A Model of Trade with Ricardian Comparative Advantage and Intra-sectoral Firm Heterogeneity

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Abstract

In this paper, we merge the heterogenous firm model of Melitz (2003) with the Ricardian model of Dornbusch, Fisher and Samuelson (DFS 1977) to explain how the pattern of international specialization and trade is determined by the interaction of comparative advantage, economies of scale, country sizes and trade barriers. The model is able to capture the existence of inter-industry trade and intra-industry trade in a single unified framework. It explains how trade openness affects the pattern of international specialization and trade. It generalizes Melitz's firm selection effect in the face of trade liberalization to a setting where the patterns of inter-industry trade and intra-industry are endogenous. Although trade openness is proved to be unambiguously welfare-improving in both countries, trade liberalization can lead to an anti-Melitz effect in the larger country if it is sufficiently uncompetitive in the sectors in which it has the strongest comparative disadvantage but in which it still produces. In this case, the operating productivity cutoff is lowered while the exporting cutoff increases in the face of trade liberalization. This is because the DFS effect dominates the Melitz effect in these sectors. Consequently, the larger country can lose from trade liberalization.

1 Introduction

There are by and large two types of international trade: inter-industry trade and intra-industry trade. It is widely recognized that the former is driven by comparative advantage and the latter by economies of scale. The most widely used models for capturing comparative advantage are of the Ricardian type (e.g. Dornbusch, Fisher and Samuelson 1977, Eaton and Kortum 2002 and Eaton, Kortum and Kramarz 2004, Bernard, Eaton, Jensen and Kortum 2003) and the Heckscher-Ohlin type. The most notable model used to capture intra-industry trade is probably attributed to Krugman (1979, 1980). More recently, Melitz (2003) extends Krugman's (1980) model to analyze intra-industry trade when there is firm heterogeneity, thus capturing the selection of firms according to productivity and profit-shifting to firms of higher productivity in the face of trade integration and trade liberalization. It stimulates much further work in this direction, notably Yeaple (2005), Baldwin (2005), Chaney (2008), and Melitz and Ottaviano (2008), to name just a few papers.

Most papers in trade focus on analyzing the effects of trade attributed to one single trade model. Thus, they ignore the interaction of the various effects when both comparative advantage and economies of scale are present and there are both inter-industry and intra-industry trade between countries. This paper proposes a unified framework to capture both intra-industry and intra-industry trade in a single model. By doing so, we have a model that explains how comparative advantage, economies of scale, firm selection and home market effect interact to sort sectors into ones in which only one of the countries produces (where there is inter-industry trade) and ones in which both countries produce (where there is intra-industry trade). In particular, we modify Dornbusch, Fischer, and Samuelson's (1977) two-country, multi-sector Ricardian framework by incorporating intra-sectoral frim heterogeneity a la Melitz (2003). A number of testable hypotheses are generated. For example, sectors in which one of the countries has strong comparative advantage would be characterized by inter-industry trade, while sectors in which neither country has strong comparative advantage would be characterized by intra-industry trade. For any given country, the fraction of firms that export is higher for a sector with stronger comparative advantage.

Furthermore, we are able to understand the interaction of the forces attributed to comparative advantage effect, productivity selection effect, home market effect and variety effect, in the face of trade openness, trade liberalization, increase in labor supply and technological progress. We find that we can always decompose the total effect into those caused by comparative advantage (we call DFS effect, to stand for Dornbusch-Fisher-Samuelson), firm selection according to productivity (we call Melitz effect), and home market effect (attributed to Krugman 1980).

Although trade integration (from autarky to trade) is always welfare-improving, the welfare effect of trade liberalization (reduction of trade barriers) depends on the relative size of the two countries, the height of trade barriers and the range of relative Ricardian technological levels of the two countries. In particular, in the case of trade liberalization, we find that the interaction of the DFS effect, the Melitz effect and home market effect can give rise to a total effect that is anti-Melitz, in the following sense. Melitz predicts that trade liberalization leads to an increase in the operating productivity cutoff but a decrease in the exporting productivity cutoff, and this gives rise to an increase in the average productivity of the firms that serve the domestic market, leading to domestic welfare gains. The anti-Melitz effect leads to a decrease in the operating productivity cutoff but an increase in the exporting productivity cutoff, and this gives rise to a decrease in the average productivity of the firms that serve the domestic market, leading to domestic welfare losses. This is because the DFS effect dominates the Melitz effect in the sectors where the larger country has the strongest comparative disadvantage and yet still produces. The existence of such sectors in the larger country is attributed to the home market effect. For this reason, they cannot exist in the smaller country. In other words, the home market effect interacts with the DFS effect to create a force so large that it overturns the Melitz effect, leading to loss from trade liberalization.

We can likewise decompose the gains from trade integration (from autarky to trade), increase in labor supply and technological progress, into the DFS effect, the Melitz effect and the home market effect. For example, uniform technological progress in the differentiated goods sectors in one country always benefits itself but has ambiguous welfare effect on the trading partner. An increase in labor supply of one country on the other hand, always raises the living standard of both countries.

Among the recent literature modelling heterogeneous firms and trade, our closest neighbors are Demidova (2008) and Okubo (2009). Demidova (2008) extends Melitz's (2003) model to a two-country, asymmetric setting. She shows that improvements in the foreign country's productivity must hurt the home country and falling trade costs can raise welfare in the technologically advanced country while the laggard country may gain or lose. She focusses only on the sectors where both countries produce, whereas in our model we allow for the existence of sectors where only one country produces. Okubo (2009) also incorporates heterogeneous firms in a Ricardian model, but the focus of his model is quite different from ours. We obtain the international pattern of specialization and trade as a function of trade barriers, relative country size and Ricardian comparative advantage. We decompose the total effect of trade liberalization, openness and labor supply increase into DFS effect, Melitz effect and home market effect and explain the condition under which one effect can dominate the others. We identify the conditions under which there is an anti-Melitz effect and a loss from trade liberalization.

Bernard, Redding and Schott (2007) incorporate firm heterogeneity into the 2x2 Heckscher-Ohlin model, and focus on how falling trade costs lead to the reallocation of resources, both within and across industries and countries. And the resource reallocation changes the ex-ante comparative advantage and provides a new source of welfare gains from trade and redistribution of income across factors. Following this line of analysis, Lu (2010) augments BRS's (2007) model and applies it to examine the exporting behavior of Chinese firms. She finds that the model can explain the observation that, in the labor-intensive sectors, the average productivity of firms that only export tend to be lower than that of firms that also sell domestically. In contrast to the work of BRS (2007) and that of Lu (2010), our paper focusses on how comparative comparative and increasing returns to scale determine inter-sectoral and intra-sectoral resource allocation as well as welfare in the face of trade liberalization and other changes. In fact, like Lu, we also obtain the result that, in the sectors in which a country has comparative advantage, firms that only export can have lower productivity than that of firms that also serve the domestic market.

Hsieh and Ossa (2010) build a Ricardo-Krugman-Melitz model with many countries and many sectors, each of which consists of heterogeneous firms engaging in monopolistic competition with each other. They then analyze how real incomes of all countries are affected by productivity growth in one of the countries. The difference with our paper is that they only focus on the case when all countries produce in all sectors. In contrast, we analyze a two-country setting, but we allow for the possibility that countries endogenously specialize in certain sectors and so do not produce in sectors where they have strong comparative disadvantage, and this gives rise to interesting possibilities. We are able to obtain closed form solution to comparative statics with regard to how operating productivity cutoff, exporting productivity cutoff, firm number and welfare are impacted by trade liberalization and technological progress.

The paper is organized as follows. Section 2 presents the model with heterogeneous firms in the closed economy and examines the properties of the equilibrium. In section 3, we carry out an analysis of the equilibrium in the open economy. We analyze the pattern of specialization and trade and identify the existence of inter-industry trade as well as intra-industry trade. In section 4, we show the impact of trade integration on the productivity cutoffs, the number of firms and the welfare per worker. In section 5, 6 and 7, we analyze the welfare effects of trade liberalization, technological progress and increase in labor supply. Section 8 is an extension that discusses how under intra-industry trade, it is possible that the firms that sell domestically can be more productive than firms that do not. The last section concludes.

2 A Closed-economy Model

The economy is composed of multiple sectors: a homogenous-good sector, and a continuum of sectors of differentiated goods. There is only one factor input called labor. The homogeneous good is produced using a constant returns to scale technology. It is freely traded with zero trade costs when the country is opened up to trade. We assume that in order to produce a differentiated good, a firm has to pay a fixed (and sunk) cost of entry. After entry, a firm decides whether or not to produce according to whether the expected present discounted value of its economic profit is non-negative after its firm-specific productivity has realized. The economic profit is determined by the following factors. There is a fixed cost of production per period, and a constant variable cost of production. The fixed cost of production is constant for all firms but the variable cost of production of a firm is partly determined by a random draw from a distribution. Upon payment of the entry cost f_e, the firm earns the opportunity to make a random draw from a distribution of firm productivity. The draw will determine the firm-specific component of the firm's productivity (i.e. reciprocal of the variable cost of production). The above features of the model are basically drawn from Melitz (2003). Unlike Melitz, there is another factor that affects the variable cost of production of a firm, which is an exogenously determined sector-specific technological level. In general, this technological level differs across sectors in the same country as well as differs across countries within the same sector. The set of sector-specifc technological levels across sectors in both countries determine the pattern of comparative advantage across sectors of each country. The above features are basically drawn from DFS (1977). Our model is therefore a hybrid of Melitz (2003) and DFS (1977).

There are L consumers, each supplying one unit of labor. Preferences are defined by a nested Cobb-Douglas function:

$$U = \alpha \ln C_h + \int_0^1 b_k \ln C_k dk \tag{1}$$

$$C_k = (\int_0^{\theta_k} c_k(j)^\rho dj)^{\frac{1}{\rho}} \text{ with } \int_0^1 b_k dk = 1 - \alpha \tag{2}$$

where α denotes the share of expenditure on homogenous goods, b_k is the share of expenditure on differentiated good $k \in [0, 1]$; θ_k is the endogenously determined mass of varieties in differentiated sector k. The homogeneous good is produced with constant unit labor requirement $\frac{1}{A_h}$. The price of the homogeneous good is $\frac{w}{A_h}$, where w is the wage, as it is produced and sold under perfect competition. For the differentiated-goods sectors, the exact price index for each sector is denoted by P_k, where

$$P_k = (\int_0^{\theta_k} p_k(j)^{1-\sigma})^{\frac{1}{1-\sigma}}, \text{ where } \sigma = \frac{1}{1-\rho} > 1$$

where $p_k(j)$ represents the price of variety j in sector k, and σ represents the elasticity of substitution between varieties. Cost minimization by firms implies that the operating revenue of firm j in sector k is given by

$$r_k(j) = b_k L \left[\frac{p_k(j)}{P_k}\right]^{1-\sigma} \tag{3}$$

where $L = wL$ denotes the total expenditure on all goods.

We shall assume that the labor productivity of a firm $A_k\varphi \in [A_k, \infty]$ in sector k follows a Pareto Distribution $P(A_k, \gamma)$, where A_k is the exogenously determined minimum productivity in differentiated sector k (the sector-specific component of productivity of the firm in sector k), and γ ($> \sigma - 1$) is the shape parameter of the distribution.[1] More precisely, the labor productivity of a firm is determined by two factors: one is firm-specific, being a random variable following a Pareto distribution $P(1, \gamma) = 1 - \left(\frac{1}{\varphi}\right)^\gamma$ where $\varphi \in [1, \infty]$; the other is A_k, which is sector-specific.[2]. Labor used in firm j in sector k is a linear function of output $y_k(j)$:

$$l_k(j) = f + \frac{y_k(j)}{A_k\varphi_k(j)},$$

[1] The assumption $\gamma > \sigma - 1$ ensures that, in equilibrium, the size distribution of firms has a finite mean.

[2] A_k represents the common development of certain industry. With identical labor input, the actual productivity will differ across the differentiated industries, e.g. an immigrate worker in China will be much more productive working in the automobile industry than farming at home. And for the same industry, the development level varies across countries. The technology uesd by New Balance, who uses automatized plants to produce sneakers in the US and UK, is more advaned than his main competitors, e.g. Adidas and Nike, who produce similar goods manually in Asia. Besides this industry-specific technology, each firm will draw an idiosyncratic technology φ from certain distribution, and here we assume that it is univeral, as $P(A_k, \gamma)$. One way to think of it is that, assuming this idiosyncraticy comes from the ability of labor, which will not vary much across industries and across countries. In the future, all the equations concerning individual firms is in fact a function of its idiosyncratic technology φ.

where f is the fixed cost of production per period, $\varphi_k(j)$ is the productivity of firm j in sector k, . Therefore, under monopolistic competition in sector k the profit-maxmimizing price is given by

$$p_k(j) = \frac{w}{\rho A_k \varphi_k(j)} \tag{4}$$

It follows that the profit of the firm is given by

$$\pi_k(j) = \frac{r_k(j)}{\sigma} - fw = \frac{b_k L \left[P_k w^{-1} \rho A_k \varphi_k(j)\right]^{\sigma-1}}{\sigma} - fw$$

If a firm draws too low a productivity, it will exit immediately, as the expected present discounted value of its economic profit is negative. To be more precise, denote the cutoff productivity of a surviving firm by $\overline{\varphi}_k$. Then, the aggregate (exact) price index can be rewritten as

$$P_k = \left[\theta_k \int_{\overline{\varphi}_k}^{\infty} p_k(j)^{1-\sigma} \frac{g(\varphi_k(j))}{1 - G(\overline{\varphi}_k)} d\varphi_k(j)\right]^{\frac{1}{1-\sigma}} = \theta_k^{\frac{1}{1-\sigma}} p_k(\widetilde{\varphi}_k),$$

where $G(.)$ is the c.d.f. of the distribution of productivity in the sector and $g(.)$ is its p.d.f. The function $G(.)$ is the same for all sectors. Moreover,

$$p_k(\widetilde{\varphi}_k) = \frac{w}{\rho A_k \widetilde{\varphi}_k}$$

where $\widetilde{\varphi}_k$ can be interpreted as the "average" productivity in sector k. It can be easily shown that

$$\begin{aligned}\widetilde{\varphi}_k &= \left[\int_{\overline{\varphi}_k}^{\infty} \varphi^{\sigma-1} \frac{g(\varphi)}{1 - G(\overline{\varphi}_k)} d\varphi\right]^{\frac{1}{\sigma-1}} \\ &= \left(\frac{\gamma}{\gamma - \sigma + 1}\right)^{\frac{1}{\sigma-1}} \overline{\varphi}_k \quad .\end{aligned} \tag{5}$$

if $G(\varphi_k)$ is the c.d.f. of a Pareto distribution $P(1, \gamma)$ so that $G(\varphi_k) = 1 - \left(\frac{1}{\varphi_k}\right)^{\gamma}$.

From now on, we shall assume for tractability that $G(.)$ is a Pareto distribution as described above. The qualitative nature of most of our results will not be affected by this assumption. There is no discounting of future, and only stationary equilibrium is considered. After making a draw from the productivity distribution, a firm may decide to exit immediately if it expects to make zero present-discounted profits in the future. The zero cutoff profit (ZCP) condition determines the productivity $\overline{\varphi}_k$ of the marginal firm that makes zero economic profits:

$$\sigma f w = r_k(\overline{\varphi}_k) = \frac{b_k L}{\theta_k}\left(\frac{\overline{\varphi}_k}{\widetilde{\varphi}_k}\right)^{\sigma-1} = \frac{b_k L}{\theta_k}\left(\frac{\gamma - \sigma + 1}{\gamma}\right) \qquad \text{(ZCP)} \tag{6}$$

We assume that in each period, an operating firm faces a constant probability δ of a bad shock that forces it to exit, and will earn a positive profit in every period until hit by the shock.

As more firms enter, the cutoff productivity increases. This in turn lowers the probability of surviving after entry. So, when the cutoff productivity becomes sufficiently high, there will be no more entry. More precisely, the free entry (FE) condition, which relates the cutoff productivity to the entry cost f_e, is given by

$$f_e w = p_{in} \widetilde{v} = \left[1 - G(\overline{\varphi}_k)\right] \frac{\widetilde{\pi}_k}{\delta} \tag{7}$$

where $p_{in} \equiv 1 - G(\overline{\varphi}_k)$ is the ex-ante probability of successful entry; $\widetilde{v} = \frac{1}{\delta}\widetilde{\pi}_k$ is the present value of the average profit flow of a surviving firm; and $\widetilde{\pi}_k = \pi_k(\widetilde{\varphi}_k)$ is the average profit flow of a surviving firm, which is equal to $fw\left[\left(\frac{\widetilde{\varphi}_k}{\overline{\varphi}_k}\right)^{\sigma-1} - 1\right] = fw\left(\frac{\sigma-1}{\gamma-\sigma+1}\right)$ according to the ZCP condition (6) and equation (5).[3]

Solving for the above system of 2 equations for 2 unknowns, we can get

$$(\overline{\varphi}_k)^\gamma = \frac{\sigma-1}{\gamma-\sigma+1}\cdot\frac{f}{\delta f_e} \equiv D_1; \qquad \theta_k = \frac{\gamma-\sigma+1}{\gamma}\cdot\frac{b_k L}{\sigma f} \equiv D_2 L$$

Proposition 1 *In the closed economy, fraction of firms that successfully enter is identical across sectors. The number of firms in each sector is proportional to the sector's share in total expenditure.*

In fact the result stated in Proposition 1 is independent of the assumption of Pareto distribution. The only unknown in equation (7) is $\overline{\varphi}_k$, as aggregate productivity $\widetilde{\varphi}_k$ is a function of $\overline{\varphi}_k$ for any distribution, thus $\overline{\varphi}_k$ is endogenously determined and unrelated to A_k.[4] Similar logic applies to the result concerning θ_k, for it is endogenously determined by $\overline{\varphi}_k$ following equation (6). And the actual cutoff productivity is $A_k\overline{\varphi}_k$, which still differs across sectors. From now on, we assume $\frac{\sigma-1}{\gamma-\sigma+1}\cdot\frac{f}{\delta f_e} \geq 1$, in order to avoid the corner solution.[5]

The intuition of this proposition is that, an increase in the sector-specific technology will cause a firm's optimal price to decrease, and following this, the aggregate price for this sector will decrease as well. These price reduction leads to two opposite effects on profits of a firm: on the one hand, the decline of sectoral aggregate price causes more demand for each firm, which increases the firm's profit; on the other hand, the decrease of price will make the profit fall. And these two effects cancel each other so that the profit of each firm does not change. As a result, the fraction of firms that can successfully enter will be the same across sectors, and the number of firms in each sector is proportional to the sector's share in total expenditure. It is also noteworthy that although the increase in sector-specific technology does not affect the number of firms, it will improve consumers' total welfare due to the increased output of each firm.

3 An Open-economy Model

In this section, we consider a global economy with two countries: Home and Foreign. We attach an asterisk to all the variables pertaining to Foreign. We index sectors such that as the index

[3] $\widetilde{\pi}_k = \pi_k(\widetilde{\varphi}_k) = \frac{r_k(\widetilde{\varphi}_k)}{\sigma} - fw = \frac{1}{\sigma}\left(\frac{\widetilde{\varphi}_k}{\overline{\varphi}_k}\right)^{\sigma-1} r_k(\overline{\varphi}_k) - fw = fw\left[\left(\frac{\widetilde{\varphi}_k}{\overline{\varphi}_k}\right)^{\sigma-1} - 1\right] = fw\left(\frac{\sigma-1}{\gamma-\sigma+1}\right)$. The third equality arises from the fact that $\left(\frac{\widetilde{\varphi}_k}{\overline{\varphi}_k}\right)^{\sigma-1} = \frac{r_k(\widetilde{\varphi}_k)}{r_k(\overline{\varphi}_k)}$. The fourth equality comes from the fact that $\sigma fw = r_k(\overline{\varphi}_k)$, which is the ZCP condition above. The fifth equality comes from (5).

[4] Certain conditon is neccesarry to ensure the uiqueness of $\overline{\varphi_k}$ in equilibrium

[5] If $\frac{\sigma-1}{\gamma_k-\sigma+1}\frac{f}{\delta f_e} < 1$, φ_k will get to the corner solution 1.Even so, Proposition 1 still holds in this case.

increases Home's comparative advantage strengthens. In other words, the sector-specific relative productivity $a(k) \equiv a_k \equiv \frac{A_k}{A_k^*}$ increases in $k \in [0, 1]$. Therefore, $a'(k) > 0$.

On the demand side, we assume that consumers in both countries have identical tastes:

$$U = \alpha \ln C_h + \int_0^1 b_k \ln C_k dk \qquad \text{with } \int_0^1 b_k dk = 1 - \alpha$$
$$\text{and } C_k = \left(\int_0^{\theta_k} c_k(j)^\rho dj + \int_0^{\theta_k^*} c_k^*(j)^\rho dj \right)^{\frac{1}{\rho}}$$

On the production side, the labor productivity in the homogeneous good sector are respectively A_h and A_h^* in Home and Foreign. In the rest of the paper, we assume that the homogeneous good sector is sufficiently large so that the homogeneous good is produced in both countries. We also assume that there is no trade cost associated with the homogeneous good. Therefore free trade of homogeneous goods implies that the wage ratio is determined by relative labor productivity in the sector, i.e. $\omega = \frac{w}{w^*} = \frac{A_h}{A_h^*}$, where w^* denotes Foreign's wage. Without loss of generality, we assume that $w = w^*$ and normalize by setting $w = w^* = 1$. Therefore, $\frac{A_h}{A_h^*} = 1$. The specification on technology in the differentiated-good sectors is the same as in autarky.

The subscript "dk" pertains to domestic firm serving domestic market in sector k, the subscript "xk" pertains to domestic firm serving foreign market in sector k, and the subscript "k" pertains to sector k without regard to which market is being served. A superscript "$*$" denotes variables pertaining to Foreign. For the differentiated-good sectors, each firm's profit-maximizing price in the domestic market is given, as before, by $p_{dk}(j) = \frac{1}{\rho A_k \varphi_k(j)}$. But Home's exporting firms will set higher prices in the Foreign market due to the existence of an iceberg trade cost, such that $\tau(> 1)$ units of goods have to be shipped from the source in order for one unit to arrive at the destination. Therefore, the optimal export price of a Home-produced good sold in Foreign is given by $p_{xk}(j) = \frac{\tau}{\rho A_k \varphi_k(j)}$. Similarly, Foreign's firms' pricing rules are given by $p_{dk}^*(j) = \frac{1}{\rho A_k^* \varphi_k^*(j)}$ and $p_{xk}^*(j) = \frac{\tau}{\rho A_k^* \varphi_k^*(j)}$. Here, we assume identical iceberg trade cost τ for both countries for simplicity.

3.1 Firm entry and exit

According to the firms' pricing rules, the gross revenue flow and net profit flow of firm j in differentiated sector k from domestic sales for Home's firms are, respectively:

$$r_{dk}(j) = b_k L \left(\frac{p_{dk}(j)}{P_k} \right)^{1-\sigma},$$
$$\pi_{dk}(j) = \frac{r_{dk}(j)}{\sigma} - f.$$

The expressions for the corresponding variables for Foreign's firms are defined analogously. The variables $r_{dk}(j)$ and $r_{dk}^*(j)$ respectively represent the gross revenue flow from domestic sales by Home's and Foreign's firms respectively; $\pi_{dk}(j)$ and $\pi_{dk}^*(j)$ respectively represent the net profit flow from the domestic sales by Home's and Foreign's firms; P_k and P_k^* are the aggregate price index in sector k of goods sold in Home and Foreign, respectively. Expressions of P_k and P_k^* are given in equation (8) below. Following the same logic, the gross exporting revenue and net profit flow of firm j in sector k for Home's firms are, respectively:

$$r_{xk}(j) = b_k L^* \left(\frac{p_{xk}(j)}{P_k^*}\right)^{1-\sigma},$$
$$\pi_{xk}(j) = \frac{r_{xk}(j)}{\sigma} - f_x.$$

The expressions for the corresponding variables for Foreign's firms are defined analogously. The variables $r_{xk}(j)$ and $r_{xk}^*(j)$ represent the gross exporting revenue flow for Home and Foreign firms respectively; $\pi_{xk}(j)$ and $\pi_{xk}^*(j)$ represent the flow of net exporting profits for Home and Foreign firms respectively. f_x is the fixed cost of exporting to be paid at each date, which is the same for all firms. Note that we can interpret $f_x = \delta f_{ex}$ as the amortized cost of entry into the export market, where f_{ex} is the one-time fixed cost of entry into the export market. Let $\overline{\varphi}_{dk}$ and $\overline{\varphi}_{xk}$ denote the cutoffs of the firm-specific productivity for domestic sales and exporting respectively of sector k for Home firms ; $\overline{\varphi}_{dk}^*$ and $\overline{\varphi}_{xk}^*$ denote the corresponding variables for Foreign. Consequently, the mass of exporting firms from Home is equal to:

$$\theta_{xk} = \frac{1 - G(\overline{\varphi}_{xk})}{1 - G(\overline{\varphi}_{dk})}\theta_{dk} = \left(\frac{\overline{\varphi}_{dk}}{\overline{\varphi}_{xk}}\right)^{\gamma} \theta_{dk}$$

where θ_{dk} denotes the mass of operating firms in Home. The corresponding expression relating the variables θ_{xk}^* and θ_{dk}^* for Foreign are defined analogously. Then, in differentiated sector k, the mass of varieties available to consumers in Home is equal to

$$\theta_k = \theta_{dk} + \theta_{xk}^*$$

and θ_k^* is defined analogously. The aggregate price indexes are given by:

$$P_k = (\theta_k)^{\frac{1}{1-\sigma}} p_{dk}(\widetilde{\varphi}_k), \qquad P_k^* = (\theta_k^*)^{\frac{1}{1-\sigma}} p_{dk}^*(\widetilde{\varphi}_k^*) \tag{8}$$

where $\widetilde{\varphi}_k$ and $\widetilde{\varphi}_k^*$ denote the aggregate productivity in differentiated sector k for goods sold in Home and Foreign, respectively. They are given respectively by:

$$(\widetilde{\varphi}_k)^{\sigma-1} = \frac{1}{\theta_k}\left\langle \theta_{dk}\left(\widetilde{\varphi}_{dk}\right)^{\sigma-1} + \theta_{xk}^*\left(\tau^{-1}\frac{1}{a_k}\widetilde{\varphi}_{xk}^*\right)^{\sigma-1}\right\rangle, \tag{9}$$
$$(\widetilde{\varphi}_k^*)^{\sigma-1} = \frac{1}{\theta_k^*}\left\langle \theta_{dk}^*\left(\widetilde{\varphi}_{dk}^*\right)^{\sigma-1} + \theta_{xk}\left(\tau^{-1}a_k\widetilde{\varphi}_{xk}\right)^{\sigma-1}\right\rangle \tag{10}$$

where $\widetilde{\varphi}_{dk}$ ($\widetilde{\varphi}_{dk}^*$) and $\widetilde{\varphi}_{xk}$ ($\widetilde{\varphi}_{xk}^*$) denote respectively the aggregate productivity level of all of Home's (Foreign's) operating firms and Home's (Foreign's) exporting firms. The relationships between $\widetilde{\varphi}_{dk}$ and $\overline{\varphi}_{dk}$, between $\widetilde{\varphi}_{dk}^*$ and $\overline{\varphi}_{dk}^*$, between $\widetilde{\varphi}_{xk}$ and $\overline{\varphi}_{xk}$, and between $\widetilde{\varphi}_{xk}^*$ and $\overline{\varphi}_{xk}^*$, are exactly the same as in the closed economy. That is, $\widetilde{\varphi}_{sk} = \left(\frac{\gamma}{\gamma-\sigma+1}\right)^{\frac{1}{\sigma-1}} \overline{\varphi}_{sk}$ and $\widetilde{\varphi}_{sk}^* = \left(\frac{\gamma}{\gamma-\sigma+1}\right)^{\frac{1}{\sigma-1}} \overline{\varphi}_{sk}^*$ for $s = x, d$. From the above equations, it is obvious that these aggregate productivity measures as well as aggregate price indexes are functions of ($\overline{\varphi}_{dk}$, $\overline{\varphi}_{dk}^*$, $\overline{\varphi}_{xk}$, $\overline{\varphi}_{xk}^*$, θ_{dk}, θ_{dk}^*). An entering firm will produce only if it can generates positive present-discounted profit by selling domestically, and export only if it can generate positive present-discounted profit by selling abroad. Then we have the following four zero cutoff profit conditions

$$r_{dk}(\overline{\varphi}_{dk}) = b_k L \left(P_k \rho A_k \overline{\varphi}_{dk}\right)^{\sigma-1} = \sigma f \tag{11}$$

$$r^*_{dk}(\overline{\varphi}^*_{dk}) = b_k L^* \left(P^*_k \rho A^*_k \overline{\varphi}^*_{dk}\right)^{\sigma-1} = \sigma f \tag{12}$$

$$r_{xk}(\overline{\varphi}_{xk}) = b_k L^* \left(\frac{P^*_k}{\tau} \rho A_k \overline{\varphi}_{xk}\right)^{\sigma-1} = \sigma f_x \tag{13}$$

$$r^*_{xk}(\overline{\varphi}^*_{xk}) = b_k L \left(\frac{P_k}{\tau} \rho A^*_k \overline{\varphi}^*_{xk}\right)^{\sigma-1} = \sigma f_x \tag{14}$$

Define $\widetilde{\pi}_k$ and $\widetilde{\pi}^*_k$ as the average profit flow of a surviving firm in sector k in Home and Foreign respectively. It can be easily shown that[6]

$$\widetilde{\pi}_k = \pi_{dk}(\widetilde{\varphi}_{dk}) + \frac{1-G(\overline{\varphi}_{xk})}{1-G\left(\overline{\varphi}_{dk}\right)} \pi_{xk}(\widetilde{\varphi}_{xk}) = \frac{\sigma-1}{\gamma-\sigma+1}\left[f + \left(\frac{\overline{\varphi}_{dk}}{\overline{\varphi}_{xk}}\right)^{\gamma} f_x\right]$$

$$\widetilde{\pi}^*_k = \pi^*_{dk}(\widetilde{\varphi}^*_{dk}) + \frac{1-G(\overline{\varphi}^*_{xk})}{1-G\left(\overline{\varphi}^*_{dk}\right)} \pi^*_{xk}(\widetilde{\varphi}^*_{xk}) = \frac{\sigma-1}{\gamma-\sigma+1}\left[f + \left(\frac{\overline{\varphi}^*_{dk}}{\overline{\varphi}^*_{xk}}\right)^{\gamma} f_x\right].$$

The potential entrant will enter if its expected post-entry present-discounted profit is above the cost of entry. Hence, the Free Entry (FE) conditions for Home and Foreign are, respectively

$$f_e = \left(1-G\left(\overline{\varphi}_{dk}\right)\right)\frac{\widetilde{\pi}_k}{\delta} = \left(\frac{\sigma-1}{\gamma-\sigma+1}\right)\frac{f\cdot\left(\overline{\varphi}_{dk}\right)^{-\gamma} + f_x\cdot\left(\overline{\varphi}_{xk}\right)^{-\gamma}}{\delta} \tag{15}$$

$$f_e = \left(1-G\left(\overline{\varphi}^*_{dk}\right)\right)\frac{\widetilde{\pi}^*_k}{\delta} = \left(\frac{\sigma-1}{\gamma-\sigma+1}\right)\frac{f\cdot\left(\overline{\varphi}^*_{dk}\right)^{-\gamma} + f_x\cdot\left(\overline{\varphi}^*_{xk}\right)^{-\gamma}}{\delta} \tag{16}$$

3.2 General equilibrium

Assuming that both countries produce in sector k, given the wage ratio $A_h/A^*_h = 1$, we can solve for $(\overline{\varphi}_{dk},\ \overline{\varphi}^*_{dk},\ \overline{\varphi}_{xk},\ \overline{\varphi}^*_{xk},\ \theta_{dk},\ \theta^*_{dk})$ from the four zero cutoff profit conditions and two free entry conditions (11) to (16) since the aggregate prices are functions of these six variables (for details, please refer to the Appendix). The solutions are given below.

[6] $\widetilde{\pi}_{dk} = \pi_{dk}(\widetilde{\varphi}_{dk}) = \frac{r_{dk}(\widetilde{\varphi}_{dk})}{\sigma} - f = \frac{1}{\sigma}\left(\frac{\widetilde{\varphi}_{dk}}{\overline{\varphi}_{dk}}\right)^{\sigma-1} r_{dk}(\overline{\varphi}_{dk}) - f = f\left[\left(\frac{\widetilde{\varphi}_{dk}}{\overline{\varphi}_{dk}}\right)^{\sigma-1} - 1\right] = f\cdot\frac{\sigma-1}{\gamma-\sigma+1}$. The third equality arises from the fact that $\left(\frac{\widetilde{\varphi}_{dk}}{\overline{\varphi}_{dk}}\right)^{\sigma-1} = \frac{r_{dk}(\widetilde{\varphi}_{dk})}{r_{dk}(\overline{\varphi}_{dk})}$. The fourth equality comes from the fact that $\sigma f = r_{dk}(\overline{\varphi}_{dk})$, which is the ZCP condition above. The fifth equality comes from one of the equations in footnote **??**. Furthermore, $\widetilde{\pi}_{xk} = f_x\left(\frac{\sigma-1}{\gamma-\sigma+1}\right)$ can be derived from similar steps as above by replacing the subscript "d" by "x" and the variable f by f_x. Finally, $1-G(\varphi) = \varphi^{-\gamma}$.

$$(\overline{\varphi}_{dk})^{\gamma} = D_1 \left[\frac{B - B^{-1}}{B - (a_k)^{\gamma}} \right] \tag{17}$$

$$(\overline{\varphi}^{*}_{dk})^{\gamma} = D_1 \left[\frac{B - B^{-1}}{B - (a_k)^{-\gamma}} \right] \tag{18}$$

$$\overline{\varphi}_{xk} = \frac{\phi}{a_k} \overline{\varphi}^{*}_{dk} \tag{19}$$

$$\overline{\varphi}^{*}_{xk} = a_k \phi \overline{\varphi}_{dk} \tag{20}$$

$$\theta_{dk} = D_2 \left[\frac{BL - \frac{B-(a_k)^{\gamma}}{B(a_k)^{\gamma}-1} L^*}{B - B^{-1}} \right] \tag{21}$$

$$\theta^{*}_{dk} = D_2 \left[\frac{BL^* - \frac{B(a_k)^{\gamma}-1}{B-(a_k)^{\gamma}} L}{B - B^{-1}} \right] \tag{22}$$

where $B = \tau^{\gamma} \left(\frac{f_x}{f} \right)^{\frac{\gamma}{\sigma-1}-1}$ and $\phi = \tau \left(\frac{f_x}{f} \right)^{\frac{1}{\sigma-1}}$. The variable B and ϕ can be interpreted as summary measures of trade barriers; a_k can be interpreted as competitiveness of Home in differentiated goods sector k. Recall that $a'_k(k) > 0$ is assumed.

The condition $\tau^{\sigma-1} f_x > f$ is needed to ensure that in both countries there exist some sectors in which some firms produce exclusively for their domestic market. This is exactly the required condition in Melitz (2003) for the same purpose. The rationale of this assumption will be explained in the Appendix.[7] In this paper, a stricter condition $\frac{f_x}{f} > \max\{\frac{L}{L^*}, \frac{L^*}{L}\}$ is adopted so as to ensure that some firms produce exclusively for their domestic market in all sectors.

According to equation (21) and (22) Home firms will exit sector k when $\theta_{dk} \leq 0$, and Foreign's firms will exit the sector if $\theta^{*}_{dk} \leq 0$. This implies that $B^{-1} \frac{B-(a_k)^{\gamma}}{B(a_k)^{\gamma}-1} < \frac{L}{L^*} < B \frac{B-(a_k)^{\gamma}}{B(a_k)^{\gamma}-1}$ is needed for both countries to produce positive outputs in this sector, or else there will be complete dominance by one country in the sector and one-way trade. Rearranging these inequalities, we can sort the sectors into 3 types according to Home's strength of comparative advantage. Home will exit sector k *iff* $k \leq k_1$, in which k_1 is the index of the sector that satisfies

$$(a_{k_1})^{\gamma} = \frac{B \left(\frac{L}{L^*} + 1 \right)}{B^2 \frac{L}{L^*} + 1};$$

and Foreign will exit sector k *iff* $k \geq k_2$, in which k_2 is the index of the sector that satisfies[8]

$$(a_{k_2})^{\gamma} = \frac{B^2 \frac{L^*}{L} + 1}{B \left(\frac{L^*}{L} + 1 \right)}.$$

[7] In fact, this condition implies $B > 1$. [If $f_x > f$, $B = \tau^{\gamma} \left(\frac{f_x}{f} \right)^{\frac{\gamma-\sigma+1}{\sigma-1}}$ is obiviously larger than 1. If not, then $B = \frac{f}{f_x} \phi^{\gamma} \geq \phi^{\gamma} > 1$, which cannot be true.] And $B > 1$ is also supported by empirical evidence. The firm's revenue in sector k, $r_k(j)$, follows a Pareto distribution with parameter $\frac{\gamma}{\sigma-1}$. According to Axtell (2001), $\frac{\gamma}{\sigma-1}$ is close to one, which in turn implies that $\frac{\gamma-\sigma+1}{\sigma-1}$ is close to 0. To be more precise, it equals 0.059 according to Axtell's estimation; therefore B approaches τ^{γ}, which must be larger than 1. [For $\gamma = 5$ (which must be larger than $\sigma - 1$), and a small $\tau = 1.1$, we need $\frac{f}{f_x} > 3220$, which is almost impossible.]

[8] Because $\frac{B^2 \frac{L^*}{L} + 1}{B\left(\frac{L^*}{L}+1\right)} > \frac{B\left(\frac{L}{L^*}+1\right)}{B^2 \frac{L}{L^*}+1}$ holds as long as $B > 1$, we always have $k_1 < k_2$.

It is also clear that all $k \in (k_1, k_2)$ will satisfy $(a_k)^\gamma \in \left(\frac{1}{B}, B\right)$ for any possible GDP ratio $\frac{L}{L^*}$, which ensures that the productivity cutoffs will never reach the corner for the sectors where both countries produce.

For the sectors where one country completely dominates, there is no interior solution to some of the equations in the system above, as the productivity cutoff and firm number for the country with zero output reach the corner. Therefore, a different set of equations need to be solved for this case. Without loss of generality, we consider the Home-dominated sectors. As there is no competition from Foreign's firms when Home's firms export their goods, the aggregate price indexes become

$$\begin{aligned} P_k &= (\theta_{dk})^{\frac{1}{1-\sigma}} \frac{1}{\rho A_k \widetilde{\varphi}_{dk}} \\ P_k^* &= (\theta_{xk})^{\frac{1}{1-\sigma}} \frac{\tau}{\rho A_k \widetilde{\varphi}_{xk}} \end{aligned}$$

Accordingly, the two zero cutoff conditions for Home (11) and (13) continue to hold.

As the Free entry condition (15) for Home firms still holds, solving the diminished system of 3 equations (11), (13), (15) for 3 unknowns, we will have

$$\begin{aligned} \theta_{dk} &= \frac{b_k L}{\sigma f}\left(\frac{\gamma - \sigma + 1}{\gamma}\right) = D_2 L \\ \theta_{xk} &= \frac{b_k L^*}{\sigma f_x}\left(\frac{\gamma - \sigma + 1}{\gamma}\right) = D_2 \frac{f}{f_x} L^* \\ (\overline{\varphi}_{dk})^\gamma &= \frac{L + L^*}{L} D_1 \end{aligned}$$

And $(\overline{\varphi}_{xk})^\gamma = \left(\frac{L+L^*}{L^*}\right)\frac{f_x}{f} D_1$ is inferred from $\theta_{xk} = \frac{1-G(\overline{\varphi}_{xk})}{1-G(\overline{\varphi}_{dk})}\theta_{dk}$. Similarly, the system can be solved for the Foreign-dominated sectors, and equations analogous to the above three are obtained for Foreign.[9]

It is noteworthy that the above equilibrium is unique, and we present the proof in the Appendix.

Proposition 2 *In sectors $k \in [k_2, 1]$, where Home has the strongest comparative advantage, only Home produces, and there is one-way trade. The same is true for Foreign in sectors $k \in [0, k_1]$. In sectors $k \in (k_1, k_2)$, where neither country has strong comparative advantage, both countries produce, and there is two-way trade.*

We show the three zones of international division of labor in Figure 1 below:[10]

[9] They are: $\theta^*_{dk} = \frac{b_k L^*}{\sigma f}\left(\frac{\gamma-\sigma+1}{\gamma}\right) = D_2 L^*$; $\theta^*_{xk} = \frac{b_k L}{\sigma f_x}\left(\frac{\gamma-\sigma+1}{\gamma}\right) = D_2 \frac{f}{f_x} L$; and $(\overline{\varphi}^*_{dk})^\gamma = \frac{L+L^*}{L^*} D_1$.

[10] The dashed curves represent the equlibrium firm numbers of the original system of equations based on the assumption that both countries produce postive outputs in all sectors.

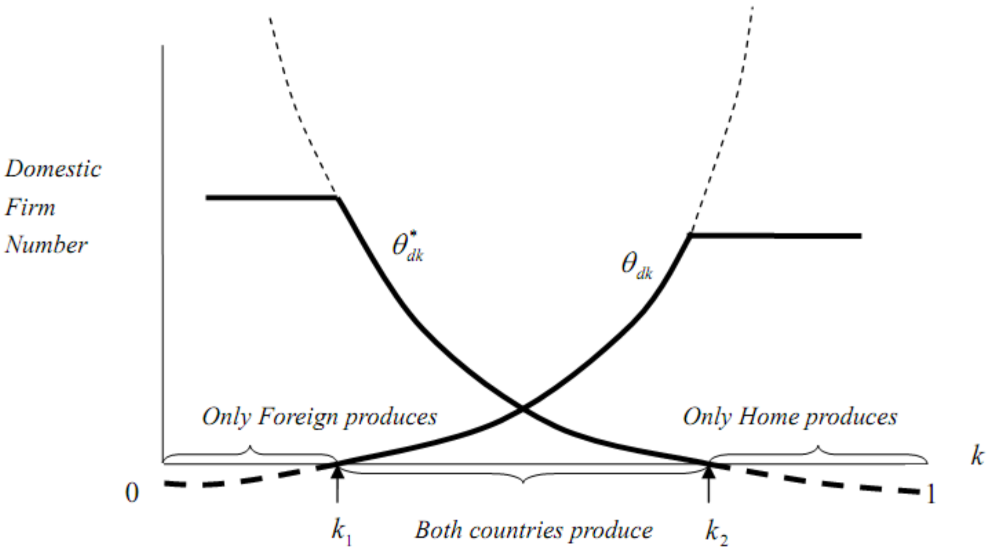

Figure 1: Three Zones of International Specialization

(Assumption: expenditure shares are equal across sectors)

Given the relative size of the two countries, the range of these three types of equilibrium depends on the the trade barrier B, as shown in equations (21) and (22). The effects of a reduction in trade barrier on these three zones depend on the GDP ratio of the two countries. If these two countries have the about the same GDP, then trade liberalization make k_1 increase, but k_2 decrease. To be more precise, trade liberalization reduces the range of sectors where both countries produce. They each specialize in a narrower range of goods. As a fall in trade cost increases the number of exporting firms, the market becomes more competitive; and this lowers the revenue of each firm. If A_k is too low in a country, then the expected revenue is less than the fixed entry cost, and so no firm will produce in that sector. If Home's GDP is sufficiently higher than Foreign's, then k_1 still increases as the trade barrier weakens, but k_2 first decreases then increases. This is the result of two counteracting forces. k_1 and k_2 approach each other as B decreases, and they converge to one point in the extreme case where $B = 1$. In this case, there is complete specialization by both countries, as in Dornbusch-Fisher-Samuelson (1977). If Foreign is much smaller than Home, it may need to give up the production in some sectors in order to completely dominate some more sectors. Thus k_2 may move to the left of the point where $B = 1$ ($k_1 = k_2$) at the early stage of trade liberalization, and must move back to the right of it eventually. If we assume that Home is North (with higher GDP) and Foreign is South, then the model predicts that the range of goods that wealthier North produces keeps shrinking as trade is liberalized, but the range of goods in which the South specializes first shrinks then expands. In other words, the zone of complete dominance by North will first expands, then shrinks. We present these results in the following proposition.

Proposition 3 *If the sizes of the two countries are equal, the range of goods in which each country specializes shrinks in the face of trade liberalization. If the sizes of the two countries are not equal,* *the range of goods in which the larger country specializes shrinks upon trade liberalization but the range of goods in which the smaller country specializes first shrinks then expands.*

4 Openness to Trade

In this section, we will analyze how opening trade impacts the economy, e.g. the productivity cutoffs, the mass of producing and exporting firms, as well as consumers' welfare. Before starting the analysis, it will be helpful to list all the related variables of the 3 types of sectors in the following table:

Type	Foreign-dominated	Two-way trade	Home-dominated
	$k < k_1$	$k_1 < k < k_2$	$k > k_2$
$(\overline{\varphi}_{dk})^\gamma$	$\varnothing$	$D_1 \frac{B-B^{-1}}{B-(a_k)^\gamma}$	$D_1 \frac{L+L^*}{L}$
$(\overline{\varphi}_{xk})^\gamma$	$\varnothing$	$D_1 \frac{B-B^{-1}}{B-(a_k)^{-\gamma}} \left(\frac{\phi}{a_k}\right)^\gamma$	$D_1 \frac{f_x}{f} \left(\frac{L+L^*}{L^*}\right)$
$(\overline{\varphi}^*_{dk})^\gamma$	$D_1 \frac{L+L^*}{L^*}$	$D_1 \frac{B-B^{-1}}{B-(a_k)^{-\gamma}}$	$\varnothing$
$(\overline{\varphi}^*_{xk})^\gamma$	$D_1 \frac{f_x}{f} \frac{L+L^*}{L}$	$D_1 \frac{B-B^{-1}}{B-(a_k)^\gamma} (a_k \phi)^\gamma$	$\varnothing$
θ_{dk}	0	$D_2 \frac{BL - \frac{B-(a_k)^\gamma}{B(a_k)^\gamma - 1} L^*}{B-B^{-1}}$	$D_2 L$
θ_{xk}	0	$\left(\frac{\overline{\varphi}_{dk}}{\overline{\varphi}_{xk}}\right)^\gamma \theta_{dk}$	$D_2 \frac{f}{f_x} L^*$
θ^*_{dk}	$D_2 L^*$	$D_2 \frac{BL^* - \frac{B(a_k)^\gamma - 1}{B-(a_k)^\gamma} L}{B-B^{-1}}$	0
θ^*_{xk}	$D_2 \frac{f}{f_x} L$	$\left(\frac{\overline{\varphi}^*_{dk}}{\overline{\varphi}^*_{xk}}\right)^\gamma \theta^*_{dk}$	0
P_k	$(D_2 L)^{\frac{1}{1-\sigma}} a_k B^{\frac{1}{\gamma}} \left(\frac{L}{L+L^*}\right)^{\frac{1}{\gamma}} \frac{1}{\rho A_k \widetilde{\varphi}_{ck}}$	$(D_2 L)^{\frac{1}{1-\sigma}} \frac{1}{\rho A_k \widetilde{\varphi}_{dk}}$	$(D_2 L)^{\frac{1}{1-\sigma}} \frac{1}{\rho A_k \widetilde{\varphi}_{dk}}$
P^*_k	$(D_2 L^*)^{\frac{1}{1-\sigma}} \frac{1}{\rho A^*_k \widetilde{\varphi}^*_{dk}}$	$(D_2 L^*)^{\frac{1}{1-\sigma}} \frac{1}{\rho A^*_k \widetilde{\varphi}^*_{dk}}$	$(D_2 L^*)^{\frac{1}{1-\sigma}} \frac{B^{\frac{1}{\gamma}}}{a_k} \left(\frac{L^*}{L+L^*}\right)^{\frac{1}{\gamma}} \frac{1}{\rho A^*_k \widetilde{\varphi}^*_{ck}}$

Table 1: Solution to the System

$$D_1 = \frac{\sigma - 1}{\gamma - \sigma + 1} \cdot \frac{f}{\delta f_e}; \quad D_2 = \frac{\gamma - \sigma + 1}{\gamma} \cdot \frac{b_k}{\sigma f}; \quad (a_{k_1})^\gamma = \frac{B\left(\frac{L}{L^*} + 1\right)}{B^2 \frac{L}{L^*} + 1}; \quad (a_{k_2})^\gamma = \frac{B^2 \frac{L^*}{L} + 1}{B\left(\frac{L^*}{L} + 1\right)}$$

4.1 Impacts on productivity cutoffs

In this subsection, we analyze how trade affects the productivity cutoffs from two aspects: within sector and across sectors. First, we look at how trade integration changes the cutoffs within a certain sector. As a result, we find that the impact of trade integration on cutoffs in Melitz (2003) holds in all sectors. Then we compare the cutoffs across sectors upon trade integration. We add a subscript c to all the parameters pertinent to autarky (c=closed economy). It has been shown in Section 2 that the autarky producing cutoff in Home and Foreign is given by $(\overline{\varphi}_{ck})^{\gamma} = (\overline{\varphi}^{*}_{ck})^{\gamma} = \left(\frac{\sigma-1}{\gamma-\sigma+1}\right)\frac{f}{\delta f_e} = D_1$. **If both countries produce**, then the equilibrium operating cutoffs are given by (17) and (18). As $(a_k)^{\gamma} \in \left(\frac{1}{B}, B\right)$, we have $\overline{\varphi}_{dk} > \overline{\varphi}_{ck}$ and $\overline{\varphi}^{*}_{dk} > \overline{\varphi}^{*}_{ck}$.

Recall that **if only one country produces**, the equilibrium operating cutoffs are given by:

$$(\overline{\varphi}_{dk})^{\gamma} = \frac{L+L^*}{L} D_1 > (\overline{\varphi}_{ck})^{\gamma} \quad \text{if only Home produces}$$
$$(\overline{\varphi}^{*}_{dk})^{\gamma} = \frac{L+L^*}{L^*} D_1 > (\overline{\varphi}^{*}_{ck})^{\gamma} \quad \text{if only Foreign produces}$$

Hence, the least productive firms in all sectors will exit the market after trade integration. As a result, resources will be reallocated to the most productive firms. Furthermore, $\overline{\varphi}_{dk} > \overline{\varphi}_{ck}$, implies that $\widetilde{\varphi}_{dk} > \widetilde{\varphi}_{ck}$, and $\overline{\varphi}^{*}_{dk} > \overline{\varphi}^{*}_{ck}$ implies that $\widetilde{\varphi}^{*}_{dk} > \widetilde{\varphi}^{*}_{ck}$. Therefore, the average productivity in any sector k is higher under trade integration than in autarky. Tus we generalize Melitz's result to a setting where there are endogenous intra-industry trade and inter-industry trade.

In the closed economy, the operating cutoffs are identical across sectors. However, this result does not hold any more in the open economy. In the sectors where both countries produce, the equilibrium operating cutoff is an increasing function of the sectoral comparative advantage. More precisely, as a_k increases, $\overline{\varphi}_{dk}$ rises but $\overline{\varphi}^{*}_{dk}$ falls. Following the free entry conditions (15) and (16), $\overline{\varphi}_{xk}$ falls but $\overline{\varphi}^{*}_{xk}$ rises. Moreover, Home is more competitive in sector k ($a_k > 1$) *iff* $\overline{\varphi}^{*}_{xk} > \overline{\varphi}_{xk} > \overline{\varphi}_{dk} > \overline{\varphi}^{*}_{dk}$. Thus, we have

Proposition 4 *In sectors where both countries produce, for a given country, a sector with stronger comparative advantage has lower probability of successful entry but higher fraction of surviving firms that export.*

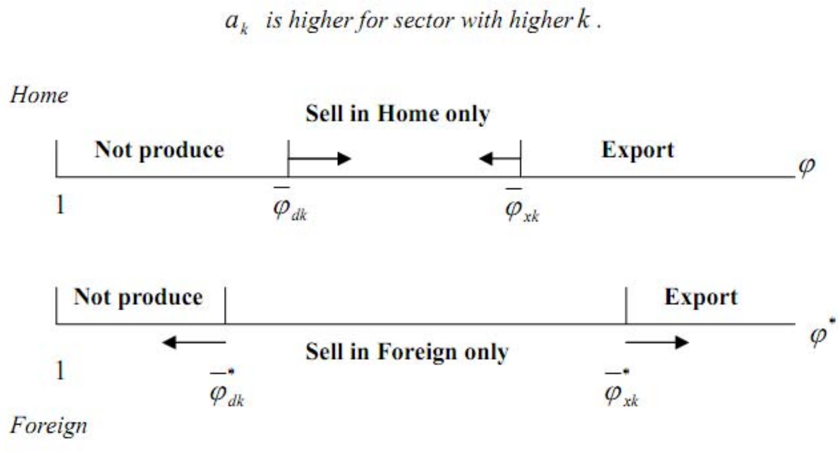

Figure 2: The effects of an increase in comparative advantage of Home on productivity cutoffs

4.2 Impacts on the masses of firms

In this subsection, we mainly focus on how trade will affect the mass of firms within and across sectors. As in the previous subsection, a subscript c denotes all the variables referring to the closed economy.

Recall that if both countries produce, then

$$\theta_{dk} = D_2 \left[\frac{BL - \frac{B-(a_k)^\gamma}{B(a_k)^\gamma - 1} L^*}{B - B^{-1}} \right] < D_2 L = \theta_{ck}; \qquad \theta^*_{dk} = D_2 \left[\frac{BL^* - \frac{B(a_k)^\gamma - 1}{B-(a_k)^\gamma} L}{B - B^{-1}} \right] < D_2 L^* = \theta^*_{ck}.$$

If only one country produces, then

$$\theta_{dk} = \frac{b_k L}{\sigma f}\left(\frac{\gamma - \sigma + 1}{\gamma}\right) = D_2 L = \theta_{ck} \quad \text{if only Home produces}$$
$$\theta^*_{dk} = \frac{b_k L^*}{\sigma f}\left(\frac{\gamma - \sigma + 1}{\gamma}\right) = D_2 L^* = \theta^*_{ck} \quad \text{if only Foreign produces}$$

We sumarize the above findings in the following proposition:

Proposition 5 *In sectors where only one country produces under trade, the number of firms in that country is the same as in autarky. In sectors where both countries produce under trade, the number of firms in each country decreases after opening up to trade.*

When both countries produce after trade, we have $\frac{\theta_{dk}}{b_k} = \left(\frac{\gamma-\sigma+1}{\gamma\sigma f}\right)\left[\frac{BL - \frac{B-(a_k)^\gamma}{B(a_k)^\gamma-1}L^*}{B-B^{-1}}\right]$. As $\frac{B-(a_k)^\gamma}{B(a_k)^\gamma-1}$ decreases with a_k, we have the following proposition:

Proposition 6 *In sectors where both countries produce under trade, the number of firms per expenditure share increases with the strength of comparative advantage for any given country.*

4.3 Impacts on welfare

If a sector in which **both countries produce**, i.e. when $(a_k)^\gamma \in \left(\frac{B\left(\frac{L}{L^*}+1\right)}{B^2\frac{L}{L^*}+1}, \frac{B^2\frac{L^*}{L}+1}{B\left(\frac{L^*}{L}+1\right)}\right)$, then we can write Home's aggregate price index in the sector as:

$$P_k = (\theta_{dk} + \theta^*_{xk})^{\frac{1}{1-\sigma}} p_{dk}(\widetilde{\varphi}_k) = \left(\theta_{dk} + \theta^*_{xk}\frac{f_x}{f}\right)^{\frac{1}{1-\sigma}} p_{dk}(\widetilde{\varphi}_{dk})$$

Substituting the equilibrium values of θ_{dk}, θ^*_{xk}, θ^*_{dk}, θ_{xk} into the above equation, we find that $\theta_{dk} + \theta^*_{xk}\frac{f_x}{f} = \theta_{ck}$. Therefore, we can simplify the price index as:

$$P_k = (\theta_{ck})^{\frac{1}{1-\sigma}} \frac{1}{\rho A_k \widetilde{\varphi}_{dk}}$$

Then, Home's real wage in terms of goods in this sector is given by:

$$\frac{1}{P_k} = (\theta_{ck})^{\frac{1}{\sigma-1}} \rho A_k \widetilde{\varphi}_{dk} = \left(\frac{B - B^{-1}}{B - (a_k)^{\gamma}} \right)^{\frac{1}{\gamma}} \frac{1}{P_{ck}} > \frac{1}{P_{ck}} \tag{23}$$

In a sector where **Foreign completely dominates**, i.e. when $(a_k)^{\gamma} \in \left(0, \frac{B\left(\frac{L}{L^*}+1\right)}{B^2 \frac{L}{L^*}+1}\right)$, Home's real wage in terms of goods is given by:

$$\frac{1}{P_k} = (\theta^*_{xk})^{\frac{1}{\sigma-1}} \rho A^*_k \widetilde{\varphi}^*_{xk} \frac{1}{\tau} = a_k^{-1} B^{-\frac{1}{\gamma}} \left(\frac{L + L^*}{L} \right)^{\frac{1}{\gamma}} \frac{1}{P_{ck}} > \frac{1}{P_{ck}} \tag{24}$$

In a sector where **Home completely dominates**, i.e. when $(a_k)^{\gamma} \in \left(\frac{B^2 \frac{L^*}{L}+1}{B\left(\frac{L^*}{L}+1\right)}, \infty \right)$, Home's real wage in terms of goods is given by:

$$\frac{1}{P_k} = (\theta_{dk})^{\frac{1}{\sigma-1}} \rho A_k \widetilde{\varphi}_{dk} = \left(\frac{L + L^*}{L} \right)^{\frac{1}{\gamma}} \frac{1}{P_{ck}} > \frac{1}{P_{ck}} \tag{25}$$

Therefore, the welfare per capita improves after trade integration. The following proposition and Figure 3 summarize the analysis above.

Proposition 7 *Welfare per capita increase in both countries after opening up to trade.*

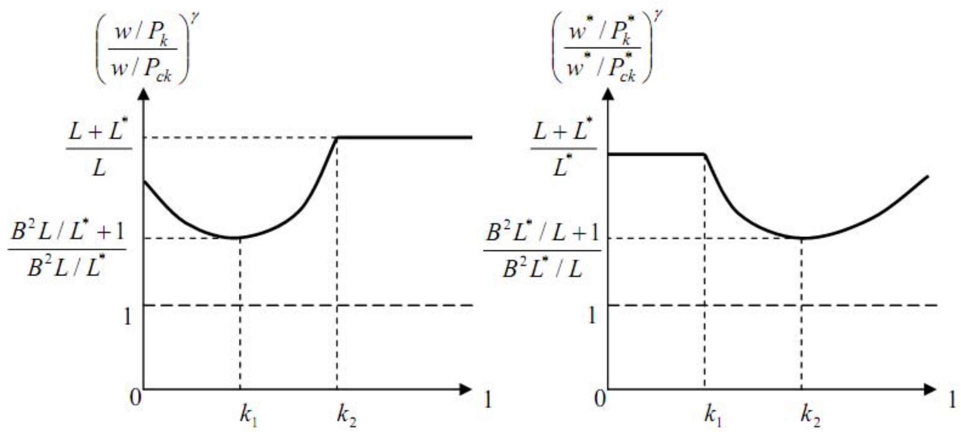

Figure 3: Welfare Impact of Trade Integration ($w = w^* = 1$ by normalization)

In the next few sections, we perform comparative statics concerning the effects of labor supply, technological progress and trade liberalization. Unlike Dornbusch et al. (1977), the relative wage is directly determined by the relative productivity in the homogeneous-good sector in our model.[11] In the following, we turn the focus to the impact of changes in trade barriers, technology and labor supply.

[11] We have also tried the version without the homogeneous sector, and relative wage is determined by balance of trade, as in Dornbusch et al. (1997). In that case, the effect on welfare is highly amibigous, but still the results are very different from Dornbusch et al. (1997), unless we made the fully symmetric assumption like Okubo (2009).

5 Trade liberalization

Trade liberalization is interpreted as a reduction of the iceberg trade cost τ, which lowers $B = \tau^{\gamma}\left(\frac{f_x}{f}\right)^{\frac{\gamma-\sigma+1}{\sigma-1}}$.

As (23) shows, welfare per capita in each country in the sectors where both countries produce just depends on the production cutoff $\overline{\varphi}_{dk}$ and $\overline{\varphi}^{*}_{dk}$ respectively, as they directly determine aggregate price index P_k and P_k^{*}. Differentiating them with respect to B, we have

$$\frac{d\left(\overline{\varphi}_{dk}\right)^{\gamma}}{dB} = \frac{2B^{-1} - \left(1 + B^{-2}\right)(a_k)^{\gamma}}{\left[B - (a_k)^{\gamma}\right]^2}$$
$$\frac{d\left(\overline{\varphi}^{*}_{dk}\right)^{\gamma}}{dB} = \frac{2B^{-1} - \left(1 + B^{-2}\right)(a_k)^{-\gamma}}{\left[B - (a_k)^{-\gamma}\right]^2}$$

which shows that $\overline{\varphi}_{dk}$ increases with B (and so does $\overline{\varphi}^{*}_{xk}$, according to equation (20)) if and only if $(a_k)^{\gamma} < \frac{2B}{1+B^2}$. Moreover, $\overline{\varphi}^{*}_{dk}$ increases with B (and so does $\overline{\varphi}_{xk}$, according to equation (19)) if and only if $(a_k)^{\gamma} > \frac{1+B^2}{2B}$.[12] Comparing $(a_{k_1})^{\gamma}$and $(a_{k_2})^{\gamma}$ with these two thresholds, we will see that the pattern of specialization (which is determined by the values of k_1 and k_2) also depends on the relative size of the two countries.

To evaluate the welfare impacts of trade liberalization, refer to equations (23) to (25) and to Appendix C. Figure 4 shows the signs of the welfare effect of trade liberalization in different sectors of Home and Foreign, corresponding to different values of L/L^{*}. The upper circle in a region of the diagram indicates the sign of Home's welfare change, and the lower circle indicates the sign of Foreign's welfare change. The diagram is defined by the curves k_1 and k_2 as a function of L/L^{*}, as well as the vertical line corresponding to $(a_k)^{\gamma} = \frac{2B}{B^2+1}$ and $(a_k)^{\gamma} = \frac{B^2+1}{2B}$. As B decreases, k_1 increases, k_2 first decreases then increases, $(a_k)^{\gamma} = \frac{2B}{B^2+1}$ increases while $(a_k)^{\gamma} = \frac{B^2+1}{2B}$ decreases. Depending on the range of $[a_0,\, a_1]$ and the value of L/L^{*}, it is possible that only a subset of the zones shown in Figure 4 is included in $k \in [0, 1]$ for any given value of L/L^{*}.

Figure 4 can be summarized by the following three propositions:

Proposition 8 *When the two country have the same size or are sufficiently similar technologically in the sense that* $\frac{2B}{1+B^2} < (a_k)^{\gamma} < \frac{1+B^2}{2B}$ *for all* k*, trade liberalization weakly improves the welfare per capita in both countries.*

Proposition 9 *Suppose Home is larger than Foreign. In the sectors where Home has the strongest comparative disadvantage but still produces, there is an anti-Melitz effect in the sense that* $\overline{\varphi}_{dk}$

[12] An increase in B as a result of an increase in τ or an increase in f_x/f would both lead to an increase in ϕ. Therefore, according to equation (20), an increase in B as a result of an increase in τ or f_x/f leads to increases in $\overline{\varphi}^{*}_{x,k}$ and $\overline{\varphi}_{d,k}$. Similarly, according to (19), an increase in B as a result of an increase in τ or f_x/f leads to increases in $\overline{\varphi}_{x,k}$ and $\overline{\varphi}^{*}_{d,k}$, following the same logic.

decreases while $\overline{\varphi}_{xk}$ increases in the face of trade liberalization, leading to welfare losses in these sectors. Foreign, the smaller country, will never lose from trade liberalization as it will never experience any anti-Melitz effect.

The last proposition deserves more discusssion, as it highlights the most important result of this paper. If Home is the larger country, the sectors in which it will lose from trade are defined by $\left\{k \mid \ (a_{k_1})^\gamma < (a_k)^\gamma < \min\{\frac{2B}{1+B^2}, (a_{k_2})^\gamma\}\right\}$. In other words, these are sectors where the larger country has strong comparative disadvantage yet still produces. In fact, there is an **anti-Melitz effect** in such circumstance, i.e. $\overline{\varphi}_{dk}$ decreases while $\overline{\varphi}_{xk}$ increases in the face of trade liberalization, leading to a decrease in the average productivity of firms serving the Home market, thus lowering welfare. We can explain this phenomenon by decomposing the total effect of trade liberalization into three effects:

1. The DFS effect (inter-sectoral resource allocation effect as B decreases)— trade liberalization leads to resources in Home (as well in Foreign) being re-allocated away from the sectors in which it has comparative disadvantage to the sectors in which it has comparative advantage. Define n_k and n_{ck} as the mass of potential entrants in the open economy and autarky respectively. Then $\theta_{dk} = n_k \left[1 - G\left(\overline{\varphi}_{dk}\right)\right]$ and $\theta_{ck} = n_{ck} \left[1 - G\left(\overline{\varphi}_{ck}\right)\right]$. The inter-sectoral resource allocation explains why, in the sectors in which Home has the strongest comparative disadvantage, the mass of potential Home entrants (n_k) decreases, while, in the same sectors, the mass of potential Foreign entrants (n_k^*) increases.[13] As n_k^* increases, Foreign's market becomes more competitive (This is because of that there will be more firms in Foreign) and so $r_{xk}(\varphi)$ decreases for all φ. **This creates pressure for an increase in $\overline{\varphi}_{xk}$** (i.e. only the more productive firms can profitably export now). As n_k decreases, θ_{dk} also decreases. This leads to the expansion of the sizes of the surviving Home firms. Thus, $r_{dk}(\varphi)$ increases for all φ. **This creates pressure for a decrease in $\overline{\varphi}_{dk}$** as some less productive firms which were expected to be unprofitable before can be expected to be profitable now. In other words, the exporting firms in Home, which are most productive, have to shrink, and so they release resources to the less productive firms. The least productive surviving firms in Home would expand, and the marginal firm that were not profitable before now become profitable.[14]

2. The Melitz effect (intra-sectoral resource allocation effect as B decreases). Here, we ignore the DFS effect, i.e. assuming that the masses of potential entrants n_k and n_k^* are fixed. That's to say, the expected toughness of competition for an exporting firm is unchanged. As a result, the export revenue of a typical exporting firm will increase as trade cost falls. **This creates**

[13] Note that if a_k is constant for all k, then n_k and n_k^* do not change for any k in the face of trade liberalization. As a_k deviates from being a constant, the DFS effect kicks in. In this cae, under trade liberalization, n_k increases (decreases) for the sectors in which Home has comparative advantage (disadvantage). The threshold value of k that demarcates the switching of an increase in n_k to a decrease is higher as L/L^* rises. This is again the home market effect of Krugman (1980).

[14] To see the effect more starkly, consider the case when L/L^* is very large. In this case, in these sectors, Home has more firms while Foreign has fewer firms. The market share of Foreign's firms in these sectors cannot be too high as Foreign's resources (L^*) is too small compared with Home's resources (L). Therefore, a decrease in n_k (as well as θ_{dk}) leads to an increase in the size and revenue of each Home firm that remains. Therefore $r_{dk}(\varphi)$ increases for all φ.

pressure for both $\overline{\varphi}_{xk}$ and $\overline{\varphi}^*_{xk}$ to decrease. Meanwhile, this will force the least productive firms in each country to exit (as there are more firms exporting to the domestic market). **This creates pressure for both $\overline{\varphi}_{dk}$ and $\overline{\varphi}^*_{dk}$ to increase.** In the sectors in which Home has the strong comparative disadvantage, the DFS effect counteracts the Melitz effect in Home. This is because, the Melitz effect causes trade liberalization to reduce Home firms' disadvantage in selling to Foreign's market. The DFS effect causes trade liberalization to increase the disadvantage of Home firms in selling to Foreign (as n_k decreases and n^*_k increases). If the DFS effect dominates, we will have an anti-Melitz effect. This will be the case in the sectors where Home has very strong comparative disadvantage yet still produce. For Foreign, the DFS effect reinforces the Melitz effect, so there cannot be anti-Melitz effect for the small country.

3. Home market effect (Krugman 1980) — the fact that there exists sectors in which the larger country (Home) has very strong comparative disadvantage yet still produces is because Home has a large demand for the differentiated goods and so it attracts Home firms to produce there to serve the large market while saving the transportation cost. This is the home market effect. In such sectors, the DFS effect dominates the Melitz effect, leading to the anti-Melitz effect in Home. This home market effect explains why there exist such sectors in the larger country only. The fact that the threshold a_k that demarcates a switching of the dominance of DFS effect versus the Melitz effect is independent of L/L^*, together with the fact that the set of goods produced by Home expands beyond this threshold a_k as L/L^* increases, explains why there exist sectors in which the DFS effect dominates the Melitz effect in the large country.[15]

If Home is sufficiently larger than Foreign, and Home's Ricardian technological levels are sufficiently lower than those of Foreign in the sectors where it has the strongest comparative disadvantage but Home still produces in those sectors, then Home will unambiguously lose from trade liberalization. For example, when $B = 2$, $L/L^* = 6$, $\gamma = 1.05$ (therefore $a_{k_1} = 0.5$ and $a_{k_2} = 0.7258$), and suppose $a_0 = 0.52$ (and therefore $k_1 < 0$, which means that there does not exist any sector in which Foreign completely dominates). Then, Home will unambiguously lose from trade liberalization, as it loses in the sectors where $k \in [0, k_2]$, and does not gain or lose in the sectors where $k \in [k_2, 1]$

[15] The fact that the set of differentiated goods produced by Home increases (i.e. k_1 decreases) while the set of differentiated goods produced by Foreign decreases (i.e. k_2 decreases) as L/L^* increases is also due to the home market effect.

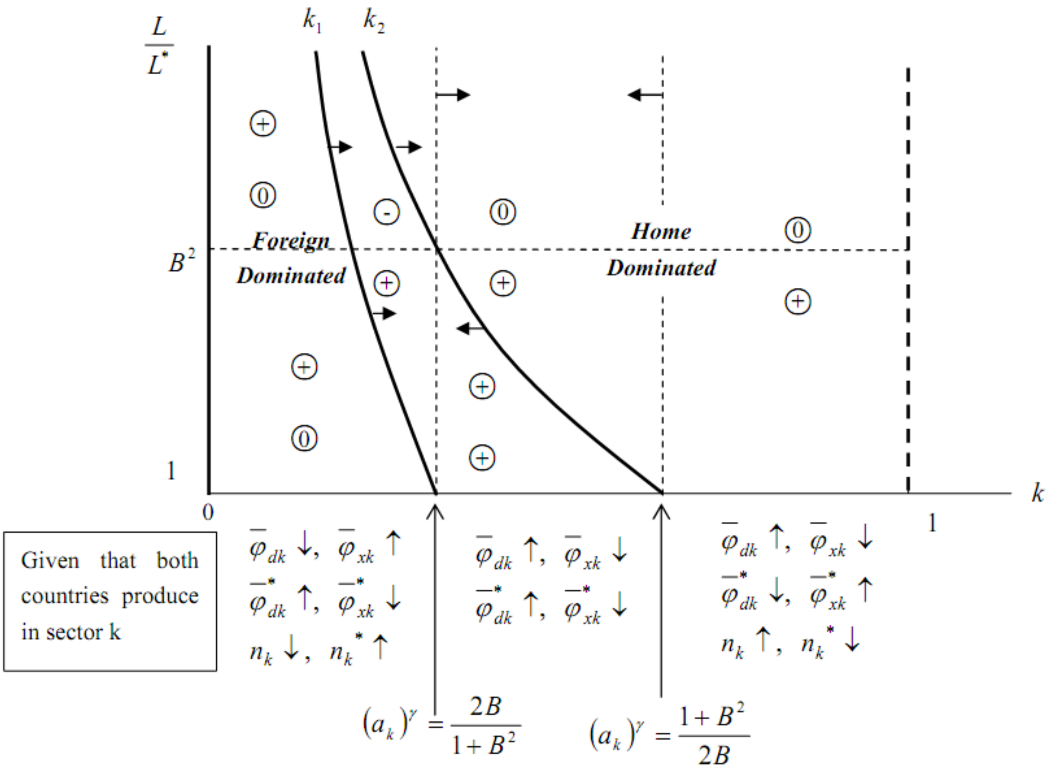

Figure 4. Welfare effects of Trade Liberalization (reduction of B).
In each cell, the upper circle indicates the welfare change of Home
and the lower circle indicates the welfare change of Foreign.
The thick-head arrows indicate the movement of lines as B falls.

6 Technological Progress

Technological progress will change not only the absolute advantage, but also the comparative advantage of these two countries. The pattern of specialization and trade as well as consumer welfare are affected.

We first consider a **uniform technological progress** in all differentiated-goods sectors, which means that A_k increases by the same proportion for all differentiated-goods sector k in Home. This can be caused by the development or adoption of a set of general-purpose technology in Home, or improvement in skills of the Home workers. To evaluate the welfare impacts of trade liberalization, refer to equations (23) to (25) and to Appendix D. In sectors where both countries produce, as the wage ratio is not affected, while A_k/A_k^* increases, it is clear that $\overline{\varphi}_{dk}$ will increase and $\overline{\varphi}^*_{dk}$ will fall. As a consequence, Home's real wage $1/P_k$ increases and Foreign's real wage $1/P_k^*$ decreases in this sector. For the Home-dominated sectors, $1/P_k$ increases due to technological progress, though the production cutoff is unchanged. Moreover, $1/P_k^*$ also increases. For a Foreign-dominated sector, both $1/P_k$ and $1/P_k^*$ remain unchanged. The above findings are summarized in the following proposition and Figure 5.[16] The detailed proof is given in the Appendix.

[16] The P_{ck} and P^*_{ck} in the figure refer to the price index of sector k in closed autarky economy, before the uniform

Proposition 10 *A uniform technological progress in all differentiated-goods sectors in Home benefits the consumers there but the welfare implications for the consumers in Foreign is ambiguous. Equiproportional technological progress in both countries benefit both countries.*

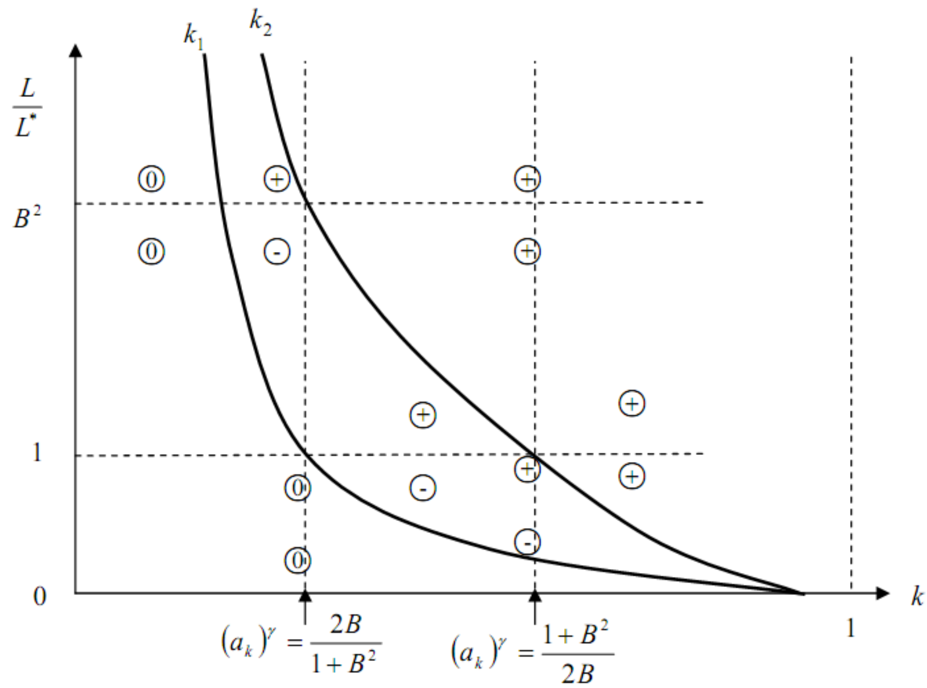

Figure 5: Welfare Impacts of Home's Uniform Technological Progress

In each cell, the upper circle indicates the welfare change of Home

and the lower circle indicates the welfare change of Foreign

From the above proposition, we know that if there is technological progress in both countries, we can decompose the total effect into an equiproportional technological progress in both plus a technological progress only in the country with faster progress. If technological improvement happens in both countries, it is clear that consumers in the country whose technology grows faster will gain, as the two effects reinforce each other. But the welfare impact on in the other country is ambiguous, as the two effects counter each other. It will depends on which of the above two effects dominates.

7 Change in labor supply

Without losing generality, suppose that L increases, which leads both $(a_{k_2})^\gamma = \frac{B^2\frac{L^*}{L}+1}{B\left(\frac{L^*}{L}+1\right)}$ and $(a_{k_1})^\gamma = \frac{B\left(\frac{L}{L^*}+1\right)}{B^2\frac{L}{L^*}+1}$ to decrease. As a result, k_1 and k_2 both fall. To evaluate the welfare im-

technology progress. Then they serve as benchmarks for P_k and P_k^* respectively, and make it easy to compare the welfare impact of the technology progress.

pacts of trade liberalization, refer to equations (23) to (25) and to Appendix E. As shown in the Appendix, Home real wage will always increase, and Foreign real wage will keep unchanged in the sectors where both countries produce and increase in other sectors. The result is summarized in Figure 6. Thus, we have the following:

Proposition 11 *An increase in labor supply of one country will benefit the workers in both countries.*

It is not surprising that one country can gain from its population growth, but it is interesting that workers in the other country also can benefit from it. This gain mainly come from the sectors where only one country produces, as the enlarged Home labor supply can provide not only more supply of the goods in the Home-dominated sector, but also more demand for the goods in the Foreign-dominated sectors. For the sectors in which both countries produce, this gain will only go to the Home country.

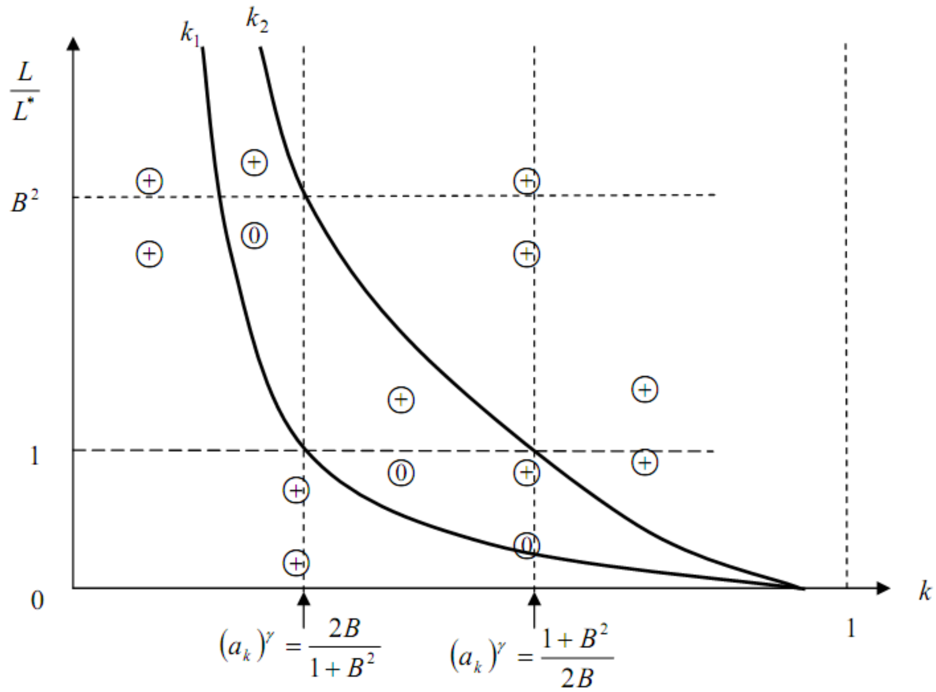

Figure 6: Welfare Impacts of an increase in Home's labor supply
In each cell, the upper circle indicates the welfare change of Home
and the lower circle indicates the welfare change of Foreign

8 Can firms that sell domestically be more productive than firms that don't?

So far, we simplified our analysis by assuming that $\frac{f_x}{f} > \max\{\frac{L}{L^*}, \frac{L^*}{L}\}$ so as to exclude the possibility that some firms only export but do not serve the domestic market. In fact, our model can accommodate this possibility, which is consistent with the recent finding based on Chinese data by Lu (2010), that in some industries, the exporters in China have a relative lower average productivity compared with the firms that serve the domestic market.

First of all, we adopt the assumption that a firm needs to incur a market entry cost if it enters a market, be it domestic or foreign. In this case, f and f_x will stand for the amortized fixed market-entry cost per period plus the overhead cost per period for serving the domestic market and foreign market respectively.[17] In this case, we can relax the assumption $\frac{f_x}{f} > \max\{\frac{L}{L^*}, \frac{L^*}{L}\}$ (only $\tau^{\sigma-1} f_x > f$ is needed), and all of our propositions still hold. The main point is that it is possible that for some sector k, we have $\overline{\varphi}_{dk} > \overline{\varphi}_{xk}$. Once we have this outcome, we can have the situation where some firms in the sectors where a country has the strongest comparative advantage may only export; thus in these sectors the firms that only export are less productive than those that also serve the domestic market. This is also consistent with Dan Lu's finding that in labor-intensive sectors, firms that serve domestic market have higher productivity than exporters, as China has comparative advantage in the labor-intensive industries.

Detailed calculation is given in the appendix. We state our result in

Proposition 12 *If the fixed entry cost for exporting is not too high, then in the sectors where a country has the strongest comparative advantage, the firms that do not serve the domestic market can have a lower average productivity than those that do.*

9 Conclusion

In this paper, we merge the heterogenous firm model of Melitz (2003) with the Ricardian model of Dornbusch et al. (1977) to explain how the pattern of international specialization and trade is determined by the interaction of comparative advantage, economies of scale, country sizes and trade barriers. The model is able to capture the existence of inter-industry trade and intra-industry trade in a single unified framework. It explains how trade openness affects the pattern of international specialization and trade. It generalizes Melitz's firm selection effect in the face of trade liberalization to a setting where the patterns of inter-industry trade and intra-industry are endogenous. Home technological progress always benefits Home but it may lower the welfare of Foreign, especially when the sets of goods produced by the two countries overlap a lot (i.e. both of them do not specialize too much). This is because it lowers the competitiveness of Foreign's firms in the two-way trade

[17] This is different from the original assumption in Melitz (2003) as well in this paper so far.

sectors which in turn lowers Foreign's welfare in those sectors. An increase in labor supply in a country, however, always benefits both countries.

Although trade openness is proved to be unambiguously welfare-improving in both countries, trade liberalization can lead to an anti-Melitz effect in the larger country if it is sufficiently uncompetitive in the sectors in which it has the strongest comparative disadvantage but in which it still produces. In this case, the operating productivity cutoff is lowered while the exporting cutoff increases in the face of trade liberalization. This is because the DFS effect dominates the Melitz effect in these sectors. Consequently, the larger country can lose from trade liberalization. The fact that only the larger country can lose from trade liberalization but not the smaller country can be attributed to the home market effect.

Finally, if the fixed entry cost for exporting is not too high compared with the fixed of entry for serving the domestic market, then in the sectors where a country has the strongest comparative advantage, the firms that do not serve the domestic market can have a lower average productivity than those that do. This is analogous to Lu's (2010) finding based on Bernard, Redding and Schott's (2007) framework of Heckscher-Ohlin model with heterogeneous firms.

Appendixes

A Solving for the System

In this appendix, we will show how to solve the model for the sectors where both countries produce. In other words, we solve for ($\overline{\varphi}_{dk}$, $\overline{\varphi}^*_{dk}$, $\overline{\varphi}_{xk}$, $\overline{\varphi}^*_{xk}$, θ_{dk}, θ^*_{dk}) from the system constituted of the four zero cutoff profit conditions and two free entry conditions. Combining the two zero cutoff conditions for firms serving the Home market, (11) and (14), we have

$$\frac{\overline{\varphi}^*_{xk}}{\overline{\varphi}_{dk}} = a_k \phi \tag{26}$$

Similarly, combining those for firms serving Foreign's market, (12) and (13), we can get

$$\frac{\overline{\varphi}_{xk}}{\overline{\varphi}^*_{dk}} = \frac{\phi}{a_k} \tag{27}$$

where we recall that $\phi = \tau \left(\frac{f_x}{f}\right)^{\frac{1}{\sigma-1}}$.

Equations (26), (27), and the FE conditions (15), and (16) now form a system of four equations and four unknowns, $\overline{\varphi}_{dk}$,$\overline{\varphi}_{xk}$,$\overline{\varphi}^*_{dk}$ and $\overline{\varphi}^*_{xk}$. Solving, we obtain (17), (18), (19) and (20).

Then recall that the aggregate price indexes are given by $P_k = \theta_k^{\frac{1}{1-\sigma}} p_{dk}(\widetilde{\varphi}_k)$ and $P^*_k = (\theta^*_k)^{\frac{1}{1-\sigma}} p^*_{dk}(\widetilde{\varphi}^*_k)$. Substituting these price indexes into Zero Cutoff Conditions (11) and (12), and with the help of equation (9) and (10), we have

$$\sigma f = \frac{b_k L}{\theta_k} \left(\frac{\overline{\varphi}_{dk}}{\widetilde{\varphi}_k}\right)^{\sigma-1} = \left(\frac{\gamma - \sigma + 1}{\gamma}\right) \cdot \frac{b_k L}{\theta_{dk} + \theta^*_{xk}\frac{f_x}{f}} \tag{28}$$

$$\sigma f = \frac{b_k L^*}{\theta^*_k} \left(\frac{\overline{\varphi}^*_{dk}}{\widetilde{\varphi}^*_k}\right)^{\sigma-1} = \left(\frac{\gamma - \sigma + 1}{\gamma}\right) \cdot \frac{b_k L^*}{\theta^*_{dk} + \theta_{xk}\frac{f_x}{f}} \tag{29}$$

From the equilibrium productivity cutoffs (17) and (18) in both countries, we get

$$\left(\frac{\overline{\varphi}_{dk}}{\overline{\varphi}^*_{dk}}\right)^{\gamma} = \frac{B - (a_k)^{-\gamma}}{B - (a_k)^{\gamma}} \tag{30}$$

Therefore, the number of exporting firms in Home and Foreign are respectively:

$$\theta_{xk} = \left(\frac{\overline{\varphi}_{dk}}{\overline{\varphi}_{xk}}\right)^{\gamma} \theta_{dk} = \left(\frac{a_k}{\phi} \cdot \frac{\overline{\varphi}_{dk}}{\overline{\varphi}^*_{dk}}\right)^{\gamma} \theta_{dk} \tag{31}$$

$$\theta^*_{xk} = \left(\frac{\overline{\varphi}^*_{dk}}{\overline{\varphi}^*_{xk}}\right)^{\gamma} \theta^*_{dk} = \left(\frac{1}{a_k \phi} \cdot \frac{\overline{\varphi}^*_{dk}}{\overline{\varphi}_{dk}}\right)^{\gamma} \theta^*_{dk} \tag{32}$$

Equations (28), (29), (30), (31), (32) then imply (21) and (22).

θ_{xk} and θ^*_{xk} can be obtained by substituting (30), (21), (22) into (31) and (32) respectively.

B The Rationale for $\tau^{\sigma-1}f_x > f$

In this appendix, we explain why we need the assumption $\tau^{\sigma-1}f_x > f$. If $B > 1$, we restrict $(a_k)^\gamma$ to be within (B^{-1}, B), in order to avoid corner solution. To make sure that there exist some sector k in which only the firms with the higher productivity will export (i.e. $\overline{\varphi}_{dk} < \overline{\varphi}_{xk}$ for some k and $\overline{\varphi}^*_{dk} < \overline{\varphi}^*_{xk}$ for some k), we need $(a_k)^\gamma$ to lie in the interval $\left(\frac{\left(\frac{f_x}{f}+1\right)B}{\frac{f_x}{f}B^2+1}, \frac{\frac{f_x}{f}B^2+1}{\left(\frac{f_x}{f}+1\right)B}\right)$ for some k. For this to be true, we need $\tau^{\sigma-1}f_x > f$. For the case $B < 1$, in order to make $\overline{\varphi}_{dk} < \overline{\varphi}_{xk}$ for some k and $\overline{\varphi}^*_{dk} < \overline{\varphi}^*_{xk}$ for some k, we also need $\frac{\left(\frac{f_x}{f}+1\right)B}{\frac{f_x}{f}B^2+1} > \frac{\frac{f_x}{f}B^2+1}{\left(\frac{f_x}{f}+1\right)B}$, which also implies that $\tau^{\sigma-1}f_x > f$ following similar argument. Hence, we assume that $\tau^{\sigma-1}f_x > f$ in order to guarantee that in both countries there exist some sectors in which some firms produce exclusively for their domestic market in both countries. And interestingly, $\tau^{\sigma-1}f_x > f$ will always imply $B > 1$, which validate our assumption on B.

C Welfare Impact of Trade Liberalization

In this appendix, we will prove how the real wage change after trade liberalization in three cases. Without loss of generality, we assume that $L > L^*$.

1. Foreign-dominated sectors: $k \in (0, k_1)$. The real wage in terms of aggregate goods of sector k in this zone in Home and Foreign are, respectively:

$$\frac{1}{P_k} = (\theta^*_{xk})^{\frac{1}{\sigma-1}} \rho A^*_k \widetilde{\varphi}^*_{xk} \frac{1}{\tau} = \rho A^*_k B^{-\frac{1}{\gamma}} \left(\frac{L+L^*}{L} D_1\right)^{\frac{1}{\gamma}} \left(\frac{\gamma}{\gamma-\sigma+1} D_2 L\right)^{\frac{1}{\sigma-1}}$$
$$\frac{1}{P^*_k} = (\theta^*_{dk})^{\frac{1}{\sigma-1}} \rho A^*_k \widetilde{\varphi}^*_{dk} = \rho A^*_k \left(\frac{L+L^*}{L^*} D_1\right)^{\frac{1}{\gamma}} \left(\frac{\gamma}{\gamma-\sigma+1} D_2 L^*\right)^{\frac{1}{\sigma-1}}$$

Since trade liberalization will increase $\frac{1}{P_k}$ as B falls, the real wage in Home will be improved. However, the real wage in Foreign, $\frac{1}{P^*_k}$, is not related to the trade barriers. That's, trade liberalization does not affect the real wage in Foreign.

2. Both countries produce: $k \in (k_1, k_2)$. The real wage in Home and Foreign are equal to:

$$\frac{1}{P_k} = (\theta_{ck})^{\frac{1}{\sigma-1}} \rho A_k \widetilde{\varphi}_{dk} = \rho A_k \left(D_1 \frac{B-B^{-1}}{B-(a_k)^\gamma}\right)^{\frac{1}{\gamma}} \left(\frac{\gamma}{\gamma-\sigma+1} D_2 L\right)^{\frac{1}{\sigma-1}}$$
$$\frac{1}{P^*_k} = (\theta^*_{ck})^{\frac{1}{\sigma-1}} \rho A^*_k \widetilde{\varphi}^*_{dk} = \rho A^*_k \left(D_1 \frac{B-B^{-1}}{B-(a_k)^{-\gamma}}\right)^{\frac{1}{\gamma}} \left(\frac{\gamma}{\gamma-\sigma+1} D_2 L^*\right)^{\frac{1}{\sigma-1}}$$

This zone is divided into two cases:

(a) Scenario A: $(a_k)^\gamma < \frac{2B}{1+B^2}$.

Note that $\frac{B-B^{-1}}{B-(a_k)^\gamma}$ decreases but $\frac{B-B^{-1}}{B-(a_k)^{-\gamma}}$ increases as trade barrier B falls, as $(a_k)^\gamma < \frac{2B}{1+B^2}$. Therefore, the real wage in Home will decline, but the real wage in Foreign rises.

(b) Scenario B: $(a_k)^\gamma \in \left(\frac{2B}{1+B^2}, \frac{1+B^2}{2B}\right)$.

Since both $\frac{B-B^{-1}}{B-(a_k)^\gamma}$ and $\frac{B-B^{-1}}{B-(a_k)^{-\gamma}}$ increase as trade barrier B falls when $(a_k)^\gamma \in \left(\frac{2B}{1+B^2}, \frac{1+B^2}{2B}\right)$, the real wages in both countries increase in this zone.

3. Home-dominated sectors: $k \in (k_2, 1)$. Real wages are given by

$$\frac{1}{P_k} = (\theta_{dk})^{\frac{1}{\sigma-1}} \rho A_k \widetilde{\varphi}_{dk} = \rho A_k \left(\frac{L+L^*}{L} D_1\right)^{\frac{1}{\gamma}} \left(\frac{\gamma}{\gamma-\sigma+1} D_2 L\right)^{\frac{1}{\sigma-1}}$$
$$\frac{1}{P_k^*} = (\theta_{xk})^{\frac{1}{\sigma-1}} \rho A_k \widetilde{\varphi}_{xk} \frac{1}{\tau} = \rho A_k B^{-\frac{1}{\gamma}} \left(\frac{L+L^*}{L^*} D_1\right)^{\frac{1}{\gamma}} \left(\frac{\gamma}{\gamma-\sigma+1} D_2 L^*\right)^{\frac{1}{\sigma-1}}$$

It is clear that real wage in Home is unchanged but that in Foreign increases as B falls.

D Welfare Impact of Uniform Technological Progress

In this appendix, we analyze how the real wage changes when there is a uniform technological progress in all differentiated-goods sectors in Home. Assume that the labor productivity in all differentiated-goods sectors become μ times the original values, where $\mu > 1$.

1. In zone $k \in (0, k_1)$, all differentiated-goods are produced by Foreign. As shown in the last section, the real wages in Home and Foreign are, respectively:

$$\frac{1}{P_k} = \rho A_k^* B^{-\frac{1}{\gamma}} \left(\frac{L+L^*}{L} D_1\right)^{\frac{1}{\gamma}} \left(\frac{\gamma}{\gamma-\sigma+1} D_2 L\right)^{\frac{1}{\sigma-1}}$$
$$\frac{1}{P_k^*} = \rho A_k^* \left(\frac{L+L^*}{L^*} D_1\right)^{\frac{1}{\gamma}} \left(\frac{\gamma}{\gamma-\sigma+1} D_2 L^*\right)^{\frac{1}{\sigma-1}}$$

Thus, the real wage in this zone in either country is not affected by the technological progress.

2. In zone $k \in (k_1, k_2)$, the real wages, as given in the last section, are:

$$\frac{1}{P_k} = \rho A_k \left(D_1 \frac{B-B^{-1}}{B-(a_k)^\gamma}\right)^{\frac{1}{\gamma}} \left(\frac{\gamma}{\gamma-\sigma+1} D_2 L\right)^{\frac{1}{\sigma-1}}$$
$$\frac{1}{P_k^*} = \rho A_k^* \left(D_1 \frac{B-B^{-1}}{B-(a_k)^{-\gamma}}\right)^{\frac{1}{\gamma}} \left(\frac{\gamma}{\gamma-\sigma+1} D_2 L^*\right)^{\frac{1}{\sigma-1}}$$

Since both A_k and $\frac{B-B^{-1}}{B-(a_k)^\gamma}$ increase with technological progress, the real wage in Home increases. However, the real wage in Foreign decreases since $\frac{B-B^{-1}}{B-(a_k)^{-\gamma}}$ decreases with technological progress.

3. In zone $k \in (k_2, 1)$, all differentiated-goods are produced by Home. The real wages, as given in the last section, are

$$\frac{1}{P_k} = \rho A_k \left(\frac{L+L^*}{L} D_1\right)^{\frac{1}{\gamma}} \left(\frac{\gamma}{\gamma-\sigma+1} D_2 L\right)^{\frac{1}{\sigma-1}}$$
$$\frac{1}{P_k^*} = \rho A_k B^{-\frac{1}{\gamma}} \left(\frac{L+L^*}{L^*} D_1\right)^{\frac{1}{\gamma}} \left(\frac{\gamma}{\gamma-\sigma+1} D_2 L^*\right)^{\frac{1}{\sigma-1}}$$

Thus, the real wage in both countries will increase.

4. Finally, the real wage in terms of homogeneous goods is unchanged. Hence, a uniform technological progress in all differentiated-goods will improve the welfare per worker in Home. However, whether the welfare per worker in Foreign increases is ambiguous.

E Welfare Impact of Labor Supply Change

1. In zone $[0, k_1)$, Foreign dominates in these sectors. We have

$$\frac{1}{P_k} = \rho A_k^* B^{-\frac{1}{\gamma}} \left(\frac{L+L^*}{L} D_1\right)^{\frac{1}{\gamma}} \left(\frac{\gamma}{\gamma-\sigma+1} D_2 L\right)^{\frac{1}{\sigma-1}}$$
$$\frac{1}{P_k^*} = \rho A_k^* \left(\frac{L+L^*}{L^*} D_1\right)^{\frac{1}{\gamma}} \left(\frac{\gamma}{\gamma-\sigma+1} D_2 L^*\right)^{\frac{1}{\sigma-1}}$$

An increase in L leads to increases in $\frac{1}{P_k}$ and $\frac{1}{P_k^*}$.

2. In zone (k_1, k_2), both countries produce in these sectors both before and after the technology progress. Hence, the real wage in terms of aggregate goods of sector k in this zone at Home and Foreign are, respectively:

$$\frac{1}{P_k} = \rho A_k \left(D_1 \frac{B-B^{-1}}{B-(a_k)^{\gamma}}\right)^{\frac{1}{\gamma}} \left(\frac{\gamma}{\gamma-\sigma+1} D_2 L\right)^{\frac{1}{\sigma-1}}$$
$$\frac{1}{P_k^*} = \rho A_k^* \left(D_1 \frac{B-B^{-1}}{B-(a_k)^{-\gamma}}\right)^{\frac{1}{\gamma}} \left(\frac{\gamma}{\gamma-\sigma+1} D_2 L^*\right)^{\frac{1}{\sigma-1}}$$

An increase in L leads to an increase in Home's real wage, but has no effect on Foreign real wage.

3. In zone $[k_2, 1]$, Home dominates in these sectors. We have

$$\frac{1}{P_k} = \rho A_k \left(\frac{L+L^*}{L} D_1\right)^{\frac{1}{\gamma}} \left(\frac{\gamma}{\gamma-\sigma+1} D_2 L\right)^{\frac{1}{\sigma-1}}$$
$$\frac{1}{P_k^*} = \rho A_k B^{-\frac{1}{\gamma}} \left(\frac{L+L^*}{L^*} D_1\right)^{\frac{1}{\gamma}} \left(\frac{\gamma}{\gamma-\sigma+1} D_2 L^*\right)^{\frac{1}{\sigma-1}}$$

It is obvious that an increase in L leads to increases in $\frac{1}{P_k}$ as well as $\frac{1}{P_k^*}$.

4. Finally, the real wage in term of the homogenous goods is unchanged. As a result, an increase in labor supply will benefit workers in both countries.

F When the assumption $\frac{f_x}{f} > \max\{\frac{L}{L^*}, \frac{L^*}{L}\}$ is relaxed

In the Home-dominated sectors $k \in [k_2, 1)$, $\overline{\varphi}_{dk} < \overline{\varphi}_{xk}$ is equivalent to $\frac{f_x}{f} > \frac{L^*}{L}$. In the two-way trade sectors $k \in (k_1, k_2)$, $\overline{\varphi}_{dk} < \overline{\varphi}_{xk}$ is equivalent to $(a_k)^\gamma > \frac{\frac{f_x}{f}B^2+1}{\left(\frac{f_x}{f}+1\right)B}$; and $\overline{\varphi}^*_{dk} < \overline{\varphi}^*_{xk}$ is equivalent to $(a_k)^\gamma < \frac{\left(\frac{f_x}{f}+1\right)B}{\frac{f_x}{f}B^2+1}$. In the Foreign-dominated sectors $k \in (0, k_1]$, $\overline{\varphi}^*_{d,k} < \overline{\varphi}^*_{x,k}$ is equivalent to $\frac{f_x}{f} > \frac{L}{L^*}$.

Here we can introduce two new thresholds k_3 and k_4, such that, $(a_{k_4})^\gamma = \frac{\frac{f_x}{f}B^2+1}{\left(\frac{f_x}{f}+1\right)B}$ and $(a_{k_3})^\gamma = \frac{\left(\frac{f_x}{f}+1\right)B}{\frac{f_x}{f}B^2+1}$. It is clear that the assumption $\frac{f_x}{f} > \max\{\frac{L}{L^*}, \frac{L^*}{L}\}$ can ensure that $k_3 < k_1 < k_2 < k_4$, thus $\overline{\varphi}_{dk} < \overline{\varphi}_{xk}$ and $\overline{\varphi}^*_{dk} < \overline{\varphi}^*_{xk}$ in all sectors $k \in [0, 1]$, and all firms either only serve the domestic market, or export and sell to domestic market at the same time. If the assumption $\frac{f_x}{f} > \max\{\frac{L}{L^*}, \frac{L^*}{L}\}$ is relaxed, some Home producers can only sell to the Foreign market if $\frac{f_x}{f} < \frac{L^*}{L}$ or $k_4 < k_2$. Furthermore, some Foreign producers can only sell to the Home market if $\frac{f_x}{f} < \frac{L}{L^*}$ or $k_1 < k_3$.

For example, $\frac{f_x}{f} < \min\{\frac{L}{L^*}, \frac{L^*}{L}\}$, we have $k_1 < k_3 < k_4 < k_2$. In zone $[0, k_1)$, only Foreign firms produce; some of them only export, while others serve both markets. In zone (k_1, k_3), firms in both countries produce; Home firms either serve both markets or domestic market only, while Foreign firms either only export or sell to both markets. In zone (k_3, k_4), firms in both countries produce; they either serve both markets or domestic market only. In zone (k_4, k_2), firms in both countries produce; Home firms either only export or sell to both markets, while Foreign firms either serve both markets or domestic market only. In zone $(k_2, 1]$,only Home firms produce; some them only export, and some serve both markets. As a result, in zone $(k_4, 1]$, the average productivity of exporting firms in Home is less than the firms that serve the domestic market and export; in zone $[0, k_3)$, the average productivity of exporting firms in Foreign is less than the firms that serve the domestic market. The opposite is true for the other zone.

In the sector with $(a_k)^\gamma \in \left(0, \frac{B\left(\frac{L}{L^*}+1\right)}{B^2\frac{L}{L^*}+1}\right)$, only Foreign produces. In the sector with $(a_k)^\gamma \in \left(\frac{B^2\frac{L^*}{L}+1}{B\left(\frac{L^*}{L}+1\right)}, \infty\right)$, only Home produces. In other sectors, both countries produce.

If both countries produce in sector k, we have

1. $(\overline{\varphi}_{dk})^\gamma = D_1\left[\frac{B-B^{-1}}{B-(a_k)^\gamma}\right] < (\overline{\varphi}_{xk})^\gamma = \left(\frac{\phi}{a_k}\right)^\gamma D_1\left[\frac{B-B^{-1}}{B-(a_k)^{-\gamma}}\right]$. That is, in the sector with $(a_k)^\gamma \in \left(0, \frac{\frac{f_x}{f}B^2+1}{\left(\frac{f_x}{f}+1\right)B}\right)$, the operating cutoff is less than the exporting cutoff in Home.

2. $(\overline{\varphi}^*_{dk})^\gamma = D_1\frac{B-B^{-1}}{B-(a_k)^{-\gamma}} < (a_k\phi)^\gamma D_1\left[\frac{B-B^{-1}}{B-(a_k)^\gamma}\right] = (\overline{\varphi}^*_{xk})^\gamma$. That is, in the sector with $(a_k)^\gamma \in \left(\frac{\left(\frac{f_x}{f}+1\right)B}{\frac{f_x}{f}B^2+1}, \infty\right)$, the operating cutoff is less than the exporting cutoff in Foreign.

Consequently, there are four thresholds (k_1, k_2, k_3, k_4) such that $\left((a_{k_1})^\gamma, (a_{k_2})^\gamma, (a_{k_3})^\gamma, (a_{k_4})^\gamma\right) =$

$$\left(\frac{B\left(\frac{L}{L^*}+1\right)}{B^2\frac{L}{L^*}+1}, \frac{B^2\frac{L^*}{L}+1}{B\left(\frac{L^*}{L}+1\right)}, \frac{\left(\frac{f_x}{f}+1\right)B}{\frac{f_x}{f}B^2+1}, \frac{\frac{f_x}{f}B^2+1}{\left(\frac{f_x}{f}+1\right)B}\right).$$

If the assumption $\frac{f_x}{f} > \max\{\frac{L}{L^*}, \frac{L^*}{L}\}$ holds, then $k_3 < k_1 < k_2 < k_4$. Hence, in the sector that both countries produce, the operating cutoff is less than the exporting cutoff in both contries. Furtherly, this assumption guarantee that $\overline{\varphi}_{dk} < \overline{\varphi}_{xk}$in the sector that only Home produce and $\overline{\varphi}^*_{dk} < \overline{\varphi}^*_{xk}$ in the sector that only Foreign produce. Consequently, in all sector, only firms with the higher productivity will export.

Now relax the assumption $\frac{f_x}{f} > \max\{\frac{L}{L^*}, \frac{L^*}{L}\}$ while maintaining the assumption $\tau^{\sigma-1}f_x > f$ (which ensures there exist some sector in which only firms with the higher productivity will export). Without loss of generality, we assume that $L^* < L$, i.e., $\frac{L^*}{L} < \frac{L}{L^*}$. Then there are the following three cases:

(a): When $\frac{L^*}{L} < \frac{f_x}{f} < \frac{L}{L^*}$and $B^2\frac{f_x}{f}\frac{L^*}{L} > 1$, then we have $k_1 < k_3 < k_2 < k_4$.

(b): When $\frac{L^*}{L}< \frac{f_x}{f} < \frac{L}{L^*}$and $B^2\frac{f_x}{f}\frac{L^*}{L} < 1$, then we have $k_1 < k_2 < k_3 < k_4$.

(c): When $\frac{f_x}{f} < \min\{\frac{L}{L^*}, \frac{L^*}{L}\}$, we have $k_1 < k_3 < k_4 < k_2$.

In all those three cases, there are some sectors in which the firms in some country that do not serve the domestic market can have a lower average productivity than those that do. In the following, we focus our attention on case (c). The explanation for case (a) and (b) is similar. The assumptions $\frac{f_x}{f} < \frac{L}{L^*}$ and $\frac{f_x}{f} < \frac{L^*}{L}$ guarantee that $k_1 < k_3$ and $k_4 < k_2$ respectively. Hence, we have $k_1 < k_3 < k_4 < k_2$. In zone (k_3, k_4), we have $\overline{\varphi}_{dk} < \overline{\varphi}_{xk}$ and $\overline{\varphi}^*_{dk} < \overline{\varphi}^*_{xk}$. In zone $(0, k_1)$, only Foreign produces. $\frac{f_x}{f} < \frac{L}{L^*}$ guarantees that $\overline{\varphi}^*_{dk} > \overline{\varphi}^*_{xk}$. In zone (k_1, k_3), both countries produce, and we have $\overline{\varphi}_{dk} < \overline{\varphi}_{xk}$ and $\overline{\varphi}^*_{dk} > \overline{\varphi}^*_{xk}$. In zone (k_4, k_2), both countries produce, and we have $\overline{\varphi}_{dk} > \overline{\varphi}_{xk}$ and $\overline{\varphi}^*_{dk} < \overline{\varphi}^*_{xk}$. In zone $(k_2, 1)$, only Home produce, and $\frac{f_x}{f} < \frac{L^*}{L}$ guarantees that $\overline{\varphi}_{dk} > \overline{\varphi}_{xk}$. **Consequently, in zone $(k_4, 1)$, where Home has comparative advantage, we have $\overline{\varphi}_{dk} > \overline{\varphi}_{xk}$, provided that $\frac{1}{\tau^{\sigma-1}} < \frac{f_x}{f} < \min\{\frac{L}{L^*}, \frac{L^*}{L}\}$.**

The following two appendixes are not intended for publication. They can be put online when the paper is published.

G Derivation of equations (9) and (10)

Let $\overline{\varphi}_{dk}$ and $\overline{\varphi}^*_{dk}$ denote the cutoff productivity of operating firms in Home and Foreign, respectively; $\overline{\varphi}_{xk}$ and $\overline{\varphi}^*_{xk}$ denote the cutoff productivity of exporting firms in Home and Foreign respectively. Then in Home, domestic producers will charge the price of $p_{dk}(j) = \frac{1}{\rho A_k \varphi_k(j)}$ and Foreign's exporting firms will charge the price of $p^*_{xk}(j) = \frac{\tau}{\rho A^*_k \varphi^*_k(j)}$. Also, the productivity of Home's operating firms will follow a distribution Ω with the density function of $\frac{g(\varphi(j))}{1-G(\overline{\varphi}_{dk})}$ on $[\overline{\varphi}_{dk}, \infty)$, and the productivity of Foreign's exporting firms will follow a distribution Ω^* with the density function of $\frac{g(\varphi(j))}{1-G(\overline{\varphi}^*_{dk})}$ on $[\overline{\varphi}^*_{dk}, \infty)$.

For each Foreign's exporting firm, $p^*_{xk}(j) = \frac{\tau}{\rho A^*_k \varphi^*_k(j)} = p_{dk}\left(\tau^{-1}\frac{1}{a_k}\varphi^*_k(j)\right)$. We know that $\tau^{-1}\frac{1}{a_k}\varphi^*_k(j)$ will follow the distribution $\Omega' = \Omega^*\left(\frac{\varphi_k(j)}{\tau^{-1}\frac{1}{a_k}}\right)$. Therefore, similar to the closed economy case, the aggregate price index for goods sold in Home in sector k is given by

$$
\begin{aligned}
P_k &= \left[\int p_{dk}(j)^{1-\sigma} d\left(\theta_{dk}\Omega + \theta^*_{xk}\Omega'\right)\right]^{\frac{1}{1-\sigma}} \\
&= \left[\int \left(\frac{1}{\rho A_k}\right)^{1-\sigma} \varphi_k(j)^{\sigma-1} d\left(\theta_{dk}\Omega + \theta^*_{xk}\Omega'\right)\right]^{\frac{1}{1-\sigma}} \\
&= \left\{\rho A_k \left[\int \varphi_k(j)^{\sigma-1} d\left(\theta_{dk}\Omega + \theta^*_{xk}\Omega'\right)\right]^{\frac{1}{\sigma-1}}\right\}^{-1} \\
&= p_{dk}\left(\left[\int \varphi_k(j)^{\sigma-1} d\left(\theta_{dk}\Omega + \theta^*_{xk}\Omega'\right)\right]^{\frac{1}{\sigma-1}}\right) \\
&= \theta_k^{\frac{1}{1-\sigma}} p_{dk}(\widetilde{\varphi}_k) \quad \text{by the definition of } P_k \text{ given by (8)}
\end{aligned}
$$

It follows that

$$
\begin{aligned}
\widetilde{\varphi}_k &= \theta_k^{\frac{1}{1-\sigma}} \left[\int \varphi_k(j)^{\sigma-1} d\left(\theta_{dk}\Omega + \theta^*_{xk}\Omega'\right)\right]^{\frac{1}{\sigma-1}} \\
&= \left[\frac{1}{\theta_k}\left\langle \theta_{dk}\left(\widetilde{\varphi}_{dk}\right)^{\sigma-1} + \theta^*_{xk}\left(\tau^{-1}\frac{1}{a_k}\widetilde{\varphi}^*_{xk}\right)^{\sigma-1}\right\rangle\right]^{\frac{1}{\sigma-1}}
\end{aligned}
$$

where the second line directly follows from the analogues of equation (5) as explained in footnote **??**. As $r_{dk}(j) = b_k L\left(\frac{p_{dk}(j)}{P_k}\right)^{1-\sigma}$ and $r^*_{xk}(j) = b_k L\left(\frac{p^*_{xk}(j)}{P_k}\right)^{1-\sigma}$, $\widetilde{\varphi}_k$ is just the average productivity of Home's and Foreign's firms that sell in Home, weighted by mass of firms in the source country. Similarly, we get

$$
\widetilde{\varphi}^*_k = \left[\frac{1}{\theta^*_k}\left\langle \theta^*_{dk}\left(\widetilde{\varphi}^*_{dk}\right)^{\sigma-1} + \theta_{xk}\left(\tau^{-1}a_k\widetilde{\varphi}_{xk}\right)^{\sigma-1}\right\rangle\right]^{\frac{1}{\sigma-1}}
$$

H Uniqueness of Equilibrium

In this appendix, we prove that there is a unique equilibrium in our model.

First, we assume that all differentiated goods are produced by Foreign. Then, the real wage in terms of sectoral aggregate good in Home and Foreign, respectively, are:

$$\frac{1}{P_k} = (\theta^*_{xk})^{\frac{1}{\sigma-1}} \rho A^*_k \widetilde{\varphi}^*_{xk} \frac{1}{\tau} = \rho a_k^{-1} A_k B^{-\frac{1}{\gamma}} \left(\frac{L+L^*}{L} D_1\right)^{\frac{1}{\gamma}} \left(\frac{\gamma}{\gamma-\sigma+1} D_2 L\right)^{\frac{1}{\sigma-1}} \tag{33}$$

$$\frac{1}{P^*_k} = (\theta^*_{dk})^{\frac{1}{\sigma-1}} \rho A^*_k \widetilde{\varphi}^*_{dk} = \rho A^*_k \left(\frac{L+L^*}{L^*} D_1\right)^{\frac{1}{\gamma}} \left(\frac{\gamma}{\gamma-\sigma+1} D_2 L^*\right)^{\frac{1}{\sigma-1}} \tag{34}$$

Is it possible for an individual Home firm to engage in production profitably in some zone? If not, then complete Foreign-dominance is an equilibrium; if so, it is not. In the following, we will show that this constitutes an equilibrium only in zone $(0, k_1)$. The operating and exporting cutoff conditions respectively for the individual Home firm are:

$$r_{dk}(\overline{\varphi}_{dk}) = b_k L \left(P_k \rho A_k \overline{\varphi}_{dk}\right)^{\sigma-1} = \sigma f$$
$$r_{xk}(\overline{\varphi}_{xk}) = b_k L^* \left(\frac{P^*_k}{\tau} \rho A_k \overline{\varphi}_{xk}\right)^{\sigma-1} = \sigma f_x$$

According to the above two equations and the real wage equations (33) and (34), we have:

$$\overline{\varphi}_{dk} = a_k^{-1} B^{-\frac{1}{\gamma}} \overline{\varphi}^*_{xk} \left(\frac{f}{f_x}\right)^{\frac{1}{\gamma}}$$
$$\overline{\varphi}_{xk} = a_k^{-1} B^{\frac{1}{\gamma}} \varphi^*_{dk} \left(\frac{f_x}{f}\right)^{\frac{1}{\gamma}}$$

Then, the total profit for the individual Home firm is:

$$\begin{aligned}
(1 - G(\overline{\varphi}_{dk})) \frac{\widetilde{\pi}_k}{\delta} &= \left(\frac{\sigma-1}{\gamma-\sigma+1}\right) \left[\frac{f(\overline{\varphi}_{dk})^{-\gamma} + f_x (\overline{\varphi}_{xk})^{-\gamma}}{\delta}\right] \\
&= \left(\frac{\sigma-1}{\gamma-\sigma+1}\right) \left[\frac{a_k^{\gamma} B (\overline{\varphi}^*_{xk})^{-\gamma} f_x + f a_k^{\gamma} B^{-1} (\overline{\varphi}^*_{dk})^{-\gamma}}{\delta}\right] \\
&= \left(\frac{\sigma-1}{\gamma-\sigma+1}\right) \left[\frac{f(\overline{\varphi}^*_{dk})^{-\gamma} + f_x (\overline{\varphi}^*_{xk})^{-\gamma}}{\delta}\right] \left[\frac{a_k^{\gamma} B (\overline{\varphi}^*_{xk})^{-\gamma} f_x + f a_k^{\gamma} B^{-1} (\overline{\varphi}^*_{dk})^{-\gamma}}{f\left(\overline{\varphi}^*_{dk}\right)^{-\gamma} + f_x \left(\overline{\varphi}^*_{xk}\right)^{-\gamma}}\right] \\
&= f_e a_k^{\gamma} \left[\frac{B^2 \frac{L}{L^*} + 1}{B\left(\frac{L}{L^*} + 1\right)}\right]
\end{aligned}$$

The last equation stems from the free entry conditions (15) and (16) and Table (1). From (15), it is obvious that there is an incentive for an individual Home firm to enter the competitive market if and only if $a_k^{\gamma} > \frac{B\left(\frac{L}{L^*}+1\right)}{B^2\frac{L}{L^*}+1}$, as $a_k^{\gamma}\left[\frac{B^2\frac{L}{L^*}+1}{B\left(\frac{L}{L^*}+1\right)}\right] > 1$ and therefore the LHS of the above equation is greater than the RHS. Equation (15) indicates that the PDV of average profit flows is greater than the entry cost, i.e., $k > k_1$. Hence, complete Foreign-dominance constitutes an equilibrium only in zone $(0, k_1)$.

Secondly, we assume that all differentiated goods are produced by Home. According to the same logic as in first case, this constitutes an equilibrium only in zone $(k_2, 1)$. In other words, there is an incentive for Foreign's firms to enter the market in zone $(0, k_2)$.

Thirdly, we assume that all differentiated goods are produced by both countries. According to the equation (21) and (22), we have shown that this constitutes an equilibrium only in zone (k_1, k_2).

Consequently, there is only a unique equilibrium: in zone $(0, k_1)$, only Foreign produces; in zone (k_1, k_2), both countries produce; in zone $(k_2, 1)$, only Home produces.

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