Domestic or global imbalances? Rising inequality and the fall in the US current account

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Abstract

This paper shows how the rise in individual income risk in the US since the 1980s could help explain the fall in its foreign asset position. The key to this result is endogenous financial deepening in an open economy where individuals can default on contracts, at the price of exclusion from financial trade. More volatile income makes default less attractive, and thus allows higher borrowing against future income. In a closed economy, this improves consumption smoothing across volatile income realisations (Krueger and Perri 2006). I show analytically how, in an open economy, relaxed default constraints from higher risk can decrease stationary asset demand with potentially no effect on consumption insurance, determined by world interest rates. In a quantitative two country general equilibrium analysis, the observed rise in individual risk in the US strongly lowers its foreign asset demand. This is reinforced by a precautionary "savings glut" from increased income volatility in an emerging economy, calibrated to the evolution of individual inequality in China, where the absence of insurance markets leaves no room for financial deepening.

JEL Classification Codes: D31, D52, E21, F21, F41
Keywords: Current Account, Global Imbalances, Heterogeneous Agents, Inequality, Incomplete Markets, Participation Constraints, Default

1 Introduction

Over the past 25 years, the US economy has experienced a significant rise in both cross-sectional income inequality and the uncertainty of individual incomes. The standard self-insurance model suggests that this should have increased individual savings at the same time as consumption inequality. But instead, as figure 1 shows, during this period of rising individual risks the US savings rate declined and the country's net foreign assets as a share of GDP fell by almost 30 percentage points, while consumption inequality increased only very little. This paper presents a simple model of an open economy where a rise in individual income risk lowers aggregate foreign assets, while leaving consumption inequality largely unchanged. The crucial assumption is that individuals have the option to default on financial contracts, at the price of exclusion from financial trade. This limits the amount they can credibly borrow against the future during periods when their income is temporarily low. Higher income risk increases individuals' incentives to maintain financial market access in order to smooth consumption. A rise in risk thus makes default less attractive, relaxes incentive constraints and allows individuals to issue more claims against their future income. In a closed economy, this increase in supply reduces the price of state-contingent assets and thus improves consumption insurance, as pointed out by Krueger and Perri (2006). In an open economy, however, the price response is muted, and the relaxed supply constraint increases aggregate issuance of claims to the rest of the world. So net foreign assets fall. For the limiting case of a given world interest rate, corresponding to a zero price response, I show that consumption volatility can be unchanged as higher income risk relaxes default constraints and increases aggregate debt. This study thus identifies changes in income risk over time as an important driver of global asset positions, but also points out the role of international variables, such as world interest rates, for domestic consumption inequality. While the main focus is on the period $1980 - 2005$, the analysis has immediate implications for the evolution of foreign asset positions in the future as long as the structure of inequality and individual income risks continues to change across developed economies.

To derive the main theoretical results, I first show how, for any given level of world interest rates, the economy has a unique consumption distribution that is stationary and does not depend on initial conditions. In this sense, individual participation-constraints provide an alternative way of "closing small open economies" (Schmitt-Grohé et al 2003) without ad hoc restrictions on preferences. Using this feature, the paper's main analytical result is to show that, in a standard economy with limited commitment to contracts (Krueger and Perri (2011), Thomas and Worrall (2007)), there is a level of individual income volatility beyond which further increases in idiosyncratic risk lead to a decline in the aggregate demand for consumption and assets. The theory also shows how financial openness significantly changes the determinants of individual consumption volatility, which, under certain conditions, is entirely determined by world asset prices, and thus unaffected by changes in domestic idiosyncratic income risk.

Using a calibrated version of the model, the quantitative section of the paper then shows that the rise in income risk experienced by the US economy over the last 25 years implies a significant reduction in the stationary demand for foreign assets. Furthermore, a two country general equilibrium model, calibrated to capture the observed rises in income heterogeneity in the US and China, shows that this effect can be augmented by a precautionary "savings glut" in emerging economies where insurance against increased income risks is unavailable. By increasing the borrowing capacity of US consumers at the same time as precautionary savings in less financially developed economies, the rise in individual income risk over the last 25 years thus leads to a reallocation of savings in line with the observed "global imbalances", including a fall in US net foreign assets on the order of one third of US GDP.
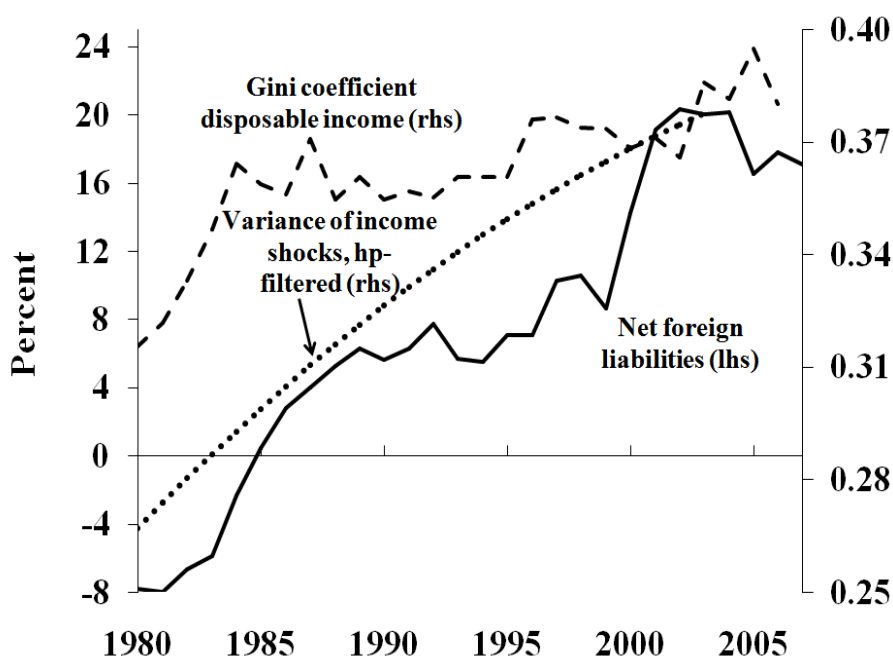

Figure 1: The series for US net foreign liabilities is taken from an updated and extended version of the dataset constructed by Lane and Milesi-Ferretti (2007). Both the Gini coefficient for US equivalent household disposable income (Heathcote et al 2010) and the hp-filtered total variance of estimated shocks to log household incomes (Krueger and Perri 2006), are based on data from the US Consumer Expenditure Survey.

Several previous studies have analysed how changes in the structure of the world economy may have lead to a permanently lower US net asset position, focusing, for example, on differences in productivity or demographic growth, or in the structure of country portfolios and the implied valuation effects.[1] Most relevant for this paper are Fogli and Perri (2006) and Mendoza et al

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[1]See for example, Backus et al (2005), Gourinchas et al (2007), Henriksen (2005), Jeanne et al (2009), Mc-

(2009), who study the role of income risk for relative precautionary savings across countries. In particular, Fogli and Perri (2006) show how the more pronounced "great moderation" in macro-volatility in the US could have reduced precautionary savings there more than in other countries. Mendoza et al (2009), on the other hand, show how countries with less developed financial markets can experience capital outflows, as previously argued by Gertler and Rogoff (1990), Boyd and Smith (1997), Matsuyama (2004) or Broer (2007). Specifically, the authors show how countries whose financial markets are less capable of insuring individual risks accumulate excess precautionary savings which they invest predominantly in bonds of the financially more developed US economy. In their model, individual risks are identical across countries, and constant over time, making exogenous differences in financial development the main determinant of foreign asset positions once countries open their capital accounts. In contrast, global imbalances in the present study are driven by the large observed changes in individual income risk in the US as well as other countries over the recent past, and the endogenous response of financial markets they cause.[2] The main contribution of this study is thus to show how, in an economy where the depth of financial markets depends endogenously on the attractiveness of default, an increase in individual risk can reduce aggregate asset demand. Again, this contrasts with Mendoza et al (2009), where an exogenously given degree of financial development would imply a positive relationship between risk and precautionary savings. The quantitative general equilibrium analysis of this paper shows how the financial deepening effect can have important consequences for aggregate foreign assets, and might possibly be augmented by a rise in precautionary savings in response to the observed increase in individual income risk in financially less developed emerging economies. The analysis does not, however, predict a sustainable US current account deficit. Rather, in the long-run, net exports finance interest payments on the foreign liabilities that the US economy accumulated during a period of adjustment, implying a balanced current account. This is similar to Fogli and Perri (2006), Mendoza et al (2009) and Jeanne et al (2009) who all focus on transitional deficits in response to a change in relative long-run equilibrium savings. The predictions are thus different to, for example, Caballero et al (2009), where a fall in the share of output that emerging economies can pledge as dividends leads to permanent net resource flows to the United States. Also, since I abstract from aggregate risk, and the gains from international asset trade are thus exhausted entirely through unconditional riskless bonds, the model cannot generate the "exorbitant privilege" (Gourinchas et al 2007) that allows the US to finance a permanent current account deficit by earning excess returns on its country-portfolio.

The results of this paper also relate to the literature on the macroeconomic role of uninsurable individual income risks and, particularly, the determinants of aggregate wealth and individ-

Grattan and Prescott (2008) or Prades and Rabitsch (2007).

[2]Mendoza et al (2009) present an extension of their model to static differences in income risk across countries, but not changes in the riskiness of incomes across time, at the end of their study.

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ual consumption inequality with participation-constrained complete markets, as in Alvarez and Jermann (2000), Kehoe and Levine (2001), Kocherlakota (1996), or Ligon et al (2002). The theoretical contribution is to show analytically that, in the standard limited commitment economy of Krueger and Perri (2011) or Thomas and Worrall (2007), a rise in individual income risk reduces the aggregate excess demand for consumption goods and assets under weak assumptions about preferences as long as the initial level of risk is high enough. This is the main theoretical result that explains falling foreign asset positions after a rise in the riskiness of individual incomes. It contrasts with traditional bufferstock savings models, where, under additional assumptions on preferences or the shape of the savings function, aggregate assets increase with earnings risk.[3] Finally, the paper shows that the relationship between income risk and consumption inequality in an open economy is fundamentally different to that in models without international asset trade, such as Krueger and Perri (2006) or Heathcote et al (2008). Particularly, in the environment of this paper, world interest rates are the key driver of changes in consumption risk and inequality. So the evolution of the international economy is a key determinant of the domestic consumption distribution and risk sharing.

The rest of the paper is structured as follows: Section II describes the environment of an open economy with debt-constrained domestic financial markets. Section III derives the analytical results. Section IV reports the quantitative results.

2 An open economy with debt-constrained domestic financial markets

This section presents a simple model of an open economy where domestic financial markets are constrained by individual default. It focuses on the impact of slow-moving changes in idiosyncratic risk on the long-run equilibrium levels of savings and consumption risk, and therefore abstracts from aggregate and transitional dynamics that are likely to dominate asset holdings in the short- and medium-run. The model is thus similar to that in Krueger and Perri (2011), amended for international asset trade. Particularly, to allow a theoretical characterisation of the relationship between income risk, aggregate assets, and consumption inequality, it presents the limit case of an open economy whose aggregate savings behaviour has no impact on world interest rates. The following section then relaxes this assumption in a general equilibrium analysis of a calibrated world economy.

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[3]See Huggett (2004) for a discussion and a review of the literature.

2.1 The general environment

The analysis focuses on a country that takes prices of goods and assets traded with the rest of the world as given. The country is populated by a large number of individuals of unit mass. Individuals are indexed by i, located on a unit-interval $i \in \mathbb{I} = [0,1]$. Time is discrete $t \in \{0, 1, 2, ..., \infty\}$ and a unique perishable endowment good is used for consumption.

The consumption endowment of agent i in period t, $z_{i,t}$, takes values in a finite set Z, $z_{i,t} \in Z = \{z_1 > z_2 > ... > z_N\}, N \geq 2$. Let $s_t : \mathbb{I} \longrightarrow Z \times R$ denote the state of the economy in period t, a measurable function that assigns income and asset values to all agents. Endowments are independent across agents, and follow a stochastic process described by a Markov transition matrix F. F has strictly positive entries, is identical across agents, monotone (in the sense that the conditional expectation of an increasing function of tomorrow's income is itself an increasing function of today's income), and has a unique ergodic distribution $\Phi_Z : \mathbb{Z} \to [0,1]$, where $\mathbb{Z}$ is the power set of Z. Thus, in the long-run, aggregate income $Y = \int_{\mathbb{I}} z_i$ is constant, while individual income fluctuates. For concreteness, the theory section mainly focuses on the case of $N = 2$, when income only takes the values $\{z^1, z^2\} = \{y_0 + \frac{1}{\nu}\varepsilon, y_0 - \frac{1}{1-\nu}\varepsilon\}$, $\varepsilon \geq 0$, where $\nu = \frac{1-q}{2-q-p}$ is the stationary mass of high-income individuals for the transition matrix $F = [p, 1-p; 1-q, q]$.[4] I define a rise in income risk in the two-income case as a mean-preserving spread to the income distribution $d\varepsilon > 0$.

Agents live forever and order consumption sequences according to the utility function

$$U = E_{s_0} \sum_{t=0}^{\infty} \beta^t u(c_{i,t}) \tag{1}$$

where E_{s_0} is the mathematical expectation conditional on s_0, $0 < \beta < 1$ discounts future utility, $c_{i,t}$ is consumption by agent i in period t, and $u : R^+ \to R$ is an increasing, strictly concave, continuously differentiable function that satifies Inada conditions and is identical for all agents in the economy.

2.2 Asset markets

I choose a specification of the economy similar to that by Alvarez and Jermann (2000), amended for the international setting. Agents engage in sequential trade of a complete set of state-contingent bonds domestically, but international asset trade is limited to non-contingent bonds.[5]

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[4]In the two income case, persistence requires $p, q > 1/2$. I also assume persistence of high and low income not to be too different: $\frac{\beta-1}{\beta} < p - q < \frac{1-\beta}{\beta}$.

[5]This is non-restrictive as there is no aggregate risk and the law of large numbers holds. It requires, however, no default on foreign debt on a country level. In a previous version of this paper I show that Broner and Ventura's (2006) result applies to my setting. So perfect secondary markets prevent governments from defaulting on agents' foreign liabilities.

Individual endowment realisations are verifiable and contractable, but asset contracts are not enforceable: at any point, individuals can default on their contractual payments at the price of eternal exclusion from financial markets. Thus the total amount an agent can borrow today against any income state z_j tomorrow is bounded by the option to default into financial autarky. There, consumption is forever equal to income. Given the Markov structure of income, the value of default as a function of the vector of current income z can be written as

$$W(z) = \sum_{t=0}^{\infty} (\beta F)^t U(z) = (I - \beta F)^{-1} U(z) \tag{2}$$

I denote holdings of bonds and Arrow-Debreu securities paying off in state s_t by b and $a(s_t)$ respectively. In any state s_t, $V(z(s_t), a(s_t), b_t)$ is the contract value as a function of income $z(s_t)$ and current asset holdings $\{a(s_t), b_t\}$.

As in Alvarez and Jermann (2000) individual i's participation constraint for any state s_{t+1} tomorrow can be written as a constraint on the claims she can issue against s_{t+1} income. This borrowing constraint is "not too tight" in the words of Alvarez and Jermann (2000) if it assures participation but does not constrain contracts otherwise

$$a_i(s_{t+1}) + Rb_{i,t+1} \geq A_i(s_{t+1}) = min\{\alpha(s_{t+1}) : V(z_i(s_{t+1}), \alpha(s_{t+1}), 0) \geq W(z_i(s_{t+1}))\} \tag{3}$$

Note that bonds are redundant in this setting, although including them facilitates somewhat the setup of the planner's problem in an open economy where aggregate bond holdings, denoted B, are potentially non-zero.

Importantly, the portfolio constraint (3) limits the issuance of assets that demand net repayments in high income periods, when the outside option of default is most attractive. On the one hand, this reduces transfers from high to low income individuals under insurance contracts. But on the other, it defines a maximum level of debt that individuals, and thus the country on aggregate, can sustain. The attractiveness of default during periods of high <u>individual</u> income, determined by the value of the outside option of financial autarky W, is thus the main determinant of the <u>aggregate</u> net asset position in stationary equilibrium.

2.3 The household's problem

Every period, households maximise their expected utility by choosing current consumption and assets subject to budget and borrowing constraints. As shown in Alvarez and Jermann (2000)

this problem has a recursive representation as

$$V(z(s), a(s), b) = max_{c,\{a(s')\},b'} \{u(c) + \beta E_s V(z', a(s'), b')\}$$
$$s.t. \ c + \sum_{s'} a(s')q(s') + b' \leq Rb + a(s) + z(s)$$
$$a(s') + Rb' \geq A(s')$$
$$A(s') = min\{\alpha(s') : V(z(s'), \alpha(s'), 0) \geq W(z(s'))\} \quad (4)$$

where c, b', a' are policy functions of the state variables $(z(s), a(s), b)$.

2.4 Definition of competitive equilibrium

Following Alvarez and Jermann (2000), for given initial distributions of assets and income, a competitive equilibrium in this economy is a set of asset prices $\{q(s')\}, R$, a set of individual decision rules $c, b', \{a'(s')\}$ with associated value functions $V(z, a, b)$
such that

1. $V(z, a, b)$ is the households maximum value function associated to the household problem given $q(s'), R$

2. $V(z, a, b)$ is attained by $c, b', a'(s')$

3. Markets for state-contingent assets clear
   $\int_{\mathbb{I}} a_i(s_t) = 0, \ \forall s_t, t$

4. The interest rate on bonds is equal to the world interest rate R.

The competitive equilibrium is called "stationary" if prices and aggregate bond holdings are constant, and the distribution of individual consumption and wealth holdings is stationary through time.

3 Individual income risk, aggregate savings, and consumption inequality in stationary equilibrium: analytical results

In this section I show how a rise in individual income risk can decrease aggregate asset holdings by making default less attractive, and thus relaxing the endogenous borrowing constraints in (4). The main challenge is to show that this effect, which is conditional on a given level of current income and directly affects only constrained individuals, also translates to aggregate assets in

the economy. For this, we have to take into account that an increase in individual risk changes both the long-run distribution of current incomes and the savings behaviour of unconstrained individuals. For example, in precautionary savings models with non-contingent assets (see e.g. Huggett (1993), Aiyagari (1993)), a rise in risk typically changes the number of individuals that exhaust their borrowing limit as well as the savings behaviour of the unconstrained. In most economies the net effect is analytically untractable. In the complete markets environment of this paper, however, even though some individuals increase their savings in response to higher income risk, we can exploit the analytical solution of the consumption distribution in Broer (2009a), which generalises results in Krueger and Perri (2011) and Thomas and Worrall (2007), to show that the net effect of relaxing constraints is to reduce aggregate assets. Working with the distribution of consumption, rather than assets directly, is convenient for two reasons. First, it turns out that unconstrained individuals' consumption can be recursively defined as a function of consumption in the most recent constrained period and thus of autarky values. In a stationary equilibrium, the resulting link between individual income risk and the aggregate of consumption across individuals then translates to aggregate net foreign assets according to the stationary budget constraint of the economy $B = \frac{C-Y}{R-1}$. Second, with the consumption distribution as a function of income risk and world interest rates in hand, we can characterise the relative importance of international asset prices and domestic income risk for consumption inequality, the second concern of this paper.

For concreteness, the section concentrates on the case where income only take two values $\{z^1, z^2\} = \{y_0 + \frac{1}{\nu}\varepsilon, y_0 - \frac{1}{1-\nu}\varepsilon\}$, which allows me to model an increase in risk as a simple rise in ε. An appendix contains the generalisation to the case of $N > 2$.

3.1 The stationary consumption distribution

Note first that arbitrage implies a link between the price of domestic Arrow-Debreu securities and international interest rates

$$q(s, s') = \frac{\pi(s, s')}{R} \tag{5}$$

where $\pi(s, s')$ is the probability of a state s' conditional on s. We can thus write the intertemporal first order condition associated to the household's problem (4) in terms of R

$$u'(c_i(s)) = R\beta u'(c_i(s'))(1 + \gamma_i(s')) \ \forall s' \tag{6}$$

where $\gamma_i(s') > 0$ if i's participation constraint binds in state s'. Strict concavity of the utility function, together with Inada conditions and a convex constraint set, imply that the unique

solution to the household's problem is thus characterised by the following set of equations

$$u'(c_i) = \beta R u'(c_i') \quad if \ \gamma_i(s') = 0 \tag{7}$$

$$V(z(s'), \alpha(s'), 0) = W(z(s')) \quad if \ \gamma_i(s') > 0 \tag{8}$$

Equation (7) is the law of motion for consumption of unconstrained agents, who share the same growth rate of marginal utility determined by the level of world interest rates. Particularly, with high interest rates ($R = \frac{1}{\beta}$), consumption never declines and "ratches up" when participation constraints in (8) bind, leading to perfect insurance in the long-run. When $R < R_{min} = \frac{u'(z_1)}{\beta u'(z_N)}$ on the other hand, even individuals with highest income z_1 do not buy insurance against the worst possible income realisation tomorrow, as state-contingent assets are too expensive (interest rates too low). So there is no unconstrained consumption transition in (7) and autarky is the only equilibrium. A constant interest rate between R_{min} and $\frac{1}{\beta}$ is thus a necessary condition for stationary non-autarkic consumption transitions with some, but not perfect, risk-sharing. The rest of this section uses simple arbitrage arguments similar to those in Thomas and Worrall (2007) to show how, with a constant interest rate $R : R_{min} < R < \frac{1}{\beta}$ the stationary consumption distribution is uniquely pinned down by the two optimality conditions (7) and (8).[6]

To see this, note that according to (7), consumption has a stationary recursive structure outside periods of binding participation constraints, when continuation values are equal to autarky values. Thus, to completely characterise the support of the stationary consumption distribution it suffices to determine the N levels of consumption $c^k, k = 1, ..N$ for agents that are participation-constrained at income z^k. For agents at lowest income this is trivial, since $c^N = z^N$ exactly solves their participation constraint. More generally, since consumption in future unconstrained states is an increasing function of that in the previous period and autarky values in constrained states only depend on current income, the expected value $V(.)$ can be expressed as a function of only current consumption and income that is strictly increasing in both its arguments.[7] Equation (8) thus defines a unique value of c^k that fulfills the participation constraint at z_k with equality. In the two-income case, we can solve for c^k analytically under the assumption that agents have constant relative risk aversion utility $u(c) = \frac{c^{1-\sigma}}{(1-\sigma)}$.[8] More generally, this can be done using a simple iterative computational procedure, which the quantitative section of this paper describes in more detail.

Figure 2 shows the resulting path of consumption in the two-income case when high and

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[6]For a derivation of the stationary consumption distribution based on an equivalent planner's problem, see Broer (2009b).

[7]Strict monotonicity of $V(.)$ in income follows from the monotonicity of F and of autarky values $W(.)$.

[8]Denoting the maximum number of unconstrained transitions by M, in the two-income case the consumption of constrained high-income individuals is $\{ \frac{(1-\sigma)(1-\beta q(\beta R)^{\frac{1-\sigma}{\sigma}})}{1+\beta(1-p-q)(\beta R)^{\frac{1-\sigma}{\sigma}} - (1-p)\beta^{M+1}q^M(\beta R)^{\frac{M+1(1-\sigma)}{\sigma}}} [\frac{1-\beta(p+q)-\beta^2(1-p-q)}{1-\beta q}W_h - (1-p)\beta^{M+1}q^{M-1}(qW_l - \frac{(1-q)W_h}{1-\beta q})] \}^{\frac{1}{1-\sigma}}$.
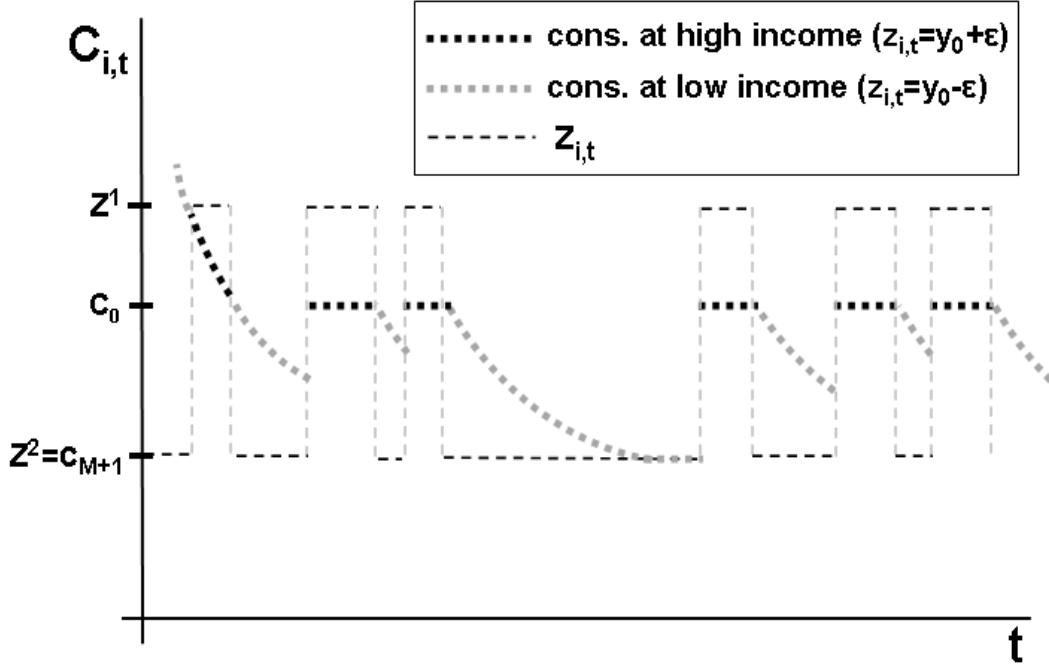
Figure 2: A path of individual consumption in the two income version of the model.

low income have equal persistence ($p = q$), for an individual whose large initial asset holdings lead to a high level of consumption and slack participation constraints in the beginning. As her consumption declines in line with (7), however, eventually a high income shock implies a binding participation constraint. Her consumption thus jumps to a level c_0 that is constant through time and depends only on income via autarky value $W(z^1)$, not initial assets. It is because of this feature of "amnesia" (Kocherlakota 1996) in periods of binding participation constraints that consumption of all agents eventually becomes independent of any finite initial wealth holdings and thus reaches a stationary path. On that path, consumption in high income periods is always lower than income ($c_0 < z^1$), which individuals are happy to accept in return for some insurance in the future. After a low income shock, consumption declines in smooth steps defined by the law of motion (7), with a lower bound equal to z^2.

Since all individuals face the same interest rate, and have identical autarky values and transition probabilities, both the bounds of the stationary consumption path c_0 and z^2, as well as the intermediate steps $c_i, i = 1, ..., M$ are common across agents. The support of the cross-sectional consumption distribution is thus identical to that of the stationary section of the consumption path in figure 2. Moreover, since $R < R_{min}$ insures that individuals who stay at low income move from highest consumption to the lower bound z^2 in a finite number of steps, the number of support points of the consumption distribution is finite. And since there is a constant, strictly positive probability of moving from any point in the distribution to any other through either

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a positive income shock or sufficiently long spells of low income, the transition function on this support has a unique ergodic distribution.[9] Finally, since income realisations are *i.i.d.* across the continuum of individuals, so are consumption transitions, and therefore the law of large numbers applies.[10] Put more simply, the cross-sectional distribution of consumption is just the "row-sum" across an infinite version of the stationary section of the consumption path in figure 2. Specifically, the unique stationary distribution associated to a particular level of world interest rates R is discrete with a finite support defined by (7) between the bounds c_0, z^2. Its frequency declines towards lower consumption realisations in a geometric sequence $\nu, \nu(1-p), \nu(1-p)q, \nu(1-p)q^2, ...$[11]

In the general case of $N > 2$ income values, the consumption distribution is very similar: the $N-1$ consumption values of constrained agents $c^k, k = 1, ..., N-1$ are the upper bounds for subdistributions whose discrete support is again defined by (7) with a lower bound z_N. The appendix shows how to derive the frequencies from the stationary mass of constrained individuals at income k and the transition probabilities of remaining at income $l < k$ for a given number of periods.

The two following sections characterise the behaviour of aggregate assets and consumption inequality as a function of international asset prices and domestic income risk in detail. However, most of the intuition behind the results is contained in figure 2: via the law of motion (7), the shape of individual consumption paths, and thus of the cross-sectional distribution, is strongly influenced by the value of interest rates, which are thus a main determinant of domestic consumption inequality. The income risk parameter ε, on the other hand, influences the shape of the distribution only through the relative values of consumption for constrained agents, c_0 and $c_{M+1} = z^2$. But by determining the upper bound of the distribution via autarky value $W(z^1)$, and with it all consumption values $c_i, i = 1, ..M$, income risk pins down the position of the distribution, and thus the level of aggregate consumption and assets.

3.2   Income risk and aggregate debt in stationary equilibrium

This section formalises the intuition that more uncertain incomes make default, with its implied exclusion from financial markets, less attractive, and thus relax individual borrowing limits and increase aggregate debt. To see why this is a challenge, note that a change in the dynamic income process not only changes the conditional variance of future incomes, but also the distribution of current and expected incomes. For example, in the two-income case, a small mean-preserving

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[9]More formally, the transition function fulfills the conditions of, for example, Theorem 11.4 (Stokey and Lucas (1989), p.332.

[10]For a version of the law of large numbers in continuum economies, see Uhlig (1996).

[11]For the two income case, Krueger and Uhlig (2006), as well as Thomas and Worrall (2007) have very similar characterisations for the case without persistence.

spread to the income distribution $d\varepsilon > 0$ increases risk but also current income inequality. As this acts to reduce resources of the income-poor, a rise in risk $d\varepsilon > 0$ always makes low-income agents worse off. But at the same time it increases the current and expected income of the income-rich, who gain more from a rise in high-income states than they loose from lower income in bad times. Figure 3 shows how, at moderate levels of risk (low ε), the net effect is to <u>raise</u>
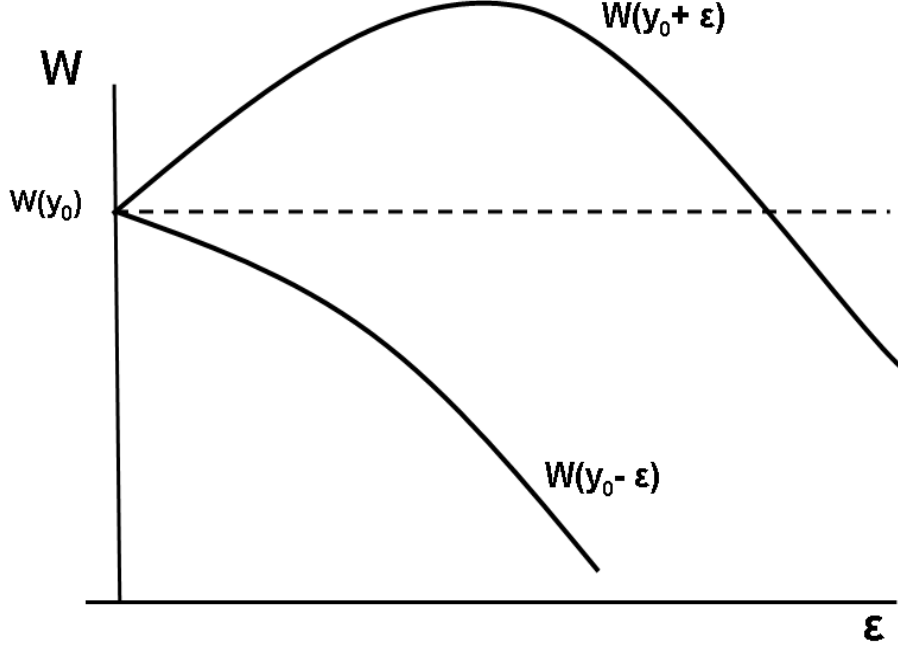

Figure 3: The figure depicts the values of autarky at high and low income in the two income version of the limited commitment continuum economy.

the attractiveness of default for the income-rich by increasing the utility value of their income stream. Only as the level of risk rises, implying falling z^2, the default-deterring second-order effect of higher risk eventually dominates. This is because the marginal utility loss from a further reduction in low income $u'(z^2)\frac{1}{1-\nu}d\varepsilon$ increases without bound, due to Inada conditions.[12] Thus, we expect the incentive improving effect of increased risk to be stronger at high levels of risk. The following proposition shows that this is indeed the case: in line with figure 3, at high enough levels of risk, a further increase relaxes participation constraints of all agents and thus

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[12]To see this algebraically, note that autarky values are

$$W_h = \frac{(1 - \beta q)u(y_0 + \frac{1}{\nu}\varepsilon) + \beta(1 - p)u(y_0 - \frac{1}{1-\nu}\varepsilon)}{1 - \beta(q + p) - \beta^2(1 - (q + p))} \tag{9}$$

$$W_l = \frac{\beta(1 - q)u(y_0 + \frac{1}{\nu}\varepsilon) + (1 - \beta p)u(y_0 - \frac{1}{1-\nu}\varepsilon)}{1 - \beta(q + p) - \beta^2(1 - (q + p))} \tag{10}$$

Given the assumptions on transition probabilities, W_l is always declining in ε, while the high income-autarky value W_h is concave in ε and falls to minus infinity as low income realisations approach zero, due to Inada conditions.

necessarily reduces aggregate assets in the economy.

Proposition 1 In the economy with two income values $\{z^1, z^2\}$, there is a level of risk $\varepsilon^\star$, with $0 < \varepsilon^\star < y_0$, such that for $\varepsilon > \varepsilon^\star$ a rise in income risk $d\varepsilon > 0$ reduces aggregate foreign assets.

Proof Note first that according to the stationary budget constraint $B = \frac{C-Y}{R-1}$ a rise in individual income risk $d\varepsilon > 0$ reduces (increases) aggregate assets across stationary equilibria if and only if it reduces (increases) aggregate consumption. Given the recursive structure of consumption and its lower bound $z^2 = y_0 - \frac{1}{1-\nu}\varepsilon$, a sufficient condition for aggregate consumption and assets to decline with ε is that c_0 does. The rest of the proof shows that c_0 declines with ε beyond some value $\varepsilon^\star$ as $\frac{dc_0}{d\varepsilon}$ crosses the zero line once from above. To see this, denote as $c(c_0, R, j)$ the unique value of consumption that results after applying the mapping defined by (7) j times to c_0 at interest rate R, with $c_1(c_0, R, j)$ the strictly positive partial derivative with respect to its first argument. We can write the effect of risk on c_0 by differentiating the participation constraint at high income, substituting in the summation $u'(c_j) = \frac{u'(c_0)}{(\beta R)^j}$ and $\frac{dc_j}{d\varepsilon} = c_1(c_0, R, j)\frac{dc_0}{d\varepsilon}$, as well as eliminating all future constrained states from both sides, to get

$$\frac{dV}{d\varepsilon} = \frac{dW}{d\varepsilon}$$

$$\Rightarrow \quad \frac{dc_0}{d\varepsilon}u'(c_0) = u'(y_0 + \frac{1}{\nu}\varepsilon) \ for \ M = 0$$

$$\frac{dc_0}{d\varepsilon}u'(c_0)\Theta = u'(y_0 + \frac{1}{\nu}\varepsilon) - (1-p)\sum_{j=1}^{M} q^{j-1}\beta^j u'(y_0 - \frac{1}{1-\nu}\varepsilon) \ for \ M > 0 \quad (11)$$

where $\Theta = 1 + (1-p)\sum_{j=\underline{}}^{M} q^{j-1}R^{-j}c_1(c_0, R, j) > 0$, and M is the number of support points between the two bounds of the consumption distribution, which increases with ε. To see that $\frac{dc_0}{d\varepsilon}$ crosses the zero line once from above, note that for $M = 0$, the right hand side is strictly positive. And for $M > 0$, which holds for $\varepsilon > \widetilde{\varepsilon} : \frac{u'(y_0+\widetilde{\varepsilon})}{u'(y_0-\widetilde{\varepsilon})} = \beta R$, the right-hand side is strictly decreasing in ε, approaching minus infinity as $\varepsilon \longrightarrow (1-\nu)y_0$. There is thus a value $\varepsilon^\star < (1-\nu)y_0$ such that $\frac{dc_0}{d\varepsilon} < 0, \forall \varepsilon > \varepsilon^\star$ and aggregate consumption and debt decline with an increase in income risk. ∎

The appendix contains a similar proof for the general case of $N > 2$ income values. Again, a sufficient condition for the stationary level of aggregate consumption, and thus assets, to decline with income risk is that the latter reduces all N levels of constrained consumption. Defining a rise in income risk as a mean-preserving spread to the support of the income distribution, Inada conditions, together with the strictly positive transition probabilities in F, again assure that there is a level of income risk beyond which this is true, as the expected utility loss from falling consumption in low-income-periods outweighs any gain in current income even for high income individuals.

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3.3 Income risk, world asset prices and domestic consumption inequality

This section shows that in an economy where limited enforcement of contracts is the key impediment to risk-sharing, movements in world asset prices have a strong impact on domestic consumption heterogeneity. Changes in individual income risk, on the other hand, can leave consumption inequality, as well as individual consumption volatility, unaffected even without perfect insurance.[13] In particular, when income risk is sufficiently high, consumption dispersion is a function of international asset prices only, not income risk. To see this, note that for any two agents that were last constrained at the same income level, (7) determines the relative marginal utilities as a function of world interest rates and the time elapsed since constraints were binding, independently of domestic income risk. More particularly, with constant relative risk aversion utility we can write the log consumption sequence of unconstrained agents $\{\widehat{c}_i\}$ as

$$\{\widehat{c}_i\} = \{\widehat{c}_0\} + \frac{i}{\sigma}\widehat{\beta R} \tag{12}$$

where hats denote logarithms. So with CRRA utility, log consumption dispersion among unconstrained agents is completely determined by world interest rates. Income risk, on the other hand, affects consumption inequality only via the relative consumption levels of constrained agents. Specifically, in the two income example, a change in income risk $d\varepsilon$ affects the cross-sectional variance of log-consumption only by its effect on the relative size of the bounds of the distribution $\hat{c}_0 - \hat{z^2}$. Although this term grows without bound as ε approaches $(1-\nu)y_0$, and z^2 thus approaches 0, its frequency weight $\nu(1-p)q^M$ declines as M rises and the number of individuals at $c_{M+1} = z^2$ falls. So as long as M grows fast enough with rising ε, the effect of income risk on the log-variance goes to zero. Since, according to (7), higher interest rates increase M by slowing down the consumption decline of unconstrained agents towards the lower bound z^2, this requires R to be above a threshold value that depends on preferences. The following corollary, proved in an appendix, makes this statement precise for $\sigma = 1$ (log-preferences).

Corollary 1 *In the economy with two income values $\{z^1, z^2\}$ and constant relative risk aversion equal to 1 (log-utility), as long as world interest rates are above a threshold $\overline{R} = \frac{q^{\frac{1+(1-p-q)\beta}{1-q\beta}}}{\beta}$, the effect of income risk on the variance of log-consumption declines to 0 as ε approaches $(1-\nu)y_0$.*

Figure 4 illustrates this effect of world interest rates and income risk on consumption inequality, as well as net foreign assets, in the two-income economy with log-utility ($\sigma = 1$), a discount factor of 0.95, $p = q = 0.95$ and $y_0 = 1$. In line with the theory, net foreign assets increase mildly with income risk at low levels but decrease strongly when incomes become more risky. At high levels of risk, although insurance is not perfect, the standard deviation of log-consumption

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[13]The rest of the section concentrates on cross-sectional dispersion measures. Since consumption processes are i.i.d. across agents, the results apply pari passu to the unconditional volatility of individuals' consumption.
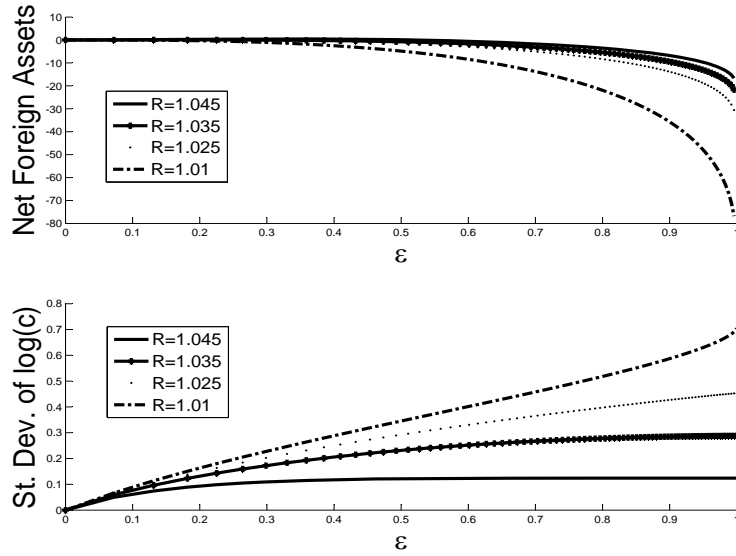
Figure 4: The figure depicts net foreign assets and the cross-sectional standard deviation of log-consumption as a function of income risk ε and world interest rates R in the stationary equilibrium of the two-income economy.

becomes independent of income risk for interest rates above the threshold level $\overline{R} = 1.017$, but not below. A fall in world interest rates, on the other hand, leads to an increase in consumption dispersion at all levels of income risk.

In the general case of $N > 2$ income values, the effect of income risk and consumption inequality and individual consumption volatility is more complex. Specifically, a rise in risk now changes the relative values of the N cutoff values c^k, and thus typically also changes consumption dispersion. But again, this effect is likely to be less important the better is insurance, as this reduces the number of individuals who are constrained at lower income values.

3.4 Extensions: Non-contingent assets and saving after default

To analytically characterise the effect of income risk on aggregate savings, the analysis so far was based on two strong assumptions: that individuals have access to a set of fully state-contingent assets until they default, and that they are completely excluded from any asset trade thereafter. The positive incentive effect of rising income risk on the attractiveness of default is, however, equally present in an alternative environment where agents only trade non-contingent bonds, subject to a borrowing limit that prevents default in all incomes states tomorrow. In this environment first analysed by Zhang (1997), default incentives are typically strongest in low-

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income periods, where marginal utility is high, making debt service very costly.[14] Since a rise in income risk always makes autarky at low income less attractive, it always improves incentives of the borrowing constrained independent of the level of income risk, unlike in the analysis with state-contingent assets above. The effect of higher risk on aggregate savings, however, is the sum of the reduction in net assets of constrained agents, who see their borrowing limit increase, and the rise in precautionary savings of unconstrained agents in response to the increase in income volatility. The net effect thus crucially depends on the number of constrained agents in the economy, but is impossible to characterise analytically. In a partial equilibrium context, the effect on incentives is more likely to dominate that on precautionary savings at low interest rates, where more agents are constrained.

Similarly, the second strong assumption, that default leads to complete financial autarky, is necessary to obtain analytical results but surely restrictive. Specifically, permanent exclusion from insurance markets seems in line with the relatively long delay in eliminating a default episode from personal credit scores, which, for example, is 10 years for chapter 7 personal bankruptcy filings in the US. But it is hard to see how agents could be prevented from storing part of their post-default income as a buffer against negative future income shocks. The possibility to draw on savings in bad times, however, may significantly reduce the punishment of exclusion from insurance markets. How important this effect is depends crucially on the rate of return on savings after default R^{aut}. The assumption of financial autarky is equivalent to $R^{aut} = -1$. As the rate of return rises, self-insurance through savings becomes more powerful, improving the trade-off between the mean and volatility of consumption after default especially for the income-rich. This is crucial, as it might break the Laffer-curve type relationship between autarky values and income risk presented, for the two income case, in figure 3. So with saving after default at high interest rates, an increase in income risk may tighten participation-constraints of the income-rich independent of the level of risk. The exact level of R^{aut} above which this is true depends on how the value function in the post-default income fluctuation problem varies with the rate of return and income risk, which has not been characterised analytically.[15] Unfortunately, therefore, the relationship between income risk, the attractiveness of default and, ultimately, aggregate debt is difficult to characterise analytically in an environment with savings after default. But the following section considers the importance of saving after default in a calibrated version of the economy.

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[14]Arellano (2008) makes this point in a slightly different context.

[15]Miao (2002) presents analytical comparative statics for this problem, but does not consider changes in income risk.

4 Individual income risk and global imbalances 1980-2003

The previous section showed that in an open economy with individual default constraints, rises in income risk can, in principle, lower aggregate savings and asset positions while leaving consumption inequality almost unaffected. This section aims to identify the quantitative importance of this effect. It first looks briefly at the comovement of income inequality and foreign assets in a cross-section of countries between 1980 and 2005. It then uses a quantitative version of the theoretical model calibrated to micro-data on individual incomes in the US to assess the contribution of the endogenous financial deepening effect to the strong rise in US liabilities during this period. For this, I first show how, in a comparison between stationary equilibria, the observed increase in income risk in the US leads to a fall in the demand for foreign assets there. I then proceed to a general equilibrium analysis of a stylised two country economy, where the US trades bonds with a large developing country, calibrated to capture the evolution of individual income inequality in China. Under the assumption that agents there can only self-insure through borrowing and saving, the analysis shows how the financial deepening effect of higher income risk in the US is augmented by a rise in precautionary savings in response to the observed increase in individual income volatility in China. Again, the driver of global imbalances in this analysis are changes in relative income risks across countries, not the opening of capital markets under different domestic financial structures as in Mendoza et al (2009).

4.1 Cross-country evidence

As argued in the theoretical analysis, endogenous financial deepening in an economy with limited commitment to contracts implies that, as income risk increases, the demand for foreign assets eventually starts falling. At sufficiently high levels of risk, the effect thus predicts a negative relationship between changes in income risk and net foreign asset growth. In the absence of comparable data on income risk across countries, figure 5 looks at a cross-section of developed countries between the early 1980s and the early 2000s, and plots average current accounts during this period against two measures of the change in domestic income inequality, namely the average yearly change in the Gini coefficient of disposable income (Brandolini et al 2007), and that in the variance of log incomes after government transfers (Krueger et al 2010), as imperfect proxies for the evolution of income risk.[16] The figure shows that there is indeed a slightly negative

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[16]The picture using the change in net foreign assets instead of average current accounts looks very similar. I use all available observations from two comparative studies on the evolution of income inequality (Brandolini et al 2007 and Krueger et al 2010), excluding Mexico, which is an outlier both in terms of its high level of income inequality and of its strong increase during the sample period. Data on current accounts is taken from the IMF World Economic Outlook database. The samples are Australia, Canada, Finland, France, Germany, Italy, the Netherlands, Sweden, the United Kingdom and the United States for the Gini coefficients, and Canada, Germany, Italy, Russia, Spain, Sweden, the United Kingdom, and the United States for the variance of log incomes. The average of current accounts as a percentage of GDP is taken between 1980 and 2005. The periods to compute

relationship between inequality and foreign assets on average, consistent with the theoretical analysis. But given the non-linearity of this relationship in theory, and the imperfect nature of the proxies for income risk, this cross-country evidence is more suggestive than conclusive. To allow a more detailed analysis of the financial deepening effect of rising income risk on aggregate assets, using a more accurate measure of the evolution of income risk over time and properly accounting for the non-linearities predicted by the theory, the following section thus focuses on a particular economy and considers a version of the theoretical model that is calibrated to match some stylised features of the US economy. Particularly, I use the stochastic process for US individual incomes estimated by Krueger and Perri (2006), and compare debt holdings and consumption inequality in stationary equilibria corresponding to the two endpoints of their sample, respectively 1980 and 2003. As a first step, a partial equilibrium analysis derives the demand function for foreign debt and consumption inequality as a function of world interest rates in both periods. I then proceed to a general equilibrium analysis of a stylised two country economy, where the US trades bonds with a large developing country, calibrated to capture the evolution of individual income inequality in China.
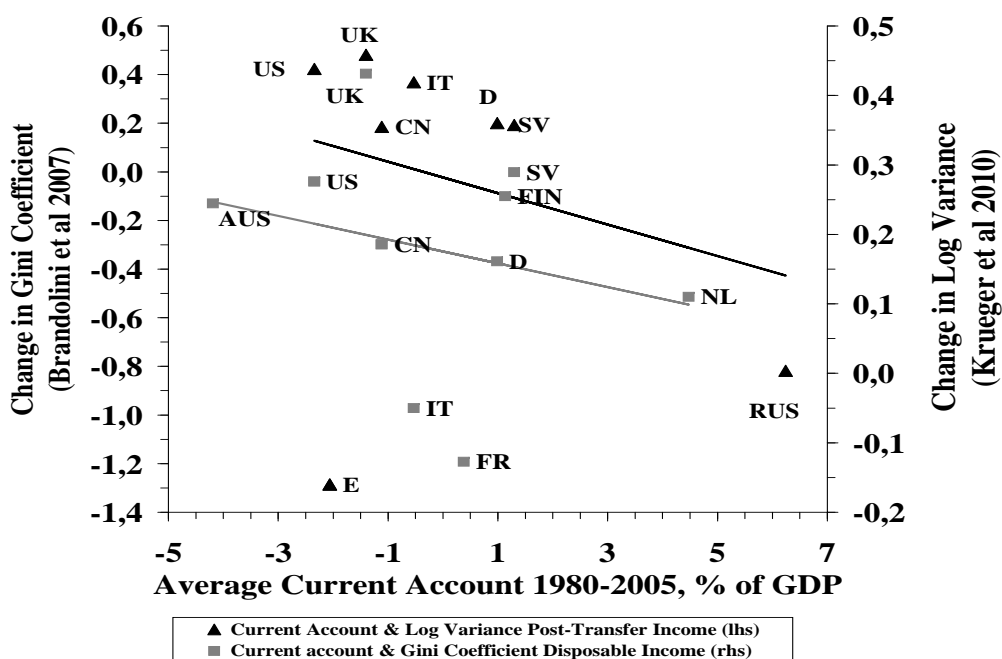

Figure 5: The figure plots the average current account as a percentage of GDP against the average yearly change in two measures of income inequality in a cross-section of developed countries: the average yearly change in the Gini coefficient of disposable income (Brandolini et al 2007), and that in the variance of log incomes after government transfers (Krueger et al 2010). The solid lines depict the fitted values of a simple least squares regression.

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changes in inequality differ with data availability across countries, see the sources for details.

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4.2 Income risk and net foreign assets in a model calibrated to the US economy

4.2.1 Calibration and model solution

To implement the theoretical model quantitatively, I calibrate the income process following Krueger and Perri (2006), using their estimates for the years 1980 and 2003, the endpoints of their sample. The authors assume the log of post tax labour income plus transfers (LEA+) $log(z_t)$ to be the sum of a group specific component α_t and an idiosyncratic part y_t. The latter, in turn, is the sum of a persistent AR(1) process m_t, with persistence parameter ρ and variance σ_m^2, plus a completely transitory component ε_t which has mean zero and variance σ_ε^2.

The process for LEA+ is thus of the form

$$log(z_t) = \alpha_t + y_t$$
$$y_t = m_t + \varepsilon_t$$
$$m_t = \rho m_{t-1} + \nu_t$$
$$\varepsilon \sim N(0, \sigma_\varepsilon^2)$$
$$\nu_t \sim N(0, \sigma_\nu^2) \tag{13}$$

Using data from the Consumer Expenditure Survey (CEX), the authors first partial out the group-specific component α_t as a function of education and other variables, identifying the variance of the idiosyncratic part of income y_t, as well as (from the short panel dimension of the CEX) its first order autocorrelation. They then fix $\rho = 0.9989$, the value estimated by Storesletten et al (2004), which allows them to identify σ_ν^2 and σ_ε^2.

The results show an increase in the variance of labour income of 18 percentage points between 1980 and 2003, the two periods I focus on. 11 percentage points are due to an increase in within-group inequality, out of which roughly two thirds are accounted for by an increase in the importance of persistent shocks, and one third by that of transitory shocks.

In my exercise I abstract from changes in the common wage rate and differences in the group specific component, which, in the present model as in that of Krueger and Perri, translate fully into consumption differences by construction.

As a baseline calibration, I choose a CRRA utility function with coefficient of relative risk aversion of 1 (log-preferences), and a discount factor of 0.96. I then look at the sensitivity of the results to changes in parameters. And I look at the case when agents who default are excluded from all financial transactions in the current period, but allowed to invest in non-contingent bonds in the future to smooth income shocks over time. This reduces the impact of higher income risk on the attractiveness of default.

To solve the model, I first approximate the persistent process for m_t with a 7-state Markov chain

using the standard Tauchen and Hussey (1991) method.[17] Following Krueger and Perri (2006) I choose a binary process for the transitory shock. I then compute the stationary consumption distribution using an iterative procedure that solves for the consumption levels of constrained agents $c^k, k = 1, ..., N$. Knowing the path of consumption in future unconstrained states via (7) and the lower bound $c_N = z_N$, as well as autarky values in future constrained states, I solve for c^{N-1} from the participation constraint at income z_{N-1}. Knowing c_N and c_{N-1} allows me to do the same for c_{N-2}, noting that individuals at z_{N-1} cannot transit below c^{N-1}. Etc. Broer (2009a) describes the recursions to derive the stationary consumption distribution in more detail.

4.2.2 Income risk and net foreign assets

Figure 6 shows the main quantitative result of this paper: a rise in income risk calibrated to the experience of the US economy between 1980 and 2003 strongly reduces the stationary demand for assets in an economy with limited commitment to contracts. For example, at an interest rate of 3.5 percent (which yields a zero foreign asset position in 1980), the observed rise in income risk leads to a fall in the stationary level of net foreign assets of more than 50 percent of annual GDP.

So far, the results of this paper were derived under the arguably strong assumption that default be punished by complete exclusion from financial transactions. Figure 7 relaxes this assumption and considers a version of the economy where individuals can invest part of their income at the world interest rate from the period following default. This option to self-insure by saving makes default more attractive, as it allows agents to smooth consumption in low income periods even without access to insurance contracts. And importantly, postponing consumption by saving becomes less costly, and therefore self-insurance more powerful, at higher interest rates. Figure 7 shows that this can break the monotonicity of stationary foreign assets in the level of income risk. Particularly, for high interest rates, self-insurance through saving is so powerful that the rise in current income for the income-rich now dominates the second-order effect of higher income risk, leading to an increase in stationary assets as more attractive default tightens borrowing constraints.

Finally, figure 8 shows that with higher risk aversion ($\sigma = 2$), which strengthens the punishment of more volatile consumption after default, the observed rise in income risk decreases stationary assets for any real interest rate below 3.5 percent.

For a more general version of the economy, where agents can save some of their income even after default, the relationship between income risk and stationary assets thus depends on the

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[17]Note that this method accords to my assumption of widening the support Z to increase risk, but leaving the transition probabilities unchanged.
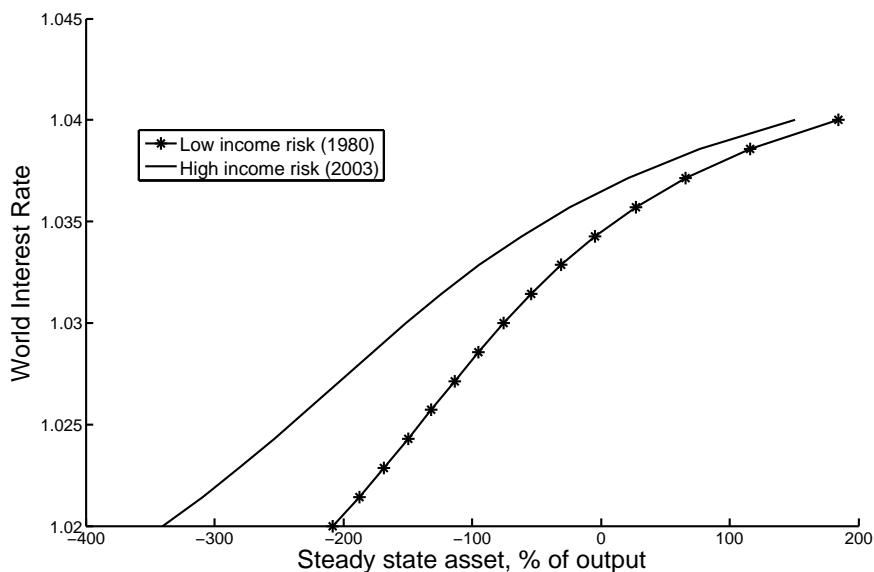
Figure 6: Asset demand function, baseline calibration.

level of world interest rates. This is why the next section endogenises the equilibrium interest rate in a general equilibrium analysis of a simple two country economy. Before, however, I analyse how consumption inequality is affected in partial equilibrium by changes in income risk and the world interest rates.

4.2.3   Income and consumption risk

In the theoretical analysis, I showed that income risk mainly determines the position of the consumption distribution, while its shape, via the law of motion (7), depended largely on the level of interest rates. Figure 5 illustrated this for the two-income case. In a more general version of the economy, however, changes in income risk affect relative autarky values, and thus also the shape of the stationary distribution. Figure 9 shows, however, that even in the calibrated economy, interest rates remain the main driver of consumption inequality: the change in consumption volatility due to a change in income risk is an order of magnitude smaller than the changes caused by movements in the world interest rate. Thus, the limited commitment economy opens a new, interesting channel of transmission from the international economy to the level of domestic inequality in individual consumption levels.
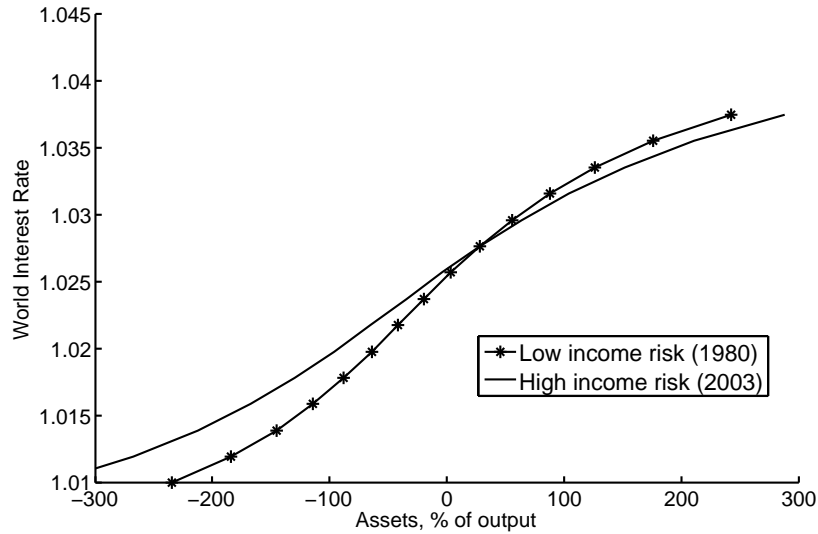
Figure 7: Asset demand function, log-preferences, saving at world interest rates permitted from the period following default.

4.3 Endogenous financial deepening and the "savings glut": A general equilibrium analysis of rising individual risks in developed and emerging economies

The partial-equilibrium analysis of the previous section was agnostic about the determinants of savings outside the US, taking as given a world interest rate. But of course, in a closed world economy, the fall in US savings caused by increased idiosyncratic risk affects equilibrium interest rates. This section thus looks at the general equilibrium in a simple economy consisting of two countries that differ both in their domestic financial market structures and the evolution of idiosyncratic risk that their agents experience over time. Rather than trying to replicate the exact path of the US foreign position, the analysis shows how the changes in individual income risk seen in the US and elsewhere may have long-run effects on foreign asset positions that are similar in magnitude to those actually observed over the last 30 years. Importantly, the analysis shows that differences in financial development, captured by the presence or absence of state-contingent assets, not only determine the relative <u>levels</u> of precautionary savings between countries, as in Mendoza et al (2009). Rather, through opposite responses of aggregate savings to comparable increases in idiosyncratic income risk, differences in the depth of domestic financial markets are shown to also determine the time <u>path</u> of global imbalances.

To make these points as simply as possible, I present a stylised world economy consisting of China
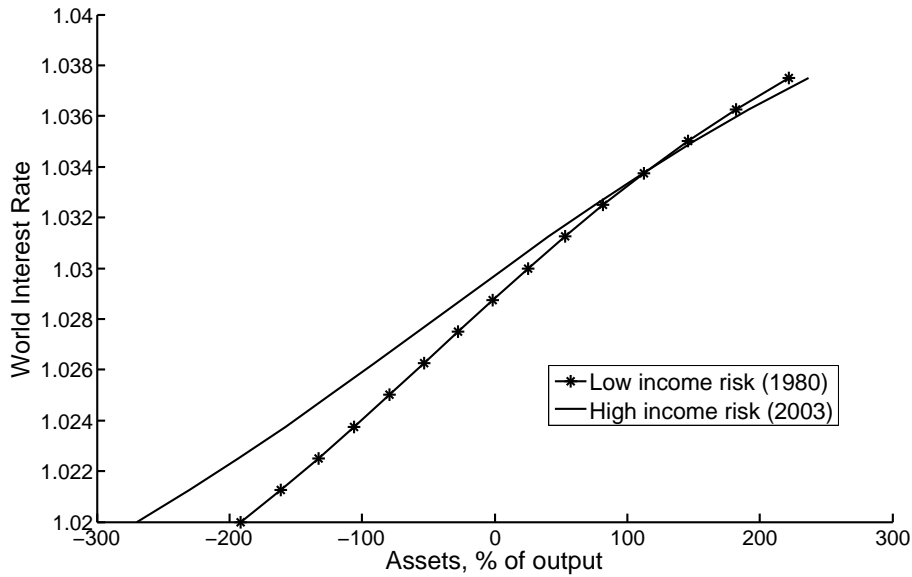
Figure 8: Asset demand function, higher risk aversion ($\sigma = 2$), saving at world interest rates permitted from the period following default.

and the US. Both countries experience a rise in idiosyncratic income uncertainty in line with their historical experience, but differ in their ability to insure against this risk through domestic financial trade. US financial markets are assumed to be complete but subject to participation constraints as before, allowing individuals to save at the world interest rate after they default on contracts. Chinese consumers, on the other hand, do not have access to complete domestic financial markets. Rather, I assume that individuals there can only engage in self-insurance through trade in bonds subject to a borrowing limit. As before, I abstract from aggregate risk and international asset trade is limited to non-contingent bonds, whose prices all agents take as given. A stationary equilibrium of the world economy is thus a process for individual consumption in both countries, an aggregate net asset position between the two countries and a market clearing interest rate.

4.3.1 Individual risk and equilibrium foreign asset positions

The analysis concentrates on the effect of changes in idiosyncratic risk on equilibrium net foreign asset positions over the last 25 years. The process of idiosyncratic risk in the US is unchanged from the previous section. Unfortunately, equivalent estimates of an income process with group-specific heterogeneity, as well as persistent and transitory within-group risk, are infeasible for China, where the necessary household panel survey is not available for the period of interest.
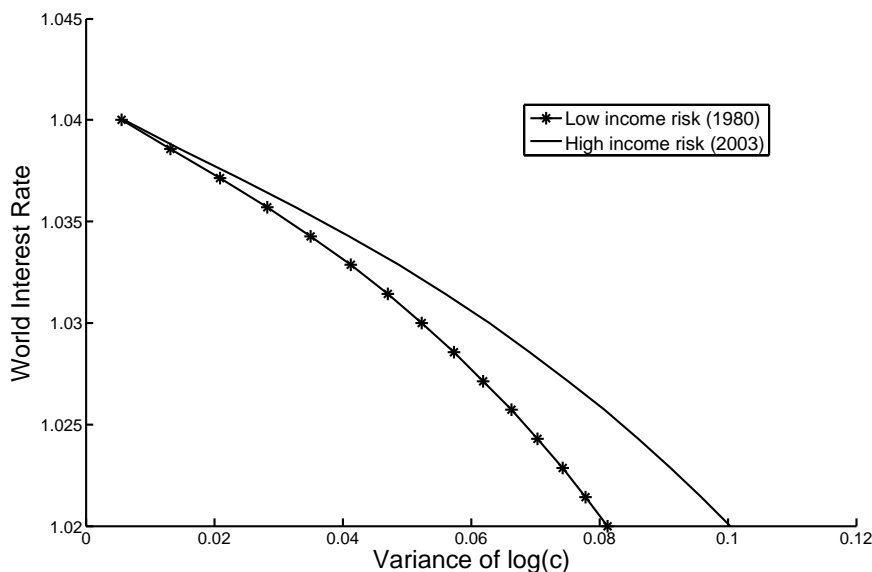
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Figure 9: Variance of log consumption, baseline calibration.

We are thus left with estimates of cross-sectional income inequality. This is a problem, as we cannot identify the different components of individual income risk from cross-sectional data alone. However, the calibrated model provides a mapping from a specific income process to the cross-sectional consumption inequality and a savings demand schedule. I thus calibrate the components of the income process to capture the Gini coefficients of consumption and income for Chinese urban regions reported in Perloff and Wu (2005) in 1985, plus a zero initial foreign asset position. Assuming that the income process in China has the same permanent-persistent-transitory structure as in the US, including the persistence parameter of 0.9989, this provides three targets for three parameters, namely the variances of the permanent, persistent and transitory component of the income process in (13).[18] The increase in idiosyncratic risk in China is then calibrated to capture the observed rise in both Gini coefficients until 2001. For this, I assume that the change in permanent income differences in China is entirely captured by the rise of Urban-Rural inequality. But I look at the sensitivity of the results to this assumption below. The results assume furthermore a relatively tight borrowing limit corresponding to average quarterly income. As country weights, I use relative GDP of both countries from the Penn World tables in 1980 and 2003.

The appendix reports the implied estimates of the income process in China. In line with the

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[18]For the permanent part of income risk, I use a uniform distribution of log-income values with 5 support points, and calibrate the support width to capture the moments of the data. For the AR(1) component of the income process I choose a 5-state discretisation using the Tauchen and Hussey (1991) method.

similar Gini coefficients for consumption and income, inequality in the 1980s is estimated to be mainly determined by permanent income differences: both the variance of persistent and transitory income shocks are small. But the observed rise in consumption and income inequality until the early 2000s, stronger for income than for consumption, is in line with a strong increase in both the variance of persistent and transitory shocks, by 7.5 and 6.6 percentage points respectively.

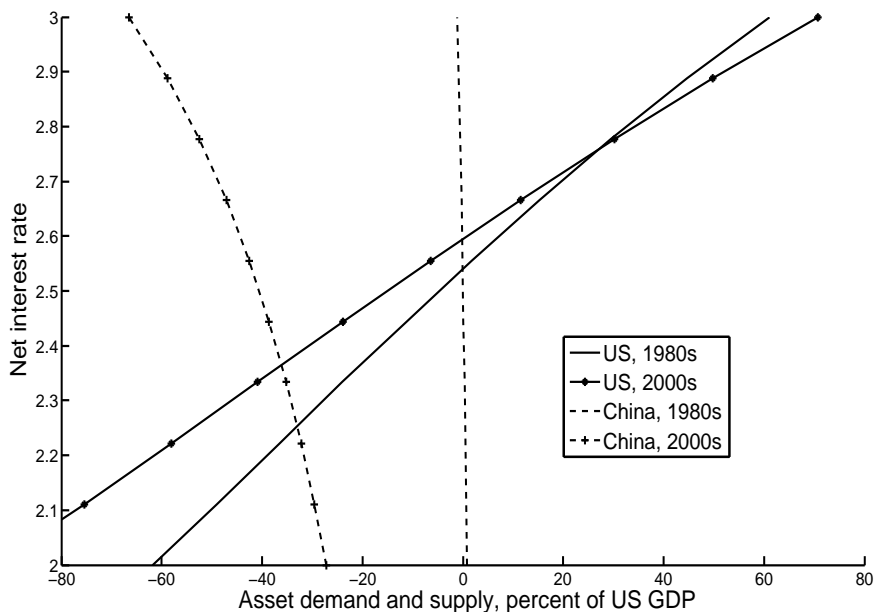Figure 10 plots the resulting equilibria for the early 1980s and the early 2000s. Chinese assets

Figure 10: Asset demand and supply in a two country world economy.

are plotted with a negative sign, such that the intersections of the demand and supply schedules give equilibrium asset positions and interest rates. The initial net interest rate of 2.5 percent is low relative to the discount factor of 0.96, as in many models of imperfect insurance. The increase in risk in the US results in the familiar fall in the savings demand schedule as a result of financial deepening. But in China, the strong rise in idiosyncratic risk after the early 1980s results in a strong <u>rise</u> in precautionary savings. This is exactly as we would expect in a self-insurance economy, where the financial deepening effect of higher income risk is absent, and the precautionary savings effect is relatively strong. The corresponding net effect is a fall in the US net foreign asset position to minus 36 percent of GDP, and a fall in the world interest rate of 17 basis points. This moderate fall in interest rates contributes by about a third to the predicted rise in consumption inequality in the US economy of 1.7 percentage points. As in the closed economy model of Krueger and Perri (2006), this increase in consumption dispersion

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is small relative to the 11 percentage point rise in income volatility observed over this period. The open economy analysis reveals, however, how precautionary savings in other countries, via implied movements in asset prices, can make important contributions to changes in domestic US consumption inequality.

4.3.2 Sensitivity analysis

As it is impossible to distinguish the effect on cross-sectional inequality of increases in permanent income difference from those of the very persistent shocks in the model, Figure 10 was based on the assumption that increases in permanent income differences are entirely captured by the difference between urban and rural regions. Since precautionary savings are largely unaffected by changes in permanent inequality but rise with persistent shocks to income, this may overstate the effect on aggregate savings. Therefore, Figure 11 shows how the results change when I make the opposite assumption of unchanged persistent shocks (which requires some recalibration also of the variance for transitory shocks, to match both Gini coefficients). As expected, the rise in equilibrium US liabilities is lower, but at 23 percent of GDP still sizeable.
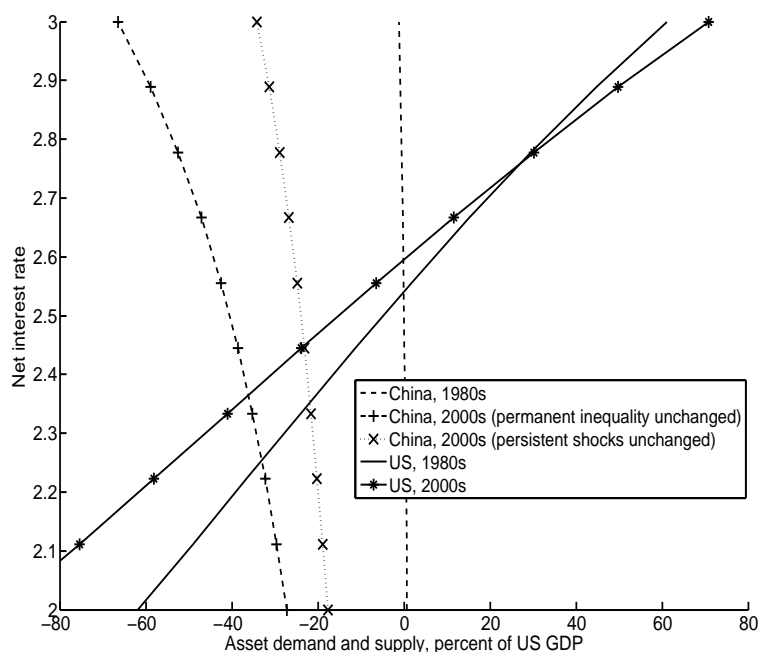

Figure 11: Asset demand and supply in a two country world economy, sensitivity.

5 Conclusion

This paper looked at the link between domestic income uncertainty, consumption inequality and net foreign asset positions in an economy where financial markets suffer from enforcement constraints. Domestic financial markets were assumed to be complete, but constrained by individuals' option to default on contracts, at the price of permanent exclusion from insurance markets. I showed that higher income risk can indeed lower aggregate savings by making the punishment of default, financial autarky, less attractive, thus endogenously "deepening" financial markets. However, for moderate to high levels of income risk further increases in income volatility have only a small effect on consumption inequality, which depends mainly on the international interest rate. A calibration of the model to the US case showed that the changes in income risk observed between 1980 and 2003 might indeed explain an important part of the fall in the net foreign asset position. This holds not only at a constant world interest rate, but also in the general equilibrium of a simple world economy where the US trades bonds with a country that has less sophisticated markets and experiences a strong increase in idiosyncratic risk similar to that seen in China. The "glut" in precautionary savings there and the endogenous financial deepening in the US, both caused by rising idiosyncratic risk, result in a significant deterioration of the US net foreign asset position, and a small fall in the world interest rate.

Future research should generalise this analysis in at least two directions: first, one should also take account of the change in aggregate macroeconomic risk, which declined over the period of analysis. Unfortunately this is still both computationally and analytically infeasible in the limited commitment environment of this paper. And second, an adequate equilibrium of the world economy should not only take into account advanced countries with deficits and emerging surplus economies, but also countries like Germany or Japan, that experienced surpluses yet have relatively developed domestic financial markets. In this context, the model's prediction of an inverse U-shape relationship between net foreign asset positions and individual income risk is especially interesting.

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7 Appendix

7.1 The consumption distribution in the general case $N > 2$

When $N > 2$, we can construct the support of the consumption distribution "bottom-up", starting from its lower bound, which we know to be z^N. For this, note that monotonicity of F ensures that both autarky values W^k and the minimum participation-compatible levels of consumption c^k increase in income z^k. Since c^k solves the participation constrained of individuals at income z^k with equality, this allows me to recursively determine c^k by substituting into the kth participation constraint the autarky values at incomes $z^j > z^k, j = k, k - 1, ..., 1$ for future states with non-negative income shocks, and the consumption values given by the law of motion (7) for unconstrained states. Starting at $i = N - 1$ and moving up income levels assures that this procedure can keep account of participation constraints that become binding as individuals move down in consumption from c^k to Z^N. Denoting the jth element of the sequence of consumption values defined by (7) starting form c^k as c_j^k, this defines the support of N subdistributions of consumption. The stationary frequency distribution can then be constructed "top-down". Particularly, all high income individuals are constrained at the minimum participation-compatible consumption for individuals with the highest income c^1. So its mass is equal to the stationary mass of individuals at z^1. To construct the rest of the frequency distribution we have to keep track of the joint distribution of consumption and income, as consumption of individuals at income z^k is bounded below by c^k. The frequency associated with c_j^1, the jth element of the subdistribution starting from c^1, is then simply $\sum \Phi_j$ where the joint consumption-income distribution is given by $\Phi_{j+1} = F_j' \Phi_j$, $\Phi_0 = [1, 0, ..., 0]'$ and F_j has 0 entries for all columns $i = 1, ...j$ that have $c^i > c_j^k$.

7.2 Idiosyncratic risk and aggregate assets in the general case $N > 2$

In order to show that, at high enough levels of individual income risk, a further rise leads to a reduction of aggregate assets even in the general case $N > 2$, I follow Kehoe and Levine (2001) and Krueger and Perri (2006), and define a rise in risk as a spread to the income support Z that leaves aggregate income unchanged. More formally, a rise in risk is a vector of changes in income values $dz \in R^N$ that has no aggregate effect $\sum_i \Phi_Z(z_i) dz_i = 0$ and strictly increases income dispersion without changing the order of income states: $dz_N \leq dz_{N-1} \leq ... \leq dz_k \leq -\vartheta < 0 \leq ... \leq dz_1$ for some $k \in \{1, ..., N\}$. Note, however, that this does not imply mean-preserving spreads to the conditional income distribution for all individuals. Rather, given persistence, dZ raises (lowers) current <u>and</u> expected future income for today's high (low) income earners.

Lemma 1 In the general economy with $N > 2$, consider a sequence of rises in income risk

$\{dz\}$ indexed by k. There is an element K of this sequence, such that dz_k causes aggregate consumption and assets to fall for $k > K$.

Proof

Given the Markov structure of income, the value of default as a function of z^k, the kth element in the sequence of income vectors implied by $\{dz\}$, can be written as

$$W(z^k) = \sum_{t=0}^{\infty} (\beta F)^k U(z^k) = (I - \beta F)^{-1} U(z^k) = \mathbb{F} U(z^k) \tag{14}$$

Due to the positive entries of F both the mass of individuals at any given income value Φ_{Z_j}, and their probability to transit to lowest income Z_N $\mathbb{F}_{jN}$ are bounded below, by $min_j \Phi_{Z_j} > \zeta$ and $min_i \mathbb{F}_{iN} > \vartheta$ respectively, for some numbers $\vartheta, \zeta > 0$. The impact of a small change in income risk on the stationary autarky values is given by

$$dW(z^k) = \mathbb{F} U'(z^k) dz \tag{15}$$

$maxdW(z^k)$ can be bounded by considering a group of individuals with minimum mass ζ at 'middle' income j (the lowest income (highest marginal utility) value that experiences a rise in consumption when risk increases) who receive a maximum transfer of resources, bounded by $dz_{jk} < Y$. Neglecting all negative terms apart from that at lowest income we get

$$max_{n \in \{1,...,N\}} dW(z_n^k) < \sum_{i=j}^{N} \mathbb{F}_{nj} U'(z_j) \frac{Y}{\zeta} - \vartheta \beta U'(z_N^k) \vartheta \tag{16}$$

Since z_N^k is strictly decreasing in k, the second term goes to minus infinity while the first is finite and independent of k. So from some K onwards, all elements of dW are necessarily negative and a rise in risk strictly relaxes all participation constraints in the economy.

The result than follows trivially from the fact that consumption of all unconstrained agents can be recursively defined from their last constrained consumption level using (7). By relaxing constraints on front-loading of consumption, a fall in all autarky values thus lowers stationary equilibrium consumption for all individuals and leads to a fall in aggregate stationary consumption and assets. ∎

7.3 Proof of Corollary 1

I first use the participation constraint to solve for $\frac{dlog(\frac{c_0}{z^2})}{d\varepsilon}$ in terms of $\frac{dlog(\frac{z^1}{z^2})}{d\varepsilon} = \frac{1}{\nu}\frac{1}{z^1} + \frac{1}{1-\nu}\frac{1}{z^2}$, and then express this as a function of m. With log-utility, we can write the participation constraint

for high-income individuals, after eliminating future constrained states, as

$$\widehat{c_0}(1 + (1-p)\beta + (1-p)q\beta^2 + (1-p)q^2\beta^3 + ...)$$
$$+q\beta\widehat{\beta R}(1 + 2(1-p)\beta + 3(1-p)q\beta^2 + 4(1-p)q^2\beta^3 ...)$$
$$= \widehat{z^1} + \widehat{z^2}((1-p)\beta + (1-p)q\beta^2 + (1-p)q^2\beta^3 + ...)$$
$$\equiv \widehat{c_0}\Theta_1 + q\beta\widehat{\beta R}\Theta_2 = \frac{\widehat{z^1}}{z^2} + \widehat{z^2}\Theta_1$$
$$\Leftrightarrow \frac{\widehat{c_0}}{z^2} = \frac{1}{\Theta_1}\frac{\widehat{z^1}}{z^2} - q\beta\widehat{\beta R}\frac{\Theta_2}{\Theta_1} \tag{17}$$

Note that M, the maximum number of unconstrained transitions along the declining path defined by (7) is the greatest integer smaller than $-\frac{\frac{\widehat{c_0}}{z^2}}{\widehat{\beta R}} > 0$ so we get

$$M = -[\frac{\frac{\widehat{z^1}}{z^2}}{\Theta_1\widehat{\beta R}} - q\beta\frac{\Theta_2}{\Theta_1}] - \xi$$
$$\Leftrightarrow \frac{z^1}{z^2} = exp(\widehat{\beta R}(q\beta\Theta_2 - (m+\xi)\Theta_1) = (\beta R)^{q\beta\Theta_2 - (m+\xi)\Theta_1} \tag{18}$$

where $0 < \xi < 1$. Noting that $z^1 = 2 - z^2$, we can solve for the effect of a change in income risk on the log-difference of the consumption bounds as

$$\frac{d\frac{\widehat{c_0}}{z^2}}{d\varepsilon} = \frac{1}{\Theta_1}\frac{d\frac{\widehat{z^1}}{z^2}}{d\varepsilon} = \frac{\frac{1}{\nu}\frac{1}{z^1} + \frac{1}{1-\nu}\frac{1}{z^2}}{\Theta_1} = \frac{1}{2}[(\beta R)^{q\beta\Theta_2 - (M+\xi)\Theta_1} + (\beta R)^{-q\beta\Theta_2 + (M+\xi)\Theta_1} + 1] \tag{19}$$

The total effect of a rise in risk on the variance of log consumption is thus

$$\nu(1-p)q^M\frac{d\frac{\widehat{c_0}}{z^2}}{d\varepsilon} = \frac{1}{2}\nu(1-p)q^M[\frac{1}{\nu}(\beta R)^{-M\Theta_1 + q\beta\frac{\Theta_2}{\Theta_1}} + \frac{1}{1-\nu}(\beta R)^{M\Theta_1 - q\beta\frac{\Theta_2}{\Theta_1} + 1}] \tag{20}$$

The effect thus goes to zero whenever $q(\beta R)^{-\Theta_1} < 1$. Noting that Θ_1 approaches $1 + \frac{(1-p)\beta}{1-q\beta}$ as M rises gives the result. ∎

8 Tables

Table 1:

Income risk and savings in a simple world economy - variances of income components for China

	permanent	persistent	transitory	Gini income	Gini consumption
1985	0.08	0.038	0.034	0.19	0.17
2001	0.08	0.13	0.10	0.27	0.21

The table reports the variances of components of an income process for Chinese urban regions that has the same structure as that reported in the text for the US: in the absence of information on group-specific attributes, (between-group) permanent income differences are modelled as a log-uniform distribution with 5 support points, while within-group income risk is the sum of a an AR(1) process with persistence parameter 0.9989 (discretised as a 5 state Markov process), plus a purely transitory binary shock (see the text for details). The parameters are chosen to target the Gini coefficients for consumption and income from Perlach and Wu (2005) for urban regions, and a zero net foreign asset position in 1980.