Markets and Other-Regarding Preferences[*]

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Abstract

The predictions of economic theory in market settings are associated with the assumption that economic agents maximize a utility function defined on their own consumption. This paper demonstrates that market outcomes will be competitive under more general assumptions about preferences. With no assumptions on the size of the population, market outcomes will be competitive if other-regarding preferences satisfy conditions that are restrictive, but substantially more general than income maximization. When the economy is large, market outcomes will be approximately competitive under weaker conditions.

1 Introduction

The earliest and most striking success of experimental economics was the confirmation of textbook behavior in simple markets.[1] Experimental markets clear at competitive prices even with small numbers of agents. The results of these experiments coincide with the predictions of equilibrium theories based on optimizing behavior by actors who seek to maximize their monetary payoff. In contrast, even in the simplest bilateral bargaining settings, such as the ultimatum game, the predictions of game-theoretic models with income-maximizing actors do not agree with experimental findings.[2] These violations are systematic and widely replicated, casting doubt on the relevance of the descriptive power of standard models in strategic settings.

Researchers responded to the bargaining experiments by proposing models that placed weaker restrictions on behavior than the joint hypotheses of subgame perfect equilibrium and income maximization. Models of bounded rationality, learning, or optimization of general utility functions all capture broad patterns of bargaining behavior.[3] On the other hand, the outcomes of market experiments are consistent with behavior far more general than income maximization. I examine a model in which agents are rational, but they may have **other regarding preferences** (ORP) that depend on more than their own monetary payoff. These models do a good job organizing some of the experimental results that are at odds with standard predictions.[4]

This paper adds to the literature by characterizing a family of preferences under which the outcomes of a specific market game are competitive. The family includes classical income-maximizing preferences, but utility may also depend on both the distribution of monetary payoffs and the intentions of others. If players have these preferences, equilibrium outcomes can be consistent with experiments that confirm standard theory and with experiments that reject it. My formal model studies an auction-style environment that has been studied in the literature. Buyers and sellers have given, known valuations. They announce bid and ask prices and the market institution dictates that transactions take place at a market-clearing price. It is well known that

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[1]See, for example, Davis and Holt [9] for a textbook treatment and Smith [26] for a seminal study.

[2]These models make unambiguous predictions only under the assumption of subgame perfection.

[3]See Camerer [6] or Sobel [27] for surveys.

[4]Bolton and Ockenfels [4] and Fehr and Schmidt [11] are two examples of this literature.

when agents avoid weakly dominated strategies and maximize their monetary gains, those individuals who have positive monetary gains from trade will transact in equilibrium and those individuals who have negative monetary gains from trade will not. I show that the same conclusion holds even when agents have preferences that place non-zero weight on the monetary payoffs of others. That is, the existence of other-regarding preferences does not change the volume of trade. The presence of these agents may lead to a larger set of possible equilibrium prices. For example, in the ultimatum game, which is a special case of my model in which there is a single seller and a single buyer, my model predicts only that trade will occur, not that the proposer will obtain all of the gains from trade. Hence my model is consistent with ultimatum game experiments in which proposers typically share the surplus equally rather than generating the unequal splits predicted by theory and consistent with results that demonstrate the robustness of market equilibria.

There is a simple intuition for the results. Imagine a situation in which an agent only decides whether to trade at a market price. If his decision does not influence the market price or the volume of trade, then he has no opportunity to change the monetary payoffs of others in the economy. Therefore, his monetary payoff alone determines behavior. In the auction markets that I study agents have limited ability to change market price or trading volume. As a result, they typically behave as if they care only about their own monetary payoff. I establish the results in two contexts. In Section 5, I make no restrictions on the number of traders in the economy. Here the conclusion that market outcomes are competitive requires strong, but informative, assumptions. The results depend critically on a replacement assumption that states, loosely, an agent would prefer to make a trade that increases his monetary payoff rather than let someone else make the same transaction. If the replacement assumption fails, market outcomes will typically not be competitive. Section 4 contains a formal description of this assumption and the others needed for the main result. Section 5 shows that these assumptions are sufficient for market outcomes to be competitive and provides examples that demonstrate that market outcomes need not be competitive if any one of the assumptions fails. In Section 6, I show that some common models of other-regarding preferences satisfy these assumptions.

In Section 7, I study the equilibrium behavior of economies as they grow large. This section describes the call market in a setting where there are, potentially, a continuum of agents and formalizes the intuition that the

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equilibrium price is approximately competitive when the economy is large. This result follows from much weaker assumptions on preferences than those needed in Section 5. I require only that agents' utility is increasing in their own monetary payoff and continuous in the distribution of payoffs in the economy. When the economy is large, individual traders lose the ability to influence prices. Consequently they act to maximize their monetary payoff.

Section 2 describes related literature. Section 3 describes the basic model. Section 4 describes a general family of preferences. Section 5 states and describes the main results. Section 6 discusses how some extended preferences used in the literature satisfy the assumptions introduced in Section 4. Section 7 studies large economies. Section 8 is a brief conclusion.

2   Related Papers

Dufwenberg, Heidhues, Kirchsteiger, Riedel, and Sobel [15] consider a general-equilibrium model in which agents have other-regarding preferences.[5] They prove that (price-taking) equilibrium outcomes in economies with separable, other-regarding preferences coincide with equilibrium outcomes with classical agents who maximize the utility derived from own consumption. The results of Dufwenberg et al. [15] extend the insight of this paper to an environment much richer than the $0 - 1$ trading setting. This paper adds to the insights of Dufwenberg et al. in two ways. First, it provides conditions under which classical outcomes arise with other-regarding preferences even when agents have market power. In this paper, it may be possible to distinguish agents with other-regarding preferences from classical agents by individual behavior (for example, an altruistic buyer may make a higher bid than a buyer who seeks to maximize monetary payoff), but nevertheless the market outcome will be competitive. On the other hand, Dufwenberg et al. show that individual agents with separable other-regarding preferences behave exactly like classical agents.

In addition, this paper gives insight into the importance of large economies for the results. Large numbers are important for three reasons: First, in limit economies, equilibrium outcomes are approximately competitive under weak conditions. In contrast to Dufwenberg et al., this result depends on the size of the population. Second, one expects the set of competitive prices to shrink

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[5]In a related paper, Benjamin [3] studies the implications of assuming that an agent has other-regarding preferences in a two-player contracting game.

as the economy grows large, making both the prediction of who will trade and the prediction of the transaction price robust to the form of preferences. Third, to prove my result I impose restrictions on a pair of traders (the "competitive fringe"). Informally, the larger the economy, the more likely it is that the economy contains such a pair of traders.

Several papers point out that predictions of economic theory do not require rational behavior. Becker [2] demonstrates that budget constraints place observable limits on demand even if otherwise behavior is random. Conlisk [8] shows that Cournot behavior in an economy with free entry and in which firms have small efficient scales will be approximately competitive for a range of boundedly rational behaviors. In a context similar to my model, Gode and Sunder [14] demonstrate through simulations and experiments that optimizing behavior of selfish agents is not necessary for double-auction markets to arrive at competitive prices and efficient allocations. Gode and Sunder assume that some of their bidders have "zero intelligence." Zero-intelligence bidders are constrained to bid no more than their valuation, but otherwise behave randomly. Still, Gode and Sunder find that markets converge rapidly to competitive outcomes. These articles suggests that standard assumptions are not necessary for classical results.[6]

This paper differs from Gode and Sunder because I establish analytical results in a static auction environment, while they provide simulation results in a less constrained dynamic setting. In contrast to Becker [2], Conlisk [8], and Gode-Sunder [14], I do not relax the assumption that agents are goal oriented. My agents optimize a general utility function that includes standard income maximization as a special case. The other papers instead assume stochastic decision making constrained by feasibility or individual rationality restrictions. A final, important, difference is that the Becker [2], Conlisk [8], and Gode and Sunder [14] results operate at the aggregate level. They demonstrate that markets work well even when there is a random component to individual behavior. My model provides conditions under which individual agents choose to behave the same way as classical agents even though they have different objectives.

There is also literature that investigates whether agents with inconsistent preferences or irrational beliefs can be exploited in markets. Laibson

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[6]While Gode and Sunder [14] establish their results through simulations, earlier work of Hurwicz, Radner, and Reiter [16] and [17] provide a theoretical foundation for their findings.

and Yariv [18] study a dynamic market and show competition between sellers prevents agents with time-inconsistent, time-separable preferences from participating in welfare-reducing trades. These agents would be subject to exploitation in non-competitive markets. In a different market environment, Rubinstein and Spiegler [23] show that agents who use behavioral rules of thumb can be exploited. In a general model, Sandroni [24] provides conditions under which agents with correct beliefs about the environment will accumulate more wealth than other agents. In the equilibria in all of these papers, one can distinguish between rational and irrational behavior. In my model, it is often the case that outcomes in markets in which agents have other-regarding preferences are indistinguishable from classical outcomes.

My results generalize observations found in Bolton and Ockenfels [4], Falk and Fischbacher [10], and Fehr and Schmidt [11]. These papers introduce models of other-regarding preferences or reciprocity. They show that their models can be consistent with both experiments that challenge and confirm predictions of standard models. My paper advances the literature by extending the results to a richer class of environments and proving the result for a general class of preferences. Further, I provide both necessary and sufficient conditions for the result. While the market institution is robust, there are limits to the environments for which one can expect competitive outcomes. These papers also have no counterpart to the asymptotic result of Section 7.

Although other-regarding preferences lead to competitive outcomes, there is no guarantee that these outcomes are efficient. In fact, one can interpret the existence of non-market transfers as ways to remedy inefficiencies that arise in market equilibrium. The existence of voluntary transfers not permitted in simple market transactions can be viewed as evidence that market outcomes are not efficient (perhaps because agents have other-regarding preferences). It is important to identify environments that lead to good outcomes when agents have other-regarding preferences. Three papers that consider this issue. Bowles and Hwang [5] study optimal government policies to induce contributions to public goods when agents have other-regarding preferences. Rauh [21] describes optimal payment schemes in a contracting environment where agents have a preference for solidarity. Rob and Zemsky [22] examine how to design incentives in firms when workers may get utility from cooperation. These papers suggest that efficient trading institutions when agents have other-regarding preferences may look different from classical markets.

3 The Model

I focus on a call-market model in which there are m buyers and n sellers. Buyers demand at most one unit of a homogeneous good. Buyer B_i has valuation v_i. Sellers can produce at most one unit of the good. Seller S_j has cost c_j. For convenience, I assume that if $j < j'$, then $c_j \leq c_{j'}$ and if $i' > i$, then $v_{i'} \leq v_i$. I also follow the notational convention that $v_{m+1} = 0$ and $c_{n+1} = 1$. These valuations are known to all participants and, for convenience, taken to be elements of $[0, 1]$.[7]

Simultaneously, each buyer makes an offer for the item (interpreted as the most he will pay to purchase an item) and each seller sets an asking price (interpreted as the least she will accept to produce the item). The market clears using the following price-formation mechanism. Put the $m+n$ bids (offers and asks) in non-decreasing order, $d_1 \leq d_2 \leq \cdots \leq d_{m+n}$. The $(m+1)^{\text{th}}$ of these quantities becomes the market price, p. Buyers who bid more than p and sellers who ask less than p transact. Those traders offering p are marginal traders. If there are equal numbers of buyers bidding at least p and sellers asking no more than p, then all marginal traders transact. If there are more agents on one side of the market, then marginal agents on the short side transact, while marginal agents on the long side of the market transact with the common probability needed to make supply equal demand. Seller S_j earns a monetary payoff of 0 if she does not transact and a monetary payoff of $p - c_j$ if she does transact. Buyer B_i earns a monetary payoff of 0 if he does not transact and a monetary payoff of $v_i - p$ if he does transact.

To see how the mechanism works, suppose that $d_{m+1} > d_m$ and there are exactly k sellers who bid less than d_{m+1}. Hence there must be $m - k$ buyers who bid less that d_{m+1}. Since there are m buyers in all, there must be k buyers who bid at least d_{m+1}. In this way, the price-formation mechanism guarantees a balance between the sellers bidding no more than the market-clearing price and buyers bidding no less than the market-clearing price. The transaction price is always p.

In general, it is possible to clear the market with bidders offering more than p trading with sellers asking less than p for any $p \in [d_m, d_{m+1}]$. Using the $(m+1)^{\text{th}}$ bid gives the buyer some power to set prices in that a marginal buyer may be able to reduce his offer, lower the market price, and still trade.

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[7]I focus on a complete-information model to sharpen my results. In general, standard double-auction environments with incomplete information have multiple equilibria even when agents maximize monetary payoffs.

If the $(m+1)^{\text{th}}$ bid, d_{m+1}, is strictly greater than d_m, then $p = d_{m+1}$ and buyers who bid at least p transact with sellers who ask less than p. When $d_m = d_{m+1}$ describing the outcome is a bit more complicated because there is the possibility that marginal traders on one side of the market trade with a probability strictly between zero and one.[8]

The outcome of the call market is a price p and a specification of the set of agents who transact.[9] Denote the outcome by (T, p) where T consists of the buyers and sellers who trade at p. The outcome is market clearing because equal numbers of buyers and sellers trade and active traders consist of exactly those agents whose bids were compatible with the market price.[10] The maximum number of compatible trades are made. If players maximize an increasing function of their monetary payoffs, then equilibrium theories predict that the market price must be **competitive** in the sense that it is market clearing and agents who have strictly positive monetary gains from trade at the market price transact while those who lose from transacting do not.

Let k^* be the maximum sustainable number of transactions with strict gains from trade – that is, $c_{k^*} < v_{k^*}$, but $c_{k^*+1} \geq v_{k^*+1}$. I assume that $k^* > 0$.

Let $E(p)$ be the excess supply function – difference between the number of sellers with costs less than p and buyers with valuations greater than p. It is clear that $E(0) < 0$, $E(1) > 0$ and $E(\cdot)$ is increasing. Consequently, the values $\underline{p}$ and $\bar{p}$ given by $\underline{p} = \min\{p : E(p) \geq 0\}$ and $\bar{p} = \max\{p : E(p) \leq 0\}$ are well defined with $\bar{p} \geq \underline{p}$. Competitive prices are those $p \in [\underline{p}, \bar{p}]$. It follows from the definition that $[\underline{p}, \bar{p}] = [c_{k^*}, c_{k^*+1}] \cap [v_{k^*+1}, v_{k^*}]$.

The equilibrium of call markets is well known when agents have classical preferences. If agents maximize strictly increasing functions of their monetary payoffs, then the dominant strategy of a seller S_j is to ask c_j. It is weakly dominated for B_i to bid more than v_i. If all of the traders use undominated strategies, then the equilibrium price will be equal to the lowest price at which market demand is equal to market supply.[11] There are many

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[8]Expressions (3) and (4) in Section 7 describe the rationing rule in a general formulation that permits a continuum of traders.

[9]One could also include a specification of the trading partner of each active agent. I assume that preferences are independent of this information and suppress it in the notation.

[10]That is, all buyers who offer more than p trade with probability one; all sellers who ask less that p trade with probability one; all buyers who offer less that p trade with probability zero; and all sellers who ask more than p trade with probability zero.

[11]There are also equilibria in weakly dominated strategies. For example, all sellers can

strategies for buyers that are compatible with equilibrium. To determine one equilibrium specification, let d_k^* denote the kth lowest valuation in the population. Let all buyers with valuations less or equal to d_m^* bid their valuation, and all other buyers bid the lowest competitive price, $\underline{p}$. It is straightforward to confirm that these are equilibrium strategies (when sellers ask their valuations) and lead to the equilibrium price of $\underline{p}$.

4   General Preferences for Market Games

The outcome of a market game determines a distribution of money over the population. Standard theory posits that a player's utility is increasing in his own monetary payoff and independent of the distribution of payoffs received by the other players. I allow preferences that are more general in two ways.

I permit preferences to depend on the entire distribution of income. So, for example, a player's utility may be based on a weighted sum of the monetary payoffs to the individuals in the game, with non-zero weights on what opponents' receive. This kind of preference relationship can exhibit altruism, spite, utilitarianism, or inequity aversion, depending on the weights. Permitting distributional preferences of this kind is fully consistent with standard game theory. The second generalization is to permit preferences to depend on the strategic context. Models of this sort have been proposed by Geanakoplos, Pearce, and Stacchetti [13], Rabin [20], and Segal and Sobel [25] among others.

Recall that in a market game an outcome is a pair (T, p) where T is the set of active traders (or, more generally, a probability distribution over traders) and p is the price. Each outcome determines a distribution of monetary payoffs, $O(T, p) \in \mathbb{R}^{m+n}$, where a component of $O(T, p)$ is a monetary payment to one of the agents in the economy. Von Neumann-Morgenstern preferences of the players are defined over elements of $\mathbb{R}^{m+n}$. I permit the preferences $\succ_{B_i, \sigma^*}$ and $\succ_{S_j, \sigma^*}$ to depend on the players' beliefs about how the game will be played, which can be summarized by a strategy profile σ^*. I assume that the derived weak preference relationships are complete and transitive.

I now present some properties of preferences that are useful in subsequent sections. In the statements below, σ^* denotes a strategy profile.

Continuity (C) For all σ^* and T, if $p > c_j$, then $\{p' : (T, p') \succ_{S_j, \sigma^*} (T, p)\}$

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ask more than the highest valuation and all buyers can bid less than the lowest valuation.

is open and if $p < v_i$, then $\{p' : (T, p') \succ_{B_i, \sigma^*} (T, p)\}$ is open.

Individual Rationality (IR) For all σ^*, if $S_j \in T$ and $p < c_j$, then for all p' and T' with $S_j \notin T', (T', p') \succ_{S_j, \sigma^*} (T, p)$ and if $B_i \in T$ and $p > v_i$, then for all p' and T' with $B_i \notin T', (T', p') \succ_{B_i, \sigma^*} (T, p)$.

Gains From Trade (GT) If $c_i < v_j$ and $T = T' \cup \{B_i, S_j\}$ for $B_i, S_j \notin T'$, then there exists $p_{i,j}^*$ such that for all σ^*, $(T, p) \succ_{S_j, \sigma^*} (T', p)$ for all $p \geq p_{i,j}^*$ and $(T, p) \succ_{B_i, \sigma^*} (T', p)$ for all $p \leq p_{i,j}^*$.

Replacement (R) For all σ^*, if $p > c_j$ and T is obtained from T' by replacing $S_{j'} \in T'$ by $S_j \notin T'$, then $(T, p) \succ_{S_j, \sigma^*} (T', p)$ and if $p < v_i$ and T is obtained from T' by replacing $B_{i'} \in T'$ by $B_i \notin T'$, then $(T, p) \succ_{B_i, \sigma^*} (T', p)$.

The continuity condition (C) guarantees that a small change in the transaction price will not destroy a strict preference.

Individual Rationality (IR) states that traders would prefer not to trade than to obtain negative monetary surplus. This assumption holds by definition in experiments designed to prevent agents from making monetary losses, the typical case. While this assumption is familiar and difficult to refute in experimental settings, it is not consistent with simple acts of charity.

Gains from Trade (GT) has two parts. First it states that whenever two traders have strictly positive monetary gains from trade, there is a price at which they would be willing to trade. Second it states that a buyer (seller) who is willing to trade at price p is also willing to trade at a lower (higher) price. If traders care only about their monetary payoff, then $p_{i,j}^*$ can be any element in (c_j, v_i). In general, one can think of $p_{i,j}^*$ as a fair price at which both B_i and S_j are happy with the division of their joint surplus.

Replacement (R) states that any trader would prefer to participate in an individually rational trade if he or she does so by replacing an active trader. (R) requires that an agent would prefer to be active rather than inactive at any market price that allows gains from trade. (R) is probably the most restrictive assumption. The notion that "if I don't do this, then someone else will" may have force as an economic argument, but it is not a convincing moral position.[12]

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[12]It also may have force as a political argument. For example, North Carolina Governor Mike Easley (quoted on front page of the New York Times, October 7, 2007) argued in

An agent who buys her book at a relatively high price from an independent bookstore rather than buying from a discount supplier or who avoids products produced by exploiting workers violates Condition (R). Nevertheless, many preferences that exhibit concerns for others will satisfy (R). The main result of the paper demonstrates that agents may not be able to express these concerns in a market setting. All buyers may agree that it is undesirable for a seller to make large profits, but if each individual would rather obtain some surplus for himself rather than see another agent get some, then the market price will be high.

To evaluate Condition (R) imagine a situation (a slight extension of the formal model) in which consumers dislike the policy of a firm and which to express their feelings through a boycott. For example, all consumers may prefer an outcome in which the price of shoes is higher, provided that the shoes are not produced under sweat-shop conditions. These preferences are not sufficient for an effective boycott. In order for the boycott to work, each consumer must be willing to pay more for shoes even if they know that their behavior will not influence the supply of shoes produced in sweat shops.

Condition (R) is both necessary and sufficient for market behavior to be competitive. It is the key behavioral property to check in order to determine whether market outcomes will be competitive.

Section 6 discusses the extent to which these conditions hold in familiar models of other-regarding preferences.

I also impose a condition on the preferences represented in the economy.

Classical Fringe (CF-1) Let $p^*_{i^*,j^*}$ be a price that satisifies (GT). There exists i^* and j^* such that S_{j^*} always bids no more than $p^*_{i^*,j^*}$.

(CF-2) For all σ^*, if $B_1 \in T$ and $p > p' > c_{k^*}$, then $(T, p') \succ_{B_1, \sigma^*} (T, p)$.

(CF-1) states that (at least one) seller will make a bid low enough to be attractive to some buyer. This condition holds in any equilibrium in which agents avoid weakly-dominated strategies. Rather than use an equilibrium refinement, I assume the condition directly. Doing so simplifies proofs and, as I show in Section 5, also identifies the role of the weak dominance refinement in the classical theorem. If S_{j^*} asks no more than $p^*_{i^*,j^*}$, then the other assumptions guarantee that there is always trade. Hence (CF-1) rules out

favor of a state lottery by stating that "our people are playing the lottery. We just need to decide which schools we should fund other states' or ours."

implausible equilibria. These implausible equilibria exist even if all traders have classical preferences.

(CF-2) states that the buyer with the highest valuation prefers to trade at lower transaction prices when the price is above the lowest competitive price. This assumption rules out certain situations in which high asking prices from sellers[13] and lack of interest in monetary payoffs on the part of B_1 can lead to an equilibrium with below-competitive volume of trade. See Example 5 in Section 5 for details. The assumption permits a wide range of other-regarding behavior (and obviously holds if B_1 has classical preferences), but is restrictive in small economies since it does not allow the buyer with the highest valuation to prefer high prices (a form of transfer to active sellers).[14] The assumption is less restrictive in large economies where it is reasonable to assume that the economy consists of a competitive fringe of agents at each valuation who are motivated solely by monetary gains. There are alternative versions (CF-2) that, when combined with the other maintained assumptions, are also sufficient for the main result in Section 5. One alternative is

(CF-2′) For all σ^* and $i = 1, \ldots, k^*$, if $v_i > p > p' \geq v_{i+1}$, then $(T, p') \succ_{B_i, \sigma^*} (T, p)$.

(CF-2′) requires that many buyers prefer to buy at lower prices, but only for an interval of prices just below their own valuation. It is weaker than (CF-2) in that it restricts preferences in a plausible way only for high prices. It is stronger than (CF-2) because it imposes restrictions on more than one buyer.

5   Competitive Outcomes

This section presents a proposition that states that competitive outcomes arise in market settings provided that the assumptions in Section 4 are satisfied. The proposition asserts that the prediction of competitive outcomes does not depend on the standard assumption that agents act to maximize their monetary payoff. I pointed out in the previous section that the assumptions are strong. It is easy to describe situations in which some of the

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[13]These asking prices can be ruled out using dominance arguments.

[14](CF-2) does not require that B_1 has classical preferences. It holds for all inequity averse agents, for example.

assumptions (specifically Conditions (IR) and (R)) do not hold. So it is also useful to note that the assumptions are necessary for the conclusion.

5.1 Equilibrium Outcomes are Competitive

Let the set of competitive prices is $[\underline{p}, \bar{p}]$. The volume of trade is competitive if there are exactly k trades, where $\bar{k} \geq k^*$ and $v_k - c_k \geq 0$. The associated composition of trade in the market is competitive if the active traders are the buyers with the highest k valuations and the sellers with the lowest k valuations. These definitions allow the existence of multiple competitive volumes and compositions if $v_{k^*+1} = c_{k^*+1}$.

Proposition 1 *In a market game in which the population satisfies (CF), if preferences satisfy (C), (IR), (GT), and (R), then, in all equilibria, the volume and composition of trade is competitive. The equilibrium price is competitive.*

Proposition 1 states that one should expect the competitive volume of trade under conditions that include income-maximizing behavior as a special case. For the call-market institution, the equilibrium price will be equal to $p = \underline{p}$ if S_j asks c_j for all j. When agents have classical (net-revenue maximizing) preferences, the only undominated strategy is for each seller to ask her cost. This will not generally be the case with ORP. Hence Proposition 1 has two implications. First, it says that the classical prediction about the volume of trade in call markets holds under more general conditions. Second, it says that the classical prediction about the equilibrium price, which depends on an equilibrium refinement, does not hold when there are ORP. ORP permit the equilibrium price to vary throughout the competitive range even when a market institution selects the price most favorable to buyers. The reason for the difference is that, when ORP are allowed, sellers may prefer not to trade rather than trade at a price slightly above their cost. These sellers might dislike giving most of the surplus to a buyer. When sellers have preferences of this kind, it may be optimal for them to ask strictly more than their cost and thereby raise the equilibrium price. Their ability to do this is constrained, however, because in equilibrium the volume of trade must be competitive. One would expect that when there are a large number of traders on at least one side of the market, the interval $[\underline{p}, \bar{p}] = [c_{k^*}, c_{k_1^*}] \cap [v_{k^*+1}, v_{k^*}]$ would shrink to a point. Hence in large economies the equilibrium price

with ORP will converge to the price that clears the market when all agents maximize monetary payoff.

The Appendix contains a proof of the proposition (and proofs of the remaining propositions in the paper), but the intuition is instructive and straightforward. (CF) and (GT) guarantee that there is at least one trade in equilibrium. Once it is known that the market is open, all traders with positive gains from trade at the equilibrium price must be active. To see this, consider a situation in which there is a buyer with positive gains from trade but is inactive in a putative equilibrium. This buyer can raise his asking price and trade with positive probability at the market price. If doing so leads to the same volume of trades, the replacement assumption guarantees that the deviation is attractive. If raising the asking price increases the number of transactions, then there is excess supply at the putative equilibrium price. Similarly, competition between active sellers will insure that the price is low enough so that the buyer will be willing to enter the market.

5.2   Necessity of Conditions

The assumptions are necessary in the sense it is possible to find examples in which the conclusion of Proposition 1 fails when only one of the assumptions does not hold. Below I give examples of economies in which all but one of the agents is a risk-neutral income maximizer while the other agent's preferences satisfy all but one of (C), (IR), (GT), (R), and (CF). In these examples, there is an equilibrium in undominated strategies with a non-competitive outcome.

Example 1 *Noncompetitive Outcomes without (GT). If (GT) fails, then the only equilibrium could be no trade, even when there are monetary gains from trade. For example, in a model in which there is a single buyer and a single seller and $b_1 > c_1$, if one of the agents always prefers no trade to trade at any price between the valuations and the other agent maximizes net income, there will be no trade in equilibrium.*

Example 2 *Noncompetitive Outcomes without (IR). If (IR) fails, the price may fall outside the competitive range. Suppose the market contains six traders, three buyers and three sellers. Two sellers have valuation 0 while the third has valuation $.2$; all maximize monetary payoffs. Two buyers have valuations $.4$ and 1, respectively, and maximize monetary payoffs. The final buyer has valuation $.1$. If this agent also had classical preferences, the*

competitive equilibrium would involve two trades at the price .2. The buyer with valuation .1 may view it as unfair for the other buyers to obtain a relatively large payoff and be willing to take a loss to lower inequality (by bidding more than .2). In such an environment, the equilibrium may involve two trades, but at a price outside the competitive range and with the buyer with valuation .1 active instead of the buyer with valuation .4.

Example 3 *Noncompetitive Outcomes without (R). Suppose there are three traders, an income-maximizing buyer with valuation 1, an income-maximizing seller with valuation .4, and a seller with valuation 0 who prefers to trade at prices greater than or equal to .5, but otherwise prefers not to trade (independent of whether the other agents trade). Hence (R) does not hold. The unique equilibrium outcome in undominated strategies involves the first two agents trading at the price .4. Hence the trading volume is the same as in the competitive equilibrium, but the identity of traders is different. If several agents have preferences that do not satisfy (R), then the equilibrium volume of trade will typically differ from competitive levels.*

Example 4 *Noncompetitive Outcomes without (C). The typical consequence of a failure of (C) is that the replacement assumption loses its power. Consider an economy in which there are two sellers with valuation 0 and two buyers who maximize their monetary payoffs with valuations .5 and 1. Suppose that the buyers bid their valuations and the sellers both bid .6. The sellers will each trade with probability one half with the higher valuation buyer at the price .6. This would be an equilibrium if the sellers strictly prefer to trade exclusively at the price .6, but prefer not to trade at lower prices. These preferences violate (C), but could satisfy the other assumptions.*

Example 5 *Noncompetitive Outcomes without (CF). Suppose that there are two sellers with cost 0 and two buyers with valuations .9 and 1, respectively. The sellers ask 0 and 1 while the buyer with valuation .9 offers 0 and the buyer with valuation 1 offers .8. .8 will be the market clearing price, but the market volume is below the competitive level. Can this be an equilibrium? The seller asking 1 can only transact if she bids 0; doing so lowers the market price to 0, so the deviation leaves her monetary payoff unchanged. It does change the distribution of payoffs, but none of the assumptions from Section 4 imply that the deviation would increase her payoffs. That is, it is possible to specify preferences so that the seller bidding 1 does not want to*

deviate. The buyer offering .8 can lower the transaction price by offering less. (CF) guarantees that this deviation will be attractive, but if (CF-2) failed, the constructed outcome could be an equilibrium.[15]

5.3 Consequences of the Main Result

Proposition 1 includes the standard ultimatum game as a special case ($n = m = 1$ and $c_1 = 0$, $v_1 = 1$). Think of the buyer's offer as the proposal. The seller accepts the proposal with a bid that is less than or equal to the buyer's offer and rejects it by asking for more. The unique equilibrium in undominated strategies is to trade at zero. Proposition 1 implies that if players have ORP, then it is still an equilibrium to trade, but that the equilibrium price may be positive. In fact, depending on preferences, the equilibrium price may be as high as the "fair" price identified in (GT), $p_{1,1}^*$. Two things may prevent the classical outcome (in which the proposer receives the entire gains from trade) from arising in ultimatum games. First, the buyer may believe that out of fairness or some other consideration, it is not appropriate to take all of the surplus for himself. Second, the seller may believe out of spite or some other consideration, that it is unacceptable to agree to an offer that gives too great a share to the buyer.

Proposition 1 makes a precise prediction about the volume of trade. It fails to make a precise prediction about the equilibrium price when $\underline{p} < \bar{p}$. I view this as a strength of the model. Recall that the precise prediction requires further assumptions (the restriction to undominated strategies) even in the classical case. Without an equilibrium refinement, the equilibrium price is determined even when all agents maximize monetary payoffs. Furthermore, empirical evidence suggests that the prediction that markets clear at $\underline{p}$ does not describe behavior.

It is worthwhile stating results for a special case of the model. A **homogeneous market game** is one in which all buyers have the same valuation, $v_i = 1$ for all i and all sellers have the same valuation, $c_j = 0$ for all j.

Corollary 1 *In a homogeneous market game in which the population satisfies (CF), if preferences satisfy (C), (IR), (GT), and (R), then in all equi-*

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[15]If one only examines equilibria in which the seller plays undominated strategies, one can weaken (CF). In that case, no seller will ask more that $\max p_{i,j}$ and the self interested buyer can be any B_{i^*} with $v_{i^*} > \max p_{i,j}$.

libria, the volume of trade is $\min\{m, n\}$. *If* $m > n$, *then the market price is* 1. *If* $m < n$, *then the market price is* 0.

Corollary 1 is an immediate consequence of Proposition 1. When $n = 1$, the homogeneous market game reduces to the proposer-competition game studied experimentally by Prasnikar and Roth [19]. They analyze a game in which there is a single seller and m buyers. The buyers are assumed to have the same valuation. The buyers each make an offer. The seller can either reject all offers or trade at the highest one. This game is equivalent to a homogeneous market. In their experiments when $m > 1$ buyer competition permitted the sole seller to obtain all of the gains from trade in equilibrium.[16] Fehr and Schmidt [11] present a special case of Corollary 1 for inequity averse agents.

6 Preferences that Satisfy The Assumptions

In this section, I discuss the relationship between the assumptions of Section 4 and several models of other-regarding preferences. This discussion is relevant to the study of markets with relatively few traders. The next section demonstrates that the conclusion of Proposition 1 holds approximately in large economies under much weaker assumptions on preferences.

I confine myself to two functional forms found in the literature.[17]

6.1 Distributional Preferences (Fehr-Schmidt)

Fehr and Schmidt look at an environment in which there are N players indexed by $i = 1, \ldots, N$ (here $N = n + m$, the total number of agents in the economy). If the monetary outcome is $(x_1, \ldots, x_N)$, then the utility function of player k is given by

$$U_k(x) = x_k - \frac{\alpha_k}{N-1} \sum_l \max\{x_l - x_k, 0\} - \frac{\beta_k}{N-1} \sum_l \max\{x_k - x_l, 0\} \quad (1)$$

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[16]Fischbacher, Fong, and Fehr [12] introduce competition among responders in the ultimatum game. In their experiments, the proposer makes low offers and these offers are accepted.

[17]An earlier version of the paper discussed conditions under which the functional forms presented by Andreoni and Miller [1], Bolton and Ockenfels [4], and Charness and Rabin [7] satisfy my assumptions.

where $\beta_k < \alpha_k$ and $0 \leq \beta_k < 1$ for all k. The first term in (1) is the monetary payoff to player k. The functional form allows the possibility that the agent experiences disutility from having less income than some agents (this is the second term) and from having more income than other agents (this is the third term). The restrictions on α_k and β_k guarantee that the agent prefers to be a unit richer than another agent than one unit poorer ($\alpha_k \geq \beta_k$) and prefers an extra dollar of income even taking into account the distributional impact ($\beta_k \leq 1$).

Fehr and Schmidt constrain the monetary outcomes to be non-negative (their applications are equivalent to models in which buyers all have valuation 1, sellers all have cost 0, and offers and asks are constrained to be in the unit interval). Consequently, (IR) follows in Fehr and Schmidt by the construction of the game. In my setting, (IR) need not hold if preferences satisfy (1). It is straightforward to construct examples in which an agent will be willing to take a monetary loss to equalize the distribution of income. This behavior, however, is typically forbidden in laboratory experiments. Hence confirming that the utility function (1) satisfies (C), (GT), and (R) demonstrates that Proposition 1 is useful for interpreting experimental evidence. (CF-1) is a restriction on behavior that need not hold even for agents who maximize their monetary payoffs (but will hold generally if players are restricted to undominated strategies).

Proposition 2 *Preferences represented by the Fehr-Schmidt function (1) with $\alpha_k \geq \beta_k$ and $\beta_k \leq 1$ for all k satisfy (C), (GT), (R), and (CF-2).*

6.2 Preferences for Fairness (Segal-Sobel)

The assumptions hold for models that exhibit intrinsic reciprocity or concern for fairness. Segal and Sobel [25] give conditions under which preferences over strategies can be represented as a weighted average of the monetary payoffs of traders in the population:

$$U_k(x; \sigma^*) = x_k + \sum_{l \neq k} a^k_{l,\sigma^*} x_l, \tag{2}$$

where the a^k_{l,σ^*} are weights that can depend on the strategic context (as represented by the strategy profile σ^*) and the distribution x of monetary gains.

Segal and Sobel demonstrate that these preferences place few limits on equilibrium outcomes unless they are constrained. The weights $\{a_{l,\sigma^*}^k\}$ are **reciprocal** if they are continuous in σ^* and buyers place non-negative weights on the utility of those traders who bid less that the market price and non-positive weight on the utility of those traders who bid more than the market price, while sellers place non-positive weights on the utility of those traders who bid less that the market price and non-negative weight on the utility of those traders who bid more than the market price. These weights respond to the kindness of strategies as compared to market behavior. For example, they require that a buyer interprets a low asking price from a seller as "nice" because it puts a downward pressure on prices.

Proposition 3 *Preferences represented by the utility function (2) satisfy (C), (IR), (GT), (R), and (CF-2) provided that the weights a_{l,σ^*}^k are reciprocal.*

7 Large Economics

Section 5 presented conditions on preferences under which equilibrium outcomes are competitive. These conditions include some interesting families of ORP as special cases, but they are restrictive. In this section I show that in large economies equilibrium outcomes are competitive under weaker conditions. The result is simple, but it requires a careful description of the call-market that generalizes the finite model studied thus far.

7.1 Call Markets for Large Economies

An abstract economy consists of a measure on an index set, $\mathcal{A}$.[18] Associate with each $a \in \mathcal{A}$ the characteristics of the agent: his valuation $D(a) \in [0,1]$, his type $\tau(a) \in \{0,1\}$ and his utility function $U_a(\cdot)$. a is the name of the agent. If $\tau(a) = 0$, Agent a is a seller. If $\tau(a) = 1$, Agent a is a buyer. An economy is a measure α on the Borel subsets of $\mathcal{A}$. If α is a simple measure (a counting measure with finite support), then α describes a simple (finite) economy. A measure α^* describes a limiting economy if there is a sequence of simple measures $\{\alpha_l\}_{l=1}^{\infty}$ that converges to α^* in the topology of weak convergence of measures.

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[18]Without loss of generality we may assume that $\mathcal{A} = [0,1]$.

An outcome is a function $x : \mathcal{A} \to \mathbb{R}_+$, where $x(a)$ is the earnings of Agent a. Given an economy α, an outcome x induces a measure μ on $\mathbb{R}_+ \times [0, 1] \times \{0, 1\}$ where $\mu(Y) = \alpha(\{a : (x(a), D(a), \tau(a)) \in Y\})$.

Agents have utility functions of the form $U_a(x(a), \mu)$ defined over their own monetary payoff and the outcome. I assume that $U_a(\cdot)$ is strictly increasing in its first argument and continuous.

In a call market, the strategy $s(a)$ of an agent a is an asking price in $\mathcal{A}$. The strategies of the agents and α give rise to a measure ρ on the subsets of $\mathcal{A}$, where $\rho(X) = \alpha(\{a : s(a) \in X\})$. ρ is the *bid distribution* induced by the strategy. It is also useful to define $\rho_t(Z) = \alpha(\{a : s(a) \in Z \text{ and } \tau(a) = t\})$ for $t = 0$ and 1. The measures ρ_t are also defined on measurable subsets of $\mathcal{A}$. Let ρ_t^* denote the corresponding measures derived from truthful bidding, $s^*(a) \equiv D(a)$.

Given α and strategies s, the call-market institution determines a market-clearing price p^* and a distribution of payoffs. The market-clearing price p^* is defined to be $p^* = \inf\{p : \rho([0, p]) \geq \rho_1([0, 1])\}$. That is, p^* is the smallest price at which the mass of bidders below the price is at least as great as the mass of buyers.

The competitive prices are

$$\{p : \rho_1^*([p, 1]) \geq \rho_0^*([0, p)) \text{ and } \rho_1^*((p, 1]) \leq \rho_0^*([0, p])\}.$$

The set of competitive prices is always a non-empty interval. There will be a unique equilibrium price if ρ_t^* is atomless for $t = 0$ and 1.

The allocation rule determines the distribution of payoffs. As in the finite case, buyers who bid more than p^* receive the item with probability one and pay the price p^* and those who bid less than p^* never receive the item. So, if $D(a) = d$ and $\tau(a) = 1$ (a is a buyer with valuation d), then the agent's monetary payoff is $d - p^*$ if $s(a) > p^*$ and the agent's monetary payoff is 0 otherwise. Similarly, sellers who bid strictly less than p^* earn $p^* - d$ and those who bid more than p^* earn 0. Those bidding exactly p^* are rationed to clear the market. The probability that a type t agent trades is γ_t, which is given by:

$$\gamma_0 = \begin{cases} 1 & \text{if } \rho_1([p^*, 1]) \geq \rho_0([0, p^*]) \\ \frac{\rho_0([0,p^*]) - \rho_1((p^*,1])}{\rho_1(\{p^*\})} & \text{otherwise} \end{cases} \tag{3}$$

and

$$\gamma_1 = \begin{cases} 1 & \text{if } \rho_0([0, p^*]) \geq \rho_1([p^*, 1]) \\ \frac{\rho_1([p^*,1]) - \rho_0([0,p^*])]}{\rho_0(\{p^*\})} & \text{otherwise} \end{cases}. \tag{4}$$

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7.2 Equilibrium Outcomes are Competitive

I show that the set of equilibrium prices of large economies must converge to a competitive price of an appropriate limit economy. The result follows because in large economies the action of a single agent has little effect on the aggregate excess demand of the economy.

The proposition requires an assumption that is analogous to the Competitive Fringe assumption that I used for finite markets.

For $\delta > 0$, a strategy s is δ-**rich** if, for all subsets of the form $I = [z, z + \delta] \subset [0, 1]$, the bid distribution induced by s satisfies $\rho(I) > 0$. This assumption will hold if an arbitrarily small fraction of the sellers in the population, having valuations uniformly selected from $[0, 1]$, seek to maximize their material payoffs and use the strategy of bidding their cost. The assumption would also hold if we added trembles to strategies so that, with positive probability, a subset of agents made any bid in $[0, 1]$. For these reasons, the richness assumption is not restrictive in large economies. As in the finite case, it is possible to weaken the assumption if one assumes instead that agents avoid weakly dominated strategies.

Proposition 4 *Let $\{\alpha_l\}_{l=1}^{\infty}$ be a sequence of economies converging to a non-atomic measure α^*. Given any $\epsilon > 0$ there is a L and $\delta_0 > 0$ such that if $l > L$ and $\delta \in (0, \delta_0)$, then any δ-rich equilibrium price of α_l, must be within ϵ of a competitive price of α^*.*

Proposition 4 states that call market equilibrium are approximately competitive when agents have other-regarding preferences. The result requires that preferences are continuous and strictly increasing in own consumption and the market satisfies a richness condition. The richness condition guarantees that small variations in bids can make a small change in the market price. Hence agents have negligible impact on prices. Agents also have negligible impact on the distribution of monetary gains, hence even though an individual may care about the distribution of income in the economy, he or she cannot do much better than optimize own monetary payoff.

8 Conclusion

Several factors contribute to the finding that competitive outcomes arise even when agents have other-regarding preferences: anonymity, limited market power, and large numbers.

The trading institution requires that transactions are anonymous. In a bilateral bargaining environment, an agent can direct his kindness to a specific individual, perhaps by offering a generous tip in addition to paying a market price. In a call market, trades must take place at a single price, consequently these transfers are not feasible. We frequently observe extra-market transfers (for example, charitable donations). These activities can be viewed as expressions of altruistic tendencies that cannot be expressed in price-mediated market transactions.

More generally, traders have limited market power in the auction setting of this paper. The ability of a trader to set the price is severely constrained by the behavior of other traders since the market price must be between the mth and $(m + 1)^{\text{th}}$ bids of the other traders. Yet a trader can influence the welfare of others only by changing the price or by changing marginally the set of active traders. The paper argues that the ability to do either of these things will be small in arbitrary economies in which assumption (R) holds or in large economies under more general conditions.

The assumption that the market is large plays an important role in the result. A direct effect of a large population is that unilateral actions have a vanishingly small impact on outcomes under mild regularity conditions. One agent can influence the identities of traders only by addition or subtracting a trading pair or replacing an active trader. Typically (under richness), the mth and $(m + 1)^{\text{th}}$ bids of the other traders are close together in large economies, so changing one's offer has a small impact on the market price. Finally, the set of competitive equilibrium prices (for the classical economy) is likely to be small when the economy is large.

I would expect the qualitative results of this paper to hold under mild assumptions under any institution in which a large number of agents with limited market power transact anonymously.

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Appendix

Proof of Proposition 1 Fix an equilibrium. Let k^* be the number of traders with strict gains from trade – that is, $c_{k^*} < v_{k^*}$, but $c_{k^*+1} \geq v_{k^*+1}$. I claim that there will be at least k^* trades in equilibrium. In order to obtain a contradiction, assume that there are $1 \leq k < k^*$ transactions at the market-clearing price p.

Note that there is at least one trade in equilibrium. Since S_{j^*} asks no more than p^*_{1,j^*} for some j, $p \leq p^*_{i^*,j^*}$ (GT) implies that B_{i^*} would bid at least p in order to make a profitable trade. Consequently, there must be at least one trade in equilibrium.

Now let S_j be the seller who trades at the smallest probability among $\{S_1, \ldots, S_{k^*}\}$. Since $k < k^*$, this seller cannot trade with probability one. Suppose first that $p \leq c_{k^*}$. In this case, there can be at most $k^* - 1$ sellers whose asking prices are less than p by (IR). On the other hand, there are at least k^* buyers who have positive gains from trade at p. At least one of these, B_i, must be bidding no more than p (by the definition of the market-clearing price) and trading with probability less than one. There are two cases.

1. B_i is bidding exactly p and trading with probability less than one.

   In this case, B_i would be able to increase his payoff by bidding slightly more. To see this, note first that increasing his bid will not increase the volume of trade. Hence by (R) he would prefer to trade with probability one at p than at probability less than one. By (C) this preference will be preserved at a slightly higher price.

2. There is an inactive buyer with valuation greater than p who is bidding less than p.

   If bidding p does not increase the volume of trade, then this buyer would be able to increase his payoff by bidding slightly more than p. Doing so will either be beneficial by (R) and (C). If bidding p does generate a new trade, then sellers must be rationed at the putative equilibrium (two or more trading with probability less than one at p). p must be equal to the valuation of these sellers (else one seller would gain by cutting her asking price by (R) and (C)). However, if p is equal to the valuation of the rationed sellers, then the inactive buyer would gain by asking p by (GT). Hence $p > c_{k^*}$.

If $p > c_{k^*}$, then there are four cases.

1. All agents asking p transact with probability one; some seller asks p.

   S_j must be asking more than p. She would gain by asking slightly less than p by (R) and (C).

2. All agents asking p transact with probability one; no seller asks p.

   If there are m bidders below p, then B_1 must be active and would gain by lowering his bid (and the market-clearing price) by (CF-2). If there are fewer than m bidders below p, then there is more than one buyer bidding p and trading with probability less than one. If p is equal to the valuation of these buyers, then S_j can gain by bidding less than p. S_j will then trade with probability one and will be better off by (GT). If p is less than the valuation of either buyer offering p, then that buyer could increase his probability of trading with a small change in price, which is attractive by (C) and (R).

3. A buyer asks p and transacts with probability less than one.

   There must be two or more buyers at p and all sellers asking p transact with probability one. By (R) and (C), this means that the price must be equal to the valuation of these buyers (otherwise one would bid more). Since $p > c_j$, S_j can increase her payoff by bidding p by (GT).

4. A seller asks p and transacts with probability less than one.

   If there is only one seller asking p, then there are exactly m lower bids. By (CF), B_1 would improve his payoff by bidding less than p because

doing so lowers the market-clearing price. If there is more than one seller asking p, then S_j (who is bidding at least p) can increase her payoff by lowering her asking price. This deviation will allow her to trade with probability one without changing the market-clearing price or the volume of trade. Consequently it is attractive by (R) since $p > c_j$.

I have shown that there must be at least k^* trades in equilibrium. Here are equilibrium strategies that support the competitive outcome. Given a competitive price p^* call buyer B_i pivotal with respect to p^* if $b_i = \min\{b_k : b_k \geq p^*\}$ and seller S_j pivotal if $c_j = \max\{c_k : c_k \leq p^*\}$. Given p^* assume that non-pivotal agents bid their valuations and pivotal agents bid p^*. Any deviation that creates trades will violate (IR). Any deviation that destroys trade will be unattractive by (GT). ∎

Proof of Proposition 2 (C) follows directly from the functional form. To verify (GT), let $p^*_{i,j} = (c_j + b_i)/2$. Fix an outcome (T, p). Let x_k be the monetary surplus of Agent k in this outcome. Assume that S_j and B_i are not in T and consider an outcome in which these agents trade at price $p \geq p^*_{i,j}$. At (T, p), S_j's utility is

$$-\frac{\alpha_j}{N-1} \sum_{l:x_l>0} x_l. \tag{5}$$

If instead S_j and B_i trade at p, then S_j's utility is

$$p - c_j - \frac{\alpha_j}{N-1} \sum_{l:x_l>p-c_j} (x_l - (p-c_j)) - \frac{\beta_j}{N-1}\left((2p - c_j - v_i) + \sum_{l:p-c_j>x_l, l\neq i} (p - c_j - x_l)\right). \tag{6}$$

The first term is the monetary surplus S_j obtains directly; the second term is the loss associated with earning less than some others; the third term is the loss associated with earning more than the trading partner (the specification of $p^*_{i,j}$ guarantees that $p - c_j \geq v_i - p$); and the fourth term is the loss associated with earning more than some others.

(GT) holds if (6) is greater than (5), but (6) minus (5) is greater than or equal to

$$p - c_j - \frac{\beta_j}{N-1} \sum_{l:p-c_j>x_l} (p - c_j) \tag{7}$$

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since $\alpha_j \geq 0$ and (7) is non-negative since $\beta_j \leq 1$ and the sum has at most $N-1$ terms.

To verify (R), (5) describes the utility of S_j prior to the replacement. Upon replacing $S_{j'}$, S_j's utility is:

$$p-c_j-\frac{\alpha_j}{N-1}\sum_{l:x_l>p-c_j,j\neq j'}(x_l-(p-c_j))-\frac{\beta_j}{N-1}\left((p-c_j)+\sum_{l:p-c_j>x_l,j\neq j'}(p-c_j-x_l)\right).$$
(8)

(R) holds if (8) is greater than (5), but (8) minus (5) is greater than

$$p-c_j-\frac{\beta_j}{N-1}\sum_{l\neq j}(p-c_j),$$
(9)

which is non-negative.

It is straightforward to check that utility is strictly decreasing in p for all active buyers, which establishes (CF-2). ∎

Proof of Proposition 3 (C) follows by definition. To confirm (GT), consider a seller who can deviate and thereby trade at the market price with a buyer who had been excluded. If this trade generates positive monetary payoff for the seller, then it will increase her utility because her trading partner's bid must be greater than or equal to the market price (and hence the seller places non-negative weight on the monetary payoff of the buyer). The same argument applies for buyers. To establish (R), consider what happens if S_j deviates in order to replace $S_{j'}$. This behavior has only two effects on the outcome: $S_{j'}$'s surplus goes down and S_j's surplus goes up. Since $\alpha_{j',\sigma^*}^{j} \leq 0$ when $S_{j'}$ is asking no more than the market price, the deviation must be attractive to S_j. Note that if a buyer is bidding more than his valuation and trading, lowering his bid can: increase his monetary payoff; reduce the market clearing price; lead the bidder to transact in addition to or in place of an existing trader. None of these possibilities can reduce the buyer's utility and the first one must increase it. Hence (IR) holds. A similar argument establishes (CF-2). ∎

Proof of Proposition 4 Suppose the conclusion of the proposition is false. In that case, there would be an $\epsilon > 0$ and a sequence of equilibrium prices $\{p_k\}$ that contained a subsequence converging to $p \notin [\underline{p} - \epsilon, \bar{p} + \epsilon]$. For concreteness, assume that $p < \underline{p} - \epsilon$. Since α_l converges to α^* there is a

L such that for $l > L$ there exists an agent a_l in the support of α_l such that $\tau(a_l) = 1$ (that is, a_l is a buyer), a_l does not trade, and $D(a_l) \geq p$. By changing his offer to a price slightly greater than p, a_l will increase the surplus from trading by at least $\epsilon/2$ and, for sufficiently high l make only a small change in the outcome. By continuity of preferences, this change is attractive. This is a contradiction to the assumption that p_l is an equilibrium price. A similar argument (applied to a seller) establishes a contradiction if there were a sequence of equilibrium prices that converged to $p > \bar{p} + \epsilon$. This argument requires richness to guarantee that a seller can trade when she lowers her asking price to just below p_l. ∎