A Simple Model of Trading and Pricing Risky Assets Under Ambiguity: Any Lessons for Policy-Makers?

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Abstract

The 2007-2008 financial crises has made it painfully obvious that markets may quickly turn illiquid. Moreover, recent experience has shown that distress and lack of active trading can jump "around" between seemingly unconnected parts of the financial system contributing to transforming isolated shocks into systemic panic attacks. We develop a simple two-period model populated by both standard expected utility maximizers and by ambiguity-averse investors that trade in the market for a risky asset. We show that, provided there is a sufficient amount of ambiguity, market breakdowns where large portions of traders withdraw from trading are endogenous and may be triggered by modest re-assessments of the range of possible scenarios on the performance of individual securities. Risk premia (spreads) increase with the proportion of traders in the market who are averse to ambiguity. When we analyze the effect of policy actions, we find that when a market has fallen into a state of impaired liquidity, bringing the market back to orderly functioning through a reduction in the amount of perceived ambiguity may cause further reductions in equilibrium prices. Finally, our model provides stark indications against the idea that policy-makers may be able to "inflate" their way out of a financial crisis.

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"The trading of legacy loans and securities continues to reveal systematic underpricing at issuance of once seemingly benign risks–credit, liquidity, counterparty, and even sovereign risks [...] Until these assessments are more clearly refined and more broadly understood, we are likely to observe elevated levels of volatility and *unwillingness by many investors to participate in certain asset markets at virtually any price*." (K. Warsh, 2009, emphasis added)

1. Introduction

The major, painful surprise of the on-going financial crisis is that established, high-volume financial markets may turn illiquid in the blink of an eye. In a matter of weeks—often days—traders, transactions,

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price discovery functions—in short liquidity—have vaporized. World financial markets, firms, households and policy-makers alike have been left hanging with trillions of financial assets at face value, whose market price is simply unknown, or often well-approximated by a bleak *zero*. As it is well known to practitioners, market break-down problems have a way to quickly become solvency problems. Perhaps institution A cannot manage a payment (say, on a derivative contract) only because it does not have the needed cash at hand. However, this can be interpreted by its creditor—institution B—as meaning that A is carrying liabilities that exceed the value of its assets. Worse, institution C, which has normally stood ready to lend short-term funds to A, may become aware of the situation and start refusing to lend to A, require dazzlingly high spreads over riskless market rates, or worse, require high percentages of collateral in the form of high-quality, liquid securities that A no longer holds in its balance sheet. At this point, a perverse spiral of cash shortage, inability to access unsecured short-term funding, and over-investment in hard-to-evaluate illiquid assets may drive a powerful financial conglomerate (A) to insolvency. Of course, even looking beyond the recent financial crisis, the history of financial markets is replete with episodes of increase in uncertainty leading to seizures of trading intensity, especially in markets such as high yield corporate bonds and bonds issued in emerging markets. For instance, it is well known that in 1998 a paralysis afflicted the junk bond markets in the U.S. and in Europe in the aftermath of the Russian and East Asian crises.[1]

Another painful realization of the 2007-2008 crisis is that the extent to which liquidity problems can jump "around" between seemingly unconnected parts of the global financial system can take a dramatic pace, often impossible to control by regulators and policy authorities and difficult to hedge in the perspective of market participants. While now the causes of the crisis have become clear and appear to be rooted in the collapse of the U.S. subprime mortgage market and in a host of poor business and risk management practices (see e.g., Sarmiento, 2009), it remains unclear how the losses, originally localized in the market for asset-backed and collateralized debt obligations (CDOs), could spread to other parts of the system (see, e.g., Dodd, 2007, and Kodres, 2008). Like an epidemic in which an invisible virus infects many unrelated people, the financial crisis spread when losses to intermediaries in one scarcely (usually over-the-counter) transparent market raised concerns about liquidity and solvency elsewhere, even in regulated and highly transparent markets. The absence of traceable linkages among the markets progressively sucked in by the mounting crisis suggests that economic analysis should focus on models in which market breakdowns may occur endogenously and involve multiple markets at the same time, irrespective of their micro-structure.

In this paper, we focus on one potential cause of market breakdowns: the presence of difficult-to-quantify uncertainty, *ambiguity* (as opposed to quantifiable *risk*), and market traders' aversion to ambiguity. For our purposes, we define a market breakdown as a situation of receding trading—possibly caused by the decision of some fraction of the potential traders not to participate in the market—accompanied by strong and persistent price declines and high premia (spreads) over equivalent but uncertainty-free securities. We

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[1]During the 1998 Russian crisis, bid-ask spreads on emerging market debt increased from 10-20 basis points to 60-80 basis points (International Monetary Fund (1998)). Bank of International Settlements (1999) describes how a number of market-makers simply withdrew from trading following the 1998 Russian default and stopped posting quotes. Sojli (2007) discusses issues of contagion spreading with specifica applications to the Russian crisis.

propose a stylized model of portfolio choice and market equilibrium with two categories of traders, standard expected utility maximizers and ambiguity-averse (henceforth, AA) agents, and show that prices, expected returns, and the decision to participate may have interesting non-linear properties such that—provided there is enough ambiguity and that this ambiguity is stronger for idiosyncratic than for systematic uncertainty— market breakdowns may endogenously occur. We use the model to characterize potential policy reactions, both formally and informally, by assessing the pros and cons of changes in the parameters of the model that have an interpretation as a result of policy actions.

Why ambiguity? For at least two reasons that it may be helpful to explain to sort out the contribution of our paper. First, because the applications of models of decision-making under ambiguity have recently witnessed such a powerful acceleration that to collect a few thoughts on one simple model connecting the 2007-2008 financial turmoil with this literature seems to be a useful effort, especially for its possible policy implications (see Gilboa, Postlewaite, and Schmeidler, 2008, and Mukerji and Tallon, 2003, for reviews of applications of ambiguity in economic models). Traditional finance theory assumes that agents are either expected or subjective expected (SEU) utility maximizers. According to Expected Utility Theory (EU), decision makers choose among different risky prospects by comparing their expected utilities. That is, they compare the weighted sums of the outcomes' utility values, using as weights the associated probabilities. Savage (1954) elaborated an alternative theory (Subjective Expected Utility Theory) for defining probabilities in absence of statistics by simply observing individuals' choices. After having derived the probability distribution, SEU theory prescribes to proceed as under EU. The main difficulty with this approach is that its applications to asset markets assume the distributions of asset payoffs are known to investors. This assumption is usually justified with the rational expectations hypothesis. For some assets and some investors this is a reasonable assumption; for others it is surely not reasonable. Do unsophisticated investors know the distribution of payoffs to even simple portfolios? Do major financial institutions know the distribution of future payoffs to all sort of asset- and mortgage-backed securities and complicated CDOs?[2]

Our second motivation lies in the fact that only a model that captures the presence of ambiguity-aversion makes a credible prediction that simple, idiosyncratic shocks that—in principle at least, according to SEU classical finance theory, ought to be fully and cheaply diversifiable—may cause systemic panic attacks across a range of differently structured and regulated markets. In a security market model that accommodates a distinction between risk and ambiguity, investors are assumed to possess a subjective knowledge about the likelihood of contingent events that is consistent with more than one probability distribution. Additionally, whatever the investor knows about the future fails to inform her of a precise probability distribution over the set of possible probabilities. In this case, we say that investors' beliefs about contingent events are characterized

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[2]Besides simple introspection on the plausibility of EU and SEU as realistic frameworks for decision-making, we have to recall that experimental research has been rather unkind to the SEU and EU frameworks. In 1953, Allais (1953) designed a choice problem to show an inconsistency of observed behaviors with the predictions of EU. In essence, experimental subjects tend to violate EU because they under-weight prospects with high potential payoff that comes with very low probability. Moreover, the experimental evidence shows that people dislike situations where they are uncertain about the probability distribution of a gamble, that is, they are *ambiguity-averse*. In particular, Ellsberg (1961) first reported that for unknown probabilities, people behave in ways that cannot be reconciled with any assignment of subjective probabilities. Section 3 discusses Ellsberg's evidence in detail.

by ambiguity. If ambiguous, the agent's beliefs are captured not by a unique probability distribution in the standard Bayesian fashion but instead by a set of probabilities. Thus not only is the outcome of an act uncertain but also the expected payoff of the action, since the payoff may be measured with respect to more than one probability. Because this attitude applies in the same way independently of market rules or other features of the micro-structure of each market, models that describe portfolio decisions and price assets under ambiguity hold a promise to generate realistically contagious crises.

In our model the AA decision maker evaluates an act by the minimum expected value that may be associated with it: the decision rule is to compute all possible expected values for each action and then choose the act which has the best minimum expected outcome. This notion of AA inspires the formal model of Choquet expected utility (CEU) preferences introduced by Schmeidler (1989). Our main findings are as follows. Provided there is a sufficient level of ambiguity, market breakdowns are endogenous and may be triggered by otherwise modest re-assessments of the range of possible scenarios concerning the perspectives of individual firms. Additionally, the more ambiguity there is, the more likely market disruptions become. In our model, market disruptions consist of equilibrium configurations in which all AA investors withdraw and stop trading for all possible asset prices consistent with that set of equilibria. We prove that uncertainty premia (spreads) increase with the proportion of traders in the market who are averse to ambiguity. This makes intuitive sense because when the market is well-functioning, there is a higher fraction of investors that asks to be compensated not only for exposure to risk, but also for bearing difficult-to-quantify uncertainty. In case markets are already impaired and participation is limited to SEU investors, the intuition is that the same supply of the security has to be absorbed by a shrinking fraction of SEU investors, who will then require to be compensated by higher spreads.

When we analyze the effect of policy actions, we find three interesting results. Reducing the amount of ambiguity perceived by the market has difficult-to-sign effects on equilibrium prices. As long as the action does not affect the participating nature of the market, prices rise as ambiguity declines. However, assuming the market had initially broken down (i.e., only SEU investor participate), by sufficiently lowering the level of ambiguity, a policy-maker may revive a well-functioning market in which all types of investors trade, but this comes at the cost of lower prices and higher premia. This is surprising: even though the policy action consists of ruling out the worst possible scenarios to reduce perceived ambiguity, for an intervention of sufficient magnitude its eventual effect on equilibrium prices may be negative and the cost of enforcing a participation equilibrium consists of higher spreads. This means that when a market has fallen into a state of low liquidity and trading disruption, bringing the market back to higher liquidity and orderly functioning through a reduction in the amount of perceived ambiguity may actually cause further reductions in equilibrium prices, which may pose a tough trade-off to policy makers. As trivial as this may sound, the implication is that it is much cheaper for policy-makers to "manage" the presence of ambiguity in well-functioning markets than in impaired ones, in the sense that in the latter case not only bigger efforts are required, but these may also confront policy officials with difficult trade-offs.

Finally, our model provides rather stark indications against the idea that policy-makers may wish to "in-

flate" their way out of a financial crisis. Even though our model treats inflation as an exogenous parameter, its implications for the effect of changes in the inflation rate are rich. A higher inflation rate in a segmented market in which all AA investors have left already, simply strengthens the segmentation, while it produces ambiguous effects on risky asset prices (negative provided there is enough subjectively perceived total uncertainty). A higher inflation rate in a non-segmented, well-functioning market produces again uncertain effects on risky asset prices and threatens to disrupt markets by forcing non-participation upon them. Therefore, higher inflation as a policy tool seems either ineffective or perverse because it cannot relax participation constraints while it may depress equilibrium prices.

At least three papers are closely related to ours. Dow and Werlang (1992) investigate some implications of decision making under AA for portfolio choice. Their results have direct application to explain market breakdowns. In their paper they analyze the simplest investment decision, namely where there is only one uncertain asset. Under standard SEU theory, an agent who must allocate her wealth between one safe and one risky asset will buy some of the risky asset if its price is less than the expected value of its future payoffs. Conversely the agent will sell the risky asset short when the price is greater than its expected payoffs. Dow and Werlang's (1992) main finding is a generalization of this result to the case of AA with one striking difference. In the case of a SEU decision maker, the threshold between optimal purchases and sales is represented by one single value for the current price. The exact amount of the asset that is bought or sold will then depend on the agent's risk aversion. On the contrary, under AA, they prove that there is an interval of prices within which the agent neither buys nor sells short the asset. At prices below the lower limit of this interval, the agent is willing to buy this asset; at prices above the upper end of this interval, the agent is willing to sell the asset short. When equilibrium forces fail to push the asset price outside this interval, there will be no willingness to trade and the market will break down (equivalently, the agent rationally chooses non participation). Section 4 provides an example that further clarifies the intuition behind Dow and Werlang's findings.

Mukerji and Tallon (2001) extend Dow and Werlang's partial equilibrium results by showing that also in equilibrium ambiguity and AA may cause investors to restrict their trading to a set of assets narrower than what would be found under SEU. Specifically, in Mukerji and Tallon's model ambiguity leads to a collapse in the trade of financial assets whose payoff is greatly affected by idiosyncratic risk when the range of variation of the idiosyncratic component in assets' payoffs is large relative to the range of variation of systematic risk, and the ambiguity of agents' beliefs about the idiosyncratic component is sufficiently large. Moreover, the effects of the presence of idiosyncratic risks cannot be simply removed by standard diversification techniques based on the law of large numbers, as it is under SEU. In fact, even under ambiguity the law of large numbers works in the usual way (that has to be); what is different is that, because the investors' knowledge is consistent with more than one probability distribution, there is more than one mean to converge to. Our model in Section 5 shares a number of features proven by Mukerji and Tallon (2001) for a rather general case.

Routledge and Zin (2001) argue that the major puzzle posed by a financial crisis is not the large change in asset prices but instead the fact that extreme market outcomes are often followed by a lack of liquidity: people seem to stop trading exactly when events ought to force them to trade more aggressively, and this

seems irrational. Routledge and Zin investigate the connection between AA and liquidity focussing on a simple market mechanism in which the demand for the derivative is summarized by the arrival of a random, exogenous willingness-to-trade signal. The bid-ask spread and the associated probability that the market maker will make a trade are treated as a measure of liquidity. The market-maker is assumed to be a monopolist in that market while the market for the underlying security is frictionless. The market maker chooses bid and ask prices for the derivative to optimally trade-off the probability of attracting a seller or buyer to affect current profits with the future utility implications implied from a trade in the derivative. The investment in the risky asset after observing a trade in the derivative allows the market maker the opportunity to hedge her position in the derivative market. When there is ambiguity about the probability distribution for the underlying security's cash flows, the market-maker is uncertain about the future, dynamic consequences of a trade, which they model through an Epstein and Wang (1994)-type recursive utility function. Routledge and Zin find that AA increases the bid-ask spread and, hence, reduces liquidity; their infinite-horizon example produces short-lived but dramatic decreases in liquidity in the face of shocks even though the underlying environment is stationary.[3]

The paper is structured as follows. Section 2 briefly presents a number of stylized features of the recent financial crisis setting the stage for the empirical features our models are geared towards. Section 3 briefly discusses the definition of ambiguity. Section 4 offers one simple but stark example of how ambiguity—specifically, ambiguity on the strength of idiosyncratic risk—may cause a collapse in trading activity. Section 5 develops our heterogeneous agent model in which assets are traded and priced by a fraction $\alpha \in (0,1)$ of SEU investors and a fraction $1 - \alpha$ of ambiguity-averse traders. After deriving results on equilibrium trading choices, prices, and risk premia, we discuss policy implications from the model. Section 6 concludes.

2. Market Break-Downs and Risk Spreads in the 2007-2008 Financial Crisis

In this Section we briefly review the main events that have characterized the recent world-wide financial crisis. Our main purpose here is not to discuss causes or resolution strategies for the financial turmoil (see, among many others, the recent papers by Gorton, 2008, Jacobs, 2009, and Mizen, 2008), but to make a Reader aware of the phenomena that have marked the outbreak of the crisis and that ought to be consistently explained by any realistic model of financial decision-making.

In the second half of 2007, the deteriorating performance of subprime mortgages in the U.S. triggered a rapid re-assessment of credit and liquidity risks across a broad range of assets, leading to widespread turbulence in financial markets. With the only exception of government-supported mortgages, securitization markets (at first those directly involving mortgage loans, and soon after all markets related to the origination

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[3]One last paper related to ours is Easley and O'Hara (2005, 2006), who investigate the role that market microstructure can play in reducing the ambiguity confronting AA traders. They develop a model with SEU-maximizing traders and naive, ambiguity-averse traders, and show how AA traders can choose not to participate in markets. They find that specific features of the microstructure (e.g., listing and delisting rules, trading halts, transparency, daily limits, clearing house rules and margin requirements, etc.) can reduce the perceived ambiguity, and induce participation by both firms and issuers. Their analysis demonstrates how designing markets to reduce ambiguity can benefit investors through greater liquidity, exchanges through greater volume and trading fees, and issuing firms through a lower cost of capital.

of asset-backed securities) shut down. Quite naturally, because the crisis originated in the mortgage market, when the strains first hit in the Summer of 2007, the primary and secondary markets for subprime mortgage-backed securities became illiquid at the very time highly leveraged investors such as hedge funds needed to trade out of losing positions. The situation was exacerbated because, without trading, there were no market prices to serve as benchmarks and no way to determine the value of the various risk tranches. Dealers in over-the-counter (OTC) markets, facing a crunch on the funding side of their balance sheets and holding an excessive amount of illiquid assets on the other, simply withdrew from the OTC markets they had contributed to create. Without dealers, trading broke down, especially in difficult-to-price securities such as CDOs, credit derivatives, and municipal bonds. By early 2008, many securities dealers and other institutions that had relied heavily on short-term financing through repurchase agreements and commercial paper were facing stringent borrowing conditions. As an example of this generalized phenomenon of market shut-down, Figure 1 presents weekly time series of origination values for newly issued commercial paper, distinguishing between all types of paper (i.e., also including unsecured paper), Aa (i.e., highly-rated) financial, and Aa asset-backed paper. The break in the upward trend in correspondence to the late Summer of 2007 is obvious.

Since the beginning of the crisis, it became evident that market collapses could spread in a contagious fashion. Hedge funds played a critical role in the cross-border spread of the rupture. When the prices of the high-risk tranches plummeted and investors could not trade out of their losing positions, then other assets— especially those with large unrealized gains, such as emerging market equities—were sold to meet margin calls or to offset losses. Equity markets fell worldwide, and most emerging market currencies similarly fell in value. The OTC market's lack of transparency aggravated the problem because investors, suddenly risk averse, did not know who was—and was not—exposed to the subprime risk.

During the first quarter of 2008, reports of increasing losses and write-downs at major financial institutions in many countries intensified concerns and resulted in a further reduction of liquidity in the interbank and money markets. Banks recognized that the difficulties in the markets for mortgages, syndicated loans, and commercial paper could lead to unanticipated funding needs. As a result, they became much less willing to provide funding to others, including other banks, especially for terms of more than a few days. Over the Summer of 2008, a weakening U.S. economy and continued financial turbulence led to a broad loss of confidence in the U.S. financial sector. Credit default swap spreads for major banks rose, several large institutions announced sharp declines in earnings, and anecdotal reports suggested that the ability of most financial firms to raise new capital was limited. In September 2008, the government-sponsored U.S. enterprises Fannie Mae and Freddie Mac were placed into conservatorship by their regulator, Lehman Brothers filed for bankruptcy, while the insurance company American International Group was rescued by massive government interventions. As a result of the Lehman Brothers bankruptcy, a number of prominent money market mutual fund suffered losses which prompted investors to withdraw large amounts; these funds responded by reducing their purchases of short-term assets, including commercial paper and by shortening the maturity of those instruments that they purchased, leading to a deterioration in the paper market.[4] Figure 1 shows an additional steep decline

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[4]Outside the U.S., although banks continued to report losses during 2008, global financial markets were relatively calm until

in origination activity in correspondence to September and lasting until early December 2008. Additionally, credit risk spreads—particularly for structured credit products—widened dramatically. For instance, the spreads of term federal funds rates and term U.S. dollar Libor over rates on comparable-maturity overnight index swaps increased significantly. Increased spreads over a credit risk-free reference are interesting because they allow the interpretation of the spreads as risk premia required to compensate the risk of the counterparty in a term deposit transaction, which is typically a idiosyncratic (i.e., transaction-specific) risk.

After the September 2008 resolution of another failing U.S. financial institution, Washington Mutual, imposed significant losses on debtholders, investors marked down their expectations regarding likely government support, which further inhibited some banks from obtain funding (e.g., Wachovia Corp., subsequently acquired by Wells Fargo). Against this backdrop, investors pulled back from risk-taking even further, funding markets for terms beyond overnight largely ceased to function, and a wide variety of financial firms experienced difficulties in raising capital. Spreads on mortgage-backed securities (MBS) and consumer asset-backed securities (ABS) also widened dramatically. Figure 2 visualizes the behavior of spreads over Treasury securities with matching duration for both a general (including heterogeneous ratings) and Aaa portfolios of MBS and ABS (securitized credit card debt pools). Starting in late 2007, all spreads increase from a historical average between 300 and 500 basis points (b.p.) to reach levels in excess of a whopping 1,000 b.p. Additionally, and this is especially obvious for MBS, starting in mid-2008 the spreads of the overall portfolio climbs well above the Aaa portfolio's, with spreads as high as 1,700 b.p. per year. These spreads reflect not only credit risk concerns on the solvency of the originators of the asset-backed securities, but presumably also liquidity concerns related to the market break-downs discussed above (see, e.g., Nguyen and Puri, 2008). Figure 3, panel A, documents a similar dynamics for both fixed- and adjustable-rate MBS (i.e., securitized pools of fixed vs. adjustable rate mortgages), comparing once more Aaa with general portfolios that also include lower-rated MBS. Although the behavior over time is rather homogeneous and similar to Figure 2, fixed-rate MBS have recently shown the steepest increases in the implied spreads over safer Treasury notes. Figure 3, panel B, concerns instead spreads for commercial paper and distinguishes between asset-backed and unsecured (non-financial) paper. The dynamics is similar to the one documented for MBS, ABS, and other money market rates, although it is remarkable that spreads have surged from less than 100 b.p. for the 2000-2006 period to more than 300 (500 for lower rated nonfinancial paper) after Lehman's crack.

In the stock market, prices tumbled and volatility soared to record levels during the Fall of 2008 as investors grew more concerned about the prospects of financial firms and about the likelihood of a deep and prolonged recession. Equity-price declines were widespread across sectors and were accompanied by substantial net outflows from equity mutual funds. During this period, the premium that investors demanded for holding equity shares—gauged by the gap between the earnings-price ratio and the yield on Treasury securities—shot

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July-August 2008. This situation changed in September, as global interbank markets seized up and lending came to a standstill. In late September, Bradford and Bingley, Fortis, and Dexia were partially or fully nationalized, and Hypo Real Estate received a large capital injection from the German government. The deepening of the crisis led many governments to announce unprecedented measures to restore credit market functioning, including large-scale capital injections into banks, expansions of deposit insurance programs, and bank debt guarantees.

up. The dynamics in the spreads of asset-backed securities and commercial paper illustrated in Figures 2 and 3 also translated in very poor performance of non-agency-sponsored subprime MBS and ABS. As an example of the typical returns generated by the recent financial crises, Figure 4 shows total return indices for MBS and ABS, distinguishing as in Figure 2 between a portfolio collecting all ratings and Aaa securities only. In the case of MBS, the substantial losses between September 2007 and December 2008 are obvious, with a combined drop in excess of 30% for Aaa securities and only slightly inferior for the overall portfolio. The loss is also visible but quantitatively inferior for ABS (approximately 15%) and concentrated during the last 6 months of 2008. Similar patterns have affected the corporate bond market. After September 2008, CDS spreads on corporate debt surged, and the yields on both investment grade and high-yield bonds rose dramatically relative to comparable-maturity Treasury yields. Figure 5 plots the classical Moody's Aaa-Baa default spread series as well as the spreads of Aaa and Baa Moody's portfolios over the yield on 10-year Treasury notes.

In summary, the recent experience of the 2007-2008 financial crisis reveals a number of stylized facts that models of financial market dynamics ought to explain:

1. The supply (origination) of newly issued securities in many primary markets (e.g., asset- and mortgage-backed securities) has completely evaporated, and in many other market it has been severely impaired.

2. At the same time, liquidity—as measured by average bid-ask spreads and by the presence of a supply of immediacy in trade execution—has disappeared in a number of secondary markets, including highly rated structured products based on subprime mortgages and other leveraged loan pools.

3. Prices have substantially dropped causing substantial and unprecedented negative returns in a wide range of fixed income markets. After September 2008 such steep declines have spread to world equity and corporate bond markets.

4. Correspondingly, yield spreads over duration-matching safe (government) securities on many categories of asset- and mortgage-backed securities and on corporate bonds have approached all-time high levels.

3. **Ambiguity and Its Implications: A Short Review**

Recent advances in decision theory have focused on models of rational decisions that are based on the intuition that individuals may attach value to a distinction between *risk* and *ambiguity* (also called Knightian uncertainty after the work by Knight, 1921). This distinction is best illustrated by the famous Ellsberg Paradox (first described in Ellsberg, 1961). Ellsberg's paradox provides a comparison of different attitudes of the same agent when facing alternative sources of uncertainty. Consider the following situation: there are two different urns with 100 colored balls each. In urn number one there are exactly 50 red balls and 50 black balls, in urn number two the proportion of black and red balls is unknown. When facing the four gambles:

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\begin{aligned}
f &= \{(\$100, B_1), (\$0, R_1)\} & g &= \{(\$100, B_2), (\$0, R_2)\} \\
m &= \{(\$0, B_1), (\$100, R_1)\} & n &= \{(\$0, B_2), (\$100, R_2)\}
\end{aligned}
$$

(where (x, A_j) indicates a gamble that pays out x dollars if a ball of color A is drawn from urn j) people generally prefer f over g and m over n. This preference implies the subjective probability belief that

$$P(B_1) > P(B_2) \quad P(R_1) > P(R_2),$$

where $P(E)$ is the probability of event E. If the experiment subjects were "sophisticated enough", then $P(B_1) + P(R_1) = P(B_2) + P(R_2) = 1$ must hold. However, this requirement cannot be satisfied by the beliefs implied by the experiment. Therefore, the experimental outcome cannot be reconciled with any assignment of subjective probabilities. Ellsberg (1961) and a vast literature after him have pointed out that the fact that investors may be averse not only to risk but also to the "uncertainty" (also called ambiguity) concerning the composition of the second urn may justify the observed choices.

In order to derive a formal definition of ambiguity and to define preferences over (dislike for) it, assume that a decision maker places bets that depend on the result of *two* coin flips, the first is a flip of a coin that she is familiar with, the second of a coin provided by somebody else. Given that she is not familiar with the second coin, it is possible that she would consider ambiguous all the bets whose payoff depends on the result of the second flip. For instance, a bet—call it f—that pays $1 if the second coin lands with heads up, or equivalently if the event $\{HH, TH\}$ obtains, represents an ambiguous gamble. If the decision maker is AA, she may see such bets as somewhat less desirable than bets that are "unambiguous," i.e., that only depend on the result of the first flip. For instance, a bet—call it g—that pays $1 if the first coin lands with heads up, or equivalently if the event $\{HH, HT\}$ obtains is unambiguous. However, suppose that we give the decision maker the possibility of buying "shares" in each bet. Then, if she is offered a bet that pays $0.50 on $\{HH\}$ and $0.50 on $\{HT\}$, she may prefer it to either of the two separate ambiguous bets that rely on the composition of the second urn. In fact, such a mixture bet has the same contingent payoffs as a bet which pays $0.50 if the first coin lands with heads up, which is unambiguous. That is, a decision maker who is averse to ambiguity may prefer the equal-probability mixture of two ambiguous bets to either of the bets. In contrast, a decision maker who is attracted to ambiguity may prefer to choose one of the ambiguous bets. Formally, assuming that the decision maker is indifferent between two ambiguous bets f and h, Schmeidler (1989) defined ambiguity aversion in terms of preference for any mixture $\alpha f + (1 - \alpha)g, \alpha \in (0, 1)$ to each of the individual bets. That is, a decision maker is AA if $\alpha f + (1 - \alpha)g \succeq f \sim g$, where "$\succeq$" is a standard preference relation and "$\sim$" indicates indifference (see MasColell et al., 1995, pp. 42-43).

To develop a more operational definition to be called into play in Sections 4 and 5, it is useful to review how decisions under ambiguity have come to be modeled by researchers in economics. The benchmark framework of choice is represented by Von Neumman and Morgenstern (1947) expected utility (EU) result, by which a rational agent will choose among alternative, uncertain prospects (acts) by maximizing the expectation of a standard, cardinal utility index function. Savage (1954) has generalized classical EU theory providing a Bayesian approach to subjective uncertainty about the outcomes deriving from acts in which individuals' subjective distributions of the resulting payoffs are derived from their preferences over stochastic payoff streams and similarly informed investors may disagree about predicted distributions. In this framework, the stochastic

payoff implied by the act f is preferred over the act g, $f \succeq g$, if and only if

$$E_p[U(f)] \equiv \sum_{j=1}^{n} p_j U(f_j) \geq E_p[U(g)] \equiv \sum_{j=1}^{n} p_j U(g_j)$$

where the p_js $(j = 1, ..., n)$ represent the subjective probabilities associated to each of the possible n discrete states of the world and $U(\cdot)$ is a Von Neumann-Morgenstern (VNM) cardinal utility function. Importantly, according to Savage's rationality (in particular, the independence or sure-thing axiom), preferences do not depend on the source of the risk. The Savage independence axiom implies that one can simply collapse the probability weighting across possible models (uncertainty) to the probabilities for payoffs (risk) to represent behavior with a single probability measure over states.

To accommodate for early evidence of behavior by experimental subjects that appeared to be inconsistent with the subjective EU (SEU) paradigm, Quiggin (1982) generalized the SEU framework by relaxing the VNM axioms and, in particular, the independence axiom. Quiggin assumed the existence of a strictly increasing and continuous probability weighting function $\nu(\cdot)$ (also called *capacity*) which reflects the "sensitivity" of people towards probability, i.e., how people react to the very size of their subjective probability assessments of alternative events. Under Quiggin's rank dependent utility (RDU), the act x with state-dependent payoffs $x_1 \geq x_2 \geq ... \geq x_n$, is evaluated according to the functional

$$\sum_{j=1}^{n} \left[\nu(p_1 + p_2 + ... + p_j) - \nu(p_1 + p_2 + ... + p_{j-1}) \right] U(x_j),$$

which implies that a gamble f is preferred to g ($f \succsim g$) if and only if

$$\sum_{j=1}^{n} \left[\nu(p_1 + ... + p_j) - \nu(p_1 + ... + p_{j-1}) \right] U(f_j) \geq \sum_{j=1}^{n} \left[\nu(p_1 + ... + p_j) - \nu(p_1 + ... + p_{j-1}) \right] U(g_j).$$

It can be shown that as long as the capacity function $\nu(\cdot)$ is convex, the above preference functional is consistent with rational choice and solves many of the experimental puzzles that had created early discomfort with the SEU choice framework, such as Allais' (1953) paradox.[5]

The point of Ellsberg's paradox is the idea that if a subject has too little information to form a prior, she will consider a set of priors as possible and that—being AA—she will take into account the minimal expected utility (over all priors in the set) when evaluating a generic gamble. Schmeidler was the first researcher to formalize and axiomatize this elementary intuition. Schmeidler (1989) started from the observation that the probability attached to an uncertain event may not reflect the full heuristic amount of information that led to the assignment of that probability. For example, when there are only two possible events H and L, they are usually—i.e., in the absence of more precise criteria of assessment (by Laplace's principle of indifference)– assigned probability 1/2 each, independently of whether the available information is meager or abundant. Motivated by this consideration, Schmeidler suggested using a capacity-weighed approach to model rational

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[5]Probabilities are special cases of capacity functions that satisfy additivity: $v(E \cup F) = v(E) + v(F)$ for two disjoint events E and F. General capacities need not satisfy additivity. Under RDU preferences, a convex capacity function is sufficient for pessimistic behavior to ensue.

choice under uncertainty by assigning non-additive probabilities (meaning that they not add up to 1), or capacities, in order to allow for recording of information that additive probabilities cannot represent. In this sense the quantity such as $1 - \nu(H) - \nu(L)$ would encode the amount of non-quantifiable uncertainty. Formally, Schmeidler (1989) proposed to evaluate a simple gamble that yields payoffs H or L according to a Choquet integral for nonadditive probabilities, which in this simple example reduces to:

$$\min_{\mu \in C(v)} [\mu \times H + (1 - \mu) \times L] \qquad C(v) = \{\mu \in [0, 1], \quad \mu \geq v(H), \; 1 - \mu \geq v(L)\}, \tag{1}$$

where v is a capacity and $C(v)$ is called the core of v.[6] Here $C(v)$ should be interpreted as the set of effective priors considered by the agents, and ambiguity is reflected by its multi-valued nature. Decision makers express ambiguity aversion by assigning higher probabilities to unfavorable states, as reflected by the min operator. Clearly, these preferences are of the multiple-prior type, in the sense that a rational decision maker in practice evaluates expected utility under many alternative sets of beliefs, focussing on the set that delivers the lowest possible expected utility.[7] Gilboa and Schmeidler (1989) proved that (1) maps into a functional representation of preferences for which a gamble f is preferred to g ($f \succeq g$) if and only if

$$\min_{P \in \wp} E_P[U(f)] \geq \min_{P \in \wp} E_P[U(g)],$$

where $E_p[U(\cdot)]$ is a standard SEU operator when the probability measure is $P \in \wp$, and the size of the set $\wp$ can be interpreted as representing the "amount" of perceived ambiguity in the decision problem.[8]

Gilboa and Schmeidler's max-min preferences leave open the problem of the specification of the set $\wp$ collecting the alternative, multiple priors held by the agents. Anderson, Hansen and Sargent (2003) and Hansen and Sargent (2001) noted that multiple-prior criteria for decision-making also appear in the robust control theory used in engineering. In particular, robust control theory specifies $\wp$ by taking a single "approximating model", that is, an approximating probability distribution, and statistically perturbing it. Often in this literature, $\wp$ is parametrized implicitly through some parameter θ such that the higher is θ, the less importance is given to alternative models deviating from the baseline approximating probability distribution. Hansen and Sargent (2001) have recognized that the uncertainty that characterizes a preference relation that reflects a concern for robustness may derive from ambiguity and, more specifically, from the poor quality of the information used to select a model. Therefore, θ can be thought of as an AA index since it measures the fear for model misspecification. Formally, Hansen and Sargent (2001) derived robustness-sensitive preferences that imply that a gamble f is preferred to g ($f \succsim g$) if and only if

$$\min_{Q \in \Delta} E_Q[U(f) + \theta R(Q||P)] \geq \min_{Q \in \Delta} E_Q[U(g) + \theta R(Q||P)],$$

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[6]The core of a capacity v consists of all finitely additive probability measures that majorize v pointwise (i.e., event-wise).

[7]AA in the sense defined by Schmeidler (1989) results when v is convex. Schmeidler (1989) and Gilboa and Schmeidler (1989) prove that evaluating uncertain gambles in this fashion represents a sound decision model for unknown probabilities, called Choquet expected utility (CEU). Intuitively, this novel non-additive expected utility theory coincides with the max-min decision rule, where the set of possible priors is the core of v. Quiggin's RDU is a special case of CEU.

[8]In the Ellsberg framework this model implies that the individual acts as if she has a set of priors for the ambiguous urn which includes a prior in which the probability of red is less than 0.5 and a prior in which the probability of black is less than 0.5. Since she acts as if she evaluates each act according to its minimum expected utility she will never chose the ambiguous urn as in her pessimistic view it will be unlikely to pay out.

where P is the approximating, baseline probability distribution and $R(Q||P)$ is the Kullback–Leibler divergence measure between Q and P. Hansen and Sargent's robustness-driven preferences have been fully "axiomatized"—i.e., shown to be consistent with rational choice under uncertainty—only later on by Maccheroni, Marinacci, and Rustichini (2006) as a specific sub-class of what they have labeled *variational preferences*. Variational preferences (VPs) are simply characterized by a utility function $U(\cdot)$ and a convex function c defined on the standard simplex that is called ambiguity index, since different values of c determine different ambiguity levels. VPs imply that a gamble f is preferred to g ($f \succeq g$) if and only if:

$$\min_{P \in \Delta} E_P[U(f)] + c(P) \geq \min_{P \in \Delta} E_P[U(g)] + c(P).$$

In words, agents consider all possible probabilistic models in Δ, giving weight $c(P)$ to each of them. Maccheroni, Marinacci, and Rustichini (2006) also show that VPs are consistent with a set of axioms that characterize rational choices under uncertainty. Moreover, VPs have the advantage of nesting many of the previously known preferences structures that (may) imply AA. For instance, setting

$$c(P) = \begin{cases} 1 & \text{if } P \in \wp \\ 0 & \text{if } P \notin \wp \end{cases}$$

with $\wp \subset \Delta$, one obtains the multiple priors preferences of Gilboa and Schmeidler (1989). Instead, noting that $R(P||Q)$ is a convex function and setting $c(P) = R(P||Q)$ one may derive Hansen and Sargent's (2001) robustness representation.

4. Ambiguity and Market Break-Downs: A Simple No-Trade Example

To provide in the simplest possible terms the intuition for how and when ambiguity aversion may cause a market to breakdown, in this Section we study a simple partial equilibrium, one-period security market economy in which two agents can trade two different securities. The asset menu is composed of a set of risky assets and a completely riskless money market account. The money market account can be thought of as an inflation-protected saving account (in which case its real return can be set to be constant and zero). The risky assets are claims to streams of profits and are heterogeneous because they are written on the output of different firms. Ex-ante, the risky securities are completely homogeneous because different firms cannot be told apart so that all stocks trade at a price q and effectively, one can speak of a single risky security.[9] The structure of the asset market is exogenously given. The two types of securities are all in zero net supply (i.e., they are endogenous assets, similar to corporate bonds or derivatives, see Mukerji and Tallon, 2001). Section 5 relaxes this assumption and considers securities in net exogenous supply. We define r^f as the real, riskless interest rate. The money market account has initial prices of 1, so that its final payoff is $1 + r^f$.

There are two investors, both averse to ambiguity, indexed by $m = 1, 2$. At time 0, each of the two investors decides the optimal composition of a self-financing portfolio, i.e., the number of shares of the risky

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[9]As an alternative, one can think of different securities as derivatives issued on the different pools of cash flows. The structure of the pools is so complex, that ex-ante is impossible to tell different pools apart. In this case, the relevant analogy is represented by asset-backed securities written on pools of loans of heterogeneous credit quality.

asset, z_m, and of the riskless bond, b_m, to be held between times 0 and 1. This is done in view of maximizing some monotone increasing (possibly concave) utility index that depends on the investor's final wealth. For concreteness and to better highlight the role played by ambiguity, we focus in this example on a risk-neutral investor that simply maximizes expected final wealth (see Epstein and Schneider, 2008, for a similar choice). Section 5 considers a more standard risk-aversion assumption. We denote by $w_m \equiv (z_m, b_m)'$ the vector of portfolio shares held by agent m.

The stochastic environment is defined by the state space Ω. For simplicity, we consider a simple, discrete state space. Uncertainty is resolved at time 1 when the *aggregate* state of the economy can be either of high output (y^H, i.e., a state of expansion) or low output (y^L, a recession), $\Omega = \{H, L\}$. Aggregate output is high with probability $\pi > 0$ and low with probability $1 - \pi > 0$, respectively. However, the payoff of the risky asset is not only influenced by the overall state of the economy—which defines systematic, aggregate *risk*—but also by circumstances peculiar to each risky asset, i.e., *idiosyncratic risk*. In practice, this means that while the realization of the systematic state $\sigma \in \Omega$ influences the pricing of all existing securities, the risky asset is also characterized by an asset-specific state $\kappa \in \{-1, 1\}$ which represents the heterogeneous realization of cash flows within each firm. While good and bad projects cannot be told apart before buying a security, and all projects are ex-ante identical and indistinguishable, after a risky asset has been purchased, its payoff will also be influenced by the realization of the idiosyncratic component. Such a component may be either good (outcome $+1$) and add to the systematic payoff with probability $p > 0$ or bad (outcome -1) and subtract from the systematic payoff with probability $1 - p > 0$. Overall, the stochastic environment is described by the product state space $\Omega' = \Omega \times \{-1, 1\}$. For concreteness, in the following we use the following parameters:

$$y^H = 3, \; y^L = 2, \; y(1) = 1, \; y(-1) = -1.$$

We later generalize our conclusions to the case in which y^H, y^L, $y(1)$, and $y(-1)$ can take any real values. The example is completed by the presence of a government, a central planner that performs two functions. First, using un-modelled economic policy tools, the government affects the inflation rate, i. For simplicity, the inflation rate can only be high or low, $i_{high} > i_{low}$. The policy makers affect the probability $\rho > 0$ of a high inflation rate. Second, the policy maker may ex-ante change the features of the environment—e.g., the state-space—in order to favor "better" overall outcomes, for instance to favor trade in securities when otherwise there would be no trade.

The possible realizations of the risky asset's payoff, expressed in nominal terms, are as follows:

$\omega \in \Omega'$	Probability	$y^\sigma + y(\kappa)$ $\;\; \sigma \in \Omega, \; \kappa \in \{-1, \, 1\}$
$H, 1$	πp	4
$H, -1$	$\pi(1 - p)$	2
$L, 1$	$(1 - \pi)p$	3
$L, -1$	$(1 - \pi)(1 - p)$	1

Figure 6

Necessary conditions for clearing all markets at an equilibrium are:

$$b_1 + b_2 = 0 \quad z_1 + z_2 = 0.$$

Assuming that the portfolio $\mathbf{w}_m$ is self-financing, the budget constraint for agent m is $b_m + z_m q = 0$, $m = 1, 2$.

4.1. Standard (subjective) expected utility analysis

Assume that all the relevant probability distributions, $(\pi, 1 - \pi)$ and $(p, 1 - p)$, on the state space Ω' are known in advance. Denote by $E_\rho[\cdot]$ the expected value operator under the probability distribution $(\rho, 1 - \rho)$. Without loss of generality, let's label as 1 the agent interested in buying the risky asset and with 2 the agent who is considering selling the risky asset. Because the risky asset is in zero net supply, these assignments are sensible. Letting $i \in \{i_{low}, i_{high}\}$, the payoff matrices (expressed in real terms) for the buyer and the seller of the risky asset are as follows. For agent 1, the prospective buyer of the asset (i.e., such that $z_1 > 0$), we have:

State at time 1	Cost of strategy at time 0	Real payoff at time 1
$H, 1$	$-q z_1 - b_1$	$E_\rho\left[\frac{4z_1}{1+i} + (1+r^f)b_1\right]$
$H, -1$	$-q z_1 - b_1$	$E_\rho\left[\frac{2z_1}{1+i} + (1+r^f)b_1\right]$
$L, 1$	$-q z_1 - b_1$	$E_\rho\left[\frac{3z_1}{1+i} + (1+r^f)b_1\right]$
$L, -1$	$-q z_1 - b_1$	$E_\rho\left[\frac{z_1}{1+i} + (1+r^f)b_1\right]$

Figure 7

The highest payoff the buyer can get is $E_\rho\left[\frac{4z_1}{1+i} + (1+r^f)b_1\right]$, in case state $(H, 1)$ realizes. Therefore, the expected payoff deriving from the investment will be guaranteed to be negative if:

$$E_\rho\left[\frac{4z_1}{1+i} + (1+r^f)b_1\right] < 0. \tag{2}$$

Using the self-financing condition we can express b_1 as a function of z_1 and q, that is, $b_1 = -z_1 q$. Plugging this into (2), we get:

$$4z_1 E_\rho\left[\frac{1}{1+i}\right] < (1+r^f)z_1 q,$$

where $E_\rho\left[\frac{1}{1+i}\right] = \rho\frac{1}{1+i_{high}} + (1-\rho)\frac{1}{1+i_{low}}$. Because $(1+r^f)z_1 > 0$, we can divide both sides of the inequality to get:

$$\frac{4}{(1+r^f)}E_\rho\left[\frac{1}{1+i}\right] < q.$$

Hence, we get that the expected payoff deriving from the investment will be negative if:

$$q > \mathrm{q}_{buy}^{SEU} \equiv \frac{4}{(1+r^f)}E_\rho\left[\frac{1}{1+i}\right].$$

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For agent 2, the prospective seller of the asset (i.e., such that $z_2 < 0$), the payoff matrix is:

State at time 1	Cost of strategy at time 0	Real payoff at time 1
$H, 1$	$-qz_2 - b_2$	$E_\rho\left[\dfrac{4z_2}{1+i} + (1+r^f)b_2\right]$
$H, -1$	$-qz_2 - b_2$	$E_\rho\left[\dfrac{2z_2}{1+i} + (1+r^f)b_2\right]$
$L, 1$	$-qz_2 - b_2$	$E_\rho\left[\dfrac{3z_2}{1+i} + (1+r^f)b_2\right]$
$L, -1$	$-qz_2 - b_2$	$E_\rho\left[\dfrac{z_2}{1+i} + (1+r^f)b_2\right]$

Figure 8

The highest payoff the seller can get is $E_\rho\left[\frac{z_2}{1+i} + (1+r^f)b_2\right]$, in case state $(L, -1)$ realizes. Therefore, the expected payoff deriving from the investment will be negative if:

$$E_\rho\left[\frac{z_2}{1+i} + (1+r^f)b_2\right] < 0. \tag{3}$$

Once more, to derive an expression for the risky asset price below which $z_2 < 0$ is guaranteed, we use the self-financing condition to express b_2 as a function of z_2 and q, $b_2 = -z_2 q$. Plugging this into (2), we obtain:

$$z_2 E_\rho\left[\frac{1}{1+i}\right] < (1+r^f)z_2 q.$$

Because $-(1+r^f)z_2 > 0$, we obtain that the expected payoff deriving from the investment will be negative if:

$$q < \bar{q}_{\text{sell}}^{SEU} \equiv E_\rho\left[\frac{1}{1+i}\right]\frac{1}{(1+r^f)}.$$

Finally, notice that,

$$\underline{q}_{\text{buy}}^{SEU} \equiv \frac{4}{(1+r^f)}E_\rho\left[\frac{1}{1+i}\right] = \frac{3}{(1+r^f)}E_\rho\left[\frac{1}{1+i}\right] + \bar{q}_{\text{sell}}^{SEU}$$

or

$$\underline{q}_{\text{buy}}^{SEU} - \bar{q}_{\text{sell}}^{SEU} = \frac{3}{(1+r^f)}E_\rho\left[\frac{1}{1+i}\right] > 0$$

which implies that $\underline{q}_{\text{buy}} > \bar{q}_{\text{sell}}$. This ranking of the two reservation prices is important because it proves that a risky price q can be found such that $\underline{q}_{\text{buy}} \geq q \geq \bar{q}_{\text{sell}}$ and trading may *always* occur in the SEU case. What the exact price q at which the market clears will be, depends on the preferences of the two individuals as well as on the jointly determined nominal bond yields and money market account real yields that clear the remaining two markets. However, what matters is that in a standard SEU framework—when all (subjective) probabilities are known in advance or, equivalently, there is a unique prior on possible states of the world—in general prices will exist such that there is trading and risky asset markets will clear. Of course, such prices may become at times very high or low; moreover, they may erratically jump, as new information arrives and/or preferences change. In any event, the security market will always express such a price as a result of buying and selling activities by traders and liquidity (the readiness to buy and sell at different prices) and trading volumes will be non-zero.

4.2. *No trading under ambiguity aversion*

Next, we assume that agents have no information on the probability distribution for idiosyncratic risk, $(1-p, p)$. Following Gilboa and Schmeidler's (1989) axiomatic foundation of AA, we model AA investors as choosing a portfolio to maximize their minimum expected utility over Ω'. This means that there are multiple priors and that each (type of) agent focusses on the states of the world that are most unfavorable. This is the sense in which an agent's subjective knowledge about the likelihood of the possible events is consistent with more than one probability distribution: what the agent knows does not inform him of a precise (second-order) probability distribution over the set of possible (first-order) probabilities. Therefore, for the agent who buys the risky asset, the relevant prior is the one that assigns zero probability ($p = 0$) to the good idiosyncratic risk state $\kappa = 1$, while the opposite is true for the seller ($p = 1$), where certainty is assigned to the good idiosyncratic risk state $\kappa = 1$. In such an ambiguous situation, the payoff matrices considered by the buyer and seller of the risky asset are as follows. For the prospective buyer of the risky asset ($z_1 > 0$):

State at time 1	Cost of strategy at time 0	Real payoff at time 1
$H, -1$	$-qz_1 - b_1$	$E_\rho \left[\frac{2z_1}{1+i} + (1+r^f)b_1 \right]$
$L, -1$	$-qz_1 - b_1$	$E_\rho \left[\frac{z_1}{1+i} + (1+r^f)b_1 \right]$

Figure 9

Following steps identical to Section 4.1, the expected payoff of the portfolio will be negative if

$$q > \underline{q}_{\text{buy}}^{AA} \equiv \frac{2}{(1+r^f)} E_\rho \left[\frac{1}{1+i} \right].$$

In this case the agent will not buy the risky asset as it will be perceived as being too expensive, given an AA assessment of its payoffs. As for the prospective seller of the risky asset ($z_2 < 0$), the matrix of payoffs is:

State at time 1	Cost of strategy at time 0	Real payoff at time 1
$H, 1$	$-qz_2 - b_2$	$E_\rho \left[\frac{4z_2}{1+i} + (1+r^f)b_2 \right]$
$L, 1$	$-qz_2 - b_2$	$E_\rho \left[\frac{3z_2}{1+i} + (1+r^f)b_2 \right]$

Figure 10

The expected payoff of the portfolio will be certainly negative if

$$q < \bar{q}_{\text{sell}}^{AA} \equiv \frac{3}{(1+r^f)} E_\rho \left[\frac{1}{1+i} \right].$$

In this case the agent will refrain from selling (shorting) the risky asset because it is considered too expensive. Given the clearing conditions of the market, it is easy to show that

$$\bar{q}_{\text{sell}}^{AA} \equiv \frac{3}{(1+r^f)} E_\rho \left[\frac{1}{1+i} \right] = \frac{1}{(1+r^f)} E_\rho \left[\frac{1}{1+i} \right] + \underline{q}_{\text{buy}}^{AA} > \underline{q}_{\text{buy}}^{AA}.$$

However, since the prospective buyer buys risky securities only for $q \leq \underline{q}_{\text{buy}}^{AA} < \bar{q}_{\text{sell}}^{AA}$ (i.e., for sufficiently low prices), while the prospective seller sells risky securities only for $q \geq \bar{q}_{\text{sell}}^{AA} > \underline{q}_{\text{buy}}^{AA}$ (i.e., for sufficiently high

prices), it is clear that the two conditions (inequalities) cannot be simultaneously satisfied, so that trading breaks down. This means that a range of market prices exist such that the prospective buyer considers the price too high while the prospective seller considers the price simultaneously too low. This is made possible by the fact that the two agents do not reduce their knowledge of the stochastic environment to a single, unique and common prior over states of the world. On the contrary, each investor takes decisions assuming as a benchmark the worst-case scenario, in this case parameterized by the bad idiosyncratic state shock. Therefore, whenever the market price enters the range $[\underline{q}_{\text{buy}}^{AA}, \bar{q}_{\text{sell}}^{AA}]$, the result is that all liquidity evaporates and trading disappears. As a result, the market will clear with $z_1 = z_2 = 0$ (no trading activity) which is equivalent to say that the market breaks down and fails to determine an equilibrium price.

The working paper version of this manuscript generalizes this example and shows that in general trading may collapse when ambiguity concerns both systematic and idiosyncratic risks, under the assumption that the "amount" (to be defined) of ambiguity concerning idiosyncratic risk exceeds the uncertainty on the systematic risk. This result holds independently of the specific values for y^H, y^L, $y(1)$, and $y(-1)$ that one may select, provided that $y(1) - y(-1) > y^H - y^L$.

5. **Parametric Ambiguity, Endogenous Participation and Equilibrium Asset Pricing**

We investigate a simple, one-period security economy in which two *types* of investors may trade in two (types of) securities, a risky asset in positive $(\bar{z})$ net exogenous supply and an endogenous bond (i.e., in zero net supply).[10] The bond is again interpretable as a riskless money market account that yields a return r^f. The bond trades for a current, unit price. Also in this case, the risky assets are *ex-post* heterogeneous, in the sense that they represent rights to the output (profits) of heterogeneous firms. *Ex-ante* however, these stocks are completely homogeneous because different firms cannot be told apart; therefore all stocks trade at a unique price q. The rest of the framework is identical to the example in Section 4. We model two kinds of risk: risk that affects the entire market (*systematic risk*) and risk that just reflects circumstances peculiar to the specific firm represented by a stock (*idiosyncratic risk*). The systematic component of the stock payoff is normally distributed, with mean μ_S and variance σ_S^2. Also the idiosyncratic component is normally distributed, with parameters μ_I and σ_I^2. The policy-maker/central planner performs two functions. First, using un-modelled economic policy tools, it sets the inflation rate at some level i. Second, the policy maker may ex-ante change features of the environment to favor "better" outcomes.

Call d the total payoff on the stock. Because systematic and idiosyncratic risks are independent, we have

$$d \sim N(\mu_I + \mu_S, \ \sigma_I^2 + \sigma_S^2).$$

Denoting by w_m^0 the initial wealth of agent m, each investor's budget constraint is given by

$$w_m^0 = q z_m + b_m,$$

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[10]This model shares a number of features with Easley and O'Hara's (2006), even though the set of questions and implications we are after differ to a large extent from theirs.

and the end-of-period-one expected *real* wealth that derives from the investment is given by:

$$w_m^1 = z_m \frac{d}{1+i} + b_m(1+r^f).$$

Since both components of the asset's payoff are normally distributed, so is w_m^1:

$$w_m^1 \sim N\left(z_m \frac{\mu_I + \mu_S}{1+i} + b_m(1+r^f), \ z_m^2 \frac{\sigma_I^2 + \sigma_S^2}{(1+i)^2}\right).$$

Finally, assume that the utility index of each agent is given by a standard exponential, CARA utility, $U(w_m^1) = -\exp(-w_m^1)$.[11] Under this condition, given the normality assumption, the expected utility of final wealth is a strictly increasing transformation of the kernel (see the working paper version of the manuscript for a proof)

$$z_m\left(\frac{\mu_I + \mu_S}{1+i} - q\right) + b_m r^f + w_m^0 - \frac{1}{2} z_m^2 \frac{\sigma_I^2 + \sigma_S^2}{(1+i)^2}, \tag{4}$$

where the budget constraint has been substituted in. Therefore, since w_m^0 is exogenously given, we obtain the equivalence

$$\arg\max_{z_m, b_m} E\left[-\exp(-w_m^1)\right] = \arg\max_{z_m, b_m}\left[z_m\left(\frac{\mu_I + \mu_S}{1+i} - q\right) + b_m r^f - \frac{1}{2} z_m^2 \frac{\sigma_I^2 + \sigma_S^2}{(1+i)^2}\right].$$

5.1. *Standard (subjective) expected utility analysis*

If agents are SEU maximizers, they consider as plausible a *unique* probability distribution for the idiosyncratic component of the asset's payoff: they consider the unique pair $(\mu_I, \sigma_I^2)'$ as sufficient statistics for the distribution of idiosyncratic risk. The expected utility of the end-of-period-one wealth is a strictly increasing transformation of the kernel (4), which is a concave function of the investment share z_m. Therefore necessary and sufficient condition for optimality is:

$$\left(\frac{\mu_I + \mu_S}{1+i} - q\right) - z_m \frac{\sigma_I^2 + \sigma_S^2}{(1+i)^2} = 0,$$

which gives an optimal risky investment of:

$$z_m^* = \frac{(1+i)\left[\mu_I + \mu_S - q(1+i)\right]}{\sigma_I^2 + \sigma_S^2}. \tag{5}$$

This is the demand for the risky asset expressed as a function of its price and the mean and the variance of its payoffs. A simple inspection of (5) reveals that the sign z_m^* depends on the relation between the price of the asset and its expected payoff (expressed in real terms):

$$z_m^* \begin{cases} > 0 \text{ if } q < \frac{\mu_I + \mu_S}{1+i} \\ = 0 \text{ if } q = \frac{\mu_I + \mu_S}{1+i} \\ < 0 \text{ if } q > \frac{\mu_I + \mu_S}{1+i} \end{cases} \tag{6}$$

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[11]This specification for preferences implies a unit CARA coefficient, but all of our qualitative implications will go through assuming a different risk aversion coefficient, at the cost of more involved algebra. The exact distribution of any initial endowment owned by the investors does not affect their demands for risky assets because of the CARA-Gaussian structure, so we do not specify it.

Note that there is a unique price $q = (\mu_I + \mu_S)/(1 + i)$ that supports the optimal decision not to invest in the risky asset. This means that for $q \neq (\mu_I + \mu_S)/(1 + i)$, investors will generally express a non-zero demand and trading may occur.

5.2. Risky asset demand under ambiguity

Assume now that at least some fraction of the agents know the exact distribution of the systematic component of the asset's payoff, but do not know the exact value of the parameters of the distribution of the idiosyncratic component. AA agents would select a portfolio to maximize expected utility if they knew the correct value of the parameters, but they do not know the parameters, and unlike expected utility maximizers they do not aggregate across parameters with a unique prior. Similarly to Section 4, we assume that AA investors act as if they had a set of distributions on returns; one distribution for each possible value of the unknown parameters. AA investors can be thought of as inexperienced potential investors who do not have enough experience in the market to reliably estimate return distributions.

In particular, AA investors only know that the mean μ_I of the distribution belongs to the set $\{\mu_1, \mu_2, ..., \mu_P\}$, and the variance σ_I^2 to the set $\left\{\sigma_1^2, \sigma_2^2, ..., \sigma_Q^2\right\}$ with $P \geq 2$ and $Q \geq 2$. To make our analysis of the equilibrium interaction between SEU and AA interesting, we assume that AA investors consider as possible mean payoffs above and below μ_I and variances above and below σ_I^2. That is, the true parameter values are convex combinations of the extreme values considered possible by the AA traders. Being AA, investors consider as relevant the prior that is less favorable to themselves, i.e., they select a portfolio to maximize their minimum expected utility over the set of possible return distributions. Therefore, the optimization problem for each agent can be rewritten as:

$$\max_{z_m, b_m} \quad \min_{(\mu_I, \sigma_I^2) \in \{\mu_1, \mu_2, ..., \mu_P\} \times \left\{\sigma_1^2, \sigma_2^2, ..., \sigma_Q^2\right\}} E\left[-\exp(-w_m^1)\right]$$
$$s.t. \quad w_m^0 = q z_m + b_m$$

Using the assumption of normality of returns, note that:

$$\arg\max_{z_m, b_m} \min_{(\mu_I, \sigma_I^2)} E\left[-\exp(-w_m^1)\right] = \arg\max_{z_m, b_m} \min_{(\mu_I, \sigma_I^2)} = \left(\frac{\mu_I + \mu_S}{1 + i} - q\right) z_m + b_m r^f - \frac{1}{2} z_m^2 \frac{\sigma_I^2 + \sigma_S^2}{(1 + i)^2}.$$

Given the definition of $V(w_m^1)$, it easy to prove that

$$z_m > 0 \iff (\mu_{\min}, \sigma_{\max}^2) \in \arg\min_{(\mu_I, \sigma_I^2)} \left(\frac{\mu_I + \mu_S}{1 + i} - q\right) z_m + b_m r^f - \frac{1}{2} z_m^2 \frac{\sigma_I^2 + \sigma_S^2}{(1 + i)^2}$$

$$z_m < 0 \iff (\mu_{\max}, \sigma_{\max}^2) \in \arg\min_{(\mu_I, \sigma_I^2)} \left(\frac{\mu_I + \mu_S}{1 + i} - q\right) z_m + b_m r^f - \frac{1}{2} z_m^2 \frac{\sigma_I^2 + \sigma_S^2}{(1 + i)^2},$$

Interestingly, for any portfolio the minimum occurs at the maximum possible level for the variance of future payoffs. Denote this variance as $\sigma_{\max}^2$. Consequently, what matters to an AA investor is not the "expected", average variance, but rather the largest variance. The fact that μ_I is replaced by $\mu_{\min}$ if the investor intends

to invest in the asset, while μ_I is replaced by $\mu_{\max}$ in case she intends to go short, makes sense because it is clear that an investor short in a security bears the maximum possible loss in case the security payoff is very high as she has to deliver such a payoff to the counterparty in the short position.

At this point, if $z_m > 0$, necessary and sufficient condition for z_m being optimal is:

$$\left(\frac{\mu_{\min} + \mu_S}{1+i} - q\right) - z_m \frac{\sigma_{\max}^2 + \sigma_S^2}{(1+i)^2} = 0$$

from which we get

$$z_m^* = \frac{(1+i)\left[\mu_{\min} + \mu_S - q(1+i)\right]}{\sigma_I^2 + \sigma_S^2}.$$

Note that $z_m^* > 0$ only if $\mu_{\min} + \mu_S - q(1+i) > 0$, that is, only if $(\mu_{\min} + \mu_S)/(1+i) > q$. Viceversa, if $z_m < 0$, necessary and sufficient condition for z_m being optimal is:

$$\left(\frac{\mu_{\max} + \mu_S}{1+i} - q\right) - z_m \frac{\sigma_{\max}^2 + \sigma_S^2}{(1+i)^2} = 0$$

from which we get

$$z_m^* = \frac{(1+i)\left[\mu_{\max} + \mu_S - q(1+i)\right]}{\sigma_I^2 + \sigma_S^2}.$$

Note that $z_m^* < 0$ only if $\mu_{\max} + \mu_S - q(1+i) < 0$, that is, only if $(\mu_{\max} + \mu_S)/(1+i) < q$. Summarizing the previous results, we get the following step-wise demand for the risky asset:

$$z_m^* = \begin{cases} \frac{(1+i)[\mu_{\min} + \mu_S - q(1+i)]}{\sigma_{\max}^2 + \sigma_S^2} > 0 & \text{if } \frac{\mu_{\min} + \mu_S}{1+i} > q \\ 0 & \text{if } \frac{\mu_{\max} + \mu_S}{1+i} \geq q \geq \frac{\mu_{\min} + \mu_S}{1+i} \\ \frac{(1+i)[\mu_{\max} + \mu_S - q(1+i)]}{\sigma_{\max}^2 + \sigma_S^2} < 0 & \text{if } \frac{\mu_{\max} + \mu_S}{1+i} < q \end{cases} \tag{7}$$

Therefore, contrary to the SEU case, there is an interval of prices, namely

$$\left[\frac{\mu_{\min} + \mu_S}{1+i}, \frac{\mu_{\max} + \mu_S}{1+i}\right], \tag{8}$$

for which it is optimal not to trade the risky asset. This occurs because each agent evaluates the possibility of buying or selling the risky asset using different probability measures (namely, distributions characterized by different mean payoffs $\mu_{\min}$ and $\mu_{\max}$, respectively). In particular, $(\mu_{\min} + \mu_S)/(1+i)$ is the highest price at which investors are willing to buy the asset: when q exceeds this bound, there will be no buyers of the risky asset. $(\mu_{\max} + \mu_S)/(1+i)$ is the lowest price at which investors are willing to sell the asset: when q goes below this bound, sellers disappear. Therefore, the effect of AA is directly observable on the lowest offer and the highest bid prices.[12] It is also interesting to stress that (8) is completely independent of $\sigma_{\max}^2$: the perceived variance does not affect the decision to trade (or to go long or short in the asset); however, once such a decision is taken, then $\sigma_{\max}^2$ will influence the size of the position.

Figure 11 illustrates this result: while under SEU the security demand function has a standard downward sloping shape and—because of the assumption of constant absolute risk aversion— it appears to be linear in

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[12]Under SEU, the agent considers only μ_I, therefore the two extremes of the interval coincide, and the switch from buying to selling occurs exactly at the price $(\mu_I + \mu_S)/(1+i)$.

q, under AA the demand function is piece-wise linear and flat at $z_m^* = 0$ over the no-trading interval (8). This is the same result obtained in Section 4: while under SEU trading generically occurs, under AA a relatively wide interval of prices may exist in which a market breaks down.[13]
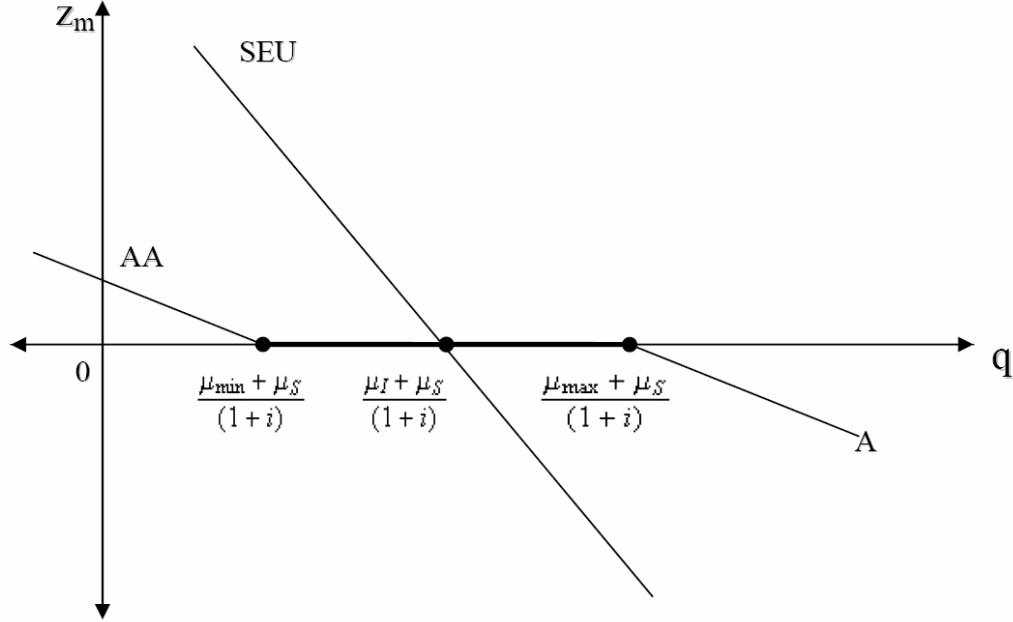

Figure 11

Notice also that because $\sigma_I \leq \sigma_{\max}$, the position in the risky asset held by AA agents is always smaller (in absolute value) than the one held by SEU agents. This is because for any given return parameters these investors evaluate the trade-off between mean and variance equivalently. They both avoid risk and require compensation in expected return in order to hold risk. But an AA investor also avoids ambiguity in the distribution of returns, and so as long as the set of possible means and variances is non-degenerate she further reduces the size of his position in the risky asset. Finally, because $\mu_{\min} \leq \mu_I \leq \mu_{\max}$, when it is optimal for an ambiguity averse agent to buy the risky asset, so it is for a SEU agent. Similarly, when it is optimal for an ambiguity averse agents to sell the risky asset, so it is for a SEU agent. This is obvious from Figure 11.

5.3. *Equilibrium asset prices*

Differently from the example in Section 4, besides showing the existence of a region in the price space for which trading collapses, in this Section we further proceed to "close" the model and compute the equilibrium price for the stock. For additional realism, let us assume that a fraction α of the agents is AA, while the complement to one, $1 - \alpha$, are SEU-maximizers. However, agents in each of the two groups are homogeneous and hold identical beliefs (multiple priors in the AA case). We denote by z_{AA}^* the asset demand by AA agents (so that αz_{AA}^* is the aggregate demand by their group), and by z_{SEU}^* the asset demand of SEU agents (so that

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[13]Interestingly, even outside the no-trading interval, the demand function under AA appears to be less sensitive to changes in q than it is under SEU. This is because under AA, the slope is given by $-(1+i)/(\sigma_I^2 + \sigma_S^2)$ while under SEU the slope is $-(1+i)/(\sigma_{\max}^2 + \sigma_S^2)$ and $\sigma_{\max}^2 > \sigma_I^2$.

$(1 - \alpha) z^*_{SEU}$ is the aggregate demand by their group). The market clearing condition for the stock market is $\bar{z} = \alpha z^*_{AA} + (1 - \alpha) z^*_{SEU}$. At this point, three possible situations can occur. First, it is possible that the market price falls in the region in which no AA agent expresses a non-zero demand for the risky asset,

$$\frac{\mu_{\min} + \mu_S}{1 + i} \leq q \leq \frac{\mu_{\max} + \mu_S}{1 + i}.$$

In this case $z^*_{AA} = 0$ and the entire supply of the stock must be absorbed by SEU agents. Therefore

$$\frac{(1 + i) (\mu_I + \mu_S - q (1 + i))}{\sigma_I^2 + \sigma_S^2} (1 - \alpha) = \bar{z},$$

from which we can derive the equilibrium price q_1^*:

$$q_1^* = \frac{\mu_I + \mu_S}{1 + i} - \frac{\bar{z} \left(\sigma_I^2 + \sigma_S^2 \right)}{(1 - \alpha) (1 + i)^2}.$$

Because $q_1^* \leq (\mu_{\max} + \mu_S)/ (1 + i)$, it follows that only if the additional condition

$$\mu_{\min} \leq \mu_I - \frac{\bar{z} \left(\sigma_I^2 + \sigma_S^2 \right)}{(1 - \alpha) (1 + i)}$$

holds, then the equilibrium price can actually be $q^* = q_1^*$. Furthermore, the market clears since $q_1^* < (\mu_I + \mu_S)/ (1 + i)$, which ensures that $z^*_{SEU} = \bar{z} > 0$. This is a limited participation equilibrium.

Second, it is possible that the current market price may fall in the region where

$$\frac{\mu_{\min} + \mu_S}{1 + i} > q.$$

In this case, both types of agents are willing to participate; as we know from (7) $z^*_{AA} > 0$, while $z^*_{SEU} \geq 0$ if $q \leq (\mu_I + \mu_S)/ (1 + i)$, and $z^*_{SEU} < 0$ otherwise. By the market clearing condition

$$\bar{z} = \alpha \frac{(1 + i) (\mu_{\min} + \mu_S - q (1 + i))}{\sigma_{\max}^2 + \sigma_S^2} + (1 - \alpha) \frac{(1 + i) (\mu_I + \mu_S - q (1 + i))}{\sigma_I^2 + \sigma_S^2},$$

we can derive the equilibrium price q_2^*:

$$q_2^* = \frac{\left[\alpha \mu_{\min} \left(\sigma_I^2 + \sigma_S^2 \right) + \mu_S \left(\alpha \sigma_I^2 + (1 - \alpha) \sigma_{\max}^2 + \sigma_S^2 \right) + (1 - \alpha) \mu_I \left(\sigma_{\max}^2 + \sigma_S^2 \right) \right]}{(1 + i) \left[\sigma_S^2 + \alpha \sigma_I^2 + (1 - \alpha) \sigma_{\max}^2 \right]} + $$
$$- \frac{\bar{z} \left(\sigma_I^2 + \sigma_S^2 \right) \left(\sigma_{\max}^2 + \sigma_S^2 \right)}{(1 + i)^2 \left[\sigma_S^2 + \alpha \sigma_I^2 + (1 - \alpha) \sigma_{\max}^2 \right]}.$$

In fact, by construction, this can be an equilibrium price only if $(\mu_{\min} + \mu_S)/ (1 + i) > q_2^*$, that is, if

$$\mu_{\min} + \mu_S > \frac{\left[\alpha \mu_{\min} \left(\sigma_I^2 + \sigma_S^2 \right) + \mu_S \left(\alpha \sigma_I^2 + (1 - \alpha) \sigma_{\max}^2 + \sigma_S^2 \right) + (1 - \alpha) \mu_I \left(\sigma_{\max}^2 + \sigma_S^2 \right) \right]}{\sigma_S^2 + \alpha \sigma_I^2 + (1 - \alpha) \sigma_{\max}^2} + $$
$$- \frac{\bar{z} \left(\sigma_I^2 + \sigma_S^2 \right) \left(\sigma_{\max}^2 + \sigma_S^2 \right)}{(1 + i) \left[\sigma_S^2 + \alpha \sigma_I^2 + (1 - \alpha) \sigma_{\max}^2 \right]}.$$

Working out the algebra of this inequality, we obtain:

$$\mu_{\min} (1 + i) (1 - \alpha) \left[\sigma_S^2 + \sigma_{\max}^2 \right] > (1 + i) \left[(1 - \alpha) \mu_I \left(\sigma_{\max}^2 + \sigma_S^2 \right) \right] - \bar{z} \left(\sigma_I^2 + \sigma_S^2 \right) \left(\sigma_{\max}^2 + \sigma_S^2 \right).$$

Dividing both sides of this inequality by $(1 + i)(\sigma_S^2 + \sigma_{\max}^2)$, we get that $(\mu_{\min} + \mu_S)/(1 + i) > q_2^*$ only if

$$\mu_{\min} > \mu_I - \frac{\bar{z}\left(\sigma_I^2 + \sigma_S^2\right)}{(1 - \alpha)(1 + i)}. \tag{9}$$

(9) ensures that q_2^* may effectively be an equilibrium price in which $z_{AA}^* > 0$. Furthermore, the sign of z_{SEU}^* depends on whether $q_2^* \leq (\mu_I + \mu_S)/(1 + i)$ or not. This is a participation equilibrium.

Third, the current market price may be such that

$$q > \frac{\mu_{\max} + \mu_S}{1 + i}.$$

In this case $z_{AA}^* < 0$, but then, as noted above, $z_{SEU}^* < 0$ because

$$q > \frac{\mu_{\max} + \mu_S}{1 + i} \geq \frac{\mu_I + \mu_S}{1 + i}.$$

Because both z_{AA}^* and z_{SEU}^* are negative, the stock market cannot clear. So, there is no equilibrium for which $q > (\mu_{\max} + \mu_S)/(1 + i)$. We summarize these results in the following:

Proposition 1 *(Equilibrium Stock Price). If the condition*

$$\mu_{\min} \leq \mu_I - \frac{\bar{z}\left(\sigma_I^2 + \sigma_S^2\right)}{(1 - \alpha)(1 + i)} \tag{10}$$

holds, then only SEU investors participate in the market and the equilibrium price is

$$q_1^* = \frac{\mu_I + \mu_S}{1 + i} - \frac{\bar{z}\left(\sigma_I^2 + \sigma_S^2\right)}{(1 - \alpha)(1 + i)^2}. \tag{11}$$

Viceversa, if

$$\mu_{\min} > \mu_I - \frac{\bar{z}\left(\sigma_I^2 + \sigma_S^2\right)}{(1 - \alpha)(1 + i)}, \tag{12}$$

both groups of investors participate, and the equilibrium price is given by

$$q_2^* = \frac{\left[\alpha\mu_{\min}\left(\sigma_I^2 + \sigma_S^2\right) + \mu_S\left(\alpha\sigma_I^2 + (1 - \alpha)\sigma_{\max}^2 + \sigma_S^2\right) + (1 - \alpha)\mu_I\left(\sigma_{\max}^2 + \sigma_S^2\right)\right]}{(1 + i)\left[\sigma_S^2 + \alpha\sigma_I^2 + (1 - \alpha)\sigma_{\max}^2\right]} +$$
$$- \frac{\bar{z}\left(\sigma_I^2 + \sigma_S^2\right)\left(\sigma_{\max}^2 + \sigma_S^2\right)}{(1 + i)^2\left[\sigma_S^2 + \alpha\sigma_I^2 + (1 - \alpha)\sigma_{\max}^2\right]}. \tag{13}$$

The equilibrium is unique.

The intuition of the result is that when the minimum possible mean payoff perceived by the AA investors is so low to fall below the threshold given by (10)—i.e., the real value of the mean idiosyncratic payoff corrected by a factor that increases in total variance, the supply volume, and the fraction of market that is made up by AA investors and declines with the inflation rate—then only SEU investors will participate. The simple explanation is that the low expected stock payoffs are insufficient to provide adequate compensation to AA investor. In this case, the pricing functional in (11) depends on objective (or, unique prior subjective) parameters, as they are (correctly) perceived by the SEU-maximizers. Interestingly, among the parameters that affect the SEU-only price, q_1^*, we also find α, the proportion of AA investors. Therefore, even when

the AA agents are not trading the stock, their mere existence will affect the equilibrium stock price. On the contrary, if the minimum possible mean payoff perceived by the AA traders is high enough to clear the same threshold, then both types of investors participate and the price has the considerably more complicated structure in (13). In this case, both the single-prior (μ_I, μ_S, σ_I^2, σ_S^2) and the multiple-prior ($\mu_{\min}$, $\sigma_{\max}^2$) parameters enter the expression for q_2^*.

Additionally, notice that while (11) has the typical mean-variance, Gaussian CARA structure in which the stock price increases with the mean expected payoffs and with the SEU-maximizer fraction, while it declines with variance and the supply volume, (13) has a complicated functional form because it allows mean payoff parameters to interact with the "variance parameters", in particular both σ_I^2 and $\sigma_{\max}^2$.[14] Also notice that when the switch from (13) to (11) occurs, in correspondence to

$$\mu_I = \mu_{\min} + \frac{\bar{z}\left(\sigma_I^2 + \sigma_S^2\right)}{(1-\alpha)(1+i)},$$

then

$$
\begin{aligned}
q_1^* &= \frac{\mu_I + \mu_S}{1+i} - \frac{\bar{z}\left(\sigma_I^2 + \sigma_S^2\right)}{(1-\alpha)(1+i)^2} \\
&= \frac{\mu_{\min}}{1+i} + \frac{\bar{z}\left(\sigma_I^2 + \sigma_S^2\right)}{(1-\alpha)(1+i)^2} + \frac{\mu_S}{1+i} - \frac{\bar{z}\left(\sigma_I^2 + \sigma_S^2\right)}{(1-\alpha)(1+i)^2} = \frac{\mu_{\min} + \mu_S}{1+i},
\end{aligned}
$$

while as long as (13) applies, it is (see the working paper version for algebra and additional details)

$$
\begin{aligned}
q_2^* &< \frac{\mu_S\left(\sigma_{\max}^2 + \sigma_S^2\right) + \mu_I\left(\sigma_{\max}^2 + \sigma_S^2\right)}{(1+i)\left[\sigma_S^2 + \sigma_{\max}^2\right]} - \frac{\bar{z}\left(\sigma_I^2 + \sigma_S^2\right)\left(\sigma_{\max}^2 + \sigma_S^2\right)}{(1+i)^2\left[\sigma_S^2 + \sigma_{\max}^2\right]} \\
&= \frac{\mu_S + \mu_I}{1+i} - \frac{\bar{z}\left(\sigma_I^2 + \sigma_S^2\right)}{(1+i)^2} = q_1^*,
\end{aligned}
$$

which means that when transitioning from mixed SEU and AA joint participation to SEU-only equilibria, the price must undergo a positive jump from q_2^* to q_1^*. This is consistent with the fact that when the equilibrium outcome is based on attracting positive demand from AA investors, the price must decline enough to compensate them for absorbing ambiguity, given their ambiguity aversion.

5.4. *Risk premia under ambiguity*

Because in our partial equilibrium set up, we have taken the real riskless interest rate as given and assumed that inflation is determined by policy-makers, we may as well set $r^f = 0$ so that the notions of expected real

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[14]The sign of the derivative of q_1^* with respect to the inflation rate is ambiguous. One can show that the stock price rises with inflation (i.e., it provides a hedge) if

$$\left[2\frac{\bar{z}\left(\sigma_I^2 + \sigma_S^2\right)}{(1-\alpha)(1+i)} - \mu_I - \mu_S\right] > 0,$$

which also depends on the inflation rate itself. Needless to say, the sign of the partial derivative of q_2^* with respect to i is even more complicated to study, but in general whether or not stocks provide a good inflation hedge will depend on the exogenous parameters in rather complicated ways.

risky return and of real risk premium coincide. Therefore the risk premium is

$$E^{(\cdot)}[1 + \tilde{R}] = \frac{E[d]}{(1+i)q^*},$$

where $\tilde{R}$ denotes the net return on the risky asset. At this point, there are two notions of risk premium that can be entertained. One is an objective notion and corresponds to an aggregate, market viewpoint that considers both the true expectations of the risky payoffs at time 1 and the current equilibrium market price. This is also the risk premium expected by an external observer that understands the structure of the model and solves for the equilibrium, as we have done.[15] Naturally, this is the notion of risk premium relevant to an econometrician interested in understanding the nature of the returns data produced by the market. The other is a subjective notion and corresponds to the expectation—obviously different across SEU and AA investors—of the premium that each individual investor will form before (or without) understanding the overall structure of the model and the outcomes generated by the interaction of SEU and AA investors.

Starting with the first, objective notion (that we and denote by setting $(\cdot)$ to $*$), in correspondence to Proposition 1 we have:

Proposition 2 (Equilibrium Risk Premia). *When (10) holds and only SEU investors participate in the market, then the real risk premium is:*

$$E_1^*\left[1 + \tilde{R}\right] = \frac{(1-\alpha)(1+i)(\mu_I + \mu_S)}{(1-\alpha)(1+i)(\mu_I + \mu_S) - \bar{z}\left(\sigma_I^2 + \sigma_S^2\right)}. \tag{14}$$

Viceversa, if (12) holds, both groups of investors participate and the real risk premium is:

$$E_2^*\left[1 + \tilde{R}\right] = \frac{(\mu_I + \mu_S)(1+i)A}{(1+i)\left[\alpha\mu_{\min}B + (1-\alpha)\mu_I C + \mu_S A\right] - \bar{z}BC}, \tag{15}$$

where $A \equiv \alpha\sigma_I^2 + (1-\alpha)\sigma_{\max}^2 + \sigma_S^2$, $B \equiv \sigma_I^2 + \sigma_S^2$, and $C \equiv \sigma_{\max}^2 + \sigma_S^2$.

The proof consists of simple algebraic manipulations given the definition of risk premium and is omitted. The two expressions make intuitive sense. $E_1^*[1 + \tilde{R}]$ increases with the total aggregate, "objective" risk $\left(\sigma_I^2 + \sigma_S^2\right)$, with the total supply to be absorbed $\bar{z}$ (because in equilibrium, a larger proportion of the overall investors' wealth will have to be exposed to the risk represented by $\sigma_I^2 + \sigma_S^2$), and with the proportion α of AA agents, since given the total supply a smaller fraction of SEU-only investors will have to absorb the entire supply and hence will demand a higher premium to compensate the diminished diversification available to them; $E_1^*[1 + \tilde{R}]$ declines with $(\mu_I + \mu_S)$ and with the inflation rate. This last result indicates that in the segmented, SEU-only equilibrium the risky asset must necessarily provide a less-than-perfect hedge, since as the inflation rate rises, the real equilibrium risk premium will decline. Similar results hold with reference to

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$E_2^*[1 + \tilde{R}]$. The intuition (as well as the formal proof) is easily obtained from the expression for the demand of AA and SEU investors in Section 5.2 when there is participation from both groups. First, the algebraic expression for $E_2^*[1 + \tilde{R}]$ shows that as one would anticipate, an increasing exogenous supply of the security will increase the risk premium while an increase in the inflation rate will decrease the risk premium. Finally, when α increases, because $\sigma_I^2 \leq \sigma_{\max}^2$ and $\mu_{\min} < \mu_I$, the numerator of the expression for q_2^*, (13) decreases; viceversa its denominator increases. Therefore, on the whole q_2^* declines so that the risk premium increases: the higher is the percentage of investors who are AA, the higher is the risk premium on the security.

(14) and (15) are derived assuming knowledge of the objective market outcome and—by the law of large numbers—correspond to the average, realized real excess returns on the risky asset if a long sequence of systematic and idiosyncratic shocks were to be drawn. However, these are different from the risk premia which are perceived by both categories of investors. The SEU investors perceive an ex-ante risk premium of $E^{SEU}[1 + \tilde{R}] = E_1^*[1 + \tilde{R}]$. This means that every time a participation equilibrium outcome obtains (i.e., (12) holds), the SEU-maximizers will be surprised by the level of the average realized real excess returns. In particular, SEU agents will systematically find that the realized real excess returns are "too high" in the light of the underlying economic environment as they perceive it. Of course, this is due to the existence of an additional source of uncertainty—ambiguity, indeed—that is priced in equilibrium and of which the SEU investors are not aware, while AA investor are.

The ambiguity averse investors perceive instead a risk premium of:[16]

$$E^{AA}[1 + \tilde{R}] = \frac{(1 - \alpha)(1 + i)(\mu_{\min} + \mu_S)}{(1 - \alpha)(1 + i)(\mu_{\min} + \mu_S) - \bar{z}(\sigma_{\max}^2 + \sigma_S^2)}.$$

It can be shown that $E^{AA}[1 + \tilde{R}] > E_2^*[1 + \tilde{R}] > E_1^*[1 + \tilde{R}] = E^{SEU}[1 + \tilde{R}]$, i.e., AA agents always demand a risk premium which is higher than what SEU agents' expect to receive. This makes sense, since they also need to be compensated for bearing ambiguity. Therefore AA investors are always negatively surprised by the realized real excess returns, even when the investors themselves participate in the market. Mechanically, this is due to the fact that AA participate only when also SEU agents do, and the resulting price will be (loosely speaking) some weighted average of the correct price for AA and SEU investors, with the latter prepared to pay a higher price, $q_1^* > q_2^*$.

5.5. *Policy implications*

5.5.1. **Affecting the diffusion of ambiguity-averse behaviors (α)**

The first type of policy a central planner may pursue is to try and affect the proportions of AA- and SEU-maximizers which compose the economy, α and $1 - \alpha$. Our results make it clear that the effect of a policy that tends to change (decrease) α will depend on the initial configuration as represented by the equilibrium price function and the level of participation. Suppose that initially both groups of investors participate in the

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[16]The following assumes that ambiguity-averse investors are expressing a positive net demand for the risky asset and are therefore *demanding* a risk premium in order to hold it. As we have seen, in equilibrium this is the only feasible outcome.

market, so that the equilibrium price is $q^* = q_2^*$. The policy maker finds a way to instantaneously *decrease* the fraction of ambiguity averse agents, α. Since $\sigma_I^2 \leq \sigma_{\max}^2$ and $\mu_{\min} < \mu_I$, the numerator of the expression for q_2^*, (13), increases; viceversa its denominator declines. Therefore, on the whole q_2^* increases. However, as this happens it is possible for the condition $(\mu_{\min} + \mu_S)/(1+i) > q_2^*$ to start being violated and q_2^* may enter the region $((\mu_{\min} + \mu_S)/(1+i), (\mu_{\max} + \mu_S)/(1+i))$; when this occurs, participation from the AA investors will cease and the pricing function will switch to (11); as we have noticed in Section 5.2, this implies an upward price jump. Therefore, an attempt to decrease α and simply increase the proportion of SEU-maximizers represents a policy approach with mixed outcomes. On the one hand, it is certain that such an intervention will increase equilibrium prices; on the other hand, it may end up penalizing the overall participation to the market and reduce liquidity and trading volume.[17] The intuition is that if very few AA agents are left in the market, the equilibrium price will mostly reflect the fundamental risk assessments expressed by SEU investor, and the resulting price will end being "too high" for AA investors to participate. On the contrary, if the policy maker were to try and *increase* α, then (again, because $\sigma_I^2 \leq \sigma_{\max}^2$ and $\mu_{\min} < \mu_I$) the numerator of the expression for q_2^*, (13) decreases; viceversa its denominator increases. Therefore, q_2^* declines so that the condition $(\mu_{\min} + \mu_S)/(1+i) > q_2^*$ continues to hold. This means that if both types of investors were trading before the policy intervention, then, after the increase in α, both types will be still both trading the stock. At the same time, this intervention will raise the real risk premium, since for AA agents to participate, more attractive conditions must be offered.

Suppose instead that initially only SEU investors were present in the market. If α is reduced, the price increases and that makes the stock even less attractive to the AA investors who are left. As a result, the limited participation equilibrium would not be affected and the risk premium would decline. However, if a policy-maker tries and increases α, from the expression for q_1^*, it is clear that this will lead to a decrease in the equilibrium price (and this in spite of the fact that AA agents are not trading). So if α increases enough, the price will fall below the threshold $(\mu_{\min} + \mu_S)/(1+i)$, so that both groups will be then be willing to trade in the stock market, which will actually increase the stock price. In fact, one may compute a threshold $\bar{\alpha}$ such that when α is raised sufficiently, then the equilibrium switches from limited participation by SEU-maximizers only to both agent types. Such an $\bar{\alpha}$ has equation (see the working paper version for a proof):

$$\bar{\alpha} = 1 - \frac{\bar{z}\left(\sigma_I^2 + \sigma_S^2\right)}{(\mu_I - \mu_{\min})(1+i)}.$$

Clearly, in the process the risk premium at first increases and then, after the switch, it declines. This means that starting from $\alpha < \bar{\alpha}$, when α is increased at first, the price will decline, the risk premium increase, and declining participation (as only the fraction $1 - \alpha$ of investors participates) will be caused; only when α is increased to the point that it reaches $\bar{\alpha}$, the price will decline even further (from $q_1^*(\alpha)$ down to $q_2^*(\bar{\alpha})$) but a new participating equilibrium will be established.

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[17]Notice that there is no contradiction in the latter claim: starting from a situation in which 100% of the potential investors (including the AA fraction α_0) participate to the market trading stocks, by sufficiently decreasing α (say, to $\alpha_0 - \Delta\alpha$) the policy-maker may induce a switch to an equilibrium in which only $1 - \alpha_0 + \Delta\alpha < 1$ of the potential investors participate. Unless $\Delta\alpha = -\alpha_0$, this implies a loss in participation.

These results reveal a clear asymmetry in the ability to use policy to affect equilibrium outcomes. While increasing α is always possible and it implies that while equilibrium prices decline, more participation may be ultimately attained, decreasing α has non-linear effects: a threshold exists, such that if α declines and therefore the equilibrium price increases enough, then the market may switch to an SEU-only participation regime and this may cause a sudden drop in trading and liquidity. This imposes severe limitations to the possibility of implementing policies through α, unless the goal of $\alpha = 0$ is attainable. At $\alpha = 0$ the AA investors disappear altogether, there is full participation, and the expression for demand and equilibrium price are the standard ones, e.g., $q^* = (\mu_I + \mu_S)/(1+i) - \bar{z}\left(\sigma_I^2 + \sigma_S^2\right)/(1+i)^2$.

5.5.2. Changing the "amount" of ambiguity-relevant uncertainty ($\mu_{\min}$ and $\sigma_{\max}^2$)

Another type of policy available to a policy maker consists in convincing AA investors that certain sensitive scenarios—as defined by the distributions $\{\mu_1, \mu_2, ..., \mu_P\}$ and $\{\sigma_1^2, \sigma_2^2, ..., \sigma_Q^2\}$—can be safely ruled out. Proposition 1 reveals that the sensitive scenarios are those defined by $\mu_{\min}$ and $\sigma_{\max}^2$. Consider first an attempt to increase $\mu_{\min}$, i.e., to "chop off" the worst possible outcomes from $\{\mu_1, \mu_2, ..., \mu_P\}$.[18] The intuition is that the policy maker tries and persuades the α-proportion of AA agents that some particularly ghastly beliefs on future idiosyncratic shocks are impossible. For instance, the policy maker may try to convey to the market the idea that $\mu_{\min} = -\mu_S < 0$ (which can be interpreted as bankruptcy) can be disregarded. Once more, we have to distinguish between the two cases implied by Proposition 1. Suppose that only SEU agents are initially trading in the market, so that (10) holds while the price is given by (11). Clearly, small variations in the worst perceived, possible mean stock payoff $\mu_{\min}$ have per se no immediate effect on the equilibrium price. Nevertheless, if $\mu_{\min}$ increases enough, ambiguity AA will decide to participate, when the condition (12) starts being satisfied. This happens when $\mu_{\min}$ exceeds

$$\bar{\mu}_{\min} = \mu_I - \frac{\bar{z}\left(\sigma_I^2 + \sigma_S^2\right)}{(1-\alpha)(1+i)}.$$

In that case, we know that the equilibrium price will jump down from q_1^* to q_2^* and the risk premium increase. The implication is that starting from situations of SEU-only participation, it will take large jumps in $\bar{\mu}_{\min}$ for the equilibrium to be significantly affected; however, when this happens, one can also expect a detrimental effect on risky asset prices. Interestingly and realistically, as the fraction of AA investors α increases, the threshold level $\bar{\mu}_{\min}$ required for inducing participation reduces, since now each AA agent has to bear a lower amount of risk. This is relatively surprising: even though the policy action consists of ruling out the worst possible scenarios increasing $\mu_{\min}$, for an action of sufficient magnitude its eventual effect on equilibrium prices will be negative and the cost of enforcing a participation equilibrium will consist of a higher risk premium. This means that when a market has fallen into a state of disruption (a SEU-only equilibrium), bringing the market back to higher liquidity and orderly functioning through a reduction in the amount of perceived ambiguity may actually go through further reductions in equilibrium prices.

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[18]No sensible policy maker will ever try and reduce $\mu_{\min}$, thus increasing the amount of ambiguity in the market and leading to reduced participation and lower prices.

Suppose instead that the market is one in which initially there is full participation by all investor types. In this case, we know that the pricing function is (13), which implies that q_2^* will increase continuously as $\mu_{\min}$ is increased (this is due to the fact that z_{AA}^* increases given z_{SEU}^*), while it is clear that increasing $\mu_{\min}$ has no effect on the participation constraint, which will still be satisfied. In fact, it can be shown algebraically that because the coefficient for $\mu_{\min}$ in q_2^* is

$$\frac{\alpha \left(\sigma_I^2 + \sigma_S^2\right)}{\sigma_S^2 + \alpha\sigma_I^2 + (1-\alpha)\sigma_{\max}^2} < 1,$$

the increase in the equilibrium price cannot prevent the participation of AA agents. Therefore in the presence of full participation, reducing the amount of ambiguity has only positive effects on prices and will reduce risk premia. It seems then that they key objective of any policy maker ought to prevent equilibria with limited participation to appear in the first instance.

Another policy action with interesting implications uses the lever of a reduction in $\sigma_{\max}^2$, i.e., the highest possible level of uncertainty in the idiosyncratic component of the shocks perceived by AA investors.[19] The intuition is that the policy maker tries and persuades the α-proportion of AA agents that some particularly appalling beliefs on the variance of future stock idiosyncratic shocks are not possible, thus effectively "chopping off" a portion of the distribution for $\left\{\sigma_1^2, \sigma_2^2, ..., \sigma_Q^2\right\}$. If initially only SEU agents were participating, then a variation in $\sigma_{\max}^2$ has no effect on the market. This because both the pricing function (11) and the participation condition (12) do not depend on $\sigma_{\max}^2$. Differently from the case of the mean idiosyncratic payoff parameter $\mu_{\min}$, when AA investors are sitting on the sidelines, their maximum perceived uncertainty on idiosyncratic shocks is irrelevant. On the other hand, if initially both groups of investors were participating, then if $\sigma_{\max}^2$ declines, the demand from the AA investors, z_{AA}^*, will increase, as shown by (7). From the market clearing condition ($\bar{z} = \alpha z_{AA}^* + (1-\alpha) z_{SEU}^*$) we know that z_{SEU}^* has to decline; however, to induce SEU agents to reduce their demand for the asset, the market price q_2^* has to increase. Additionally, participation is not affected. This is a very interesting result. First, reducing $\sigma_{\max}^2$ is completely ineffective if the market is already segmented to include SEU investors only. This means that if a policy maker lets the situation deteriorate enough, to the point that all AA agents leave the market, then changing their perceived uncertainty concerning idiosyncratic shocks becomes irrelevant. As we have seen, when a market has already broken down in terms of participation, trying to affect the perceived uncertainty by increasing $\mu_{\min}$ may actually depress prices and inflate risk premia, although this policy can eventually raise liquidity and induce full participation. Second, when the action is taken while segmentation has not yet occurred, its effect is not to change the participation incentives, but to simply provide support for equilibrium prices. In this sense, policies that rely on interventions on the perceived uncertainty of future idiosyncratic shocks only have price effects but no liquidity effects.

It is possible to check that these effects are magnified if it is assumed that the single-priors of the SEU investors are probability-weighted averages of the multiple priors held by the AA investors, i.e.,

$$\mu_I = \sum_{j=1}^{P} \beta_j^\mu \mu_j = \beta_{\min}^\mu \mu_{\min} + \sum_{j=2}^{P} \beta_j^\mu \mu_j$$

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[19]No sensible policy maker will ever try and increase $\sigma_{\max}^2$, because this would simply lead to lower market prices.

$$\sigma_I^2 = \sum_{j=1}^{Q} \beta_j^\sigma \sigma_j^2 = \sum_{j=1}^{Q-1} \beta_j^\sigma \sigma_j^2 + \beta_{\max}^\sigma \sigma_{\max}^2$$

where $\{\beta_j^\mu\}_{j=1}^{P}$ and $\{\beta_j^\sigma\}_{j=1}^{Q}$ are subjective priors (common to all SEU agents) over alternative levels of mean and variance of the risky payoff. Clearly, "chopping off" $\mu_{\min}$ and/or $\sigma_{\max}^2$ from the multi-prior distribution perceived by the AA investors can be interpreted as equivalent to setting $\beta_{\min}^\mu = 0$ and/or $\beta_{\max}^\sigma = 0$ re-scaling all other weights to equal $\tilde{\beta}_j^\mu = \beta_j^\mu/(1 - \beta_{\min}^\mu)$ and $\tilde{\beta}_j^\sigma = \beta_j^\sigma/(1 - \beta_{\max}^\sigma)$. As a result, we have:

$$\tilde{\mu}_I = \sum_{j=2}^{P} \tilde{\beta}_j^\mu \mu_j > \mu_I = \sum_{j=1}^{P} \beta_j^\mu \mu_j \qquad \tilde{\sigma}_I^2 = \sum_{j=1}^{Q-1} \beta_j^\sigma \sigma_j^2 < \sigma_I^2 = \sum_{j=1}^{Q} \beta_j^\sigma \sigma_j^2.$$

This creates a natural link between the multiple priors of AA investors and the single, unique prior of the SEU agents. At this point, an increase in $\mu_{\min}$ causes an increase in μ_I (to $\tilde{\mu}_I$). In this case, there is an effect even in the case in which initially only SEU investors are trading in the market: the security price increases while the risk premium declines, and this independently of whether the increase in $\mu_{\min}$ may be large enough to lead AA agents to participate. If instead the market is one in which initially both SEU and AA investors are trading, the market price will increase both because $\mu_{\min}$ increases and also because the same effect is exercised on μ_I. Therefore the effects previously argued are simply magnified. An identical conclusion applies to the case in which the policy-maker reduces $\sigma_{\max}^2$: if initially only SEU agents were participating in the economy, then the reduction in σ_I^2 (to $\tilde{\sigma}_I^2$) will increase the demand from SEU investors and lead to an increase in price and a decline in the risk premium. If initially both groups of investors were participating, then if $\sigma_{\max}^2$ declines, the demand from the AA investors, z_{AA}^*, will increase, and the same happens to z_{SEU}^*. Therefore results are the same but they are magnified by the newly instituted link between multiple and single, SEU-type priors.

5.5.3. Inflation (i)

One last lever in the hands of the policy maker is the inflation rate, i. Suppose that i is increased in an environment in which only SEU investors participate. Because the quantity

$$\frac{1}{(1+i)^2}\left[-(\mu_I + \mu_S) + \frac{2\bar{z}\left(\sigma_I^2 + \sigma_S^2\right)}{(1-\alpha)(1+i)}\right], \tag{16}$$

(which determines the sign of the derivative of q_1^* with respect to i) has an ambiguous sign that depends on all the parameters, the equilibrium market price q_1^* changes non-monotonically with i. In fact, when the initial inflation rate is sufficiently high, simulations reveal that (16) is likely to be negative so that the risky asset price declines when inflation increases; however, for $i \simeq 0$—which is rather realistic in a deep recession caused by a financial crisis—one cannot establish any sign restrictions, because the outcome depends on the relative magnitude of the factors $(\mu_I + \mu_S)$ and $\bar{z}\left(\sigma_I^2 + \sigma_S^2\right)$. However, it is clear that if initially the non-participation constraint (10) held, an increase in i cannot perturb the inequality. Therefore a higher inflation rate in a segmented market in which all AA investors have left already, simply strengthens the segmentation, while produces ambiguous effects on risky asset prices (negative provided there is enough subjectively perceived

total variance, $\sigma_I^2 + \sigma_S^2$). Therefore, inflation as a policy tool seems either ineffective or perverse because it cannot relax participation constraints while it may depress real equilibrium prices.

In the same situation, consider now a reduction of the inflation rate. The effect on asset prices remains ambiguous, although an increase in prices is the most likely outcome if the initial inflation rate is not close to zero. On the other hand, if the reduction in the inflation rate is substantial, there is now a chance that constraint (10) may stop being satisfied, i.e., that (provided $\mu_I - \mu_{\min} > [\bar{z}\left(\sigma_I^2 + \sigma_S^2\right)]/\left(1-\alpha\right)$) an inflation rate $\bar{\iota}$ may be found such that $1 + \bar{\iota} = \bar{z}\left(\sigma_I^2 + \sigma_S^2\right) / \left[(1-\alpha)\left(\mu_I - \mu_{\min}\right)\right]$ and the constraint (12) is satisfied. If this happens, a reduction in the inflation rate will bring increased participation but lower asset prices.

Suppose that i is increased in an environment in which both SEU and AA investors participate. If one differentiates the expression for (13) with respect to i, a reasoning similar to the one applied to the SEU-only market leads to conclude that the market price q_2^* may change non-monotonically when i is increased. In fact, when the initial inflation rate is high, then the reaction of the real risky asset price is likely to be negative so that the real price declines when inflation increases and hedging is imperfect; however, for $i \simeq 0$ one cannot establish any sign restrictions, because the outcome depends on the relative magnitude of the factors $[\alpha\mu_{\min}\left(\sigma_I^2 + \sigma_S^2\right) + \mu_S\left(\alpha\sigma_I^2 + (1-\alpha)\sigma_{\max}^2 + \sigma_S^2\right) + (1-\alpha)\mu_I\left(\sigma_{\max}^2 + \sigma_S^2\right)]$ and $\bar{z}\left(\sigma_I^2 + \sigma_S^2\right)\left(\sigma_{\max}^2 + \sigma_S^2\right) / \left[\sigma_S^2 + \alpha\sigma_I^2 + (1-\alpha)\sigma_{\max}^2\right]$. However, it is clear that if initially (12) held, it may stop being satisfied. Therefore a higher inflation rate in a non-segmented market produces ambiguous effects on risky asset prices (negative provided there is enough subjectively perceived total variance, $\sigma_I^2 + \sigma_S^2$ and/or the perceived maximum idiosyncratic variance by AA investors, $\sigma_{\max}^2$, is sufficiently high) and even threatens to disrupt markets by forcing segmentation upon them.[20] Once more, inflation as a policy tool seems either ineffective or perverse because it may induce limited participation and depress equilibrium real asset prices.

In the same situation, consider now a reduction of the inflation rate. The effect on equilibrium asset prices remains ambiguous, although an increase in asset prices remains the likely outcome if the initial inflation rate is not close to zero. On the other hand, there is no a chance that the participation constraint (12) may stop being satisfied: if both SEU and AA agents were initially trading in the market, this remains the case in a regime with lower inflation. Therefore low inflation has only virtues, leading to steady, widespread participation, steady liquidity, and (with high probability) even to higher risky asset prices in real terms. Therefore, it seems that a sensible policy-maker interested in supporting a well-functioning, non-segmented asset market ought to reduce inflation.

5.6. *Generalizations*

We ask now to what degree our conclusion that AA may lead to a market break-downs depends on the fact that so far the AA investors have "suffered" from ambiguity only with reference to idiosyncratic risk. Also in the case of this parametric model, it is possible to show that a sufficient condition for ambiguity to induce market break-downs is that the spread between the highest and the lowest possible return of the

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[20]Intuitively, this happens when inflation becomes so high that the real expected payoffs from the risky asset become insufficient to compete with the real, riskless rate of return guaranteed to AA investors by the money market account.

idiosyncratic risk component is larger than the spread between the highest and the lowest possible return of the systematic component. In general, the algebra and the related sufficient conditions are rather involved. In the interest of intuition and simplicity, we assume from now on that $\bar{z} = 0$, i.e., that the risky asset is again of a derivative type in endogenous zero net supply. Assume that AA investors do not know the exact value for the parameters μ_S and σ_S^2 but—similarly to what happens to the idiosyncratic payoff components—they perceive that they will be drawn from the sets $\{\mu_{S,1}, \mu_{S,2}, ..., \mu_{S,P'}\}$ and $\{\sigma_{S,1}^2, \sigma_{S,2}^2, ..., \sigma_{S,Q'}^2\}$, respectively. Without loss of generality, let $w_m^0 = 0$. As always, there shall be trade in the risky asset only if there is one agent who is willing to buy the asset, and another agent who is willing to sell it. As before, the end-of-period-one expected real wealth that derives from the investment is given by:

$$w_m^1 = z_m \frac{d}{1+i} + b_m(1+r^f).$$

where z_m can be positive or negative, depending on whether agent m is buying or selling the asset. The problem solved by agent m can be written as:

$$\arg\max_{z_m, b_m} \left[z_m \left(\frac{\mu_I + \mu_S}{1+i} - q \right) + b_m r^f - \frac{1}{2} z_m^2 \frac{\sigma_I^2 + \sigma_S^2}{(1+i)^2} \right]$$

subject to an obvious budget constraint. Initially, let's consider the case in which ambiguity-aversion only concerns the systematic component of payoffs. This means that—rather oddly, one has to admit—investors perceive a unique prior on the idiosyncratic component represented by $N\left(\mu_I, \sigma_I^2\right)$. Therefore they maximize an expected utility objective with kernel (4), where $\left(\mu_S, \sigma_S^2\right) \in \{\mu_{S,1}, \mu_{S,2}, ..., \mu_{S,P'}\} \times \{\sigma_{S,1}^2, \sigma_{S,2}^2, ..., \sigma_{S,Q'}^2\}$. The necessary and sufficient condition for optimality yields an optimal risky investment identical to (5) from which the same analysis on the sign z_m^* and its dependence on the relationship between the price of the asset and its expected payoff in (6) can be performed. Without loss of generality, assume now $\mu_{S,\min} = \mu_{S,1} \le \mu_{S,2} \le ... \le \mu_{S,P'} = \mu_{S,\max}$. For any prior $(\mu_S, \sigma_S^2) \in \{\mu_{S,1}, \mu_{S,2}, ..., \mu_{S,P'}\} \times \{\sigma_{S,1}^2, \sigma_{S,2}^2, ..., \sigma_{S,Q'}^2\}$ the agents can form, the two following implications are obvious:

$$\begin{aligned} q &< \frac{\mu_I + \mu_S}{1+i} \implies q < \frac{\mu_I + \mu_{S,\max}}{1+i} \\ q &> \frac{\mu_I + \mu_S}{1+i} \implies q > \frac{\mu_I + \mu_{S,\min}}{1+i}. \end{aligned}$$

Hence there is one agent who is willing to buy the asset only if $q < (\mu_I + \mu_{S,\max})/(1+i)$. Viceversa, there is one agent who is willing to sell the asset only if $q > (\mu_I + \mu_{S,\min})/(1+i)$. Therefore there will be trade in the risky asset (that is, $z_m^* \neq 0$, $m = 1, 2$) only if

$$\mu_I + \mu_{S,\min} < \mu_I + \mu_{S,\max},$$

which is clearly satisfied. Therefore, if the ambiguity concerns only the systematic risk component, then there will be always trade. This creates the suspicion that when ambiguity aversion affects both systematic and idiosyncratic risk, for trading to fail there must be "more ambiguity" on the idiosyncratic component than on the systematic one.

As a next step, we re-introduce ambiguity aversion in the idiosyncratic component. As before, AA investors only know that the mean μ_I of the distribution belongs to the set $\{\mu_1, \mu_2, ..., \mu_P\}$, and the variance σ_I^2 to the set $\{\sigma_1^2, \sigma_2^2, ..., \sigma_Q^2\}$ with $P \geq 2$ and $Q \geq 2$. Hence, for any $(\mu_S, \sigma_S^2) \in \{\mu_{S,1}, \mu_{S,2}, ..., \mu_{S,P'}\} \times \{\sigma_{S,1}^2, \sigma_{S,2}^2, ..., \sigma_{S,Q'}^2\}$ it is easy to check that:

$$z_m > 0 \iff (\mu_{I,\min}, \sigma_{I,\max}^2) \in \underset{(\mu_I, \sigma_I^2)}{\arg\min} \left(\frac{\mu_I + \mu_S}{1+i} - q \right) z_m + b_m r^f - \frac{1}{2} z_m^2 \frac{\sigma_I^2 + \sigma_S^2}{(1+i)^2}$$

$$z_m < 0 \iff (\mu_{I,\max}, \sigma_{I,\max}^2) \in \underset{(\mu_I, \sigma_I^2)}{\arg\min} \left(\frac{\mu_I + \mu_S}{1+i} - q \right) z_m + b_m r^f - \frac{1}{2} z_m^2 \frac{\sigma_I^2 + \sigma_S^2}{(1+i)^2}.$$

Let

$$\mu_{I\min} = \min\{\mu_1, \mu_2, ..., \mu_P\} \qquad \mu_{I\max} = \max\{\mu_1, \mu_2, ..., \mu_P\} \qquad \sigma_{I\max} = \max\{\sigma_1^2, \sigma_2^2, ..., \sigma_Q^2\}$$

For any $(\mu_S, \sigma_S^2) \in \{\mu_{S,1}, \mu_{S,2}, ..., \mu_{S,P'}\} \times \{\sigma_{S,1}^2, \sigma_{S,2}^2, ..., \sigma_{S,Q'}^2\}$, we get the following step-wise demand for the risky asset:

$$z_m^* = \begin{cases} \frac{(1+i)[\mu_{I,\min} + \mu_S - q(1+i)]}{\sigma_{I,\max}^2 + \sigma_S^2} > 0 & \text{if } \frac{\mu_{I,\min} + \mu_S}{1+i} > q \\ 0 & \text{if } \frac{\mu_{I,\max} + \mu_S}{1+i} \geq q \geq \frac{\mu_{I,\min} + \mu_S}{1+i} \\ \frac{(1+i)[\mu_{I,\max} + \mu_S - q(1+i)]}{\sigma_{I,\max}^2 + \sigma_S^2} < 0 & \text{if } \frac{\mu_{I,\max} + \mu_S}{1+i} < q \end{cases}.$$

For any prior $(\mu_S, \sigma_S^2) \in \{\mu_{S,1}, \mu_{S,2}, ..., \mu_{S,P'}\} \times \{\sigma_{S,1}^2, \sigma_{S,2}^2, ..., \sigma_{S,Q'}^2\}$ the agents may have formed, the following implications are easy to check:

$$q < \frac{\mu_{I,\min} + \mu_S}{1+i} \implies q < \frac{\mu_{I,\min} + \mu_{S,\max}}{1+i}$$

$$q > \frac{\mu_{I,\max} + \mu_S}{1+i} \implies q > \frac{\mu_{I,\max} + \mu_{S,\min}}{1+i}.$$

Hence there is one agent who is willing to buy the asset only if $q < (\mu_{I,\min} + \mu_{S,\max})/(1+i)$. Viceversa, there is one agent who is willing to sell the asset only if $q > (\mu_{I\max} + \mu_{S\min})/(1+i)$. As a result, there can be trade (that is, $z_m^* \neq 0$, $m = 1, 2$) only if $\mu_{I,\max} + \mu_{S,\min} < \mu_{I,\min} + \mu_{S,\max}$, that is only if $\mu_{I,\max} + \mu_{S,\min} < \mu_{I,\min} + \mu_{S,\max}$ or $(\mu_{S,\max} - \mu_{S,\min}) > (\mu_{I,\max} - \mu_{I,\min})$. In conclusion, sufficient condition for trade to fail is that the spread between the highest and the lowest possible return of the idiosyncratic component is larger than the spread between the highest and the lowest possible return of the systematic component. This confirms that market breakdowns due to AA may occur if and only if the ambiguity concerns the idiosyncratic risk components and if this ambiguity exceeds the ambiguity on the systematic components.

5.7. *Can policy making improve welfare?*

Up to this point, we have simply adopted a heuristic approach to policy interventions in the presence of ambiguity. This meant that we have evaluated as "useful" the policies able to induce participation, with the constraint that in general they should not achieve this goal through a reduction in equilibrium prices. In this Section we go beyond this simplistic approach and show that—even when the utility indices of both groups

of investors are formally considered and the fact that resources may be distributed by a central planner only after introducing taxes or other forms of redistributions is kept into account—welfare improving policies that exploit the possibility of increasing $\mu_{\min}$ and reducing $\sigma_{\max}^2$ may exist.

Consider the following policy aimed at inducing AA investors to trade: the policy maker guarantees a subsidy in the measure $s_c > 0$ based on the occurrence of the events in the partition $\{\mu_1, \mu_2, ..., \mu_P\}$ perceived by the AA investor. In particular, if the event that finds realization is such that the corresponding mean μ_c is below or equal to some threshold (i.e., if the actual state of the world is sufficiently "bad"),

$$\mu_c \leq \mu_I - \frac{\bar{z} \left(\sigma_I^2 + \sigma_S^2 \right)}{(1 - \alpha) (1 + i)^2},$$

then the subsidy is paid in the amount

$$s_c = \frac{\mu_I}{(1 + i)} - \frac{\mu_c}{(1 + i)} - \frac{\bar{z} \left(\sigma_I^2 + \sigma_S^2 \right)}{(1 - \alpha) (1 + i)^2}.$$

Otherwise, i.e., for sufficiently good states, no subsidy is paid. For concreteness, let's suppose that the policy maker sets $\mu_c = \mu_{\min}$. In practice, this implies that the policy consists of increasing $\mu_{\min}$ up to $\tilde{\mu}_{\min} = \mu_{\min} + s_{\min}$, where

$$s_{\min} = \frac{\mu_I}{(1 + i)} - \frac{\mu_{\min}}{(1 + i)} - \frac{\bar{z} \left(\sigma_I^2 + \sigma_S^2 \right)}{(1 - \alpha) (1 + i)^2}.$$

Since the policy effectively "chops off" a portion of $\{\mu_1, \mu_2, ..., \mu_P\}$ thus reducing the uncertainty effectively perceived by AA agents, it is sensible to also assume that AA agents also perceive the policy as a reduction of the maximal possible variance to $\tilde{\sigma}_{\max}^2 = \sigma_{\max}^2 - \Delta$ for some $0 < \Delta < \sigma_{\max}^2$. This is realistic because compensating AA for the worst possible scenarios must also reduce to some extent their perceived uncertainty on the variance of the idiosyncratic payoff they are facing. SEU agents are not affected by the policy because they do not believe that a state characterized by $\mu_{\min}$ will ever occur. To support the policy, the central authority introduces a tax rate $\tau \in (0, 1)$ on the systematic payoff. Notice that the tax must be collected at time 0 and as such it hits the expected stock payoffs, not their realization. As a result of the policy notice that $\mu_{\min}$ is replaced by $\tilde{\mu}_{\min} = \mu_{\min} + s_{\min}$, $\sigma_{\max}^2$ by $\tilde{\sigma}_{\max}^2 = \sigma_{\max}^2 - \Delta$, and μ_S by $\tilde{\mu}_S = \mu_S(1 - \tau)$. On the contrary, μ_I, σ_I^2, and σ_S^2 are left unchanged, although we have to notice that the total variance perceived by AA investors also changes to

$$\tilde{\sigma}_{AA,TOT}^2 = \sigma_S^2 + \tilde{\sigma}_{\max}^2 = \sigma_{AA,TOT}^2 - \Delta.$$

Finally, define $\xi \equiv \sigma_S^2 + \alpha \sigma_I^2 + (1 - \alpha) \sigma_{\max}^2$ and notice that after the policy is enacted, ξ switches to

$$\tilde{\xi} \equiv \tilde{\sigma}_S^2 + \alpha \tilde{\sigma}_I^2 + (1 - \alpha) \tilde{\sigma}_{\max}^2 = \left[\sigma_S^2 + \alpha \sigma_I^2 + (1 - \alpha) \left(\sigma_{\max}^2 \right) - (1 - \alpha) \Delta \right] = \xi - (1 - \alpha) \Delta < \xi.$$

Finally, call the weighted sum—with weights α and $1 - \alpha$, respectively—of the utility indices of both types of agents before the policy is implemented $W^{NP}(\sigma_I^2, \sigma_{\max}^2, \sigma_S^2, \mu_I, \mu_S, \mu_{\min}, i)$ (where W means "welfare") and the sum of the utility indices under the policy $W^P(\tilde{\sigma}_I^2, \tilde{\sigma}_{\max}^2, \tilde{\sigma}_S^2, \tilde{\mu}_I, \tilde{\mu}_S, \tilde{\mu}_{\min}, i)$. It is tedious but straightforward to compute

$$W^{NP}(\sigma_I^2, \sigma_{\max}^2, \sigma_S^2, \mu_I, \mu_S, \mu_{\min}, i) = r^f w^0 + \left[\left(1 + \alpha + 2r^f \right) \frac{\bar{z}^2 \left(\sigma_I^2 + \sigma_S^2 \right)}{2 (1 + i)^2} - \frac{\bar{z} r^f \left(\mu_I + \mu_S \right) (1 - \alpha)}{1 + i} \right],$$

and

$$
W^P(\tilde{\sigma}_I^2, \tilde{\sigma}_{\max}^2, \tilde{\sigma}_S^2, \tilde{\mu}_I, \tilde{\mu}_S, \tilde{\mu}_{\min}, i) = r^f w^0 + \left(1 + 2r^f\right) \frac{\bar{z}^2 \left(\tilde{\sigma}_I^2 + \tilde{\sigma}_S^2\right) \left(\tilde{\sigma}_{\max}^2 + \tilde{\sigma}_S^2\right)}{2 \left(1 + i\right)^2 \left[\tilde{\sigma}_S^2 + \alpha \tilde{\sigma}_I^2 + \left(1 - \alpha\right) \tilde{\sigma}_{\max}^2\right]} +
$$

$$
- \frac{\bar{z} r^f \left[\alpha \tilde{\mu}_{\min} \left(\tilde{\sigma}_I^2 + \tilde{\sigma}_S^2\right) + \tilde{\mu}_S \left(\alpha \tilde{\sigma}_I^2 + \left(1 - \alpha\right) \tilde{\sigma}_{\max}^2 + \tilde{\sigma}_S^2\right) + \left(1 - \alpha\right) \tilde{\mu}_I \left(\tilde{\sigma}_{\max}^2 + \tilde{\sigma}_S^2\right)\right]}{\left(1 + i\right) \left[\tilde{\sigma}_S^2 + \alpha \tilde{\sigma}_I^2 + \left(1 - \alpha\right) \tilde{\sigma}_{\max}^2\right]} + \frac{\left(1 - \alpha\right) \alpha \left(\tilde{\mu}_I - \tilde{\mu}_{\min}\right)^2}{\tilde{\sigma}_S^2 + \alpha \tilde{\sigma}_I^2 + \left(1 - \alpha\right) \tilde{\sigma}_{\max}^2}.
$$

Then the following result obtains:

Proposition 3 *(Welfare gains). Assume $\mu_S > \mu_I$ and that, for the given α, the following condition holds:*

$$
\alpha \mu_I < \left(1 - \alpha\right) \mu_S.
$$

Then $W^P(\tilde{\sigma}_I^2, \tilde{\sigma}_{\max}^2, \tilde{\sigma}_S^2, \tilde{\mu}_I, \tilde{\mu}_S, \tilde{\mu}_{\min}, i) > W^{NP}(\sigma_I^2, \sigma_{\max}^2, \sigma_S^2, \mu_I, \mu_S, \mu_{\min}, i)$, which means that a policy intervention that changes $\tilde{\mu}_{\min}$ and $\sigma_{\max}^2$ in the ways illustrated before will improve overall welfare.

The condition—which can be re-written as $\alpha/(1-\alpha) < \mu_S/\mu_I$—can be interpreted as saying that either α (the "weight" of AA investors) is small relative to the proportion of SEU investors or that the mean impact of idiosyncratic payoff risk is small compared to the mean impact of systematic risk. One way or the other, this condition implies that either ambiguity has a moderate role in composition terms or in payoff structure terms. This would sufficient for a central planner to devise a simple "state-contingent" lump-sum subsidy scheme that increases welfare, in the definition given above. The appendix proves this proposition.

Notice that the policy examined in this Section is potentially a tail-event one, i.e., one that can be announced but has very small chances to be actually implemented, in the sense that setting $\mu_c = \mu_{\min}$ corresponds to the intuition that the public subsidy $s_{\min}$ is paid out only in correspondence of truly catastrophic event, similarly to public insurance provided to cover against large natural catastrophes. This is the same phenomenon noticed in a different application by Easley and O'Hara (2005): simply changing the perception of extreme models/events can potentially have large effects on equilibrium outcomes. These effects arise because AA individuals attach great importance to worst case scenarios, with the result that they can choose not to participate in the market. The effects are large relative to those that could be expected in an economy with only SEU traders. In such an economy the effect of extreme scenarios on asset prices is multiplied by the prior on the model which would most naturally be small. With AA investors the effect of extreme scenarios is direct—it is not multiplied by any prior belief.

5.8. *Discussion: recent policy strategies*

It is tempting to use the implications of our simple model to evaluate the policies that have been recently enacted to deal with the financial crisis. In doing so, we have to keep in mind at least two caveats. First, the model in this Section is obviously too rudimentary to provide an exhaustive framework within which to assess the effectiveness of any policy. Second, our discussion will mostly focus on policy strategies enacted in the U.S. Even though many countries/areas seem to have quite explicitly followed the steps undertaken by

the Federal Reserve and the U.S. Treasury in dealing with financial turbulence, clearly 2008 has witnessed a range of heterogeneous policy reactions throughout the world, from which we will largely abstract here.

Policy-makers have reacted to the spiralling crises in three ways. First, they have switched to an extremely accommodative monetary policy stance, with fast-paced reductions in key policy target rates between 400 and 500 b.p. between the Summer of 2007 and the end of 2008. Even though these efforts are also directed at contrasting the global recession that has swept through developed and emerging market economies since early 2008, one can also interpret—this certainly has been the case in the U.S. in the first part of the crisis, up to the Summer of 2008—these actions as attempts to simply reduce the spread of AA behaviors (i.e., α) in the economy: by fighting off the impending recession and clearly communicating goals and tools, policy makers may create an environment in which any residual, difficult-to-quantify uncertainty is easier to disregard and in which it may be rational to behave as typical SEU investors would. Notice that in this interpretation, clear communication of feasible goals and effective tools is as important as the actual strategies that aim at stimulating production, employment and therefore financial solvency.[21]

Second, it may argued that policy makers have explicitly and implicitly dealt with financial turmoil and impaired market functioning in the most direct way, i.e., by "jump-starting" a number of credit markets by direct intervention. For instance, already in August 2007, the Federal Reserve announced changes in discount window policies to facilitate the orderly functioning of short-term credit markets. In December 2007, the Federal Reserve introduced the Term Auction Facility (TAF), through which predetermined amounts of discount window credit are auctioned every two weeks. The TAF appears to have overcome the reluctance to borrow associated with standard discount window lending because of its competitive auction format, the certainty that a large amount of credit would be made available, and the fact that it is not designed to only meet urgent needs, thus avoiding stigma effects. To address the increasing demand for dollar funding in foreign jurisdictions, in December 2007 the Federal Reserve has arranged a number of reciprocal currency arrangements (swap lines) with the European Central Bank and the Swiss National Bank; similar arrangements exist with the Bank of England. In November 2008, the Federal Reserve announced a Term Asset-Backed Securities Loan Facility (TALF) facility designed to increase credit availability at normal interest rate spreads. Under its current design, the Federal Reserve Bank of New York lends to holders of certain Aaa-rated asset-backed securities, such as commercial MBS and private-label residential MBS.[22] These policies all imply that policy-makers directly intervene in the collapsed markets by either playing a market-making role, or by providing liquidity to other market investors. Clearly, these are strategies that fail to match with any precise feature of the model and that are sensible only ex-post, when markets have collapsed already and dealing with limited participation requires strong measures.

Finally, a handful of recent policy measures—in fact, the most hotly debated for their political and fiscal implications—may also be interpreted as attempts at lowering $\mu_{\min}$ and/or $\sigma^2_{\max}$, i.e., to reduce the "amount"

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[21]Notice that monetary policy cannot represent a tool through which $\mu_{\min}$ and/or $\sigma^2_{\max}$ may be directly affected because general policies influence by construction systematic, not idiosynchratic factors. As we have seen, at least in our simple model, it is only the latter type of risk that matters for optimal decisions under AA.

[22]Other, related programs are briefly described in the working paper version of this manuscript.

of ambiguity. In the Spring of 2008 the U.S. Treasury announced a temporary guarantee of the share prices of money market mutual funds and, beginning in October 2008, it used authority granted under the Emergency Economic Stabilization Act to purchase preferred shares in a large number of depository institutions. Similar policies have been enacted in Canada and in a number of European countries (see Section 2 for specific cases). In particular, a handful of conglomerate financial firms have been rescued in the U.S. and the U.K. For instance, in mid-March of 2008, Bear Stearns was pushed to the brink of failure after finding itself without access to short term financing. In early September 2008, the conditions of American International Group (AIG) deteriorated rapidly. In view of the likely systemic implications and the potential for significant adverse effects of a disorderly failure of AIG, the Federal Reserve lent \$85 billion to the firm. In September 2008, to maintain the GSEs' ability to purchase home mortgages, the Treasury established a backstop lending facility to purchase up to \$100 billion of preferred stock in Fannie Mae and Freddie Mac, and to initiate a program to purchase agency MBS. Market anxiety about the condition of Citigroup intensified in November 2008. To support financial market stability, the U.S. government entered into an agreement with Citigroup to provide a package of capital, guarantees, and liquidity access. As part of the agreement, the Treasury and Federal Deposit Insurance Corporation (FDIC) are providing capital protection against outsized losses on a pool of about \$306 billion in residential and commercial real estate and other assets, Citigroup has issued preferred shares to the Treasury and FDIC, and the Treasury has purchased an additional \$20 billion in Citigroup preferred stock.[23] Besides the specifics and the obvious goal of preventing a devastating financial meltdown punctuated by multiple runs to financial institutions, all of these operations can also be interpreted as ways in which policy makers have cooperated to persuade financial markets that worst-case, catastrophic scenarios in the form of extremely low $\mu_{\min}$ values or excessively large $\sigma^2_{\max}$ values would be simply impossible because of the vigilant presence of the policy authorities.

Even though we have seen that shrinking the amount of ambiguity may lead to good outcomes, it is a current topic of debate whether: (i) these policies have been reasonably effective in supporting asset prices and in limiting the credit risk spreads incorporated in market valuations (see Section 2), and (ii) the policies have managed to prevent fully-fledged market collapse (with very low participation). In fact, as for (i), Section 5.3 and 5.4 have shown that reducing $\mu_{\min}$ may actually depress prices when the intervention takes place within an already nonparticipating equilibrium (especially for high values of α). As for (ii) the empirical matter of assessing whether interventions have been successful is currently clouded by the fact that a number of parallel policy programs have actually injected massive amounts of liquidly in the collapsing markets.

6. Conclusions

Our model has stressed that understanding financial market behavior taking the presence of ambiguity-aversion into account may deliver important insights, both to understand recent events during the 2007-2008 financial

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[23]The Bank of England and the British Treasury have been involved in a number of similar rescue operations (e.g., to the benefit of Northern Rock, Royal Bank of Scotland, and Lloyds) throughout 2008. In some cases, fully-fledged nationalization has been used to tame market anxieties as to the solvency of the financial institutions.

crisis and to improve the effectiveness of policy actions. The main lesson seems to be that—due to the powerful threshold effects associated to market breakdowns—the key task for policy makers may be in fact not to remedy to poor market liquidity and collapsing participation to return markets to viable participation and liquidity standards, but in fact to avoid in the first instance and at all costs that a market breakdown may occur. Of course, this is almost (it is always true that the crises that are easier to resolve are the ones that do not occur) trivial while—more importantly, in the light of the situation in the world financial market at the time this paper is being written—we may be too late. However, our model does imply that many obvious policy tools—e.g., simply reducing the level of perceived ambiguity by market participants—may often produce counter-intuitive effects while under many parameter configurations an unpalatable trade-off between risk premia and participation may emerge. Another interesting implications is that higher inflation does not seem to be an appealing strategy to deal with distress in financial markets. In fact, even though its price effects may be difficult to track, if any, reducing the inflation rate may cause more attractive effects.

There are a number of directions in which our results could be extended. On the one hand, our investors face a set of return distributions and those who are AA fail to aggregate these distributions to produce a predicted return distribution. However, there are, at least, two other reasonable ways to view the decision problem faced by our AA decision makers. First, they could be thought of as choosing robust portfolios. That is they could search for portfolios that are robust to their uncertainty about the correct model for returns. Hansen and Sargent (2001) follow this approach to evaluating macroeconomic models. Second, they could be thought of as behavioral traders who either have biased beliefs or who do not maximize expected utility (see Barberis and Thaler, 2000). It would be interesting to examine the same or a similar model set up as the one in this paper and see whether our implications for policy-making are robust to approaching the problem of modelling optimal portfolio decisions under these alternative approaches.

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Appendix

After the adoption of the policy, AA agents are willing to invest in the risky asset only if

$$\tilde{\mu}_{\min} > \tilde{\mu}_I - \frac{\bar{z}\left(\tilde{\sigma}_I^2 + \tilde{\sigma}_S^2\right)}{(1-\alpha)(1+i)}.$$

Since $r^f w^0$ appears in both $W^{NP}(\sigma_I^2, \sigma_{\max}^2, \sigma_S^2, \mu_I, \mu_S, \mu_{\min}, i)$ and $W^P(\tilde{\sigma}_I^2, \tilde{\sigma}_{\max}^2, \tilde{\sigma}_S^2, \tilde{\mu}_I, \tilde{\mu}_S, \tilde{\mu}_{\min}, i)$, to evaluate the effect of the policy, it suffices to compare:

$$\left(1 + 2r^f\right) \frac{\bar{z}^2\left(\tilde{\sigma}_I^2 + \tilde{\sigma}_S^2\right)\left(\tilde{\sigma}_{\max}^2 + \tilde{\sigma}_S^2\right)}{2(1+i)^2\left[\tilde{\sigma}_S^2 + \alpha\tilde{\sigma}_I^2 + (1-\alpha)\tilde{\sigma}_{\max}^2\right]} - \frac{\bar{z}r^f\left[\alpha\tilde{\mu}_{\min}\left(\tilde{\sigma}_I^2 + \tilde{\sigma}_S^2\right) + \tilde{\mu}_S\left(\alpha\tilde{\sigma}_I^2 + (1-\alpha)\tilde{\sigma}_{\max}^2 + \tilde{\sigma}_S^2\right)\right]}{(1+i)\left[\tilde{\sigma}_S^2 + \alpha\tilde{\sigma}_I^2 + (1-\alpha)\tilde{\sigma}_{\max}^2\right]} +$$
$$+ \frac{\bar{z}r^f\left[(1-\alpha)\tilde{\mu}_I\left(\tilde{\sigma}_{\max}^2 + \tilde{\sigma}_S^2\right)\right]}{(1+i)\left[\tilde{\sigma}_S^2 + \alpha\tilde{\sigma}_I^2 + (1-\alpha)\tilde{\sigma}_{\max}^2\right]} + \frac{(1-\alpha)\alpha\left(\tilde{\mu}_I - \tilde{\mu}_{\min}\right)^2}{\left[\tilde{\sigma}_S^2 + \alpha\tilde{\sigma}_I^2 + (1-\alpha)\tilde{\sigma}_{\max}^2\right]}$$

with

$$\left(1 + \alpha + 2r^f\right) \frac{\bar{z}^2\left(\sigma_I^2 + \sigma_S^2\right)}{2(1+i)^2} - \frac{\bar{z}r^f\left(\mu_I + \mu_S\right)(1-\alpha)}{(1+i)}.$$

Recall that $\tilde{\sigma}_I^2 + \tilde{\sigma}_S^2 = \sigma_I^2 + \sigma_S^2$.

Sufficient condition for the induced trading to increase welfare are:

$$\text{(i)} \quad \left(1 + 2r^f\right) \frac{\bar{z}^2\left(\sigma_I^2 + \sigma_S^2\right)\left(\tilde{\sigma}_{\max}^2 + \sigma_S^2\right)}{2(1+i)^2\left[\sigma_S^2 + \alpha\sigma_I^2 + (1-\alpha)\tilde{\sigma}_{\max}^2\right]} > \left(1 + 2r^f\right) \frac{\bar{z}^2\left(\sigma_I^2 + \sigma_S^2\right)}{2(1+i)^2}$$

$$\text{(ii)} \quad \frac{\bar{z}r^f\left[\alpha\tilde{\mu}_{\min}\left(\sigma_I^2 + \sigma_S^2\right) + \tilde{\mu}_S\left(\alpha\sigma_I^2 + (1-\alpha)\tilde{\sigma}_{\max}^2 + \sigma_S^2\right) + (1-\alpha)\mu_I\left(\tilde{\sigma}_{\max}^2 + \sigma_S^2\right)\right]}{(1+i)\left[\sigma_S^2 + \alpha\sigma_I^2 + (1-\alpha)\tilde{\sigma}_{\max}^2\right]} < \frac{\bar{z}r^f\left(\mu_I + \mu_S\right)(1-\alpha)}{1+i}$$

$$\text{(iii)} \quad \alpha \frac{\bar{z}^2\left(\sigma_I^2 + \sigma_S^2\right)}{2(1+i)^2} < \frac{(1-\alpha)\alpha\left(\mu_I - \tilde{\mu}_{\min}\right)^2}{\tilde{\sigma}_S^2 + \alpha\tilde{\sigma}_I^2 + (1-\alpha)\tilde{\sigma}_{\max}^2}.$$

As for (i), recall that: $\xi - (1-\alpha)\Delta = \sigma_S^2 + \sigma_I^2 + (1-\alpha)\left(\sigma_{\max}^2 - \Delta\right)$ and $\tilde{\sigma}_{AA,TOT}^2 - \Delta = \sigma_S^2 + \sigma_{\max}^2 - \Delta$. Choose Δ such that $\sigma_{\max}^2 - \Delta > \sigma_I^2$. Hence

$$\frac{\tilde{\sigma}_{AA,TOT}^2}{\xi - (1-\alpha)\Delta} > 1.$$

Then inequality (i) holds. With reference to (ii), for any σ_I^2, $\sigma_{\max}^2$, and σ_S^2, notice that:

$$\alpha\left(\sigma_I^2 + \sigma_S^2\right) + (1-\alpha)\left(\sigma_{\max}^2 + \sigma_S^2\right) = \alpha\sigma_I^2 + (1-\alpha)\sigma_{\max}^2 + \sigma_S^2$$

Therefore

$$0 \le \lambda = \frac{\alpha\left(\sigma_I^2 + \sigma_S^2\right)}{\alpha\sigma_I^2 + (1-\alpha)\sigma_{\max}^2 + \sigma_S^2} \le 1$$

and

$$1 - \lambda = \frac{(1-\alpha)\left(\sigma_{\max}^2 + \sigma_S^2\right)}{\alpha\sigma_I^2 + (1-\alpha)\sigma_{\max}^2 + \sigma_S^2}.$$

As result

$$\frac{\left[\alpha\tilde{\mu}_{\min}\left(\sigma_I^2 + \sigma_S^2\right) + \tilde{\mu}_S\left(\alpha\sigma_I^2 + (1-\alpha)\tilde{\sigma}_{\max}^2 + \sigma_S^2\right) + (1-\alpha)\mu_I\left(\tilde{\sigma}_{\max}^2 + \sigma_S^2\right)\right]}{(1+i)\left[\sigma_S^2 + \alpha\sigma_I^2 + (1-\alpha)\tilde{\sigma}_{\max}^2\right]}$$

can be rewritten as the sum between $\tilde{\mu}_S = \mu_S(1-\tau)$ and a convex combination of $\mu_{\min}$ and μ_I with weights λ and $1-\lambda$ as above. Let

$$\beta = \frac{\alpha\left(\sigma_I^2 + \sigma_S^2\right)}{\xi + (1-\alpha)(-\Delta)}, \quad 1 - \beta = \frac{(1-\alpha)\left(\sigma_S^2 + \sigma_{\max}^2 - \Delta\right)}{\xi + (1-\alpha)(-\Delta)}.$$

To verify (ii), we need to check that

$$(\mu_I + \mu_S)(1-\alpha) > \beta\left(\mu_{\min} + s_{\min}\right) + \mu_S(1-\tau) + (1-\beta)\mu_I.$$

The right-hand side of the inequality is maximized at $\beta = 0$, hence consider $(\mu_I + \mu_S)(1-\alpha) > \mu_S(1-\tau) + \mu_I$. If there exists a $\tau \in (0,1)$ that satisfies this condition, then it satisfies also the condition with $\beta \ne 0$.

In particular, notice that $0 < -\mu_I\frac{\alpha}{\mu_S} + (1-\alpha) < 1$. Therefore, there exists a τ as prescribed. Finally, as for (iii),

$$\alpha\frac{\bar{z}^2\left(\sigma_I^2 + \sigma_S^2\right)}{2(1+i)^2} < \frac{(1-\alpha)\alpha\left(\mu_I - \mu_{\min} - s_{\min}\right)^2}{\xi + (1-\alpha)(-\Delta)}$$

$$\implies \frac{\bar{z}^2\left(\sigma_I^2 + \sigma_S^2\right)}{2(1+i)^2} < \frac{(1-\alpha)\left(\mu_I - \mu_{\min} - s_{\min}\right)^2}{\xi + (1-\alpha)(-\Delta)}$$

$$\implies \frac{\bar{z}^2\left(\sigma_I^2 + \sigma_S^2\right)}{2(1-\alpha)(1+i)^2}\left[\xi + (1-\alpha)(-\Delta)\right] < \left(\mu_I - \mu_{\min} - s_{\min}\right)^2.$$

By the definition of $s_{\min}$,

$$\frac{\bar{z}^2\left(\sigma_I^2 + \sigma_S^2\right)}{2(1-\alpha)(1+i)^2}\left[\xi + (1-\alpha)(-\Delta)\right] < \frac{\bar{z}^2\left(\sigma_I^2 + \sigma_S^2\right)\left(\sigma_I^2 + \sigma_S^2\right)}{(1-\alpha)(1-\alpha)(1+i)^2}$$

$$\implies \frac{1}{2}\left[\xi + (1-\alpha)(-\Delta)\right] < \frac{\sigma_I^2 + \sigma_S^2}{1-\alpha}$$

$$\implies (1-\alpha)\left[\sigma_S^2 + \sigma_I^2 + (1-\alpha)\left(\sigma_{\max}^2 - \Delta\right)\right] < 2\left(\sigma_I^2 + \sigma_S^2\right)$$
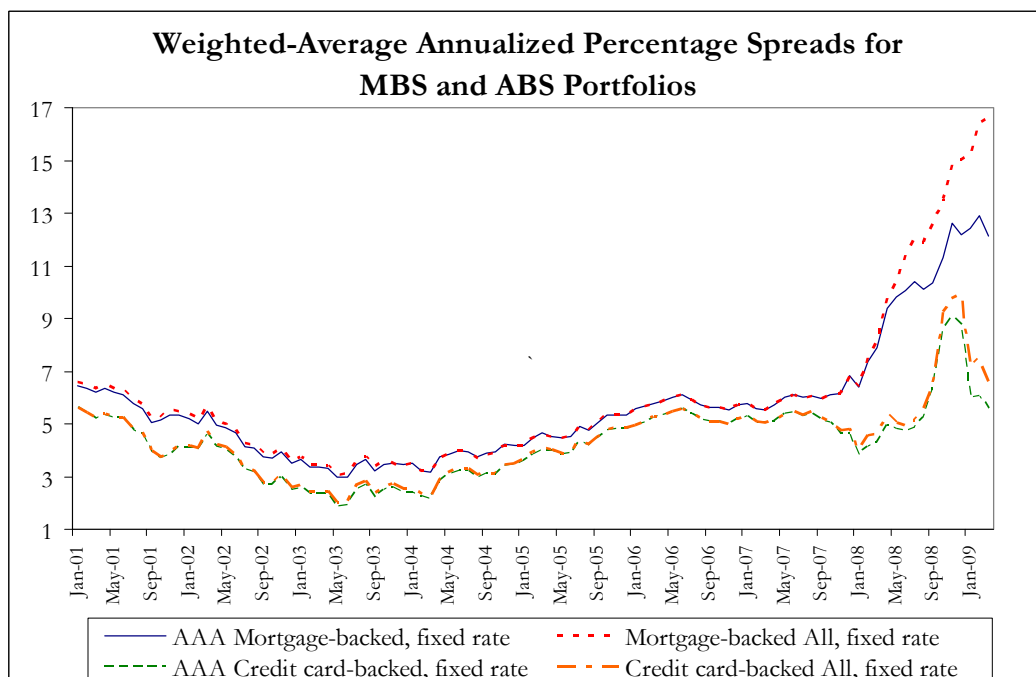
holds in general.

Figure 1

Dynamics over Time of Origination and Market Value for U.S. Commercial Paper
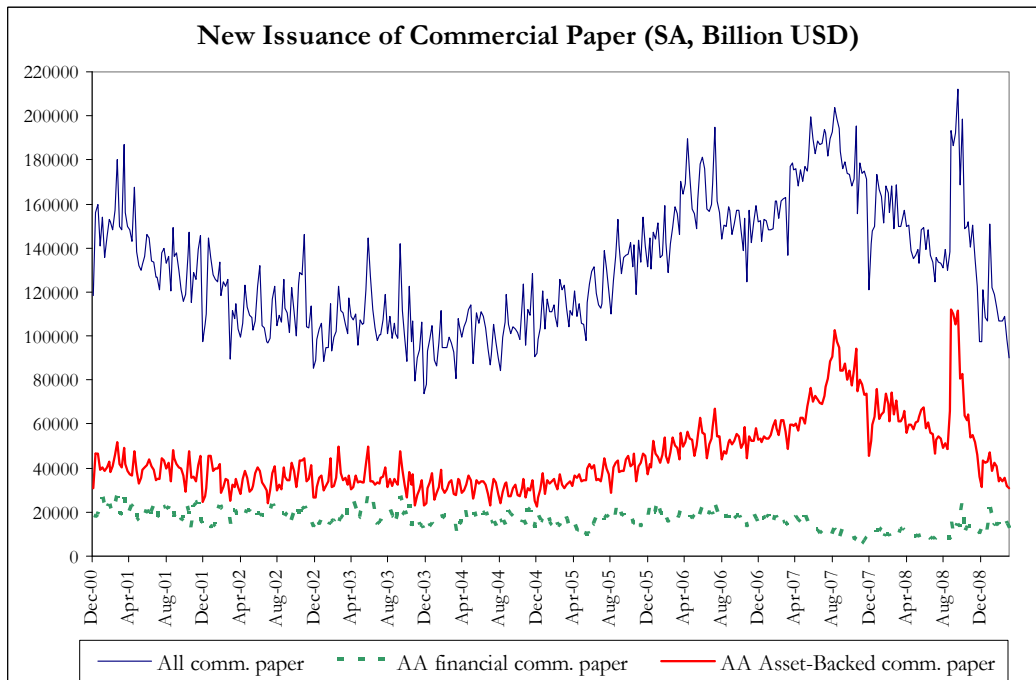
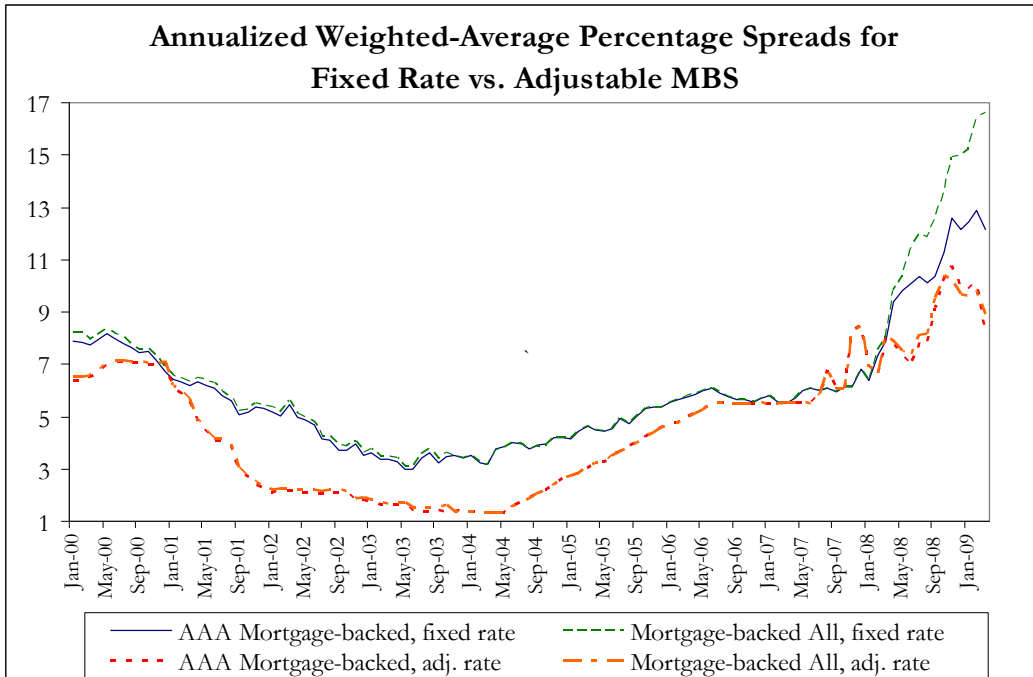Source of the data: Federal Reserve Board of Governors

Figure 2

Dynamics of Risk Spreads inAsset-Backed Markets

Source of the data: Federal Reserve Board of Governors and Bloomberg/Bear Sterns
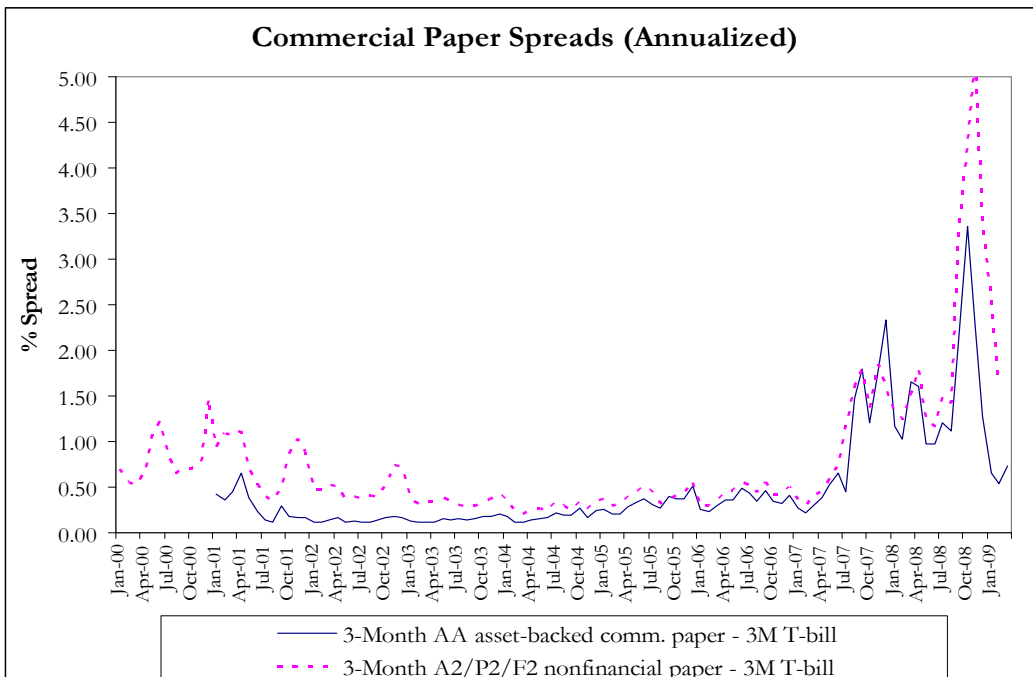
Figure 3

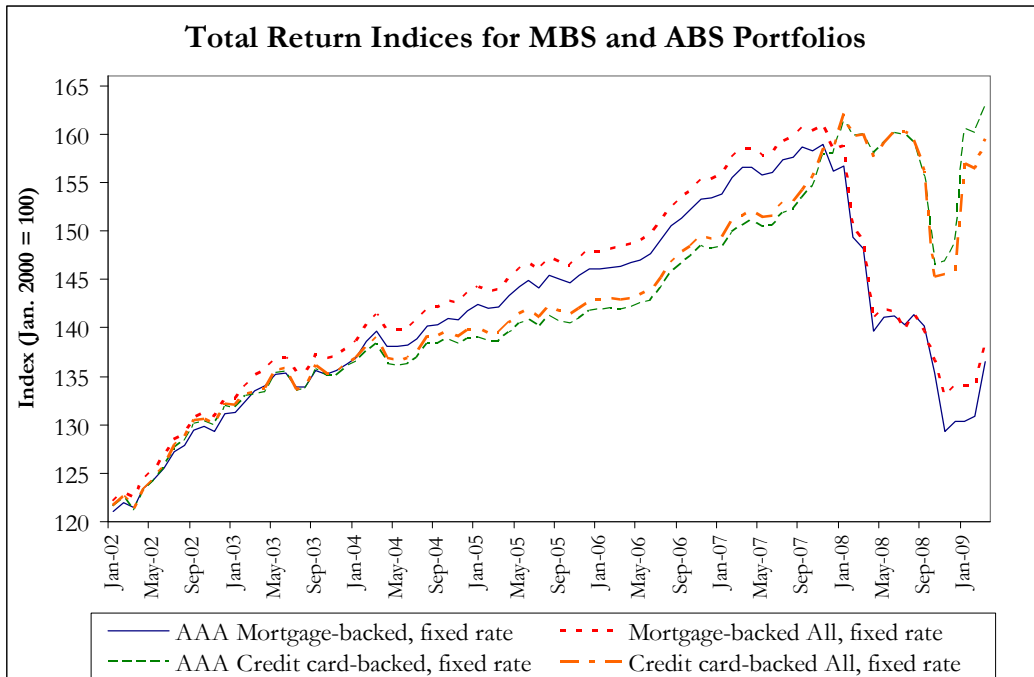**Dynamics of Risk Spreads in MBS and Commercial Paper Markets**

Panel A

Annualized Weighted-Average Percentage Spreads for Fixed Rate vs. Adjustable MBS

Legend:
— AAA Mortgage-backed, fixed rate
- - - Mortgage-backed All, fixed rate
- - - AAA Mortgage-backed, adj. rate
-·- Mortgage-backed All, adj. rate

Source of the data: Federal Reserve Board of Governors and Bloomberg/Bear Sterns

Panel B

Commercial Paper Spreads (Annualized)

Legend:
— 3-Month AA asset-backed comm. paper - 3M T-bill
- - - 3-Month A2/P2/F2 nonfinancial paper - 3M T-bill

Source of the data: Federal Reserve Board of Governors

Figure 4

Total Index Returns for MBS and ABS

Total Return Indices for MBS and ABS Portfolios

Index (Jan. 2000 = 100)

Legend:
— AAA Mortgage-backed, fixed rate
···· Mortgage-backed All, fixed rate
--- AAA Credit card-backed, fixed rate
-·-· Credit card-backed All, fixed rate

Source of the data: Bloomberg/Bear Sterns

Figure 5

Corporate Bond Spreads

AAA-Baa Seasoned Corporate Bond Spread (Annualized)

% Spread

Legend:
— Aaa-Baa default spread
···· Baa spread over 10Y Treasury
--- Aaa spread over 10Y Treasury
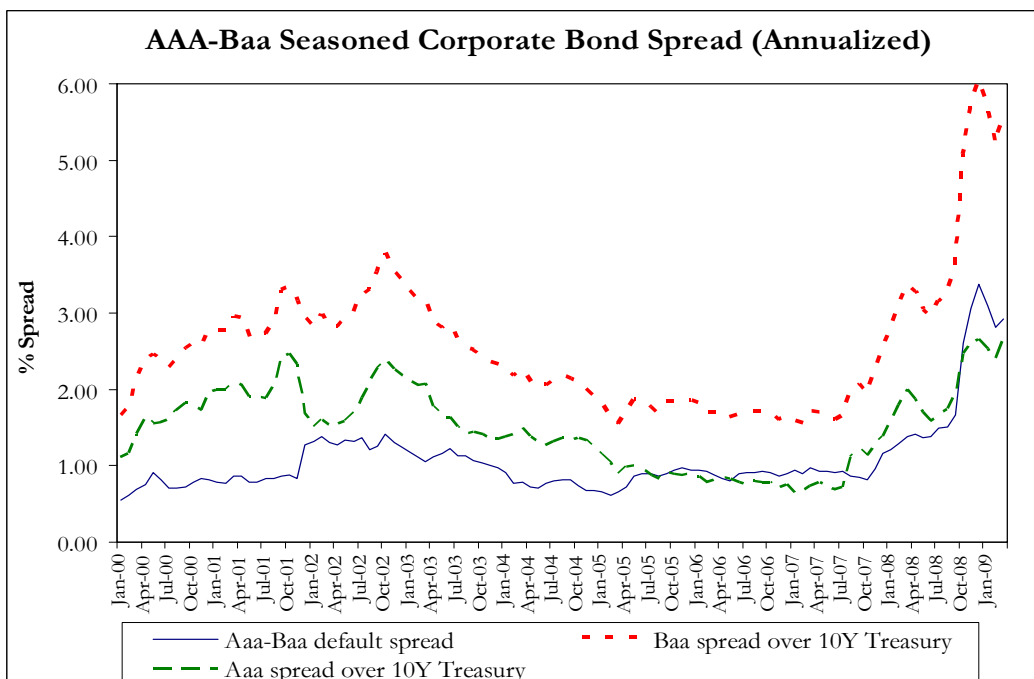
Source of the data: Federal Reserve Board of Governors and Moody's