Credit Risk and Disaster Risk

Preliminary and Incomplete

François Gourio*

November 2010

Abstract

Macroeconomic models with financial frictions typically imply that the excess return on a well-diversified portfolio of corporate bonds is close to zero. In contrast, the empirical finance literature documents large and time-varying risk-premia in the corporate bond market (the "credit spread puzzle"). This paper introduces a parsimonious real business cycle model where firms issue defaultable debt and equity to finance investment. The mix between debt and equity is determined by a trade-off between tax savings and bankruptcy costs. By their very nature, corporate bonds, while safe in normal times, are highly exposed to the risk of economic depression. This motivates introducing a small, time-varying risk of large economic disaster. This simple feature generates large, volatile and countercyclical credit spreads as well as novel business cycle implications. An increase in disaster risk leads to higher risk premia, and raises expected discounted bankruptcy costs, hence worsening financial frictions. This leads to a reduction in investment, output, and leverage. Financial frictions amplify significantly the effects of disaster risk: the response of investment and output is about three times larger than in the frictionless model.

Keywords: financial frictions, financial accelerator, bubbles, asset pricing, credit spread puzzle, business cycles, equity premium, time-varying risk premium, disasters, rare events, jumps.

JEL: E32, E44, G12.

1 Introduction

A large literature in macroeconomics shows that financial frictions can amplify or propagate shocks and hence contribute to the business cycle.[1] This analysis is almost uniformly conducted in linearized DSGE models, where asset prices are much less volatile than in the data, and aggregate risk premia are small and nearly constant. The mechanisms of these models often feature asset prices: for instance, Kiyotaki and Moore (1997) emphasize the amplification effect of changes in collateral value, while Bernanke,

*Department of Economics, Boston University and NBER. Address: 270 Bay State Road, Boston MA 02215. Email: fgourio@bu.edu. Phone: (617) 353 4534. Michael Siemer provided outstanding research assistance. I thank Simon Gilchrist and participants in presentations at Boston University and SED 2010 for comments. NSF funding under grant SES-0922600 is gratefully acknowledged.

[1] Bernanke and Gertler (1989), Kiyotaki and Moore (1997), Bernanke, Gertler and Gilchrist (1999), Carlstrom and Fuerst (1997), Cooley, Quadrini and Marimon (2004).

Gertler and Gilchrist (1999) focus on the default premium. The question is then: what is the effect of financial frictions on macroeconomic dynamics in a model where asset prices more closely mimic the data? In particular, is there an amplification effect of financial frictions in such a model?

In this paper I focus on the behavior of credit spreads, which have received much attention both in DSGE models and in the empirical literature. Credit spreads are highly correlated, and forecast, investment and output.[2] Studies that estimate versions of the Bernanke-Gertler-Gilchrist model[3] also find an important role for the financial accelerator and for financial shocks. However, these models are at odds with the "credit spread puzzle", documented in the finance literature.[4] In the Bernanke, Gertler and Gilchrist model, a portfolio of corporate bonds would earn essentially the risk-free rate, since idiosyncratic risk is diversified, and aggregate risk premia are small. In contrast, in the data the probability of default of an investment grade bond is small, about 0.4% per year, (and there is substantial recovery upon default, around 50%), but the spreads are large, on average around 100bp.[5] These large spreads suggest the importance of a large, potentially time-varying risk premium. These spreads are moreover quite volatile, with a standard deviation around 40bp per year, and they are countercyclical. The level of spreads was particularly elevated during the recent financial crisis,[6] but the cyclicality of spreads is a recurring feature of U.S. business cycles.

This paper addresses theses questions in the context of a simple dynamic general equilibrium model, by embedding a trade-off model of capital structure, where the choice of defaultable debt is driven by taxes and bankruptcy costs, into a real business cycle (RBC) model. More precisely, the capital structure choice modifies the standard RBC model equilibrium in two ways. First, the standard Euler equation is adjusted to reflect that investment is financed using both debt and equity, and the user cost of capital hence takes into account expected discounted bankruptcy costs as well as the tax savings generated by debt finance. Second, an additional equation determines the optimal leverage choice, by equating the marginal expected discounted (tax) benefits and (bankruptcy) costs of debt. Hence, the model remains highly tractable and intuitive, which allows to evaluate the role of defaultable debt and leverage choice on quantities and prices in a parsimonious model and in a transparent fashion. In particular, the model encompasses the standard real business cycle model as a special (limiting) case.

By their very nature, corporate bonds are very sensitive to tail risk: while safe in normal times, they are highly exposed to the risk of economic depression. This motivates introducing a small, time-varying risk of large economic disaster, following the work of Rietz (1988), Barro (2006), Gabaix (2007), and Gourio (2010). The first result of this paper is that this simple feature generates large, volatile and countercyclical credit spreads as well as novel business cycle implications. The trade-off model, a

[2] Philippon (2008), Gilchrist, Yankov and Zakrajek (2009), Mueller (2009), among others.

[3] For instance, Christiano, Motto and Rostagno (2009), or Gilchrist, Ortiz and Zakrajek (2009).

[4] Huang and Huang (2003), Hackbarth, Miao and Morellec (2006), Chen (2008), Chen, Collin-Dufresne and Goldstein (2009), Kuehn and Strebulaev (2008) among others.

[5] This is the spread of a BAA-rated corporate bond over a AAA-rated corporate bond (rather than a Treasury), so as to net out differences in liquidity.

[6] Some of this variation in credit spreads could be driven by time-varying liquidity spread (but note that I focus on the BAA-AAA spread), or to the deteriorating balance sheets of banks and other financial institutions, who may be the marginal investors in these markets. However, corporate bonds are not exotic assets: any household can buy a mutual fund or an ETF of corporate bonds.

staple of corporate finance, interacts with the large risk and correspondingly large risk premia created by disaster risk. The second main result is that financial frictions amplify substantially (by a factor of around three) the response of the economy to a shock to the disaster probability. Consistent with the extant literature, this amplification effect does not arise if the economy is subjected only to TFP shocks. Hence, it is the interaction between financial friction and time-varying disaster risk which generates novel, quantitatively appealing implications for both asset prices and quantities.

The key mechanism is as follows. When the probability of economic disaster exogenously increases, the probability of default rises (holding debt policy fixed). This raises expected discounted bankruptcy costs directly. However, expected discounted bankruptcy costs also rise through a discount rate channel: agents anticipate that defaults are now more systematic, i.e. more likely to be triggered by a bad aggregate shock rather than a bad idiosyncratic shock. More systematic defaults lowers the value of the corporate bond. Higher expected discounted bankruptcy costs increase the user cost of capital, leading to a reduction in investment. In equilibrium, firms also cut back on debt, but since debt is cheaper due to the tax advantage, the user cost of capital has to rise.

In contrast to most of the literature, which focuses on small firms which cannot raise equity easily and rely on bank financing, this model is designed to capture the richer margins that large U.S. corporations use to raise capital. In my model, firms always pay dividends, and no borrowing constraint binds. The relative attractiveness of debt and equity finance varies over time, leading to variation in the user cost of capital. My model thus is not subject to a standard critique of financial frictions models, that most firms do pay dividends and are "thus" unconstrained. Nor does my model rely on a significant heterogeneity between small, productive, constrained firms on the one hand, and large, unproductive, unconstrained firms on the other hand. Incorporating these realistic elements would of course be interesting, but it is not required. This suggests that the model mechanism is quite robust.

My model can be interpreted either as a purely rational model, with an objective probability of economic disaster, but it also admits a "behavioral" interpretation: the probability of disaster may not be a strictly rational belief, but rather a time-varying pessimism. For instance, during the recent financial crisis, many commentators highlighted the possibility that the U.S. economy could fall into another Great Depression. The model studies the macroeconomic effect of such time-varying beliefs. This simple modeling device captures the idea that aggregate uncertainty is sometimes high, i.e. people sometimes worry about the possibility of a deep recession. It also captures the idea that there are some asset price changes which are not obviously related to current or future productivity, i.e. "bubbles", "animal spirits", and which in turn affect the macroeconomy.

Organization of the paper

Section 2 sets up the model. Section 3 studies its quantitative implications. Section 4 considers some extensions of the baseline model. Section 5 concludes. An appendix details the numerical method, some extensions, and discusses a two-period partial equilibrium version of the model.

Related literature

This paper is related to four different branches of literature. First, the paper draws from the recent literature on "disasters" or rare events (Rietz (1988), Barro (2006), Barro and Ursua (2008), Gabaix (2007), Gourio (2008a, 2008b, 2010), Martin (2008), Santa-Clara and Yan (2008), Wachter (2008), Weitzmann (2007), and the criticisms of Julliard and Ghosh (2008) and Backus, Chernov and Martin (2009)). However it is important to realize that the key model mechanism is not specific to increases in the probability of disasters, but would also hold for general shocks to aggregate uncertainty.

Second, the paper builds on the large macroeconomic literature studying general equilibrium business cycle models with financing constraints (Bernanke and Gertler (1989) and Kiyotaki and Moore (1997)). Some recent studies in this vain are Chugh (2010), Gomes and Schmid (2008), Jermann and Quadrini (2008), Mendoza (2010), Miao and Wang (2010), and Liu , Wang and Zha (2009). Hennessy and Levy (2007) and Covas and den Haan(2009) consider the evolution of capital structure over the business cycle.

Third, the paper studies the real effects of a shock to uncertainty, a channel recently emphasized by Bloom (2009). In Bloom's model, increases in uncertainty about idiosyncratic productivity create "wait-and-see" effects through lumpy hiring and investment. In contrast, in my model, an increase in aggregate uncertainty lowers desired investment through general equilibrium (risk premia) and by exacerbating financial frictions. Also, Bloom's model generates endogenously a reduction in actual TFP, while the dynamics of my model are orthogonal to TFP. This paper hence provides an alternative mechanism through which higher uncertainty depresses economic activity. A related mechanism has recently been explored in the studies of Arellano, Bai and Kehoe (2010) and Gilchrist, Sim and Zakrajek (2010), who consider uncertainty shocks as in Bloom (2009), but in a setup with credit frictions. One important difference is that these authors focus on shocks to microeconomic risk, i.e. a shock to the variance of idiosyncratic shocks, while I consider a shock to aggregate uncertainty (the probability of economic disaster). The two effects are quite different, since in my case the increase in uncertainty is amplified as expected discounted bankruptcy costs rise through an increase in the discount rate as I show. These authors also do not attempt to reproduce the behavior of credit spreads (e.g. the large and volatile excess return on corporate bonds). On the other hand, they consider a richer microeconomic setting with heterogeneity. Arellano et al. also assume that labor must be financed in advance, so that their model generates a labor wedge.

Fourth, the paper relates to the vast literature on the "credit spread puzzle" (e.g. Huang and Huang (2003), Hackbardt, Miao and Morellec (2006), Chen (2010), Chen, Collin Dufresne and Goldstein (2009), and Bhamra, Kuehn and Strebulaev (2009a, 2009b)). As discussed in the introduction, this literature documents that the prices of corporate bonds are too low to be accounted for in a risk-neutral model, and considers various risk adjustments, borrowed either from the long-run risk literature, the habits literature or the disaster risk literature, to improve the fit of prices. This literature does not consider investment and is not set in general equilibrium, making it difficult to evaluate the macroeconomic impact of the financial frictions. On the other hand, this literature considers long-term debt and more detailed asset pricing implications.

2 The Model

We first present the household problem, then the firm problem, and finally define the equilibrium and asset prices.

2.1 Household

There is a representative household who has recursive preferences over consumption and leisure, following Epstein and Zin (1989):

$$U_t = \left((1-\beta)(C_t^{\upsilon}(1-N_t)^{1-\upsilon})^{1-\psi} + \beta E_t \left(U_{t+1}^{1-\gamma} \right)^{\frac{1-\psi}{1-\gamma}} \right)^{\frac{1}{1-\psi}}, \tag{1}$$

Here ψ is the inverse of the intertemporal elasticity of substitution (IES) over the consumption-leisure bundle, and γ measures risk aversion towards static gambles over the bundle. When $\psi = \gamma$, the model collapses to expected utility. The additional flexibility of recursive utility will prove useful, because the IES plays an important role in the analysis.

The household supplies labor in a competitive market, and trades in stocks and bonds issued by the corporate sector.[7] The budget constraint reads

$$C_t + n_t^s P_t + q_t B_t \leq W_t N_t + \varrho_t B_{t-1} + n_{t-1}^s \left(P_t + D_t \right) - T_t, \tag{2}$$

where W_t is the real wage, B_t is the quantity of debt issued by the corporate sector in period t at price q_t, each unit of which is redeemed in period $t+1$ for ϱ_{t+1}, n_t^s is the quantity of equity shares, P_t is the price of equity, D_t is the dividend, and T_t is a lump-sum tax. We will normalize the number of equity shares n_t^s to one. In the absence of default, $\varrho_{t+1} = 1$, but $\varrho_{t+1} < 1$ if some bonds are not repaid in full. The household takes the process of ϱ_{t+1} as given, but it is determined in equilibrium by default decisions of firms, as we will see later.

Intertemporal choices are determined by the stochastic discount factor (a.k.a. marginal rate of substitution), which prices all assets:

$$M_{t,t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{\upsilon(1-\psi)-1} \left(\frac{1-N_{t+1}}{1-N_t} \right)^{(1-\upsilon)(1-\psi)} \frac{U_{t+1}^{\psi-\gamma}}{E_t \left(U_{t+1}^{1-\gamma} \right)^{\frac{\psi-\gamma}{1-\gamma}}}. \tag{3}$$

The labor supply decision is governed by the familiar condition:

$$W_t = \frac{1-\upsilon}{\upsilon} \frac{C_t}{1-N_t}. \tag{4}$$

2.2 Firms

We first describe the general structure of the firm problem, then we fill in the details.

[7] It is possible to introduce government bonds as well. If the government finances its spending using lump-sum taxes, Ricardian equivalence holds, and government policy does not affect the equilibrium allocation and prices.

2.2.1 Summary

There is a continuum of firms, which are all identical ex-ante and differ ex-post only in their realization of an idiosyncratic shock. For simplicity, we assume that firms live only for two periods. Firms purchase capital at the end of period t in a competitive market, for use in period $t+1$. This investment is financed through a mix of equity and debt. In period $t+1$, the aggregate shocks and the idiosyncratic shock are revealed, firms decide on employment and production, and then sell back their capital. Two cases arise at this point: (1) the firm value is larger than outstanding debt: the debt is then repaid in full and the residual value goes to shareholders; or (2) the firm value is smaller than outstanding debt: in this case the firm declares default, equityholders receive nothing, and bondholders capture the firm's value, net of some bankruptcy costs. In all cases, the firms disappear after production in period $t+1$ and new firms are created, which will raise funds and invest in period $t+1$, and operate in period $t+2$.[8]

The timing assumption implies that a default *realization* does not affect employment, output and profits. Ex-ante however, default *risk* affects the cost of capital to the firm and hence its investment decision. This investment decision in turns affects employment and output, and in general equilibrium all quantities and prices. In section 4, we consider an extension where default affects production.

Since firms are ex-ante identical, they will all make the same choices. Because both production and financing technologies exhibit constant return to scales, the size distribution of firms is indeterminate, and has no effect on the aggregate outcomes.

2.2.2 Production

All firms operate the same technology. The production function has constant returns to scale and is Cobb-Douglas in capital and labor. The output of firm i is hence

$$Y_{it} = K_{it}^{\alpha}(z_t N_{it})^{1-\alpha},$$

where z_t is aggregate total factor productivity (TFP), K_{it} is the individual firm capital stock, and N_{it} is labor. Both input and output markets are competitive, and the labor market is frictionless.

2.2.3 Productivity shocks

Following Gourio (2010), I assume that the aggregate TFP process in this economy is a unit root process, with "small" normally distributed shocks as well as rare disasters. Formally,

$$\log z_{t+1} = \log z_t + \mu + \sigma e_{t+1} + x_{t+1} \log(1 - b_{tfp}),$$

where $\{e_t\}$ is *i.i.d.* $N(0,1)$, and $x_t = 1$ (a disaster) with probability p_t, and $x_t = 0$ with probability $1-p_t$. Hence, with probability p_t, the level of TFP falls permanently by a factor b_{tfp}. I will also assume that the realization of disaster affects the capital stock (see the next paragraph). The probability of disaster p_t follows itself a Markov chain with transition matrix Q. The *three* aggregate shocks $\{e_{t+1}, x_{t+1}, p_{t+1}\}$ are assumed to be independent, conditionally on p_t.

[8] The assumption that firms live two periods, while obviously unrealistic, leads to substantial simplification of the analysis, which is useful to solve the model but also to clarify its implications. Moreover, even in a model where firms are long-lived, the model mechanism would likely still play an important role (see section 3.1).

2.2.4 Investment shocks

Firms decide on investment at time t, but the actual quantity of capital that they will have to operate at time $t+1$ is random, and is affected both by realizations of aggregate disasters x_{t+1} as well as the idiosyncratic shock ε_{t+1}. Specifically, firm i picks $K^w_{i,t+1}$ at time t (w for wish), and it actually has $K_{i,t+1} = K^w_{i,t+1}(1 - x_{t+1}b_k)\varepsilon_{it+1}$ to operate in period $t+1$, and it will have $(1-\delta)K_{i,t+1}$ units of capital to resell. The idiosyncratic shock ε_{it+1} is *i.i.d.* across firms and across time, and drawn from a cumulative distribution function H, with mean unity.

2.2.5 Discussion of the assumptions regarding disasters

Disasters are large macroeconomic shocks such as wars and economic depressions. In a standard neo-classical model there are two simple ways to model them – as destruction of the capital stock, or as a reduction in total factor productivity. Similarly, TFP plays an important role during economic depressions (e.g., Kehoe and Prescott, 2007). While economists do not understand well the sources of fluctuations in total factor productivity, large and persistent declines in TFP may be linked to poor government policies.

The introduction of capital destruction deserves some discussion. First, in some cases, this simple modeling may be realistic: a war may well physically destroy a large share of equipment and structures. But there are alternative interpretations; for instance, b_k could reflect expropriation of capital holders (if the capital is taken away and then not used as effectively), or it could be a "technological revolution" that makes a large share of the capital worthless. It could also be that even though physical capital is not literally destroyed, some intangible capital (such as matches between firms, employees, and customers) is lost. Finally, one can imagine a situation where demand shifts lead to capital idleness: for instance, factories which were built to produce luxury boats or private aircrafts might never be used at full capacity following a deep recession. From the point of view of the theory, what is important is that, realistically, the return on capital is low during a disaster. Capital destruction generates this in a simple, tractable way, but the same result can be obtained using adjustment costs – in a disaster, investment falls significantly, leading to a reduction in the price of capital.

2.2.6 Capital structure choice

The choice of equity versus debt is driven by a standard trade-off between the bankruptcy costs and the tax advantage of debt (cite). Specifically, bondholders recover a fraction θ of the firm value upon default, where $0 < \theta < 1$, and that a firm which issues debt at a price q receives χq, where $\chi > 1$. That is, for each dollar that the firm raises in the bond market, the government gives a subsidy $\chi - 1$ dollar. In reality, interest on corporate debt is deductible from the corporate income tax, hence the implicit subsidy takes place when firms' earnings are taxed. For simplicity, I assume instead that the subsidy takes place at issuance.

The price q is determined at time of issuance, taking into account default risk, and hence depends on the firm's choice of debt and capital as well as the economy's state variables. Equity issuance is assumed to be costless. When $\chi = \theta = 1$, the capital structure is indeterminate and the Modigliani-

Miller theorem holds. When $\chi = 1$, the firm finances only through equity, since debt has no advantage. As a result, there is no default, and we obtain the standard RBC model. When $\theta = 1$, or more generally $\theta\chi \geq 1$, the firm finances only through debt, since default is not costly enough. We assume $\chi\theta < 1$, a necessary assumption to generate an interior choice for the capital structure.

2.2.7 Employment, Output, Profits, and Firm Value

To solve the optimal financing choice, we first need to determine the profits and the firm value at the end of the period. The labor choice is determined through the standard static profit maximization problem, given the realized values of both productivity and capital stock, and given the aggregate wage:

$$\pi\left(K_{it}, z_t; W_t\right) = \max_{N_{it} \geq 0} \left\{K_{it}^{\alpha}(z_t N_{it})^{1-\alpha} - W_t N_{it}\right\},$$

which leads to the labor demand

$$N_{it} = K_{it}\left(\frac{z_t^{1-\alpha}(1-\alpha)}{W_t}\right)^{\frac{1}{\alpha}},$$

and the output supply

$$Y_{it} = K_{it}^{\alpha}(z_t N_{it})^{1-\alpha} = K_{it}\left(\frac{(1-\alpha)}{\frac{W_t}{z_t}}\right)^{\frac{1-\alpha}{\alpha}}.$$

These equations can then be aggregated. Define aggregates through $K_t = \int_0^1 K_{it}di$, $Y_t = \int_0^1 Y_{it}di$, etc., we obtain that

$$Y_t = K_t^{\alpha}(z_t N_t)^{1-\alpha}, \tag{5}$$

i.e. an aggregate production function exists, and it has exactly the same shape as the microeconomic production function.[9] The wage satisfies the usual condition

$$W_t = (1-\alpha)\frac{Y_t}{N_t}. \tag{6}$$

The law of motion for capital is obtained by summing over i the equation $K_{i,t+1} = K_{i,t+1}^{w}(1 - x_{t+1}b_k)\varepsilon_{it+1}$. As noted above, all firms are identical ex-ante, and they will make the same investment choice $K_{i,t+1}^{w} = K_{t+1}^{w}$, hence idiosyncratic shocks average out and the aggregate capital is

$$K_{t+1} = K_{t+1}^{w}(1 - x_{t+1}b_k).$$

To take into account default, we need to know the firm value next period, since it determines the lending terms it can get. First, note that profits at time $t+1$ are given by

$$\pi_{it+1} = Y_{it+1} - W_{t+1}N_{it+1} = \alpha Y_{it+1} = \alpha K_{it+1}\left(\frac{(1-\alpha)}{\frac{W_{t+1}}{z_{t+1}}}\right)^{\frac{1-\alpha}{\alpha}} = K_{it+1}\alpha\frac{Y_{t+1}}{K_{t+1}},$$

i.e. each firm receives factor payments proportional to the quantity of capital it has, and to the aggregate marginal product of capital $\alpha\frac{Y_{t+1}}{K_{t+1}}$. The total firm value at the end of the period is

$$\begin{aligned} V_{it+1} &= \pi_{it+1} + (1-\delta)K_{it+1}, \\ &= K_{it+1}\left(1 - \delta + \alpha\frac{Y_{t+1}}{K_{t+1}}\right). \end{aligned} \tag{7}$$

[9] This is a consequence of the frictionless labor market, as firms equate their marginal product of labor, and hence given the Cobb-Douglas formulation their marginal product of capital.

We define the return on capital as $R^K_{t+1} = (1 - x_{t+1} b_k)\left(1 - \delta + \alpha \frac{Y_{t+1}}{K_{t+1}}\right)$. The individual return on capital is $R^K_{it} = \varepsilon_{it+1} R^K_{t+1}$. The firm value is thus

$$V_{it+1} = R^K_{it+1} K^w_{t+1} = \varepsilon_{it+1} R^K_{t+1} K^w_{t+1}.$$

From now on, I abstract from the firm subscript i.

2.2.8 Investment and Financing Decisions

As noted above, all firms make the same choices for capital, debt, and hence equity issuance, which are linked through the budget constraint $\chi q_t B_{t+1} + S_t = K^w_{t+1}$. To find the optimal choice of investment and financing, we first need to find the likelihood of default, and the loss-upon-default, for any possible choice of investment and financing. This determines the price of corporate debt. Taking as given this bond price schedule, the firm can then decide on optimal investment and financing.

More precisely, the firm will default if its realized value V_{t+1}, which is the sum of profits and the proceeds from the sale of undepreciated capital, is too low to repay the debt B_{t+1}. This will occur if the firm's idiosyncratic shock ε is smaller than a cutoff value, which itself depends on the realization of aggregate states $(e_{t+1}, p_{t+1}, x_{t+1})$. Mathematically, at time $t+1$, the value of firms which finish operating is $V_{t+1} = \varepsilon_{t+1} R^K_{t+1} K^w_{t+1}$, hence default occurs if and only if

$$\varepsilon_{t+1} < \frac{B_{t+1}}{R^K_{t+1} K^w_{t+1}} \stackrel{def}{=} \varepsilon^*_{t+1}.$$

Given this default rule, the bond issue is priced ex-ante using the representative agent's stochastic discount factor:

$$q_t = E_t \left(M_{t+1} \left(\int_{\varepsilon^*_{t+1}}^{\infty} dH(\varepsilon) + \frac{\theta}{B_{t+1}} \int_0^{\varepsilon^*_{t+1}} \varepsilon R^K_{t+1} K^w_{t+1} dH(\varepsilon) \right) \right).$$

In this equation, the first term gives the value of the debt in the full repayment states. These states depend on the realization of shocks occurring at time $t+1$, notably disasters, through ε^*_{t+1}. The second term gives the average recovery during default states, divided among all the bondholders and net of bankruptcy costs. We can rewrite the bond price as

$$q_t = E_t \left(M_{t+1} \left(1 - H\left(\varepsilon^*_{t+1}\right) + \frac{\theta R^K_{t+1} K^w_{t+1}}{B_{t+1}} \Omega\left(\varepsilon^*_{t+1}\right) \right) \right), \tag{8}$$

where I have defined $\Omega(x) = \int_0^x s dH(s)$. Note the following properties of Ω, given that H has mean unity: (i) $\Omega(x) = 1 - \int_x^\infty s dH(s)$; (ii) $\lim_{x \to \infty} \Omega(x) = 1$; (iii) $\Omega'(x) = x h(x)$.

We can now state the firm's problem at time t: it must decide how much to invest, how much debt to issue, and hence how much of the investment is financed through equity, so as to maximize the expected discounted equity value:

$$\max_{B_{t+1}, K^w_{t+1}, S_t} E_t \left(M_{t+1} \max\left(V_{t+1} - B_{t+1}, 0\right)\right) - S_t, \tag{9}$$

s.t.:

$$\chi q_t B_{t+1} + S_t = K^w_{t+1}, \tag{10}$$

$$V_{t+1} = \varepsilon_{t+1} R^K_{t+1} K^w_{t+1}. \tag{11}$$

Equation (10) is the funding constraint: investment must come out of equity S_t, or the sale of bonds (including the subsidy) $\chi q_t B_{t+1}$. The objective function (9) takes into account the option of default. Given that the firm defaults if $\varepsilon_{t+1} < \varepsilon^*_{t+1}$, we have

$$\begin{aligned} E_t\left(M_{t+1}\max\left(V_{t+1}-B_{t+1},0\right)\right) &= E_t\left(M_{t+1}\int_{\varepsilon^*_{t+1}}^{\infty}\left(\varepsilon R^K_{t+1}K^w_{t+1}-B_{t+1}\right)dH(\varepsilon)\right) \\ &= E_t\left(M_{t+1}\left(\left(1-\Omega(\varepsilon^*_{t+1})\right)R^K_{t+1}K^w_{t+1}-\left(1-H\left(\varepsilon^*_{t+1}\right)\right)B_{t+1}\right)\right), \end{aligned}$$

which allows to rewrite the firm's program by decomposing the expectation, and substituting in the constraint (10), which after some algebra leads to

$$\max_{B_{t+1},K^w_{t+1}} E_t\left(M_{t+1}\left(R^K_{t+1}K^w_{t+1}+\left(\chi\theta-1\right)R^K_{t+1}K^w_{t+1}\Omega(\varepsilon^*_{t+1})+\left(\chi-1\right)B_{t+1}\left(1-H\left(\varepsilon^*_{t+1}\right)\right)\right)\right)-K^w_{t+1} \tag{12}$$

$$s.t. \quad : \qquad \varepsilon^*_{t+1} = \frac{B_{t+1}}{R^K_{t+1}K^w_{t+1}}.$$

In this expression, the first term is the expected discounted firm value, $E_t\left(M_{t+1}R^K_{t+1}K^w_{t+1}\right)$; the second term is expected discounted bankruptcy costs (and hence is negative since $\chi\theta < 1$); and the third term is the expected discounted tax shield. The last term is simply the cost of investment. By contrast, in a frictionless model, the firm would simply maximize $E_t\left(M_{t+1}R^K_{t+1}K^w_{t+1}\right) - K^w_{t+1}$. Here the firm also takes into account the value of tax subsidies and default costs in making its decisions. While default costs are not born by equity holders ex-post, debt is issued at a lower price which reflects the expected, discounted default costs, so equity holders actually bear the default costs ex-ante.

To solve this program, we simply take the first-order conditions with respect to K^w_{t+1} and B_{t+1}. The first-order condition with respect to K^w_{t+1} yields,

$$E_t\left(M_{t+1}R^K_{t+1}\left(1+\left(\chi\theta-1\right)\Omega(\varepsilon^*_{t+1})+\left(\chi-1\right)\varepsilon^*_{t+1}\left(1-H\left(\varepsilon^*_{t+1}\right)\right)\right)\right)=1. \tag{13}$$

Recall that $R^K_{t+1} = \left(1-x_{t+1}b_k\right)\left(1-\delta+\alpha\frac{Y_{t+1}}{K_{t+1}}\right)$ is the familiar expression for the unlevered physical return on capital, adjusted to reflect the possibility of disasters. Equation (13) is the usual Euler equation, modified to take into account the financing friction: financing costs are higher because of the bankruptcy costs (the second term), but lower because of the tax shield (the third term). Overall the firm has always access to cheaper financing than in the standard (equity-financed only) model, since it always has the possibility to not take any debt. Hence, the capital stock is always higher when $\chi > 1$ than in the frictionless version.

The first order condition with B_{t+1} is

$$\chi\left(1-\theta\right)E_t\left(M_{t+1}\varepsilon^*_{t+1}h\left(\varepsilon^*_{t+1}\right)\right)=\left(\chi-1\right)E_t\left(M_{t+1}\left(1-H\left(\varepsilon^*_{t+1}\right)\right)\right). \tag{14}$$

This equation determines the optimal financing choice between debt and equity. The left-hand side is the marginal cost of debt, i.e. an extra dollar of debt will increase the likelihood of default, and the associated bankruptcy costs. The right-hand side is the marginal benefit of debt, i.e. the higher tax shield in non-default states. Importantly, both the marginal cost and the marginal benefit are discounted using the stochastic discount factor M_{t+1}. This risk-adjustment is consistent with the empirical work by Almeida and Philippon (2007), who note that corporate defaults are more frequent in "bad times" and the ex-ante marginal cost of debt must take this into account. This risk-adjustment will play a substantial role in the analysis below: for a given debt level, an increase in the probability of disaster increases expected discounted default costs, not only because defaults become more likely, but also because they are more likely to occur during bad aggregate times.

For this equation to generate a unique threshold, some regularity condition must be imposed on the distribution H.[10]

We can define desired leverage $L_{t+1} = B_{t+1}/K^w_{t+1}$, which is decided at time t. The firm defaults if $\varepsilon R^K_{t+1} < L_{t+1}$ i.e. if the return on capital is too low relative to the leverage.

Overall, equations 13 and 14 are the only departures of our model from the standard real business cycle model: first, the Euler equation needs to be adjusted to reflect the tax shield and bankruptcy costs; second, the optimal leverage is determined by the trade-off between costs and benefits of debt finance.

2.3 Equilibrium

The equilibrium definition is standard. First, the labor market clears, i.e. the marginal product of labor equals the wage equals the marginal rate of substitution,

$$(1-\alpha)\frac{Y_t}{N_t} = W_t = \frac{(1-\upsilon)C_t}{\upsilon\,(1-N_t)}.$$

Second, the goods market clears, i.e. total consumption plus investment plus bankruptcy costs equals output,

$$C_t + I_t + (1-\theta)R(\varepsilon_t^*)V_t = Y_t. \tag{15}$$

Under a slight change of interpretation of the model, this resource constraint can be significantly simplified. Assume that the default cost is a tax, i.e. it is transferred to the government, which then rebates it to household. Then, the resource constraint is simply

$$C_t + I_t = Y_t. \tag{16}$$

2.3.1 Recursive Representation

It is useful, both for conceptual clarity and to implement a numerical algorithm, to present a recursive formulation of this equilibrium. This can be done in three steps.

First, we make the simplifying assumption that the bankruptcy cost is a tax, instead a of a real resource cost. This tax is then rebated to households through the lump-sum transfers T_t. As a result, bankruptcy costs do not appear in the resource constraint (see section 2.2.3).

Second, we note that the equilibrium can be entirely characterized from time t onwards given the values of the realized aggregate capital stock K_t, the probability of disaster p_t, and the level of total factor productivity z_t, i.e. these are the three state variables.[11]

[10] The technical condition (which we assume from now on) is that the function $z \to \frac{zh(z)}{1-H(z)}$ is increasing. Bernanke, Gertler and Gilchrist (1999) make the same assumption in the context of a related model. Most distributions (such as the log-normal distribution) satisfy this assumption.

[11] The level of outstanding debt B_t at the beginning of period is not a state variable, since it does not affect production or investment possibilities. It does affect default, but because bankruptcy costs are not in the resource constraint, the

Third, examination of the first-order conditions shows that they can be rewritten solely as a function of the detrended capital $k_t = K_t/z_t$ and p_t. This is a standard simplification in the stochastic growth model when technology follows a unit root, which also applies to the framework of this paper.

Overall, this analysis shows that the equilibrium policy functions can be expressed as functions of two state variables only, k and p. Hence, the model has the same states as the frictionless real business cycle (RBC) model. There is an additional equilibrium policy function to solve for, the desired leverage $L(k,p)$, and correspondingly, we have an additional first-order condition (equation (14)), Last, the first-order condition determining optimal investment, i.e. the standard Euler equation (equation 13)), is modified to take into account the marginal financing costs.

Mathematically, define detrended output, consumption, investment, etc. as $y = Y/z$, $c = C/z$, $i = I/z$, $w = W/z$, etc. and detrended utility as $g(k,p) = U/z^{\upsilon(1-\psi)}$. A recursive equilibrium is a list of functions, $y(k,p)$, $N(k,p)$, $c(k,p)$, $i(k,p)$, $L(k,p)$, $g(k,p)$, $w(k,p)$, and the realized return on capital $R^K(k', p', x')$, such that

$$y(k,p) = k^{\alpha} N(k,p)^{1-\alpha},$$

$$w(k,p) = (1-\alpha)\frac{y(k,p)}{N(k,p)} = \frac{1-\upsilon}{\upsilon}\frac{c(k,p)}{1-N(k,p)},$$

$$c(k,p) + i(k,p) = y(k,p),$$

$$k' \;=\; k'(k,p,x',e') = \frac{(1-x'b_k)\left((1-\delta)\,k + i(k,p)\right)}{(1-x'b_{tfp})e^{\mu+\sigma e'}},$$

$$\varepsilon^*(k,p,p',x',e') \;=\; \frac{L(k,p)}{R^K(k',p',x')} = \frac{L(k,p)}{R^K(k'(k,p,x',e'),p',x')},$$

$$R^K(k',p',x') = (1-x'b_k)\left(1-\delta+\alpha\frac{y(k',p')}{k'}\right),$$

$$E_{p',e',x'}\left(M\left(k,p,p',x',e'\right) R^K(k,p,x')\left(\begin{array}{c} 1+(\chi\theta-1)\,\Omega\left(\varepsilon^*(k,p,p',x',e')\right) \\ +\,(\chi-1)\left(1-H(\varepsilon^*(k,p,p',x',e')\right) \end{array}\right)\right) \;=\; 1,$$

$$E_{p',e',x'}\left(M\left(k,p,p',x',e'\right)\left(\begin{array}{c} \chi\left(\theta-1\right)\varepsilon^*(k,p,p',x',e')h\left(\varepsilon^*(k,p,p',x',e')\right) \\ +\,(\chi-1)\left(1-H\left(\varepsilon^*(k,p,p',x',e')\right)\right) \end{array}\right)\right) \;=\; 0,$$

with the stochastic discount factor given by the formula,

$$\begin{aligned} M(k,p,p',\varepsilon',x') \;=\; & \beta\left(e^{\mu+\sigma\varepsilon'}\right)^{(1-\gamma)\upsilon-1}(1-x'b_{tfp})^{(1-\gamma)\upsilon-1} \\ & \times\left(\frac{c(k',p')}{c(k,p)}\right)^{\upsilon(1-\psi)-1}\left(\frac{1-N(k',p')}{1-N(k,p)}\right)^{(1-\upsilon)(1-\psi)} \\ & \times\frac{g(k',p')^{\frac{\psi-\gamma}{1-\psi}}}{E_{z',p',x'}\left(\left(\frac{z'}{z}\right)^{\upsilon(1-\gamma)} g(k',p')^{\frac{1-\gamma}{1-\psi}}\right)^{\frac{\psi-\gamma}{1-\gamma}}}, \end{aligned}$$

realization of default does not matter in itself – what matters is the possibility of default going forward. Here we rely on two assumptions: (1) the default cost is a tax; (2) default takes place after production.

and utility by the formula

$$g(k,p) = c(k,p)^{\upsilon(1-\psi)}(1-N(k,p))^{(1-\upsilon)(1-\psi)}+\beta e^{\mu\upsilon(1-\psi)}\left(E_{p',\varepsilon',x'}e^{\sigma\varepsilon'\upsilon(1-\gamma)}(1-x'b)^{\upsilon(1-\gamma)}g(k',p')^{\frac{1-\gamma}{1-\psi}}\right)^{\frac{1-\psi}{1-\gamma}}.$$

A computational advantage of this formulation is that the variables are stationary since we take out the stochastic trend, which of course facilitates the numerical implementation. The numerical method is detailed in the appendix.

2.3.2 Asset Prices

Any payoff can be priced using the stochastic discount factor, which is given by the representative agent's marginal rate of substitution. I focus here on four assets: a pure risk-free asset, a short-term government bond which may default during disasters, the corporate bond, and the equity. All these assets last only one period. The price of the risk-free asset can be calculated as the expectation of the stochastic discount factor,

$$P_t^{rf} = E_t(M_{t+1}) = P^{rf}(k,p).$$

The government bond is assumed to default by a factor Δ during disasters, leading to the following price:

$$P_t^{gov} = E_t(M_{t+1}(1-x_{t+1}\Delta)) = P^{gov}(k,p).$$

As discussed in section 2.2.3, the corporate bond price is

$$P_t^{corp} = q_t = E_t\left(M_{t+1}\left(1-H(\varepsilon_{t+1}^*)+\frac{\theta R_{t+1}^K K_{t+1}^w}{B_{t+1}}\Omega(\varepsilon_{t+1}^*)\right)\right) = P^{corp}(k,p).$$

Note that the payoff to a diversified portfolio of corporate bonds, used in the household budget constraint (equation (2)), is $\varrho_{t+1} = 1-H(\varepsilon_{t+1}^*)+\frac{\theta R_{t+1}^K K_{t+1}^w}{B_{t+1}}\Omega(\varepsilon_{t+1}^*)$. Last, the equity price satisfies

$$P_t^{eq} = E_t\left(M_{t+1}\left(R_{t+1}^K K_{t+1}^w\left(1-\Omega(\varepsilon_{t+1}^*)\right)-B_{t+1}\left(1-H(\varepsilon_{t+1}^*)\right)\right)\right).$$

This price can be rewritten in a stationary form as follows,

$$p^{eq}(k,p) = \frac{P_t^{eq}}{K_{t+1}^w} = E_t\left(M_{t+1}\left(R_{t+1}^K\left(1-\Omega(\varepsilon_{t+1}^*)\right)-L_{t+1}\left(1-H(\varepsilon_{t+1}^*)\right)\right)\right).$$

Given constant return to scale, the equity price P_t^{eq} is exactly equal to equity issuance S_t: since there is free entry, pure profits are zero.

3 Quantitative Analysis

This section studies the implications of the model presented in the previous section. First, I present some comparative statics to illustrate the workings of the model. Then, I parametrize the model and discuss the quantitative implications of the model for business cycle quantities, for asset returns, and especially for the volatility and cyclicality of credit spreads. The model incorporates shocks to uncertainty, hence in the absence of analytical solution, it is necessary to solve it using nonlinear methods. I use projection methods to find the policy functions $c(k,p), N(k,p), g(k,p), L(k,p)$, by approximating these functions with Chebychev polynomials. The numerical method is detailed in the appendix.

3.1 Steady-state comparative statics

We consider here a simplified version of the model, where the probability of disaster is constant, and there are no TFP shocks ($\sigma = 0$). This corresponds to a "steady-state analysis", but one that takes into account the risk of disasters.[12] I first consider the effect of idiosyncratic risk σ_ε, tax subsidy χ, and recovery rate θ, which determine the choice of leverage, then I turn to the effect of the probability of disaster and the comparison with the frictionless model.

3.1.1 Choice of leverage

Figure 1 illustrates the effect of several key parameters on the steady-state values of capital, leverage, default probability and credit spreads. Each column corresponds to one parameter. The first column shows the effect of idiosyncratic volatility σ_ε. Holding debt policy constant, higher idiosyncratic risk leads to more default and hence higher credit spreads, increasing the user cost of capital. This leads firms to reduce investment. In equilibrium, firms also endogenously reduce leverage, which mitigates the increase in default and in credit spreads. Firms hence rely more heavily on equity issuance, which is more costly.

The second column shows the effect of the tax subsidy χ. A higher χ directly reduces the user cost of capital, since holding debt policy constant, the firm is able to raise more capital. Second, a higher χ makes debt relatively more attractive than equity, leading firms to take on more debt and increase leverage. This higher leverage leads to a higher probability of default and higher credit spreads. It is interesting that even a very small χ leads to significant leverage. This is, in part, driven by the assumption that ε is log-normally distributed, making very low realizations of ε highly unlikely.

The third column shows the effect of the recovery rate parameter θ. Since the expected cost of bankruptcy falls, the user cost of investment falls and investment rises. Holding debt policy constant, a higher θ leads to a lower credit spread, since the recovery value is higher. However, in equilibrium firms take on more debt, which leads to higher probability of default and higher credit spreads.

3.1.2 User cost, financial frictions and probability of disaster

To understand the key mechanism of the model, it is useful to use this simplified ("steady-state") version of the model, and perform the comparative static of a change in the probability of disaster. The discount factor is

$$M(x') = \beta e^{\mu((1-\psi)\upsilon-1)} \times \frac{(1-x'b_{tfp})^{(1-\gamma)\upsilon-1}}{\left(1-p+p(1-b_{tfp})^{(1-\gamma)\upsilon}\right)^{\frac{\psi-\gamma}{1-\gamma}}},$$

[12] More precisely, suppose that the probability of disaster p is fixed, and there are no TFP shocks. Consider a long sample where no disaster gets realized. Then, over time, the allocation will converge to a certain "steady-state" limit.

where $x' = 1$ if a disaster occurs, and 0 if not. The economy's steady-state capital $k = K/z$ and leverage $L = B/K^w$ are then determined by the two equations:

$$
\begin{aligned}
1 \quad = \quad & \frac{\beta e^{\mu((1-\psi)\upsilon-1)}}{\left(1-p+p(1-b_{tfp})^{(1-\gamma)\upsilon}\right)^{\frac{\psi-\gamma}{1-\gamma}}} \left(1-\delta+\alpha k^{\alpha-1}\right) \times \\
& \left(\begin{array}{c} (1-p)\left(1+(\chi\theta-1)\,\Omega\left(\varepsilon^*_{nd}\right)+(\chi-1)\,\varepsilon^*_{nd}\left(1-H\left(\varepsilon^*_{nd}\right)\right)\right) \\ +p\left(1-b_{tfp}\right)^{\upsilon(1-\gamma)-1}\left(1-b_k\right)\left(1+(\chi\theta-1)\,\Omega\left(\varepsilon^*_{d}\right)+(\chi-1)\,\varepsilon^*_{d}\left(1-H\left(\varepsilon^*_{d}\right)\right)\right) \end{array} \right),
\end{aligned} \tag{17}
$$

and

$$
\begin{aligned}
0 \quad = \quad & (1-p)\left(\chi\left(\theta-1\right)\varepsilon^*_{nd}h\left(\varepsilon^*_{nd}\right)+(\chi-1)\left(1-H\left(\varepsilon^*_{nd}\right)\right)\right) \\
& +p\left(1-b_{tfp}\right)^{\upsilon(1-\gamma)-1}\left(\chi\left(\theta-1\right)\varepsilon^*_{d}h\left(\varepsilon^*_{d}\right)+(\chi-1)\left(1-H\left(\varepsilon^*_{d}\right)\right)\right),
\end{aligned} \tag{18}
$$

with $\varepsilon^*_d = \frac{L}{(1-b_k)\phi}$ and $\varepsilon^*_{nd} = \frac{L}{\phi}$, and $\phi = 1-\delta+\alpha k^{\alpha-1}$. Equation (18) first determines leverage $\frac{L}{\phi}$, and equation (17) then determines the marginal product of capital ϕ and hence k. When there is neither disaster risk nor financial frictions, i.e. $p = 0$ and $\chi = \theta = 1$, the first equation collapses to the standard user cost equation,

$$\beta e^{\mu((1-\psi)\upsilon-1)}\left(1-\delta+\alpha k^{\alpha-1}\right) = 1.$$

When there is disaster risk but no financial frictions, we have $\chi = \theta = 1$ and the steady-state capital is determined through the equation

$$\beta e^{\mu((1-\psi)\upsilon-1)}\left(1-\delta+\alpha k^{\alpha-1}\right)\left((1-p)+p\left(1-b_{tfp}\right)^{\upsilon(1-\gamma)}\right)^{\frac{1-\psi}{1-\gamma}} = 1,$$

hence a higher probability of disaster p leads to a lower capital stock provided that the IES is greater than unity (see Gourio (2010) for a general analysis).

Finally, in the model with financial friction, the probability of disaster will lead to a reduction in leverage in equation (18), and hence an increase in the user cost (adjusted for the tax shield and bankruptcy costs) in equation (17). To illustrate this effect, figure 2 displays the effect of a rise in p on capital, leverage, credit spreads and the user cost $\alpha k^{\alpha-1}$, which is $r+\delta$ in the standard neoclassical model. The figure allows to compare the frictionless model ($\chi = \theta = 1$) and the model with the friction ($\chi > 1$). The percentage response of the steady-state capital stock to a change in the probability of disaster is substantially larger in the model with the financial friction, reflecting that the user cost is much more affected by an increase in disaster risk.[13]

An increase in risk in itself increases the probability of default, but also makes the risk of default more likely to be driven by a bad *aggregate* realization, hence increases the cost of debt significantly (as reflected by the credit spread). Overall, the probability of disaster p has an effect similar to that of σ_ε, in that it increases risk, as in Arellano, Bai and Kehoe (2010) or Gilchrist, Sim and Zakrajek (2010), but the important difference is that bad states now also have higher state-contingent prices, making default and the ensuing bankruptcy deadweight losses even more costly in a risk-adjusted sense.

[13] For high values of the probability of disaster p, the credit spread is decreasing in p. This counterintuitive result simply reflects that firms reduce debt significantly to avoid bankruptcy in disaster states.

3.2 Calibration

Parameters are listed in Table 1. The period is one year. Many parameters follow the business cycle literature (and Prescott (1995)). The risk aversion parameter is set equal to four. Note that this is the risk aversion over the consumption-hours bundle. Since the share of consumption in the utility index is .3, the effective risk aversion to a consumption gamble is 1.33 (Swanson (2010)).

The intertemporal elasticity of substitution of consumption (IES) is set at 2. There is a large debate regarding the value of the IES. Most direct estimates using aggregate data find low numbers (e.g. Hall (1988)), but this view has been challenged by several authors (see among others Bansal and Yaron (2004), Guvenen (2006), Mulligan (2004), Vissing-Jorgensen (2002)). As emphasized by Bansal and Yaron (2004), a low IES has the counterintuitive effects that higher expected growth lowers asset prices, and higher uncertainty increases asset prices. Section 4.4 analyzes how the results are affected by this intertemporal elasticity of substitution.[14]

One crucial element of the calibration is the probability and size of disaster. Following Barro (2006, 2009) and Barro and Ursua (2008), I use the historical distribution of disasters rather than a single value.[15] I summarize the historical distribution in the Barro (2006) paper using a five-point distributions, with disaster sizes ranging from 15% to 57%, and probabilities confirming to historical probabilities. The probability of a disaster is .017 per year on average. In my model, with $b_k = b_{tfp} = .25$ (say), both consumption and output fall by 25% if there is a disaster. Note that since the Solow residual is $z^{1-\alpha}$, the actual drop in productivity is smaller than 25%. While these disaster sizes may seem very large, they are the ones estimated by Barro and Barro and Ursua (2007) in a large international panel data set. Also, the results of the paper are largely unchanged if the disaster size is set to be smaller – e.g., perhaps the US faces lower disaster risk than most other countries – and risk aversion is higher.

Whether one should model a disaster as a capital destruction or a reduction in TFP is an important question. Clearly some disasters, e.g. in South America since 1945, or Russia 1917, affected TFP, perhaps by introducing an inefficient government and poor policies. On the other hand, World War II led in many countries to massive physical destructions and losses of human capital. It would be interesting to gather further evidence on disasters, and measure b_k and b_{tfp} directly. This is beyond the scope of this paper. I concentrate on the parsimonious benchmark case $b_k = b_{tfp}$. In section 4, I relax this assumption.

The second crucial element is the persistence and volatility of movements in this probability of disaster. I assume that the log of the probability follows an AR(1) process:

$$\log p_{t+1} = \rho_p \log p_t + (1 - \rho_p) \log \overline{p} + \sigma_p \varepsilon_{p,t+1},$$

[14] In the frictionless model (Gourio (2010)), the IES determines whether investment and output go up or down in response to a shock to the probability of disaster. In themodel of this paper, even for IES lower than unity, investment and output go down, due to the higher expected discounted bankrutpcy costs, see section 4.4. Gourio (2010) also shows that standard IES estimates in the frictionless model may be substantially downward biased.

[15] Using a single value yields similar results, however it is more realistic to use a distribution of disaster sizes. Given the importance of the distribution of payoffs for the pricing of equity and debt, having a more realistic distribution of disaster size seems useful.

where $\varepsilon_{p,t+1}$ is *i.i.d.* $N(0,1)$.[16] The parameter $\overline{p}$ is picked so that the average probability is .017 per year, and I set $\rho_p = .75$ and the unconditional standard deviation $\frac{\sigma_p}{\sqrt{1-\rho_p^2}} = 1.50$ in order to roughly match the volatility of credit spreads.

I use a log-normal distribution for H, the distribution of idiosyncratic shocks. Last, we need to pick the parameters which determine the leverage choice: χ, θ and σ_ε, the variance of idiosyncratic shocks. Following the corporate finance literature, I set $\theta = .4$, consistent with estimates of recovery rates in "bad times". As argued in the next section, σ_ε and χ determine the average probability of default and the average leverage. To calibrate these parameters, I set a target for the probability of default, which is approximately 0.5% per year. I also set a target for leverage equal to 0.55. In the data leverage is somewhat smaller, perhaps 0.45. The motivation is that targeting a leverage of 0.45 leads to an unrealistically large variance of idiosyncratic shocks σ_ε, and reduces the implications of aggregate shocks. Further work may make progress on this by either making profits more volatile in the model, or by introducing a distribution for idiosyncratic shocks with more skewness. These two targets imply reasonable values for σ_ε and χ, $\sigma_\varepsilon = 0.227$ and $\chi = 1.062$. (Note that the targets are not exactly matched because the calibration is done using the "steady-state" of the model).

Last, I will consider a variant of the model, where the leverage ratio is fixed: the firm is free to pick any debt and capital, but they must be linked through $B_{t+1} = \overline{L} K_{t+1}^w$ where $\overline{L}$ is the average leverage ratio of the benchmark model. The appendix derives the first-order condition for this variant of the model in detail.

3.3 Impulse response functions

I first illustrate the dynamics of the model in response to the three fundamental shocks: the standard TFP shock, the disaster, and a change in the probability of disaster. I next discuss how the model fits quantitatively stylized facts about leverage, credit spreads, and investment.

3.3.1 The effect of a TFP shock

Figure 3 shows the response of quantities and returns to a one standard-deviation shock (i.e. 2%) to the level of total factor productivity. For clarity, this picture, as well as the ones following, assumes that no other shock is realized. The response of quantities is similar to that of the standard real business cycle model: investment rises as firms desire to accumulate more capital, employment rises because of the higher labor demand, and consumption adjusts gradually, leading to temporarily high interest rates. The equity return is high on impact, reflecting the sensitivity of firms' dividends to TFP shocks due to leverage, but corporate bonds are largely immune to small TFP shocks - the default and recovery rates are barely affected.[17] As a result, the path for the bond return mirrors that of the risk-free return. There is essentially no dynamics in leverage or credit spreads, since the trade-off determining optimal debt is hardly affected by the slightly higher TFP.

[16] This equation allows the probability to be greater than one, however I will approximate this process with a finite Markov chain, which ensures that $0 < p_t < 1$ for all $t \geq 0$.

[17] I define the default rate as the share of firms in default. Because some of the capital is recovered in defaults, this is not the realized loss for debholders.

3.3.2 The effect of a disaster

Figure 4 shows the response of quantities and returns to a disaster which hits at $t = 5$. The disaster realization leads capital and TFP to fall by the factors b_k and b_{tfp} respectively. In this simulation, $b_k = b_{tfp} = 25\%$. The calibration assumes that these parameters are equal. As a result, the transitional dynamics are very simple, as seen in the figure, and as proved in the following proposition.

Proposition 1 *Assume that $b_k = b_{tfp}$. Then, a disaster leads consumption, investment, and output to drop by a factor $b_k = b_{tfp}$, while hours do not change. The return on physical capital is also reduced by the same factor. There is no further effect of the disaster on quantities or prices, i.e. all the effect is on impact.*

Proof. The equilibrium is characterized by the policy functions $c(k,p), i(k,p), N(k,p)$ and $y(k,p) = k^{\alpha}N(k,p)^{1-\alpha}$ which express the solution as a function of the probability of disaster p (the exogenous state variable) and the detrended capital k (the endogenous state variable). The detrended capital evolves according to the shocks ε', x', p' through

$$k' = \frac{(1 - x'b_k)\left((1-\delta)k + i(k,p)\right)}{e^{\mu+\sigma\varepsilon'}\left(1 - x'b_{tfp}\right)}.$$

Since $b_k = b_{tfp}$, $k' = \frac{((1-\delta)k+i(k,p))}{e^{\mu+\sigma\varepsilon'}}$is independent of the realization of disaster x'. As a result, the realization of a disaster does not affect c, i, N, y, since k is unchanged, and hence it leads consumption $C = cz$, investment $I = iz$, and output $Y = yz$ to drop by a factor $b_k = b_{tfp}$ on impact. Furthermore, once the disaster has hit, it has no further effect since all the endogenous dynamics are captured by k, which is unaffected. The statement regarding returns follows from the expression of the physical return, $R^K_{t+1} = (1 - x_{t+1}b_k)\left(1 - \delta + \alpha\frac{y_{t+1}}{k_{t+1}}\right)$. The second parenthesis is unaffected, so the only effect of the disaster is to multiply the return by the factor $(1 - b_k)$. ■

This low return on physical capital is divided among equity and debt. But it is also further reduced by default, which leads to losses since $\theta < 1$. In this simulation, approximately 12% of firms are in default. As a result, for the parameter values used to produce figure 4, the equity return is approximately -52% and the bond return is -4.5%. Note that the returns we compute are the average across all the firms, as defined in section 2.3: there are always some firms with very high idiosyncratic shocks which do not default. But figure 4 illustrates that both equity and corporate debt are risky assets, since their returns are very low precisely in the states (disasters) when marginal utility is high (consumption growth is low). The figure illustrates that a disaster does not generate any transitional dynamics in quantities or risk premia.

3.3.3 The effect of an increase in the probability of a disaster

The important shock in this paper is the shock to the probability of disaster. Figure 5 presents the responses to an unexpected increase in the probability of disaster at time $t = 5$. The higher risk leads to a sharp reduction in investment. Simultaneously, the higher risk pushes down the risk-free interest rate, as the demand for precautionary savings increases. This lower interest rate decreases employment through an intertemporal substitution effect. Hence, output decreases because both employment and the capital stock decrease, even though there is no change in current or future total factor productivity. Intuitively, there is less demand for investment and this reduces the need for production.[18]

Consumption increases on impact since people want to invest less in the now more risky capital. Consumption then falls over time. Qualitatively, these dynamics are fairly similar to that in the frictionless version, but the quantitative results are quite different. To illustrate this clearly, figure 6 superimposes the responses to a shock to the probability of disaster for the frictionless model and for the current model. The response of macro quantities on impact is 2 to 3 times larger in the model with financial frictions. The model with constant leverage produces even slightly more amplification.

As argued in section 3.1.2, the mechanism through which disaster risk affects the economy is by changing the expected discounted bankruptcy costs. These become significantly higher, since default is (i) more likely and (ii) more likely to occur in "bad times", so that the deadweight losses associated with bankruptcy are heavily discounted. This increases the user cost for a given financial policy, leading to cut back on investment. Moreover, firms also adjust their financial policy, reducing debt and leverage.

Because risk increases, risk premia rise as the economy enters this recession: the difference between equity returns and risk-free returns becomes smaller, and the spread of corporate bonds over risk-free bonds also rises (see the bottom panel of figure 5.) This result is specific to these parameters, however. The equilibrium level of credit spreads depends on the *endogenous* quantity of debt, or leverage that firms decide to take on. For certain parameter values, the endogenous decrease in leverage leads, paradoxically, to lower credit spreads in response to a higher probability of disaster. However, for the parameter values that we use, firms do not decide to cut back on debt too much, and spreads rise with the probability of disaster. The model hence generates the required negative correlation between credit spreads and the probability of disaster. More generally, the model implies that risk premia are larger in recessions, consistent with the data.

3.4 Business cycle and financial statistics

Tables 2 through 4 report standard business cycle and asset return statistics as well as default rates and leverage ratios.[19] To illustrate the role of disaster risk and time-varying disaster risk, I solve the model with the benchmark calibration for three different sets of shocks: (i) only TFP shocks, (ii) TFP shocks and disasters, but a constant probability of disaster; (iii) the full model, with time-varying risk of disaster. I also consider three variant of the model: (a) with the financial friction, (b) with constant leverage, and (c) with no financial friction. The benchmark model results (i.e. model a-iii) are indicated in bold in these tables.

The models with only TFP shocks (rows 1 through 3) generate a decent match for quantity dynamics, as is well known from the business cycle literature. This model, however, generates rather small spreads for corporate bonds, and these spreads simply account for the average default of corporate bonds,

[18] In the frictionless model (Gourio (2010)), shocks to the probability of disaster are shown to be equivalent, in terms of quantities, to preference shocks. This result, however, does not apply to the current framework.

[19] The leverage and default probability data are taken from Chen, Collin-Dufresne, and Goldstein (2009). The other data (GDP, consumption, investment, and credit spreads) are from FRED. I use BAA-AAA as the credit spread measure, and obtain similar results as Chen, Collin-Dufresne, and Goldstein. All series are annualized.

because aggregate risk premia are very small. Moreover, these spreads are essentially constant. The risk premium for equity is also very small and equity returns are not volatile. Note that except for investment, which is somewhat less volatile in the model with financial friction, the quantity moments are largely unchanged as we go from row 1 to row 3. Hence, financial frictions do not amplify the response to TFP shocks.[20] The slower volatility of investment may be explained by the higher steady-state capital stock (as in Santoro and Wei (2010)).

When constant disaster risk is added to the model (rows 4 through 6), the quantity dynamics are unaffected (table 2). Table 3 reveals that credit spreads are significantly larger however, because defaults are much more likely during disasters, when marginal utility is high. The model generates a higher equity risk premium and a plausible credit spread. However, the volatility of spreads is close to zero. This motivates turning to the model with time-varying risk of disaster.

Rows 7 through 9 display the results for the models with time-varying disaster risk. The variation in the disaster risk does indeed lead to volatile credit spreads, roughly in line with the data. The levels of equity and risk premia are similar to that of the model with constant probability of disaster. Introducing the time-varying risk of disaster also generates new quantity dynamics: investment becomes more volatile, and output becomes slightly more volatile too. Moreover, spreads are strongly countercyclical. Overall, the model fits well many stylized facts. The volatility of credit spreads in the data suggests an important role for this shock to the disaster probability, which has appealing quantity implications.

In comparison, the model with constant leverage generates even more volatility of quantities and especially of credit spreads. Because firms cannot delever easily when p rises, the model generates more movements in spreads and investment.

The amplification effect of disaster risk shock through financial frictions is visible in table 2: while the financial friction model exhibits less volatility than the RBC model when disaster risk is constant, it has more volatility than the RBC model when disaster risk is added. This is especially true for investment volatility, which nearly doubles as time-varying disaster risk is introduced.

Finally, the model implies that leverage is somewhat volatile, though it falls somewhat short of the data. Firms, in the model, are able to adjust leverage instantaneously at no cost, hence leverage falls when the probability of disaster rises as the economy enters a recession. In reality, it seems likely that leverage for new firms or new investment falls, but firms with ongoing operations may have higher leverage as their value falls. The key point is that this does not invalidate the model mechanism, i.e. firms which are investing, face a higher user cost because of higher expected discounted bankruptcy costs.

Overall, the model has two main deficiencies: first, the correlation of consumption and output is too low; second, the equity return is not volatile enough. The latter point is, however, driven by the fact that equities are only a one-period asset here, implying that the conditional volatility of equity returns equals the conditional volatility of dividends (i.e. there is only a cash flow effect and no discount rate effect)

[20] The appendix presents a comparison of the impulse response functions to a TFP shock for the different models, which confirms this result.

4 Extensions and Sensitivity analysis

This section considers some extensions of the baseline model, and the sensitivity of the quantitative results to some changes in the parameters.

4.1 Capital adjustment costs

While the benchmark model abstracts from adjustment costs in the interest of simplicity, introducing them is useful to generate further volatility in the value of capital. In particular, the model implies that an increase in the probability of disaster has essentially no effect on realized equity returns or bond returns.[21] This implication is overturned if there are adjustment costs, because the price of capital then falls following an increase in the probability of disaster (since investment falls). It is simplest to consider an external adjustment cost formulation. Suppose that capital goods are produced by a competitive investment sector which takes I_t consumption goods at time t, and K_t capital goods at time t, and generates $K_{t+1} = (1-\delta)K_t + \Phi\left(\frac{I_t}{K_t}\right) K_t$ capital goods next period. These capital goods are then sold to firms which are, as in the benchmark model, then faced with the shocks x_{t+1} and ε_{it+1}. As a result, the price of capital goods at end of time t is given by:

$$P_t^K = \frac{1}{\Phi'\left(\frac{I_t}{K_t}\right)}.$$

The same formulas as in the model then apply, with the proviso that the return on capital R_{t+1}^K must now be interpreted as the value of one unit of capital at the $t+1$, and is equal to

$$R_{t+1}^K = \left((1-\delta)\, P_{t+1}^K + \alpha \frac{Y_{t+1}}{K_{t+1}}\right)(1 - x_{t+1} b_k),$$

and the first-order condition for investment needs to be modified to take into account the price of capital (the right-hand-side):

$$E_t\left(M_{t+1} R_{t+1}^K \left(1 + (\chi\theta - 1)\,\Omega\left(\varepsilon_{t+1}^*\right) + (\chi - 1)\,\varepsilon_{t+1}^*\left(1 - H\left(\varepsilon_{t+1}^*\right)\right)\right)\right) = P_t^K,$$

and the other expressions can similarly be computed. Following Jermann (1998), I set $\Phi(x) = a_0 + a_1 \frac{x^{1-\eta}}{1-\eta}$, where a_0 and a_1 are picked to make the steady-state investment rate and marginal Q independent of η. Tables 5 through 7 report model moments for two values of η, and figure 7 compares the impulse response function of the benchmark model (without adjustment costs) and the model with adjustment costs ($\eta = .1$), when the shock is an increase in the probability of disaster.[22]

As expected, adjustment costs smooth the response of investment and output, reducing their volatility. The qualitative dynamics, as well as the asset prices, remain similar. On impact, the return on equity is now lower, and the return on the corporate bond is also slightly lower, reflecting the fall in the value of capital and the ensuing higher default rate.

[21] Technically, the only effect is through a decrease in the supply for labor which pushes the wage up, leading to slightly lower profits and hence slightly higher default rates.

[22] The appendix provides the comparison of impulse response functions to the other two shocks (i.e.the TFP shock and the disaster realization shock).

4.2 Welfare cost of the tax shield

Following a large literature in corporate finance, the model features a tax subsidy to debt, or tax shield. The tax shield is inefficient in the standard "nonstochastic" sense that capital investment decisions are distorted. But the tax shield also leads to larger fluctuations in quantities, which are undesirable. Removing the tax shield would reduce volatility.

More precisely, the tax shield lowers the user cost of capital and hence encourages capital accumulation. However, the competitive equilibrium of the model without taxes is already Pareto optimal, hence the subsidy leads to *overaccumulation* of capital. Moreover, the subsidy leads to an increase in the volatility of business cycles, because the user cost of capital becomes more sensitive to shocks to the probability of disaster. Table 8 illustrates this effect by displaying the volatility of output, investment and employment, for various values of χ. Both in terms of steady-states and in terms of fluctuations then, the tax subsidy generates deadweight losses. Figure 8 presents the estimate of the welfare cost of the tax subsidy, which is substantial. For our benchmark calibration of $\chi = 1.062$, removing the tax shield entirely would increase welfare substantially, equivalent to a permanent increase of consumption of approximately 3.52%.

4.3 Time-varying resilience of the economy (Incomplete)

In this section, we extend the model to make default realization matter for output, capturing the intuition that firms in default are less productive as they need to reorganize and are constrained in their relations with suppliers and customers. The model then implies that a low probability of disaster, by leading to high leverage, makes the economy less resilient, i.e. its investment and output fall more should a bad shock (whatever its nature) occur. This is consistent with the popular story that during the 2000s, perception of risk fell, leading firms to increase leverage and making the 2008 recession worse.

We make the following two changes to the model. The first is to relax the simplifying assumption that bankruptcy costs are a tax: we now take into account that bankruptcy costs are a real resource cost. The second is that firms in default have lower productivity. We model this in a simple way as a reduction of the effective capital that is available for production: if a firm has K units of physical capital available, but it is in default, only ζK units of capital can actually be used for production. (This is equivalent to a drop of TFP in the firms in default.) These two changes do not affect the expression for the default threshold, which remains $\varepsilon^*_{t+1} = \frac{B_{t+1}}{K^w_{t+1} R^K_{t+1}}$.We can define the capital available for production, which is now

$$\begin{aligned} K^{op}_{t+1} &= K^w_{t+1}(1 - x_{t+1} b_k) \left(\int_{\varepsilon^*_{t+1}}^{\infty} \varepsilon dH(\varepsilon) + \zeta \int_0^{\varepsilon^*_{t+1}} \varepsilon dH(\varepsilon) \right), \\ &= K^w_{t+1}(1 - x_{t+1} b_k) \left(1 - (1 - \zeta)\Omega(\varepsilon^*_{t+1}) \right), \end{aligned}$$

The production function is then obtained by aggregation as $Y_t = \left(K^{op}_t\right)^{\alpha} \left(z_t N_t\right)^{1-\alpha}$. We also need to modify consequently the firm value and bond price equations. The equations are available in the appendix. As a result of this change, the quantity of debt B is now an additional state variable. *(Results to be added.)*

4.4 State-contingent debt (Incomplete)

In this section, I allow firms to issue debt which repayments are contingent on the disaster realization x'. (This state-contingent debt might in practice be implemented through government bailouts in disaster.) I then reconsider the effect of disaster risk on investment. The budget constraint now reads,

$$K_{t+1}^{w} = S_t + \chi q_t^{nd} B_{t+1}^{nd} + \chi q_t^{d} B_{t+1}^{d},$$

where B_{t+1}^{nd} (resp. B_{t+1}^{d}) is the face value of the debt to be repaid in non-disaster (resp. disaster) states, and q_t^{nd} (resp. q_t^{d}) the associated price:

$$q_t^{nd} = E_t \left((1 - x_{t+1}) M_{t+1} \left(\int_{\varepsilon_{t+1}^{*}}^{\infty} dH(\varepsilon) + \frac{\theta}{B_{t+1}} \int_{0}^{\varepsilon_{t+1}^{*}} \varepsilon R_{t+1}^{K} K_{t+1}^{w} dH(\varepsilon) \right) \right),$$

where $(1 - x_{t+1})$ is a dummy equal to 1 if no disaster happens. Similarly,

$$q_t^{d} = E_t \left(x_{t+1} M_{t+1} \left(\int_{\varepsilon_{t+1}^{*}}^{\infty} dH(\varepsilon) + \frac{\theta}{B_{t+1}} \int_{0}^{\varepsilon_{t+1}^{*}} \varepsilon R_{t+1}^{K} K_{t+1}^{w} dH(\varepsilon) \right) \right).$$

(Results to be added.)

4.5 Comparison with idiosyncratic uncertainty shocks (Incomplete)

This section compares the effect of an increase in disaster risk and the effect of an in increase in idiosyncratic uncertainty, as considered for instance in Bloom (2009), Arellano, Bai and Kehoe (2009), and Gilchrist, Sim and Zakrajek (2010). The two increases in risk lead to qualitatively similar results, however they act in part through different channels and hence have potentially quite different quantitative effects. The increase in disaster risk not only changes the distribution of payoff, but also affects discount rates. This can be illustrated in two ways: (i) the implications of disaster risk depend significantly by aggregate risk aversion, unlike the implications of idiosyncratic risk; (ii) in the frictionless version, an increase in disaster risk still leads to a recession, whereas an increase in idiosyncratic risk has either no effect, or a positive effect on economic activity.*(Results to be added.)*

4.6 Samples with disasters

Tables 5 through 7 report the model moments in the benchmark model if the sample includes disasters. Most of the results are largely unaffected by the inclusion of disaster, though quantities and returns are of course more volatile since they include some large realizations. The average excess returns on equities is 1.38% (vs. 2.30% in a sample without disasters): this average return is a compensation for a risk not a pure sample selection problem. Similarly, the average return on corporate bonds is 0.44% (vs. 0.60% in a sample without disasters). The dynamics of credit spreads are completely unaffected.

4.7 Role of the IES and Expected utility

While the model setup assumes recursive utility, the model can also be solved with expected utility. When the elasticity of substitution is kept equal to 2, and the risk aversion is lowered to .5 to reach expected utility, the *qualitative* implications are largely unaffected. Because risk aversion is lower, all risk premia are lower, and the response of quantities to a probability of disaster shock (a risk shock) is also smaller since agents care less about risk. Moreover, a shock to the probability of disaster increases consumption, hence with expected utility it is a "good state" (while this is not the case with Epstein-Zin utility, since the future value is lower, making a high probability of disaster state a "bad state"). This implies that assets which pay off well in that state have higher risk premia, not lower risk premia. Tables 5 through 7 report the model moments with this specification.

When the elasticity of substitution is small, a shock to the probability of disaster may lead to different qualitative effects. In the frictionless model, the threshold value is one: if the IES is below unity, investment, output and employment rise (rather than fall) as the probability of disaster rises. In this case, the demand for savings goes up, despite the low risk-adjusted return on capital. In the model of this paper, the threshold value for the IES is now lower than unity. This is consistent with the discussion above that higher uncertainty has more impact in a model with financial frictions. Hence, for a certain range of values of IES below unity, the financial friction model implies that higher disaster risk lowers economic activity, while the frictionless model implies the opposite – an extreme example of the potential importance of financial frictions. Tables 5 through 7 report the model moments with a low IES (.25), which generates the opposite comovement. This specification is unattractive, as it implies that risk premia are procyclical, contrary to the data.

4.8 Equity Issuance costs (To be added)

4.9 Model without capital destruction (To be added)

5 Conclusion

The contribution of the paper is twofold. First, it introduces a simple financial friction in a standard equilibrium business cycle model, by considering a trade-off between the tax savings and bankruptcy costs. This friction is attractive, since it applies to all firms, large and small, and does not rely on binding borrowing constraints. The key mechanism is variation in risk-adjusted bankruptcy costs which leads to a variation in the ex-ante user cost of capital. Second, the paper considers how the economy reacts to an increase in disaster risk (or more generally an increase in aggregate uncertainty). The disaster risk shock is essential to replicating the level, volatility and countercyclicality of credit spreads. Moreover, the financial friction substantially amplifies (by a factor of about three) the response of macroeconomic aggregates to disaster risk. These two findings suggest that disaster risk may play a significant role in some recessions.

There are two main limitations to the model. First, on the quantitative side, the model implies a correlation of consumption and output and a volatility of equity returns that are too low, compared to the data. The later largely reflects the short-term nature of equity in this model: the conditional volatility of equity returns equals the conditional volatility of dividends. This leads to the second limitation of the model, which is that debt is short-term, in contrast to the data. This device, often used in the financial frictions literature, simplifies and clarifies substantially the analysis, but makes it harder to compare some implications of the model to the data, and also shuts down some potentially important channels for persistence, such as the dynamics of net worth.

References

[1] Almeida, Hetor, and Thomas Philippon,. 2007, "The Risk-Adjusted Costs of Financial Distress", Journal of Finance.

[2] Arellano, Cristina, Yan Bai and Patrick Kehoe, 2010, "Financial Markets and Fluctuations in Uncertainty", Mimeo, U of Minnesota.

[3] Aruoba, S. Boragan , Jesus Fernandez-Villaverde and Juan F. Rubio-Ramirez, 2006, "Comparing Solution Methods for Dynamic Equilibrium Economies", Journal of Economic Dynamics and Control, 30(12), 2477-2508.

[4] Backus, David, Mikhail Chernov and Ian Martin, 2009. "Disasters Implied by Equity Index Options", Mimeo NYU.

[5] Bansal, Ravi, and Amir Yaron, 2004. "Risks for the long run: A potential explanation of asset pricing puzzles." Journal of Finance 59, 1481–1509.

[6] Barro, Robert, 2006. "Rare disasters and asset markets in the twentieth century." Quarterly Journal of Economics 121, 823–866.

[7] Barro, Robert, Emi Nakamura, Jon Steinsson, and Jose Ursua, 2009. "Crises and Recoveries in an Empirical Model of Consumption Disasters". Mimeo.

[8] Barro, Robert, and Jose Ursua, 2008. "Macroeconomic Crisis since 1870." Brookings Paper on Economic Activity.

[9] Bhamra, Harjoat, Lars-Alexander Kuehn and Ilyan Strebulaev. "The Aggregate Dynamics of Capital Structure and Macroeconomic Risk", 2009, forthcoming Review of Financial Studies.

[10] Bhamra, Harjoat, Lars-Alexander Kuehn and Ilyan Strebulaev. "The Levered Equity Risk Premium and Credit Spreads: A Unified Framework", 2009 forthcoming Review of Financial Studies.

[11] Bloom, Nicholas, 2009. "The effect of uncertainty shocks." Econometrica, 77(3):623–685.

[12] Boldrin, Michele, Lawrence J. Christiano, and Jonas Fisher, 2001. "Habit Persistence, Asset Returns, and the Business Cycle." American Economic Review, 91(1), pp. 149-166.

[13] Campanale, Claudio, Rui Castro, and Gian Luca Clementi. 2007. "Asset Pricing in a Production Economy with Chew-Dekel Preferences". Mimeo, NYU.

[14] Campbell, John, and John Cochrane, 1999. By force of habit: A consumption-based explanation of aggregate stock market behavior. Journal of Political Economy 107, 205–251.

[15] Campbell, John Y., and Robert Shiller. "The Dividend-Price Ratio and Expectations of Future Dividends and Discount Factors." Review of Financial Studies, 1988, 1, pp. 195-227.

[16] Caldara, Dario, Jesus Fernandez-Villaverde, , Juan F. Rubio-Ramirez, and Yao Wen. "Computing DSGE Models with Recursive Preferences." Mimeo, UPenn.

[17] Carlstrom, Charles T. and Timothy S. Fuerst (1997), "Agency Costs, Net Worth, and Business Fluctuations: A Computable General Equilibrium Analysis, American Economic Review 87, 893-910

[18] Chen, Hui. 2010 "Macroeconomic Conditions and the Puzzles of Credit Spreads and Capital Structure". Journal of Finance.

[19] Chugh, Sanjay. 2010. "Firm risk and leverage-based business cycles." U of Maryland, Mimeo.

[20] Cooley, Thomas and Edward Prescott, 1995. "Frontiers of Business Cycle Research". Princeton University Press.

[21] Cooley, T., R. Marimon, and V. Quadrini, 2004, Aggregate consequences of limited contract enforceability, Journal of Political Economy 112, 817-847.

[22] Covas, Francesco and Wouter den Haan (2009). "The Cyclical Behavior of Debt and Equity Finance", American Economic Review.

[23] Epstein, Larry G. and Zin, Stanley E. "Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework." Econometrica, 1989, 57(4), pp. 937-969.

[24] Fernandez-Villaverde, Jesus, Pablo Guerron-Quintana, Juan Rubio-Ramirez, and Martin Uribe, 2009. "Risk matters: the real effect of volatility shock", NBER working paper 14875.

[25] Gabaix, Xavier, 2007. "Variable rare disasters: An exactly solved framework for ten puzzles in macro-finance." Mimeo, NYU.

[26] Simon Gilchrist and Egon Zakrajsek, 2008. "Investment and the cost of capital: New evidence from the corporate bond market." Mimeo, BU.

[27] Simon Gilchrist, Jae Sim and Egon Zakrajsek, 2010. "Uncertainty, Financial Frictions, and Investment Dynamics." Mimeo, BU.

[28] Simon Gilchrist, Vladimir Yankov, Egon Zakrajšek, 2009. "Credit market shocks and economic fluctuations: Evidence from corporate bond and stock markets." Journal of Monetary Economics, 56(4):471-493.

[29] Joao F. Gomes and Lukas Schmid. 2009. "Equilibrium credit spreads and the macroeconomy." Mimeo, Wharton.

[30] Gourio, Francois. 2008. "Disasters and Recoveries." American Economic Review - Papers and Proceedings, 98(2): 68-73.

[31] Gourio, Francois, 2008b. "Time Series Predictability in the Disaster Model." Finance Research Letters, 5(4): 191-203.

[32] Gourio, Francois, 2010. "Disaster risk and business cycles." Mimeo.

[33] Hackbarth Dirk, Jianjun Miao, and Erwan Morellec, 2006. ““Capital Structure, Credit Risk, and Macroeconomic Conditions” Journal of Financial Economics, 82, 519–550.

[34] Hall, Robert, 2009. “The High Sensitivity of Economic Activity to Financial Frictions”, Mimeo, Stanford.

[35] Hennessy, Christopher and A. Levy (2007). “Why Does Capital Structure Choice Vary with Macroeconomic Conditions?”, Journal of Monetary Economics..

[36] Huang, J., and M. Huang, 2003, “How Much of the Corporate-Treasury Yield Spread is Due to Credit Risk?,” Working paper, Stanford University.

[37] Jermann, Urban J., 1998. “Asset Pricing in Production Economies.” Journal of Monetary Economics, 41(2):257-275.

[38] Julliard, Christian and Anita Gosh, 2008. “Can Rare Events Explain the Equity Premium Puzzle?”, Mimeo LSE.

[39] Justiniano, Alejandro, Giorgio Primicieri and Andrea Tambalotti, 2009. “Investment Shocks and the Relative Price of Investment”. Journal of Monetary Economics.

[40] Liu Zheng, Pengfei Wang, and Tao Zha, 2009, Do Credit Constraints Amplify Macroeconomic Fluctuations?, working paper, Hong Kong University of Science and Technology.

[41] Martin Lettau and Sydney Ludvigson. Time-varying risk premia and the cost of capital: An alternative implication of the q theory of investment. Journal of Monetary Economics, 49(1):31 –66, 2002.

[42] Philippon, Thomas, 2009. “The Bond Market’s Q”. Forthcoming, Quarterly Journal of Economics.

[43] Martin, Ian, 2008. “Consumption-Based Asset Pricing with Higher Cumulants”. Mimeo, Stanford.

[44] Mendoza, Enrique, 2010. “Sudden Stops, Financial Crises and Leverage” American Economic Review.

[45] Merton, Robert C., 1974, On the Pricing of Corporate Debt: The Risk Structure of Interest Rates, Journal of Finance 29, 449-47.

[46] Miao, Jianjun, and Pengfei Wang. 2010. “Credit risk and business cycles.” Mimeo, Boston University.

[47] Rietz, Thomas, 1988. “The equity premium: A solution.” Journal of Monetary Economics 22, 117–131.

[48] Santa-Clara and Yan, 2008. “Crashes, Volatility, and the Equity Premium: Lessons from S&P 500 Options”. Review of Economics and Statistics, forthcoming.

[49] Santoro, Marika and Chao Wei, 2010. “Taxation, investment and asset pricing.” Forthcoming, Review of Economic Dynamics.

[50] Swanson, Eric and Glenn Rudebusch, 2008. "The Bond Premium in a DSGE Model with Long-Run Real and Nominal Risks", Mimeo.

[51] Swanson, Eric, 2010. "Risk Aversion and the Labor Margin in Dynamic Equilibium Models", Mimeo FRS.

[52] Tallarini, Thomas D. Jr., 2000. "Risk-Sensitive Real Business Cycles." Journal of Monetary Economics, 45(3), pp. 507-532.

[53] Wachter, Jessica, 2008. "Can time-varying risk of rare disasters explain aggregate stock market volatility?" Mimeo, Wharton.

[54] Weil, Philippe, 1989. "Nonexpected Utility in Macroeconomics Nonexpected Utility in Macroeconomics", Quarterly Journal of Economics, 105(1):29-42.

[55] Weitzmann, Martin, 2007. "Subjective Expectations and Asset-Return Puzzles," American Economic Review, 97(4):1102-1130.

Parameter	Symbol	Value
Capital share	α	.3
Depreciation rate	δ	.08
Share of consumption in utility	υ	.3
Discount factor	β	.98
Trend growth of TFP	μ	.01
Standard deviation of TFP shock	σ	.02
Intertemporal elasticity of substitution	$1/\psi$	2
Risk aversion over the consumption-leisure bundle	γ	4
Mean probability of disaster		.017
Distribution of $b_{tfp} = b_k$: values		(.15,.25,.35,.45,.57)
Distribution of $b_{tfp} = b_k$: probabilities		(.333,.267,233,.033,.133)
Persistence of $\log(p)$	ρ_p	.75
Unconditional std. dev. of $\log(p)$	$\frac{\sigma_p}{\sqrt{1-\rho_p^2}}$	1.5
Idiosyncratic shock volatility	σ_ε	0.2267
Tax subsidy	$\chi - 1$	0.0616
Recovery rate	θ	0.4

Table 1: **Parameter values for the benchmark model.** The time period is one year.

		$\sigma(\Delta \log Y)$	$\frac{\sigma(\Delta \log C)}{\sigma(\Delta \log Y)}$	$\frac{\sigma(\Delta \log I)}{\sigma(\Delta \log Y)}$	$\frac{\sigma(\Delta \log N)}{\sigma(\Delta \log Y)}$	$\rho_{C,Y}$	$\rho_{I,Y}$
	Data	2.78	0.65	2.52	0.96	0.61	0.61
No disaster risk	Benchmark	1.80	0.55	1.88	0.33	0.97	0.99
	Constant leverage	1.80	0.55	1.89	0.33	0.97	0.99
	RBC	1.82	0.56	2.47	0.35	0.96	0.98
Constant Disaster risk	Benchmark	1.81	0.55	1.97	0.34	0.96	0.99
	Constant leverage	1.81	0.55	1.97	0.34	0.96	0.99
	RBC	1.83	0.56	2.50	0.35	0.96	0.98
Time-varying Disaster risk	**Benchmark**	**2.11**	**0.77**	**3.38**	**0.83**	**0.12**	**0.86**
	Constant leverage	2.46	0.89	4.62	1.05	-0.23	0.86
	RBC	1.86	0.60	2.89	0.46	0.79	0.91

Table 2: **Business cycle statistics (annual).** Second moments implied by the model, for different versions of the model. The statistics are computed in a sample without disasters. rho(A,B) is the correlation of the growth rate of time series A and B. The benchmark model is in bold.

		$E(R_f)$	$E(R_e)$	E(spread)	σ(spread)	ρ(Spread,GDP)	$\sigma(R_f)$	$\sigma(R_e)$
	Data	0.80	7.60	0.94	0.41	-0.37	2.50	16.20
No disaster risk	Benchmark	2.46	2.44	0.51	0.00	-0.56	0.23	0.39
	Constant leverage	2.46	2.39	0.55	0.01	0.64	0.23	0.39
	RBC	2.59	2.54	-0.01	0.00	0.11	0.27	0.33
Constant	Benchmark	1.21	3.84	1.29	0.00	-0.65	0.22	0.41
Disaster risk	Constant leverage	1.21	3.81	1.32	0.01	0.65	0.22	0.41
	RBC	1.32	2.65	-0.01	0.00	-0.03	0.26	0.33
Time-varying	**Benchmark**	**1.31**	**3.60**	**0.98**	**0.46**	**-0.53**	**2.33**	**1.18**
Disaster risk	Constant leverage	1.31	3.85	1.21	1.40	-0.80	2.58	1.88
	RBC	1.41	2.65	-0.01	0.00	-0.07	2.05	0.34

Table 3: **Financial Statistics, 1.** Mean and standard deviation of the risk-free return, the equity return, and the spread between the corporate bonds and the risk-free bond. The statistics are calculated in a sample without disasters. The correlation is the correlation between the spread BAA-AAA and HP-filtered GDP.

		E(Lev)	Std(Lev)	E(ProbDef)	Std(ProbDef)
	Data	0.45	0.09	0.39	NA
No disaster risk	Benchmark	0.56	0.00	0.79	0.01
	Constant leverage	0.57	0.00	0.86	0.03
	RBC	0.00	0.00	0.00	0.00
Constant	Benchmark	0.54	0.00	0.50	0.01
Disaster risk	Constant leverage	0.55	0.00	0.53	0.02
	RBC	0.00	0.00	0.00	0.00
Time-varying	**Benchmark**	**0.54**	**0.04**	**0.58**	**0.23**
Disaster risk	Constant leverage	0.54	0.00	0.49	0.02
	RBC	0.00	0.00	0.00	0.00

Table 4: **Financial Statistics, 2.** Mean and volatility of leverage and of probability of default. The statistics are calculated in a sample without disasters. Data from Chen, Collin-Dufrense and Goldstein (2009).

	$\sigma(\Delta \log Y)$	$\frac{\sigma(\Delta \log C)}{\sigma(\Delta \log Y)}$	$\frac{\sigma(\Delta \log I)}{\sigma(\Delta \log Y)}$	$\frac{\sigma(\Delta \log N)}{\sigma(\Delta \log Y)}$	$\rho_{C,Y}$	$\rho_{I,Y}$
Data	2.78	0.65	2.52	0.96	0.61	0.61
Benchmark model	2.11	0.77	3.38	0.83	0.12	0.86
Samples with disasters	5.47	1.00	1.58	0.32	0.81	0.80
Adjustment costs ($\eta = .1$)	1.75	0.83	2.60	0.62	0.54	0.83
Adjustment costs ($\eta = .2$)	1.58	0.90	2.11	0.48	0.73	0.80
IES = .5	1.59	0.78	1.58	0.23	0.96	0.96
IES = .25	1.57	0.94	2.11	0.51	0.70	0.80
Risk aversion = .5	1.97	0.69	2.83	0.67	0.39	0.88

Table 5: **Extensions of the model: business cycle statistics (annual).**

	$E(R_f)$	$E(R_e)$	E(spread)	σ(spread)	ρ(Spread,GDP)	$\sigma(R_f)$	$\sigma(R_e)$
Data	0.80	7.60	0.94	0.41	-0.37	2.50	16.20
Benchmark model	1.31	3.60	0.98	0.46	-0.53	2.33	1.18
Samples with disasters	1.30	2.68	0.98	0.46	-0.52	2.34	7.05
Adjustment costs ($\eta = .1$)	1.31	3.62	0.98	0.46	-0.46	2.17	1.75
Adjustment costs ($\eta = .2$)	1.32	3.62	0.98	0.46	-0.37	2.09	2.07
IES = .5	1.61	3.90	0.98	0.47	-0.10	2.36	1.11
IES = .25	1.97	4.27	0.98	0.46	0.21	2.40	1.11
Risk aversion = .5	2.28	3.68	0.77	0.28	-0.63	1.50	0.91

Table 6: **Extensions of the model: Financial Statistics, 1.**

	E(Lev)	Std(Lev)	E(ProbDef)	Std(ProbDef)
Data	0.45	0.09	0.39	0.51
Benchmark model	0.54	0.04	0.58	0.23
Samples with disasters	0.54	0.04	0.81	2.55
Adjustment costs ($\eta = .1$)	0.54	0.04	0.58	0.24
Adjustment costs ($\eta = .2$)	0.54	0.05	0.58	0.24
IES = .5	0.54	0.04	0.58	0.23
IES = .25	0.54	0.04	0.58	0.23
Risk aversion = .5	0.55	0.03	0.65	0.18

Table 7: **Extensions of the model: Financial Statistics, 2.**

	$\sigma(\Delta \log Y)$	$\frac{\sigma(\Delta \log I)}{\sigma(\Delta \log Y)}$	$\frac{\sigma(\Delta \log N)}{\sigma(\Delta \log Y)}$	$E(\text{Lev})$
$\chi = 1.062$ (Benchmark)	2.11	3.38	0.83	0.54
$\chi = 1.06$	2.11	3.41	0.82	0.54
$\chi = 1.05$	2.05	3.41	0.77	0.53
$\chi = 1.04$	2.00	3.37	0.71	0.51
$\chi = 1.03$	1.96	3.30	0.64	0.50
$\chi = 1.02$	1.91	3.14	0.57	0.47
$\chi = 1.01$	1.89	3.00	0.51	0.43
$\chi = 1.005$	1.87	2.94	0.48	0.39
$\chi = 1.002$	1.86	2.92	0.47	0.34
$\chi = 1.001$	1.86	2.91	0.46	0.30
$\chi = 1$ (RBC)	1.86	2.89	0.43	0.00

Table 8: **Effect of tax shield parameter on mean leverage and volatilities of quantities.**

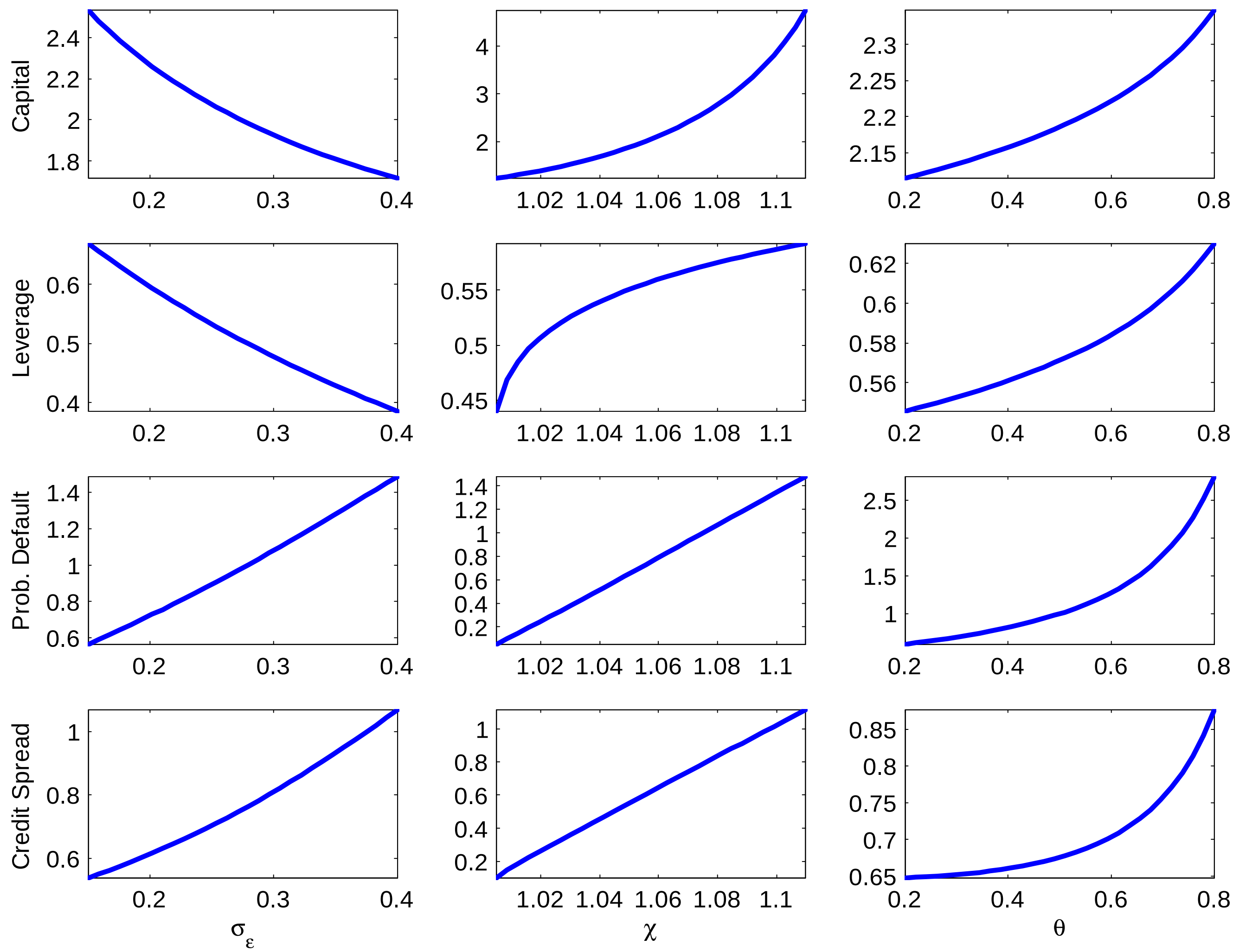

Figure 1: **Comparative statics on steady-state.** Effect of idiosyncratic volatility σ_ε, tax subsidy χ, and recovery rate θ, on capital, leverage, probability of default (in %), and credit spread (in %).

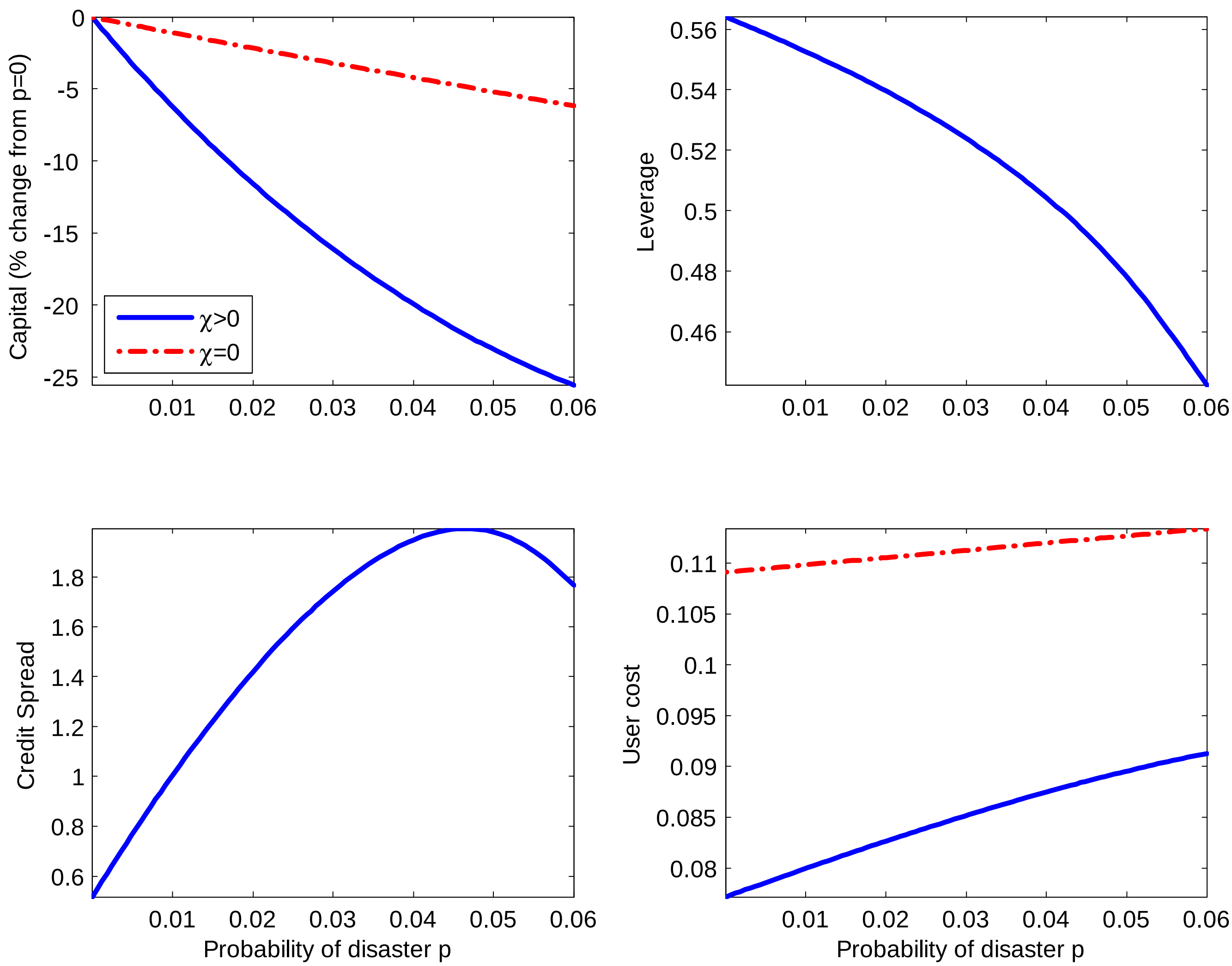

Figure 2: **Comparative statics on steady-state.** Effect of an increase in the probability of disaster on capital, leverage, credit spreads (in %), and the user cost of capital, for the frictionless model ($\chi = 0$, red dot-dashed line) and the benchmark model ($\chi > 0$, blue full line).

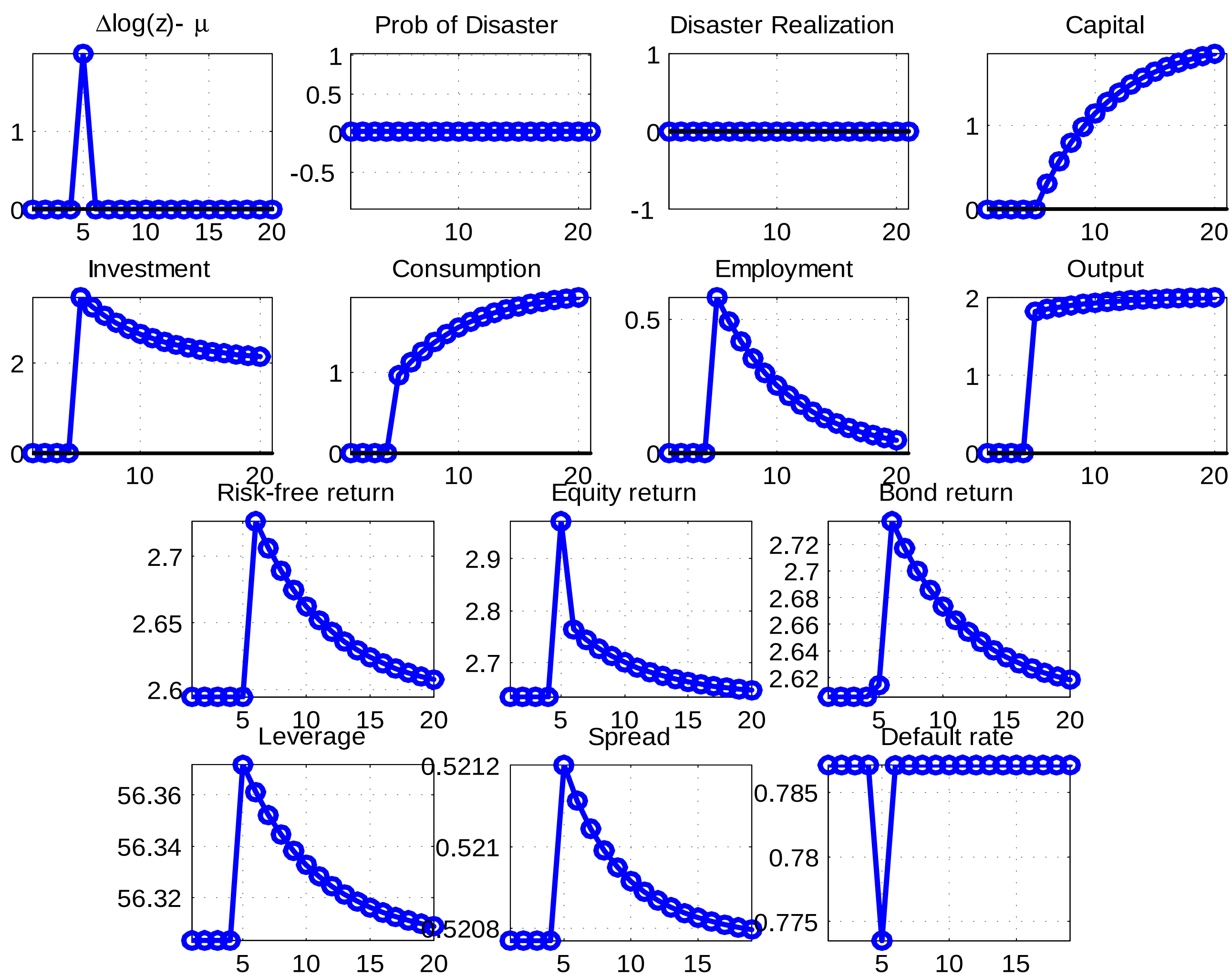

Figure 3: Impulse response function of model quantities and returns to a one standard deviation shock to total factor productivity ($e_{t+1} = 1$). Quantity responses are shown in % deviation from balanced growth path. Returns, default rates, credit spreads, leverage and the probability of disaster are annual, in % per year.

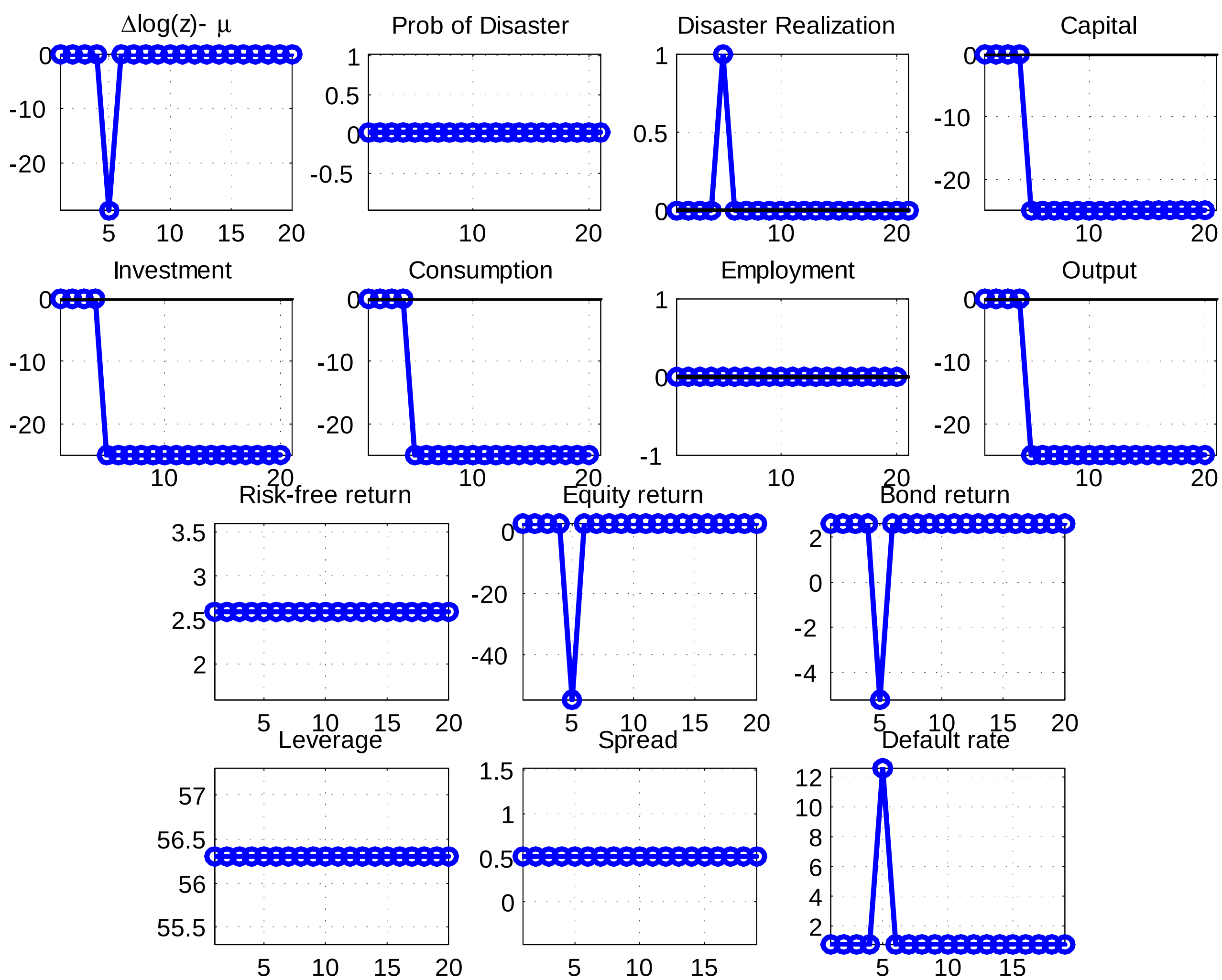

Figure 4: Impulse response function of model quantities and returns to a disaster realization (x_{t+1}). Quantity responses are shown in % deviation from balanced growth path. Returns, default rates, credit spreads, leverage and the probability of disaster are annual, in % per year.

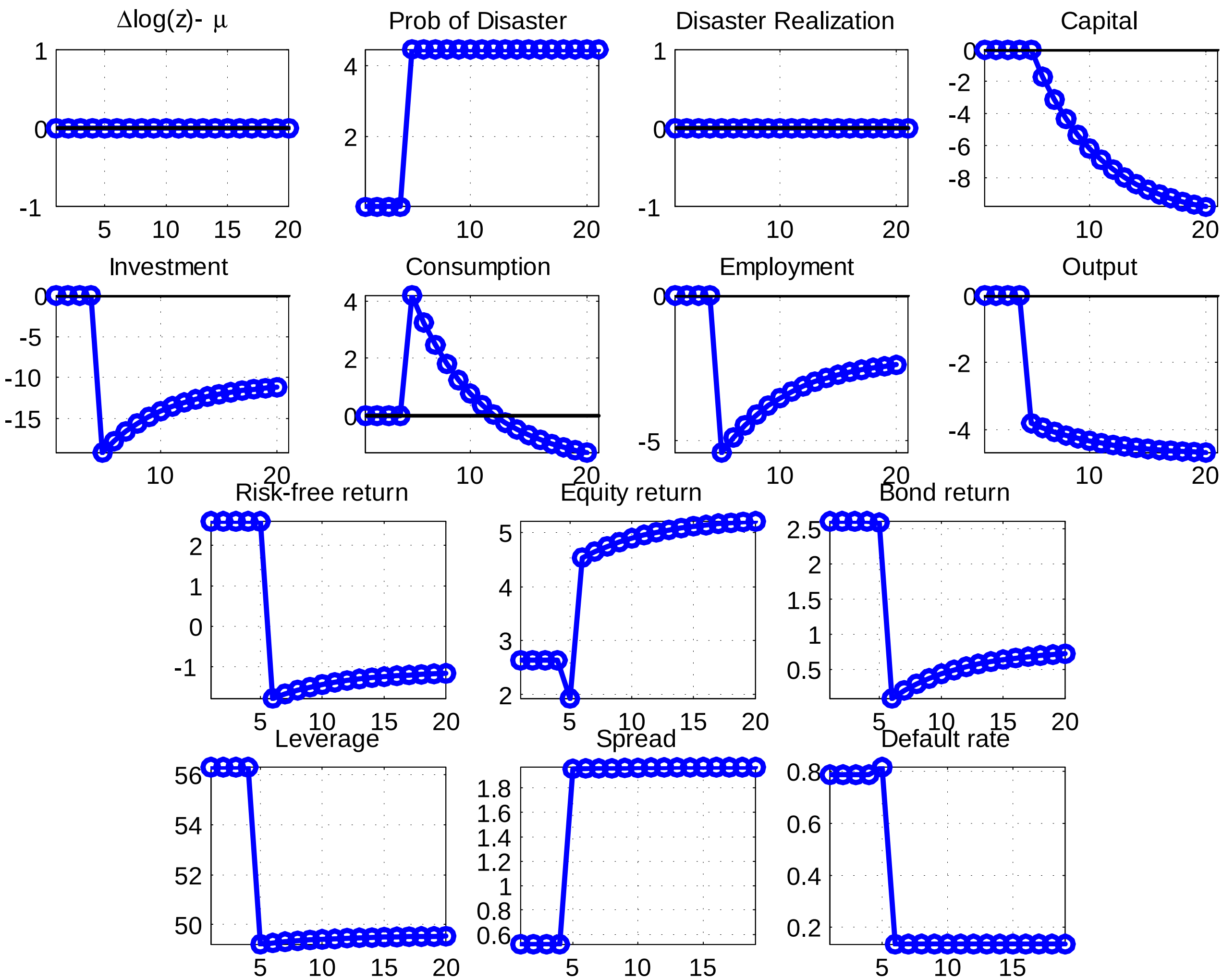

Figure 5: Impulse response function of model quantities and returns to a shock to the probability of disaster (p_{t+1}). Quantity responses are shown in % deviation from balanced growth path. Returns, default rates, credit spreads, leverage and the probability of disaster are annual, in % per year.

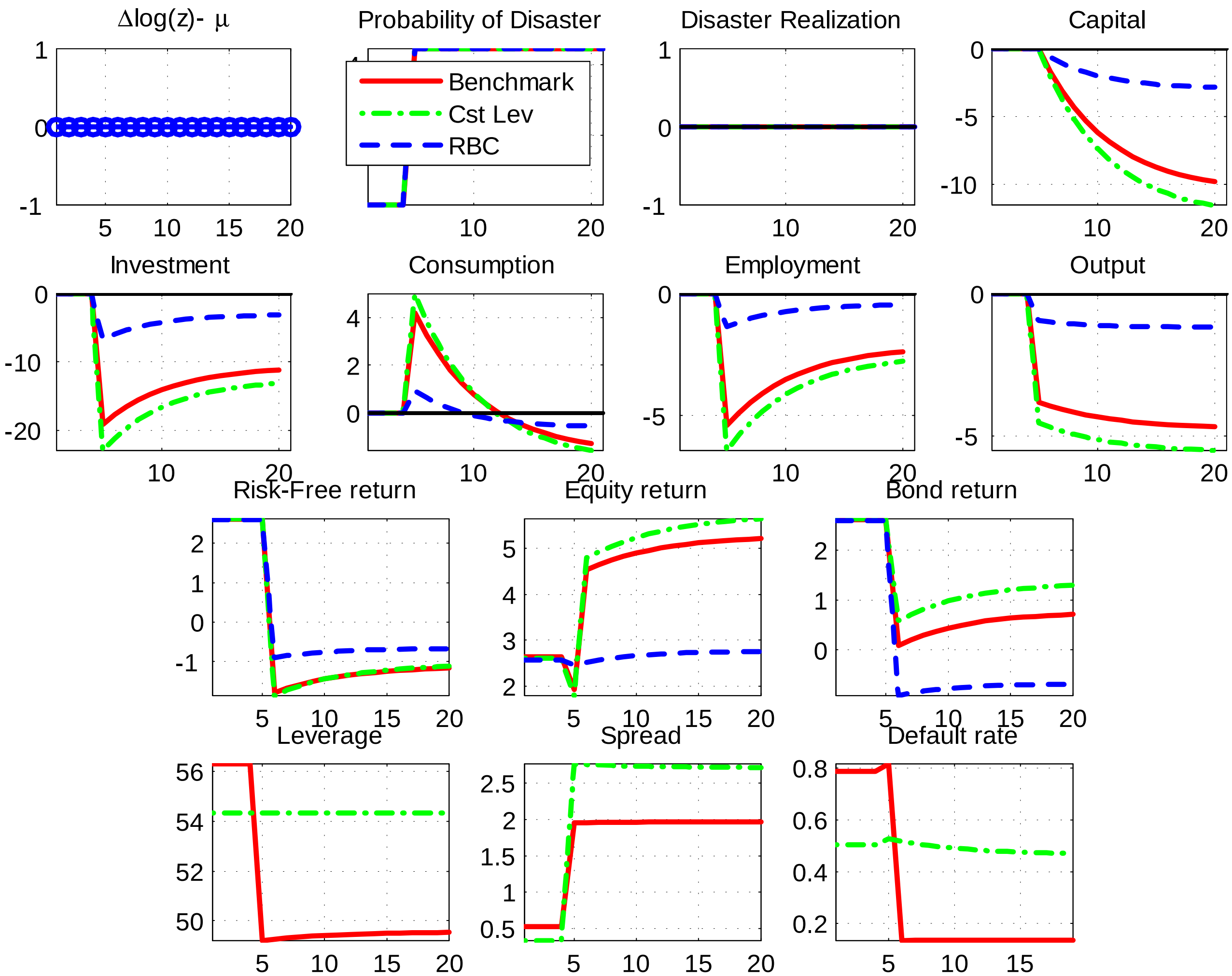

Figure 6: This figure compares the impulse response of three models to a probability of disaster shock: the benchmark model (red full line), the model with constant leverage (green dot-dashed line), and the frictionless RBC model (blue dashed line). Quantity responses are shown in % deviation from balanced growth path. Returns, default rates, credit spreads, leverage and the probability of disaster are annual, in % per year.

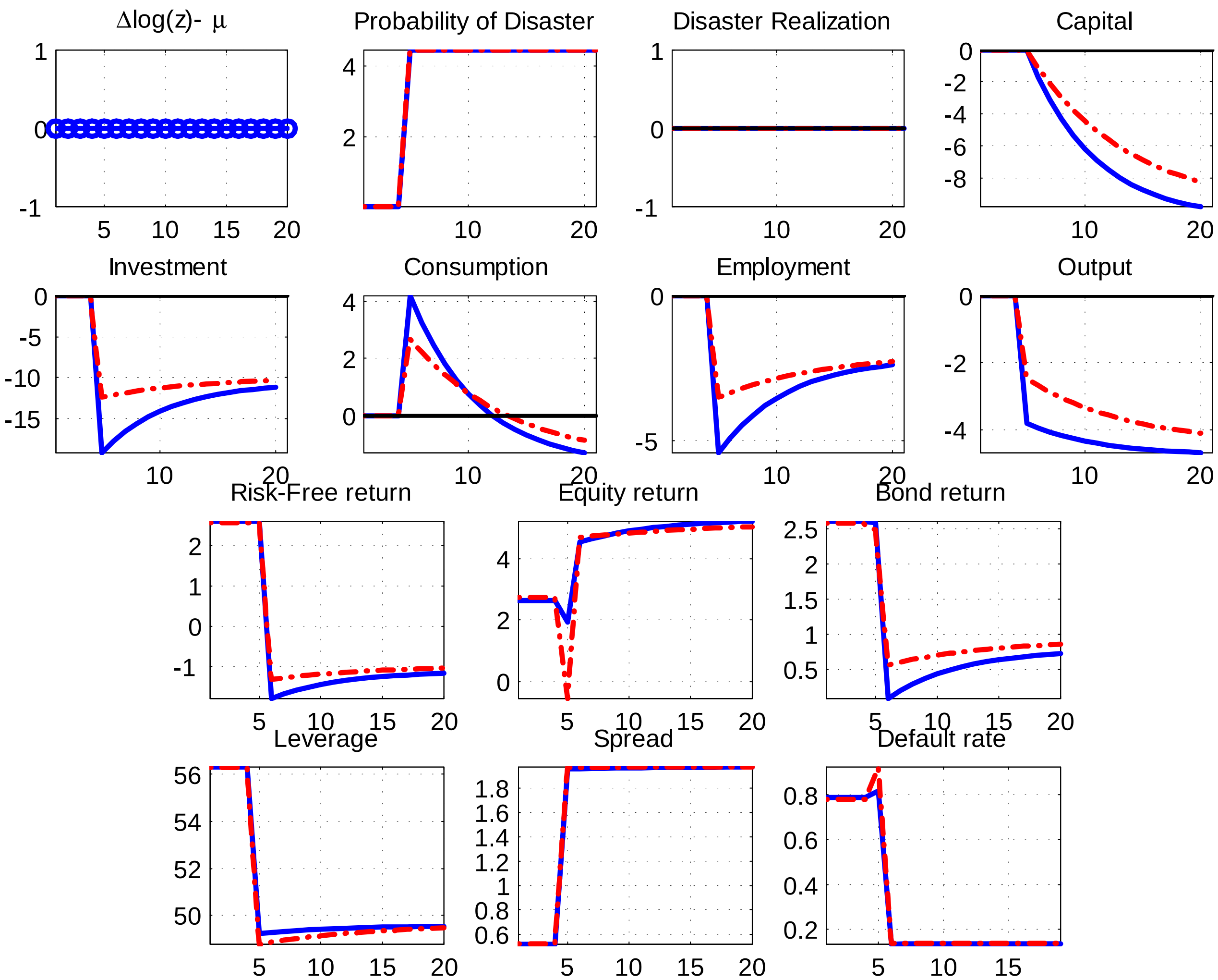

Figure 7: Impulse response function of model quantities and returns to a shock to the probability of disaster (p_{t+1}). Blue full line = benchmark model, red dashed line = model with capital adjustment costs. Quantity responses are shown in % deviation from balanced growth path. Returns, default rates, credit spreads, leverage and the probability of disaster are annual, in % per year.

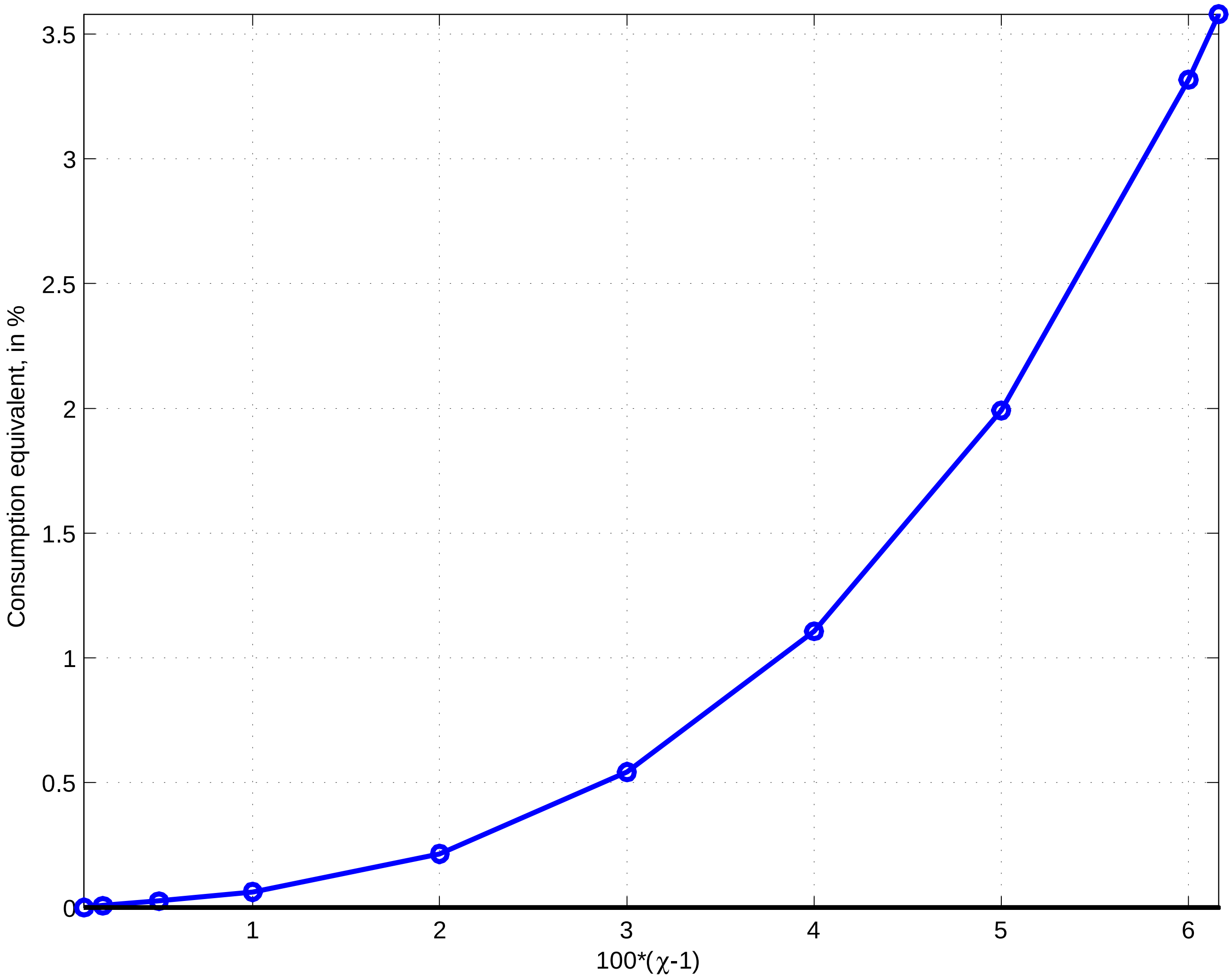

Figure 8: Welfare cost of removing the tax shield, i.e. setting $\chi = 0$, as a function of χ.