

Financial Crises, Bank Risk Exposure and Government Financial Policy

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Abstract

A macroeconomic model with financial intermediation is developed in which the intermediaries (banks) can issue outside equity as well as short term debt. This makes bank risk exposure an endogenous choice. The goal is to have a model that can not only capture a crisis when banks are highly vulnerable to risk, but can also account for why banks adopt such a risky balance sheet in the first place. We use the model to assess quantitatively how perceptions of fundamental risk and of government credit policy in a crisis affect the vulnerability of the financial system *ex ante*. We also study the effects of macro-prudential policies designed to offset the incentives for risk-taking.

1 Introduction

A distinguishing feature of the recent U.S. recession - known now as the Great Recession - was the significant disruption of financial intermediation. The meltdown of the shadow banking system along with the associated strain placed on the entire financial system led to an extraordinary increase in financing costs. This increase in financing costs, which peaked in the wake of the Lehman Brothers collapse, is considered a major factor in the collapse of durable goods spending in the fall of 2008 that in turn triggered the huge contraction in output and employment.

The challenge for macroeconomists has been to build models that can not only capture this phenomenon but also be used to analyze the variety of unconventional measures pursued by the monetary and the fiscal authorities to stabilize credit markets. In this regard, there has been a rapidly growing literature that attempts to incorporate financial factors within the quantitative macroeconomic frameworks that had been the workhorses for monetary and fiscal policy analysis up until this point. Much of this work is surveyed in Gertler and Kiyotaki (2010). A common feature of many of these papers has been to extend the basic financial accelerator mechanism developed by Bernanke and Gertler (1989) and Kiyotaki and Moore (1997) to financial intermediaries ("banks") in order to capture the disruption of intermediation.

Key to motivating a crisis within these frameworks is the heavy reliance of banks on short term debt. This feature makes these institutions highly exposed to the risk of adverse returns to their balance sheet in way that is consistent with recent experience. Within these frameworks and most others in this class, however, by assumption the only way banks can obtain external funds is by issuing short term debt.¹ Thus, in their present form, these models are not equipped to address how the financial system found itself so vulnerable in the first place. This question is of critical importance for designing policies to ensure the economy does not wind up in this position again. For example, a number of authors have suggested that such a risky bank liability structure was ultimately the product of expectations the gov-

¹Some quantitative macro models with financial sectors include: Bernanke, Gertler and Gilchrist (1999), Brunnermeier and Sannikov (2009), Christiano, Motto and Rostagno (2009), Gilchrist, Ortiz and Zakresjek (2009), Mendoza (2010) and Jermann and Quadrini (2009). Only the latter considers both debt and equity finance, though they restrict attention to borrowing constraints faced by non-financial firms.

ernment will intervene to stabilize financial markets in a crisis, just as it just did recently. With the existing macroeconomic frameworks it is not possible to address this issue.

In this paper we develop a macroeconomic model with an intermediation sector that allows banks to issue outside equity as well as short term debt. This makes bank risk exposure an endogenous choice. Here the goal is to have a model that can not only capture a crisis when financial institutions are highly vulnerable to risk, but also account for why these institutions adopt such a risky portfolio structure in the first place. The basic framework builds on Gertler and Karadi (2011) and Gertler and Kiyotaki (2010). It extends the agency problem between banks and savers within these frameworks to allow intermediaries a meaningful trade-off between short term debt and equity. Ultimately, a bank's decision over its balance sheet will depend on its perceptions of risk, which will in turn depend on both the fundamental disturbances to the economy and expectations about government policy.

We first use the model to analyze how different degrees of fundamental risk in the economy affect the balance sheet structure of banks and the aggregate equilibrium. We then analyze the vulnerability of the economy to a crisis in each kind of risk environment. When perceptions of risk are low, banks opt for greater leverage. But this has the effect of making the economy more vulnerable to a crisis.

We next turn to analyzing credit policy during a crisis. Following Gertler and Karadi (2011), we analyze large scale asset purchases of the type the Federal Reserve used to help stabilize financial markets following the Lehman collapse. Within this framework, the central bank has an advantage during a crisis that it can easily obtain funds by issuing short term government debt, in contrast to private intermediaries that are constrained by the weakness of their respective balance sheets. Thus this kind of credit policy is effective in mitigating a crisis even if the central bank is less efficient in acquiring assets than is private sector.

What is new in the present framework is that it is possible to capture the side effect of the credit policy on moral hazard. In particular, as we show, the anticipated credit policy will induce banks to adopt a riskier balance sheet, which will in turn require a larger scale credit market intervention during a crisis. This sets the stage for an analysis of macro-prudential policy designed to offset the effects of anticipated credit policy on the incentives for bank risk-taking.

To be sure, there is lengthy theoretical literature that examines the

sources of vulnerability of a financial system. For example, Fostel and Geanakoplos (2009) stress the role of investor optimism in encouraging risk taking. Others such as Diamond and Rajan (2009), Fahri and Tirole (2009) and Chari and Kehoe (2009) stress moral hazard consequences of bailouts and other credit market interventions. Our paper differs mainly by couching the analysis within a full blown macroeconomic model to provide a step toward assessing the quantitative implications.

There is as well a related literature that analyzes macro-prudential policy. Again, much of it is qualitative (e.g. Lorenzoni, 2009, Korinek, 2009, and Stein 2010). However, there are also quantitative frameworks, e.g., Bianchi (2009) and Nikolov (2009). Our framework differs partly by endogenizing the financial friction and partly by exploring the interaction between credit policies used to stabilize the economy *ex post* and macro-prudential policy used to mitigate risk taking *ex ante*.

Finally relevant are the literatures on international risk sharing and on asset pricing and business cycles.² Conventional quantitative models used for policy evaluation typically examine linear dynamics within a local neighborhood of a deterministic steady state. In doing so they abstract from an explicit consideration of uncertainty. Because bank liability structure will depend on perceptions of risk, however, accounting for uncertainty is critical. Here we borrow insights from these literatures by considering second order approximations to pin down determinate bank liability shares.

2 The Baseline Model

2.1 Physical Setup

Before introducing financial frictions, we present the basic physical environment. There are a continuum of firms of mass of unity. Each firm produces output using an identical constant returns to scale Cobb-Douglas production function with capital and labor as inputs. We can express aggregate output Y_t as a function of aggregate capital K_t and aggregate labor hours L_t as:

$$Y_t = A_t K_t^\alpha L_t^{1-\alpha}, \quad 0 < \alpha < 1, \quad (1)$$

where A_t is aggregate productivity which follows a stationary Markov process.

²See, for example, Campbell (1994), Devereux and Sutherland (2009), Lettau (2003) and Tille and Van Wincoop (2007).

Let S_t be the aggregate capital stock "in process" for period $t+1$. Capital in process at t for $t+1$ is the sum of current investment I_t and the stock of undepreciated capital, $(1 - \delta)K_t$.

$$S_t = (1 - \delta)K_t + I_t. \quad (2)$$

Capital in process for period $t+1$ is transformed into capital for production after the realization of a multiplicative shock to capital quality, ψ_{t+1} ,

$$K_{t+1} = \psi_{t+1}S_t. \quad (3)$$

Following the finance literature (e.g., Merton (1973)), we introduce the capital quality shock as a simple way to introduce an exogenous source of variation in the value of capital. As will become clear later, the market price of capital will be endogenous within our framework. In this regard, the capital quality shock will serve as an exogenous trigger of asset price dynamics. The random variable ψ_{t+1} is best thought of as capturing some form of economic obsolescence, as opposed to physical depreciation. Appendix B in the working paper version of this paper provides an explicit micro-foundation. We assume the capital quality shock ψ_{t+1} follows an i.i.d. process, with an unconditional mean of unity. In addition, we allow for occasional disasters in the form of sharp contractions in quality, as describe later. These disaster shocks serve to instigate financial crises.³

Firms acquire new capital from capital goods producers. There are convex adjustment costs in the gross rate of change in investment for capital goods producers. Aggregate output is divided between household consumption C_t , investment expenditures, and government consumption G_t ,

$$Y_t = C_t + [1 + f(\frac{I_t}{I_{t-1}})]I_t + G_t, \quad (4)$$

where $f(\frac{I_t}{I_{t-1}})I_t$ reflects physical adjustment costs, with $f(1) = f'(1) = 0$ and $f''(I_t/I_{t-1}) > 0$.

Our preference structure follows Miao and Wang (2010), which is in turn based on Guvenen (2009) and Greenwood, Hercowitz and Huffman (GHH,

³Other recent papers that make use of this kind of disturbance include, Gertler and Karadi (2011), Brunnermeier and Sannikov (2009) and Gourio (2009). An alternative but more cumbersome approach would be to introduce a "news" shock that affects current asset values. Gertler and Karadi (2011) illustrate the similarities between the two.

1988):

$$E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \frac{1}{1-\gamma} \left(C_{\tau} - hC_{\tau-1} - \frac{\chi}{1+\varphi} L_{\tau}^{1+\varphi} \right)^{1-\gamma}, \quad (5)$$

where E_t is the expectation operator conditional on date t information and $\gamma > 0$. The preference specification allows for habit formation and, as in GHH, abstracts from wealth effects on labor supply.

Flow adjustment costs of investment and habit formation are standard features of many quantitative macro models. We include them here because they improve the quantitative performance of the model considerably and can be added at relatively little cost in terms of model complexity. However, we abstract from other standard features that help account for employment volatility, such as price and wage rigidities since doing so would complicate the model considerably. Instead, allowing for GHH preferences provides a simple way for the model to produce reasonably sized fluctuations in hours in the absence of either nominal price rigidities or labor market frictions.⁴

If there were no financial frictions, the competitive equilibrium would correspond to a solution of the planner's problem that involves choosing aggregate quantities $(Y_t, L_t, C_t, I_t, S_t)$ as a function of the aggregate state $(C_{t-1}, I_{t-1}, \psi_t S_{t-1}, A_t)$ in order to maximize the expected discounted utility of the representative household subject to the resource constraints. This frictionless economy will serve as a benchmark to which we may compare the implications of the financial frictions.

In what follows we will introduce banks that intermediate funds between households and nonfinancial firms. We will also introduce financial frictions that may impede credit flows.

2.2 Households

Following Gertler and Karadi (2011), we formulate the household sector in way that permits maintaining the tractability of the representative agent approach. In particular, there is a representative household with a continuum of members of measure unity. Within the household there are $1-f$ "workers" and f "bankers". Workers supply labor and return their wages to the household. Each banker manages a financial intermediary (which we will call

⁴Another advantage of GHH preferences is that it permits increasing the degree risk aversion without introducing counterfactual hours fluctuations. The degree of risk aversion is relevant for welfare comparisons of different policies.

a "bank") and transfers nonnegative dividends back to the household subject to its flow of fund constraint. Within the family there is perfect consumption insurance.

Households do not acquire capital nor do they provide funds directly to nonfinancial firms. Rather, they supply funds to banks. (It may be best to think of them as providing funds to banks other than the ones they own). Banks offer two types of liabilities to households: non-contingent riskless short term debt (deposits) and equity, which may be thought of as perfectly state contingent debt. We refer to equity issued by banks and held by households as "outside" equity. This contrasts with the accumulated retained earnings of a banker who manages an intermediary and is involved in the operation. We refer to the latter as "inside" equity. The distinction between outside and inside equity will become important later since banks will face constraints in obtaining external funds. In addition, households may acquire short term riskless government debt. Both bank deposits and government debt are one period real riskless bonds and thus are perfect substitute in the equilibrium we consider. Thus we impose this condition from the onset and assume that both pay the same gross real return R_t from $t - 1$ to t .

We normalize the units of outside equity so that each equity is a claim to the future returns of one unit of the asset that the bank holds. Let Z_t be the flow returns at t generated by one unit of the bank's assets and q_t the price of the outside equity at t . Then the payoff at t for a share of outside equity acquired at $t - 1$ equals $[Z_t + (1 - \delta)q_t] \psi_t$. Note that the payoff is adjusted for both the physical depreciation and the quality adjustment of the capital that underlies bank assets.

The household chooses consumption, labor supply, riskless debt, and outside equity $(C_t, L_t, D_{ht}, \bar{e}_t)$ to maximize expected discounted utility (5) subject to the flow of funds constraint,

$$C_t + D_{ht} + q_t \bar{e}_t = W_t L_t + \Pi_t - T_t + R_t D_{ht-1} + [Z_t + (1 - \delta)q_t] \psi_t \bar{e}_{t-1}. \quad (6)$$

Here W_t is the wage rate, T_t is lump sum taxes, and Π_t is net distributions from ownership of both banks and capital producing firms. Let u_{Ct} denote the marginal utility of consumption and $\Lambda_{t,t+1}$ the household's stochastic discount factor. Then the household's first order conditions for labor supply and consumption/saving are given by

$$E_t u_{Ct} W_t = \chi L_t^\varphi (C_t - h C_{t-1} - \frac{\chi}{1 + \varphi} L_t^{1+\varphi})^{-\gamma}, \quad (7)$$

$$E_t(\Lambda_{t,t+1})R_{t+1} = 1, \quad (8)$$

$$E_t(\Lambda_{t,t+1}R_{et+1}) = 1, \quad (9)$$

with

$$u_{Ct} \equiv (C_t - hC_{t-1} - \frac{\chi}{1+\varphi}L_t^{1+\varphi})^{-\gamma} - \beta h(C_{t+1} - hC_t \frac{\chi}{1+\varphi}L_{t+1}^{1+\varphi})^{-\gamma},$$

$$\Lambda_{t,\tau} \equiv \beta^{\tau-t} \frac{u_{C\tau}}{u_{Ct}},$$

$$R_{et+1} = \frac{[Z_{t+1} + (1-\delta)q_{t+1}]\psi_{t+1}}{q_t}.$$

Because banks may be financially constrained, bankers will retain earnings to accumulate assets. Absent some motive for paying dividends, they may find it optimal to accumulate to the point where the financial constraint they face is no longer binding. In order to limit bankers' ability to save to overcome financial constraints, we allow for turnover between bankers and workers. In particular, we assume that with i.i.d. probability $1 - \sigma$, a banker exits next period. Upon exiting, a banker transfers accumulated retained earnings to the household and becomes a worker. Note that the expected survival time $\frac{1}{1-\sigma}$ may be quite long. It is critical, however, that the expected horizon is finite, in order to motivate payouts while the financial constraints are still binding.

Each period, $(1 - \sigma)f$ workers randomly become bankers, keeping the number in each occupation constant. Finally, because in equilibrium bankers will not be able to operate without any financial resources, each new banker receives a "start up" transfer from the family as a small constant fraction of the aggregate assets of bankers.⁵ Accordingly, Π_t is net funds transferred to the household, i.e., funds transferred from exiting bankers minus the funds transferred to new bankers (aside from profits of capital producers).

2.3 Nonfinancial Firms

There are two types of nonfinancial firms: goods producers and capital producers.

⁵Because the balance sheet of each bank is small relative to the aggregate assets, each banker will not take into account the effect of its choice on the size of future start-up.

2.3.1 Goods Producers

Competitive goods producers operate a constant returns to scale technology with capital and labor inputs, given by equation (1). Firms choose labor to satisfy

$$W_t = (1 - \alpha) \frac{Y_t}{L_t}. \quad (10)$$

It follows that we may express gross profits per unit of capital Z_t as,

$$Z_t = \frac{Y_t - W_t L_t}{K_t} = \alpha A_t \left(\frac{L_t}{K_t} \right)^{1-\alpha}. \quad (11)$$

A goods producer can commit to pay all the future gross profits to the creditor bank. In particular, we suppose that the bank is efficient at evaluating and monitoring nonfinancial firms and also at enforcing contractual obligations with these borrowers. That is why these borrowers rely exclusively on banks to obtain funds. Then a goods producer who invests can obtain funds from a bank without any financial friction by issuing new state-contingent securities at the price Q_t . The producer then uses the funds to buy new capital goods from capital goods producers. Each unit of the security is a state-contingent claim to the future returns from one unit of investment:

$$\psi_{t+1} Z_{t+1}, (1 - \delta) \psi_{t+1} \psi_{t+2} Z_{t+2}, (1 - \delta)^2 \psi_{t+1} \psi_{t+2} \psi_{t+3} Z_{t+3}, \dots$$

Through perfect competition, the price of new capital goods is equal to Q_t , and goods producers earn zero residual profits state-by-state.

2.3.2 Capital Producers

Capital producers make new capital using input of final output and subject to adjustment costs, as described in section 2.2. They sell new capital to firms at the price Q_t . Given that households own capital producers, the objective of a capital producer is to choose I_t to solve:

$$\max E_t \sum_{\tau=t}^{\infty} \Lambda_{t,\tau} \left\{ Q_{\tau} I_{\tau} - \left[1 + f \left(\frac{I_{\tau}}{I_{\tau-1}} \right) \right] I_{\tau} \right\}.$$

From profit maximization, the price of capital goods is equal to the marginal cost of investment goods production as,

$$Q_t = 1 + f \left(\frac{I_t}{I_{t-1}} \right) + \frac{I_t}{I_{t-1}} f' \left(\frac{I_t}{I_{t-1}} \right) - E_t \Lambda_{t,t+1} \left(\frac{I_{t+1}}{I_t} \right)^2 f' \left(\frac{I_{t+1}}{I_t} \right). \quad (12)$$

Profits (which arise only outside of steady state), are redistributed lump sum to households.

2.4 Banks

To provide funds to goods producers in each period, banks raise funds both internally from retained earnings and externally from households. Every period each bank raises deposits and outside equity from households. In addition the bank has its own net worth - accumulated from retained earnings (which we refer to as inside equity). The bank then uses all its available funds to make loans to goods producers. As noted earlier, there is no friction in transferring funds between a bank and goods producers. As in case of investment banks, banks finance goods producers by purchasing perfectly state-contingent security. Their total value of loans is equal to the number s_t times the price Q_t of the state-contingent security (or "asset") - i.e. the bank's claim on the future returns from one unit of a goods producer's capital at the end of period t (i.e. capital at t in process for $t + 1$).

For an individual bank, the flow-of-funds constraint implies the value of loans funded within a given period, $Q_t s_t$, must equal the sum of bank net worth n_t , and funds raised from households, consisting of outside equity $q_t e_t$ and deposits d_t .

$$Q_t s_t = n_t + q_t e_t + d_t. \quad (13)$$

Note that in general, Q_t need not equal q_t . As will become clear, when the bank is financially constrained, the price Q_t of a bank's claim on a unit of capital of nonfinancial firm will in general be lower than the price q_t of outside equity, given that only banks can provide funds costlessly to goods producers.

While banks may issue new outside equity, they raise inside equity only through retained earnings.⁶ Since inside equity involves management and control of the firm's assets, we suppose it is prohibitively costly for the existing insiders to bring in new ones with sufficient wealth. In particular, the bank's net worth n_t at t is the gross payoff from assets funded at $t - 1$, net

⁶As a crude first pass, one can think of inside equity as common stock and outside equity as preferred stock or subordinate debt. Insiders of a firm are more likely to hold common stocks because they are the firm's ultimate residual claimants. Outsiders are likely to hold preferred stocks or subordinate debts. In general, common stock is thought to be more costly to issue than preferred stock or subordinate debt.

of returns to outside equity holders and depositors. Let R_{kt} denote the gross rate of return on a unit of the bank's assets from $t - 1$ to t . Then:

$$n_t = R_{kt}Q_{t-1}s_{t-1} - R_{et}q_{t-1}e_{t-1} - R_t d_{t-1}, \quad (14)$$

with

$$R_{kt} = \frac{[Z_t + (1 - \delta)Q_t]\psi_t}{Q_{t-1}},$$

and where the rate of return on outside equity R_{et} is given by equation (9). Observe that outside equity permits the bank to hedge against fluctuations in the return on its assets. It is this hedging value that makes it attractive to the bank to issue outside equity.

Given the bank faces a financing constraint, it is in its interest to retain all earnings until the time it exits, at which point it pays out its accumulated retained earnings as dividends. Accordingly, the objective of the bank at the *end* of period t is the expected present value of the future terminal dividend,

$$V_t = E_t \left[\sum_{\tau=t+1}^{\infty} (1 - \sigma) \sigma^{\tau-t-1} \Lambda_{t,\tau} n_{\tau} \right]. \quad (15)$$

To motivate an endogenous constraint on the bank's ability to obtain funds, we introduce the following simple agency problem: We assume that after a bank obtains funds, the banker managing the bank may transfer a fraction of assets to his or her family. It is the recognition of this possibility that has households limit the funds they lend to banks.

In addition, we assume that the fraction of funds the bank may divert depends on the composition of its liabilities. In particular, we assume that at the margin it is more difficult to divert assets funded by short term deposits than by outside equity. Short term deposits require the bank to continuously meet a non-contingent payment. Dividend payments, in contrast, are tied to the performance of the bank's assets, which is difficult for outsiders to monitor. By giving banks less discretion over payouts, short term deposits offer more discipline over bank managers than does outside equity.⁷

Let x_t denote the fraction of bank assets funded by outside equity:

$$x_t = \frac{q_t e_t}{Q_t s_t}. \quad (16)$$

⁷The idea that short term debt serves as a disciplining device over banks is due to Calomiris and Kahn (1991).

Then we assume that after the bank has obtained funds it may divert the fraction $\Theta(x_t)$ of assets, where $\Theta(x_t)$ is the convex function of x_t :

$$\Theta(x_t) = \theta \left(1 + \varepsilon x_t + \frac{\kappa}{2} x_t^2 \right). \quad (17)$$

We allow for the possibility that there could be some efficiency gains in monitoring the bank from having at least a bit of outside equity participation in funding the bank (i.e. ε can be negative). However, we restrict attention to calibrations where the bank's ability to divert assets increases when outside equity replaces deposits for funding: at the margin $\theta(\varepsilon + \kappa x_t)$ is positive. Finally, we assume that the banker's decision over whether to divert funds must be made at the end of the period t but before the realization of aggregate uncertainty in the following period. Here the idea is that if the banker is going to divert funds, it takes time to position assets and this must be done between the periods (e.g., during the night).

If a bank diverts assets for its personal gain, it defaults on its debt and is shut down. The creditors may re-claim the remaining fraction $1 - \Theta(x_t)$ of funds. Because its creditors recognize the bank's incentive to divert funds, they will restrict the amount they lend. In this way bank may face an external financing constraint.

Let $V_t(s_t, x_t, n_t)$ be the maximized value of the bank's objective V_t , given an asset and liability configuration (s_t, x_t, n_t) at the *end* of period t . Then in order to ensure the bank does not divert funds, the incentive constraint must hold:

$$V_t \geq \Theta(x_t) Q_t s_t. \quad (18)$$

Equation (18) states that for households to be willing to supply funds to a bank, the bank's franchise value V_t must be at least as large as its gain from diverting funds.

Combining (13) and (14) yields the evolution of n_t as a function of s_{t-1}, x_{t-1} and n_{t-1} as,

$$n_t = [R_{kt} - x_{t-1} R_{et} - (1 - x_{t-1}) R_t] Q_{t-1} s_{t-1} + R_t n_{t-1}. \quad (19)$$

It follows that in general the franchise value of the bank at the end of period $t - 1$ satisfies the Bellman equation

$$V_{t-1}(s_{t-1}, x_{t-1}, n_{t-1}) = E_{t-1} \Lambda_{t-1,t} \left\{ (1 - \sigma) n_t + \sigma \underset{s_t, x_t}{Max} [V_t(s_t, x_t, n_t)] \right\}. \quad (20)$$

where the right side takes into account that the bank exits with probability $1 - \sigma$ and continues with probability σ . Accordingly, at each time t , the bank bank chooses s_t and x_t to maximize $V_t(s_t, x_t, n_t)$ subject to the incentive constraint (18) and the law of motion for net worth (19). We conjecture the value function V_t is the function of the balance sheet components as:

$$V_t(s_t, x_t, n_t) = (\mu_{st} + x_t\mu_{et})Q_t s_t + \nu_t n_t. \quad (21)$$

In Appendix A of our companion working paper, we provide a detailed derivations and verify this conjecture.

We begin with the solution for s_t . Let ϕ_t be the maximum ratio of bank assets to net worth (leverage ratio) that satisfies the incentive constraint. Then by construction,

$$Q_t s_t = \phi_t n_t. \quad (22)$$

Equation (22) is a key relation of the banking sector: It indicates that when the borrowing constraint binds, the total quantity of private assets that a bank can intermediate is limited by its net worth, n_t . From the bank's optimization problem, ϕ_t is given by

$$\phi_t = \frac{\nu_t}{\Theta(x_t) - (\mu_{st} + x_t\mu_{et})}, \quad (23)$$

with

$$\nu_t = E_t(\Lambda_{t,t+1}\Omega_{t+1})R_{t+1}, \quad (24)$$

$$\mu_{st} = E_t[\Lambda_{t,t+1}\Omega_{t+1}(R_{kt+1} - R_{t+1})], \quad (25)$$

$$\mu_{et} = E_t[\Lambda_{t,t+1}\Omega_{t+1}(R_{t+1} - R_{et+1})]. \quad (26)$$

Ω_{t+1} is the shadow value of a unit of net worth to the bank at $t + 1$ and is given by

$$\Omega_{t+1} = 1 - \sigma + \sigma[\nu_{t+1} + \phi_{t+1}(\mu_{st+1} + x_{t+1}\mu_{et+1})]. \quad (27)$$

The relation is intuitive: The leverage ratio ϕ_t is increasing in two factors which raise the franchise value of the bank: the discounted excess value of bank assets $(\mu_{st} + x_t\mu_{et})$ and the saving in deposit costs from another unit of net worth ν_t . Because both these factors raise the bank's franchise value of bank, they reduce the incentive for the bank to divert funds, making its creditors willing to lend more. Conversely, ϕ_t is decreasing in $\Theta(x_t)$, the fraction of funds banks are able to divert.

The leverage ratio also varies inversely with risk perceptions. In particular, the bank values its expected returns using an "augmented stochastic discount factor," which is the product of the household stochastic discount factor $\Lambda_{t,t+1}$ and the stochastic shadow marginal value of net worth Ω_{t+1} . Note that the latter varies countercyclically: because the bank's incentive constraint is tighter in recessions than in booms, an additional unit of net worth is more valuable in bad times than in good times. Accordingly, since both Ω_{t+1} and $\Lambda_{t,t+1}$ are counter-cyclical, the augmented stochastic discount factor is countercyclical. It follows that since the realized excess return to assets $R_{kt+1} - R_{t+1}$ varies procyclically, increased volatility in the bank's stochastic discount factor reduces the excess value of the bank's assets and thus its continuation value: The leverage ratio drops as a result. In this way, uncertainty affects the bank's ability to obtain funds.

We next turn to the choice of the liability structure. As we show in our companion working paper, the fraction of assets financed by outside equity, x_t , is increasing in the ratio of the excess value from substituting outside equity for deposit finance (μ_{et}) to the excess value on assets over the deposit (μ_{st}) as follows:

$$\begin{aligned} x_t &= -\frac{\mu_{st}}{\mu_{et}} + \left[\left(\frac{\mu_{st}}{\mu_{et}} \right)^2 + \frac{2}{\kappa} \left(1 - \varepsilon \frac{\mu_{st}}{\mu_{et}} \right) \right]^{\frac{1}{2}} \\ &= x \left(\frac{\mu_{et}}{\mu_{st}} \right), \text{ where } x' > 0. \end{aligned} \quad (28)$$

The excess value to the bank from outside equity issues arises because the financing constraint effectively makes it more risk averse than households. From (26), μ_{et} is the expected value of the product of the augmented stochastic discount factor and the difference in the rate of returns on deposits and outside equity $R_{t+1} - R_{et+1}$. On the other hand, the household's portfolio decision yields the following arbitrage relation between the deposit rate and the return on outside equity:

$$E_t[\Lambda_{t,t+1}(R_{t+1} - R_{et+1})] = 0. \quad (29)$$

Observe that the household discounts the returns by the stochastic factor $\Lambda_{t,t+1}$ while the banker uses a discount factor $\Lambda_{t,t+1}\Omega_{t+1}$. Since the latter is more volatile and countercyclical than the former, the bank obtains hedging value by switching from deposits to outside equity: Accordingly, the excess value of outside equity issue $\mu_{et} = E_t[\Lambda_{t,t+1}\Omega_{t+1}(R_{t+1} - R_{et+1})]$ is positive.

Absent any cost of issuing state-contingent liabilities, the bank would move to one hundred percent outside equity finance. However, increasing the fraction of outside equity enhances the incentive problem by making it easier for bankers to divert funds, as equation (17) suggests. Thus the bank faces a trade-off in issuing outside equity. In general, there will be an interior solution for outside equity finance.

While outside equity improves banks' ability to hedge fluctuations in net worth, what matters for the overall outside credit they can obtain is their inside equity, or net worth, along with the maximum feasible leverage ratio ϕ_t . Since ϕ_t does not depend on bank-specific factors, we can aggregate equation (22) to obtain a relation between the aggregate demand for securities by banks S_{pt} and aggregate net worth in the banking sector N_t .

$$Q_t S_{pt} = \phi_t N_t. \quad (30)$$

The evolution of N_t accordingly plays an important role in the dynamics of the model economy. We turn to this issue next.

2.5 Evolution of Aggregate Bank Net Worth

Total net worth in the banking sector banks, N_t , equal the sum of the net worth of existing bankers N_{ot} (*o* for old) and of entering bankers N_{yt} (*y* for young):

$$N_t = N_{ot} + N_{yt}. \quad (31)$$

Net worth of existing bankers equals earnings on assets held in the previous period net the cost of outside equity finance and deposit finance, multiplied by the fraction that survive until the current period, σ :

$$N_{ot} = \sigma \{ [Z_t + (1 - \delta)Q_t] \psi_t S_{pt-1} - [Z_t + (1 - \delta)q_t] \psi_t \bar{e}_{t-1} - R_t D_{t-1} \}. \quad (32)$$

Because we assumed that the family transfers to each new banker a constant fraction, say $\xi/(1 - \sigma)$, of the total assets of exiting bankers, where ξ is a small number, we have aggregate net worth of new bankers as:

$$N_{yt} = \xi [Z_t + (1 - \delta)Q_t] \psi_t S_{pt-1}. \quad (33)$$

Total net worth of banks is now:

$$N_t = (\sigma + \xi) [Z_t + (1 - \delta)Q_t] \psi_t S_{pt-1} - \sigma [Z_t + (1 - \delta)q_t] \psi_t \bar{e}_{t-1} - \sigma R_t D_{t-1}. \quad (34)$$

Observe that a deterioration of capital quality (a decline in ψ_t) directly reduces the rate of return on assets and net worth. Further, the higher the leverage of the bank is, the larger will be the percentage impact of return fluctuations on net worth. The use of outside equity, however, reduces the impact of return fluctuations on net worth.⁸

2.6 Credit Policy

Earlier we characterized how the total value of privately intermediated assets, $Q_t S_{pt}$, is determined. We now suppose that the central bank is willing to facilitate lending. This kind of credit policy corresponds to the central bank's large scale purchase of high grade private securities, which was at the center of its attempt to stabilize credit markets during the peak of the financial crisis.⁹ Let $Q_t S_{gt}$ be the value of assets intermediated via the central bank and let $Q_t S_t$ be the total value of intermediated assets:

$$Q_t S_t = Q_t S_{pt} + Q_t S_{gt}. \quad (35)$$

To conduct credit policy, the central bank issues short-term government debt to households that pays the riskless rate R_{t+1} and then lends the funds to non-financial firms at the market lending rate R_{kt+1} . We suppose that government intermediation involves efficiency costs: in particular, the central bank credit consumes resources of $\Gamma_t(Q_t S_{gt})$, where the function Γ_t is increasing in the quantity of government assets intermediated. This dead-weight loss could reflect the administrative costs of raising funds via government deb or perhaps the costs to the central bank of identifying preferred private sector investments. On the other hand, the government always honors its debt: Thus, unlike the case with private financial institutions there is

⁸The net profit transfer from banks and capital goods producers to the representative household is

$$\begin{aligned} \Pi_t = & Q_t I_t - I_t \left[1 + f \left(\frac{I_t}{I_{t-1}} \right) \right] - \xi [Z_t + (1 - \delta) Q_t] \psi_t S_{t-1} \\ & + (1 - \sigma) \{ [Z_t + (1 - \delta) Q_t] \psi_t S_{t-1} - [Z_t + (1 - \delta) q_t] \psi_t \bar{e}_{t-1} - R_t D_{t-1} \}. \end{aligned}$$

⁹Accordingly, this analysis concentrates on the central bank's direct lending programs which we think were the most important dimension of its balance sheet activities. See Gertler and Kiyotaki (2010) for a formal characterization of the different types of credit market interventions that the Federal Reserve and Treasury pursued in the current crisis.

no agency conflict that inhibits the central bank from obtaining funds from households.¹⁰

Accordingly, suppose the central bank is willing to fund the fraction ζ_t of intermediated assets:

$$S_{gt} = \zeta_t S_t. \quad (36)$$

As we will show, by increasing ζ_t after the onset of a financial crisis, the central bank can reduce the excess return ($R_{kt+1} - R_{t+1}$). In this way credit policy can reduce the cost of capital, thus stimulating investment. Later we describe how the central bank may choose the path of ζ_t to combat a financial crisis.

The government together with central bank must satisfy the budget constraint. Government expenditures consist of government consumption G , which we hold fixed, and monitoring costs from central bank intermediation:

$$G_t = G + \Gamma_t (Q_t S_{gt}). \quad (37)$$

The government budget constraint, in turn is

$$G_t + Q_t S_{gt} + R_t D_{gt-1} = T_t + [Z_t + (1 - \delta)Q_t] \psi_t S_{gt-1} + D_{gt}, \quad (38)$$

where D_{gt} is government debt and T_t is lump-sum taxes on the household.

2.7 Equilibrium

To close the model, we require market clearing in the market for securities, outside equity, deposits and labor. Market clearing for securities requires that the total supply (given by equation (2)) net government security purchases must equal private demand S_{pt} (given by equations (30) and (23)). This implies,

$$Q_t(S_t - S_{gt}) = \frac{\nu_t}{\theta(1 + \varepsilon x_t + x_t^2) - (\mu_{st} + x_t \mu_{et})} N_t. \quad (39)$$

¹⁰An equivalent formulation of credit policy has the central bank sell government debt to financial intermediaries. Intermediaries in turn fund their government debt holdings by issuing deposits to households that are perfect substitutes. Assuming the agency problem applies only to the private assets it holds, only the funding of private assets by financial institutions is balance sheet constrained. As in the baseline scenario, the central bank is able to elastically issue government debt to fund private assets. It is straightforward to show that the equilibrium conditions in this scenario are identical to those in the baseline case. One virtue of this scenario is that the intermediary holdings of government debt are interpretable as interest bearing reserves, which is how the central bank has funded its assets in practice.

Similarly in the market equilibrium for outside equity, the demand by households \bar{e}_t equals the supply by banks e_t

$$q_t \bar{e}_t = x_t \cdot Q_t S_{pt}, \quad (40)$$

where the fraction of total assets funded by outside equity x_t is given by equation (28). Finally, given the flow of funds constraint, equilibrium deposits must equal aggregate bank assets net outside equity and net worth:

$$D_t = D_{ht} - D_{gt} = (1 - x_t) Q_t S_{pt} - N_t. \quad (41)$$

The final equilibrium condition is that labor demand equals labor supply, which requires

$$(1 - \alpha) \frac{Y_t}{L_t} \cdot E_t \left[\frac{u_{Ct}}{(C_t - hC_{t-1} - \frac{\chi}{1+\varphi} L_t^{1+\varphi})^{-\gamma}} \right] = \chi L_t^\varphi. \quad (42)$$

To close the model we need to describe the exogenous processes for the productivity shock A_t and the capital quality shock ψ_t . Since we wish to concentrate on the impact of the capital quality shock, we simply fix A_t to a constant value A .

The capital quality shock, in contrast, follows an i.i.d. process that allows for randomly arriving infrequent "disasters". In particular: ψ_t is the product of a process that evolves in normal times, $\tilde{\psi}_t$, and one that arises in "disasters" $\tilde{\psi}_t^D$

$$\psi_t = \tilde{\psi}_t \tilde{\psi}_t^D, \quad (43)$$

with

$$\log \tilde{\psi}_t = \varsigma \epsilon_{\psi t}; \quad \log \tilde{\psi}_t^D = \eta_t,$$

where $\epsilon_{\psi t}$ is distributed $N(0, 1)$, and η_t is distributed binomial:

$$\eta_t = \begin{cases} -(1 - \pi)\Delta & \text{with prob. } \pi \\ \pi\Delta & \text{with prob } 1 - \pi \end{cases},$$

where Δ is a positive number, implying the disaster innovation $-(1 - \pi)\Delta$ is negative. We normalize the process so that the mean of η_t is zero and the variance is equal to $(1 - \pi)\Delta^2$.

In practice, assuming the disaster shock Δ is not too large, we will be able to capture risk with a second order approximation of the model. In this

instance, what will matter for capturing risk is the overall standard deviation of the combined ψ_t shock. We introduce the disaster formulation to enrich the economic interpretation of the model.

The equations (1, 2, 3, 4, 6, 8, 9, 11, 12, 24, 25, 26, 28, 34, 39, 40, 41, 42) determine the seventeen endogenous variables ($Y_t, K_t, S_t, C_t, I_t, L_t, Z_t, R_{t+1}, q_t, \nu_t, \mu_{st}, \mu_{et}, x_t, Q_t, N_t, \bar{e}_t, D_t$) as a function of the state variables ($S_{t-1}, C_{t-1}, I_{t-1}, \bar{e}_{t-1}, R_t D_{t-1}, R_t D_{gt-1}, S_{gt-1}, \psi_t$), together with the exogenous stochastic process of ψ_t and the government policy vector of (G_t, T_t, S_{gt}, D_{gt}). One of these eighteen equations is not independent by Walras Law.

Absent credit market frictions, the model reduces to a real business cycle framework modified with habit formation and flow investment adjustment costs. With the credit market frictions, however, balance sheet constraints on banks may limit real investment spending, affecting aggregate real activity. A crisis is possible where weakening of bank balance sheets significantly disrupts credit flows, depressing real activity.

3 Crisis Simulations and Policy Experiments

In this section we present several numerical experiments designed to illustrate how the model may capture some key features of a financial crisis and how credit policy and also macro-prudential policy might work to mitigate the crisis. We consider both a low risk and a high risk economy. For each case we examine the implications of both credit and macro-prudential policies.

3.1 Calibration

Not including the standard deviations of the exogenous disturbances, there are thirteen parameters for which we need to assign values. Eight are standard preference and technology parameters. These include the discount factor β , the coefficient of relative risk aversion γ , the habit parameter h , the utility weight on labor χ , the inverse of the Frisch elasticity of labor supply φ , the capital share parameter α , the deterministic depreciation rate δ and the elasticity of the price of capital with respect to investment η . For these parameters we use reasonably conventional values, as reported in Table 1.

The five additional parameters are specific to our model: σ the quarterly survival probability of bankers; ξ the transfer parameter for new bankers, and the three parameters that help determine the fraction of gross assets

that banker can divert: θ , ε and κ . We set $\sigma = 0.968$, implying that bankers survive for eight years on average.

Finally, we choose ξ , θ , ε and κ to hit four targets. The first three involve characteristics of the low risk economy, which is meant to capture the period of macroeconomic tranquility just prior to the recent crisis (i.e., the "Great Moderation"). In particular, we target: an average credit spread of one hundred basis points per year, an aggregate leverage ratio of four (assets to the sum of inside and outside equity), and a ratio of outside to inside equity of two thirds. The last target is having the aggregate leverage ratio fall by a third as the economy moves from low risk to high risk. The choice of an aggregate leverage ratio of four reflects a crude first pass attempt to average across sectors with vastly different financial structures. For example, before the beginning of the crisis, most housing finance was intermediated by financial institutions with leverage ratios between twenty (commercial banks) and thirty (investment banks.) The total housing stock, however, was only about a third of the overall capital stock. Leverage ratios are clearly smaller in other sectors of the economy. We base the steady state target for the spread on the pre-2007 spreads as a rough average of the following spreads over the Great Moderation period: mortgage rates versus government bond rates, BAA corporate bond rates versus government bonds, and commercial paper rates versus T-Bill rates. The target ratio of outside to inside equity approximates the ratio of common equity to the sum of preferred equity and subordinate debt in the banking sector just prior to the crisis. The drop in the aggregate leverage ratio of a third as the economy moves from low risk to high risk, is a rough estimate of what would occur if the financial system undid the buildup of leverage over the past decade.

The standard deviations of the shock processes are picked so that standard deviation of annual output growth in the low risk economy corresponds roughly to that in the Great Moderation period, while that for the high risk economy corresponds to the period of volatility in the two decades prior (from the early 1960s through the early 1980s).

A key feature of the model is that the bank balance sheet structure depends on risk perceptions. It is thus important to take account of risk in the computation of the model. To do so, we borrow insights from the literature on international risk sharing (e.g., Devereux and Sutherland (2009) and Tille and Van Wincoop (2007)) and on asset pricing and business cycles (e.g. Campbell (1994) and Lettau (2003)) by working with second order approximations of the equations where risk perceptions matter. We then construct

a "risk-adjusted" steady state, where given agents perceptions of second moments, variables remain unchanged if the realization of the (mean-zero) exogenous disturbance is zero. The risk-adjusted steady state differs from the non-stochastic state only by terms that are second order. These second order terms, which depend on variances and covariances of the endogenous variables pin down bank balance sheet. To analyze model dynamics, we then look at a first order log-linear approximation around the risk-adjusted steady state.

To calculate the relevant second moments we use an iterative procedure. We first log-linearize the model around the non-stochastic steady state. We then use the second moments calculated from this exercise to compute the risk-adjusted steady state. We repeat the exercise, this time calculating the moments from the risk-adjusted steady state. We keep iterating until the moments generated by the first order dynamics around the risk-adjusted steady state are consistent with the moments used to construct it.

One point to note about our model is that banks' outside equity will depend not only on second moments (the hedging value of outside equity) but also first moments, due to the cost stemming from the tightening of the incentive constraint (which is increasing in the excess return to capital). Thus, though we treat the second moment effect as constant, the first order effect will lead to cyclical variation in the use of outside equity.

3.1.1 No Policy Response

We begin by considering the behavior of the model economy without any kind of policy response for the low risk economy and the high risk economy.

The first two columns of Table 2 shows the relevant statistics for the risk adjusted steady states for the low and high risk economies. Note first that outside equity as a share of total bank liabilities, x , is nearly fifty percent greater in the high risk economy than in the low risk economy - roughly fifteen percent instead of ten percent. This occurs, of course, because equity has greater hedging value in the high risk economy. The primary leverage ratio ϕ (assets to inside equity) declines as risk increases. This occurs for two reasons: First, the more extensive use of outside equity tightens the banks' borrowing constraint by intensifying the agency problem. Second, the excess value of bank assets, μ_{st} , falls as the covariance of the excess return on banks assets, $R_{kt+1} - R_{t+1}$, with the augmented stochastic discount factor, $\Lambda_{t,t+1}\Omega_{t+1}$, becomes more negative. This also tightens the bank's borrowing

constraint by reducing its franchise value (and thus increasing its incentive to divert assets.).

The net effect of the increase in outside equity x and the decline in bank asset-net worth ratio ϕ is that the leverage ratio, measured by the ratio of assets to total equity (inside plus outside) declines as risk increases. This inverse relation between risk and leverage is consistent with conventional wisdom and has been stressed by a number of others (e.g., Fostel and Geanakoplos, 2009). Further, because it affects the ability of banks to intermediate funds, this behavior of the leverage ratio has consequences for the real equilibrium. In the risk-adjusted steady state, the capital stock is roughly two percent lower in the high risk economy. As a consequence, output and consumption are roughly one percent and a half lower.

The sensitivity of the leverage ratio to risk perceptions also has consequences for the dynamics outside the steady state and, in particular, for the response to a crisis. We illustrate this point next.

In particular, suppose the economy is hit by a decline in capital quality by five percent of the existing stock. (The shock is i.i.d., as we noted earlier). We fix the size of the shock simply to produce a downturn of roughly similar magnitude to the one observed over the past year. Within the model economy, the initial exogenous decline is then magnified in two ways. First, because banks are leveraged, the effect of the decline in assets values on bank net worth is enhanced by a factor equal to the leverage ratio. Second, the drop in net worth tightens the banks' borrowing constraint inducing effectively a fire sale of assets that further depresses asset values. The crisis then feeds into real activity as the decline in asset values leads to a fall in investment.

Figure 1 displays the responses of the key variables for both the low and high risk economies. For comparison we also plot the response of a frictionless economy (denoted RBC for "real business cycle"). Note that the contraction in real activity is greatest in the low risk economy. The reason is straightforward: The perception of low risk induces banks to make more extensive use of short term debt to finance assets and rely less on outside equity. The high leverage ratio in the low risk economy makes banks' inside equity highly susceptible to the declines in asset values initiated by the disturbance to capital quality. As a consequence, in the wake of the shock, the spread jumps roughly six hundred basis points. This in turn increases the cost of capital, which leads to a sharp contraction in investment, output and employment. Note the contraction in output in the low risk economy

is at the peak of the trough nearly fifty percent greater than in the case of the model without financial frictions. The difference of course is due to the sharp widening of the spread that arises in the model with financial frictions. The spread further is slow to return to its norm as it takes time for banks to rebuild their stocks of inside equity. In the frictionless model, by contrast, the excess return is fixed at zero.

In the high risk economy the output contraction is more modest than in the low risk economy. The anticipation of high risk induces banks to substitute outside equity for short term debt. Outside equity then acts as a buffer in two ways. First, it moderates the drop in inside equity induced by the decline in assets values. Second, as the crisis unfolds after the initiating disturbance, banks are able to relax their borrowing constraint a bit by shortening their maturity structure by substituting short term debt for outside equity. (Recall that short term debt permits creditors greater discipline over bankers). While outside equity moderates the downturn - there is a modest increase in the spread of one hundred basis points, which is far less than what occurs in the low risk economy -, it is not a perfect buffer as it is sufficient to induce a noticeably larger contraction than in the frictionless economy.

3.1.2 Credit Policy Response

Here we analyze the impact of direct central bank lending as a means to mitigate the impact of the crisis. Symptomatic of the financial distress in the simulated crisis is a large increase in the spread between the expected return on capital and the riskless interest rate. In practice, further, it was the appearance of abnormally large credit spreads in various markets that induced the central bank to intervene with credit policy. Accordingly we suppose that the Fed adjusts the fraction of private credit it intermediates, ζ_t , to the difference between spread ($E_t R_{kt+1} - R_{t+1}$), and its steady state value ($ER_k - R$), as:

$$\zeta_t = v_g[(E_t R_{kt+1} - R_{t+1}) - (ER_k - R)]. \quad (44)$$

To parametrize the rule, we pick the smallest value of the feedback coefficient v_g such that under a simulated crisis credit policy produces a moderation in spreads what is observed (within a rough ballpark). Under this criteria a value of v_g equal to 100 works reasonably well.

Because the introduction of systematic credit policy will affect bank's balance sheet structure, we first examine the impact of the policy rule on the steady states for the low and high risk economies. The second two columns of Table 2 reports how the anticipation of government intervention affects the stochastic steady state of the low risk and high risk economies. The anticipation of government intervention leads to a reduction in risk perceptions. Banks thus rely more heavily on short term debt, relative to the case with no policy. The effect is most dramatic in the high risk economy, as the anticipation of policy intervention leads to a reduction in outside equity issuances of twelve percent, as compared to only five percent in the low risk economy. Note that in each case there is a positive first order effect on the quantity variables. This is due to the combined effect of reduced outside equity issuance and reduced risk perceived by the private sector, which work to relax bank borrowing constraints.

Figure 2 reports the response of the economy to a crisis shock for the low risk economy. In the low risk economy, credit policy has a significant stabilizing effect on the economy. The increase in central bank credit significantly reduces the rise in the spread, which in turn reduces the overall drop in investment. At its peak, central bank credit increases to over fifteen percent of the capital stock.

Figure 3 reports the impact of credit policy for the high risk economy. The gain from credit policy is small in the high risk economy. It is limited in this case because, absent credit policy, banks hold a greater buffer of outside equity to absorb the disturbance. The anticipation of policy induces moral hazard, as banks rely less on their capital structure and more on public credit policy to absorb risks. Interestingly, even though gain in output stabilization is relatively modest, the size of the credit market intervention ζ_t is roughly the same as in the low risk economy, as comparison of Figures 2 and 3 makes clear. Again, the reason is that anticipated policy intervention crowds out outside equity issuance in favor of short term debt. This necessitates a more intense credit policy intervention to stabilize credit spreads.

Another way to see the issue is to suppose the private sector does not anticipate credit policy. Then consider how intense an unanticipated credit policy would need to be, as measured by the feedback parameter, v_g , to provide the same degree of stabilization as an anticipated policy of the same intensity as our baseline policy of $v_g = 100$. As Figure 4 illustrates, if the policy is unanticipated, a significantly more modest intervention will provide the same degree of stabilization. In this case an unanticipated intervention

with $v_g = 50$ would provide identical stabilization to the anticipated baseline policy. As the figure shows, in this instance, the fraction of credit the central bank needs to intermediate is only about half of its value under perfectly anticipated policy.

The problem is that absent some form of commitment, it is not credible for the central bank to claim that it will not intervene during a crisis. Further, the ex post benefits to intervention are clearly greater the more highly leveraged is the banking sector, which increases the incentives of the central bank to intervene. Thus, it is rational for banks to anticipate credit policy intervention in a crisis, leading banks to raise their risk exposure.

We emphasize that how much moral hazard may reduce the net effectiveness of credit policy is a quantitative issue. In the low risk economy, for example, outside equity issuance is low because of risk perceptions and not because of anticipated policy. Because the likelihood of crises is low, anticipated interventions during crises do not have much impact on private capital structure decisions.

3.2 Macro-prudential Policy

Within our framework there are two related motives for a macro-prudential policy that encourages banks to use outside equity and discourages the use of short term debt. First, due to the role of asset prices in affecting borrowing constraints, there exists a pecuniary externality which banks do not properly internalize when deciding their balance sheet structure. In particular, individual banks do not take account of the fact that if they were to issue outside equity in concert, they would make the banking sector better hedged against risk, thus dampening fluctuations in asset prices and economic activity. Given that the financial market frictions induce countercyclical movement in the wedge between the rates of returns on investment and saving, the failure of banks to internalize external benefits of outside equity issuance leads to a reduction in welfare. A number of papers have emphasized how this kind of externality might induce the need for some form of ex ante regulation or, equivalently Pigouvian taxation and/or subsidies. Examples include Lorenzoni (2009), Korinek (2009), Bianchi (2009) and Stein (2010).

Second, as we noted in the previous section, the anticipation of credit market interventions during a crisis may induce banks to hedge by less than they otherwise would, tilting their liability structure toward short term debt. How this factor might introduce a need for ex ante macro-prudential policy

has also been emphasized in the literature. Recent examples that focus on this kind of time consistency problem include Diamond and Rajan (2009), Fahri and Tirole (2010), and Chari and Kehoe (2009).

We now proceed to use our model to illustrate the impact of macro-prudential policy that works to offset banks' incentive to adjust their liability structure in favor of short term debt. In particular we suppose that the government offers banks a subsidy of τ_t^s per unit of outside equity issued and finances the subsidy with a tax τ_t on total assets.¹¹ The flow of funds constraint for a bank is now given by

$$(1 + \tau_t) Q_t s_t = n_t + (1 + \tau_t^s) q_t e_t + d_t \quad (45)$$

where the bank takes τ_t^s and τ_t as given. In equilibrium the tax is set to make the subsidy revenue neutral, so that the net impact on bank revenues is zero. However, the subsidy will clearly raise the relative attractiveness to the bank of issuing outside equity.

In addition we suppose that the subsidy is set to make the net gain to outside equity from reducing deposits constant in terms of consumption goods. Hence we set τ_t^s equal to a constant τ^s divided by the shadow cost of deposits ν_t , as follows

$$\tau_t^s = \frac{\tau^s}{\nu_t} \quad (46)$$

As we show in Appendix A of our companion working paper, the marginal benefit to the bank from issuing equity is now the sum of the excess value from issuing outside equity and the subsidy: $\mu_{et} + \tau^s$.

The subsidy/tax scheme we propose has the flavor of a countercyclical capital requirement (for outside equity issue). The subsidy increases the steady state level of x_t : In this respect it is a capital requirement. At the same time, x_t will vary countercyclically as it does in the decentralized equilibrium.

The benefit from the macro-prudential policy is the reduction in aggregate volatility. There is however a cost: The required increase in outside equity is costly for the bank due to the effect on the incentive constraint.¹² This cost, further, is increasing at the margin given that the diversion rate $\Theta(x_t)$

¹¹We restrict attention to policies that affect the incentive for banks to raise outside equity since within our framework inside equity can be raised only through retained earnings. In later work we plan to allow for a richer specification of inside equity accumulation.

¹²Nikolov (2009) also emphasizes this trade-off.

is increasing and convex in x_t . This suggests that a subsidy that pushes the steady state level of x above its decentralized level but not all the way to full equity finance is desirable. From simulating the model we find that a subsidy to outside equity finance of forty basis points per quarter maximizes steady state welfare for the high risk economy and is very close to optimal in the low risk economy. This policy implies that the steady state value of x increases roughly sixty six percent in the high risk (from 13 percent to 22 percent) and doubles in the low risk economy (from 9.6 percent to 19 percent).

Figure 5 then illustrates the effect of a crisis in the high risk economy when the macro-prudential policy described above is in place. The key point to note is that in this instance a more modest intervention by credit policy can achieve a similar degree of stabilization of the economy. At the peak the fraction of government lending in the crisis is only a third of its level in the case with macro-prudential policy. Intuitively, the extra cushion of outside equity required by the macro-prudential policy reduces the need for central bank lending during the crisis. In addition, the two policies combined appear to offer slightly greater stabilization: The contraction of output is persistently smaller by roughly a quarter percent per year.

In the low risk economy anticipated credit policy does not have much effect on bank risk taking ex ante. As we noted earlier, short term debt is high because perceptions of risk are low. Nonetheless, macro-prudential policy is still potentially useful given the pecuniary externality that leads banks to not properly internalize the aggregate effects of their individual leverage decisions, - especially when credit policy is not available as a stabilizing tool, either because it is too costly or not a politically viable option. Figure 6 considers a crisis in the low risk economy in the absence of credit policy. As the figure illustrates, the macro-prudential policy by itself leads to a considerable stabilization of the economy during the crisis. Again, the high initial buffer of outside equity provides the stabilizing mechanism.

Finally, we examine the net benefits from macro-prudential policy more formally considering the welfare effects under different scenarios. The welfare criterion we consider is the unconditional steady state value of lifetime utility of the representative agent, given by (5). We consider a second order approximation of the utility function around the risk-adjusted steady state and then evaluate welfare under different policy scenarios. We restrict attention to the high risk economy, since it is in this instance that the potential benefits from macro-prudential policy are highest.

Table 4 presents measures of welfare gains in consumption equivalents un-

der various different policy scenarios. In particular, we compute the percent increase in consumption per period needed for the household in the regime with no policy to be indifferent with being in the regime with the policy under consideration. We suppose that efficiency costs are the following quadratic function of the quantity of assets the central bank intermediated:

$$\Gamma(Q_t S_{gt}) = \tau_1 Q_t S_{gt} + \tau_2 (Q_t S_{gt})^2 \quad (47)$$

We use a quadratic formulation to capture the idea that costs are larger when the government has a long position in assets ($S_{gt} > 0$) than when it is short ($S_{gt} < 0$) an equal amount in absolute value. Because we have little direct information about the efficiency costs of credit policy, we consider a variety of different values. In each case we pick τ_1 and τ_2 so that (i) after a disaster shock, efficiency costs per unit of central bank assets intermediated hits a given target measured in annual basis points per year and (ii) efficiency costs average zero in the wake of a symmetric positive shock to the economy. The percent costs are measured in annual basis points. Since in the wake of the crisis shock government holding of private assets persists for many years, the efficiency costs cumulate over time. A rough estimate is that the present value efficiency costs are about ten times the amount in the first year.

The table considers values ranging from 10 per to 50. To be clear, these costs are meant to reflect the total costs of the variety of different credit market interventions used in practice. For some programs, such as the large scale asset purchases of commercial paper and mortgage-backed securities, the efficiency costs were probably quite low and likely less than 10 basis points per year per unit of credit intermediated. The equity injections under the Troubled Asset Relief Program likely involved higher costs, particularly when one takes account of the redistributive effects (which is beyond the scope of this model.)¹³

The first row of Table 3 considers the welfare gains from the credit policy studied in the previous section under different assumptions about efficiency costs. Under no costs, there is a welfare gain equal to 0.26 percent of steady state consumption per period. This gain declines monotonically as efficiency costs increase, falling to near zero as these costs reach 50 basis points per unit of credit per year.

The next row considers macro-prudential policy in the absence of credit policy. There is a net gain of 0.26 percent of steady state consumption,

¹³See, for example, Veronesi and Zinagales (2010) for an analysis of the costs of the TARP.

roughly equal to the gain from credit policy absent efficiency costs. As efficiency costs increase, macro-prudential policy dominates credit policy.

In the last row we examine macro-prudential policy in conjunction with credit policy. With the combined policy, the gains increase to 0.32 percent of steady state consumption in the case absent efficiency costs and decline just to 0.29 percent as efficiency costs reach 50 basis points. With macro-prudential policy in place, credit policy is less aggressive during a crisis, making the associated efficiency costs less a factor than otherwise. Finally, we note that the gains from policy come from gains in risk reduction. We have employed relatively modest degrees of risk aversion, By raising risk aversion to levels that could account for the equity premium, for example, we would expect the measured gains from policy to increase significantly.

4 Conclusion

We have developed a macroeconomic framework with an intermediation sector where the severity of a financial crises depends on the riskiness of banks' balance sheet structure, which is endogenous. It is possible to use the model to assess quantitatively how perceptions of fundamental risk and government credit policy affect the vulnerability of the financial system. It is also possible to study the quantitative effects of macro-prudential policies designed to offset the incentives for risk-taking.

As with recent theoretical literature, we find that the incentive effects for risk taking may reduce the net benefits of credit policies that stabilize financial markets. However, by how much the benefits are reduced is ultimately a quantitative issue. Within our framework it is possible to produce examples where the moral hazard costs are not consequential to the overall benefits from credit policy, (especially when the disaster shock is rare). Of course, one can also do the reverse. In addition, an appropriately designed macro-prudential policy can also mitigate moral hazard costs. Clearly, more work on pinning down the relevant quantitative considerations is a priority for future research.

References

- [1] Bernanke, B., and Gertler, M., 1989. Agency Costs, Net Worth and Business Fluctuations. *American Economic Review* 79, 14-31.
- [2] Bernanke, B., Gertler, M., and Gilchrist, S., 1999. The Financial Accelerator in a Quantitative Business Cycle Framework. In Taylor, J., and Woodford, M. (Eds.), *Handbook of Macroeconomics*. Elsevier, Amsterdam, Netherlands.
- [3] Bianchi, J., 2009. Overborrowing and Systematic Externalities in the Business Cycle. Mimeo, University of Maryland.
- [4] Brunnermeier, M. K., and Sannikov, Y., 2009. A Macroeconomic Model with a Financial Sector. Mimeo, Princeton University.
- [5] Calomiris, C., and Kahn, C., 1991. The Role of Demandable Debt in Structuring Banking Arrangements. *American Economic Review* 81, 497-513.
- [6] Campbell, J., 1994. Inspecting the Mechanism. *Journal of Monetary Economics* 33, 463-506.
- [7] Chari. V.V., and Kehoe, P., 2010. Bailouts, Time Consistency and Optimal Regulation. Mimeo, University of Minnesota.
- [8] Christiano, L., Motto, R., and Rostagno, M., 2009. Financial Factors in Business Fluctuations. Mimeo, Northwestern University.
- [9] Devereux, M., and Sutherland, A., 2009. Country Portfolios in Open Economy Macro Models. Mimeo, University of British Columbia.
- [10] Diamond, D., and Rajan, R., 2009. Illiquidity and Interest Rate Policy. Mimeo, University of Chicago.
- [11] Fahri, E., and Tirole, J., 2009. Collective Moral Hazard, Systematic Risk and Bailouts. Mimeo, Harvard University and University of Toulouse.
- [12] Fostel A., and Geanakoplos, J., 2009. Leverage Cycles and the Anxious Economy. *American Economic Review* 94, 1211-1244.

- [13] Gertler, M., and Karadi, P., 2011. A Model of Unconventional Monetary Policy, *Journal of Monetary Economics*, January.
- [14] Gertler, M., and Kiyotaki, N., 2010. Financial Intermediation and Credit Policy in Business Cycle Analysis. In Friedman, B., and Woodford, M. (Eds.), *Handbook of Monetary Economics*. Elsevier, Amsterdam, Netherlands.
- [15] Gilchrist, S., Yankov, V., and Zakrasjek, E., 2009. Credit Market Shocks and Economic Fluctuations: Evidence from Corporate Bond and Stock Markets. Mimeo, Boston University.
- [16] Gourio, F., 2009. Disaster Risk and Business Cycles. Mimeo, Boston University.
- [17] Greenwood, J., Hercowitz, Z., and Huffman, G.W., 1988. Investment, Capacity Utilization, and the Real Business Cycle. *American Economic Review* 78, 402-417.
- [18] Guvenen, F., 2009. A Parsimonious Macroeconomic Model for Asset Pricing. *Econometrica* 77, 1711-1750.
- [19] Jermann, U., and Quadrini, V., 2009. Macroeconomic Effects of Financial Shocks. Mimeo, University of Pennsylvania and University of Southern California.
- [20] Kiyotaki, N., and Moore, J., 1997. Credit Cycles. *Journal of Political Economy* 105, 211-248.
- [21] Korinek, A., 2009. Systematic Risk-Taking Amplification Effects, Externalities and Regulatory Responses. Mimeo, University of Maryland.
- [22] Lettau, M., 2003. Inspecting the Mechanism: Closed-Form Solutions for Asset Prices in Real Business Cycle Models. *Economic Journal* 113, 550-575.
- [23] Lorenzoni, G., 2008. Inefficient Credit Booms. *Review of Economic Studies* 75, 809-833.
- [24] Mendoza, E., 2008. Sudden Stops, Financial Crises and Leverage: A Fisherian Deflation of Tobin's Q. Mimeo, University of Maryland.

- [25] Merton, R., 1973. An Intertemporal Capital Asset Pricing Model. *Econometrica* 41, 867-887.
- [26] Miao, J., and Wang, P., 2010. Credit Risk and Business Cycles. Mimeo, Boston University.
- [27] Nikolov, Kalin, 2009. "Is Private Leverage Excessive?" mimeo, LSE.
- [28] Stein, J.C., 2010. Monetary Policy as Financial-Stability Regulation. Mimeo, Harvard University.
- [29] Tille, C., and Von Wincoop, E., 2010. International Capital Flows. *Journal of International Economics* 80, 157-175.
- [30] Veronesi, P., and Zingales, L., 2010. Paulson's Gift. *Journal of Financial Economics* 97, 339-368.

5 Appendix

5.1 Appendix A

Insert the conjectured solution (21) for $V_t(s_t, x_t, n_t)$ into the Bellman equation (20). Then maximize this objective with respect to the to the incentive constraint (18). Using the Lagrangian,

$$\mathcal{L} = [(\mu_{st} + x_t\mu_{et})Q_t s_t + \nu_t n_t](1 + \lambda_t) - \lambda_t \theta \left(1 + \varepsilon x_t + \frac{\kappa}{2} x_t^2\right),$$

where λ_t is the Lagrangian multiplier with respect to the incentive constraint, the first order necessary conditions for x_t, s_t and λ_t yield:

$$(1 + \lambda_t) \mu_{et} = \lambda_t \theta (\varepsilon + \kappa x_t), \quad (48)$$

$$(1 + \lambda_t) (\mu_{st} + x_t \mu_{et}) = \lambda_t \theta \left(1 + \varepsilon x_t + \frac{\kappa}{2} x_t^2\right), \quad (49)$$

$$(\mu_{st} + x_t \mu_{et}) Q_t s_t + \nu_t n_t = \Theta(x_t) Q_t s_t. \quad (50)$$

The left side of equation (48) is the marginal benefit to the bank from substituting outside equity finance for unit of short term debt. The right side is the marginal cost, equal to the increase in the fraction of assets the bank can divert times the shadow value of the incentive constraint λ_t . The first order condition for Equation (49) implies that the marginal benefit from increasing asset, $\mu_{st} + x_t \mu_{et}$ is equal to the marginal cost of tightening the incentive constraint by $\theta \left(1 + \varepsilon x_t + \frac{\kappa}{2} x_t^2\right)$. Finally, the first order condition for λ_t yields the incentive constraint. From (49), we learn that the incentive constraint binds (λ_t is positive) only if the adjusted excess value of bank assets $\mu_{st} + x_t \mu_{et}$ is positive.

Combing equations (48, 49) yields a relation for x_t the is increasing in the ratio of excess values μ_{et}/μ_{st} :

$$\begin{aligned} x_t &= -\left(\frac{\mu_{et}}{\mu_{st}}\right)^{-1} + \left[\left(\frac{\mu_{et}}{\mu_{st}}\right)^{-2} + \frac{2}{\kappa} \left(1 - \varepsilon \left(\frac{\mu_{et}}{\mu_{st}}\right)^{-1}\right) \right]^{\frac{1}{2}} \\ &\equiv x \left(\frac{\mu_{et}}{\mu_{st}}\right), \text{ where } x' > 0 \text{ given } \kappa > \frac{1}{2} \varepsilon^2. \end{aligned} \quad (51)$$

This is equation (28) in the text.

From (19, 20, 21), we have

$$\begin{aligned} & (\mu_{st} + x_t \mu_{et}) Q_t s_t + \nu_t n_t \\ = & E_t \Lambda_{t,t+1} \Omega_{t+1} \{ [R_{kt+1} - R_{t+1} + x_t (R_{t+1} - R_{et+1})] Q_t s_t + R_{t+1} n_t \}, \end{aligned}$$

where Ω_{t+1} is defined in (27). Comparing the terms of n_t , s_t and x_t , we verify that the conjectured form of value function satisfies Bellman equation for any (n_t, s_t, x_t) if (24, 25, 26) holds.

When we have macro-prudential policy as in (45), the value function (21) is modified to

$$V_t(s_t, x_t, n_t) = [(\mu_{st} - \tau_t \nu_t) + (\mu_{et} + \tau_t^s \nu_t) x_t] Q_t s_t + \nu_t n_t.$$

Then the first order necessary condition for x_t is changed to

$$(1 + \lambda_t) (\mu_{et} + \tau^s) = \lambda_t \theta (\varepsilon + \kappa x_t).$$

Because of the balanced budget constraint in equilibrium $\tau_t = \tau_t^s x_t$ in the aggregate, there is no change in the first order conditions for s_t and λ_t . Thus we have

$$\mu_{et} + \tau^e = \frac{\lambda_t}{1 + \lambda_t} \theta (\varepsilon + \kappa x_t).$$

5.2 Appendix B

One way to motivate capital quality shock is to assume that final output is produced from composite of a continuum of intermediate goods $Y_t(\omega)$, $\omega \in [0, 1]$, according to a constant returns to scale production function

$$Y_t = \left\{ \int_0^1 \vartheta_t(\omega) [Y_t(\omega)]^{\frac{\varsigma-1}{\varsigma}} d\omega \right\}^{\frac{\varsigma}{\varsigma-1}},$$

where $\varsigma > 1$. $\vartheta_t(\omega)$ is parameter of technology shock: $\vartheta_t(\omega) = 1$ if the variety ω is productive and $\vartheta_t(\omega) = 0$ if the variety is no longer productive at date t .

At the beginning of period all the varieties $\omega \in [0, 1]$ are equally likely to be productive. During the period t , however, a random fraction $1 - \psi_t \in (0, 1)$ of varieties becomes obsolete and is replaced by new varieties. The new variety is not available for production until period $t+1$ and is equally

likely to obsolete with old surviving varieties in period $t+1$. Each variety is produced by employing capital and labor according to a common Cobb-Douglas production technology:

$$Y_t(\omega) = A_t [S_{t-1}(\omega)]^\alpha [L_t(\omega)]^{1-\alpha}.$$

The goods producers must allocate capital stock $S_{t-1}(\omega)$ at the beginning of period before shock $\vartheta_t(\omega)$ realizes. Furthermore, the capital allocated to the production of obsolete varieties becomes worthless and will be no longer usable for future production. Concerning labor $L_t(\omega)$, the producers can allocate after the shock realizes. The resource constraint is

$$\int_0^1 S_{t-1}(\omega) d\omega = S_{t-1}, \text{ and } \int_0^1 L_t(\omega) d\omega = L_t.$$

The optimal allocation of producers implies

$$\begin{aligned} S_{t-1}(\omega) &= S_{t-1}, \text{ for all } \omega \in [0, 1], \\ L_t(\omega) &= \frac{L_t}{\psi_t}, \text{ for } \omega \text{ such that } \vartheta_t(\omega) = 1, \\ L_t(\omega) &= 0, \text{ for } \omega \text{ such that } \vartheta_t(\omega) = 0. \end{aligned}$$

Then the aggregate output of final goods becomes

$$\begin{aligned} Y_t &= \left\{ \psi_t \left[A_t (S_{t-1})^\alpha \left(\frac{L_t}{\psi_t} \right)^{1-\alpha} \right]^{\frac{\varsigma-1}{\varsigma}} \right\}^{\frac{\varsigma}{\varsigma-1}} \\ &= A_t \psi_t^{\frac{1}{\varsigma-1}} (\psi_t S_{t-1})^\alpha L_t^{1-\alpha}. \end{aligned}$$

The aggregate effective capital stock will then evolve according to equation (3) in text. Output at date t is additionally affected by obsolescence unless different varieties are perfect substitute. The text is special case in which $\varsigma \rightarrow \infty$.¹⁴

¹⁴In order to normalize the mean of ψ_t to be equal to 1 as in text, we need to adjust deterministic depreciation δ and I_t accordingly.

Table 1: Parameter Values

γ	2	Risk aversion
β	0.99	Discount factor
α	0.33	Capital share
δ	0.02	Depreciation rate
χ	0.25	Utility weight of labor
φ	1/3	Inverse Frisch elasticity of labor supply
$I f''/f'$	1	Inverse elasticity of investment to the price of capital
h	0.75	Habit parameter
σ	0.9685	Survival rate of bankers
ξ	0.0289	Transfer to entering bankers
		Asset diversion parameters:
θ	0.264	
ϵ	-1.21	
κ	13.41	

Table 2: Steady States

	No Policy		Credit Policy		Macroprudential Policy ($\tau^s = 0.0061$)	
	Low Risk	High Risk	Low Risk	High Risk	Low Risk	High Risk
Output	23.81	23.44	24.18	23.75	24.04	23.71
C	18.58	18.31	18.82	18.52	18.73	18.49
L	8.16	8.05	8.25	8.13	8.22	8.12
K	209.42	204.98	214.28	209.18	212.42	208.88
N	31.82	38.43	30.08	37.52	30.88	38.07
Risk Free Rate (%)	4.08	3.72	4.06	3.68	4.05	3.56
Spread (%)	0.99	1.43	0.89	1.36	0.94	1.47
x (%)	10.12	15.13	9.63	13.30	18.78	22.08
ν	1.62	1.36	1.75	1.40	1.81	1.52
μ_e (%)	0.05	0.14	0.03	0.12	0.03	0.08
μ_s (%)	0.29	0.15	0.33	0.18	0.37	0.26
ϕ	6.58	5.33	7.12	5.57	6.88	5.49
QK/(N+xQK)	3.95	2.95	4.22	3.20	3.00	2.48
N/xQK	1.50	1.24	1.46	1.35	0.77	0.83
SD shock (%)	0.69	2.07	0.69	2.07	0.69	2.07
SD output growth (%)	1.09	2.51	0.84	2.37	0.79	2.31

Table 3: Welfare Effects of Policy

<i>Efficiency cost of credit policy* (bps)</i>	Welfare gain from no policy in consumption equivalents (%)			
	0	10	25	50
Credit Policy	0.264	0.216	0.145	0.026
Macroprudential Policy	0.265	0.265	0.265	0.265
Macroprudential and Credit Policy	0.316	0.311	0.303	0.291

*The corresponding values of (τ_1, τ_2) for efficiency costs of credit policy equal to 10, 25 and 50 bps are, respectively: (0.000125, 0.0000062), (0.0003125, 0.0000156) and (0.0006250, 0.0000312).

Figure 1. Crisis Experiment: Low Risk vs. High Risk Economy

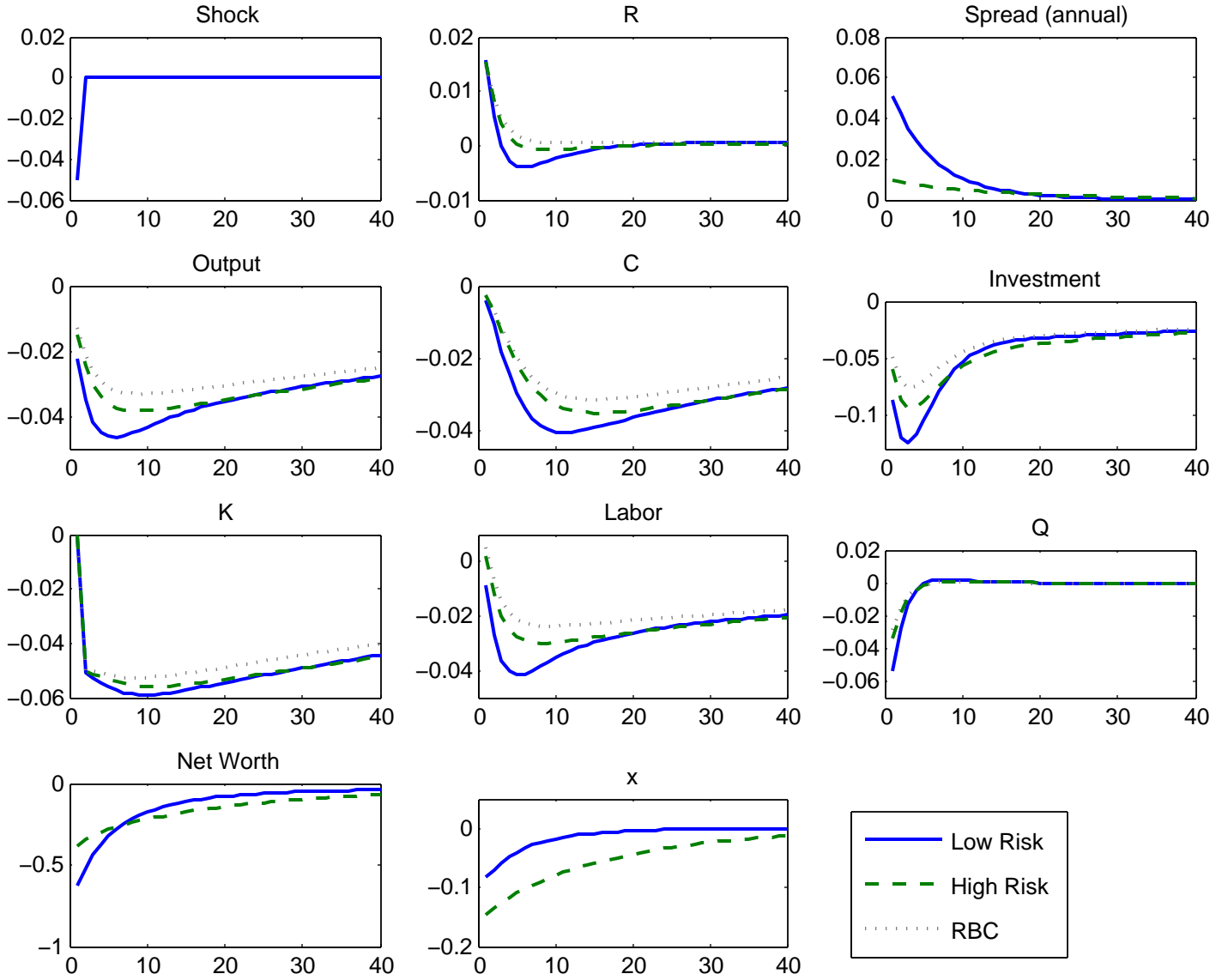


Figure 2. Credit Policy Response to Crisis: Low Risk Economy

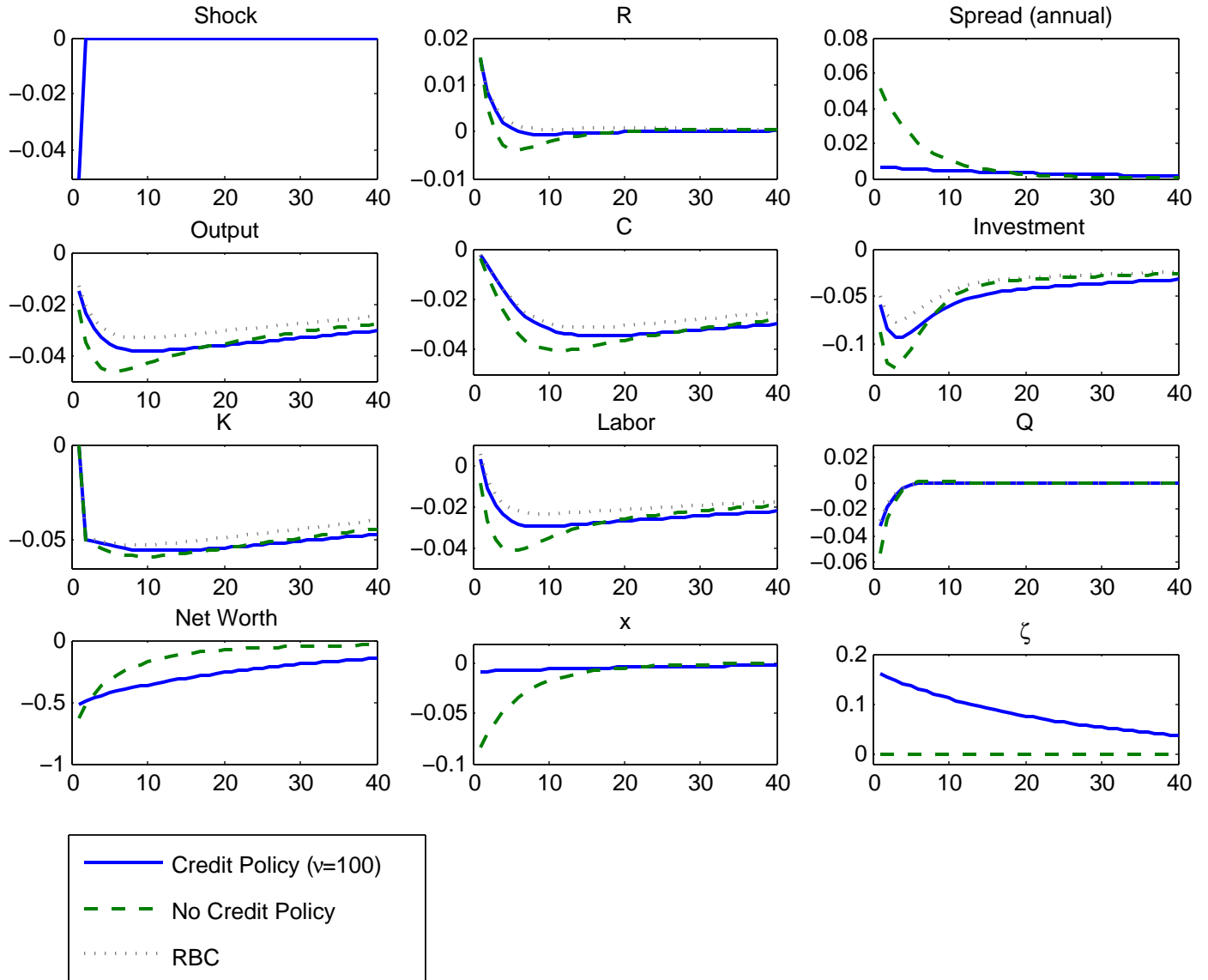


Figure 3. Credit Policy Response to Crisis: High Risk Economy

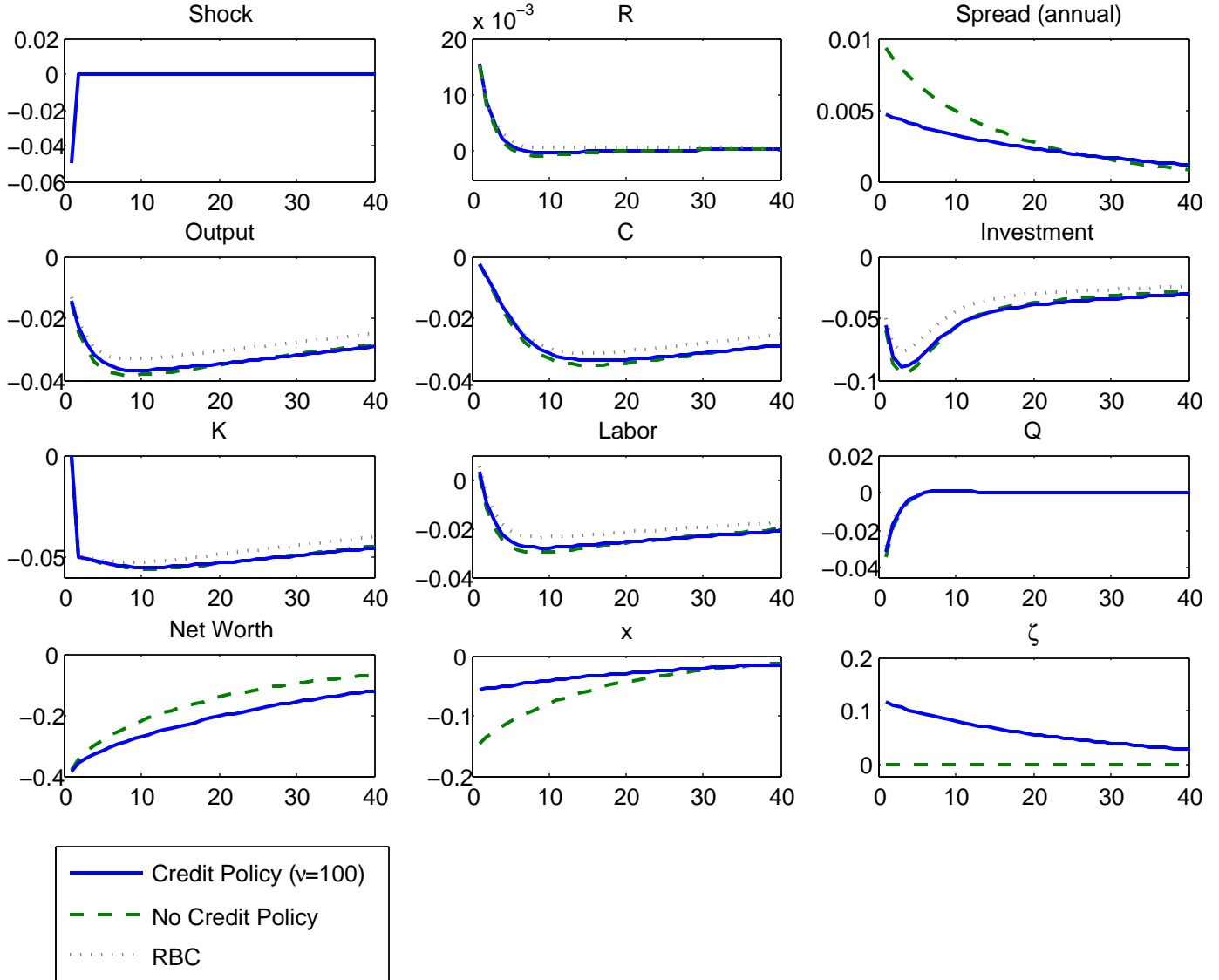


Figure 4. Anticipated vs. Unanticipated Policy in High Risk Economy

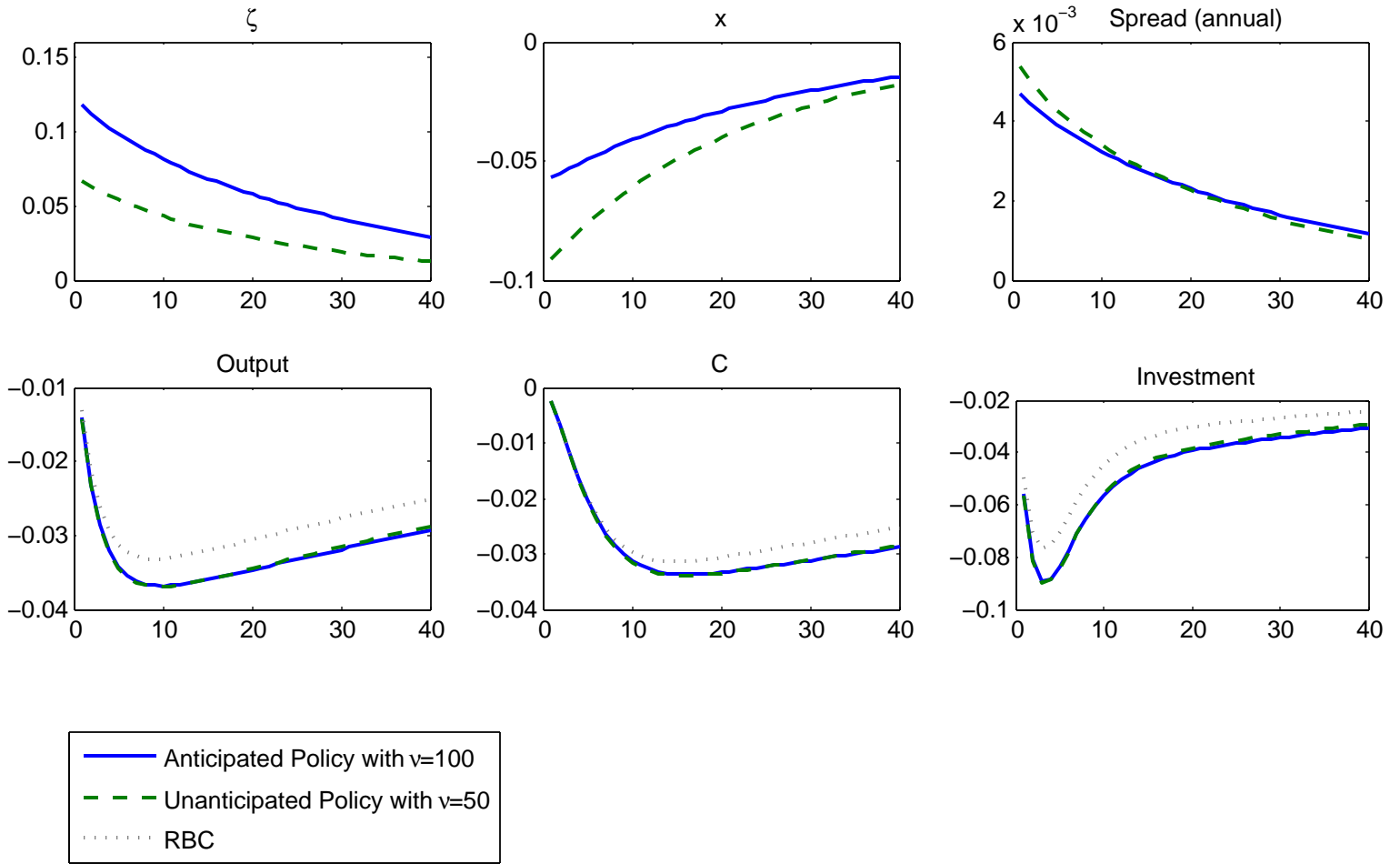


Figure 5. Macroprudential along with Credit Policy in the High Risk Economy

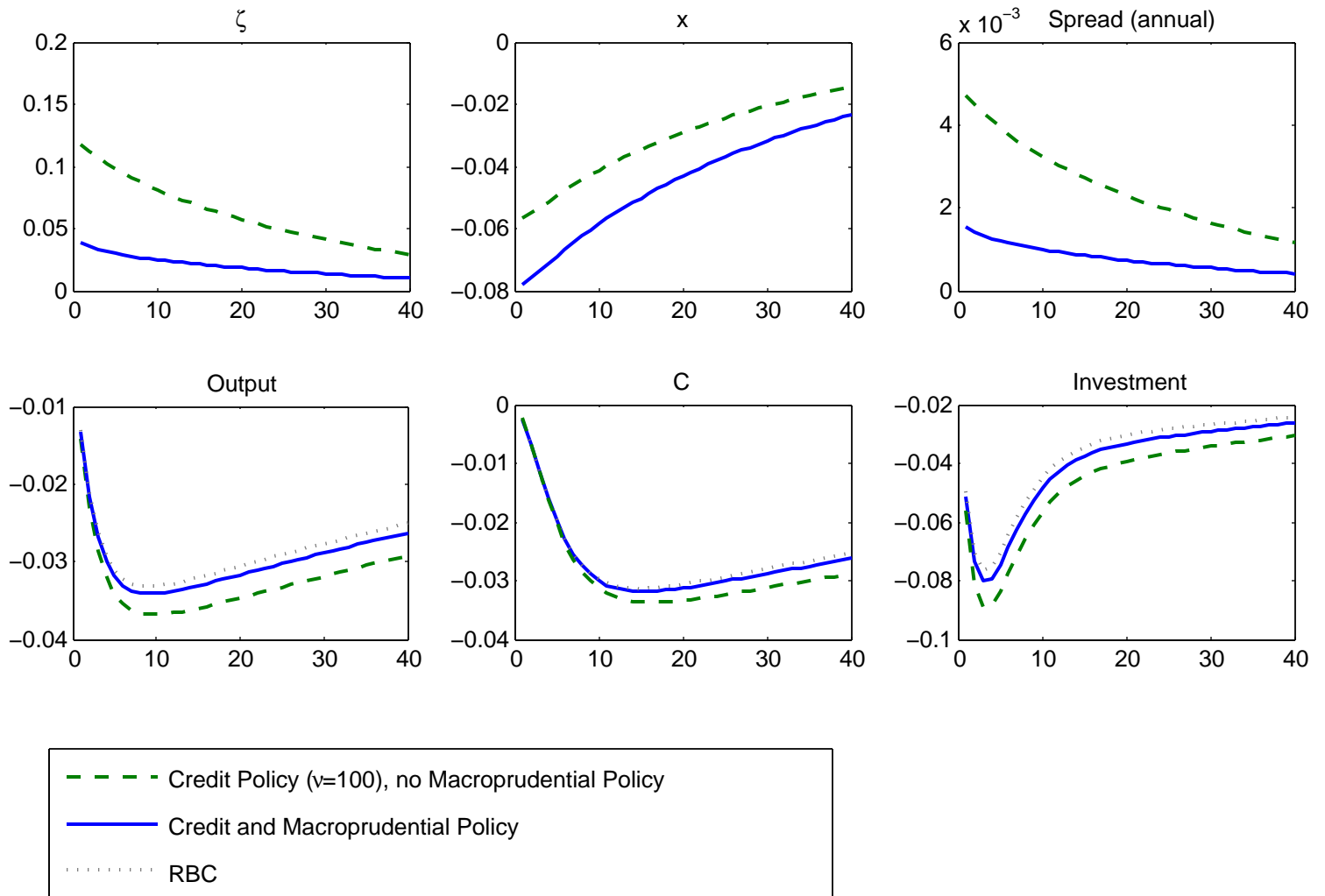


Figure 6: Macroprudential Policy without Credit Policy in the Low Risk Economy

