Macroeconomic Effects of Financial Shocks*

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Abstract

We document the cyclical properties of U.S. firms' financial flows. Equity payouts are procyclical and debt payouts are countercyclical. We develop a model with explicit roles for debt and equity financing and explore how the observed dynamics of real and financial variables are affected by 'financial shocks'. Standard productivity shocks can only partially explain the movements in real and financial variables. The addition of financial shocks brings the model much closer to the data. The recent events in the financial sector show up clearly as a tightening of firms' financing conditions causing the GDP decline in 2008-09.

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1 Introduction

Recent economic events starting with the subprime crisis in the summer of 2007 suggest that the financial sector plays an important role as a source of business cycle fluctuations. While there is a long tradition in macroeconomics to model financial frictions, most of the literature has focused on the role played by the financial sector in propagating shocks that originate in other sectors of the economy. For example in the propagation of productivity shocks. Instead, the importance of financial shocks—that is, perturbations that originate directly in the financial sector—has not been fully explored in the literature. Moreover, most of the previous studies have not tried to replicate *simultaneously* real aggregate variables and aggregate flows of financing, in particular, debt and equity. In this paper we attempt to make some progress along these dimensions.

We start by documenting the cyclical properties of firms' equity and debt flows at an aggregate level. We then build a business cycle model with explicit roles for firms' debt and equity financing. Debt contracts are not fully enforceable and the ability to borrow is limited by an enforcement constraint. The enforcement constraint is subject to random disturbances which we call 'financial shocks'. To examine the model's empirical implications, we propose a method for recovering the financial shocks directly from the model's enforcement constraint using time series data for debt, capital and output. Our method parallels the standard approach for measuring productivity shocks as residuals from the production function using empirical measurements of output, capital and labor.

Using the constructed series of financial and productivity shocks, we show that the model driven solely by productivity shocks fails to capture the key dynamic features of U.S. business cycles as well as the behavior of equity and debt flows. The model augmented with financial shocks is much closer to the data—not only for capturing the dynamics of financial flows but also for generating the dynamics of the real business cycle quantities, especially labor. The simulation of the model show a worsening of firms' ability to borrow in 2008-09 with a sharp economic downturn. This is in line with the standard interpretation of the economic events beginning in the summer of 2007. The simulation of the model also shows that the economic downturns in 1990-91 and 2001 were strongly influenced by changes in credit conditions.

In our model firms finance investment with equity and debt. The firms' ability to borrow is limited by an enforcement constraint that depends on the lifetime profitability of the firm. As the profitability varies with the business cycle, so does a firm's ability to borrow. In this regard our model is related to Kiyotaki & Moore (1997), Bernanke, Gertler & Gilchrist (1999), and Mendoza & Smith (2005). Our model, however, differs in two important dimensions. First, we allow firms to have negative equity payouts, which can be interpreted as new equity issues. Therefore, in choosing the stock of equity the firm is not limited to reinvesting profits.[1] Second, we consider shocks that affect directly the financial sector of the economy. Therefore, the financial sector can act as a source of the business cycle, in addition to affecting the propagation of shocks that originate in other sectors of the economy. In this respect our paper is related to Benk, Gillman & Kejak (2005) who also consider shocks affecting the financial sector. However, the nature of the financial shocks and the structure of the model considered in these papers are very different from ours. Recent contributions by Christiano, Motto & Rostagno (2008), Gilchrist, Yankov & Zakrajsek (2009) and Kiyotaki & Moore (2008), have also considered shocks that originate in the financial sector and suggest that these shocks could play an important role as a source of macroeconomic fluctuations.

The paper is structured as follows. In Section 2 we consider some empirical evidence on real and financial cycles in the US economy. Section 3 presents the model and characterizes some of the properties analytically. Model calibration and quantitative findings are presented in Section 4.

2 Real and financial cycles in the U.S.

This section presents the main empirical observations that motivate the paper. It describes the properties of real and financial business cycles. We start by reporting the business cycle properties of firms' aggregate financial flows. To our knowledge, these properties have not been previously documented and explored in the macro literature.

Figure 1 plots the net payments to equity holders and the net debt repurchases in the nonfinancial business sector (corporate and noncorporate).

[1] There are other studies that allow for equity issuance over the business cycle. See, for example, Choe, Masulis & Nanda (1993), Covas and den Haan (2005), Leary and Roberts (2005), and Hennessy & Levy (2005). The main focus of these studies, however, is on the financial behavior of firms, not in the quantitative relevance of financial frictions for the macroeconomic propagation of aggregate shocks.

Financial data is from the Flow of Funds Accounts of the Federal Reserve Board. Equity payout is defined as dividends and share repurchases minus equity issues of nonfinancial corporate businesses, minus net proprietor's investment in noncorporate businesses. This captures the net payments to business owners (shareholders of corporations and noncorporate business owners). Debt is defined as 'Credit Market Instruments' which include only liabilities that are directly related to credit markets transactions. It does not include, for instance, tax liabilities. Debt repurchases are simply the reduction in outstanding debt (or increase if negative). Both variables are expressed as a fraction of nonfarm business GDP. See Appendix A for a more detailed description.

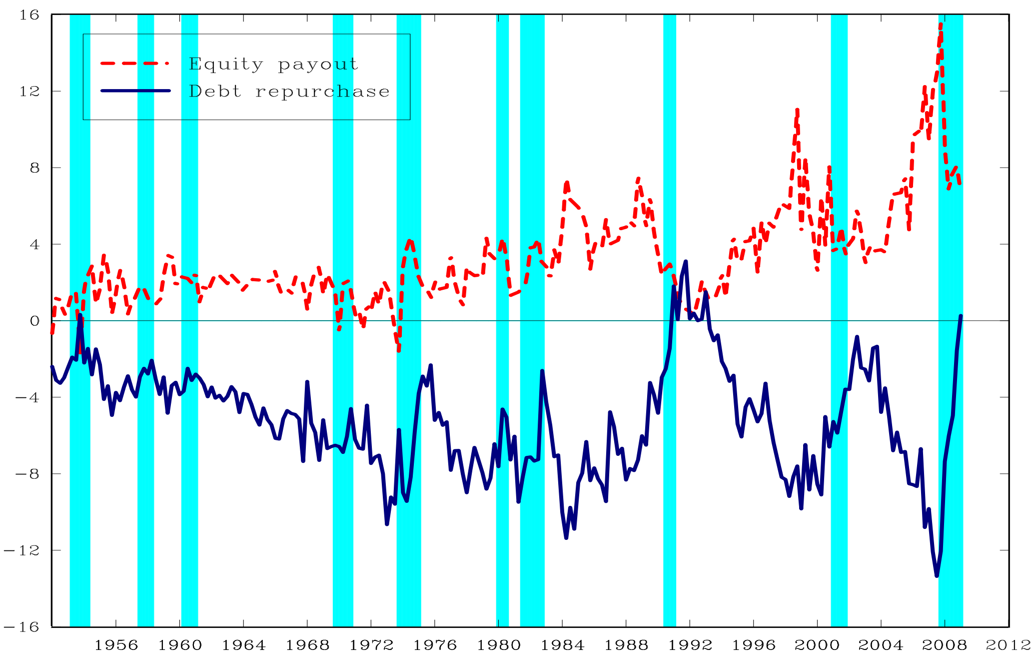

Figure 1: Financial flows in the nonfinancial business sector (corporate and noncorporate): 1952.1-2009.1. Source: Flow of Funds, Federal reserve Board.

Two patterns are visible in the figure, very strongly so for the second half of the period considered. First, equity payouts are negatively correlated with debt repurchases. This suggests that there is some substitutability between equity and debt financing. Second, while equity payouts tend to increase in booms, debt repurchases increase during or around recessions. This suggests that recessions lead firms to restructure their financial positions by cutting debt and reducing the payments made to shareholders.

The properties of real and financial cycles are further characterized in Table 1. The table reports the standard deviations and correlations with GDP for equity payouts and debt repurchases in the nonfinancial sector. Statistics for a number of key business cycle variables are also presented. Equity payouts and debt repurchases are normalized by the value added produced in the sector. For these two variables we do not take logs because some observations are negative. All variables are detrended with a band-pass filter that preserves cycles of 1.5-8 years (Baxter and King (1999)).

Table 1: Business cycles properties of macroeconomic and financial variables, 1984:1-2009:1.

	Std(Variable)	Corr(Variable,GDP)
Macroeconomic variables		
GDP	0.84	
Consumption (N.D.& S.)	0.49	0.83
Investment	5.98	0.87
Hours	1.18	0.81
TFP	0.60	0.38
Financial variables		
EquPay/GDP	1.05	0.41
DebtRep/GDP	1.29	-0.61

Notes: Financial data is from the Flow of Funds Accounts of the Federal Reserve Board. *Equity payout* is net dividends minus net issue of corporate equity (net of share repurchases) minus proprietor's net investment in the noncorporate sectors. *Debt repurchase* is the negative of the net increase in credit market instruments. Both variables are divided by their sectoral GDP. The macroeconomic variables have been logged. All variables are detrended with a band-pass filter that preserves cycles of 1.5-8 years (Baxter and King (1999). See Appendix A for more details.

We focus on the period after 1984 for two reasons. First, it has been widely documented in relation with the so called Great Moderation that 1984 corresponds to a break in the volatility in many business cycle variables (quite possibly the downturn in 2008 might mark a further break). Second, as documented in Jermann and Quadrini (2008), this time period also saw major changes in U.S. financial markets compared to the previous period. In particular, spurred by regulatory changes, share repurchases have become more common, and this seems to have had a major impact on firms' payout policies and financial flexibility. This is apparent in Figure 1 where the volatility of the financial flows changes after the early 1980s. Therefore, by concentrating on the period after 1984 we do not have to address the causes of the structural break that arose in the early 1980s.

The correlations of equity payouts and debt repurchases with output confirm the properties we highlighted in Figure 1. Equity payouts are procyclical and debt repurchases are countercyclical for the nonfinancial business sector. These properties also hold if we exclude the noncorporate business and consider only the nonfinancial corporate sector (not reported in the table to economize on space). The business cycle properties of the real variables reported in Table 1 are well known, and we will get back to them when comparing the model to the data.

3 Model

We start with the description of the environment in which an individual firm operates as this is where our model diverges from a more standard business cycle model. We then present the household sector and define the general equilibrium.

3.1 Financial and investment decisions of firms

There is a continuum of firms, in the $[0, 1]$ interval, with a gross revenue function $F(z_t, k_t, l_t) = z_t k_t^{\theta} l_t^{1-\theta}$. The variable z_t is the stochastic level of productivity, k_t is the input of capital depreciating at rate δ, l_t is the input of labor. Consistent with the typical timing convention, k_t is chosen at time $t-1$, and therefore, it is predetermined at time t. Instead, the input of labor l_t can be flexibly changed in period t.

Firms use equity and debt. Debt, denoted by b_t, is preferred to equity (pecking order) because of its tax advantage as, for example, in Hennessy and Whited (2005). Given r_t the interest rate, the *effective* gross rate for the firm is $R_t = 1 + r_t(1 - \tau)$, where τ represents the tax benefit.

The ability to borrow is bounded by the limited enforceability of debt contracts as firms can default on their obligations. Let $\overline{V}_t(k_{t+1}, b_{t+1})$ be the equity value of the firm at the end of the period, after investing and paying the shareholders. At this point the *individual* states of the firm are given by the new stock of capital, k_{t+1} and the new debt, b_{t+1}. The equity value is defined as the expected discounted value of equity payouts starting from the next period, that is,

$$\overline{V}_t(k_{t+1}, b_{t+1}) = E_t \sum_{j=1}^{\infty} m_{t+j} d_{t+j},$$

where m_{t+j} is the relevant stochastic discount factor, which we will derive later, and d_{t+j} is the payment to the shareholders (equity payout). We have made explicit that the equity value at the end of the period depends on the individual state variables, capital and debt. The subscript t captures the dependence on the aggregate states. The firm's value is typically decreasing in the end-of-period debt because, everything else equal, debt reduces the future payments that can be made to the shareholders.

Default arises after the realization of revenues. In case of default, the firm (equity owners) has the ability to retain the revenues $F(z_t, k_t, l_t)$, as these are liquid funds that can be easily diverted, and renegotiates the debt. In case of renegotiation the lender can sell the firm but can recover only a fraction $\xi_t < 1$ of the equity value $\overline{V}_t$. The variable ξ_t is stochastic and takes the same value for all firms. The stochastic nature of this variable derives from changes in the liquidity of the firm's assets (resale value) and/or the relative bargaining powers of firm owners and lenders in case of renegotiation. This is described in details in Appendix B.

Because of the loss of value in liquidating the firm ($\xi_t < 1$), both parties have an interest in renegotiating the debt. Appendix B describes in details the intra-period transactions and timing of the renegotiation game and show that this leads to the following enforcement constraint:

$$\xi_t \overline{V}_t(k_{t+1}, b_{t+1}) \geq F(z_t, k_t, l_t). \tag{1}$$

The variable ξ_t plays a crucial rule in determining the borrowing capacity of the firm and we refer to its stochastic innovations as "financial shocks". Since ξ_t is the same for all firms, the stochastic innovations are aggregate shocks. Therefore, we have two sources of aggregate uncertainty: productivity, z_t, and financial, ξ_t. Since these are not idiosyncratic shocks, we will concentrate on the symmetric equilibrium where all firms are alike (representative firm).

To see more clearly how ξ_t affects the financing and production decisions of the firm, we rewrite the enforcement constraint in a slightly modified fashion. First, let's define $V_t(k_t, b_t) = d_t + \overline{V}_t(k_{t+1}, b_{t+1})$ the equity value of the firm at the beginning of the period, before making any payment to the shareholders. At the beginning of the period the *individual* states are the capital and the debt chosen in the previous period. The enforcement constraint (1) can then be rewritten as:

$$\xi_t \Big[V_t(k_t, b_t) - d_t \Big] \geq F(z_t, k_t, l_t).$$

At the beginning of the period k_t and b_t are given. The only variables that can be changed to balance the enforcement constraint are the input of labor, l_t, and the equity payout, d_t. Therefore, if we start from a pre-shock state in which the enforcement constraint is binding and the firm wants to keep the production plan unchanged, a negative financial shock (lower ξ_t) requires a reduction in equity payout d_t. In other words, the firm is forced to increase its equity, and therefore, to reduce the new debt. However, if the firm cannot reduce d_t, it has to cut employment. Thus, whether the financial shock affects employment depends on the flexibility with which the firm can change its financial structure, that is, the composition of debt and equity financing.

To formalize the rigidities affecting the substitution between debt and equity, we assume that the firm's payout is subject to a quadratic cost:

$$\varphi(d_t) = d_t + \kappa \cdot (d_t - \bar{d})^2$$

where $\kappa \geq 0$, and $\bar{d}$ is a coefficient whose value is equal to the long-run payout target (steady state).

The equity payout cost should not be interpreted necessarily as a pecuniary cost. It is a simple way of modeling the speed with which firms can change the source of funds when the financial conditions change. Of course, the possible pecuniary costs associated with share repurchases and equity issuance can also be incorporated in the function $\varphi(.)$. The convexity assumption would then be consistent with the work of Hansen & Torregrosa (1992) and Altinkilic & Hansen (2000), showing that underwriting fees display increasing marginal cost in the size of the offering.

Another way of thinking about the adjustment cost is that it captures the preferences of managers for dividend smoothing. Lintner (1956) showed first that managers are concerned about smoothing dividends over time, a fact further confirmed by subsequent studies. This could derive from agency problems associated with the issuance or repurchase of shares as emphasized by several studies in finance. The explicit modeling of these agency conflicts, however, is beyond the scope of this paper.[2]

The parameter κ is key for determining the impact of financial frictions. When $\kappa = 0$ the economy is almost equivalent to a frictionless economy. In this case, debt adjustments triggered by the financial shocks can be quickly accommodated through changes in firm equity. When $\kappa > 0$, the substitution between debt and equity becomes costly and firms readjust the sources of funds slowly. In this case, financial shocks can have non-negligible effects, in the short-run, on the production decision of firms.

Firm's problem: We now write the problem of the firm recursively. The individual states are the capital stock, k, and the debt, b. The aggregate states, which we will make precise later, are denoted by $\mathbf{s}$.

The firm chooses the input of labor, l, the equity payout, d, the new capital, k', and the new debt, b'. The optimization problem is:

$$V(\mathbf{s}; k, b) = \max_{d,l,k',b'} \left\{ d + Em'V(\mathbf{s}'; k', b') \right\} \tag{2}$$

subject to:

$$(1-\delta)k + F(z,k,l) - wl + \frac{b'}{R} = b + \varphi(d) + k'$$

$$\xi Em'V(\mathbf{s}'; k', b') \geq F(z,k,l)$$

The problem is subject to the budget and the enforcement constraints. The function $V(\mathbf{s}; k, b)$ is the cum-dividend (fundamental) market value of the firm and m' is the stochastic discount factor. The variables w and R are, respectively, the wage rate and the effective gross interest rate. The stochastic discount factor, the wage and interest rate are determined in the general equilibrium and are taken as given by an individual firm.

[2] As an alternative to the adjustment cost on equity payouts, we could use a quadratic cost on the change of debt, which would lead to similar properties. Therefore, our model can be interpreted more broadly as capturing the rigidities in the adjustment of all sources of funds, not only equity.

Denote by μ the Lagrange multiplier associated with the enforcement constraint. The first-order conditions are:

$$F_l(z,k,l) = w \cdot \Big(\frac{1}{1-\mu\varphi_d(d)}\Big), \tag{3}$$

$$(1+\xi\mu)E\,\tilde{m}'\Big[1-\delta+\Big(1-\mu'\varphi_d(d')\Big)F_k(z',k',l')\Big] = 1, \tag{4}$$

$$(1+\xi\mu)RE\,\tilde{m}' = 1. \tag{5}$$

where $\tilde{m}' = m' \cdot \Big(\frac{\varphi_d(d)}{\varphi_d(d')}\Big)$ is the 'effective' stochastic discount factor. Subscripts denote derivatives. The detailed derivation is in Appendix C.

Especially important for understanding the key findings of the paper is the optimality condition for labor, equation (3). As usual, the marginal productivity of labor is equalized to the marginal cost. However, the marginal cost is the wage rate augmented by a wedge that depends on the 'effective' tightness of the enforcement constraint, that is, $\mu\varphi_d(d)$. A tighter enforcement constraint increases the effective cost of labor and reduces the firm's demand for labor. Similarly, when the enforcement constraint becomes less tight, the effective cost of labor declines, increasing its demand. Therefore, the main channel through which financial shocks are transmitted to the real sector of the economy is by affecting the demand of labor.

To get further insights, it will be convenient to consider the special case without any cost of equity payout, that is, $\kappa = 0$. Thus, $\varphi_d(d) = \varphi_d(d') = 1$, $\tilde{m}' = m'$ and condition (5) becomes $(1+\xi\mu)REm = 1$. Taking as given the aggregate prices R and Em', this condition implies that there is a negative relation between the financial shock, ξ, and the multiplier μ. In other words, lower liquidation values of the firm's equities make the enforcement constraint more binding. Then from condition (3) we see that a higher value of μ implies a lower demand of labor.

This mechanism is reinforced when $\kappa > 0$. In this case it will be costly to re-adjust the financial structure and the change in ξ induces a larger movement in μ. Of course, the change in the policies of all firms will also affect the equilibrium prices, with some feedback on the individual policies. To derive the response of prices we have to close the model and derive the general equilibrium.

3.2 Households sector and general equilibrium

There is a continuum of homogeneous households maximizing the expected lifetime utility $E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, l_t)$, where c_t is consumption, l_t is labor, and β is the discount factor. Households are the owners (shareholders) of firms. In addition to equity shares, they hold non-contingent bonds issued by firms.

The household's budget constraint is:

$$w_t l_t + b_t + s_t(d_t + q_t) = \frac{b_{t+1}}{1+r_t} + s_{t+1}q_t + c_t + T_t$$

where w_t and r_t are the wage and interest rates, b_t is the one-period bond, s_t the equity shares, d_t the equity payout received from the ownership of shares, q_t is the market price of shares, and $T_t = B_{t+1}/[1+r_t(1-\tau)] - B_{t+1}/(1+r_t)$ are lump-sum taxes financing the tax benefits received by firms on debt.

The first order conditions with respect to labor, l_t, next period bonds, b_{t+1}, and next period shares, s_{t+1}, are:

$$w_t U_c(c_t, l_t) + U_h(c_t, l_t) = 0 \tag{6}$$

$$U_c(c_t, l_t) - \beta(1+r_t)EU_c(c_{t+1}, l_{t+1}) = 0 \tag{7}$$

$$U_c(c_t, l_t)q_t - \beta E(d_{t+1} + q_{t+1})U_c(c_{t+1}, l_{t+1}) = 0. \tag{8}$$

The first two conditions are key to determine the supply of labor and the risk-free interest rate. The last condition determines the market price of shares. After re-arranging and using forward substitution, this price is:

$$q_t = E_t \sum_{j=1}^{\infty} \left(\frac{\beta^j \cdot U_c(c_{t+j}, l_{t+j})}{U_c(c_t, l_t)} \right) d_{t+j}.$$

Firms' optimization is consistent with households' optimization. Therefore, the stochastic discount factor is equal to $m_{t+j} = \beta^j U_c(c_{t+j}, l_{t+j})/U_c(c_t, l_t)$.

We can now provide the definition of a recursive general equilibrium. The set of aggregate states $\mathbf{s}$ are given by the current value of productivity z, the current value of the financial variable ξ, the aggregate capital K, and the aggregate bonds B. Therefore, $\mathbf{s} = (z, \xi, K, B)$.

Definition 3.1 (Recursive equilibrium) *A recursive competitive equilibrium is defined as a set of functions for (i) households' policies $c(\mathbf{s}; b)$ and*

$l(\mathbf{s}; b)$; (ii) firms' policies $d(\mathbf{s}; k, b)$, $l(\mathbf{s}; k, b)$, $k(\mathbf{s}; k, b)$ and $b(\mathbf{s}; k, b)$; (iii) firms' value $V(\mathbf{s}; k, b)$; (iv) aggregate prices $w(\mathbf{s})$, $r(\mathbf{s})$ and $m(\mathbf{s}, \mathbf{s}')$; (v) law of motion for the aggregate states $\mathbf{s}' = H(\mathbf{s})$. Such that: (i) household's policies satisfy the optimality conditions (6)-(7); (ii) firms' policies are optimal and $V(\mathbf{s}; k, b)$ satisfies the Bellman's equation (2); (iii) the wage and interest rates are the equilibrium clearing prices in the labor and bond markets and $m(\mathbf{s}, \mathbf{s}') = \beta U_c(c_{t+1}, l_{t+1})/U_c(c_t, l_t)$; (iv) the law of motion $H(\mathbf{s})$ is consistent with individual decisions and the stochastic processes of z and ξ.

3.3 Some characterization of the equilibrium

To illustrate some of the properties of the model, it will be convenient to look at two special cases in which the equilibrium can be characterized analytically. First, we show that for a deterministic steady state with constant z and ξ, the default constraint is always binding. Second, if $\tau = 0$ and $\kappa = 0$, changes in ξ (financial shocks) have no effect on the real sector of the economy.

Proposition 3.1 *If $\tau > 0$ the enforcement constraint binds in the steady state.*

In a deterministic steady state $m = 1/(1 + r)$ and $\varphi_d(d) = \varphi_d(d') = 1$. Therefore, the first order condition for debt, equation (5), simplifies to $(1 + \bar{\xi}\mu)Rm = 1$, where $\bar{\xi}$ is the steady state value. Substituting the above definition of m, we get $(1 + \bar{\xi}\mu)R = 1 + r$. Because $R = 1 + r(1 - \tau)$, we must have that $\mu > 0$ if $\tau > 0$. Thus, as long as there is a tax benefit from issuing debt, the enforcement constraint is binding in a steady state.

With uncertainty, however, the constraint may not be binding at all times because firms may reduce their borrowing in anticipation of future shocks. However, the constraint is always binding if τ is sufficiently large and the shocks are sufficiently small. This will be the case in the quantitative exercises we will present in the next sections.

Let's consider now the stochastic economy concentrating on the special case in which $\tau = 0$ and $\kappa = 0$. We have the following proposition:

Proposition 3.2 *With $\tau = 0$ and $\kappa = 0$, changes in ξ have no effect on l and k'.*

When $\kappa = 0$ we have that $\varphi_d(d) = \varphi_d(d') = 1$ and $\tilde{m}' = m'$. Therefore, the first order condition (5) can be written as $(1 + \xi\mu)REm' = 1$. From the household's first order condition (7) we have that $(1+r)Em' = 1$. Combining these two conditions we get $(1 + \xi\mu)[1 + r(1 - \tau)] = 1 + r$. When $\tau = 0$ this condition implies $\xi\mu = 0$. Therefore, μ is always zero and, assuming that the aggregate prices do not change, the firm's choice of l and k' will not be affected by the change in ξ.

What we have to show next is that the sequence of prices do remain constant if firms do not change l and k'. This becomes obvious once we recognize that changes in debt issuance and equity payout associated with fluctuations in ξ cancel out in the household's budget constraint. Therefore, the sequence of prices do not change in equilibrium and financial shocks are fully neutral for the real sector of the economy.

We have then established that, when $\tau = 0$ and $\kappa = 0$, business cycle movements are only driven by fluctuations in aggregate productivity z. The model becomes a standard RBC model. In fact, the key first order conditions become:

$$wU_c(c, l) + U_l(c, l) = 0,$$

$$F_l(z, k, l) = w,$$

$$E\left\{\frac{\beta U_c(c', l')}{U_c(c, l)}\right\}\Big[1 - \delta + F_k(z', k', l')\Big] = 1.$$

which are the first order conditions for the standard RBC model. In this particular version of the model, the financial structure of firms is undetermined, that is, firms are indifferent between debt and equity.

4 Quantitative analysis

In this section we show first that the model with only productivity shocks cannot replicate the business cycle movements of real and financial flows experienced by the U.S. economy since the mid 1980s. We then show that adding financial shocks not only improves the model's predictions for the financial flows, but also for the key macroeconomic variables, especially working hours. In particular, the consideration of financial shocks allows the model to capture the GDP downturn of 2008-09, as well as the downturns in the previous two recessions, 1990-91 and 2001. This suggests that tighter credit conditions have played an important role in all the recessions experienced by the U.S. economy since the mid 1980s. Let's first describe the parametrization.

4.1 Parametrization

The parameters can be grouped into two sets. The first set includes parameters that can be calibrated using steady state targets, some of which are typical in the business cycle literature. The second group includes parameters that cannot be calibrated using steady state targets. The model doesn't admit a closed form solution in general, and therefore, we use numerical methods. In the computation we conjecture that the enforcement constraints are always binding and solve a linear approximation of the dynamic system (see Appendix D). The model solution is then used to check the initial conjecture of bonding constraints. We have also solved the model with a more general non-linear approach that accommodates occasionally binding constraints. Appendix E describes the computational method and shows that the solution based on the linear approximation is quite accurate.

4.1.1 Parameters set with steady state targets

The period in the model is a quarter. We set $\beta = 0.9825$, implying that the annual steady state return from holding shares is 7.32 percent. The utility function takes the form $U(c, l) = \ln(c) + \alpha \cdot \ln(1 - l)$ where $\alpha = 1.9265$ is chosen to have steady state hours equal to 0.3. The Cobb-Douglas parameter in the production function is set to $\theta = 0.36$ and the depreciation to $\delta = 0.025$. The mean value of z is normalized to 1. These values are standard in the literature and they are based on the typical steady state targets. The quantitative properties of the model are not very sensitive to this first group of parameters.

The tax wedge is set to $\tau = 0.35$. This corresponds to the benefit of debt over equity if the marginal tax rate is 35 percent. This parameter is important for the quantitative performance of the model because it determines whether the enforcement constraint is binding. As we will see, with this value of τ (and the remaining parametrization of the model), the enforcement constraint is almost always binding in all simulations conducted in the paper.

The mean value of the financial variable, $\bar{\xi}$, is chosen to match the average leverage, that is, the ratio of debt, b, over the capital stock, k. We impose a steady state leverage of 0.46 which is the average value over the period 1984-2009:1 for the Nonfinancial Business sector in the Flow of Funds data.

4.1.2 Parameters that cannot be set with steady state targets

The parameters that cannot be set with steady state targets are those determining the stochastic properties of the shocks and the cost of equity payout, the parameter κ. Of course, in a steady state equilibrium, the stochastic properties of the shocks do not matter and the equity payout is always equal to the long-term target. Therefore, we have to use an alternative procedure.

For the productivity shock z_t we follow the standard Solow residuals approach. This recovers the realizations of the productivity shock from the production function using aggregate times series for output and factor inputs (capital and labor). To construct the series of financial shocks ξ_t, we follow a similar approach but using the enforcement constraint: given empirical times series for output, productivity, capital and debt, we derive the financial shocks as residuals from the enforcement constraint. This approach implies that we do not directly force any endogenous variable in the model to perfectly match an individual data series. Once we have constructed the series for the two shocks over the period 1984:1-2009:1, we estimate a two dimensional autoregressive system.

Let's start with the productivity shocks. From the production function used in the model we have

$$\hat{z}_t = \hat{y}_t - \theta\,\hat{k}_t - (1-\theta)\,\hat{l}_t$$

where $\hat{z}_t$, $\hat{y}_t$, $\hat{k}_t$ and $\hat{l}_t$ are the percentage (or log-) deviations from the deterministic trend. Given the parameter θ chosen above and the empirical series for $\hat{y}_t$, $\hat{k}_t$ and $\hat{l}_t$, we construct the series for $\hat{z}_t$.

For the financial variable ξ_t we use the enforcement constraint under the assumption that it is always binding, that is,

$$\xi_t \overline{V}_t = y_t.$$

The assumption that the enforcement constraint is always satisfied with equality will then be verified ex-post.

The construction of the series for ξ_t using the enforcement constraint requires empirical series for $\overline{V}_t$ and y_t. While GDP is the natural empirical counterpart for output y_t, choosing the empirical counterpart of $\overline{V}_t$ is more challenging. At least for publicly traded firms, we could use stock market valuations. However, given the noise in stock market prices and the well known difficulties for small scale models to match stock prices data, we do not follow this route. Instead, we use the model to derive a representation of $\overline{V}_t$ in terms of variables that can be measured more conveniently from the data.

Given the recursive structure of the model, $\overline{V}_t$ can be expressed as a function of the following four variables: $(z_t, \xi_t, k_{t+1}, b^r_{t+1})$. The variable $b^r_{t+1} \equiv b_{t+1}/R_t$ is the end-of-period value of the debt. The reason we use b^r_{t+1} instead of b_{t+1} is because the former corresponds more directly to the value reported in the data (end-of-period balance sheet).[3] Then, the linearized version of the enforcement constraint can be written as:

$$\hat{\xi}_t = c_z \widehat{z}_t + c_y \widehat{y}_t + c_k \widehat{k}_{t+1} + c_b \widehat{b}^r_{t+1},$$

where the hat sign denotes log-deviations from the steady state. The terms (c_z, c_y, c_k, c_b) are coefficients that are functions of the model parameters. Therefore, given empirical series for $\widehat{z}_t$, $\widehat{y}_t$, $\widehat{k}_{t+1}$ and $\widehat{b}^r_{t+1}$, we can construct the series for the financial variable $\hat{\xi}_t$.

For $\widehat{y}_t$ and $\widehat{k}_{t+1}$, we use the same empirical counterparts as for the construction of the productivity shocks z_t. For $\widehat{b}^r_{t+1}$ we use the debt series for nonfinancial businesses consistent with the debt data discussed earlier in the paper. All series are in real terms and the log value is linearly detrended. A more detailed description is provided Appendix A.

Once we have constructed the time series for the productivity and financial shocks, we estimate the two dimensional system

$$\begin{pmatrix} \hat{z}_{t+1} \\ \hat{\xi}_{t+1} \end{pmatrix} = A \begin{pmatrix} \hat{z}_t \\ \hat{\xi}_t \end{pmatrix} + \begin{pmatrix} \varepsilon_{z,t+1} \\ \varepsilon_{\xi,t+1} \end{pmatrix}, \tag{9}$$

where A is a two-by-two matrix.

At this point we are left with the equity cost parameter κ. This is chosen so that the standard deviation of equity payout (normalized by output) is as in the data over the period 1984-2009:1. Note that the coefficients used

[3]We could have expressed the equity value as a function of (z_t, ξ_t, k_t, b_t). However, the specification in terms of k_{t+1} and b_{t+1}/R_t is preferable because the terms k_{t+1} and b_{t+1}/R_t represents the end-of-period value of capital and debt, which correspond to the balance sheet values reported in the data.

to derive the financial shocks, c_z, c_y, c_k, c_b, are themselves a function of the estimated transition matrix A and the payout cost parameter κ. We therefore have to use an iterative procedure. We start with initial guesses for A and κ, solve the model to derive c_z, c_y, c_k, c_b, and then we estimate the shock process to get a new value for A. The process is repeated until convergence. Convergence is extremely fast and it is not sensitive to different initial conditions. Because payout volatility is decreasing in the cost parameter κ, this parameter is also easily determined.[4]

Figure 2 displays the constructed series for productivity and financial shocks. The full set of parameters values are reported in Table 2.

Table 2: Parametrization.

Description	
Discount factor	$\beta = 0.9825$
Tax advantage	$\tau = 0.3500$
Utility parameter	$\alpha = 1.8991$
Production technology	$\theta = 0.3600$
Depreciation rate	$\delta = 0.0250$
Enforcement parameter	$\bar{\xi} = 0.1965$
Payout cost parameter	$\kappa = 0.2460$
Std of productivity innovations	0.0044
Std of credit innovations	0.0111
Correlation of shock innovations	0.357
Matrix for the shocks process	$A = \begin{bmatrix} 0.928 & -0.004 \\ 0.053 & 0.971 \end{bmatrix}$

[4]An alternative approach to construct the financial shocks that does not require an iterative procedure is based on the approximation of $\overline{V}_t$ by the book value of equity $k_{t+1} - b_{t+1}/R_t$. In the model, $\overline{V}_t$ tracks quite closely the book value $k_{t+1} - b_{t+1}/R_t$. In the special case where $\tau = 0$ and $\kappa = 0$, $\overline{V}_t$ is exactly equal to $k_{t+1} - b_{t+1}/R_t$. As it turns out, the shocks measured through the model implied $\overline{V}_t$, are very similar to those obtained by simply replacing the market value of equity $\overline{V}_t$ by the book value $k_{t+1} - b_{t+1}/R_t$ (see Figure 2). It would also be possible to change the model by specifying the enforcement constraint as $\xi_t\,(k_{t+1} - b_{t+1}/R_t) \geq y_t$. The quantitative properties are almost unaffected by this change.

4.2 Findings

We now study the dynamics of the model induced by the constructed series of shocks. In particular, starting with the initial values $\hat{z}_{1984:1}$ and $\hat{\xi}_{1984:1}$, we use the estimated parameters of the autoregressive structure (9) to generate the sequences of innovations $\varepsilon_{z,t}$ and $\varepsilon_{\xi,t}$ that replicate exactly the series $\hat{z}_t$ and $\hat{\xi}_t$, for t=1984:2,..,2009:1. We then feed these innovations into the model and compute the responses for the key macroeconomic and financial variables. Although we feed into the model the actual sequence of shocks, they are not perfectly anticipated by the agents. They forecast future values of z_t and ξ_t using the autoregressive system (9).

4.2.1 Productivity shocks

We consider first the dynamics of the model induced by the series of productivity shocks $\varepsilon_{z,t}$ only. The financial shocks $\varepsilon_{\xi,t}$ are set to their unconditional mean of zero and the initial value of ξ to its mean $\bar{\xi}$.

Figure 3 plots the series of output, hours worked and financial flows. To highlight the importance of financial frictions, the figure also reports the responses generated by the model without financial frictions. This version of the model is obtained by setting $\tau = 0$ and $\kappa = 0$. With these two parameter values, the real sector of the model becomes equivalent to the standard real business cycle model. The financial flows become indeterminate because firms are indifferent between debt and equity financing. Thus, the financial flows are plotted only for the model with financial frictions.

As can be seen from the figure, the series generated by the two models, with and without financial frictions, poorly match their empirical counterparts. In particular, while the data shows an output boom during the 1990s, the simulated series displays a decline. This pattern is also present in working hours, a fact that has been emphasized in McGrattan and Prescott (2007). It is also worth noting that the model does not generate the large output drop observed during the last two quarters of 2008 and the first quarter of 2009. Also, the drops in output experienced by the US economy during the previous two recessions, 1990-91 and 2001, are significantly smaller than in the data.

The performance of the model in terms of labor is even worse. The model is now also unable to generate enough volatility of hours. In the sensitivity analysis we show that this finding is robust to the alternative specification of preferences based on indivisible labor as in Hansen (1985).

The model does also a poor job in capturing the movements in debt and equity flows, which are plotted in the lower panels of Figure 3. The properties of the model with financial frictions are further illustrated by the impulse responses to a one-time productivity shock reported in Figure 6.

One conclusion we draw from Figure 3 is that, with only productivity shocks, financial frictions do not improve the performance of the model. In fact, as shown in the upper left panel, output is now slightly less volatile than in the case without financial frictions. To understand why financial frictions dampen the response of output to productivity shocks, we should look at the enforcement constraint (1). Consider a positive productivity shock. This affects directly the right hand side of the enforcement constraint, making it tighter. The fact that the enforcement constraint becomes tighter counterbalances the expansionary impulse induced by the productivity improvement. As a result, output increases less.

4.2.2 Financial shocks

Figure 4 plots the series of output, hours worked, and financial flows in response to the sequence of financial shocks. With financial shocks only, the dynamics of output and labor is now much closer to the data. In particular we now see a boom in output and hours during the 1990s. Furthermore, financial shocks generate sharp drops in output and labor in all three recessions: 1990-91, 2001 and 2008-09.

The improved performance of the model relies on the direct impact that financial shocks have on the demand for labor. As shown in the upper right panel, financial shocks generate much larger fluctuations in working hours than the productivity shocks. More importantly, they generate large drops in labor during the three recessions, as well as an upward trend during the 1990s.

The importance of the financial shocks for the demand of labor can be seen from the first order condition (3), which for convenience we rewrite here:

$$F_l(z, k, l) = w \cdot \left(\frac{1}{1 - \mu\varphi_d(d)}\right)$$

The variable μ is the multiplier for the enforcement constraint. A negative financial shock makes the enforcement constraint tighter, increasing the multiplier μ and the wedge on the cost of labor. This effect becomes bigger if the change in equity payout is costly (in which case $\varphi_d(d) \neq 1$). Intuitively, if the firm wants to keep the same production scale and hire the same number of workers, it has to reduce the equity payout. Because this is costly, the firm chooses in part to reduce the equity payout and in part to reduce the input of labor. Impulse response functions to a one-time financial shock are displayed in Figure 7.

The model with financial shocks also performs much better in capturing the dynamics of the financial flows as shown in the lower panels of Figure 4. The series generated by the model closely mimic the main features of the empirical series for debt and equity flows. Of course, we wouldn't expect this parsimonious model to fit the data perfectly. In particular, the volatility of debt repurchases is somewhat higher than in the data.

Figure 5 plots the series generated by the model in response to both shocks: productivity and financial. Overall, the model does a reasonable job matching output and hours worked as well as the financial flow series. For financial flows and labor, the performance of the model is very similar to the case with only financial shocks. In fact, the movements of these variables are mostly driven by financial shocks. For output, the performance is somewhat worse than the case with only financial shocks, but still better than in the case with only productivity shocks. In particular, the model continues to predict sharp drops in output during each of the three major recessions.

4.2.3 Second moments

Business cycle models have traditionally been evaluated by comparing the second moments of the simulated series with the moments of the empirical series. Clearly, comparing the whole paths of the time series as we did in the previous subsections represents a tougher test than just looking at second moments. Nevertheless, we report here how the model performs along this dimension. As shown in Table 3, the model matches many of the moments quite well.

Consider first the model driven by productivity shocks only and without financial frictions. This is the standard RBC model. All moments presented in the table are based on band-passed filtered series that contain fluctuations from 6 to 32 quarters. This model generates a standard deviation of output of 0.77% compared to the empirical standard deviation of 0.84%. Still, the standard deviation of hours worked is substantially lower than in the data.

Focusing on second moments masks the failure of the model to match

Table 3: Business cycles properties of macroeconomic and financial variables.

	Data 1984:1-2009:1	With Frictions Productivity & Financial Shocks $\kappa = 0.246$ $\tau = 0.35$	With Frictions Productivity Shocks Only $\kappa = 0.246$ $\tau = 0.35$	Without Frictions Productivity Shocks Only $\kappa = 0$ $\tau = 0$
Standard deviations				
GDP	0.84	0.84	0.48	0.77
Consumption	0.49	0.20	0.15	0.23
Investment	5.98	3.34	1.62	2.95
Hours	1.18	0.87	0.28	0.40
EquPay/Y	1.05	0.85	0.49	
DebtRep/Y	1.29	1.82	0.75	
Correlation with GDP				
Consumption	0.83	0.36	0.93	0.85
Investment	0.87	0.98	1.00	0.99
Hours	0.81	0.81	0.12	0.98
TFP	0.38	0.74	0.90	1.00
EquPay/Y	0.41	0.67	-0.25	
DebtRep/Y	-0.61	-0.77	0.07	

Notes: All variables are detrended with a band-pass filter that preserves cycles of 1.5-8 years (Baxter and King (1999).

output and labor time series over the past 25 years as we have shown above. In the model, productivity shocks move output mainly directly and through hours worked, generating a correlation of 1 between productivity and output. In the data, this correlation is only 0.38. The addition of financial frictions reduces the amplitude of output fluctuations and the model is unable to match the correlations of debt and equity flows with output.

Adding financial shocks to the model with financial frictions dramatically improves the model's ability to match labor hours volatility, as well as the correlations of debt and equity flows with output. One dimension along which the model remains weak is the volatility of consumption. The volatility of consumption generated by the model is less than half the empirical value. This is even slightly worse than the model with productivity shocks only. Among other things, this is due to the fact that in the model financial shocks and financial frictions affect consumers only indirectly. Overall, this suggests that adding some type of financial friction and financial shocks to the consumption side of the model would be a fruitful avenue. We leave this for future work.

4.2.4 Sensitivity analysis

Since Hansen (1985), it has become common in business cycle studies to use a linear specification of the disutility of labor. Here we consider the utility function $\log(c_t) - \alpha l_t$. We maintain a steady state value of hours worked equal to 0.3, which requires $\alpha = 2.713$. All remaining parameters are kept unchanged and we use the same sequence of shocks we constructed earlier.[5]

As shown in Figure 8, the performance of the model is similar to the benchmark case: with only productivity shocks the model does poorly but the addition of financial shocks brings the model much closer to the data. The main difference with the benchmark case is that hours worked (and output) become now somewhat more volatile, particularly in the case with financial shocks (this is what we would expect with linear disutility of leisure).

Let's look now at the sensitivity with respect to the equity payout cost parameter κ. In the benchmark case, κ was set to 0.246 to match the empirical standard deviation of equity payouts. Here we reduce its value to one tenth. With lower κ, output and hours worked become less responsive to financial shocks and financial flows become more volatile. As shown in Figure 9, the paths for output and hours worked are now somewhere in between the benchmark case shown in Figure 5, and the case with only productivity shocks shown in Figure 3. However, despite having reduced κ to one tenth its benchmark value, financial shocks still contribute significantly to the volatility of output and working hours.

5 Conclusion

Are financial frictions and shocks that affect firms' ability to borrow important for macroeconomic fluctuations? The analysis of this paper suggests that they are. Models driven solely by productivity shocks have a number of known shortcomings in replicating key macroeconomic variables. We propose a model that incorporates explicitly the financial flows associated with firms' debt and equity financing. Within this model we show that shocks to firms' ability to borrow, combined with some rigidities in the adjustment of their

[5]In principle we should re-calibrate κ and generate a new series of financial shocks. Doing so would not change the key findings.

financial structure, can bring the model closer to the data. This is possible thanks to the impact that financial shocks have on the demand of labor.

When we use the model to interpret recent economic events, the following picture emerges: The recent crisis shows up clearly in our model as a tightening of firms' financing conditions that have caused a sharp downturn in GDP (and labor) starting in the second half of 2008. Tight financial conditions have also been important in the previous macroeconomic downturns of 1990-91 and 2001.

Appendix

A Data sources

Financial data is from the Flow of Funds Accounts of the Federal Reserve Board.

Debt is 'Credit Market Instruments' of Nonfinancial Business (F. 101 line 17). Because the net increases in liabilities (Flow Tables) are seasonally adjusted while the levels (Balance Sheet Tables) are not, we construct the seasonally adjusted series for the stock of liabilities using the flow data. The initial stock at the end of 1983 is taken from the Balance Sheet data. The credit market liabilities includes mainly corporate bonds (for the corporate part), mortgages and bank loans (for corporate and noncorporate). It does not include trade and tax payables. The series is deflated with the Price Index for Gross Value added in the Business Sector (National Income and Product Accounts, NIPA, Table 1.3.4). *Debt Repurchases* are defined as the negative of the change of debt divided by the Gross Value Added for the Business Sector (NIPA Table 1.3.5.), adjusted by the price index.

To compute the historical average *Leverage Ratio* we use 'Credit Market Liabilities' divided by 'Assets' of the Nonfinancial Corporate Business (B.102, line 1 and 21) and Noncorporate Business (B.103, line 1 and 23).

Equity payout is defined as 'Net Dividends' in the Nonfinancial Corporate Business (F.102, line 3) minus 'Net New Equity Issue' in the Nonfinancial Corporate Business (F.102, line 38), minus 'Proprietors Net Investment' in the Nonfinancial Noncorporate Business (F103, line 29, F104, line 23).

The *Capital Stock* is constructed from adding up Capital Expenditures of the Nonfinancial Business (F.101 line 4) minus Consumption of Fixed Capital of Domestic Business (F8. line 15). The initial stock value is from the Fixed Asset Table (Table 6.1) for the nonfinancial corporate businesses, sole proprietorships and partnerships, including the stock of inventory from (NIPA Table 5.7.5) at the end of 1983. Nominal values are deflated by the Price Index for Fixed Nonresidential Investment (NIPA Table 1.1.4.). *Investment* is defined as the Capital Expenditures of the Nonfinancial Business (F.101 line 4). The alternative of constructing the capital stock assuming a constant depreciation rate with the same investment data does not significantly alter the results.

Aggregate series for *GDP* and *Consumption* (nondurables and services) is from NIPA Table 1.1.3. *Hours worked* are 'Total Private Aggregate Weekly Hours' from the Current Employment Statistics national survey. All series are seasonally adjusted.

B Enforcement constraint

Firms start the period with liabilities b_t. Before producing they choose the labor input, l_t, investment, $i_t = k_{t+1} - (1-\delta)k_t$, equity payout, $\tilde{d}_t$, and the next period debt, b_{t+1}. The tilde over the equity payout captures the costs associated with payouts as specified in the main text of the paper. The payments to workers, suppliers of investments, shareholders and holders of previous debt is made before the realization of revenues. In order to make these payments the firm contracts an intra-period loan to cover the cash flow mismatch during the period. The intra-period loan is equal to $L_t = w_t l_t + i_t + \tilde{d}_t + b_t - b_{t+1}/R_t$ and it is fully repaid at the end of the period after the realization of revenues. Because it is repaid within the period, there is no interest. Given the budget constraint $b_t + w_t l_t + i_t + \tilde{d}_t = F(z_t, k_t, l_t) + b_{t+1}/R_t$, it can be verified that the intra-period loan is equal to the firm's revenues, that is, $L_t = F(z_t, k_t, l_t)$.

The decision to default on the intra-period loan arises after the realization of revenues $F(z_t, k_t, l_t)$. Because the firm has the ability to divert these revenues, the intra-period lender can only access the residual equity of the firm. Suppose that the liquidation value of the firm is a fraction ψ_t of the residual equity value, that is, $\psi_t \overline{V}_t$. The variable ψ_t is stochastic and depends on (unspecified) markets conditions.[6] Notice that at the end of the period the intertemporal debt is not due until the next period. Therefore, it is only the residual equity of the firm (net of the intertemporal loan) that guarantees the intra-period loan.

Because $\psi_t < 1$, there is a loss of value in the liquidation of the firm. Therefore, both the lender and the firm have an interest in renegotiating.

Bargaining is over the repayment of the intra-period debt. Denote by e_t the payment agreed upon by the contractual parties. By reaching an agrement the firm makes the payment e_t but continues operation. Therefore, the firm gets $F(z_t, k_t, l_t) - e_t + \overline{V}_t$ and the intra-period lender gets e_t. Without agreement, the firm gets the threat value $F(z_t, k_t, l_t)$ and the lender gets the liquidation value $\psi_t \overline{V}_t$. Therefore, the net value of reaching an agreement for the firm is $\overline{V}_t - e_t$ and for the lender is $e_t - \psi_t \overline{V}_t$.

Denote by η the bargaining power of the firm and $1-\eta$ the bargaining power

[6] This fraction can result from the assumption that the sale of the firm requires the search for a buyer with which the lender bargains the price. The fraction ψ_t can then being interpreted as the probabilities of finding the buyer and/or the bargaining power of the lender in the determination of the selling price. The probability of finding a buyer and/or the price extracted in bargaining increase when the market conditions improve.

of the lender. The bargaining problem solves:

$$\max_{e_t} \left\{ (\overline{V}_t - e_t)^\eta (e_t - \psi_t \overline{V}_t)^{1-\eta} \right\}$$

Taking the first order conditions and solving we get $e_t = \overline{V}_t[1 - \eta(1 - \psi_t)]$.

Incentive-compatibility requires that the value of not defaulting, $\overline{V}_t$, is not smaller than the value of defaulting, $F(z_t, k_t, l_t) - e_t + \overline{V}_t$. Therefore, the enforcement constraint is $\overline{V}_t \geq F(z_t, k_t, l_t) - e_t + \overline{V}_t$. Using $e_t = \overline{V}_t[1 - \eta(1 - \psi_t)]$ derived above, the enforcement constraint can be written as:

$$\overline{V}_t \geq F(z_t, k_t, l_t) + \eta(1 - \psi_t)\overline{V}_t$$

Collecting terms and rearranging we get:

$$\xi_t \overline{V}_t \geq F(z_t, k_t, l_t),$$

where $\xi_t = 1 - \eta(1 - \psi_t)$.

If we assume that the bargaining power of the firm is $\eta = 1$, we have that ξ_t is the fraction recovered in the sale of the firm, that is, $\xi_t = \psi_t$. However, we will get the same functional form for any value of the bargaining power $\eta > 0$. Also notice that, although we have not done it explicitly here, it is not difficult to see that the stochastic nature of ξ_t can also derive from a shock to the bargaining power of the firm η.

We would like to point out that the particular timing about the payments and decision to default is only made for analytical convenience. For example, the assumption that the firm contracts an intra-period loan is a short cut to the fact that firms carry 'cash' or 'liquidity' to the next period. The cash is then used to pay the equity holders (including dividends) and to finance working capital (wages and investment). When interpreted this way, the payments of dividends comes from previous period earnings, which is a more natural interpretation of reality. All of this can be formalized by explicitly adding cash. However, to avoid the introduction of an additional state variable, the assumption of the intra-period loan achieves the same outcome without complicating the analysis.

C First order conditions

Consider the optimization problem (2) and let λ and μ be the Lagrange multipliers associate with the two constraints. Taking derivatives we get:

$$d: \qquad 1 - \lambda \varphi_d(d) = 0$$

$$
\begin{aligned}
l: &\quad \lambda F_l(z,k,l) - \lambda w - \mu F_l(z,k,l) = 0 \\
k': &\quad (1+\xi\mu) E m' V_k(\mathbf{s}'; k', b') - \lambda = 0 \\
b': &\quad (1+\xi\mu) E m' V_b(\mathbf{s}'; k', b') + \frac{\lambda}{R} = 0
\end{aligned}
$$

The envelope conditions are:

$$
\begin{aligned}
V_k(\mathbf{s}; k, b) &= \lambda\Big[1 - \delta + F_k(z,k,l)\Big] - \mu F_k(z,k,l) \\
V_b(\mathbf{s}; k, b) &= -\lambda
\end{aligned}
$$

Using the first condition to eliminate λ and substituting the envelope conditions we get:

$$
\begin{aligned}
& F_l(z,k,l) = w\left(\frac{1}{1-\mu\varphi_d(d)}\right) \\
& (1+\xi\mu) E m' \left(\frac{\varphi_d(d')}{\varphi_d(d)}\right)\Big[1 - \delta + (1-\mu'\varphi_d(d'))F_k(z',k',l')\Big] = 1 \\
& (1+\xi\mu) R\, E m' \left(\frac{\varphi_d(d')}{\varphi_d(d)}\right) = 1
\end{aligned}
$$

Defining $\tilde{m}' = m'\varphi_d(d')/\varphi_d(d)$ and substituting we get (3)-(5).

D Numerical solution: linear approximation

If the enforcement constraint is always binding, we can solve the model by log-linearizing the dynamic system around the steady state. Of course, whether the enforcement constraint is always binding needs to be verified. This will be done in the next section (Appendix E). In that section will also show more generally the accuracy of the linear approximation compared to the solution obtained with a nonlinear approximation.

The equilibrium of the model is characterized by the following system of dynamic equations:

$$
wU_c(c,l) + U_l(c,l) = 0 \tag{10}
$$

$$
U_c(c,l) = \beta\left(\frac{R-\tau}{1-\tau}\right) EU_c(c',l') \tag{11}
$$

$$wl + b - \frac{b'}{R} + d - c = 0 \tag{12}$$

$$F_l(z,k,l) = w\Big(\frac{1}{1-\mu\varphi_d(d)}\Big) \tag{13}$$

$$(1+\xi\mu)E\,\tilde{m}(c,l,d,c',l',d')\Big[1-\delta+(1-\mu'\varphi_d(d'))F_k(z',k',l')\Big] = 1 \tag{14}$$

$$(1+\xi\mu)RE\,\tilde{m}(c,l,d,c',l',d') = 1 \tag{15}$$

$$F(z,k,l) - wl - b + \frac{b'}{R} - k' - \varphi(d) = 0 \tag{16}$$

$$\xi Em(c,l,c',l')V' = F(z,k,l) \tag{17}$$

$$V = d + Em(c,l,c',l')V' \tag{18}$$

Equations (10)-(12) are the first order conditions for households and their budget constraint. Notice that in equilibrium the tax payments of households in the budget constraint is accounted for by a lower interest earned on bonds, R. Notice also that $1 + r = (R - \tau)/(1 - \tau)$, which is the term used in the first order condition for the choice of bonds. Equations (13)-(15) are the first order conditions for firms and (16)-(18) are the budget constraint, the enforcement constraint and the value function for firms.

We have nine dynamic equations. After linearizing around the steady state, we can solve the nine equations system for the nine variables c_t, d_t, l_t, w_t, R_t, V_t, μ_t, k_{t+1}, b_{t+1}, as linear functions of the states, z_t, ξ_t, k_t, b_t.

E Linear and nonlinear solution

The simulations presented in the paper are based on the first order approximations of the dynamic system. In order to apply this method we have conjectured that the enforcement constraint is always binding during the simulated period. There are two ways of checking whether the enforcement constraint is in fact binding.

As a first approximation, we can check whether the Lagrange multiplier μ implied by the linearized system is always positive during the simulated period.

Because the linearized system is based on the assumption that the enforcement constraint is always binding, there is no guarantee that the linearized solution for the multiplier μ is positive. Therefore, if μ takes negative values over some periods, we would question the accuracy of the solution based on the log-linear approximation. Fortunately, in our simulation with both productivity and financial shocks, the multiplier touches zero in only one of the 101 simulated quarters (from 1984:1 to 2009:1). In all other quarters it is always positive. This is also the case when the simulation is done with only one of the two shocks.

The fact that the Lagrange multiplier in the linear solution does not take negative values during the simulation period is not a proof that the enforcement constraint has always been binding during the period. A more accurate test would require solving the model using a higher order approximation that allows for the enforcement constraint to be occasionally non-binding. We have then solved the model using a nonlinear approximation.

The nonlinear approach approximates the four conditional expectations in equations (11), (14), (15) and (18) as functions of the four-dimensional state vector (z, ξ, k, b) through a finite element representation that interpolates linearly between the grid points of the state space. Starting with initial guesses for the conditional expectations, we can compute all variables of interest by solving simple systems of nonlinear equations. We first solve for a system including a binding enforcement constraint (condition (17)). If the solution for the multiplier is negative, we set it equal to zero, and then solve the system containing the rest of the equations (and therefore, ignoring condition (17)). New values for the conditional expectations are produced through Gauss-Hermite quadrature (z_t, ξ_t are assumed to be lognormal). We iterate on the four functions for conditional expectations until convergence.

Figure 10 compares the simulated paths for a set of variables generated with the linear and nonlinear solutions. The nonlinear method uses 18 grid points for each of the 4 state variables for a total of 18^4 grid points. As shown in the upper left panel, the multipliers obtained from the simulation of the model using the two methods are almost indistinguishable after an initial period of adjustment. The same holds for the variables in the second and third rows of the figure. Small differences are visible in the first 20 periods. This is due to the fact that we start the simulation at the deterministic steady state, when the average debt level in the nonlinear method is somewhat higher than in the deterministic steady state.

As shown in the upper right panel, even at the point where the multiplier touches zero (this is the third quarter of 2007), the optimal solution does not require an effective loosening of the constraint. Based on the normal probability distribution, the probability that the multiplier equals zero is only 0.0009 for the benchmark calibration. As we discussed earlier, lowering the tax subsidy for debt τ, increasing the volatility of the shocks, and increasing the equity cost κ would all contribute to increasing the probability that the multiplier goes to zero. But as long as the changes in these parameters are not too big, the properties of the simulated variables would not change significantly.

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Fig 2: Productivity Shocks and Financial Shocks

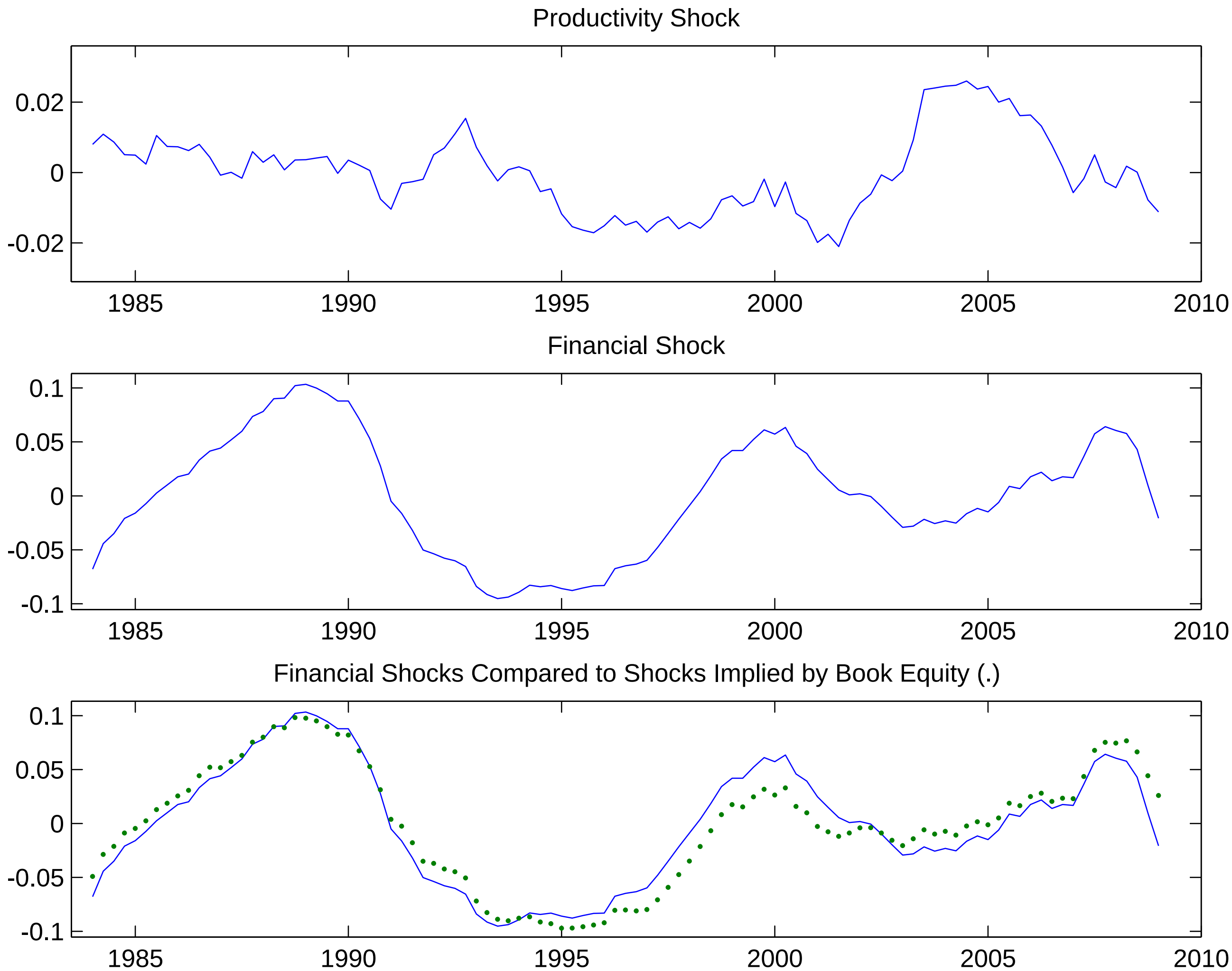

Fig 3: Productivity Shocks only: Data (thick line), No Financial Frictions (-), Financial Frictions (--)

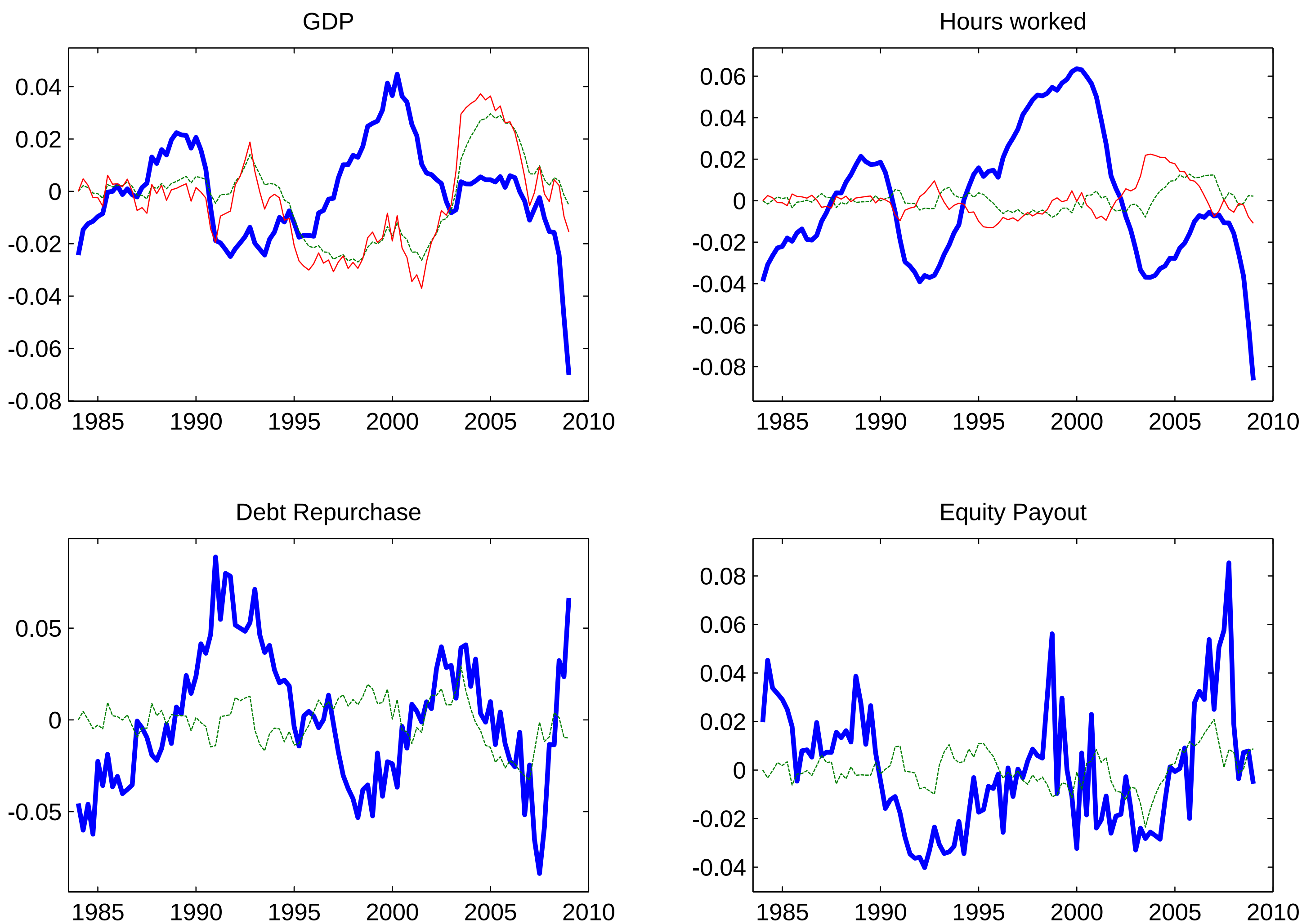

Fig 4: Model with Financial Shocks versus Data (thick line)

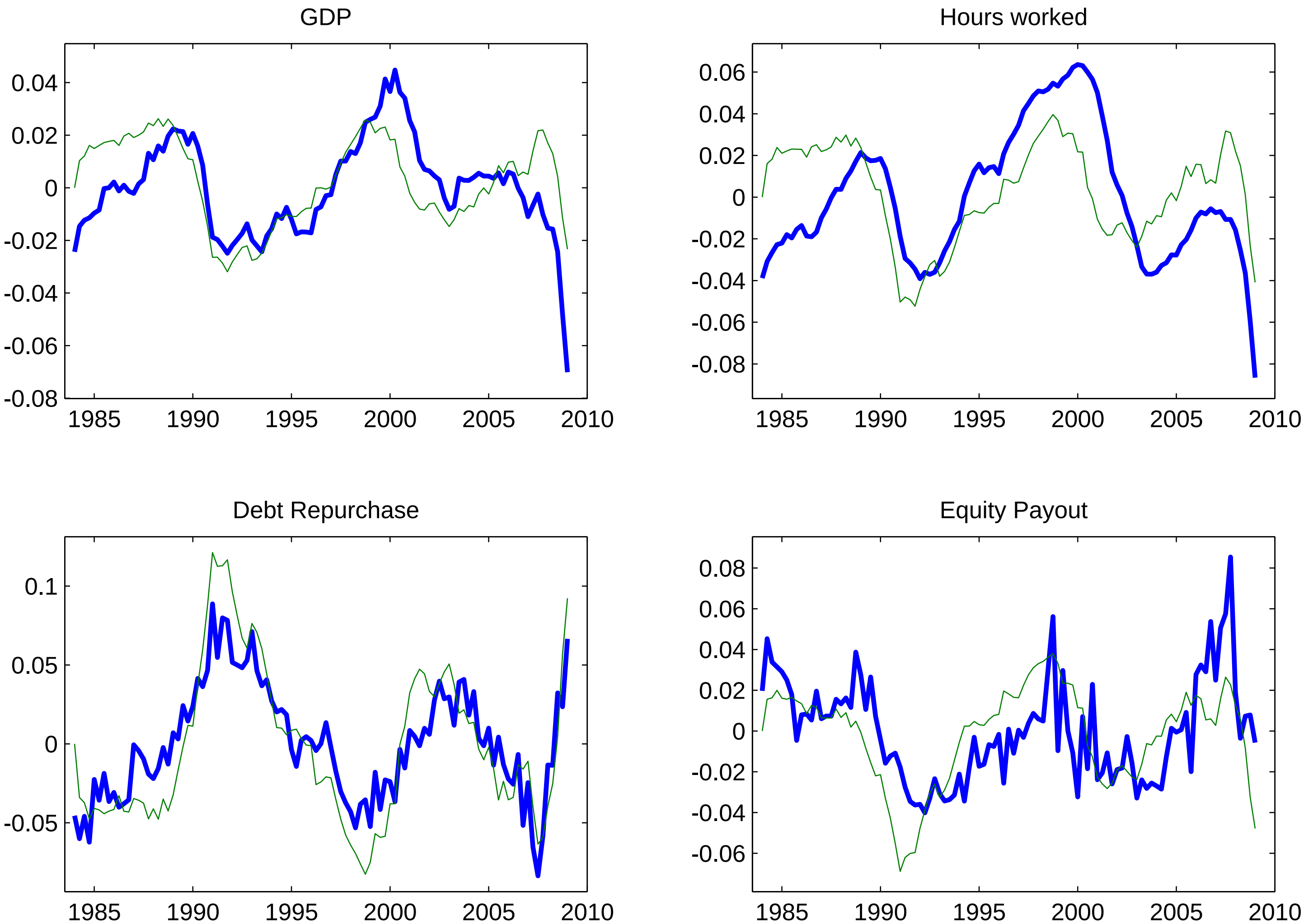

Fig 5: Model with both Shocks and Data (thick line)

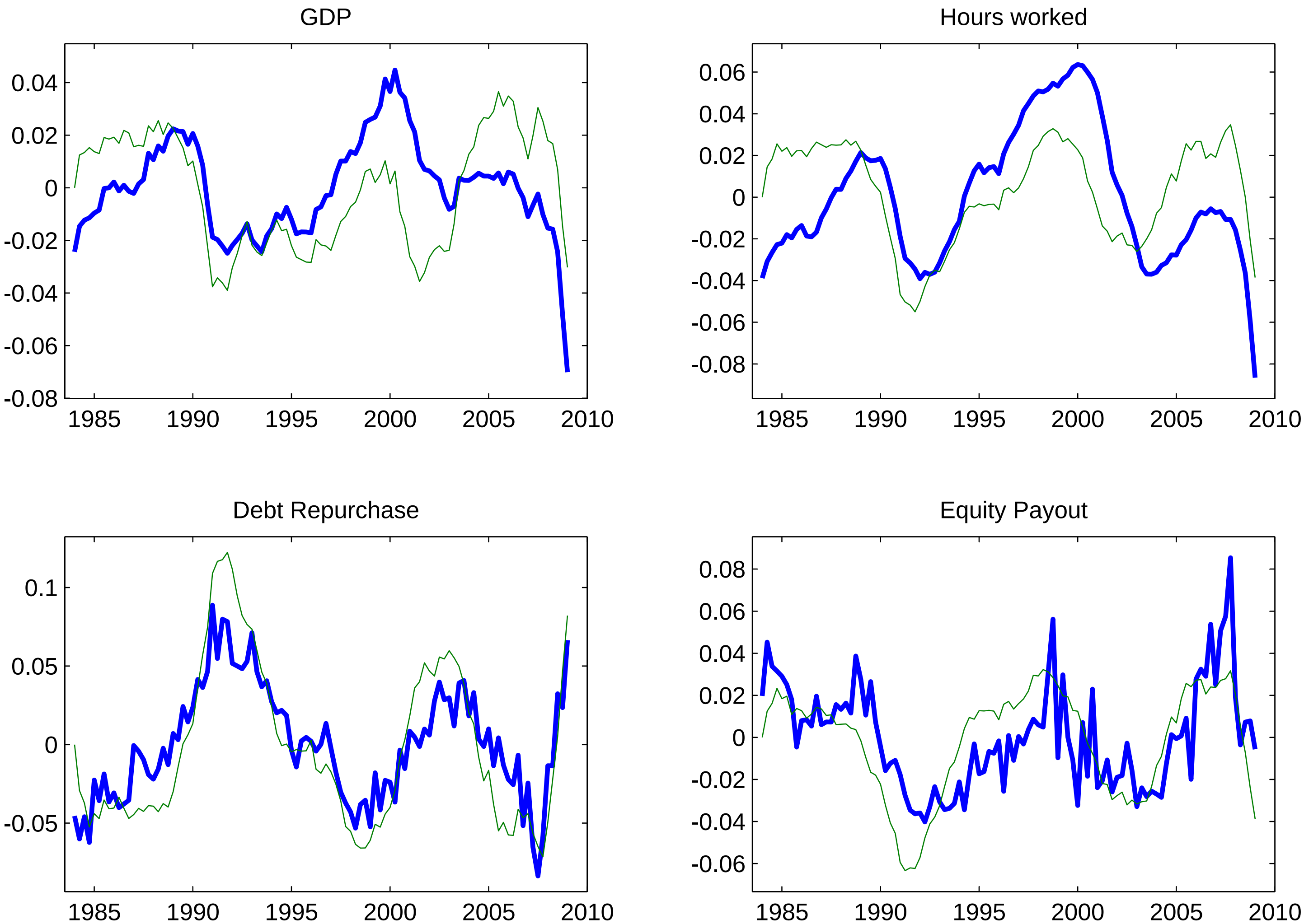

Fig 6: Impulse Responses to Productivity Shock

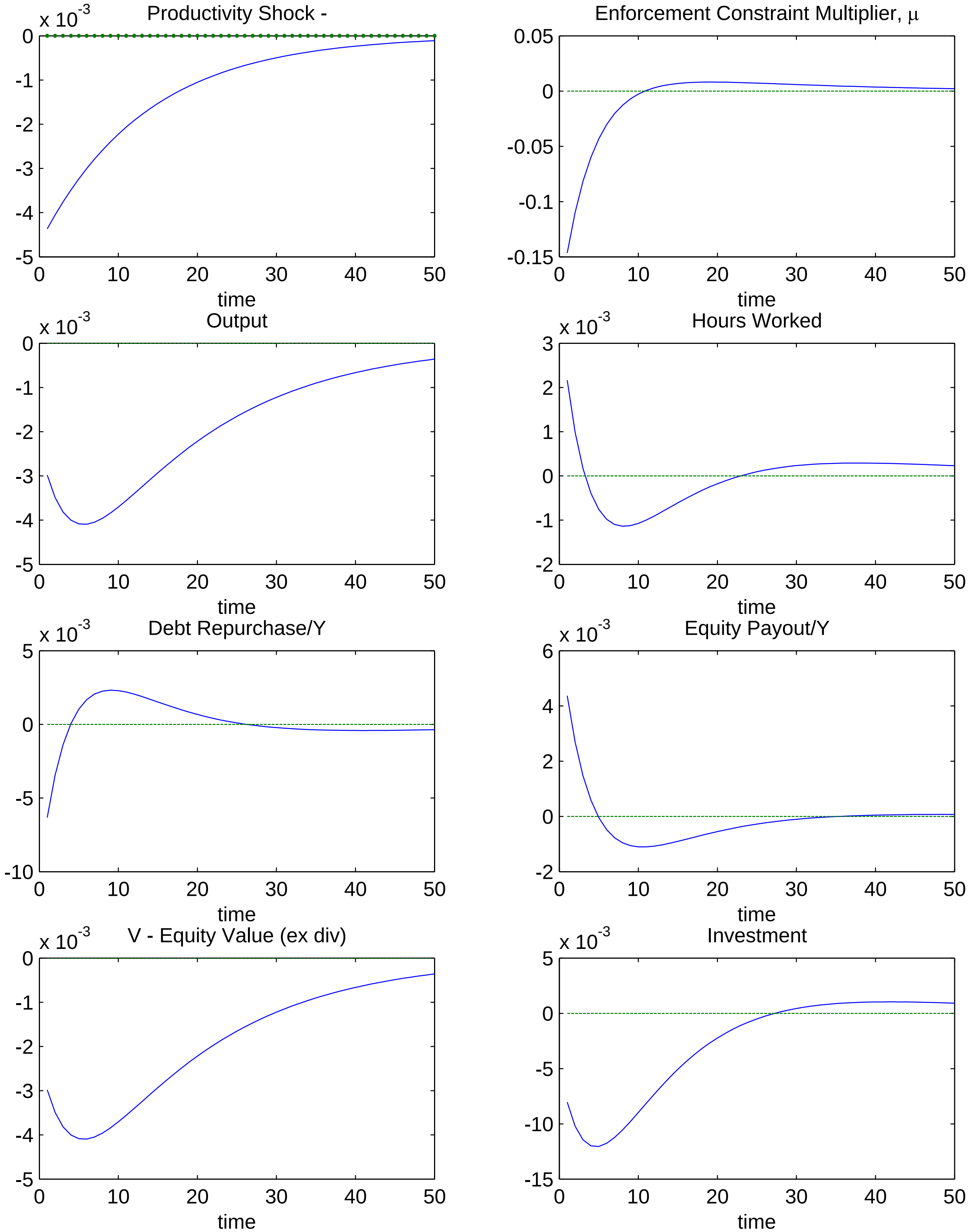

Fig 7: Impulse Responses to Financial Shock

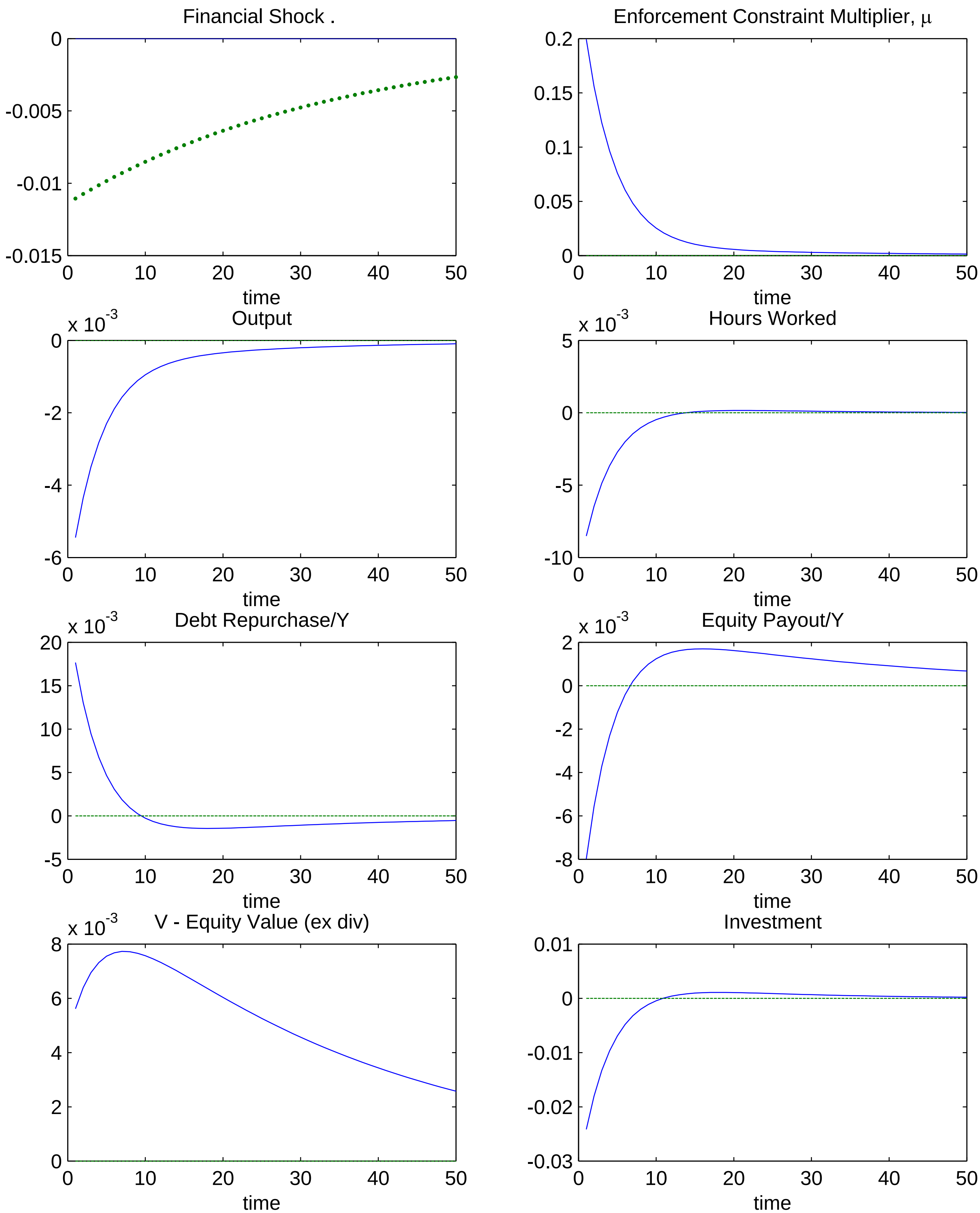

Fig 8: Sensitivity analysis: Hansen (1985) utility (Both shocks (-), productivity shocks only (--))

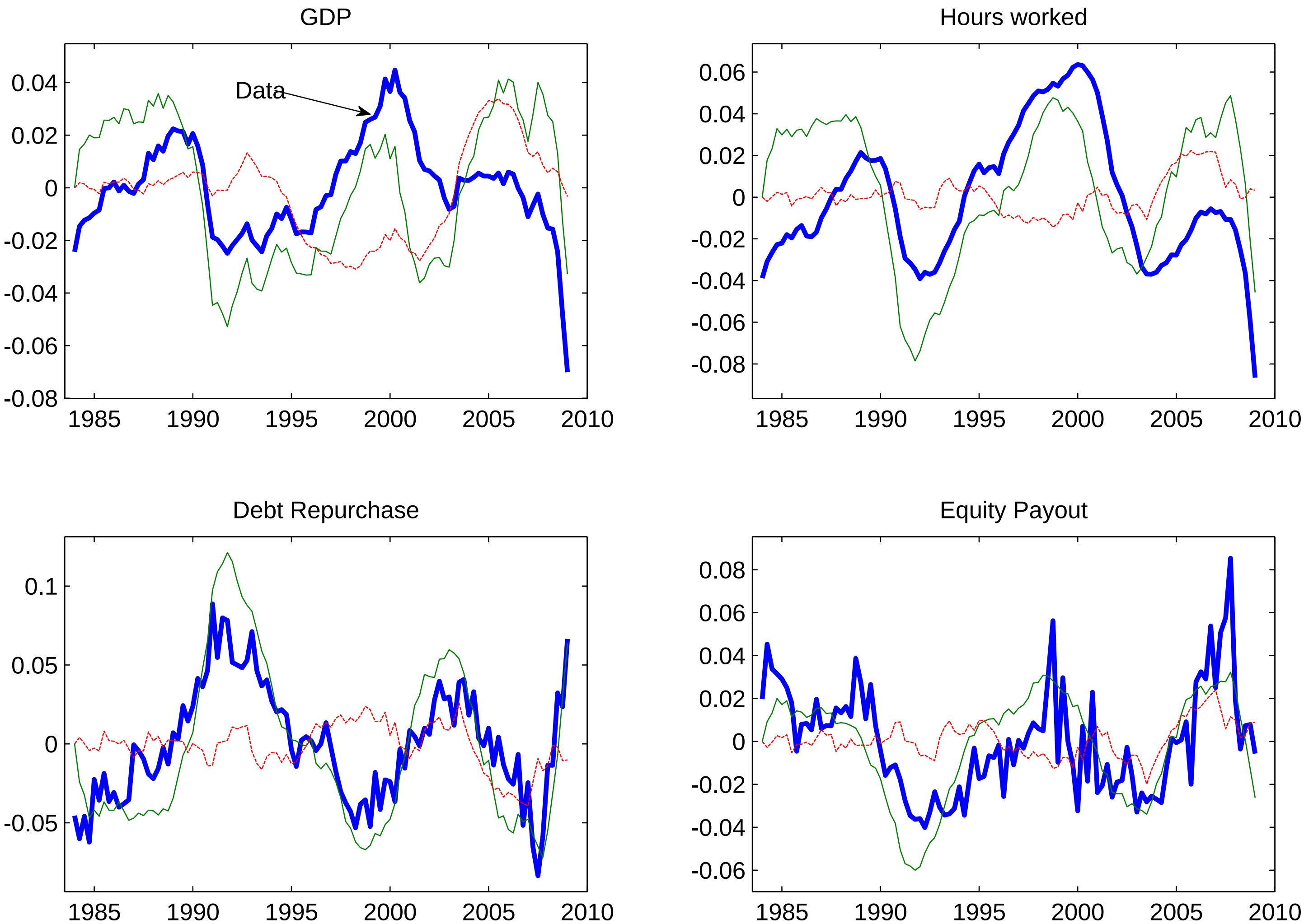

Fig 9: Sensitivity analysis: Equity Payout Cost at one tenth of benchmark value (both shocks)

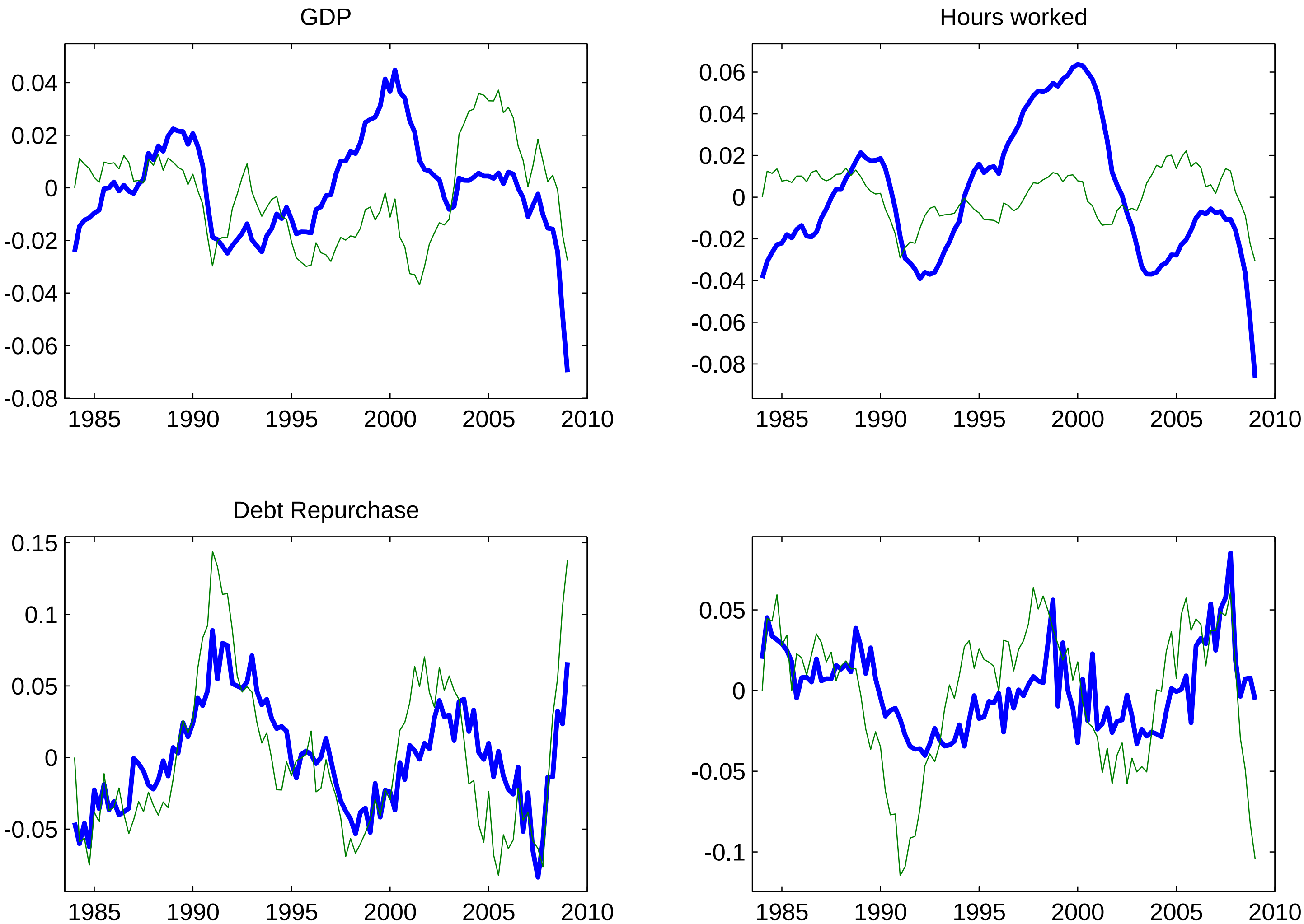

Fig 10: Linear and Nonlinear Solutions