

# House Price Booms and the Current Account

Klaus Adam  
Mannheim University and CEPR

Albert Marcet  
London School of Economics, CEP and CEPR

Pei Kuang  
University of Frankfurt

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## Abstract

A simple open economy asset pricing model can account for the house price and current account dynamics in the G7 over the years 2001-2008. The model features rational households, but assumes that households entertain subjective beliefs about price behavior and update these using Bayes' rule. The resulting beliefs dynamics considerably propagate economic shocks and crucially contribute to replicating the empirical evidence. Belief dynamics can temporarily delink house prices from fundamentals, so that low interest rates can fuel a house price boom. House price booms, however, are not necessarily synchronized across countries and the model correctly predicts the heterogeneous response of house prices across the G7, following the fall in real interest rates at the beginning of the millennium. The response to interest rates depends sensitively on agents' beliefs at the time of the interest rate reduction, which are a function of the prior history of disturbances hitting the economy. According to the model, the US house price boom could have been largely avoided, if real interest rates had decreased by less after the year 2000.

JEL Class. No: F41, F32, E43

## 1 Introduction

We present a stylized open economy asset pricing model with rationally investing households that can quantitatively replicate the house price dynamics in the G7 economies over the years 2001-2008, as well as the dynamics of the current account.

Our model predicts that boom and bust dynamics in house prices are triggered by macro-fundamentals, e.g., changes in real interest rates or housing preferences, but not necessarily tightly linked to these. And similar to the data,

the resulting price booms are associated with an expansion of the housing stock, a deterioration of the current account, and a consumption boom, while the subsequent house price declines are associated with current account improvements and subdued consumption.

To study the relationship between house price movements, housing construction, consumption and international borrowing, we generalize the closed economy asset pricing models developed previously in Adam and Marcet (2010a, 2010b) and Adam Marcet and Nicolini (2010) along three dimensions. First, we consider a setting with two assets, namely a domestically traded risky asset - the housing stock - and an internationally traded riskless bond. Second, we incorporate as a new feature a borrowing constraint that limits household leverage and the overall amount of borrowing, following Kiyotaki and Moore (1997). Third, we consider a production economy with endogenous asset supply by explicitly incorporating a construction sector. Despite these extensions, our model is relatively parsimonious.

The quantitative success of the model crucially rests on the assumption that we allow for households that are uncertain about how house prices relate to economic fundamentals. Similar to academic economists, our households fail to know exactly what is the correct model linking house prices with fundamentals. We incorporate this feature by putting to work the concept of ‘internal rationality’, as developed previously in Adam and Marcet (2010a, 2010b). Internally rational investors are utility maximizers in the standard sense and entertain fully specified and dynamically consistent beliefs about all payoff-relevant variables that are external to them (including competitive market prices). Internally rational agents, however, do not fully understand how market prices are formed, so that their subjective probability distribution about prices may not be exactly equal to the true equilibrium distribution. Agents’ beliefs, however, are assumed to be close to the rational expectations equilibrium (REE) beliefs typically attributed to agents in the literature.

Given agents’ subjective uncertainty about price, optimal behavior implies that they update beliefs about house price behavior by applying Bayes’ rule to market outcomes. Agents’ price beliefs thus become a state variable and (Bayesian) learning giving rise to a dynamic feedback between price beliefs and actual price outcomes. As we show, this generates a considerable amount of additional propagation in the model and can fuel boom and bust dynamics in housing prices. This is of interest because the momentum in house price changes that can be observed in the data has proven difficult to explain with the help of rational expectations models, see Glaeser and Gyourko (2006). Therefore, Glaeser et al. (2008) previously suggested that models of learning can help resolving this empirical puzzle.

Learning about price dynamics turns out to be important for explaining the persistent rise and fall in house prices occurring over the years 2001-2008 in the G7. The learning model thereby suggests that the strong fall in real interest rates after the year 2000 contributed significantly to the subsequent housing boom in some of the G7 economies. In line with the empirical evidence, however, the model predicts that these movements are not necessarily synchronized across

countries. While some G7 countries experienced house price booms (US, UK, Canada, Italy, France), they did so to very different degrees, and some countries (Japan and Germany) even displayed stagnant house prices over this period. The model successfully replicates this feature of the data because the predicted house price boom is highly dependent on agents' price beliefs at the time of the interest rate reduction, which in turn depend on the (adverse) shocks hitting the respective economies prior to the fall in the real rate.

Our learning model is also compatible with observed house price expectations. Piazzesi and Schneider (2009) use the Michigan Survey of Consumers to document that the share of agents believing prices to increase further comoves positively with the house price level and in the last housing boom. Specifically, the share of optimistic agents reached its peak precisely at the time when house prices peaked. This fact is consistent with the learning approach proposed in this paper.

The paper also suggests important policy lessons. Since the world interest rate is an exogenous parameter in the model, we can study the effects of alternative interest rate scenarios. For the U.S. economy, the model predicts that the recent US house price boom would have been largely avoided and the current account been considerably smaller, if interest rates had fallen by less at the beginning of the 2000's. Although such a link between real interest rates and house price booms is frequently discussed in the press<sup>1</sup>, to the best of our knowledge we present the first formal model in which a sizable *and* persistent house price boom can arise from a persistent reduction in the level of real interest rates.<sup>2</sup>

The paper is structured as follows. The next section discusses the related literature on house price fluctuations and current account dynamics. Section 3 presents stylized facts we seek to explain about the behavior of house prices, the current account and private consumption in the G7. Section 4 introduces a stylized open economy model, derives the household optimality conditions and the equations determining the equilibrium outcomes for a general set of subjective price beliefs. In section 5 we show that the model has difficulties in replicating salient features of the data under RE. Section 6 introduces subjective price beliefs that are close to the RE beliefs (in distribution). In section 7 we show how the learning model can qualitatively account for the observed dynamics in house prices, current accounts and consumption in the G7. Sections 8 and 9 then explore the quantitative model performance. A conclusion briefly summarizes.

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<sup>1</sup>For example, in The Economist (August 23, 2007): 'Does America need a recession?', Economic Focus.

<sup>2</sup>Himmelberg et al. (2005) show that with low real interest rates a further reduction in rates can give rise to a large house price increase under rational expectations. It fails, however, to give rise to a persistent increase in house prices.

## 2 Related Literature

Few papers study house price dynamics within dynamic macroeconomic models before the recent recession. An important exception is Iacoviello (2005) who develops a monetary business cycle model with housing and collateral constraints.

A variety of recent papers use models of learning to explain observed house price data. Burnside, Eichenbaum and Rebelo (2011), for example, present a model in which a temporary house price boom emerges from infectious optimism that eventually dissipates once investors become more certain about fundamentals.

Laibson and Mollerstrom (2010) present a model in which aggregate wealth fluctuates because agents learn something about the expected future productivity of capital goods. The authors show how positive news about future productivity can then generate an increase in asset prices, a consumption boom and a current account deficit. The small volatility of macroeconomic fundamentals, however, poses problem for such news driven explanations for asset price fluctuations, as first noted some time ago by Shiller (1981). In our setup, (Bayesian) learning about house price behavior amplifies the effects of changes in fundamentals, thereby generating considerably larger movements in house prices with only small changes in fundamentals and interest rates.

Other papers also account for some of the empirical features that we describe. Matsuyama (1990) provides a theoretical analysis of the income effect of government spending, housing subsidies and sector-specific productivity change on residential investment and the current account. He shows that anticipated government spending shocks lead to a decline in house prices and residential investment, but that the effect on the current account depends on whether housing and nondurable consumption are substitutes or complements. Punzi (2006) evaluates the quantitative impact of the housing market on the current account using a two-sector, two-country DSGE model with heterogenous agents and a housing collateral constraint. In her setup, housing preference shocks generate a negative correlation between house prices and the current account. Gete (2010) seeks to explain current account and housing price dynamics through cross-country heterogeneity in the evolution of housing demand. If the desire to smooth consumption across housing services and nondurable consumption goods is strong or if households' preferences feature low intra-temporal substitution elasticity, then an increase in housing demand can give rise to a house price increase and a current account deficit.

## 3 Stylized Facts

### 3.1 U.S. House Prices and the Current Account 1996-2008

A variety of house price measures indicate that house prices in the United States increased considerably over the years 1996-2006. Figure 1 depicts indices of the real house price (RHP), the price-to-income (PIR) ratio, and the price-to-rent

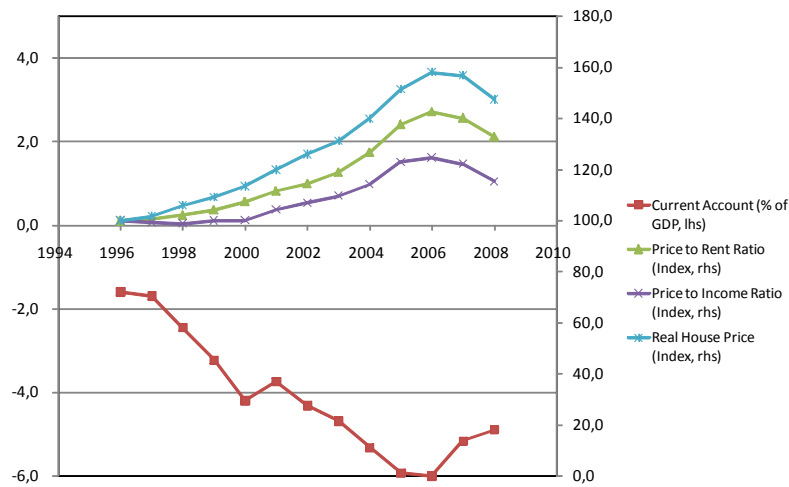


Figure 1: United State: House Price Measures and the Current Account

(PRR) ratio normalizing indices to a value of 100 in the year 1996.<sup>3</sup> Prices increased - depending on the preferred house price measure - between 24% and 58% in the subsequent 10 years. While house prices started increasing well before the year 2000, all house price measures indicate that over 70% of the total increase takes place after the year 2000.

Figure 1 also depicts the U.S. current account deficit (in % of GDP).<sup>4</sup> The current account and the house price are strongly negatively associated over time, with the deficit widening considerably throughout the period 1996 - 2006, except for a slight and temporary improvement in the recession year 2001. Once house prices started to revert direction in the years 2007 and 2008, however, the U.S. current account deficit also started to narrow, a development that accelerated in the year 2009 (see the current account data in table 1 below).

Table 1 below reports the annual change in the value of the U.S. housing stock (at market prices), together with the size of the current account deficit.<sup>5</sup> The bottom row in the table reveals a surprising fact: in the period 1996-2005, any 1\$ increase in the value of the U.S. housing stock was on average associated with a 0.26\$ increase in international borrowing in the form of a current account deficit. This ratio was remarkably stable in this period (the standard deviation is just 0.04), but breaks down after 2006 when current account deficits retreat

<sup>3</sup>The data is taken from the OECD Economic Outlook No. 87, 2010, Annex Tables 59 and 60. The real house price index is the nominal house price index deflated by CPI price index.

<sup>4</sup>The data is from the OECD Economic Outlook No. 87, 2010, Annex Table 51.

<sup>5</sup>The change in housing value is computed using the Federal Reserve Board Flow of Funds Statistics, Table B.100, Release 2010-12-9. The current account numbers are taken from NIPA tables, as downloaded through the FRB St. Louis FRED database.

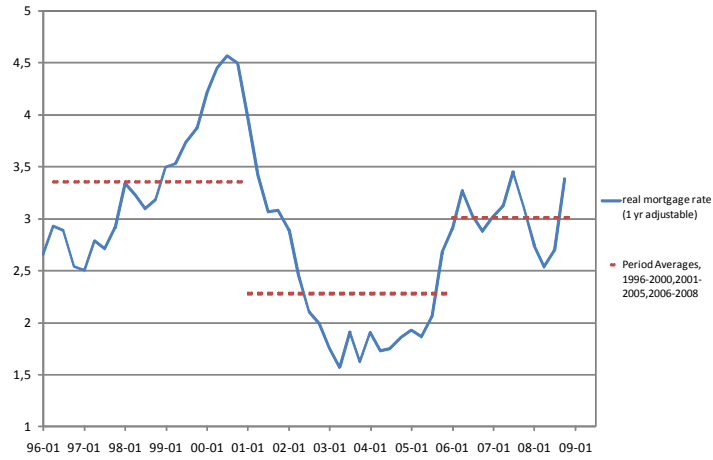


Figure 2: United States: Ex-ante real mortgage rate

only mildly despite the domestic housing value either increasing less strongly (in 2006) or even decreasing substantially in value (after 2007).

Year:	96	97	98	99	00	01	02	03	04	05	06	07	08	09
Value change														
U.S. house stock (vs prev. year, trn dollars)	+0.4	+0.5	+0.9	+1.0	+1.7	+1.4	+1.3	+1.7	+2.6	+3.5	+1.0	-1.7	-3.8	-0.8
U.S. CA deficit (bn dollars)	114	129	205	292	410	392	452	516	625	741	798	717	670	380
CA deficit / ch. in house value	0.27	0.24	0.22	0.28	0.25	0.28	0.34	0.30	0.23	0.21	0.84	-0.41	-0.18	-0.45

Table 1: U.S. Housing Value Appreciation and the Current Account

### 3.2 U.S.: Real Rates, Consumption, and Construction

The house price and current account developments in the U.S. described in the previous section were accompanied by a number of other broad macroeconomic trends. We focus on three variables: real interest rates, private consumption and construction activity. We now describe the observed behavior of these variables.

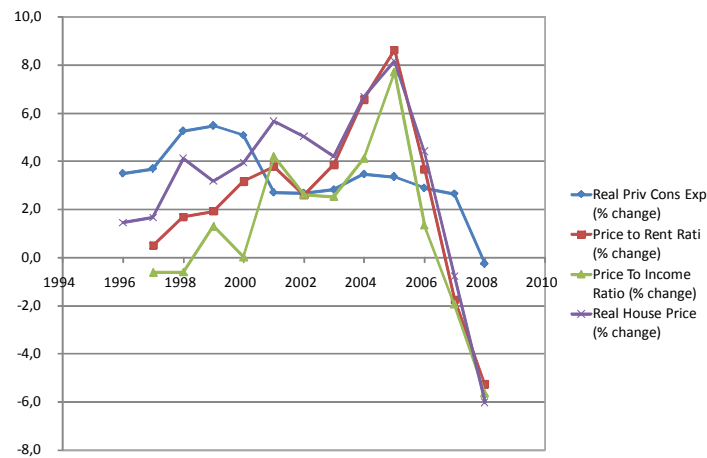


Figure 3: United States: House Price Changes and Consumption Growth

The acceleration of the U.S. house price increase and the widening of current account deficits after the years 2000/2001 coincided with a considerable fall in ex-ante real interest rates. Figure 2 illustrates this fact, depicting the one year adjustable mortgage rate subtracting from it the median expected 1 year ahead CPI inflation rate from the survey of professional forecasters.<sup>6</sup> The figure shows that ex-ante real interest rates considerably dropped around the beginning of 2001 and stayed low for an extended period of time, before rising again around the year 2006. We divide the evolution of real interest rates into three subperiods. A period of relatively high and rising real rates over the years 1996-2000, a period of falling low real rates over the years 2001-2005, and a period of rising and moderately high rates in 2006-2008. Figure 2 also shows the average interest rate for each subperiod.

Figure 3 depicts real private consumption growth together with various measures of house price growth.<sup>7</sup> Private consumption strongly expanded over the years 1996-2006 (around 3% or even higher) but came down somewhat after house prices reverted direction in 2007 and 2008.

Over the period 1996-2006 the number of new houses built also strongly expanded. Figure 4 reports the number of new housing units completed in the U.S. jointly with a variety of house price measures.<sup>8</sup> House prices and the number of housing completions are strongly positively correlated.

<sup>6</sup>The mortgage rate is the '1-year adjustable rate mortgage average in the United States' from Freddie Mac (SeriesID: MORTGAGE1US). The 1 year treasury rate is the 'market yield on U.S. treasury securities at 1-year constant maturity, quoted on investment basis' from the Federal Reserve Board (SeriesID: TCMNOMY1).

<sup>7</sup>The real private consumption growth data is from the OECD Economic Outlook No.87, 2010, Annex Table 3. The house price series employ the same data as used for figure 1.

<sup>8</sup>The housing units data is from U.S. Census Bureau, using the series 'new privately owned housing units completed'. The house price series are the same as the ones shown in figure 1.

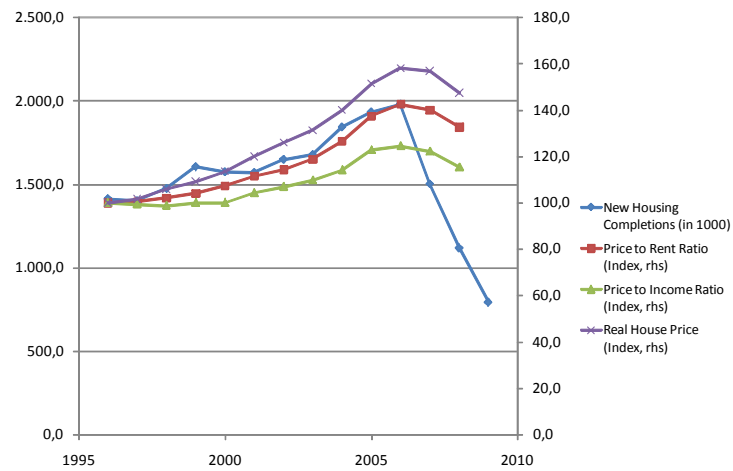


Figure 4: United States: House Prices and New Housing Completions

<b>House Price Change 2001-07</b>	<b>Current Account Surplus (2001-07, cum. sum in % of GDP)</b>	<b>Real Priv. Cons. Increase (2001-2007, cum. sum in % )</b>
Real house price	-0.55	0.72
Price-rent ratio	-0.42	0.75
Price-income ratio	-0.52	0.61

**Table 2:** Cross-Sectional Correlations in the G7

### 3.3 Cross-Sectional Evidence from G7 Economies

These observations we have mentioned for the U.S. Economy can similarly be documented in the cross section of G7 economies.<sup>9</sup>

Table 2 shows that over the period 2001-2007 house price increases and current account surpluses are negatively correlated in the cross-section of G7 coun-

<sup>9</sup>Data limitations prevent us from discussing also the construction of new houses or the relationship between value changes of the housing stock and the current account in all G7 countries.



tries.<sup>10</sup> Countries with larger house price booms thus tended to have also larger current account deficits. This holds independently of the considered house price measure (RHP, PIR, PRR). Furthermore, house price increases are strongly positively correlated with real private consumption growth over the same period, showing that countries with larger house price booms also tended to have larger consumption booms. Finally, as shown in table 3, the house price reversals in 2007-2008 are similarly strongly negatively correlated with changes in the current account surplus. These cross-sectional relationships are consistent with the correlation over time that can be observed for U.S. data.

<b>House Price Change 2007-2008</b>	<b>Change in Current Account Surplus 2008 vs. 2007 (in % of GDP)</b>
Real house	-0.75
Price-rent ratio	-0.90
Price-income ratio	-0.83

**Table 3:** Cross-Sectional Correlations in the G7

Although the G7 evidence confirms the comovements between house price, current account and consumption dynamics documented for the U.S., there exists a considerable amount of cross-sectional heterogeneity across the G7 economies. Figure 5 illustrates this fact by depicting the real house price indices for the G7 economies, normalizing the house price indices to 100 for the year 2000. It is clear that house prices show high volatility and high persistence in all countries. Table 4 shows that the serial correlation of housing prices is on average extremely high for the G7 countries.

What is interesting however is that these large low frequency movements are not synchronized. While four countries experienced sustained house price increases even larger than the U.S. after the year 2000, Japan and Germany experienced real price decreases. Besides illustrating the considerable cross-sectional heterogeneity, figure 5 also shows the very high persistence of house

<sup>10</sup>Aizenman and Jinjark (2009) also provide evidence on the strong positive association between current account deficits and the appreciation of real estate prices across a number of countries.

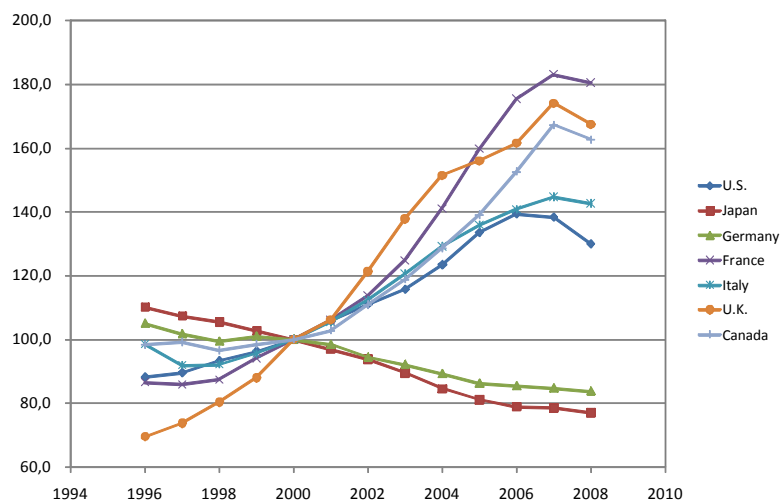


Figure 5: Real house prices in the G7 (indices, normalized to 100 for the year 2000)

price movements in each country.

<b>G7 average autocorrelation</b> 1996-2008	
<b>House Prices Measure</b>	
real house prices	0.98
price-to-rent ratio	0.97
price-to-income	0.97

**Table 4:** Autocorrelation of G7 House Prices

## 4 An Open Economy Model with Housing

This section introduces a small open economy model with endogenous housing supply in which households can internationally borrow for consumption and investment purposes. Household borrowing is thereby subject to a borrowing constraint, as in Kiyotaki and Moore (1997).

**Preferences and Beliefs.** We consider an economy populated by a unit mass of utility maximizing households. Households are identical in terms of

preferences and beliefs - a fact that is not known to agents<sup>11</sup> - with the representative household maximizing

$$E_0^{\mathcal{P}} \sum_{t=0}^{\infty} \delta^t (\xi_t h_t + c_t) \quad (1)$$

where  $c_t \geq 0$  denotes consumption of goods,  $h_t \geq 0$  consumption of housing services,  $\delta \in (0, 1)$  the time discount factor, and  $\xi_t > 0$  a housing preference shock. We assume that the preference shock evolves according to

$$\ln \xi_t = \ln \xi_{t-1} + \ln \varepsilon_t \quad (2)$$

with  $\varepsilon_t$  being an *iid* innovation satisfying  $E[\ln \varepsilon_t] = 0$  and  $E[(\ln \varepsilon_t)^2] = \sigma_\varepsilon^2$ . The preference  $\xi_t$  shock captures changes in the population's preferences for housing services relative to consumption.

The household's expectation in (1) is computed using a (potentially subjective) probability measure  $\mathcal{P}$ , which is defined over the space of payoff-relevant outcomes  $\Omega$ . The measure  $\mathcal{P}$  assigns probabilities to all Borel subsets of  $\Omega$ , so that agents entertain a standard probability space  $(\mathcal{P}, \mathcal{B}, \Omega)$ , with  $\mathcal{B}$  denoting the sigma-algebra containing all Borel. Importantly, the set  $\Omega$  includes all sequences of payoff-relevant variables that agents take as given. This includes fundamental shocks, but also competitive market prices. Agents' choices in some period  $t$  are then functions of the realization of these payoff-relevant variables up to  $t$ . While the measure  $\mathcal{P}$  itself is time-invariant, i.e., dynamically consistent, it will often imply that rational agents are learning about the house price process. This is the case, for example, if  $\mathcal{P}$  is generated by a model that agents entertain about the stochastic process of house prices and by some prior beliefs about unknown parameters describing this process. Further details about the underlying probability space is given in section 4.1 below.

**Budget Constraint and Collateral Constraint.** We let  $H_t \geq 0$  denote the stock of houses owned by the household in period  $t$ . The housing stock yields housing services  $h_t$  according to

$$h_t = G(H_t) \quad (3)$$

for some twice continuously differentiable and (weakly) concave function  $G(\cdot)$  satisfying the conditions  $\lim_{H \rightarrow 0} G'(H) = \infty$  and  $\lim_{H \rightarrow \bar{H}} G'(H) = -\infty$  for some  $\bar{H} \leq \infty$ . We impose this latter condition for technical convenience: it insures the existence of an optimal house holding plan for all beliefs about house prices satisfying the restriction that house prices cannot become negative. The above assumptions imply that there is a bliss point  $H^B$  such that  $G'(H^B) = 0$ , with  $G'(H) \geq 0$  for  $H < H^B$  and  $G'(H) \leq 0$  for  $H > H^B$ . For reasons that

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<sup>11</sup>As explained in Adam and Marcet (2010b), common knowledge of agents' preferences and beliefs might place additional restrictions on the house price beliefs that rational agents can entertain.

will become apparent below, the housing stock may exceed this bliss point in equilibrium.

Using the consumption good as numeraire and letting  $q_t$  denote the price of houses, the agent's flow budget constraint is

$$c_t + (H_t - (1 - d)H_{t-1})q_t + Rb_{t-1} + k_t = y_t + b_t + \pi_t + k_{t-1}p_t \quad (4)$$

where  $y_t \geq 0$  denotes an exogenous income process,  $b_t$  the household's new loans,  $R$  the gross real interest rate on maturing loans  $b_{t-1}$ ,  $d \in [0, 1)$  the rate at which the housing stock depreciates,  $\pi_t$  profits from the ownership of (housing development) firms, and  $k_t \geq 0$  capital sold to competitive housing developers who use this capital as an input for the production of new houses. This capital stock fully depreciates in one period. To capture time lags in housing production and for simplicity we assume that the price  $p_t$  remunerating  $k_{t-1}$  is a competitive forward price that is fixed in period  $t - 1$ .

Note that we do not explicitly model a competitive rental market. This is without loss of generality, as the rental market only determines the market clearing rental price  $\xi_t$ . The remaining part of the paper, thus interprets  $\xi_t$  as the rental price for housing services.

We assume that consumers are also subject to a borrowing constraint of the form

$$b_t \leq \theta \frac{E_t^{\mathcal{P}} q_{t+1}}{R} H_t \quad (5)$$

as in Kiyotaki and Moore (1997). The parameter  $\theta \in [0, 1 - d]$  captures the share of houses owned by the household today that can serve as collateral to lenders. A value of  $\theta < 1$  thereby incorporates the effects of physical depreciation of houses, as well as the possibility that seizing the collateral in case of default is costly for lenders.

The borrowing constraint (5) will be key to understand the model-implied relation between house prices and current account dynamics. In a situation with high expected future house price, current house prices will tend to be high as well, and the borrowing constraint relatively loose. Agents can thus increase international borrowing precisely at a time where house prices are high, thereby establishing a connection between house price booms and current account deficits.<sup>12</sup>

We assume that the market for collateralized loans is internationally fully integrated, so that the interest rate for such loans is determined by the exogenous world interest rate  $R \in (1, \frac{1}{\delta})$ . The latter implies that international lenders are more patient than domestic households. In addition to simplifying the analysis, it captures the presence of China and other emerging economies as large and patient international lenders in the global economy. For simplicity, foreign lenders are assumed to hold the same beliefs  $\mathcal{P}$  as domestic agents.

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<sup>12</sup>This connection would be even more direct, if the collateral constraint determined the maximum amount of borrowing as a function of the current instead of expected future housing wealth. For our analysis such a change in the borrowing constraint would make virtually no difference.

**Housing Supply.** We now turn to the determinants of housing supply. There exists a competitive housing development sector consisting of a unit mass of housing development firms. The representative firm operates a decreasing returns technology for constructing new houses. We assume that the amount of new housing produced at  $t$  is given by

$$(\alpha\delta)^{-1} k_{t-1}^\alpha$$

with  $k_{t-1} \geq 0$  denoting the amount of development capital used by housing developers and  $\alpha \in (0, 1)$ . To capture time lags in housing construction we assume that firms choose the level of input  $k_{t-1}$  in period  $t - 1$ , i.e., one period in advance.

Firms in the housing sector are owned by the consumer in the small open economy, who receive profits as lump sum transfers. Since firms do not have a true intertemporal maximization problem (there is no state variable in the firms' problem), we can assume that they maximize expected profits from housing construction by choosing

$$\max_{k_{t-1} \geq 0} E_{t-1}^{\mathcal{P}} \left( \frac{1}{\alpha\delta} k_{t-1}^\alpha q_t - p_t k_{t-1} \right)$$

where  $p_t$  is the price of period  $t$  inputs purchased from households in period  $t - 1$  in a competitive forward market. The profit-maximizing input choice is given by

$$k_{t-1}^* = \left( \frac{E_{t-1}^{\mathcal{P}} q_t}{\delta p_t} \right)^{\frac{1}{1-\alpha}}$$

and determines a supply function for new houses of the form

$$S(E_{t-1}^{\mathcal{P}} q_t, p_t) = \frac{1}{\alpha\delta} \left( \frac{E_{t-1}^{\mathcal{P}} q_t}{\delta p_t} \right)^{\frac{\alpha}{1-\alpha}} \quad (6)$$

with  $\frac{\alpha}{1-\alpha}$  denoting the elasticity of housing supply with respect to the expected selling price  $E_{t-1}^{\mathcal{P}} q_t$ . The housing stock then evolves according to

$$H_t = (1 - d)H_{t-1} + S(E_{t-1}^{\mathcal{P}} q_t, p_t) \quad (7)$$

and developers realized profits in period  $t$  are given by

$$\pi_t = \frac{1}{\alpha\delta} \left( \frac{E_{t-1}^{\mathcal{P}} q_t}{\delta p_t} \right)^{\frac{\alpha}{1-\alpha}} q_t - p_t \left( \frac{E_{t-1}^{\mathcal{P}} q_t}{\delta p_t} \right)^{\frac{1}{1-\alpha}} \quad (8)$$

## 4.1 The Underlying Probability Space

We now describe details of the underlying the probability space  $(\mathcal{P}, \mathcal{B}, \Omega)$ . We let the state space of outcomes be given by

$$\Omega \equiv \Omega_p \times \Omega_q \times \Omega_\xi \times \Omega_y \times \Omega_\pi$$

where  $\Omega_X = \prod_{t=0}^{\infty} R_+$  is the space of possible infinite sequences for the variable  $X \in \{p, q, \xi, y, \pi\}$ . The probability space thus contains all possible sequences of all payoff-relevant variables that agents take as given, including prices, profits and exogenous shocks. The set of all possible histories up to period  $t$  for some variable  $X$  is denoted by  $\Omega_X^t$ , and its typical element  $X^t \in \Omega_X^t$ , except for  $p$  where  $\Omega_p^t$  denotes histories up to  $t + 1$ . Furthermore, we let  $\Omega^t = \Omega_p^t \times \Omega_q^t \times \Omega_\xi^t \times \Omega_y^t \times \Omega_\pi^t$  denote the set of histories of all exogenous variables up to period  $t$ , and  $\omega^t \in \Omega^t$  its typical element. These beliefs  $\mathcal{P}$  and the set  $\Omega$  are assumed common to all agents, including firms, domestic consumers and foreign agents.

The previous setup implies that rational agents condition their decisions on the history of observed realizations, i.e., consumers choose for each  $t$

$$(c_t, h_t, H_t, b_t, k_t) : \Omega^t \rightarrow R^5 . \quad (9)$$

A key feature of this formulation is that agents do not treat prices  $(p_t, q_t)$  and profits  $\pi_t$  as if they were given known function of the history of fundamentals and therefore redundant condition variables, as is the case under rational expectations. Instead, agents treat prices just as another random variable among all exogenous variables, because they do not necessarily know how a given history of fundamentals  $(y, \xi)^t$  maps into prices and profits. Agents express their uncertainty about the joint distribution of prices and fundamentals using the probability measure  $\mathcal{P}$ . In the spirit of studying small deviations from rational expectations, we will specify below a probability measure  $\mathcal{P}$  that is close - but not exactly equal - to the rational expectations beliefs.

## 4.2 Household Optimality Conditions

We now derive the conditions characterizing optimal household behavior. We thereby proceed by assuming that a maximum for the household problem exists.<sup>13</sup> First order conditions are then necessary and sufficient for household optimality because the objective function is concave and the constraints are linear in the households' choices.

Households maximize the objective (1) subject to the constraints (3), (4) and (5). Taking explicitly into account the non-negativity constraints for  $c_t$  and  $k_t$ , the Lagrangian of the household problem is given by

$$\max_{\{c_t, H_t, b_t, k_t\}} E_0^{\mathcal{P}} \sum_{t=0}^{\infty} \delta^t \left( \begin{array}{l} \xi_t G(H_t) + c_t \\ -\lambda_t \left( \begin{array}{l} c_t + (H_t - (1-d)H_{t-1})q_t + b_{t-1}R \\ +k_t - y_t - b_t - \pi_t - k_{t-1}p_t \end{array} \right) \\ +\gamma_t (\theta E_t^{\mathcal{P}} q_{t+1} H_t - Rb_t) \\ +\mu_t c_t + \kappa_t k_t \end{array} \right)$$

where  $p_0$ ,  $k_{-1}$  and  $b_{-1}$  are given initial conditions.

<sup>13</sup>Existence of a maximum can be insured, for example, by imposing that the utility from consumption ( $c_t$ ) is bounded at some very high level. See Appendix A.1 in Adam and Marcet (2010b) for how this works in a related model.

The household's first order conditions (FOCs) are

$$c_t : \lambda_t = 1 + \mu_t \text{ with } \mu_t \geq 0 \text{ \& } c_t \mu_t = 0 \quad (10)$$

$$H_t : \xi_t G'(H_t) - \lambda_t q_t + \delta(1-d)E_t^{\mathcal{P}} \lambda_{t+1} q_{t+1} + \gamma_t \theta E_t^{\mathcal{P}} q_{t+1} = 0 \quad (11)$$

$$b_t : \lambda_t = \delta R E_t^{\mathcal{P}} \lambda_{t+1} + \gamma_t R \\ \text{with } \gamma_t \geq 0 \text{ \& } (\theta E_t^{\mathcal{P}} q_{t+1} H_t - R b_t) \gamma_t = 0 \quad (12)$$

$$k_t : \lambda_t = \delta p_{t+1} E_t^{\mathcal{P}} \lambda_{t+1} + \kappa_t \text{ with } \kappa_t \geq 0 \text{ \& } k_t \kappa_t = 0 \quad (13)$$

for all  $t \geq 0$ . Equation (10) implies that  $\lambda_t \geq 1$ .

We now describe the evolution of equilibrium variables. We focus on the case where the collateral constraint (5) is binding for all periods  $t \leq T+1$ , where  $T$  is the sample size, and the non-negativity constraints are never binding.<sup>14</sup> In this case,  $\mu_t = 0$  for all  $t = 1, \dots, T$  so that the FOC (10) is satisfied for

$$\lambda_t = 1$$

and the FOC (12) for

$$\gamma_t = \frac{1}{R} - \delta > 0$$

for all  $t$ . Using these results and equation (11) one obtains

$$q_t = \rho E_t^{\mathcal{P}} q_{t+1} + \xi_t G'(H_t) \quad (14)$$

where

$$\rho \equiv \delta(1-d-\theta) + \frac{\theta}{R} < 1 \quad (15)$$

Given  $q_t$  and  $E_t^{\mathcal{P}} q_{t+1}$ , equation (14) determines the optimal amount of houses demanded by the household. Since  $G'(\cdot)$  continuously varies between  $+\infty$  and  $-\infty$ , this equation always has a solution for the optimal housing stock  $H_t$ , for any given pair  $(q_t, E_t^{\mathcal{P}} q_{t+1})$ . Moreover, since  $H_t > 0$  it confirms our initial conjecture of positive housing demand.

Importantly, for  $q_t < (\delta(1-d-\theta) + \frac{\theta}{R}) E_t^{\mathcal{P}} q_{t+1}$ , equation (14) implies  $G'(H_t) < 0$ , so that housing demand exceeds the bliss point level  $H^B$ . This is individually optimal because housing generates capital gains and relaxes the households' borrow constraint, which allows to increase consumption. Therefore, whenever houses are supposed to appreciate strongly in the future, it can become individually optimal to purchase housing above the bliss point.

When the collateral constraint is binding in the first  $T+1$  periods, the optimal level of borrowing follows from the binding collateral constraint and is given by

$$b_t R = \theta E_t^{\mathcal{P}} q_{t+1} H_t \quad (16)$$

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<sup>14</sup>We discuss the equilibrium outcomes when the zero bound on consumption is binding in appendix A.1. Since zero consumption does not accord well with the data, we focus in the text on equilibria with positive consumption. In our model simulations we use parameterizations for which the non-negativity constraint is not binding.

The capital offered by the consumer to housing developers is only restricted to satisfy

$$(1 - \delta p_{t+1}) k_t = 0$$

so that either  $p_t = \delta^{-1}$  or  $k_t = 0$ . This means that if non-negativity constraints are non-binding capital and consumption are not uniquely determined, the agent is indifferent between increasing slightly the capital sold to firms at  $t$  in exchange for  $\delta^{-1}$  more units of consumption tomorrow. Since firms will have a positive demand for  $k$  market clearing occurs at

$$p_t = \delta^{-1} \tag{17}$$

with capital supply by consumer being perfectly elastic, so that  $k_t$  is determined by firms' demand.

Finally, consumption can be obtained residually from the flow budget

$$c_t = y_t + b_t + \pi_t - (H_t - (1 - d)H_{t-1}) q_t - b_{t-1}R - k_t + k_{t-1}\delta^{-1} \tag{18}$$

where we imposed (17).

### 4.3 Equilibrium Dynamics for General Beliefs $\mathcal{P}$

For arbitrary and given beliefs  $\mathcal{P}$ , the equilibrium evolution of the house price  $q_t$  and the housing stock  $H_t$  must satisfy equations (14) and (7), rewritten here as

$$q_t = \rho E_t^{\mathcal{P}} q_{t+1} + \xi_t G'(H_t) \tag{19}$$

$$H_{t+1} = (1 - d)H_t + S(E_t^{\mathcal{P}} q_{t+1}, \delta^{-1}) \tag{20}$$

These equations can be solved for the process  $\{q_t, H_t\}_{t=0}^{\infty}$ . Borrowing then follows from equation (16), housing supply from (6), profits from (8) and equilibrium consumption from (18).

## 5 Rational Expectations Equilibrium (REE)

We now assume that agents entertain rational expectations ( $E_t^{\mathcal{P}}[\cdot] = E_t[\cdot]$ ) and derive the resulting REE in the case that the collateral constraint is always binding.<sup>15</sup> We start by determining the deterministic steady state, then analyze the effects of preference and income shocks, and finally discuss the effects of changes in international real interest rates. As will become clear from the discussion below, under RE the model has great difficulties in replicating the observed house price dynamics.

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<sup>15</sup>This can be insured by choosing sufficiently high values for the endowments  $\{y_t\}$  or sufficiently low values for  $\theta$ .



## 5.1 Deterministic Steady State

We start out by determining the deterministic steady state, i.e., the REE in which  $\xi_t = \xi$  and  $y_t = y$  for all  $t$ . Letting variables without time subscripts denote steady outcomes, equations (34) and (35) imply

$$q^{ss} = \frac{\xi G'(H^{ss})}{1 - \rho} \quad (21)$$

$$H^{ss} = \frac{1}{\alpha \delta d} (q^{ss})^{\frac{\alpha}{1-\alpha}} \quad (22)$$

which jointly determine a unique steady state value for  $q^{ss}$  and  $H^{ss}$ .<sup>16</sup> Steady state capital, borrowing and consumption are given by

$$\begin{aligned} k^{ss} &= (q^{ss})^{\frac{1}{1-\alpha}} \\ b^{ss} &= \theta \frac{q^{ss} H^{ss}}{R} \\ c^{ss} &= y + \theta \left( \frac{1}{R} - 1 \right) q^{ss} H^{ss} - (q^{ss})^{\frac{1}{1-\alpha}} \end{aligned}$$

## 5.2 Stochastic Equilibrium

We now analyze the effects of shocks to housing preferences  $\xi_t$  and household income  $y_t$ . In the interest of deriving closed form approximate solutions we consider solutions of (34) and (35) when the function  $G(\cdot)$  is linearized around its steady state. We discuss the additional effects arising from the concavity of  $G(\cdot)$  separately below.

Substituting  $G'(H_t)$  by  $G'(H^{ss})$  in equation (34) implies that the REE house price to rent ratio is (approximately) given by

$$\frac{q_t^{RE}}{\xi_t} = \frac{G'(H^{ss})}{1 - \rho} \quad (23)$$

so that log house price growth evolves according to

$$\ln \frac{q_t^{RE}}{q_{t-1}^{RE}} = \ln 1 + \ln \varepsilon_t \quad (24)$$

For the linear approximation we thus have  $E_t q_{t+1}^{RE} = q_t^{RE}$  and  $p_t = \delta^{-1}$ , so that the housing stock approximately evolves according to:<sup>17</sup>

$$H_{t+1}^{RE} = (1 - d)H_t^{RE} + \frac{1}{\alpha \delta} (q_t^{RE})^{\frac{\alpha}{1-\alpha}}$$

<sup>16</sup>Existence and uniqueness follow from the following considerations. Equation (21) defines  $q$  as a continuous and (weakly) decreasing function of  $H$  which approaches  $+\infty$  as  $H \rightarrow 0$  and  $-\infty$  as  $H \rightarrow \bar{H}$ . From (22) we have that  $q$  is a strictly increasing function of  $H$ . As a result there exist a unique intersection.

<sup>17</sup>The subsequent equation reveals that sufficiently small housing preference shocks will indeed imply that  $H_t$  stays in the neighborhood of  $H$  with high likelihood, as initially assumed.

The previous findings show that preference and income shocks both fail to affect the price-to-rent ratio (23) and that the real house price follows a unit root in this approximate REE. Therefore, under RE preference and income shock can neither explain the large swings in the price-to-rent ratio and are unlikely to explain the persistent boom and bust patterns in real house prices observed in the data.

Preference shocks also cannot explain the observed house price dynamics when taking into account the concavity in  $G$  ( $G''(H^{SS}) < 0$ ). Concavity implies that any expansion of the housing stock, following a positive innovation to housing preferences, leads to a reduction in  $G'$  over time. Therefore, the house price will increase less on impact and also decreases with time as the housing stock expands. Moreover, equation (34) implies that on impact and in the long-run the price-to-rent ratio decreases. Real house prices and the price-to-rent ratio will thus move in opposite directions, while the data both measures are positively correlated.<sup>18</sup>

### 5.3 The Effects of Changes in Real Interest Rates

We now consider the effects of unexpected changes in the real interest rate for the RE house price. Equation (23) shows that a reduction in real interest rates generates an increase in the real house price and the price-to-rent ratio. Yet, as we document below, a RE setting it is unlikely that changes in real interest rate can properly account for the observed house price dynamics.

To analyze the effects of real interest rate changes we assume that the economy starts from a steady state position in the year 2000. We then subject it to the stylized changes in the real interest rate indicated by the dashed line in figure 2. Specifically, we consider a persistent and unexpected decrease in the real rate in the year 2001, followed by an equally unexpected real rate increase in the year 2006.<sup>19</sup> The effects of anticipated real rate changes will be discussed separately below.

We parameterize the rest of the model as follows. We set  $\theta = 0.26$ , which is the 1996-2005 average of the annual value change change the U.S. housing stock over the current account deficit, see table 1. For the annual discount factor we choose  $\delta = 0.96$ , so that the discount factor is slightly below the real interest rate path that we feed into the model.<sup>20</sup> Finally, we set the annual house depreciation rate equal to  $d = 3\%$ . None of the shown results proves particularly sensitive to the assumed parameter values.

<sup>18</sup>We consider this case formally in appendix A.2, which derives the REE for a quadratic approximation to  $G$ .

<sup>19</sup>The initial real interest rate is set equal to the average ex-ante gross real mortgage rate over the periods 1996-2000, i.e.,  $R_{96-00} = 1.0335$ . In the year 2001 we then consider an expected and permanent fall in the real interest rate that lasts for 5 years to a value of  $R_{01-05} = 1.0228$ , which is equal to the average ex-ante U.S. real mortgage rate for this period. Thereafter, we consider an unexpected and permanent upward shift in real rates to  $R_{06-08} = 1.0301$ , which is again taken from the data.

<sup>20</sup>This is required to insure optimality of the binding collateral constraint.

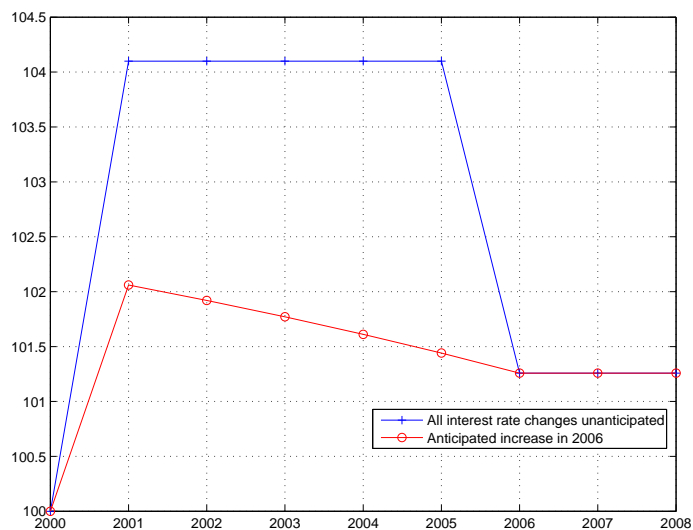


Figure 6: RE dynamics of the real house price (stylized real interest rate path from figure 2)

To simplify the analysis, we first consider again solution with a linear approximation, i.e., we assume  $G'(H_t) = G'(H^{ss})$  over the relevant range of housing stock values  $H_t$ . The behavior arising with  $G'' < 0$  is still to be written. We keep housing preference shocks constant at  $\xi_t = \xi$  and normalize  $\xi G'(H)$  such that the initial steady state real house price in the year 2000 (prior to any change in the real interest rate) is equal to 100.

The resulting REE real house price dynamics from unexpected changes in the interest rate are illustrated by the upper line in figure 6.<sup>21</sup> The figure reveals that RE imply that house price changes occur simultaneously with unanticipated changes in real interest rates. For the U.S. economy, however, one cannot find a close simultaneous association between changes in the real mortgage rates and house prices changes. Mortgage rates, for example, stayed approximately constant between the beginning of 2003 until the end of 2005, see figure 2, while house prices increased strongly over these two years. Likewise, real mortgage rates were roughly constant over the years 2006-2008, while house prices decreased considerably over these years. Due to this close association with interest rates, house prices under RE do not exhibit the persistence that can be observed for house price fluctuations in the data.

Furthermore, the amplitude of the fluctuations generated by interest rate shocks tends to be small compared to the data. The RE model justifies a 4% appreciation between 2000 and 2005, while the U.S. experienced a tenfold

<sup>21</sup>Since  $\xi_t = \xi$ , the price-to-rent ratio is simply proportional to the real house price series.

increase over this period. From a RE viewpoint, it thus appears difficult to account for the observed house price dynamics using changes in real interest rates as a driving force.

Even greater difficulties arise if one assumes instead that agents fully anticipate future changes in real interest rates, instead of assuming that any given change is considered permanent. If agents anticipate the 2006 real interest rate increase, then house prices evolve according to the lower line shown in figure 6. The initial house price increase in 2001 is then even smaller and followed by a gradual decrease, due to the anticipated real rate increase (and house price decrease) in the year 2006. In the data, however, house prices increased strongly after the year 2001.

We can conclude that under RE it is difficult to account for the U.S. house price dynamics using the observed interest rate dynamics. Rather than predicting house price increases over the years 2001-2006, RE predicts that house prices move together with interest rates, that fluctuations are fairly small and that house price persistence is relatively low.

## 6 Specifying Near REE Beliefs

We now consider rational agents who hold subjective beliefs about the house price process. Under the rational expectations hypothesis, agents are assumed to know that the joint distribution over exogenous shocks and market prices has a singularity and where exactly this singularity is located.<sup>22</sup> Yet, even expert economists rarely agree on the correct economic model linking fundamentals to prices. Therefore, it appears of interest to relax the assumption that agents know the correct model of prices and to consider instead agents who do not know exactly how prices behave.

We assume that agents express uncertainty about the true price process by formulating a perceived joint distribution over prices and fundamentals. We formulate this joint distribution with the aid of a perceived model of price behavior for which agents do not know exactly the parameters. And in the spirit of analyzing small departures from rational expectations beliefs, we parameterize beliefs so that beliefs of learning agents approach (in distribution) those entertained by agents in a linearized REE in which interest rates are constant.

We now describe the probability beliefs and then show how these approach the beliefs held in the corresponding REE.

The probability space  $(\mathcal{B}, \Omega)$  is as defined in section 4.1. To simplify the analysis we assume that agents have very good information about all variables except housing prices, which is the variable we are interested in. In particular, we assume that agents hold rational expectations about the exogenous processes  $\{y_t, \xi_t\}_{t=0}^{\infty}$  and about  $\{p_t = \delta^{-1}\}_{t=0}^{\infty}$ .<sup>23</sup> We relax, however, the assumption of rational house price expectations.

<sup>22</sup>Models of learning about fundamentals also often make this assumption implicitly.

<sup>23</sup>Since the process for  $y_t$  and  $\xi_t$  are exogenous to the model, it is straightforward to relax this assumption for these variables.

Specifically, we relax the assumption that agents believe that average house price growth is constant over time, as in the corresponding REE outcome (24). Instead, we allow for a persistent house price growth component  $\beta_t$ , in addition to a transitory component  $v_t$ , so that our agents believe that house prices evolve according to

$$\ln \frac{q_t}{q_{t-1}} = \ln \beta_t + \ln v_t \quad (25)$$

This relaxation relative to the REE outcome (24) is motivated by the empirical evidence on house price behavior, which displays periods of persistently increasing prices ( $\ln \beta_t > 0$  for a number of periods) and periods of persistently falling prices ( $\ln \beta_t < 0$  for a number of periods). Agent's beliefs about house price growth implied by (25) will be close to REE beliefs (in distribution) whenever agents believe that  $\ln \beta_t \approx \ln 1$  with high likelihood.

For simplicity, we shall assume that the persistent component follows a random walk<sup>24</sup>

$$\ln \beta_t = \ln \beta_{t-1} + \ln \eta_t \quad (26)$$

and that the innovations are given by

$$\begin{pmatrix} \ln v_t \\ \ln \eta_t \end{pmatrix} \sim iiN \left( \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_v^2 & 0 \\ 0 & \sigma_\eta^2 \end{pmatrix} \right) \quad (27)$$

The views implied by (25)-(27) then represent a small deviation from the REE beliefs, whenever the variance of the drift term  $\sigma_\eta^2$  is very small, so that  $\ln \beta_t$  behaves almost like a constant, and if agents' initial beliefs about  $\ln \beta_0$  is centered with high precision at its corresponding REE value, i.e.,  $\ln \beta_0 \approx \ln 1$  with high likelihood, see equation (24). We shall impose conditions insuring this property below.

A learning problem arises in the present setting because agents observe the history of realized house price growth  $\frac{q_t}{q_{t-1}}$ , but not the terms  $\beta_t$  and  $v_t$  separately. Therefore, agents optimally update their beliefs about  $\beta_t$  in the light of new data about house price growth.

Agents' prior beliefs about the persistent component at time zero is assumed normal with

$$\ln \beta_0 \sim N(\ln m_0, \sigma_0^2) \quad (28)$$

and  $\sigma_0^2$  denoting then steady state (Kalman filter) uncertainty, i.e.,

$$\sigma_0^2 = \frac{-\sigma_v^2 + \sqrt{(\sigma_v^2)^2 + 4\sigma_v^2\sigma_\eta^2}}{2}$$

The prior beliefs (28) together with the process (25)-(27) completely specify agents' beliefs about house price behavior.

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<sup>24</sup>The fact that  $\beta_t$  is non-stationary is not important for our results. The model outcome are almost the same when specifying instead a stationary process

$$\ln \beta_t = (1 - \rho) + \rho \ln \beta_{t-1} + \ln \eta_t$$

and choosing some value  $\rho < 1$  that is sufficiently close to one.

We can complete the overall belief description by assuming that agents know how to map a history of prices into profits. In other words, agents know that profits in  $t$  are given by a function  $\pi(E_{t-1}^{\mathcal{P}}q_t, q_t)$  equal to the right-hand side of (8) for  $p_t = \delta^{-1}$ . This completes the description of  $\mathcal{P}$ .

We now impose restrictions on  $\mathcal{P}$  so that agents' beliefs approach (in distribution) the REE beliefs about the price process under the linearization considered. We do this in two steps. First, we center initial beliefs so as to be consistent with no growth in real house prices by choosing  $\ln m_0 = \ln 1$ . Second, we consider the case when beliefs are such that  $\sigma_\eta^2 \rightarrow 0$ . As a result of this second assumption, prior uncertainty  $\sigma_0$  about price growth vanishes ( $\sigma_0^2 \rightarrow 0$ ). Agents thus become increasingly certain about the fact that log house price growth is equal to zero. Formally, as  $\sigma_0^2 \rightarrow 0$  their beliefs about prices converge to REE beliefs in distribution (or 'in law'). Below, we shall consider small but positive values for  $\sigma_\eta$  so that our agents entertain prior beliefs that are close to (but not equal) to the beliefs that give rise to the REE.

## 6.1 Internal Rationality and Discounted Sums

It appears to be a commonly held view among academic economists that rational behavior leaves no room for independent beliefs about prices, because 'rationality implies that agents know how to formulate prices as discounted sums of future fundamentals'. As discussed extensively in Adam and Marcet (2010a), this widespread view is incorrect, once agents do not know exactly the features of all other agents in the economy.

This view is equally incorrect for the housing model studied in the present paper. As we show below, agents fail to hold enough knowledge to formulate prices as a discounted sum involving only beliefs about fundamentals. In other words, agents cannot derive a mapping linking prices to the history of dividends through a pure process of deductive reasoning. This can be most easily demonstrated for the case where the non-negativity constraint on consumption is never binding, so that (14) holds each period. Forward iteration on this equation then yields a discounted sum formulation for the housing price

$$q_t = E_t^{\mathcal{P}} \sum_{j=0}^{\infty} \rho^j \xi_{t+j} G'(H_{t+j}) \quad (29)$$

which holds under internal rationality. Importantly, this discounted sum involves beliefs about future housing decisions  $H_{t+j}$  in addition to beliefs about future fundamentals. And the agent's optimal housing plan will depend on the agents price beliefs. Since the belief system  $\mathcal{P}$  does not allow to express these price beliefs as a function of the agent's beliefs about future fundamentals, price beliefs play an independent role for determining the value of the discounted sum (29).

## 7 Equilibrium Dynamics with Learning

We now explore the equilibrium dynamics in an economy in which agents hold the near REE beliefs  $\mathcal{P}$  specified in the previous section. We first derive the evolution of the conditional house price growth expectations  $m_t$  implied by the probability measure  $\mathcal{P}$ , then discuss the resulting price dynamics

### 7.1 Belief Updating

Bayesian updating of beliefs implies that agents' posterior beliefs about  $\beta_t$  at time  $t$  are given by

$$\ln \beta_t \sim N(\ln m_t, \sigma_0^2)$$

where  $\ln m_t$  evolves recursively according to

$$\ln m_t = \ln m_{t-1} + g \left( \ln \frac{q_t}{q_{t-1}} - \ln m_{t-1} \right) \quad (30)$$

with the 'gain' parameter given by

$$g = \frac{\sigma_0^2}{\sigma_\eta^2} > 0$$

Agents' conditional expectations of house price growth are then given by

$$E_t^{\mathcal{P}} \frac{q_{t+1}}{q_t} = m_t e^{\frac{1}{2}(\sigma_0^2 + \sigma_\eta^2 + \sigma_v^2)} \approx m_t$$

and  $m_t$  evolves approximately according to equation (30). Furthermore, to avoid simultaneity between price expectations and price outcomes, it is convenient to assume that information on prices is introduced with a delay in  $m_t$ , so that we actually use

$$\ln m_t = \ln m_{t-1} + g \left( \ln \frac{q_{t-1}}{q_{t-2}} - \ln m_{t-1} \right) \quad (31)$$

A microfounded belief system justifying this delay is provided in proposition 2 in Adam and Marcet (2010a).

### 7.2 Qualitative Behavior of Equilibrium Prices under Learning

This section discusses the qualitative behavior of equilibrium house prices under learning. As before, we consider again the approximate solution when linearizing  $G$  around its steady state value, so that take  $G'$  is constant. The asset pricing equation (34) then implies that the equilibrium asset price under learning is (approximately) given by

$$q_t = \frac{\xi_t G'(H^{ss})}{1 - \rho m_t} \quad (32)$$

so that realized log house price growth is

$$\ln \frac{q_t}{q_{t-1}} = \ln \frac{1 - \rho m_{t-1}}{1 - \rho m_t} + \ln \varepsilon_t \quad (33)$$

For the case with vanishing prior uncertainty ( $\sigma_\eta^2, \sigma_0^2 \rightarrow 0$ ), the gain  $g$  is small so that  $m_t$  changes only slowly from period to period. Beliefs then remain close to  $m_t = 1$  for all  $t$  and the above price is well defined because  $\rho < 1$ .

The key feature of house prices under learning is that there is feedback between expectations of price growth and actual price growth. Equation (32) shows that higher expected growth  $m_t$  leads to higher price and thus realized price growth, which in turn increases the expectations tomorrow via the belief updating rule. Therefore, the model has the potential to generate price booms that are fueled by the interaction between expectations and realized prices.

This can formally be shown by combining (33) and (31) to obtain a non-linear second order difference equation governing the behavior of  $m_t$ . The dynamics of this difference equation are very similar to those described in section 4.2.1 of Adam, Marcet and Nicolini (2010) for stock prices. The previous paper shows that price changes display momentum locally around the REE value, i.e., once prices start growing (falling), there is a tendency for prices to continue growing (falling), as well as there being mean reversion in the longer-run.

House price increases will come to an end when realized house price growth falls short of the expected price growth. Equation (33) shows that this occurs whenever the *increase* in price growth optimism becomes too weak to sustain the high *level* of price growth expectations. For example, if  $m_t$  is very high, but stays constant from one period to the next ( $m_t = m_{t-1}$ ), then equation (33) implies that realized price growth is equal to 1 on average, i.e., falls short of expectations. This sets in motion a sequence of downward belief revisions that lead to a price bust. Alternatively, upward price dynamics can come to an end if there is an increase in real interest rates that causes house prices to increase less than initially expected, or a negative shock to housing preferences.<sup>25</sup> As a result of the initial disappointment, there will be a decrease in price growth expectations, thereby a further fall in house prices, triggering a sequence of downward belief revisions.

The model thus has the potential to generate a house price boom which eventually will lead to a bust.

### 7.3 The Qualitative Response to Interest Rates Changes

We now explore the effects of an unanticipated decrease in real interest rates in period  $t$ .<sup>26</sup> Equation (33) implies that realized house price growth in period  $t$  increases as a result of a reduction in real interest rates.<sup>27</sup> The price increase is

<sup>25</sup>For the general case where  $G$  is strictly concave, the upward dynamics may also come to a halt if the increase in the housing stock associated with high house prices levels leads to a fall in  $G'$ .

<sup>26</sup>Technically, the change in the real interest rate is a probability zero event under the postulated beliefs.

<sup>27</sup>The interest rate enters in the definition of  $\rho$ , see equation (15)



thereby stronger for an economy in which agents in period  $t$  are more optimistic about future price growth (in which  $m_t$  is higher).

After a fall in real interest rates, the initial increase in realized price growth will feed into future beliefs about price growth via the belief updating equation (31). Due to momentum this leads to a sequence of further increase in realized price growth.

Importantly, interest rates are not the only determinant of whether or not a price boom occurs. If the house price in a given country has been increasing already before the reduction in interest rates, then the interest rate reduction will make it more likely that the house price boom will continue in this country. Conversely, in a country where the house prices have been decreasing, the interest rate reduction may only ameliorate the decrease in house prices. Therefore the model is consistent with the observation that house price booms are not synchronized across countries, even though interest rates behave in a similar way.

A house price boom also relaxes the collateral constraint and leads to an increase in total borrowing, which is given by

$$b_t = \frac{\theta q_t m_t}{R} H_t$$

House price increases are thus associated with increased international borrowing, i.e., a current account deficit. Provided investment in new houses is not too elastic, the house price boom will also be associated with a consumption boom.

Finally, from equation (6) follows that an increase in expected house price growth leads to an increase in the production of new houses, thereby qualitatively matching the observation about new housing supply displayed in Figure 4. Admittedly, the model cannot reproduce the asymmetric and sharp decline in new houses after the year 2006. Given the simplicity of the model, however, this should be hardly surprising.

Qualitatively, the model thus has the potential to explain a housing boom, that is associated with a current account deficit, a consumption increase and an increase in the production of new housing units. The next sections explore the ability of the learning model to quantitatively account for the real house price and current account dynamics in the U.S. and the remaining G7 economies.

## 8 The U.S. Experience: 2001-2008

We now calibrate the learning model to the U.S. economy and show that it can quantitatively replicate the real house price and current account developments for the U.S. economy over the years 2001-2008. The performance for the remaining G7 economies is analyzed in the next section.

We use as data inputs the history of real house prices over the years 1996-2000 and the stylized path for real interest rates of the years 2001-2008, as captured by the dashed line in figure 2.<sup>28</sup> Except for the stylized information

<sup>28</sup>Specifically, for the years 1996-2000 we set real interest rates equal to the average ex-ante

about the real interest rate, the predictions we show below do not use any data after the year 2000.

As in section 5.3, we choose  $\theta = 0.26$ ,  $\delta = 0.96$  and  $d = 3\%$ . We set the initial price growth expectations in 1996 to be consistent with stable house prices, as in a REE, i.e., we choose  $m_{1996} = 1$ . We then use the belief updating equation (31) from the model to impute house price beliefs for the year 2000 ( $m_{2000}$ ). We do this using as inputs the assumed initial belief and the real house price growth observation from the U.S. data. The choice of the gain parameter  $g$  is explained below.

We then use  $m_{2000}$  and the real interest rate  $R_{1996-2000}$  to compute the equilibrium real house price for the year 2000. We thereby normalize the equilibrium real house price in the year 2000 to 100 by choosing the value of  $\xi G'(H)$  correspondingly.<sup>29</sup> We then use the model to generate the predicted real house price for the years 2001-2008, using as inputs only the interest rate decrease for the years 2001-2005 and the increase for the years 2006-2008.

We then choose the gain parameter  $g$  in the belief updating equation (31) to minimize the absolute distance between the model implied prediction for the real house price and the data. This leads to a annual gain of  $g = 0.06$ , which implies that agents believe that on average 94% of any observed annual house price increase is transitory in nature.

Figure 7 reports the model-predicted outcome jointly with the U.S. real house price series for the years 2000-2008. The model shows that the decrease in real interest rates in the year 2001 gives rise to an initial increase in the real house price. Since realized price increases feed positively into future beliefs via the updating equation, the initial increase will be followed by further upward price movements, giving rise to a house price boom. The increase comes to an end in the year 2006 when interest rates move up again, causing the house price to slowly revert direction, in line with the data. The resulting downward revision in beliefs then sets in motion a sequence of price reductions.

Figure 7 also depicts the model predicted counterfactual house price path that would be obtained if real interest rates in the years 2001-2008 remained at their pre 2001 average.<sup>30</sup> House prices would then have increased only very mildly. A small increase would have occurred nevertheless, simply because of the positive price momentum that existed already prior to the year 2000. Overall, the model is consistent with the view that the US housing price boom was mostly caused by interest rates being too low for too long.

The current account dynamics implied by the housing boom in figure 7 depend partly on the long-run housing supply elasticity ( $\frac{\alpha}{1-\alpha}$ ), because housing

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gross real mortgage rate, i.e.,  $R_{96-00} = 1.0335$ . To capture the real interest rate decrease following in years 2001-2005, we set real interest to  $R_{01-05} = 1.0228$ , which is again the average ex-ante U.S. real mortgage rate for this period in the data. Finally, we capture the upward shift in real rates in the years 2006-2008 by setting  $R_{06-08} = 1.0301$ , which is again taken from the data.

<sup>29</sup>We keep  $\xi G'(H)$  fixed at this calibrated value in all subsequent model periods. The value for  $\xi G'(H)$  only normalizes the house price level, but has no impact on the dynamics.

<sup>30</sup>Gross ex-ante real interest rates are then assumed to stay constant at their 1996-2000 average, which is  $R_{96-00} = 1.0335$ .

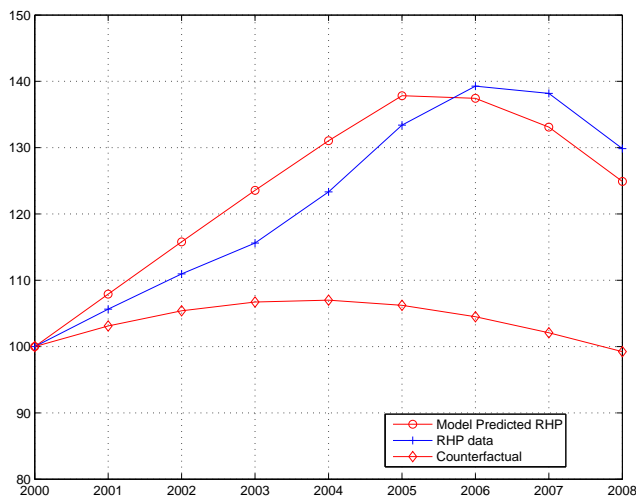


Figure 7: US real house prices: model predictions, data and counterfactual

can be used as collateral in international borrowing. For their preferred specification, Topel and Sherwin (1988) estimate a long-run housing supply elasticity of 3 for the United States. Since there is considerable uncertainty about this parameter, we allow for values between 1 and 5 and choose the long-run elasticity that minimizes the absolute distance between the model predicted current account deficit ratio and the current account deficit ratio in the data.<sup>31</sup> The model then prefers a relative elastic supply with  $\frac{\alpha}{1-\alpha} = 5$ . Figure 8 depicts the current account ratio in the data and the one implied by the model. The model predicts well the deterioration of the U.S. current account over the years 2001-2005, but overpredicts the improvements following the house price collapse after 2005. Overall, the quantitative performance of the model is surprisingly good, given that it abstracts from many other factors relevant for the current account, for example, fiscal borrowing.

Figure 8 also depicts the counterfactual reaction of the current account if real interest rate had stayed at their 1996-2000 average. The model predicts that large part of the current account surplus would not have occurred, had interest rates not decreased after the year 2000. This is the result of a lower volume and a lower value of collateral in the absence of a price and construction boom.

<sup>31</sup>To obtain a model-implied current account to GDP ratio, one also has to take a stand on the exogenous income process. We choose a time-invariant income, i.e.,  $y_t = y$ , so that income fluctuations do not contribute to explaining variations in the model-implied current account to GDP ratio. We then choose the level of  $y$  so as to minimize the sum of absolute distances between model-predicted current account to GDP ratio and the data.

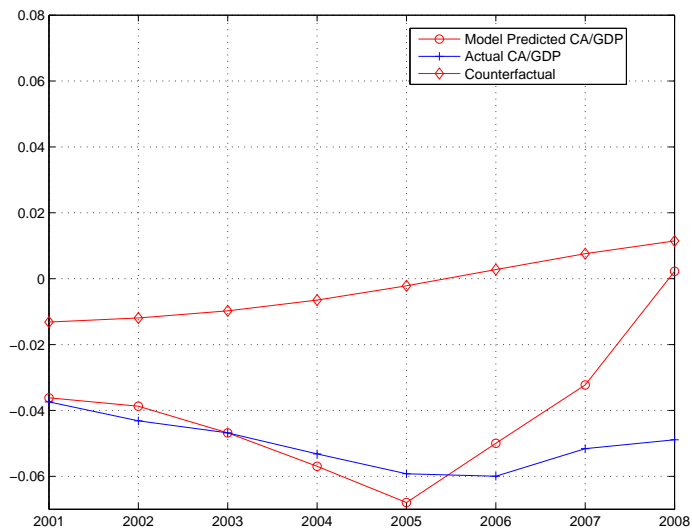


Figure 8: US current account deficit ratio: model predicted and data

## 9 Other G7 Economies: 2001-2008

We now evaluate the ability of the learning model to explain the real house price and current dynamics over the years 2001-2008 in the remaining G7 economies.

### 9.1 Real House Price Dynamics

We tie our hands by using the same model parameterization as for the U.S. economy.<sup>32</sup> We also subject each of the G7 economies to the same stylized interest rate path as the U.S. economy, which amounts to interpreting the U.S. real mortgage rate as a proxy for international real interest rates. Clearly, this approach biases results against us, as we could instead choose to parameterize the model for each country so as to achieve the best possible fit with the data. As we show below, the model nevertheless performs surprisingly well.

It is important to note that countries differ although we use the same parameterization as for the US economy. This is the case because agents entertain different beliefs at the time of the real interest rate decrease. As before, these beliefs are computed using the belief updating equation (31) and by assuming that initial beliefs in 1996 are consistent with no house price growth ( $m_{1996} = 1$  in each country). The different country-specific house price histories then lead to different imputed beliefs for the year 2000. We thereby use the same gain

<sup>32</sup>This is true, except for the value of  $\xi G'$ , which is chosen in each country to normalize the model-implied real house prices in the year 2000 to 100. As is apparent from equations (32) and (33), the choice of  $\xi G'$  does not affect the model dynamics.

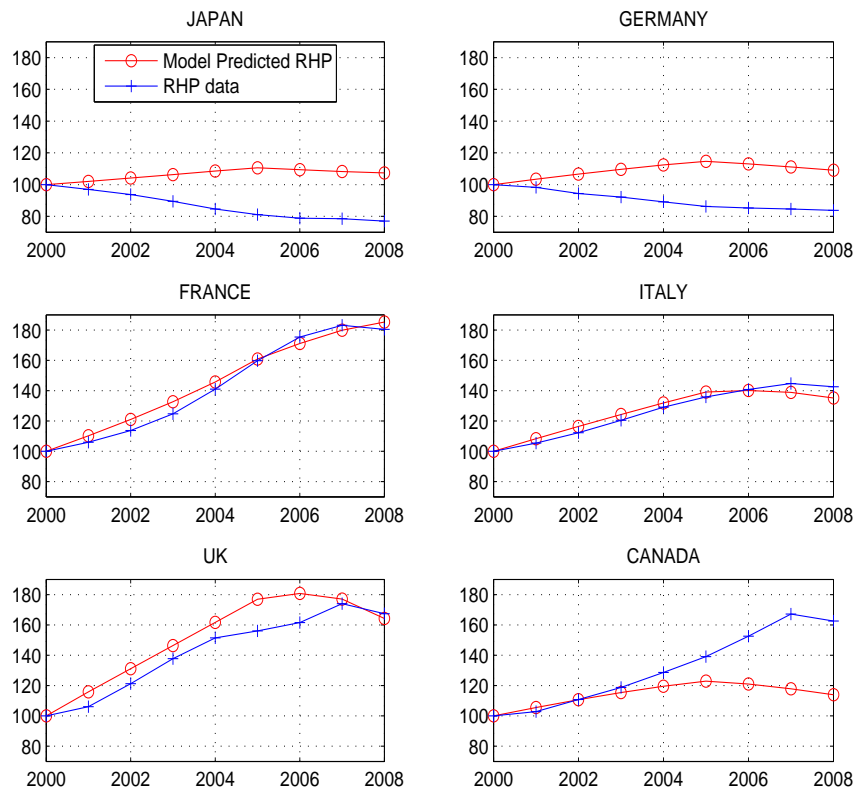


Figure 9: Other G7 economies: models-predicted real house prices and real house price data

parameter ( $g = 0.06$ ) as for the US economy for all countries, except for the U.K. where we use a slightly smaller gain value of  $g = 0.046$  because house price dynamics become unstable for the U.S. parameterization.<sup>33</sup> For none of the countries (except for the UK), we use information on price behavior after the year 2000.

The outcome of this exercise is depicted in 9. The figure illustrates that our model predicts strong house price increases for France, Italy, and the U.K., in line with the empirical evidence. The model also predicts much more muted increases for Germany and Japan than for these first countries, albeit it does not replicate the observed fall in house prices in these countries. Real house prices in Germany and Japan have been falling prior to the year 2000, so that momentum implies that agents' price growth expectations tend to decrease further. The interest rate reduction turns this negative momentum into some slight positive momentum. For the case with Canada, the model predicts a house price boom, but underpredicts its size, especially at the end of the sample period.

The second row of table 5 below reports the annual cross-sectional correlation between the model-predicted real house price and the real house price in the data. The table shows that this correlation is very high in each year. The model thus accounts surprisingly well for the asynchronous low frequency movements in house prices, even though we subject all economies to the same interest rate shocks.

Year	2001	2002	2003	2004	2005	2006	2007	2008
Real House Price	0.83	0.91	0.92	0.90	0.85	0.81	0.79	0.77

Table 5: Yearly cross-sectional correlation between model-predicted and actual real house price for the G7 economies

## 9.2 Current Account Dynamics

We now evaluate the ability of the learning model to explain the current account dynamics for the remaining G7 economies. As is clear from figure 9, the model will have difficulties in explaining the *level* of the current account for Germany, Japan and Canada. For these countries the model predicts house price booms (albeit small ones for Germany and Japan) and therefore initially current account deficits, while the data display current account surpluses for these economies throughout the considered period. For these reasons, we explore below also the ability of the model to account for the changes in the G7 current accounts over time.

Figure 10 reports the model-predicted outcomes, when seeking to matching the level of the current account to GDP ratio in each country. We thereby employ the same method for matching the data as for the US economy. As

<sup>33</sup>We choose the gain parameter for the U.K. so as to minimize the sum of absolute differences between the model predicted house price and the data over the years 2001-2008.

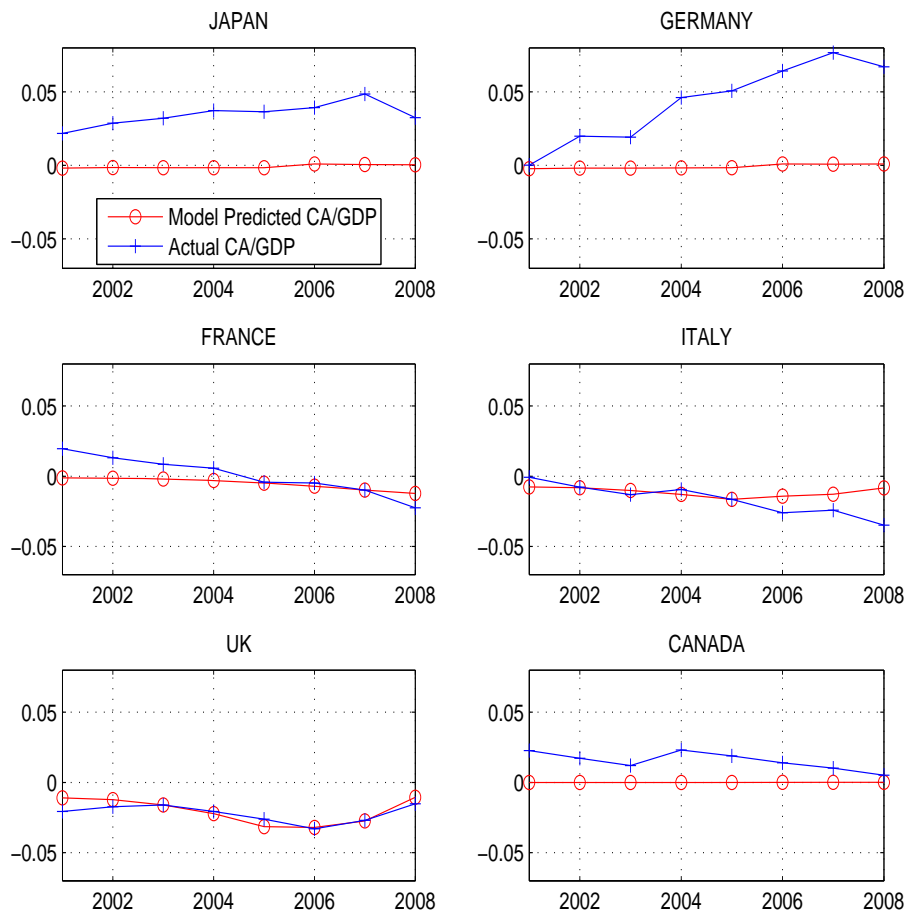


Figure 10: Model-predicted and actual current account ratios: matching the levels

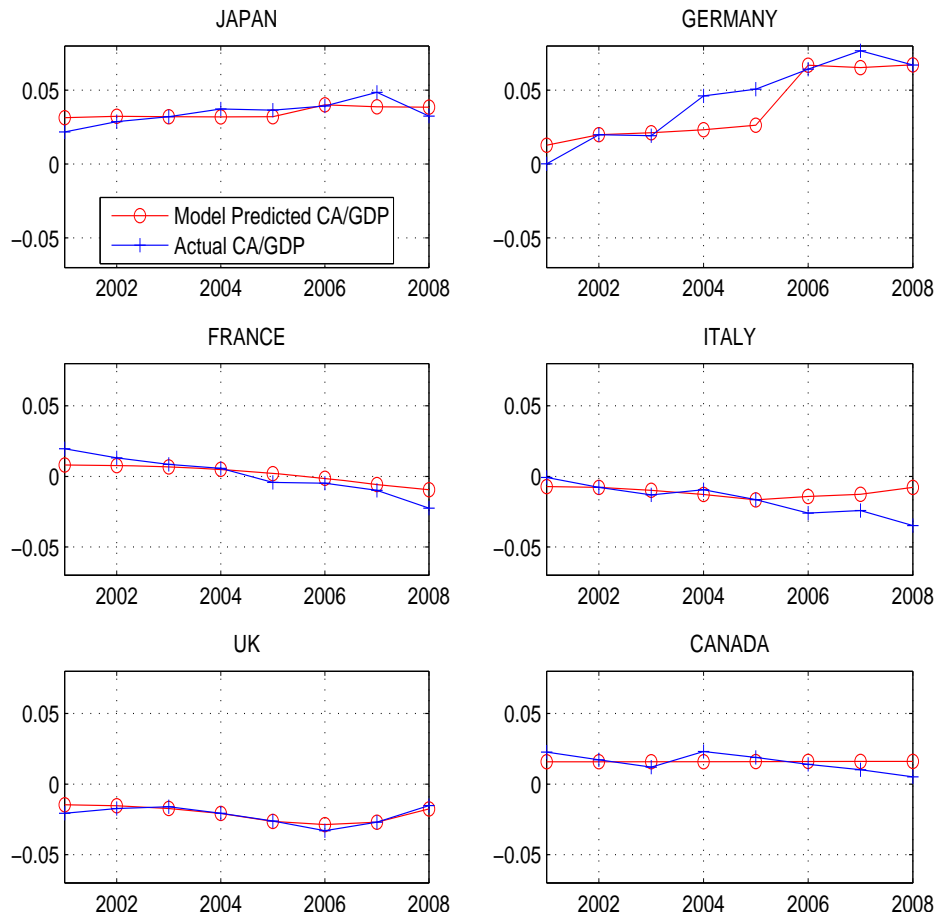


Figure 11: Model-predicted and actual current account ratios: matching the trend



expected, the figure shows that the model fails to replicate the evidence for Germany, Japan and Canada.<sup>34</sup> At the same time, the model does a fairly good job in replicating the evidence for France, Italy and the U.K.<sup>35</sup>

Figure 11 reports the outcome when we seek to match instead the *trend* in the current account ratios. We achieve this by allowing for a time-invariant unexplained level component  $c$  in the current account ratio of each country.<sup>36</sup> Figure 11 shows that the model then performs surprisingly well for all G7 economies, especially the performance for Germany, Japan and Canada improves considerably. Table 6 below reports the parameters values for the long-run supply elasticity and the unexplained level component used in figure figure 11. For France, Italy and the U.K. the model-implied supply elasticities are almost unchanged and the level variable is relatively small. For Germany, Japan and Canada there is a positive (unexplained) level component in the current account and housing supply is estimated to be rather inelastic.

Overall, our simple model can successfully account for the trends in the G7 current account ratios over the years 2001-2008, and reasonably well for the level in all countries but Japan, Germany and Canada.

	Japan	Germany	France	Italy	U.K.	Canada
$\alpha/(1-\alpha)$	1	1	5	5	4.4	1
$c$ (relative to GDP)	0.037	0.051	0.010	0.001	-0.010	0.016

Table 6: Long-run supply elasticity and unexplained level in CA/GDP ratio

## 10 Conclusions

This paper shows how a simple model of learning can quantitatively account for the G7 house price developments over the recent housing boom (2001-2008), as well as for the associated current account dynamics. While the model has difficulties in explaining the level of the current account ratios of some countries, it does surprisingly well in replicating the G7 trends.

Our model predicts that a fall in the level of the real interest rate can fuel a persistent and long lasting increase in real house prices. The house price effects depend, however, crucially on the degree to which agents expect already capital gains to materialize, i.e., on the past price dynamics and shocks hitting the economy.

<sup>34</sup>Since the matching procedure seeks to minimize the sum of absolute distances, it chooses a very high values for the income process  $y$ , so that the model predicted current account to GDP ratio is zero for these countries.

<sup>35</sup>The implied long-run housing supply elasticity,  $\alpha/(1-\alpha)$ , is equal to 5 in France and Italy and equal to 4 in the U.K.

<sup>36</sup>We then choose in each country the long-run supply elasticity  $\frac{\alpha}{1-\alpha} \in [1, 5]$ , the time-invariant income  $y$ , and a time-invariant current account level variable  $c$ , so as to minimize the sum of the absolute distance between the model predicted current account ratio and the current account ratio in the data, when adding  $c$  to each difference term in the sum.

The model also suggests that house price booms can give rise to important welfare distortions because they lead to an over-extension of the housing stock. It thus appears of interest to explore to what extent policy instruments, e.g., adjustments in the permissible leverage ratio, could be used to prevent an overly large build-up of the housing stock. As emphasized in Glaeser et al. (2008), the welfare effects of a house price boom thereby depend not only on the size and duration of the house price increase, but also on the elasticity of housing supply.

## A Appendix

### A.1 Non-negativity constraints on consumption

We now determine the behavior of the model when the non-negativity constraints on consumption are binding. For expositional clarity we discuss here the non-stochastic case only.

In the main text we supposed that the collateral constraint is binding in all periods. As we show now, this could lead to a violation of the non-negativity constraint on consumption whenever income  $y_t$  is not high enough. Consider, for example, a case when  $E_j^P q_{j+1} H_j$  is very high at  $j = t$  and very low at  $j = t + 1$ . If borrowing is at the collateral limit in both periods, then this implies a large decrease in debt at  $t + 1$ . And if income  $y_{t+1}$  is not high enough, this would require negative consumption in  $t + 1$ . The optimal solution then cannot have the feature that the collateral constraint is binding periods  $t$  and  $t + 1$  simultaneously.

In such a situation one can determine the optimal solution as follows. One can suppose that the non-negativity constraint on consumption is binding only in period  $t + 1$  but not binding in period  $t$ , so that we have  $\gamma_t = 0$ . From the non-binding zero limit in  $t$  we have  $\lambda_t = 1$  and from (12) we obtain  $\lambda_{t+1} > 1$ , as  $\delta R < 1$ , so that indeed  $c_{t+1} = 0$ . The binding borrowing constraint at  $t + 1$  then determines  $b_{t+1}$ . Using this and the fact that  $c_{t+1} = 0$  one obtains  $b_t$  from the budget constraint at  $t + 1$ . The value for  $c_t$  then follows from the budget constraint at  $t$ . Moreover, since  $\gamma_{t+1} > 0$  we can have (12) holding and  $\lambda_{t+2} = 1$  so that  $\mu_{t+2} = 0$  and  $c_{t+2} > 0$ , so that from  $t + 2$  onwards we are back in the case analyzed in the main text where consumption is positive and the collateral constraint is binding.

If the previous solution would still imply negative consumption in  $t$ , then one would have to extend the approach to a setting where consumption is zero for more than one period, say between periods  $t + 1, \dots, t + n$ . In this case we would have that the collateral constraint being non-binding for  $n$  periods, i.e., for periods  $t, \dots, t + n - 1$  and one could work backwards to derive a candidate solution in the same manner as described above. Again, after period  $t + n$  one would be back in the setting analyzed in the main text.

## A.2 Linear-quadratic approximation

The main text studies the approximate equilibrium outcomes when linearizing  $G(\cdot)$  around its steady state value. This simplifies the analysis and allows us to relate the analysis to results derived in Adam, Marcet and Nicolini (2010). We now extend the analysis to a quadratic approximation of  $G(\cdot)$ . Besides increasing the order of the approximation, this is of interest because it introduces an interaction between housing prices and the level of housing construction. Considering concavity in  $G(\cdot)$  is also useful because it makes it less likely that explosive paths for prices will arise under learning: as house prices and new construction increases, the marginal value of housing services  $G'(H_t)$  decreases, which exerts a dampening effect on the upward prices dynamics under learning.

We show below that the unique locally non-explosive rational expectations (RE) solution then takes the form

$$\begin{aligned}\hat{q}_t &= a^{RE}\hat{\xi}_t + b^{RE}\hat{H}_t \\ \hat{H}_{t+1} &= c^{RE}\hat{H}_t + d^{RE}\hat{\xi}_t\end{aligned}$$

where hatted variables denote deviations from the steady state and  $(a^{RE}, b^{RE}, c^{RE}, d^{RE})$  are given coefficients satisfying  $a^{RE} > 0$ ,  $b^{RE} < 0$ ,  $0 < c^{RE} < 1$  and  $d^{RE} > 0$ .

We now derive a first order accurate approximation to the rational expectations (RE) solution of the equation system

$$q_t = \left( \frac{\theta}{R} + (1-d-\theta)\delta \right) E_t^P q_{t+1} + \xi_t G'(H_t) \quad (34)$$

$$H_{t+1} = (1-d)H_t + S(E_t^P q_{t+1}, \delta^{-1}) \quad (35)$$

We linearize these equations around some steady state  $(q, H, \xi)$ , i.e., around a point solving the above system of equation for  $q_t = q$ ,  $H_t = H$  and  $\xi_t = \xi$  for all  $t$ . Letting hatted variables again denote deviations from steady state values, a first order approximation to (34) delivers

$$\hat{q}_t = \rho E_t^P \hat{q}_{t+1} + G'\hat{\xi}_t + \xi G''\hat{H}_t \quad (36)$$

where, as in the text,  $\rho = \left( \frac{\theta}{R} + (1-d-\theta)\delta \right)$  and with all derivatives evaluated at the steady state. A linearization of (35) delivers

$$\hat{H}_{t+1} = (1-d)\hat{H}_t + S'E_t^P q_{t+1} \quad (37)$$

We now conjecture a perceived law of motion (PLM) of the form

$$\hat{q}_t = a\hat{\xi}_t + b\hat{H}_t$$

With RE and using the law of motion for  $\xi_t$  we have

$$E_t \hat{q}_{t+1} = a\hat{\xi}_t + bE_t \hat{H}_{t+1} \quad (38)$$

Substituting into (37) delivers

$$\widehat{H}_{t+1} = (1-d)\widehat{H}_t + S'(a\widehat{\xi}_t + bE_t\widehat{H}_{t+1})$$

Taking the expectations  $E_t$  of this equation delivers

$$E_t\widehat{H}_{t+1} = \frac{(1-d)}{(1-S'b)}\widehat{H}_t + \frac{S'a}{(1-S'b)}\widehat{\xi}_t \quad (39)$$

so that (38) implies

$$E_t\widehat{q}_{t+1} = a\widehat{\xi}_t + \frac{(1-d)b}{(1-S'b)}\widehat{H}_t + \frac{S'ab}{(1-S'b)}\widehat{\xi}_t \quad (40)$$

Substituting this into (36) delivers the actual law of motion (ALM)

$$\widehat{q}_t = \left( \rho a + \rho \frac{S'ab}{(1-S'b)} + G' \right) \widehat{\xi}_t + \left( \rho \frac{(1-d)b}{(1-S'b)} + \xi G'' \right) \widehat{H}_t$$

Equation coefficients in the ALM and PLM delivers two conditions for  $a^{RE}$  and  $b^{RE}$  given by

$$\begin{aligned} a^{RE} &= \rho a^{RE} + \rho \frac{S'a^{RE}b^{RE}}{1-S'b^{RE}} + G' \\ b^{RE} &= \rho \frac{(1-d)b^{RE}}{1-S'b^{RE}} + \xi G'' \end{aligned}$$

The second equation depends only on  $b^{RE}$  but is quadratic, the first is linear in  $a^{RE}$ , conditional on  $b^{RE}$ .

$$0 = S' (b^{RE})^2 + (-1 + \rho(1-d) - S'\xi G'') b^{RE} + \xi G''$$

which has two solutions

$$\begin{aligned} b_1^* &= \frac{(1 - \rho(1-d) + S'\xi G'') + \sqrt{(-1 + \rho(1-d) - S'\xi G'')^2 - 4S'\xi G''}}{2S'} \\ b_2^* &= \frac{(1 - \rho(1-d) + S'\xi G'') - \sqrt{(-1 + \rho(1-d) - S'\xi G'')^2 - 4S'\xi G''}}{2S'} \end{aligned}$$

The corresponding solution for  $a$  is

$$a_i^* = \frac{G'}{1 - \frac{\rho}{1-S'b_i^*}} \quad (41)$$

From (37) and (40) follows that the dynamics for  $\widehat{H}_t$  are given by

$$\begin{aligned} \widehat{H}_{t+1} &= (1-d)\widehat{H}_t + S'E_t^P q_{t+1} \\ &= (1-d)\widehat{H}_t + S' \left( a_i^* \widehat{\xi}_t + \frac{(1-d)b_i^*}{(1-S'b_i^*)} \widehat{H}_t + \frac{S'a_i^*b_i^*}{(1-S'b_i^*)} \widehat{\xi}_t \right) \\ &= \frac{1-d}{1-S'b_i^*} \widehat{H}_t + \frac{S'a_i^*}{1-S'b_i^*} \widehat{\xi}_t \end{aligned} \quad (42)$$

Since  $1 > d > 0$ , we have that  $S'b_i^* < 0$  is a sufficient condition for the dynamics for  $\widehat{H}_t$  to be locally non-explosive. It is easy to show that the solution  $(a_2^*, b_2^*)$  is non-explosive while  $(a_1^*, b_1^*)$  implies locally explosive dynamics. Therefore,  $(a^{RE}, b^{RE}) = (a_2^*, b_2^*)$  and  $b^{RE} < 0$ . The values for  $(c^{RE}, d^{RE})$  follow from equation (42). As we show in the next section, show there exists no other locally non-explosive RE equilibrium.

### A.2.1 Local Uniqueness of the RE Solution

We now show that there exists no other locally non-explosive RE solution than the one derived in the previous section. We bring the linearized equation (37) and (36) in vector notation:

$$\begin{pmatrix} 1 & -S' \\ 0 & \rho \end{pmatrix} \begin{pmatrix} \widehat{H}_{t+1} \\ E_t^P \widehat{q}_{t+1} \end{pmatrix} = \begin{pmatrix} 1-d & 0 \\ -\xi G'' & 1 \end{pmatrix} \begin{pmatrix} \widehat{H}_t \\ \widehat{q}_t \end{pmatrix} + \begin{pmatrix} 0 \\ -G' \end{pmatrix} \xi_t$$

Inverting the matrix on the left, which is always invertible, we get

$$\begin{pmatrix} \widehat{H}_{t+1} \\ E_t^P \widehat{q}_{t+1} \end{pmatrix} = \begin{pmatrix} 1 - \frac{\xi G'' S'}{\rho} - d & \frac{1}{\rho} S' \\ -\frac{\xi G''}{\rho} & \frac{1}{\rho} \end{pmatrix} \begin{pmatrix} \widehat{H}_t \\ \widehat{q}_t \end{pmatrix} + \begin{pmatrix} -\frac{1}{\rho} G' S' \\ -\frac{1}{\rho} G' \end{pmatrix} \begin{pmatrix} 0 \\ -G' \end{pmatrix} \xi_t$$

which is a system with one predetermined and one ‘jump’ variable. It has a locally unique REE if the first matrix on the right-hand side has one explosive and one stable eigenvalue. The eigenvalues are

$$\begin{aligned} \lambda_1 &= \frac{1}{2\rho} \left( \rho - d\rho - \xi G'' S' + 1 + \sqrt{(\rho - d\rho - \xi G'' S' + 1)^2 + 4\rho(d-1)} \right) \\ \lambda_2 &= \frac{1}{2\rho} \left( \rho - d\rho - \xi G'' S' + 1 - \sqrt{(\rho - d\rho - \xi G'' S' + 1)^2 + 4\rho(d-1)} \right) \end{aligned}$$

It is straightforward to show that  $\lambda_1$  is unstable ( $\lambda_1 > 1$ ) while  $\lambda_2$  is a stable eigenvalue ( $-1 < \lambda_2 < 1$ ).

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