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Utilitarianism and Unequal Longevities: A Remedy?*

Marie-Louise Leroux† and Gregory Ponthiere‡

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Abstract

Classical utilitarianism, if coupled with standard assumptions such as the expected utility hypothesis and additive lifetime welfare, has the undesirable corollary to recommend a redistribution of resources from short-lived to long-lived agents, against any intuition of compensation. This paper proposes a remedy to that undesirable property of utilitarianism. This remedy consists in imputing, when solving the social planner's problem, the consumption equivalent of a long life to the consumption of long-lived agents. Provided the consumption equivalent is positive, the modified first-best problem exhibits a compensation of short-lived agents, under the form of a higher consumption. Then, in a general framework where agents differ in survival prospects, we compare the *ex ante* remedy (compensating agents with a lower life expectancy) and the *ex post* remedy (compensating short-lived agents), and show their incompatibility.

Keywords: utilitarianism, differential longevity, compensation, redistribution, consumption equivalent.

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1 Introduction

Although widely used by taxation theorists, utilitarianism exhibits nonetheless a quite counterintuitive corollary in the particular context of unequal longevities. Actually, under standard assumptions such as the expected utility hypothesis and additive lifetime welfare, utilitarianism recommends nothing less than the redistribution of resources from *short-lived* agents to *long-lived* agents.

That corollary, which is in full contradiction with any intuition of compensation, can be explained as follows. Under additive lifetime welfare, a social planner cannot distinguish between, on the one hand, one life of x periods, and, on the other hand, x lives of one period. Hence, provided Gossen's first law (1854) - i.e. the law of declining marginal utility of consumption per period - holds, it is always optimal, for a utilitarian social planner, to give the same consumption per period to all agents, whatever their length of life is. As a consequence, long-lived agents do not only live longer: they benefit also, at the social optimum, from more resources. Hence, provided living long is a good thing (or, at least, not a bad thing *per se*), short-lived people are, in a sense, penalized *twice*: once by Nature and once by Bentham.

This double penalization is quite counterintuitive, especially when longevity differentials are exogenous. Clearly, in that case, one would like short-lived agents to be compensated for their short life, as they cannot be regarded as responsible for this. Note that the intuition for compensation may also be strong even when longevity differentials are partly endogenous. For instance, shorter lives due to a strong taste for sin goods, or a large disutility from physical activity may be also regarded as caused by factors that are exogenous to the agent, and, as such, which would support some compensation.[1]

Classical utilitarianism can hardly do justice to such intuitions. All this does not really come as a surprise: as shown by Mirrlees (1982), utilitarianism can, at best, serve as an ethical standard in the special case of a society of *identical* individuals, because, in that case, the totality of all individuals can be regarded as a single individual. However, once some heterogeneity is introduced in the fundamentals (e.g. preferences, handicap, etc.), utilitarianism can only be used as a useful approximation, and may lead to counterintuitive results.[2]

[1] Note that the introduction of individual effort variables raises tensions between the compensation of inequalities due to non-transferable individual characteristics (i.e. equal treatment of agents with equal efforts) and the normative ideal of responsibility (i.e. equal treatment of agents with equal characteristics), as shown by Fleurbaey and Maniquet (2006).

[2] For instance, Arrow (1971) and Sen (1973) showed that, given that a handicaped person is likely to have

Given that a variation in the length of life can hardly be regarded as non-fundamental, it is not surprising that utilitarianism yields here some counterintuitive results.

But even if the difficulties faced by classical utilitarianism under longevity differentials could be expected, this leaves us nonetheless with a quite uncomfortable position. The origin of this discomfort lies in the universality of longevity differentials. Actually, as shown by demographers, longevity differentials within a given cohort have always been large, and remain significant today. For instance, according to the United Nations Development Program (2008), the life expectancy of women is, in the U.S., about 5.2 years larger than the one for men in 2007 (80.4 years against 75.2). Moreover, there exist also large disparities in survival conditions according to characteristics such as the education, the income, the ethnicity, and the employment status.[3] Hence, if the mere existence of longevity differentials suffices to reject the use of classical utilitarianism, there seems to remain little room for using it as an ethical doctrine.

Should we then abandon utilitarianism when considering policy discussions in which agents have unequal lengths of life, that is, in almost all policy issues? Whereas one may be tempted to answer affirmatively, it should be stressed that various solutions can be brought, in order to keep the utilitarian framework, but *without* the undesirable redistribution from short-lived to long-lived agents.

A first solution consists in relaxing the assumption of additive lifetime welfare, and in representing lifetime utility by a concave transform of the sum of temporal utilities. That solution, proposed by Bommier (2005) and Bommier *et al* (2007a, 2007b), introduces a distinction between one life of x periods and x lives of one period, so that a utilitarian social planner becomes less likely to redistribute from the short-lived to the long-lived. Another solution, explored in Leroux and Ponthiere (2009), consists of relaxing the expected utility hypothesis, which is another way to avoid the double penalization. However, those approaches, which rely on complex representations of individual preferences, do not lead to analytically tractable solutions, neither at the laissez-faire, nor at the first-best. Hence, these can save utilitarianism only at the cost of much larger analytical complexity.

As an alternative to those solutions, this paper proposes to keep the utilitarian framework, as well as the standard additive lifetime welfare and expected utility hypotheses, but

a lower marginal utility of consumption than other persons, utilitarianism would give him fewer resources than to much better off persons, which is quite paradoxical.

[3] See Rogot *et al* (1992).

to apply a kind of "remedy", in order to avoid the undesirable redistribution from short-lived to long-lived agents.

The remedy consists in counting, when solving the social planner's problem, the (lifetime spread) *consumption equivalent of a long life* as part of the (per period) consumption bundle of long-lived agents, in such a way that their longer life is counted by the planner as something that they enjoy.[4] The intuition behind the introduction of that consumption equivalent is that this allows the utilitarian planner to take into account the value of continuing life, and, thus, to distinguish, despite additive lifetime welfare, between one life of x periods and x lives of one period.

Moreover, by introducing the consumption equivalent of a long life in the social planner's problem, the possibility of compensating short-lived agents - rather than penalizing them twice - is allowed, and the undesirable redistribution from short-lived to long-lived agents is contradicted, and may, for a large value of longevity, be turned into a redistribution from long-lived to short-lived agents.

The goal of this paper is to examine how that remedy allows the compensation of short-lived agents or, at least, reduces the - counterintuitive - transfers from short-lived to long-lived agents. For that purpose, we study a two-period model, where agents face distinct survival conditions, and contrast the standard utilitarian allocation of resources with the modified utilitarian problem in which the consumption equivalent of a long life is imputed to long-lived agents' consumption. We will also characterize various manners in which the remedy can be applied, the consumption equivalent being used to compensate either differences in longevity prospects, or differences in actual longevities.

At this stage, several specificities of our approach should be stressed.

Firstly, the present study remains *utilitarian stricto sensu*, in the sense that this still relies on the three pillars of utilitarianism: (1) welfarism, (2) sum-ranking and (3) consequentialism. Thus, our approach differs from ethical frameworks relying on a broader informational basis (e.g. primary goods, functionings), on an non-aggregative objective (e.g. maximin), and paying attention to the relation between means and ends (e.g. responsibility-based approaches). Note that our exclusive focus on a strict utilitarian framework does not reveal any unconditional adherence to any of the three pillars, whose ethical plausibility has

[4] That remedy is close to what Broome (2004) proposes in his attempt to account for the value of longevity in a utilitarian framework, but in a goods metrics (and not utility metrics).

been largely questioned by philosophers.[5] On the contrary, this paper focuses on a specific problem of utilitarianism under unequal longevities, and aims at proposing a solution *without* having to get rid of utilitarianism (and of its analytical convenience for taxation theory). Hence, our approach can be regarded as a complement to alternative, non-utilitarian frameworks, whose properties under unequal longevities remain to be explored.

Secondly, the approach developed in this paper can be interpreted as a utilitarianism constrained by some *compensation constraint*, in the sense that the social planner's allocation is now chosen under an additional restriction, which specifies the level of the consumption equivalent of a long life. But this constrained utilitarianism can also, in a particular sense, be regarded as *a full or complete utilitarianism*. Actually, the imputation of a long life through the addition of a consumption equivalent in the consumption bundle of long-lived agents contributes to fully describe the agents' baskets, unlike what used to be the case under standard utilitarianism, where some good (or bad) - a long life - was enjoyed (or overcome) by some agents - and not by others - but remained outside the planner's calculations. Classical utilitarianism was incomplete, and, thus, necessarily *partial*. The remedy can be regarded as a way to make it impartial again, despite longevity differentials.[6]

Thirdly, the remedy proposed here is *operational*, in the sense that the correction carried out relies on existing empirical evidence. While knowing the consumption equivalent of a longer life is a prerequisite for the application of the remedy proposed here, the large literature dedicated to the estimation of the value of a statistical life (VSL) on the basis of risk-wage studies, consumption behaviour studies or contingent valuation methods can serve as a basis for computing the consumption equivalent of a longer life.[7] Actually, many recent papers, such as Nordhaus (2003) and Becker *et al* (2005), are concerned with the empirical estimation of consumption equivalents of a longer life.[8] Hence the remedy proposed here has an empirical counterpart, and is fully operational.

This paper is organized as follows. Section 2 presents the standard utilitarian redistribution problem under unequal longevities. Section 3 proposes the remedy, and contrasts the modified first-best problem with the standard utilitarian problem. Section 4 introduces risk about the length of life, and compares the remedy allowing the *ex ante* compensation

[5] On utilitarianism's weaknesses, see Sen and Williams (1982).

[6] In other words, the remedy makes classical utilitarianism impartial again, by taking into account a morally relevant piece of information: the differences in longevity prospects.

[7] On the VSL, see the survey by Viscusi and Aldy (2003).

[8] Note that the empirical estimation, on the basis of VSL statistics, of consumption equivalents of a longer life dates back to the pioneer works by Usher (1973).

(i.e. for unequal life expectancies) with the one allowing the *ex post* compensation (i.e. for unequal actual lengths of life). Numerical illustrations are provided in Section 5. Section 6 concludes.

2 The basic model

Let us first consider the standard utilitarian problem of redistribution under differential longevities. For that purpose, we shall consider here two types of agents, $i = 1, 2$, with different longevities. Type-1 agents live one period (of length normalized to 1), while type-2 agents live two periods.

As usually assumed in the literature, the utility of death is fixed to zero. Hence, under additive lifetime welfare, the lifetime utility of agents is

$$\begin{aligned} U^1 &= u\left(c^1\right) \\ U^2 &= u\left(c^2\right) + u\left(d^2\right) \end{aligned}$$

where c^i and d^i are first and second period consumptions for agent $i = 1, 2$.[9] We assume, as usual, $u'(.) > 0$ and $u''(.) < 0$.[10] Moreover, for simplicity, the utility function takes the form $u(c) = v(c) + \beta$, where $v(0) = 0$, and $\lim_{c\to\infty} v(c) = \bar{v}$, with $\beta < \bar{v} < \infty$.

2.1 The laissez-faire

At the laissez-faire, and assuming that each agent has one half of the total endowment W of resources, the optimal consumptions are

$$\begin{aligned} c^1 &= \frac{W}{2} \\ c^2 &= d^2 = \frac{W}{4} \end{aligned}$$

Contrary to common beliefs, nothing guarantees, in general, that the long-lived agent is, at the laissez-faire, better off than the short-lived agent. However, under mild conditions on individual temporal utility functions $u(.)$ and the available resources W, long-lived people are necessarily better off than short-lived persons, and, thus, advantaged by Nature, despite the equality of resources available for each of them.

[9] For simplicity, we abstract here from pure time preferences. See Section 4 for a more complete model with (natural) time discounting.

[10] Note the temporal utility functions $u(.)$ are assumed to be the same for all agents.

Proposition 1 *If $u(0) \geq 0$ (i.e. $\beta \geq 0$), type-2 agents are, at the laissez-faire, better off than type-1 agents, whatever the total amount of resources W is. If $u(0) < 0$ (i.e. $\beta < 0$), type-2 agents are, at the laissez-faire, better off than type-1 agents if and only if $W > W^S$, where W^S is such that $2v\left(\frac{W^S}{4}\right) + \beta = v\left(\frac{W^S}{2}\right)$.*

Proof. The first part of the Proposition follows from the concavity of $u(.)$**:** under $u(0) \geq 0$ (i.e. $\beta \geq 0$), we necessarily have, under $u''(\cdot) < 0$,

$$u\left(\frac{W}{2}\right) < u\left(\frac{W}{4}\right) + u\left(\frac{W}{4}\right)$$

However, under $u(0) < 0$ (i.e. $\beta < 0$), that inequality is satisfied only if

$$2v\left(\frac{W}{4}\right) - v\left(\frac{W}{2}\right) + \beta > 0$$

The LHS of that expression is negative at $W = 0$, but tends to $\bar{v} + \beta > 0$ when W tends to infinity. Hence, by continuity, there must exist a resource level W^S at which that expression equals zero. For higher resource levels, the above inequality is strictly satisfied, so that short-lived agents are, at the laissez-faire, worse off than long-lived agents, despite the equality of endowment $W/2$. ■

Hence, it follows from Proposition 1 that, under either $u(0) \geq 0$ or $W > W^S$, short-lived agents enjoy, at the laissez-faire, a lower welfare level than the one of long-lived agents. Thus, under those conditions, short-lived agents are said to be disadvantaged by Nature, as they enjoy, for an *equal* amount of resources, a lower lifetime utility level than long-lived agents. Given that it is only under a very low level of resources that short-lived are advantaged by Nature, we shall, throughout this paper, pay a larger attention to the case where short-lived are disadvantaged, and leave the other, less plausible case, aside.

2.2 The utilitarian optimum

Let us now examine how a social planner would distribute a given amount W of resources. The problem of the social planner can be written as:

$$\max_{c^1,c^2,d^2} u\left(c^1\right) + u\left(c^2\right) + u\left(d^2\right)$$
$$\text{s.to } c^1 + c^2 + d^2 \leq W$$

Note that the objective function takes the same form as in a problem where there would be three agents living one period, or one agent living three periods, and, thus, does not do justice to the longevity differentials across people.

From the FOCs of that simple optimization problem,

$$u'\left(c^1\right) = u'\left(c^2\right) = u'\left(d^2\right)$$

it can be seen that, under a decreasing marginal utility of consumption, the optimal allocation is such that $c^1 = c^2 = d^2 = W/3$. Thus, total consumption for a type-1 agent is $W/3$ while it is $2W/3$ for a type-2 agent.

Hence, utilitarianism redistributes resources from type-1 agents (i.e. short-lived agents) to type-2 agents (i.e. long-lived agents), contrary to any intuition of compensation.[11] Clearly, given that the agent of type 1 lives a shorter life, and is, under the mild conditions of Proposition 1, disadvantaged by Nature, one may be tempted to give him some compensation, that is, some additional consumption. But utilitarianism yields the opposite result: the long-lived agent will also benefit from a higher total consumption. That double penalization is summarized in Proposition 2.

Proposition 2 *Under the conditions of Proposition 1, classical utilitarianism yields a double penalization, as it reinforces the welfare inequalities between short-lived and long-lived agents induced by Nature.*

Proof. Under the conditions of Proposition 1, the inequality in lifetime welfare between the long-lived and the short-lived agents is, at the laissez-faire:

$$2u\left(\frac{W}{4}\right) - u\left(\frac{W}{2}\right) > 0$$

At the utilitarian optimum, that inequality becomes

$$2u\left(\frac{W}{3}\right) - u\left(\frac{W}{3}\right) > 0$$

whose first term is larger than the first term at the laissez-faire, while the second term is smaller, so that the lifetime welfare inequality under utilitarianism must be strictly larger than under the laissez-faire. ■

The undesirable tendency of utilitarianism to reinforce, under the conditions of Proposition 1, the welfare inequalities induced by Nature is illustrated on Figure 1. Under the laissez-faire, each agent allocates his endowment $W/2$ optimally, so that short-lived agents

[11] Note that this redistribution violates, under the conditions of Proposition 1, what Sen (1973) called the Weak Equity Axiom: when an agent has a lower utility level than another for all levels of income, the optimal allocation must not give less income to him than to the other.

consume their whole endowment during period 1 ($c^1 = W/2$) and long-lived agents smooth their consumption across periods ($c^2 = d^2 = W/4$). But such an allocation does not maximize the sum of individual utilities, as the marginal utility of consumption of type-1 agents is lower than the marginal utility of consumption of type-2 agents at their two periods of life. Thus, from a utilitarian point of view, it is necessary to redistribute resources from short-lived agents to long-lived agents, in such a way as to equalize consumption per period. This redistribution makes type-1 agents penalized twice: once by Nature, and once by Bentham.

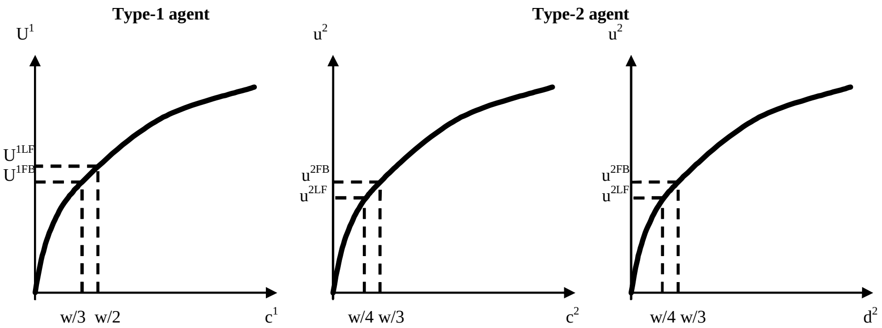


Figure 1: The basic utilitarian problem

Note that this undesirable corollary of utilitarianism follows from the conjunction of additive utilities across people and time: this double-additivity makes the social planner treat the second period of the life of type-2 agents *as if* this was the life of another person. This is the precise reason why utilitarianism has here an *anti-redistributive* feature.[12] Otherwise, if some means was found to distinguish the two periods contained in a long life from the two periods contained in two short lives, the social planner could treat the unique life-period of a type-1 agent differently from the first or second life-period of a type-2 agent. Section 3 proposes one way to carry out such a distinction.

[12] A similar argument is made in Bommier (2005) and in Bommier *et al.* (2007a).

3 The modified utilitarian problem

3.1 The consumption equivalent of a long life

In order to distinguish periods of life depending on the number of periods lived by agents, one remedy consists in introducing, within the social planner's problem, a fictive consumption equivalent of a long life, and to solve the so-constructed modified utilitarian problem.

For that purpose, let us first denote by α the fictive consumption equivalent to a long life. The consumption equivalent to a long life α corresponds to the value, expressed in the (unique) consumption good, of enjoying a long life, that is, in the present context, a life of two periods. Alternatively, α can also be interpreted as reflecting the consumption equivalent of the continuity of life across periods. In the rest of this paper, we shall assume that such a consumption equivalent exists, in the sense that, for any longevity differential, it is possible to find a compensation in terms of the consumption good.[13]

The consumption equivalent of a long life α makes the agent indifferent between, on the one hand, a short life with that additional consumption, and, on the other hand, a long life:

$$u\left(c^{1*}+\alpha\right)=u\left(c^{1**}\right)+u\left(d^{1**}\right) \tag{1}$$

where c^{1*}, c^{1**} and d^{1**} are the consumptions under the laissez-faire, when the agent faces one period of life [i.e. problem (*)] or two periods of life [i.e. problem (**)].[14] From Section 2, we have $c^{1*}=W/2$ and $c^{1**}=d^{1**}=W/4$ (as a type-1 agent with two periods of life would behave as a type-2 agent).

The level of α depends on the utility functions of agents. If, for instance, $u(.)$ was linear and if $u(0)=0$, we would have $u(c^{1*}+\alpha)=W/2+\alpha$, and $u\left(c^{1**}\right)+u\left(d^{1**}\right)=W/2$, so that α would be equal to zero: to make an agent indifferent between a short life and a long life, no compensation is required, as it suffices, for this short-lived agent, to transfer second-period consumption to the first period. Alternatively, if $u(.)$ is affine with an intercept β, we have $u(c^{1*}+\alpha)=W/2+\alpha+\beta$, and $u\left(c^{1**}\right)+u\left(d^{1**}\right)=W/2+2\beta$, which yields $\alpha=\beta$. However, if $u(.)$ is concave, and if $u(0)$ is not too low or if resources W are sufficiently large, we have $u(W/2)<u(W/4)+u(W/4)$, so that a positive compensation $\alpha>0$ is required for the short-lived.

[13] That assumption is far from weak, especially if the longevity differentials considered are large. In that case, a consumption equivalent may not exist.

[14] Note that this would also be true for type-2 agents.

Given that temporal utility is widely acknowledged to be concave in consumption, and that resources W can be regarded as sufficiently large, the intuition tends to assign to α a positive sign: this captures the idea that it is better to have, *ceteris paribus*, a long life rather than a short life, that is, it is better, *for a given amount of resources*, to live long. Actually, the conditions guaranteeing a positive α are, by construction, the same as the ones that lead to a double penalization by Nature and by Bentham, i.e. the conditions of Proposition 1.[15]

At this stage, it is also crucial to notice that, in the special case of *identical* temporal utility functions for all agents, the consumption equivalent of a long life α does not only have the capacity, by construction, to equalize the lifetime utility under a short life and a long life, but it has also the capacity to *compensate* a short-lived agent by giving him as much utility as a long-lived agent.

To see this, note first that, given that agents have the same utility functions, the RHS of expression (1) is also equal to $u\left(c^{2**}\right) + u\left(d^{2**}\right)$, because all agents would solve the consumption program similarly if put in the same situation under identical utility functions. It follows from this that the consumption equivalent α does not only bring the equality of utility between the short-life and the long-life for a given individual, but it also *equalizes the lifetime utilities* of agents having different lengths of life:

$$u\left(c^{1*} + \alpha\right) = u\left(c^{2**}\right) + u\left(d^{2**}\right) \tag{2}$$

as $c^{1**} = c^{2**}$ and $d^{1**} = d^{2**}$ for agents solving the same problem (**). The advantage of expression (2) over expression (1) is that (2) relies on empirically observable choices: type-2 agents' consumptions at the laissez-faire.

The construction of the consumption equivalent of a long life is illustrated on Figure 2. To find the value of α, we compute the total utility of a type-2 agent under the laissez-faire, equal to $U^2 = u(c^2) + u(d^2)$, and look for the level of c^1 that yields $U^1 = U^2$. On the left graph, that level is equal to $W/2 + \alpha$. Hence, the consumption equivalent of a long life, α, corresponds to the thick horizontal segment on the left graph of Figure 2.

[15] To see this, note first that, under $u(0) \geq 0$, or $u(0) < 0$ and $W > W^S$, we have, at the laissez-faire, $u\left(\frac{W}{2}\right) < 2u\left(\frac{W}{4}\right)$, so that only a positive α could make the LHS equal to the RHS: $u\left(\frac{W}{2} + \alpha\right) = 2u\left(\frac{W}{4}\right)$.

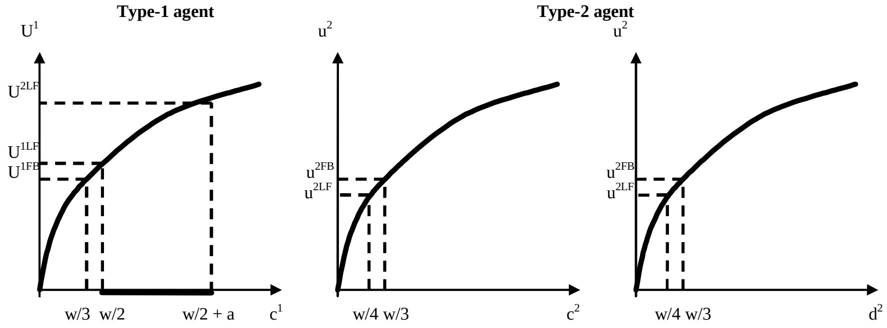


Figure 2: The consumption equivalent

In the light of its inherent capacity to compensate agents for facing shorter lives, the consumption equivalent of a long life α consists in an adequate tool for a utilitarian social planner facing an allocation problem of the kind described above. Clearly, the social planner, instead of solving the planning problem and redistributing resources from short-lived to long-lived agents, may rather include the consumption equivalent of a long life α in his calculations, to take into account the fact that long-lived agents have been advantaged by Nature.[16] One way to proceed is to treat α as a fictive consumption of long-lived agents, whose value is obtained by solving (2). The next subsection develops that alternative social planning problem.

3.2 The modified planner's problem

The modified utilitarian problem differs from the standard one (see Section 2) in a single aspect: the planner, instead of defining the consumption of agents while not taking into account longevity differentials, incorporates now the consumption equivalent of a long life as part of the long-lived agents' consumption.

Note that there exist several ways to introduce α in the consumption of the long-lived agent. *A priori*, the social planner might count the whole consumption equivalent of a long life as a part of the second-period consumption, or, alternatively, as a part of the first-period consumption. However, if α captures the lifetime - rather than instantaneous - value of the continuation of life as a whole, it makes more sense to spread that consumption equivalent of a long life *equally* on all periods lived by a long-lived agent. By proceeding in that way,

[16] Note that if $\alpha < 0$, then the social planner would rather take into account the fact that the long-lived are *disadvantaged* by Nature.

the social planner will distinguish the two periods of life of a long-lived agent from the unique period of life lived by a short-lived agent. The two alternative ways to spread the consumption equivalent of a long life (either entirely on the first or on the second period) would not imply such a distinction, so that this equal division of α on all life-periods seems more appropriate.[17]

Provided the consumption equivalent of a long life is introduced in the consumption of the long-lived agent in that manner, the planner's problem becomes:

$$\begin{array}{l} \max_{c^1,c^2,d^2} u\left(c^1\right) + u\left(c^2 + \frac{\alpha}{2}\right) + u\left(d^2 + \frac{\alpha}{2}\right) \\ \text{s.to } c^1 + c^2 + d^2 \leq W \end{array}$$

where α is obtained from equation (2).[18] The introduction of α can be regarded here as the addition of a *compensation constraint*, aimed at making the planner internalize longevity differentials as an ethically relevant piece of information.

In the present case, the FOCs of that modified problem are

$$u'\left(c^1\right) = u'\left(c^2 + \frac{\alpha}{2}\right) = u'\left(d^2 + \frac{\alpha}{2}\right)$$

which yield, under the conditions of Proposition 1:

$$c^1 > c^2 = d^2$$

Using both the FOCs and the budget constraint, it is easy to see that, under this constrained first-best,

$$\begin{aligned} c^1 &= W/3 + \alpha/3 \\ c^2 &= d^2 = W/3 - \alpha/6 \end{aligned}$$

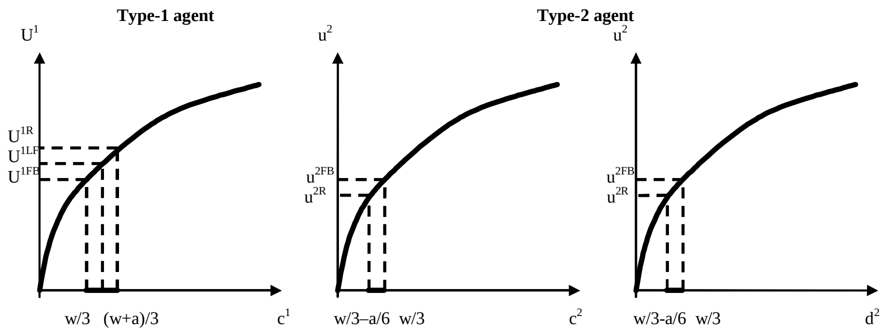
First, note that under standard utilitarianism, we had $c^1 = c^2 = d^2 = W/3$. Thus, under the condition that $\alpha > 0$, the social planner now gives more per period consumption to short-lived agents than to long-lived agents.[19] Indeed, in addition to the first-best allocation $W/3$, short-lived agents now also receive, as a compensation, a third of the consumption

[17] Note that, in the case where $\alpha < 0$, it makes also sense to spread the *cost* of living longer on the two periods of life.

[18] Note that α cannot be seen as a satisfaction parameter for having a long life, as α is in good metrics, not utility metrics. This point is worth being stressed, as an additive satisfaction parameter expressed in utility terms would not affect at all the problem of the social planner.

[19] Alternatively, under $\alpha < 0$, we would have $c^1 < c^2 = d^2$.

equivalent of a long life. On the contrary, the long-lived agents now undergo a reduction of their consumption in the first and second periods, equal to $\alpha/6$. That modified utilitarian problem is illustrated by Figure 3.



The modified utilitarian problem

The size of that compensation (i.e. the extent to which $c^1 > c^2 = d^2$) depends on the value α (i.e. the thick segment on the left graph of Figure 2). In the case of linear utility functions, α is equal to 0, and we have $c^2 = d^2 = c^1$. Clearly, in that extreme case, there is no value given to a long life, and so no compensation should be given to the short-lived. However, if α tends to infinity, c^2 and d^2 tend towards 0, while c^1 tends to W. In that extreme case, the social planner gives the whole resources to the short-lived, as the long life of type-2 agents makes them insensitive to the pleasure of goods consumption. Between those two extreme cases, there exist many intermediate cases, which can yield, for a low α, $c^1 < c^2 + d^2$, or, for a large α, $c^1 > c^2 + d^2$.

Note also that thanks to the equal division of α on the two periods of life, the modified utilitarian solution keeps the fundamental properties of utilitarianism: *even if* one abstracts from the fictive consumption equivalent of a long life α, there is an equalization of the marginal utilities of consumption over time, and, thus, a perfect smoothing of consumption over time. Other ways to spread α across periods would not yield this property, as they would lead to a non-equalization of marginal utilities of consumption across periods.

Finally, the modified utilitarian problem has also the interesting property of *compensating* short-lived agents, as stated in Proposition 3.

Proposition 3 *Under the conditions of Proposition 1, the remedy reduces welfare inequal-*

ities in comparison with the utilitarian optimum.

Proposition 3 states that the inequality in lifetime welfare between the long-lived and the short-lived under the remedy is reduced in comparison with utilitarianism. Under the latter, the gap between type-1 and type-2 utilities is

$$V^{2FB} - V^{1FB} = 2u\left(\frac{W}{3}\right) - u\left(\frac{W}{3}\right)$$

while, under the remedy, this gap becomes

$$V^{2R} - V^{1R} = 2u\left(\frac{W}{3} - \frac{\alpha}{6}\right) - u\left(\frac{W}{3} + \frac{\alpha}{3}\right)$$

where V^{iFB} and V^{iR} are the indirect utility functions of individual i under standard utilitarianism and under the remedy.

This discussion illustrates how the introduction of the consumption equivalent of a long life suffices to avoid the undesirable corollary of utilitarianism present in the standard problem, where short-lived agents were penalized twice: once for a shorter life, and once for enjoying less consumption. Here, at least, the introduction of α allows for a different treatment of life-periods, depending on whether these are lived by long-lived or by short-lived agents.

However, the modified problem does not necessarily yield the equalization of lifetime utilities across agents, as

$$V^{1R} \gtrless V^{2R} \iff u\left(\frac{W}{3} + \frac{\alpha}{3}\right) \gtrless 2u\left(\frac{W}{3} - \frac{\alpha}{6}\right)$$

Thus, nothing guarantees the equalization of lifetime utilities under the remedy: whether short-lived agents end up better-off or worse-off than long-lived agents depends on the level of α. For instance, on Figure 3, it appears that, given the shape of the temporal utility function, type-1 agents are, despite the compensation associated with the remedy, at a lower level of lifetime utility V^{1R} than type-2 agents. The proposed remedy is thus distinct from a Maximin solution equalizing agents' utilities $V^1 = V^2$: we remain here under utilitarianism, even though it is a constrained kind of utilitarianism.

4 *Ex ante versus ex post* compensation

4.1 An economy where longevity is risky

A major limitation of the preceding analysis was to focus on a purely deterministic world, where short-lived and long-lived agents can be identified *ex ante* by the social planner. As a consequence, the planner could use the consumption equivalent of a long life α to provide some compensation, knowing exactly all agents' longevity.

In reality, it is quite difficult to proceed in that way, as the length of life is inherently risky. Some agents have, because of some characteristics, a higher propensity to die, but this does not guarantee that each of those agents will necessarily enjoy a shorter life. In other words, the enjoyment of a higher *expected* length of life or life expectancy *ex ante* does not necessarily imply the enjoyment of a longer life *ex post*.[20]

In order to take into account the difference between the expected length of life and the actual length of life, we now assume that agents of type $i = 1, 2$ all live a first period of life with certainty, but reach the second period with a probability π^i.[21] Agents are thus not equal, and differ with respect to an exogenous characteristic influencing their longevity prospects, so that their life expectancies $1 + \pi^i$ are unequal. As above, we assume that type-1 agents suffer from some disadvantage with respect to type-2 agents:

$$\pi^1 < \pi^2$$

In comparison with the previous sections, the introduction of a probability of survival creates, implicitly, a third and a fourth type of agents: besides long-lived type-2 agents and short-lived type-1 agents, we now have also the *lucky* type-1 agents and the *unlucky* type-2 agents. The former enjoy, despite their belonging to a disadvantageous group, a long life, while the latter have a short life, despite being members of a group *a priori* advantaged by Nature.

The existence of lucky and unlucky agents within each group raises the question of the optimal form of compensation: do we want to compensate people *ex ante*, that is, to compensate them for belonging to a group *a priori* disadvantaged by Nature (i.e. type-1 agents)? Or, on the contrary, do we want to compensate people *ex post*, that is, to compensate them for having a shorter life, whatever the group to which they belong (i.e. unlucky type-1 and type-2 agents)? Undoubtedly, the form taken by the compensation - and, thus, the remedy to be implemented - depends on whether one adopts an *ex ante* or an *ex post* approach. This section contrasts those two compensation problems, and examines

[20] For instance, although women exhibit a higher life expectancy than men, there exist some women who have a shorter life than some men, so that the groups with distinct survival prospects do not correspond to the groups with distinct actual longevities.

[21] For simplicity, we assume here that this probability is exogenous. On optimal tax policy under endogenous survival probabilities through health spending, see Leroux *et al* (2008).

how the remedy can be implemented in each case.

4.2 The laissez-faire

Under the laissez-faire, each agent of type i, who is assumed to be an expected utility maximizer, chooses consumptions in such a way as to maximize

$$u(c^i) + \pi^i u(d^i)$$

subject to the budget constraints

$$\begin{aligned} c^i &\leqslant \frac{W}{2} - s^i \\ d^i &\leqslant s^i R^i \end{aligned}$$

where s^i denotes savings and R^i is the return on savings. It is assumed, for simplicity, that the pure interest rate is zero, and that a perfect, class-specific, annuity market exists, which yields an actuarially fair return, so that the return on savings R^i is $1/\pi^i$.

First order conditions yield that

$$u'(c^i) = u'(d^i)$$

so that consumption should be smoothed over time. Substituting into the individual's lifetime budget constraint, laissez-faire levels of consumptions are

$$\begin{aligned} c^1 &= d^1 = \frac{W}{2(1+\pi^1)} \\ c^2 &= d^2 = \frac{W}{2(1+\pi^2)} \end{aligned}$$

In the laissez-faire, agents with a shorter life expectancy consume more than agents with a high life expectancy. By consuming more in the first period (and eventually in the second period), they partially insure themselves against the risk of incuring a low level of utility both because they had a shorter life and because they consumed less.

Regarding the issue of who is disadvantaged by Nature, it follows from the above FOC that type-1 agents, who face a lower life expectancy, can be said to be disadvantaged by Nature with respect to type-2 agents only under particular circumstances. Indeed, the expected lifetime welfare of type-1 agents at the laissez-faire is, for a *given* amount of resources, smaller than the one of type-2 agents only if some conditions are satisfied, as stated in Proposition 4.

Proposition 4 *Suppose* $\pi^2 > \pi^1$*. If* $u(0) \geq 0$ *(i.e.* $\beta \geq 0$*) and* $v'(c)\,c/v(c) < 1$*, type-2 agents have, at the laissez-faire, a higher expected utility than type-1 agents, whatever the total amount of resources* W *is. If* $u(0) < 0$ *(i.e.* $\beta < 0$*), type-2 agents are, at the laissez-faire, better off than type-1 agents if and only if* $W > W^s$*, where* W^s *is such that*

$$\left(1+\pi^2\right) v\left(\frac{W^s}{2}\frac{1}{1+\pi^2}\right) - \left(1+\pi^1\right) v\left(\frac{W^s}{2}\frac{1}{1+\pi^1}\right) + \beta\left(\pi^2-\pi^1\right) = 0$$

Note that, as $\pi^1 \to 0$ *and* $\pi^2 \to 1$*, we have* $W^s \to W^S$*. Otherwise, we have* $W^s \lesseqgtr W^S$*.*

Proof. Substituting for the laissez-faire consumptions in the inequality

$$u(c^1) + \pi^1 u(d^1) < u(c^2) + \pi^2 u(d^2)$$

we obtain

$$\left(1+\pi^2\right) v\left(\frac{W}{2\left(1+\pi^2\right)}\right) - \left(1+\pi^1\right) v\left(\frac{W}{2\left(1+\pi^1\right)}\right) + \beta\left(\pi^2-\pi^1\right) > 0$$

If $u(0) \geq 0$ (i.e. $\beta \geq 0$) and $v'(c)\,c/v(c) < 1$, that inequality is always satisfied. If $u(0) < 0$ (i.e. $\beta < 0$), it depends on the level of W. Under $W = 0$, the LHS is negative (as $\pi^2 > \pi^1$), but as W tends to infinity, the LHS tends to $(\bar{v}+\beta)(\pi^2-\pi^1)$, which is positive. Hence, by continuity, there must exists some critical level of endowment W^s such that

$$\left(1+\pi^2\right) v\left(\frac{W^s}{2\left(1+\pi^2\right)}\right) - \left(1+\pi^1\right) v\left(\frac{W^s}{2\left(1+\pi^1\right)}\right) + \beta\left(\pi^2-\pi^1\right) = 0$$

Comparing this with

$$2v\left(\frac{W^S}{4}\right) - v\left(\frac{W^S}{2}\right) + \beta = 0$$

we see that the first term in the latter is higher, while the second is smaller, while the third term is more negative. Hence we have $W^s \gtrless W^S$. ■

Whereas Proposition 4 states the conditions under which type-1 agents are disadvantaged, in the sense that these enjoy a smaller expected lifetime utility than type-2 agents for a given initial endowment, one may also be interested in the conditions under which unlucky, short-lived agents (whatever these are of type-1 or of type-2) are disadvantaged in comparison with lucky, long-lived agents. This is the object of Proposition 5 below.

Proposition 5 *If* $u(0) \geq 0$ *(i.e.* $\beta \geq 0$*), long-lived agents are, at the laissez-faire, better off than short-lived agents, whatever the total amount of resources* W *is. If* $u(0) < 0$ *(i.e.* $\beta < 0$*), long-lived agents are, at the laissez-faire, better off than short-lived agents if and only if* $W > W^{S\prime}$*, where* $W^{S\prime}$ *is such that* $2v\left(\frac{W^{S\prime}}{2(1+\pi^2)}\right) - v\left(\frac{W^{S\prime}}{2(1+\pi^1)}\right) + \beta = 0$*. We have* $W^{S\prime} < W^S$*.*

Proof. At the laissez-faire, the short-lived agents are disadvantaged only if

$$u\left(\frac{W}{2(1+\pi^1)}\right) < u\left(\frac{W}{2(1+\pi^2)}\right) + u\left(\frac{W}{2(1+\pi^2)}\right)$$

that is, provided

$$2v\left(\frac{W}{2(1+\pi^2)}\right) - v\left(\frac{W}{2(1+\pi^1)}\right) + \beta > 0$$

This is always true under $u(0) \geq 0$ (i.e. $\beta \geq 0$), but not necessarily under $u(0) < 0$. Indeed, under $\beta < 0$, the LHS is negative at $W = 0$. However, as W tends to infinity, the LHS tends to $\bar{v} + \beta > 0$. Hence, by continuity, there must exist a critical level of total endowment $W^{S\prime}$ such that a strict equality holds.

Comparing that critical level $W^{S\prime}$ with the expression defining W^S,

$$2v\left(\frac{W^S}{4}\right) - v\left(\frac{W^S}{2}\right) + \beta = 0$$

we see that the first term is here smaller, while the second one is larger, and the third one is the same. Hence $W^{S\prime} < W^S$. ■

Thus, the conditions for the disadvantage of being short-lived under risky longevity, as stated in Proposition 5, are much weaker than the conditions necessary for the disadvantage of the short-lived under deterministic longevity (Proposition 1), as the threshold of resources $W^{S\prime}$ is inferior to W^S. The intuition behind this is that, under deterministic longevities, short-lived people can avoid some part of the damage from being short-lived by consuming their all endowment in the first period. However, in a risky world, such a behaviour is not optimal, and leads to a higher damage from having a short life.

4.3 The classical utilitarian problem

The social planner aims at maximizing the social expected utility, subject to the budget constraint of the economy:

$$\max_{c^1,d^1,c^2,d^2} u\left(c^1\right) + \pi^1 u\left(d^1\right) + u\left(c^2\right) + \pi^2 u\left(d^2\right)$$
$$\text{s.to } c^1 + \pi^1 d^1 + c^2 + \pi^2 d^2 \leq W$$

From the first order conditions, $u'\left(c^i\right) = u'(d^i)$, we obtain that

$$c^1 = d^1 = c^2 = d^2 = \frac{W}{2 + \pi^1 + \pi^2}$$

Under the conditions of Proposition 4, we find that utilitarianism reinforces welfare inequalities as consumptions are now smaller than in the laissez-faire for type-1 individuals and higher for type-2 individuals. This is also the case, under the conditions of Proposition 5, for the inequalities in actual lifetime welfare between long-lived and short-lived agents.

Proposition 6 *Under the conditions of Proposition 4, classical utilitarianism reinforces the inequalities in expected lifetime utility between type-2 and type-1 agents which hold at the laissez-faire. Under the conditions of Proposition 5, classical utilitarianism reinforces the inequalities in actual lifetime utility between lucky (long-lived) agents and unlucky (short-lived) agents.*

Hence, utilitarianism reinforces the inequalities induced by Nature and it does not bring any compensation to the agents with a lower life expectancy or a shorter life, which is quite counterintuitive. We shall now see how a remedy could bring some compensation to disadvantaged agents.

4.4 The *ex ante* compensation problem

Let us first focus on the *ex ante* approach to compensation, and leave the *ex post* compensation for the next subsection. Thus, compensation will here consist in modifying the utilitarian planner's problem to compensate agents of type 1, who face a lower life expectancy. To illustrate this, one can regard type-1 agents as men and type-2 as women. *Ex ante* compensation consists in compensating men for facing worse survival perspectives, even though some of them will live longer than women.

4.4.1 The *ex ante* consumption equivalent defined

In the context of a risky length of life, the consumption equivalent needs to be redefined: it becomes a consumption-equivalent for a larger life expectancy. It makes, by construction, the agent indifferent between two scenarios regarding the length of his life: a "short-life" scenario and a "long-life" scenario, but those scenarios take here the form of two lotteries with different life expectancies. In this more general model, α is such that

$$u\left(c^{1*}+\frac{\alpha}{2}\right)+\pi^{1}u\left(d^{1*}+\frac{\alpha}{2}\right)=u\left(c^{1**}\right)+\hat{\pi}^{1}u\left(d^{1**}\right) \tag{3}$$

where $\hat{\pi}^1$ is an hypothetical survival probability larger than π^1. Note that this *ex ante* consumption equivalent α depends on the consumptions chosen in the case of good longevity prospects c^{1**} and d^{1**} [i.e. problem (**)], as well as on the ones under bad survival prospects [i.e. problem (*)]: $c^{1*}$and d^{1*}. As in the previous section, it depends on the consumption levels under laissez-faire, on the shape of temporal utility functions and also on the survival probabilities π^1 and $\hat{\pi}^1$ (i.e. the differential in life expectancy).

Regarding the precise form of the compensation, it should be stressed that α is here assumed to bring indifference when spread *equally* on the two periods of life, on the grounds that agents tend, at the laissez-faire, to divide their resources equally on the two periods of (expected) life. Hence it makes sense to spread the consumption equivalent equally on the two periods.[22]

Note also that, given that agents have the *same* utility functions, and thus would solve the consumption program similarly if put in the same situation (i.e. $\pi^1 = \pi^2$), α equalizes also the expected lifetime utilities of agents:

$$u\left(c^{1*} + \frac{\alpha}{2}\right) + \pi^1 u\left(d^{1*} + \frac{\alpha}{2}\right) = u\left(c^{2**}\right) + \pi^2 u\left(d^{2**}\right) \tag{4}$$

as the $c^{1**} = c^{2**}$ and $d^{1**} = d^{2**}$ for agents solving the same problem (**).

Thus, the *ex ante* consumption equivalent of a long life α has here, given that agents share the same utility functions, the property to *equalize the expected lifetime utilities* of agents facing different survival prospects. Under $\alpha > 0$, α compensates agents of type 1 for facing a lower life expectancy than type-2 agents.[23] Note that it is only under the conditions of Proposition 4 that the consumption equivalent of a larger life expectancy α is strictly positive.[24] Otherwise, if the conditions of Proposition 4 are not satisfied, one may have a negative or zero consumption equivalent of a higher life expectancy, in which case one could not say that type-1 agents are disadvantaged for having a short life.

4.4.2 The *ex ante* problem

Let us now turn back to the utilitarian planner's problem. We assume that the social planner takes α as given, and spreads it equally on the two periods of life of the agents facing better

[22] Note that, in the special case where $\pi^1 = 0$ and $\hat{\pi}^1 = 1$, the risk disappears, so that $\alpha/2$ plays exactly the same role as α did in the previous section, and compensates for a certain shorter life. In the other cases, where longevity is risky, α can be interpreted as a compensation premium for facing worse survival conditions, due for instance to a handicap or some characteristic reducing survival prospects.

[23] Here again, if $u(0) < 0$ and W is very low, so that it is better to live a short life, then $\alpha < 0$, so that α brings here a compensation to the long-lived for having a long and miserous life. As above, we shall here concentrate on the more plausible case where $\alpha > 0$.

[24] Indeed, at the laissez-faire, we have, under the conditions of Proposition 4, $u\left(c^1\right) + \pi^1 u\left(d^1\right) < u\left(c^2\right) + \pi^2 u\left(d^2\right)$. Hence only a positive α can restaure the equality, so that $u\left(c^1 + \frac{\alpha}{2}\right) + \pi^1 u\left(d^1 + \frac{\alpha}{2}\right) = u\left(c^2\right) + \pi^2 u\left(d^2\right)$.

survival prospects (type-2 agents),

$$\max_{c^1,d^1,c^2,d^2} u\left(c^1\right) + \pi^1 u\left(d^1\right) + u\left(c^2 + \frac{\alpha}{2}\right) + \pi^2 u\left(d^2 + \frac{\alpha}{2}\right)$$
$$\text{s.to } c^1 + \pi^1 d^1 + c^2 + \pi^2 d^2 \leq W$$

where α is defined by expression (4) estimated at the laissez-faire. As in Section 3, the addition of α can be regarded as a compensation constraint, which requires that the social planner takes into account, in his allocation problem, the fact that type-2 agents face better survival prospects than type-1 agents.

FOCs can be rearranged as

$$u'\left(c^1\right) = u'(d^1) = u'\left(c^2 + \frac{\alpha}{2}\right) = u'\left(d^2 + \frac{\alpha}{2}\right)$$

Hence, we have, under $\alpha > 0$:

$$c^1 = d^1 > c^2 = d^2$$

Substituting for the above FOCs in the budget constraint yields

$$c^1 = d^1 = \frac{W + \frac{\alpha}{2}(1+\pi^2)}{2+\pi^1+\pi^2} > c^2 = d^2 \frac{W - \frac{\alpha}{2}(1+\pi^1)}{2+\pi^1+\pi^2}$$

Thus, the introduction of risk in the modified utilitarian problem does not affect the major virtue of the remedy: there is still some compensation given to type-1 agents, in the sense that they benefit from a higher consumption in comparison with the consumption of type-2 agents. Moreover, the remedy preserves the efficiency property of utilitarianism: the marginal utilities of consumption are equalized across periods for the different agents, so that there is still a smoothing of consumption over time.

Proposition 7 *Under the condition of Proposition 4, the ex ante remedy reduces the inequalities in expected lifetime welfare between type-2 and type-1 agents with respect to the utilitarian optimum.*

To see this, we compute the difference in expected lifetime utility of type-2 and type-1 agents under utilitarianism

$$V^{2FB} - V^{1FB} = u\left(\frac{W}{2+\pi^1+\pi^2}\right)(1+\pi^2) - u\left(\frac{W}{2+\pi^1+\pi^2}\right)(1+\pi^1)$$

and compare it with the difference under the *ex ante* remedy

$$V^{2R} - V^{1R} = u\left(\frac{W - \frac{\alpha}{2}(1+\pi^1)}{2+\pi^1+\pi^2}\right)(1+\pi^2) - u\left(\frac{W + \frac{\alpha}{2}(1+\pi^2)}{2+\pi^1+\pi^2}\right)(1+\pi^1)$$

It is straightforward to see that the first expression exceeds the second one.

4.4.3 The limits of *ex ante* compensation

Whereas the compensation carried out here may appear appealing, it should be stressed that such a compensation, by treating all type-1 agents similarly, and all type-2 agents similarly, tends to neglect an important aspect of the picture: all members of a group, although facing the same expected length of life, do not necessarily enjoy the same actual length of life. Some members are lucky, and enjoy a long life, while others are unlucky, and die after period 1. Such longevity differentials may also require their own correction. But the *ex ante* approach, by concentrating on the compensation of agents *a priori* disadvantaged by Nature, has little to say on that, and may even reinforce the injustices.

To see the tendency of the *ex ante* remedy to exacerbate *some* inequalities *ex post*, let us have a closer look at individual *ex post* lifetime utilities. In the following table, we write the lifetime utility of each category of individual, under the *ex ante* remedy and utilitarianism:

		ex ante remedy		utilitarianism
Type 1	Lucky	$2u\left(\frac{W+\frac{\alpha}{2}(1+\pi^2)}{2+\pi^1+\pi^2}\right)$	$>$	$2u\left(\frac{W}{2+\pi^1+\pi^2}\right)$
	Unlucky	$u\left(\frac{W+\frac{\alpha}{2}(1+\pi^2)}{2+\pi^1+\pi^2}\right)$	$>$	$u\left(\frac{W}{2+\pi^1+\pi^2}\right)$
Type 2	Lucky	$2u\left(\frac{W-\frac{\alpha}{2}(1+\pi^1)}{2+\pi^1+\pi^2}\right)$	$<$	$2u\left(\frac{W}{2+\pi^1+\pi^2}\right)$
	Unlucky	$u\left(\frac{W-\frac{\alpha}{2}(1+\pi^1)}{2+\pi^1+\pi^2}\right)$	$<$	$u\left(\frac{W}{2+\pi^1+\pi^2}\right)$

Table 1: Welfare comparison between ex ante remedy and utilitarianism

This table shows that, while type-1 agents are always better-off with the *ex ante* remedy than under utilitarianism, type 2 are always worse-off, independantly of being lucky or unlucky *ex post*. Thus, with the *ex ante* remedy, unlucky type 2 are penalized twice *ex post* (because they live only one period and receive lower consumption), whereas lucky type-1 agents have a double advantage, i.e. living two periods and having higher consumption levels.

It follows that, in comparison with the standard utilitarian problem, the *ex ante* compensation reduces *ex post* welfare inequalities between the lucky type-2 agents and the unlucky type-1 agents.[25] However, it is exactly the opposite for unlucky type-2 agents and lucky type-1 agents as, under the compensation, the former end up with a consumption -

[25] To see this, we simply compute the difference in lifetime utility between the lucky type-2 agents and the unlucky type-1 agents, and compare them under each scenario.

and a utility - that is quite low, while the latter enjoy not only a longer life, but, also, a consumption that is larger than under utilitarianism. In the absence of compensation, this inequality would be less sizeable. Thus, the *ex ante* compensation reduces the *ex post* utility differentials between some agents, but raises them between others.

Proposition 8 *In comparison with utilitarianism and under the conditions of Proposition 5, the ex ante remedy:*

a) *increases the lifetime welfare of type-1 agents and decreases the lifetime welfare of type-2 agents, independently from their actual survival.*

b) *reduces lifetime welfare inequalities between lucky type-2 agents and unlucky type-1 agents, but raises lifetime welfare inequalities between lucky type-1 agents and unlucky type-2 agents.*

The latter property of the *ex ante* remedy may be regarded as somewhat counter-intuitive. Actually, one may want all short-lived agents to be treated equally, that is, to be compensated for their shorter life in the same way, whatever their life expectancy was. The next subsection explores that alternative approach.

4.5 The *ex post* compensation problem

So far, the question of compensation was addressed by considering life expectancy as the unique dimension relevant for compensation. As emphasized in the last section, such an *ex ante* approach involves a strong simplification: some type-1 agents enjoy a long life, while some type-2 agents die after period 1. The existence of lucky type-1 agents and unlucky type-2 agents raises some important questions for a social planner concerned with compensation through the implementation of a remedy.

Clearly, under the modified problem considered above, there is, *ex post*, an unequal treatment of unlucky type-1 agents and unlucky type-2 agents, as the former receives a higher first-period consumption than the latter, while both of them enjoy a life of unitary length. Thus, from an *ex post* perspective, it may be argued that the unlucky type-2 agents are penalized twice: once by Nature, and once by the remedy. Such a double penalization seems hardly justifiable: unlucky type-2 agents are, from an *ex post* perspective, equal to type-1 agents, so why should these be treated differently by the social planner? Why should only short-lived type-1 agents be compensated?

The goal of this subsection is to explore the *ex post* problem, and to show how the compensation of unlucky type-2 agents can be carried out by means of a remedy that is close to the one discussed above.

4.5.1 The *ex post* consumption equivalent defined

The *ex post* consumption equivalent can be defined so as to equalize the utility of a short-lived and of a long-lived agent. As in Section 3, the *ex post* consumption equivalent, denoted by α, is such that

$$u\left(c^{i*}+\alpha^{i}\right)=u\left(c^{i*}\right)+u\left(d^{i*}\right) \tag{5}$$

where c^{i*} and d^{i*} are the consumptions chosen by a type-i agent under the laissez-faire (i.e. before knowing whether he survives to period 2 or not). Consumption equivalents are here type-specific, as agents make, at the laissez-faire, distinct consumption choices (because they face different survival prospects).

Regarding the sign of the *ex post* equivalent, it is no surprise that it depends on the conditions guaranteeing that short-lived agents have a lower actual lifetime utility than long-lived agents, that is, on the conditions of Proposition 5.[26] As far as the difference between the levels of α^1 and α^2 is concerned, we do not know whether $\alpha^1 \lesseqgtr \alpha^2$.

4.5.2 The *ex post* problem

In the *ex ante* compensation problem, the undesirable double penalization of unlucky type-2 agents comes from the equal division of the consumption equivalent of a long life on all periods of life. Thus, the equal division of α seems to yield an unequal treatment of unlucky type-1 and type-2 agents. In this section, we show that only an unequal division, where the whole consumption equivalent is imputed to second period consumption, can avoid this penalization.

To see how this modified problem can yield a compensation for the lack of luck, let us rederive the modified problem under that alternative implementation of the remedy. Assigning the entire consumption equivalent of consumption on the second period, the problem of a social planner is:

[26] To see this, note that, at the laissez-faire, we have, under the assumptions that $\beta > 0$ or $\beta < 0$ but $W > W^s$, $u\left(c^i\right) < u\left(c^i\right) + u\left(d^i\right)$. Hence only a positive α^i can restaure the equality, so that $u\left(c^i+\alpha^i\right) = u\left(c^i\right)+u\left(d^i\right)$.

$$\max_{c^1,d^1,c^2,d^2} u\left(c^1\right) + \pi^1 u(d^1+\alpha^1) + u\left(c^2\right) + \pi^2 u\left(d^2+\alpha^2\right)$$
$$\text{s.to } c^1 + c^2 + \pi^1 d^1 + \pi^2 d^2 \leq W$$

where α^i is estimated from expression (5), at the laissez-faire. Here again, the introduction of a consumption equivalent can be regarded as the addition of compensation constraints to the planner's problem. The unique difference here is that the *ex post* compensation constraints aim at taking into account not the membership of a group disadvantaged by Nature and facing worse survival prospects (i.e. being of type 1), but, rather, the simple fact that some persons, either type 1 or 2, will turn out to be unlucky, and will die after the first period, whatever their type is.

FOCs can be rearranged as

$$u'\left(c^1\right) = u'\left(c^2\right) = u'\left(d^1+\alpha^1\right) = u'\left(d^2+\alpha^2\right)$$

Hence, when $\alpha^i > 0$,

$$c^1 = c^2 > d^i$$

but whether $d^1 \lesseqgtr d^2$ depends on the levels of α^1 and α^2. Given the budget constraint, we have

$$\begin{aligned} c^1 &= c^2 = \frac{W + \pi^1\alpha^1 + \pi^2\alpha^2}{2+\pi^1+\pi^2} \\ d^1 &= \frac{W + \pi^2\alpha^2 - \alpha^1\left(2+\pi^2\right)}{2+\pi^1+\pi^2} \\ d^2 &= \frac{W + \pi^1\alpha^1 - \alpha^2\left(2+\pi^1\right)}{2+\pi^1+\pi^2} \end{aligned}$$

Note that unlucky type-1 agents and unlucky type-2 agents are now treated equally. As $c^1 = c^2$, they both enjoy a first period of life with the same consumption, unlike what prevailed under the *ex ante* modified problem. This consumption level is also, under either $\alpha^1 > 0$ and/or $\alpha^2 > 0$, higher than under standard utilitarianism, so that unlucky individuals are better off. However, second-period consumptions d^1 and d^2, which are now different for type-1 and type-2 agents, are, in general, smaller than under standard utilitarianism. Actually, this is the case provided $\pi^2\left(\alpha^2-\alpha^1\right) - 2\alpha^1 < 0$ and $\pi^1\left(\alpha^1-\alpha^2\right) - 2\alpha^2 < 0$, which are mild conditions (as the difference between α^1 and α^2 cannot be extremely large, given that the difference between π^1 and π^2 must belong to $[0,1]$).

Let us now compare *ex post* welfare inequalities by proceeding as before and by comparing lifetime utilities with the utilitarian case:

		ex post remedy		utilitarianism
Type 1	Lucky	$u\left(\frac{W+\pi^1\alpha^1+\pi^2\alpha^2}{2+\pi^1+\pi^2}\right)+u\left(\frac{W+\pi^2\alpha^2-\alpha^1\left(2+\pi^2\right)}{2+\pi^1+\pi^2}\right)$	$\lessgtr$	$2u\left(\frac{W}{2+\pi^1+\pi^2}\right)$
	Unlucky	$u\left(\frac{W+\pi^1\alpha^1+\pi^2\alpha^2}{2+\pi^1+\pi^2}\right)$	$>$	$u\left(\frac{W}{2+\pi^1+\pi^2}\right)$
Type 2	Lucky	$u\left(\frac{W+\pi^1\alpha^1+\pi^2\alpha^2}{2+\pi^1+\pi^2}\right)+u\left(\frac{W+\pi^1\alpha^1-\alpha^2\left(2+\pi^1\right)}{2+\pi^1+\pi^2}\right)$	$\lessgtr$	$2u\left(\frac{W}{2+\pi^1+\pi^2}\right)$
	Unlucky	$u\left(\frac{W+\pi^1\alpha^1+\pi^2\alpha^2}{2+\pi^1+\pi^2}\right)$	$>$	$u\left(\frac{W}{2+\pi^1+\pi^2}\right)$

Table 2: Welfare comparison between ex post remedy and utilitarianism

Thus, as this table shows, the *ex post* remedy increases the *ex post* welfare of unlucky agents (independently of their type), while this may not be the case for lucky agents in comparison with the situation under utilitarianism, depending on the values of α^1 and α^2.

Regarding the inequalities between lucky and unlucky agents inside a given group (either type-1 or type-2), we find that inequalities inside group 1 are reduced as compared to utilitarianism if $\pi^2\left(\alpha^2-\alpha^1\right)-2\alpha^1<0$ and inequalities inside group 2 are also reduced if $\pi^1\left(\alpha^1-\alpha^2\right)-2\alpha^2<0$. This is due to the simple fact that the *ex post* remedy raises first-period consumptions and reduces, under mild conditions, second-period consumptions with respect to utilitarianism, so that this reduces *ex post* inequalities of lifetime welfare between the short-lived and the long-lived of a given type i. This reduction of lifetime welfare inequalities within each group occurs under general conditions, that is, if one excludes extreme cases where the differential between the two consumption equivalents α^1 and α^2 is extremely large (i.e. cases where survival probabilities differ strongly).[27] These results are summarized in the following proposition:

Proposition 9 *In comparison with utilitarianism and under the conditions of Proposition 5, the ex post remedy*

a) *increases the ex post welfare of unlucky agents, but may or may not increase the ex post welfare of lucky agents*

[27] Under extreme differentials between α^1 and α^2, the *ex post* welfare inequalities within one group may be increased under the *ex post* remedy (but not in the other group). However, such a large gap is implausible, especially if the groups under study do not exhibit large differences in group-specific life expectancies.

b) *reduces lifetime welfare inequalities within each group $i = 1, 2$ between lucky (i.e. long-lived) agents and unlucky (i.e. short-lived) agents, provided* $\pi^2 \left(\alpha^2 - \alpha^1\right) - 2\alpha^1 < 0$ *and* $\pi^1 \left(\alpha^1 - \alpha^2\right) - 2\alpha^2 < 0$.

The intuition behind these two latter conditions is the following. If, for instance, the consumption equivalent for one group is extremely large, let us say, if $\alpha^1 >> \alpha^2$, then, under the *ex post* remedy, there would be a reduction of lifetime welfare inequalities between short-lived and long-lived type-1 agents, but, because of redistributions across types favouring type-2 agents, a rise of lifetime welfare inequalities between the short-lived and the long-lived type-2 agents. But such a specific case is hardly plausible, so that, in general, the *ex post* remedy involves a compensation of short-lived agents in each group.

Besides the compensation of unlucky (short-lived) agents, the change in the way in which α^i is spread on the lifecycle has also important consequences for other aspects of the social optimum. The second-period consumption of agents d^i is now lower than their first-period consumption c^i, unlike under the *ex ante* approach. This results from the planner's will to compensate unlucky agents. The non-equalization of marginal utilities of consumption over time can be regarded as an efficiency loss: from an efficiency perspective, consumption should be smoothed, for each individual, across all his life-periods. However, the *ex post* compensation requires to give a higher first-period consumption to first-period agents than to survivors in the second period, which implies that consumption cannot be smoothed across periods for long-lived agents. Therefore a tension arises between compensation and efficiency concerns.

All in all, the modified *ex post* problem differs strongly from the standard utilitarian problem and the *ex ante* problem. While standard utilitarianism implies $c^1 = c^2 = d^1 = d^2$, the *ex ante* remedy compensates agents who face worse survival prospects, but maintains the consumption smoothing for each agent: $c^1 = d^1 > c^2 = d^2$, and the *ex post* problem does the opposite: it maintains first-period consumptions equalization, but gets rid of consumption smoothing: $c^1 = c^2 > d^i$. It should be stressed here that the *ex ante* and *ex post* compensations are not only different, but these are also *incompatible*: one cannot compensate both *ex ante* and *ex post*. The reason why this is not possible is obvious once one looks at the solutions of the two problems: if one compensates *ex ante*, one must have $c^1 = d^1$ and $c^2 = d^2$, which, if combined with the *ex post* condition $c^1 = c^2$, implies, by transitivity of equality, $c^1 = c^2 = d^1 = d^2$, which is nothing else than the standard utilitarian solution, that is, the absence of compensation.

Hence, if one compensates *both* agents with a lower life expectancy (i.e. *ex ante* compensation) and unlucky (short-lived) agents (i.e. *ex post* compensation), this amounts to make no compensation at all. This incompatibility can be explained as follows. One cannot redistribute resources towards type-1 agents without favouring some long-lived agents (i.e. lucky type-1 agents) and disfavouring some short-lived agents (i.e. unlucky type-2 agents), contrary to the intuition behind *ex post* compensation. Similarly, one cannot redistribute resources towards unlucky agents without favouring some agents with a high life expectancy (i.e. unlucky type-2 agents) and disfavouring agents with a low life expectancy (i.e. lucky type-1 agents), contrary to the intuition behind *ex ante* compensation.

Thus a choice is required between *ex ante* and *ex post* compensations. That dilemma is close to the ones studied by Fleurbaey and Maniquet (2006), who stressed the impossibility to compensate agents on one dimension (e.g. luck) without interfering with the treatment of agents differing in another dimension (e.g. effort), so that a tension exists between compensation and responsibility. The two characteristics are here actual longevity and life expectancy, and it is impossible to compensate agents disadvantaged on one dimension without influencing the treatment of agents differing in the other dimension.

5 A numerical illustration

Let us now illustrate the different solutions numerically. For that purpose, we assume that the temporal utility function $u(c)$ has a CES form:

$$u(c^i) = \frac{c^{i1-\sigma}}{1-\sigma} + \beta$$

where σ is the elasticity of intertemporal substitution, while β is an intercept. Note the crucial role played by those two preference parameters for the computation of consumption equivalent of a long life. If, for instance, utility is linear in consumption, we have $\sigma = \beta = 0$, so that the consumption equivalent is zero. However, for other values of preference parameters, α is likely to vary significantly.[28] Note that, under $\sigma < 1$, if β is too large (i.e. the utility from mere survival is large), there cannot be any consumption equivalent of a long life, as no consumption can compensate for the fact of facing a shorter life.

[28] For an empirical study on that topic, see Ponthiere (2008).

Regarding the calibration of σ and β, we shall proceed as follows. As a benchmark case, we start with a value of $\sigma = 0.5$ and let β take three distinct values: 0, -2 and 2.[29] This will allow us to highlight the crucial influence of the intercept of the temporal utility function on the planner's problem. A finer, more realistic calibration of the intercept is left for the end of this section.

To illustrate the heterogeneity, suppose that type-1 agents are males and type-2 agents are females. In the U.S., life expectancy at birth for males is about 75 years, equal here to $1 + \pi^1 = 1.25$.[30] For women, life expectancy is 80.5 years, equal here to $1 + \pi^2 = 1.40$. Thus, one has $\pi^1 = 0.25$ and $\pi^2 = 0.40$.

Table 3 below shows the laissez-faire, the utilitarian optimum and the *ex ante* and *ex post* utilitarian optima in the benchmark case where $\beta = 0$, under a total endowment $W = 20$. These results confirm our previous theoretical findings. At the laissez-faire, consumption is smoothed across periods for all agents, but consumption is larger for type-1 agents, who face a lower chance of surviving till the second period. At the utilitarian optimum, that advantage given by Nature is reinforced: consumption is equalized at all periods for all agents, so that type-2 agents end up with a higher expected utility than under the laissez-faire, whereas type-1 agents end up worse off. This is the second penalization, due to Bentham.

$\beta = 0, \sigma = 0.5$	c^1	c^2	d^1	d^2	U^1	U^2	$u(c^1)$	$u(c^1)$ $+u(d^1)$	$u(c^2)$	$u(c^2)$ $+u(d^2)$
Laissez-faire	8.00	7.14	8.00	7.14	7.07	7.48	5.66	11.31	5.35	10.69
Utilitarian optimum	7.55	7.55	7.55	7.55	6.87	7.69	5.49	10.99	5.49	10.99
Remedy, ex ante $\alpha = 1.92$	8.05	7.09	8.05	7.09	7.10	7.46	5.68	11.35	5.33	10.65
Remedy, ex post $\alpha^1 = 24.00$, $\alpha^2 = 21.43$	13.05	13.05	$\to 0$	$\to 0$	7.22	7.22	7.22	7.22	7.22	7.22

Table 3: Laissez-faire and Optimum values for Beta=0

The remedy goes against that tendency to punish disadvantaged agents. In the *ex ante* case, type-1 agents receive higher consumptions at the two periods than type-2 agents. As a consequence, the *ex ante* utility gap (i.e. the differential in expected lifetime welfare) is

[29] Note that, while empirical studies of σ yield an estimate of 0.83 (see Blundell *et al*, 1994), such estimates cannot be used here, as these rely on a model where the period is a year, unlike in the present model.

[30] Indeed, if we consider that a period is of length 40 years, and starting at the age of 25, we obtain that a life expectancy of 75 years involves 65 years (i.e. 25 years + the first period) + 10 years, equal to 0.25 period.

reduced as compared with the laissez-faire and standard utilitarianism. Thus, the remedy brings a solution, where disadvantaged agents receive some compensation, and which reduces the double penalization. In the *ex post* case, the modified optimum involves large and equal first-period consumptions, as well as low consumptions in the second period, so as to compensate equally all unlucky agents dying at the end of the first period.[31] Note also that, because $\beta = 0$ and $d^i \to 0 \; \forall i$, the expected utilities, the utility of the unlucky (i.e. short-lived) agents and of the lucky (i.e. long-lived) ones are equal. Thus, in this case, the *ex post* remedy treats all agents equally, independently from their type and from their actual longevity.

Let us now examine how sensitive those results are to the calibration of the intercept β. For that purpose, Tables 4 and 5 compare the laissez-faire and the various utilitarian optima under $\beta = -2$ and $\beta = 2$. First note that the value of the intercept does not play any role on consumption levels at the laissez faire and at the utilitarian solutions.[32] Only utility levels and consumption equivalents α are affected by the level of β. As shown in Table 4, the utilitarian optimum again reinforces the inequalities induced by Nature. However, the *ex ante* remedy is less helpfull than under $\beta = 0$, as the consumption equivalent of a long life is low due to the negative value of β; the utility gap $U^2 - U^1$ is thus less reduced than before. Regarding the *ex post* case, we now find that the expected utility of type-2 agents is lower than the one of type-1 agents, which is due to the difference in survival and to the negative intercept. However, because consumptions are equal across agents in the second period and because *ex post* utilities do not depend on the agent's type π^i, those utility levels are the same for lucky and unlucky agents. This is a result that carries over when $\beta \neq 0$.

$\beta = -2, \sigma = 0.5$	c^1	c^2	d^1	d^2	U^1	U^2	$u(c^1)$	$u(c^1)$ $+u(d^1)$	$u(c^2)$	$u(c^2)$ $+u(d^2)$
Laissez-faire	8.00	7.14	8.00	7.14	4.57	4.68	3.66	7.31	3.35	6.69
Utilitarian optimum	7.55	7.55	7.55	7.55	4.37	4.89	3.49	6.99	3.49	6.99
Remedy, ex ante α=0.512	7.68	7.43	7.68	7.43	4.43	4.83	3.54	7.09	3.45	6.90
Remedy, ex post α^1=13.69, α^2=11.74	10.61	10.61	$\to$0	$\to$0	4.01	3.71	4.51	2.51	4.51	2.51

Table 4: Laissez-faire and Optimum values for Beta=-2

[31] If an agent survives, he derives utility only from mere survival.

[32] Note that first order conditions are independant of β so that the laissez faire and the utilitarian levels of consumption are identical across the four tables.

Table 5, which considers the case where $\beta = 2$, confirms the above findings. The utilitarian optimum reinforces welfare inequalities prevailing under the laissez-faire, and the *ex ante* remedy tends to correct this. When $\beta \geq 0$, type-1 agents are better-off under the *ex ante* remedy than under the laissez-faire, while type-2 agents are worse off. Thus, as soon as the utility of mere survival is positive, the remedy does more than playing against the Benthamite tendency to favour (potentially) long-lived agents: it operates a redistribution that favours (potentially) short-lived people. Hence, the remedy does not only remove the double penalization, but provides a net compensation to the agents disadvantaged by Nature. Turning now to the *ex post* remedy, we still observe large first-period consumptions, zero second-period consumptions, and the equality of the *ex post* utilities of the lucky and of the unlucky agents (whatever their initial survival chance).

$\beta = 2, \sigma = 0.5$	c^1	c^2	d^1	d^2	U^1	U^2	$u\left(c^1\right)$	$u\left(c^1\right)$ $+u\left(d^1\right)$	$u\left(c^2\right)$	$u\left(c^2\right)$ $+u\left(d^2\right)$
Laissez-faire	8.00	7.14	8.00	7.14	9.57	10.28	7.66	15.31	7.35	14.69
Utilitarian optimum	7.55	7.55	7.55	7.55	9.37	10.49	7.49	14.99	7.49	14.99
Remedy, ex ante $\alpha = 3.3856$	8.44	6.75	8.44	6.75	9.76	10.07	7.81	15.62	7.20	14.39
Remedy, ex post α^1=36.31, α^2=33.12	15.97	15.97	$\rightarrow 0$	$\rightarrow 0$	10.49	10.79	9.99	11.99	9.99	11.99

Table 5: Laissez-faire and Optimum values for Beta=2

All this tends to highlight the significant sensitivity of the modified utilitarian optimum to the level of β. In front of such a sensitivity, a natural question to be asked is: to what extent would the remedy affect the utilitarian optimum under an *empirically-based* estimate of the intercept β? In other words, what could be a plausible value for β? In the Appendix, we show, on the basis of empirical estimates of the value of a statistical life, that a plausible value for the intercept of the temporal utility function is $\beta = 4.472$.

Table 6 below shows the laissez-faire, and various utilitarian optima under that empirically-grounded value of β. The laissez-faire and utilitarian levels of consumption are identical to the previous simulations. However, we now observe bigger utility differentials than under a lower β. As usual, the utilitarian optimum, by equalizing consumptions across periods for all agents, tends to reinforce the utility gap. However, the *ex ante* remedy brings a significant compensation to agents facing worse survival prospects: the consumption of type-1 agents should be about 42 % larger than the one of type-2 agents, in both periods. If one regards type-1 agents as men and type-2 agents as women, those figures would support a massive redistribution across genders, aimed at compensating differentials in survival prospects.[33] As a consequence, males would be, under the *ex ante* remedy, better off than under the laissez-faire - and, *a fortiori*, than under utilitarianism -, whereas the opposite would hold for women.

$\beta = 4.472, \sigma = 0.5$	c^1	c^2	d^1	d^2	U^1	U^2	$u(c^1)$	$u(c^1) + u(d^1)$	$u(c^2)$	$u(c^2) + u(d^2)$
Laissez-faire	8.00	7.14	8.00	7.14	12.66	13.74	10.13	20.26	9.82	19.63
Utilitarian optimum	7.55	7.55	7.55	7.55	12.46	13.95	9.97	19.93	9.97	19.93
Remedy, ex ante α =5.277	8.94	6.30	8.94	6.30	13.07	13.29	10.45	20.90	9.49	18.99
Remedy, ex post α^1=54.30, α^2=50.33	20.27	20.27	$\rightarrow 0$	$\rightarrow 0$	14.59	15.26	13.48	17.95	13.48	17.95

Table 6: Laissez-faire and Optimum values for Beta=4.472

As usual, the *ex post* remedy involves a large first-period consumption, equal for all types of agents, as well as zero second-period consumptions. Expected utilities of both type-1 and type-2 individuals are also increased in comparison with the laissez-faire, the utilitarian optimum and the *ex ante* remedy case. However, the expected utility is higher for the long-lived agent than for the short-lived, simply because $\pi^2 > \pi^1$, and the gap in expected utility is also larger than under the *ex ante* remedy. Finally, the *ex post* utility of an unlucky individual is higher than in any other situation while it is the reverse for any lucky individual. This confirmes and completes the results of Table 2.

In sum, this section illustrates the tendency of classical utilitarianism to exacerbate Nature-based inequalities, and the capacity of the remedy to operate some compensation. But the *ex ante* and *ex post* remedies differ significantly. The former remedy favours agents facing worse survival prospects (e.g. men over women) by a +/- 42 % additional consumption extra per period, and reduces inequalities in expected utility terms. On the contrary, the latter remedy compensates all the unlucky, short-lived agents (e.g. short-lived men and women). This is achieved by more than doubling the first-period consumption with respect

[33] Note that here, we do not introduce any other differences between genders (such as differences in productivity for instance). Such differences might affect our results significantly.

to the laissez-faire (but giving almost nothing in the second period).

6 Concluding remarks

This paper starts from a paradoxical result of classical utilitarianism: a tendency to redistribute resources from short-lived to long-lived agents, implying, under mild conditions, a double penalization of short-lived agents: one penalty by Nature, one by Bentham.

In order to avoid that paradoxical result, this paper proposes a remedy: the imputation of the consumption equivalent of a long life within the consumption of long-lived agents. Such an imputation can be justified as either a compensation, or, more simply, as a correction of the - so far incomplete - informational basis used by the social planner in his allocation problem.

In a basic model with deterministic longevity, the imputation of the consumption equivalent of a long life (estimated at the laissez-faire) to the consumption of long-lived agents at all periods was shown to yield, with respect to utilitarianism, a compensation to the short-lived agents. Hence, whereas utilitarianism tends generally to exacerbate welfare inequalities caused by Nature, the remedy implies, on the contrary, a compensation of agents disadvantaged by Nature. Note, however, that the modified utilitarian optimum is distinct from a Maximin approach, as this does not equalize lifetime utilities. Moreover, the remedy preserves the property of equalization of consumptions across periods for long-lived agents, which is in conformity with basic efficiency intuitions.

Then, in a more general framework where longevity is risky, and where agents face different life expectancies, we distinguished between two ways in which the remedy can be used, to allow either for the *ex ante* compensation of agents facing worse survival prospects, or for the *ex post* compensation of agents having a shorter life. While the *ex ante* approach requires the remedy to be equally spread on the life of agents advantaged by Nature, the *ex post* approach requires, on the contrary, the equivalent to be concentrated on the old age of the surviving people, whatever these were initially advantaged or disadvantaged by Nature. We showed that each of those two compensation techniques, which aims at reducing welfare inequalities between some specific groups, are incompatible.

Regarding the - necessary - choice between the *ex ante* and the *ex post* compensations, there seems to be no clear support for one particular criterion, as these satisfy quite different properties (see the table below).[34]

	Equal cons./p for equal life expe.	Equal cons./p for equal longevity	Equal ELT cons. for equal life expe.	Equal ELT cons. for equal longevity	Equal LT cons. for equal life expe.	Equal LT cons. for equal longevity
Utilitarianism	yes	yes	yes	no	no	yes
Modified Utilit. (ex ante)	yes	no	yes	no	no	no
Modified Utilit. (ex post)	yes	(yes)	yes	(yes)	(yes)	(yes)

Table 7: Properties of Utilitarianism and of Modified Utilitarianism (ex ante, ex post)

However, if one approach had to be chosen, the *ex post* compensation might have a stronger support, as what matters, at the end of the day, is the utility enjoyed by people, rather than the expected utility of people. Hence, if a choice is to be made, the *ex post* compensation approach seems more plausible, even though, as shown above, the *ex post* compensation approach has a larger cost in terms of total welfare (no consumption smoothing), so that this discussion remains largely open.

But whatever the chosen remedy is, this paper, by its generality, has the merit to cast a new light on a variety of problems of redistribution involving longevity inequalities. Longevity differentials are present in many policy debates, concerning pensions, long-term care, etc. Hence, in all those issues, if one acts as a standard utilitarian policy maker, there will be large transfers towards long-lived agents, in opposition with basic ethical intuition. Thus, even if this paper just provides a remedy, this seems to be better than the disease alone.

Finally, three extensions of this paper should be mentionned. First, while the social planner takes here the consumption equivalent of a long life as a constant, which is estimated on the basis of laissez-faire choices, one may argue that the social planner should solve his modified problem while taking the consumption equivalent as a *variable*, which depends on his *own* allocation of resources.[35] Second, whereas this paper assumes that at least

[34] "cons./p" stands for consumption per period, "ELT cons." stands for expected lifetime consumption, and "LT cons." stands for lifetime consumption. The "(yes)" in several columns reflects that the property is satisfied by the *ex post* approach only when a corner solution where $d^i = 0$ prevails, which is a special, but numerically plausible case (see Section 5).

[35] That alternative approach, which captures the idea that the consumption equivalent of a long life

some part of the heterogeneity in longevity-affecting characteristics could be observed by the social planner, this is not always the case in reality, so that one may also want to explore the second-best problem, under *non observability* of types 1 and 2, and, thus, of the consumption equivalents. Third, although this paper concentrates on a disease (and a cure) for utilitarianism under unequal longevities, it would be worth considering whether other ethical frameworks suffer from the same kind of problem, and, more generally, to consider the issue of compensation of unequal longevities *outside utilitarianism*. Hence much work remains to be done.

In any case, taking longevity differentials into account properly is not, for a government, optional. In the light of the central position of longevity as a determinant of human lifetime welfare, we tend to believe that it is a necessity.

7 Appendix

Regarding the calibration of the intercept β, note first that the value of statistical life is, in the present model, equal to

$$VL\left(c^{i}\right) \equiv \frac{u\left(c^{i}\right) - u'\left(c^{i}\right) c^{i}}{u'\left(c^{i}\right)}$$

Indeed, the expected lifetime utility of a type-i agent is $U^{i} = \left(1+\pi^{i}\right) u\left(c^{i}\right)$, where $c^{i} = w/\left(1+\pi^{i}\right)$ when consumption is smoothed across periods and where w is the initial wealth. The value of life can be interpreted as the amount of wealth the agent is willing to give up in order to increase his survival probability, for a given level of expected utility, i.e.

$$\begin{aligned} VL\left(c^{i}\right) &\equiv \left.\frac{dw}{d\pi^{i}}\right|_{\bar{U}} = -\left.\frac{\partial U^{i}/\partial \pi^{i}}{\partial U^{i}/\partial w}\right|_{\bar{U}} \\ &= \frac{u\left(c^{i}\right) - u'\left(c^{i}\right) c^{i}}{u'\left(c^{i}\right)} \end{aligned}$$

Substituting for $u(c^{i})$ in $VL(c^{i})$ yields:

$$VL\left(d^{i}\right) \equiv \beta\left(c^{i}\right)^{\sigma} + c^{i}\left(\frac{1}{1-\sigma} - 1\right)$$

Assuming that the initial endowment equals 10 and that consumption is smoothed across periods, the VSL is

$$VL\left(5\right) = \beta\left(5\right)^{0.5} + 5\left(\frac{1}{1-0.5} - 1\right) = \beta\left(5\right)^{0.5} + 5$$

depends on what life is, would be worth being pursued, but is not trivial, as the endogeneity of α requires an additional constraint to be imposed, in order to avoid a multiplicity of optima.

Given that the VSL amounts to about 120 times income per head per year (see Miller, 2001), which amounts to $\frac{120}{40}$ income per period of 40 years, we have, given the two-period structure, $VL(5) = \frac{120}{40}(5) = 15$, from which it follows that $\beta = 4.472$.

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