Monetary Dominance and Government Default

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Abstract

I present an intertemporal optimizing model in which government debt initially follows an unsustainable path, a situation that can be resolved in three possible ways: (i) a fiscal adjustment; (ii) a government default; or (iii) higher inflation. The central bank can respond to a government debt rollover crisis by letting the government default or by rescuing the government from a default and letting inflation increase (monetary accommodation). I show that if the fiscal adjustment requires a minimum amount of time to be implemented, it may be impossible to implement without a minimum level of accommodation on the side of the central bank. In general, the more accommodative the monetary authorities are expected to be, the more likely the fiscal adjustment. The likelihood of inflation is minimized when the central bank is either very accomodative or very unaccomodative. Monetary accommodation, however, does not provide a free lunch, because an accomodative central bank may increase inflation in equilibrium.

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1 Introduction

Why have government debt markets been so much more turbulent in the euro area than elsewhere (given that the government deficits or debt levels do not seem much worse on average than in, say, the US, Japan or the UK)[1]? One view is that this is because the euro area lacks the mechanisms of fiscal discipline that are necessary to make future fiscal consolidation credible. Another view is that this is because the monetary authorities provide less support to government debt markets in the euro area than elsewhere (De Grauwe (2011)). Those views have very different implications for what the European Central Bank should do. According to the first view, increased ECB support for government debt markets will only make matters worse by weakening further fiscal discipline and creating moral hazard. According to the second view, the lack of ECB support is largely to blame for the crisis.

The purpose of this paper is to provide a theoretical perspective on this debate. I present a model in which a government must implement a fiscal consolidation in order to put its debt on a sustainable path. Implementing the fiscal adjustment takes time. The public expects the government to implement the fiscal adjustment (given the time), but there is also a certain probability that bad fiscal news arrive, triggering a sudden realization that the adjustment will not be implemented, and a debt rollover crisis. If there is a rollover crisis, the central bank must choose between letting the government default (be hard) and intervening to prevent a default (be soft). Since the central bank intervenes when the government is insolvent rather than illiquid, its intervention resembles more a bailout than lending-in-last resort. The monetary authorities, if they choose to intervene, close the fiscal gap by increasing the inflation rate and seigniorage.

A key determinant of the debt dynamics—and of the likelihood of a fiscal adjustment—is how the central bank is expected to respond to a rollover crisis. The expectation that the monetary authorities will let the government default raises the default risk premium and the interest rate on government debt, leading to a more rapid accumulation of debt and making a rollover crisis more likely. Indeed, for some parameter values, the debt dynamics are such that a rollover crisis must occur with certainty before the fiscal adjustment has any chance of being implemented. In this case, the rollover crisis, being anticipated by all agents, occurs immediately and there is no

[1]The evidence supporting this statement will be reviewed in section 2

fiscal adjustment. More generally, the likelihood of a fiscal adjustment is decreasing with the probability that the central bank will let the government default in a rollover crisis.

The relationship between the likelihood of inflation and the central bank's reputation is somewhat subtle. The likelihood of inflation is the lowest if the central bank is believed to be either very hard or very soft. That a more inflation-averse central bank makes inflation less likely is intuitive, but the second part of the result (that a very soft central bank also makes inflation less likely) is more surprising. The reason is that a very soft central bank, by slowing down the buildup in debt, makes a rollover crisis less likely and a fiscal adjustment more likely, so that raising seigniorage is less likely to be necessary. The likelihood of inflation is maximum when the central bank's reputation is intermediate. Then, there is a significant probability of a rollover crisis, and there is a significant probability that the rollover crisis, if it occurs, will be followed by inflation.

Having a soft central bank, however, does not offer a free lunch. A soft central bank increases the probability of a fiscal adjustment, but it does not increase it all the way to one, implying that inflation may occur in equilibrium. A calibration of the model suggests that although the probability of inflation may not be very high with a soft central bank (around 10 percent, against a 90 percent probability for the fiscal adjustment), the probability of inflation is not zero. The only way of guaranteeing a zero probability of inflation is to have a central bank that is committed to never provide a monetary backstop to the government debt crisis, but the problem in this case is that a default is likely.

This paper is related to several strands of literature. There is a long line of literature about the relationship between monetary policy and fiscal policy, and the ''unpleasant arithmetics" of government debt monetization. A distinction that has become central in that literature is between fiscal dominance and monetary dominance. Monetary dominance means, broadly speaking, that the monetary authorities do not let monetary policy be influenced by the needs of the fiscal authorities. Monetary dominance is often modeled by assuming a fiscal rule that ensures that the government is always solvent conditional on monetary policy following its own objectives (what (Leeper (1991)) calls a passive fiscal policy). In this case, a conflict between monetary dominance and fiscal dominance is always avoided by the fiscal authorities, and monetary dominance is never challenged. By contrast, I consider here situations in which although monetary dominance is the norm, it may be challenged and the monetary authorities are then faced with a choice between letting the government default and letting inflation increase. More generally, it is useful to make a distinction between soft—or ex ante—monetary dominance (the fiscal authorities avoid a conflict) and hard—or ex post— monetary dominance (when challenged, the monetary authorities let the government default). Most of the literature has looked at ex ante monetary dominance, but this paper is about the implications of the exercise of hard monetary dominance in a crisis.

Monetary dominance was by and large taken for granted in advanced economies until the global financial crisis. With the rising level of government debt in advanced economies, however, monetary dominance is put in question. Environments in which monetary dominance is less than certain have been studied in the recent literature. For example, Davig, Leeper and Walker (2010) use a rational expectations framework to assess the implications of rising debt in an environment with a "fiscal limit", i.e., a point where the government no longer has the ability to finance higher debt levels by increasing taxes, so either an adjustment to fiscal spending or monetary policy must occur to stabilize debt. Davig et al, however, do not consider *default* as a third option.[2] With the exception of Uribe (2006), the literature has not considered fiscal consolidation, inflation and default in the context of the same model. To my knowledge, this paper is the first one to look at the impact of hard monetary dominance on the likelihood of a fiscal adjustment (or to put it differently, whether the hard form of monetary dominance induces the soft form).

2 Facts

Viewed from Mars, it may seem surprising that the government debt crisis occurred, of all places, in the euro area. The fiscal fundamentals seem better in the euro area than in the US, Japan or the UK. Figure 1 plots the primary balance to GDP ratio against debt to GDP ratio in 2009 for a sample of advanced economies. The fiscal balance was higher and/or the debt lower in many euro area countries than in the US, Japan or the UK. In fact, the latter countries had fiscal fundamentals that are comparable to the euro area countries with the worst fundamentals (Greece, Ireland, Portugal, Spain).

[2]Bi (2011) presents an intertemporal optimizing model in which default is an alternative to a fiscal consolidation, but there is no money.

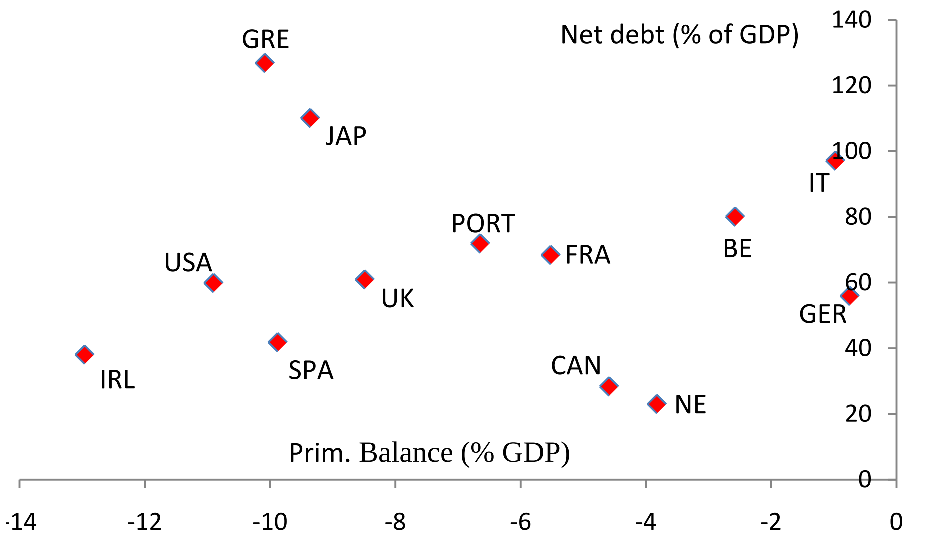

Figure 1: Ratio of fiscal balance and net government debt to GDP in selected advanced economies (2009). Source: WEO.

In addition, the ECB has more firing power if it wanted to support the debt of the governments that are most affected by the crisis. Figure 2 compares the size of the assets of the European System of Central Banks (excluding gold) to the size of the government debt of selected euro area countries. It would seem feasible for the ESCB to buy the whole Greek government debt and sterilize the impact on money supply—although a massive purchase of Italian debt would be more problematic. For the sake of comparison, the outstanding stock of US Treasury securities is about five times larger than the US Fed balance sheet. To take an extreme case of government debt monetization, the US Fed would need to multiply the monetary base (and presumably the price level, in the long run) by a factor 5 in order to buy the whole Treasury debt. A monetary backstop for a government debt crisis seems much more costly to implement in the US than in the euro area.

One area in which the euro area is different is the perception of hard monetary dominance. It is difficult to know what the US Fed would do if the US Treasury were unable to roll over its debt, but the impression of most economists and market participants is that in such an extreme scenario the

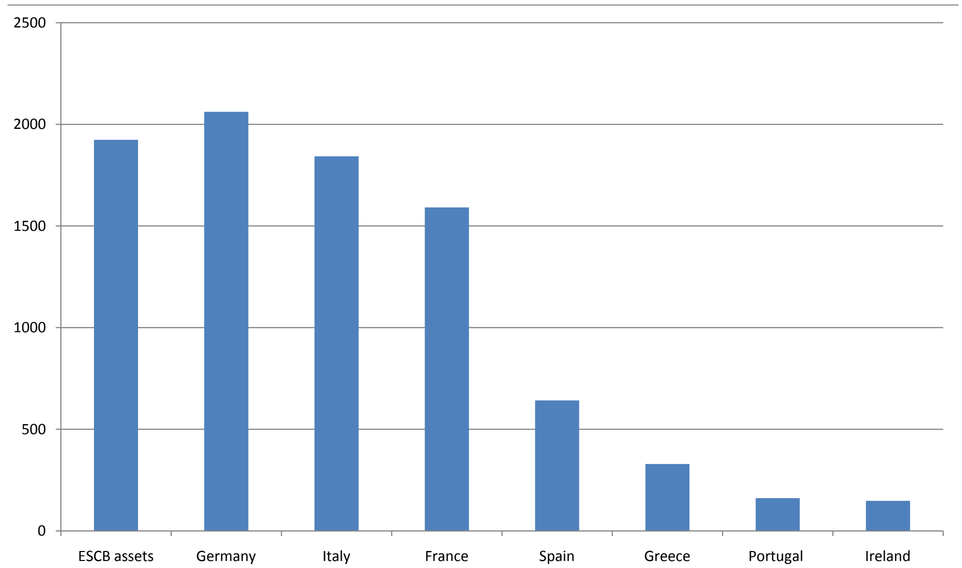

Figure 2: ESCB assets and government debt of selected euro area countries (2010, bn euros). Source: ECB and eurostats.

US Fed would intervene to prevent a default.[3] The mandate of the US Fed is relatively broad, would not clearly preclude such an action, and it can be changed by Congress.[4] By contrast, the ESCB has been explicitly designed, under the Maastricht Treaty, to minimize the risk of government debt monetization. Safeguards against debt monetization have been enshrined in an international treaty that is very cumbersome to change. In addition, the balance of power is changed by the mere fact that it involves one strong and independent monetary authority against 17 weakly coordinated fiscal authorities. This does not mean that the ECB cannot provide support for government debt markets,[5] and one might speculate that it might do so to

[3]As reported in Jeanne (2011), I conducted an informal survey with a few former Fed economists and none of them thought that the Fed would let the Treasury default.

[4]According to the 1977 Federal Reserve Reform Act, monetary policy should be conducted so as to "promote effectively the goals of maximum employment, stable prices and moderate long term interest rates." The so-called dual mandate, thus, requires the Fed to care not only about inflation and unemployment, but also about the level of interest rates. Preventing a government default could also be justified, in the short run, as a way of maintaining financial and banking stability.

[5]It has already started to do so, although reluctantly and in a deliberately temporary

a much larger extent if this were the only way of avoiding a collapse of the euro area. But overall, monetary dominance is "harder" in the euro area than elsewhere. It is very credible that the ECB would let a euro area government default, and it is perhaps this possibility that has made the euro area different.

3 Theory

The theoretical analysis is based on a model that captures the tension between monetary and fiscal dominance in the most tractable possible way (subject to the constraint that the model be realistic enough to lend itself to numerical simulations). I use a continuous-time model with money in the utility function. The government has to continuously refinance its debt and the central bank may (or not) provide seigniorage. Output is exogenous and there is no nominal stickiness.

3.1 Assumptions

The economy is populated by a continuum of mass 1 of identical individuals who consume one homogeneous good. The utility of the representative consumer is

$$U(t) = \int_t^{+\infty} \left[c(s) + u(m(s))\right] e^{-\rho(s-t)} ds,$$

where $m(s) = M(s)/P(s)$ is real money holdings. The budget constraint of the consumer is

$$c(t) + \tau(t) + \frac{M'(t)}{P(t)} + \frac{B'(t)}{P(t)} = y(t) + i(t)\frac{B(t)}{P(t)},$$

where $\tau(t)$ is net taxes, $B(t)$ is the consumer's holdings of nominal bonds issued by the government (and $i(t)$ is the nominal interest rate on those bonds), and $y(t)$ is an exogenous endowment stream (the economy's output). In general nominal variables are denoted with upper-case letters and real variables are denoted with lower-case letters.

The government is composed of the treasury and the central bank. The treasury issues bonds and levies taxes. The central bank issues money. For

manner, with the creation of the Securities Market Programme (SMP) in May 2010 most of my results I do not need to keep track of the flows between the treasury and the central bank and can use only the budget constraint for the consolidated government sector,[6]

$$\tau(t) + \frac{M'(t)}{P(t)} + \frac{B'(t)}{P(t)} = i(t)\frac{B(t)}{P(t)}. \tag{1}$$

Note that there are no government expenditures. Equivalently, government expenditures are a lump-sum transfer to the consumer, and $\tau(t)$ is then redefined as the net transfer. It would not change anything of substance to the model to assume that the government spends some fiscal resources on an exogenous stream of public goods.[7] Then $\tau(t)$ would be redefined as the government's primary balance. Hereafter $\tau(t)$ will be called the primary balance.

Fiscal policy and monetary policy are respectively controlled by the treasury and the central bank. The instrument of fiscal policy is the government primary balance, $\tau(t)$. Monetary policy will be characterized by the path for the inflation rate, $\pi(t) = P'(t)/P(t)$. For now, I will restrict the attention to equilibria in which the price level is a continuous function of time, so starting from an the initial price level $P(0)$, the price path $(P(t))_{t\geq 0}$ is uniquely determined by the inflation path, and the path for money supply $(M(t))_{t\geq 0}$ is then uniquely determined by the first-order equation for money demand, $u'(M(t)/P(t)) = \rho + \pi(t)$. It is more problematic here than in standard models to define the instrument of monetary policy as the nominal interest rate $i(t)$, as the interest rate that the treasury pays on its debt includes an endogenous default premium.

I assume that the central bank's objectives is to maintain the inflation rate at zero.[8] Monetary dominance, thus, means that the treasury must set the primary balance at a level that ensures solvency conditional on no seigniorage. I want to look at situations in which monetary dominance is the norm but it can also be unexpectedly challenged. I do so by making the following assumptions.

[6]The flows between the treasury and the central bank will matter in section 5.1, which discusses sterilized interventions by the central bank in the government debt market.

[7]This would not change anything provided that the public good yields no utility to the consumer, or that the utility from the public good is additive and so does not change the demand for consumption good or real money holdings.

[8]One could assume that the inflation target is positive, but this makes it more difficult to obtain closed-form solutions without yielding significant new insights.

First, I normalize the primary balance by the level of debt,

$$\widetilde{\tau}(t) = \frac{\tau(t)}{b(t)}.$$

I consider a situation in which a fiscal adjustment is necessary because $\widetilde{\tau}$ is initially too low. A fiscal adjustment means that the primary balance is increase to a constant level that ensures solvency conditional on zero seigniorage. If the treasury pays the riskless real interest rate, ρ, on its debt, a fiscal adjustment means raising the primary balance to $\tau = \rho b$, or the normalized balance to $\widetilde{\tau} = \rho$. The government's (real) debt then remains constant after the fiscal adjustment.

I assume that the fiscal adjustment is necessary because the normalized primary balance is initially set at a constant level that is too low, $\widetilde{\tau}_0 < \rho$, which (given zero seigniorage) implies an exploding path for government debt. The difference $\phi = \rho - \widetilde{\tau}_0$ will be called the *fiscal gap*.

A fiscal adjustment may be implemented to close the fiscal gap but this takes time. There are many reasons that a fiscal adjustment takes time in the real world but they will be left outside of the model.[9] I will simply assume that the fiscal adjustment is implemented (if it is) at a future time T. In general, the timing of the fiscal adjustment could be uncertain but such uncertainty is not necessary for my results. I will assume that T is known in period 0 for the sake of simplicity.

If the fiscal adjustment occurred with certainty in period T, we would be in a model with monetary dominance and no risk of government default. But I want to look at situations in which monetary dominance may be challenged, and the monetary authorities then have a choice between saving the treasury from a default or letting it default. Thus, I assume that with some probability it may become common knowledge before time T that the fiscal adjustment will not occur at time T. This event will be called *bad fiscal news*.

The stochastic process governing the arrival of bad fiscal news is specified as follows. It stands to reason that bad fiscal news are more likely if the fiscal adjustment is more demanding. Thus, I assume that bad fiscal news occur when the size of the required fiscal adjustment, $(\rho - \tau_0)b(t)$, exceeds a threshold. Since the only time-varying variable in this expression is the level of real debt, one can equivalently assume that bad fiscal news occur when real debt $b(t)$ exceeds a threshold b^*. This threshold is exogenous, but

[9] See for example **?**.

I assume that it is not known ex ante. This uncertainty (realistically) implies that in the eyes of the public, bad fiscal news could occur at any time before time T. All agents in the economy share the same prior belief about the debt threshold. It will be convenient, for tractability, to describe this prior belief by a probability distribution function that is defined over the *logarithm* of the debt threshold, $d^* = \log(b^*)$. The probability distribution is denoted by $f(\cdot)$, and the cumulative distribution by $F(\cdot)$.

Because of the risk of bad fiscal news the treasury could be faced with a *rollover crisis*, i.e., a situation in which it is unable to roll over its outstanding stock of debt. If there is a rollover crisis, the central bank has a choice between two options:

- let the treasury default: then real debt is reduced by the "haircut" h and the primary balance is set to a level that keeps the post-default real debt constant;[10] the central bank keeps the inflation rate equal to zero.

- increase seigniorage to prevent a default: the inflation rate is increased to a positive constant level such that seigniorage covers the gap $\phi = \rho - \widetilde{\tau}_0$.[11] (The inflation rate discontinuously jumps up but the price level does not.)

In other words, a government debt rollover crisis challenges monetary dominance, to which the central bank can responds in two ways: by upholding monetary dominance and letting the treasury default, or by coming to the rescue of the treasury and deviating from its inflation target. The determinants of the central bank's decision is a complex problem, which I leave outside of the model's reach for now. I simply assume that conditional on a rollover crisis, the central bank lets the treasury default with probability μ. Variable μ is an exogenous parameter of the model which captures the degree of "hard" monetary dominance. The standard model with monetary dominance corresponds to the polar case $\mu = 1$.

[10]The post-default level of the primary balance is given by $\widetilde{\tau} = \rho(1 - h)$. No fiscal adjustment is required in a default if $\widetilde{\tau}_0 = \rho(1 - h)$, i.e., if $h = \phi/\rho$.

[11]I assume that the gap can be covered by seigniorage revenues. This is not always true as the maximum level of seigniorage may be bounded. Seigniorage revenues may be increased by financial repression.

To conclude, there are three possible outcomes in equilibrium:

- Fiscal adjustment: the treasury can roll over its debt until time T; monetary dominance is not challenged, the fiscal adjustment is implemented, there is no default and the inflation rate remains equal to zero.

- Default: there is a debt rollover crisis before time T and the central bank lets the treasury default rather than letting inflation increase.

- Inflation: there is a debt rollover crisis before time T and the central bank increases inflation rather than letting the treasury default.

The question is how the probability of those three outcomes depends on the degree of hard monetary dominance, μ.

3.2 Equilibrium dynamics of debt

The dynamics of debt are influenced by the feedback loop described in Figure 3. The arrow labeled with the number 1 reflects the causality from the rate of increase in government debt to the risk of default. A higher rate of increase in debt makes it more likely that the debt level is about to exceed the threshold for bad fiscal news, triggering a rollover crisis and possibly a default. The causality relationships represented by arrows 2 and 3 are straightforward. The default risk raises the default premium and so the interest rate (arrow 2) which in turn raises the rate of increase in the level of debt (arrow 3).

I now derive equations capturing those causality relationships. First, the consolidated government budget constraint (1) can be rewritten

$$b'(t) = \left[i(t) - \pi(t) - \widetilde{\tau}(t)\right] b(t) - \left[m'(t) + \pi(t)m(t)\right]. \tag{2}$$

Before the fiscal adjustment, and as long as there is no rollover crisis, inflation is equal to zero and this equation simplifies to,

$$d'(t) = i(t) - \widetilde{\tau}(0), \tag{3}$$

where d_t is the logarithm of the real debt level, $d(t) = \log(b(t))$. This equation captures the causality going from the interest rate to the rate of increase in the debt level (arrow 3 in Figure 3).

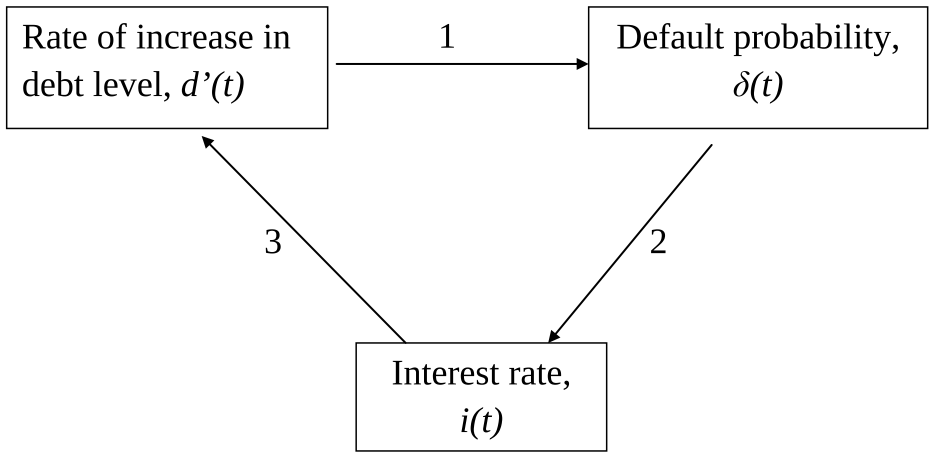

Figure 3: Feedback loop in government debt dynamics

Given the risk neutrality of lenders, if there is no inflation (and no expected jump in the price level), the nominal interest on government debt satisfies

$$i(t) = \rho + \delta(t)h, \tag{4}$$

where $\delta(t)$ is the flow probability of default. This equation represents the causality going from the default risk to the interest rate (arrow 2 in Figure 3).

Using (4) to substitute out $i(t)$ from (3), we obtain

$$d'(t) = \phi + h\delta(t), \tag{5}$$

where $\phi = \rho - \widetilde{\tau}(0)$ is the fiscal gap. Debt increases for two reasons: because the primary balance is initially too low ($\phi > 0$) and because of the default premium ($\delta(t) > 0$).

Let us come to the third causality relationship, from the rate of increase in debt to the default probability. Let us consider a time t before the fiscal adjustment. If bad fiscal news have not arrived at that time, the public has learnt that the threshold d^* is above the current level of debt $d(t)$. Thus, the probability that bad fiscal news are going to arrive in the infinitesimal time interval $[t, t + dt]$ is equal to the probability that d^* lies in that interval, knowing that it is higher than $d(t)$, that is (based on a Bayesian revision of the prior beliefs)

$$\frac{f(d(t))}{1-F(d(t))}d'(t)^{+}dt.$$

The superscript "+" is used to denote that we consider only the positive values of the derivative $d'(t)$ (since there is a positive default probability only if debt increases).

If we assume that a rollover crisis is triggered by bad news, then the probability of a default is equal to the probability of bad news times the probability μ that the central bank will not come to the rescue of the treasury, that is

$$\delta(t) = \frac{f(d(t))}{1-F(d(t))}d'(t)^{+}\mu. \tag{6}$$

This equation captures the causality from the rate of increase in the debt level to the default probability (arrow 1 in Figure 3). Putting things together, rapidly increasing debt raises the probability of default, which in turn increases raises the interest rate and leads to a faster accumulation of debt.

Using (6) to substitute out $\delta(t)$ from (5) gives

$$d'(t) = \phi + \mu h \frac{f(d(t))}{1-F(d(t))}d'(t)^{+}. \tag{7}$$

This equation can in general be solved to obtain $d'(t)$ as a function of $d(t)$, giving us a first-order differential equation for $d(t)$. However one can solve for $d'(t)$ if and only if the following condition is satisfied,

$$\mu h \frac{f(d(t))}{1-F(d(t))} < 1. \tag{8}$$

If this condition is not satisfied, then the right-hand side of (7) is higher than the left-hand side for all values of $d'(t)$ and the equation has no solution. There is no level of the interest rate that can compensate lenders for the default risk, because a one percent increase in the interest rate raises the default premium by more than one percent.

Thus, there are two cases to consider. The first case is if there exists a continuously differentiable function $d(t)$ defined over the time interval $[0, T]$ that satisfies the differential equation equation (7) as well as the initial condition $d(0) = d_0$. In this case, the government can roll over its debt until time T, unless bad fiscal news arrive before that. In the absence of bad fiscal news, the fiscal adjustment is implemented and the debt ratio is stabilized at time T.

The second case is where equation (7) does not have a well-defined solution over the time interval $[0, T]$ because condition (8) is violated for a level of debt d that is reached before time T. In this case, there is a rollover crisis before time T, and the crisis must in fact occur at time 0. There is no way that the rollover crisis could occur after time 0 since creditors, expecting a discrete loss with a positive probability (if $\mu > 0$) at the time of the crisis, would stop rolling over the debt just before the time of the crisis.

The following proposition states that the second case occurs when the index of monetary dominance μ is above a threshold.

Proposition 1 *There exists a threshold μ^* in the monetary dominance index such that,*

(i) if $\mu > \mu^*$, there is a rollover crisis at time 0;

(ii) if $\mu \leq \mu^*$, there is a rollover crisis if and only if bad fiscal news arrive before the fiscal adjustment.

Proof. Let us define $\hat{d}(\mu)$ as the highest level of d for which condition (8) is satisfied, that is

$$\hat{d}(\mu) = \sup\{d \geq d(0), \mu h \frac{f(d)}{1 - F(d)} < 1\}, \tag{9}$$

with $\hat{d}(\mu) = +\infty$ if condition (8) is satisfied for all d larger than $d(0)$ (and $\hat{d}(\mu) = d(0)$ if the condition is satisfied for no d larger than $d(0)$). If $\hat{d}(\mu)$ is finite it is decreasing with μ.

Assume $d(t) < \hat{d}$, so that (7) can be solved for $d'(t)$,

$$d'(t) = \frac{\phi}{1 - \mu h \frac{f(d(t))}{1 - F(d(t))}}. \tag{10}$$

If $d(t)$ is continuous, it follows from this equation (and the continuity of $f(\cdot)$) that $d(t)$ is continuously differentiable. This differential equation can be written for the inverse function $t(d)$,

$$\phi t'(d) = 1 - \mu h \frac{f(d)}{1 - F(d)}, \tag{11}$$

which can be integrated in closed form to give,

$$\phi t = d - d_0 + \mu h \log(1 - F(d)). \tag{12}$$

This equation implicitly defines d as a function of t. For a given d, t is decreasing with ϕ and μh, so for a given t, d is increasing with ϕ or μh. If monetary dominance is harder (μ is higher), the government debt diverges more quickly because of a higher default risk premium.

Then define

$$\hat{t}(\mu) = \frac{\hat{d}(\mu) - d(0) + \mu hlog(1 - F(\hat{d}(\mu)))}{\phi}. \tag{13}$$

This is the time at which the debt level reaches $\hat{d}(\mu)$. There are two cases to consider.

First, one could have $\hat{t}(\mu) < T$. In this case, there is a rollover crisis at time $\hat{t}(\mu) < T$ at the latest, and it occurs before the fiscal adjustment is implemented. Since T and $\hat{t}(\mu)$ are both known ex ante, it is common knowledge that there will be a rollover crisis before the fiscal adjustment, and if $\mu > 0$ that the treasury will default with a nonzero probability. Anticipating this, the lenders will not wait until time $\hat{t}(\mu) < T$ to stop rolling over the debt, and by backward induction there will be a rollover crisis at time 0.

The second case is if $\hat{t}(\mu) > T$. In this case, the fiscal adjustment is implemented unless bad fiscal news arrive before time T.

Then define μ^* as the value of μ such that $\hat{t}(\mu^*) = T$. ■

Monetary dominance, thus, is not conducive to the fiscal adjustment, which is never implemented if $\mu > \mu^*$. This is because hard monetary dominance leads government debt to increase at a higher pace, and to reach earlier the level where it is not possible to roll debt over any more.

If $\mu < \mu^*$, the probability of a fiscal adjustment is equal to the probability that the threshold d^* is above $d(T)$. This probability is $1 - F(d(T))$. Note that since $F(d(T))$ is increasing with μ, the probability of a fiscal adjustment is decreasing with μ. Thus we have:

Corollary 2 *The ex ante probability that the fiscal adjustment is implemented decreases with μ.*

Proof. See discussion above ■

If $\mu < \mu^*$ and there is a rollover crisis because of bad news, there is a default with probability μ and there is inflation with probability $1 - \mu$. The probabilities of the three possible outcomes (fiscal adjustment, default and inflation) are thus given by the table below.

Probability of	$\mu \leq \mu^*$	$\mu > \mu^*$
Fiscal Adjustment	$1 - F(d(T))$	0
Default	$F(d(T))\mu$	μ
Inflation	$F(d(T))(1 - \mu)$	$1 - \mu$

3.3 A tractable specification

Let us assume that the distribution of the threshold d^* is exponential, that is

$$f(d) = \lambda e^{-\lambda(d-d_0)} \tag{14}$$

for $d \geq d_0$ and $f(0) = 0$ for $d < d_0$. The corresponding probability distribution function for b^* is $\lambda b_0^{\lambda}/b^{1+\lambda}$ which implies that the expected value of b^* is given by

$$E_0(b^*) = \frac{\lambda}{\lambda - 1} b_0. \tag{15}$$

The dynamics of debt are especially simple in this case since $F(d) = 1 - e^{-\lambda d}$, which implies

$$\frac{f(d)}{1 - F(d)} = \lambda. \tag{16}$$

Thus, the condition for the government to be able to roll over its debt, equation (8), does not depend on the level of debt d, but only on the parameters μ, h and λ. The government is able to roll over its debt if and only if

$$\mu h \lambda < 1. \tag{17}$$

If this condition is not satisfied the government has a rollover crisis at time 0. If it is satisfied the government has a rollover crisis only if bad fiscal news arrive, and equation (7) implies that (the logarithm of) debt grows linearly with time:

$$d(t) = d_0 + \frac{\phi}{1 - \mu h \lambda} t. \tag{18}$$

The probabilities of fiscal adjustment, inflation and default can then be computed in closed form.

4 Numerical simulation

This section illustrates the quantitative properties of the model with a numerical illustration. I use the tractable specification presented in the previous section with the parameter values indicated in the table below.

h	ϕ	y	$b(0)$	T	λ
0.5	0.05	1	1	1	3

The haircut is assumed to be 50 percent of the debt face value. Output is normalized to 1 so that the values for the fiscal balance and the level of debt can be interpreted as share of GDP. The initial fiscal gap is assumed to be 5 percent of GDP, and the initial debt level is 100 percent of GDP. The fiscal adjustment is expected to be implemented rather quickly, after one year. Finally the parameter of the exponential distribution is set to $\lambda = 3$. Given equation (15), this implies that the debt threshold triggering the arrival of bad fiscal news is expected to be 150 percent of GDP on average.

Figure 4 shows the variations of the ex ante probability of the three possible equilibrium outcomes with the index of monetary dominance μ. The probability of fiscal adjustment is represented in blue, that of inflation in red, and the probability of default in green. The three probabilities sum to 1 since adjustment, inflation and default are the only possible outcomes.

Several results are worth highlighting. First, the fiscal adjustment is possible only if the monetary dominance index μ is lower than 2/3, that is if conditional on a rollover crisis the central bank saves the treasury from a default with a probability of at least 1/3. The probability of fiscal adjustment is the highest when the treasury can rely on a monetary backup with certainty ($\mu = 0$) because then it does not have to pay a default risk premium: the debt of the government increases at a lower pace and is less likely to cross the threshold triggering bad fiscal news. This effect is large: the probability of fiscal adjustment goes from 0 to 86.1 percent when the monetary dominance index decreases from 2/3 to 0 percent.

Interesting results on the probability of inflation. In particular, the probability of inflation is increasing with the monetary dominance index for μ lower than 2/3. That is, decreasing the probability that the central bank will raise seigniorage conditional on a rollover crisis raises the probability that there will be inflation in equilibrium. This is because the probability of a crisis is an endogenous variable. Decreasing the likelihood of monetary

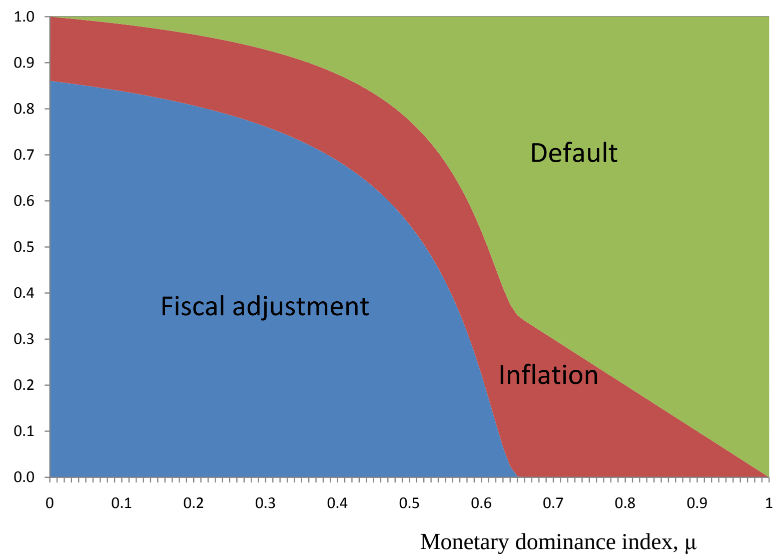

Figure 4: Variation of probability of fiscal adjustment, inflation and default with the monetary dominance index μ

backup conditional on a rollover crisis reduces the likelihood of a fiscal adjustment makes the rollover crisis more likely. The probability of inflation reaches its maximum (about 35 percent) when μ is equal to $2/3$. From the point of view of expected inflation, an intermediate level of monetary dominance brings about the worst of both worlds: μ is too high from the point of view of inducing fiscal adjustment, but still not high enough to guarantee that a rollover crisis will not be followed by inflation.

Note however that there is no free lunch in terms of inflation. The only way that inflation can be made impossible is by setting the maximum level of monetary dominance ($\mu = 1$) and in this case a default is certain. The probability of a fiscal adjustment is maximized if $\mu = 0$ but in this case the probability of inflation is not zero, it is about 13.9 percent. In fact, there cannot be a free lunch in this model since the monetary backstop reduces but does not remove completely the flow probability that the fiscal adjustment will be abandoned.

5 Extensions [incomplete]

5.1 Crisis lending

So far the central bank was helping the government by increasing the flow of seigniorage revenue, not by lending to the treasury. The balance sheet of the central bank was not involved. We now look at the impact of central bank lending to the treasury (directly or through open market operations). We distinguish two types of operation: sterilized lending to the treasury before a rollover crisis, and lending to the treasury at the time of the crisis (or unsterilized lending accompanied with money creation).

Sterilized lending is an open market operation by which the central bank sells or buys government debt in exchange for another asset. Thus one needs to introduce another asset into the model. Let us assume that it is possible to invest in a riskless storage technology that yields a real return ρ.[12] The consolidated government budget constraint becomes

$$\tau(t) + \rho a(t) + \frac{M'(t)}{P(t)} + \frac{B'(t)}{P(t)} = a'(t) + i(t)\frac{B(t)}{P(t)}, \tag{19}$$

where $a(t)$ is the real asset holdings of the central bank (reserves). Conditional on zero inflation the budget constraint may be rewritten

$$b'(t) - a'(t) = [\rho - \widetilde{\tau}(t)]\,[b(t) - a(t)] + h\delta(t)b(t), \tag{20}$$

where $\widetilde{\tau}(t)$ is the ratio of the primary balance to *net* debt $b(t) - a(t)$.

The central bank could buy government debt and sell its other assets, that is reduce the quantity of debt held by the private sector $b(t)$ and $a(t)$ by the same amounts, keeping the level of net debt $b(t) - a(t)$ the same. This will slow down the progression of net debt because the term $h\delta(t)b(t)$ on the right-hand side of (20). So other things equal, sterilized interventions and help preventively. Note that this is not because the treasury gains from defaulting on the central bank. This is because the high interest rate it pays to the central bank is rebated as a high dividend (or capitalized as bank equity).

Another possible intervention is for the central bank to buy the government debt in a rollover crisis. Sterilized buying does not matter at this

[12] This assumption, by itself, does not change anything to the previous model, except that the consumption path is no longer determined (the return on the storage technology being the same as the consumers' discount rate).

point, so let's look at an unsterilized intervention that raises money supply. Now the price level jumps up if the central bank rescues the treasury from a default. So even during the zero-inflation regime, the interest rate on government debt will have to include an inflation premium in addition to the default premium. This does not really help this makes debt increase at a high rate.[13]

5.2 Multiple equilibria

Central bank support is often presented as lending-in-last-resort in the context of multiple equilibria (see, e.g., De Grauwe (2011)).

Multiple equilibria in government debt crises have been modeled in two ways. In Calvo (1988), this involves a feedback loop between the probability of a default and the level of the interest rate. An increase in the default probability can be self-fulfilling because it raises the interest rate and the service of debt, making it more likely that the government default. Calvo (1988) presents this point in the context of a two-period model.

This approach to equilibrium multiplicity is not obvious to capture in my model with continuous time and zero-maturity debt. It can be captured, however, by relaxing the assumption that $d(t)$ is a continuous function of time, and allowing $d(t)$ to jump up at a point in time. (The jump can be interpreted as the interest rate increasing to infinity at this point in time.) In equilibrium, the level of debt can jump up at any time, as we now show.

First, assume that the time of the jump t is known. The real level of debt jumps up from $d(t^-)$ to $d(t^+) > d(t^-)$ at time t. The expectation of the jump does not generate a rollover crisis before t is the expected loss that creditors have to suffer conditional on a default is exactly offset by the profit that they make conditional on no default. There is a default at time t if there is bad fiscal news and the central bank does not rescue the treasury. There is bad news about the fiscal adjustment at time t with probability $[F(d(t^+) - F(d^-)]/F(d^-)$, in which case the government creditors lose a fraction h of their claim with probability μ. The equilibrium condition, then, is

$$\frac{F(d(t^+) - F(d^-)}{F(d^-)}\mu = \left[1 - \mu\frac{F(d(t^+) - F(d(t^-))}{F(d(t^-))}\right]\frac{d(t^+) - d(t^-)}{d(t^-)}. \qquad (21)$$

[13]Things might be different with long-term debt and/or nominal stickiness.

The left-hand side is the probability of a default times the haircut and the right-hand side is the probability of no default times the gain if there is no default.

The debt level could jump up at any time t such that given $d(t^-)$, the equation above admits a solution $d(t^+) > d(t^-)$. If this is the case, the jump could deterministic or random and the expectation of the jump will not affect the pre-jump dynamics of debt. The probability of a jump would be indeterminate. (If the left-hand side is larger than the right-hand side, then the jump could occur with a finite flow probability and the expectation of the jump will generate a default risk premium.)

This multiplicity exists, however, only if μ is not too low. In the extreme case where $\mu = 0$, it obviously does not exist because the default probability is zero, but it is not necessary to have $\mu = 0$. Thus, monetary backup can remove this multiplicity [develop].

The second way of modeling equilibrium multiplicity is as a run a la Diamond-Dybvig. This is the approach followed for example by Cole and Kehoe (2000). It is assumed that government debt is marketed in such a way that individual lenders are trapped in a default if they lend when the total number of lenders is insufficient to allow the government to roll over its debt. Then again, monetary backup can remove this multiplicity. In this case it is necessary to have $\mu = 0$, since a small expected loss is sufficient to trigger a run.

Both approaches to multiple equilibria thus provide a justification for monetary backup as lending in last resort. Lending in last resort is very compelling because it is a free lunch (it removes the bad equilibrium without any lending in equilibrium). However, the theory behind the multiplicity of equilibria is more problematic or contrived in the case of government debt crises than in the case of banking for reasons well explained by Chamon (2007). The government is a large strategic agent that could issue its debt in such a way that the multiplicity disappears.

5.3 Incentives

The model's basic idea is that implementing a fiscal adjustment takes time and that a fiscal adjustment is less likely if debt increases too quickly to a level where monetary dominance is likely to be challenged. However, one could argue that a rapid pace of increase in government debt might generate a sense of urgency, a need to do the adjustment more quickly. Since the timing of the adjustment T was taken as exogenous, this type of incentives was not captured by the model so far.

In order to talk about incentives one must look at the marginal benefits of reducing T for the fiscal policy-maker. I will compare two assumptions about the objectives of the policy-maker: that he maximizes social welfare, and that he maximizes the expected flow benefit of staying in power. Social welfare is the utility of the representative consumer, assuming that output decreases by a fraction γ if there is a default (standard assumption in the sovereign debt literature). In the alternative approach, the policy-maker is assumed to lose power if there is a default (and inflation?)

Calibration. Figure [.] shows the variations of the policymaker's objective with T, under the two alternatives and for different values of μ. The policymaker's objective is higher for lower T. However what matters for incentives is the slope and how it varies with μ. I assume that there is a cost to reducing T, decreasing with T and concave (so that the marginal cost of decreasing T is larger for low T). The impact of μ on incentives is ambiguous in general.

6 Conclusion [incomplete]

Making probability of default more endogenous. Having long-term debt. Nominal stickiness. Real business cycle shocks. The model will have to be solved for numerically.

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