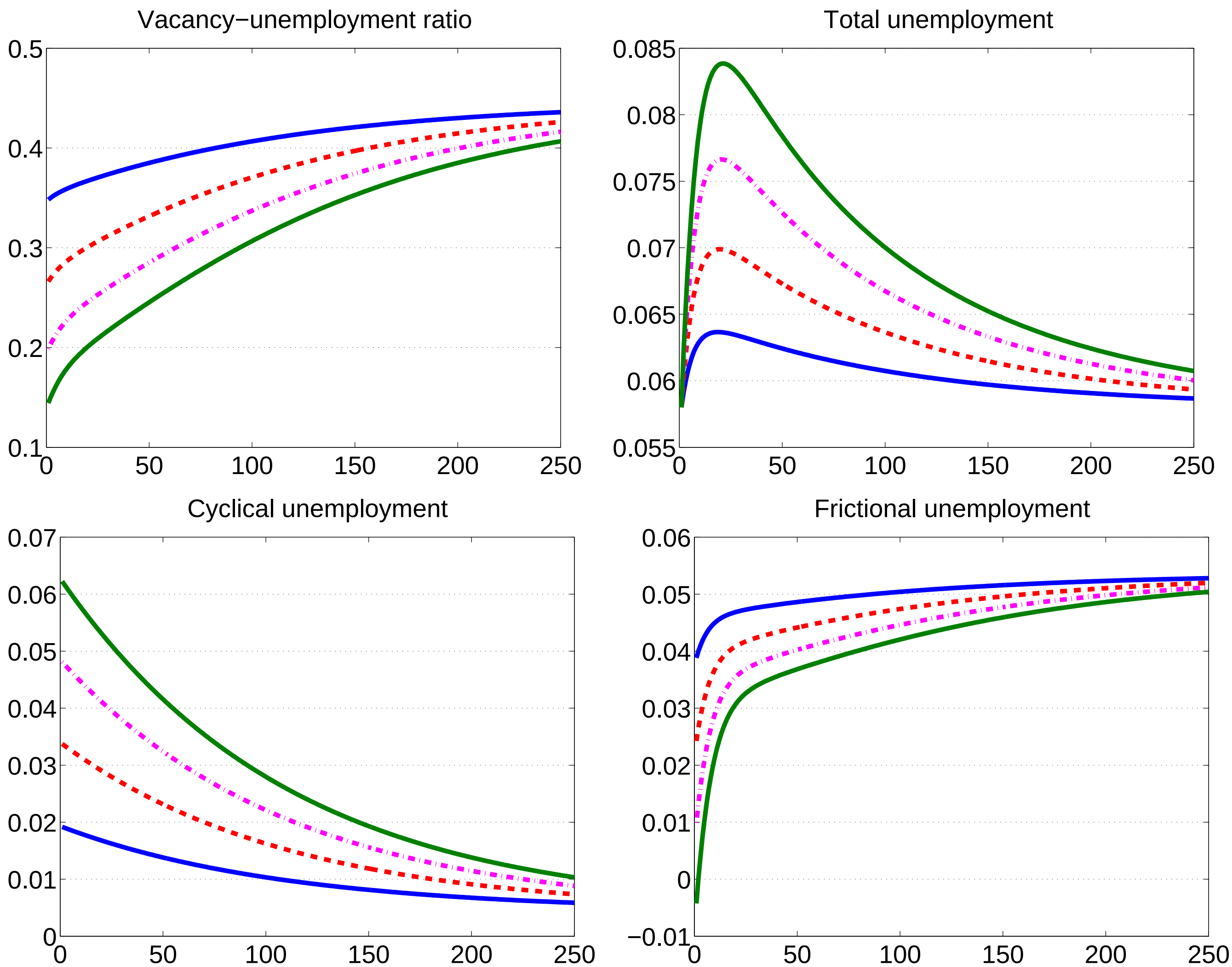


FIGURE 9: EXACT RESPONSE OF LABOR MARKET VARIABLES TO NEGATIVE PRODUCTIVITY SHOCKS (5, 10, 15, AND 20 STANDARD DEVIATIONS)

Notes: Response functions represent the evolution of labor market tightness, unemployment and its components (in percentage of the labor force) when a negative productivity shock hits the economy. The dark (blue) solid line is the response to a 5-standard-deviation shock; the dashed line to a 10-standard-deviation shock; the dot-and-dash line to a 15-standard-deviation shock; and the light (green) solid line to a 20-standard-deviation shock. A standard deviation for productivity shock is $\sigma = 0.0026$. The time period on the x-axis is a week. The response functions are obtained with a shooting algorithm, as described in Section 8.7.

TABLE 1: PARAMETER VALUES IN SIMULATIONS

	Interpretation	Value	Source
s	Separation rate	0.95%	JOLTS, 2000–2009
δ	Discount factor	0.999	Corresponds to 5% annually
ω	Efficiency of matching	0.23	JOLTS, 2000–2009
η	Elasticity of job-filling	0.5	Petrongolo and Pissarides (2001)
$\mathbb{E}[a]$	Mean productivity	1	Normalization
ρ	Autocorrelation of productivity	0.991	MSPC, 1964–2009
σ	Conditional variance of productivity	0.0026	MSPC, 1964–2009
		Job-rationing model	
w_0	Steady-state real wage	0.67	Matches unemployment $= 5.8\%$
α	Returns to labor	0.74	Matches labor share$= 0.66$
$\mathcal{M}$	Markup	1.11	Christiano et al. (2005)
γ	Real wage rigidity	0.70	Haefke et al. (2008)
c	Recruiting costs	0.21	$0.32 \times \overline{w}$
		Benchmark models	
MP model:			
c	Recruiting costs	0.32	$0.32 \times \overline{w}$
β	Worker's bargaining power	0.86	Matches unemployment $= 5.8\%$
MPS model:			
c	Recruiting costs	0.32	$0.32 \times \overline{w}$
w_0	Steady-state real wage	0.991	Matches unemployment $= 5.8\%$
SZ model:			
c	Recruiting costs	0.22	$0.32 \times \overline{w}$
α	Returns to labor	0.21	Matches labor share$= 0.66$
β	Worker's bargaining power	0.86	Matches unemployment $= 5.8\%$

Notes: Section 7 and Appendix D provide details on the calibration strategy. All parameters are calibrated at weekly frequency.

TABLE 2: ESTIMATION OF THE WAGE SCHEDULE WITH U.S. DATA

$\log(w_t)$	(1)	(2)	(3)	(4)	(5)	(6)	(7)
$\log(a_t)$	0.44	0.28	0.22	0.62	0.30	0.42	0.45
	(0.07)	(0.10)	(0.04)	(0.08)	(0.25)	(0.07)	(0.05)
R^2	0.19	0.18	0.04	0.27	0.10	0.15	0.32
Number obs.	182	34	182	182	33	182	182

Notes: This table presents the results from regressions of log real wage on log labor productivity. Standard errors of the estimates are in parenthesis. All series used are seasonally adjusted. Column (1) is the preferred specification. w_t is average hourly earning in the nonfarm business sector, constructed by the Bureau of Labor Statistics (BLS) Current Employment Statistics (CES) program, and deflated by the Consumer Price Index (CPI) for all urban households constructed by BLS. Average hourly earning is a quarterly series. CPI is a quarterly average of monthly series. $\log(a_t)$ is computed as the residual $\log(y_t) - \alpha \cdot \log(n_t)$. y_t and n_t are quarterly real output and employment in the nonfarm business sector, respectively, and are constructed by the BLS Major Sector Productivity and Costs (MSPC) program. The sample period is 1964:Q1–2009:Q2. Columns (2)–(7) perform robustness checks. (2) and (3) estimate the regression with alternative measures of real wage. In (2), w_t is the compensation of private industry workers, which is part of the Employment Cost Index (ECI) constructed by the BLS, deflated by the CPI. The ECI is a measure of the change in the cost of labor, free from the influence of employment shifts among occupations and industries over the business cycle. Compensation of private industry workers is a quarterly series. The sample period is 2001:Q1–2009:Q2 (the longest period for which ECI is available). In (3), w_t is real compensation constructed by the BLS MSPC program. This is a quarterly series, and the sample period is 1964:Q1–2009:Q2. Columns (4)–(6) estimate the regression with alternative measures of labor productivity. In (4), a_t is purified TFP at yearly frequency, constructed by Basu et al. (2006). The sample period is 1964–1996. In (5), $\log(a_t)$ is computed as $\log(y_t) - \alpha \cdot \log(h_t)$, in which h_t is quarterly hours worked in the nonfarm business sector, constructed by the BLS MSPC program. The sample period is 1964:Q1–2009:Q2. In (6), $\log(a_t)$ is simply computed as $\log(y_t) - \log(n_t)$. The sample period remains 1964:Q1–2009:Q2. The quarterly series $\log(w_t)$ and $\log(a_t)$ are detrended using an HP filter with smoothing parameter 10^5 in all regressions, except in (4) and (7). In (4), I use a smoothing parameter of 500 because the series are at yearly frequency. In (7), I use a smoothing parameter of 1,600 in a regression otherwise similar to (1), as a robustness check.

TABLE 3: SUMMARY STATISTICS, QUARTERLY U.S. DATA, 1964–2009

	U	V	θ	W	Y	a
Standard Deviation	0.168	0.185	0.344	0.021	0.029	0.019
Autocorrelation	0.914	0.932	0.923	0.950	0.892	0.871
	1	-0.886	-0.968	-0.239	-0.826	-0.478
	–	1	0.974	0.191	0.785	0.453
Correlation	–	–	1	0.220	0.828	0.479
	–	–	–	1	0.512	0.646
	–	–	–	–	1	0.831
	–	–	–	–	–	1

Notes: All data are seasonally adjusted. The sample period is 1964:Q1–2009:Q2. Unemployment rate U is quarterly average of monthly series constructed by the BLS from the CPS. Vacancy rate V is quarterly average of monthly series constructed by merging data constructed by the BLS from the Job Openings and Labor Turnover Survey (JOLTS), and data from the Conference Board, as explained in Section8.3. Labor market tightness θ is the ratio of vacancy level to unemployment level. W is quarterly, average hourly earning in the nonfarm business sector, constructed by the BLS CES program, and deflated by the quarterly average of monthly CPI for all urban households, constructed by BLS. Y is quarterly real output in the nonfarm business sector constructed by the BLS MSPC program. $\log(a)$ is computed as the residual $\log(Y) - \alpha \cdot \log(N)$, as explained in Section 7.1. N is quarterly employment in the nonfarm business sector constructed by the BLS MSPC program. All variables are reported in log as deviations from an HP trend with smoothing parameter 10^5.

TABLE 4: SIMULATED MOMENTS IN THE LOG-LINEARIZED MODEL

	U	V	θ	W	Y	a
Standard Deviation	0.133	0.159	0.287	0.013	0.024	0.018
	(0.020)	(0.021)	(0.041)	(0.002)	(0.004)	(0.003)
Autocorrelation	0.928	0.830	0.900	0.870	0.888	0.870
	(0.024)	(0.051)	(0.033)	(0.042)	(0.037)	(0.042)
	1	-0.922	-0.978	-0.985	-0.993	-0.985
		(0.022)	(0.007)	(0.005)	(0.002)	(0.005)
	–	1	0.985	0.933	0.921	0.933
			(0.004)	(0.020)	(0.024)	(0.020)
	–	–	1	0.974	0.971	0.974
				(0.008)	(0.009)	(0.008)
Correlation	–	–	–	1	0.997	1.000
					(0.001)	(0.000)
	–	–	–	–	1	0.997
						(0.001)
	–	–	–	–	–	1

Notes: Results from simulating the log-linearized model with stochastic productivity. All variables are reported as logarithmic deviations from steady state. Simulated standard errors (standard deviations across 200 model simulations) are reported in parentheses. Section 8.3 provides details on the simulation. Appendix C describes the log-linearized equilibrium conditions.

Table 5: Simulated moments in the MP model

	U	V	θ	W	Y	a
Standard Deviation	0.008	0.010	0.018	0.018	0.019	0.018
	(0.001)	(0.001)	(0.002)	(0.002)	(0.003)	(0.002)
Autocorrelation	0.929	0.837	0.902	0.866	0.869	0.866
	(0.020)	(0.043)	(0.027)	(0.036)	(0.035)	(0.036)
	1	-0.927	-0.978	-0.983	-0.984	-0.983
		(0.019)	(0.006)	(0.004)	(0.004)	(0.004)
	–	1	0.985	0.935	0.935	0.935
			(0.004)	(0.017)	(0.018)	(0.017)
	–	–	1	0.975	0.975	0.975
				(0.007)	(0.007)	(0.007)
Correlation	–	–	–	1	1.000	1.000
					(0.000)	(0.000)
	–	–	–	–	1	1.000
						(0.000)
	–	–	–	–	–	1

Notes: Results from simulating the MP model with stochastic productivity. All variables are reported as logarithmic deviations from steady state. Simulated standard errors (standard deviations across 200 model simulations) are reported in parentheses. Section 8.3 provides details on the simulation.

Table 6: Simulated moments in the MPS model with $\gamma = 0$

	U	V	θ	W	Y	a
Standard Deviation	1.026	3.923	4.708	0.000	0.306	0.018
	(0.157)	(1.029)	(1.140)	(0.000)	(0.149)	(0.002)
Autocorrelation	0.938	0.716	0.791	0.995	0.923	0.866
	(0.022)	(0.075)	(0.055)	(0.000)	(0.041)	(0.036)
	1	-0.703	-0.806	-0.000	-0.867	-0.925
		(0.052)	(0.039)	(0.000)	(0.034)	(0.013)
	–	1	0.987	0.000	0.644	0.726
			(0.005)	(0.000)	(0.064)	(0.047)
	–	–	1	-0.000	0.728	0.809
				(0.000)	(0.051)	(0.031)
Correlation	–	–	–	1	-0.000	-0.000
					(0.000)	(0.000)
	–	–	–	–	1	0.727
						(0.040)
	–	–	–	–	–	1

Notes: Results from simulating the MPS model with stochastic productivity, when $\gamma = 0$. All variables are reported as logarithmic deviations from steady state. Simulated standard errors (standard deviations across 200 model simulations) are reported in parentheses. Section 8.3 provides details on the simulation.

TABLE 7: SIMULATED MOMENTS IN THE MPS MODEL WITH $\gamma = 0.7$

	U	V	θ	W	Y	a
Standard Deviation	0.308	0.402	0.691	0.013	0.042	0.018
	(0.059)	(0.086)	(0.141)	(0.002)	(0.010)	(0.002)
Autocorrelation	0.931	0.814	0.894	0.866	0.906	0.866
	(0.022)	(0.045)	(0.029)	(0.036)	(0.031)	(0.036)
	1	-0.895	-0.966	-0.966	-0.992	-0.966
		(0.024)	(0.009)	(0.006)	(0.003)	(0.006)
	–	1	0.980	0.898	0.881	0.898
			(0.004)	(0.016)	(0.027)	(0.016)
	–	–	1	0.953	0.954	0.953
				(0.008)	(0.011)	(0.008)
Correlation	–	–	–	1	0.962	1.000
					(0.017)	(0.000)
	–	–	–	–	1	0.962
						(0.017)
	–	–	–	–	–	1

Notes: Results from simulating the MPS model with stochastic productivity, when $\gamma = 0.7$. All variables are reported as logarithmic deviations from steady state. Simulated standard errors (standard deviations across 200 model simulations) are reported in parentheses. Section 8.3 provides details on the simulation.

TABLE 8: SIMULATED MOMENTS IN THE SZ MODEL

	U	V	θ	W	Y	a
Standard Deviation	0.008	0.010	0.018	0.018	0.018	0.018
	(0.001)	(0.001)	(0.003)	(0.003)	(0.003)	(0.003)
Autocorrelation	0.926	0.836	0.901	0.863	0.865	0.865
	(0.024)	(0.048)	(0.031)	(0.043)	(0.042)	(0.043)
	1	-0.928	-0.978	-0.983	-0.984	-0.984
		(0.021)	(0.007)	(0.005)	(0.005)	(0.005)
	–	1	0.985	0.937	0.936	0.936
			(0.004)	(0.018)	(0.018)	(0.018)
	–	–	1	0.975	0.975	0.975
				(0.008)	(0.008)	(0.008)
Correlation	–	–	–	1	1.000	1.000
					(0.000)	(0.000)
	–	–	–	–	1	1.000
						(0.000)
	–	–	–	–	–	1

Notes: Results from simulating the log-linearized SZ model with stochastic productivity. All variables are reported as logarithmic deviations from steady state. Simulated standard errors (standard deviations across 200 model simulations) are reported in parentheses. Section 8.3 provides details on the simulation.

TABLE 9: SIMULATED MOMENTS IN THE NONLINEAR MODEL

	Mean	Std. dev.	Autoc.	Correlation		
				U	U^C	U^F
U	0.059	0.013	0.901	1	0.969	-0.789
	(0.004)	(0.002)	(0.028)		(0.023)	(0.162)
U^C	0.020	0.021	0.888	–	1	-0.914
	(0.006)	(0.004)	(0.034)			(0.075)
U^F	0.039	0.008	0.843	–	–	1
	(0.002)	(0.001)	(0.048)			

Notes: Results from simulating the nonlinear model with stochastic productivity. Simulated standard errors (standard deviations across 200 model simulations) are reported in parentheses. Section 8.6 provides details on the simulation algorithm, and the stochastic process for labor productivity.