

A Monetary Analysis of Balance Sheet Policies¹

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Abstract

We augment a standard macroeconomic model to analyze the effects and limitations of balance sheet policies. We show that the central bank can stimulate real activity by changing the size or the composition of its balance sheet, when interest rate policy reaches its limits. Increased lending against eligible collateral allows implementing optimal discretionary policy at the zero lower bound, while changing the balance sheet composition can neutralize increases in firms' borrowing costs. These policies are non-neutral if collateral is scarce, which is reflected by a liquidity premium. We further examine long-term bond purchases and quantify limits of balance sheet policies.

JEL classification: E32; E52; E58.

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1 Introduction

Central banks in industrialized countries have responded to the recent financial crisis with unconventional monetary policies (see Borio and Disyatat, 2009, for an overview). The Bank of England (BoE) and the US Federal Reserve (Fed), for example, have set the policy rate at its zero lower bound (ZLB) and introduced various lending facilities and direct asset purchases.³ These policies, which have been summarized by the term "balance sheet policy" (see Borio and Disyatat, 2009), were aimed at reducing spreads attributable to illiquidity (see Blinder, 2010, and Kocherlakota, 2011), stabilizing stressed credit markets (see Yellen, 2009), and stimulating spending and activity (see Bean, 2009). However, they have been implemented with only little theoretical or empirical guidance available. In particular, conventional macroeconomic models are unable to explain how central bank's balance sheet policies can be effective at the ZLB (see Walsh, 2010).

In this paper, we augment a standard monetary model to be applicable for the analysis of balance sheet policies in addition to pure interest rate policy, on which the New Keynesian paradigm has focussed. Given that we aim at providing a basic framework that facilitates a generic analysis of the effects and the limitations of balance sheet policies, we abstract from institutional details and specify the model in a sufficiently simple way to derive analytical results. In contrast to related studies, we focus on monetary policy implementation and the provision of liquidity by the central bank at the ZLB, while we disregard the possibility of central banks to mitigate disruptions of private financial intermediation.⁴ We show that changing the size and the composition of the central bank balance sheet can be non-neutral, even when financial intermediation is not disrupted, as long as assets eligible for open market operations are scarce, which is reflected by the existence of a liquidity premium. Within

³Under the Asset Purchase Facility the Bank of England purchased commercial papers, corporate bonds, and government bonds. The US Federal Reserve, for example, introduced the Term Auction Facility, which provided short-term credit to depository institutions, the Commercial Paper Funding Facility, where three-month commercial paper are purchased, and the Treasury Securities Lending Facility, which provides Treasury securities in exchange for mortgage-backed securities and commercial paper.

⁴This has been analyzed in related studies, where central banks provide financial intermediation (e.g. direct central bank lending) in situations where private financial intermediation is more costly due to severe financial frictions (see Curdia and Woodford, 2011, Gertler and Karadi, 2011, and Gertler and Kiyotaki, 2011). Our paper complements this literature, which abstracts from the role of money as a means of payment (see Dellas, 2011).

this framework, balance sheet policies are shown to be especially useful when a pure interest rate policy reaches its limits, which is demonstrated for demand shocks at the ZLB and for changes in firms' lending rates. We further examine the limitations of balance sheet policies, in particular, when liquidity premia shrink until collateral becomes abundant. The analysis can rationalize the types of liquidity providing lending facilities that were introduced by the BoE or the Fed in 2008-2009 and shut down in 2010 or by the Bank of Japan (BoJ) in 2001,⁵ while is less suited to assess large scale asset purchases by the central bank or the treasury.⁶

The term quantitative easing refers to an increase in the supply of reserves via purchases of securities, such as government bonds (see Bernanke et al., 2004), while credit easing has been defined in a broader way by Bernanke (2009) as: "the Federal Reserve's credit easing approach focuses on the mix of loans and securities that it holds and on how this composition of assets affects credit conditions for households and businesses". Conducting these policies when the policy rate is at its ZLB should be ineffective according to conventional macroeconomic models since private agents will demand money up to satiation (see Walsh, 2010). Specifically, quantitative easing in terms of treasury securities should be irrelevant as long as they do not change expectations about future conduct of monetary and fiscal policy (see Eggertsson and Woodford, 2003, or Curdia and Woodford, 2011). Moreover, a policy that exclusively changes the composition of the central bank's balance sheet, which will be called collateral policy in this paper, is obviously neutral in single interest rate models, where assets are perfect substitutes. Hence, standard macroeconomic models are hardly able to account for recent evidence, which suggests that the lending facilities of the BoE, the BoJ, and the Fed have been effective, in particular, by easing liquidity supply and by reducing liquidity premia (see e.g. Ashcraft et al., 2011, Bowman et al., 2011, Christensen et al., 2009, D'Amico and King, 2010, Duygan-Bump et al., 2010, Gangon et al., 2010, Joyce,

⁵Willardson and Pederson (2010) provide an overview of the lending facilities implemented by the Fed in 2008-2010. Shiratsuka (2010) summarizes the balance sheet policies of the BoJ in 2001-2006.

⁶Krishnamurthy and Vissing-Jorgensen (2011) examine effects of the Fed's large scale purchases of long-term treasuries in late 2010 (often called "quantitative easing 2") on interest rates on treasuries and corporate bonds. They find substantial effects on treasury yields and smaller effects on corporate bond yields, which accord to Swanson's (2011) analysis of the US Treasury's Operation Twist in 1961. These effects can hardly be explained within our framework and seem to rely on policy induced changes in expectations and on preferred-habitat investors, like in Vayanos and Vila (2009).

2010, or Sarkar and Schrader, 2010, among others).

We apply a macroeconomic model that mainly differs from a canonical New Keynesian model by a collateral constraint in open market operations and that is sufficiently simple to derive the main results in an analytical way. Multiple assets are considered that differ with regard to their ability to serve as collateral for money. The central bank sets the policy rate, i.e. the price of money in terms of eligible assets, and decides on the size and the composition of its balance sheet. Private agents rely on money for goods market purchases, while money is supplied only in exchange for eligible securities, e.g. for short-term government bonds. This requirement leads to a spread between the interest rate on non-eligible and eligible assets, i.e. a liquidity premium. It implies that interest rates on non-eligible securities are positive, even if the policy rate is at the ZLB, which accords to the empirical evidence that – as emphasized by Ohanian (2011) – interest rates on non-money market securities tend to be positive. Given that liquidity is then positively valued, an expansionary monetary policy which extends the central bank’s balance sheet can be non-neutral, even when a pure interest rate policy has reached its limits.

Firms are assumed to demand loans for working capital and to issue standard debt before idiosyncratic productivity shocks are realized, which gives rise to a positive fraction of defaulting firms. Higher costs of borrowing tend to raise the marginal costs of production and thereby exert downward pressure on production. We further consider demand shocks, e.g. contractionary shocks to the rate of time preference and liquidity demand shocks, which can be as large such that an endogenously adjusted policy rate can hit its ZLB. In this framework, we examine *quantitative easing* (i.e. increasing the amount of eligible assets), which can ease households’ access to cash like a conventional money injection, and *collateral policy* (i.e. accepting loans as collateral while keeping the size of the balance sheet constant), which can lower the firms’ cost of borrowing by reducing the (il-)liquidity premium on loans. We show that quantitative easing and collateral policy affect the equilibrium allocation and prices only if eligible assets are scarce or, phrased in technical terms, if the collateral constraint in open market operations is binding. This is the case when there exists a liquidity premium on eligible assets, i.e. when eligible assets can be exchanged against money at a price (i.e. the policy rate) that is lower than the consumption Euler rate, which

measures the marginal valuation of money.⁷ If, however, quantitative easing, which tends to increase money supply and therefore consumption, is conducted in an excessive way, it can be ineffective when collateral becomes abundant.

Our main results can be summarized as follows. When collateral is scarce, quantitative easing and collateral policy are not equivalent to a policy rate adjustment and can enhance the ability of the central bank to respond to aggregate shocks compared to a pure interest rate policy. We show that a collateral policy can neutralize an increase in borrowing costs of firms induced by (small) default risk shocks. Quantitative easing can enable the central bank to implement optimal policy under demand shocks, even when it takes private sector expectations as given and the policy rate is at the ZLB. Increasing the fraction of eligible government bonds then reduces both, the interest rates on treasuries as well as the loan rate, due to a shrinking liquidity premium. We further present numerical results for a calibrated version of the model to explore the limits of quantitative easing and collateral policy at the ZLB.⁸ These limits are reached when a stimulating policy drives down the Euler rate until it equals the policy rate. We find that a maximum quantitative easing policy can substantially reduce interest rate spreads and can stimulate output at the ZLB like a reduction of the policy rate by 80 basis points (when it is not at the ZLB). The inflation responses are much smaller and differ for both policies: Quantitative easing increases inflation, whereas collateral policy reduces inflation. We further consider a large exogenous increase in the liquidity demand for investments that drives down the policy rate to its ZLB. It further leads to a pronounced output contraction and a strong increase in the liquidity premium. Even a maximum quantitative easing policy cannot neutralize this shock, though it can mitigate the output contraction by 50% and thereby helps to escape from the ZLB. Finally, we show that central bank purchases of long-term treasury bonds are equivalent to purchases of short-term treasury bills, except for the effects on the respective asset prices. For example, a collateral policy in terms of long-term treasuries reduces long-term yields.

There exists a large literature on monetary policy options at the ZLB. Most of them

⁷Canzoneri et al. (2007) provide evidence for US data in favor of a positive average spread between a standard consumption Euler rate and the policy rate, which they identify with the Federal Funds rate.

⁸In a companion paper, Schabert (2010) applies a closely related model and shows that the additional monetary policy instruments can help to overcome the well-known monetary policy trade-off between stabilizing prices and closing output-gaps when the policy rate is above its ZLB.

advocate the possibility of providing monetary stimulus at the ZLB through shaping interest-rate expectations. The basic idea is that a monetary expansion, if perceived as permanent, can stimulate the economy by creating expected inflation and reducing the real rate of interest (see Krugman, 1998). Eggertsson and Woodford (2003) and Jung et al. (2005) show that a commitment to keep nominal interest rates low in future can provide an effective way of escaping a liquidity trap. Levin et al. (2010) examine large, persistent shocks and find that a policy relying on shaping interest rate expectations might not be sufficient to stabilize the economy. Auerbach and Obstfeld (2005) analyze open market bond purchases and find that under distortionary taxation this policy can lift the economy out of the liquidity trap if the monetary base is permanently increased.

According to conventional wisdom, lump-sum injections of money such as helicopter drops are ineffective at the ZLB (see Krugman, 1998). The reason is that standard macroeconomic models consider only a single interest rate. Once the policy rate reaches the ZLB, the opportunity costs of holding money fall to zero such that money demand is indetermined or private agents demand money up to satiation (see Walsh, 2010). Moreover, open market operations that aim at easing money supply, like a quantitative easing policy, are ineffective at the ZLB as long as they do not change expected future policy paths. Then, neither the size nor the composition of the central bank's balance sheet are relevant as long as financial market are frictionless (see Eggertsson and Woodford, 2003).

Motivated by central bank responses to the recent financial crisis, a literature on non-standard policies under financial market imperfections is now developing. Gertler and Karadi (2011) analyze direct central bank lending when financial intermediaries need collateral to attract deposits. When financial institutions deleverage due to a decline in asset prices, central bank interventions such as borrowing directly to firms can be a powerful tool. Based on Gertler and Karadi's (2011) model augmented by idiosyncratic investment risks and constraints on the resaleability of assets, Gertler and Kiyotaki (2011) show that direct central bank lending is beneficial in crisis situations when private intermediaries are financially constrained. Del Negro et al. (2010) consider entrepreneurs facing a borrowing and a resaleability constraint (like in Kiyotaki and Moore, 2008) and add these frictions to a medium scale macroeconomic model (see Christiano et al., 2005). They calibrate the model

and a negative shock to the resaleability of assets to match the U.S. in late 2008, and show that the Fed’s policy interventions prevented a second Great Depression. Ashcraft et al. (2011) develop an overlapping generations model, where investments in assets are subject to margin requirements. The required return on an asset falls when the central bank accepts this assets as collateral at a lower haircut, which corresponds to evidence from the Fed’s Term Asset-Backed Securities Loan Facilities. Curdia and Woodford (2011) augment a New Keynesian model by considering segmented asset markets and costly financial intermediation. They show that targeted asset purchases by the central bank can be effective when financial markets are sufficiently disrupted. In contrast to our result, they conclude that quantitative easing is likely to be ineffective.⁹

The paper is organized as follows. Section 2 presents the model. In Section 3, we describe when quantitative easing and collateral policy are effective, and we show that monetary policy instruments are not equivalent. In Section 4, we show how balance sheet policies can be applied in response to default risk shocks and in the case where the ZLB on the policy rate is binding. In Section 5, we quantitatively examine the limits to quantitative easing and collateral policy, and we assess the ability of the central bank to mitigate effects of large liquidity demand shocks. Section 5 briefly discusses the effects of long-term bond purchases. Section 6 concludes.

2 The model

In this section, we present a sticky price model where households face a cash-in-advance constraint and firms require working capital, like in Christiano et al. (2005). Money is supplied by the central bank only in exchange for eligible collateral (like in Reynard and Schabert, 2010), in particular, government bonds and/or corporate debt, where the latter is associated with default risk. The central bank sets the policy rate and decides on the size and the composition of its balance sheet, which will be called quantitative easing and collateral policy, respectively. In particular, it controls the fractions of assets that are eligible in open market operations, which can also be interpreted as haircuts on assets

⁹A main reason for this difference is that private agents in their model do not internalize the collateral constraint, i.e. that money is only supplied in exchange for eligible assets, which corresponds to the case where eligible assets are not scarce (i.e. the collateral constraint is slack).

under discount window lending (see Ashcraft et al., 2011). Households take these policies into account when they invest in assets, which gives rise to different interest rates due to liquidity premia. Quantitative easing and collateral policy can lower these liquidity premia and can stimulate aggregate demand as long as collateral is scarce. To present the problems of households and firms in a transparent way, we introduce indices for individual households and firms.

2.1 Timing of events

Households enter the period t with money, government bonds, and household debt, $M_{i,t-1}^H + B_{i,t-1} + D_{i,t-1}$. Households further dispose of a time-invariant time endowment. They supply labor to intermediate goods producing firms, which do not hold any financial wealth. At the beginning of the period, aggregate shocks are realized. Then, the central bank sets its instruments, i.e. it announces the fractions of government bonds and corporate loans that are accepted as collateral in open market operations, $\kappa_t^B \in (0, 1]$ and $\kappa_t \in [0, 1]$, and the policy rate $R_t^m \geq 1$. The remainder of the period can be divided into four subperiods.

1. The labor market opens, where a perfectly competitive intermediate goods producing firm j hires workers $n_{j,t}$. We assume that it has to pay workers their wages in cash before goods are sold. Since the firm does not hold any financial wealth, it has to borrow cash. Firm j thus faces the constraint

$$L_{j,t}/R_{j,t}^L \geq P_t w_t n_{j,t}, \quad (1)$$

where w_t denotes the real wage rate, P_t denotes the final goods price and $L_{j,t}/R_{j,t}^L$ the amount received by the borrowing firm. Firms draw idiosyncratic productivity shocks from the same distribution and do not fully commit to repay the loans at the end of the period. Lenders sign standard debt contracts with ex-ante identical firms at the same price $1/R_t^L$, taking into account that a fraction κ_t of all loans can be used as collateral for repurchase agreements and that a fraction δ_t^e of firms default.

2. The money market opens where the central bank sells or purchases assets outright or supplies money via repurchase agreements against collateral at the rate R_t^m . In contrast to household debt, corporate loans and government bonds can be eligible,

where only the latter can be purchased outright by the central bank. In period t , household i receives new money (injections) from the central bank $I_{i,t}$, which consists of money received from the central bank's outright bond purchases, as well as money received via repos with bonds $M_{i,t}^R$ and loans $M_{i,t}^L$. Specifically, the central bank supplies money against a fraction κ_t^B of randomly selected bonds and a fraction κ_t of randomly selected loan contracts, such that $I_{i,t}$ is constrained by the following collateral constraint:

$$I_{i,t} \leq \kappa_t^B (B_{i,t-1}/R_t^m) + \kappa_t (L_{i,t}/R_t^m). \quad (2)$$

After receiving money injections from the central bank, household i delivers the amount $L_{i,t}/R_t^L$ to firms according to the loan contract. Its holdings of money, bonds, and loans are then $M_{i,t-1}^H + I_{i,t} - (L_{i,t}/R_t^L)$, $B_{i,t-1} - \Delta B_{i,t}^c$, and $L_{i,t} - L_{i,t}^R$, where $\Delta B_{i,t}^c$ are bonds received by the central bank and $L_{i,t}^R$ are loans under repos, such that $I_{i,t} = (\Delta B_{i,t}^c/R_t^m) + (L_{i,t}^R/R_t^m)$.

3. Wages are paid and intermediate goods as well as final goods are produced. Then, the final goods market opens, where purchases of consumption goods require cash holdings. Hence, household i faces a cash-in-advance constraint in the goods market:

$$P_t c_{i,t} \leq I_{i,t} + M_{i,t-1}^H - (L_{i,t}/R_t^L) + P_t w_t n_{i,t}. \quad (3)$$

Final goods producing firms receive cash for their sales and pay for intermediate goods. All firms further pay out dividends to their owners (households), which sum up to $P_t v_{i,t}$ for household i , such that its money holdings is given by $\widetilde{M}_{i,t} = I_{i,t} + M_{i,t-1}^H - (L_{i,t}/R_t^L) + P_t w_t n_{i,t} - P_t c_{i,t} + P_t v_{i,t}$.

4. Before the asset market opens, repurchase agreements are settled: Household i buys back loans $L_{i,t}^R = R_t^m M_{i,t}^L$ and government bonds $B_{i,t}^R = R_t^m M_{i,t}^R$, such that its bond holdings equals $\widetilde{B}_{i,t} = B_{i,t-1} - \Delta B_{i,t}^c + B_{i,t}^R$. This implies that there are no central bank losses due to default, since loans are repurchased by households before maturity. In the asset market, households receive payoffs from maturing assets as well as government transfers $P_t \tau_{i,t}$. Further, the government issues new bonds at the price $1/R_t$.

Household i can thus carry wealth into period $t + 1$ in form of bonds, state-contingent claims, or money, such that its asset market constraint is

$$\begin{aligned} & (B_{i,t}/R_t) + E_t[\varphi_{t,t+1}D_{i,t}] + M_{i,t}^H \\ & \leq \widetilde{B}_{i,t} + \widetilde{M}_{i,t} - R_t^m (M_{i,t}^R + M_{i,t}^L) + (1 - \delta_t^e) L_{i,t} + D_{i,t-1} + P_t\tau_{i,t}, \end{aligned} \quad (4)$$

where $\varphi_{t,t+1}$ denotes a stochastic discount factor (which will be defined in section 2.3).

The central bank reinvests its payoffs from maturing bonds into new government bonds and leaves money supply unchanged, $\int_0^1 M_{i,t}^H di = \int_0^1 (M_{i,t-1}^H + I_{i,t} - M_{i,t}^R - M_{i,t}^L) di$.

2.2 Firms

There are intermediate goods producing firms, which are perfectly competitive and sell their goods to monopolistically competitive retailers. The latter sell a differentiated good to bundlers who assemble final goods using a Dixit-Stiglitz technology.

There is a continuum of *intermediate goods producing firms* indexed with $j \in [0, 1]$. They are perfectly competitive, owned by the households, and produce an identical intermediate good with labor. Production depends on random idiosyncratic productivity levels $\omega_{j,t} \geq 0$, which materialize after the labor market closes. Firm j produces according to the production function

$$IO_{j,t} = \omega_{j,t} n_{j,t}^\alpha,$$

where $\alpha \in (0, 1)$ and sells the intermediate good to retailers who pay the price Z_t in cash (after they received cash from households' goods purchases). We assume that wages have to be paid in advance, i.e. before the intermediate goods are sold. For this, firm j borrows cash $L_{j,t}$ from households at the price $1/R_{j,t}^L$ and repays the loan at the end of the period. To account for credit default risk in a simple way, we assume that the realizations of the idiosyncratic productivity levels can freely be observed by borrower, while the lenders can only observe the realized idiosyncratic productivity level at proportional monitoring costs $\varrho \geq 0$. We then consider the following standard debt contract with limited liability: Firm j offers a loan contract at the price $1/R_{j,t}^L$ that leads to a pay-off of 1 when its productivity level is sufficiently high $\omega_{j,t} \geq \bar{\omega}_{j,t}$, where $\bar{\omega}_{j,t}$ is the minimum productivity level that enables full repayment. Otherwise, if $\omega_{j,t} < \bar{\omega}_{j,t}$, firm j goes bankrupt and the lender can seize total

revenues. The maximization problem of firm j can be written as

$$\max E_t[(Z_t/P_t)\omega_{j,t}n_{j,t}^\alpha - w_t n_{j,t} - l_{j,t}(R_{j,t}^L - 1)/R_{j,t}^L], \quad \text{s.t. (1),} \quad (5)$$

where $l_{j,t} = L_{j,t}/P_t$ and the expectations operator E_t is based upon the information at the beginning of the period after aggregate state variables, but not the productivity levels $\omega_{j,t}$, are realized. These idiosyncratic productivity levels are drawn from the same potentially time-varying distribution with density function $f_t(\omega_{j,t})$ and a mean of one, $E_t(\omega_{j,t}) = 1$. Since firms are ex-ante identical, loan contracts for different firms are signed at the same rate $R_{j,t}^L = R_t^L$. The first order conditions to the problem (5) are therefore given by $R_t^L - 1 = \mu_{j,t}$, $(Z_t/P_t)\alpha n_{j,t}^{1-\alpha} = w_t + \mu_{j,t}w_t$, (1), and $\mu_{j,t}[(l_{j,t}/R_t^L) - w_t n_{j,t}] = 0$, where $\mu_{j,t} \geq 0$ is the multiplier on (1). Hence, intermediate goods producing firms do not borrow more than required to pay wages $w_t n_{j,t}$ if $R_t^L > 1 \Rightarrow \mu_{j,t} > 0$, which will be satisfied throughout the analysis. Given that $\mu_{j,t} = \mu_t$ and $n_{j,t} = n_t$, all firms behave in an identical way and the following conditions describe labor demand and the volume of loans:

$$(Z_t/P_t)\alpha n_t^{1-\alpha} = w_t R_t^L, \quad (6)$$

$$l_t/R_t^L = w_t n_t. \quad (7)$$

After idiosyncratic productivity shocks are realized, firm j fully repays loans $l_t = \alpha(Z_t/P_t)a_t n_t^\alpha$ if $\omega_{j,t} \geq \alpha$ or, if $\omega_{j,t} < \alpha$, lenders get $(1-\varrho)\omega_{j,t}(Z_t/P_t)a_t n_t^\alpha$, where $\varrho\omega_{j,t}(Z_t/P_t)a_t n_t^\alpha$ denotes the monitoring costs. Firms drawing a productivity level that exceeds α transfer their profits to the households. The expected pay-off for a lender is then $\int_\alpha^\infty \alpha(Z_t/P_t)a_t n_t^\alpha f_t(\omega_{j,t})d\omega_{j,t} + (1-\varrho)\int_0^\alpha \omega_{j,t}(Z_t/P_t)a_t n_t^\alpha f_t(\omega_{j,t})d\omega_{j,t}$ and the expected rate of repayment $1 - \delta_t^e \in [0, 1)$ on firm loans is exogenous and equals

$$1 - \delta_t^e = \int_\alpha^\infty f_t(\omega_{j,t})d\omega_{j,t} + \alpha^{-1}(1-\varrho)\int_0^\alpha \omega_{j,t}f_t(\omega_{j,t})d\omega_{j,t}. \quad (8)$$

We assume that the distribution of the idiosyncratic productivity shocks can vary stochastically over time in a mean preserving way. Hence, these (aggregate) shocks to the distribution, which will be called default risk shocks, shift the mass of defaulting firms $\int_0^\alpha f_t(\omega_{j,t})d\omega_{j,t}$ over time (e.g. by changing the variance of idiosyncratic productivity levels) without affecting the expected productivity level. Realizations of default risk shocks are revealed at

the beginning of the period t , and will therefore randomly alter the current period expected rate of repayment $1 - \delta_t^\varepsilon$. These shocks will be considered in section 4.1.

Monopolistically competitive *retailers* buy intermediate goods $IO_t = \int_0^1 IO_{j,t} dj$ at the common price Z_t . A retailer $k \in [0, 1]$ relabels the intermediate good to $y_{k,t}$ and sells it at the price $P_{k,t}$ to perfectly competitive bundlers, who bundle the goods $y_{k,t}$ to the final consumption good y_t with the technology, $y_t^{\frac{\varepsilon-1}{\varepsilon}} = \int_0^1 y_{k,t}^{\frac{\varepsilon-1}{\varepsilon}} dk$, where $\varepsilon > 1$. The cost minimizing demand for $y_{k,t}$ is therefore given by $y_{k,t} = (P_{k,t}/P_t)^{-\varepsilon} y_t$.

Retailers set their prices to maximize profits. We assume that each period a measure $1 - \phi$ of randomly selected retailers may reset their prices independently of the time elapsed since the last price setting, while a fraction $\phi \in [0, 1)$ of retailers do not adjust their prices. A fraction of $1 - \phi$ retailers set their price to maximize the expected sum of discounted future. The first order condition for their price \tilde{P}_t is given by (where we use that Z_t/P_t are real marginal cost, mc_t) $\tilde{Z}_t = \frac{\varepsilon}{\varepsilon-1} Z_t^1/Z_t^2$, where $\tilde{Z}_t = \tilde{P}_t/P_t$, $Z_t^1 = c_t^{-\sigma} y_t mc_t + \phi \beta E_t \pi_{t+1}^\varepsilon Z_{t+1}^1$ and $Z_t^2 = c_t^{-\sigma} y_t + \phi \beta E_t \pi_{t+1}^{\varepsilon-1} Z_{t+1}^2$. With perfectly competitive bundlers and the homogenous bundling technology, the price index P_t for the final consumption good satisfies $P_t^{1-\varepsilon} = \int_0^1 P_{k,t}^{1-\varepsilon} dk$. Using the demand constraint, we obtain $1 = (1 - \phi) \tilde{Z}_t^{1-\varepsilon} + \phi \pi_t^{\varepsilon-1}$. Aggregate intermediate output is then given by $IO_t = n_t^\alpha$, where $n_t = \int_0^1 n_{j,t} dj$, since idiosyncratic shocks satisfy $E_t(\omega_{j,t}) = 1$, while there is a production inefficiency due to price dispersion across retailers. Specifically, the market clearing condition in the intermediate goods market, $IO_t = \int_0^1 y_{k,t} dk$, gives $n_t^\alpha = \int_0^1 (P_{k,t}/P_t)^{-\varepsilon} y_t dk \Leftrightarrow y_t = n_t^\alpha/s_t$, where $s_t \equiv \int_0^1 (P_{k,t}/P_t)^{-\varepsilon} dk$ and $s_t = (1 - \phi) \tilde{Z}_t^{-\varepsilon} + \phi s_{t-1} \pi_t^\varepsilon$ given s_{-1} .

2.3 Households

There is a continuum of infinitely lived households indexed with $i \in [0, 1]$. Households have identical preferences and potentially different asset endowments. Household i maximizes the expected sum of a discounted stream of instantaneous utilities

$$E_0 \sum_{t=0}^{\infty} \beta^t \xi_t u(c_{i,t}, n_{i,t}), \quad \text{where } u(c_{i,t}, n_{i,t}) = \frac{c_{i,t}^{1-\sigma} - 1}{1-\sigma} - \theta \frac{n_{i,t}^{1+\sigma_n}}{1+\sigma_n} \quad (9)$$

and $\theta > 0, \sigma \geq 1, \sigma_n \geq 0$.

Further, E_0 is the expectation operator conditional on the time 0 information set, and $\beta \in (0, 1)$ is the subjective discount factor. The term ξ_t is a stochastic preference parameter, which shifts households' discounting as in other studies on policy options at the ZLB (see e.g. Eggertsson, 2011, or Christiano et al., 2011). These shocks will be considered in section 4.2.

A household i is initially endowed with money $M_{i,-1}^H$, government bonds $B_{i,-1}$, and privately issued debt $D_{i,-1}$. In each period, it supplies labor, lends out funds to intermediate goods producing firms, trades assets with the central bank in open market operations, and can reinvest. Before household i enters the goods market, where it needs money as the only accepted means of payment, it can get additional money in open market operations in exchange for government bonds. When it invests in loans, they can eventually be refinanced at the central bank. Given that idiosyncratic productivity shocks are not realized at this moment and that the choice which particular loan contract is eligible is made after loan contracts are signed, the price of loan is $1/R_t^L$ for all firms j .

The household faces the collateral constraint (2), where we disregard the case where money is withdrawn, $I_{i,t} \geq 0$, by considering a sufficiently large fraction of repos. In the goods market, household i can use wages, money holdings, and additional cash net of lending from current period open market operations for its consumption expenditures (see 3). Before the asset market opens, it buys back assets under repos. In the asset market, it further receives payoffs from maturing assets (including loans which are repaid according to the debt contract), it can buy bonds from the government, it can trade all assets with other households, and it can borrow and lend using a full set of nominally state contingent claims. Dividing the period t price of one unit of nominal wealth in a particular state of period $t + 1$ by the period t probability of that state gives the stochastic discount factor $\varphi_{t,t+1}$. The period t price of a payoff $D_{i,t}$ in period $t + 1$ is then given by $E_t[\varphi_{t,t+1}D_{i,t}]$. Substituting out the stock of bonds and money held before the asset market opens, $\tilde{B}_{i,t}$ and $\tilde{M}_{i,t}$, in (4), the asset market constraint of household i can be rewritten as

$$0 \leq M_{i,t-1}^H - M_{i,t}^H + B_{i,t-1} - (B_{i,t}/R_t) + (1 - \delta_t^e) L_{i,t} - (L_{i,t}/R_t^L) \quad (10)$$

$$+ D_{i,t-1} - E_t[\varphi_{t,t+1}D_{i,t}] - (R_t^m - 1) I_{i,t} + P_t w_t n_{i,t} - P_t c_{i,t} + P_t v_{i,t} + P_t \tau_{i,t},$$

where household i 's borrowing is restricted by $M_{i,t}^H \geq 0$, $B_{i,t} \geq 0$, and the no-Ponzi game

condition $\lim_{s \rightarrow \infty} E_t \varphi_{t,t+s} D_{i,t+s} \geq 0$. The term $(R_t^m - 1) I_{i,t}$ in (10) measures the costs of money acquired in open market operations, i.e. household i receives new cash $I_{i,t}$ in exchange for $R_t^m I_{i,t}$ assets. Maximizing the objective (9) subject to $\kappa_t^B B_{i,t-1} + \kappa_t L_{i,t} \geq R_t^m I_{i,t}$ (see 2), the goods market constraint (3), the asset market constraints (10) and the borrowing constraints, for given initial values $M_{i,-1}$, $B_{i,-1}$, and $D_{i,-1}$ leads to the following first order conditions for consumption, working time, additional money, and loans

$$\xi_t c_{i,t}^{-\sigma} = \lambda_{i,t} + \psi_{i,t}, \quad (11)$$

$$\theta \xi_t n_{i,t}^{\sigma_n} = w_t (\lambda_{i,t} + \psi_{i,t}), \quad (12)$$

$$\psi_{i,t} = (R_t^m - 1) \lambda_{i,t} + R_t^m \eta_{i,t}, \quad (13)$$

$$(\lambda_{i,t} + \psi_{i,t}) / R_t^L = (1 - \delta_t^e) \lambda_{i,t} + \eta_{i,t} \kappa_t, \quad (14)$$

as well as for investments in government bonds, money, and contingent claims

$$\lambda_{i,t} = \beta R_t E_t \frac{\lambda_{i,t+1} + \kappa_{t+1}^B \eta_{i,t+1}}{\pi_{t+1}}, \quad (15)$$

$$\lambda_{i,t} = \beta E_t \frac{\lambda_{i,t+1} + \psi_{i,t+1}}{\pi_{t+1}}, \quad (16)$$

$$\varphi_{t,t+1} = \frac{\beta}{\pi_{t+1}} \frac{\lambda_{i,t+1}}{\lambda_{i,t}}, \quad (17)$$

where $\lambda_{i,t} \geq 0$ denotes the multiplier on (10), $\eta_{i,t} \geq 0$ the multiplier on $\kappa_t^B B_{i,t-1} + \kappa_t L_{i,t} \geq R_t^m I_{i,t}$, and $\psi_{i,t} \geq 0$ the multiplier on (3). Further, (2), (3),

$$\psi_{i,t} [I_{i,t} + M_{i,t-1}^H - (L_{i,t} / R_t^L) + P_t w_t n_{i,t} - P_t c_{i,t}] \geq 0, \quad (18)$$

$$\eta_{i,t} [\kappa_t^B B_{i,t-1} + \kappa_t L_{i,t} - R_t^m I_{i,t}] \geq 0, \quad (19)$$

and (10) with equality hold as well as the transversality conditions. The debt rate R_t^D , which slightly differs in the short-run from a standard consumption Euler rate due to the multiplier on the cash-in-advance constraint $\psi_{i,t}$ (see 11), is defined as follows

$$E_t \varphi_{t,t+1} = 1 / R_t^D. \quad (20)$$

Comparing the first order conditions with regard to investments in bonds (15) and contingent claims (17) shows that the debt rate (20) can differ from the bonds rate $1/R_t$ by a liquidity

premium, which relies on a binding collateral constraint, $\eta_{i,t+1} > 0$, and increases with the future fraction of eligible bonds κ_{t+1}^B . Combining (13) and (14) to $R_t^m (\lambda_{i,t} + \eta_{i,t}) = R_t^L ((1 - \delta_t^e) \lambda_{i,t} + \eta_{i,t} \kappa_t)$, shows that the loan rate tends to decrease with the expected repayment rate $1 - \delta_t^e$ and with the fraction of firm loans eligible as collateral in open market operations, κ_t , if $\eta_{i,t} > 0$. As long as loans are not fully eligible $\kappa_t < 1$, there will be spread between the policy rate and the loan rate due to a liquidity premium, even if there is no default risk, $\delta_t^e = 0$. Combining the optimality conditions (13), (15), and (16) to

$$R_t E_t [(\lambda_{i,t+1} + \kappa_{t+1}^B \eta_{i,t+1}) / \pi_{t+1}] = E_t [R_{t+1}^m (\lambda_{i,t+1} + \eta_{i,t+1}) / \pi_{t+1}], \quad (21)$$

further shows that households are indifferent between investing in money or investing in government bonds and converting these (partially) into cash in the next period at the rate R_{t+1}^m . For $\kappa_{t+1}^B = 1$, the interest rate on government bonds is closely linked to next period's expected policy rate, i.e. R_t equals $E_t R_{t+1}^m$ up to first order. If not all bonds are accepted in open market operations, $\kappa_t^B < 1$, bonds are less liquid and akin to household debt.

2.4 Public sector

The central bank transfers seigniorage revenues $P_t \tau_t^m$ to the treasury, which issues one-period bonds and pays a transfer $P_t \tau_t$ to households. Government bonds grow at a constant rate, $B_t^T = \Gamma B_{t-1}^T$, where $\Gamma \geq 1$. The treasury's budget constraint reads $(B_t^T / R_t) + P_t \tau_t^m = B_{t-1}^T + P_t \tau_t$, where government bonds B_t^T are either held by households, B_t , or the central bank, B_t^C : $B_t^T = B_t + B_t^C$. Note that B_t^T summarizes the total supply of short-term government bonds, which are typically considered to be eligible for open market operations in normal times. In section 6, we additionally introduce long-term government bonds and examine the price effects of accepting them in open market operations. To avoid fiscal policy effects via tax distortions, we assume that the government has access to lump-sum transfers, which adjust to balance the budget.

The central bank supplies money outright $M_t^H = \int_0^1 M_{i,t}^H di$, and under repos against bonds, $M_t^R = \int_0^1 M_{i,t}^R di$, and loans, $M_t^L = \int_0^1 M_{i,t}^L di$. It transfers its interest earnings to the treasury at end of period, $P_t \tau_t^m = B_t^C - (B_t^C / R_t) + (R_t^m - 1) (M_t^R + M_t^L)$, and reinvests its wealth exclusively in new government bonds, which accords to common central bank

practice (see Meulendyke, 1998). Its budget constraint thus reads $(B_t^C/R_t) - B_{t-1}^C + P_t\tau_t^m = R_t^m (M_t^H - M_{t-1}^H) + (R_t^m - 1)(M_t^R + M_t^L)$. Substituting out central bank transfers, its bond holdings evolve according to

$$B_t^C - B_{t-1}^C = R_t^m (M_t^H - M_{t-1}^H). \quad (22)$$

We mainly consider three central bank instruments. Like in standard models, the central bank controls the policy rate $R_t^m \geq 1$. It can further adjust the fraction of eligible loans $\kappa_t \in [0, 1]$ and the fractions of eligible bonds $\kappa_t^B \in (0, 1]$, which both affect the size and the composition of the central bank balance sheet. We consider two particular balance sheet policies for the subsequent analysis, i.e. quantitative easing and collateral policy, which are defined as follows.¹⁰

- *Quantitative easing* increases money supply by additionally accepting collateral in open market operations. Quantitative easing can be conducted in terms of public debt or corporate debt and is implemented by an increase in κ_t or κ_t^B , respectively.
- *Collateral policy* changes the composition of the central bank's balance sheet without affecting its size. It is implemented by a change in κ_t , accompanied by a neutralizing reduction in κ_t^B .¹¹

The central bank further sets the inflation target π and it controls whether money is supplied in exchange for bonds in repos or outright (while loans are only traded under repos). We assume that it controls the share of bond repos $\Omega \geq 0$, defined as $M_t^R = \Omega M_t^H$. In the sections 3.2 and 4.2, we consider the limiting case $\Omega \rightarrow \infty$ to facilitate the derivation of analytical results.

¹⁰Among the liquidity facilities created by the Bank of England or the US Federal Reserve during 2008-09, many had elements of both quantitative easing and collateral policy, as defined above. Under the Asset Purchase Facility, the Bank of England purchased commercial papers, corporate bonds, and government bonds, which corresponds to our definition of quantitative easing in terms of public debt and corporate debt. The purchases of treasury securities by the US Federal Reserve and the extension of credit to depository institutions through the Term Auction Facility come closest to a policy of quantitative easing by increasing κ_t^B , whereas programs such as the Term Securities Lending Facility and the Commercial Paper Funding Facility, correspond to our definition of collateral policy.

¹¹The sterilization is conducted such that the nominal monetary base ceteris paribus remains unchanged, i.e. $\Delta\kappa_t^B$ and $\Delta\kappa_t$ satisfy $\Delta\kappa_t^B = \frac{l_t}{b_t/\pi_t} \Delta\kappa_t$.

3 Equilibrium properties

In this section, we present some main equilibrium properties. In particular, we show when central bank instruments are in principle effective and how they affect the allocation and equilibrium prices. In a rational expectations (RE) equilibrium all plans and constraints of households and firms are satisfied and consistent with monetary and fiscal policy, for given initial asset endowments. A definition of the RE equilibrium is given in appendix A.

3.1 When are balance sheet policies effective?

The goods market constraint, which reads $P_t c_t \leq M_t^H + M_t^R + M_t^L$ in equilibrium, is relevant for non-neutrality of monetary policy. Only if it is binding, changes in money supply can affect prices and the allocation. Further, the collateral constraint, which in equilibrium reads

$$M_t^H - M_{t-1}^H + M_t^R + M_t^L \leq \kappa_t^B (B_{t-1}/R_t^m) + \kappa_t (L_t/R_t^m), \quad (23)$$

is decisive for the effectiveness of quantitative easing and collateral policy. The instruments κ_t^B and κ_t can affect the equilibrium allocation only by relaxing the collateral constraint (23) (see corollary 1 in appendix A). Hence, if $\eta_t = 0$, such that the collateral constraint (23) is slack, neither quantitative easing nor collateral policy will affect the equilibrium allocation or the associated price system. To see when this is the case, we first use the conditions (11) and (16), which imply $\xi_t c_t^{-\sigma} = \beta E_t \frac{\xi_{t+1} c_{t+1}^{-\sigma}}{\pi_{t+1}} + \psi_t$ and that the multiplier on the goods market constraint ψ_t satisfies in equilibrium

$$\psi_t (c_t^\sigma / \xi_t) = 1 - (1/R_t^{Euler}) \geq 0. \quad (24)$$

The Euler rate R_t^{Euler} is defined as $1/R_t^{Euler} = \beta E_t \frac{\xi_{t+1} c_{t+1}^{-\sigma}}{\xi_t c_t^{-\sigma} \pi_{t+1}}$ and implies that households are indifferent between $1/R_t^{Euler}$ units of the means of payment for consumption goods in period t and one unit in period $t + 1$. Hence, they are willing to pay a maximum price $R_t^{Euler} - 1$ to transform one unit of an illiquid asset, i.e. an asset that is not accepted as a means of payment today and delivers one unit of money tomorrow, into one unit of money today. An Euler rate larger than one therefore indicates a positive valuation for money and implies that households will not hold more money than for consumption expenditures. Then, $\psi_t > 0$ (see 24) and the goods market constraint is binding (see 18). If, however, the

Euler rate equals one the marginal valuation of money today is zero and the goods market constraint is slack, $\psi_t = 0$, such that changes in money supply are neutral. Notably, the Euler rate, rather than the policy rate, determines whether the goods market constraint is binding or not.

We further use that the conditions (11), (13), and (16) imply $\xi_t c_t^{-\sigma} = R_t^m (\lambda_t + \eta_t)$ and $\lambda_t = \beta E_t \frac{\xi_{t+1} c_{t+1}^{-\sigma}}{\pi_{t+1}}$. Eliminating λ_t , shows that the multiplier for the collateral constraint η_t satisfies

$$\eta_t (c_t^\sigma / \xi_t) = (1/R_t^m) - (1/R_t^{Euler}) \geq 0, \quad (25)$$

in equilibrium. Condition (25) shows that when the policy rate is strictly smaller than the Euler rate, $R_t^m < R_t^{Euler}$, the multiplier is positive $\eta_t > 0$ and the collateral constraint is binding (see 19). In this case, the goods market constraint is binding as well, $\psi_t > 0$ (see 24), given that $R_t^m \geq 1$. Households can then get money in exchange for an eligible asset at a price, $R_t^m - 1$, which is below their marginal valuation of money, $R_t^{Euler} - 1$. Hence, they use eligible assets as much as possible to get money in open market operations, such that (23) is binding. In this case, there will be an (il-)liquidity premium on non-eligible assets.

If, however, the policy rate equals the Euler rate, households are indifferent between transforming eligible assets into money or holding them until maturity. Thus, if $R_t^m = R_t^{Euler}$, the collateral constraint is not binding, $\eta_t = 0$ (see 25). In this case, the model reduces to a standard model where the (real) policy rate affects aggregate demand via the consumption Euler equation (see definition 5 in appendix A). Then, the policy instruments κ_t and κ_t^B do neither affect the allocation nor the price system, such that quantitative easing and collateral policy are ineffective, which accords to the conventional view on quantitative easing (see e.g. Eggertsson and Woodford, 2003). These results are summarized in the following proposition.

Proposition 1 *For a given sequence $\{c_t\}_{t=0}^\infty$, the demand for real balances is uniquely determined iff the Euler-rate satisfies $R_t^{Euler} > 1$. Quantitative easing and collateral policy can then affect the equilibrium allocation and the associated price system iff the policy rate is smaller than the Euler-rate, $R_t^{Euler} > R_t^m$.*

Proof. In equilibrium, the cash constraint (3) implies $c_t \leq m_t^H + m_t^R + m_t^L$, which is binding iff $\psi_t > 0$ (see 18). According to (24), this is the case iff $R_t^{Euler} > 1$. Then, the demand

for real balances $m_t^H + m_t^R + m_t^L$ is determined for a given sequence $\{c_t\}_{t=0}^\infty$. The collateral constraint (23) is further binding, iff $\eta_t > 0$ (see 19), which is the case iff $R_t^{Euler} > R_t^m$ (see 25). Then, $[c_t - \kappa_t (l_t/R_t^m)]\pi_t = \kappa_t^B (b_{t-1}/R_t^m) + m_{t-1}^H$ holds and changes in κ_t and κ_t^B can affect consumption and the inflation rate for a given policy rate R_t^m and asset endowments, $b_{t-1} > 0$ and $m_{t-1}^H > 0$. ■

Notably, money demand can be uniquely determined, even if the policy rate is at the ZLB $R_t^m = 1$ as long as $R_t^{Euler} > 1$, which is not possible in standard macroeconomic models where only a single nominal interest rate is considered. For $R_t^m = 1$, both multiplier η_t and ψ_t are identical in our model (see 24 and 25), since eligible assets can costlessly be transformed into money, while quantitative easing and collateral policy can still affect aggregate demand and prices if the Euler rate exceeds one, $R_t^{Euler} > 1$. Given that the latter is endogenous, the effectiveness of balance sheet policies depends on the state of the economy and on monetary policy itself, which will be examined in section 5.

3.2 Are balance sheet policies equivalent to interest rate policy?

In this section, we examine the relevance of the balance sheet policies, in the sense that if the conditions for their effectiveness can be satisfied in equilibrium and that they are not redundant. Specifically, we show that a central bank can induce effectiveness of its balance sheet policies and that they are not equivalent to policy rate adjustments. For this preliminary analysis, we apply a simplified version of the model, where we disregard preference shocks and idiosyncratic productivity shocks, $\xi_t = 1$ and $\omega_{j,t} = 1$. We further assume that prices are perfectly flexible, $\phi = 0$, that the utility function satisfies, $\sigma = 1$, and that money is only supplied under repos, $\Omega \rightarrow \infty$, and not held outright $M_t^H = 0$, such that the central bank holds government bonds only temporarily.¹² A RE equilibrium with a binding collateral constraint ($\eta_t > 0$), which requires the policy rate to be lower than the Euler rate (see 25), can then be reduced to a set of sequences in output, inflation, household bond holdings, and the loan rate (see appendix B).

¹²Given that this is consistent with initial money holdings and initial central bank bond holdings equal to zero, the total stock of government bonds will be held by households, $B_t = B_t^T$.

Definition 1 For $\sigma = 1$, $\xi_t = \omega_{j,t} = 1$, $\phi = 0$, $\Omega \rightarrow \infty$, a RE equilibrium with a binding collateral constraint is a set of sequences $\{y_t, \pi_t, R_t^L, b_t\}_{t=0}^{\infty}$ and $P_0 > 0$ satisfying

$$y_t = [(\mu/\theta) (1/R_t^L)]^{\alpha/(1+\sigma_n)}, \quad (26)$$

$$\frac{1}{R_t^L} = \kappa_t \frac{1}{R_t^m} + (1 - \kappa_t) \beta E_t \frac{y_t}{y_{t+1} \pi_{t+1}}, \quad (27)$$

$$y_t = [\kappa_t^B b_{t-1} \pi_t^{-1} + \kappa_t \mu y_t] / R_t^m, \quad (28)$$

$$b_t = \Gamma b_{t-1} \pi_t^{-1} \forall t \geq 1 \quad \text{and} \quad \Gamma P_0 b_0 = B_{-1}, \quad (29)$$

where $\mu = \frac{\varepsilon-1}{\varepsilon} \alpha < 1$, for a monetary policy setting $1 \leq R_t^m < 1 / [y_t \beta E_t (y_{t+1}^{-1} \pi_{t+1}^{-1})]$, κ_t , and κ_t^B for a given initial stock of bonds $B_{-1} > 0$.

Condition (26) is derived from equating labor supply with labor demand and using the production function as well as goods market clearing. It shows that the costs of loans R_t^L reduce aggregate output. Condition (27), which is based on (11), (13), (14), and (16), shows that the loan price $1/R_t^L$ is a linear combination of the inverses of the policy rate $1/R_t^m$ and of the Euler rate $1/R_t^{Euler} = \beta E_t [y_t / (y_{t+1} \pi_{t+1})]$, where the former is weighted with the fraction of eligible loans κ_t and the latter with $1 - \kappa_t$. If loans are fully eligible, $\kappa_t = 1$, the loan rate equals the policy rate. If they are not eligible, $\kappa_t = 0$, loans cannot be liquidated and the loan rate equals the Euler rate. Combining the cash-in-advance constraint and the collateral constraint leads to (28), which shows that bonds and loans can serve as collateral for money in repos. The evolution of privately held government bonds is further determined by the total supply of bonds, which grow with the rate Γ (see 29).

The policy instruments R_t^m , κ_t , and κ_t^B enter the equilibrium conditions (26)-(29) in different ways. The effects of changes in these instruments are therefore in general not equivalent. An exception is the case where loans are not eligible, $\kappa_t = 0$, for which the instruments R_t^m and κ_t^B become virtually equivalent, since they jointly appear solely in (28). Using fraction of eligible bonds as an instrument will, nonetheless, be useful for the central bank, in particular, when the policy rate is at its ZLB (see section 4.2).

Proposition 2 *The central bank can set its instruments such that a RE equilibrium as given in definition 1 exists. Then, all monetary policy instruments R_t^m , κ_t , and κ_t^B are non-neutral, and permanent changes of R_t^m , κ_t , and the growth rate of κ_t^B exert qualitatively different effects on the equilibrium allocation and the associated price system.*

Proof. See appendix B. ■

The policy instruments R_t^m , κ_t , and κ_t^B lead to non-equivalent macroeconomic effects, which are derived in the proof of proposition 2. An increase in the policy rate R_t^m , for example, leads to a decline in inflation, since a larger amount of nominal bonds are required for a given amount of nominal consumption expenditures. For a positive fraction of eligible loans, $\kappa > 0$, this further leads to a higher loan rate R_t^L (see 27), which raises the marginal costs of production such that total output declines. A change in the growth rate of κ_t^B alters the inflation rate like a change in the money growth rate in a conventional flexible price model. Output then increases with a smaller growth rate of κ_t^B , due to a reduction in the inflation rate, which tends to reduce the Euler rate and the loan rate according to (27). The central bank can further directly induce a lower loan rate by raising the fraction of eligible loans κ_t if the policy rate is smaller than the Euler rate (see 27).

The long-run inflation rate π further depends on the supply of eligible assets, when the collateral constraint is binding. As the stock of government bonds grows at the rate Γ , the price level tends to grow with the same rate when government bonds are eligible (see 29). In order to control the long-run supply of money and thus the long-run inflation rate, which will also be relevant for the subsequent analysis where prices are sticky, the central bank can reduce the fraction of accepted bonds κ_t^B accordingly.

Proposition 3 *Consider a steady state of the RE equilibrium given in definition 1. When the central bank sets R_t^m and κ_t in a stationary way, it can control the steady state inflation rate via the growth rate $\gamma_t = \kappa_t^B / \kappa_{t-1}^B$, and it can implement long-run price stability by setting γ_t equal to the inverse of the growth rate of government bonds Γ^{-1} .*

Proof. See appendix B. ■

According to proposition 3, the central bank can neutralize the impact of the supply of bond on the long-run inflation rate. Specifically, the central bank can implement its inflation target independent of fiscal policy and can ensure long-run price stability by setting $\gamma_t = \Gamma^{-1}$. This result will be repeatedly used in the subsequent sections to simplify the analysis.

4 Limits to conventional monetary policy

Based the non-equivalence of monetary policy instruments (see proposition 2), we now show that quantitative easing and collateral policy can be used by the central bank to implement

preferred allocations in cases where a conventional monetary policy, i.e. a pure interest rate policy, reaches its limits. For this, we consider the more realistic case of imperfectly flexible prices, $\phi > 0$. In the first part, we examine default risk shocks and show that the central bank can fully neutralize these shocks with collateral policy. In the second part, we consider preference shocks and examine the ability of quantitative easing to implement an optimal discretionary policy at the ZLB.

4.1 Collateral policy and default risk shocks

Here, we analyze policy responses to default risk shocks (see 8), i.e., to mean preserving changes in the distribution of idiosyncratic productivity shocks, while we disregard preference shocks $\xi_t = 1$. Default risk shocks alter the expected repayment rate of loans, which will induce lenders to demand a higher loan rate. Given that changes in the loan rate affect the marginal costs of firms (see 6), we examine if monetary policy can offset default risk shocks. It will be shown that (small) shocks to default risk, which tend to distort the allocation, can be completely neutralized by collateral policy.

When the variance of idiosyncratic productivity shocks is positive, a non-zero fraction of intermediate goods producing firms default and lender take the repayment rate into account (see 14). Combining (11), (13), (14), and (16), the equilibrium loan rate then satisfies

$$\frac{1}{(1 - \delta_t^e) R_t^L} = \frac{\kappa_t}{1 - \delta_t^e} \frac{1}{R_t^m} + \left(1 - \frac{\kappa_t}{1 - \delta_t^e}\right) \beta E_t \frac{c_{t+1}^{-\sigma}}{\pi_{t+1} c_t^{-\sigma}}, \quad (30)$$

instead of (28). Under a time varying distribution of idiosyncratic productivity shocks, the expected repayment rate $1 - \delta_t^e$ varies over time (see 8). According to the assumption that changes in the distribution of idiosyncratic productivity shocks are revealed at the beginning of the period, shocks to the expected default rate δ_t^e affect the loan rate in the same period. In particular, the loan rate then tends to increase with expected default rate (see 30).¹³

The right hand side of (30) shows that the central bank can in principle offset default risk shocks, i.e. changes in δ_t^e that are revealed at the beginning of the period, by adjusting its instruments κ_t or R_t^m . Suppose that the central bank only uses the policy rate R_t^m as an instrument and that the fraction of eligible loans is positive $\kappa_t > 0$. It can then offset a

¹³Even when all loans are eligible, $\kappa_t = 1$, the default rate tends to increase the loan rate, $1/R_t^L = (1/R_t^m) - \delta_t^e(1/R_t^{Euler})$ (see 30), given that the central bank is assumed not to take over the risks of default.

decrease in the repayment rate by lowering the policy rate. Alternatively, the central bank can lower the loan rate by accepting (more) loans as collateral in open market operations, i.e. by raising κ_t (see 30), when the policy rate is lower than the Euler rate, which ensures the collateral constraint to be binding (see proposition 1). In both cases, monetary policy would impact on aggregate demand under a binding collateral constraint, either by reducing the price of money or by supplying more money against loans in open market operations.

If the central bank however adjusts the fraction of eligible government bonds κ_t^B in a suited way, it can react to the change in κ_t or in R_t^m such that money supply is held constant. Given that all monetary policy instruments κ_t , κ_t^B , and R_t^m enter the set of equilibrium conditions only via the collateral constraint and the asset pricing conditions for the loan rate and the bond rate (see definition 3 in appendix A), the central bank can completely neutralize a change in default risk by applying collateral policy. Such a policy would only alter the bond price, which has not further impact on the allocation or the price system (see definition 4 in appendix A).

Proposition 4 *Suppose that the collateral constraint is binding. Default risk shocks will in general alter the equilibrium allocation and the price system if the central bank uses only one instrument. If these shocks are sufficiently small, the central bank can fully neutralize these shocks by collateral policy, such that the equilibrium allocation under risk-free loans prevails.*

Proof. See appendix C ■

It should be noted that the success for this policy is limited to small default risk shocks. Specifically, the maximum size of default rate changes that can be neutralized by collateral policy is determined by the liquidity premium and equals $(\bar{R}_t^{Euler} / \bar{R}_t^m) - 1$ (see proof of proposition 4).¹⁴ The central bank can nevertheless reduce the effects of larger default risk shocks by collateral policy, i.e. by reducing the illiquidity premium on loans. Of course, the incentives of borrowers, which can be affected by this policy, have not been taken into account and is beyond the scope of this analysis.

4.2 Quantitative easing at the zero lower bound

In this section, we examine policy options at the ZLB on the policy rate. For this, we consider shocks to the preference parameter ξ_t to facilitate comparisons with other studies

¹⁴According to Longstaff et al. (2005), this roughly equals 50 b.p. (see section 5.1 for further details).

on policy options at the ZLB, like Eggertsson and Woodford (2003), while we disregard idiosyncratic productivity shocks, $\omega_{j,t} = 1$. It will be shown that a central bank, which acts under discretion and takes expectations as given, can implement a stabilization policy by quantitative easing, even when the ZLB is binding. To demonstrate this in a transparent way, we disregard central bank lending against corporate debt, $\kappa_t = 0$, such that only treasury securities are eligible. Changes in the policy rate and in the fraction of eligible bonds will then exert equivalent effects on the equilibrium allocation. Nevertheless, quantitative easing will be useful for the central bank in cases where the policy rate cannot be adjusted due to its ZLB. We assume that government bonds are initially not fully eligible, $\kappa^B < 1$, leaving quantitative easing room for maneuver. Equivalently, we can set κ_t^B equal to one and allow for long-term bonds to be accepted as collateral (see section 6).

To facilitate the derivation of analytical results, we apply a local analysis of the economy at a steady state with a binding collateral constraint. In the steady state, which is described in appendix C, all real variables are constant and are denoted by small letters without a time index. The steady state Euler rate satisfies $R^{Euler} = \pi/\beta$, as usual. The loan rate equals the Euler rate (see 27) and the debt rate R^D as well (see 17 and 20), $R^L = R^D = R^{Euler}$. The central bank sets the policy rate below the Euler rate in the steady state. Hence, there is a liquidity premium, as revealed by steady state version of (25) $\eta = c^{-\sigma}[(1/R^m) - (\pi/\beta)] \geq 0$. We further assume that the central bank inflation target is consistent with long-run price stability, $\pi = 1$, which ensures absence of average price dispersion. Precisely, the central bank implements long-run price stability by long-run adjustments of κ_t^B contingent on the supply of government bonds (see proposition 3). We can therefore disregard a growing supply of bonds $\Gamma > 1$, which can be neutralized by a shrinking fraction of eligible bonds, and we assume – without loss of generality – that $\Gamma = 1$. Given that $\pi = 1$ implies $R^{Euler} > 1$ and that $R^m < R^{Euler}$, the goods market constraint as well as the collateral constraint are binding in the steady state.

In a neighborhood of this steady state, the equilibrium sequences are approximated by the solutions to linearized equilibrium conditions (see appendix C). An equilibrium is then defined as follows, where \hat{a}_t denotes the percent deviation of a generic variable a_t from its steady state value a : $\hat{a}_t = \log(a_t) - \log(a)$.

Definition 2 For $\Omega \rightarrow \infty$, $\Gamma = \pi = \omega_{j,t} = 1$, $R^m \in [1, 1/\beta)$, $\kappa^B < 1$, and $\kappa_t = 0$, a RE equilibrium is a set of convergent sequences $\{\hat{y}_t, \pi_t, \hat{b}_t, \hat{R}_t^L\}_{t=0}^\infty$ satisfying

$$\hat{y}_t = \hat{b}_{t-1} - \hat{\pi}_t - \widehat{R}_t^m + \hat{\kappa}_t^B, \quad (31)$$

$$\sigma \hat{y}_t = \sigma E_t \hat{y}_{t+1} - \widehat{R}_t^L + E_t \hat{\pi}_{t+1} + (1 - \rho_\xi) \hat{\xi}_t, \quad (32)$$

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \chi(\varpi - 1) \hat{y}_t + \chi \widehat{R}_t^L, \quad (33)$$

$$\hat{b}_t = \hat{b}_{t-1} - \hat{\pi}_t, \quad (34)$$

where $\varpi = \frac{1+\sigma_n}{\alpha} + \sigma > 1$ for monetary policy setting $\{\hat{\kappa}_t^B, \widehat{R}_t^m\}_{t=0}^\infty$ and preference shocks satisfying $\hat{\xi}_t = \rho_\xi \hat{\xi}_{t-1} + \varepsilon_t$, $E_{t-1} \varepsilon_t = 0$ and $\rho_\xi \in [0, 1)$, given an initial value $b_{-1} > 0$.

The linear model summarized in definition 2 is similar to a New Keynesian model with the "cost channel" (see Ravenna and Walsh, 2007). In particular, the conditions (32) and (33) resemble standard conditions for aggregate demand and for aggregate supply, where the latter is affected by the costs of loans due to the working capital assumption. The crucial difference to the canonical New Keynesian model is, however, that this is not a single interest rate framework. Specifically, the policy rate, which is not identical to the loan rate (since $\kappa_t = 0$, see 27), neither enters (32) nor (33). Nevertheless, the policy rate affects the equilibrium allocation via the reduced version of the money supply constraint (31). Here, an increase in the policy rate tends – for a given amount of eligible bonds – to reduce the amount of money and thereby aggregate demand. Inflation, output, real bonds, and the loan rate, which equals the Euler rate (see 27 with $\kappa_t = 0$), will simultaneously be determined, given both monetary policy instruments, i.e. the policy rate \widehat{R}_t^m and the fraction of eligible bonds $\hat{\kappa}_t^B$.

We consider the case where the central bank aims at stabilizing the economy, which is hit by preference shocks. We thereby closely follow Ravenna and Walsh's (2007) and apply a linear-quadratic approximation of households' welfare in the neighborhood of an efficient steady state of an isomorphic model, which requires long-run price stability and (unmodelled) fixed fiscal transfers that compensate for average price mark-ups and the average lending rate. Following Ravenna and Walsh (2007), we further consider an efficient output level that would be realized under flexible prices (see 26 for the $\sigma = 1$ case) and a policy rate pegged at one, $y^* = (\alpha/\theta)^{\alpha/(\sigma_n + \sigma\alpha - 1 + \alpha)}$, to define output gaps in the usual way, $x_t = y_t/y^*$, which implies $\hat{x}_t = \hat{y}_t$. We assume that the central bank cannot fully commit to

a once-and-for-all policy plan (and can, thus, not control private sector expectations) and minimizes the following intertemporal loss function in a discretionary way

$$L = E \sum_{t=0}^{\infty} \beta^t \frac{1}{2} (\widehat{\pi}_t^2 + \Lambda \widehat{y}_t^2), \quad (35)$$

where $E \sum_{t=0}^{\infty} \beta^t u_t \approx [u/(1-\beta)] - \Omega L$, $\Omega > 0$ and $\Lambda = (\chi/\varepsilon)(\varpi - 1)$, as shown by Ravenna and Walsh (2007). Minimization of the loss function (35) is subject to the private sector equilibrium conditions (32) and (33), whereas (31) and (34) describe how policy instruments can be adjusted to implement a particular set of sequences $\{\widehat{\pi}_t, \widehat{y}_t\}_{t=0}^{\infty}$. By eliminating the lending rate, (32) and (33) can be combined to a single constraint to the policy problem,

$$\widehat{\pi}_t = (\beta + \chi) E_t \widehat{\pi}_{t+1} + \chi \eta \widehat{y}_t + \chi \sigma E_t \widehat{y}_{t+1} + \chi (1 - \rho_{\xi}) \widehat{\xi}_t, \quad (36)$$

where we defined $\eta = \frac{1+\sigma_n}{\alpha} > 1$ for convenience. Notably, the central bank faces a trade-off between closing the output-gap and stabilizing the price level even if only preference shocks are present, because both constraints (32) and (33) are binding.¹⁵ Minimizing (35) with respect to $\widehat{\pi}_t$ and \widehat{y}_t subject to the constraint (36) in a discretionary way, leads to following first order condition (or the targeting rule)

$$\widehat{\pi}_t = -[\Lambda/(\chi\eta)] \widehat{y}_t. \quad (37)$$

The optimal discretionary plan of the central bank is then a set of sequences $\{\widehat{\pi}_t, \widehat{y}_t\}_{t=0}^{\infty}$ satisfying (36) and (37). When preference shocks $\widehat{\xi}_t$ are sufficiently small, the central bank can implement this optimal plan solely by adjusting the policy rate according to (31) and (34) for a given fraction of eligible assets κ^B . In particular, a decline in ξ_t , which leads to a fall in inflation and an increase in the output-gap under the optimal plan, calls for a reduction in the policy rate R_t^m according to (31).¹⁶ Hence, if the economy is hit by a large contractionary ξ_t -shock, the ZLB can hinder the central bank to implement the optimal plan by policy rate adjustments. In this case, the central bank can still implement the plan via quantitative easing, i.e. by increasing κ_t^B (see 31). This is shown for the parameter restrictions $\sigma < (1 + \sigma_n)/\alpha$ and $\varepsilon > 2$, which ensure equilibrium uniqueness and

¹⁵This effect of the "cost channel" for central banks trade-offs is stressed by Ravenna and Walsh (2007).

¹⁶This is shown in the proof of proposition 5.

unambiguous responses under the optimal plan.

Proposition 5 *Consider the model given in definition 2 for $\sigma < (1 + \sigma_n) / \alpha$ and $\varepsilon > 2$. The optimal policy plan under discretion is uniquely determined and implies a binding ZLB under pure interest rate policy for large contractionary shocks to $\hat{\xi}_t$. The central bank can then still implement its optimal policy plan by quantitative easing, i.e. by increasing κ_t^B .*

Proof. See appendix C. ■

Proposition 5 implies that quantitative easing can increase the range, i.e. the set of states, for which the central bank can implement its optimal plan. Quantitative easing, however, also reaches its limits either when κ_t^B equals one or when the collateral constraint becomes slack (see proposition 1). Easing money supply tends to increase current aggregate demand, which implies a decreasing Euler rate. When the latter falls to a level that equals the policy rate, collateral is abundant and quantitative easing becomes neutral. The following proposition summarizes the effects of quantitative easing on the loan rate and on the bond rate for the case where the policy rate is pegged at the ZLB, $R_t^m = 1$.

Proposition 6 *Consider the model given in definition 2 for $\sigma < (1 + \sigma_n) / \alpha$. When the policy rate is at the ZLB, the equilibrium is uniquely determined. Quantitative easing in terms of treasuries, i.e. an exogenous and autocorrelated increase κ_t^B , then leads to a decline in the bond rate as well as in the loan rate.*

Proof. See appendix C. ■

According to proposition 6, quantitative easing in terms of government bonds does not only lead to a lower bond rate but also to a lower loan rate, since it reduces the marginal valuation of liquid assets. Given that the Euler rate (which equals the loan rate for $\kappa_t = 0$) falls, the collateral constraint can get slack before κ_t^B reaches one. For a quantitative analysis of these limits to quantitative easing (and collateral policy) we extend and calibrate the model in the subsequent section. There, we will also introduce large liquidity demand shocks, which we view as more relevant for the application of quantitative easing than the preference shocks (which have been examined in this section), given that a fall in ξ_t tends to reduce the liquidity premium for a given monetary policy (see proof of proposition 5).

5 Limits to quantitative easing and collateral policy

It has been shown in the previous section that quantitative easing and collateral policy can be useful additional tools for the central bank, when pure interest rate policy has reached its limits. Here, we explore how the effectiveness of quantitative easing and collateral policy is bounded, in particular, by the scarcity of collateral in open market operations. We apply a numerical analysis, where we disregard preference and productivity shocks ($\xi_t = \omega_{j,t} = 1$) that have been examined in the previous section for demonstrative purposes, and analyze exogenous central bank actions and monetary policy responses to a large liquidity demand shocks at the ZLB, where the liquidity premium between eligible and non-eligible assets can be particularly large. To facilitate the calibration of the model, we consider investments in physical capital and calibrate money demand for consumption and investments purposes.

5.1 Extension and calibration

We extend the model presented in section 2 by introducing physical capital. Households own the stock of capital, $k_t = \int k_{i,t} di$, and rent it to firms at the rate r_t^k . The capital stock of household i evolves according to $k_{i,t} = (1 - \delta) k_{i,t-1} + x_{i,t} S(x_{i,t}/x_{i,t-1})$, where $\delta \in (0, 1)$ denotes the depreciation rate and x_t investment expenditures. The function $S(x_t/x_{t-1}) = 1 - \frac{\vartheta}{2} (x_t/x_{t-1} - 1)^2$, with $\vartheta > 0$, specifies adjustment costs of capital. We assume that households rely on cash for purchases of investment goods up to an exogenous fraction $\omega_t \geq 0$. We further introduce $\nu > 0$, which describes the fraction of purchases of consumption goods that require cash. Thus, the cash constraint (3) is replaced by

$$vP_t c_{i,t} + \omega_t P_t x_{i,t} \leq I_{i,t} + M_{i,t-1}^h - (L_{i,t}/R_t^L) + P_t w_t n_{i,t}, \quad (38)$$

such that money demand of households is increasing in ν and ω_t . These parameters allow relating expenditures to the monetary base in accordance with empirical counterparts. In section 5.3, we further analyze a shock to ω_t , which captures increased liquidity demand for purchases of investment goods. Intermediate goods producing firms rent capital from households. Firm j produces with technology $IO_{j,t} = n_{j,t}^\alpha k_{j,t-1}^{1-\alpha}$ and pays the rental rate on capital after their goods are sold, such that the constraint (1) of firm j is unchanged. The first order conditions for $R_t^L > 1$ are given by $mc_{j,t} \alpha (n_{j,t}/k_{j,t-1})^{\alpha-1} = w_t R_t^L$,

$mc_{j,t}(1-\alpha)(n_{j,t}/k_{j,t-1})^\alpha = r_t^k$, and (7). To calibrate the model in a consistent way, we further introduce government spending so that goods market clearing requires $y_t = c_t + x_t + g_t$. The full set of equilibrium conditions can be found in Appendix D.

For the numerical analysis, we use standard parameter values as far as possible. The parameters of the utility function equal $\sigma = 2$ and $\sigma_n = 1$, the labor share equals $\alpha = 0.66$, the steady state markup $1/mc = 11\%$ ($\varepsilon = 10$), steady state working time $n = 1/3$, the fraction of non-optimally price adjusting firms $\phi = 0.75$, the share of government spending $g_t/y_t = 0.19$, and the value for the adjustment cost parameter $\vartheta = 2.5$ (see e.g. Christiano et al., 2005). The steady state values of ω_t and v are calibrated to the observed ratios $Px/M0 = 1.15$ and $Pc/M0 = 2.71$, and the depreciation rate is set to $\delta = 0.023$ to match the observed ratio of consumption to investment, $c/x = 2.36$.¹⁷ The long-run policy rate is set at $R^m = 1.0133$ (or 5.41% in terms of annualized rates), which equals the average of the federal funds rate for the QIV/1982–QIII/2008 (to exclude the pre-Volcker period), and the inflation target is set at its average value $\pi = 1.00647$ (or 2.61% at an annual rate) for the same period (where we used data from the U.S. Bureau of Economic Analysis).

The policy rate is either pegged at the ZLB, $R^m = 1$, or set according to a simple Taylor rule $\widehat{R}_t^m = \rho_\pi \widehat{\pi}_t + \rho_y \widehat{y}_t$ with $\rho_\pi = 1.5$ and $\rho_y = 0.5^{1/4}$ (in section 5.3), where both policies are consistent with equilibrium determinacy (see proposition 4 in the previous version of this paper, Hörmann and Schabert, 2010). We consider the case where loans are not eligible in the steady state, i.e. $\kappa = 0$, which accords to the Fed's pre-crisis "Treasury only" regime. In contrast, government debt is fully eligible, $\kappa^B = 1$, where we assume – without explicitly specifying – that long-run growth in government bonds and in κ_t^B is consistent with the long-run inflation rate (see proposition 3). We further set the repo share to $\Omega = 1.5$ to match the observed ratio between total reserves and reserves supplied under repurchase agreements, which was almost constant in the 2000s before the crisis.¹⁸

The spread between the policy rate and the loan rate, which equals the Euler rate

¹⁷Data on (nominal) consumption, investment, government spending and Gross Domestic Product are taken from NIPA Table 1.15, where durable consumption goods are included into investment, and data on the monetary base are from the Federal Reserve Board's H3 Statistical Release. All data are seasonally adjusted and refer to averages for 25 years over the period QI/1981–QIV/2006.

¹⁸See Federal Reserve Bank of New York, Domestic Open Market Operations, various issues, and FRED database.

$R^L = R^{Euler} = \pi/\beta$ in a steady state with $\kappa = 0$, matters for the size of monetary policy effects. According to the literature on the "corporate bond credit spread puzzle" (see Christensen, 2008), the yield spread between treasury securities and corporate bonds can be attributed to a default risk component and a liquidity component (see e.g. Longstaff et al., 2005). Given that we disregard default risk shocks in this section, we focus on the liquidity component. Specifically, we refer to Longstaff et al.'s (2005) conservative estimate of the liquidity premium for the spread between corporate bonds and treasury securities, who report that, for AAA rated corporate bonds, 51% of the credit spread can be explained by default risk.¹⁹ Given that the average short-term spread among AAA corporate bonds equals 104 basis points at annualized rates (see Longstaff et al., 2005), we consider a liquidity premium of $(1 + 49\% \cdot 0.0104)^{1/4} - 1 = 13$ basis points (in terms of quarterly rates), which implies the discount factor to equal $\beta = \frac{\pi}{R^m + 13 \cdot 10^{-4}} = 0.992$ and a (quarterly) loan rate of 1.0146. Given that the central bank sets its targets according to $\pi > \beta$ and $1 \leq R^m \in [1, \pi/\beta)$, the collateral constraint and the cash constraint are binding in the steady state (see 24 and 25).

5.2 Maximum effects of balance sheet policies

The calibrated model is solved by applying a first-order approximation at the deterministic steady state. We compute impulse responses to unexpected and autocorrelated increases in the fraction of eligible loans, κ_t . Most variables are given in terms of percentage deviations from steady state, $\hat{a}_t = \log(a_t) - \log(\bar{a})$, as defined earlier. Further, we consider deviations expressed in percentage points, $\tilde{a}_t = 100(a_t - \bar{a})$, for κ_t , κ_t^B , ω_t , and interest rates, e.g., $\tilde{R}_t^m = 1$ denotes an increase in the policy rate by 100 basis points.

The policy rate is held at its ZLB, where quantitative easing can be effective as long as the collateral constraint is binding, i.e. the Euler rate exceeds one (see 25). Easing money supply will however lead to the point where households' and firms' cash demand will be satiated such that collateral becomes abundant. Specifically, the multiplier on the collateral constraint η_t (see 24 and 25) has to satisfy

$$\eta_t = (c_t^{-\sigma}/R_t^m) - \beta E_t c_{t+1}^{-\sigma} \pi_{t+1}^{-1} > 0, \quad (39)$$

¹⁹Collin-Dufresne et al. (2001) attribute only 25% of the variation in credit spreads to credit default risk.

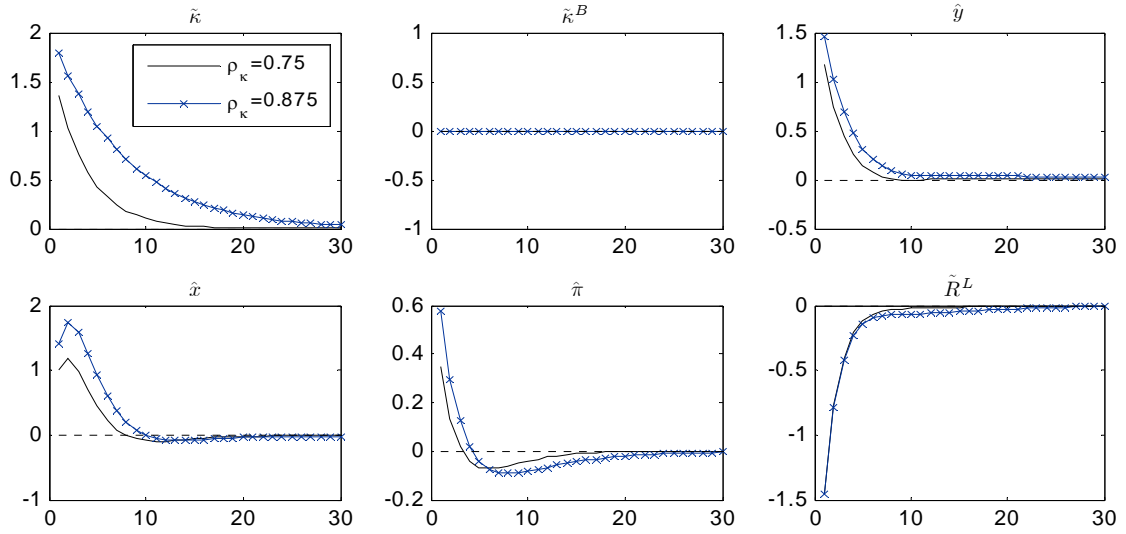


Figure 1: Maximum effects of quantitative easing

for quantitative easing (and collateral policy) to be effective. Thus, the range over which the collateral constraint is binding is particularly large at its ZLB, $R_t^m = 1$ (see 39). A closer look at (39) shows that the multiplier approaches zero if an increase in current consumption is sufficiently large and not too persistent. Beyond this point, quantitative easing and collateral policy are neutral.

We first consider isolated effects of quantitative easing in terms of corporate debt, i.e. an increase in κ_t , where $\hat{\kappa}_t$ exhibits an autocorrelation of $\rho_\kappa = 0.75$ (0.875) which accords to an expected duration of the policy of one year (two years). Figure 1 shows the impulse response to the maximum quantitative easing policy, which is defined as a quantitative easing policy which just lets the collateral constraint bind, $\eta_t > 0$. The maximum quantitative easing policy with $\rho_\kappa = 0.75$ (see solid line) implies an increase of $\Delta\kappa_t = 0.0136$ (or $\tilde{\kappa}_t = 1.36$). This induces the loan rate to fall to its ZLB and a rise in output by 1.18%, while inflation increases by 35 basis points. When the policy is more persistent, $\rho_\kappa = 0.875$ (see starred line) a larger intervention is possible according to (39). Quantitative easing can then be conducted at a larger scale ($\Delta\kappa_t = 0.0179$ or $\tilde{\kappa}_t = 1.79$) such that output rises by 1.46% and inflation by 58 basis points. Hence, supplying additional money against 1% of all loans in open market operations raises output on impact by 0.87% (0.81%) in case of the less (more)

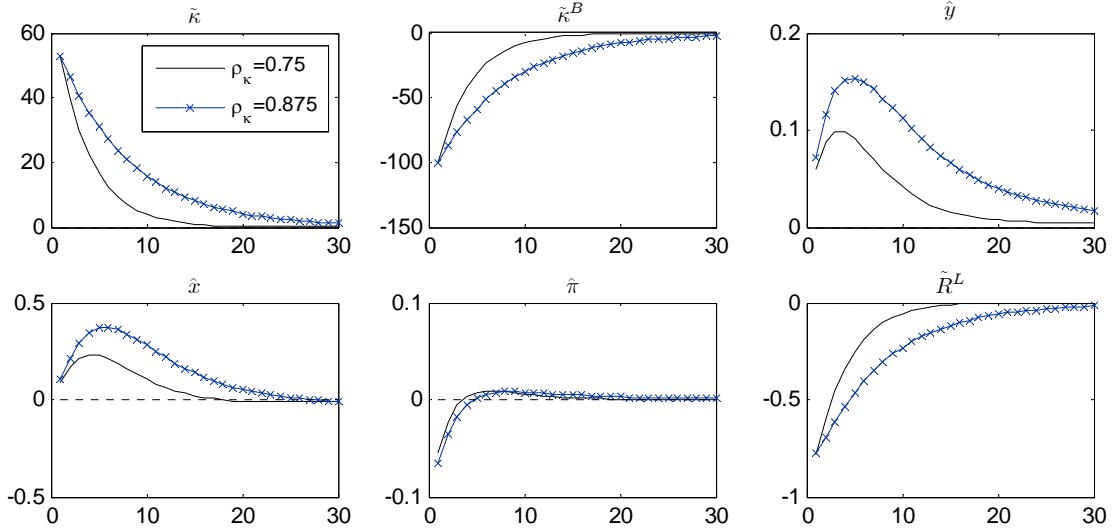


Figure 2: Maximum effects of collateral policy

persistent intervention. Compared to conventional monetary policy, the maximum output effect of quantitative easing (for $\rho_{\kappa} = 0.875$) corresponds to the output effect of a reduction in the policy rate of about 80 basis points (when it is not at the ZLB).

Next, we examine isolated effects of collateral policy, which is limited not only by (39) but also by the size of the central bank's balance sheet and the availability of collateral, i.e. by $\kappa_t^B \geq 0$ and $\kappa_t \leq 1$. For our calibration, these restrictions are more severe than (39) and a maximum collateral policy is given by $\Delta\kappa_t^B = -1$ and $\Delta\kappa_t = -\frac{b/\pi}{T}\Delta\kappa_t^B = 0.53$ (or $\tilde{\kappa}_t = 53$ and $\tilde{\kappa}_t^B = -100$). Hence, more than the half of all loans are exchanged against government bonds. As corporate debt is now eligible, R_t^L declines, so that marginal cost and inflation fall (see Figure 2). Reduced goods prices and increased real balances allow households to increase consumption and investments. For an autocorrelation of 0.75 (see solid line), output exhibits a peak response of 0.1% in the third quarter and inflation declines by five basis points. For a more persistent collateral policy, $\rho_{\kappa} = 0.875$ (see starred line), the loan rate declines more persistently, which leads to a more pronounced decline in inflation and an increase in output by a maximum of 0.15%. Hence, even a maximum collateral policy has relatively small effects, which implies that the effects of quantitative easing in terms of treasuries and of quantitative easing in terms of loans (see figure 1) are virtually identical.

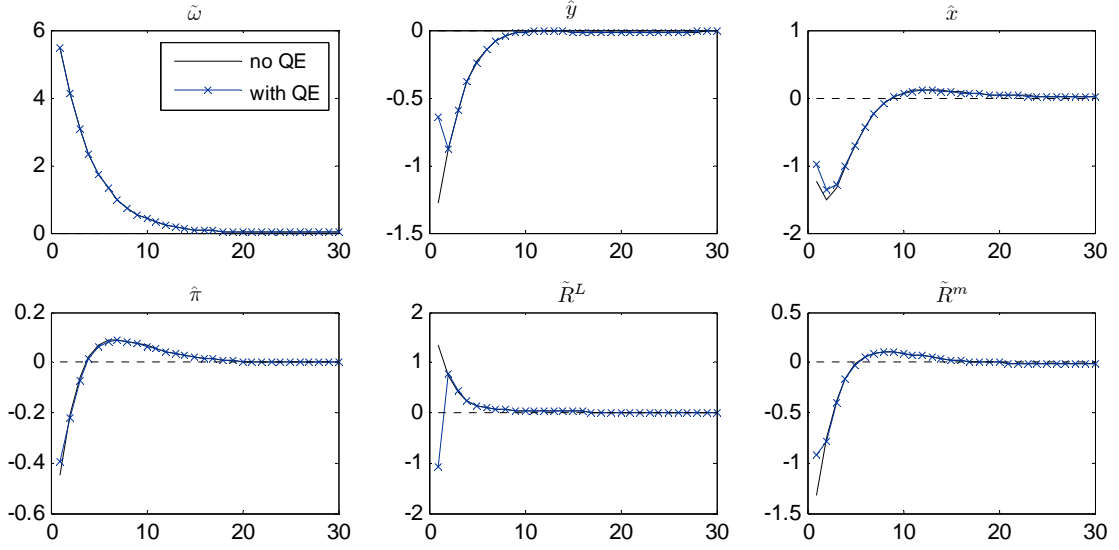


Figure 3: Liquidity demand shock with and without quantitative easing

5.3 Quantitative easing under liquidity demand shocks

We now consider a liquidity demand shock, i.e. an unexpected increase in the fraction ω_t of investment goods that have to be purchased with cash (38). This shock, which implies that less investments can be financed on credit, can for example be interpreted as an increase in financial distress that lowers the extent to which investment goods can be pledged as collateral. We assume an AR(1) process for ω_t with an autocorrelation of 0.75. The policy rate is initially at its steady state value $R^m = 1.0133$ and otherwise governed by the Taylor rule. We consider a shock that drives the policy rate to the ZLB in the impact period, which requires $\Delta\omega_t = 0.0549$ (or $\tilde{w}_t = 5.49$). The solid line in Figure 3 shows the impulse responses to this shock without quantitative easing. Investment and consumption fall, so that output declines by 1.29% despite the endogenous reduction of the policy rate. The inflation rate falls, while the spread between the policy rate and the loan rate increases.

The starred line shows the responses for the case where the central bank applies a maximum quantitative easing policy in terms of corporate debt at the ZLB, which is again determined by the multiplier on the collateral constraint (see 39). Though, the central bank can accommodate the increase in money demand by raising κ_t , it can not completely neutralize the liquidity demand, even when the collateral constraint is binding. The reason

is that an increase in ω_t does not only affect money demand via (38), but also tends to raise the required return on investments in physical capital, given that investments – as a "cash good" – get more costly. This can be seen from the first order condition for investments in physical capital (see definition 6 in appendix D), which for the simplified case where $S_t = 1$ and $S'_t = 0$ can be written as $\lambda_t + \omega_t \psi_t = \beta E_t [\lambda_{t+1} r_{t+1}^k + (1 - \delta) (\lambda_{t+1} + \omega_{t+1} \psi_{t+1})]$, where the RHS measures the marginal costs of investment in physical capital.

Quantitative easing is only conducted in the first period, since the policy rate increases afterwards. For this policy, the contractionary effects are mitigated and output falls by only 0.65%. Inflation is 5 basis points larger than without intervention, falling only by 40 basis points. Given that the impact output contraction is reduced by 50%, our analysis shows that the central bank can substantially reduce output effects of liquidity demand shocks via a quantitative easing policy, which helps escaping from the ZLB.

6 Long-term bonds

In this section, we extend the set of instruments by introducing long-term public debt, which can also be eligible for open market operations. Specifically, we assume that the government additionally issues two-period bonds (at a growth rate $\Gamma^L > 0$), which can be traded before maturity. In period t , the central bank can accept different types of government bonds as collateral: one-period bonds issued in $t - 1$ and two-period bonds, which are either issued in $t - 2$ with maturity date t , $B_{t-2,t}$, or issued in $t - 1$ with maturity date $t + 1$, $B_{t-1,t+1}$. It can therefore conduct balance sheet policies in terms of loans, short-term bonds, or long-term bonds. In particular, accepting long-term bonds as collateral allows to extend a quantitative easing policy in terms of treasury securities, even if all short-term bonds are already accepted as collateral, $\kappa_t^B = 1$. Let $q_{t-1,t}$ be the period t price of long-term debt issued in $t - 1$ and maturing in $t + 1$. The collateral constraint is then given by

$$I_t \leq \kappa_t^B (B_{t-1}/R_t^m) + \kappa_t^{LB} [(q_{t-1,t} B_{t-1,t+1} + B_{t-2,t})/R_t^m] + \kappa_t (L_t/R_t^m),$$

instead of (2), where κ_t^{LB} denotes the fraction of maturing and outstanding eligible two-period bonds, which are treated in an identical way by the central bank. Given that lump-sum transfers are still available for the government, accepting long-term bonds are equivalent

with regard to the allocation and the price system to an increase in the fraction of eligible short-term bonds κ_t^B (see appendix E), except for the impact on the particular bond rates. To examine the price of two-period bonds, we introduce two-period bonds into the household budget constraint for period t : $(B_{t,t+2}/R_{t,t+2}) - B_{t-2,t} + q_{t-1,t}B'_{t-1,t+1} - q_{t-1,t}B_{t-1,t+1} \leq \Upsilon_t$, where Υ_t denotes the RHS of (10) and $B'_{t-1,t+1}$ denotes the end-of-period t stock of two-period bonds issued in $t-1$ and held until $t+1$. The period t first order conditions for two-period bonds issued in t and held until $t+1$, $B_{t,t+2}$, is given by $(1/R_{t,t+2})(\lambda_t/P_t) = \beta E_t \left[\left((\lambda_{t+1}/P_{t+1}) + (\kappa_{t+1}^{LB} \eta_{t+1}/P_{t+1}) \right) q_{t,t+1} \right]$ and for debt issued in $t-1$ and held until maturity $t+1$, $B'_{t-1,t+1}$, is $q_{t-1,t}(\lambda_t/P_t) = \beta E_t \left[(\lambda_{t+1}/P_{t+1}) + (\kappa_{t+1}^{LB} \eta_{t+1}/P_{t+1}) \right]$. Using the latter for period $t+1$ and substituting out $q_{t,t+1}$, we get

$$(1/R_{t,t+2}) = \beta^2 E_t \left[\left((\lambda_{t+1} + \kappa_{t+1}^{LB} \eta_{t+1}) / \lambda_t \right) \cdot \left((\lambda_{t+2} + \kappa_{t+2}^{LB} \eta_{t+2}) / \lambda_{t+1} \right) / \pi_{t,t+2} \right], \quad (40)$$

where $\pi_{t,t+2} = \pi_{t+1}\pi_{t+2}$. Condition (40) shows that the price of two-period bonds $1/R_{t,t+2}$ tends to increase with the fraction of eligible two-period bonds in period $t+1$ and $t+2$. The effects of a collateral policy in terms of long-term and short-term government bonds are then obvious: Given that lump-sum transfers/taxes are available, additionally issuing two-period bonds does not affect the equilibrium allocation when they are not eligible, $\kappa_t^{LB} = 0$. If the central bank accepts a fraction of two-period bonds as collateral $\kappa_t^{LB} > 0$, the equilibrium allocation can be affected due to an eased money supply. These effects are, however, neutralized under a collateral policy, i.e. if the central bank keeps money supply constant by a reduction in the fraction of eligible one-period bonds. Then, all terms on the RHS of (40), except of the κ_t^{LB} s, are unchanged, such that the price of two-period bonds unambiguously increases if the collateral policy is expected to last in $t+1$ and $t+2$ and if collateral is scarce. Neglecting second and higher order terms, the maximum effect of collateral policy on the annualized yield of two-period bonds approximately equals the mean of the liquidity premia in the two subsequent periods.²⁰ Hence, this policy can substantially

²⁰When two-period bonds are not accepted as collateral $\kappa_{t+i}^{LB} = 0 \forall i \geq 0$, (40) leads to a standard pricing condition for two-period debt: $1/R_{t,t+2} = \beta^2 E_t [(\lambda_{t+2}/\lambda_t) / \pi_{t,t+2}] = E_t [\varphi_{t,t+1} \varphi_{t+1,t+2}]$, which can be written as $(1/R_{t,t+2})|_{\kappa_{t+1}^{LB}=\kappa_{t+2}^{LB}=0} = (1/R_t^D) \cdot E_t (1/R_{t+1}^D) + \mathcal{O}^2$, where \mathcal{O}^2 collects terms of higher than first order. If two-period bonds are fully eligible, $\kappa_{t+i}^{LB} = 1 \forall i \geq 0$, the price $1/R_{t,t+2}$ satisfies $(1/R_{t,t+2})|_{\kappa_{t+1}^{LB}=\kappa_{t+2}^{LB}=1} = E_t (1/R_{t+1}^m) \cdot E_t (1/R_{t+2}^m) + \mathcal{O}^2$, where we used (13) and (16).

reduce the term premium, i.e. the spread between long-term and short-term treasury yields, while it does neither affect the allocation nor interest rates on other assets.

7 Conclusion

Balance sheet policies have recently been introduced by several central banks while policy rates were set at its ZLB. At the same time, conventional macroeconomic analysis of monetary policy predicts that balance sheet policies at the ZLB are irrelevant as long as they do not affect expectations about future policies. In this paper, we augment a standard monetary model to be applicable for the analysis of the effects as well as the limitations of balance sheet policies. We show that they can be non-neutral, even when expectations are unchanged and financial intermediation is not disrupted. As a main principle, we show that this relies on the scarcity of eligible securities, which is reflected by a liquidity premium.

We show that quantitative easing (i.e. increasing the balance sheet's size) and collateral policy (i.e. changing the balance sheet's composition) are not equivalent to a policy rate adjustment and are particularly useful when pure interest rate policy reaches its limits: Collateral policy can neutralize an increase in firms' borrowing costs, while quantitative easing allows to implement optimal policy when the policy rate is at the ZLB. A numerical analysis shows that a maximum quantitative easing policy can stimulate output at the ZLB like a reduction of the policy rate by 80 basis points (when it is not at the ZLB), and it can mitigate the output contraction of large liquidity demand shocks by 50%, which suffices to escape from the ZLB. Finally, we demonstrate that purchases of long-term treasuries are equivalent to purchases of short-term treasuries, except for the effects on long-term treasury yields.

Overall, the paper provides a rationale for the introduction of liquidity providing lending facilities by the BoE or the Fed in 2008-2010 and by the BoJ in 2001-2006. The results imply that balance sheet policies are particularly helpful in response to a surge in liquidity demand, regardless whether it originates in a shortage of market liquidity or a hoarding of reserves.

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Appendix

A Rational expectations equilibrium

In equilibrium, there will be no arbitrage opportunities and markets clear, $n_t = \int_0^1 n_{jt} dj = \int_0^1 n_{it} di$, $y_t = \int_0^1 y_{jt} dj = \int_0^1 c_{it} di = c_t$, and $\int_0^1 L_{i,t} di = \int_0^1 L_{j,t} dj$, and aggregate asset holdings satisfy $\forall t \geq 0$: $\int_0^1 D_{i,t} di = 0$, $\int_0^1 M_{i,t}^H di = M_t^H$, $\int_0^1 M_{i,t}^R di = M_t^R$, $\int_0^1 M_{i,t}^L di = M_t^L$, $\int_0^1 B_{i,t} di = B_t$, $\int_0^1 I_{i,t} di = I_t = M_t^H - M_{t-1}^H + M_t^R + M_t^L$, and $B_t^T = B_t + B_t^C$. The latter and (22) imply $B_t - B_{t-1} = B_t^T - B_{t-1}^T - R_t^m (M_t^H - M_{t-1}^H)$. A RE equilibrium can then be defined as.

Definition 3 A RE equilibrium is given by a set of sequences $\{c_t, y_t, n_t, \lambda_t, m_t^R, m_t^H, m_t^L, b_t, b_t^T, l_t, w_t, mc_t, \tilde{Z}_t, s_t, \pi_t, R_t, R_t^{Euler}, R_t^L\}_{t=0}^\infty$ satisfying

$$\theta n_t^{\sigma_n} = w_t c_t^{-\sigma}, \quad (41)$$

$$[(1 - \delta_t^e) R_t^L]^{-1} = \kappa_t [(1 - \delta_t^e) R_t^m]^{-1} + [1 - \kappa_t / (1 - \delta_t^e)] \beta c_t^\sigma E_t [\xi_{t+1} c_{t+1}^{-\sigma} / (\xi_t \pi_{t+1})], \quad (42)$$

$$\lambda_t = \beta E_t [\xi_{t+1} c_{t+1}^{-\sigma} / \pi_{t+1}], \quad (43)$$

$$\lambda_t = \beta R_t E_t [(\lambda_{t+1} (1 - \kappa_{t+1}^B) + \kappa_{t+1}^B \xi_{t+1} c_{t+1}^{-\sigma} / R_{t+1}^m) \pi_{t+1}^{-1}], \quad (44)$$

$$1/R_t^{Euler} = \beta E_t [(\xi_{t+1} c_{t+1}^{-\sigma}) / (\xi_t c_t^{-\sigma} \pi_{t+1})], \quad (45)$$

$$c_t = m_t^H + m_t^R + m_t^L, \text{ if } R_t^{Euler} > 1, \quad (46)$$

$$\text{or } c_t \leq m_t^H + m_t^R + m_t^L, \text{ if } R_t^{Euler} = 1,$$

$$\kappa_t^B b_{t-1} / (R_t^m \pi_t) = m_t^H - m_{t-1}^H \pi_t^{-1} + m_t^R, \text{ if } R_t^{Euler} > R_t^m, \quad (47)$$

$$\text{or } \kappa_t^B b_{t-1} / (R_t^m \pi_t) \geq m_t^H - m_{t-1}^H \pi_t^{-1} + m_t^R, \text{ if } R_t^{Euler} = R_t^m,$$

$$\kappa_t l_t / R_t^m = m_t^L \text{ if } R_t^{Euler} > R_t^m \text{ or } \kappa_t l_t / R_t^m \geq m_t^L \text{ if } R_t^{Euler} = R_t^m, \quad (48)$$

$$b_t - b_{t-1} \pi_t^{-1} = (\Gamma - 1) b_{t-1}^T \pi_t^{-1} - R_t^m (m_t^H - m_{t-1}^H \pi_t^{-1}), \quad (49)$$

$$mc_t \alpha n_t^{\alpha-1} = w_t R_t^L, \quad (50)$$

$$l_t / R_t^L = w_t n_t, \quad (51)$$

$$\tilde{Z}_t (\varepsilon - 1) / \varepsilon = Z_t^1 / Z_t^2, \quad (52)$$

$$\text{where } Z_t^1 = c_t^{-\sigma} y_t mc_t + \phi \beta E_t \pi_{t+1}^\varepsilon Z_{t+1}^1 \text{ and } Z_t^2 = c_t^{-\sigma} y_t + \phi \beta E_t \pi_{t+1}^{\varepsilon-1} Z_{t+1}^2,$$

$$1 = (1 - \phi) (\tilde{Z}_t)^{1-\varepsilon} + \phi \pi_t^{\varepsilon-1}, \quad (53)$$

$$m_t^R = \Omega_t m_t^H, \quad (54)$$

$$b_t^T = \Gamma b_{t-1}^T / \pi_t, \quad (55)$$

$$y_t = n_t^\alpha / s_t, \quad (56)$$

$$y_t = c_t, \quad (57)$$

$$s_t = (1 - \phi) \tilde{Z}_t^{-\varepsilon} + \phi s_{t-1} \pi_t^\varepsilon, \quad (58)$$

the transversality conditions, a monetary policy setting $\{R_t^m \geq 1, \kappa_t^B \in (0, 1], \kappa_t \in [0, 1]\}_{t=0}^\infty$, $\Omega_t > 0$ and $\pi \geq \beta$, and a fiscal policy setting $\Gamma \geq 1$, for a given sequence $\{\xi_t, \delta_t^e\}_{t=0}^\infty$, initial values $M_{-1}^H > 0$, $B_{-1} > 0$, $B_{-1}^T > 0$, and $s_{-1} \geq 1$.

Note that the price of government bonds enters the set of equilibrium conditions in definition 3 only via (44) and is irrelevant for the equilibrium allocation. We therefore redefine the RE equilibrium for the case where the cash and collateral constraints are binding, which requires that the Euler rate R_t^{Euler} exceeds the policy rate $R_t^m \geq 1$ (see 46, 47, and 48).

Definition 4 *A RE equilibrium with binding cash and collateral constraints is given by a set of sequences $\{c_t, y_t, n_t, m_t^R, m_t^H, m_t^L, b_t, b_t^T, l_t, w_t, mc_t, \tilde{Z}_t, s_t, \pi_t, R_t^L\}_{t=0}^\infty$ satisfying (41), (42), (49)-(58), $c_t = m_t^H + m_t^R + m_t^L$,*

$$\kappa_t^B b_{t-1} / (R_t^m \pi_t) = m_t^H - m_{t-1}^H \pi_t^{-1} + m_t^R, \quad (59)$$

$$\kappa_t l_t / R_t^m = m_t^L, \quad (60)$$

the transversality conditions, a monetary policy setting $\{R_t^m, \kappa_t^B, \kappa_t\}_{t=0}^\infty$, where

$$R_t^m \in [1, 1 / \{ \beta E_t [\xi_{t+1} c_{t+1}^{-\sigma} / (\xi_t c_t^{-\sigma} \pi_{t+1})] \}],$$

$\Omega_t > 0$ and $\pi \geq \beta$, and a fiscal policy setting $\Gamma \geq 1$, for a given sequence $\{\xi_t, \delta_t^e\}_{t=0}^\infty$, initial values $M_{-1}^H > 0$, $B_{-1} > 0$, $B_{-1}^T > 0$, and $s_{-1} \geq 1$.

When money supply is not effectively rationed due to a non-binding collateral constraint and there are no idiosyncratic productivity shocks, the model reduces to a standard sticky price model and a RE equilibrium can be reduced and redefined as follows.

Definition 5 *A RE equilibrium for $\eta_t = 0$ and $\omega_{j,t} = 1$ is a set of sequences $\{c_t, y_t, n_t, l_t, w_t, mc_t, \tilde{Z}_t, s_t, \pi_t, R_t^L\}_{t=0}^\infty$ satisfying*

$$\mu_t \theta n_t^{\sigma_n} = w_t c_t^{-\sigma}, \quad (61)$$

$$\xi_t c_t^{-\sigma} = \beta R_t^m E_t [\xi_{t+1} c_{t+1}^{-\sigma} \pi_{t+1}^{-1}], \quad (62)$$

$$R_t^L = R_t^m, \quad (63)$$

(50)-(53), (56)-(58), the transversality conditions, a monetary policy setting $\{R_t^m \geq 1\}_{t=0}^\infty$ and the inflation target $\pi \geq \beta$, for a given sequence $\{\xi_t\}_{t=0}^\infty$ and $s_{-1} \geq 1$.

The policy instruments κ_t and κ_t^B enter the set of equilibrium conditions listed in definition 4 only via (42), (59), and (60), while they do not appear in definition 5. Using definition 5, which shows that $R_t^L = R_t^m$ if $R_t^m = R_t^{Euler}$ (45) and (47), we can summarize some equilibrium properties as follows.

Corollary 1 *The policy instruments κ_t and κ_t^B can affect the equilibrium allocation and the associated price system only via the collateral constraints (47) and (48) and, in case of κ_t , by altering the firms' costs of borrowing, R_t^L . Both instruments are neutral with regard to the equilibrium allocation and the price system if the constraints (47) and (48) are slack.*

B Appendix to section 3

Consider the model given in definition 4 for the simplifying case where $\xi_t = \omega_{j,t} = 1$, $\phi = 0$, $\sigma = 1$, and $\Omega \rightarrow \infty$, such that $M^H \rightarrow 0$. Then, a RE equilibrium given in definition 4 can be redefined as a set of sequences $\{c_t, y_t, n_t, b_t, l_t, w_t, \pi_t, R_t^L\}_{t=0}^\infty$ satisfying (51), (57),

$$\frac{1}{R_t^L} = \frac{\kappa_t}{R_t^m} + (1 - \kappa_t)\beta E_t \frac{c_t}{c_{t+1}\pi_{t+1}}, \quad c_t = \kappa_t^B b_{t-1} / (R_t^m \pi_t) + \kappa_t l_t / R_t^m, \quad (64)$$

$$\theta n_t^{\sigma_n} = w_t c_t^{-1}, \quad w_t R_t^L = \frac{\varepsilon - 1}{\varepsilon} \alpha n_t^{\alpha-1}, \quad c_t = n_t^\alpha, \quad b_t = \Gamma b_{t-1} \pi_t^{-1}, \quad (65)$$

and $R_t^m < 1 / [c_t \beta E_t (c_{t+1}^{-1} \pi_{t+1}^{-1})]$. Eliminating n_t , w_t , c_t and l_t from (64)-(65) gives the set of equilibrium conditions listed in definition 1.

Proof of proposition 2. Consider the set of equilibrium conditions given in definition 1. Use (28) or $\kappa_t^B b_{t-1} / \pi_t = (R_t^m - \kappa_t \mu) y_t$ as well as its time $t + 1$ version and $b_t = \Gamma b_{t-1} \pi_t^{-1}$, to eliminate y_t and y_{t+1} in (27):

$$\frac{1}{R_t^L} = \frac{\kappa_t}{R_t^m} + (1 - \kappa_t) \frac{\beta}{\Gamma} E_t \left[\frac{R_{t+1}^m - \kappa_{t+1} \mu}{R_t^m - \kappa_t \mu} \frac{\kappa_t^B}{\kappa_{t+1}^B} \right], \quad (66)$$

where $\mu = \frac{\varepsilon-1}{\varepsilon} \alpha \in (0, 1)$. We further divide both sides of $\kappa_t^B b_{t-1} / \pi_t = (R_t^m - \mu \kappa_t) y_t$ by its period $t - 1$ version $\kappa_{t-1}^B b_{t-2} / \pi_{t-1} = (R_{t-1}^m - \mu \kappa_{t-1}) y_{t-1}$, and use $b_{t-1} = \Gamma b_{t-2} \pi_{t-1}^{-1}$, to express inflation as $\pi_t = \Gamma \frac{\kappa_t^B}{\kappa_{t-1}^B} \frac{R_{t-1}^m - \mu \kappa_{t-1}}{R_t^m - \mu \kappa_t} \frac{y_{t-1}}{y_t}$. We further replace output with (28) in the latter and in $R_t^m < 1 / [y_t \beta E_t (y_{t+1}^{-1} \pi_{t+1}^{-1})]$, and real bonds with (29), to get the following four equilibrium conditions: (66),

$$y_t = (\mu / \theta)^{\alpha / (1 + \sigma_n)} (1 / R_t^L)^{\alpha / (1 + \sigma_n)}, \quad (67)$$

$$\pi_t = \Gamma \frac{\kappa_t^B}{\kappa_{t-1}^B} \frac{R_{t-1}^m - \mu \kappa_{t-1}}{R_t^m - \mu \kappa_t} \left(\frac{R_t^L}{R_{t-1}^L} \right)^{\alpha / (1 + \sigma_n)}, \quad (68)$$

$$\frac{\eta_t}{y_t^{-1}} = \frac{1}{R_t^m} - \frac{\beta}{\Gamma} E_t \frac{\kappa_t^B}{\kappa_{t+1}^B} \frac{R_{t+1}^m - \mu \kappa_{t+1}}{R_t^m - \mu \kappa_t} > 0, \quad (69)$$

which can be solved sequentially. For a given solution for output and inflation, we can solve for real bonds and for the initial price level P_0 using (28) and (29). We consider the case where the instruments are initially set at $\kappa \geq 0$, $\kappa_t^B = \gamma \kappa_{t-1}^B$, where $\kappa_0^B > 0$ and $\gamma > \beta / \Gamma$, and $R^m \in [1, \gamma \Gamma / \beta)$. Then, $\frac{\eta_t}{y_t^{-1}} = \frac{1}{R^m} - \frac{\beta}{\Gamma} \gamma > 0$, such that the collateral constraint is binding. *First*, consider a marginal increase in the policy rate in period t from R^m to

$R_{t+i}^m = \bar{R}^m \forall i \geq 0$ where $\bar{R}^m \in [R^m, \gamma\Gamma/\beta)$, while $\kappa_t = \kappa > 0$ and $\kappa_t^B = \gamma\kappa_{t-1}^B$. Condition (66) then reads $\frac{1}{R_t^L} = \frac{\kappa}{R_t^m} + (1 - \kappa)\frac{\beta}{\Gamma}\frac{1}{\gamma}$, such that

$$\frac{\partial(1/R_t^L)}{\partial R_t^m} = -\frac{1}{(R^m)^2}\kappa < 0 \text{ and } \frac{\partial y_t}{\partial R_t^m} = -\frac{\alpha}{1 + \sigma_n}y\frac{R^L}{(R^m)^2}\kappa < 0,$$

where we used $\partial y_t / \partial (1/R_t^L) = [\alpha / (1 + \sigma_n)]yR^L[\partial(1/R_t^L) / \partial R_t^m]$ (see 67). Inflation satisfies $\pi_t = \Gamma\gamma$ (see 68), such that

$$\frac{\partial \pi_t}{\partial R_t^m} = -\Gamma\gamma \left(\frac{\kappa}{R^m} + (1 - \kappa)\frac{\beta}{\Gamma}\frac{1}{\gamma} \right)^{-1} \left(\frac{\kappa}{(R^m)^2} \left[\frac{R^m}{R^m - \mu\kappa} - \frac{\alpha}{1 + \sigma_n} \right] + \frac{(1 - \kappa)}{R^m - \mu\kappa} \frac{\beta}{\Gamma}\frac{1}{\gamma} \right) < 0$$

where the term in the square bracket is non-negative. Since $\frac{\eta_t}{c_t^{-1}} = \frac{1}{R_t^m} - \frac{\beta}{\Gamma}\gamma$, it follows that $\eta_t > 0$ if $\bar{R}^m < \gamma\Gamma/\beta$. *Second*, consider a marginal decrease in the growth rate of κ^B from γ to $\gamma_{t+i} = \bar{\gamma} < \gamma \forall i \geq 0$, while $R_t^m = R^m$ and $\kappa_t = \kappa$. Then, (66) and (68) reduce to $\frac{1}{R_t^L} = \frac{\kappa}{R^m} + (1 - \kappa)\frac{\beta}{\Gamma}\frac{1}{\gamma_t}$ and $\pi_t = \Gamma\gamma_t\frac{y_{t-1}}{y_t}$, such that

$$\frac{\partial(1/R_t^L)}{\partial \gamma_t} = -(1 - \kappa)\frac{\beta}{\Gamma}\frac{1}{\gamma^2} < 0, \quad \frac{\partial y_t}{\partial \gamma_t} = -\frac{\alpha}{1 + \sigma_n}yR^L(1 - \kappa)\frac{\beta}{\Gamma}\frac{1}{\gamma^2} < 0 \quad \text{and} \quad \frac{\partial \pi_t}{\partial \gamma_t} > 0.$$

where we used $\frac{\partial \pi_t}{\partial y_t} < 0$ for the last inequality. Hence, a lower γ_t reduces the loan rate R_t^L and increases output if $\kappa < 1$. Since $\frac{\eta_t}{c_t^{-1}} = \frac{1}{R^m} - \frac{\beta}{\Gamma}\gamma_t$, it follows that $\eta_t > 0$ if $\gamma_t > \frac{R^m\beta}{\Gamma}$. *Third*, consider a marginal increase in the fraction of eligible loans κ from κ to $\kappa_{t+i} = \bar{\kappa} > \kappa \forall i \geq 0$, while $R_t^m = R^m$ and $\gamma_t = \gamma$. Then, (66) reduces to $\frac{1}{R_t^L} = \frac{\kappa_t}{R^m} + (1 - \kappa_t)\frac{\beta}{\Gamma}\gamma$, such that

$$\frac{\partial(1/R_t^L)}{\partial \kappa_t} = \frac{1}{R^m} - \frac{\beta}{\Gamma}\gamma > 0, \quad \frac{\partial y_t}{\partial \kappa_t} = y\frac{\alpha}{1 + \sigma_n}(1/R^L)^{-1} \left(\frac{1}{R^m} - \frac{\beta}{\Gamma}\gamma \right) > 0.$$

indicating that a higher κ_t unambiguously reduces the loan rate and raises output. The impact on inflation (see 68), is $\frac{\partial \pi_t}{\partial \kappa_t} = \Gamma\gamma \left(\frac{\mu}{R^m - \kappa\mu} - \frac{\alpha}{1 + \sigma_n}R^L \left(\frac{1}{R^m} - \frac{\beta}{\Gamma}\gamma \right) \right)$ and therefore ambiguous due to changes in κ_t and y_t . Since $\frac{\eta_t}{c_t^{-1}} = \frac{1}{R^m} - \frac{\beta}{\Gamma}\gamma > 0$, the collateral constraint remains binding. Hence, a RE equilibrium as given in definition 1 exists and permanent changes of R_t^m , κ_t , and κ_t^B exert qualitatively different effects on R_t^L , π_t and y_t . ■

Proof of proposition 3. Consider a steady state, where all endogenous variables grow with a constant rate. Substituting out $1/R_t^L$ in (67) with (66), gives

$$y_t = (\mu/\theta)^{\alpha/(1+\sigma_n)} \left(\frac{\kappa_t}{R_t^m} + (1 - \kappa_t)\frac{\beta}{\Gamma}E_t \left[\frac{R_{t+1}^m - \kappa_{t+1}\mu}{R_t^m - \kappa_t\mu} \frac{\kappa_t^B}{\kappa_{t+1}^B} \right] \right)^{\alpha/(1+\sigma_n)}. \quad (70)$$

Suppose that the central bank sets R_t^m and κ_t in a stationary way. Then, (68) and (70) imply that steady state inflation only depends on the growth rate of κ_t^B and on $\Gamma : \pi = \Gamma\gamma$, where $\kappa_t^B = \gamma\kappa_{t-1}^B$. The central bank can therefore control the steady state inflation rate by setting γ contingent on Γ , while the steady state price level is stable, $\pi = 1$, iff $\gamma = \Gamma^{-1}$. ■

C Appendix to section 4

Proof of proposition 4. According to (30), changes in δ_t^e tend to alter the loan rate and therefore labor demand (see 6) if they are not offset by monetary policy. Define \bar{x}_t as the level of x_t that prevails in the case, where $\sigma_{\omega,t} = 0$ (such that $\delta_t^e = 0$). Then, $\bar{R}_t^L = \{(1 - \kappa_t) \beta E_t[(\bar{c}_{t+1}^{-\sigma}/\bar{c}_t^{-\sigma}) \bar{\pi}_{t+1}^{-1}] + \kappa_t/\bar{R}_t^m\}^{-1}$ holds for some $\kappa_t \in [0, 1]$. Rewriting (30), we can condition the fraction of eligible loans κ_t in the following way

$$\tilde{\kappa}_t = \left[1 - (1 - \delta_t^e) (\bar{R}_t^L / \bar{R}_t^{Euler}) \right] / \left[(\bar{R}_t^L / \bar{R}_t^m) - (\bar{R}_t^L / \bar{R}_t^{Euler}) \right] \in [0, 1], \quad (71)$$

where $\bar{R}_t^{Euler} = (\beta E_t[(\bar{c}_{t+1}^{-\sigma}/\bar{c}_t^{-\sigma}) \bar{\pi}_{t+1}^{-1}])^{-1} \geq \bar{R}_t^L$. By setting $\kappa_t = \tilde{\kappa}_t$, such that κ_t is a function of δ_t^e , the central bank can offset the immediate impact of δ_t^e on the loan rate. Likewise, it can set the policy rate R_t^m contingent on δ_t^e . Given that κ_t and R_t^m affect money supply via the collateral constraint (23), the equilibrium allocation will in general be altered. Setting $\kappa_t = \tilde{\kappa}_t$ is however consistent with the equilibrium allocation and prices under $\delta_t^e = 0$ if κ_t^B is adjusted according to $\tilde{\kappa}_t^B = [-\tilde{\kappa}_t \bar{l}_t + \bar{R}_t^m (\bar{c}_t - \bar{m}_{t-1}^H \bar{\pi}_t^{-1})] / (\bar{b}_{t-1} / \bar{\pi}_t)$, for which we combined the cash constraint, $P_t c_t \leq M_t^H + M_t^R + M_t^L$, and (23). Hence, if δ_t^e is sufficiently small such that $\tilde{\kappa}_t \in [0, 1]$, and κ_t and κ_t^B are set equal to $\tilde{\kappa}_t$ and $\tilde{\kappa}_t^B$, default risk shocks only alter the bond rate according to (21), which is irrelevant for the equilibrium allocation as shown by definition 4 in appendix A. ■

We now characterize the steady state. The central bank determines $\kappa \in [0, 1]$ and the long-run (target) values for the inflation rate $\pi \geq \beta$ and the policy rate $R^m \geq 1$. In a steady state, all endogenous variables grow with a constant rate. Thus, to be consistent with a long-run equilibrium, the time-invariant policy targets have to be consistent with the steady state. In what follows we examine properties of all other endogenous variables in a steady state with $\delta^e = 0$. Given steady state inflation π , (53) implies that $\tilde{Z} = ((1 - \phi\pi^{\varepsilon-1}) / (1 - \phi))^{1/(1-\varepsilon)}$, and (52) that Z^1/Z^2 is constant. The price dispersion term s_t satisfying (58), thus converges

in the long-run to $s = \frac{1-\phi}{1-\phi\pi^\varepsilon} \tilde{Z}^{-\varepsilon}$, given that $\phi\pi^\varepsilon < 1 \Leftrightarrow \pi < (1/\phi)^{1/\varepsilon}$. Since s is bounded from below and neither productivity nor labor supply exhibit trend growth, real resources cannot permanently grow with a non-zero rate, $y = c = n^\alpha/s$. Then, Z_t^2 converges to $Z^2 = yc^{-\sigma}/(1-\phi\beta\pi^{\varepsilon-1})$ if $\phi\beta\pi^{\varepsilon-1} < 1 \Leftrightarrow \pi < [1/(\phi\beta)]^{1/(\varepsilon-1)}$. Given that Z^1/Z^2 and \tilde{Z} are constant and $Z_t^1 = Z^1 = \frac{yc^{-\sigma}mc}{1-\phi\beta\pi^\varepsilon}$ holds (since $Z_t^1/Z_t^2 = Z^{1,2}$), real marginal costs are also constant and given by $mc = \tilde{Z}(\varepsilon-1)\varepsilon^{-1}(1-\phi\beta\pi^\varepsilon)/(1-\phi\beta\pi^{\varepsilon-1})$. Since steady state consumption is constant, (45) determines the steady state Euler rate in the usual way, $R^{Euler} = \pi/\beta$, which equals the steady state debt rate, $R^D = \pi/\beta$ (see 17 and 20). Condition (42) further implies for the steady state loan rate

$$(1/R^L) = \kappa(1/R^m) + (1-\kappa)\beta/\pi. \quad (72)$$

Given that the loan rate, marginal cost, and working time are constant, (50) implies a constant steady state wage rate, $w = mc\alpha n^{\alpha-1}/R^L$. Moreover, the steady state is characterized by $\theta n^{\sigma_n} = wc^{-\sigma}$, $c = n^\alpha$, and $l = R^L w n = R^L \theta c^{\sigma+(1+\sigma_n)/\alpha} = \frac{\varepsilon-1}{\varepsilon} \alpha c$ (see 41, 56, 57, and 51).

We now consider the simplified case, where $\Omega \rightarrow \infty$ and $\Gamma = \pi = 1$. Log-linearizing (41), (50), (52), (53), (56), (57), and $b_t = \Gamma b_{t-1} \pi_t^{-1}$ at the steady state gives

$$\sigma \hat{c}_t + \sigma_n \hat{n}_t = \hat{w}_t, \quad \widehat{mc}_t = \hat{w}_t + \widehat{R}_t^L + (1-\alpha)\hat{n}_t, \quad \hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \chi \widehat{mc}_t, \quad \alpha^{-1} \hat{y}_t = \hat{n}_t, \quad (73)$$

$$\hat{c}_t = \hat{y}_t, \quad \hat{b}_t = \hat{b}_{t-1} - \hat{\pi}_t, \quad (74)$$

where $\chi = (1-\phi)(1-\beta\phi)/\phi$. Further, log-linearizing (64) for $\delta_t^\varepsilon = 0$, using (72), and defining $\varkappa = \frac{\kappa}{1-\kappa} \frac{\pi/\beta}{R^m}$, we get

$$\sigma E_t \hat{c}_{t+1} - \sigma \hat{c}_t + (1-\rho_\xi) \hat{\xi}_t + E_t \hat{\pi}_{t+1} - (\varkappa - \kappa/(1-\kappa)) \hat{\kappa}_t + \varkappa \widehat{R}_t^m = (1+\varkappa) \widehat{R}_t^L, \quad (75)$$

where we used that the preference shock is autocorrelated: $E_t \hat{\xi}_{t+1} = \rho_\xi \hat{\xi}_t$. Combining (47) and (48) with (51), to get $\kappa_t R_t^L w_t n_t + \kappa_t^B b_{t-1}/\pi_t = R_t^m m_t^R$ and eliminating wages, gives in linearized form

$$(\varsigma\varpi - 1) \hat{c}_t + \varsigma \hat{\kappa}_t + \varsigma \widehat{R}_t^L + (1-\varsigma) \hat{\kappa}_t^B + (1-\varsigma) \hat{b}_{t-1} - (1-\varsigma) \hat{\pi}_t = \widehat{R}_t^m, \quad (76)$$

where $\varsigma = \kappa \frac{\varepsilon-1}{\varepsilon} \alpha / R^m > 1$ and $\varpi = \frac{1+\sigma_n}{\alpha} + \sigma > 1 + \sigma$. Eliminating \hat{w}_t , \hat{n}_t , and \widehat{mc}_t in

(73)-(76), we can summarize the RE equilibrium as a set of sequence $\{\widehat{R}_t^L, \widehat{y}_t, \pi_t, \widehat{b}_t\}_{t=0}^\infty$ that converge to the steady state and satisfy (74), (75), (76), and

$$\widehat{\pi}_t = \beta E_t \widehat{\pi}_{t+1} + \chi(\varpi - 1)\widehat{c}_t + \chi \widehat{R}_t^L, \quad (77)$$

for $\{\widehat{\kappa}_t, \widehat{\kappa}_t^B, \widehat{R}_t^m\}_{t=0}^\infty$ set by the central bank, given $\{\widehat{\xi}_t\}_{t=0}^\infty$ and $b_{-1} > 0$. Eliminating \widehat{c}_t , the conditions (74), (75), (76), and (77), reduce to (31)-(34) for $\kappa = 0 \Rightarrow \varsigma = \varkappa = 0$.

Proof of proposition 5. Consider the RE equilibrium as in definition 2. To establish the claims made in the proposition, we first combine (36) with the targeting rule (37) to get

$$((\beta + \chi)\varepsilon^{-1}(1 + \sigma/\eta) - \chi\sigma)\widehat{y}_{t+1} = (\chi\eta + \varepsilon^{-1}(1 + \sigma/\eta))\widehat{y}_t + \chi\widehat{u}_t,$$

where $\widehat{u}_t = (1 - \rho_\xi)\widehat{\xi}_t$ and we used $\Lambda = \chi\left(\frac{\sigma+\eta}{\varepsilon}\right)$. There is a unique solution if $(\beta + \chi)\frac{1}{\varepsilon}\frac{\sigma+\eta}{\eta} = \chi\sigma$, i.e. $\widehat{y}_t = \{\chi/[\chi\eta + \varepsilon^{-1}(1 + \sigma/\eta)]\}\widehat{u}_t$ and $\widehat{\pi}_t = -\{\varepsilon^{-1}(1 + \sigma/\eta)\chi/[\chi\eta + \varepsilon^{-1}(1 + \sigma/\eta)]\}\widehat{u}_t$ or, otherwise, if the eigenvalue lies outside the unit circle, $\left|\frac{\chi\eta + \varepsilon^{-1}(1 + \sigma/\eta)}{(\beta + \chi)\varepsilon^{-1}(1 + \sigma/\eta) - \chi\sigma}\right| > 1$. If $(\beta + \chi)\varepsilon^{-1}\left(\frac{\sigma+\eta}{\eta}\right) < \chi\sigma$, this requires $\chi(\eta - \sigma) + \varepsilon^{-1}(1 + \sigma/\eta)(1 + \beta + \chi) > 0$, which is satisfied if $\eta > \sigma$. If $(\beta + \chi)\varepsilon^{-1}(1 + \sigma/\eta) > \chi\sigma$, uniqueness requires $\varepsilon^{-1}(1 + \sigma/\eta)[\chi(\varepsilon\eta - 1) + (1 - \beta)] > 0$, which is always satisfied given that $\varepsilon > 1$ and $\eta > 1$. The unique solutions for the output gap and inflation take the form $\widehat{y}_t = \delta_x \widehat{u}_t$ and $\widehat{\pi}_t = \delta_\pi \widehat{u}_t$, where the coefficients δ_x and δ_π can easily be identified via the method of undetermined coefficients. For $(\beta + \chi)\varepsilon^{-1}(1 + \sigma/\eta) \neq \chi\sigma$, the coefficients are given by

$$\delta_x = -\varepsilon\chi/\Theta < 0 \text{ and } \delta_\pi = (1 + \sigma/\eta)\chi/\Theta > 0, \quad (78)$$

where $\Theta = (1 - \beta\rho)(1 + \sigma/\eta) + \chi(\rho(\varepsilon\eta - 1)(1 + \sigma/\eta) + \varepsilon\eta(1 - \rho)) > 0$. To implement this solution, the central bank has to set its instruments R_t^m and κ_t^B according to

$$\widehat{R}_t^m - \widehat{\kappa}_t^B = \widehat{b}_{t-1} - [\delta_\pi + \delta_x](1 - \rho_\xi)\widehat{\xi}_t, \quad (79)$$

where we used (31). For $\varepsilon > (\sigma/\eta) + 1$, which is satisfied if $\varepsilon > 2$ for $\eta > \sigma$, the term in the square brackets in (79) is negative, such that a decline in $\widehat{\xi}_t$ demands the central bank either to lower R_t^m or to raise κ_t^B . The former is possible as long as $R_t^m \geq 1$ and thus for

shocks satisfying $\widehat{\xi}_t \geq \frac{1-1/R^m}{\chi[(\sigma/\eta)+1-\varepsilon](1-\rho_\xi)} < 0$ (see 78 and 79). For

$$\widehat{\xi}_t < \frac{(1-1/R^m)\Theta}{\chi[(\sigma/\eta)+1-\varepsilon](1-\rho_\xi)}, \quad (80)$$

the policy plan cannot be implemented by a policy rate reduction. The central bank can then implement the plan by raising $\widehat{\kappa}_t^B$, which requires that $\kappa^B < 1$ and that the collateral constraint is binding, $R_t^{Euler} = R_t^L > R_t^m = 1$. Using (32) and the solutions (78), it can easily be shown that this holds if $\widehat{\xi}_t \geq -\frac{(1-\beta/\pi)\Theta}{[(\sigma+\eta)\varepsilon\chi+(1-\beta\rho)(1+\sigma/\eta)](1-\rho_\xi)}$. Combining the latter with (80) shows that the central bank can implement the policy plan at the ZLB with quantitative easing if the spread between the Euler rate and the policy rate satisfies $\frac{1-\beta/\pi}{1-1/R^m} > \frac{(\sigma+\eta)\varepsilon\chi+(1-\beta\rho)(1+\sigma/\eta)}{\chi[\varepsilon-(1+\sigma/\eta)]}$. ■

Proof of proposition 6. To establish the claims made in the proposition, we consider the effects of an exogenous and autocorrelated increase in κ_t^B (where ρ is the coefficients of autocorrelation) when the policy rate pegged at its ZLB, $R_t^m = 1$. Using (13) and (17), we can rewrite (21) in equilibrium as $\frac{1}{R_t} = E_t[(1-\kappa_{t+1}^B)\varphi_{t,t+1}] + \beta E_t[\kappa_{t+1}^B \frac{1}{R_{t+1}^m} \frac{\psi_{t+1} + \lambda_{t+1}}{\lambda_t \pi_{t+1}}]$. Further using (16) and (20), we get $(1/R_t) = (1 - E_t \kappa_{t+1}^B) \cdot (1/R_t^D) + E_t \kappa_{t+1}^B \cdot E_t(1/R_{t+1}^m) + \mathcal{O}^2$, where \mathcal{O}^2 summarizes higher order terms. Since $R_t^D > R_t^m$ is initially satisfied in a steady state (where $R^D = R^{Euler} = \pi/\beta$) with a binding collateral constraint, the bond rate R_t is, up to first order, decreasing in the expected fraction of eligible bonds $E_t \kappa_{t+1}^B$ for $R_t^m = 1$. Now consider the model given in definition 2 for $\widehat{\xi}_t = 0$, which can further be reduced to (34), $\widehat{\pi}_t = (\beta + \chi) E_t \widehat{\pi}_{t+1} + \chi(\varpi - 1 - \sigma) \widehat{y}_t + \chi \sigma E_t \widehat{y}_{t+1}$ and $\widehat{y}_t = \widehat{b}_{t-1} - \widehat{\pi}_t + \widehat{\kappa}_t^B$. Substituting out output, leads to $\zeta_1 E_t \widehat{\pi}_{t+1} + \zeta_2 \widehat{b}_t = \widehat{\pi}_t - \zeta_3 \widehat{\kappa}_t^B$, which together with (34) can be written as

$$\begin{pmatrix} E_t \widehat{\pi}_{t+1} \\ \widehat{b}_t \end{pmatrix} = A \begin{pmatrix} \widehat{\pi}_t \\ \widehat{b}_{t-1} \end{pmatrix} + \begin{pmatrix} \zeta_1 & \zeta_2 \\ 0 & 1 \end{pmatrix}^{-1} \begin{pmatrix} -\zeta_3 \\ 0 \end{pmatrix} \widehat{\kappa}_t^B, \text{ where } A = \begin{pmatrix} \zeta_1 & \zeta_2 \\ 0 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}.$$

where $\zeta_1 = \beta + \chi(1 - \sigma)$, $\zeta_2 = \chi(\varpi - 1) > 0$, and $\zeta_3 = \chi(\varpi - 1 - \sigma(1 - \rho)) > 0$. Given that there exists exactly one predetermined variable, \widehat{b}_{t-1} , local determinacy requires one stable and one unstable eigenvalue. The characteristic polynomial of A is given by $F(X) = X^2 - (1 + \zeta_1^{-1} + \zeta_2 \zeta_1^{-1})X + \zeta_1^{-1}$, where $F(0) = 1/\zeta_1$ and $F(1) = -\zeta_2/\zeta_1$ and $sign F(0) = -sign F(1)$. Hence, there exists at least one real stable eigenvalue between zero and one. Fur-

ther, $F(X)$ at $X = -1$ is given by $F(-1) = \{-\chi(\sigma - [1 - \alpha + \sigma_n]/\alpha) + 2(1 + \beta + \chi)\}/\zeta_1$, where the term in the curly brackets is strictly positive, such that $\text{sign}F(0) = \text{sign}F(-1)$, if and only if $\frac{1}{2}(1 + \sigma - \frac{1+\sigma_n}{\alpha}) < 1 + \frac{1+\beta}{\chi}$, which is ensured by $\sigma < (1 + \sigma_n)/\alpha$. Then, there exists exactly one stable eigenvalue, between zero and one, and one unstable eigenvalue, indicating local determinacy.

Given that the stable eigenvalue is strictly positive, we know that the unique solution to the system (31)-(34), is given by the generic form $\hat{\pi}_t = \delta_1 \hat{b}_{t-1} + \delta_2 \hat{\kappa}_t^B$, $\hat{y}_t = \delta_3 \hat{b}_{t-1} + \delta_4 \hat{\kappa}_t^B$, $\hat{R}_t^L = \delta_5 \hat{b}_{t-1} + \delta_6 \hat{\kappa}_t^B$, and $\hat{b}_t = (1 - \delta_1) \hat{b}_{t-1} - \delta_2 \hat{\kappa}_t^B$, where the stable eigenvalue is $1 - \delta_1 \in (0, 1)$. Applying the method of undetermined coefficients, it can easily be shown that the coefficients satisfy $\delta_3 = 1 - \delta_1 \in (0, 1)$, $\delta_2 = (\chi\eta + \sigma\chi\rho)/\Xi > 0$, and $\delta_4 = [(1 - \rho\beta) + \delta_1\beta + (\sigma - 1)\chi\delta_3 + \chi(1 - \rho)]/\Xi > 0$, where $\Xi = (1 - \rho\beta) + \delta_1(\beta + \chi) + \chi\eta + (\sigma - 1)\chi\rho + \sigma\chi\delta_3 > 0$. Inserting these solutions in (32) and combining terms, leads to the following coefficients for the loan rate solution: $\delta_5 = -\delta_1(1 - \delta_1)(\sigma - 1) < 0$ and $\delta_6 = -[(\sigma - 1)\delta_3\delta_2 + (\delta_2 + \sigma\delta_4)(1 - \rho)] < 0$. Given that $\partial\hat{R}_t^L/\partial\hat{\kappa}_t^B = \delta_6 < 0$, an increase in $\hat{\kappa}_t^B$ reduces the loan rate on impact. ■

D Appendix to section 5

In this section, we define the RE equilibrium of the extended version with physical capital, where we disregard preference shocks and default risk shocks, for convenience.

Definition 6 *A RE equilibrium with physical capital is given by a set of sequences $\{c_t, y_t, k_t, x_t, n_t, \lambda_t, \psi_t, \eta_t, q_t, m_t^R, m_t^H, m_t^L, b_t, b_t^T, l_t, w_t, r_t^k, mc_t, \tilde{Z}_t, s_t, \pi_t, R_t^L\}_{t=0}^\infty$ satisfying (47)-(49), (51)-(55), as well as*

$$c_t^{-\sigma} = \lambda_t + v\psi_t, \quad (81)$$

$$1/R_t^L = \kappa_t(1/R_t^m) + (1 - \kappa_t)\beta E_t \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}\pi_{t+1}} \quad (82)$$

$$\theta n_t^{\sigma_n} = (\lambda_t + \psi_t)w_t, \quad (83)$$

$$\lambda_t + \psi_t = R_t^m(\lambda_t + \eta_t), \quad (84)$$

$$\lambda_t + \psi_t = R_t^L(\lambda_t + \eta_t\kappa_t), \quad (85)$$

$$\begin{aligned} \lambda_t + \omega_t\psi_t &= c_t^{-\sigma}q_t[S(x_t/x_{t-1}) + (x_t/x_{t-1})S'(x_t/x_{t-1})] \\ &\quad - \beta E_t c_{t+1}^{-\sigma}q_{t+1}[(x_{t+1}/x_t)^2 S'(x_{t+1}/x_t)], \end{aligned} \quad (86)$$

$$c_{i,t}^{-\sigma} q_t = \beta E_t [\lambda_{t+1} r_{t+1}^k + (1 - \delta) q_{i,t+1} c_{i,t+1}^{-\sigma}], \quad (87)$$

$$\lambda_t = \beta E_t \frac{\lambda_{t+1} + \psi_{t+1}}{\pi_{t+1}}, \quad (88)$$

$$w_t R_t^L = m c_t \alpha (n_t / k_{t-1})^{\alpha-1}, \quad (89)$$

$$r_t^k = m c_t (1 - \alpha) (n_t / k_{t-1})^\alpha, \quad (90)$$

$$k_t = (1 - \delta) k_{t-1} + x_t S(x_t / x_{t-1}), \quad (91)$$

$$y_t = n_t^\alpha k_{t-1}^{1-\alpha} / s_t, \quad (92)$$

$$y_t (1 - g/y) = c_t + x_t, \quad (93)$$

$$v c_t + \omega_t x_t = m_t^H + m_t^R + m_t^L, \text{ if } \psi_t > 0, \quad (94)$$

$$\text{or } v c_t + \omega_t x_t \leq m_t^H + m_t^R + m_t^L, \text{ if } \psi_t = 0,$$

(where q_t denotes the value of installed capital relative to consumption goods and the adjustment cost function is given by $S(x_t/x_{t-1}) = 1 - \frac{\vartheta}{2} (x_t/x_{t-1} - 1)^2$) as well as the transversality conditions, a monetary policy setting $\{R_t^m \geq 1, \kappa_t^B, \kappa_t \in [0, 1]\}_{t=0}^\infty$, $\Omega_t > 0$, and $\pi \geq \beta$, and a fiscal policy setting $\Gamma \geq 1$ and $\{g_t/y_t > 0\}_{t=0}^\infty$, for a given sequence $\{\omega_t\}_{t=0}^\infty$ and initial values $M_{-1}^H > 0$, $B_{-1} > 0$, $B_{-1}^T > 0$, $k_{-1} > 0$, and $s_{-1} \geq 1$.

E Appendix to section 6

We refer to the RE equilibrium given in definition 6 and introduce two-period bonds that can be eligible for repurchase agreements, $\kappa_t^{LB} \in [0, 1]$. Notably, the price of two-period government bonds is not relevant for the equilibrium allocation.

Definition 7 A RE equilibrium where two-term bonds can be eligible for repos is given by a set of sequences $\{c_t, y_t, k_t, x_t, n_t, \lambda_t, \psi_t, \eta_t, q_t, m_t^R, m_t^H, m_t^L, b_t, b_t^T, l_t, w_t, r_t^k, m c_t, \tilde{Z}_t, s_t, \pi_t, R_t^L, b_{t,t+2}\}_{t=0}^\infty$ satisfying (47)-(49), (51)-(55), (81)-(93) as well as

$$v c_t + \omega_t x_t = m_t^H + m_t^R + m_t^L + \frac{\kappa_t^{LB}}{R_t^m} \left[\frac{q_{t-1,t} b_{t-1,t+1}}{\pi_t} + \frac{b_{t-2,t}}{\pi_{t-1} \pi_t} \right], \text{ if } R_t^{Euler} > R_t^m, \quad (95)$$

$$\text{or } v c_t + \omega_t x_t \leq m_t^H + m_t^R + m_t^L + \frac{\kappa_t^{LB}}{R_t^m} \left[\frac{q_{t-1,t} b_{t-1,t+1}}{\pi_t} + \frac{b_{t-2,t}}{\pi_{t-1} \pi_t} \right], \text{ if } R_t^{Euler} = R_t^m,$$

$$b_{t,t+2} = \Gamma^L b_{t-2,t} / (\pi_{t-1} \pi_t), \quad (96)$$

(where $b_{t,t+2} = B_{t,t+2}/P_t$) as well as the transversality conditions, a monetary policy setting $\{R_t^m \geq 1, \kappa_t^B, \kappa_t^{LB} \in [0, 1], \kappa_t \in [0, 1]\}_{t=0}^\infty$, $\Omega_t > 0$, and $\pi \geq \beta$, and a fiscal policy setting $\Gamma^L \geq 1$, $\Gamma \geq 1$, and $\{g_t/y_t > 0\}_{t=0}^\infty$, for a given sequence $\{\omega_t\}_{t=0}^\infty$ and initial values $M_{-1}^H > 0$, $B_{-1} > 0$, $B_{-2,0} > 0$, $B_{-1,1} > 0$, $B_{-1}^T > 0$, $k_{-1} > 0$, and $s_{-1} \geq 1$.

Given that the fraction of eligible long-term bonds κ_t^{LB} enters the set of equilibrium condition only via (95), a change in κ_t^{LB} is equivalent to a change in κ_t^B (see 47).

F Parameter values

Table A1 Benchmark parameter values	
Subjective discount factor	$\beta = 0.992$
Inverse of intertemporal substitution elasticity	$\sigma = 2$
Inverse of Frisch elasticity of labour supply	$\sigma_n = 1$
Substitution elasticity	$\varepsilon = 10$
Steady state working time	$n = 0.33$
Labour share	$\alpha = 0.66$
Investment adjustment cost	$\vartheta = 2.48$
Rate of depreciation of capital stock	$\delta = 0.03$
Government expenditure share (constant)	$g = 0.19$
Fraction of non-price adjusting firms	$\phi = 0.75$
Steady state interest rate	$R^m = 1.0133$
Steady state share of repos to outright purchases	$\Omega = 1.5$
Steady state share of loans eligible in open market operations	$\kappa = 0$
Steady state share of gov. bonds eligible in open market operations	$\kappa^B = 1$
Steady state inflation	$\pi = 1.00647$
Steady state cash requirement for consumption	$\nu = 0.7399$
Steady state cash requirement for investment	$\omega = 0.4292$