Legal Institutions, Innovation and Growth

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Abstract. We build a stylized model of endogenous technological change and analyze the relationship between legal institutions, innovation and growth. Two legal systems are analyzed: a rigid system, where an uncontingent law is written ex ante (before knowing the current technology) and a flexible system where law-makers select the law ex post (after observing the current technology). We show that flexible legal systems dominate in terms of welfare, amount of innovation and output growth in economies at intermediate stages of technological development – which are periods when legal change is more needed – while rigid legal systems are preferable at early stages of technological development, when commitment problems are more severe. For mature technologies the two legal systems are shown to be equivalent. Surprisingly, we find that rigid legal systems may induce excessive (greater than first-best) R&D investment and output growth.

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1. Introduction

This paper analyzes the link between legal institutions, innovation and growth. In particular, it investigates how legal institutions deal with the challenges presented by technological innovation. Technology changes the world we live in and may alter the facts that justify existing rules. An early example of technology that gave rise to legal problems is railroads. The railroad industry brought up a variety of unprecedented cases and placed novel demands on the legal system.[1] Other technological inventions that also demanded legal innovation include medicine (e.g., in vitro fertilization and genetic testing), automobiles, computing, and communication (e.g., telegraphy and, more recently, the internet).[2]

In dealing with technological change, a legal system faces (at least) two challenges. First, since new technologies may require different legal rules, a legal system must adapt to changing conditions. Second, a legal system should also be judged by its capacity to provide incentives to innovate: an adaptable legal system is of no use if in the absence of innovation the economy does not change.

To address these issues, we analyze a stylized model of endogenous technological progress. More specifically, as in Aghion and Howitt (1992) we consider a model where innovations improve the quality of existing products and make old products obsolete.[3] In the context of our model, the amount of R&D investment (and, consequently, the probability that new technologies are discovered) depends on the law expected in the new technological environment.

We study two legal systems: a rigid regime and a flexible regime. In the former legal system, courts or regulators have no discretion and are bound to enforce the existing rule (statute or administrative regulation). Statutes and regulations are written ex-ante, before knowing the technology that will prevail. Following the incomplete contract literature (e.g., Grossman and Hart, 1986, and Hart and Moore, 1990) we assume that ex ante it is not possible to accurately describe future contingencies, so that in the rigid regime the existing

[1] See for instance Ely (2001).

[2] See Khan (2004) and Friedman (2002) for a discussion of how the US legal institutions responded to various examples of technological innovation.

[3] In the growth literature, quality-improving innovations are known as "vertical" innovations. Aghion and Howitt (1992) and Grossman and Helpman (1991) are the seminal papers on growth with vertical innovation. See Romer (1987, 1990) for growth models where innovation is horizontal (i.e., innovators expand the *variety* of available goods).

rule is assumed to be uncontingent. In other words, the same law is applied in all technological environments. In Section 4.4, we weaken this assumption and consider the possibility that the statute or regulation can be changed at a cost. In our model of the rigid regime we also assume that the legislator (or regulator) understands how the law affects the incentives to innovate and knows the payoff consequences of the law in each technological environment.[4] In the flexible legal regime courts and regulators have discretion and choose the law ex post, after observing the current state of the technology. The law is then state-contingent.

Leaving discretion ex post seems a sensible choice, especially in periods of rapid and ongoing technological change. Are there instances where the lack of flexibility of the rigid regime is preferable? The answer is "yes" since in our model law-makers suffer from credibility problems: the ex-ante optimal law, which is the law that provides better incentives to innovate, is not always time consistent once innovation has taken place. The rigid regime does not suffer from commitment problems because rules, which law-enforcers are bound to follow, are written ex ante. However, as discussed above, the drawback of the rigid regime is that it cannot perfectly adapt to changing conditions because the statute (or regulation) prescribes the same law in all contingencies.

It is then apparent that the choice between the two legal systems involves a trade-off between commitment and flexibility. In this paper, we argue that the terms of this trade-off change over time as technology matures. Consequently, legal institutions that may be appropriate in the early stages of technological development may not be optimal at later stages.

Needless to say, the trade-off between commitment and flexibility (or, in other words, between rules and discretion) has long been studied in macroeconomics. However, we want to emphasize one important point of distinction from the rule-versus-discretion literature. That literature assumes that the degree of uncertainty, which is the crucial parameter to evaluate the trade-off, is exogenous.[5] Instead, in this paper the degree of uncertainty (which

[4]Similarly, the incomplete contract literature assumes that the contracting parties cannot write a fully-contingent contract but they correctly anticipate the consequences of their actions in all future states of the world.

[5]For example, Rogoff (1985) compares rigid targeting systems and flexible monetary regimes. The key parameter in his comparison is the variance of aggregate productivity shocks: intuitively, one obtains that rigid regimes are preferable if uncertainty is low. More recently, Amador *et al.* (2006) study the optimal trade-off between commitment and flexibility in an intertemporal consumption/savings model with time inconsistent preferences and show that the optimal amount of flexibility depends negatively on the degree of disagreement

is related to the speed of technological change) is *endogenous* and depends on the chosen rule.

The assumptions that the legal rule is incomplete (i.e., not contingent on the state) and that the underlying uncertainty is endogenous have important implications in our model of the rigid regime. To see this, consider the problem of a legislator who has to write an uncontingent law before knowing whether or not the status-quo technology will be replaced by a more advanced technology. When the likelihood of discovering the new technology is either very low or very high, the incompleteness constraint that the legislator faces in the rigid regime matters less: in either case, the legislator will simply select the rule that optimally regulates the most likely state. Since the probability of replacing the status-quo technology depends on the law that is selected ex ante and since, as explained above, a rigid system has a comparative advantage in a certain environment (where the incompleteness constraint does not bind), the legislator has an incentive to choose a law that reduces the underlying uncertainty in the economy. In particular, he may end up selecting a rule that either discourages or, more surprisingly, strongly encourages R&D investment. As a result, we obtain that the rate of growth in a rule-based system is either very low or excessive (greater than first-best). Conversely, overinvestment in R&D never occurs when legal institutions are flexible.

The goal of this paper is to study which legal system is better suited to maximize welfare. In the context of a simple model with only two technological states, we show that flexible legal systems dominate (in terms of welfare, amount of innovation and output growth) in economies at intermediate stages of technological development, these are periods when legal change is more needed. Instead, rigid legal systems are preferable at the early stages of technological development, when commitment problems are more severe. Indeed, in the early stages considerations about consumers' health and safety are more likely to matter. Since R&D firms correctly foresee that law-makers in the flexible regime would heavily regulate ex-post, investment in research is suboptimally low and the economy is likely to remain with the old, inefficient, technology. Finally, we show that when technology is mature, the two legal systems lead to the same economic outcomes.

(which measures the severity of the commitment problem) relative to the dispersion of taste shocks. In both papers the degree of uncertainty is exogenously given.

In Section 4, we study a dynamic model where technology undergoes continual change and obtain similar results to the ones obtained in the basic model. In the stationary equilibrium of the rigid regime we show that the speed of technological change is either very low or very high. In the flexible regime, because of lack of commitment, we obtain that investment in R&D is especially low at the early stages of technological development.

Our paper is organized as follows. Section 2 briefly discuss the related literature. In Section 3 we present the basic model with two possible technologies and studies the optimal laws for each technology. Section 4 compares the rigid and flexible regimes. Section 5 analyzes the dynamic case where technology undergoes continual change. Section 6 concludes. All proofs are in the Appendix.

2. Related Literature

To the best of our knowledge, Anderlini *et al.* (2010) is the first paper to analyze credibility problems in judicial decision making. More specifically, the authors consider a model of Case Law where the probability that judges are constrained by a rule (the current precedent) is endogenous and evolve over time depending on judicial decisions. Their main result is that the rule of precedent (*stare decisis*), whereby current decisions increase the probability that future judges will be constrained, partially solves credibility problems. In their dynamic model of Case Law, the state variable is given by the current precedent. In this paper, we completely abstract from *stare decisis*; the state variable of our model is given by the current state of the technology.

Comin and Hobijn (2009) analyze a model of lobbying and technology adoption and argue that countries where the legislative authorities have more flexibility, the judicial system is not effective, or the regime is not very democratic, new technologies replace old technologies more slowly. This happens because rigidity in lawmaking makes lobbying for protecting the old technology more difficult. The mechanism that explains why in their paper a rigid system may favor technological progress relatively to a flexible system is completely different from ours. In our model, the channel is twofold. First, flexibility may harm technological progress because of time consistency problems. This explains why law-makers in a flexible system may choose ex-post a law that is less favorable to inventors than the one in the first-best solution. Second, for the reasons explained above rigid systems may choose a law that is more favorable to investors compared to the first-best solution.

Acemoglu *et al.* (2007) study the relationship between contractual incompleteness, technological complementarities, and technology adoption. In their model, a firm chooses its technology and investment levels in contractible activities with suppliers of intermediate inputs. Suppliers then choose investments in noncontractible activities, anticipating payoffs from an ex post bargaining game. Their paper argues that greater contractual incompleteness leads to the adoption of less advanced technologies, and that the impact of contractual incompleteness is more pronounced when there is greater complementary among the intermediate inputs.[6]

Finally, Immordino *et al.* (2009) analyze optimal policies when firms' research activity leads to innovations that may be socially harmful. Public intervention, affecting the expected profitability of innovation, may both thwart the incentives to undertake research and guide the use of each innovation. In our setting we abstract from the enforcement problem, and we judge the optimality of a legal system by studying the trade off between its adaptability to technological change and its capacity to provide incentives to innovate.

Before moving on, we briefly discuss the large legal literature that has studied the interaction between law and technology. More specifically, various legal scholars have investigated how legislation (a relatively inflexible system) and common law adjudication (a relatively flexible system) deal with technological change.[7] According to this literature, the main limitation of legislation is that rules, which are set in advance, likely suffer from either overinclusiveness or underinclusiveness. Conversely, being more gradual, common law adjudication may benefit from the society's experience with a technology. However, the literature has also pointed out that legislation has several merits. Legislatures have greater democratic legitimacy. Moreover, in drafting the law they can take a broader perspective since, unlike courts, they do not focus on the case at hand. Furthermore, legislation can act in advance and does not have to wait until the issue is litigated. As a result, it can act at a stage where technology is still capable of being shaped and technology is not an accomplished fact.[8]

[6]See also Acemoglu (2009, p. 801) for a discussion of the possibility of an hold-up problem in technology adoption.

[7]For instance, see Tribe (1973), Furrow (1982), Jasanoff (1995), Dworkin (1996), and Bennett Moses (2003).

[8]It bears mentioning that the distinction between legislation and common law is often blurred: common law has become less flexible over time and, at the same time, legislatures are increasingly delegating to administrative agencies in order to enhance flexibility (Calabresi, 1982).

3. The Basic Model

As in Aghion and Howitt (1992), we consider a model of endogenous technological change where new products provide greater quality than existing goods. The economy consists of three sectors: the R&D sector, the intermediate good sector and the final good sector. As discussed below, we assume that the law regulates the production process of the intermediate good. To keep our setting tractable and focus attention on the interaction between legal systems and innovation, our model of technological change is simplified along various dimensions: for instance, the input prices in the R&D sector and in the intermediate good sector are assumed to be exogenous.

3.1. Technology and Market Structure

The final good is produced competitively using the intermediate good. The production function of the final good is assumed to be Cobb-Douglas:

$$y\left(i\right)=A\left(i\right)x\left(i\right)^{\frac{1}{2}}, \tag{1}$$

where $x\left(i\right)$ is the intermediate good and $A\left(i\right)$ is a parameter that measures the productivity of the intermediate good. To keep our setting as tractable as possible, we assume that the output elasticity with respect to $x\left(i\right)$ is equal to 1/2.[9] The index $i\in\{0,1\}$ denotes the state of technology sophistication of the intermediate good which is available in the current period.[10] Technology 0 is assumed to be strictly less productive than technology 1: that is, $A\left(1\right)=\gamma_{A}A(0)$, with $\gamma_{A}>1$. The invention process (which will be described shortly) is stochastic. In particular, technology 1 is available only if the investment made by the *R&D* firm is successful. If R&D investment succeeds, the old technology becomes obsolete. In other words, we assume that the innovation is drastic.[11]

The intermediate good sector is assumed to be a monopoly. Monopoly power derives from

[9] Under this assumption, the indirect utility of the representative agent in the economy has a very simple form (see Subsection 3.6). This will allow us to obtain closed form solutions for the equilibrium laws in the two legal regimes. However, we expect that the main thrust of our results would not change if we allow for a more general specification.

[10] In Section 5 we take i to be such that $i=0,1,...,\infty$.

[11] Innovation is nondrastic if and only if the firm that uses the status-quo technology can make positive profits when the firm that produces the most advanced technology is charging the monopolistic price. It can be shown (see Aghion and Howitt, 1992, Section V) that innovations are drastic if γ_{A} is sufficiently high.

intellectual property: the intermediate good firm holds a patent that it has purchased from the R&D firm. We assume that the marginal cost of the intermediate good firm is constant in $x(i)$ and decreasing in a,

$$MC(a) = \frac{1}{a}, \tag{2}$$

where a is the activity regulated by the law. We assume $a \in [\underline{a}, \overline{a}]$, with $\overline{a} > \underline{a} > 0$. For example, a can be thought as the inverse of the level of precaution in the production process. When a is high, the firm is taking *low* levels of precaution and, consequently, its marginal cost is low. To abide by the law, the intermediate good firm must choose the activity level that the law prescribes. We also assume that a is observable at no cost, so that the law is perfectly enforced.[12] The price of the intermediate good (relatively to the final good) is denoted by $p(i)$.

3.2. Research

The R&D firm chooses how much to invest in research. The amount of investment affects the probability of discovering the new technology for the intermediate good. Denoting by z the number of researchers hired by the R&D firm, we assume that the new technology is discovered with probability θz, where $\theta > 0$ (the probability is equal to one if $z \geqslant 1/\theta$). The patent of the new technology is sold to a firm in the intermediate good sector. With probability $1 - \theta z$ there is no innovation and the old technology is maintained.

3.3. Preferences

The utility of the representative agent of this economy is

$$u\left(c(i), a, i\right) = c(i) - \lambda\left(i\right) a. \tag{3}$$

Utility linearly depends on the consumption of the final good $c(i)$ and, due to a production externality from the intermediate good firm, on the activity level a. Note that this externality is reduced if the intermediate good firm uses more precaution (that is a is close to $\underline{a}$). To motivate (3) consider, for example, the case where the final good, for instance biscuits, is produced with genetically modified corn and a is the amount of regulation in the intermediate

[12] We abstract from the enforcement issue in the belief that this problem is somewhat similar in the two legal regimes.

good sector. The emissions of sparks and cinder caused by railroads is another classic example of externality.[13] We assume that $\lambda(1) = \gamma_\lambda \lambda(0)$ where $\gamma_\lambda \geqslant 0$. For simplicity, we normalize $\lambda(0)$ to 1. If $\gamma_\lambda > 1$, the consumer faces a more dangerous innovation. In this case, the innovation makes it more costly for the consumer to have a more permissive legislation. If instead $0 \leqslant \gamma_\lambda < 1$, the negative externality from the production sector is less severe under the new technology.

3.4. The Maximization Problem of the Intermediate Good Firm

We denote by $\pi(a,i)$ the profit function of the monopolist that produces the intermediate good according to technology i,

$$\pi(a,i) = \max_{x(i)\geq 0} \left[p(i) - MC(a)\right] x(i). \tag{4}$$

Since the final-good producer is competitive, the inverse demand of the intermediate good is

$$p(i) = \frac{1}{2} A(i)\, x(i)^{-\frac{1}{2}}. \tag{5}$$

That is, $p(i)$ is equal to the marginal product of the intermediate good. The monopolist consequently chooses to produce

$$x(i) = \left[\frac{A(i)}{4MC(a)}\right]^2. \tag{6}$$

After substituting (6) in (4), we obtain

$$\pi(a,i) = a\Phi(i), \tag{7}$$

where

$$\Phi(i) = \left[\frac{A(i)}{4}\right]^2. \tag{8}$$

[13]See Grady (1988) and Ely (2001) for an account of early decided cases that addressed these issues. At the end of the 19th century, for instance, typical allegations of negligence included the failure to have a spark arrester, to keep it functioning, to use the appropriate type of fuel, to keep the roadway free of weeds, or the failure to build fire guards on the edge of the roadway.

Note that $\Phi(i)$ does not depend on the level of activity a, but only on the state of the technology i. More importantly, notice from (7) that profits are increasing in a.

3.5. Optimal Investment in Research

We assume that the R&D firm that discovers the new technology has all the bargaining power and can sell its patent for a price equal to $\pi(a,1)$. Then, the optimal choice of z solves the following problem:

$$\max_{z\in\left[0,\frac{1}{\theta}\right]}\left[\theta z\pi(a,1)-\frac{1}{2}z^2\right], \tag{9}$$

where, for simplicity we have assumed a quadratic cost of hiring z researchers.[14]

Assuming an interior solution, the optimal number of researchers $\widetilde{z}$ is

$$\widetilde{z}=\theta a\Phi(1), \tag{10}$$

which is increasing in a. Note that the amount of investment defined by (10) is not, in general, socially optimal because the R&D firm chooses z in order to maximize profits, not the consumers' surplus.[15]

The decision problem of the R&D firm makes transparent the mechanism through which the law affects the probability of successful innovation in our model: a pro-business law (which allows higher levels of activity a) increases the profits of the intermediate good firm and makes R&D investment more profitable, thereby increasing the probability of discovering the more productive technology.

It is important to notice that in order to determine the amount of R&D investment, the R&D firm needs to predict the law that will be enforced under technology 1. Notice in fact that the law that is enforced under the status-quo technology does not enter in (9) and, consequently, does not affect the decision of the R&D firm.

Before concluding this section, we compute the expected rate of growth of the economy

[14]Notice that because the probability of successful innovation has constant returns to scale, the number of firms is indeterminate. Throughout this section, we assume that there is a single R&D firm.

[15]This is the standard appropriability effect emphasized by the literature on innovation.

using (1), (6), (2) and (10):

$$g = \theta \widetilde{z} \left[\frac{y(1) - y(0)}{y(0)} \right] = \left(\gamma_A^2 - 1 \right) \theta^2 \Phi(1) a. \tag{11}$$

Note that the more permissive the regulation, the higher the rate of output growth in the economy.[16]

3.6. *Ex-Post Optimal Laws*

As discussed above, activity a is regulated by the law. Law-makers are benevolent in the following sense: they choose the law in order to maximize the utility of the representative consumer. In order to solve the legislator's problem, we derive the indirect utility of the representative consumer in each state i. Using (1), (3), (6) and the equilibrium condition $c(i) = y(i)$, we obtain that the indirect utility is linear in a:

$$u(a, i) = a\vartheta(i), \tag{12}$$

where

$$\vartheta(i) = \frac{1}{4} A(i)^2 - \lambda(i) \tag{13}$$

From (12) note that an increase of a has two effects on utility. First, it has a direct (and negative) effect due to its externality. The higher $\lambda(i)$, the higher this effect. Second, a higher a decreases the marginal cost of the intermediate good producer and increases the production of the final good. A more pro-business law has then an indirect (and positive) effect on utility because consumption increases; the higher $A(i)$, the higher the marginal benefit of increasing a due to this second effect. Since $A(i)$ and $\lambda(i)$ both depend on i, the law that optimally solves the trade-off between the two effects is potentially different under the two technologies.

We now compute the law that law-makers would choose ex-post (that is, after observing the current technological state). We shall refer to this law as the *ex-post optimal law* and we denote it by $a^*(i)$.[17] Given the linearity of (12), it is straightforward to find for each

[16] The negative consequences of regulation on productivity in the US manufacturing industry have been empirically studied by Gray (1987).

[17] As we will see in Section 4, this law may not coincide with the law that law-makers would choose ex-ante,

technological environment the level of $a \in [\underline{a}, \overline{a}]$ that maximizes (12). In particular, we obtain

$$a^*(i) = \begin{cases} \overline{a} & \text{if} \quad \vartheta(i) \geqslant 0 \\ \underline{a} & \text{if} \quad \vartheta(i) < 0 \end{cases} \tag{14}$$

From the definition of $\vartheta(i)$, note that $\vartheta(i) \geqslant 0$ when $A(i)$, the productivity of the intermediate good, is relatively high compared to its externality $\lambda(i)$. In this case the ex-post optimal law will be a pro-business law (a law that minimizes the marginal cost of the intermediate good firm). When instead $\vartheta(i) < 0$, the direct effect of a on consumers' utility is relatively large. In this case the ex-post optimal law will be punitive for the intermediate good firm.

Throughout this paper we will assume that innovation, besides increasing the productivity of the intermediate good, is welfare-improving.

Assumption 1: *We assume that innovation increases consumers' utility:* $\vartheta(1) > \vartheta(0)$.

An implication of Assumption 1 is that the ex-post optimal law is weakly increasing in i. In other words, the ex-post optimal law is more favorable to the firm producing the intermediate good after the innovation than before.

As discussed above, the ex-post optimal law is either $\underline{a}$ or $\overline{a}$ depending on the value of $A(i)$ relatively to $\lambda(i)$. Notice that if $A(0)$ is sufficiently low (in relative terms), we have that $a^*(0)$ and $a^*(1)$ are *both* equal to $\underline{a}$. If instead the productivity of the status-quo technology is relatively high, we have that $a^*(0)$ and $a^*(1)$ are both equal to $\overline{a}$. A common feature in both cases is that the ex-post optimal laws under the two technologies coincide. When instead the starting value of $A(0)$ belongs to an intermediate range, we have that $a^*(0)$ is $\underline{a}$ while $a^*(1)$ is $\overline{a}$.

In the early stages of their life cycle, most technologies are likely characterized by low productivity and important consequences on consumers' safety. As technologies become more developed, we expect productivity to increase and the negative externality on consumers to matter relatively less. Consequently, we now propose the following classification:

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when the uncertainty about the technology has not yet been resolved.

Definition 1: *Suppose that Assumption 1 is satisfied. A technology is said to be at an early stage of development when $\vartheta(0) < \vartheta(1) < 0$. This occurs when the productivity of the status-quo technology is sufficiently low:*

$$A(0) < \frac{2\sqrt{\gamma_\lambda}}{\gamma_A}. \tag{15}$$

A technology is said to be at an intermediate stage of development when $\vartheta(1) \geqslant 0 > \vartheta(0)$. This occurs when

$$\frac{2\sqrt{\gamma_\lambda}}{\gamma_A} \leqslant A(0) < 2. \tag{16}$$

Finally, a technology is said to be mature when $\vartheta(1) > \vartheta(0) \geqslant 0$. This occurs when

$$A(0) \geqslant 2. \tag{17}$$

It is important to stress that we are not saying that in the early stages technological innovation is not welfare improving. By Assumption 1 innovation always increases consumers' utility. Our point is that in the early stages considerations concerning consumers' protection matter relatively more: that is, the marginal benefit from a more permissive law is lower than its marginal cost. The opposite holds true when technology is mature.

As discussed in the previous section, the probability of successful innovation depends on the law that will be enforced if the new technology is discovered. Since innovation is welfare improving, law-makers want to promote innovation beyond the suboptimal level chosen by the R&D firm. Law-makers have an effective instrument to promote research: choosing a pro-business law. This increases the profit of the intermediate good firm, raises the price of a patent and provides stronger incentives to invest in R&D. The goal of fostering innovation, however, is not the only goal that law-makers want to achieve. The law must also optimally regulate the technological environment. As we will see in the next section, the two goals often do not coincide.

4. Commitment vs. Flexibility with Endogenous Uncertainty

In this section we finally compare our two legal regimes. First, consider a *flexible* regime (denoted by F) where the law-maker chooses the law ex-post, after knowing the current state of the technology. In this case, it is easy to find out the law that is implemented in each state i: it coincides with $a^*(i)$, the ex-post optimal law in that state. Second, consider a *rigid* regime (denoted by C, which stands for commitment) where the law is chosen ex-ante, before knowing the current state of the technology. In the rigid regime, law-makers are bound to enforce ex-post the law that was chosen ex-ante. We crucially assume that in the rigid regime, the law cannot be made contingent on the technological environment. To justify this, one may assume that the two environments are difficult to describe ex-ante. However, as is standard in the incomplete contract literature, we also assume that the law-maker understands how the law affects the probability of successful innovation and knows the payoff consequences of the law in the two technological states. This assumption is necessary to make the legislator in the rigid regime able to optimally write the law before uncertainty is realized. Let a_C denote the law that will be enforced under both technologies in the rigid regime.

Note that, in general and for different reasons, the two legal systems that we have just described are bounded away from efficiency. On the one hand, the flexible regime is adaptable but it lacks commitment. As a result, it may not provide sufficient incentives to innovate. To see this, assume for instance that $\vartheta(1) < 0$. In this case, the R&D firm correctly foresees that ex-post the law in the flexible legal regime will be costly for the intermediate good firm. As a result, discovering the new technology is not very profitable. This depresses investment and reduces the probability that welfare-improving innovation occurs. On the other hand, note that in the rigid regime the law-maker is able to commit but is bound to choose a single law and, consequently, he cannot adapt to changing conditions. The incompleteness of the law is the source of inefficiency of the rigid regime.

Under the rigid regime, the timing is as follows. First, the legislator chooses the legal level of activity a_C. Then, the R&D firm chooses the investment level. Investment is either a success or a failure. Regardless of the current state of the technology, the intermediate good firm exerts a precautionary level equal to a_C. Finally, the production of the intermediate good and of the final good take place. The legislator chooses a_C in order to maximize the expected utility of the representative agent. Using (10), welfare level in the rigid regime can be written as:

$$W_C = \max_{a_C \in [\underline{a}, \overline{a}]} \left[\theta^2 \Phi(1) a_C\right] a_C \vartheta(1) + \left[1 - \theta^2 \Phi(1) a_C\right] a_C \vartheta(0). \tag{18}$$

Note that ex-ante (before uncertainty is realized) the law has another effect on consumers' utility besides the ones discussed in the previous section: it affects the probabilities of the two technological states. It is then apparent that law-makers have one instrument (the law a_C) to pursue two goals: to provide incentives to innovate and to optimally regulate the new technological environment. In general, the legislator cannot achieve both goals with a single instrument and welfare in the rigid regime is then suboptimal.

Under the flexible regime, the timing is as follows. First, R&D firms choose how much to invest. In making this choice, they correctly foresee the choice that law-makers will select ex-post. Investment is either a success or a failure. Law-makers observe the current technological environment and have discretion to choose the law. As discussed above, in each state i, law-makers choose $a^*(i)$, the law that maximizes consumers' ex-post welfare in that state. Finally, the production of the intermediate good and of the final good take place. Welfare in the flexible regime can then be written as:

$$W_F = \left[\theta^2 \Phi(1) a^*(1)\right] a^*(1)\vartheta(1) + \left[1 - \theta^2 \Phi(1) a^*(1)\right] a^*(0)\vartheta(0). \tag{19}$$

Note that the probability of successful innovation depends on $a^*(1)$ because the R&D firm correctly expects that $a^*(1)$ will be enforced in state 1.

4.1. Exogenous Innovation

In comparing the rigid and the flexible regimes, it is instructive to begin by considering the benchmark case where the probability of successful innovation is exogenous. Let ι denote the probability that innovation occurs.

When technological innovation is exogenous, the law cannot (obviously) provide incentives to innovate. Therefore, legal systems differ only with respect to their ability to choose the best law for each technology. Given this premise, it is entirely straightforward to conclude that when innovation is exogenous the flexible regime weakly dominates the rigid one. The two regimes are equivalent only in two cases: when there is no uncertainty and when the ex-post optimal laws are the same under both technologies.[18] This occurs because in both instances the incompleteness constraint is not binding.

[18]The latter possibility arises when the economy is at the early or at the advanced stage of development

Let $W_j(\iota)$ denote maximized welfare when the probability of innovation is equal to ι, where $j = C, F$. We state without proof the following results.

Proposition 1. Exogenous Innovation: *When technology is either at an early stage or is mature, for all $\iota \in [0,1]$ we have $W_F(\iota) = W_C(\iota)$. When instead technology is at an intermediate stage, $W_F(\iota) > W_C(\iota)$ for all $\iota \in (0,1)$ and $W_F(\iota) = W_C(\iota)$ when $\iota = 0, 1$.*

It is instructive to compute $W_F(\iota)$ and $W_C(\iota)$ when technology is at an intermediate stage. This is the most interesting case since when the signs of $\vartheta(0)$ and $\vartheta(1)$ coincide, we know from Proposition 1 that the two legal systems yield the same outcomes. Recalling that in the flexible regime law-makers choose the ex-post optimal laws (14), we have

$$W_F(\iota) = \iota \overline{a} \vartheta(1) + (1-\iota) \underline{a} \vartheta(0). \tag{20}$$

We now discuss the rigid regime. It is easy to verify that in the rigid regime the legislator chooses $\underline{a}$ (resp. $\overline{a}$) when ι is below (resp. above) a certain threshold. To understand this result, recall that the legislator must choose a single (uncontingent) law. Therefore, he will choose the law that better regulates the status-quo technology (which is equal to $\underline{a}$, when technology is at an intermediate stage) if and only if successful innovation is not very likely. One can verify that this threshold, which is denoted by $\overline{\iota}$, is given by

$$\overline{\iota} = \frac{(\underline{a} - \overline{a})\,\vartheta(0)}{(\overline{a} - \underline{a})\,(\vartheta(1) - \vartheta(0))}. \tag{21}$$

Then, one obtains

$$W_C(\iota) = \begin{cases} \iota \underline{a} \vartheta(1) + (1-\iota) \underline{a} \vartheta(0) & \text{if} \quad \iota \leqslant \overline{\iota} \\ \iota \overline{a} \vartheta(1) + (1-\iota) \overline{a} \vartheta(0) & \text{otherwise} \end{cases} \tag{22}$$

In Figure 1 below, we draw (20) and (22). Both $W_F(\iota)$ and $W_C(\iota)$ are increasing in ι by Assumption 1. It is important to notice that the welfare loss of the rigid regime *vis-à-vis* the flexible one is relatively large for intermediate values of ι. In this region of parameters, in fact, it is relatively more costly to have an uncontingent law (the Lagrange multiplier associated with the incompleteness constraint is high). This observation helps explain why in a model where R&D investment is endogenous, the legislator in the rigid regime may have an incentive to select a law that leads to either overinvestment or underinvestment in R&D.

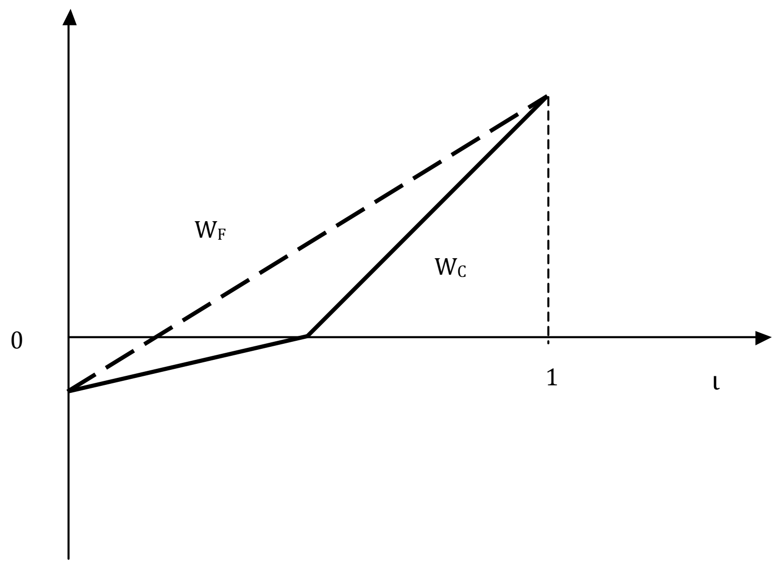

Figure 1: Welfare levels with exogenous innovation for $\vartheta(0) < 0$ and $\vartheta(1) > 0$

4.2. Endogenous R&D investment

When R&D investment is endogenously chosen, the probability of discovering the new technology depends on the law. This assumption has two important implications. First, it now matters whether or not law-makers are able to commit. Credibility problems arise because the first order conditions in the ex-ante problem (before R&D investment is chosen) and in the ex-post problem (after knowing whether R&D investment was a success) are now different. This occurs because at the ex-ante stage, but not at the ex-post stage, law-makers take into account the effect of the law on the incentives to invest through (10). Because of credibility problems, we will show that in some cases committing to a rule (choosing the rigid regime) is preferable to leaving ex-post discretion to law-makers (choosing the flexible regime). A second implication is that the legislator in the rigid regime may now have an incentive to select a rule that reduces the underlying uncertainty in the economy. As shown below, this result can be achieved by either strongly encouraging or strongly discouraging

R&D investment.

To determine the law in the rigid regime, we rewrite (18) as:

$$W_C = \max_{a_C \in [\underline{a}, \overline{a}]} \vartheta(0) a_C + \theta^2 \Phi(1) a_C^2 [\vartheta(1) - \vartheta(0)]. \tag{23}$$

Note that by Assumption 1 we have that $\vartheta(1) - \vartheta(0) > 0$. The objective function is then convex in a_C. This implies that in (23) we have a bang-bang solution: the chosen law a_C is either $\underline{a}$ or $\overline{a}$. As a result, the probability of discovering the new technology is either the lowest or the highest possible one.

$$a_C = \begin{cases} \overline{a} & \text{if} \quad \frac{\vartheta(0)}{\vartheta(1)-\vartheta(0)} + \theta^2 \Phi(1) (\overline{a} + \underline{a}) \geqslant 0 \\ \underline{a} & \text{otherwise} \end{cases} \tag{24}$$

We now discuss the optimal law chosen by the legislator in each of the three stages of technological development.

Mature stage. When $\vartheta(0) \geqslant 0$, using (24) we have that $a_C = \overline{a}$.[19] Choosing law $\overline{a}$ allows to achieve two goals at the same time: it provides the right incentive to conduct research and it optimally regulates the two technological environments that we may observe ex post.

Intermediate Stage. As always, the goal of favoring innovation is achieved by selecting $\overline{a}$. However, notice that when technology is at an intermediate stage, $\overline{a}$ is ex-post optimal in case innovation is successful but it is suboptimal if innovation is not successful. From (24) we know that the legislator chooses $\underline{a}$ when $\vartheta(1) - \vartheta(0)$ is small and, consequently, providing incentives to innovate is not very valuable. Another case in which we expect $\underline{a}$ to be selected is when $\vartheta(0) << 0$. In this case, it would be too risky to choose $\overline{a}$. Notice in fact that a pro-business law would be extremely inefficient if the new intermediate good is not discovered. Finally, one can verify that if the probability of successful innovation can be made close to 1 by choosing $\overline{a}$, the legislator will likely choose $\overline{a}$ because the eventuality of having an inefficient law ex-post can be made sufficiently low.

Early Stage. At this stage, providing incentives (choosing $\overline{a}$) is suboptimal when R&D investment fails but also when it succeeds. However, condition (24) establishes that in some

[19]Note from (23) that welfare in the rigid regime is increasing in a_C when $\vartheta(0) \geqslant 0$.

cases the legislator may select $\overline{a}$. To understand this result consider Figure 2 below. It shows the indirect utility of the representative consumer as a function of the law. Points A and B (resp. points C and D) indicate the agent's utility associated to law $\underline{a}$ (resp. $\overline{a}$) in state 1 and 0. Since at this technological stage we have that in both states law $\underline{a}$ is ex-post optimal, from a welfare point of view point A dominates C and point B dominates D. To understand why the legislator may sometimes choose $\overline{a}$ note from (18) that welfare in the rigid regime is a sum with endogenous weights. Even if A dominates C and B dominates D, the weighted sum of A and B may be smaller than the weighted sum of C and D. This may happen when choosing $\overline{a}$ raises the probability of state 1 by a large amount.[20]

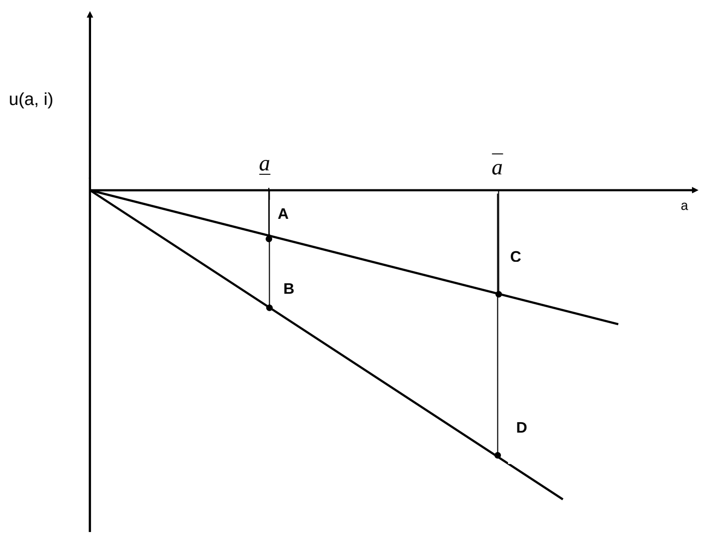

Figure 2: Utility of the representative consumer in the rigid regime when $\vartheta(1)$ and $\vartheta(0) < 0$

After deriving the laws that are enforced in the two regimes, it is straightforward to compare the two legal institutions. Proposition 2 establishes that in contrast to the previous section, when R&D investment is endogenous the flexible regime is not necessarily optimal

[20]Notice that a necessary condition to choose $\overline{a}$ is to have $\overline{a}\vartheta(1) \geqslant \underline{a}\vartheta(0)$. In Figure 2 this implies that utility C must be greater than B.

in all circumstances. In particular, when technology is at an early stage we have that the rigid regime may actually dominate the flexible regime because of its ability to provide better incentives to innovate. Notice, in fact, that at the early stage of technological development the flexible regime would choose a law that protects public safety and provides weak incentives to innovate. Also, notice that by choosing $a_C = \underline{a}$ the legislator in the rigid regime can achieve the same welfare that is obtained in the flexible regime. However, if the legislator decides so, he can also choose $a_C > \underline{a}$ in order to provide incentives to innovate. This possibility is not available in the flexible regime and this explains why we obtain that the commitment regime weakly dominates the flexible one. When technology matures, we obtain that the two systems yield the same outcomes. Finally, in economies at an intermediate stage of development – which are periods when legal change is needed and there are no commitment problems – the flexible system is strictly better than the rigid regime because of its ability to choose the best law for each technology.

Let g_i, with $i = F, C$, denote the rate of output growth under legal regime i. The next proposition states the main result of the section:

Proposition 2. *Welfare Comparison: (i) When technology is mature, we have that $W_C = W_F$ and $g_C = g_F$.(ii) When technology is at an intermediate stage of development, we have that $W_C < W_F$ and $g_C \leqslant g_F$.(iii) When technology is at an early stage of development, we have that $W_C \geqslant W_F$ and $g_C \geqslant g_F$.*

4.3. Rigidity and Overinvestment

As a benchmark, we now define and derive the law in the *first-best*. Similarly to the flexible regime, the first-best law specifies a law for each technology environment and, similarly to the rigid regime, the first-best law is written ex ante under commitment. Let a_i^{FB} denote the first-best law that will be enforced under technology $i = 0, 1$. Welfare in the first-best, which is denoted by W^*, is equal to:

$$W^* = \max_{a_0^{FB},\ a_1^{FB} \in\ [\underline{a}, \overline{a}]} \left[\theta^2 \Phi(1) a_1^{FB}\right] a_1^{FB} \vartheta(1) + \left[1 - \theta^2 \Phi(1) a_1^{FB}\right] a_0^{FB} \vartheta(0). \tag{25}$$

To compute the first-best law, first notice that a_0^{FB} has no effect on the amount of R&D investment. Then, we have that a_0^{FB} is equal to $a^*(0)$, the ex-post optimal law in state 0. To find a_1^{FB}, two cases must be considered. First, assume that $\vartheta(1) \geqslant 0$. In this case, we have that $a_1^{FB} = a^*(1) = \overline{a}$. To see this, note that this law fosters innovation and at the same time optimally regulates the more advanced technological environment. Second, when $\vartheta(1) < 0$, it is immediate to verify that the objective is concave in a_1^{FB} on the interval $[\underline{a}, \overline{a}]$. Therefore, to find a_1^{FB} we have to study the sign of the derivative at $\underline{a}$ and $\overline{a}$. We obtain that $a_1^{FB} = \underline{a}$ if and only if $2\vartheta(1) - \vartheta(0) \leqslant 0$, while $a_1^{FB} = \overline{a}$ if and only if $2\overline{a}\vartheta(1) - \underline{a}\vartheta(0) \geqslant 0$. In the remaining cases, the chosen a_1^{FB} is an interior solution. The interpretation is quite straightforward. When $\vartheta(1) < 0$ and $\vartheta(1) - \vartheta(0)$ is small, innovation does not increase welfare by much and, consequently, a_1^{FB} coincides with $\underline{a}$, the ex-post optimal law at this stage.

In what follows, we compare the probability of successful innovation under the first-best law with the one obtained under the rigid and the flexible regimes. Surprisingly, we find that in some cases $a_C > a_1^{FB}$. This implies that in the rigid regime there may be overinvestment in R&D (hence, too much growth) compared to what would be socially optimal.[21]

It is easy to verify that the possibility of overinvestment occurs only when technology is at the early stage.[22] At this stage, two reasons push the legislator in the rigid regime to choose a pro-business law: to increase the probability that welfare improving innovation occurs and to reduce the probability of staying with the status-quo technology and suffering from inefficient regulation. Notice in fact that at an early stage a high a_C is always suboptimal but is relatively more inefficient under the old than under the new technology (see Figure 2 which shows that $C > D$). In the first-best problem, the second reason is not present because the law is state-contingent and, consequently, the status-quo technology is optimally regulated. This is why rigid legal system may induce overinvestment in R&D compared to the optimal level.

As summarized in the following proposition, in the rigid legal regime we may have either overinvestment (if $a_C = \overline{a}$ while $a_1^{FB} < \overline{a}$) or underinvestment (when $a_C = \underline{a}$ while $a_1^{FB} > \underline{a}$). In the flexible legal regime investment is never larger than the efficient one.

[21] Usually, the literature on incomplete contracts has focused on the possibility of underinvestment due to ex-post exploitation (see Grout, 1984). The overinvestment result is also obtained in the incomplete contract literature: see for instance, Chung (1995). The underlying reason is however different from ours: in that literature, some parties may overinvest to strategically affect their bargaining power ex-post.

[22] When instead $\vartheta(1) \geqslant 0$ we obtained $a_1^{FB} = \overline{a}$. Then, it is not possible to observe $a_C > a_1^{FB}$.

Proposition 3. *The Possibility of Overinvestment: The rate of output growth in the flexible regime is always smaller or equal than first-best. In the early stage of technological development, economies adopting the rigid regime may grow faster than first-best.*

At first glance, it may seem paradoxical that the rate of output growth in the rigid regime may be inefficiently high. However, recall that welfare depends on the consumption of final good but also on the activity level of the intermediate good firm. A high rate of growth may be suboptimal when it is obtained by committing to a high a, which implies a low use of precaution in the intermediate good sector.

4.4. Costly Change of the Law

We now assume that the law in the rigid regime can be changed by incurring an exogenous cost $\kappa > 0$ after knowing which technology is available. Let $W_C(\kappa)$ denote welfare in the rigid regime when the cost of changing the statute is equal to κ. Note that $W_C(\infty) = W_C$ and $W_C(0) = W_F$ where W_C and W_F were defined in (18) and (19), respectively.

The timing is as follows. At the beginning, the legislator selects the law a_C. The R&D firm chooses the amount of investment. Note that in making this decision the R&D firm understands the legislator's incentives to change the law ex-post. After knowing the current technological environment, the legislator decides whether to enforce the existing law a_C or to change it by incurring the cost κ. Finally, production and consumption take place.

In this section, we obtain two main results. First, we show that for all $\kappa > 0$ the possibility of changing the statute does not alter the welfare rankings that we have established in Proposition 2. Second, we show that $W_C(\kappa)$ varies in κ. In some cases, it varies in a non-monotone way.

Proposition 4. *Costly Change: (i) When technology is mature, $W_C(\kappa)$ is constant in κ and for all $\kappa > 0$ we have $W_C(\kappa) = W_F$; (ii) When technology is at an intermediate stage, $W_C(\kappa)$ is decreasing in κ and for all $\kappa > 0$ we have $W_C(\kappa) < W_F$. (iii) When technology is at an early stage, $W_C(\kappa)$ is not necessarily monotone in κ and for all $\kappa > 0$ we have $W_C(\kappa) \geqslant W_F$.*

To understand part (i) of Proposition 4, note that when technology is mature both legal regimes attain the first-best by selecting $\overline{a}$. Since it is never optimal to change the law ex-post, κ does not affect welfare. When technology is at an intermediate stage, flexibility is needed and a pro-business law is also credible: the rigid regime would then benefit from choosing a low κ. As long as κ is strictly positive, however, the flexible regime remains superior at this stage. In the early stage, $W_C(\kappa)$ weakly dominates the flexible regime because the rigid regime has the possibility of selecting $\underline{a}$ and reproducing the flexible regime.

Interestingly, Proposition 4 also states that in the early stage of technological development, $W_C(\kappa)$ may not be monotone in κ. In particular, for some finite κ it may happen that

$$W_C(\kappa) > W_C(\infty) \geqslant W_C(0). \tag{26}$$

In other words, choosing a positive, but finite, cost of changing the law would further improve welfare in the rigid regime. The second inequality in (26) follows from result (iii) in Proposition 2. We now prove that the first inequality is verified for some parameter values. For instance, assume that when $\kappa = \infty$ the parameters in (24) are such that the optimal law in the fully rigid regime is $\overline{a}$. Moreover, pick a κ that satisfies the following two inequalities:[23]

$$\vartheta\,(0)\,\overline{a} < -\kappa + \vartheta\,(0)\,\underline{a}, \tag{27}$$

$$\vartheta\,(1)\,\overline{a} \geqslant -\kappa + \vartheta\,(1)\,\underline{a}. \tag{28}$$

Note that this partially rigid legal system would provide credible incentives to innovate since inequality (28) establishes that $\overline{a}$ is not changed ex-post in state 1. Moreover, given that κ satisfies (27), the law $\overline{a}$ would be changed ex-post in case innovation fails (an event occurring with strictly positive probability). This indicates that an intermediate value of κ would provide flexibility and also commitment. Therefore, it would be preferable to having either $\kappa = \infty$ or $\kappa = 0$.

5. The Dynamic Model

We now consider a dynamic model. We assume that time, which is indexed by $t \geqslant 0$, is continuous and infinite. Moreover, we assume that the number of feasible innovations is also infinite. In other words, technological progress never settles. Let i denote the current technological state. The productivity of the intermediate good increases as follows: $A\,(i+1) = \gamma_A A(i)$,

[23] By looking at Figure 2, it is easy to see that such a κ always exists. Just notice that $\vartheta\,(0)\,\overline{a}$ corresponds to point D, $\vartheta\,(0)\,\underline{a}$ to point B, $\vartheta\,(1)\,\overline{a}$ to point C and $\vartheta\,(1)\,\underline{a}$ to point A. A value of κ that satisfies (27) and (28) always exists because $B - D > A - C$.

with $\gamma_A > 1$. As before, we assume that innovations are drastic: if the more productive intermediate good $i+1$ is discovered, the intermediate good i becomes obsolete. Flow production of consumption good is given by (1). To simplify the algebra, we assume that the externality from the intermediate good sector is constant: $\lambda(i+1) = \lambda(i) = \lambda$.

The representative consumer has the following intertemporal preferences

$$U(c,a) = \int_0^\infty e^{-rt}(c_t - \lambda a_t)\,dt, \tag{29}$$

where r is the constant rate of time preference, also equal to the interest rate.[24] We denote by c_t and a_t the time-t consumption and intermediate good producer's activity, respectively. Flow consumption at time t is stochastic since it depends on the technological state that is available at that time. The economy is similar to the one analyzed in Section 3. As before, the marginal cost is affected by the law according to equation (2).

Using (1), (3), (6) and the equilibrium condition $c(i) = y(i)$, we obtain that when the technological state is i, the instantaneous utility of the representative agent is equal to $a_i\vartheta(i)$, where

$$\vartheta(i) = \frac{1}{4}A(0)^2\gamma_A^{2i} - \lambda, \tag{30}$$

and a_i is the law which is enforced under technology i.

As in Section 3, innovation is welfare improving. Note in fact that $\vartheta(i+1) > \vartheta(i)$. It is also important to notice that in the long-run, after a possibly long sequence of innovations, $\vartheta(i)$ will become positive and, consequently, $\overline{a}$ will be the ex-post optimal law. Let $N \geqslant 0$ denote the lowest technological state where $\vartheta(i)$ is weakly positive: that is, $\vartheta(N) \geqslant 0$ but $\vartheta(N-1) < 0$. ($N = 0$ if the initial $A(0)$ is sufficiently high and λ is sufficiently low).

The innovation process is assumed to be a continuous time Markov Chain, as in Aghion and Howitt (1992). The economy passes through a sequence of states $0 \to 1 \to ...$ staying with innovation k for a sojourn time X_k with density $\theta z(k+1)e^{-\theta z(k+1)}$, where $z(k+1)$ are the R&D expenditures that are incurred in order to discover the intermediate product of quality $k+1$.[25] We assume $0 \leqslant z(k+1)\theta < \infty$.

[24] This is because the marginal utility of consumption is constant. The linearity of the utility removes any incentive to either save or borrow for consumption-smoothing or risk-sharing purposes. Then, $c(i) = y(i)$.

[25] This stochastic process is known in the statistical literature under the name of pure birth process. See

Throughout this section, the term *transition* will denote the interval of time from 0 to the time when innovation N is discovered. Knowing the equilibrium investment levels, we compute the expected duration of the transition. Since sojourn times are independent, this is given by

$$\textstyle\sum_{i=1}^{N} \frac{1}{\theta \widetilde{z}(i)}. \tag{31}$$

Expression (31) clearly indicates that the transition is fast when R&D expenditures and θ are high and when N is small.

In the dynamic model, the intermediate good firm owns a life-time patent. However, since innovations are assumed to be drastic, the intermediate good firm stops making profits as soon as a more advanced technology is discovered. Therefore, the value of a patent for invention i is

$$\Pi(i) = \frac{\pi\left(a_i, i\right)}{r + \theta z(i+1)}. \tag{32}$$

Expression (32) is the expected present value of the flow of monopoly profits $\pi\left(a_i, i\right)$ generated by the i-product over an interval of time that is exponentially distributed with parameter $\theta z(i+1)$. Note that $\Pi(i)$ is decreasing in future R&D expenditures, since higher values for $z(i+1)$ shortens the expected tenure of the monopolist producing the i-intermediate product. In computing (32), we assume perfect foresight: in each t all agents correctly foresee the R&D expenditures that will be incurred and the laws that will be enforced in all subsequent technological environments.

As before, we consider two legal regimes. In the flexible regime, at any instant courts and regulators select the ex-post optimal law.[26] Note that the law enforced in the flexible regime does not depend on t, but only on i. This occurs because the trade-off that law-makers face only depends on i. In the rigid regime, the law a_C is chosen at $t = 0$ and is never changed. In this case, regardless of the current technological state and of the current time, courts and regulators are bound to enforce a_C.

Since new innovations increase the productivity of the intermediate good, from (7) we have that profits $\pi\left(a_i, i\right)$ are increasing in i. This implies that over time R&D firms have

Feller (1966). Notice that a pure birth process is a generalization of a Poisson process in which the arrival rate is not constant but is allowed to depend on the current state (in our case, the current i).

[26]This is because even in this dynamic model law-makers in the flexible regime solve a static problem since their choice only affects the current payoff.

stronger incentive to invest. In order to have a balanced growth path for R&D investment, we modify (9) as follows:

$$\max_{z\geqslant 0}\left[\theta z\Pi\left(i\right)-\Omega(i)\frac{1}{2}z^{2}\right], \tag{33}$$

where $\Omega(i+1)=\gamma_{\omega}\Omega(i)$, with $\gamma_{\omega}>1$. That is, we implicitly assume that the cost of hiring researchers is increasing as technology matures (possibly due to increasing wages). Therefore, the optimal amount of R&D investment to discover the intermediate good i, where $i>0$, is

$$\widetilde{z}(i)=\frac{\theta\pi\left(a_{i},i\right)}{\Omega(i)\left(r+\theta\widetilde{z}(i+1)\right)}. \tag{34}$$

Since the LHS of equation (34) is strictly decreasing in $\widetilde{z}(i+1)$, (34) clearly illustrates the negative dependency between current and future research. It is also important to note that R&D investment for the i^{th} invention depends on future laws. The channel through which this occurs is twofold. On the one hand, the law that is enforced in the technological state i affects flow profits; on the other hand, the law that regulates the technological state $i+1$ changes the expected duration of the monopoly for the intermediate good firm.

We now make the following assumption in order to have closed form solutions for maximized welfare in the rigid regime.

Assumption 2: *We assume that $\gamma_{\omega}=\gamma_{A}^{2}$. Then, the ratio between $\pi\left(a,i\right)$ and $\Omega(i)$ is constant for all i.*

5.1. Dynamics: Rigid Regime

We now characterize the rigid regime. For any given law a_C, we find out the equilibrium R&D investment for innovation i. As shown in Aghion and Howitt (1992), there exist many sequences $\{\widetilde{z}(i)\}_{i=1}^{\infty}$ satisfying (34). However, in this paper we focus on the unique stationary equilibrium. Using Assumption 2, the stationary R&D investment $\overline{z}$ must satisfy the functional relationship

$$\overline{z}=\frac{A\left(0\right)^{2}a_{C}\theta}{4^{2}\Omega(0)\left(r+\theta\overline{z}\right)}. \tag{35}$$

One can verify that $\overline{z}$ is increasing in a_C and $A(0)$ and decreasing in $\Omega(0)$.

In our stationary equilibrium, the expected number of innovations per unit interval is then constant and equal to $\theta\overline{z}$. Moreover, the probability that there will be exactly i innovations from time 0 to time t is given by

$$\frac{(\theta\overline{z}t)^i e^{-t\theta\overline{z}}}{i!}. \tag{36}$$

In other words, in a stationary equilibrium the number of innovations up to time t is a random variable following a Poisson distribution of constant rate $\theta\overline{z}$.

Expected welfare in the rigid regime can be easily computed:

$$W_C = \max_{a_C \in [\underline{a},\overline{a}]} \int_0^\infty e^{-rt} \sum_{i=0}^{\infty} \frac{e^{-\theta\overline{z}t} (\theta\overline{z}t)^i}{i!} a_C \vartheta(i) dt. \tag{37}$$

Since Assumption 2 implies that $\overline{z}$ does not depend on i, using (30) one can write

$$W_C = \max_{a_C \in [\underline{a},\overline{a}]} \int_0^\infty e^{-(r+\theta\overline{z})t} a_C \left[\frac{1}{4} A(0)^2 e^{\theta\overline{z}\gamma_A^2 t} - \lambda e^{\theta\overline{z}t}\right] dt, \tag{38}$$

or

$$W_C = \max_{a_C \in [\underline{a},\overline{a}]} a_C \left[\frac{A(0)^2}{4\left(r - \left(\gamma_A^2 - 1\right)\theta\overline{z}\right)} - \frac{\lambda}{r}\right]. \tag{39}$$

Note that the law a_C enters (39) twice. The law has a direct effect on welfare: it multiplies the term in square brackets. Also, the law has an indirect effect through $\overline{z}$; in particular, recall that $\overline{z}$ is increasing in a_C.[27]

Next, we compute the optimal law in the rigid regime. It is obvious that when $\vartheta(0) \geqslant 0$, the legislator will choose $a_C = \overline{a}$ since this law is ex-post optimal for all i. When $\vartheta(0) < 0$ the trade-off is less trivial: choosing a pro-business law is costly along the transition but optimal in the long-run. One can show (see the proof of Proposition 5 in the Appendix) that if $\gamma_A^2 > 2$ the objective in (38) is convex in a_C. In the remainder of this paper, we assume that this condition is met.[28] Then, as in the static model we obtain a bang-bang solution: a_C is either $\overline{a}$ or $\underline{a}$.

The next proposition provides some sufficient conditions that guarantee that the law in the rigid regime is equal to $\overline{a}$.

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[27]Note that for welfare to have an upper bound, the following condition must be met: for all $a_C \in [\underline{a}, \overline{a}]$ we need $r - \left(\gamma_A^2 - 1\right)\theta\overline{z}(a_C) > 0$. If this is not the case, criteria for evaluating infinite utility streams (such as the over-taking criterion) must be used.

[28]Note that this condition, which is sufficient but not necessary for the objective to be convex, is defendable in the context of our model: a low γ_A would not square with the assumption that innovations are drastic. See footnote 11 above.

Proposition 5: *(Rigid Regime: Dynamic Model) The law in the rigid regime is equal to $\overline{a}$ when θ, γ_A and $A(0)$ are sufficiently high.*

Proposition 5 states that when γ_A and θ are sufficiently high, the legislator will select $\overline{a}$. The intuition is the following. First, note that high values of γ_A and θ give the legislator more incentives to use the law to increase R&D investment. Second, also notice that when γ_A and θ are high, the transition will likely be short.

It is important to emphasize that the fact that the speed of technological change is endogenous gives the legislator an additional incentive to choose a pro-business law. By providing strong incentives to innovate the length of the transition becomes shorter, thereby reducing the cost of writing a rigid statute specifying a pro-business law. As we found in Subsection 4.3, this suggests that rigid regimes may sometimes grow at a rate greater than the optimal one.

Finally, by looking at (39) it is also intuitive that the lower $A(0)$, the weaker the incentives to choose a pro-business law. The reason is simply that if the initial productivity is low, the transition from state 0 to state N is longer.

5.2. Dynamics: Flexible Regime

Unlike the commitment regime, the enforced law in the flexible regime depends on the current technological state. In particular, the law is $\overline{a}$ for all $i \geqslant N$ and $\underline{a}$ for all $i < N$.

Knowing the laws that are enforced for all i, we can determine $\widetilde{z}(i)$ using (34). We proceed backwards. For all $i \geqslant N$ we solve for the stationary solution which solves (35) from N onwards, where in (35) a_C is replaced by $\overline{a}$. Next, we determine $\widetilde{z}(N-1)$. We expect $\widetilde{z}(N-1)$ to be low. The reason is twofold. First, R&D investment for invention $N-1$ is low because the law that is enforced in case invention $N-1$ occurs is equal to $\underline{a}$, which is costly for the intermediate good firm. The second reason that depresses investment at this state is that the monopoly power of the producer of the $N-1$ intermediate good is expected to be short-lived. This is because the law in state N will be $\overline{a}$ and, consequently, R&D investment for innovation N is expected to be high.[29] Knowing $\widetilde{z}(N-1)$ and proceeding backwards one can then compute all expenditures in R&D along the transition.

[29]This suggests that growth in the flexible regime may slow down exactly before taking off. A similar "no-growth trap" has been obtained in Aghion and Howitt (1992).

In Figure 3 below, we compute and draw for specific parameter values the sequence $\{\widetilde{z}(i)\}_{i=0}^{\infty}$. Note that R&D investment is constant for all $i \geqslant N$.[30] In the transition, we observe innovation cycles: low innovation when i is odd stimulates innovation when i is even. As discussed above, the research investment for product $N-1$ is especially low.

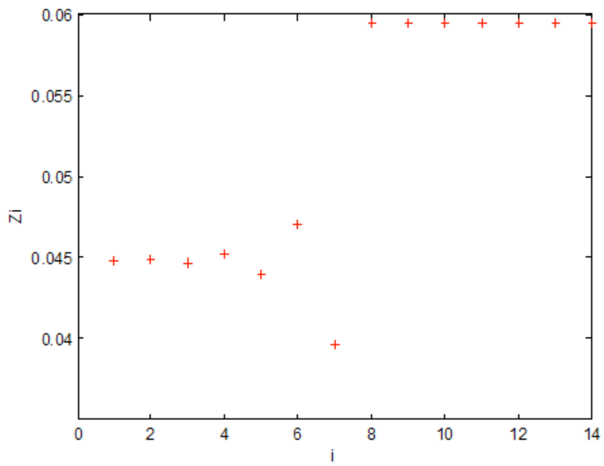

Figure 3: R&D Investment in the Flexible Regime

Given the sequence of equilibrium investment levels, we determine expected welfare in the flexible regime:

$$W_F = \int_0^{\infty} e^{-rt} \sum_{i=0}^{N-1} P_i(t)\underline{a}\vartheta(i)dt + \int_0^{\infty} e^{-rt} \sum_{i=N}^{\infty} P_i(t)\overline{a}\vartheta(i)dt, \tag{40}$$

where $P_i(t)$ is the probability that at time t the most advanced innovation is exactly i. In particular, we have

$$P_0(t) = e^{-\theta\widetilde{z}(1)t} \tag{41}$$

and for $i \geqslant 1$

$$P_i(t) = \theta\widetilde{z}(i)e^{-\theta\widetilde{z}(i+1)t} \int_0^t e^{\theta\widetilde{z}(i+1)s} P_{i-1}(s)ds. \tag{42}$$

[30]In Figure 3 we obtain $N = 8$.

For example, solving the above recursive relation we obtain (assuming that $\widetilde{z}(1)$ is different from $\widetilde{z}(2)$)

$$P_1(t) = \theta\widetilde{z}(1)\left(\frac{1}{\theta\widetilde{z}(2) - \theta\widetilde{z}(1)}e^{-\theta\widetilde{z}(1)t} + \frac{1}{\theta\widetilde{z}(1) - \theta\widetilde{z}(2)}e^{-\theta\widetilde{z}(2)t}\right), \tag{43}$$

which shows, for instance that $P_1(t)$ is low if $\theta\widetilde{z}(2)$ is high and/or $\theta\widetilde{z}(1)$ is low.[31]

5.3. Discussion

First, we compare the speed of technological change in the two regimes.[32] Two cases are possible. First, assume that the law in the rigid regime is $\overline{a}$. In this case, the rate of output growth in the rigid regime will be greater than in the flexible one. If instead the rigid regime chooses $\underline{a}$, comparing the rates of output growth during the transition is not straightforward. However, as soon as invention N is discovered, we obtain that in the flexible regime the enforced law will be $\overline{a}$ and, consequently, the economy without commitment will grow at a faster pace.

We now compare welfare in the two regimes. First, we expect the rigid regime to dominate when $\underline{a}$ is small because this reduces the investment in R&D along the transition and slows down growth. Also, we expect the rigid regime to dominate when θ and γ_A are high since in this case, from Proposition 5, we know that the rigid regime is likely to choose $\overline{a}$, which speeds up the transition and make the lack of flexibility of the rigid regime less costly.

Which system is preferable when N is high is less straightforward. On the one hand, a long transition makes credibility problems more serious and commitment more desirable. This should give an edge to the rigid regime. But on the other hand, a large N may induce the legislator in the rigid regime to select $\underline{a}$. This implies that in both regimes the transition will be slow. However, as soon as the transition is over, the flexible regime will dominate the rigid one by growing faster and by choosing a more efficient law.

6. Conclusion

This paper investigates whether a flexible legal system is preferable to a rigid system in keeping up with technological progress. To answer this question we developed a simple model

[31]For the derivation of (41), (42) and (43), see, for instance, Karlin and Taylor (1975, p. 121) and Feller (1966, p. 41).

[32]As discussed above, for each legal regime we picked the equilibrium in which the economy eventually moves to a balanced growth path, where all expected rates of growth are constant.

of endogenous technological change where innovations are vertical (that is, new products provide greater quality and replace existing ones) and we analyze the two legal regimes.

We argue that the comparison between the two institutions involves a trade-off between commitment and flexibility. In this paper, this trade-off is is far from trivial since the degree of uncertainty, which is a key parameter in the rules-versus-discretion literature, is not exogenous but depends on R&D firms' investment decisions, which are endogenous to the model.

In the context of a model with only two technological states, we show that rigid legal systems are preferable (in terms of welfare and rate of output growth) in the early stages of technological development. In the intermediate stages we obtain that flexible legal systems are preferable: output grows faster and welfare is greater. Finally, when technology is mature, the two legal systems are shown to be equivalent.

The amount of innovation in the rigid regime may be either inefficiently low or, under some conditions, inefficiently high.

The welfare comparison summarized above holds even when we assume that in the rigid regime the statute (or regulation) can be changed ex-post at a cost.

We then extend our analysis to a model where technology undergoes continual change, we show that similar results to the ones obtained in the simple setting with only two technologies hold. In the stationary equilibrium of the rigid regime we show that the speed of technological change is either very low or very high. In the flexible regime, we find that because of commitment problems technological change is relatively slow in the early stages of technological development.

A final question would be to ask how our conclusions would change in an economy where R&D investment increases the *variety* of available goods (for instance as in Romer, 1990). Various results obtained in the current setting would likely survive. However, we expect the legislator in a rigid regime (where the law is not contingent on each variety) to discourage innovation, but not to induce overinvestment. Indeed, contrary to our conclusions, horizontal innovations always increase the complexity of the economy since new varieties coexist with old varieties. Therefore, the legislator in the rigid regime would likely have a bias against such innovations. Everything else being equal, we expect the rigid regime to grow at a slower pace than in the setting we have analyzed here.

Appendix

Proof of Proposition 2: i) Using Definition 1, when technology is mature we have that $\vartheta(0) \geqslant 0$ and $\vartheta(1) > 0$. In the flexible regime, we know from (14) that law-makers select $\overline{a}$ in both states. In the rigid regime, from (24) we conclude that $a_C = \overline{a}$. This implies that welfare in the two regimes is the same and that $g_C = g_F$.

ii) Suppose now that technology is at an intermediate stage, as defined in Subsection 3.6. In the flexible regime, from (14) we conclude that the law enforced in state 1 (resp. 0) is $\overline{a}$ (resp. $\underline{a}$). In the rigid regime, using (24) we know that a_C is either $\underline{a}$ or $\overline{a}$. First, assume that $a_C = \underline{a}$. Given (10), this implies that the probability that state 1 occurs in the rigid regime is lower than in the flexible one. Consequently, from (11) we have that $g_F > g_C$. Moreover, we also obtain that $W_F > W_C$. The reason is twofold: because $\underline{a}$ does not maximize $u(a,1)$ and because state 1 (which by Assumption 1 provides greater utility than state 0) is more likely in the flexible regime. Second, assume that $a_C = \overline{a}$. In this case, the probability that state 1 occurs in the two regimes is the same: then, $g_F = g_C$. Since we assumed an interior solution for R&D investment, the probability that state 1 occurs is strictly smaller than one. Then, with some positive probability state 0 occurs. Since $\overline{a}$ does not maximize $u(a,0)$, this implies that when $a_C = \overline{a}$ we also have that $W_F > W_C$.

iii) When technology is at an early stage, using (14) we obtain that the ex-post optimal law is $\underline{a}$ in both states so that the flexible regime provides weak incentives to innovate. Since the rigid regime can replicate the flexible one by choosing $a_C = \underline{a}$ and since the rigid regime can also choose $a_C = \overline{a}$, it must be the case that $W_C \geqslant W_F$ and $g_C \geqslant g_F$. ■

Proof of Proposition 3: We derive a_0^{FB} and a_1^{FB}. Since a_0^{FB} does not affect the amount of R&D investment, the first-best law in state 0 is easy to compute:

$$a_0^{FB} = \begin{cases} \underline{a} & \text{if} \quad \vartheta(0) < 0 \\ \overline{a} & \text{if} \quad \vartheta(0) \geqslant 0 \end{cases} \tag{A.1}$$

To find a_1^{FB} two cases must be considered. First, assume that $\vartheta(1) \geqslant 0$. In this case a_1^{FB} is obviously equal to $\overline{a}$. Second, assume $\vartheta(1) < 0$. In this case, since the second derivative is $2\vartheta(1)$, the objective is concave in a_1^{FB}. Therefore, one obtains:

$$a_1^{FB} = \begin{cases} \overline{a} & \text{if} \quad 2\overline{a}\vartheta(1) - a^{FB}(0)\vartheta(0) \geqslant 0 \\ \underline{a} & \text{if} \quad 2\underline{a}\vartheta(1) - a^{FB}(0)\vartheta(0) \leqslant 0 \\ \underline{a}\ \frac{\vartheta(0)}{2\vartheta(1)} & \text{otherwise} \end{cases} \tag{A.2}$$

To show that R&D investment in the flexible regime cannot be larger than the one under the first-best law, we must show that $a^*(1) \leqslant a_1^{FB}$. Two cases are possible. When $\vartheta(1) \geqslant 0$ in the flexible regime as well as under the first-best law we have that the law for the more advanced technology is equal to $\overline{a}$, so that investment in the flexible regime is identical to the first-best level. When $\vartheta(1) < 0$, law-makers in the flexible regime choose $\underline{a}$. The rate of growth under the first-best can only be larger than g_F.

In order to prove that the commitment regime may induce overinvestment in research, we must show that there exists a region of parameter values where $a_C = \overline{a}$ and at the same time $a_1^{FB} < \overline{a}$. To show this, assume that $\vartheta(1) < 0$. (When $\vartheta(1) \geqslant 0$ innovation in the first-best is already at the maximum level and the commitment regime can at most grow at the same rate). Moreover, consider the following parameter values: take $\overline{a} = 1$ and $\theta^2\Phi(1) = 1$ so that if the law is $\overline{a}$, state 1 occurs with probability one. In this case, we have

$$W_C = \max_{a_C \in [\underline{a}, \overline{a}]} (1 - a_C)\vartheta(0)\, a_C + a_C\vartheta(1)\, a_C. \tag{A.3}$$

Using (24), one can show that the law in the rigid regime is 1 if

$$\underline{a} > \frac{\vartheta(1)}{\vartheta(0) - \vartheta(1)}. \tag{A.4}$$

Using (A.1) and (A.2) we know that when $\vartheta(1) < 0$ we have that $a^{FB}(1) < 1$ if and only if

$$\underline{a} < \frac{2\vartheta(1)}{\vartheta(0)}. \tag{A.5}$$

One can verify that when $\vartheta(0) < 2\vartheta(1)$ it is always possible to find a value for $\underline{a}$, with $0 < \underline{a} < 1$, such that both (A.4) and (A.5) are satisfied, which proves our claim that at least for some parameter values the rigid regime induces overinvestment. Note that since (A.4) and (A.5) are strict inequalities, the same argument would also go through if $\theta^2\Phi(1)$ is strictly below but sufficiently close to one so that, as we assumed in the paper, the probability of state 1 is strictly lower than one. ■

Proof of Proposition 4: We introduce some notation. Let $a_C(\kappa)$ denote the law that is initially chosen in the partially rigid regime. Moreover, we denote by $a(i, \kappa, a_C(\kappa))$ the law that is chosen ex-post in state i given that the cost of changing the law is κ and that $a_C(\kappa)$ was initially chosen.

i) When $A(0) \geqslant 2$, it is immediate to verify that $a_C(\kappa) = \overline{a}$ for all κ. Moreover, for $i = 1, 2$, we have $a(i, \kappa, \overline{a}) = \overline{a}$. Welfare in the rigid regime does not depend on κ since the law is never changed ex-post. Then we have that $W_C(\kappa) = W_C(0) = W_F$ for all κ.

ii) Suppose now that technology is at an intermediate stage, as defined in Subsection 3.6. In this case, for all κ and all $a_C(\kappa) \in [\underline{a}, \overline{a}]$ we have that

$$a(1, \kappa, a_C(\kappa)) = \begin{cases} a_C(\kappa) & \text{if} \quad a_C(\kappa) \geqslant \frac{\vartheta(1)\overline{a} - \kappa}{\vartheta(1)} \\ \overline{a} & \text{otherwise} \end{cases} \tag{A.6}$$

It is easy to verify that at this stage (since $\vartheta(1) \geqslant 0$) for any given $a_C(\kappa)$ we have that $a(1, \kappa, a_C(\kappa))$ is weakly decreasing in κ. Next, we define the following expressions:

$$\widetilde{W_1}(\kappa, a_C(\kappa)) = \max\left\{\vartheta\,(1)\,a_C; -\kappa + \max_{a'} \vartheta\,(1)\,a'\right\} \tag{A.7}$$

and

$$\widetilde{W_0}(\kappa, a_C(\kappa)) = \max\left\{\vartheta\,(0)\,a_C; -\kappa + \max a' \vartheta\,(0)\,a'\right\}. \tag{A.8}$$

It can be shown that for a given $a_C(\kappa)$, both $\widetilde{W_0}(\kappa, a_C(\kappa))$ and $\widetilde{W_1}(\kappa, a_C(\kappa))$ are weakly decreasing in κ.

Finally, we define maximized welfare in the (partially) rigid regime where the cost of changing the law ex-post is κ

$$W_C(\kappa) = \max_{a_C(k)\in[\underline{a},\overline{a}]} \theta^2\Phi(1)a(1,\kappa,a_C(\kappa))\left[\widetilde{W_1}(\kappa, a_C(\kappa)) - \widetilde{W_0}(\kappa, a_C(\kappa))\right] + \widetilde{W_0}(\kappa, a_C(\kappa)) \tag{A.9}$$

We now show that for all $\kappa'' > \kappa' > 0$, we have that $W_C(\kappa'') \leqslant W_C(\kappa')$. To see this, note that

$$\begin{aligned}
W_C(\kappa') &\geqslant \theta^2\Phi(1)a(1,\kappa',a_C(\kappa''))\left[\widetilde{W_1}(\kappa', a_C(\kappa'')) - \widetilde{W_0}(\kappa', a_C(\kappa''))\right] + \widetilde{W_0}(\kappa', a_C(\kappa'')) \\
&\geqslant \theta^2\Phi(1)a(1,\kappa'',a_C(\kappa''))\left[\widetilde{W_1}(\kappa'', a_C(\kappa'')) - \widetilde{W_0}(\kappa'', a_C(\kappa''))\right] + \widetilde{W_0}(\kappa'', a_C(\kappa'')) \\
&= W_C(\kappa'')
\end{aligned}$$

The first inequality follows from the fact that $a_C(\kappa')$ is the optimal strategy when the cost is κ'. Hence it cannot provide less utility than choosing $a_C(\kappa'')$. To understand the second inequality, just recall that for a given law $a_C(\kappa)$, we have that $a(1,\kappa,a_C(\kappa))$, $\widetilde{W_1}(\kappa, a_C(\kappa))$ and $\widetilde{W_0}(\kappa, a_C(\kappa))$ are all weakly decreasing in κ.

We now show that for all $\kappa > 0$ we have $W_C(\kappa) < W_F$. Consider any $\kappa > 0$. Two cases are possible: $a_C(\kappa) = \overline{a}$ or $a_C(\kappa) < \overline{a}$.

First, assume that $a_C(\kappa) = \overline{a}$. In this case, $a(1,\kappa,a_C(\kappa)) = \overline{a}$ and the probability that state 1 occurs when $\kappa = 0$ and when $\kappa > 0$ is the same. (Recall in fact that when $\kappa = 0$ we have $a(1,0,a_C(0)) = \overline{a}$) Then, when $a_C(\kappa) = \overline{a}$ we obtain that $\widetilde{W_1}(0, a_C(0)) = \widetilde{W_1}(\kappa, \overline{a})$. However, since in state 0, $\overline{a}$ is suboptimal, we also have that $\widetilde{W_0}(0, a_C(0)) > \widetilde{W_0}(\kappa, \overline{a})$. Since the probability that state 0 occurs is assumed to be strictly positive, from (A.9) we conclude that $W_C(\kappa) < W_C(0) = W_F$.

Second, assume that $a_C(\kappa) < \overline{a}$. Two further cases are possible: either $a(1,\kappa,a_C(\kappa)) = \overline{a}$ or $a(1,\kappa,a_C(\kappa)) < \overline{a}$. In the former case, the probabilities of state 1 occurring is the same when $\kappa = 0$ and when $\kappa > 0$. However, it must be that $\widetilde{W_0}(0, a_C(0)) \geqslant \widetilde{W_0}(\kappa, a_C(\kappa))$ and, because the law is changed in state 1, $\widetilde{W_1}(0, a_C(0)) > \widetilde{W_1}(\kappa, a_C(\kappa))$. This implies that $W_C(\kappa) < W_F$. Second, assume that we have that $a(1,\kappa,a_C(\kappa)) < \overline{a}$. Then, $\widetilde{W_1}(0, a_C(0)) > \widetilde{W_1}(\kappa, a_C(\kappa))$. Moreover, the probability that state 1 occurs when $\kappa = 0$ is strictly greater than the same probability when $\kappa > 0$. Then, we also have that $W_C(\kappa) < W_F$.

iii) When technology is at an early stage the ex-post optimal law is always $\underline{a}$ so that the flexible regime does not provide any incentive to innovate. The rigid regime, on the contrary, can choose to provide incentives. Therefore, exactly as in case iii) of Proposition 2, the rigid regime could replicate the flexible one by picking $\underline{a}$, so that it must be the case that $W_C(\kappa) \geqslant W_F$. We refer to the main text (see Section 4.4) for an example that shows that $W_C(\kappa)$ may not be monotone in κ when technology is at the early stage. ■

Proof of Proposition 5: We rewrite the legislator's problem in (39) as

$$W_C = \max_{a_C \in [\underline{a}, \overline{a}]} f(a_C) a_C, \tag{A.10}$$

where

$$f(a_C) = \frac{A(0)^2}{4\left(r - \left(\gamma_A^2 - 1\right)\theta \overline{z}(a_C)\right)} - \frac{\lambda}{r} \tag{A.11}$$

and $\overline{z}(a_C)$ is obtained using (35).

Step 1: *A sufficient (but not necessary) condition to insure that the objective in problem (A.10) is convex is that* $\gamma_A^2 > 2$.

Proof: To show this, since $f'(a_C) > 0$ and $a_C > 0$, it is enough to show that $f''(a_C) > 0$. For simplicity, we denote $\left(\gamma_A^2 - 1\right)$ by $\boldsymbol{\gamma}$.

The first and second derivatives of $f(a_C)$ are, respectively,

$$f'(a_C) = \frac{\boldsymbol{\gamma}\theta \overline{z}'(a_C) A\left(0\right)^2}{4\left(r - \boldsymbol{\gamma}\theta\overline{z}(a_C)\right)^2} \tag{A.12}$$

and

$$f''(a_C) = \left(\frac{\boldsymbol{\gamma}\theta A^2}{4}\right) \frac{\overline{z}''(a_C)\left(r - \boldsymbol{\gamma}\theta\overline{z}(a_C)\right) + 2\boldsymbol{\gamma}\theta(\overline{z}'(a_C))^2}{\left(r - \boldsymbol{\gamma}\theta\overline{z}(a_C)\right)^3} \tag{A.13}$$

Given that in order for welfare to have an upper bound, we assumed $r - \boldsymbol{\gamma}\theta\overline{z}(a_C) > 0$ for all laws, we obtain that $f''(a_C) > 0$ if and only if

$$\overline{z}''(a_C)\left(r - \boldsymbol{\gamma}\theta\overline{z}(a_C)\right) + 2\boldsymbol{\gamma}\theta(\overline{z}'(a_C))^2 > 0. \tag{A.14}$$

After obtaining $\overline{z}'(a_C)$ and $\overline{z}''(a_C)$ from equation (35), inequality (A.14) can be written (after denoting $A(0)$ and $\Omega(0)$ simply by A and Ω, respectively) as

$$\boldsymbol{\gamma}\frac{1}{32}\theta^3 A^4 \left[\Omega(4r^2\Omega + \theta^2 A a_c)\right]^{-1} - \frac{1}{16}\theta^3 A^4 \Omega \left[\Omega(4r^2\Omega + \theta^2 A a_c)\right]^{-\frac{3}{2}} \left(r - \boldsymbol{\gamma}\theta\overline{z}(a_C)\right) > 0. \tag{A.15}$$

After some algebra, it can be shown that a sufficient condition to have (A.15) strictly positive is that $\gamma_A^2 > 2$.

Step 2: *If* $\gamma_A^2 > 2$, *the law in the rigid regime is equal to* $\overline{a}$ *when* θ, γ_A *and* $\vartheta(0)$ *are sufficiently high.*

Proof: First, note that given that the objective in (A.10) is convex, a sufficient condition to insure that the optimal law is $\overline{a}$ is that

$$\frac{A\left(0\right)^{2}}{4\left(r-\left(\gamma_{A}^{2}-1\right)\theta\overline{z}(\overline{a})\right)}-\frac{\lambda}{r}>0. \tag{A.16}$$

Since $\overline{z}(\overline{a})$ does not depend on γ_A, it is easy to verify that when γ_A is sufficiently large, (A.16) is satisfied. Therefore, the solution of problem (39) is $a_C = \overline{a}$. After verifying that $\overline{z}(a_C)$ is increasing in θ and $A\left(0\right)$, a similar argument is used to show that when either θ or $A\left(0\right)$ are sufficiently high, we have $a_C = \overline{a}$. ■

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